Higher order Differential equation.

Homogeneous — non-Homogeneous

An equation ~~of the form~~

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_n y = Q(x)$$

Where $a_0, a_1 \ldots a_n$ are Constant.

$\Rightarrow (a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n) y = Q(x)$

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Where $D = \frac{d}{dx}$ is called higher Order linear non-homogeneous equation with Constant Co-efficient.

If $Q(x) = 0$, Then the equation $(a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n) y = 0$ is called higher Order linear homogeneous D.E with Constant Co-efficient.

$\Rightarrow x^2 y'' + 5x y' + 7y = e^x$.

This is a Second Order first degree linear non-homogeneous differential equation with Variable Co-efficient.

$\Rightarrow y''' + 3y = 0$

This is 3rd Order first degree linear homogeneous D.E with Constant Co-effic

⇒ Solve if $y'' + 7y' + 12y = 0$

Solution: Given that

$$y'' + 7y' + 12y = 0 \longrightarrow (1)$$

This is a 2nd Order 1st degree linear homogeneous D.E with Constant Co-efficient.

Let $y = e^{mx}$ be a solution of (1)

$$y' = m e^{mx}$$
$$y'' = m^2 e^{mx}$$

putting these value in equation (1), we get

$$m^2 e^{mx} + 7m e^{mx} + 12 e^{mx} = 0$$
$$e^{mx}(m^2 + 7m + 12) = 0$$
$$m^2 + 7m + 12 = 0$$

The auxilary equation is

$$m^2 + 7m + 12 = 0$$
$$m^2 + 4m + 3m + 12 = 0$$
$$m = -3, -4$$

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Hence the general solution is

$$y = c_1 e^{-4x} + c_2 e^{-3x}$$

where c_1 ans c_2 are Arebtrary Cos

Solve $y'' - 4y' + 4y = 0$

Solution: Given that

$$y'' - 4y' + 4y = 0 \quad \text{--- (1)}$$

This is 2nd order first degree linear homogeneous equation with constant co-efficient.

Let $y = e^{mx}$ be a solution of (1)

$$y' = me^{mx}$$

$$y'' = m^2 e^{mx}$$

putting these value of (1), we get

$$m^2 e^{mx} - 4me^{mx} + 4e^{mx} = 0$$

$$m^2 - 4m + 4 = 0$$

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The auxilary equation is

$$m^2 - 4m + 4 = 0$$

$$m = 2, 2$$

the general solution is

$$y = c_1 e^{2x} + x c_2 e^{2x} = (c_1 + x c_2) e^{2x}$$

Solve it

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0 \quad \text{--- (1)}$$

let $y = e^{mx}$

the auxilary equation is

$$m^2 + 2m + 2 = 0$$

$$m = \frac{-2 \pm \sqrt{4 - 4.2}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= -1 \pm i$$

Hence the General Solution is

$$y = e^{-x}[c_1 \cos x + c_2 \sin x]$$

$y_c = e^{(\text{Real part})x}[c_1 \cos(\text{imaginary}) + c_2 \sin]$

Higher order Homogeneous D.E with constant Co-efficient.

The higher order homogeneous DE is

$$(a_0D^n + a_1D^{n-1} + a_2D^{n-2} + \cdots + a_n)y = 0 \longrightarrow (1)$$

Let $y = e^{mx}$ be the trial solⁿ of (1).

$Dy = me^{mx}$

$D^2y = m^2e^{mx}$

$\vdots$

$D^ny = m^n e^{mx}$

putting these value in (1), we get

$$a_0 m^n e^{mx} + a_1 m^{n-1} e^{mx} + \cdots + a_n e^{mx} = 0$$

This is called auxiliary equation.

The above equation can be written as

$$(m-m_1)(m-m_2)\cdots(m-m_n) = 0$$

$$m = m_1, m_2, \cdots, m_n$$

Hence the general solution

$$y = c_1e^{m_1x} + c_2e^{m_2x} + \cdots + c_ne^{m_nx}$$

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Case 1: For distinct roots

$(D^3 + 6D^2 + 11D + 6)y = 0 \longrightarrow (1)$

Let $y = e^{mx}$ be the trial solⁿ of (1).

Then A.E is

$$m^3 + 6m^2 + 11m + 6 = 0$$

$$m^3 + m^2 + 5m^2 + 5m + 6m + 6 = 0$$

$$m^2(m+1) + 5m(m+1) + 6(m+1) = 0$$

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$(m+1)(m+2)(m+3) = 0$

$m = -1, -2, -3,$

Hence the general Solution is

$$y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{-3x}$$

Case (II) For equal roots

$$m = \alpha, \alpha, \alpha.$$

$$y = c_1 e^{\alpha x} + x c_2 e^{\alpha x} + x^2 c_3 e^{\alpha x}$$

$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0 \longrightarrow (1)$

Let $y = e^{mx}$

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$m^3 - 3m^2 + 3m - 1 = 0$

$(m-1)^3 = 0 \quad m = 1, 1, 1.$

Hence the general Solution is

$$y = c_1 e^{x} + c_2 x e^{x} + c_3 x^2 e^{x}$$

Case (III) For Complex roots

$$m = \alpha \pm i\beta$$

$$m = \alpha + i\beta, \ m = \alpha - i\beta$$

$$y = Ae^{(\alpha+i\beta)x} + Be^{(\alpha+i\beta)x}$$

$$= \left(Ae^{i\beta x} + Be^{-i\beta x}\right)e^{\alpha x}$$

$$= e^{\alpha x}\{A(\cos\beta x + i\sin\beta x) + B(\cos\beta x - i\sin\beta x)\}$$

$$= e^{\alpha x}\{(A+B)\cos\beta x + (A-B)\sin\beta x\}$$

$$= e^{\alpha x}\{c_1\cos\beta x + c_2\sin\beta x\}$$

Example (i) $m = 2 \pm 3i$

$$y = e^{2x}(c_1\cos 3x + c_2 \sin 3x)$$

(ii) $m = \pm i$

$$y = e^{0.x}(c_1\cos x + c_2 \sin x)$$

$$= c_1 \cos x + c_2 \sin x$$

$(D^4 + 5D^2 + 6)y = 0$

$$y = e^{mx}$$

$$m^4 + 5m^2 + 6 = 0$$

$$(m^2+2)(m^2+3) = 0$$

$m = \pm i\sqrt{2}$ $\quad m = \pm i\sqrt{3}$

Hence the General Solution is

$$y = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x + c_3 \sin\sqrt{3}x + c_4 \cos\sqrt{3}x$$

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Higher order non-homogeneous D, E

Non-homogeneous D, E

The equation higher order non-homogeneous D, E

$$(a_0D^n + a_1D^{n-1} + \cdots + a_n)y = Q(x) \rightarrow (1)$$

The corresponding homogeneous equation of (1) is

$$(a_0D^n + a_1D^{n-1} + \cdots + a_n)y = 0 \rightarrow (2)$$

The solution of (2) called complementary function and its denoted by y_c

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$$y'' - 7y' + 12y = e^{3x}$$

$$y'' - 7y' + 12y = 0$$

Complementary function

C.F $\longrightarrow y_c$

Particular Integral $\longrightarrow$ P.I $\longrightarrow y_p$

Hence the General Solution

$$y = y_c + y_p$$

(i) Method of undetermined co-efficient

(ii) Method of variation of parameters

(iii) Operator method (Short method)

Method of undetermine Co-efficient

$F(x)$ ~~~~~~ y_p

① $x^2 \longrightarrow y_p = A_0 + A_1 x + A_2 x^2$

② $e^{mx} \longrightarrow y_p = A e^{mx}$

③ $\left.\begin{array}{c} \sin x \\ \downarrow \\ \cos x \end{array}\right\} \longrightarrow y_p = A \sin x + B \cos x$

④ $x^2 e^{3x} \longrightarrow y_p = (A_0 + A_1 x + A_2 x^2) e^{3x}$

⑤ $x^2 \cos 3x \longrightarrow y_p = (\cancel{A_0 + A_1 x + A_2 x^2) e^{3x}}$

$= (A_0 + A_1 x + A_2 x^2)(A \cos 3x + B \sin 3x)$

⑥ $e^{3x} \sin 4x \longrightarrow y_p = e^{3x}(A \sin 4x + B \cos 4x)$

Note: If any point of y_p is already in y_c

Then multiply y_p by x or x^2

$m = 2, 2$

$y = c_1 e^{2x} + c_2 x e^{2x}$

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Case (I) $Q(x) =$ polynomial function

x^3

$$y_p = A + Bx + Cx^2 + Ex^3$$

x

$$y_p = A + Bx$$

x^2

$$y_p = A + Bx + Cx^2$$

Case (II) $Q(x) =$ exponential function

$$= e^{ax}$$

$$y_p = A e^{ax}$$

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Case (III) $Q(x) =$ sin / cos function

i.e $Q(x) = \sin ax / \cos ax$

$$y_p = A \cos ax + B \sin ax$$

$\boxed{\text{N.B}} \Rightarrow$ Complementary function (y_c) এর কোন term Particular Integral এর কোন term এর মত হলে Particular Integral এর term কে x দ্বারা গুণ করতে হবে।

Case (IV) $Q(x) = e^{ax}(x^2+x+1)$

e^{ax} = exponential function

(x^2+x+1) → polynomial function.

$$y_p = c_1 e^{ax}(c_2 + c_3 x + c_4 x^2)$$

$$y_p = (A + Bx + Cx^2)e^{ax}$$

Case (V) $Q(x) = e^{ax}(\sin bx / \cos bx)$

$$y_p = (A\cos bx + B\sin bx)e^{ax}$$

Case (VI) $Q(x) =$ polynomial function $\times$ sin/cos function

$$= (x^2 + x + 1)\sin ax$$

$$= (A + Bx + Cx^2)\cos ax$$

$$+ (E + Fx + Gx^2)\sin ax$$

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$$\frac{d^2y}{dx^2}+y = x\sin x$$

Solution: Given that

$$\frac{d^2y}{dx^2}+y = x\sin x \longrightarrow (1)$$

let $y=e^{mx}$ be the solution of $y''+y=0$

Then the A.E is $m^2+1=0$

$m=\pm i$

The complementary function

$$y_c = c_1\cos x + c_2 \sin x.$$

By the method of undetermined Co-efficient we get,

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$$y_p = (A+Bx)\cos x + (E+Fx)\sin x$$

$$y_p' = -(A+Bx)\sin x + B\cos x + (E+Fx)\cos x + F\sin x$$

$$y_p'' = -(A+Bx)\cos x - B\sin x - B\sin x - (E+Fx)\sin x + F\cos x + F\cos x$$

$$\therefore y_p'' = -(A+Bx)\cos x - 2B\sin x - (E+Fx)\sin x + 2F\cos x$$

putting these Value in (1), we get

$$-\cancel{(A+Bx)\cos x} - 2B\sin x - \cancel{(E+Fx)\sin x} + 2F\cos x + \cancel{(A+Bx)\cos x} + \cancel{(E+Fx)\sin x} = x\sin x$$

$$\Rightarrow -2B\sin x + 2F\cos x = x\sin x$$

$$y_p = Ax\sin x + Bx\cos x + Cx^2\sin x + Dx^2\cos x$$

$$y_p' = A\sin x + Ax\cos x + B\cos x - Bx\sin x + 2Cx\sin x + Cx^2\cos x + 2Dx\cos x - Dx^2\sin x$$

$$= (A+2Cx-Bx-Dx^2)\sin x + (Ax+B+Cx^2+2Dx)\cos x$$

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$$y_p'' = (A+2Cx - Bx - Dx^2)\cos x + (2C - B - 2Dx)\sin x$$
$$+ (A+2Cx+2D)\cos x - (B+Ax+2Dx+Cx^2)\sin x$$

putting this value of (1), we get

$$\therefore (A+2Cx-Bx-Dx^2)\cos x + (2C-B-2Dx)\sin x$$
$$+ (A+2Cx+2D)\cos x - (B+Ax+2Dx+Cx^2)\sin x$$
$$+ Ax\sin x + Bx\cos x + Cx^2 \sin x + Dx^2\cos x = x\sin x$$

$$\Rightarrow (2A+2D)\cos x + 4Cx\cos x + (2C-2B)\sin x$$
$$- 4Dx\sin x = x\sin x$$

Comparing on both sides, we get

$2C-2B=0$ $\quad -4D = 1 \therefore D = -\frac{1}{4}$ $\quad 2A+2D \therefore A = \frac{1}{4}$ $\quad C=0$

$B = 0$ $[\because C=0]$

Thus the particular integral $y_p = \frac{1}{4}x\sin x - \frac{1}{4}x^2\cos x$

Hence the General solution is

$$\boxed{y = y_c + y_p}$$

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$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x} \quad \text{(1)}$$

Solution: let $y = e^{mx}$ be the trial soln of

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0.$$ Then the

A.E is

$$m^2 - 3m + 2 = 0$$
$$(m-1)(m-2) = 0$$
$$\therefore m = 1, 2$$

$$y_c = c_1e^x + c_2e^{2x}$$

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By the method of undetermined co-efficient, we set

$$y_p = \underline{A + Bx + Cx^2} + (Dx + Ex^2)e^x + Fe^{3x}$$

$$y_p = \underline{A + Bx} + Cx^2 + Dxe^x + \underline{Ex^2e^x} + Fe^{3x}$$

$$y_p' = B + 2Cx + De^x + Dxe^x + 2xEe^x + 3Fe^{3x} + Ex^2e^x$$

$$y_p'' = -$$

A, B, C, D, E, F

$$\boxed{y = y_c + y_p}$$

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$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2xe^{2x} + Ce^x \longrightarrow (1)$$

Let $y = e^{mx}$ be the trial solution of

$y'' - 2y' + y = 0$. Then the A.E

$$m^2 - 2m + 1 = 0$$

$$m = \frac{2 \pm \sqrt{4 - 4.1.1}}{2.1}$$

$$= 1 \pm 0$$

$$= 1, 1$$

$$y_c = c_1e^x + c_2xe^x$$

By the undetermine co-efficient, we

$$y_p = (A + Bx)e^{2x} + Cx^2e^x$$

$$= \underline{Ae^{2x} + Bxe^{2x} + Cxe^x}$$

$A, B, C = ?$

$$y = y_c + y_p$$

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$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{x} - 10\sin x \longrightarrow (1)$$

Let $y = e^{mx}$ be the trial solution of $y'' - 2y' - 3y = 0$ Then the A.E is

$$m^2 - 2m - 3 = 0$$

$$m = \frac{+2 \pm \sqrt{4 - 4\cdot(-3)}}{2}$$

$$= \frac{2 \pm \sqrt{4+12}}{2}$$

$$= 1 \pm 2$$

$$= 3, -1$$

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The complementary function

$$\boxed{y_c = c_1 e^{3x} + c_2 e^{-x}}$$

By the method of undetermine co-efficients we get

$$\boxed{y_p = Ae^{x} + B\cos x + C\sin x}$$

$$y_p' = Ae^{x} - B\sin x + C\cos x$$

$$y_p'' = Ae^{x} - B\cos x - C\sin x$$

putting these value in (1), we get

$$Ae^{x} - B\cos x - C\sin x - 2Ae^{x} + 2B\sin x - 2C\cos x - 3Ae^{x} - 3B\cos x - 3C\sin x = 2e^{x} - 10\sin x$$

$$\Rightarrow -4Ae^{x} - (4B+2C)\cos x + (2B-4C)\sin x = 2e^{x} - 10\sin x$$

Comparing on both sides, we get

$-4A = 2$	$-(4B+2C) = 0$	$2B - 4C = -10$
$A = -1/2$	$C = 2$	$2B + 8B = -10$
		$B = -1$

$\boxed{4.38}$
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$$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 3x^2 + 4\sin x - 2\cos x \longrightarrow (1)$$

Let $y = e^{mx}$ be the solution of $y^{iv} + y'' = 0$ Then the A.E

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$$m^4 + m^2 = 0$$

$$m^2(m^2+1) = 0 \qquad c_1 e^{0.x}$$

$$m = 0, \quad m^2 = -1$$
$$m = \pm\sqrt{-1}$$
$$= 0 \pm i$$

Then the Complementary function

$$y_c = e^{(0)x}(c_1\cos x + c_2\sin x)$$

By the method of undetermined Co-efficient, ~~we~~

$$y_p = A + Bx + Cx^2 +$$

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$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{4x}$$

$$m^2 - 2m - 3 = 0$$

$$m^2 - 3m + m - 3 = 0$$

$$m(m-3) + 1(m-3) = 0$$

$$m = -1, 3$$

$$y_c = c_1 e^{-x} + c_2 e^{3x}$$

By the undetermine coefficient, we set

$$\boxed{y_p = Ae^{4x}}$$

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$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^{3x}$$

$$m^2 - 2m - 3$$

$$y_c = c_1 e^{-x} + c_2 e^{3x}$$

$$\boxed{y_p = xAe^{3x}}$$

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$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2 e^x$$

$$m^2 - 3m + 2 = 0$$

$$m^2 - 2m - m + 2 = 0$$

$$m(m-2) - 1(m-2) = 0$$

$$m = 1, 2 \qquad y_c = c_1 e^x + c_2 e^{2x}$$

$$y_p = (A + Bx + Cx^2)e^x$$

$$= Ae^x + Bxe^x + Cx^2e^x$$

$$\boxed{y_p = Ax^3e^x + Bxe^x + Cx^2e^x}$$

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$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x - 4e^{3x}$$

$$y_c = c_1 e^x + c_2 e^{2x}$$

$$y_p = Ax^2 + Bx + C + (A + Bx)e^x + Ae^{3x}$$
$$= Ax^2 + Bx + C + Ae^x + Bxe^x + Ae^{3x}$$
$$\boxed{y_p = \underline{Ax^2 + Bx + C} + \underline{Ax^2e^x + Bxe^x + Ae^{3x}}}$$

$\boxed{4.38}$

$$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 3x^2 + 4\sin x - 2\cos x$$

$$m^4 + m^2 = 0$$
$$m^2(m^2 + 1) = 0$$
$$m^2 = 0 \qquad m^2 = -1$$
$$m = \pm i$$
$$\underline{m = 0, 0}$$

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$$y_c = c_1e^{0.x} + c_2xe^{0.x} + c_3\cos x + c_4\sin x$$
$$= c_1 + xc_2 + c_3\cos x + c_4\sin x$$

$$y_p = \underline{Ax^2} + \underline{Bx} + C + D\cos x + E\sin x$$
$$\boxed{y_p = Ax^2 + Bx^3 + Cx^4 + Dx\cos x + Ex\sin x}$$

$\boxed{4.39}$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$$

$$m^2 - 2m - 3 = 0$$
$$m^2 - 3m + m - 3 = 0$$
$$m = -1, 3$$
$$y_c = c_1e^{-x} + c_2e^{3x}$$
$$\boxed{y_p = Ae^x + B\cos x + C\sin x}$$

Method of variation of parameters

Parameter $\longrightarrow$ arbitrary Constant.

$\longrightarrow$ c_1, c_2, A, B etc

$\Rightarrow a_0(x)y'' + a_1(x)y' + a_2(x)y = F(x) \longrightarrow ①$

then the Complementary function is

$$y_c = c_1 y_1(x) + c_2 y_2(x)$$

Where c_1 and c_2 are any arbitrary.

By the method of variation of parameters, we get

$$y_p = v_1(x)\, y_1(x) + v_2(x)\, y_2(x) \longrightarrow ②$$

$$y_p' = v_1 y_1' + y_1 v_1' + v_2 y_2' + v_2' y_2$$

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let $\quad v_1' y_1 + v_2' y_2 = 0 \longrightarrow ③$

$$\left.\begin{aligned} y_p' &= v_1 y_1' + v_2 y_2' \\ y_p'' &= v_1' y_1' + v_1 y_1'' + v_2' y_2' + v_2 y_2'' \end{aligned}\right\} \longrightarrow ④$$

putting ② and (4), in (1).

$$v_1' y_1' + v_2' y_2' = \frac{F(x)}{a_0(x)} \longrightarrow ⑤$$

Solving (3) and ⑤, we get

$$v_1' = \frac{\begin{vmatrix} 0 & y_1 \\ \frac{F(x)}{a_0(x)} & y_2 \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}} \qquad v_2' = \frac{\begin{vmatrix} y_1 & 0 \\ y_1' & \frac{F(x)}{a_0(x)} \end{vmatrix}}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}}$$

Variation of Parameters

$\frac{d^2y}{dx^2} + y = \tan x$

<u>Soln</u>: Let $y = e^{mx}$ be the trial solution of $\frac{d^2y}{dx^2} + y = 0$, Then the A.E is

$$m^2 + 1 = 0$$
$$m = \pm i$$

Then Complementary function

$$y_c = c_1\cos x + c_2\sin x$$

By Variation of Parameters,

$$y_p = v_1(x)\cos x + v_2(x)\sin x$$

$$\therefore y_p' = -v_1\sin x + v_2\cos x + v_1'\cos x + v_2'\sin x$$

We impose the condition,

$$v_1'\cos x + v_2'\sin x = 0$$
$$v_1'(-\sin x) + v_2'\cos x = \tan x$$

$$v_1' = \frac{\begin{vmatrix} 0 & \sin x \\ \tan x & \cos x \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} = \frac{0 - \sin x \tan x}{\cos^2 x + \sin^2 x} = -\frac{\sin^2 x}{\cos x} \quad \text{(i)}$$

$$v_2' = \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} = \frac{\cos x \tan x}{1} = \sin x \quad \text{(ii)}$$

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Integrating (1), we get

$$\int v_1' = \int -\frac{\sin^2 x}{\cos x}\,dx = -\int \frac{1-\cos^2 x}{\cos x}$$

$$= \int (\sec x - \cos x)\,dx = -\log(\sec x + \tan x) + \sin x$$

Integrating (2), we get

$$\int v_2' = \int \sin x\,dx =$$

$$v_2 = \cos x$$

Therefore, Particular Integral

$$y_p = -\cos x \log(\sec x + \tan x) + \sin x \cos x$$
$$- \cos x \sin x$$

$$= -\cos x \log(\sec x + \tan x)$$

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Hence the G.S is

$$y = y_c + y_p$$

Q) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x} \quad \text{———} (1)$$

let $y = e^{mx}$ be $\longrightarrow$ $m = \pm 1$

$$y_c = c_1 e^x + c_2 e^{-x}$$

By the variation of parameter, we get

$$y_p = v_1 e^x + v_2 e^{-x}$$

$$y_p' = v_1 e^x - v_2 e^{-x} + v_1' e^x + v_2' e^{-x}$$

we propose the condition

$$\left.\begin{aligned} v_1' e^x + v_2' e^{-x} &= 0 \\ v_1' e^x - v_2' e^{-x} &= \frac{2}{1+e^x} \end{aligned}\right\}$$

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By the Crammer's rule, we get

$$v_1' = \frac{\begin{vmatrix} 0 & e^{-x} \\ \frac{2}{1+e^x} & -e^{-x} \end{vmatrix}}{\begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}} = \frac{0 - e^{-x}\frac{2}{1+e^x}}{-e^x e^{-x} - e^{-x}e^x}$$

$$= \frac{e^{-x}}{1+e^x} = \frac{e^{-x}}{e^x(e^{-x}+1)} \quad (i)$$

$$v_2' = \frac{\begin{vmatrix} e^x & 0 \\ e^x & \frac{2}{1+e^x} \end{vmatrix}}{\begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}} = \frac{e^x \frac{2}{1+e^x}}{-e^x e^{-x} - e^x e^{-x}} = -\frac{e^x}{1+e^x} \quad (ii)$$

Integrating,

$$v_1 = \int \frac{e^{-x} e^{-x}}{e^{-x}+1} dx$$

put $e^{-x}+1 = z$

$e^{-x} dx = -dz$

$e^{-x} = z-1$

$$= -\int \frac{(z-1)\,dz}{z}$$

$$= -\int \left(1 - \frac{1}{z}\right) dz$$

$$= -z + \log z$$

$$= -(e^{-x}+1) + \log(e^{-x}+1)$$

Integrating

$$v_2 = -\int \frac{e^x}{1+e^x} dx$$

$$= -\log(1+e^x)$$

$$\frac{d^2y}{dx^2} + y = \sec x$$

Sol^n: let $y = e^{mx}$ be a trial sol^n of $y'' + y = 0$

then the A.E is $m^2 + 1 = 0$ $m = \pm i$

$$y_c = c_1 \sin x + c_2 \cos x$$

By the method of variation of Parameter

$$y_p = v_1(x) \sin x + v_2(x) \cos x$$

$$y_p' = v_1 \cos x - v_2 \sin x + v_1' \sin x + v_2' \cos x$$

we impose the Condition

$$v_1' \sin x + v_2' \cos x = 0$$
$$v_1' \cos x + v_2'(-\sin x) = \sec x$$

By Cramer's rule, we get

$$v_1' = \frac{\begin{vmatrix} 0 & \cos x \\ \sec x & -\sin x \end{vmatrix}}{\begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}} = \frac{-\cos x \cdot \sec x}{-\sin^2 x - \cos^2 x} = 1 \quad \text{—— (1)}$$

$$v_2' = \frac{\begin{vmatrix} \sin x & 0 \\ \cos x & \sec x \end{vmatrix}}{\begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}} = \frac{\sin x \sec x}{-\sin^2 x - \cos^2 x} = -\tan x \quad \text{—— (2)}$$

Integrating,

$$v_1 = \int dx = x$$

Integrate $v_2 = -\int \tan x \, dx = \log(\cos x)$

$$y = y_c + y_p$$

Short Methods / operator method

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$$(a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \cdots + a_n) y = Q(x)$$

i.e $\quad f(D) y = Q(x)$

$$\boxed{\therefore y_p = \frac{1}{f(D)} Q(x)}$$

Case-1:

$Q(x) =$ exponential function

$= e^{ax}$

$$\therefore y_p = \frac{1}{f(D)} e^{ax}$$

$$= \frac{1}{f(a)} e^{ax}; \quad f(a) \neq 0$$

if $f(a) = 0$, then $y_p = x \frac{1}{f'(D)} e^{ax}$

Case-II

$Q(x) =$ sin / cos function

i.e $\quad Q(x) = \sin ax / \cos ax$

$$\therefore y_p = \frac{1}{f(D)} \sin ax / \cos ax$$

$$= \frac{1}{f(D^2)} \sin ax / \cos ax$$

$$= \frac{1}{f(-a^2)} \sin ax / \cos ax$$

if $f(-a^2) \neq 0$

Case (III)

$Q(x) =$ polynomial function.

i.e $Q(x) = x^2 + x + 1$

$Q(x) = x^3$

* $(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$
* $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$
* $(1+x)^{-2} = 1 - 2x + 3x^2 - \dots$
* $(1-x)^{-2} = 1 + 2x + 3x^2 + \dots$
* $(1-x^2)^{-1} = 1 + x^2 + x^4 + \dots$

Case (IV)

$Q(x) =$ exponential $\times$ polynomial f^n

$= e^{ax}(x^2 + x + 1)$

For particular Integral:

$$y_p = \frac{1}{f(D)} e^{ax}(x^2 + x + 1)$$

$$= e^{ax} \frac{1}{f(D+a)} (x^2 + x + 1)$$

$$= e^{ax} \frac{1}{a} (1 + D/a)^{-1} (x^2 + x + 1)$$

$$= e^{ax} \frac{1}{a} \left(1 - \frac{D}{a} + (D/a)^2 - \dots\right) (x^2 + x + 1)$$

$$= e^{ax} \frac{1}{a} \left(x^2 + x + 1 - \frac{2x}{a} - \frac{1}{a} + \frac{2}{a^2}\right)$$

$\downarrow$

Case (III) same as

Case (V)

$Q(x) =$ exponential $\times$ sin/cos function.

$= e^{ax}(\sin bx / \cos bx)$

For particular integral

$$y_p = \frac{1}{f(D)} e^{ax}(\sin bx/\cos bx)$$

$$= e^{ax} \frac{1}{(D+a)} (\sin bx/\cos bx)$$

$\downarrow$

Case (II) Same as.

Case (VI)

$Q(x) = (x^2+x+1)[\sin ax/\cos ax]$

For particular integral.

$$y_p = \frac{1}{f(D)} (x^2+x+1)(\sin ax/\cos ax)$$

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now

$$e^{iax} = \underset{\text{real}}{\cos ax} + i \underset{\text{imaginary}}{\sin ax}$$

real part of $e^{iax} = \cos ax$ and

Immaginary part of $e^{iax} = \sin ax$

$$y_p = \frac{1}{f(D)}(x^2+x+1)\sin ax$$

$= \text{Imaginary part of } \frac{1}{f(D)}(x^2+x+1)e^{iax}$

$\downarrow$

Case ty (IV)

ans

$$y_p = \frac{1}{f(D)} (x^2+x+1) \cos ax$$

$$= \text{real part of } \frac{1}{f(D)} (x^2+x+1) e^{iax}$$

Case (1)

$$Q(x) = \text{exponential function} = e^{ax}$$

$$y_p = \frac{1}{f(D)} e^{ax}$$

$$= \frac{1}{f(a)} e^{ax}, \quad f(a) \neq 0$$

p-73   If $f(a) = 0$ then $y_p = x \frac{1}{f'(D)} e^{ax}$

B.D

6(a) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{2x} + e^{-2x}$

Solution: $(D^2 + 4D + 4)y = e^{2x} + e^{-2x} \longrightarrow (1)$

Let $y = e^{mx}$ be the trial soln of $(D^2+4D+4)y = 0$

Then the A.E is $m^2 + 4m + 4 = 0$

$$m = -2, -2$$

$$y_c = c_1 e^{-2x} + c_2 x e^{-2x}$$

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For particular integral:

$$y_p = \frac{1}{D^2+4D+4}(e^{2x} + e^{-2x})$$

$$= \frac{1}{(D+2)^2} e^{2x} + \frac{1}{(D+2)^2} e^{-2x} \quad [f(a) = 0]$$

$$= \frac{1}{(2+2)^2} e^{2x} + x \frac{1}{2(D+2)} e^{-2x}$$ → হিসেবে x গুণ হবে নিচে D এর সাপেক্ষে differentiation

$$= \frac{1}{16} e^{2x} + x \cdot x \frac{1}{2 \cdot 1} e^{-2x}$$

$$= \frac{1}{16} e^{2x} + \frac{x^2}{2} e^{-2x}$$

Hence the general solution

$$\boxed{y = y_c + y_p}$$

I would like to do this

Case(II)

$$Q(x) = \sin/\cos x \text{ function}$$

i.e $Q(x) = \sin ax/\cos ax$

$$y_p = \frac{1}{f(D)} \sin ax/\cos ax$$

$$= \frac{1}{f(D^2)} \sin ax/\cos ax$$

$$= \frac{1}{f(-a^2)} \sin ax/\cos ax$$

if $(f(-a^2) \neq 0)$

Exam 1:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$$

Solution: ~~Given that~~

$$(D^2+D+1)\,y = \sin 2x \longrightarrow (1)$$

let $y = e^{mx}$ be the trial sol^n of

$$(D^2+D+1)\,y = 0$$

Then the auxilary equation is

$$m^2+m+1=0$$

$$\therefore m = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= -\tfrac{1}{2} \pm i\,\sqrt{3}/2$$

$ax^2+bx+c=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$y_c = e^{-\frac{1}{2}x}\left[c_1 \cos \tfrac{\sqrt{3}}{2}x + c_2 \sin \tfrac{\sqrt{3}}{2}x\right]$$

For particular Integral:

$$y_p = \frac{1}{D^2+D+1} \sin 2x$$

$$= \frac{1}{-2^2+D+1} \sin 2x$$

$$= \frac{1}{D-3} \sin 2x$$

$$= \frac{D+3}{D^2-9} \sin 2x$$

$$= \frac{D+3}{-2^2-9} \sin 2x$$

$$= -\frac{1}{13}(D+3) \sin 2x$$

$$= -\frac{1}{13}(2\cos 2x + 3\sin 2x)$$

Hence the General Solution is

$$\boxed{y = y_c + y_p} \text{ Ans}$$

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Case (III)

$Q(x) =$ polynomial function.

i.e $Q(x) = x^2+x+1$.

$$(D^3+2D^2+D)y = e^{2x} + x^2 + x \longrightarrow (1)$$

Let $y = e^{mx}$ be the trial soln of

$(D^3+2D^2+D)y = 0$, then the A.E is

$$m^3+2m^2+m = 0$$

$$m(m^2+2m+1) = 0$$

$$m = 0, -1, -1,$$

$$y_c = c_1 + c_2 e^{-x} + c_3 x e^{-x}$$

For particular Integral:

$$y_p = \frac{1}{D^3+2D^2+D}\left(e^{2x}+x^2+x\right)$$

$$= \frac{1}{D(D+1)^2}e^{2x} + \frac{1}{D(D+1)^2}(x^2+x)$$

$$= \frac{1}{2(2+1)^2}e^{2x} + \frac{1}{D}(1+D)^{-2}(x^2+x)$$

$$= \frac{1}{18}e^{2x} + \frac{1}{D}(1-2D+3D^2-\cdots)(x^2+x)$$

$$= \frac{1}{18}e^{2x} + \frac{1}{D}(x^2+x-4x-2+6)$$

$$= \frac{1}{18}e^{2x} + \frac{1}{D}(x^2-3x+4)$$

$$= \frac{1}{18}e^{2x} + \frac{x^3}{3} - \frac{3}{2}x^2 + 4x$$

Hence the General Solution

$$y = y_c + y_p$$

Case (IV)

$$Q(x) = \underbrace{e^{ax}}_{\text{exponential}} \underbrace{(x^2+x+1)}_{\text{polynomial } f^n}$$

For particular Integral:

$$y_p = \frac{1}{f(D)}e^{ax}(x^2+x+1)$$

$$= e^{ax}\frac{1}{f(D+a)}(x^2+x+1)$$

$$= e^{ax}\frac{1}{a}\left(1+\frac{D}{a}\right)^{-1}(x^2+x+1)$$

$$= e^{ax}\frac{1}{a}\left(1-\frac{D}{a}+\left(\frac{D}{a}\right)^2-\cdots\right)(x^2+x+1)$$

$$= e^{ax}\frac{1}{a}\left(x^2+x+1-\frac{2x}{a}-\frac{1}{a}+\frac{2}{a}\right)$$

case (III)

Exam-3

$$(D^3-7D-6)y = e^{2x}x^2$$

let $y=e^{mx}$ be the trial soln of

$$(D^3-7D-6)y=0$$

$$m^3-7m-6=0$$

$$(m+1)(m-3)(m+2)=0$$

$$\therefore m=-1,-2,3$$

$$y_c = c_1e^{-x}+c_2e^{-2x}+c_3e^{3x}$$

For particular integral:

$$y_p = \frac{1}{D^3-7D-6}e^{2x}x^2$$

$$= e^{2x}\frac{1}{(D+2)^3-7(D+2)-6}x^2$$

$$= e^{2x}\frac{1}{D^3+6D^2+12D+8-7D-14-6}x^2$$

$$= e^{2x}\frac{1}{D^3+6D^2+5D-12}x^2$$

$$= -\frac{1}{12}e^{2x}\frac{1}{\left(1-\frac{D^3+6D^2+5D}{12}\right)}x^2$$

$$= -\frac{1}{12}e^{2x}\left(1-\frac{D^3+6D^2+5D}{12}\right)^{-1}x^2$$

$$= -\frac{1}{12}e^{2x}\left\{1+\frac{D^3+6D^2+5D}{12}+\left(\frac{D^3+6D^2+5m}{12}\right)^2+\cdots\right\}x^2$$

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$$= -\frac{1}{12}e^{2x}\left\{x^2 + \frac{12}{12} + \frac{10x}{12} + \frac{50}{144}\right\}$$

Hence the General Solⁿ:

$$y = y_c + y_p$$

Case (V) $Q(x)$ = exponential function × $\sin x/\cos x$ funct

$$Q(x) = e^{ax}(\sin bx/\cos bx$$

For P.I

$$y_p = \frac{1}{f(D)}e^{ax}(\sin bx/\cos bx)$$

$$= e^{ax}\frac{1}{(D+a)}(\sin bx/\cos bx)$$

Case II same as

Exam: 6(a)

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \sin x$$

i.e $(D^2 - 2D + 2)y = e^x \sin x$

let $y = e^{mx}$ be ______

$$y_c = e^x(c_1 \cos x + c_2 \sin x)$$

For particular integral

$$y_p = \frac{1}{D^2 - 2D + 2}e^x \sin x$$

$$= e^{x} \frac{1}{(D+1)^2 - 2(D+1) + 2} \sin x$$

$$= e^{x} \frac{1}{D^2 + 2D + 1 - 2D - 2 + 2} \sin x$$

$$= e^{x} \cdot \frac{1}{D^2 + 1} \sin x$$

$$= x e^{x} \frac{1}{2D} \sin x$$

$$= -\frac{1}{2} x e^{x} \cos x$$

Hence the General Solution $y = y_c + y_p$

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Case (VI)

$Q(x) =$ polynomial $\times$ sin/cos f^n

$Q(x) = (x^2+x+1)\ (\sin ax/\cos ax)$

$\therefore y_p = \frac{1}{f(D)} (x^2+x+1)\ (\sin ax/\cos ax)$

now,

$$e^{iax} = \cos ax + i \sin ax$$

$\cos ax$ real, $\sin ax$ imaginary

real part of $e^{iax} = \cos ax$ and

imaginary part of $e^{iax} = \sin ax$

$$y_p = \frac{1}{f(D)} (x^2+x+1) \sin ax$$

$= $ imaginary part of $\frac{1}{f(D)} (x^2+x+1) e^{iax}$.

case by (

and $y_p = \frac{1}{f(D)} (x^2+x+1) \cos ax$

$= \text{Real part of } \frac{1}{f(D)} (x^2+x+1) e^{iax}$

Exam 2

$$\frac{d^2y}{dx^2} + 4y = x \sin x$$

i.e $(D^2+4)y = x \sin x \longrightarrow (1)$

let $y = e^{mx}$ be the trial soln of

$(D^2+4)y = 0$ then the A.E

is

$$m^2+4=0$$

i.e $m = \pm 2i$

$$y_c = c_1 \cos 2x + c_2 \sin 2x$$

For particular integral:

$$y_p = \frac{1}{D^2+4} x \sin x$$

$= \text{imaginary part of } \frac{1}{D^2+4} x e^{ix}$

$= \;'' \quad '' \quad '' \; e^{ix} \frac{1}{(D+i)^2+4} x$

$= \;'' \quad '' \quad '' \; e^{ix} \frac{1}{D^2+2iD-1+4} x$

$= \;'' \quad '' \quad '' \; e^{ix} \frac{1}{3\left(1+\frac{D^2+2Di}{3}\right)} x$

$= \;'' \quad '' \quad '' \; e^{ix} \frac{1}{3} \left(1+\frac{D^2+2Di}{3}\right)^{-1} x$

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$= \text{"} \quad \text{"} \quad \text{"} \quad e^{ix}\frac{1}{3}\left(1-\frac{D^2+2Di}{3}+\cdots\right)x$

$= \text{"} \quad \text{"} \quad \text{"} \quad e^{ix}\frac{1}{3}\left(x-\frac{2}{3}i\right)$

$= \text{"} \quad \text{"} \quad \text{"} \quad (\cos x + i\sin x)\left(\frac{x}{3}-2/3\, i\right)$

$= \frac{x}{3}\cos x - \frac{2}{3}i\cos x + \frac{x}{3}i\sin x + \frac{2}{3}\sin x$

$= x/3\sin x - \frac{2}{3}\cos x$ [Imaginary part]

$= \frac{1}{9}(3\sin x - 2\cos x)$

$= \frac{x}{3}\sin x - \frac{2}{3}\cos x$

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Hence the G. S

$y = y_c + y_p$

3 (a) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x}\sin 2x$

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Soln: $y_c = c_1e^{2x} + xc_2e^{2x}$

For P.I By the operator method we set

$$y_p = \frac{1}{D^2-4D+4}\cdot 3x^2e^{2x}\sin 2x$$

$$= \frac{3x^2e^{2x}\sin 2x}{(D-2)^2}$$

$$= 3e^{2x}\frac{1}{(D+2-2)^2}x^2\sin 2x$$

$$= 3e^{2x}\frac{1}{D^2}x^2\sin 2x$$

$= 3e^{2x} \times$ imaginary part of $\frac{1}{D^2} x^2 e^{i2x}$

$= 3e^{2x} \times$ " " " $e^{i2x} \frac{1}{(D+2i)^2} x^2$

$= 3e^{2x} \times$ " " " $e^{i2x} \frac{1}{D^2+4Di-4} x^2$

$= 3e^{2x} \times$ " " " $e^{i2x} \left(-\frac{1}{4}\right) \left[1-\left(Di+\frac{D^2}{4}\right)^{-1}\right] x^2$

$= 3e^{2x} \times$ " " " $e^{i2x} \frac{1}{-4} \left\{1+\left(Di+\frac{D^2}{4}\right)+\left(Di+\frac{D^2}{4}\right)^2+\cdots\right\} x^2$

$= 3e^{2x} \times$ " " " $e^{i2x} \frac{1}{-4} \left\{x^2+2xi+\frac{1}{2}+2\right\}$

$= -3e^{2x} e^{2x} (\cos 2x + i \sin 2x) \cdot \frac{1}{4} (x^2+2xi+5/2)$

$= -\frac{3}{8} e^{2x} (\cos 2x + i \sin 2x) (2x^2+4xi+5)$

$= -3/8\, e^{2x} (4x \cos 2x + 2x^2 \sin 2x + 5 \sin 2x)$

$= -3/8\, e^{2x} [(2x^2+5) \sin 2x + 4x \cos 2x]$

Hence the Complete Solution is

$y = y_c + y_p$

Solve $(D^2-3D+2)y = x^2$

$m = 1, 2$

$y_c = c_1 e^{2x} + c_2 e^{x}$ where c_1 and c_2 are arbitrary constant.

By operator method, we have

$$y_p = \frac{1}{D^2-3D+2} x^2$$

$$= \frac{1}{2}\left\{1 - \left(\frac{3D-D^2}{2}\right)\right\}^{-1} x^2$$

$$= \frac{1}{2}\left\{1 + \left(\frac{3D-D^2}{2}\right) + \left(\frac{3D-D^2}{2}\right)^2 + \cdots\right\} x^2$$

$$= \frac{1}{2}\left\{x^2 + 3x - 1 + 9/2 \cdot 2\right\}$$

$$= \frac{1}{2}(x^2 + 3x + 8)$$

$$\boxed{y = y_c + y_p}$$

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Exponential function

(i) $\frac{1}{f(D)} e^{mx} = \frac{1}{f(m)} e^{mx}$ if $f(m) \neq 0$

(ii) $\frac{1}{f(D)} e^{mx} = e^{mx} \frac{1}{f(D+m)}$ if $f(m) = 0$

(iii) $\frac{1}{f(D)} e^{mx} \cdot F(x) = e^{mx} \frac{1}{f(D+m)} F(x)$

$\downarrow$

polynomial function

এক্ষেত্রে Binomial theorem ব্যবহার করতে হবে

$(D^2+9)y = e^{2x}$

$m = \pm 3i$

$y_c = c_1 \cos 3x + c_2 \sin 3x$

By the operator method, we get

$$y_p = \frac{1}{D^2+9} e^{2x}$$

$$= \frac{1}{13} e^{2x}$$

$y = y_c + y_p$

$(D^2-4)y = e^{-2x}$

$m = \pm 2$

$$y_c = c_1 e^{2x} + c_2 e^{-2x}$$

By operator method, we get

$$y_p = \frac{1}{D^2-4} e^{-2x}$$

$$= e^{-2x}\frac{1}{(D-2)^2-4}\cdot 1$$

$$= e^{-2x}\frac{1}{D^2-4D}\cdot 1$$

$$= e^{-2x}\frac{1}{4D}\left(1-\frac{D}{4}\right)^{-1}\cdot 4$$

$$= -\frac{1}{4}e^{-2x}\frac{1}{D}\left\{1+D/4+\frac{D^2}{16}+\cdots\right\}\cdot 1$$

$$= -\frac{1}{4}e^{-2x}x$$

$$\boxed{y = y_c + y_p}$$

$(D^2+4D+4)y = x^3 e^{-2x}$

$m = -2, -2$

$$y_c = c_1 e^{-2x} + x c_2 e^{-2x}$$

By operator method, we get

$$y_p = \frac{1}{D^2+4D+4}\, x^3 e^{-2x}$$

$$= \frac{1}{(D+2)^2} x^3 e^{-2x}$$

$$= e^{-2x}\frac{1}{(D-2+2)^2} x^3 = e^{-2x}\frac{1}{D^2}x^3$$

$$= e^{-2x}\frac{x^5}{20} = \frac{1}{20}x^5 e^{-2x}$$

$$y = y_c + y_p$$

$(D^2 - D + 1)y = x^3 - 3x^2 + 1$

$$m = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$y_c = e^{\frac{1}{2}x}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right]$$

By operator method we get

$$y_p = \frac{1}{D^2 - D + 1}\, x^3 - 3x^2 + 1$$

$$= \{1 - (D - D^2)\}^{-1}(x^3 - 3x^2 + 1)$$

$$= \{1 + (D - D^2) + (D - D^2)^2 + \cdots\}(x^3 - 3x^2 + 1)$$

$$= \{1 + D - 2D^3 + \cdots\}(x^3 - 3x^2 + 1)$$

$$= x^3 - 3x^2 + 1 + 3x^2 - 6x - 12$$

$$= x^3 - 6x - 11$$

$$y = y_c + y_p$$

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* যদি ডানদিকে sin/cos থাকে

<u>rule</u> (i) $\frac{1}{f(D^2)} \sin ax = \frac{1}{f(-a^2)} \sin ax$ if $f(-a^2) \neq 0$

(ii) $\frac{1}{f(D^2)} \cos bx = \frac{1}{f(-b^2)} \cos bx$ if $f(-b^2) \neq 0$

<u>rule:</u> (i) $\sin ax = \frac{e^{iax} - e^{-iax}}{2i}$

(ii) $\cos ax = \frac{e^{iax} + e^{-iax}}{2}$

$(D^2+1)y = \sin 3x$

$m = \pm i$ $\quad y_c = c_1 \cos x + c_2 \sin x$

By operator method, we have

$$y_p = \frac{1}{D^2+1}\sin 3x$$

$$= -\frac{1}{8}\sin 3x \qquad y = y_c + y_p$$

$(D^2+4)y = \cos 2x$

$m = \pm 2i$ $\quad y_c = c_1 \cos 2x + c_2 \sin 2x$

By operator method, we get

$$y_p = \frac{1}{D^2+4}\cos 2x$$

$$= \frac{1}{D^2+4}\cdot \frac{e^{i2x}+e^{-i2x}}{2}$$

$$= \frac{1}{2}\left[\frac{1}{D^2+4}e^{i2x} + \frac{1}{D^2+4}e^{-i2x}\right]$$

$$= \frac{1}{2}\left[e^{i2x}\frac{1}{(D+2i)^2+4}\cdot 1 + e^{-i2x}\frac{1}{(D-2i)^2+4}\cdot 1\right]$$

$$= \frac{1}{2}\left[e^{i2x}\frac{1}{D^2+4iD}\cdot 1 + e^{-i2x}\frac{1}{D^2-4iD}\cdot 1\right]$$

$$= \frac{1}{2}\left[e^{i2x}\frac{1}{4iD}\left(1+\frac{D}{4i}\right)^{-1} + e^{-i2x}\frac{1}{-4iD}\left(1-\frac{D}{4i}\right)^{-1}\right]$$

$$= \frac{e^{i2x}}{8i}\frac{1}{D}\left(1-\frac{D}{4i}+\left(\frac{D}{4i}\right)^2-\cdots\right)\cdot 1 - \frac{e^{-i2x}}{8i}\frac{1}{D}\left(1-\frac{D}{4i}\cdots\right)$$

$$= \frac{e^{i2x}}{8i}\frac{1}{D}(1) - \frac{e^{-i2x}}{8i}\frac{1}{D}(1)$$

$$= \frac{x}{4}\left(\frac{e^{i2x}-e^{-i2x}}{2i}\right)$$

$$= \frac{x}{4}\sin 2x$$

$(D^2-3D+2)y = \sin 3x$

$m = 1, 2 \qquad y_c = c_1 e^{x} + c_2 e^{2x}$

By operator method, we have

$$y_p = \frac{1}{D^2-3D+2}\sin 3x$$

$$= \frac{1}{-9-3D+2}\sin 3x$$

$$= -\frac{1}{3D+7}\sin 3x$$

$$= -\frac{3D-7}{9D^2-49}\sin 3x$$

$$= -\frac{3D-7}{-81-49}\sin 3x$$

$$= \frac{1}{130}(3D-7)\sin 3x$$

$$= \frac{1}{130}(3\cdot 3\cos 3x - 7\sin 3x)$$

$$= \frac{1}{130}(9\cos 3x - 7\sin 3x)$$

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$y = y_c + y_p$

$(D^2+4D+4)y = x^3 e^{-3x}$

$m = -2, -2 \qquad y_c = c_1 e^{-2x} + x c_2 e^{-2x}$

By operator method, we have

$$y_p = \frac{1}{(D+2)^2} x^3 e^{-3x}$$

$$= e^{-3x}\frac{1}{(D-3+2)^2}x^3$$

$$= e^{-3x}\frac{1}{(D-1)^2}x^3$$

$$= e^{-3x}\frac{1}{(1-2D+D^2)}x^3$$

$= e^{-3x}\{1-(2D-D^2)\}^{-1} x^3$

$= e^{-3x}\{1+(2D-D^2)+(2D-D^2)^2+\cdots\}x^3$

$= e^{-3x}\{1+2D-D^2+4D^2-4D^3+\cdots\}x^3$

$= e^{-3x}[x^3+6x^2-6x+24x-24]$

$= e^{-3x}(x^3+6x^2+18x-24)$

$y = y_c + y_p$

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