

## Z –transformation

Formula:

$$Z\{a^n u[n]\} = \frac{1}{1 - a z^{-1}}$$

So,

$$Z^{-1}\left\{\frac{1}{1 - a z^{-1}}\right\} = a^n u[n]$$

Formula in difference equation:

$$Z\{x[n - 1]\} = z^{-1}[X[z] + x[-1]z]$$

$$Z\{x[n + 1]\} = z[X[z] - x[0]]$$

Problem: Find  $x[n]$  for  $X(z) = \frac{1}{(1-0.75 z^{-1})(1-0.5 z^{-1})}$

Solution:

$$\frac{1}{(1 - 0.75 z^{-1})(1 - 0.5 z^{-1})} = \frac{A}{1 - 0.75 z^{-1}} + \frac{B}{1 - 0.5 z^{-1}}$$

$$A = \frac{1}{1-0.5 \times \frac{4}{3}} = 3, B = \frac{1}{1-0.75 \times 2} = -2$$

So,

$$\begin{aligned}
 X(z) &= 3 \frac{1}{1 - 0.75 z^{-1}} - 2 \frac{1}{1 - 0.5 z^{-1}} \\
 \Rightarrow Z^{-1}\{X(z)\} &= 3 Z^{-1}\left\{\frac{1}{1 - 0.75 z^{-1}}\right\} - 2 Z^{-1}\left\{\frac{1}{1 - 0.5 z^{-1}}\right\} \\
 \Rightarrow x[n] &= 3 (0.75)^n u[n] - 2 (0.5)^n u[n]
 \end{aligned}$$

Problem:  $X(z) = \frac{1}{(1+0.5 z^{-1})(1-0.5 z^{-1})(1-z^{-1})}$

Solution:  $\frac{1}{(1+0.5 z^{-1})(1-0.5 z^{-1})(1-z^{-1})} = \frac{A}{1+0.5 z^{-1}} + \frac{B}{1-0.5 z^{-1}} + \frac{C}{1-z^{-1}}$

$$A = \frac{1}{(1 - 0.5 \times (-2)) (1 - (-2))} = \frac{1}{6}, B = \frac{1}{(1 + 0.5 \times 2) (1 - 2)} = -\frac{1}{2}$$

$$C = \frac{1}{(1 + 0.5) (1 - 0.5)} = \frac{4}{3}$$

$$X(z) = \frac{1}{6} \frac{1}{1 + 0.5 z^{-1}} - \frac{1}{2} \frac{1}{1 - 0.5 z^{-1}} + \frac{4}{3} \frac{1}{1 - z^{-1}}$$

$$\Rightarrow Z^{-1}\{X(z)\} = \frac{1}{6} Z^{-1}\left\{\frac{1}{1 - (-0.5) z^{-1}}\right\} - \frac{1}{2} Z^{-1}\left\{\frac{1}{1 - 0.5 z^{-1}}\right\} + \frac{4}{3} Z^{-1}\left\{\frac{1}{1 - z^{-1}}\right\}$$

$$\Rightarrow x[n] = \frac{1}{6} (-0.5)^n u[n] - \frac{1}{2} (0.5)^n u[n] + \frac{4}{3} (1)^n u[n]$$

H.W: Problem:  $X(z) = \frac{1}{(1+z^{-1})(1-0.75z^{-1})}$

Problem: Solve  $2y[n] - 3y[n-1] = 2^n u[n]; y[-1] = 2$

Solve:

$$2Z\{y[n]\} - 3Z\{y[n-1]\} = Z\{2^n u[n]\}$$

$$\Rightarrow 2Y(z) - 3z^{-1}[Y(z) + y[-1]z] = \frac{1}{1-2z^{-1}}$$

$$\Rightarrow 2Y(z) - 3z^{-1}Y(z) - 6 = \frac{1}{1-2z^{-1}}$$

$$\Rightarrow (2-3z^{-1})Y(z) = \frac{1}{1-2z^{-1}} + 6$$

$$\Rightarrow Y(z) = \frac{1}{(1-2z^{-1})(2-3z^{-1})} + 6 \frac{1}{2-3z^{-1}} \dots \dots \dots (i)$$

Now,

$$\frac{1}{(1-2z^{-1})(2-3z^{-1})} = \frac{A}{1-2z^{-1}} + \frac{B}{2-3z^{-1}}$$

$$A = \frac{1}{2 - 3 \times \frac{1}{2}} = 2, B = \frac{1}{1 - 2 \times \frac{2}{3}} = -3$$

Form equation (i),

$$Y(z) = 2 \frac{1}{1 - 2z^{-1}} - 3 \frac{1}{2 - 3z^{-1}} + 6 \frac{1}{2 - 3z^{-1}}$$

$$\Rightarrow Y(z) = 2 \frac{1}{1 - 2z^{-1}} + 3 \frac{1}{2 - 3z^{-1}}$$

$$\Rightarrow Y(z) = 2 \frac{1}{1 - 2z^{-1}} + 3 \frac{1}{2 \left(1 - \frac{3}{2} z^{-1}\right)}$$

$$\Rightarrow Z^{-1}\{Y(z)\} = 2 Z^{-1}\left\{\frac{1}{1 - 2z^{-1}}\right\} + \frac{3}{2} Z^{-1}\left\{\frac{1}{1 - \frac{3}{2} z^{-1}}\right\}$$

$$\Rightarrow y[n] = 2 2^n u[n] + \frac{3}{2} \left(\frac{3}{2}\right)^n u[n]$$

$$\Rightarrow y[n] = 2^{n+1} u[n] + \left(\frac{3}{2}\right)^{n+1} u[n]$$

H.W: Problem: Solve  $6 y[n] - 5 y[n - 1] = 4^n u[n]; y[-1] = 0$

Problem:  $-4 y[n + 1] + 2 y[n] = 3^n u[n]; y[0] = 0$

Solution:

$$-4 Z\{y[n + 1]\} + 2 Z\{y[n]\} = Z\{3^n u[n]\}$$

$$\Rightarrow -4 z[Y(z) - y[0]] + 2 Y(z) = \frac{1}{1 - 3 z^{-1}}$$

$$\Rightarrow -4z Y(z) + 2Y(z) = \frac{1}{1 - \frac{3}{z}}$$

$$\Rightarrow (2 - 4z) Y(z) = \frac{1}{\frac{z - 3}{z}}$$

$$\Rightarrow Y(z) = \frac{z}{(z - 3)(2 - 4z)}$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{1}{(z - 3)(2 - 4z)} \dots \dots \dots (i)$$

Now,

$$\frac{1}{(z - 3)(2 - 4z)} = \frac{A}{z - 3} + \frac{B}{2 - 4z}$$

$$A = \frac{1}{2 - 4 \times 3} = -\frac{1}{10}, \quad B = \frac{1}{\frac{1}{2} - 3} = -\frac{2}{5}$$

Form equation (i),

$$\begin{aligned} \frac{Y(z)}{z} &= -\frac{1}{10} \frac{1}{z-3} - \frac{2}{5} \frac{1}{2-4z} \\ \Rightarrow Y(z) &= -\frac{1}{10} \frac{z}{z-3} - \frac{2}{5} \frac{z}{2-4z} \\ \Rightarrow Y(z) &= -\frac{1}{10} \frac{z}{z\left(1-\frac{3}{z}\right)} - \frac{2}{5} \frac{z}{-4z\left(1-\frac{2}{4z}\right)} \\ \Rightarrow Y(z) &= -\frac{1}{10} \frac{1}{1-3z^{-1}} + \frac{1}{10} \frac{1}{1-\frac{1}{2}z^{-1}} \\ \Rightarrow Z^{-1}\{Y(z)\} &= -\frac{1}{10} Z^{-1}\left\{\frac{1}{1-3z^{-1}}\right\} + \frac{1}{10} Z^{-1}\left\{\frac{1}{1-\frac{1}{2}z^{-1}}\right\} \\ \Rightarrow y[n] &= -\frac{1}{10} 3^n u[n] + \frac{1}{10} \left(\frac{1}{2}\right)^n u[n] \end{aligned}$$