2. From the ODE of all circle passing through the origin and having center on the x-axis. Find out the solution.

Solve : If the center is on X-axis then the point will be (a, b) → (a, 0)

So, $(x-h)^2 + (y-b)^2 = a^2$

$(x-a)^2 + (y-0)^2 = a^2$

$x^2 – 2ax + a^2 + y^2 - a^2 = 0$

$x^2 – 2ax + y^2 = 0$........(i) Differentiating (i) no. equation-

$$2x + 2y\frac{dy}{dx} - 2x = 0$$

a = $x + y\frac{dy}{dx}$

$x^2 – 2x[x + y\frac{dy}{dx}] + y^2$ which is the required general derivatives.

3. Given that,

$y = ax^2 + bx$.........(i) By differentiating-

$$\frac{dy}{dx} = 2ax + b$$

Again differentiating-

$$\frac{d^2 y}{dx^2} = 2a$$

a = $\frac{1}{2}\frac{d^2 y}{dx^2}$ and b = $\frac{dy}{dx}$ - 2ax

= $\frac{dy}{dx} - 2.x.\frac{1}{2}\frac{d^2 y}{dx^2}$

= $\frac{dy}{dx} - x\frac{d^2 y}{dx^2}$

Putting a and b in equation (i) then we have-

$$y = \frac{x2}{2}\frac{d^2 y}{dx^2} + x[\frac{dy}{dx} - x\frac{d^2 y}{dx^2}]$$

Which is the required O.D.E

#Example: y = Acos x + Bsinx

Solution : Differentiating- $\frac{dy}{dx} = -Asin\ x + Bcosx$

So, B = $\frac{\frac{dy}{dx}+Asin\ x}{Cos\ x}$ Let's differentiate once more - $\frac{d^2 y}{dx^2} = -Acos\ x - Bsin\ x$

$$\Rightarrow A = \frac{\frac{d^2 y}{dx^2}+Bsin\ x}{Cos\ x}$$

Finally, y = $\frac{d^2 y}{dx^2} + Bsin\ x + \tan x(\frac{dy}{dx}) + \tan x\ \ A\ Sin\ x$