CS51 - Decision Making with LBA

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Decision Making: Scenario Analysis

Background

Tomorrow I have two important things to prepare for: a test and a presentation with my group partner and I only have time to prepare for one; so I need to choose between either of these option. The presentation is with my group partner, who also has the same exam tomorrow and so he also has the same two option to choose between like me. In this game-situation, the involved players are me and my partner. The choice for both of us is to prepare either for the test or presentation.

Assumptions

I take the following assumptions to make the scenario simpler to analyze with game theory. We have time to prepare for only one of them: either test or presentation. There is no communication between us, so I don't know his choice, which makes it a simultaneous game. Both test and presentation grades are out of 5 and total grade is calculated taking the average of the two grades. We have some prior knowledge on the test topics. So, even if we don't study today I could answer some of the question and get a grade 2 out of 5. We don't have any preparation on presentation, so we can't do anything if noone is prepared: result 0 out of 5. As a joint project, one of us alone can give the whole presentation from the team. So, if only one prepare well, we both would have 5 out of 5. For the test, a good preparation will result in an individual grade of 5 out of 5. Also we both are rational player, means we choose the choice with most logical payoff.

Payoff Matrix

According to our assumptions, I created the payoff matrix for me and my partner (Table 1). It is a non-zero sum symmetric matrix, as its quantities are not sum up to 0 and the payoff for a specific strategy against another strategy is same for both player.

Me/My Partner	Test	Presentation
Test	2.5,2.5	5*,3.5*
Presentation	3.5*,5*	3.5,3.5

Table 1: The payoff matrix for me and my partner, where the value of the matrix indicates the average grade of the player. The best responses of each player against other's strategies are marked by an asterisk (*).

The numerical values of the matrix represents the average grade of the players. They are calculated based on our assumptions. If both prepare for the test, they will get a 5 in the test but 0 in presentation, resulting an average of 2.5. If both prepare for presentation, they will have 5 in presentation and 2 in the test, resulting in 3.5 average. If one of them prepare for the presentation and other for the test, the former would get 5 and 2, while the later would have 5 and 5 in presentation and test respectively resulting in 3.5 and 5. There is no strongly or weakly dominated strategy in the matrix.

Nash Equilibrium

One way to find pure nash equilibrium is to check the best responses for each player against every strategies, which is marked in the payoff matrix with an asterisk (Table 1). From the best responses, we can see that the top-right and bottom-left cell are representing two pure nash equilibrium.

Another way to define nash equilibrium is to a set of strategies, where no player would want to change his choice given that he knows other players' choice. Here, in the top-right one, for me, the payoff for test against presentation is larger than presentation against presentation, $EU_{TP} > EU_{PP}$ $(5 > 3.5)$, so I will not change my decision. Similarly, for my partner, $EU_{PT} > EU_{TT}$ $(3.5 > 2.5)$, so he would also strict with his solution given that I chose test. Exactly same is applicable for the bottom-left theory.

As there must be odd number of nash equilibrium and we found only two, there must be one mixed nash equilibrium. I assume that the probability of me and my partner to prepare for test is p and q respectively (Table 2).

So, my average payoff for choosing test, $EV_T = q\,(EU_{TT}) + (1-q)(EU_{TP}) = 2.5q + 5 - 5q$

My average payoff for choosing preparation, $EV_P = q\,(EU_{PT}) + (1-q)(EU_{PP}) = 3.5$

For mixed nash equilibrium, they both are equal:

$2.5q + 5 - 5q = 3.5 \Rightarrow 2.5q = 1.5 \Rightarrow q = 0.6$

As it is a symmetric matrix, the probability, p, for which my partner's average payoff for both strategy would be same is equal to q, 0.6.

Me/My Partner	Test, q = 0.6	Presentation, (1-q) = 0.4
Test, p = 0.6	2.5,2.5	5*,3.5*
Presentation, (1-p) = 0.4	3.5*,5*	3.5,3.5

Table 2: Denoting the probability to choose a strategy to calculate the mixed nash equilibrium and their values found by calculation.

So, the mixed nash equilibrium would be when me and my partner choose test 40% time and presentation 60% time. If they choose with these probabilities, no one would want to change their choice given the choice of the other player.

Decision Tree

From the value of the payoff matrix and the probability of the mixed nash equilibrium, I created a decision tree from my perspective (Fig. 1) using Essy Tree software (2019). It has a major decision node (square one) for decide between test or presentation and two event nodes (circular one) showing the possible outcome and their probability for each choice and lastly the terminal nodes (triangle one) are showing my payoff for the specific outcomes. The probabilities are chosen from the mixed nash equilibrium since as a 'rational' player, my partner would stick with these probabilities.

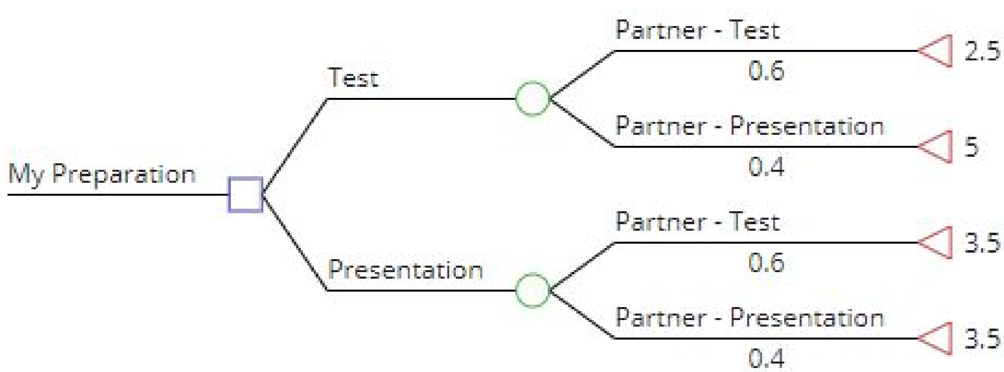

Figure 1: Decision tree showing my choice between preparing for test or presentation, the probability of different possible events for a rational player and my payoff for each possible outcomes.

Different Strategies

According to the maximax strategy, I would choose the maximum value of the all best possible outcome. For choosing test, my best possible payoff is 5, while for presentation it is 3.5. As 5 > 3.5, I would choose the test in maximax strategy (Table 3). As a optimist I would think that my partner would prepare for the presentation and I would not face any problem.

For maximin strategy, I would like the maximum value of the all worst possible outcomes. Here, the worst payoffs for choosing test and presentation are 2.5 and 3.5 respectively. So, I would choose presentation according to maximin strategy. As a pessimist, I would imagine the worst case that my partner would not prepare the presentation, so I would prepare for presentation despite of the chance to get bad grade in test.

Choice/Outcome	Partner - Test	Partner - Presentation	Maximum	Minimum
Test	2.5	5	5 (Maximax)	2.5
Presentation	3.5	3.5	3.5	3.5 (Maximin)

Table 3: The best and worst possible payoffs for each of my options and my choice in maximax and maximin strategy.

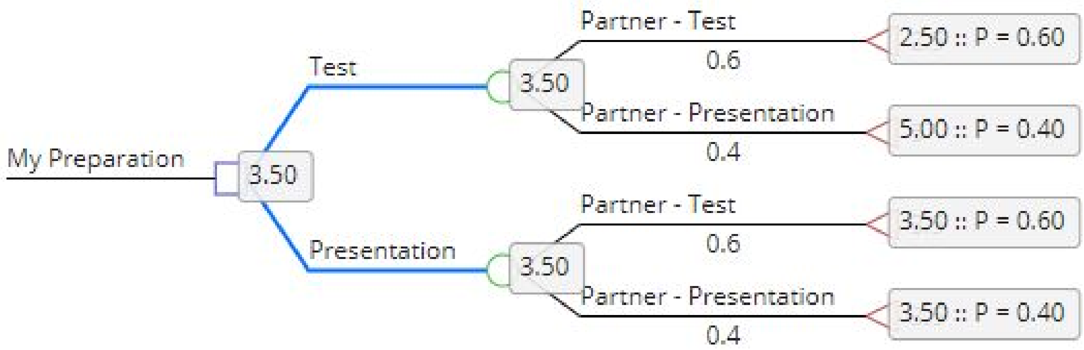

Figure 2: The expected value of my both choices corresponding to the probability of my partner's choice, which gives an equal payoff for both choices.

In expected value strategy, we need to compare the expected values for both of my options according to their probability. As my partner, being a rational decision maker, choose the probability for test and presentation similar to the mixed nash equilibrium, both option gives me an equal payoff of 3.5 (Fig. 2). So, I cannot decide any choice over another based on the expected value strategy.

Now in the minimax regret theory, we would compare my payoffs for a specific outcomes with the best possible cases for that outcome and calculate how regretful I would be for choosing one choice over another. Then from the list of maximum regret for any choices, I would choose the minimum one to minimize my regret as much as possible. In this case, my maximum possible regret values for choosing test and presentation are 1 and 1.5, so I would choose test to minimize my regret (Table 4). I would be more regretful if I choose presentation and end up losing grade in my test.

Regret values of my choices for a specific outcome:			Maximum Regret	Expected Opportunity Loss
Choice/Outcome	Partner-Test	Partner-Presentation		
Probability	0.6	0.4		
Test	1	0	1 (Minimax Regret)	0.6
Presentation	0	1.5	1.5	0.6

Table 4: Regret values of both of my choices for specific outcomes and maximum regret values and expected opportunity lost for each of my choices to calculate my best decision in minimax regret and expected opportunity loss strategies.

I would go with the same approach to calculate the regret values for expected opportunity loss, but instead of applying a minimax strategy with the regret values, I would use the probability values to calculate my expected opportunity loss for each choice. It ended up being the same loss for both choices as my partner chooses the probability rationally (Table 4).

Perfect Information

In the normal scenario, as a simultaneous game, I do not know the choices of my partner before making my decision. But if I can get this knowledge before choosing, I can increase my payoff according to the payoff matrix. But as the information will not come with any trade off, I need to calculate how much will my payoff increase due to this perfect information before making a rational trade-off for information. The value of perfect information is simply the difference between my payoff using the perfect information and my 2nd best payoff.

In previous decision tree (Fig. 1), we chose our decision and then waited to see which outcome would happen. But in the branch of perfect information, we would know the outcome (my partner's choice) before my decision making. In the new tree (Fig. 3), we can see the event node is earlier then my decision node in the branch of perfect information.

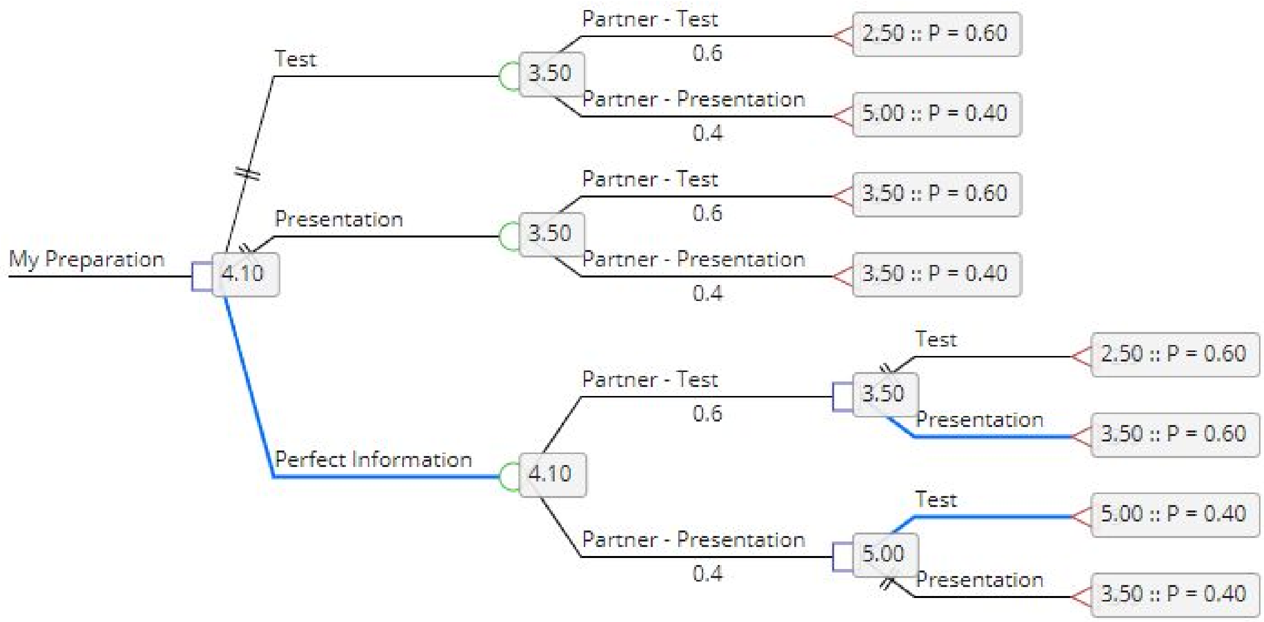

Figure 3: The updated tree with the branch for perfect information where outcome is known earlier than the decision making. The expected values are also calculated to measure the value of perfect information.

Still my partner is choosing test and presentation 60% and 40% of time, but every time I knew exactly what decision he made due to the perfect information. The payoff with the perfect information is 4.10, while the 2nd best payoff is the 3.50. So, the value of the perfect information is equivalent to the utility I have when I received an increase of 0.60 in my academic grade. So, I would spend at most that much money, which I willing to pay in response of a 0.60 increase in grade. If grades are very important to me, it'll be more monetary value or vice-versa.

Bayesian Probability

We both can use the bayesian approach if this is a repeated scenario, that is, very often we both have to decide between a test or presentation under these same assumptions. For the very first time, I would randomly assign the probability value of my partner's choice in my decision tree. Either I can guess that he is neutral and put 50% and 50% for test and presentation or I can guess that he is rational and would follow mixed nash equilibrium, that is 60%,40%. Then each time we both face the situation and know about other's personality, it will change my perspective towards him and thus change the probability value. If I see, mostly he is choosing presentation over test, the probability of his choices would change in my decision tree, thus change my decision. For example, if I experienced that he chose presentation 10 times out of 15 (67%), I will decide to prepare for test using expected value in my new decision tree (Fig. 4).

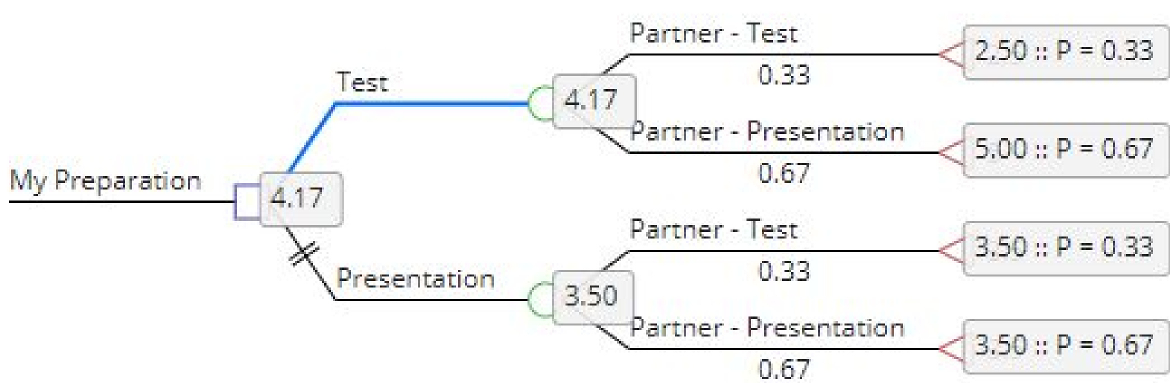

Figure 4: Using bayesian approach I may start with a guessed or reasoned probability of my partner's choice which would be updated every time we faced each other and thus would be affect my decision tree and decision as this tree used 33% probability which is way less than the rational probability..

Cognitive Bias

Very often, people don't think rationally due to some cognitive bias. Many people tend to avoid any type of risk and act risk-averse. With this cognitive bias, they tend to underestimate the rational outcomes. Their utility is less than the outcome and it can be described as following equation: Utility, $U(x) = x^a,\ where\ 0 < a < 1$. My partner could be risk-averse with $U(x) = x^{0.5} = \sqrt{x}$, which would change the initial payoff matrix (Table 1) into a new one, where the payoffs are our utility (Table 5). Due to his underestimation of payoff, my probability in the mixed nash equilibrium would be changed. Also, he would always use pessimist's choice (maximin strategy) to avoid any worst case and always choose the presentation which is less beneficial pure nash equilibrium to him than the bottom-left one. Also due to his risk-averse behaviour and preference of presentation, using the bayesian probability his probability to choose presentation will increase in my decision tree (like Fig.4) and he will stuck in the top-left cell.

Me/My Partner	Test	Presentation
Test	2.5,1.58	5*,1.87*
Presentation	3.5*,2.23*	3.5,1.87

Table 5: The payoff matrix with utility changed due to my partner's risk-averse behaviour, which leads to a change in probability in mixed nash equilibrium.

Comparing different strategy, thinking rationally and showing risk-neutral behaviour could help him to get rid of choosing always risk-averse pessimist's approach, which would lead more payoff for him in the long run.

Reflection

In this scenario, we saw the choice between the preparation for a test or presentation and the possible payoff matrix as well as the decision of mine in a different strategy. That section gives me insight on how to use game theory in real-life scenarios with appropriate assumptions. Many often, I faced the similar kind of situation with my academic life, but now I have better tool analyze and come up with the best idea. I also understood that very often we became risk-averse or cognitively biased and choose irrational decision, but a rational decision always gives the better payoff in the long run, though it looks like a risky or odd thing to do.

Another big thing is the value of perfect information. From the decision tree, I saw how largely the payoffs can increase due to a perfect information. Maybe that’s why the betting and bribes are so hard to wipe totally from the sport field.

In summary, though game theory restricts us by various assumptions and do not let us to work with the full picture, the game theory can explain and increase our understanding on the very specific and detailed behaviours in a day to day life situations.

References

Essy LLC. (2019). Essy Decision Tree Software. Retrieved from http://essytree.com/