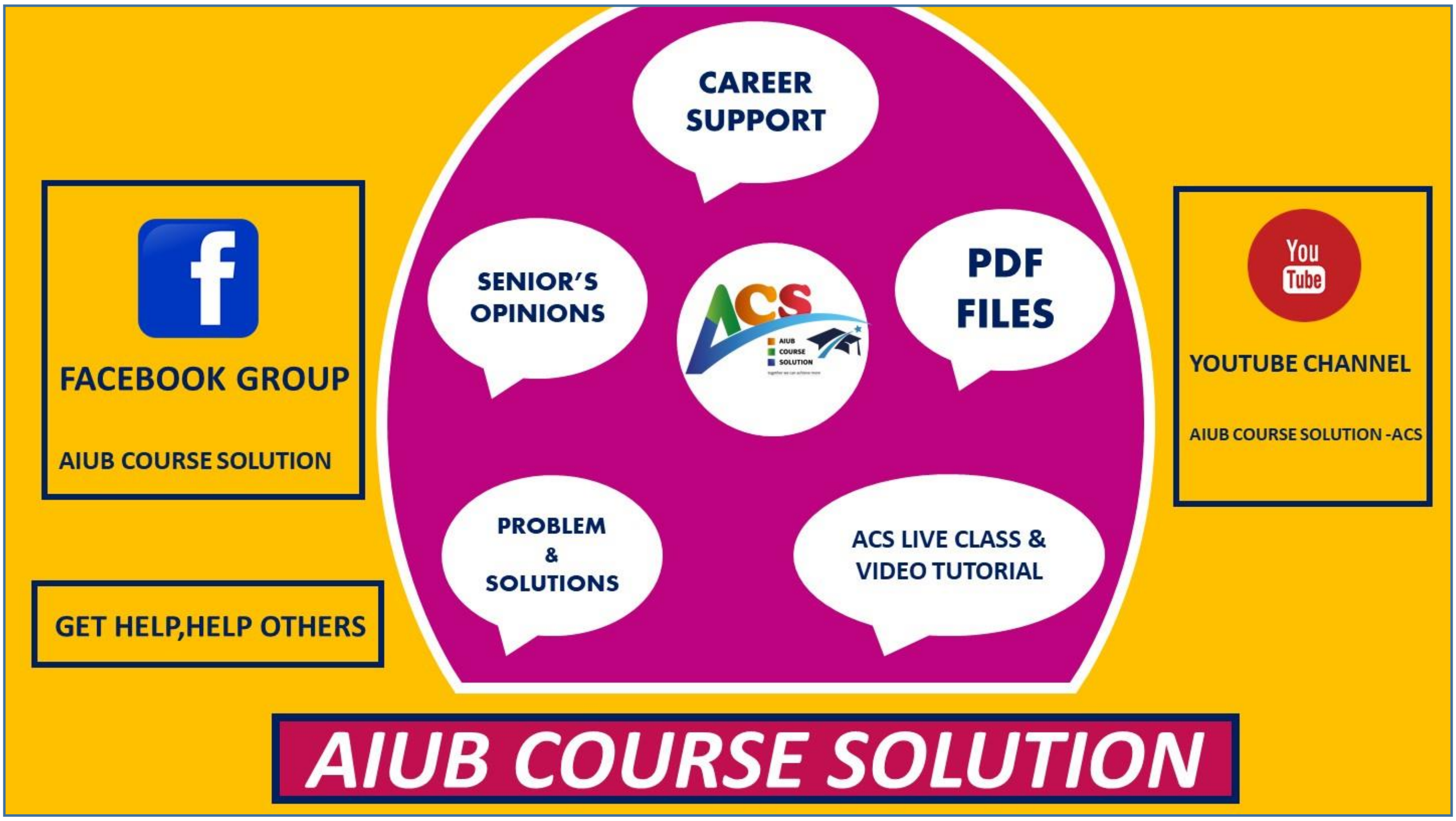

TOGETHER WE CAN ACHIEVE MORE

COURSE NAME: Math-3

CHAPTER: 7.3

SOLVED BY

NAME: Rupa Paul

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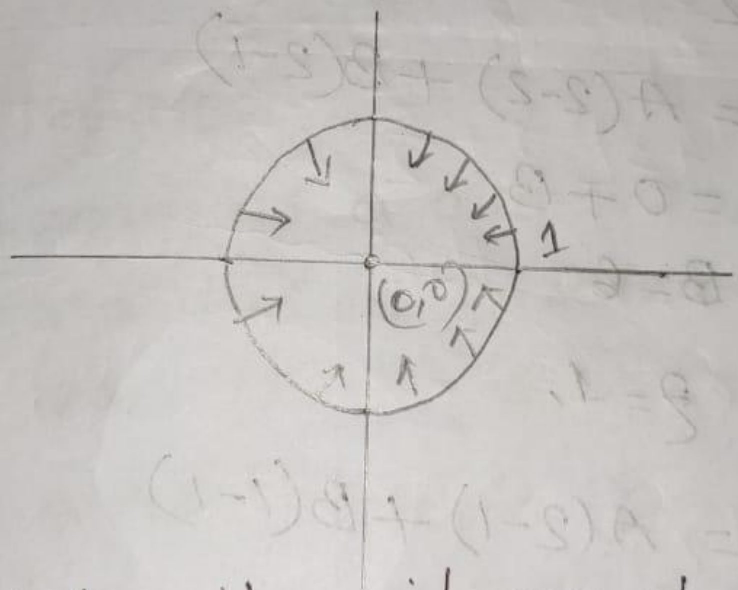
Chapter: 7.3

1. (a) $f(z) = \dfrac{3z}{(z-1)(z-2)}$

(a) $|z| < 1$

that means a circle centre(0,0) which radius is less than 1.



If $|z| < 1$, then it must be $|z| < 2$

$\|z\| < 1$	$\|z\| < 2$
so, $\dfrac{\|z\|}{1} < 1$	so, $\dfrac{\|z\|}{2} < 1$

2

Given, $f(z) = \frac{3z}{(z-1)(2-z)}$

So, $\frac{3z}{(z-1)(2-z)} = \frac{A}{z-1} + \frac{B}{2-z}$ ——(1)

$\Rightarrow 3z = A(2-z) + B(z-1)$

For,
$z = 2$

$6 = A(2-2) + B(2-1)$

$6 = 0 + B$

$\therefore B = 6$

For $z = 1$,

$3 = A(2-1) + B(1-1)$

$3 = A + 0$

$\therefore A = 3$

So, $f(z) = \frac{3}{z-1} + \frac{6}{2-z}$

(3)

$$\Rightarrow f(z) = \frac{3}{-1(1-z)} + \frac{6}{2\left(1-\frac{z}{2}\right)}$$

$$= -3(1-z)^{-1} + 3\left(1-\frac{z}{2}\right)^{-1}$$

$$= -3\left[1+z+z^2+z^3+\cdots\right] + 3\left[1+\left(\frac{z}{2}\right)+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3+\cdots\right]$$

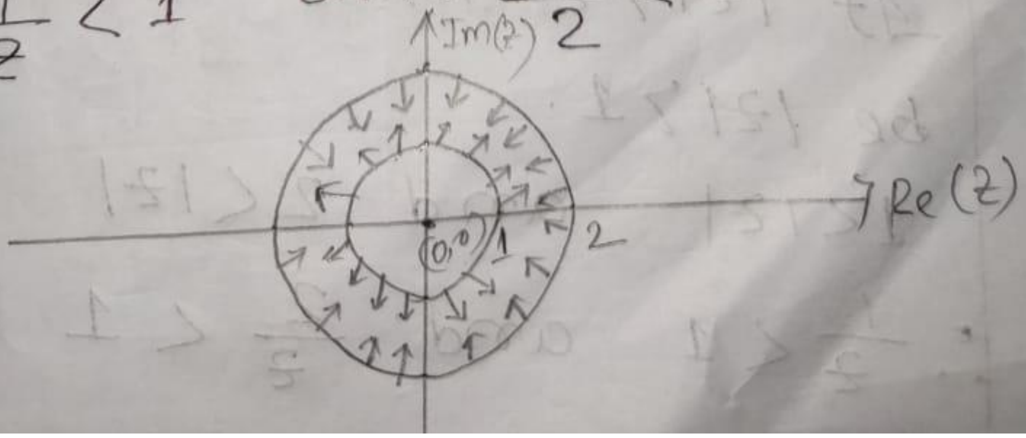
which is required Laurent series.

(b) $1<|z|<2$

which means a circle center $(0,0)$ and radius is greater than 1 and less than 2.

$1<|z|$ and $|z|<2$

$\therefore \frac{1}{z}<1$ and $\frac{|z|}{2}<1$.



(4)

so, $f(z) = \frac{3}{z-1} + \frac{6}{2-z}$

$= \frac{3}{z(1-\frac{1}{z})} + \frac{6}{2(1-\frac{z}{2})}$

$= \frac{3}{z}\left(1-\frac{1}{z}\right)^{-1} + 3\left(1-\frac{z}{2}\right)^{-1}$

$= \frac{3}{z}\left[1+\frac{1}{z}+\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3+\cdots\right]$

$+3\left[1+\frac{z}{2}+\left(\frac{z}{2}\right)^2+\left(\frac{z}{2}\right)^3+\cdots\right]$

which is the required Laurent series.

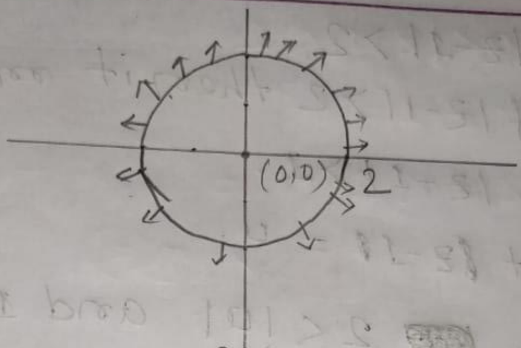
(c) $|z| > 2$

If $|z| > 2$ then it must be $|z| > 1$.

$1 < |z|$ and $2 < |z|$

$\therefore \frac{1}{z} < 1$ and $\frac{2}{z} < 1$

(5)



$$f(z)=\frac{3}{z-1}+\frac{6}{2-z}$$

$$=\frac{3}{z\left(1-\frac{1}{z}\right)}+\frac{6}{-z\left(1-\frac{2}{z}\right)}$$

$$=\frac{3}{z}\left(1-\frac{1}{z}\right)^{-1}-\frac{6}{z}\left(1-\frac{2}{z}\right)^{-1}$$

$$=\frac{3}{z}\left[1+\frac{1}{z}+\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3+\cdots\right]-\frac{6}{z}\left[1+\frac{2}{z}+\left(\frac{2}{z}\right)^2 \cdots\right]$$

which is the required laurent series.

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(d) $|z-1| > 2$

If $|z-1| > 2$ then it must me

$|z-1| > 1$.

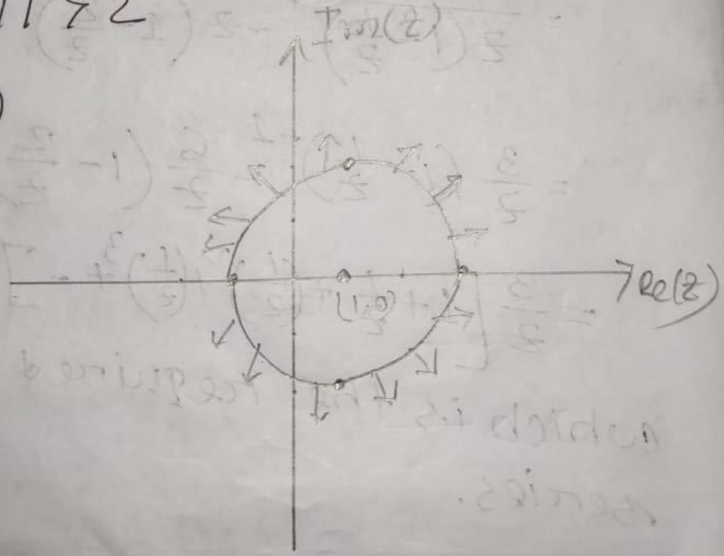
Let $|z-1| = u$.

So, $2 < |u|$ and $1 \leq |u|$

$\therefore \frac{2}{u} < 1$ and $\frac{1}{u} < 1$.

$|z-1| > 2$

$(1,0)$



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$$f(z) = \frac{3}{z-1} + \frac{6}{2-z}$$

$$= \frac{3}{u} + \frac{6}{-(z-2)}$$

$$= \frac{3}{u} - \frac{6}{(z-1)-1}$$

$$= \frac{3}{u} - \frac{6}{u-1}$$

$$= \frac{3}{u} - \frac{6}{u\left(1-\frac{1}{u}\right)}$$

$$= \frac{3}{u} - \frac{6}{u}\left(1-\frac{1}{u}\right)^{-1}$$

$$= \frac{3}{u} - \frac{6}{u}\left[1 + \frac{1}{u} + \left(\frac{1}{u}\right)^2 + \left(\frac{1}{u}\right)^3 + \cdots\right]$$

$$= \frac{3}{z-1} - \frac{6}{z-1}\left[1 + \frac{1}{z-1} + \left(\frac{1}{z-1}\right)^2 + \left(\frac{1}{z-1}\right)^3 + \cdots\right]$$

which is required Laurent series.

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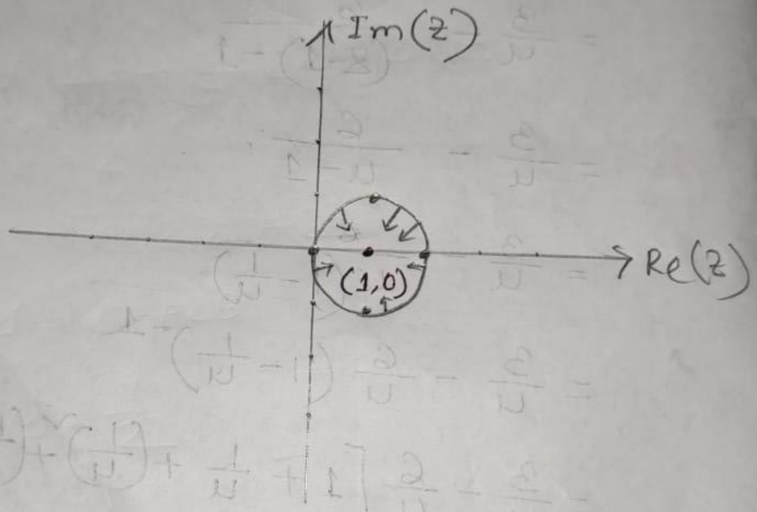
(e) $0 < |z-1| < 1$

Let $\underset{(1,0)}{z-1} = u$.

$0 < u < 1$.



So,

$0 < |u|$ and $|u| < 1$

Not necessary | $\frac{|u|}{1} < 1$

$$f(z) = \frac{3}{z-1} + \frac{6}{2-z}$$

$$= \frac{3}{u} + \frac{6}{-(z-2)}$$

$$= \frac{3}{u} - \frac{6}{(z-1)-1}$$

$$= \frac{3}{u} - \frac{6}{u-1}$$

$$= \frac{3}{u} - \frac{6}{-1(1-u)}$$

$$= \frac{3}{u} + 6(1-u)^{-1}$$

$$= \frac{3}{u} + 6\left[1 + u + u^2 + u^3 + \cdots\right].$$

$$= \frac{3}{z-1} + 6\left[1 + (z-1) + (z-1)^2 + (z-1)^3 + \cdots\right]$$

which is required Laurent series

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2. $f(z) = \dfrac{1}{z(z-2)}$

Let, $\dfrac{1}{z(z-2)} = \dfrac{A}{z} + \dfrac{B}{z-2}$

$\Rightarrow 1 = A(z-2) + B.(z).$

If $z = 2.$

$1 = A(2-2) + 2B$

$\Rightarrow 1 = 0 + 2B$

$\therefore B = \dfrac{1}{2}$

If $z = 0.$

$1 = A(0-2) + B.0$

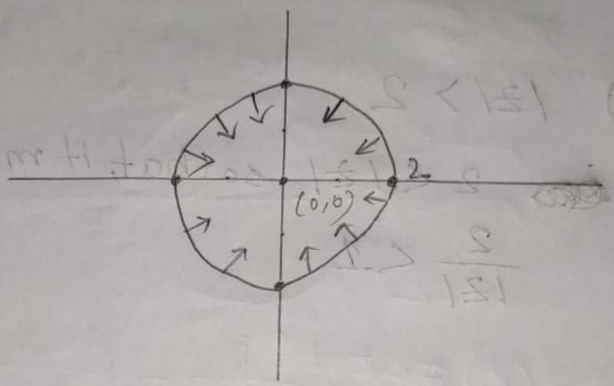
$\Rightarrow 1 = -2A$

$\therefore A = -\dfrac{1}{2}.$

$\therefore f(z) = \dfrac{-1/2}{z} + \dfrac{1/2}{z-2}$

(11)

(a) $0<|z|<2$



$|z|<2$

so, $\frac{|z|}{2}<1$.

$$f(z)=\frac{-1/2}{z}+\frac{1/2}{z-2}$$

$$=-\frac{1}{2z}+\frac{1/2}{-2\left(1-\frac{z}{2}\right)}$$

$$=-\frac{1}{2z}-\frac{1}{4}\left(1-\frac{z}{2}\right)^{-1}$$

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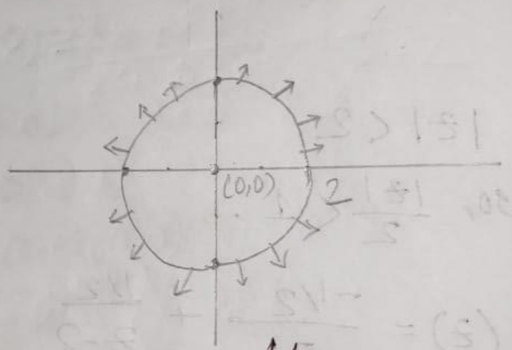
$$= \frac{1}{-2z} - \frac{1}{4}\left[1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + \cdots\right]$$

(b) $|z| > 2$.

$2 < |z|$ so that, it must me $1 < |z|$

so $\frac{1}{|z|} < 1$

$\frac{2}{|z|} < 1$.



$$f(z) = \frac{-1/2}{z} + \frac{1/2}{z-2}$$

$$= -\frac{1}{2z} + \frac{\frac{1}{2}}{z\left(1-\frac{2}{z}\right)}$$

$$= -\frac{1}{2z} + \frac{1}{2z}\left(1-\frac{2}{z}\right)^{-1}$$

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$$= \frac{1}{-2z} + \frac{1}{2z}\left[1 + \frac{2}{z} + \left(\frac{2}{z}\right)^2 + \left(\frac{2}{z}\right)^3 + \cdots\right].$$

3. $f(z) = \dfrac{5z}{(z^2+1)(z+2)}$

Let,

$$\frac{5z}{(z^2+1)(z+2)} = \frac{Az+B}{(z^2+1)} + \frac{C}{z+2}$$

$\Rightarrow 5z = (Az+B)(z+2) + C(z^2+1)$

$\Rightarrow 5z = Az^2 + Bz + 2Az + 2B + Cz^2 + C$ —— (1)

For, $z = -2$

$\Rightarrow -10 = (-2A+B)(-2+2) + C(4+1)$

$\Rightarrow -10 = 0 + 5C$

$\therefore C = -2.$

For, $z = 0$

$0 = (0+B)(0+2) + C(1)$

$\Rightarrow 2B + C = 0.$

$\Rightarrow 2B = -C$

(14)

$\Rightarrow B = \frac{-C}{2}$

$= \frac{2}{2} = 1.$

From equation (i)

$A + C = 0$

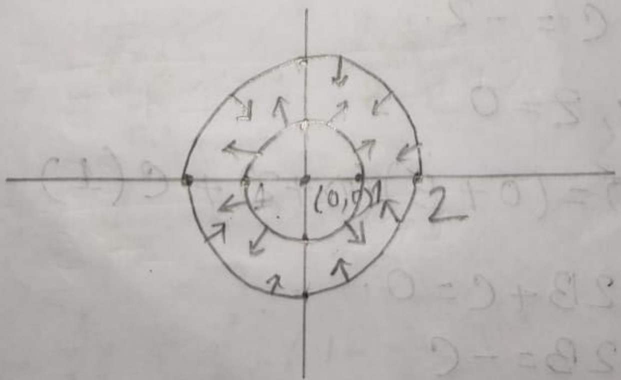
$\Rightarrow A = -C$

$\therefore A = 2$

So, $f(z) = \frac{2z+1}{z^2+1} - \frac{2}{z+2}$

(a) $1 < |z| < 2$

center (0,0) radius less than 2 greater than 1.



(15)

$1 < |z|$ and $|z| < 2$

so, $\frac{1}{z} < 1$ so, $\frac{|z|}{2} < 1$

$\therefore \frac{1}{|z^2|} < 1$

$$f(z) = \frac{2z+1}{z^2+1} - \frac{2}{z+2}$$

$$= \frac{2z+1}{z^2+1} - \frac{2}{z+2}$$

$$= \frac{2z}{z^2+1} + \frac{1}{z^2+1} - \frac{2}{z+2}$$

$$= \frac{2z}{z^2\left(1+\frac{1}{z^2}\right)} + \frac{1}{z^2\left(1+\frac{1}{z^2}\right)} - \frac{2}{2\left(1+\frac{z}{2}\right)}$$

$$= \frac{2}{z}\left(1+\frac{1}{z^2}\right)^{-1} + \frac{1}{z^2}\left(1+\frac{1}{z^2}\right)^{-1} - \left(1+\frac{z}{2}\right)^{-1}$$

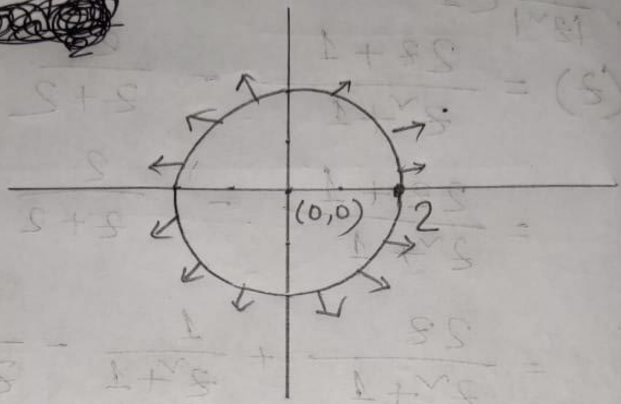
$$= \frac{2}{z}\left[1-\left(\frac{1}{z^2}\right)+\left(\frac{1}{z^2}\right)^2-\left(\frac{1}{z^2}\right)^3+\cdots\right] + \frac{1}{z^2}\left[1-\left(\frac{1}{z^2}\right)^2\right.$$

$$\left.+\left(\frac{1}{z^2}\right)^2-\left(\frac{1}{z^2}\right)^3+\cdots\right] - \left[1-\left(\frac{z}{2}\right)+\left(\frac{z}{2}\right)^2-\left(\frac{z}{2}\right)^3+\cdots\right]$$

which is required Laurent series

(b) $|z| > 2$

here, $|z| > 2$ so it also $|z| > 1$.



$2 < |z|$ and $1 < |z|$

so, $\frac{2}{z} < 1$ and $\frac{1}{z} < 1$

$\frac{1}{|z^2|} < 1$.

$$f(z) = \frac{2z+1}{z^2+1} - \frac{2}{z+2}$$

$$= \frac{2z}{z^2+1} + \frac{1}{z^2+1} - \frac{2}{z+2}$$

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$$= \frac{2z}{z^2\left(1+\frac{1}{z^2}\right)} + \frac{1}{z^2\left(1+\frac{1}{z^2}\right)} - \frac{2}{z\left(1+\frac{2}{z}\right)}$$

$$= \frac{2}{z}\left(1+\frac{1}{z^2}\right)^{-1} + \frac{1}{z^2}\left(1+\frac{1}{z^2}\right)^{-1} - \frac{2}{z}\left(1+\frac{2}{z}\right)^{-1}$$

$$= \frac{2}{z}\left[1-\left(\frac{1}{z^2}\right)+\left(\frac{1}{z^2}\right)^2-\left(\frac{1}{z^2}\right)^3+\cdots\right] + \frac{1}{z^2}\left[1-\left(\frac{1}{z^2}\right)+\left(\frac{1}{z^2}\right)^2 - \left(\frac{1}{z^2}\right)^3+\cdots\right] - \frac{2}{z}\left[1-\left(\frac{2}{z}\right)+\left(\frac{2}{z}\right)^2-\left(\frac{2}{z}\right)^3+\cdots\right]$$