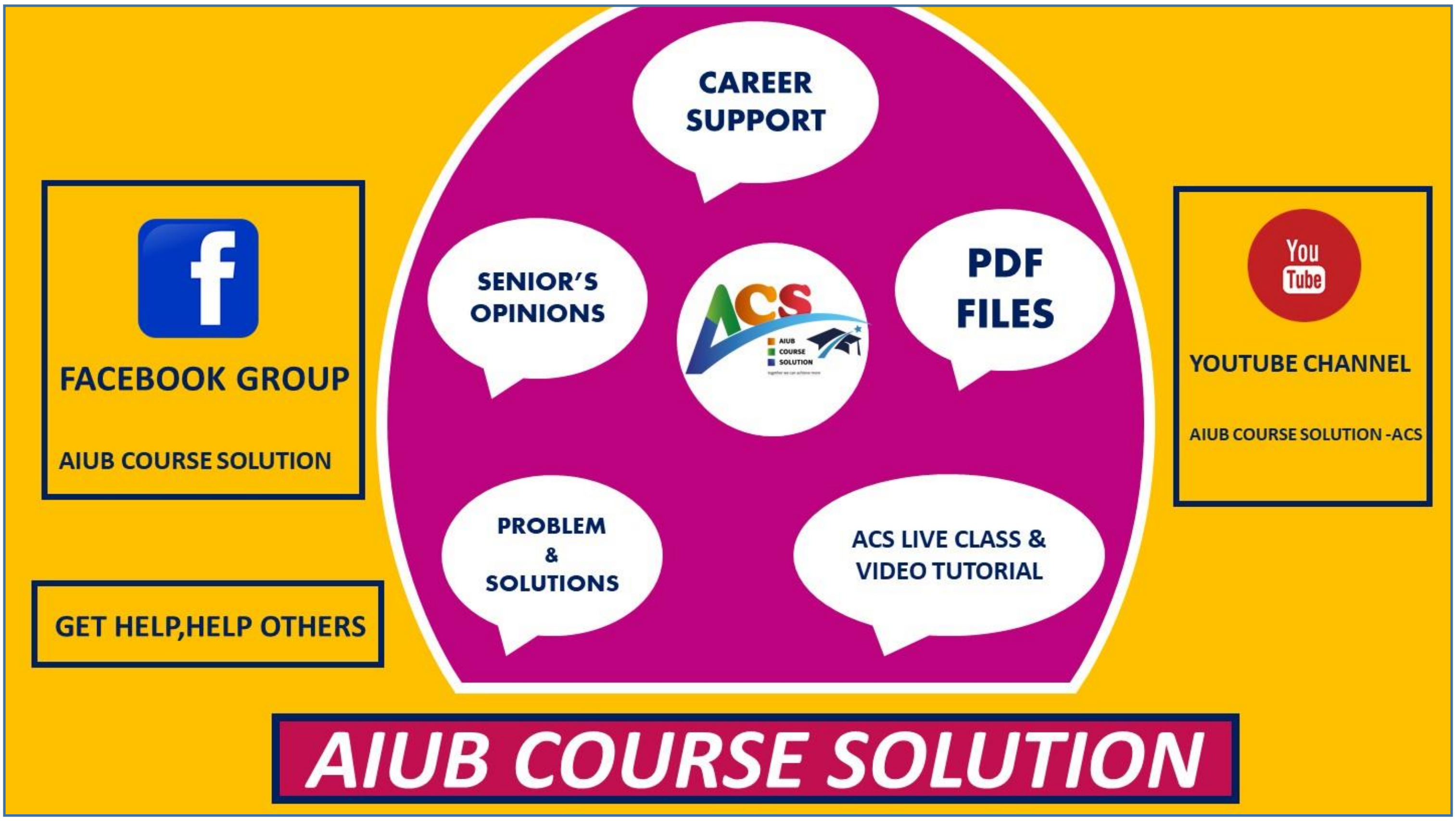


TOGETHER WE CAN ACHIEVE MORE

COURSE NAME: Math-3

CHAPTER: 7.1

SOLVED BY

NAME: Rupa Paul

AIUB COURSE SOLUTION-ACS

LINK = https://www.youtube.com/channel/UCC3KjA8kstFtM-2CxVr-jcg

AIUB COURSE SOLUTION

LINK= https://www.facebook.com/groups/aiubcoursesolution/

Chapter-07 (01)

Exercise-7.1

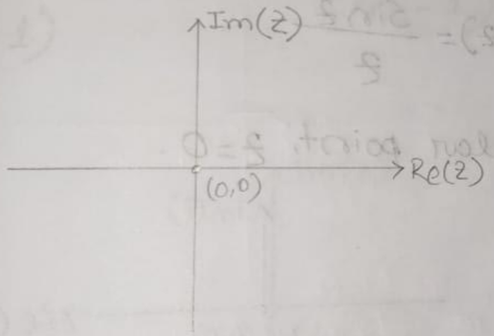
1. (i) Singular points of the function,

$f(z) = \frac{1}{2z}$

For singularity,

$2z = 0$.

$\therefore z = 0$.



$f(z) = \frac{1}{z^2-4}$.

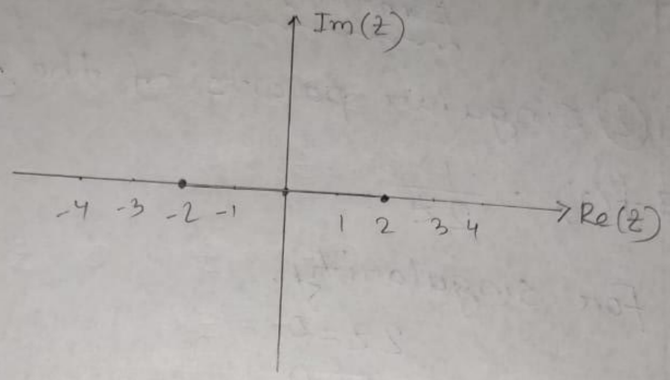
$z^2 - 4 = 0$

$\Rightarrow (z)^2 - (2)^2 = 0$

$\Rightarrow (z+2)(z-2) = 0$

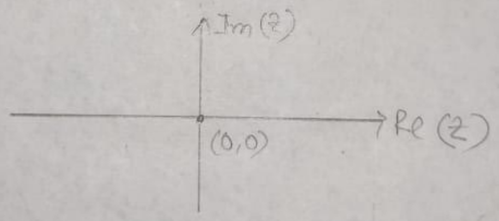
$\therefore z = 2, -2$

$(2,0), (-2,0)$



$$f(z) = \frac{\sin z}{z}$$

Singular point, $z=0$.



$$f(z) = \frac{1}{z^6+1}$$

For singularity $z^6+1=0$.

03

$$\Rightarrow (z^3)^2 - (i)^2 = 0$$

$$\Rightarrow (z^3+i)(z^3-i) = 0$$

$z^3 + i = 0$

$\Rightarrow z^3 = -i$

$\Rightarrow z^3 = i^3$

$\therefore z = i$

$(0, 1)$

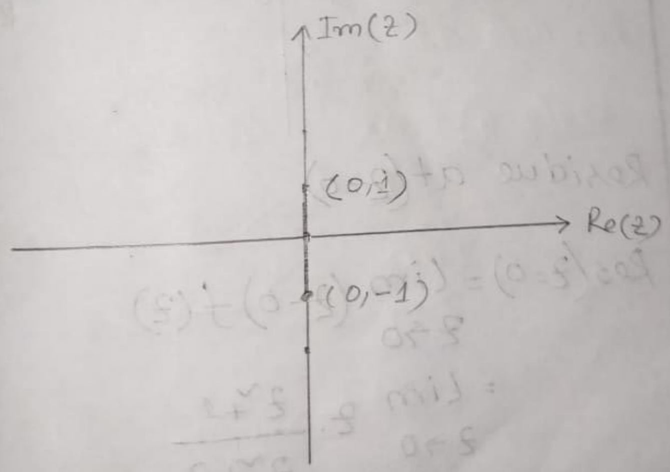
$z^3 - i = 0$

$\Rightarrow z^3 = i$

$\Rightarrow z^3 = -(i)^3$

$\therefore z = -i$

$(0, -1)$



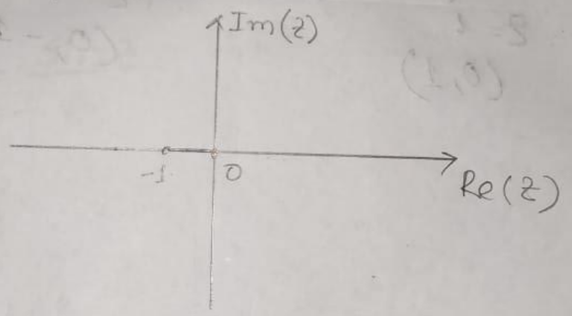
(ii) $f(z) = \dfrac{z^2+1}{z^2+z}$

For Singularity, $z^2+z=0$.

$\Rightarrow z(z+1)=0$.

So, $z=0$, $z+1=0$

$\Rightarrow z=-1$



Residue at $(z=0)$

$$\text{Res}(z=0) = \lim_{z\to 0} (z-0) f(z)$$

$$= \lim_{z\to 0} z \cdot \frac{z^2+1}{z^2+z}$$

$$= \lim_{z\to 0} z \cdot \frac{z^2+1}{z(z+1)}$$

$$= \lim_{z \to 0} \frac{z^2+1}{z+1}$$

$$= \frac{0+1}{0+1} = 1.$$

Residue at $(z=-1)$.

$$Res(z=-1) = \lim_{z \to -1} (z+1) f(z)$$

$$= \lim_{z \to -1} (z+1) \frac{z^2+1}{z^2+z}$$

$$= \lim_{z \to -1} (\cancel{z+1}) \frac{z^2+1}{z(\cancel{z+1})}$$

$$= \lim_{z \to -1} \frac{z^2+1}{z}$$

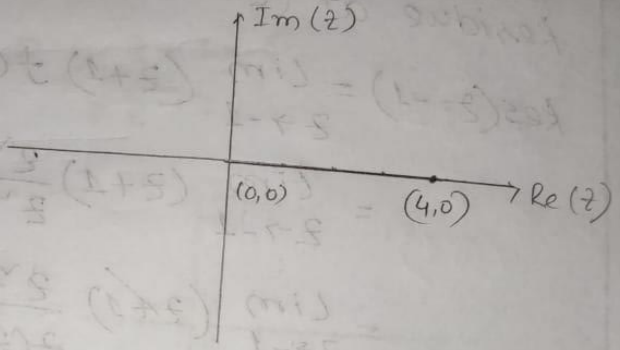
$$= \frac{(-1)^2+1}{-1} = \frac{2}{-1} = -2.$$

B) $f(z) = \dfrac{z^2+2}{z-4}$

For singularity,

$$z-4=0$$

$$\therefore z=4$$



Residue at $(z=4)$.

$$\text{Res}(z=4) \lim_{z\to 4} (z-4) \cdot \frac{z^2+2}{(z-4)}$$

$$= \lim_{z\to 4} z^2+2$$

$$= 16+2$$

$$= 18.$$

07

$f(z) = \dfrac{1}{z^3+i}$

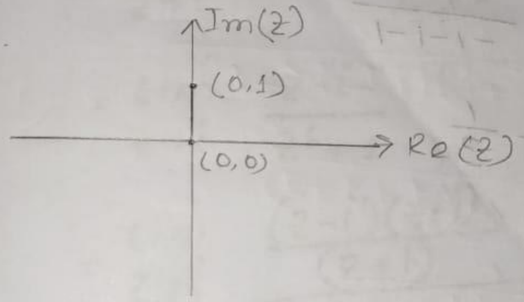
For singularity, $z^3+i=0$

$$\Rightarrow z^3 = -i$$

$$\Rightarrow z^3 = i^3$$

$$\therefore z = i$$

$(0, 1)$



Residue at $(z=i)$

$$\text{Res}(z=i) = \lim_{z\to i}\,(z-i)\,\frac{1}{z^3+i}$$

$$= \lim_{z\to i} \frac{z-i}{z^3+i}$$

$$= \lim_{z\to i} \frac{z-i}{(z)^3-(i)^3}$$

$$= \lim_{z \to i} \frac{\cancel{(z-i)}}{\cancel{(z-i)}(z^2+zi+i^2)}$$

$$= \lim_{z \to i} \frac{1}{z^2+iz+i^2}$$

$$= \frac{1}{i^2+i^2+i^2}$$

$$= \frac{1}{-1-1-1}$$

$$= -\frac{1}{3}$$

(09)

$f(z) = \frac{1}{z^6+1}$

For singularity, $z^6+1=0$.

From (i) we get that,

$z = i, \quad z = -i$

Residue at $(z=i)$

$$\text{Res}(z=i) = \lim_{z\to i} (z-i)\frac{1}{z^6+1}$$

$$= \lim_{z\to i} \frac{z-i}{z^6+1}$$

$$= \lim_{z\to i} \frac{\frac{(z-i)(z+i)}{(z+i)}}{z^6+1}$$

$$= \lim_{z\to i} \frac{\frac{z^2-i^2}{z+i}}{z^6+1}$$

$$= \lim_{z\to i} \frac{z^2+1}{(z+i)(z^6+1)}$$

$$= \lim_{z\to i} \frac{z^2+1}{(z+i)\{(z^2)^3+(1)^3\}}$$

(10)

$$= \lim_{z\to i} \frac{z^2+1}{(z+i)(z^2+1)(z^4-z^2+1)}$$

$$= \lim_{z\to i} \frac{\cancel{(z^2+1)}}{(z+i)\cancel{(z^2+1)}(z^4-z^2+1)}$$

$$= \lim_{z\to i} \frac{1}{(z+i)(z^4-z^2+1)}$$

$$= \frac{1}{(i+i)(i^4-i^2+1)}$$

$$= \frac{1}{2i(1+1+1)}$$

$$= \frac{1}{6i}$$

Residue at $(z=-i)$

$$\lim_{z\to -i} \frac{z+i}{z^6+1}$$

$$= \lim_{z\to -i} \frac{\frac{(z+i)(z-i)}{(z-i)}}{z^6+1}$$

$$= \lim_{z\to +i} \frac{z^2+1}{(z-i)(z^6+1)}$$

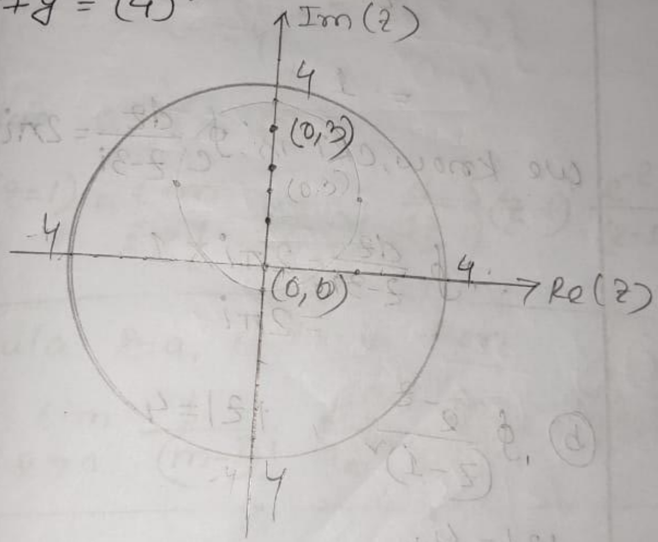
(11)

2. (a) $\oint_C \frac{dz}{z-3i}$, C is the circle $|z|=4$

$|z|=4$

$\Rightarrow |x+iy| = 4$

$\Rightarrow x^2+y^2 = (4)^2$



For Singularity,

$z-3i=0$

$\Rightarrow z = 3i$

$(0,3)$

(12)

Residue at $(z=3i)$

$$\operatorname{Res}(z=3i) = \lim_{z\to 3i} (z-3i) f(z)$$

$$= \lim_{z\to 3i} \cancel{(z-3i)} \frac{1}{\cancel{(z-3i)}}$$

$$= 1$$

We know, CRT is, $\oint_C \frac{dz}{z-3i} = 2\pi i \left[\operatorname{Res}(z=3i)\right]$

$$\therefore \oint \frac{dz}{z-3i} = 2\pi i \times 1$$
$$= 2\pi i$$

(b) $\oint \frac{e^{-z}}{(z-1)^2}$, $|z|=4$.

$|z| = 4$.

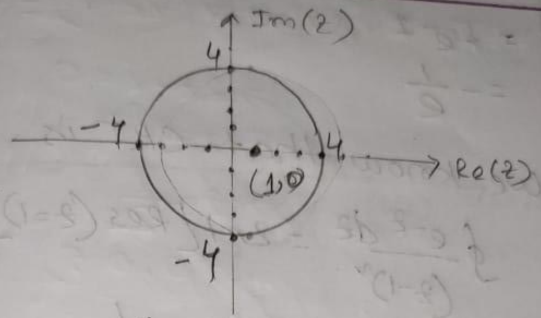
$\therefore x^2 + y^2 = (4)^2$

For singularity, $(z-1)^2 = 0$.

$\therefore z = 1$.

order $= 2$

(13)



Residue at $(z=1)$.

$$Res(z=1) = \lim_{z\to 1} \frac{1}{(2-1)!} \frac{d}{dz}\left\{(z-1)^2 \frac{e^{-z}}{(z-1)^2}\right\}.$$

Formula. $z=a$, order m then.

$$\lim_{z\to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}}\left\{(z-a)^m f(z)\right\}.$$

$$= \lim_{z\to 1} \frac{d}{dz}\left\{(z-1)^2 \cdot \frac{e^{-z}}{(z-1)^2}\right\}.$$

$$= \lim_{z\to 1} \frac{d}{dz}\left(e^{-z}\right)$$

$$= \lim_{z\to 1}\left[-e^{-z}\right]$$

$= -e^{-1}$

$= -\frac{1}{e}$

We know that CRT is,

$$\oint \frac{e^{-z}\,dz}{(z-1)^2} = 2\pi i\,[\text{Res}\,(z=1)].$$

$$= 2\pi i \times -\frac{1}{e}$$

$$= -\frac{2\pi i}{e}$$

(c) $\oint_C \frac{dz}{(z-6)^{10}}$, $|z|=4$.

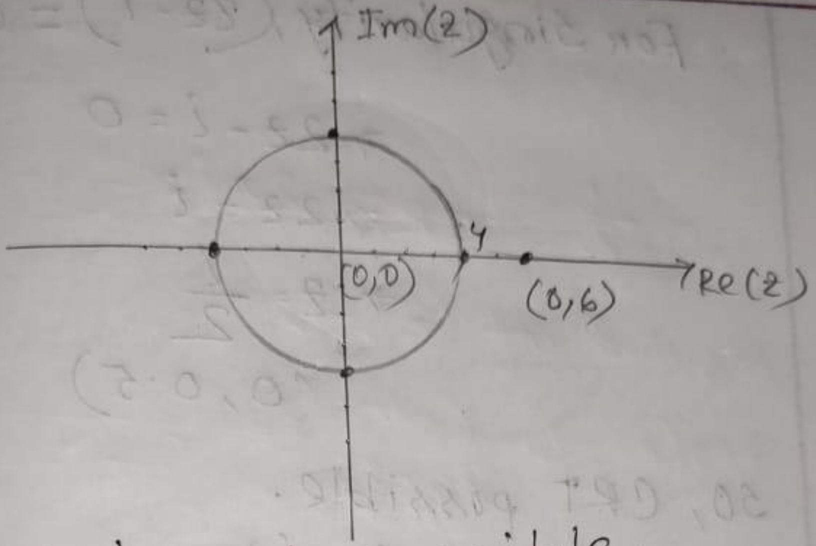
$|z|=4$.

$x^2+y^2=(4)^2$.

For singularity, $(z-6)^{10}=0$

$\Rightarrow z=6$.

15

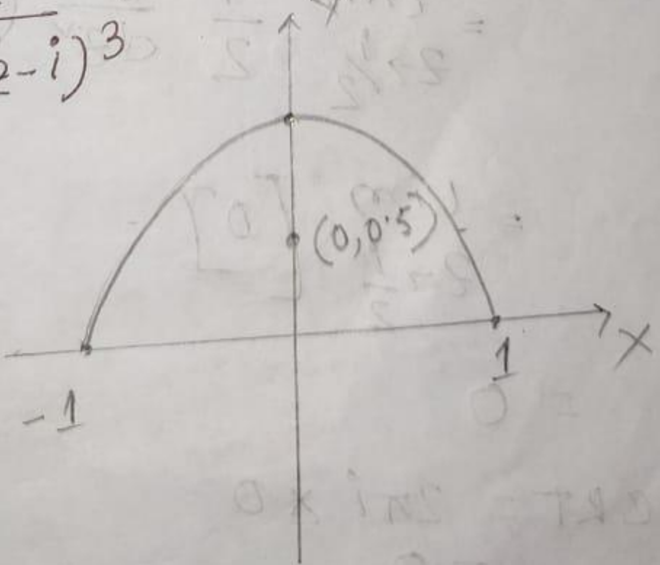


which is not possible.

As, $z=6$ lie outside C, so CRT is not possible.

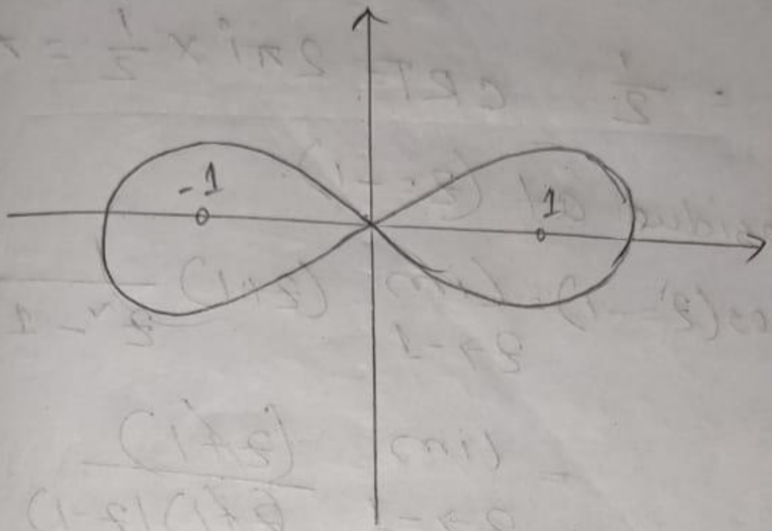
3(a)

(i) $\oint_C \frac{2z}{(2z-i)^3}$



17

(ii) $\oint \frac{dz}{z^2-1}$



For singularity, $z^2-1=0$.

$\Rightarrow (z)^2-(1)^2=0$

$\Rightarrow (z+1)(z-1)=0$.

$\therefore z=1,-1$.

So, CRT possible.

Residue at, $(z=1)$.

$$\text{Res}(z=1)=\lim_{z\to 1}(z-1)\frac{1}{z^2-1}$$

$$=\lim_{z\to 1}\frac{\cancel{(z-1)}}{(z+1)\cancel{(z-1)}}$$

(18)

$$= \lim_{z \to 1} \frac{1}{z+1}$$

$$= \frac{1}{2} \cdot \quad CRT = 2\pi i \times \frac{1}{2} = \pi i$$

Residue at $(z=-1)$.

$$Res(z=-1) = \lim_{z \to -1} (z+1) \frac{1}{z^2-1}$$

$$= \lim_{z \to -1} \frac{\cancel{(z+1)}}{\cancel{(z+1)}(z-1)}$$

$$= \lim_{z \to -1} \frac{1}{z-1}$$

$$= \frac{1}{-2}.$$

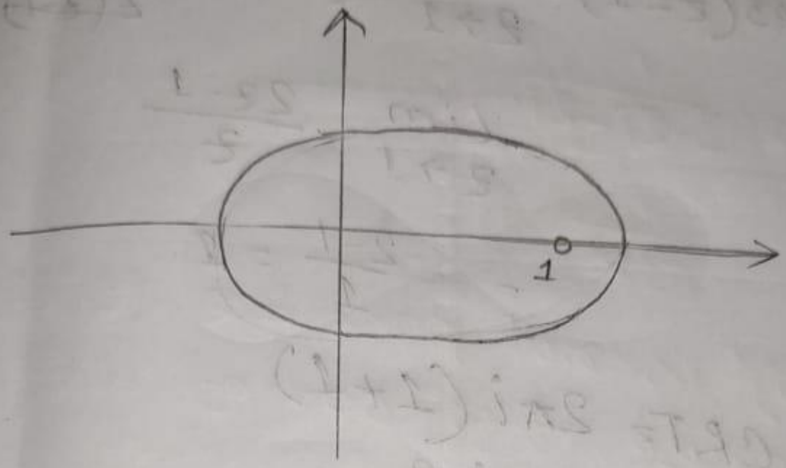
$$CRT = 2\pi i \times \left(-\frac{1}{2}\right)$$

$$= -\pi i$$

$$\therefore CRT = \pi i - \pi i$$

$$= 0.$$

(iii) $\oint \frac{2z-1}{z^2-z}\,dz$



For singularity, $z^2 - z = 0$

$$\Rightarrow z(z-1) = 0.$$

$$\therefore z = 0,\ z = 1.$$

So, CRT possible.

Residue at $(z=0)$

$$Res(z=0) = \lim_{z \to 0} \cancel{z} \cdot \frac{2z-1}{\cancel{z}(z-1)}$$

$$= \lim_{z \to 0} \frac{2z-1}{(z-1)}$$

$$= \frac{-1}{-1}$$

$$= 1$$

Residue at $(z=1)$

$$Res(z=1)=\lim_{z\to 1} \cancel{(z-1)}\cdot\frac{2z-1}{z\cancel{(z-1)}}$$

$$=\lim_{z\to 1}\frac{2z-1}{z}$$

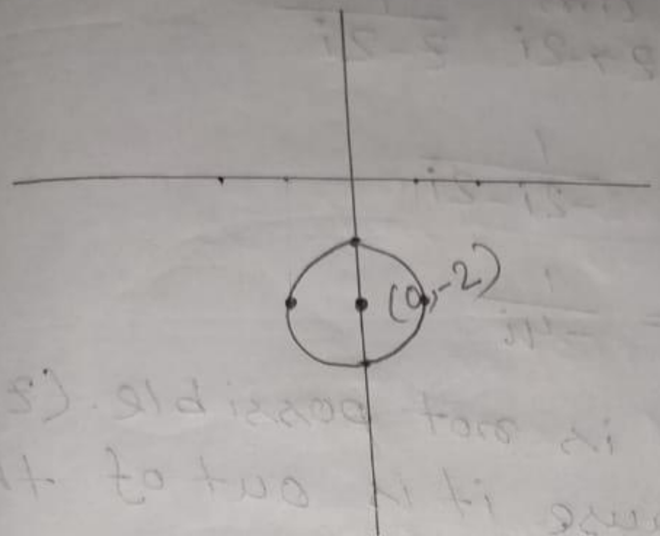
$$=\frac{2-1}{1}=1.$$

$$\therefore CRT = 2\pi i(1+1)$$
$$=2\pi i\cdot 2$$
$$=4\pi i$$

4. a) $\oint_C \frac{dz}{z^2+4}$ i) $|z+2i|=1$.

$$|z+2i|=1$$
$$\Rightarrow |x+iy+2i|=1$$
$$\Rightarrow \sqrt{(x)^2+(y+2)^2}=1$$
$$\Rightarrow (x)^2+(y+2)^2=(1)^2$$

$(0,-2)$.



For singularity, $z^2+4=0$

$$\Rightarrow z^2-(2i)^2=0$$

$$\Rightarrow (z+2i)(z-2i)=0$$

$\therefore z=-2i$, $(0,-2)$ | $z=2i$, $(0,2)$

Residue at $(z=-2i)$

$$\text{Res}(z=-2i)=\lim_{z\to -2i}(z+2i)\frac{1}{z^2+4}$$

$$=\lim_{z\to -2i}\cancel{(z+2i)}\frac{1}{\cancel{(z+2i)}(z-2i)}$$