Math-3 (Sample Questions for students)

1. Evaluate $\int_C \bar{z}\, dz$; where C is the line segment from $z = 0$ to $z = 3$. (4 points)

(a) $\frac{9}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{1}{2}$

2. Evaluate $\int_c \frac{1}{z-i}\, dz$; $C: |z - i| = 3$, clockwise. (4 points)

(a) $2\pi i$ (b) $-2\pi i$ (c) 0 (d) -1

3. The path of $z(t) = 2\sin t + i\, 3\cos t$ represents- (3 points)

(a) Ellipse (b) Hyperbola (c) Circle (d) parabola

4. $|z - 2| = 4$; center of the circle is (1 point)

(a) $(-2,0)$ (b) $(2,0)$ (c) $(0,2)$ (d) $(0,1)$

5. What is the equation of the path C, passing through the points $z = 0$ to $z = 2$. (2 points)

(a) $y = 0$ (b) $x = 0$ (c) $y = 2$ (d) $x + y = 2$

6. . Mention whether the point $(1,2)$ are interior, exterior or boundary of $|z - 5 + i| = 4$. (2 points)

(a) interior (b) exterior (c) on boundary (d) none

7. Parametric representation of line segment from $z = 1 + i$ to $z = 4 - 2i$ is: (4 points)

(a) $x = t, y = t + 2$

(b) $x = -t, y = -t + 2$

(c) $x = t, y = -t + 2$.

(d) $x = 2t, y = -2 + t$

8. Which of the following line parallel to imaginary axis? (2 points)

(a) $z = 1$ to $z = 1 + i$

(b) $z = 1$ to $z = 1 + 2i$

(c) $z = 1 + i$ to $z = i$

(d) $z = -1$ to $z = 1$.

9. Evaluate by CRT $\oint_C \frac{dz}{(z-6)^{10}}; C: |z| = 4$. (2 points)

(a) not determined (b) 0 (c) $2\pi i$ (d) $-2\pi i$

10. For $\oint_C \frac{2z\,dz}{(2z-i)^3}$, $C: |z| = 1$, lower half circle including the real line from -1 to $+1$; evaluate residue at singular point: (4 points)

(a) $\frac{i}{2}$ (b) 0 (c) not determined (d) $\frac{1}{2}$.

11. Evaluate $\oint_C \frac{dz}{z(z-1)(z-2)}; C: |z| = r, 3 < r < 4.$ (2 points)

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi i$ (d) none.

12. In which of the following improper integrals contour integral can be applied? (2 points)

a. $\int_{-\infty}^{\infty} \frac{x}{x^2+2x+1} dx$

b. $\int_{-\infty}^{\infty} \frac{1}{x^2+2x+1} dx$

c. $\int_{-\infty}^{\infty} \frac{x^3}{\left(x^2+1\right)^2} dx$

d. $\int_{-\infty}^{\infty} \frac{1}{(x+1)(x-3i)} dx$.

13. To evaluate improper integral the contour is considered as: (1 point)

a. Closed circle
b. Semi-circle of radius R in lower half plane
c. Semi-circle of radius R in upper half plane
d. Semi-circle in upper half plane including the real line from -R to +R .

14. If $\int_{-\infty}^{\infty} \frac{x^2}{\left(x^2+4\right)} dx = -2\pi$ then what is the value of $\int_0^{\infty} \frac{x^2}{x^2+4} dx$? (1 point)

a. $-\pi$ b. π c. 4π d. -4π .

15. Which one is the Laurent Series expansion for $f(z)=\frac{1}{z-2}$ valid for $|z|<2$.

(3 points)

a. $f(z)=\frac{1}{z}\left(1+\frac{2}{z}+\frac{4}{z^2}+\frac{8}{z^3}+...\right)$

b. $f(z)=\frac{1}{-2}\left(1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+...\right)$

c. $f(z)=\frac{1}{-2}\left(1-\frac{z}{2}+\frac{z^2}{4}-\frac{z^3}{8}+...\right)$

d. $f(z)=\frac{1}{z}\left(1-\frac{2}{z}+\frac{4}{z^2}-\frac{8}{z^3}+...\right)$.

16. Which one is the Laurent Series expansion for $f(z)=\frac{1}{z-1}$ valid for $|z|>1$.

(3 points)

a. $f(z)=\frac{1}{z}\left(1-\frac{1}{z}+\frac{1}{z^2}-\frac{1}{z^3}+...\right)$

b. $f(z)=-\left(1-z+z^2-z^3+...\right)$

c. $f(z)=\frac{1}{z}\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+...\right)$

d. $f(z)=-\left(1+z+z^2+z^3+...\right)$.

17. Which one is the Laurent Series expansion for $f(z)=\frac{1}{z(z-1)}$ valid for $0<|z|<1$.

(3 points)

a. $f(z)=\frac{1}{z}\left(1+z+z^2+z^3+...\right)$

b. $f(z)=-\frac{1}{z}\left(1+z+z^2+z^3+...\right)$

c. $f(z)=\frac{1}{z^2}\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+...\right)$

d. $f(z)=\frac{1}{z}\left(1+z+z^2+z^3+...\right)$.

18.Which one is the Z-transformation of the discrete valued function $x[n]=\delta[n-3]$

(2 points)

a. $X(z)=z^3$ b. $X(z)=\frac{1}{1-z^{-1}}$ c. $X(z)=z^{-3}$ d. $X(z)=3$.

19. Which one(s) is the Z-transformation of the discrete valued function $x[n]=2^n u[n], \text{for } |z|>2$ where $u[n]$ is the discrete time unit step function ?

(2 points)

a. $X(z)=\dfrac{1}{1+2z^{-1}}$

b. $X(z)=\dfrac{1}{1-2z^{-1}}$

c. $X(z)=\dfrac{z}{z+1}$

d. $X(z)=\dfrac{z}{z+2}$.

20. Which one is the inverse Z-transform of the function $X(z)=\dfrac{1}{1+2z^{-1}}, |z|>2$?

(2 points)

a. $x[n]=2^n u[n]$

b. $x[n]=\left(\dfrac{1}{2}\right)^n u[n]$

c. $x[n]=(-2)^n u[n]$

d. $x[n]=-(2)^n u[n]$.

21. Which one is the inverse Z-transform of the function $X(z)=\dfrac{1}{z^2}$?

(2 points)

a. $x[n]=\delta[n-2]$

b. $x[n]=\delta[n+2]$

c. $x[n]=-\delta[n+2]$

d. $x[n]=-\delta[n-2]$.

22. If $X(z)=\dfrac{1}{\left(1-z^{-1}\right)\left(1+z^{-1}\right)}, |z|>1$, then which one is the inverse Z-transform of this function? (4 points)

a. $x[n]=\dfrac{1}{2}u[n]+\dfrac{1}{2}(-1)^n u[n]$

b. $x[n]=-\dfrac{1}{2}u[n]+\dfrac{1}{2}(-1)^n u[n]$

c. $x[n]=-\frac{1}{2}u[n]-\frac{1}{2}(-1)^n u[n]$

d. $x[n]=\frac{1}{2}u[n]-\frac{1}{2}(-1)^n u[n]$.

23. Which of the following is the solution of the difference equation :

$y[n]+y[n-1]=0, y[-1]=-1,\ n\geq 0.$ (4 points)

a. $y[n]=1$

b. $y[n]=-1$

c. $y[n]=(-1)^n$

d. $y[n]=-(-1)^n$.

24. Which of the following is the solution of the difference equation :

$y[n]-2y[n-1]=0, y[-1]=1,\ n\geq 0.$ (4 points)

a. $y[n]=2$

b. $y[n]=-2$

c. $y[n]=(2)^{n+1}$

d. $y[n]=(2)^n$.

25. Which of the following is the solution of the difference equation :

$y[n]-y[n-2]=0, y[-1]=0, y[-2]=1,\ n\geq 0.$ (4 points)

a. $y[n]=\frac{1}{2}+\frac{1}{2}(-1)^n$

b. $x[n]=-\frac{1}{2}+\frac{1}{2}(-1)^n$

c. $x[n]=-\frac{1}{2}-\frac{1}{2}(-1)^n$

d. $x[n]=\frac{1}{2}-\frac{1}{2}(-1)^n$