

**Title:** Study of Superposition Theorem.

**Abstract:** The superposition theorem states that in a linear bilateral multi-source DC circuit, the current through or voltage across any particular element may be determined by considering the contribution of each source independently, with the remaining sources replaced with their internal resistance. The contributions are then summed, paying attention to polarities, to find the total value. Superposition cannot in general be applied to non-linear circuits or to non-linear functions such as power.

**Introduction:**

The objectives of this exercise were to-

1. Investigating the application of the superposition theorem to multiple DC source circuits in terms of both voltage and current measurements.
2. Examining the power measurement.

**Theory and Methodology:**

The principle of superposition is applicable only for linear systems. The concept of superposition can be explained mathematically by the following response and excitation principle:

$$\begin{array}{l} i_1 \rightarrow v_1 \quad i_2 \\ \rightarrow v_2 \\ i_1 + i_2 \rightarrow v_1 + v_2 \end{array}$$

Then, the quantity to the left of the arrow indicates the excitation and to the right, the system response. Thus, we can state that a device, if excited by a current  $i_1$  will produce a response  $v_1$ . Similarly, an excitation  $i_2$  will cause a response  $v_2$ . Then if we use an excitation  $i_1 + i_2$ , we will find a response  $v_1 + v_2$ .

The principle of superposition has the ability to reduce a complicated problem to several easier problems each containing only a single independent source.

Superposition theorem states that,

**In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as algebraic sum of the individual contributions of each source acting alone.**

All the remaining independent sources were disabled when determining the contribution due to a particular independent source. Then, all the remaining voltage sources were made zero by replacing them with short circuits, and all remaining current sources were made zero by replacing them with open circuits. There were no dependent source but if there was any that should active during the process of superposition.

**Action Plan:**

1. Only one source was allowed to be active in the circuit comprising of many independent sources and the rest were deactivated
2. A voltage source was deactivated by replacing it with a short circuit and a current source was deactivated by replacing it with an open circuit
3. The response was obtained by applying each source one at a time and then was added algebraically.

**Limitations:** Superposition is a fundamental property of linear equations and, therefore, can be applied to any effect that is linearly related to the cause. That is, we want to point out that, superposition principle applies only to the current and voltage in a linear circuit but it cannot be used to determine power because power is a non-linear function.

**Apparatus:**

1. Trainer board
2. Digital multimeter
3. DC source
4. Resistors
5. Connecting wires

**Precautions:**

1. All the apparatus were checked.
2. To consider the effect of one voltage source the other was replaced with a wire.
3. Before connecting DC source in the trainer board that was checked.
4. The DC source was not switched on while implementing the circuit in the trainer board.
5. Voltmeter was connected in the parallel through the resistor. Ammeter was connected in the series through the resistor.

Circuit Diagram:

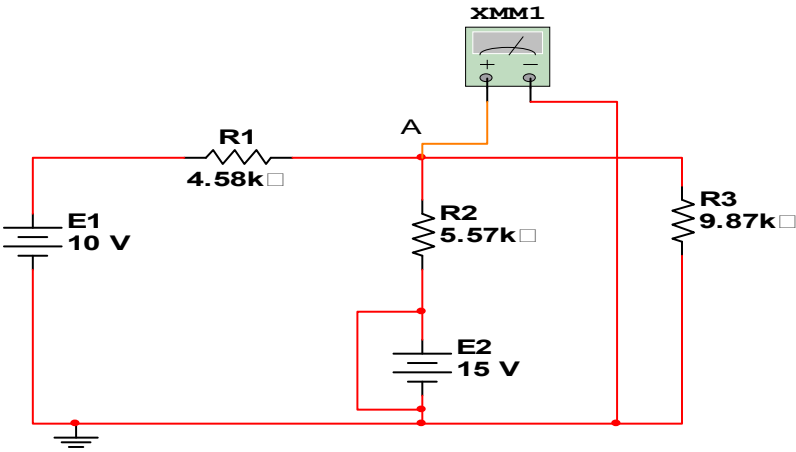


Figure:6.1 ( E1 active )

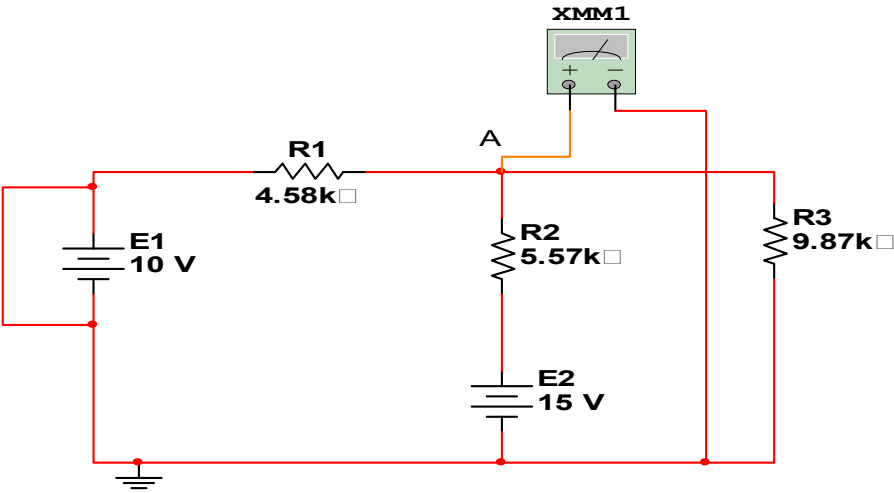


Figure:6.1 ( E2 active )

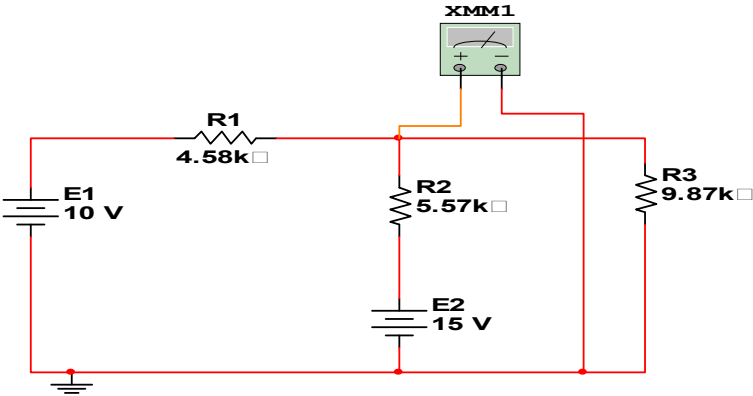


Figure:6.1 (E1 and E2 active)

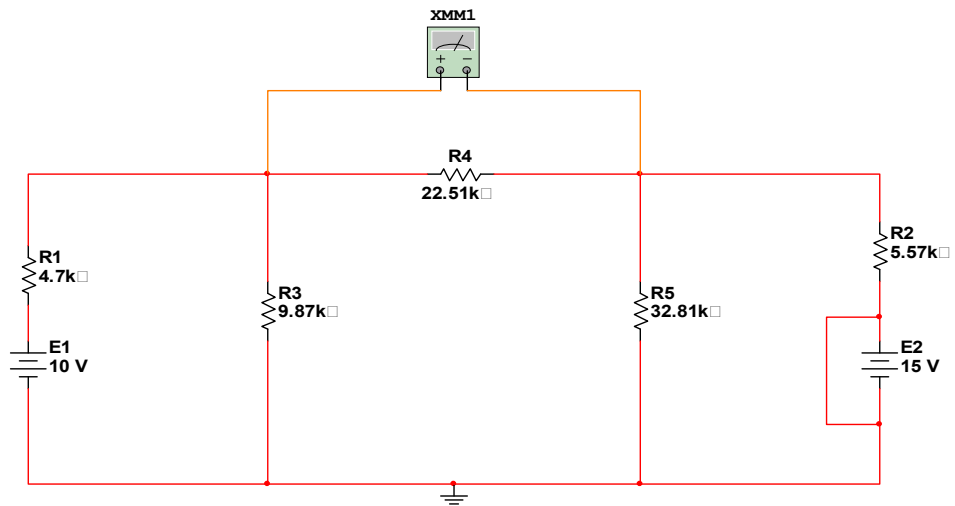


Figure:6.2 (E1 active )

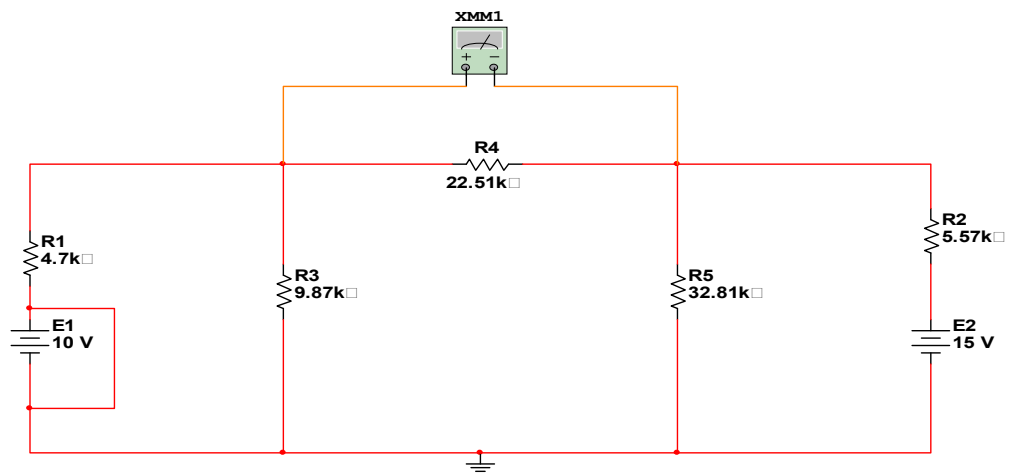


Figure:6.2 (E2 active )

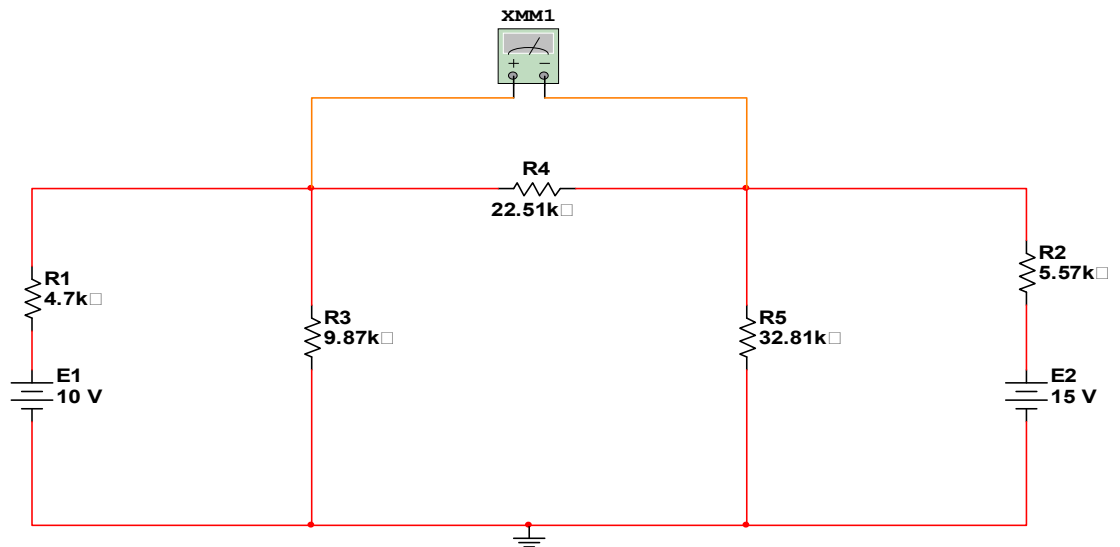


Figure:6.2 (E1 and E2 active)

## **Experimental Procedure:**

### **Voltage Application**

1. The dual supply circuit of Figure 6.1 was considered using  $E1 = 10$  volts,  $E2 = 15$  volts,  $R1 = 4.58\text{ k}$ ,  $R2 = 5.57\text{ k}$  and  $R3 = 9.87\text{ k}$ . Superposition was used to find the voltage from node A to ground. First source  $E1$  was considered by assuming that  $E2$  was replaced with its internal resistance (a short). Then the voltage at node A was determined using standard series-parallel techniques and recorded that in Table 6.1. The process was repeated using  $E2$  while shorting  $E1$ . Finally, those two voltages were summed and recorded in Table 6.1.
2. To verify the superposition theorem, the process was implemented directly by measuring the contributions. The circuit of Figure 6.1 was built with the values specified in step 1, then  $E2$  was replaced with a short.
3. The voltage was measured at node A and recorded in Table 6.1
4. Then the shorting wire was removed and source  $E2$  was inserted. Also, source  $E1$  was replaced with a short. Then the voltage at node A was measured and recorded in Table 6.1
5. The shorting wire was removed and re-inserted source  $E1$ . Both sources were then in the circuit. The voltage was measured at node A and recorded in Table 6.1

### **Current Application**

6. The dual supply circuit of Figure 6.2 was considered using  $E1 = 10$  volts,  $E2 = 15$  volts,  $R1 = 4.7\text{ k}$ ,  $R2 = 5.57\text{ k}$ ,  $R3 = 9.87\text{ k}$ ,  $R4 = 22.51\text{ k}$  and  $R5 = 32.81\text{ k}$ . Superposition was used to find the current through  $R4$  flowing from node A to B. Each source was again treated independently with the remaining sources replaced with their internal resistances. The current through  $R4$  was calculated first considering  $E1$  and then considering  $E2$ . These results were summed and recorded in Table 6.2.
7. The circuit of Figure 6.2 was assembled using the specified values. Source  $E2$  was replaced with a short and the current was measured through  $R4$
8. The short was replaced with source  $E2$  and swapped source  $E1$  with a short. The current through  $R4$  was measured.
9. The shorting wire was removed and re-inserted source  $E1$ . Both sources were then in the circuit. The current through  $R4$  was measured and recorded in Table 6.2

**Simulation:**

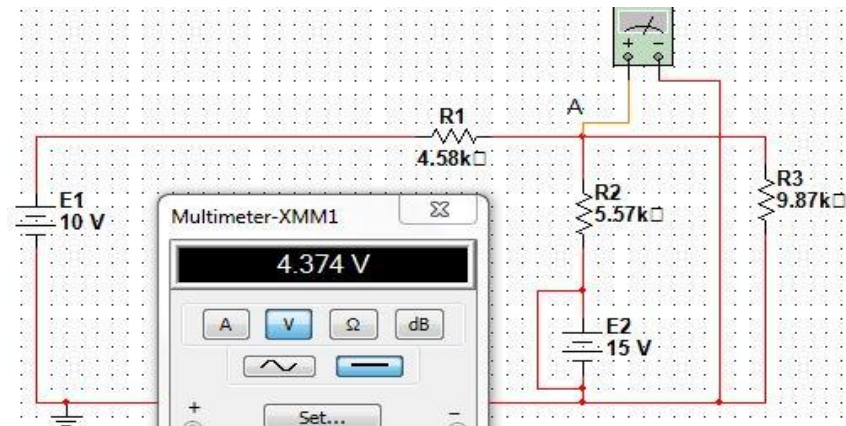


Figure:6.1( E2 shorted)

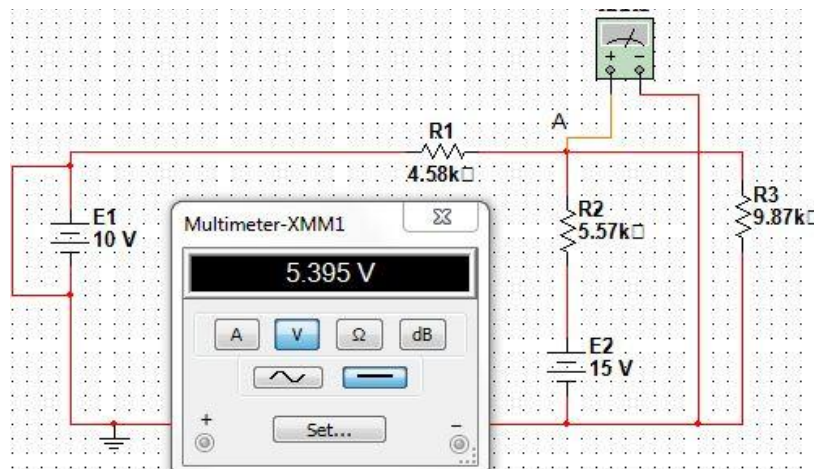


Figure:6.1( E1 shorted)

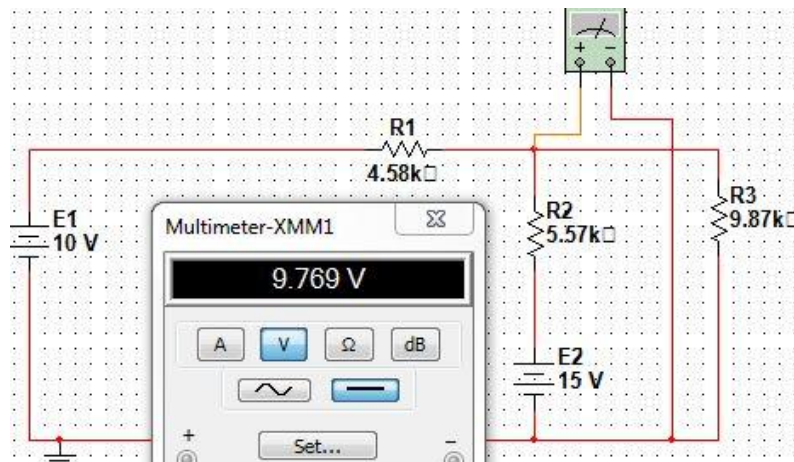


Figure:6.1( E1 and E2 active)

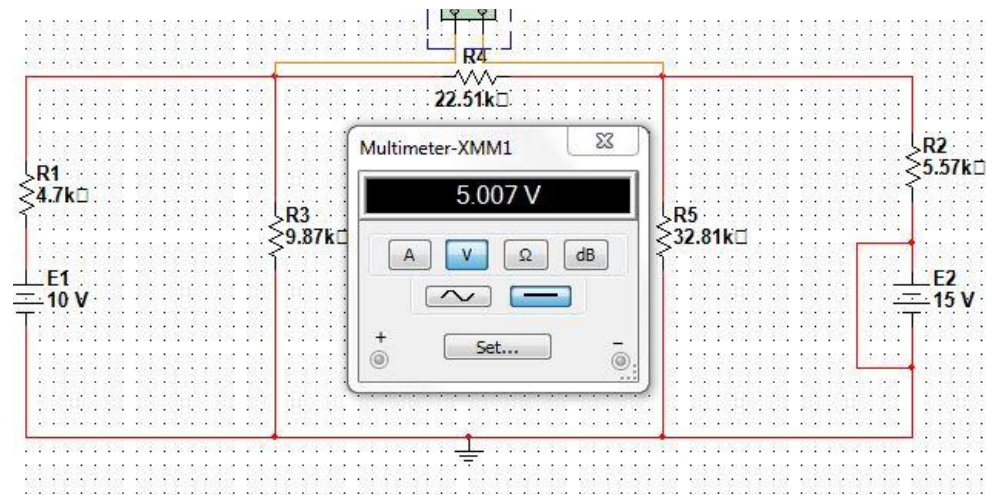


Figure:6.2( E2 shorted)

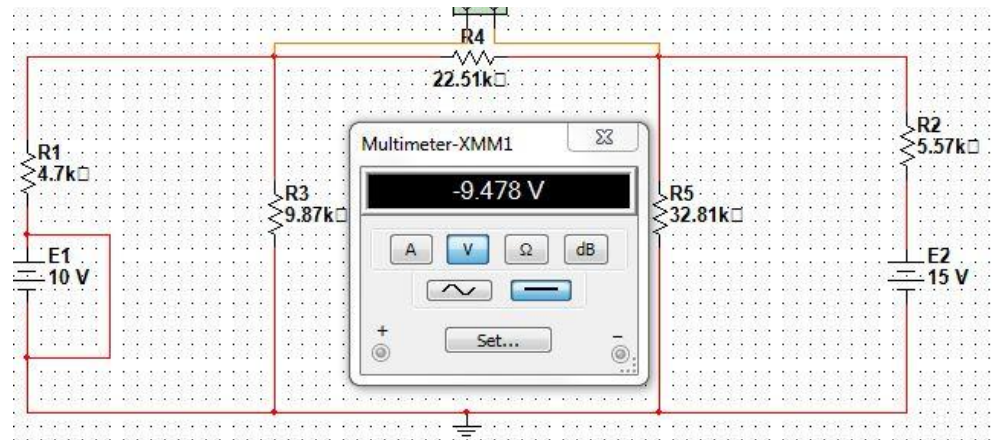


Figure:6.2( E1 shorted)

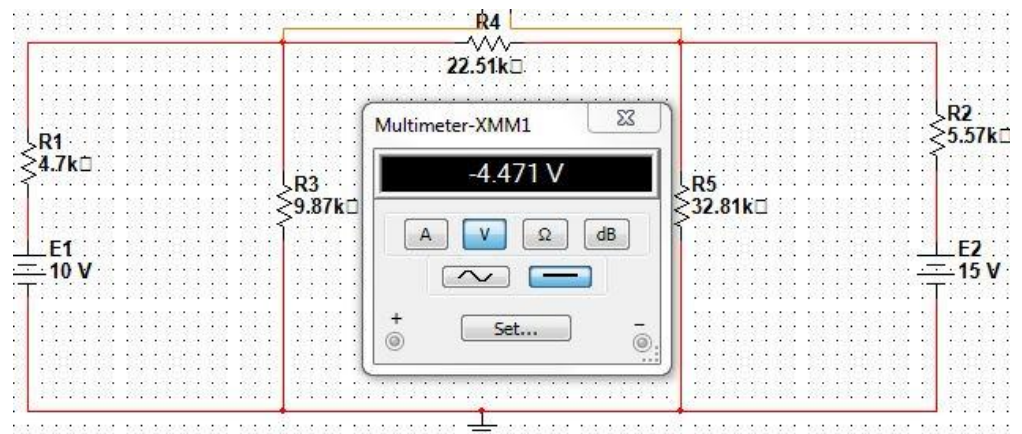


Figure:6.2( E1 and E2 )

**Measurement:**

**Data Tables:**

Source	V <sub>A</sub> Theory	V <sub>A</sub> Experimental	Deviation
E1 only	4.37 V	4.374 V	0.004
E2 only	5.386 V	5.395 V	0.009
E1 & E2	9.756 V	9.769 V	0.013

Table 6.1

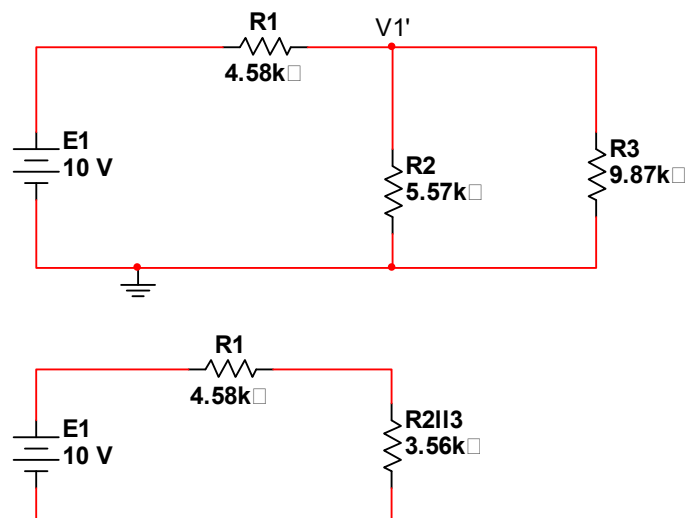
Source	I <sub>R4</sub> Theory	I <sub>R4</sub> Experimental	Deviation
E1 only	0.2227 mA	0.22243 mA	0.00027
E2 only	-0.354 mA	-0.42 mA	0.066
E1 & E2	-0.131 mA	-0.19 mA	0.059

Table 6.2

**Calculation:**

For figure 6.1:

When E1 Active,

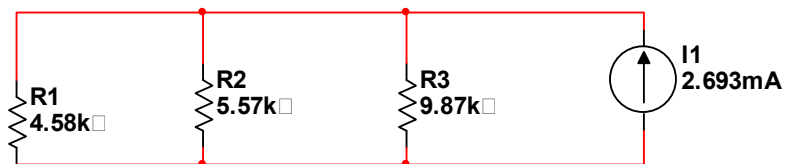
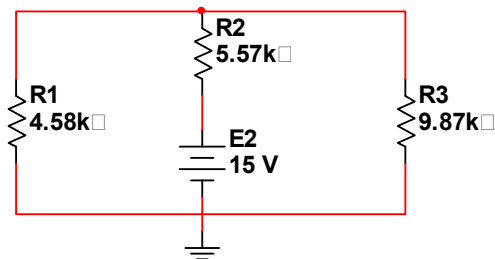


$$R = 4.58 + 3.56 = 8.141K \text{ and } E1 = 10V$$

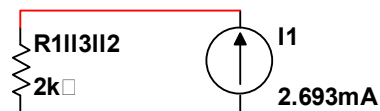


$$\therefore V_A' = \frac{10 \times R(2||3)}{R(2||3) + R1} = 4.37 \text{ V}$$

When E2 Active,



$$I1 = E2/R2 = 15/5.57 = 2.693\text{mA}$$



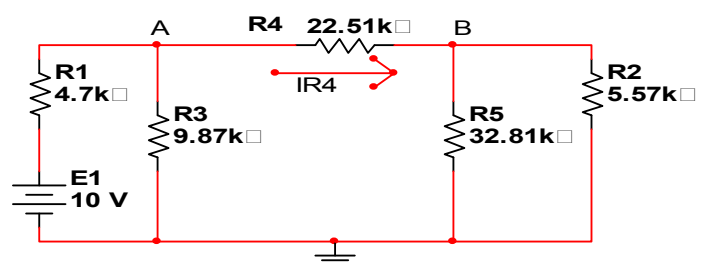
$$V_A'' = I1 \times 2 = 2.693 \times 2 = 5.386 \text{ V}$$

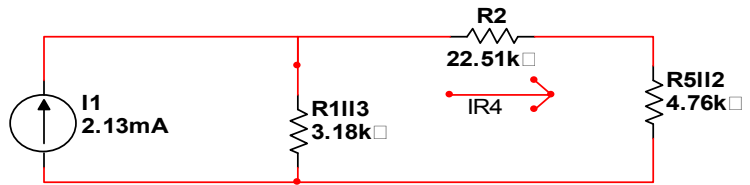
When E1 and E2 active:

$$V_A = V_A' + V_A'' = 4.37 + 5.386 = 9.756 \text{ V}$$

For figure 6.2:

When active E1,





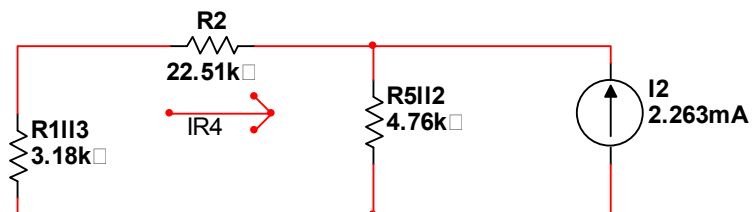
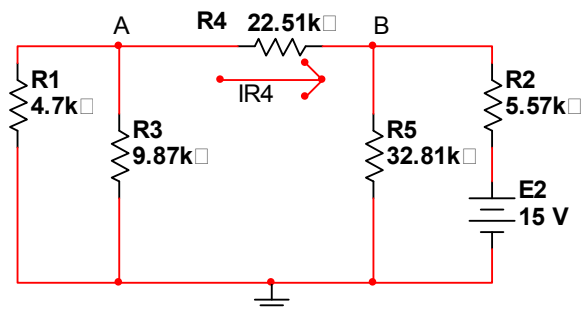
$$I_1 = E_1/R_1 = 10/4.7 = 2.13\text{mA}$$

$$R(5||2)=4.76\text{k} \text{ and } R(1||3)=3.18\text{k}$$

$$R=R(5||2)+R(1||3)+R_4 = 4.76+3.18+22.51= 30.45\text{k}$$

$$I_{R4}' = \frac{2.13 \times R(1||3)}{R} = 0.2227\text{mA}$$

When active E2,



$$I_2 = E_2/R_2 = 15/5.57 = -2.263\text{mA}$$

$$R(5||2)=4.76\text{k} \text{ and } R(1||3)=3.18\text{k}$$

$$R=R(5||2)+R(1||3)+R_4 = 4.76+3.18+22.51= 30.45\text{k}$$

$$I_{R4}'' = \frac{-2.263 \times R(2||5)}{R} = -0.354\text{mA}$$

When E1 and E2 active,

$$I_{R4} = I_{R4}' + I_{R4}'' = (0.2227-0.354)\text{mA} = -0.131\text{mA}$$

### **Results:**

For figure 6.1: E1 only 4.374 V  
E2 only 5.395 V  
E1 & E2 9.769 V

For figure 6.2: E1 only 0.22243 mA  
E2 only -0.42 mA  
E1 & E2 -0.19 mA

### **Discussion:**

1. The trainer board and the multimeter was checked before the start of the experiment.
2. The resistor was placed properly according to the figure.
3. The value of the voltage was increased gradually as applying a large voltage can damage the resistors.
4. Finally all the data was placed in the data table. For the given equation, a result was obtained.

**Conclusions:** In this experiment the data/findings were interpreted and determine to the extent to which the experiment was successful in complying. The goal was initially set. The ways of the study was improved, investigated and described by measuring, converting and calculating the circuit of super position theorem.

### **Reference(s):**

1. Robert L. Boylestad, "Introductory Circuit Analysis", Prentice Hall, 12<sup>th</sup> Edition, New York, 2010, ISBN 9780137146666