

Z-Transform

$$\textcircled{*} \quad x[n] = 3 \cdot 2^n \cdot u[n]$$

$$Z\{x[n]\} = 3 \cdot \frac{1}{(1 - 2z^{-1})}$$

$$\textcircled{*} \quad x[n] = -3^n u[n]$$

$$= -\frac{1}{1 - 3z^{-1}}$$

$$\textcircled{*} \quad x[n] = (-5)^n u[n]$$

$$= \frac{1}{1 - (-5)^n}$$

$$\textcircled{*} \quad x[n] = (-2)^n u[-n-1]$$

$$x(z) = \frac{1}{1 - (-2)^{z-1}}$$

$$\textcircled{*} \quad x(z) = \frac{(z+1)}{1 + 1/3^{z-1}} \quad |z| > 1/3$$

$$x[n] = \left(-\frac{1}{3}\right)^n u[n]$$

$$\textcircled{*} \quad x[-z] = z^5$$

$$x[n] = 5(n+5)$$

$$\textcircled{*} \quad x(z) = z^{-1}$$

$$\Rightarrow x[n] = 5(n-1)$$

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By Pracchad

$$\textcircled{*} \quad x[n] = e^{-nat} u[n]$$

$$x(z) = \sum_{n=-\infty}^{\infty} e^{-nat} u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} 0^{e^{-nat}} z^{-n}$$

$$= \sum_{n=0}^{\infty} (e^{at} z)^{-n}$$

$$= \frac{1}{1 - e^{at} z^{-1}}$$

$$= \frac{z}{z - e^{-at}}$$

$$\textcircled{*} \quad x[n] = \cos(\omega_0 n) u[n]$$

$$x[n] = \frac{1}{2} (e^{i\omega_0})^n u[n] + \frac{1}{2} (e^{-i\omega_0}) u[n]$$

$$= \frac{1}{2} (e^{i\omega_0})^n u[n] \xrightarrow{z} \frac{1/2}{1 - e^{i\omega_0} z^{-1}} \quad |z| > 1$$

$$= \frac{1}{2} (e^{-i\omega_0})^n u[n] \xrightarrow{z} \frac{1/2}{1 - e^{-i\omega_0} z^{-1}} \quad |z| > 1$$

$$\textcircled{*} \quad x[n] = \sin(\omega_0 n) u[n]$$

$$\Rightarrow s[n] = \begin{cases} (1+z)^{-n} & n=0 \\ 0 & n \neq 0 \end{cases}$$

$$\textcircled{*} \quad \cancel{x(n)} \in \{s[n]\}$$

$$x(z) = \sum_{n=-\infty}^{\infty} s[n] z^{-n} = (1) z^0 = 1.$$

Exercise 8.1

①

$$(a) \boxed{S[n] \rightarrow 1}$$

Now,

$$\begin{aligned} Z\{S[n]\} &= \sum_{n=-\infty}^{\infty} S[n] \cdot z^{-n} \\ &= S[0] \cdot z^0 \\ &= 1 \quad (\text{Ans}) \end{aligned}$$

$$(b) \boxed{S[-n-m] \rightarrow z^{-m}}$$

$$\begin{aligned} Z\{S[-n-m]\} &= \sum_{n=-\infty}^{\infty} S[-n-m] \cdot z^{-m} \\ &= S[m] \cdot z^{-m} \\ &= S[0] \cdot z^{-m} \quad (\text{writing } \frac{1}{z} = z^{-1}) \\ &= 1 \cdot z^{-m} \\ &= z^{-m} \quad (\text{Ans}) \end{aligned}$$

$$(c) na^n u[n] \leftrightarrow \frac{az}{(z-a)^2} \quad (\text{writing } \frac{1}{z-a} = [n]x) \quad (*)$$

$$Z\{na^n u[n]\} \leftrightarrow \frac{az(z+a)}{(z-a)^3} = [n]z^2 \quad (=)$$

$$\Rightarrow \{[n]z^2\} z - \text{Ans} \quad (*)$$

$$\Delta = \sum_{n=0}^{\infty} n^2 z^n = -S[n]z^2 \sum_{n=0}^{\infty} z^n = (-z)^2 x$$

② $\boxed{n^m u[n] \leftrightarrow \frac{a^z}{(z-a)^m}}$
 $z \{ n^m u[n] \} = -z \cdot \frac{d}{dz} \left(\frac{z}{z-a} \right)$
 $= -z \left[\frac{z-a-z}{(z-a)^2} \right]$
 $= \frac{a^z}{(z-a)^m} \quad (\text{Ans})$

③ $\boxed{n^m a^n u[n] \leftrightarrow \frac{az(z+a)}{(z-a)^{m+1}}}$
 $z \{ n^m a^n u[n] \} = -z \cdot \frac{d}{dz} \left\{ -z \cdot \frac{d}{dz} \left(\frac{z}{z-a} \right) \right\}$
 $= -z \cdot \frac{d}{dz} \left\{ -z \left[\frac{z-a-z}{(z-a)^2} \right] \right\}$
 $= z \cdot \frac{d}{dz} \left(\frac{az^2 + a^2 z + 2az}{(z-a)^3} \right)$
 $= \frac{az(z+a+2z)}{(z-a)^3}$
 $= \frac{az(z+a)}{(z-a)^3}$

$$(d) \boxed{x(z) = \frac{1 + 1/3z^{-1}}{1 - 1/3z^{-2}} ; |z| > \frac{1}{3}}$$

$$= \frac{z^2 + 1/3z}{z^2 - 1/3}$$

$$= \frac{z(z + 1/3)}{(z + 1/3)(z - 1/3)}$$

$$= \frac{z}{(z - 1/3)}$$

inverse z transform

$$\therefore x[n] = \left(\frac{1}{3}\right)^n u[n] \quad (\text{Ans})$$

$$(e) \boxed{x(z) = \frac{1 - az^{-1}}{z^{-1} - a}, |z| > |a|}$$

$$x(z) = \frac{z - a}{1 - az} \quad \frac{\overline{z}}{\overline{z} - \overline{a}} = \frac{(\text{f})x}{z}$$

$$\frac{x(z)}{z} = \frac{z - a}{-a(z - 1/a)} \quad \frac{\overline{z}}{\overline{z} - \overline{1/a}} = \frac{(\text{f})x}{z}$$

$$= -\frac{1}{a} \left[\frac{A}{z - \frac{1}{a}} + \frac{B}{z} \right]$$

$$(\text{f}): A = \frac{(2-a)(z-1/a)}{z(z-1/a)} - \int_{\gamma} e^{zt} \frac{1}{z} dz = [n]x$$

$$= \frac{\frac{1}{a} - a}{\frac{1}{a}}$$

$$= \frac{1-a^2}{a} \times \frac{a}{1}$$

$$= 1-a^2$$

$$\therefore B = \left[\frac{(z-a) \neq}{z(z-\frac{1}{a})} \right] z=0$$

$$= \frac{-a}{-1/a} = a \times a = a^2$$

$$\therefore \frac{x(z)}{z} = \frac{1}{-a} \left[\frac{1-a^2}{z-\frac{1}{a}} + \frac{a^2}{z} \right] \frac{1}{(1-s)(1+s)}$$

$$\Rightarrow x(z) = \left(\frac{1-a^2}{-a} \right) \cdot \left(\frac{z}{z-\frac{1}{a}} \right) + \frac{a^2}{-a} \cdot \frac{1}{(s-1)}$$

$$\Rightarrow x(z) = \left(\frac{1-a^2}{-a} \right) \frac{z}{z-\frac{1}{a}(1-s)(1+s)} \Big| z=0$$

inverse Z-transformation

$$x[n] = \left(\frac{1-a^2}{-a} \right) \left(\frac{1}{a} \right)^n u[n] - a^2 s^n$$

$$= -a s^n - \left(\frac{1-a^2}{a} \right) \left(\frac{1}{a} \right)^n u[n]$$

more

$$x[n] = \frac{1}{a} \left(\sum_{k=0}^{n-1} (-1)^k \right) \frac{1}{(1-s)(1+s)}$$

(Ans)

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$$\frac{A}{s(1-s)} + \frac{B}{1+s} = \frac{(s)X}{s^2+1}$$

Inverse Z transformation

$$Z^{-1} \left[X(z) = A \left[3z^{-1} \left(\frac{1}{1-0.75z^{-1}} \right) \right]^{-2} z^{-1} \left(\frac{1}{1-0.5z^{-1}} \right)^n \right] = 2 (0.75)^n u[n]$$

(C)
$$\boxed{X(z) = \frac{z^{-1}}{(1+0.75z^{-1})(1-0.75z^{-1})^n}}$$

$$\frac{z^{-1}}{(1+0.75z^{-1})(1-0.75z^{-1})^n} = \frac{A(1-0.75z^{-1})}{(1+0.75z^{-1})} + \frac{B(1-0.75z^{-1})}{(1-0.75z^{-1})^2} + \frac{C(1-0.75z^{-1})}{(1-0.75z^{-1})^3}$$
$$\Rightarrow z^{-1} = A(1-0.75z^{-1})^n + B(1-0.75z^{-1})^{n-1} + C(1-0.75z^{-1})^{n-2}$$
$$\Rightarrow z^{-1} = A \left\{ \frac{1-2.75z^{-1}}{(1-0.75z^{-1})} + (0.75z^{-1})^2 \right\} + B \left\{ \frac{1-2.75z^{-1}}{(1-0.75z^{-1})^2} + (0.75z^{-1})^3 \right\} + C \left\{ \frac{1-2.75z^{-1}}{(1-0.75z^{-1})^3} + (0.75z^{-1})^4 \right\}$$

Put $z^{-1} = \frac{1}{2f \cdot 0.75}$

$$\Rightarrow \frac{1}{0.75} = C(1+1)$$

$$\Rightarrow 2C = \frac{1}{0.75}$$

$$\frac{1}{(1-0.75)} + \frac{1}{(1-0.75)^2} = \frac{1}{0.75}$$

Put $z^{-1} = \frac{1}{0.75}$

$$\Rightarrow -\frac{1}{0.75} = A(2) + 0 + 0$$

$$-\frac{1}{0.75} = -\frac{1}{3}$$

From equation ⑪

$$A+B+C=0$$

$$\Rightarrow B = -(A+C)$$

$$= -\left(-\frac{1}{3} + \frac{2}{3}\right)$$

$$= -\left(\frac{-1+2}{3}\right) = \frac{1}{3}$$

$$\therefore X(z) = -\frac{1}{3} \cdot \frac{1}{(1+0.75z^{-1})} - \frac{1}{3} \cdot \frac{1}{(1-0.75z^{-1})} + \dots$$

Inverse Z transform formation

$$x[n] = -\frac{1/3(-0.75)^n u[n] - 1/3^2 \left[\frac{1}{1-0.75z^{-1}} \right] + \frac{2}{3} [0.75]^n u[n]}{(1-0.75z^{-1})(1-0.5z^{-1})}$$

$$= \frac{1}{(-0.75z^{-1}) (1-0.5z^{-1})} = \frac{B}{(1-0.75z^{-1})} + \frac{A}{(1-0.5z^{-1})}$$

$$\Rightarrow 1 = A(-0.5)^{-n} + B(-0.75)^{-n}$$

$$\Rightarrow 1 = A \cdot \frac{1}{(-0.5)} + B \cdot \frac{1}{(-0.75)}$$

$$z^{-1} = \frac{1}{0.75}$$

$$z^{-1} = \frac{1}{0.5}$$

$$\Rightarrow B = \frac{1}{3}$$

$$\Rightarrow A = 3$$

$$\therefore A = 3$$

$$\therefore 2 = \frac{1}{0.5}$$

$$\therefore 3 = \frac{1}{0.75}$$

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$$(e) \boxed{(n+1) u[n] \leftrightarrow \frac{z}{(z-1)^2}}$$

$$= n+1 u[n]$$

$$= nu[n] + u[n]$$

$$z \{ [nu[n]] + u[n] \} =$$

$$= -z \cdot \frac{d}{dz} \left(\frac{z}{z-1} \right) + \frac{z}{z(z-1)^2} \quad \rightarrow [alpha form] \textcircled{1}$$

$$= -z \cdot \left[\frac{(z-1)-z}{(z-1)^2} \right] + \frac{z}{z-1}$$

$$= -z \cdot \left[\frac{-1}{(z-1)^2} \right] + \frac{z}{z-1}$$

$$= \frac{z}{(z-1)^2} + \frac{z}{(z-1)} \quad \frac{b}{z^2 b} \cdot \frac{1}{z} =$$

$$= \frac{z+z(z-1)}{(z-1)^2}$$

$$= \frac{z+z-z}{(z-1)^2}$$

$$= \frac{z}{(z-1)^2} \quad (\cancel{z})$$

Determine the sequence $x[n]$ with z -transform

and hence sketch $x[n]$

$$X(z) = (1+2z^{-1}) (1+3z^{-1}) (1-z^{-1})$$

$$x(z) = (1+2z^{-1} + 3z^{-2} + 6z^{-3}) (1-z^{-1})$$

$$x(z) = (1+2z^{-1} + 3z^{-2} + 6z^{-3} - z^{-1} - 2z^{-2} - 3z^{-3} - 6z^{-4})$$

$$= (1+2z^{-1} + 3z^{-2} + 6z^{-3} - z^{-1} - 2z^{-2} - 3z^{-3} + 2z^{-4})$$

$$= (5 - 4z^{-1} - 3z^{-2} + 2z^{-4})$$

inverse of z transformation

$$z^{-1} \{ X(z) \} = 5s[n] - 4s[n-1] - 3s[n-2] + 2s[n-4]$$

$$x[n] = 5s[n] + 2s[n-1] - 4s[n-1] - 3s[n-2]$$

Equivalently,

$$x[n] = \begin{cases} 2 & n=1 \\ 5 & n=0 \\ -1 & n=-1 \\ -3 & n=2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow \frac{(s+1)}{(s+1-5)} = \frac{(s+1)}{(s+1-5)(s+1+5)}$$

$$\frac{1}{s+1-5} = \frac{A}{s+5} + \frac{B}{s-5} \quad \frac{(s+1)X}{(s+1-5)(s+1+5)} = \frac{(s+1)X}{s^2-25}$$

3.

$$(a) \boxed{x(z) = \frac{1}{1 + 1/4z^{-1}}, |z| > 1/4}$$

$$\therefore x[n] = (\frac{1}{4})^n u[n]$$

An2

$$(b) \boxed{x(z) = \frac{1}{1 + 1/4z^{-1}}, |z| < 1/4}$$

$$x[n] = -(-1/4)^n u[-n-1]$$

An3

$$(c) \boxed{x(z) = \frac{1/2 z^{-1}}{1 + 2/5 z^{-1} - 1/20 z^{-2}}, |z| > 1/2}$$

$$= \frac{z^{-1}/2}{z^2 + 2/5 z - 1/20}$$

$$= \frac{z^{-1}/2}{z^2 + 2/5 z - 1/20}$$

$$= \frac{z(z - 1/2)}{z(z + 1/2) - 1/20(z - 1/2)}$$

$$= \frac{z(z - 1/2)}{(z + 1/2)(z - 1/20)}$$

$$\Rightarrow \frac{x(z)}{z} = \frac{A}{z + 1/2} + \frac{B}{z - 1/20}$$

$$A = \left[\frac{(z - 1/2)(z + 1/2)}{(z + 1/2)(z - 1/10)} \right]_{z=1/2}$$

$$= \frac{-1/2 - 1/2}{-1/2 - 1/10} = \frac{5}{3}$$

$$B = \left[\frac{(z - 1/2)(z - 1/10)}{(z + 1/2)(z - 1/10)} \right]_{z=1/10}$$

$$= \frac{\frac{1/10 - 1/2}{1/10 + 1/2}}{\frac{2}{5}} = \frac{-2}{3/5}$$

$$= -\frac{2}{3} \times \frac{5}{3} = -\frac{10}{9}$$

$$\therefore \frac{x(z)}{z} = \frac{\frac{5}{3}z}{z + 1/2} = \frac{\frac{2}{3}z}{z - 1/10} = (\frac{2}{3})z$$

$$\therefore x(z) = \frac{\frac{5}{3}z}{z + 1/2} = \frac{\frac{2}{3}z}{z - 1/10}$$

inverse

$$x[n] = \frac{5}{3} \left[(-1/2)^n u[n] - \frac{2}{3} \left(\frac{1}{10} \right)^n u[n] \right]$$

(Ans)

$$X(z) = \frac{10z^2}{(1+z^{-1})(1-z^{-1})^2} ; |z| > 1$$

$$X(z) = \frac{10z^2}{(z+1)(z-1)^2}$$

$$\frac{X(z)}{10z^2} = \frac{A}{(z+1)} + \frac{B}{(z-1)} + \frac{C}{(z-1)^2}$$

$$A \left[\frac{(z+1)}{(z+1)(z-1)^2} \right]_{z=1} = -1$$

$$= \frac{1}{(-2)^2} = \frac{1}{4} + \left(\frac{1}{-2} \right) \cdot \left(\frac{1}{-2-1} \right) = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

$$B = \left[\frac{(z-1)}{(z+1)(z-1)^2} \right]_{z=1} = \frac{1}{(-2)^2} = \frac{1}{4}$$

$$= \left[\frac{1}{(z+1)(z-1)} \right]_{z=1} = \frac{1}{(-2)(-2-1)} = \frac{1}{6}$$

$$C = \left[\frac{(z-1)}{(z+1)(z-1)^2} \right]_{z=1} = \left[\frac{1}{(z+1)} \right]_{z=1} = \frac{1}{(-2)} = -\frac{1}{2}$$

$$\frac{X(z)}{10z^2} = \frac{1/4}{z+1} + \frac{1/2}{(z-1)^2}$$

$$x(z) = \frac{5}{2} z^{-2} (z+1) + 5 z^{-2} (z-1)$$

$$= 5/2 (z^{-1} + 1/z) + 5 (1 - \frac{1}{z})$$

$$x(z) = \frac{5}{2} z^{-1} + \frac{5}{2} \cdot \frac{1}{z} + 5 (1 - \frac{1}{z})$$

inverse of z transform

$$x[n] = \frac{5}{2} (-1)^n u[n] + 5 n u[n]$$

(Ans)

(b)

$$x(z) = \frac{1}{(1+0.5z^{-1})(1-0.5z^{-1})(1-z^{-1})} \rightarrow \frac{1}{z-1}$$

$$x(z) = \frac{z^3}{(z+1/2)(z-1/2)(z-1)}$$

$$\frac{x(z)}{z} = \frac{z^2}{(z+1/2)(z-1/2)(z-1)} \rightarrow \frac{(z+1/2)(z-1/2)}{(z+1/2)(z-1/2)(z-1)} = 1$$

$$= \frac{A}{z+1/2} + \frac{B}{z-1/2} + \frac{C}{z-1}$$

$$A = \left[\frac{z^2(z+1/2)}{(z+1/2)(z-1/2)(z-1)} \right]_{z=-1/2} = \frac{1}{6}$$

$$B = \left[\frac{z^2}{(z+1/2)(z-1)} \right]_{z=1/2} \Rightarrow \frac{1}{9} = A = -\frac{1}{2}$$

$$1 = \left[\frac{z^n}{(z+1/2)(z-1/2)} \right]_{z=1} = \frac{(1+1)^n - (1-1)^n}{(1+1/2)(1-1/2)} = \frac{2^n}{3/4} = \frac{8}{3}$$

$$\frac{x(z)}{z} = -\frac{\frac{1}{10}(z-1)}{(z+1/2)} - \frac{\frac{1}{2}}{(z-1/2)} + \frac{\frac{4}{3}}{z-1}$$

$$\therefore x(z) = -\frac{1}{6}z(z+1/2)^{-1} - \frac{1}{2}z(z-1/2)^{-1} + \frac{4}{3}z(z-1)^{-1}$$

Inverse Z transform

$$x[n] = \frac{1}{6}(-1/2)^n u[n] - \frac{1}{2}(1/2)^n u[n] + \frac{4}{3}u[n]$$

$$= \frac{1}{6}(-0.5)^n u[n] - \frac{1}{2}(0.5)^n u[n] + \frac{4}{3}u[n]$$

$$(e) \boxed{x(z) = \frac{1}{(1-z^{-1})(1-0.75z^{-1})}, |z| > 1}$$

$$\Rightarrow x(z) = \frac{z^n}{(z-1)(z-0.75)}$$

$$\Rightarrow \frac{x(z)}{z-0.75} = \frac{z}{(z-1)(z-0.75)}$$

$$= \frac{A}{z-1} + \frac{B}{(z-0.75)(z-1)} = \frac{A}{z-1} + \frac{B}{(z-0.75)(z-1)} = A$$

$$H(z) = \frac{z}{(z-1)(z-0.75)} \quad z=1 \Rightarrow \frac{1}{(1-0.75)(1-1)} = \frac{1}{0} \text{ (undefined)}$$

$$A = \frac{1}{(1-0.75)} = \frac{1}{0.25} = 4$$

$$B = \frac{1}{(1-0.75)} = \frac{1}{0.25} = 4$$

$$\text{so, } \frac{x(z)}{z} = \frac{4}{z-1} - \frac{3}{z-0.75}$$

$$x(z) = 4\left(\frac{z}{z-1}\right) - 3\left(\frac{2}{z-0.75}\right)$$

inverse z transform

$$x[n] = 4u[n] - 3(0.75)^n u[n]$$

$$x[n] = 4u[n] - 3(0.75)^n u[n] \quad (\text{Ans})$$

$$(d) \boxed{x(z) = \frac{A}{(1-0.75z^{-1})(1-0.5z^{-1})}}$$

$$\frac{x(z)}{A} = \frac{1}{(1-0.75z^{-1})(1-0.5z^{-1})}$$

$$\frac{x(z)}{(1-0.75z^{-1})(1-0.5z^{-1})} = \frac{A}{(1-0.75z^{-1})} + \frac{B}{(1-0.5z^{-1})}$$

$$\Rightarrow 1 = A(1-0.5z^{-1}) + B(1-0.75z^{-1})$$

$$z^{-1} = \frac{1}{0.5}$$

$$\Rightarrow 1 = B\left(1 - \frac{0.75}{0.5}\right)$$

$$\Rightarrow B = \frac{1}{-0.5}$$

$$\therefore B = -2$$

$$3 = \frac{1}{(z-1/2)(z-1)} \cdot \frac{1}{z-0.5} + \frac{1}{z-1} \cdot \frac{1}{z-1}$$

$$z^{-1} = \frac{1}{0.5} = 2$$

$$1+1 \Rightarrow 1 = \frac{A}{3}$$

$$\therefore A = 3$$

$$\therefore \frac{x(z)}{A} = \frac{3}{(1-0.75z^{-1})} + \frac{1}{(1-0.5z^{-1})}$$