Final Term Sample Question -2022

Solution

1.(a) sketch the path and its orientation given by

$$z(t) = 3e^{it} \quad (0 \le t \le \pi)$$

$\Rightarrow x(t) + iy(t) = 3(\cos t + i\sin t)$

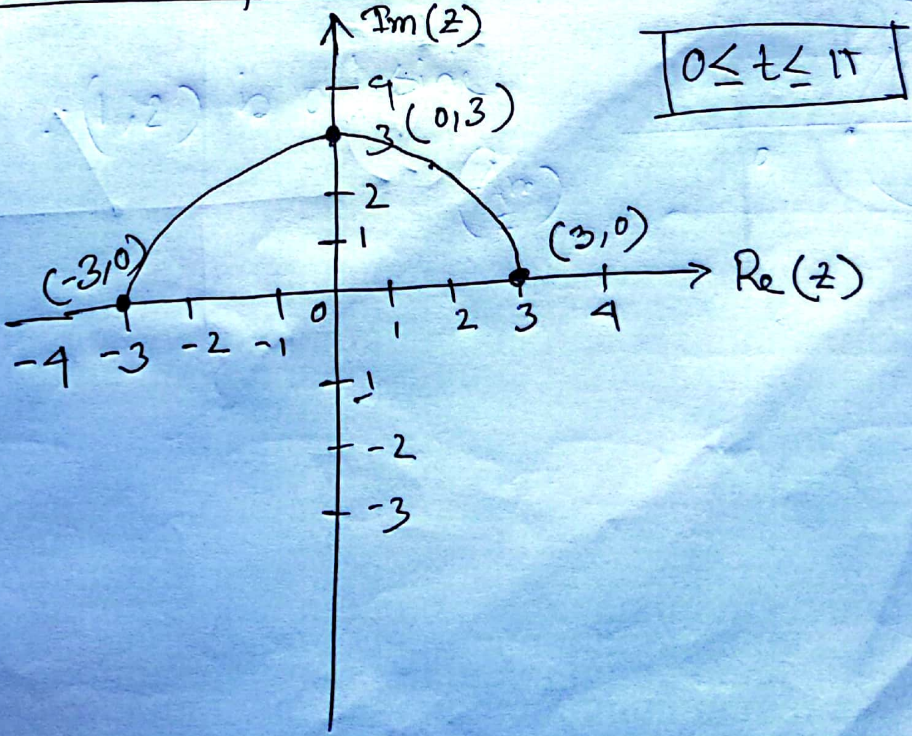
$\Rightarrow x(t) + iy(t) = 3\cos t + i\,3\sin t.$

Seperating Real & Imaginary Part;

$x(t) = 3\cos t$

$y(t) = 3\sin t$

t	$x(t)$	$y(t)$	(x,y)
0	3	0	(3,0)
$\pi/2$	0	3	(0,3)
π	-3	0	(-3,0)



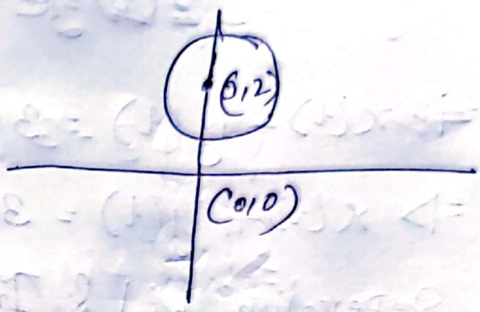
(b) Sketch and represent $|z-2i|=5$ Parametrically, counterclockwise

$\Rightarrow \quad |z-2i|=5$

$\Rightarrow |x+iy-2i|=5$

$\Rightarrow |x+i(y-2)|=5$

$\Rightarrow x^2+(y-2)^2=5^2$



$r=5;$
Center$(0,2)$

Here; $x=r\cos t \quad Y=r\sin t$

$\therefore x=5\cos t \quad y=5\sin t$

$\therefore x(t)=5\cos\theta$

$\therefore y(t)=2+5\sin\theta$ (Ans)

2) Test wheather the point (1,2) is interior, extenion or boundary of $|z-3|=2$

$\Rightarrow \quad |z-3|=2$

$\Rightarrow |x+iy-3|=2$

$\Rightarrow |(x-3)+iy|=2$

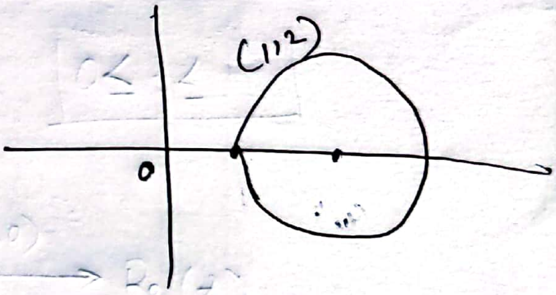
$\Rightarrow (x-3)^2+y^2=2^2$

$\Rightarrow (1-3)^2+(2)^2=2^2$

Here; $\Rightarrow 9+4>4$ ($\therefore$ 1,2 is extenion)

$\Rightarrow 13>4$

$\therefore$ (1,2) is out side.



Ans

(d) Find the equation of the path c, which is the line segment from $z = 1$ to $z = 2+i$.

$\Rightarrow$ Let, $z = 1$ to $z = 2+i$

$(1, 0)$ — x_1, y_1

$(2, 1)$ — x_2, y_2

$X = t$

$Y = t - 1$

Here;

$x_1 = 1$

$x_2 = 2$

$Y_1 = 0$

$Y_2 = 1$

Here,

$$\frac{x - x_1}{y - y_1} = \frac{x_1 - x_2}{y_1 - y_2}$$

$$\Rightarrow \frac{x-1}{y-0} = \frac{1-2}{0-1}$$

$$\Rightarrow \frac{x-1}{y} = \frac{-1}{-1}$$

$$= 1$$

$\Rightarrow y = x - 1$

$\Rightarrow x - 1 = y$

(An)

(e) Find the singular point (pole) and corresponding order of

$$f(z) = \frac{3z}{(z+1)^3}$$

Here;

$(z+1)^3 = 0$

$\Rightarrow \cancel{(z)^3 + (1)^3 = 0}$

$\Rightarrow \cancel{z^3} = (x + iy + 1)^3 = 0$

$\Rightarrow (x+1)^3 + iy^3 = 0$

$\Rightarrow (x+1)^3 + y^3 = 0$

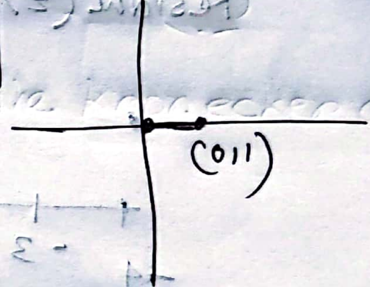
$\therefore r = 0$

center $= (0,1)$

(Ans)

$3z = 0$

a.



(f) Find the residue at singular point of $f(z) = \frac{z+1}{(z-4)}$ for $|z|$ (

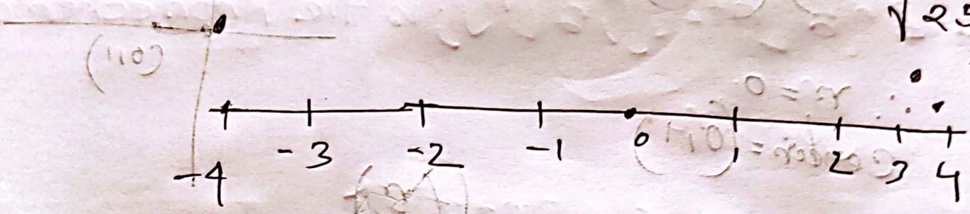
Here, $f(z) = \frac{z+1}{z-4}$; $|z| = 5$

$\therefore \{ \frac{1}{z-4} = 5;\ z = 4;\ |z| = 5 \}$

$\therefore \{ z = \frac{21}{5};\ z = 4$

Residue $(z) \geq 5 \wedge Re(z) \leq 5 \wedge (\sqrt{25} - Re(z)^2 = -Im(z) \vee$

$\sqrt{25 - Re(z)^2} = Im($



$\Rightarrow \frac{1}{z-4} = 5$

$\Rightarrow z+1 = 5$

$\Rightarrow |z| = 5$

(Ans)

(i) Evaluate $Z\{(-1)^n u[n]\}$; $u[n]$ is the discreate time unit step function.

$\Rightarrow L_t [\{Z\{(-1)^r = n ; 0 < t < 1 \; [u(n) ; t \geq 1\}] (s)$

$\Rightarrow L_t [\{\{Z(-1)^r = n) ; Z(0 \leq t < 1)\} \{u(n) ; t \geq 1\}] (s)$

(Ans)

(h) Evaluate $Z\{\delta[n+2]\}$, $\delta[n]$; is the kronecker delta function.

$\Rightarrow Z\{\delta[n+2]\}$

$\Rightarrow X(z) = \sum_{-\infty}^{\infty} \delta[n+2] z^{-n}$

$= (1) z^2$

$= z^2$

(Ans)

i) Evaluate $Z\{(2^n - 3)\, u[n]\}$. Also find ROC.

$\Rightarrow Z(2^n - 3)\, u[n]$

$\Rightarrow Z\, (2^n - 3) \cdot un$

$\Rightarrow Z(nu \cdot 2^n - 3nu)$

$\Rightarrow nuz \cdot 2^n - 3nuz$

(Ans)

J) Evaluate $Z^{-1}\{Z^{-4}\}$

$\Rightarrow Z^{-1}\{Z^{-4}\}$

$\Rightarrow Z^{-1} \cdot Z^{-4}$

$\Rightarrow Z^{-5}$

(Ans)

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k) Evaluate $Z^{-1}\left\{\frac{1}{1-2z^{-1}}\right\}; |z|>2$

$\Rightarrow Z^{-1}\left\{\frac{1}{1-2z^{-1}}\right\}$

$\therefore x^{n} = (-2)^{n} u[n]$ (Ans)

l) Evaluate $Z^{-1}\left\{\frac{z^{-1}}{1+3z^{-1}}\right\}; |z|>3$

$\Rightarrow \cancel{Z^{-1}}\left\{\frac{\cancel{z^{-1}}}{1+3z^{-1}}\right\}$

$\Rightarrow \frac{1}{1+3z^{-1}}$

$\Rightarrow x^{n} = (+3)^{n} u[n]$ (Ans)

m) Evaluate $Z^{-1}\frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{9}z^{-2}}; |z|>\frac{1}{3}$

$\Rightarrow \frac{z^{2}+\frac{1}{3}z}{z^{2}-\frac{1}{9}}$

$= \frac{z\cancel{(z+1/3)}}{\cancel{(z+1/3)}(z-1/3)} = \frac{z}{(z-1/3)}$

$\therefore x[n] = (1/3)^{n} u[n]$

(Ans)

(n) Sketch the path and its orientation given by $z(t) = 2\sin(t) + i\,3\cos(t) + 3 + 2i$; $(0 \le t \le 2\pi)$

$\Rightarrow z(t) = 2\sin(t) + i\,3\cos(t) + 3 + 2i$; $(0 \le t \le 2\pi)$

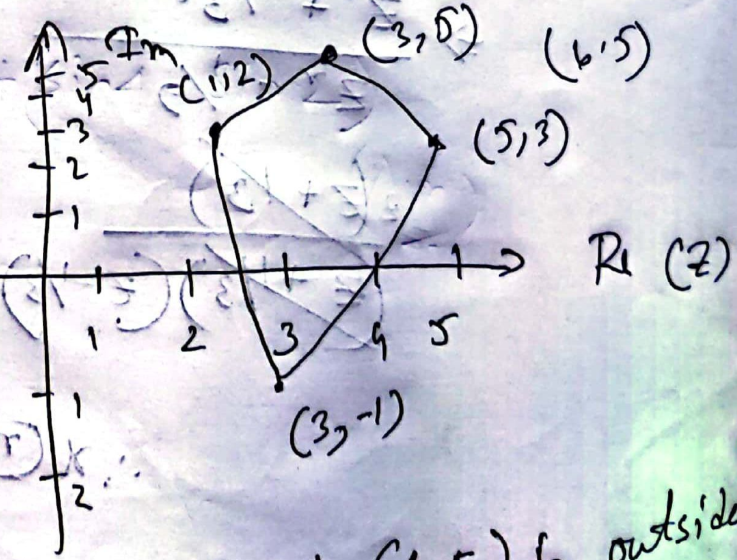
$\Rightarrow x(t) + iy(t) = 2\sin t + 3 + i\,(3\cos(t) + 2)$

Real and imaginary Part;

$x(t) = 2\sin t + 3$

$y(t) = 3\cos t + 2$

t	$x(t)$	$y(t)$	(x,y)
0	3	5	(3,5)
$\pi/2$	5	3	(5,3)
π	3	-1	(3,-1)
$3\pi/2$	1	2	(1,2)
2π	3	5	(3,5)



Point (6,5) is outside

(0) Sketch and represent the line segment from $1+i$ to $4-2i$ Parametrically.

$\Rightarrow$ The equation of straight line passing though the point is $(1,1)$ to $(4,-2)$ is

$$y-1 = \frac{(-2-1)}{(4-1)}(x-1)$$

That is $y = -x+2$

Let; $x = t$, then $y = -t+2$ where t varies from $t=1$ to $t=4$

So, the parametric equation of line segment from $1+i$ to $4-2i$ is

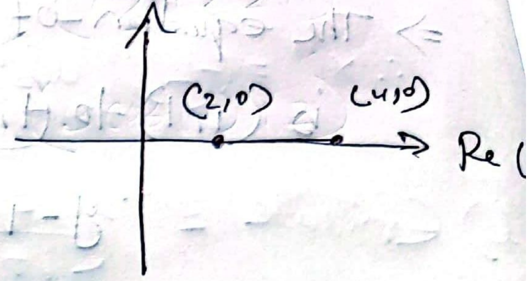
$x(t) = t$; $y(t) = -t+2$ $(1 \leq t \leq 4)$

(Ans)

2 (c) Sketch the path C, where C consists the line segment $z=2$ to $z=4$ and hence evaluate $\int_C (e^{2z}+\cos z)\,dz$.

$(2,0)$ $(4,0)$

$\Rightarrow$



$z=(x+iy)$

$\Rightarrow x+i.0$

$\therefore z=x$

$$\frac{dz}{dx}=\frac{d}{dx}(x)$$

$\therefore dz = dx$

So, $\int_C (e^{2z}+\cos z)\,dz$

$$=\int_2^4 (e^{2x}+\cos x)\,dx$$

$$\Rightarrow \left[\frac{e^{2x}}{2}\right]_2^4+\left[\sin x\right]_2^4$$

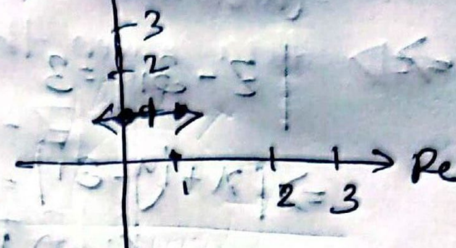
$$\Rightarrow \frac{1}{2}(e^8-e^4)+(\sin 4-\sin 2)$$

(Ans)

(4)

2) Sketch the path C, where C consists of line segment from $z = i$ to $z = i+1$ and hence evaluate $\int_C (3z-2)\,dz$

$(1,0) \quad (1,1)$



$\Rightarrow z = x + iy$

$\Rightarrow 1 + iy$

$\therefore z = iy$

$dz = i\,dy$

So, $\int_C (3z-2)\,dz$

$= \int_0^1 (3 \cdot iy - 2)\, i\,dy$

$= \int_0^1 (3iy - 2)\, i\,dy$

$= \dfrac{i \cdot (3i-4)}{2}$

$= -\dfrac{4i+3}{2}$

$= -1.5 - 2i$ (Ans)

(b) Sketch the path C, where C is the circle $|z-3i|=3$ (clockwise) and hence evaluate $\int_C \left[\frac{3}{(z-3i)^2}\right] dz$ Expansi

$\Rightarrow$

$\Rightarrow \quad |z-3i|=3$ $\qquad (3,0) \quad (3,3)$

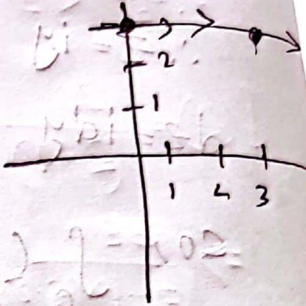
$\Rightarrow |x+iy-3i|=3$

$\Rightarrow |x+i(y-3)|=3$

$\Rightarrow \sqrt{x^2+(y-3)^2}=3$

$= x^2+(y-3)^2=(3)^2$

$\therefore r=3$; Center $(0,3)$



$Z = x+iy$

$= 3+iy$

$= 3+y$

$x = r\cos\theta$

$y = r\sin\theta$

$dz = \frac{dx}{dx}(x)$

$\therefore x(t) = 3\cos\theta \; (3,0)$

$\therefore dz = dx$

$y(t) = 3 + 3\sin\theta$ Ans

$$\int_C \left[\frac{3}{(z-3i)^2}\right] dz \quad (3,3)$$

$$= \int_0^3 \left[\frac{3}{(z-3i)^2}\right] dx$$

$$= 3\left(\frac{i}{3} - \frac{i+1}{6}\right)$$

$$= \frac{i-1}{2}$$

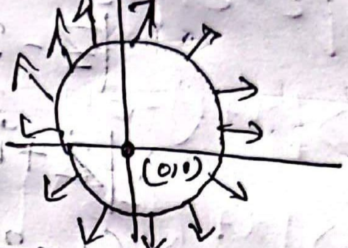
$= -0.5 + 0.5i$ (Ans)

Expand $f(z) = \frac{3z}{(z-1)(2-z)}$ in a laurent Series Valid for $|z|>2$ (b)

$\Rightarrow$ If $|z|>2$ then it must be $|z|>1$

$1<|z|$ and $2<|z|$

$\therefore \frac{1}{z}<1$ and $\frac{2}{z}<1$



$$f(z) = \frac{A}{(z-1)} + \frac{B}{2-z}$$

$$\Rightarrow \frac{3z}{(z-1)(2-z)} = \frac{A}{(z-1)} + \frac{B}{(2-z)}$$

$$\Rightarrow 3z = A(2-z) + B(z-1)$$

$\therefore$ for $z=2$;

$$3\cdot \times 2 = A(2-2) + B(2-1)$$

$$6 = 0 + B \rightarrow B = 6$$

$$A = 3$$

$$\therefore f(z) = \frac{3}{z-1} + \frac{z}{2-z}$$

$$= \frac{3}{z\left(1-\frac{1}{z}\right)} + \frac{2}{-z\left(1-\frac{2}{z}\right)}$$

$$\Rightarrow \frac{3}{z}\left(1+\frac{1}{z}+\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3+\cdots\right) - \frac{6}{z}\left(1+\frac{2}{z}+\left(\frac{2}{z}\right)^2+\left(\frac{2}{z}\right)^3+\cdots\right)$$

which is required laurent Series

(Ans)

(d) Evaluate $\oint_C \frac{2z-1}{(z^2-9)} dz$; $C: |z-3| = 2$ using the Cauchy residue theorem.

$\Rightarrow$ Here given that;

$$|z-3| = 2$$

$$\Rightarrow |x+iy-3| = 2$$

$$\Rightarrow \cancel{(x-3)+i}$$

$$\Rightarrow |(x) + i(y-3)| = 2$$

$$= (x)^2 + (y-3)^2 = (2)^2$$

$\therefore r = 2$; Center $(0, 3)$

For singularity;

$$z^2 - 9 = 0$$

$$\Rightarrow z^2 = 9$$

$$\Rightarrow z^2 = 3^2$$

$$\Rightarrow z = 3$$

Residue at $z = 3$

$$\text{Res}(z=3) = \lim_{z\to 3} \frac{1}{z-3} f(z)$$

$$= \lim_{z\to 3} \left(\frac{1}{z-3} \cdot \frac{2z-1}{z^2-9} \cdot dz \right)$$

$$\Rightarrow \frac{1}{3-3} \left(\frac{2\times 3-1}{(3)^2-9} \cdot dz \right)$$

$$\Rightarrow 0 \times \frac{6-1}{0}$$

$$= 5$$

(Ans)

② Expand $f(z) = \frac{2z+5}{(z-2)(1+z)}$ in a laurunt series valid for $1<|z|<2$

$\Rightarrow 1<|z|<2$

If $|z|<1$, then it must be $|z|<2$

$|z|<1 \qquad |z|<2$

So, $\frac{|z|}{1}<1 \qquad \frac{|z|}{2}<1$

$f(z) = \frac{2z+5}{(z-2)(1+z)}$

So; $\frac{2z+5}{(z-2)(1+z)} = \frac{A}{(z-2)} + \frac{B}{(1+z)}$

$\Rightarrow 2z+5 = A(1+z) + B(z-2)$

$\Rightarrow$ For $z=2$;

$2\times2+5 = A(1+2) + B(2-2)$

$\Rightarrow 9 = A\times3 + B\times0$

$\Rightarrow 9 = 3A$

$\Rightarrow 3A = 9$

$\Rightarrow A = 9-3$

$\Rightarrow A = 6$

For, $z=1$;

$2\times1+5 = A(1+2) + B(1-2)$

$\Rightarrow 7 = 2A - B$

$\Rightarrow -2A = -B-7$

$\Rightarrow -B = 2A-7$

so; $f(z) = \frac{6}{z-2} + \frac{2A-7}{1+z}$

$= \frac{6}{z(1-\frac{z}{2})} + \frac{2A-7}{z(1+\frac{1}{z})}$

$= \frac{6}{z}\left[1+\frac{1}{z}+\left(\frac{2}{z}\right)^2+\left(\frac{2}{z}\right)^3+\cdots\right] + \frac{2A}{z}\left(1+\frac{1}{z}\ldots\right.$

which is required laurunt series.

(d) Evaluate ... $2z-1$

3.
(a) ii)

$$X(z) = \frac{1}{(1+0.5z^{-1})(1-0.50z^{-1})(1-z^{-1})}; |z|>1$$

$$X(z) = \frac{z^3}{(z+1/2)(z-1/2)(z-1)}$$

$$\frac{X(z)}{z} = \frac{z^2}{(z+1/2)(z-1/2)(z-1)}$$

$$= \frac{A}{(z+1/2)} + \frac{B}{(z-1/2)} + \frac{C}{(z-1)}$$

$$\therefore A = \left[\frac{z^2\cancel{(z+1/2)}}{\cancel{(z+1/2)}(z-1/2)(z-1)}\right]_{z=-1/2} = \frac{1}{6}$$

$$\therefore B = \left[\frac{z^2\cancel{(z-1/2)}}{(z+1/2)\cancel{(z-1/2)}(z-1)}\right]_{z=1/2} = \frac{\frac{1}{4}}{1\times(-1/2)} = -\frac{1}{2}$$

$$\therefore C = \left[\frac{z^2\cancel{(z-1)}}{(z+1/2)(z-1/2)\cancel{(z-1)}}\right]_{z=1}$$

$$= \frac{1}{3/4} = \frac{4}{3}$$

$$\therefore \frac{X(z)}{z} = -\frac{\frac{1}{6}}{(z+1/2)} - \frac{\frac{1}{2}}{(z-1/2)} + \frac{\frac{4}{3}}{(z-1)}$$

$$\therefore X(z) = -\frac{1}{6} z \left(z+1/2\right)^{-1} - 1/2\, z \left(z-1/2\right)^{-1} + 4/3\, z\,(z-1)^{-1}$$

inverse of z transformation;

$$x[n] = \frac{1}{6}\left(-1/2\right)^n u[n] - 1/2\left(1/2\right)^n u[n] + 4/3\, u[n]$$

$$= \frac{1}{6}(-0.5)^n u[n] - 1/2\,(0.5)^n u[n] + 4/3\, u[n]$$

(Ans)

(1) $X(z) = \dfrac{5}{(1+z^{-1})(1+0.25z^{-1})}$; $|z|>1$

$\Rightarrow X(z) = \dfrac{z^2}{(z+1)(z-0.25)}$

$\Rightarrow \dfrac{X(z)}{z} = \dfrac{z}{(z+1)(z-0.25)}$

$= \dfrac{A}{(z+1)} + \dfrac{B}{(z-0.25)}$

$\therefore A = \left[\dfrac{z}{z-0.25}\right]_{z=1} = \dfrac{1}{1-0.25} = \dfrac{1}{0.75}$

$\therefore B = \left[\dfrac{z}{z+1}\right]_{z=0.25} = \dfrac{0.25}{0.25+1} = \dfrac{0.25}{1.25}$

$\therefore \dfrac{X(z)}{z} = \dfrac{\frac{1}{0.75}}{z+1} + \dfrac{\frac{0.25}{1.25}}{z-0.25}$

$\therefore X(z) = 1/0.75\left(\dfrac{z}{z+1}\right) + 0.25/1.25\left(\dfrac{z}{z-0.75}\right)$

inverse z transformat

$x[n] = 1/0.75\, u[n] + 0.25/1.25\,(0.25)^n u[n]$

(Ans)

(b) $X(z) = (1+2z)(1-z^{-1})(2+z^{-2})$

$= (1+2z+z^{-2}+4)(1-z^{-1})$

$= (1+2z+\cancel{z^{-2}}+4-z^{-1}-2-\cancel{z^{-2}}-4z^{-1})$

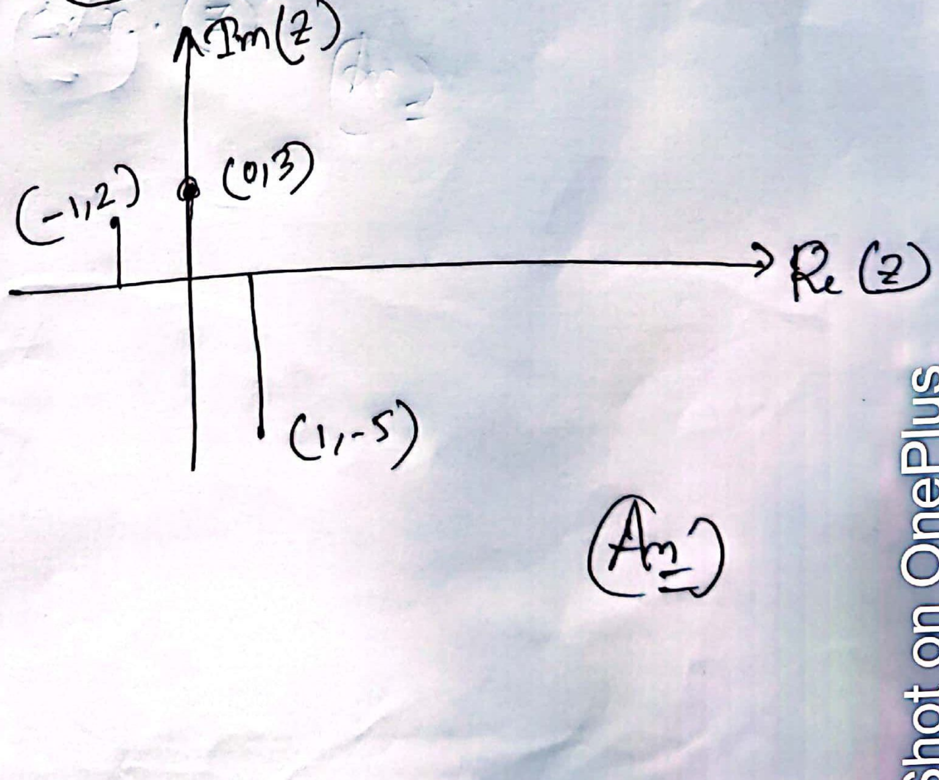
$= (3-5z^{-1}+2z)$

$\therefore z^{-1}\{x(z)\} = 3\delta[n]-5\delta[n-1]+2\delta(n+1]$

$\Rightarrow \therefore x(n) = 3\delta[n]+2\delta[n+1]-5\delta[n-1]$

equivalently,

$$x[n] = \begin{cases} 2 & ; n=1 \\ 3 & ; n=0 \\ -5 & ; n=1 \end{cases}$$



(Ans)

(d) $y[n+2] - 4y[n+1] + 4y[n] = 0$; $y[0] = 0$; $y[1] = 6$;

$\Rightarrow z^2[Y(z) - Y[0] - Y(1)z^{-1}] - 4z[Y(z) - Y[0]] + 4Y(z$

$\Rightarrow (z^2 - 4z + 4)\,Y(z) - 6z = 0$

$\Rightarrow Y(z) = \dfrac{6z}{z^2 - 4z + 4}$

$\Rightarrow Y(z) = \dfrac{6z}{(z-2)^2}$

$\Rightarrow Y(z) = \dfrac{3 \cdot 2z}{(z-2)^2}$

$\therefore y[n] = 3 \cdot n\,(2)^n$

(Ans)

$$y[n] - 4y[n-1] - 12y[n-2] = 0 \; ; \; y(-1) = 0 \; ; \; y(-2) = 1$$

$$\Rightarrow Y[z] - 4z[Y(z) - Y[0] - Y(1)z^{-1}] - 12z[Y(z) - Y(0) - Y(z)z^{-1}] = 0$$

$$= (z^2 - 4z - 12)Y(z) - 1z = 0$$

$$\Rightarrow Y(z) = \frac{z}{z^2 - 4z - 12}$$

$$\Rightarrow Y(z) = \frac{z}{z^2 - 6z + 2z - 12}$$

$$\Rightarrow Y(z) = \frac{z}{z(z-6) + 2(z-6)}$$

$$\Rightarrow Y(z) = \frac{z}{(z-6)(z+2)}$$

$$\Rightarrow y[n] = 1 \cdot n \cdot (z)^n$$

Ans

(e) (a) $Y[n] + Y[n-1] = X[n+1]$

$\Rightarrow Y[z] + z^{-1}[Y(z) + Y(-1)z^{1}] = \cancel{X} z^{1}[Y[z] + Y[z] + Y[-1]z^{1}$

$Y[z] + Y[-1]z^{1}] + z^{-1}$

$\Rightarrow Y[z] + z^{-1}[Y[z] = \cancel{X[z} z^{1} X[z]$

$\Rightarrow Y[z] + (1 + z^{-1}) = \cancel{x[1 - z^{+1}]} z^{1} X[z]$

$\therefore \dfrac{Y[z]}{X[z]} = \dfrac{z^{1}}{1 + z^{-1}}$

$\therefore H(z) = \dfrac{z^{1}}{1 + z^{-1}}$ (Ans)

(b) impulse response

$h[n] = (-1)^{n} u[n]$ (Ans)

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