MATH ASSIGNMENT

MD.IBRAHIM HOSSAIN

ROLL: 230525

DEPARTMENT OF

ELECTRICAL, ELECTRONIC AND COMMUNICATION ENGINEERING

<u>SEPARABLE EQUATIONS:</u>

DEFINATION

An equation of the form

$$F(x)\,G(y)\ dx + f(x)\,g(y)\ dy = 0$$

is called an equation with variables separable or simply a separable equation

For example, the equation $(x-4)\,y^4\,dx\ - x^3\left(y^2-3\right)dy = 0$ is a separable equation.

$\therefore$ The separable equation,

$f_2(y) f_1(x)\,dx + f_2(x) f_1(y)\,dy = 0$

Separating variables and we get,

$\dfrac{f_1(x)}{f_2(x)}dx + \dfrac{f_1(y)}{f_2(y)}dy = 0$

Integrating it,

$\displaystyle\int \frac{f_1(x)}{f_2(x)}dx + \int \frac{f_1(y)}{f_2(y)}dy = 0$

$F(x) + F(y) = C$

Where C is an arbitrary constant which is the required general solution.

$x=1, x=2$ then $C=3$

Therefore,

$F(x) + F(y) = 3$

It is particular solution which we get from general equation.

MATH ASSIGNMENT

PROBLEM-1:

$(x-4)\,y^4\,dx - x^3\left(y^2-3\right)dy = 0$

Solution:

Given that,

$(x-4)\,y^4\,dx - x^3\left(y^2-3\right)dy = 0$hat,

This is a separable equation

Separating variables and we get,

$$\frac{x-4}{x^3}dx - \frac{y^2-3}{y^4}dy = 0$$

Integrating and we get,

$$\int \frac{x-4}{x^3}dx - \int \frac{y^2-3}{y^4}dy = 0$$

$$\Rightarrow \int \left(x^{-2} - 4x^{-3}\right)dx - \int \left(y^{-2} - 3y^{-4}\right)dy = 0$$

$$\Rightarrow -\frac{1}{x} + \frac{2}{x^2} + \frac{1}{y} - \frac{1}{y^3} = C$$

Where C is an arbitrary constant which is the required general solution.

PROBLEM-2:

$x\sin(y)\,dx + \left(x^2+1\right)\cos(y)\,dy = 0$

And the initial condition $y(1) = \dfrac{\pi}{2}$

Solution:

Given that,

$x\sin(y)\,dx + \left(x^2+1\right)\cos(y)\,dy = 0$

This is a separable equation

Separating variables, we get

MATH ASSIGNMENT

$$\frac{x}{x^2+1}dx+\frac{\cos(y)}{\sin(y)}dy=0$$

Integrating and we get,

$$\int\frac{x}{x^2+1}dx+\int\frac{\cos(y)}{\sin(y)}dy=0$$

$$\Rightarrow\frac{1}{2}\log\left(x^2+1\right)+\log(\sin(y))=C$$

Or, $\frac{1}{2}\log\left(x^2+1\right)+\log(\sin(y))=\log C$

Where C is an arbitrary constant which is the required general solution.

Exercise:

2. $(xy+2x+y+2)\,dx+\left(x^2+2x\right)dy=0$

Solution:

Since,

$$xy+2x+y+2$$
$$=x(y+2)+1(y+2)$$
$$=(x+1)(y+2)$$

The DE can be rewritten as, $(x+1)(y+2)\,dx+\left(x^2+2x\right)dy=0$

This is a separable equation

Separating variables, we get

$$\frac{(x+1)\,dx}{x^2+2x}+\frac{dy}{y+2}=0$$

Integrating and we get,

$$\int\frac{(x+1)\,dx}{x^2+2x}+\int\frac{dy}{y+2}$$

$$\Rightarrow\frac{1}{2}\ln\left(x^2+2x\right)+\ln(y+2)=C$$

Where C is an arbitrary constant which is the required general equation.

3. $2r\left(s^2+1\right)dr+\left(r^4+1\right)ds$

Solution:

$$2r\left(s^2+1\right)dr+\left(r^4+1\right)ds$$

MATH ASSIGNMENT

This is a separable equation.

Separating variable and we get,

$$\frac{2r\,dr}{\left(r^4+1\right)}+\frac{ds}{\left(s^2+1\right)}=0$$

Integrating it and we get,

$$\int\frac{2r\,dr}{\left(r^4+1\right)}+\int\frac{ds}{\left(s^2+1\right)}=0$$

$$\Rightarrow arc\ \tan\left(r^2\right)+arc\ \tan(s)=C$$

Where C is the arbitrary constant which is the required general solution.

4. $\cos(y)\,dx+\sin(x)\,dy=0$

Solution:

$\cos(y)\,dx+\sin(x)\,dy=0$

This is a separable equation.

Separating variable and we get,

$$\frac{dx}{\sin(x)}+\frac{dy}{\cos(y)}=0$$

Integrating it and we get,

$$\int\frac{dx}{\sin(x)}+\int\frac{dy}{\cos(y)}=0$$

$$\Rightarrow \ln\left(\left|\operatorname{cosec}x-cot x\right|\right)+\ln\left(\left|sec y+\tan y\right|\right)=C$$

where C is the arbitrary constant which is the required general solution.

5.$\tan(\theta)\,dr+2r\,d\theta=0$

Solution:

$\tan(\theta)\,dr+2r\,d\theta=0$

This is a separable equation.

Separating variable and we get,

$$\frac{dr}{2r}+\frac{d\theta}{\tan(\theta)}=0$$

Integrating it and we get,

MATH ASSIGNMENT

$$\int \frac{dr}{2r} + \int \frac{d\theta}{\tan(\theta)} = 0$$

$$\Rightarrow \frac{1}{2}\ln(r) + \ln(\sin(x)) = C$$

where C is the arbitrary constant which is the required general solution.

6.$(e^v + 1)\cos u\, du + e^v(\sin u + 1)\, dy = 0$

Solution:

$(e^v + 1)\cos u\, du + e^v(\sin u + 1)\, dy = 0$

This is a separable equation.

Separating variable and we get,

$$\frac{\cos u\, du}{\sin u + 1} + \frac{e^v dv}{e^v + 1} = 0$$

Integrating it and we get,

$$\int \frac{\cos u\, du}{\sin u + 1} + \int \frac{e^v dv}{e^v + 1} = 0$$

$$\Rightarrow \ln(\sin u + 1) + \ln(e^v + 1) = \ln C$$

$$\Rightarrow (\sin u + 1) + (e^v + 1) = C$$

where C is the arbitrary constant which is the required general solution.

7.$(x + 4)\left(y^2 + 1\right)dx + y\left(x^2 + 3x + 2\right)dy = 0$

Solution:

$(x + 4)\left(y^2 + 1\right)dx + y\left(x^2 + 3x + 2\right)dy = 0$

This is a separable equation.

Separating variable and we get,

$$\frac{(x + 4)\, dx}{\left(x^2 + 3x + 2\right)} + \frac{ydy}{\left(y^2 + 1\right)} = 0$$

next we integrate,

$$\int \frac{ydy}{y^2 + 1} = \frac{1}{2}\ln\left(y^2 + 1\right)$$

To integrate the dx term, we use the fractions. we get,

MATH ASSIGNMENT

$$\frac{x+4}{x^2+3x+2}=\frac{x+4}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2},$$

$$x+4=A(x+2)+B(x+1)$$

$$\text{then } x=-1 \text{ gives } A=3 \text{ and } x=-2 \text{ gives } B=-2.$$

$$\text{thus we find}$$

$$\int\frac{(x+4)\,dx}{x^2+3x+2}=3\int\frac{dx}{x+1}-2\int\frac{dx}{x+2}$$

$$=3\ln(x+1)-2\ln(x+2)$$

$$=\ln\frac{(x+1)^3}{(x+2)^2}$$

Now,

$$\int\frac{(x+4)\,dx}{x^2+3x+2}+\int\frac{ydy}{y^2+1}$$

$$=\ln\frac{(x+1)^3}{(x+2)^2}+\frac{1}{2}\ln\left(y^2+1\right)+C$$

where C is the arbitrary constant which is the required general solution.