American International University Bangladesh (AIUB)

Final Term Assignment

ON Chapter- 6,7,8

Submitted to

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Section : D

Course : Math 3

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Chapter - 6

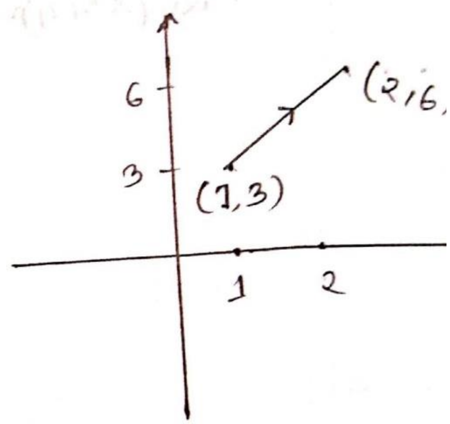
Example : 1

$$z(t) = (1+3i)t \; ; \; (1 \le t \le 2)$$

$\Rightarrow x(t) + i\,y(t) = t + i\,3t$

Comparing both sides,

$x = t \quad ; \quad y = 3t$



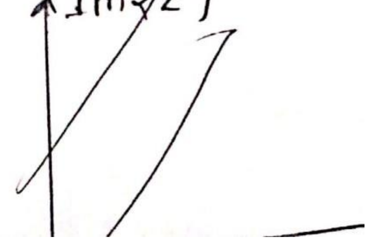
$\underline{\underline{t=1}}$;

$(x, y) = (1, 3)$

$\underline{\underline{t=2}}$

$(x, y) = (2, 6)$

So, $z(t) = (1+3i)t$ represents the line segment from $(1,3)$ to $(2,6)$ in complex plane.



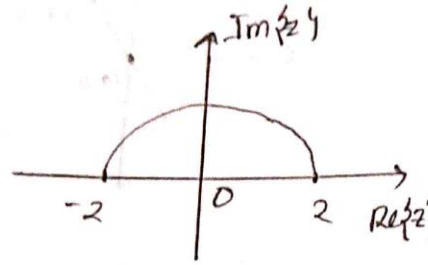
Example 2 $z(t) = 2e^{it}$; $0 \leq t \leq \pi$

Here, $r = 2$

& t has limit from 0 to π. So, $z(t) = 2e^{i\theta}$

represents upper semicircle of radius 2

with center $(0,0)$



$z(t) = 2e^{it}$

$\Rightarrow x + iy = 2\cos t + i2\sin t$

$\therefore x = 2\cos t$, $y = 2\sin t$

$\therefore$ center: $(0,0)$

Example: 3 line segment from $1+i$ to $4-2i$

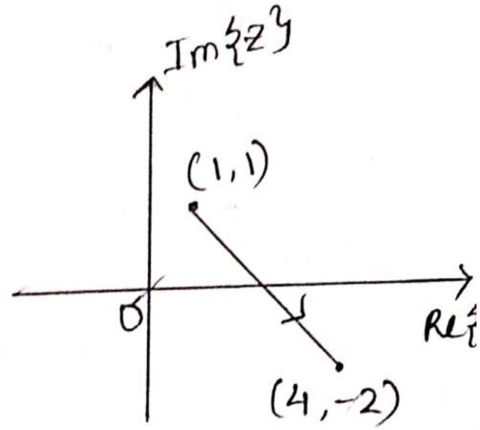
Equation of the line: Points $(1,1)$ & $(4,-2)$

$$\frac{x-1}{1-4} = \frac{y-1}{1+2}$$

$$\Rightarrow \frac{x-1}{-3} = \frac{y-1}{3}$$

$$\Rightarrow y - 1 = -x + 1$$

$$\Rightarrow y = -x + 2$$



Let,

$x = t$ then, $y = -t + 2$ where t varies from 1 to 4

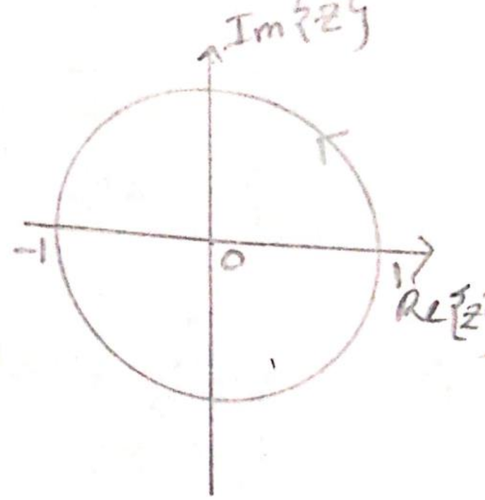
So, the parametric equation of line segment $1+i$ to $4-2i$ is

$x(t) = t$, $y(t) = -t + 2$ $(1 \le t \le 4)$

Example: 4 Sketch & represent unit circle counter-clockwise [parametrically]

Solve

Unit step circle is $|z| = 1$



$\Rightarrow |x + iy| = 1$

$\Rightarrow \sqrt{x^2 + y^2} = 1$

$\Rightarrow x^2 + y^2 = 1$

Let,

$x = \cos t$ & $y = \sin t$

then,

$\cos^2 t + \sin^2 t = 1$

where t varies from 0 to 2π.

Example : 5 $\int_{c} f(z)\,dz$; $z=0$ to $z=2$ & other from $z=2$ to $z=3+i$,

$f(z)=z^2$

For Path C_1

$y=0$

$f(z) = z^2$

$= (x+iy)^2$

$= x^2$

$z = x+iy$

$\Rightarrow z = x$

$\Rightarrow dz = dx$

$\therefore \int_{C_1} f(z)\,dz = \int_0^2 x^2\,dx$

$= \left[\frac{x^3}{3}\right]_0^2$

$= 8/3$

For Path C_2

Passes through $(2,0)$ & $(3,1)$

$\therefore y-0 = \frac{1-0}{3-2}(x-2)$

$\Rightarrow y = x-2 \Rightarrow x = y+2$

$f(z) = z^2$

$= (x+iy)^2$

$= (y+2+iy)^2$

$z = x+iy$

$= y+2+iy$

$dz = (1+i)\,dy$

$\therefore \int_{C_2} f(z)\,dz = \int_0^1 (y+2+iy)^2 (1+i)\,dy$

$= i\int_0^1 (4+4i-2y^2+2iy^2 + 8iy)\,dy$

$= 10/3 + \frac{26i}{3}$

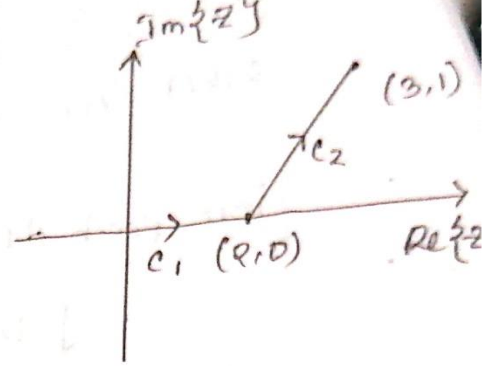
$\therefore \oint_c$

$$\therefore \int_C f(z) = \int_{C_1} f(z)\,dz + \int_{C_2} f(z)\,dz$$

$$= \cancel{60}\ 8/3 + 10/3 + 26i/3$$

$$= 6 + \frac{26}{3} i$$

(Ans)



Chapter- 6

Complex Integration:

1 ① $z(t) = (1+3i)t$ $\boxed{1 \leq t \leq 4}$

$\Rightarrow x(t) + iy(t) = t + i3t$

$\therefore x(t) = t$

$y(t) = 3t$

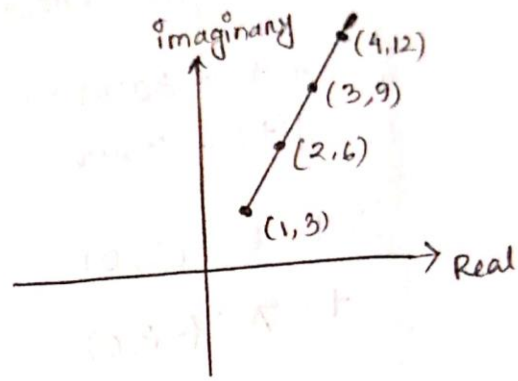
For.

$t=1$; point : $(1,3)$

$t=2$; point : $(2,6)$

$t=3$; point : $(3,9)$

$t=4$; point : $(4,12)$



(IV) $z(t) = 3\sin t + i3\cos t$ $\boxed{-\pi \leq t \leq \pi}$

$\Rightarrow x(t) + iy(t) = 3\sin t + i3\cos t$

$\therefore x(t) = 3\sin t$

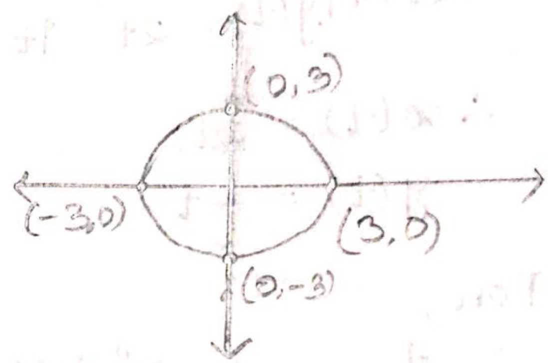
$y(t) = 3\cos t$

$\therefore t = -\pi$; $(0, 3)$

$t = \pi$; $(0, -3)$

$t = -\pi/2$; $(-3, 0)$

$t = \pi/2$; $(3, 0)$



v) $z(t) = 3 + i + 4e^{it}$ $\boxed{0 \leq t \leq 2\pi}$

$\Rightarrow x(t) + iy(t) = 3 + i + 4(\cos t + i\sin t)$

$\Rightarrow x(t) + iy(t) = 3 + i + 4\cos t + i4\sin t$

$\Rightarrow x(t) + iy(t) = (3 + 4\cos t) + i(1 + 4\sin t)$

$\therefore x(t) = 3 + 4\cos t$ & $y(t) = 1 + 4\sin t$

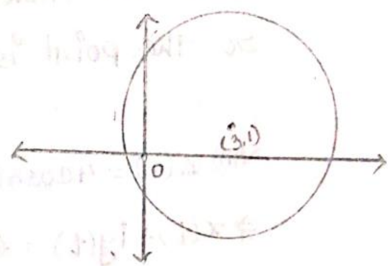
$\Rightarrow \frac{x-3}{4} = \cos t$ — (I) $\Rightarrow \frac{y-1}{4} = \sin t$ — (II)

Now, $\{(I)^2 + (II)^2\} \Rightarrow$

$$\frac{(x-3)^2}{4^2} + \frac{(y-1)^2}{4^2} = \cos^2 t + \sin^2 t$$

$$\Rightarrow \frac{(x-3)^2}{4^2} + \frac{(y-1)^2}{4^2} = 1$$

$$\Rightarrow (x-3)^2 + (y-1)^2 = 4^2$$



<u>Circle</u>

center: $(3, 1)$

radius: 4

vii) $z(t) = 2\sin(t) + i3\cos(t) + 3+2i$ $\boxed{(0 \le t \le 2\pi)}$

$\Rightarrow x(t) + iy(t) = (2\sin(t)+3) + i(2 + 3\cos(t))$

$x(t) = 2\sin(t)+3$

$y(t) = 3\cos t + 2$

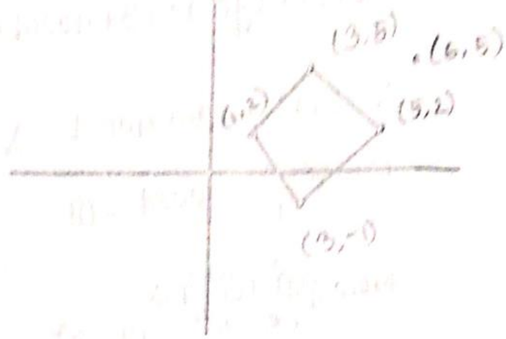
$t = 0$; $(3,5)$

$t = \pi/2$; $(5,2)$

$t = \pi$; $(3,-1)$

$t = 3\pi/2$; $(1,2)$

$t = 2\pi$; $(3,5)$



From the sketch, (6,5) is outside the boundary.
So, the point is exterior.

viii) $z(t) = 4\cosh(t) + 3i\sinh(t)$

$\Rightarrow x(t) + iy(t) = 4\cosh(t) + i3\sinh(t)$

$\therefore x(t) = 4\cosh(t)$ & $y(t) = 3\sinh(t)$

$\Rightarrow \cosh(t) = \frac{x}{4}$ — (I) $\Rightarrow \sinh(t) = \frac{y}{3}$ — (II)

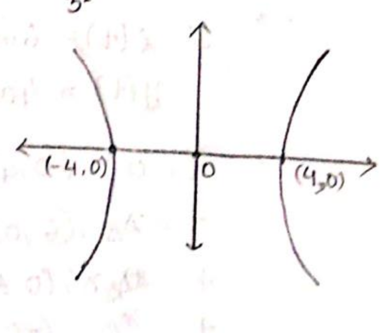
$\{(I)^2 - (II)^2\} \Rightarrow \cosh^2 t - \sinh^2 t = \frac{x^2}{4^2} - \frac{y^2}{3^2}$

$= \frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$

Hyperbola

center: (0,0)

vertices: (4,0) & (-4,0)

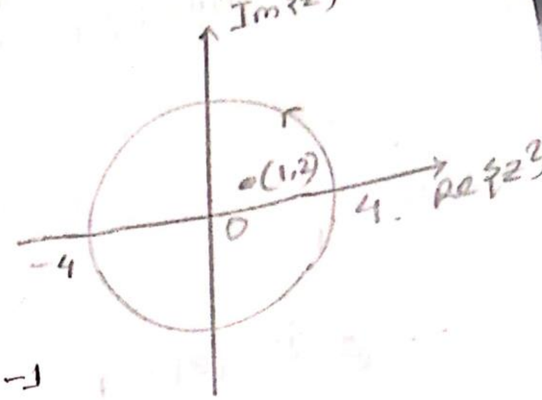


2 (iv) $|z-5+i|=4$

$\Rightarrow |x+iy-5+i|=4$

$\Rightarrow |(x-5)+i(y+1)|=4$

$\Rightarrow (x-5)^2+(y+1)^2=4$



Let,

$x=5+\cos t$ & $y = \sin t - 1$

then,

$\cancel{\cos^2 t + \sin^2 t}$

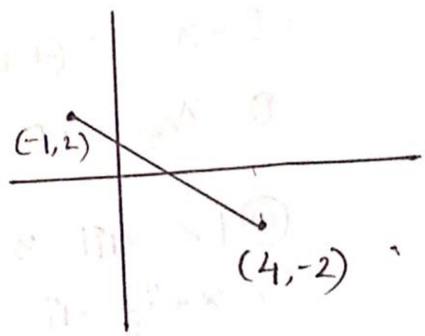
$(\cos t+5)^2+(\sin t-1)^2=4$

where t varies from $t=0$ to $t=2\pi$ (Ans)

And the point $(1,2)$ is interior.

2 (i) $-1+2i$ to $4-2i$

points: $(-1,2)$ & $(4,-2)$



$\therefore$ Equation: $\dfrac{x+1}{-1-4}=\dfrac{y-2}{2+2}$

$\Rightarrow \dfrac{x+1}{-5}=\dfrac{y-2}{4}$

$\Rightarrow 4x+4=-5y+10$

$\Rightarrow 4x+5y=6$

Let, $x=t$

$\therefore 4t+5y=6$

$\Rightarrow y=\frac{1}{5}(6-4t)$ (Ans)

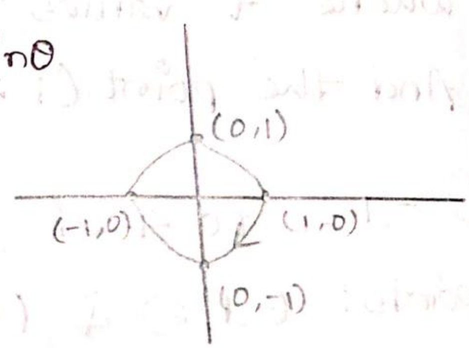
↓ (4, -2)

2 (ii) $|z| = 1$

$\Rightarrow x + iy = e^{i\theta}$

$\Rightarrow x + iy = 1(\cos\theta + i\sin\theta)$

$\therefore x = \cos\theta$ & $y = \sin\theta$



$\theta = 0$; $(1, 0)$

$\theta = 3\pi/2$; $(0, -1)$

$\theta = \pi$; $(-1, 0)$

$\theta = \pi/2$; $(0, 1)$

3. Sketch path c from $z = 0$ to $3i$ & evaluate $\int_c z^2 dz$

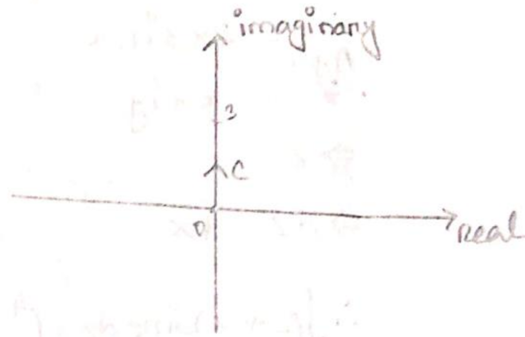
For path c:

$$x = 0$$

$$f(z) = z^2$$

$$= (x+iy)^2$$

$$= (iy)^2 = -y^2$$



$$z = x+iy$$

$$\Rightarrow z = iy$$

$$\Rightarrow dz = idy$$

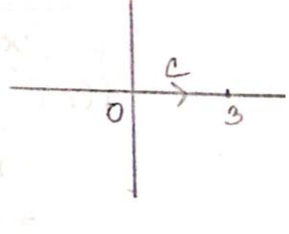
$$\therefore \int_c z^2 dz = i\int_0^3 -y^2 dy$$

$$= -i\ \tfrac{1}{3}[y^3]_0^3$$

$$= -9i \text{ (Ans)}$$

4. For path c:

$$y = 0$$



$$f(z) = \bar{z}$$

$$= x - iy$$

$$= x$$

$$z = x+iy$$

$$\Rightarrow z = x$$

$$\Rightarrow dz = dx$$

$$\therefore \int_c \bar{z} dz = \int_0^3 x dx$$

$$= \tfrac{1}{2}[x^2]_0^3$$

$$= \tfrac{1}{2}(9)$$

$$= 9/2 \text{ (Ans)}$$

(Ans)

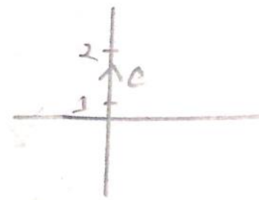
6. For path c

$x = 0$

$$f(z) = \ln(z)$$
$$= \ln(x+iy)$$
$$= \ln(iy)$$

$$z = x+iy$$
$$\Rightarrow dz = i\,dy$$

$$\therefore \int_c \ln(z)dz = i\int_1^2 \ln(iy)dy$$
$$= i^2\left[1/y\right]_1^2$$
$$= -1\left(1/2 - 1\right)$$
$$= 1/2 \text{ (Ans)}$$



7.

$|z| = 2$

$\Rightarrow z = 2e^{i\theta}$

$\Rightarrow \dfrac{dz}{d\theta} = 2ie^{i\theta}$

$\Rightarrow dz = 2ie^{i\theta} \cdot d\theta$

$$\therefore \int_C (z + z^{-1})\, dz = \int_0^{2\pi} \{2e^{i\theta} + (2e^{i\theta})^{-1}\} 2ie^{i\theta} \cdot d\theta$$

$$= \int_0^{2\pi} (4e^{i\theta} + e^{-i\theta}) ie^{i\theta} \cdot d\theta$$

$$= 4i\int_0^{2\pi} \left(e^{i2\theta} d\theta + \int_0^{2\pi} d\theta\right)$$

$$= 4i\int_0^{2\pi} e^{i2\theta} d\theta + i\int_0^{2\pi} d\theta$$

$$= \frac{4i}{2i}\left[e^{i2\theta}\right]_0^{2\pi} + i\left[\theta\right]_0^{2\pi}$$

$$\cancel{= 2(e^{i4\pi} - 1) + i8\pi} \qquad = 2(e^{i4\pi} - e^{0}) + i(2\pi)$$

$$\cancel{= 2(\cos 4\pi + i\sin 4\pi)} \qquad = 2(e^{i4\pi} - 1) + 2\pi i$$

$$= 2(\cos 4\pi + i\sin 4\pi - 1) + \cancel{i8\pi}\, 2\pi i$$

$$= \cancel{i8\pi}\, 2\pi i$$

(Ans)

9. For path C_1

$y = 0$

$f(z) = z \cdot \bar{z}$

$= (x+iy)(x-iy)$

$= x^2$

$z = x + iy$

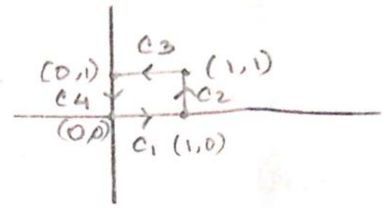
$\Rightarrow z = x$

$\Rightarrow dz = dx$

$\therefore \int_{C_1} z \cdot \bar{z} \, dz = \int_0^1 x^2 dx$

$= 1/3 \left[x^3\right]_0^1$

$= 1/3$



For path C_2

$x = 1$

$f(z) = z \cdot \bar{z}$

$= (x+iy)(x-iy)$

$= x^2 + y^2$

$= 1 + y^2$

$z = x + iy = 1 + iy$

$dz = i\,dy$

$$\therefore \int_{C_2} z\cdot\bar{z}\,dz = i\int_0^1 (1+y^2)\,dy$$
$$= i[y]_0^1 + i\,\tfrac{1}{3}[y^3]_0^1$$
$$= i + i/3 = 4i/3$$

For path C_3

$y = 1$

$$f(z) = z\cdot\bar{z}$$
$$= (x+iy)(x-iy)$$
$$= x^2 + 1$$

$$z = x + iy = x + i$$
$$dz = dx$$

$$\therefore \int_{C_3} z\cdot\bar{z}\,dz = \int_1^0 (x^2+1)\,dx$$
$$= 1/3\,[x^3]_1^0 + [x]_1^0$$
$$= -1/3 - 1 = -4/3$$

For path C_4

$x = 0$

$$f(z) = z\cdot\bar{z}$$
$$= (x+iy)(x-iy)$$
$$= y^2$$
$$z = x+iy = 0+iy$$
$$\therefore dz = i\,dy$$

$$\therefore \int_{C_4}^{0} z\cdot\bar{z}\,dz = i\int_1^0 y^2\,dy$$
$$= i\,1/3\,[y^3]_1^0$$
$$= -i/3$$

$$\therefore \int_C z\cdot\bar{z}\,dz = 1/3 + 4i/3 - 4/3 - i/3$$
$$= -1 + i \quad \text{(Ans)}$$

10. $|z - i| = 3$

$\Rightarrow z - i = 3e^{i\theta}$

$\Rightarrow z = 3e^{i\theta} + i$

$\Rightarrow dz/d\theta = 3ie^{i\theta} \cancel{d\theta}$

$\Rightarrow dz = 3ie^{i\theta} d\theta$

$$\therefore \int_c \left\{\frac{1}{z-i} - \frac{2}{(z-i)^2}\right\} dz = \int_{2\pi}^{0} \left\{\frac{1}{3e^{i\theta}} - \frac{2}{(3e^{i\theta})^2}\right\} 3ie^{i\theta} d\theta$$

$$= \int_{2\pi}^{0} i\,d\theta - \int_{2\pi}^{0} \frac{2i}{3e^{i\theta}} d\theta$$

$$= i[\theta]_{2\pi}^{0} - \frac{2i}{3}\int_{2\pi}^{0} e^{-i\theta} d\theta$$

$$= i[\theta]_{2\pi}^{0} - \frac{2i}{3}(-i)\left[e^{-i\theta}\right]_{2\pi}^{0}$$

$$= i(-2\pi) + 2/3\,(1 - e^{-i2\pi})$$

$$= -i2\pi + 2/3\{1 - (\cos 2\pi - i\sin 2\pi)\}$$

$$= -i2\pi + 2/3\,(1 - 1)$$

$$= -i2\pi$$ (Ans)

Chapter-7

Example: 7.2 $\oint_c \frac{\sin \pi z}{(z-2)^2} dz$, $|z|=3$

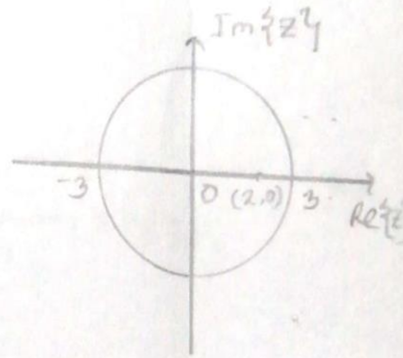
$f(z)$ has singularity at $z=2$ of order 2 which lies inside the circle $|z|=3$.

$$Res(z=2) = \lim_{z \to 2} \frac{d}{dz} \left\{ (z-2)^2 \frac{\sin \pi z}{(z-2)^2} \right\}$$

$$= \lim_{z \to 2} \frac{d}{dz} \{ \sin \pi z \}$$

$$= \lim_{z \to 2} \pi \cos \pi z$$

$$= \pi$$



$$\therefore \oint_c \frac{\sin \pi z}{(z-2)^2} = 2\pi i \{ Res(z=2) \}$$

$$= 2\pi^2 i \quad \text{(Ans)}$$

~~Example~~ :

Example: 7.3 $\oint_C \frac{dz}{z^3}$; $|z+1|=3$

$f(z)$ has singularity at $z=0$ of order 3 which lies inside the region.

$$\therefore Res(z=0) = \lim_{z=0} \frac{1}{(3-1)!} \frac{d^2}{dz^2}\left\{ z^3 \frac{1}{z^3} \right\}$$

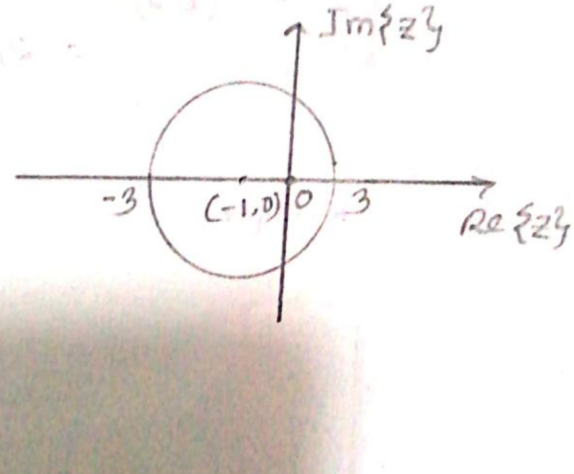
$$= 0$$

$$\therefore \oint_C \frac{dz}{z^3} = 2\pi i \{Res(z=0)\}$$

$$= 2\pi i(0)$$

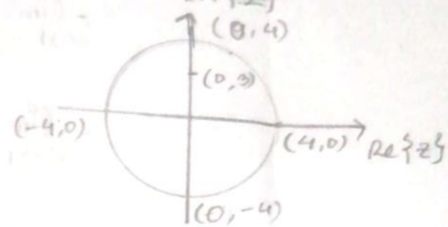
$$= 0$$

(Ans)



$\underline{\underline{7'1}}$

2(a) $\oint_C \frac{dz}{z-3i}$; $|z|=4$



Here,

$z-3i=0$

$\Rightarrow z=3i$

$\therefore f(z)$ has singularity at $z=3i$ of order 1

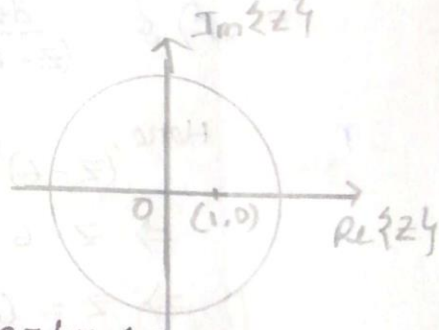
$$\therefore \text{Res}\{z=3i\} = \lim_{z\to 3i} \frac{1}{(1-1)!} \left\{(z-3i)\frac{1}{z-3i}\right\}$$

$$= \lim_{z\to 3i} 1 = 1$$

$$\therefore \oint_C \frac{dz}{\cancel{3}z-3i} = 2\pi i\{\text{Res}(z=3i)\}$$

$$= 2\pi i(1) = 2\pi i \quad \text{(Ans)}$$

b) $\oint_C \frac{e^{-z}}{(z-1)^2}$; $|z|=4$



Here,

$(z-1)^2=0$

$\Rightarrow z=1$

$\therefore f(z)$ has singularity at $z=1$ of order 2

$$\therefore \text{Res}\{z=1\} = \lim_{z\to 1} \frac{d}{dz}\left\{(z-1)^2 \frac{e^{-z}}{(z-1)^2}\right\}$$

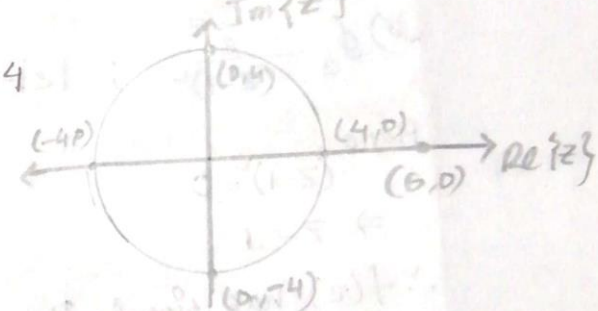
$$= \lim_{z\to 1} \frac{d}{dz}\left\{e^{-z}\right\}$$

$$= \lim_{z \to 1} -e^{-z}$$

$$= -e^{-1} = -1/e$$ ~~(Ans)~~

$$\therefore \oint_C \frac{e^{-z}}{(z-1)^2}\,dz = 2\pi i \{Re(z=1)\}$$
$$= 2\pi i\,(-1/e) = -\frac{2\pi i}{e} \text{ (Ans)}$$

c) $\oint \frac{dz}{(z-6)^{10}}$; $|z|=4$



Here, $(z-6)^{10} = 0$

$\Rightarrow z-6=0$

$\Rightarrow z=6$

So, $f(z)$ has singularity at $z=6$ of order 10. which lies outside the circle; $|z|=4$

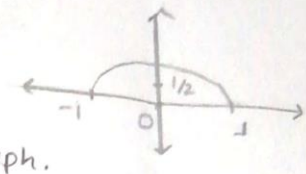
$\therefore Res\{z=6\} = 0$

$$\therefore \oint_C \frac{dz}{(z-6)^{10}} = 2\pi i \{Res(z=6)\}$$
$$= 0$$ (Ans)

3(a) $\oint_C \frac{2z}{(2z-i)^3}\,dz$

(1)

singular point, $z = i/2$ of order 3 which lies inside the graph.



$\therefore Res(z = i/2) = \cancel{\lim_{z \to 1/2} \frac{d^2}{dz^2}\{z(-1/2)\ 2z}$

$$= \lim_{z \to 1/2} \frac{1}{(3-1)!} \frac{d^2}{dz^2}\left[(z - i/2)^3 \frac{2z}{(2z-i)^3}\right]$$

$$= \lim_{z \to 1/2} \frac{1}{2!} \frac{d^2}{dz^2}\left\{\frac{(2z)}{2^3}\right\}$$

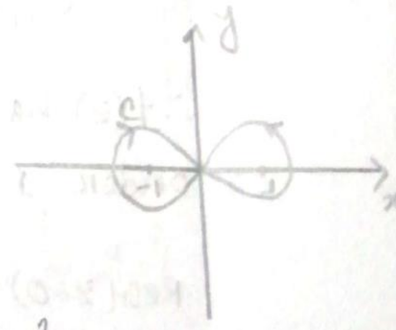
$$= \lim_{z \to 1/2} \frac{1}{16} \frac{d^2}{dz^2}(2z)$$

$$= 1/16 \lim_{z \to 1/2} (0) = 0$$

$$\therefore \oint_C \frac{2z}{(2z-i)^3} = 2\pi i\{Re(z = i/2)\} = 0 \text{ (Ans)}$$

b(ii) $\oint \frac{dz}{z^2 - 1}$

singular point, $z = \pm 1$ of order 1 which lies inside the graph



$$\therefore Res(z = -1) = \lim_{z \to -1}\left\{(z+1)\frac{1}{z^2-1}\right\}$$

$$= \lim_{z \to -1}\left\{(z+1)\frac{1}{(z+1)(z-1)}\right\}$$

$$= \lim_{z \to -1} \frac{1}{z-1}$$

$$= \frac{1}{-1-1} = -1/2$$

$$Res(z=1) = \lim_{z \to 1} \left\{ (z-1) \frac{1}{z^2-1} \right\}$$

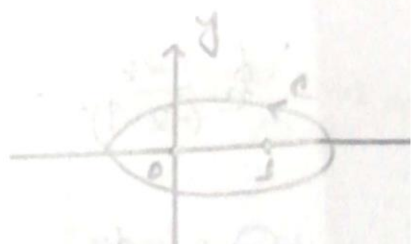
$$= \lim_{z \to 1} \frac{1}{z+1}$$

$$= 1/2$$

$$\therefore \oint \frac{dz}{z^2-1} = 2\pi i \left\{ Res(z=-1) + Res(z=1) \right\}$$

$$= 2\pi i \left(1/2 - 1/2 \right) = 0 \quad \text{(Ans)}$$

(iii) $\oint \frac{2z-1}{z^2-z} dz$



Here, $z^2 - z = 0$

$\Rightarrow z(z-1) = 0$

$\therefore f(z)$ has singularity at $z = 0, 1$ of order 1 which lay inside the region.

$$Res(z=0) = \lim_{z \to 0} \left\{ z \cdot \frac{2z-1}{z^2-z} \right\}$$

$$= \lim_{z \to 0} \frac{2z-1}{z-1}$$

$$= \frac{2 \cdot 0 - 1}{0 - 1} = 1$$

$$\text{Res}(z=1) = \lim_{z \to 1} \left\{ (z-1) \frac{2z-1}{z(z-1)} \right\}$$

$$= \lim_{z \to 1} \frac{2z-1}{z}$$

$$= \frac{2 \times 1 - 1}{1} = 1$$

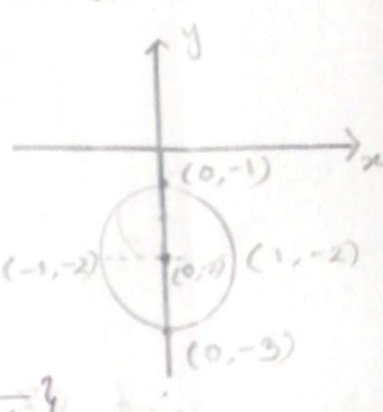
$$\therefore \oint \frac{2z-1}{z^2-z} dz = 2\pi i \{ \text{Res}(z=0) + \text{Res}(z=1) \}$$

$$= 2\pi i (1+1) = 4\pi i \quad \text{(Ans)}$$

4(a) $\oint \frac{dz}{z^2+4}$; $C: |z+2i| = 1$

(1)

C is the ~~center~~ circle of center $(0,-2)$ & radius 1



f(z) has singularity at $z = -2i$ of order 1 which lies inside the circle.

$$\therefore \text{Res}(z=-2i) = \lim_{z \to -2i} \left\{ (z+2i) \frac{1}{z^2+4} \right\}$$

$$= \lim_{z \to -2i} \left\{ (z+2i) \frac{1}{(z+2i)(z-2i)} \right\}$$

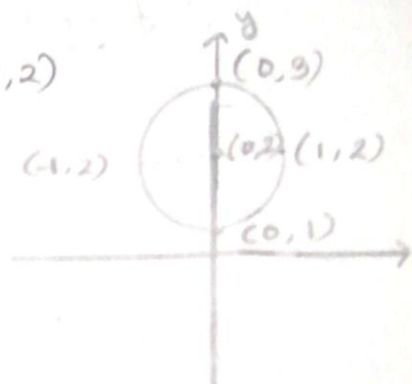
$$= \lim_{z \to -2i} \frac{1}{z-2i}$$

$$= \frac{1}{-2i-2i} = -1/4i$$

$$\therefore \oint \frac{dz}{z^2+4} = 2\pi i\{Res(z=-2i)\}$$
$$= 2\pi i\left(-\tfrac{1}{4i}\right) = -\pi/2 \quad \text{(Ans)}$$

(ii) $\oint \frac{dz}{z^2+4}$; $|z-2i|=1$

C is the circle of center $(0,2)$ & radius 1.



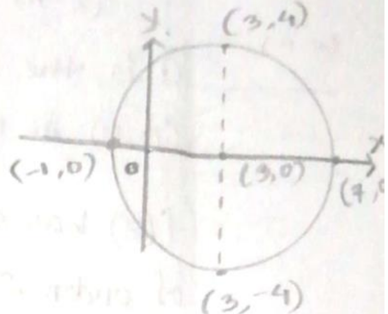
$f(z)$ has singularity at $z=2i$ of order 1.

$$\therefore Res(z=2i) = \lim_{z\to 2i}\left\{(z-2i)\frac{1}{z^2+4}\right\}$$
$$= \lim_{z\to 2i}\left\{(z-2i)\frac{1}{(z+2i)(z-2i)}\right\}$$
$$= \lim_{z\to 2i}\left\{\frac{1}{z+2i}\right\}$$
$$= \frac{1}{2i+2i} = 1/4i$$

$$\therefore \oint \frac{dz}{z^2+4} = 2\pi i\{Res(z=2i)\}$$
$$= 2\pi i\{1/4i\}$$
$$= \pi/2$$
(Ans)

(b) $\oint \frac{\cos(\pi z^3)}{(z-1)(z-2)}\,dz \; ; \; |z-3| = 4$

C is the ~~…~~circle of center (3,0) & radius 4



f(z) has singularity at $z=1,2$ of order 1.

$$\text{Res}(z=1) = \lim_{z\to 1}\left\{(z-1)\frac{\cos(\pi z^3)}{(z-1)(z-2)}\right\}$$

$$= \lim_{z\to 1} \frac{\cos(\pi z^3)}{z-2}$$

$$= \frac{\cos\pi}{1-2} = 1$$

$$\text{Res}(z=2) = \lim_{z\to 2}\left\{(z-2)\frac{\cos\pi z^3}{(z-1)(z-2)}\right\}$$

$$= \lim_{z\to 2} \frac{\cos \pi z^3}{z-1}$$

$$= \frac{\cos 8\pi}{2-1} = 1$$

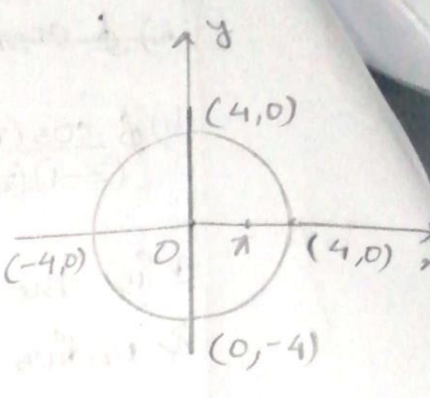
$$\therefore \oint \frac{\cos(\pi z^3)}{(z-1)(z-2)}\,dz = 2\pi i\left\{\text{Res}(z=1)+\text{Res}(z=2)\right\}$$

$$= 2\pi i\,(1+1)$$

$$= 4\pi i$$

(Ans)

c) $\oint \frac{\sin 3z}{(z-\pi)^2} dz$; $|z|=4$



c is the circle of center $(0,0)$

& radius 4

$f(z)$ has singularity at $z=4$ of

order 2.

$$\therefore Res(z=\pi) = \lim_{z\to\pi} \frac{1}{(2-1)!} \frac{d}{dz} \left\{(z-\pi)^2 \cdot \frac{\sin 3z}{(z-\pi)^2}\right\}$$

$$= \lim_{z\to\pi} \frac{d}{dz}\{\sin 3z\}$$

$$= \lim_{z\to\pi} 3\cos 3z$$

$$= 3\cos 3\pi$$

$$= -3$$

$$\therefore \oint \frac{\sin 3z}{(z-\pi)^2} dz = 2\pi i\{Res(z=\pi)\}$$

$$= 2\pi i(-3)$$

$$= -6\pi i$$

(Ans)

7.2

Example 7.5 $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)^2}$

Now,

$$\oint_C \frac{dz}{(z^2+4)^2} = \int_{-R}^{R} \frac{dx}{(x^2+4)^2} + \int_{C_R} \frac{dz}{(z^2+4)^2} \quad \text{--- (1)}$$

Now,
We have to evaluate $\displaystyle\oint_C \frac{dz}{(z^2+4)^2}$

Singular point, $z = 2i$ of order 2 which lies in the upper half circle.

$$z^2+4=0$$
$$z^2=-4$$
$$\Rightarrow z = \sqrt{4}\, i$$
$$\Rightarrow z = \pm 2i$$

$$\therefore \operatorname{Res}(z=2i) = \lim_{z\to 2i} \frac{d}{dz}\left\{(z-2i)^2 \frac{1}{(z^2+4)^2}\right\}$$

$$= \lim_{z\to 2i} \frac{d}{dz}\left\{(z-2i)^2 \frac{1}{(z+2i)^2(z-2i)^2}\right\}$$

$$= \lim_{z\to 2i} \frac{d}{dz}\left\{\frac{1}{(z+2i)^2}\right\}$$

$$= \lim_{z\to 2i} \frac{-2}{(z+2i)^3}$$

$$= \frac{-2}{(2i+2i)^3} = \frac{-2}{(4i)^3} = \frac{-2}{-64i}$$

$$= 1/32i$$

$$\therefore \oint_C \frac{dz}{(z^2+4)^2} = 2\pi i \{Re(z=2i)\}$$
$$= 2\pi i \left(\frac{1}{32i}\right)$$
$$= \pi/16$$

Now equ① becomes,

$$\int_{-R}^{R} \frac{dx}{(x^2+4)^2} + \int_{C_R} \frac{dz}{(z^2+4)^2} = \pi/16$$

$$\Rightarrow \lim_{R\to\infty} \int_{-R}^{R} \frac{dx}{(x^2+4)^2} + \lim_{R\to\infty} \int_{C_R} \frac{dz}{(z^2+4)^2} = \lim_{R\to\infty} \pi/16$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{dx}{(x^2+4)^2} = \pi/16$$ [using Jordan's lemma

(Ans)

Exercise

1 (i) $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$

Here we consider,

$$\oint_C \frac{dz}{z^2+2z+2} = \int_{-R}^{R} \frac{dx}{x^2+2x+2} + \int_{C_R} \frac{dz}{z^2+2z+2} \quad \text{---(1)}$$

Now we have to evaluate,

$$\oint_C \frac{dz}{z^2+2z+2}$$

$$z = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

Here singular point $z = -1+i$ of order 1 which lies inside the upper half circle.

$$Res(z=-1+i) = \lim_{z \to -1+i} \left[\{z-(-1+i)\} \frac{1}{z^2+2z+2} \right]$$

$$= \lim_{z \to -1+i} \left[\{z-(-1+i)\} \frac{1}{\{z-(-1+i)\}\{z-(-1-i)\}} \right]$$

$$= \lim_{z \to -1+i} \frac{1}{z-(-1-i)}$$

$$= \frac{1}{-1+i+1+i} = 1/2i$$

$$\therefore \oint_C \frac{dz}{z^2+2z+2} = 2\pi i\{Res(z=-1+i)\}$$
$$= 2\pi i\{1/2i\}$$
$$= \pi$$

~~From~~ Now equ (1) becomes,

$$\int_{-R}^{R} \frac{dx}{x^2+2x+2} + \int_{C_R} \frac{dz}{z^2+2z+2} = \pi$$

$$\Rightarrow \lim_{R\to\infty} \int_{-R}^{R} \frac{dx}{x^2+2x+2} + \lim_{R\to\infty} \frac{dz}{z^2+2z+2} = \lim_{R\to\infty} \pi$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2} = \pi \text{ [Using Jordan's Lemma]}$$

(Ans)

(ii) $\int_0^\infty \frac{dx}{x^2+1}$

Consider,

$$\oint_C \frac{dz}{z^2+1} = \int_{-R}^{R} \frac{dx}{x^2+1} + \int_{C_R} \frac{dz}{z^2+1} \quad \text{---} (1)$$

Now we have to evaluate,

$$\oint_C \frac{dz}{z^2+1}$$

Singular point, $z = i$ of order 1

$z^2+1=0$
$\Rightarrow z^2 = -1$
$\Rightarrow z = \pm\sqrt{-1} = \pm i$

which lies inside the upper half circle.

$$\therefore Res\{(z=i)\} = \lim_{z\to i}\left\{(z-i)\frac{1}{z^2+1}\right\}$$

$$= \lim_{z\to i}\left\{(z-i)\frac{1}{(z+i)(z-i)}\right\}$$

$$= \lim_{z\to i}\frac{1}{z+i} = \frac{1}{2i}$$

$$\therefore \oint_C \frac{dz}{z^2+1} = 2\pi i\{Res\,(z=i)\}$$

$$= 2\pi i\,(1/2i) = \pi$$

Now equ ① becomes,

$$\int_{-R}^{R}\frac{dx}{x^2+1} + \int_{C_R}\frac{dz}{z^2+1} = \pi$$

$$\Rightarrow \lim_{R\to\infty}\int_{-R}^{R}\frac{dx}{x^2+1} + \lim_{R\to\infty}\int_{C_R}\frac{dz}{z^2+1} = \lim_{R\to\infty}\pi$$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{dx}{x^2+1} = \pi$$ [Using Jordan's lemma]

$$\Rightarrow 2\int_0^{\infty}\frac{dx}{x^2+1} = \pi$$ [Using Integration Properties]

$$\therefore \int_0^{\infty}\frac{dx}{x^2+1} = \pi/2$$ (Ans)

Example 7.7 $f(z) = \frac{1}{(1+z^2)(z+2)}$

(i) $1<|z|<2$

Since, $1<|z|<2$

$\Rightarrow \cancel{1/z > 1}$

$1/z < 1$ & $z/2 < 1$

$\therefore \frac{1}{|z^2|} < 1$ & $\frac{|z|}{2} < 1$

Let,

$$\frac{1}{(1+z^2)(z+2)} = \frac{Az+B}{1+z^2} + \frac{C}{z+2} \quad \text{—(1)}$$

$$\therefore 1 = (Az+B)(z+2) + C(1+z^2)$$

At, $z=-2$; $5C = 1$ $\therefore C = 1/5$

Equating co-efficients of $z^2 \Rightarrow A+C=0$

$\Rightarrow A = -1/5$

Equating co-efficients of $z \Rightarrow 2A+B=0$

$\Rightarrow B = -2A = 2/5$

Equ① becomes,

$$\frac{1}{(1+z^2)(z+2)} = \frac{-\frac{1}{5}z+\frac{2}{5}}{1+z^2} + \frac{\frac{1}{5}}{z+2}$$

$$= \frac{2}{5(1+z^2)} - \frac{1}{5}\frac{1}{(1+z^2)} + \frac{1}{5}\frac{1}{(z+2)}$$

$$= \frac{2}{5}\frac{1}{z^2(1+1/z^2)} - \frac{1}{5}\frac{1}{z^2(1+1/z^2)} + \frac{1}{5}\frac{1}{2(1+z/2)}$$

$$= \frac{2}{5z^2}(1+1/z^2)^{-1} - \frac{1}{5z^2}(1+1/z^2)^{-1} + \frac{1}{10}(1+z/2)^{-1}$$

$$= \frac{2}{5z^2}(1 - 1/z^2 + 1/z^4 - 1/z^6 + \dots) - 1/5z^2(1 - 1/z^2 + 1/z^4 - 1/z^6 + \dots) + \frac{1}{10}(1 - z/2 + z^2/4 - \dots)$$

which is the required Laurent series.

(ii) $|z| > 2$

$\Rightarrow \frac{|z|}{2} > 1$

$\Rightarrow \frac{2}{|z|} < 1$ also $1/z^4 < 1$

$$\therefore \frac{1}{(1+z^2)(z+2)} = \frac{2}{5}\frac{1}{1+z^2} - \frac{1}{5}\frac{1}{1+z^2} + \frac{1}{5}\frac{1}{z+2}$$

$$= \frac{2}{5z^2}\left(1+z^2\right)^{-1} - \frac{1}{5z}\left(1+z^2\right)^{-1} + \frac{1}{5z}\left(1+2/z\right)^{-1}$$

$$= \frac{2}{5z^2}\left(1-1/z^2+1/z^4-1/z^6+\dots\right) - \frac{1}{5z}\left(1-1/z^2+1/z^4-1/z^6+\dots\right)$$

$$+ \frac{1}{5z}\left(1-2/z+4/z^2+\dots\right)$$

which is the required Laurent series.

7.3

1. $f(z) = \dfrac{3z}{(z-1)(2-z)}$

(a) $|z|<1$

Since, $|z|<1$, $|z|<2$

$$\Rightarrow \frac{|z|}{2} < 1$$

$$\frac{3z}{(z-1)(2-z)} = \frac{A}{z-1} + \frac{B}{2-z}$$

$$A = \frac{3}{2-1} = 3$$

$$B = \frac{6}{2-1} = 6$$

$$\therefore \frac{3z}{(z-1)(2-z)} = \frac{3}{z-1} + \frac{6}{2-z}$$

$$= \frac{3}{(z-1)} + \frac{6}{2}\,\frac{1}{(1-z/2)}$$

$$= -3(1-z)^{-1} + 6/2\,(1-z/2)^{-1}$$

$$= -3(1+z+z^2+z^3+\ldots) + 6/2\,(1+z/2+z^2/4+z^3/8+\ldots)$$

which is the required Laurant series.

(b) (c) $|z|>2$

$$\Rightarrow \frac{|z|}{2} > 1$$

$$\Rightarrow \frac{2}{|z|} < 1 \ \& \ \frac{1}{|z|} < 1$$

$$f(z) = \frac{3}{z-1} + \frac{6}{2-z} \quad [\text{From } a]$$

$$= \frac{3}{z(1-1/z)} + \frac{6}{-z(1-2/z)}$$

$$= \frac{3}{z}(1-1/z)^{-1} - \frac{6}{z}(1-2/z)^{-1}$$

$$= \frac{3}{z}(1+1/z+1/z^2+\ldots) - \frac{6}{z}(1+2/z+4/z^2+\ldots)$$

which is the required Laurant series.

(d) $|z-1|>2$

Let,

$u = z-1$

So,

$|u|>2$ & also $|u|>1$

$\Rightarrow \frac{|u|}{2} > 1$ & $\Rightarrow \frac{1}{|u|} < 1$

$\Rightarrow \frac{2}{|u|} < 1$

Now,

$$f(z) = \frac{3(u+1)}{u\{2-(u+1)\}}$$

$$= \frac{3(u+1)}{u(1-u)}$$

$$\frac{3(u+1)}{u(1-u)} = \frac{A}{u} + \frac{B}{1-u}$$

$A = 3$; $B = 6$

$$\therefore f(z) = \frac{3}{u} + \frac{6}{1-u}$$

$$= \frac{3}{u} - \frac{6}{u}\frac{1}{(1-1/u)}$$

$$= \frac{3}{u} - \frac{6}{u}(1-1/u)^{-1}$$

$$= \frac{3}{u} - \frac{6}{u}(1 + 1/u + 1/u^2 + \dots)$$

$$= \frac{3}{z-1} - \frac{6}{z-1}\left(1 + \frac{1}{z-1} + \frac{1}{(z-1)^2} + \dots\right)$$

(Ans)

2(b) $f(z) = \frac{1}{z(z-2)}$

$|z| > 2$

$\Rightarrow \frac{|z|}{2} > 1$

$\Rightarrow \frac{2}{|z|} < 1$

~~f(z) =~~

$\frac{1}{z(z-2)} = \frac{A}{z} + \frac{B}{z-2}$

$A = -1/2 \ , \ B = 1/2$

$\therefore \frac{1}{z(z-2)} = -\frac{1}{2z} + \frac{1}{2} \frac{1}{z-2}$

$= -\frac{1}{2z} + \frac{1}{2} \frac{1}{z(1-2/z)}$

$= -\frac{1}{2z} + \frac{1}{2z} (1-2/z)^{-1}$

$= -\frac{1}{2z} + \frac{1}{2z} (1 + 2/z + 4/z^2 + \cdots)$

(Ans)

Chapter-8

Formula

1. $Z\{\delta[n]\} = 1$ & $Z^{-1}[1] = \delta[n]$; All Z
2. $Z\{u[n]\} = \frac{1}{1-z^{-1}}$ & $Z^{-1}\{\frac{1}{1-z^{-1}}\} = u[n]$; $|z|>1$
3. $Z\{\delta[n-m]\} = z^{-m}$ & $Z^{-1}\{z^{-m}\} = \delta[n-m]$
4. $Z\{a^n u[n]\} = \frac{1}{1-az^{-1}}$ & $Z^{-1}\{\frac{1}{1-az^{-1}}\} = a^n u[n]$
5. Formula in difference equation:

$$Z\{x[n-1]\} = z^{-1}\{X[z] + x[-1]z\}$$

$$Z\{x[n+1]\} = z^{0}\{X[z] - x[0]z\}$$

Problem Find $x[n]$ for $X(z) = \dfrac{1}{(1-0.75z^{-1})(1-0.5z^{-1})}$

Solve

Let,

$$\frac{1}{(1-0.75z^{-1})(1-0.5z^{-1})} = \frac{A}{(1-0.75z^{-1})} + \frac{B}{(1-0.5z^{-1})}$$

$$A = \frac{1}{1-0.5\times 4/3} = 3$$

$$B = \frac{1}{1-.75\times 2} = -2$$

So,

$$X(z) = \frac{3}{1-0.75z^{-1}} - \frac{2}{1-0.5z^{-1}}$$

$$\Rightarrow Z^{-1}\{X(z)\} = 3Z^{-1}\left\{\frac{1}{1-0.75z^{-1}}\right\} - 2Z^{-1}\left\{\frac{1}{1-0.5z^{-1}}\right\}$$

$$\Rightarrow x[n] = 3(0.75)^n u[n] - 2(0.5)^n u[n]$$

(Ans)

Problem $X(z) = \dfrac{1}{(1+0.5z^{-1})(1-0.5z^{-1})(1-z^{-1})}$

Let,

$$X(z) = \frac{A}{1+0.5z^{-1}} + \frac{B}{1-0.5z^{-1}} + \frac{C}{1-z^{-1}}$$

$$A = \frac{1}{\{1-0.5(-2)\}\{1-(-2)\}} = 1/6$$

$$B = \frac{1}{(1+0.5 \times 2)(1-2)} = -1/2$$

$$C = \frac{1}{(1+0.5)(1-0.5)} = 4/3$$

$$\therefore X(z) = 1/6 \frac{1}{1+0.5z^{-1}} - \frac{1}{2} \frac{1}{1-0.5z^{-1}} + \frac{4}{3} \frac{1}{1-z^{-1}}$$

$$\Rightarrow Z^{-1}\{X(z)\} = \frac{1}{6} Z^{-1}\left\{\frac{1}{\cancel{1+0.5z^{-1}}\ 1-(-0.5)z^{-1}}\right\} - \frac{1}{2} Z^{-1}\left\{\frac{1}{1-0.5z^{-1}}\right\} + \frac{4}{3} Z^{-1}\left\{\frac{1}{1-z^{-1}}\right\}$$

$$= \frac{1}{6}(-0.5)^n u[n] - \frac{1}{2}(0.5)^n u[n] + \frac{4}{3} u[n]$$

(Ans)

<u>Problem</u> $X(z) = \frac{1}{(1+z^{-1})(1-0.75z^{-1})}$

Let,

$$X(z) = \frac{A}{1+z^{-1}} + \frac{B}{1-0.75z^{-1}}$$

$$\Rightarrow \frac{1}{(1+z^{-1})(1-0.75z^{-1})} = \frac{A}{1+z^{-1}} + \frac{B}{1-0.75z^{-1}}$$

$$A = \frac{1}{1-0.75(-1)} = 4/7$$

$$B = \frac{1}{1+4/3} = 3/7$$

$$\therefore X(z) = \frac{4}{7}\frac{1}{1+z^{-1}} + \frac{3}{7}\frac{1}{1-0.75z^{-1}}$$

$$\Rightarrow Z^{-1}\{X(z)\} = \cancel{\frac{4}{7} Z^{-1}\left\{\frac{1}{1-(-1)z^{-1}}\right\}}$$

$$= \frac{4}{7} Z^{-1}\left\{\frac{1}{1-(-1)z^{-1}}\right\} + \frac{3}{7} Z^{-1}\left\{\frac{1}{1-0.75z^{-1}}\right\}$$

$$\Rightarrow x[n] = \frac{4}{7}(-1)^n u[n] + \frac{3}{7}(0.75)^n u[n]$$

(Ans)

Problem $2y[n] - 3y[n-1] = 2^n u[n]$; $y[-1] = 2$

Solve

$$2y[n] - 3y[n-1] = 2^n u[n]$$

$$\Rightarrow 2Z\{y[n]\} - 3Z\{y[n-1]\} = Z\{2^n u[n]\}$$

$$\Rightarrow 2Y(z) - \cancel{3Y(}\ 3z^{-1}[Y(z) + y[-1]z] = \frac{1}{1-2z^{-1}}$$

$$\Rightarrow 2Y(z) - 3z^{-1}Y(z) - 6 = \frac{1}{1-2z^{-1}}$$

$$\Rightarrow (2-3z^{-1})Y(z) = \frac{1}{1-2z^{-1}} + 6$$

$$\Rightarrow Y(z) = \frac{1}{(1-2z^{-1})(2-3z^{-1})} + 6\frac{1}{2-3z^{-1}} \quad \text{--- (1)}$$

$$\frac{1}{(1-2z^{-1})(2-3z^{-1})} = \frac{A}{1-2z^{-1}} + \frac{B}{2-3z^{-1}}$$

$$A = \frac{1}{2-3(1/2)} = 2$$

$$B = \frac{1}{1-2\times 2/3} = -3$$

From equ①,

$$Y(z) = \frac{2}{1-2z^{-1}} - 3\frac{1}{2-3z^{-1}} + 6\frac{1}{2-3z^{-1}}$$

$$\Rightarrow Y(z) = 2\frac{1}{1-2z^{-1}} + 3\frac{1}{2(1-\frac{3}{2}z^{-1})}$$

$$\Rightarrow z^{-1}\{Y(z)\} = 2z^{-1}\left\{\frac{1}{1-2z^{-1}}\right\} + \frac{3}{2}z^{-1}\left\{\frac{1}{1-\frac{3}{2}z^{-1}}\right\}$$

$$\Rightarrow \cancel{y(z)}\, y[n] = 2(2)^n u[n] + \frac{3}{2}(3/2)^n u[n]$$

$$= 2^{n+1}u[n] + \left(\frac{3}{2}\right)^{n+1}u[n]$$

(Ans)

<u>Problem</u> $6y[n]-5y[n-1]=4^n u[n]$; $y[-1]=0$

<u>Solve</u>

$$6y[n]-5y[n-1]=4^n u[n]$$

$$\Rightarrow 6Z\{y[n]\}-5Z\{y[n-1]\}=Z\{4^n u[n]\}$$

$$\Rightarrow 6Y(z)-5z^{-1}[Y(z)+y(-1)]=\frac{1}{1-4z^{-1}}$$

$$\Rightarrow 6Y(z)-5z^{-1}Y(z)=\frac{1}{1-4z^{-1}}$$

$$\Rightarrow (6-5z^{-1})Y(z)=\frac{1}{1-4z^{-1}}$$

$$\Rightarrow Y(z)=\frac{1}{(1-4z^{-1})(6-5z^{-1})} \quad -(1)$$

$$\frac{1}{(1-4z^{-1})(6-5z^{-1})}=\frac{A}{1-4z^{-1}}+\frac{B}{6-5z^{-1}}$$

$$A=\frac{1}{6-5(1/4)}=4/19$$

$$B=\frac{1}{1-4(6/5)}=-5/19$$

Now equ(1) becomes,

$$Y(z) = \frac{4}{19}\frac{1}{1-4z^{-1}} - \frac{5}{19}\frac{1}{6-5z^{-1}}$$

$$\Rightarrow Y(z) = \frac{4}{19}\frac{1}{1-4z^{-1}} - \frac{5}{19}\frac{1}{6(1-\frac{5}{6}z^{-1})}$$

$$\Rightarrow z^{-1}\{Y(z)\} = \frac{4}{19}z^{-1}\left\{\frac{1}{1-4z^{-1}}\right\} - \frac{5}{6\times19}z^{-1}\left\{\frac{1}{1-\frac{5}{6}z^{-1}}\right\}$$

$$\Rightarrow y[n] = \frac{4}{19}(4)^n u[n] - \frac{5}{95}(5/6)^n u[n]$$

(Ans)

Problem $-4y[n+1] + 2y[n] = 3^n u[n]$; $y[0] = 0$

Solve:

$$-4y[n+1] + 2y[n] = 3^n u[n]$$

$$\Rightarrow -4z\{y[n+1]\} + 2z\{y[n]\} = z\{3^n u[n]\}$$

$$\Rightarrow -4z[Y(z) - y[0]] + 2Y(z) = \frac{1}{1-3z^{-1}}$$

$$\Rightarrow -4zY(z) + 2Y(z) = \frac{1}{1-3z^{-1}}$$

$$\Rightarrow (2-4z)Y(z) = \frac{1}{1-3/z} = \frac{1}{\frac{z-3}{z}}$$

$$\Rightarrow Y(z) = \frac{z}{(z-3)(2-4z)}$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{1}{(z-3)(2-4z)} \quad \text{—} ①$$

$$\frac{1}{(z-3)(2-4z)} = \frac{A}{z-3} + \frac{B}{2-4z}$$

$$A = \frac{1}{2-4\times 3} = -1/10 \quad ; \quad B = \frac{1}{1/2 - 3} = -2/5$$

From equ①,

$$\frac{Y(z)}{z} = -\frac{1}{10}\,\frac{1}{z-3} - \frac{2}{5}\,\frac{1}{2-4z}$$

$$\Rightarrow Y(z) = -\frac{1}{10}\,\frac{z}{z-3} - \frac{2}{5}\,\frac{z}{2-4z}$$

$$\Rightarrow Y(z) = -\frac{1}{10}\,\frac{z}{z(1-3z^{-1})} - \frac{2}{5}\,\frac{z}{-4z\left(1-\frac{2}{4z}\right)}$$

$$\Rightarrow Y(z) = -\frac{1}{10}\,\frac{1}{1-3z^{-1}} + \frac{2}{\cancel{5}20}\,\frac{1}{1-\frac{1}{2}z^{-1}}$$

$$\Rightarrow z^{-1}\{Y(z)\} = -\frac{1}{10}\,z^{-1}\left\{\frac{1}{1-3z^{-1}}\right\} + \frac{1}{10}\,z^{-1}\left\{\frac{1}{1-\frac{1}{2}z^{-1}}\right\}$$

$$\Rightarrow y[n] = -\frac{1}{10}(3)^n u[n] + \frac{1}{10}(1/2)^n u[n]$$

(Ans)