

Final Term Sample Question -2022 ①

Solution

1.(a) sketch the path and its orientation given by

$$z(t) = 3e^{it} \quad (0 \leq t \leq \pi)$$

$$\Rightarrow x(t) + iy(t) = 3(\cos t + i \sin t)$$

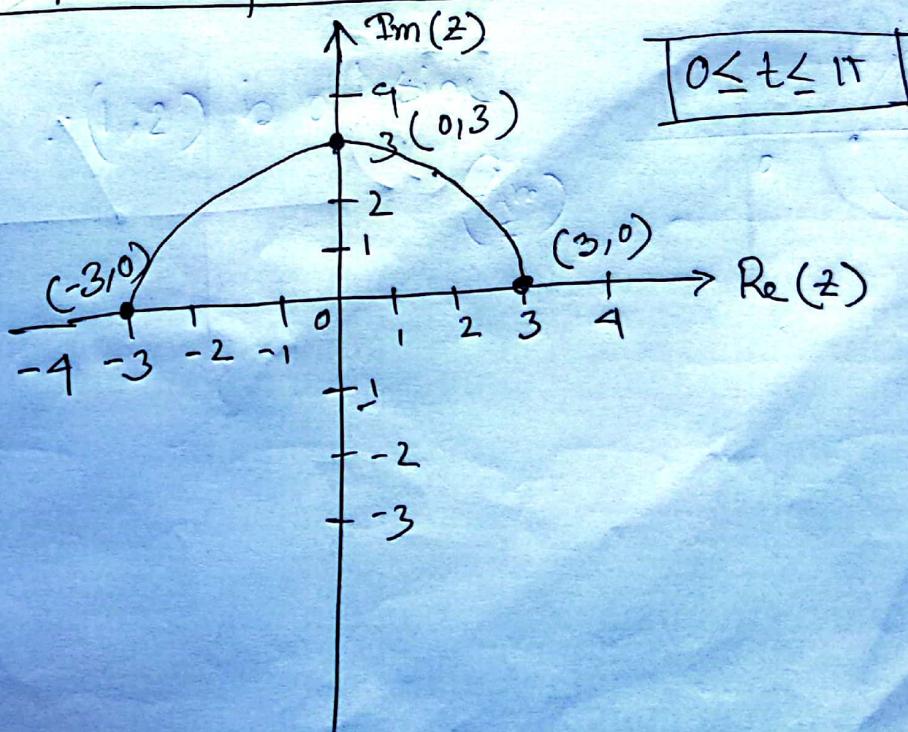
$$\Rightarrow x(t) + iy(t) = 3 \cos t + i 3 \sin t.$$

Separating Real & Imaginary Part;

$$x(t) = 3 \cos t$$

$$y(t) = 3 \sin t$$

t	$x(t)$	$y(t)$	(x, y)
0	3	0	(3, 0)
$\pi/2$	0	3	(0, 3)
π	-3	0	(-3, 0)



(b) Sketch and represent $|z - 2i| = 5$ parametrically, counter-clockwise.

$$\Rightarrow |z - 2i| = 5$$

$$\Rightarrow |x + iy - 2i| = 5$$

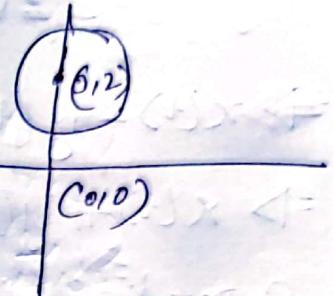
$$\Rightarrow |x + i(y-2)| = 5$$

$$\Rightarrow x^2 + (y-2)^2 = 25$$

$r = 5$;
Centre $(0, 2)$

Here; $x = r \cos \theta$ $y = r \sin \theta$

$$\therefore x = 5 \cos \theta \quad y = 5 \sin \theta$$



$$\therefore x(t) = 5 \cos \theta$$

$$\therefore y(t) = 2 + 5 \sin \theta$$

$$[\pi \leq \theta \leq 0]$$



Ex 1) Test whether the point $(1, 2)$ is interior, exterior or boundary of $|z - 3| = 2$

$$\text{of } |z - 3| = 2$$

\Rightarrow

$$|z - 3| = 2$$

$$\Rightarrow |x + iy - 3| = 2$$

$$\Rightarrow |(x-3) + iy| = 2$$

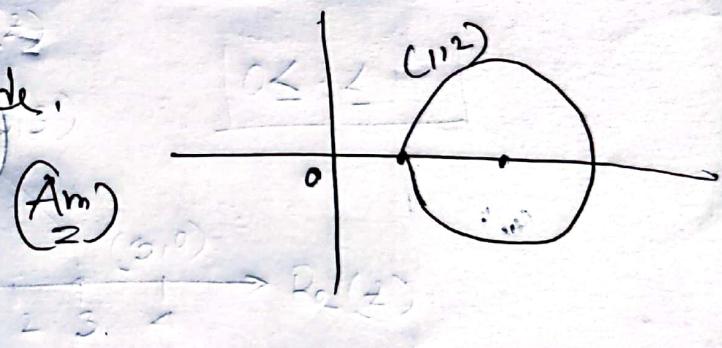
$$\Rightarrow (x-3)^2 + y^2 = 2^2$$

$$\Rightarrow (1-3)^2 + (2)^2 = 2^2$$

Here; $\Rightarrow 9 + 4 > 4$ ($\therefore 1, 2$ is exterior)

$$\Rightarrow 13 > 4$$

$\therefore (1, 2)$ is outside.



(d) Find the equation of the path C , which is the line segment from $z = 1$ to $z = 2 + i$.

$$\Rightarrow \text{Let, } z = 1 \text{ to } z = 2 + i \quad \begin{matrix} (1, 0) \\ x_1, y_1 \end{matrix}$$

$$x = t \quad \begin{matrix} (2, 1) \\ x_2, y_2 \end{matrix}$$

$$y = t - 1 \quad s = |z - p| + \begin{matrix} \text{Here,} \\ x_1 = 1 \\ x_2 = 2 \\ y_1 = 0 \\ y_2 = 1 \end{matrix}$$

$$\text{Here, } \frac{x - x_1}{y - y_1} = \frac{x_2 - x_1}{y_2 - y_1} \quad \begin{matrix} s = |z - p| + (z - p) \\ x_1 = 1 \\ x_2 = 2 \\ y_1 = 0 \\ y_2 = 1 \end{matrix}$$

$$\Rightarrow y = x - 1 \quad \Rightarrow \frac{x - 1}{y - 0} = \frac{1 - 2}{0 - 1} \quad \begin{matrix} x - 1 \\ y \\ -1 \end{matrix}$$

$$\Rightarrow x - 1 = y \quad \Rightarrow \frac{x - 1}{y} = \frac{-1}{-1} \quad \begin{matrix} x \\ y \\ -1 \end{matrix}$$



(2)

(e) Find the singular point (pole) and corresponding order of it

$$f(z) = \frac{z^2}{(z+1)^3}$$

Here,

$$(z+1)^3 = 0$$

$$\Rightarrow (z)^3 + (1)^3 = 0$$

$$\Rightarrow z^3 = -(z+iy+1)^3 = 0$$

$$\Rightarrow (x+1)^3 + iy^3 = 0$$

$$\Rightarrow (x+1)^3 + y^3 = 0$$

$$z = \frac{1}{y} \leftarrow \therefore n=0$$

$$\text{center} = (0, 1)$$

$$z = 1 + iy \leftarrow$$

$$z = |z| \leftarrow$$

(Ans)

$$z \in \mathbb{C} \setminus \{(0, 1)\}$$

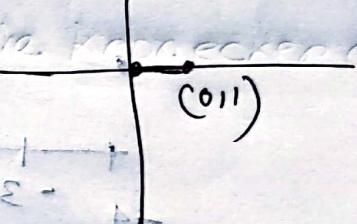
(Ans)

$$\Rightarrow x(z) = \sum_{n=0}^{\infty} [a_n z^n]$$

$$= 0$$

$$= z^2$$

(Ans)



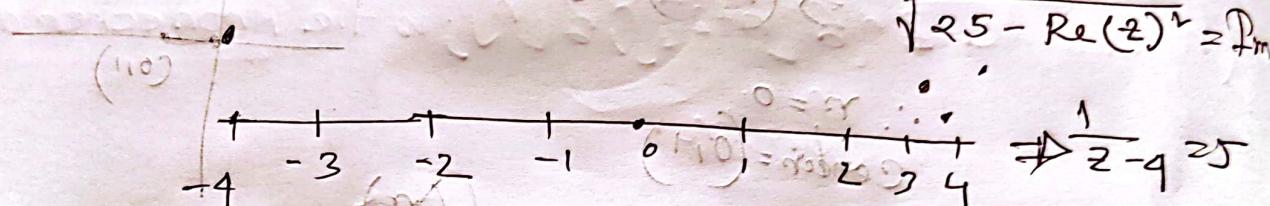
(f) find the residue at singular point $Off(z) = \frac{z+1}{(z-1)^2}$ for (13)

$$\text{Here, } f(z) = \frac{z+1}{z-1}; |z| = 5$$

$$\therefore \left\{ \frac{1}{z-4} = 5; z=4; |z|=5 \right\}$$

$$\therefore \{ z = \frac{21}{5}; z = 4$$

$$\text{Residue}(z) \geq 5 \wedge \text{Re}(z) \leq 5 \wedge \sqrt{\sqrt{25} - \text{Re}(z)} = -\text{Im}(z)$$



$$\Rightarrow z+1=5$$

(Ans)

3) Evaluate $Z\{(-1)^n u[n]\}$; $u[n]$ is the discrete time unit step function.

$$\Rightarrow L_z \left[(-1)^n u[n] \right] = \sum_{n=0}^{\infty} (-1)^n u(n) z^{-n}$$

$$\Rightarrow L_z \left[(-1)^n u[n] \right] = \sum_{n=0}^{\infty} (-1)^n z^{-n} u(n)$$

$$= \frac{1}{1 - \frac{1}{z}}$$

(n) Evaluate $Z\{s[n+2]\}$; $s[n]$ is the Kronecker delta function.

$$\Rightarrow Z\{s[n+2]\}$$

$$\Rightarrow X(z) = \sum_{n=-\infty}^{\infty} s[n+2] z^{-n}$$

$$= (1) z^2$$

$$= z^2$$

$$(A_m)$$

i) Evaluate $Z\{ (2^n - 3) u[n] \}$. Also find ROC.

$$\Rightarrow Z(2^n - 3) u[n]$$

$$\Rightarrow Z(2^n) \cdot Z(-3)$$

$$\Rightarrow Z(nu, 2^n - 3nu)$$

(ii) $\boxed{\Rightarrow nu z + 2^n - 3nu z} \quad (\text{Ans})$

j) Evaluate $Z^{-1}\{z^4\}$

$$\Rightarrow Z^{-1}\{z^{-4}\}$$

$$z^{-1} z^{-4}$$

$$\Rightarrow z^{-5}$$

$$(\text{Ans})$$

$$z^{-5} = z^{-2} + \sum_{n=0}^{\infty} x^n$$

$$z^{-2} = 1 -$$

$$z^{-5} =$$

(Ans)

k) Evaluate $z^{-1} \left\{ \frac{1}{1-2z^{-1}} \right\}; |z| > 2$

$$\Rightarrow z^{-1} \left\{ \frac{1}{1-2z^{-1}} \right\}$$

$$\therefore x^n = (-2)^n u[n] \quad (\text{Ans})$$

l) Evaluate $z^{-1} \left\{ \frac{z^{-1}}{1+3z^{-1}} \right\}; |z| > 3$

$$\Rightarrow z^{-n} \left\{ \frac{z^{-1}}{1+3z^{-1}} \right\} \stackrel{x = \frac{1}{z}}{\rightarrow} \frac{1}{1+3x^2} = \frac{1}{(1+\sqrt{2})^2}$$

$$\Rightarrow \frac{1}{1+3z^{-1}}$$

$$\Rightarrow x^n = (+3)^n u[n] \quad (\text{Ans})$$

m) Evaluate $\frac{z^{-1} + 1/3z^{-1}}{1 - 1/9z^{-2}} \rightarrow j; |z| > 3$

$$(C.V) \Rightarrow \frac{z^{-n} + 1/3z^{-n}}{z^{-n} - 1/9}$$

$$(E.C) \quad \cancel{z} \frac{z(z+1/3)}{(z+1/3)(z-1/3)} = \frac{z}{(z-1/3)}$$

$$(F) \quad \therefore x[n] = (1/3)^n u[n] \quad (\text{Ans})$$

Shot on OnePlus

By Pracchad

(n) Sketch the path and its orientation given by $z(t) = 2\sin(t)i + 3\cos(t) + 3 + 2i$; ($0 \leq t \leq 2\pi$)

$$\Rightarrow z(t) = 2\sin(t)i + 3\cos(t) + 3 + 2i; (0 \leq t \leq 2\pi)$$

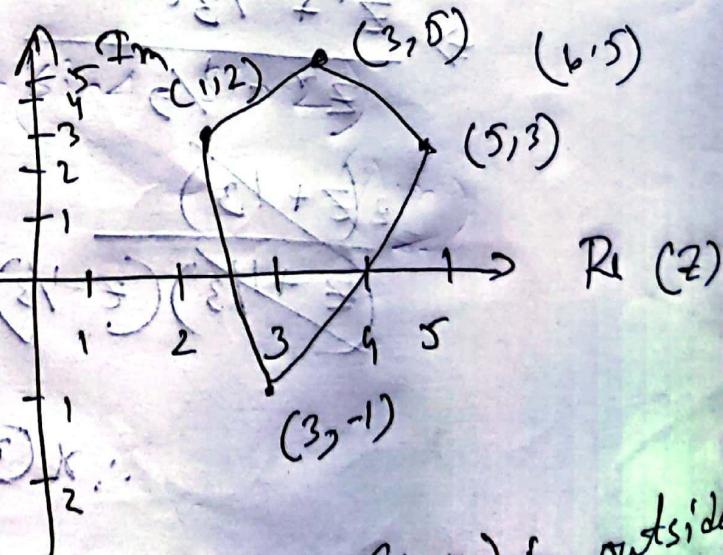
$$\Rightarrow x(t) + iy(t) = 2\sin t + 3 + i(3\cos t + 2)$$

Real and imaginary Part;

$$x(t) = 2\sin t + 3$$

$$y(t) = 3\cos t + 2$$

t	$x(t)$	$y(t)$	(x, y)
0	3	5	(3, 5)
$\pi/2$	5	3	(5, 3)
π	3	-1	(3, -1)
$3\pi/2$	1	2	(1, 2)
2π	3	5	(3, 5)



Point $(6, 5)$ is outside

(0) Sketch and represent the line segment from $1+i$ to $4-2i$ parametrically.

⇒ The equation of straight line passing through the point is $(1, 1)$ to $(4, -2)$ is

$$y-1 = \frac{(-2-1)}{(4-1)} (x-1) \quad (line) = 5$$

$$\text{That is } y = -x + 2$$

Let; $x = t$, then $y = -t + 2$ where t varies

from $t = 1$ to $t = 4$

So, the parametric equation of line segment

from $1+i$ to $4-2i$ is

$$x(t) = t; y(t) = -t + 2 \quad (1 \leq t \leq 4) \quad (\text{Ans})$$

$$(z = 1+3i) + (3-2i) \frac{1}{3}$$

$$(w) z = 13$$

$$(z = 13)(1-13) = \overline{(2-13)}$$

2. Sketch the path C , where C consists of the line segments

$z=2$ to $z=4$ and hence evaluate $\int_C (e^{2z} + \cos z) dz$.

(2, 0) (4, 0)



$$z = (x+iy)$$

$$\Rightarrow x+i0$$

$$\therefore z = x$$

$$\frac{dz}{dx} = \frac{d}{dx}(x) = e^{2x} + \cos x = e^{2x} - 1$$

$$\therefore dz = dx = [dx] \text{ with } x = t \text{ (let)}$$

$$\text{so, } \int (e^{2z} + \cos z) dz$$

$$= \int_2^4 (e^{2x} + \cos x) dx$$

$$\Rightarrow \left[\frac{e^{2x}}{2} \right]_2^4 + [\sin x]_2^4 = (f) \times$$

$$\Rightarrow \frac{1}{2} (e^8 - e^4) + (\sin 4 - \sin 2)$$

(Ans)

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(4)

Sketch the path C , where C consists of line segment from $z = i$ to $z = i+1$ and hence evaluate $\int_C (3z^2 - 2) dz$

$$(1, 0) \rightarrow (1, 1)$$

$$\Rightarrow z = x + iy$$

$$\Rightarrow 1 + iy$$

$$\therefore z = iy$$

$$dz = idy$$

$$\text{So, } \int_C (3z^2 - 2) dz$$

$$\Rightarrow \int_0^1 (3iy^2 - 2) idy$$

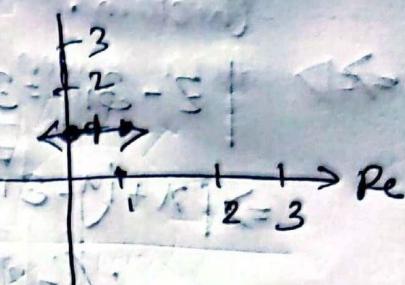
$$= \int_0^1 (3iy^2 - 2) idy$$

$$= \frac{i(3i^2 - 4)}{2} \Big|_0^1$$

$$= 3$$

$$= \frac{-4i + 3}{2}$$

$$= -1.5 - 2i \quad (\text{Ans})$$



(b) Sketch the path C , where C is the circle $|z - 3i| = 3$ (clockwise) and hence evaluate $\int_C \left[\frac{3}{(z - 3i)^n} \right] dz$ (Expand)

$$\Rightarrow |z - 3i| = 3$$

$$\Rightarrow |x + iy - 3i| = 3$$

$$\Rightarrow |x + i(y - 3)| = 3$$

$$\Rightarrow \sqrt{x^2 + (y - 3)^2} = 3$$

$$= x^2 + (y - 3)^2 = 3^2$$

$\therefore r = 3$; Center: $(0, 3)$

$$\text{By } x = r \cos \theta$$

$$y = r \sin \theta$$

$$\therefore x(t) = 3 \cos \theta (3, 0)$$

$$y(t) = 3 + 3 \sin \theta$$

$$\int_C \left[\frac{3}{(z - 3i)^n} \right] dz$$

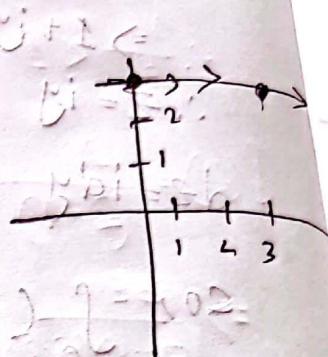
$$= \int_0^3 \left[\frac{3}{(z - 3i)^n} \right] dz$$

$$= 3 \left(\frac{i}{3} - \frac{i+1}{6} \right)$$

$$= \frac{i-1}{2}$$

$$= -0.5 + 0.5i$$

$$(3, 0) \quad (3, 3)$$



$$z = x + iy \\ z = 3 \text{ for } y = 3$$

$$= 3 + iy$$

$$dz = (x - 3 + iy)' dt = 3i dt$$

$$dz = \frac{dx}{dt} (t) dt$$

$$dz = dt$$

$$z + i\theta$$

$$= \frac{1}{z - 3 + i\theta}$$

AM

1) Expand $f(z) = \frac{3z}{(z-1)(z-2)}$ in a Laurent Series Valid for $|z| > 2$

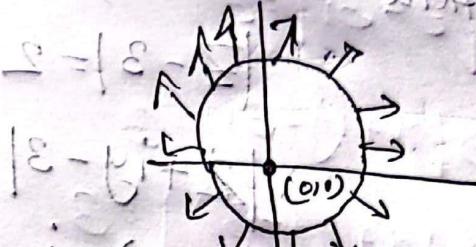
\Rightarrow If $|z| > 2$ then it must be $|z| > 1$

$$1 < |z|$$

$$\text{and } 2 < |z|$$

$$\therefore \frac{1}{z} < 1$$

$$\text{and } \frac{2}{z} < 1$$



$$f(z) = \frac{A}{z-1} + \frac{B}{z-2}$$

$$\Rightarrow \frac{3z}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$\Rightarrow 3z = A(z-2) + B(z-1)$$

$$\therefore \text{for } z=2;$$

$$3 \times 2 = A(2-2) + B(2-1)$$

$$6 = 0 + B \Rightarrow B = 6$$

$$A = 3$$

$$\therefore f(z) = \frac{3}{z-1} + \frac{6}{z-2}$$

$$= \frac{3}{z\left(\frac{1}{z}-\frac{1}{z}\right)} + \frac{6}{z\left(\frac{1}{z}-\frac{2}{z}\right)}$$

$$\Rightarrow \frac{3}{z} \left(\frac{1}{\frac{1}{z}-\frac{1}{z}} + \frac{1}{\frac{1}{z}-\frac{2}{z}} + \frac{1}{\frac{1}{z}-\frac{3}{z}} + \dots \right) - \frac{6}{z} \left(\frac{1}{\frac{1}{z}-\frac{1}{z}} + \frac{1}{\frac{1}{z}-\frac{2}{z}} + \frac{1}{\frac{1}{z}-\frac{3}{z}} + \dots \right)$$

which is required Laurent Series

(Ans)

③ Evaluate $\oint_C \frac{2z-1}{(z-3)^n} dz$, $C: |z-3|=2$ using the Cauchy residue theorem

Theorem:

\Rightarrow Here given that;

$$|z-3|=2$$

$$|z+iy-3|=2 \Rightarrow \frac{2}{\pi} \text{ bmo}$$

$$\Rightarrow (z-3)^{n+1}$$

$$\Rightarrow |(z)+i(y-3)|=2 \Rightarrow \frac{2}{|z-y|} + \frac{A}{|z-y|} = (S)$$

$$= (x)^n + (y-3)^n = (2)^n \frac{2^n}{(1-\frac{y}{2})^n} = \frac{2^n}{(1-\frac{y}{2})(1-\frac{y}{2})^{n-1}}$$

$\therefore n=2$, Center $(0, 3)$

for singularity;

$$z^n = 0$$

$$\Rightarrow z^n = 9$$

$$\Rightarrow z^n = 3^2$$

$$\Rightarrow z = 3$$

Residue at $z=3$

$$\text{Res}(z=3) = \lim_{z \rightarrow 3} \frac{1}{z-3} \frac{f(z)}{(z-3)^n} = \frac{1}{1-0} = (S)$$

$$= \lim_{z \rightarrow 3} \left(\frac{1}{z-3} \cdot \frac{2z-1}{(z-3)^n} dz \right)$$

$$\Rightarrow \frac{1}{3-3} \cdot \frac{2 \times 3 - 1}{(3-3)^n} dz$$

Since trivial $\Rightarrow 0 \times \frac{6-1}{0}$ or nothing

Q) Expand $f(z) = \frac{2z+5}{(z-2)(1+z)}$ in a Laurent series valid for $1 < |z| <$

$$\Rightarrow 1 < |z| < 2$$

If $|z| < 1$, then it must be $|z| < 2$

$$|z| < 1$$

$$\text{So, } \frac{|z|}{1} < 1$$

$$|z| < 2$$

$$\frac{|z|}{2} < 1$$

$$f(z) = \frac{2z+5}{(z-2)(1+z)}$$

$$\text{So, } \frac{2z+5}{(z-2)(1+z)} = \frac{A}{(z-2)} + \frac{B}{(1+z)}$$

$$\Rightarrow 2z+5 = A(1+z) + B(z-2)$$

$$\Rightarrow \text{For } z=2$$

$$2 \times 2 + 5 = A(1+2) + B(2-2)$$

$$\Rightarrow 9 = A \times 3 + B \times 0$$

$$\Rightarrow 9 = 3A$$

$$\Rightarrow 3A = 9 \Rightarrow A = 3$$

$$\Rightarrow A = 3$$

$$\Rightarrow A = 3$$

$$\text{So, } f(z) = \left[\frac{6}{z-2} + \frac{2A-7}{1+z} \right] = 9 \dots$$

$$= \frac{6}{z(1-\frac{2}{z})} + \frac{2A-7}{z(1+\frac{1}{z})}$$

$$= \frac{6}{z} \left[1 + \frac{1}{z} + \left(\frac{2}{z}\right)^2 + \frac{2A-7}{z^2} \right] + \frac{2A-7}{z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right)$$

which is required Laurent Series.

(d) Evaluate $\mathcal{P}[X_{2-1}]$

3.
(a) ii')

$$x(z) = \frac{1}{(1+0.5z^{-1})(1-0.5z^{-1})(1-z^{-1})}; |z| > 1$$

$$x(z) = \frac{z^3}{(z+1/2)(z-1/2)(z-1)}$$

$$\frac{x(z)}{z} = \frac{z^2}{(z+1/2)(z-1/2)(z-1)}$$

$$= \frac{A}{(z+1/2)} + \frac{B}{(z-1/2)} + \frac{C}{(z-1)}$$

$$(z-1) \therefore A = \left[\frac{z^2(z+1/2)}{(z+1/2)(z-1/2)(z-1)} \right]_{z=-1/2} = \frac{1}{6}$$

$$\therefore B = \left[\frac{z^2(z-1/2)}{(z+1/2)(z-1/2)(z-1)} \right]_{z=1/2} = \frac{1}{4}$$

$$\therefore x(z) = -0.5 \left(\frac{1}{z+1/2} + \frac{1}{z-1/2} + \frac{1}{z-1} \right)$$

$$\therefore C = \left[\frac{z^2(z-1)}{(z+1/2)(z-1/2)(z-1)} \right]_{z=1} = -\frac{1}{2}$$

$$= \frac{-1}{(\frac{1}{2}+j\frac{3}{4})^2} = \frac{4}{(-\frac{3}{4})^2}$$

$$\therefore \frac{x(z)}{z} = -\frac{1}{6} - \frac{\frac{1}{2}(1 + \frac{1}{z})}{(z + \frac{1}{2})} - \frac{\frac{1}{2}(1 - \frac{1}{z})}{(z - \frac{1}{2})} + \frac{\frac{4}{3}}{(z - 1)^{-1}}$$

$$\therefore x(z) = -\frac{1}{6}z(z + \frac{1}{2})^{-1} - \frac{1}{2}z(z - \frac{1}{2})^{-1} + \frac{4}{3}z^{-1}$$

inverse of z transformation ;

$$x[n] = \frac{1}{6}(-\frac{1}{2})^n u[n] - \frac{1}{2}(\frac{1}{2})^n u[n] + \frac{4}{3}u[n]$$

$$\Rightarrow x[n] = \frac{1}{6}(-0.5)^n u[n] - \frac{1}{2}(0.5)^n u[n] + \frac{4}{3}u[n]$$

(Ans)

$$\frac{2s+10}{2s+5} = \frac{2s+10}{1+2s+5} = \frac{2s+10}{2s+5} = \frac{2s+10}{2s+5}$$

$$\Rightarrow s = -\frac{2s+10}{3+5}$$

$$\Rightarrow s = \frac{2s+10}{2s+5} + \frac{1}{2s+5} s = \frac{2s+10}{2s+5} + \frac{1}{2s+5}$$

$$\Rightarrow -s = \frac{2s+10}{2s+5} + \frac{1}{2s+5} s = \frac{2s+10}{2s+5} + \frac{1}{2s+5}$$

$$\Rightarrow -s = \frac{2s+10}{2s+5} + \frac{1}{2s+5} s = \frac{2s+10}{2s+5} + \frac{1}{2s+5}$$

inverse transform is required

$$\Rightarrow x[n] = 0.5^n u[n] - 1.5^n u[n]$$

$$\textcircled{1} \quad X(z) = \frac{5}{(1+z^{-1})(1+0.25z^{-1})} \quad ; \quad |z| > 1$$

$$\Rightarrow X(z) = \frac{z}{(z+1)(z-0.25)}$$

$$\Rightarrow \frac{X(z)}{z} = \frac{1}{(z+1)(z-0.25)}$$

$$[x(n)]^H + [x(n)] = \frac{A}{(z+1)} + \frac{B}{(z-0.25)}$$

$$\therefore A = \left[\frac{z}{z-0.25} \right]_{z=1} = \frac{1}{1-0.25} = \frac{1}{0.75}$$

$$\therefore B = \left[\frac{z}{z+1} \right]_{z=0.25} = \frac{0.25}{0.25+1} = \frac{0.25}{1.25}$$

$$\therefore \frac{X(z)}{z} = \frac{\frac{1}{0.75}}{z+1} + \frac{\frac{0.25}{1.25}}{z-0.25}$$

$$\therefore X(z) = \frac{1}{0.75} \left(\frac{z}{z+1} \right) + \frac{0.25}{1.25} \left(\frac{z}{z-0.25} \right)$$

inverse z-transform

$$x[n] = \frac{1}{0.75} u[n] + \frac{0.25}{1.25} (0.25)^n u[n]$$

(Ans)

$$\begin{aligned}
 (b) \quad x(z) &= (1+2z) (1-z^{-1}) (2+z^{-2}) \\
 &= (1+2z+z^{-2}+4) (1-z^{-1}) \\
 &= (1+2z+z^{-2}+4-z^{-1}-2-2z^{-2}-4z^{-1}) \\
 &= (3-5z^{-1}+2z)
 \end{aligned}$$

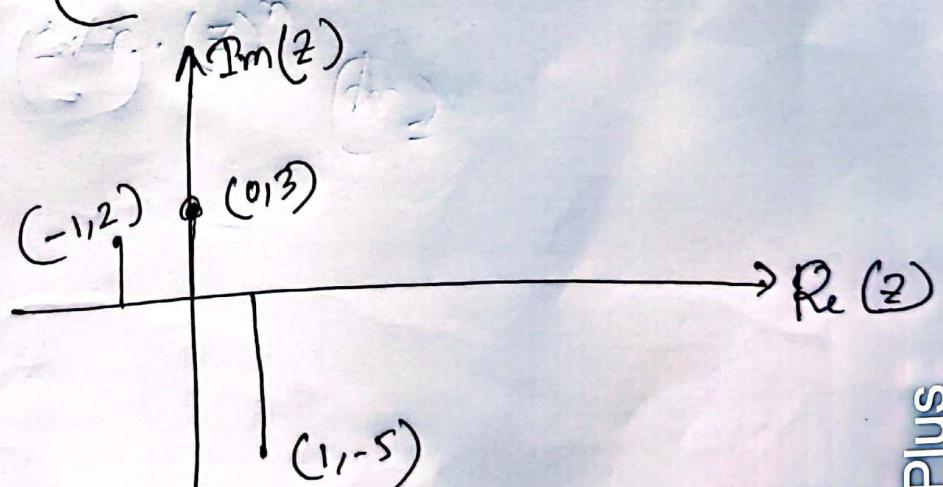
$$\therefore z^{-1} \{x(z)\} = 3S[n] - 5S[n-1] + 2S[n+1]$$

$$\Rightarrow \therefore x(n) = 3s[n] + 2s[n+1] - 5s[n-1]$$

equivalently,

$$x[n] = \begin{cases} 2 & ; n=1 \\ 3 & ; n=0 \\ -5 & ; n=-1 \end{cases}$$

$$\Rightarrow y[n] =$$



(Res)

$$(d) Y[n+2] - 4Y[n+1] + 4Y[n] = 0, \quad Y[0] = 0; \quad Y[1] = 6;$$

$$\Rightarrow z^2[Y(z) - Y[0] - Y[1]z^{-1}] + 4z[Y(z)] = Y[0] + 4Y[1]$$

$$\Rightarrow (z^2 - 4z + 4)Y(z) - 6z = 0$$

$$\Rightarrow Y(z) = \frac{6z}{z^2 - 4z + 4}$$

$$\Rightarrow Y(z) = \frac{6z}{(z-2)^2}$$

$$\Rightarrow Y(z) = \frac{3 \cdot 2^z}{(z-2)^2}$$

$$\therefore Y[n] = 3 \cdot n \cdot (2)^n$$

(b) Impulse Response

Ara)

$$h[n] = (-1)^n \cdot n! \cdot 2^n$$

$$(s_1, s_2, \dots, s_n) = (s_1, s_2, \dots)$$

$$(s_{10}, s_{11}, \dots)$$

$$(z-1)$$

CA

$$\begin{aligned} & \because Y[n] - 4Y[n-1] - 12Y[n-2] = 0, \quad Y(-1) = 0, \quad Y(-2) = 1 \\ & \Rightarrow Y[2] - 4z[Y(z) - Y(0)] - Y(1)z^2 = 12z[Y(z) - Y(0)Y(z)z^{-1}] \\ & \Rightarrow (z^2 - 4z - 12)Y(z) + 12 = 0 \\ & \Rightarrow Y(z) = \frac{z}{z^2 - 4z - 12} \\ & \Rightarrow Y(z) = \frac{2}{z^2 - 6z + 2z - 12} = \frac{2}{z(z-6) + 2(z-6)} \\ & \Rightarrow Y(z) = \frac{2}{(z-6)(z+2)} \\ & \Rightarrow Y[n] = 2 \cdot n \cdot (z)^n \end{aligned}$$

$$\textcircled{e} @ Y[n] + Y[n-1] = X[n+1]$$

$$\Rightarrow Y[z] + z^{-1}[Y(z) + Y(-1)]z^1 = \frac{z^1[Y(z) + Y(z) + Y(-1)]}{[Y(z) + Y(-1)]z^1}$$

$$\Rightarrow Y[z] + z^{-1}[Y(z)] = \cancel{z^1} \cancel{z^1} \times [z^1]$$

$$\Rightarrow Y[z] + (1 + z^{-1}) = \cancel{\left(\frac{z^1 + 1}{z^1} \right)} z^1 \times [z^1]$$

$$\therefore \frac{Y[z]}{X[z]} = \frac{z^1}{1 + z^{-1}}$$

$$Y(z) + H(z) = \frac{z^1}{1 + z^{-1}} \quad (\textcircled{d})$$

① impulse response

$$h[n] = (-1)^n u[n]$$

(Ans)