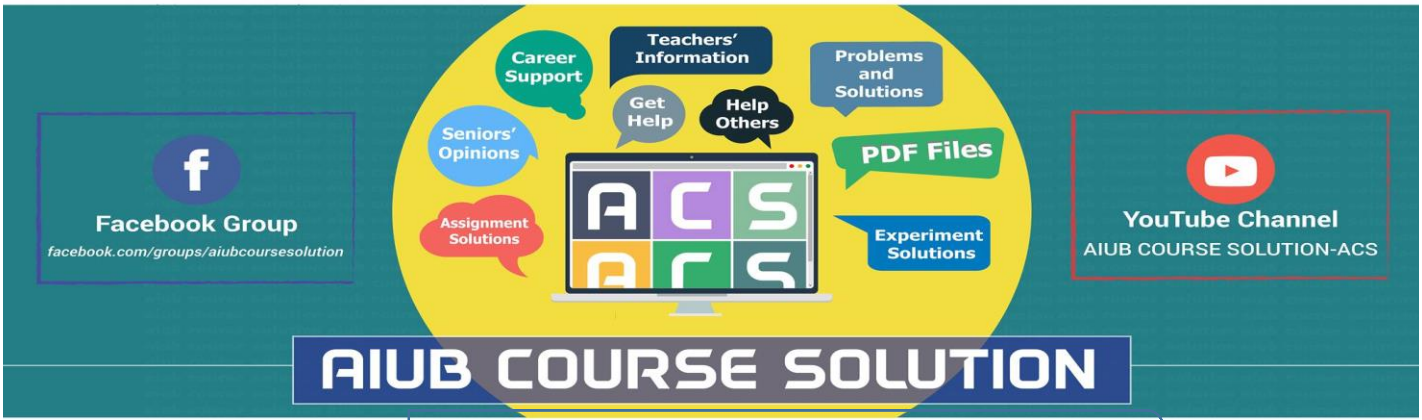

TOGETHER WE CAN ACHIEVE MORE

COURSE: MATH 3

CHAPTER: CHAPTER 6

SOLVED BY:

RUPA PAUL

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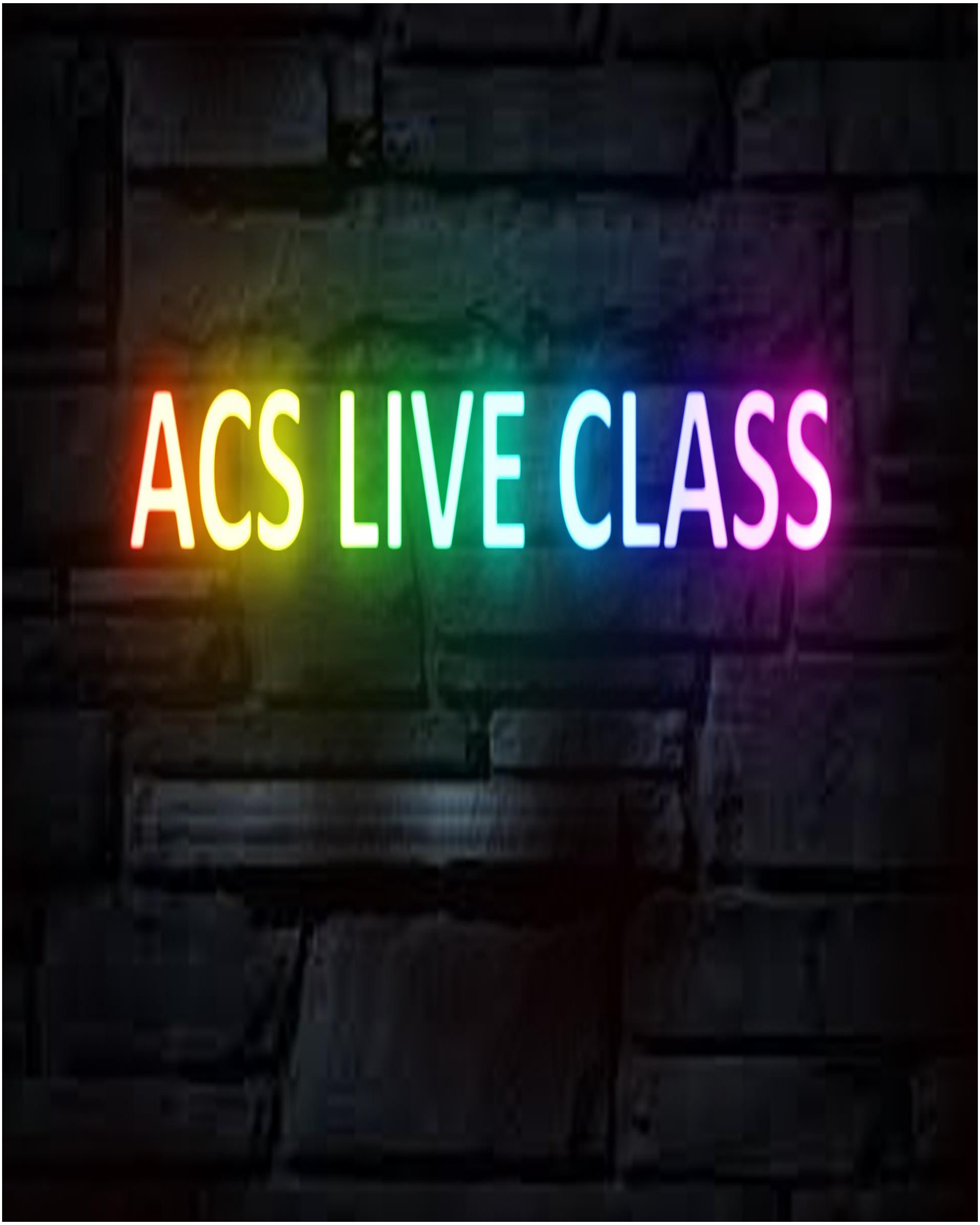


ACS LIVE CLASS

Chapter-06

(1)

1. (i) $z(t) = (1+3i)t \quad (1 \le t \le 4)$.

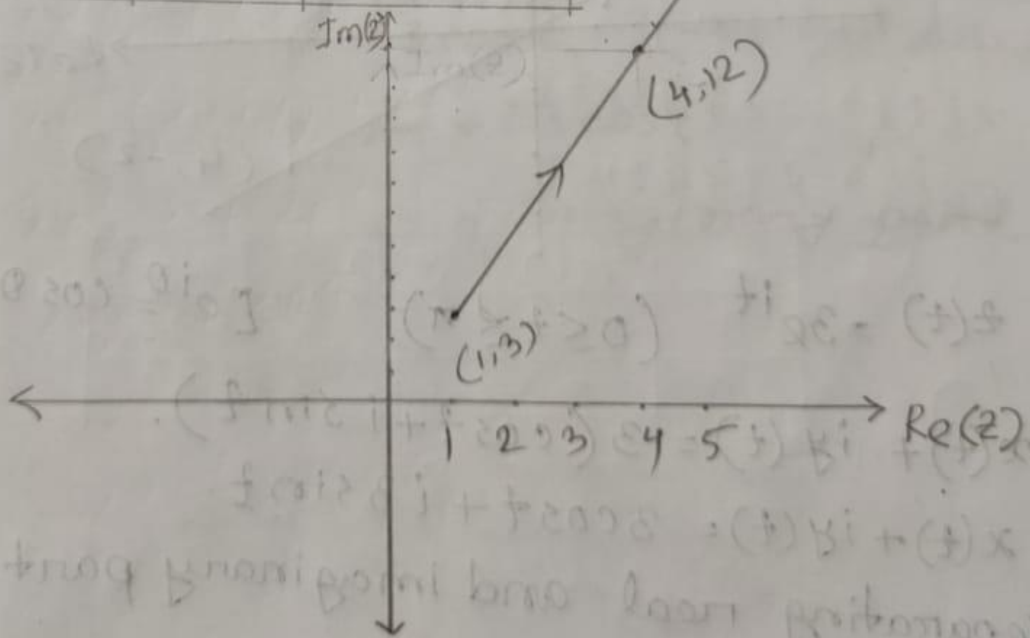
$\Rightarrow x(t) + iy(t) = t + i3t \qquad [z = x+iy]$

Separating real and imaginary part-

$x(t) = t$

$y(t) = 3t$

t	$x(t)$	$y(t)$	(x,y)
1	1	3	(1,3)
2	2	6	(2,6)
4	4	12	(4,12)



(ii) $z(t) = (2-i)t \quad (-2 \le t \le 2)$

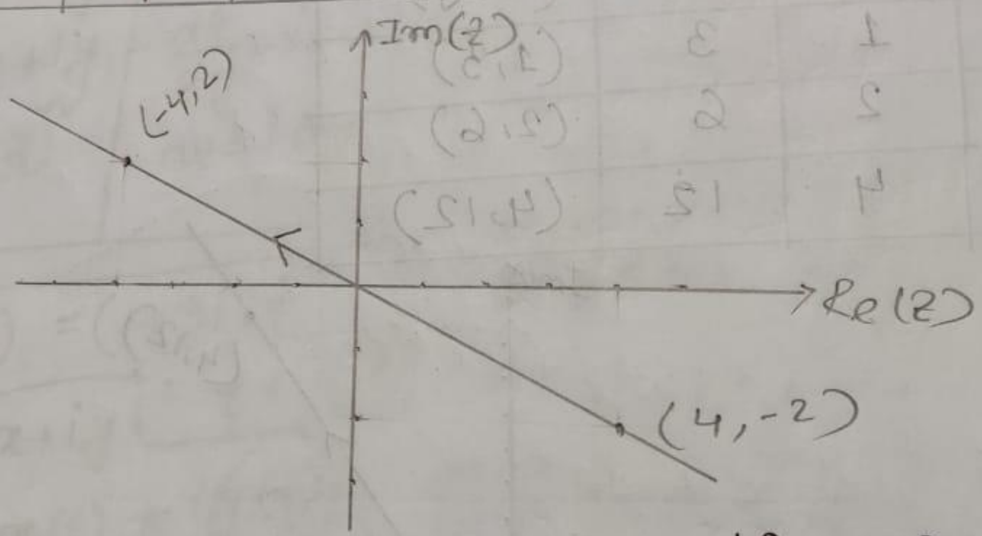
$\Rightarrow x(t) + iy(t) = 2t - it$

separating real and imaginary part,

$x(t) = 2t$

$y(t) = -t$

t	$x(t)$	$y(t)$	(x, y)
-2	-4	2	$(-4, 2)$
2	4	-2	$(4, -2)$



(iii) $z(t) = 3e^{it} \quad (0 \le t \le \pi)$ $\quad [e^{i\theta} = \cos\theta + i\sin\theta]$

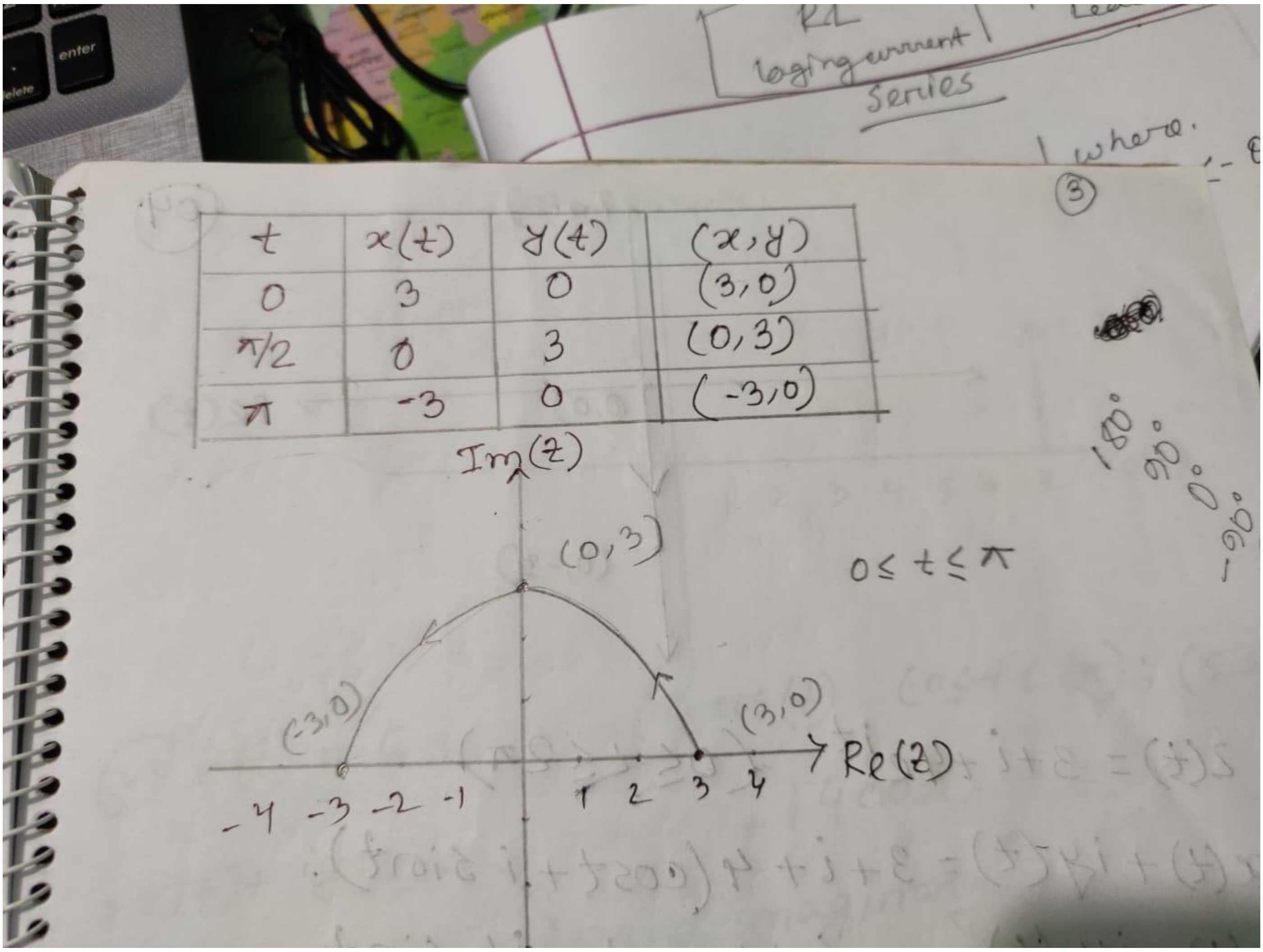
$\Rightarrow x(t) + iy(t) = 3(\cos t + i\sin t).$

$\Rightarrow x(t) + iy(t) = 3\cos t + i\,3\sin t.$

separating real and imaginary part.

$x(t) = 3\cos t$

$y(t) = 3\sin t$



t	$x(t)$	$y(t)$	(x,y)
0	3	0	(3,0)
$\pi/2$	0	3	(0,3)
π	-3	0	(-3,0)

04

(iv) $z(t) = 3\sin t + i3\cos t \quad (-\pi \le t \le \pi)$

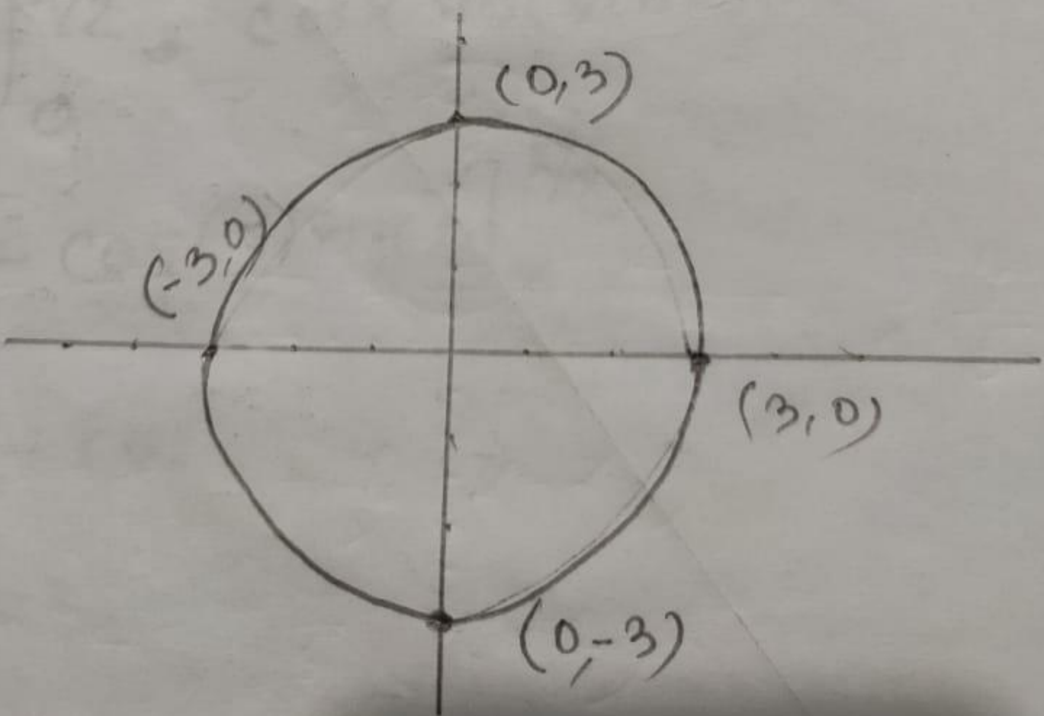
$\Rightarrow x(t) + iy(t) = 3\sin t + i3\cos t$

Separating real and imaginary part-

$x(t) = 3\sin t$

$y(t) = 3\cos t$

t	$x(t)$	$y(t)$	(x,y)
π	0	-3	(0,-3)
$\pi/2$	3	0	(3,0)
0	0	3	(0,3)
$-\pi/2$	-3	0	(-3,0)
$-\pi$	0	-3	(0,-3)



(v) $z(t) = 3 + i + 4e^{it}$ $(0 \le t \le 2\pi)$ (05)

$\Rightarrow x(t) + iy(t) = 3 + i + 4e^{it}$

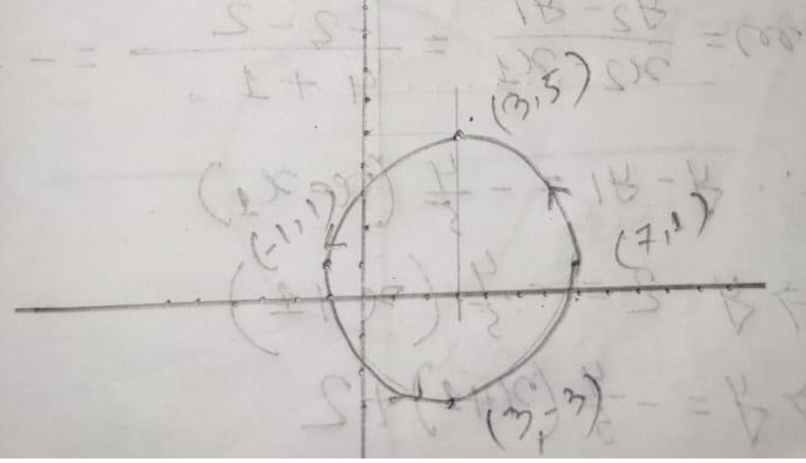
$\Rightarrow x(t) + iy(t) = 3 + i + 4\cos t + i4\sin t$

Separating real and imaginary part-

$x(t) = 3 + 4\cos t.$

$y(t) = 1 + 4\sin t$

t	$x(t)$	$y(t)$	(x, y)
0	7	1	(7,1)
$\pi/2$	3	5	(3,5)
π	-1	1	(-1,1)
$3\pi/2$	3	-3	(3,-3)
2π	$\cancel{8}$ 7	1	(7,1)



(06)

(vi) $z(t) = 6\sin(t) + i\,4\cos(t),\ \boxed{0 \le t \le 2\pi}$ (5,1)

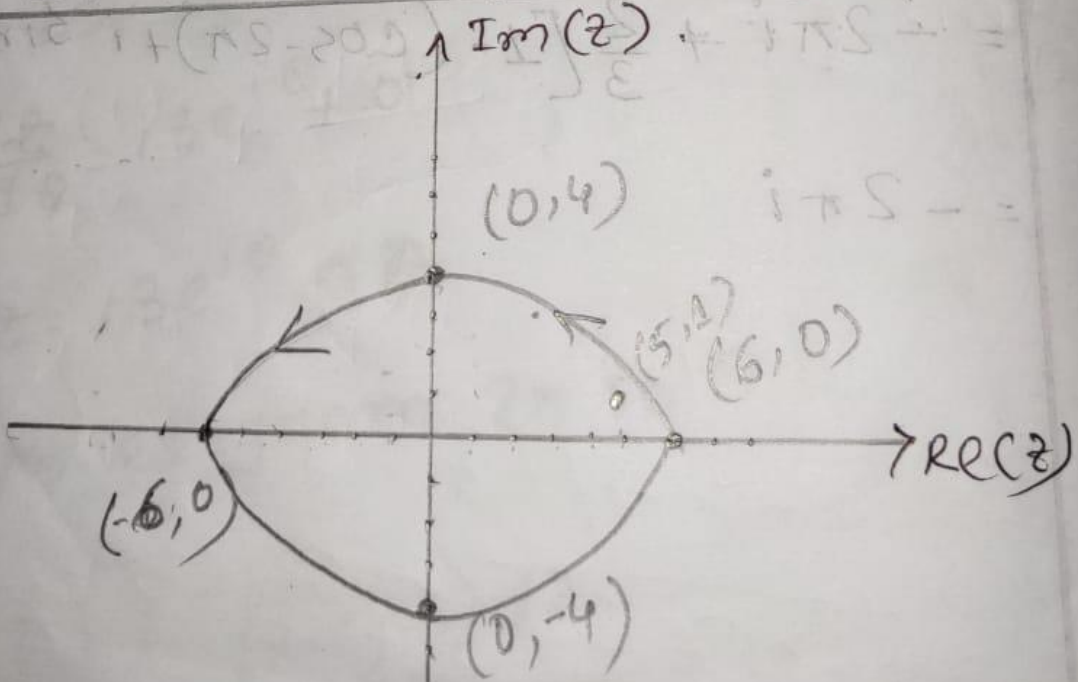
$\Rightarrow x(t) + iy(t) = 6\sin t + i\,4\cos t.$

Separating real and imaginary part.

$x(t) = 6\sin t$

$y(t) = 4\cos t.$

t	$x(t)$	$y(t)$	(x,y)
0	0	4	$(0,4)$
$\pi/2$	6	0	$(6,0)$
π	0	-4	$(0,-4)$
$3\pi/2$	-6	0	$(-6,0)$
2π	0	4	$(0,4)$



(5,1) interior

(07)

(vii) $z(t) = 2\sin t + i3\cos(t) + 3 + 2i$

$(0 \le t \le 2\pi)$; $(6,5)$

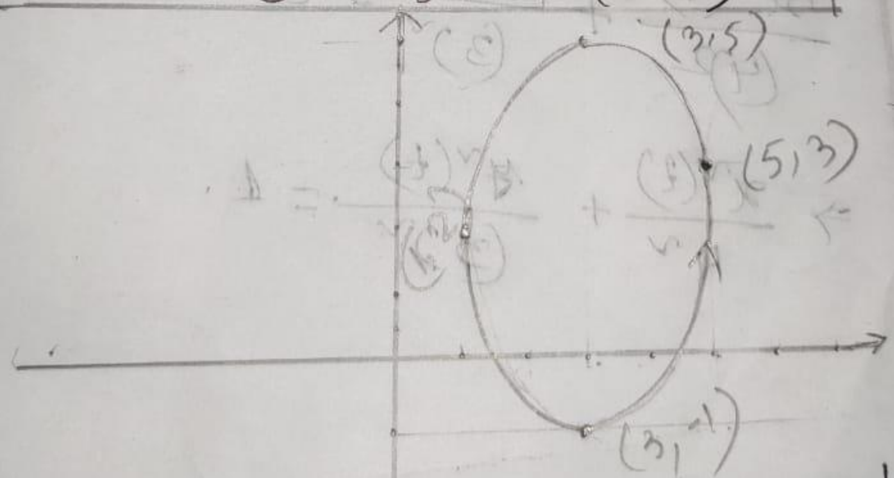
$\Rightarrow x(t) + iy(t) = 2\sin t + 3 + i(3\cos t + 2)$.

Real and imaginary part,

$x(t) = 2\sin t + 3$.

$y(t) = 3\cos t + 2$.

t	$x(t)$	$y(t)$	(x, y).
0	3	5	$(3,5)$.
$\pi/2$	5	3	$(5,3)$
π	3	-1	$(3,-1)$
$3\pi/2$	1	2	$(1,2)$
2π	3	5	$(3,5)$



point $(6,5)$ outside

(08)

(viii) $z(t) = 4\cosh(t) + 3i\sinh(t)$.

$\Rightarrow x(t) + iy(t) = 4\cosh t + 3i\sinh t$

real and imaginary part.

$x(t) = 4\cosh t$.

$\Rightarrow \cosh t = \frac{x(t)}{4}$. —— (i)

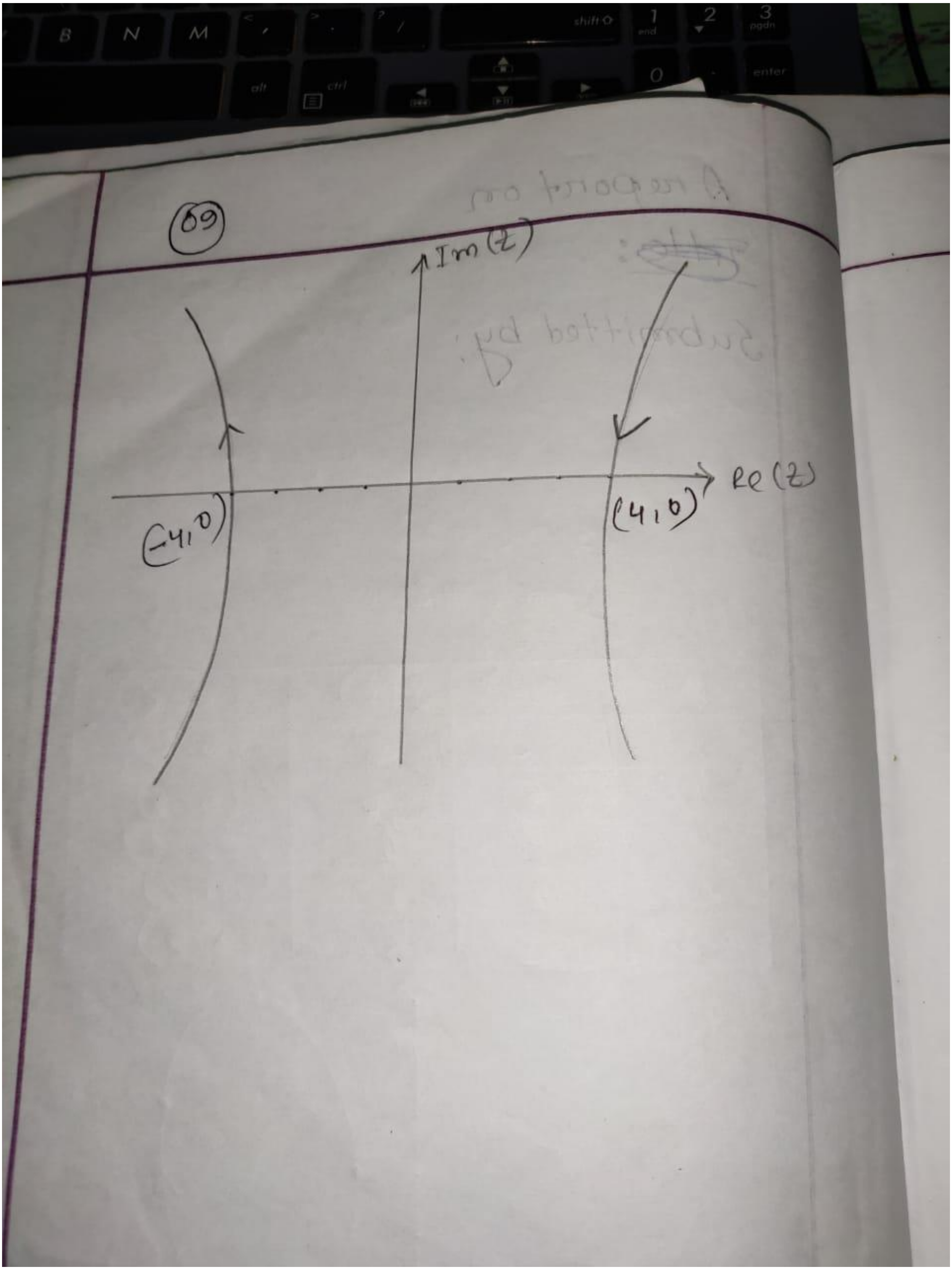
$y(t) = 3\sinh t$

$\Rightarrow \sinh t = \frac{y(t)}{3}$. —— (ii).

(i) − (ii)

$$\frac{x^2(t)}{(4)^2} + \frac{y^2(t)}{(3)^2} = \cosh^2(t) - \sinh^2(t)$$

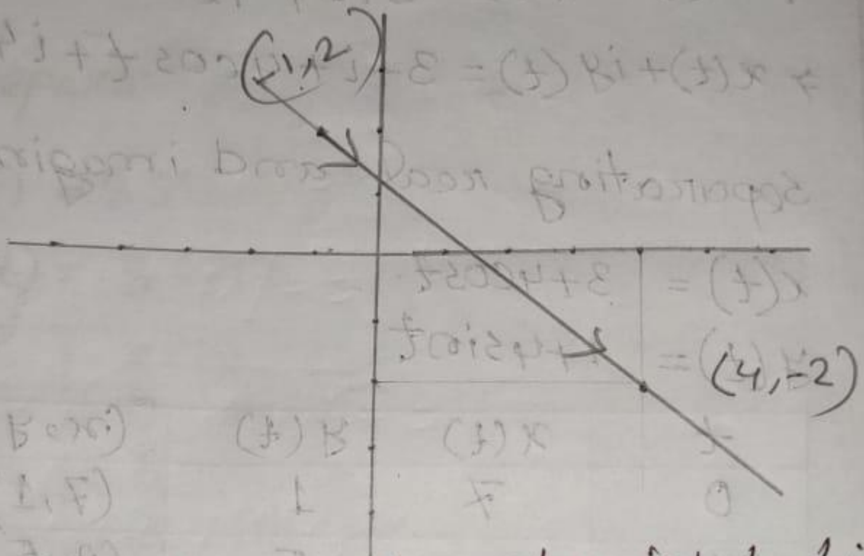
$$\Rightarrow \frac{x^2(t)}{4^2} + \frac{y^2(t)}{(3)^2} = 1.$$


09
Im(z)
Re(z)
(-4,0)
(4,0)

①

2. (i) $-1+2i$ to $4-2i$

$(-1,2)$ $(4,-2)$



The equation of straight line passing through $(-1,2)$ and $(4,-2)$

$$y-y_1 = m(x-x_1)$$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-2}{4+1} = -\frac{4}{5}.$$

$$\therefore y-y_1 = -\frac{4}{5}(x-x_1)$$

$$\Rightarrow y-2 = -\frac{4}{5}(x+1)$$

$$\Rightarrow y = -\frac{4}{5}(x+1)+2$$

(2)

Let, $x = t$.

$\therefore y(t) = -\frac{4}{5}(t+1)+2$

$(-1,2)$ and $(4,-2)$

so, $-1 \leq t\ 4$

(ii) $|z| = 1$

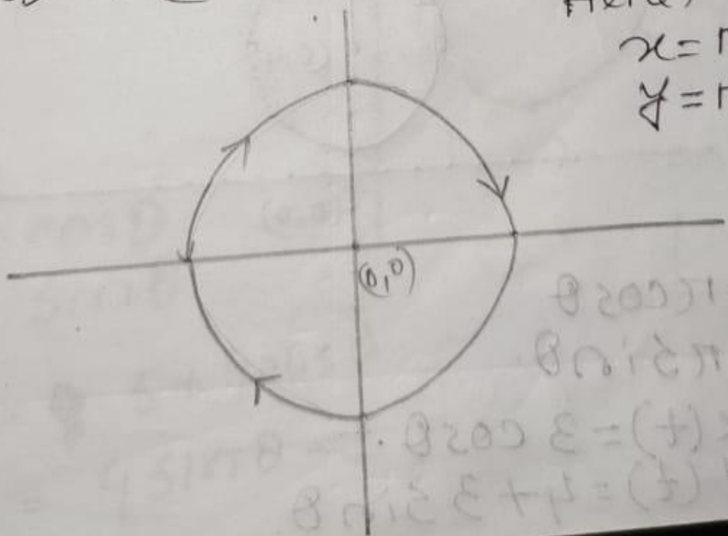
$\Rightarrow |x+iy| = 1$

$\Rightarrow \sqrt{(x)^2+(y)^2} = 1$

$\Rightarrow x^2 + y^2 = (1)^2$.

$\Rightarrow (x-0)^2 + (y-0)^2 = (1)^2$

Here,
$x = r\cos\theta$
$y = r\sin\theta$



$x(t) = \cos\theta$

$y(t) = \sin\theta$

(iii) $|z-4i| = 3$

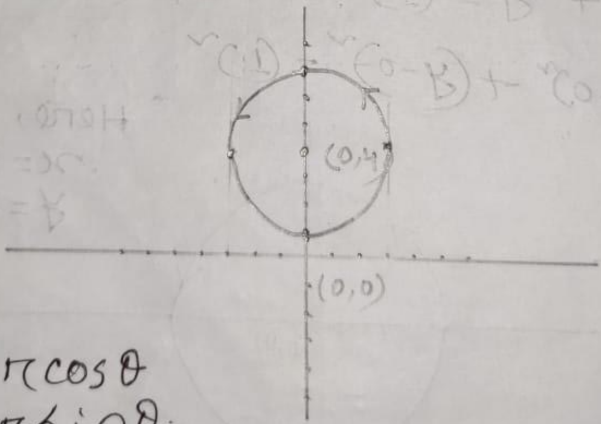
$\Rightarrow |x+iy-4i| = 3.$

$\Rightarrow |x+i(y-4)| = 3.$

$\Rightarrow \sqrt{(x)^2 + (y-4)^2} = 3.$

$\Rightarrow x^2 + (y-4)^2 = (3)^2.$

$r=3$, center $(0,4)$



$x = r\cos\theta$

$y = r\sin\theta$

$x(t) = 3\cos\theta.$

$y(t) = 4 + 3\sin\theta.$

(iv) $|z-5+i|=4$.

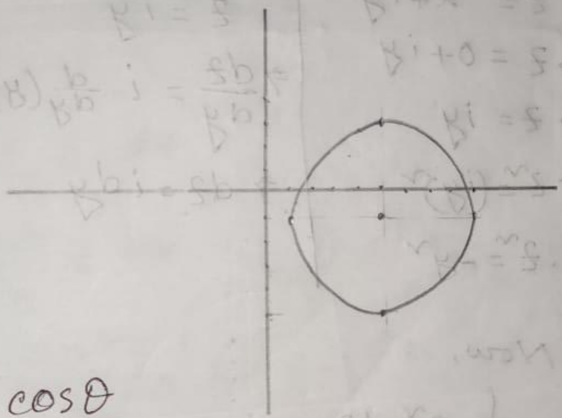
$\Rightarrow |x+iy-5+i|=4$.

$\Rightarrow |x-5+i(y+1)|=4$.

$\Rightarrow \sqrt{(x-5)^2+(y+1)^2}=4$.

$\Rightarrow (x-5)^2+(y+1)^2=(4)^2$.

$r=4$, center$(5,-1)$.



$x=r\cos\theta$

$y=r\sin\theta$

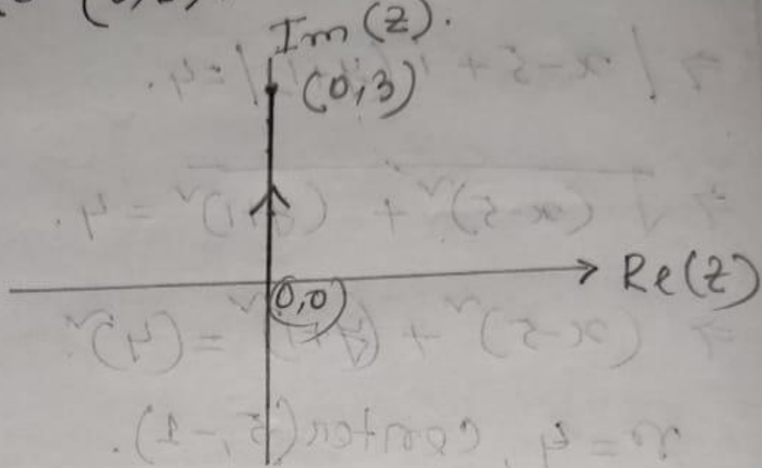
$x(t)=5+4\cos\theta$

$y(t)=4\sin\theta-1$

(5)

3. $z=0$ to $z=3i$ and evaluate $\int_C z^2 dz$

(0,0) to (0,3).



$z = x+iy$

$\Rightarrow z = 0+iy$

$\Rightarrow z = iy$

$\Rightarrow z^2 = (iy)^2$

$\therefore z^2 = -y^2$

$z = iy$

$\Rightarrow \frac{dz}{dy} = i \frac{d}{dy}(y)$.

$\Rightarrow dz = i\,dy$.

Now,

$\int_C z^2\, dz$

$= \int_0^3 -y^2 i\, dy$

$= -i\left[\frac{y^3}{3}\right]_0^3$

(6)

$= -i\left[\frac{27}{3} - 0\right]$.

$= -9i$

4. $z=0$ to $z=3$ and evaluate $\int_c \bar{z}\,dz$.

(0,0) to (3,0).

$\bar{z} = x - iy$.
$= x - i.0$
$= x$

$\bar{z} = x$.

$\frac{d\bar{z}}{dx} = \frac{d}{dx}(x)$.

$\Rightarrow d\bar{z} = dx$.

Now,

$\int_c \bar{z}\,dz$

$= \int_0^3 x\,dx$.

$= \left[\frac{x^2}{2}\right]_0^3$

$= \frac{9}{2}$

7. $\int_C (z+z^{-1})\,dz$ $\qquad C: |z|=2$

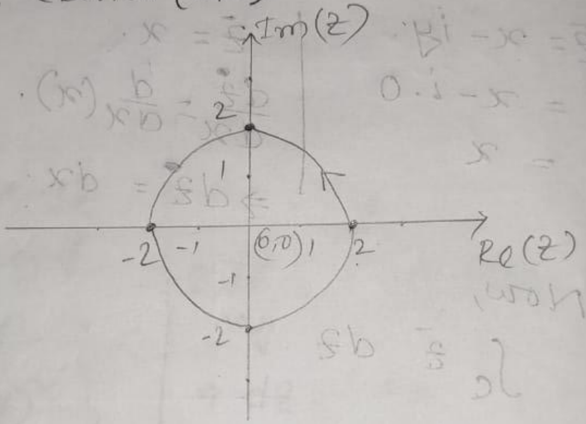
$\qquad\qquad\qquad\qquad\qquad\qquad \downarrow$

$\qquad\qquad\qquad\qquad\qquad\qquad z = re^{i\theta}$

$|z| = 2.$ $\qquad\qquad\qquad\qquad z = 2e^{i\theta}.$

$\Rightarrow x^2+y^2 = (2)^2.$

$r = 2.$ center $(0,0)$.



$z = 2e^{i\theta}$

$z^{-1} = \frac{1}{2e^{i\theta}}$

$\frac{dz}{d\theta} = 2ie^{i\theta}$

$\Rightarrow dz = 2ie^{i\theta}d\theta$

Here θ varies from 0 to 2π.

So, $\int_C (z+z^{-1})\,dz.$

$= \int_0^{2\pi} \left(2e^{i\theta} + \frac{1}{2}e^{-i\theta}\right) 2ie^{i\theta}\,d\theta$

(8)

$$= \int_0^{2\pi} 4i e^{i\theta} \cdot e^{i\theta} \, d\theta + \int_0^{2\pi} i e^{-i\theta} \cdot e^{i\theta} \, d\theta.$$

$$= 4i \int_0^{2\pi} e^{2i\theta} \, d\theta + \int_0^{2\pi} i \cdot 1 \, d\theta.$$

$$= 4i \left[\frac{e^{2i\theta}}{2i} \right]_0^{2\pi} + i[\theta]_0^{2\pi}$$

$$= \frac{4i}{2i} \left[e^{i4\pi} - 1 \right] + i[2\pi - 0].$$

$$= 2\left[e^{i4\pi} - 1\right] + i2\pi$$

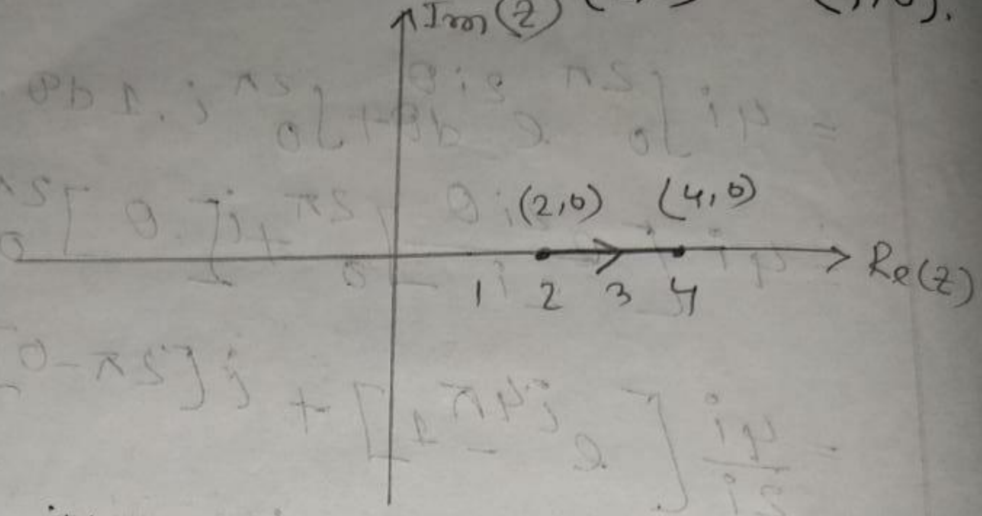
$$= 2\left[\{\cos 4\pi + i \sin 4\pi\} - 1\right] + i2\pi.$$

$$= 2(\cancel{1} - 0 - \cancel{1}) + 2\pi i$$

$$= 2\pi i$$

9

8. $\int_C (e^{2z}+\cos z)\,dz$ from $z=2$ to $z=4$
$(2,0)$ $(4,0)$.



$z = x+iy$

$\Rightarrow z = x$

$\Rightarrow \dfrac{dz}{dx} = \dfrac{d}{dx}(x)$

$\Rightarrow dz = dx.$

So, $\int_C (e^{2z}+\cos z)\,dz$

$= \int_2^4 (e^{2x}+\cos x)\,dx$

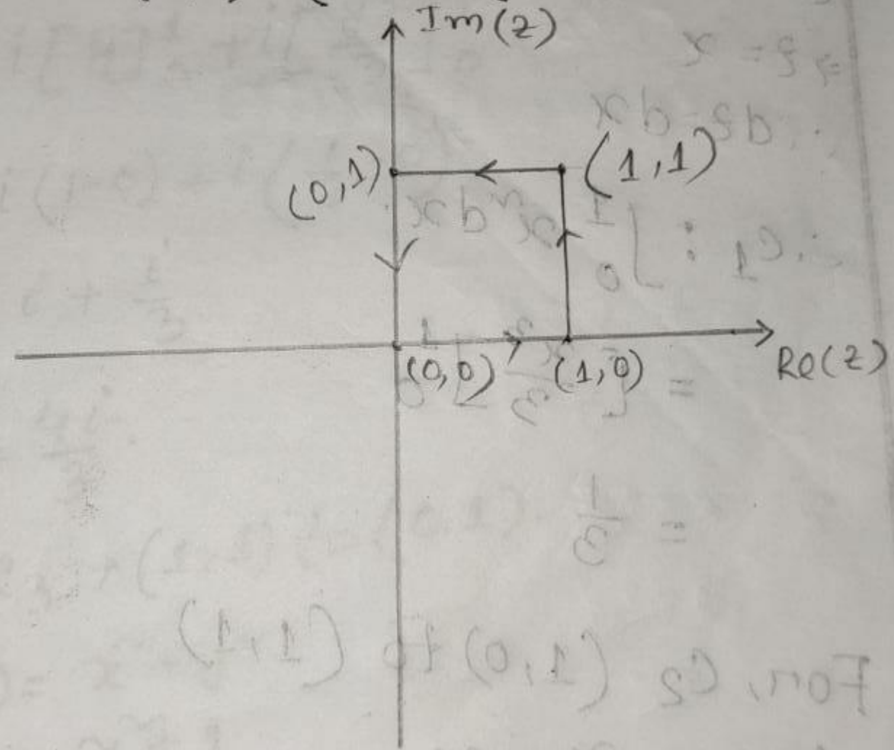
$= \left[\dfrac{e^{2x}}{2}\right]_2^4 + \left[\sin x\right]_2^4$

$= \frac{1}{2}(e^8 - e^4) + (\sin 4 - \sin 2)$

9. $\int_C (z.\bar{z})\,dz$

square vertices $0, 1, 1+i, i$

$(0,0), (1,0), (1,1), (0,1)$



$z = x+iy$

$\bar{z} = x-iy$

$z.\bar{z} = (x+iy)(x-iy)$

$= x^2+y^2$

For, C_1
$(0,0)$ to $(1,0)$

$f(z) = x^2 + (0)^2$
$= x^2$

$z = x + iy$
$\Rightarrow z = x$
$\therefore dz = dx$

$\therefore C_1 : \int_0^1 x^2 dx$
$= \left[\frac{x^3}{3}\right]_0^1$
$= \frac{1}{3}$

For, C_2 $(1,0)$ to $(1,1)$
$f(z) = x^2 + y^2$
$= (1)^2 + y^2$
$= 1 + y^2$

12

$z = x + iy$

$\Rightarrow z = 1 + iy.$

$\Rightarrow dz = i\,dy$

$\therefore C_2 : \int_0^1 (1+y^2)\, i\,dy$

$= i[y]_0^1 + i\left[\frac{y^3}{3}\right]_0^1$

$= i(1-0) + i\left(\frac{1}{3} - 0\right)$

$= i + \frac{i}{3}$

$= \frac{4i}{3}.$

For $C_3 \rightarrow (1,1)$ to $(0,1).$

$f(z) = x^2 + y^2$

$= x^2 + 1$

$z = x + iy.$

$= x + i$

$\therefore dz = dx$

13

$C_3 : \int_1^0 (x^2+1)\,dx.$

$= \left[\frac{x^3}{3}\right]_1^0 + [x]_1^0$

$= -\frac{1}{3} - 1.$

$= -4/3.$

For C_4: $(0,1)$ to $(0,0)$.

$f(z) = x^2 + y^2$

$= 0 + y^2$

$= y^2.$

$z = x + iy$

$= 0 + iy$

$= iy.$

$\therefore dz = i\,dy.$

$C_4 : \int_1^0 y^2\, i\,dy$

$= i\left[\frac{y^3}{3}\right]_1^0$

$= -\frac{i}{3}$.

50. $\int_C (z \cdot \bar{z})\,dz$

$= \frac{1}{3} - \frac{4}{3} + \frac{4i}{3} - \frac{i}{3}$.

$= \frac{1-4}{3} + \frac{4i-i}{3}$

$= -\frac{3}{3} + \frac{3i}{3}$

$= (-1+i)$ Ans.

10. $\int_C \left(\frac{1}{z-i} - \frac{2}{(z-i)^2}\right) dz$

here,

$|z-i| = 3$

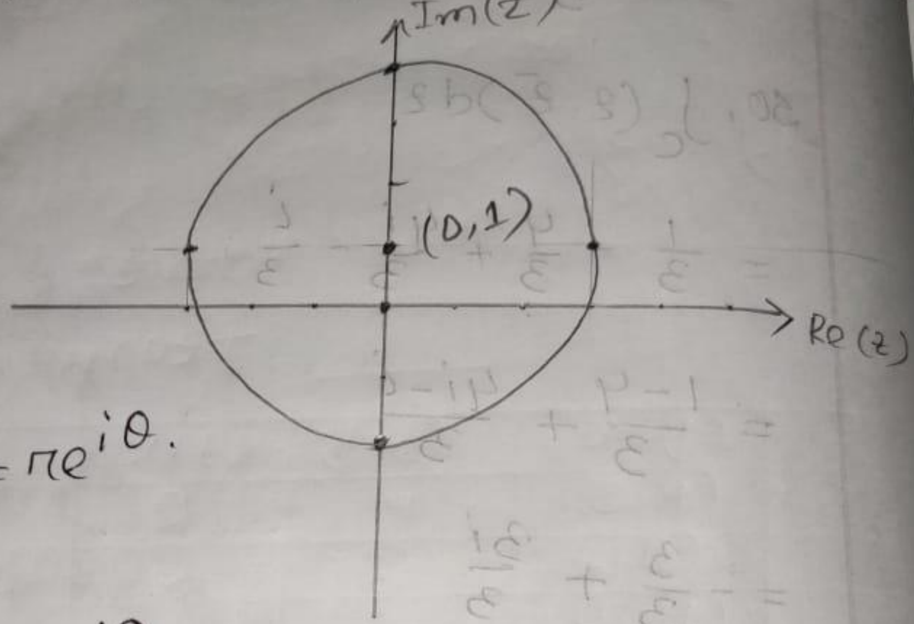
$\Rightarrow |x+iy-i| = 3$.

$\Rightarrow \sqrt{(x)^2 + (y-1)^2} = 3$

$\Rightarrow x^2 + (y-1)^2 = (3)^2$.

$\Rightarrow$

so, $r = 3$, center$(0, 1)$.



Here, $z = re^{i\theta}$.

$z - i = 3e^{i\theta}$

$\Rightarrow z = 3e^{i\theta} + i$

$\Rightarrow \dfrac{dz}{d\theta} = i3e^{i\theta} + 0$

$dz = i3e^{i\theta}\, d\theta$

θ varies from 2π to 0

$\therefore \int_c \left(\dfrac{1}{z-i} - \dfrac{2}{(z-i)^2}\right) dz$

(16)

$$=\int_{2\pi}^{0}\left(\frac{1}{3e^{i\theta}}-\frac{2}{9e^{2i\theta}}\right)3ie^{i\theta}\,d\theta.$$

$$=\frac{3i}{3}\int_{2\pi}^{0}\frac{e^{i\theta}}{e^{i\theta}}\,d\theta-\frac{6i}{9}\int_{2\pi}^{0}\frac{e^{i\theta}}{e^{2i\theta}}\,d\theta$$

$$=i\int_{2\pi}^{0}d\theta-\frac{2i}{3}\int_{2\pi}^{0}e^{i\theta-2i\theta}\,d\theta$$

$$=i(0-2\pi)+\frac{2i}{3}\cdot\frac{1}{i}\left[e^{-i\theta}\right]_{2\pi}^{0}$$

$$=-2\pi i+\frac{2}{3}\left[e^{0}-e^{-2\pi i}\right].$$

$$=-2\pi i+\frac{2}{3}\left[1-(\cos-2\pi)+i\sin(-2\pi)\right].$$

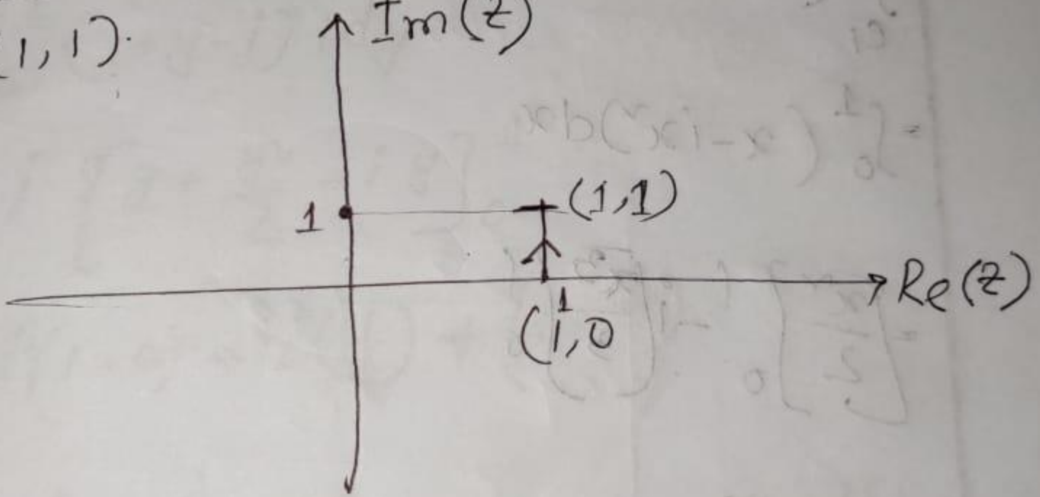
$$=-2\pi i$$

11. $\int_C (x+y-ix^2)\,dx$.

c is the shortest path ---- from $z=0$ to $z=1$. and then ------ $z=1$ to $z=1+i$

$z=1$ $(1,0)$

$z=1+i$ $(1,1)$.



$\int_C f(z)\,dz$

$= \int_{C_1} f(z)\,dz + \int_{C_2} f(z)\,dz$.

For C_1: $z=0$ to $z=1$

$(0,0)$ $(1,0)$

$z = x + iy$

$f(x) = x + y - ix^2$

$= x - ix^2$

$z = x$

$dz = dx$

$$\int_{C_1} (x - ix^2)\,dx$$

$$= \int_0^1 (x - ix^2)\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - i\left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{i}{3}$$

For C_2: $z = 1$ to $z = 1 + i$

$z = x + iy$

$z = 1 + iy$

$dz = 0 + i\,dy \Rightarrow dz = i\,dy$

Here,
on

(19)

$f(x) = x+y-ix^2$

$= 1+y-i$

$\int_{C_2} (1+y-i)\, i\, dy$

$= \int_0^1 (1+y-i)\, i\, dy$

$= i\left[y+\frac{y^2}{2}-iy\right]_0^1$

$= i(1-0)+i\left(\frac{1}{2}\right)+(1-0)$

$= i+\frac{i}{2}+1$

$= \frac{3i}{2}+1$

So, $\int_C (x+y-ix^2)\, dz$

$= \frac{1}{2}-\frac{i}{3}+\frac{3i}{2}+1$

$= \frac{3}{2}+\left(\frac{9i-2i}{6}\right)$

$= \frac{3}{2}+\frac{7i}{6}$ Ans.