2(b) Expand $f(z) = \frac{1}{z\,(z-2)}$ in Laurent series for $|z| > 2$

Solution: $|z| > 2 \Longrightarrow \frac{|z|}{2} > 1 \Longrightarrow \frac{2}{|z|} < 1 \Longrightarrow \left|\frac{2}{z}\right| < 1$

$\frac{1}{z\,(z-2)} = \frac{A}{z} + \frac{B}{z-2}$............(i)

$$A = \frac{1}{0-2} = -\frac{1}{2}$$

$$B = \frac{1}{2}$$

Form equation (i)

$$\frac{1}{z\,(z-2)} = -\frac{1}{2}\,\frac{1}{z} + \frac{1}{2}\,\frac{1}{z-2}$$

$$= -\frac{1}{2}\,\frac{1}{z} + \frac{1}{2}\,\frac{1}{z\,\left(1-\frac{2}{z}\right)}$$

$$= -\frac{1}{2}\,\frac{1}{z} + \frac{1}{2}\,\frac{1}{z}\left(1-\frac{2}{z}\right)^{-1}$$

$$\frac{1}{z\,(z-2)} = -\frac{1}{2z} + \frac{1}{2z}\left(1 + \frac{2}{z} + \frac{4}{z^2} + \frac{8}{z^3} + \cdots \dots \dots\right)$$

1(e) $f(z) = \frac{3z}{(z-1)(2-z)}; 0 < |z-1| < 1$

Solution: Let, $z - 1 = u$

So,

$$|u| < 1$$

$$f(u) = \frac{3(u+1)}{u(2-u-1)}$$

$$= \frac{3(u+1)}{u(1-u)}$$

Now,

$\frac{3(u+1)}{u(1-u)} = \frac{A}{u} + \frac{B}{1-u}$.........(i)

$$A = \frac{3(0+1)}{(1-0)} = 3$$

$$B = \frac{3(1+1)}{1} = 6$$

So, form (i) we get,

$$\frac{3(u+1)}{u(1-u)} = \frac{3}{u} + \frac{6}{1-u}$$

$$= \frac{3}{u} + 6\,(1-u)^{-1}$$

$$= \frac{3}{u} + 6(1 + u + u^2 + u^3 + \cdots \ldots \ldots .)$$

So, $f(z) = \frac{3}{z-1} + 6[1 + (z-1) + (z-1)^2 + (z-1)^3 + \cdots]$ Ans.