Linear Regression

1. Introduction to Linear Regression

Linear regression is a fundamental statistical and machine learning algorithm that models the relationship between a dependent variable (target) and one or more independent variables (features) by fitting a linear equation to the observed data.

2. Types of Linear Regression

2.1 Simple Linear Regression

- One independent variable
- Equation: $y = mx + b$
    - y: dependent variable
    - x: independent variable
    - m: slope
    - b: y-intercept

2.2 Multiple Linear Regression

- Multiple independent variables
- Equation: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n$
    - β_0: intercept
    - β_i: coefficients
    - x_i: independent variables

2.3 Polynomial Regression

- Non-linear relationship
- Uses polynomial terms
- Equation: $y = \beta_0 + \beta_1 x + \beta_2 x^2 + ... + \beta_n x^n$

3. Mathematical Foundation

3.1 Ordinary Least Squares (OLS)

- Minimizes sum of squared residuals
- Residual = Actual value - Predicted value
- Cost function: $J(\theta) = \Sigma(y_i - \hat{y}_i)^2$

3.2 Assumptions

1. Linearity
2. Independence
3. Homoscedasticity
4. Normality
5. No multicollinearity
6. No autocorrelation

4. Applications

4.1 Business Applications

- Sales forecasting
- Price prediction
- Risk assessment
- Market analysis
- Resource planning

4.2 Scientific Applications

- Experimental analysis
- Environmental modeling
- Medical research
- Demographics studies
- Economic forecasting