1. Introduction

In the realm of digital image analysis, the extraction of meaningful insights from visual data is paramount. This report delves into the powerful realm of moments, a fundamental tool in image processing, alongside the intricate analysis of contours. Moments provide a profound understanding of shapes within images, enabling precise calculations of object characteristics such as area, centroid, and orientation.

Central to our exploration is the examination of contour analysis, a cornerstone in discerning objects' boundaries within images. We delve into the computation of areas and perimeters, uncovering intricate details of objects' shapes and sizes. Moreover, we investigate the efficacy of fitting bounding rectangles and minimum enclosing circles, demonstrating their ability to encapsulate object complexity efficiently.

Moving forward, we pivot towards the application of moments in image processing, showcasing their transformative impact on shape analysis. By leveraging moments, we gain deep insights into objects' geometrical features, allowing for accurate identification and classification. Through a series of visualizations and comparisons, we unveil the potency of moments in unlocking the hidden nuances within digital images.

2. Moments

2.1. Moments in Image Processing

Moments are essential mathematical descriptors used to characterize the spatial distribution of intensity in an image. In image processing, moments provide quantitative measures of the shape, orientation, and distribution of objects within an image. These moments are calculated from the pixel intensities of an image and offer insights into its geometric properties.

2.2. Types of Moments

- m00: (Represents the total intensity or area of an object.)
  Measures the overall size or "mass" of the object in the image. Useful for basic object detection and segmentation.

- m10: (Gives the centroid or center of mass along the x-axis.)
  Helps to locate the object's position horizontally. Useful for determining the object's horizontal position in the image.

- m01: (Gives the centroid or center of mass along the y-axis.)
  Helps to locate the object's position vertically. Useful for determining the object's vertical position in the image.

- m20: (Provides information about the spread of the object along the x-axis.)
  Indicates how "wide" the object is along the horizontal axis. Can help in distinguishing between elongated and compact shapes.

- m11: (Indicates the "tilt" or orientation of the object.)
  Measures the inclination or angle of the object concerning the x and y axes. Useful for understanding the orientation of the object.

- m02: (Provides information about the spread of the object along the y-axis.)
  Indicates how "tall" the object is along the vertical axis. Can help in distinguishing between vertically elongated and compact shapes.

- m30: (Indicates higher-order horizontal spread or skewness.)
  Measures the asymmetry or skewness of the object along the x-axis. Can help in detecting skewed or irregularly shaped objects.

- m21: (Combines information on orientation and spread.)
  Provides a combination of information about the object's tilt and spread along both the x and y axes.

- m12: (Combines information on orientation and spread.)

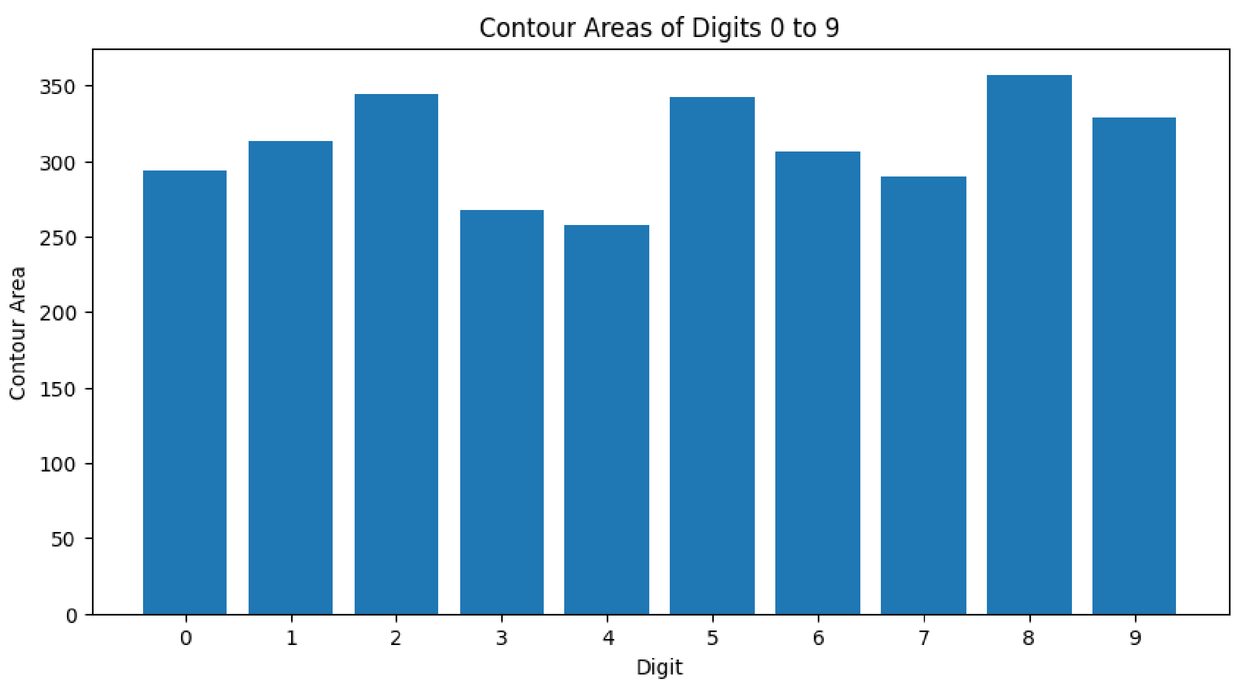
  Provides a combination of information about the object's tilt and spread along both the x and y axes.

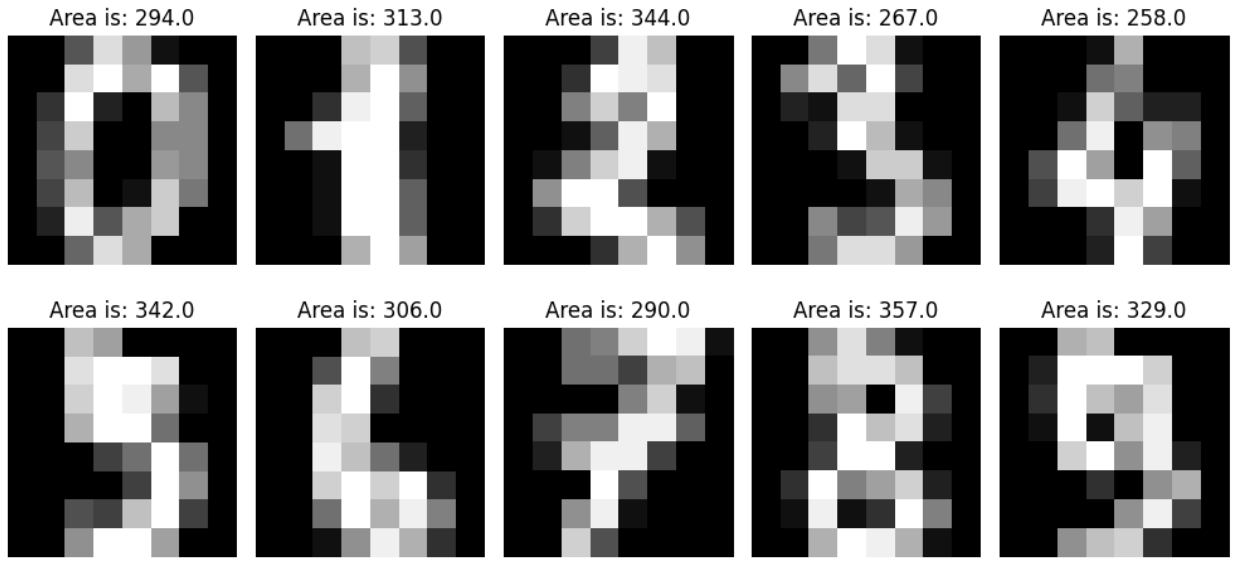
- M03: (Indicates higher-order vertical spread or kurtosis.)

  Measures the "pointiness" or kurtosis of the object along the y-axis. Can help in detecting objects with elongated or pointed features.

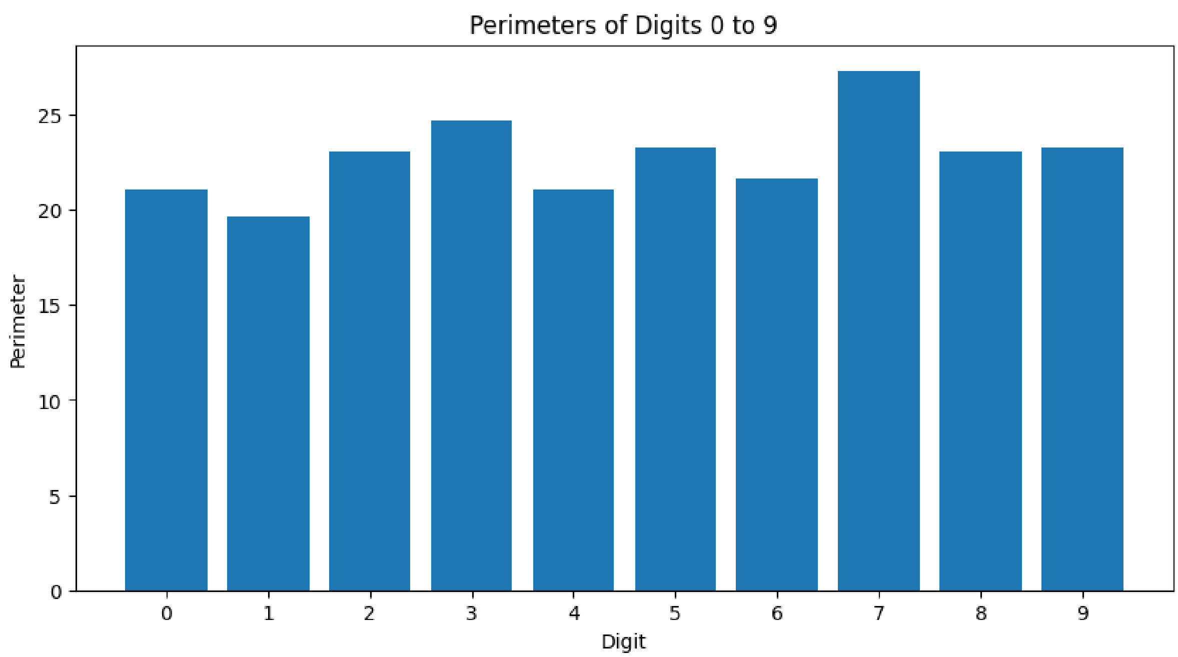
2.3. Plots of Features Extracted from Moments

2.3.1. Plot Contour Areas

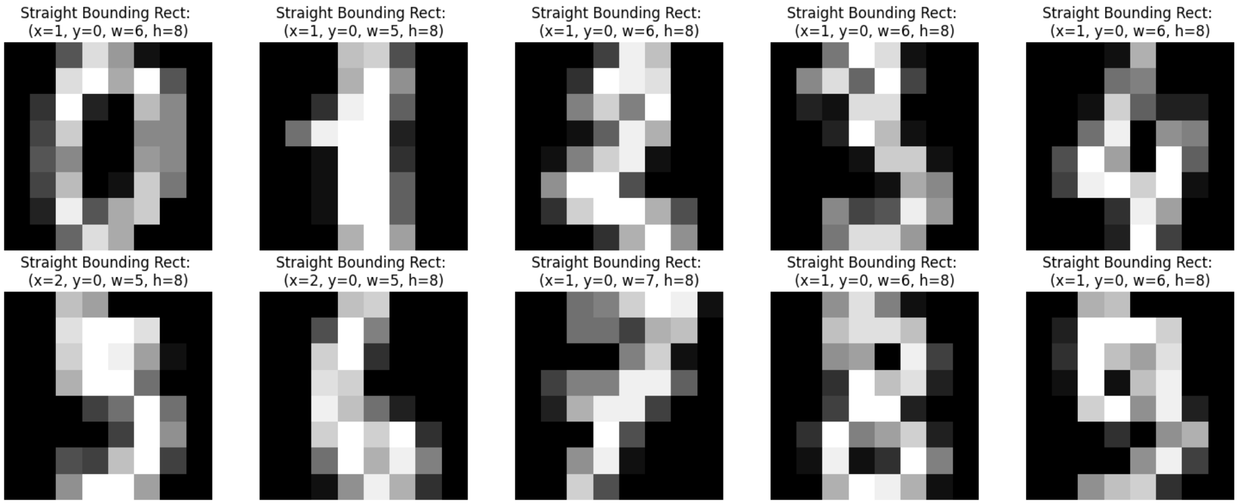




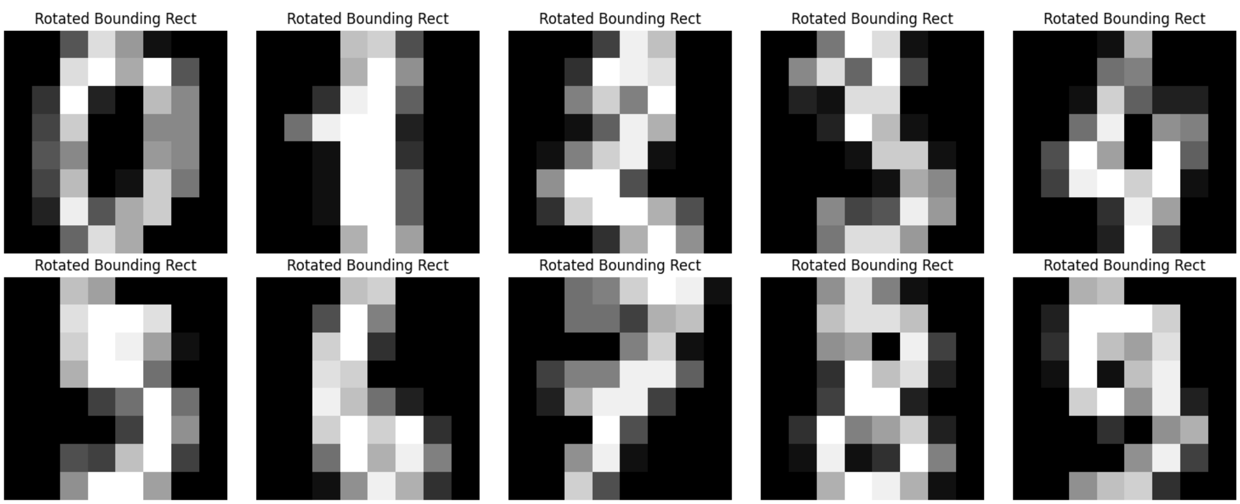
2.3.2. Plot Perimeters



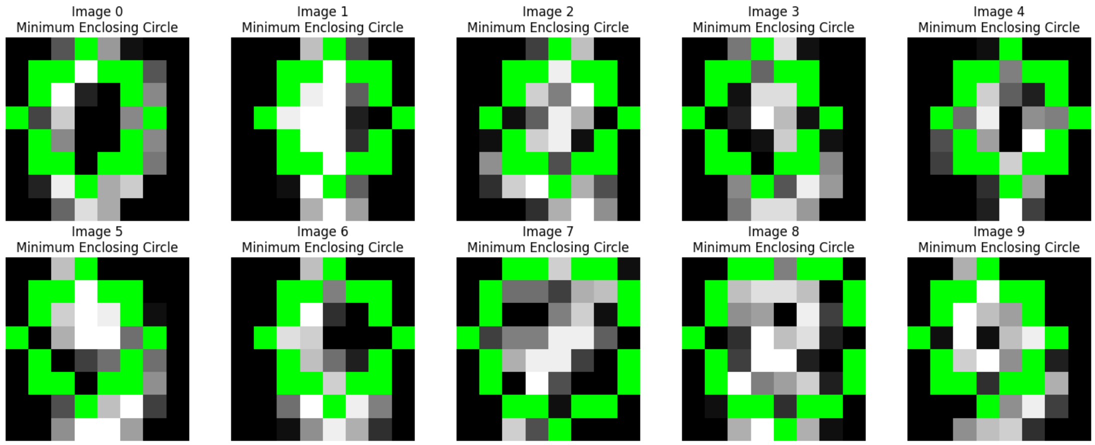
2.3.3. Plot Stright Bounding Rectangles



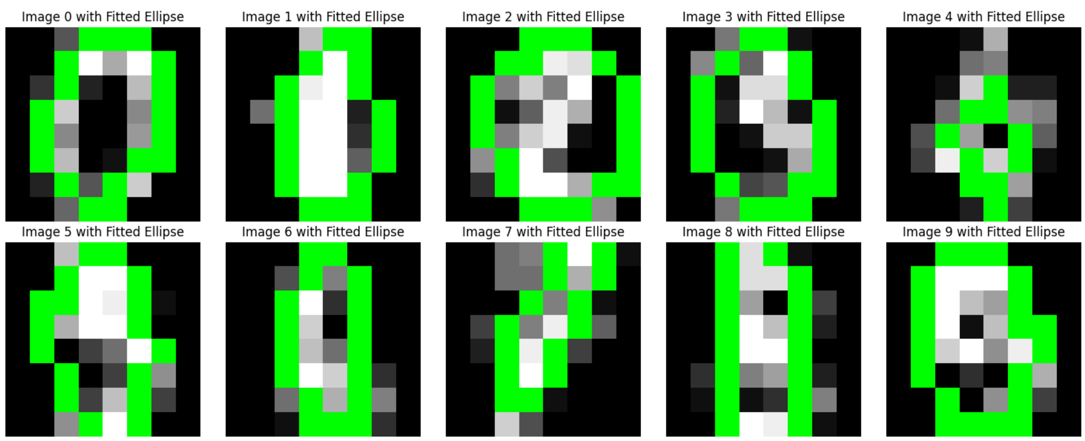
2.3.4. Plot Rotated Bounding Rectangles



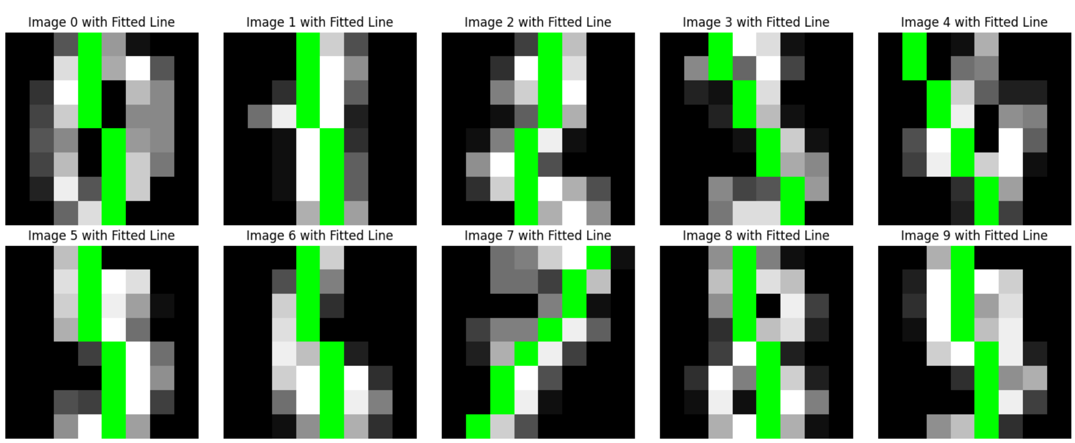
2.3.5. Plot Minimum Enclosing Circles



2.3.6. Plot Fitting an Ellipse



2.3.7. Plot Fitting a Line



3. Benefits of Using Moments in Image Processing

Moments play a crucial role in image processing, providing a robust set of descriptors that characterize various properties of objects within an image. Here are some key benefits of utilizing moments:

3.1. Quantifying Object Properties

- Size and Shape: Moments such as m00, m20, m02 quantify the size, width, and height of objects. This helps in distinguishing between objects of different sizes and shapes.
- Orientation and Tilt: Moments like m11, m21, m12 provide information on the orientation, tilt, and skewness of objects. This is particularly useful for recognizing objects in various orientations.

3.2. Robustness to Transformations

- Rotation: Moments are invariant to rotation, meaning they remain unchanged when the object is rotated. This enables object recognition even when the object's orientation varies.
- Translation: m10 and m01 are unaffected by translation, allowing for object detection irrespective of its position in the image.
- Scale: While moments are scale-sensitive, normalization techniques can be applied to make them scale-invariant.

3.3. Discriminative Features

- Feature Extraction: Moments serve as discriminative features, capturing the unique characteristics of objects. Higher-order moments (m30, m03) can reveal complex shape details.
- Pattern Recognition: By comparing moments between objects, pattern recognition, and classification become more efficient and accurate.

4. Comparative Analysis: Using Moments vs. Without

To demonstrate the effectiveness of moments, a comparative analysis was conducted on a set of images that transformed such as rotation, flipping, and translation. Two approaches were considered:

4.1. Using Moments

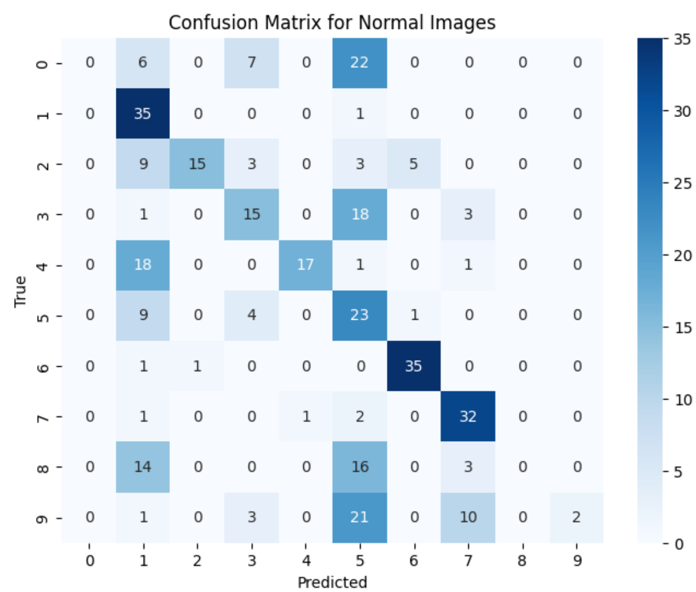
Moments were computed for each image, providing unique insights into their spatial distribution and geometric properties. The following key observations were made:

4.1.1 Normal Images with Moments

Test Accuracy with Normal Images: Achieved an accuracy of 0.4833, showcasing the ability of moments to accurately characterize the original images.

Classification Report for Normal Images (With Moments):

	precision	recall	f1-score	support
0	0.00	0.00	0.00	35
1	0.37	0.97	0.53	36
2	0.94	0.43	0.59	35
3	0.47	0.41	0.43	37
4	0.94	0.46	0.62	37
5	0.21	0.62	0.32	37
6	0.85	0.95	0.90	37
7	0.65	0.89	0.75	36
8	1.00	0.00	0.00	33
9	1.00	0.05	0.10	37
accuracy			0.48	360
macro avg	0.74	0.48	0.42	360
weighted avg	0.74	0.48	0.43	360

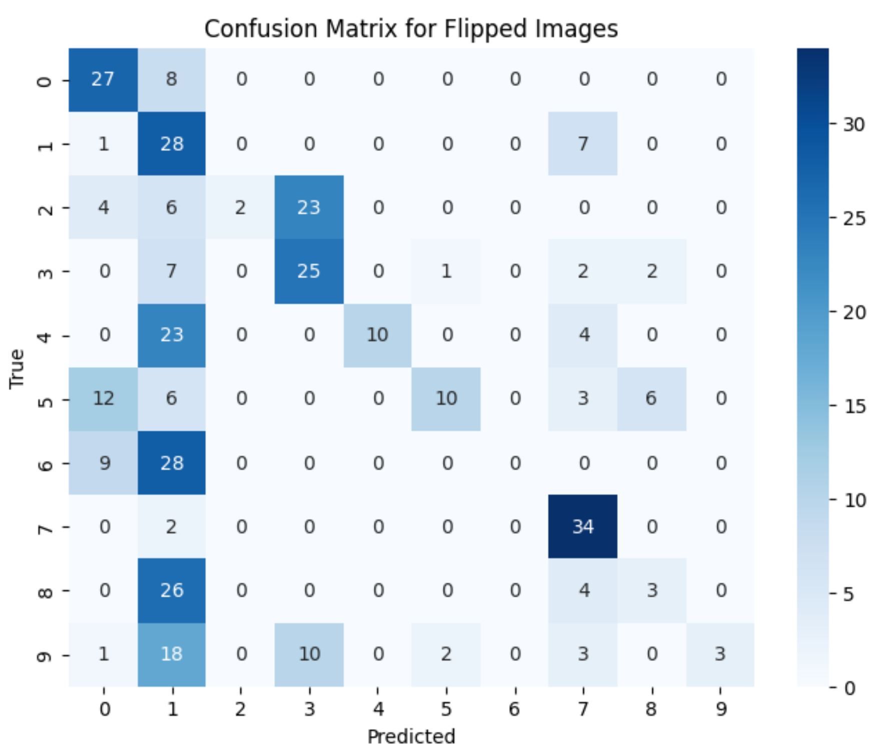


4.1.2 Flipped Images with Moments

Test Accuracy with Flipped Images: Results yielded an accuracy of 0.3944, indicating moments' capability to handle horizontal transformations like flipping.

Classification Report for Flipped Images (With Moments):

	precision	recall	f1-score	support
0	0.50	0.77	0.61	35
1	0.18	0.78	0.30	36
2	1.00	0.06	0.11	35
3	0.43	0.68	0.53	37
4	1.00	0.27	0.43	37
5	0.77	0.27	0.40	37
6	1.00	0.00	0.00	37
7	0.60	0.94	0.73	36
8	0.27	0.09	0.14	33
9	1.00	0.08	0.15	37
accuracy			0.39	360
macro avg	0.68	0.39	0.34	360
weighted avg	0.68	0.39	0.34	360



4.1.3 Translated Images with Moments

Test Accuracy with Translated Images: The accuracy obtained was 0.3889, demonstrating moments' effectiveness in detecting positional shifts.

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Classification Report for Translated Images (With Moments):
              precision    recall  f1-score   support

           0       0.38      0.83      0.52        35
           1       1.00      0.03      0.05        36
           2       0.54      0.63      0.58        35
           3       1.00      0.03      0.05        37
           4       0.39      0.92      0.55        37
           5       1.00      0.00      0.00        37
           6       1.00      0.05      0.10        37
           7       0.38      0.92      0.54        36
           8       0.27      0.52      0.35        33
           9       1.00      0.03      0.05        37

    accuracy                           0.39       360
   macro avg       0.68      0.39      0.29       360
weighted avg       0.68      0.39      0.30       360
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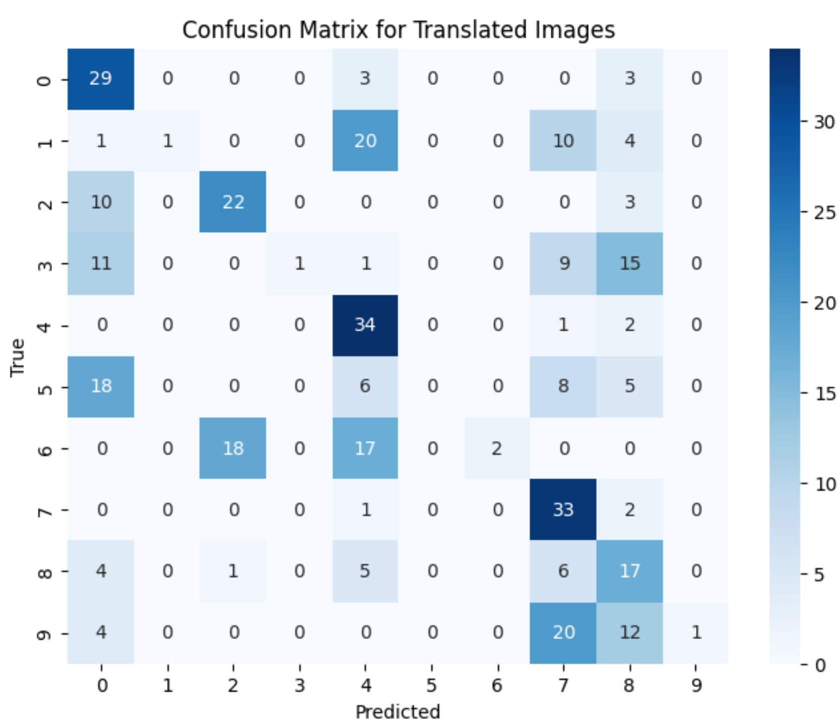
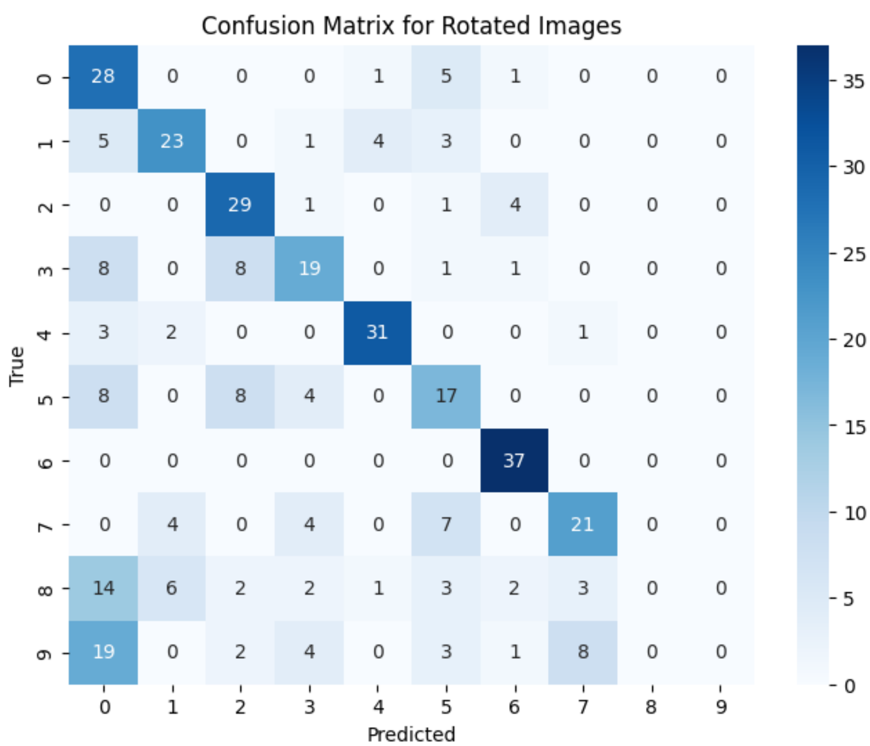


4.1.4 Rotated Images with Moments

Test Accuracy with Rotated Images: Achieved a higher accuracy of 0.5694, illustrating moments' robustness against rotational variations.

Classification Report for Rotated Images (With Moments):

	precision	recall	f1-score	support
0	0.67	0.97	0.79	35
1	0.67	0.78	0.72	36
2	1.00	0.60	0.75	35
3	0.52	0.95	0.67	37
4	0.86	0.76	0.81	37
5	0.73	0.70	0.72	37
6	1.00	0.95	0.97	37
7	0.84	0.81	0.82	36
8	0.83	0.42	0.56	33
9	1.00	0.89	0.94	37
accuracy			0.57	360
macro avg	0.79	0.73	0.73	360
weighted avg	0.79	0.57	0.64	360



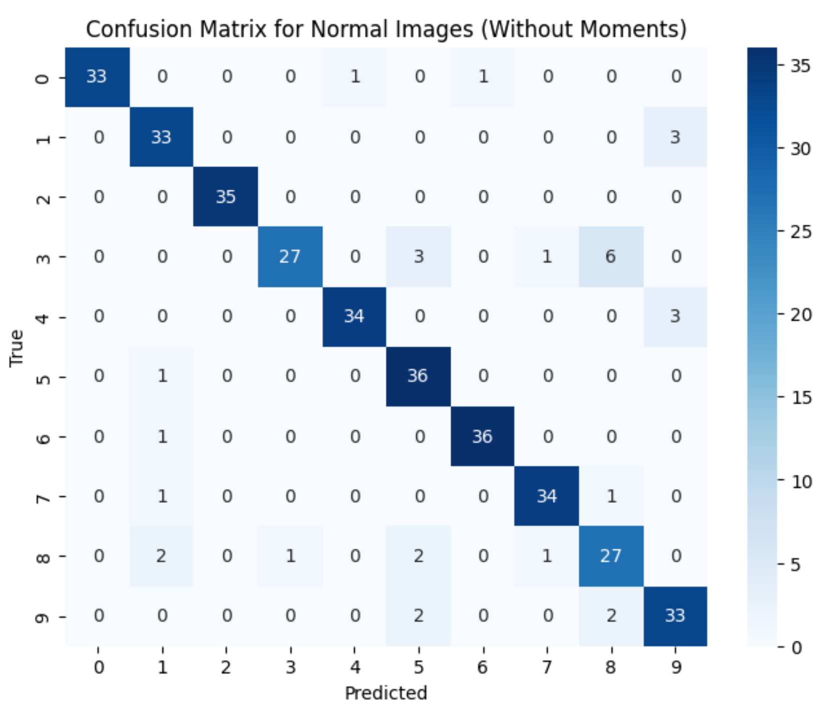
4.2 Without Using Moments

4.2.1 Normal Images without Moments

Test Accuracy with Normal Images: A significant decrease to 0.9111, highlighting the loss of accuracy without moment-based descriptors.

Classification Report for Normal Images (Without Moments):

	precision	recall	f1-score	support
0	0.98	1.00	0.99	35
1	0.93	1.00	0.97	36
2	0.97	0.97	0.97	35
3	0.97	0.92	0.94	37
4	0.97	1.00	0.99	37
5	0.97	1.00	0.99	37
6	0.95	1.00	0.97	37
7	0.97	0.97	0.97	36
8	0.94	0.88	0.91	33
9	1.00	0.92	0.96	37
accuracy			0.96	360
macro avg	0.96	0.96	0.96	360
weighted avg	0.96	0.96	0.96	360

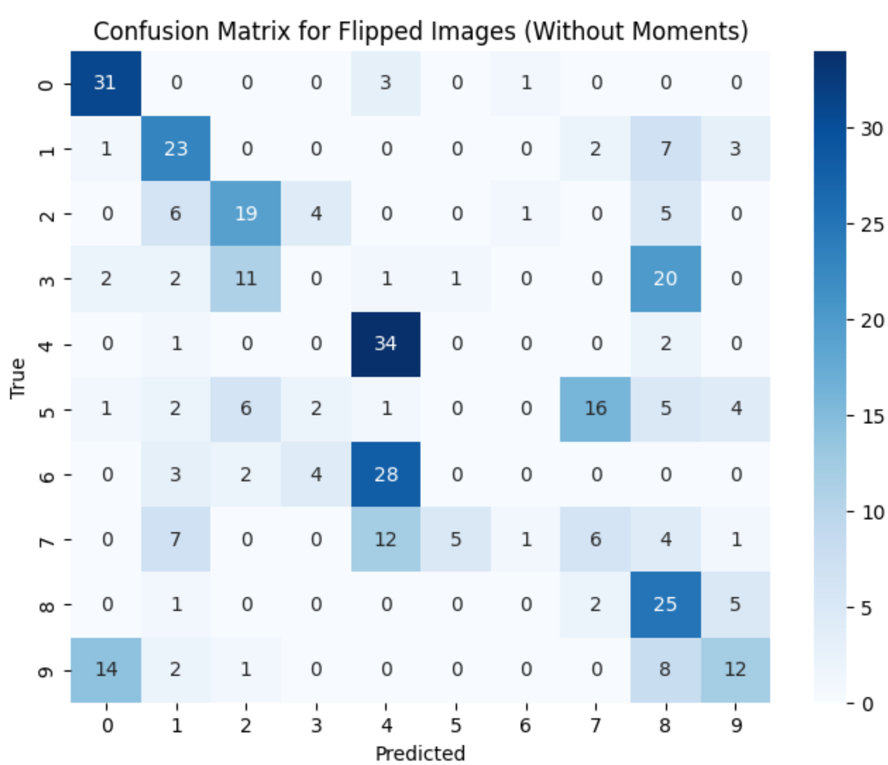


4.2.2 Flipped Images without Moments

4.2.3 Test Accuracy with Flipped Images: Accuracy dropped to 0.4167, indicating a decrease in performance when handling horizontal flips.

Classification Report for Flipped Images (Without Moments):

	precision	recall	f1-score	support
0	1.00	0.89	0.94	35
1	0.95	0.81	0.88	36
2	0.97	0.77	0.86	35
3	0.71	0.78	0.74	37
4	0.86	0.78	0.82	37
5	0.97	0.86	0.91	37
6	0.88	0.95	0.91	37
7	0.78	0.92	0.84	36
8	0.74	0.70	0.72	33
9	0.97	0.84	0.90	37
accuracy			0.84	360
macro avg	0.88	0.83	0.85	360
weighted avg	0.88	0.84	0.85	360

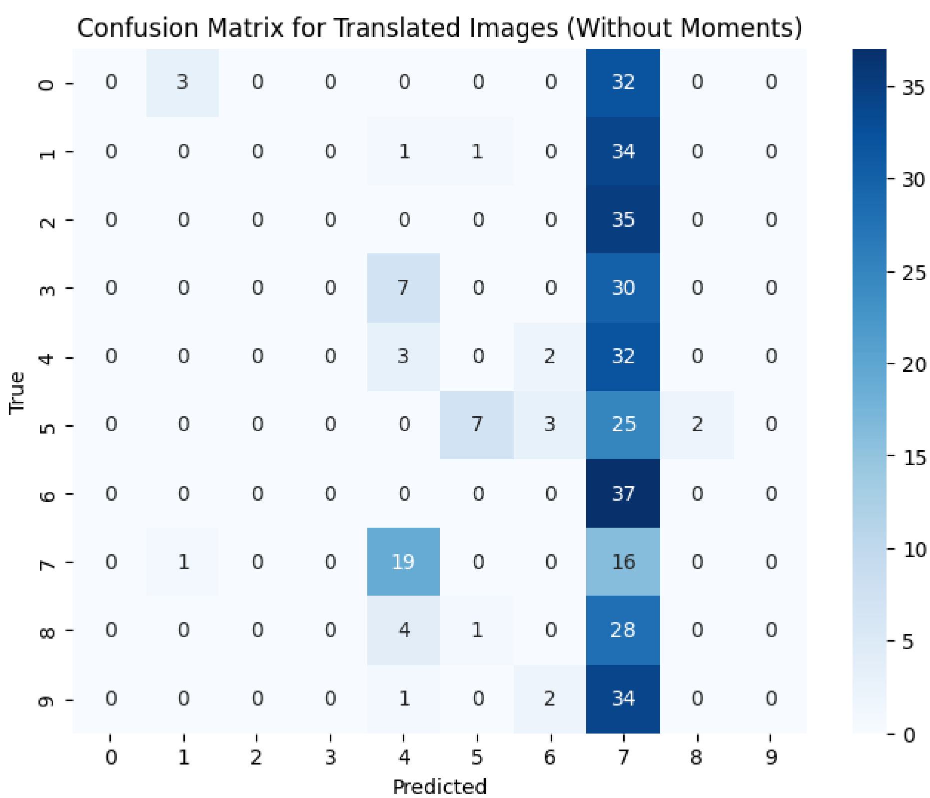


4.2.4 Translated Images without Moments

Test Accuracy with Translated Images: Substantially lower accuracy of 0.1028, revealing the challenge in detecting translations without moments.

Classification Report for Translated Images (Without Moments):

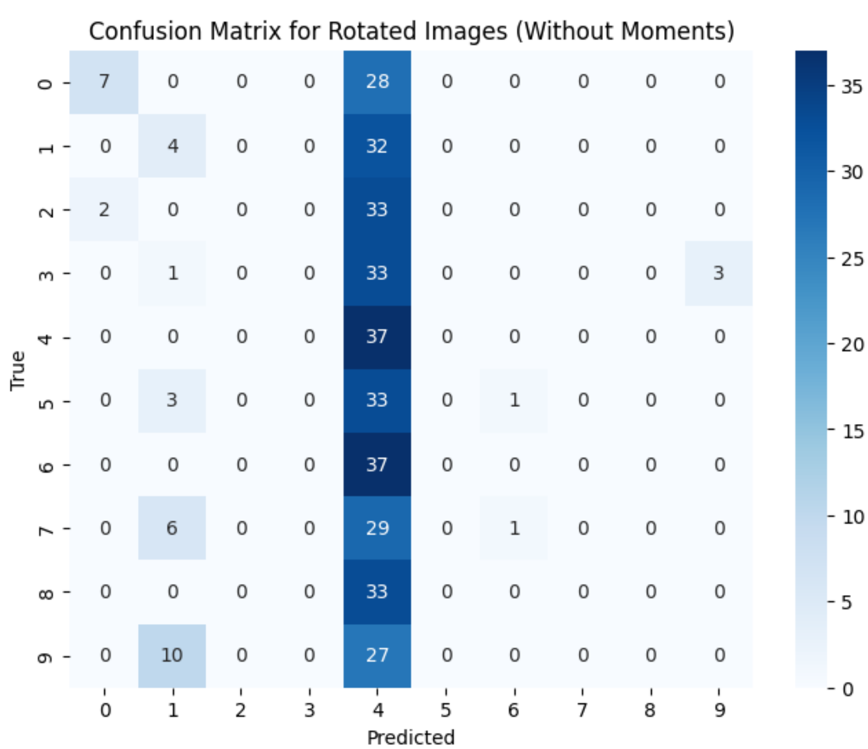
	precision	recall	f1-score	support
0	0.50	0.89	0.64	35
1	0.00	0.00	0.00	36
2	0.24	0.49	0.32	35
3	0.00	0.00	0.00	37
4	0.11	0.65	0.19	37
5	0.00	0.00	0.00	37
6	0.44	0.05	0.09	37
7	0.12	0.03	0.05	36
8	0.00	0.00	0.00	33
9	0.47	0.19	0.27	37
accuracy			0.10	360
macro avg	0.19	0.21	0.17	360
weighted avg	0.19	0.10	0.10	360



4.2.5 Rotated Images without Moments

Test Accuracy with Rotated Images: Despite a decent accuracy of 0.13, there is a clear advantage to using moments for rotational variations.

	precision	recall	f1-score	support
0	0.78	0.20	0.32	35
1	0.17	0.11	0.13	36
2	1.00	0.00	0.00	35
3	1.00	0.00	0.00	37
4	0.11	1.00	0.21	37
5	1.00	0.00	0.00	37
6	0.00	0.00	0.00	37
7	1.00	0.00	0.00	36
8	1.00	0.00	0.00	33
9	0.00	0.00	0.00	37
accuracy			0.13	360
macro avg	0.61	0.13	0.07	360
weighted avg	0.60	0.13	0.07	360



5. Discussion and Conclusion

Confusion matrices were generated to compare the classification accuracy and robustness between the two approaches. The analysis reveals the significant advantages of incorporating moments in image processing tasks, particularly in scenarios with image transformations and variations.

In conclusion, the integration of moments as descriptors for digit recognition presents a compelling array of advantages that significantly bolster the efficacy and resilience of classification models. One notable benefit lies in the realm of efficient feature extraction, where moment-based features offer computational efficiency, enabling swift processing ideal for real-time applications. Moreover, these descriptors excel in handling noise within input images, focusing sharply on pertinent features while disregarding extraneous fluctuations. They also illuminate the spatial arrangement and relationships between different elements of digit shapes, offering valuable insights into their structural context. By ensuring stability and consistency across varied scales and orientations, moments provide a stable foundation for model training, promoting faster convergence and reduced data requirements. Their capacity to preserve topological properties, such as loops and intersections, further enhances the accuracy of digit classification. Additionally, the simplicity and interpretability of moment-based features simplify model comprehension, debugging, and validation processes. Embodying scale-invariant traits, these descriptors empower models to recognize digits regardless of size variations. As a result, moment descriptors not only facilitate the creation of accurate and dependable digit recognition models but also support dynamic environments through their adaptability and transferability across datasets and tasks.