



AMERICAN INTERNATIONAL UNIVERSITY-BANGLADESH
Faculty of Science & Technology
Department of Mathematics
MAT1102: Differential Calculus and Coordinate Geometry (Sections: All)

SAMPLE QUESTIONS

1. Answer the following questions:

- (a) $\vec{A} = 2\hat{x} + 3\hat{y} + 5\hat{z}$, $\vec{B} = 4\hat{x} + 6\hat{y} + 10\hat{z}$,
- (i) Find $\vec{B} \cdot \vec{A}$ and $\vec{A} \times \vec{B}$.
 - (ii) Find the length of $2\vec{A} + 4\vec{B}$.
 - (iii) Determine if the vectors are parallel, perpendicular or neither.
- (b) $\vec{A} = \hat{x} - \hat{y} + 3\hat{z}$ and $\vec{B} = 3\hat{x} + 2\hat{y} + \hat{z}$, $\vec{C} = \hat{x} + \hat{y} + 3\hat{z}$,
- (i) Find the area of the parallelogram if \vec{A} and \vec{B} are edges of parallelogram.
 - (ii) Find the volume of parallelepiped whose adjacent sides are \vec{A} , \vec{B} and \vec{C} .
 - (iii) Evaluate $\vec{A} \times \vec{B} \times \vec{C}$.
 - (iv) Determine the direction of \vec{B} .
- (c) $\vec{P} = \hat{x} - 3\hat{y} - 6\hat{z}$ and $\vec{Q} = 6\hat{x} + 2\hat{y} + 4\hat{z}$.
- (i) Find a unit vector which is perpendicular to both \vec{P} and \vec{Q} .
 - (ii) Determine whether \vec{P} and \vec{Q} are parallel or not?
 - (iii) Find the cosine and the sine angles between vectors \vec{A} and \vec{B} .
 - (iv) Find the projection of \vec{Q} on \vec{P} .

2. Answer the following questions:

(a) Given

$$\frac{x-4}{2} = \frac{y+5}{5} = \frac{z+1}{8},$$
$$\frac{x-2}{7} = \frac{y+1}{3} = \frac{z-1}{6}.$$

Determine whether the lines are parallel, intersecting or skew. If intersects, then find the point of intersection.

(b) Find the parametric, and symmetric forms of the lines passes through the points $(1, -2, 3)$ and $(2, 4, -1)$.

(c) Find the equation of straight line which passes through the point $(5, 2, 1)$ and parallel to the vector $\vec{b} = 2\hat{x} + \hat{y} + \hat{z}$ in vector, symmetric and parametric forms.

(d) Find the angle between two straight lines

$$\frac{x+1}{2} = \frac{y-3}{4} = \frac{z-1}{5} \text{ and } \frac{x}{3} = \frac{y+1}{2} = \frac{z-5}{4}.$$

3. Answer the following questions:

(a) Find the angle between the planes $x + 2y + 3z = 5$ and $2x - y + 3z = 10$.

(b) Find the equation of the plane which passes through the point $(2, 5, -4)$ and parallel to the plane $3x - 3y - 2z = 12$.

(c) Find the point(s) of intersection between the planes $4x + y + 10z = -2$ and $-8x + 2y + 3z = -8$.

(d) Find the point(s) of intersection of three planes $-2x + 7y - 5z = 8$, $x - y = 1$ and $5x + 5y + 9z = -32$.

4. Answer of the following questions:

(a) Sketch the following functions. Also find the domain and range:

(i) $f(x) = \sqrt{x-5}$, (ii) $f(x) = [x]$, (iii) $u(t) = t - 3$, (iv) $f(x) = x^2 + 2x$.

(v) $y = |x - 3| + 5$, (vi) $y = (x + 2)^2$, (vii) $y = 5 - |x - 2|$, (viii) $y = x^2 + 5$

(b) (i) Derive the general equation of parabola.

(ii) Find the polar transform of the $(-1, \sqrt{3})$. Also locate them in (r, θ) plane.

(iii) Identify and determine the vertices and foci of the curve

$$9x^2 - 4y^2 - 8y - 40 = 0, \text{ or } \frac{(x-2)^2}{3^2} - \frac{(y-4)^2}{4^2} = 1$$

Also sketch the curve.

(c) (i) Derive the general equation of Ellipse and Hyperbola.

(ii) Identify and determine the vertices and foci of the curve

$$(a) y^2 - 2y - 4x + 3 = 0,$$

$$(b) 9x^2 - 16y^2 - 18x - 64y - 199 = 0.$$

Also sketch the curve.

(iii) Find the vertices, and foci of the following conic

$$\frac{(x-3)^2}{2^2} + \frac{(y+4)^2}{5^2} = 1. \text{ Also sketch its graph.}$$

- (d) (i) Find the angle of rotation to remove the xy term from the curve

$$4x^2 + 8xy - 4y^2 - 3x = 5,$$

- (ii) Reduce the equation $7x^2 + 5xy + 7y^2 - 9 = 0$ to standard form when the axes are rotated by an angle $\theta = \frac{\pi}{4}$. And hence sketch the curve.