Math#3

Mid Term Solution Question 2021-2022

(1) i) Evaluate $\mathcal{L}^{-1}\{5t^2 - e^{5i}\}$

Here, $\mathcal{L}^{-1}[f(s)]$ (t) is the inverse laplace transform of $f(s)$ with positive real variable t and i is the imaginary part

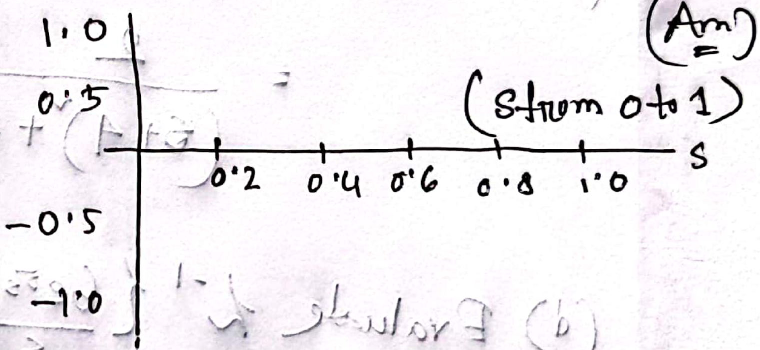
$\therefore$ The laplace of the given function is $5\delta''(s) - ie^{5}\delta(s)$
Here, (δ) is the dirac delta function.

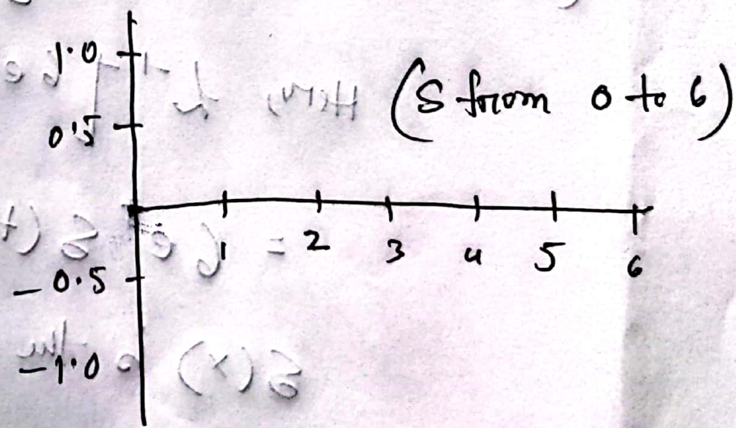
(Ans)

ii) Evaluate $\mathcal{L}^{-1}[5t^2 - e6t]$

$\therefore$ The laplace of the given function is $5\delta''(s) - e^{6}\delta'(s)$
Here (δ) is the dirac Delta function

(Ans)





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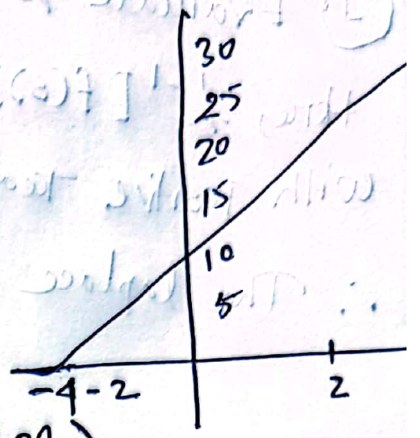
By Pracchad

(b) i) Evaluate $L(4\delta(t+3))$



$= \frac{d}{dt}\{4\delta(t+3)\}$

$= 4\delta\left(\frac{d}{dt}(3+t)\right)$

$= 4\delta \frac{d}{dt}(3) + \frac{d}{dt}(t)$

$\int 4\delta(t+3)\,dt = 4\delta\left(\frac{t^2}{2}+3t\right)$ (Ans)

(c) Evaluate $L\{e^{-4t}\sin 2t\}$

$= \frac{2}{(s+4)^2+4}$ (Ans)

(d) Evaluate $L^{-1}\left\{\frac{6e^{5s}}{s}\right\}$

Here, $L^{-1}\left[6e^{5} \times \frac{s}{s}\right]$

$= 6e^{5}\delta(t)$

$\delta(x)$ is the dirac delta funtion.

(Ans)

(e) Evaluate $\mathcal{L}\{(t+5)\,u(t-1)\}$, where $u(t-1)$ is the step function.

$\Rightarrow$ Here, $u(t-1)$ is the unit ~~of~~ step function.

$\therefore \mathcal{L}\{(t+5)\,u(t-1)$

$\Rightarrow Lt^2u + 4tuL - 5Lu$ (Ans)

(f) Evaluate $\mathcal{L}^{-1}\left\{5 + \dfrac{s}{s^2+9}\right\}$

$= \mathcal{L}^{-1}\left\{5 + \left(\dfrac{s}{s^2} + 9\right)\right\}$

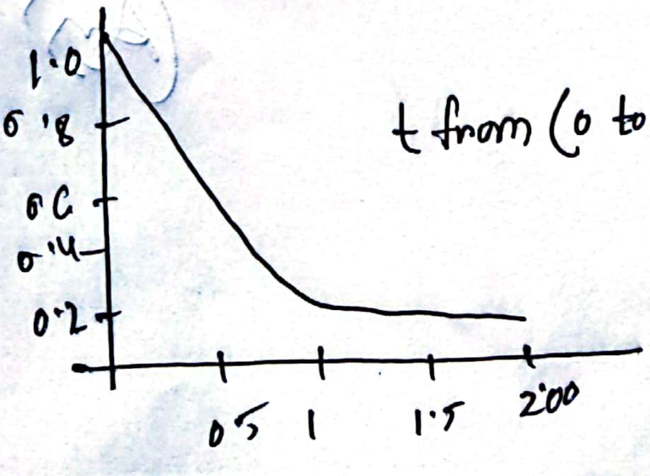
$= 14\,\delta(t) + 1$ (Ans)

(g) Evaluate $\mathcal{L}^{-1}\left\{\dfrac{s+1}{(s+1)^2+6}\right\}$

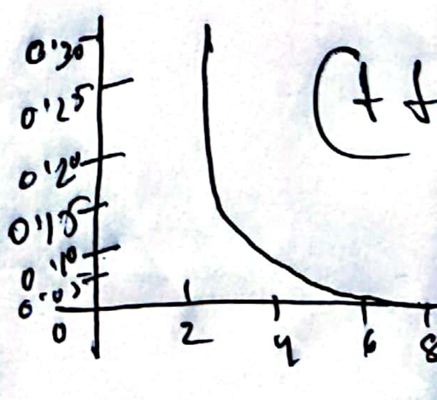
$= \mathcal{L}^{-1}\left[\dfrac{s+1}{(s+1)^2} + 6\right]$

$= 6\,\delta(t) + e^{-t}$

(Ans)



t from (0 to 2)



(t f

h) Evaluate $Re\{(2+i)(3-3i)\}$

Here, Complex Number,

$(2+i)(3-3i)$

$=(2+i)\times(3-3i)$

$= 2*3+2*(-3i)+i*3+i*(-3i)$

$=6-6i+3i-3i^2$

$=6-6i+3i+3$

$=6+3+i(-6+3)$

$=9-3i$

Rectangular form : $Z=9$

Angle notation $Z=9\angle 0^\circ$

Polar form $= Z\times(\cos 0^\circ + i\sin 0^\circ)$

Exponential form $= Z = 9\times e^{i0}$

$= 9\times e^{i0}$

(Ans)

i) Find the rectangular form of $z = 2e^{i\pi/3}$

We know $z = a + bi$

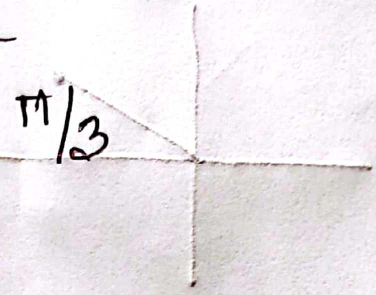
Given that $z = 2e^{i\pi/3}$

$r = 2$ & $\theta = \pi/3$

$a = r\cos\theta$

$= 2\cos \pi/3$

$= 1$



$b = r\sin\theta$

$= 2\sin \pi/3$

$= 1.7320$

$\therefore z = a + bi$

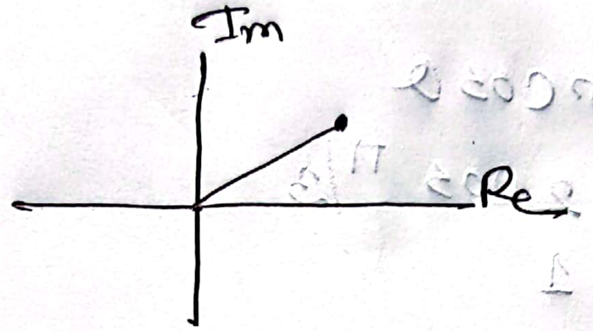
$= 1 + 1.7320 i$ (Ans)

J) Find the polar form of $z = 2+2\sqrt{3}i$

Here,

$r = 2\sqrt{4+\sqrt{6}}$ & $\theta = \tan^{-1}(3-\sqrt{6})$

$\therefore$ The polar form is $= 2\sqrt{4+\sqrt{6}}\, e^{i\tan^{-1}(}$



(Ans)

K) Find the Principal Arguement of $z = (\sqrt{}$

Here, $r = \sqrt{3}$

$\theta = -\frac{\pi}{12}$

$z = \sqrt{3}\, e^{\frac{1}{12} i (-1)\pi}$

$= \sqrt{3}\, e^{-i\pi/12}$

(Ans)

i) Identify the complex equation $z + i = 4$

The solution of the equation

$z + i = 4$ is $[4 - i)$

$\therefore$ the complex equation of $z + i = 4$ is

$[4 - i]$

(Ans)

ii) Identify the complex equation $|z + i| = 4$

The solution of the equation is

$[4 - i ; -(4 + i)]$

(Ans)