MATLAB software version 2015

Problem Set 1.1

Sample MATLAB Code for problems 1-15:

Question: Find the Laplace transforms and also sketch (if free hand sketching is getting complex then use MATLAB) the following functions:

$$\boldsymbol{f(t) = (t^2 + 2t)\sin t}\,.$$

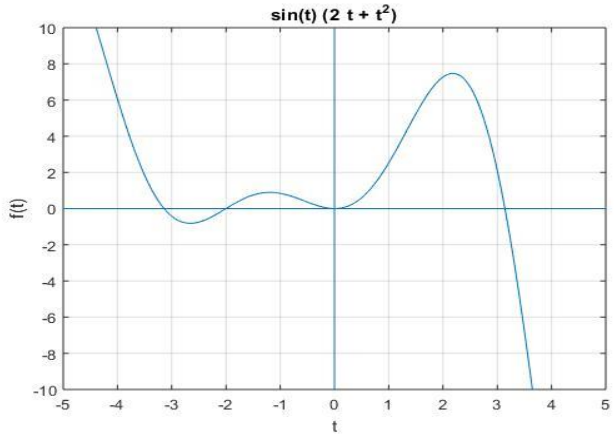
MATLAB Code:

```
syms t s; % defining symbolic variables
f=(t^2+2*t)*sin(t); % defining function
simplify(laplace(f,t,s)) % finding laplace transform
ezplot(f,[-5,5,-10,10]) % command to plot symbolic function
xlabel('t'); % labeling x-axis
ylabel('f(t)'); % labeling y-axis
line(xlim,[0 0]) % making x-axis visible
line([0 0],ylim) % making y-axis visible
grid. % showing grid
```

MATLAB Output:

ans =

(4*s^3 + 6*s^2 + 4*s - 2) / (s^2 + 1)^3



Sample MATLAB Code for problems 16-20:

Question: Find the Laplace transforms and also sketch (if free hand sketching is getting complex then use MATLAB) the following functions:

$$\textbf{20. } f(t) = \begin{cases} 1 - t\,; 0 \le t \le 1 \\ 0\,; \quad t > 1 \end{cases}$$

MATLAB Code:

```
syms t s;
i1=int((1-t)*exp(-s*t),0,1); %defining first part of the integral of the form
```

$$\int_0^1 (1-t)\, e^{-st}\, dt$$

```
i2=int(0*exp(-s*t),1,inf); %defining second part of the integral of the form
```

$$\int_1^\infty 0.\, e^{-st}\, dt$$

```
ans=simplify(i1+i2) summation of the previous two answers ans sampling.
```

MATLAB Output:

ans =

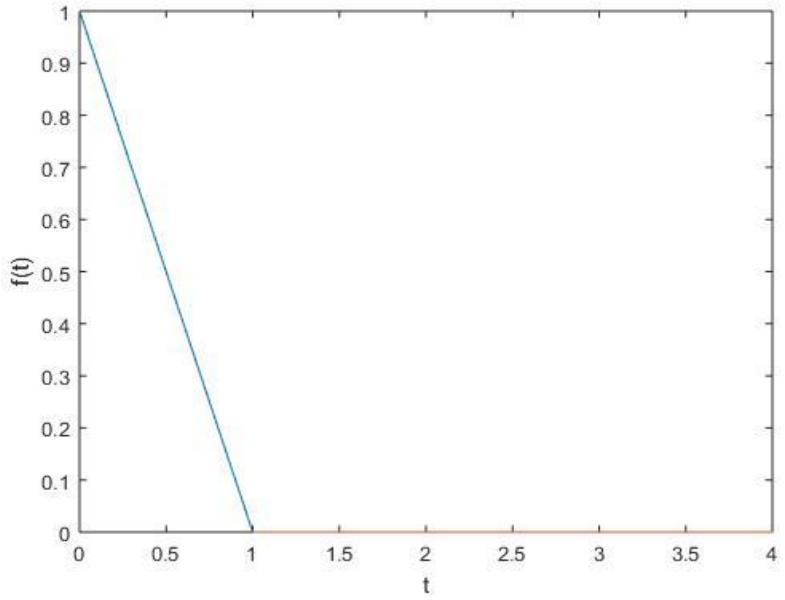
(s + exp(-s) - 1)/s^2

Plotting function:

MATLAB Code:

```
f=@(t) (t>=0 & t<1).*(1-t)+(t>1).*0; % defining piecewise function
t1=linspace(0,1,1000);% defining interval of first piece
t2=linspace(1,4,1000);% defining interval of second piece
plot(t1,f(t1),t2,f(t2))
xlabel('t');
ylabel('f(t)');
```

MATLAB Output:



Problem Set 1.2

Sample MATLAB Code for problems 21-29:

Question: Sketch the following function and find their Laplace transforms:

25. $f(t) = 4\cos t \;\; u(t-\pi)$

Matlab Code:

```
syms t s;
f=4*cos(t)*heaviside(t-pi); % defining Heaviside function
ans=laplace(f,t,s)
```

MATLAB Output:

ans =

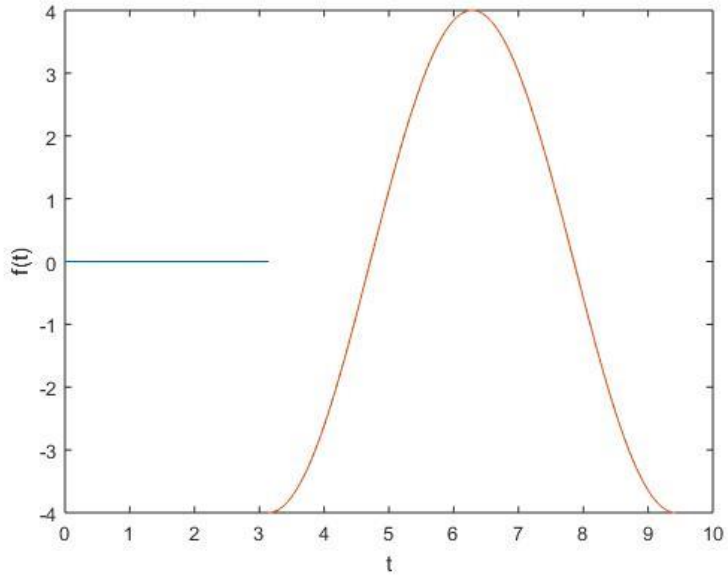
-(4*s*exp(-pi*s))/(s^2 + 1)

Plotting function:

MATLAB Code:

```
f=@(t)(t>0 & t<=pi).*0 + (t>pi).*(4*cos(t));
t1=linspace(0,pi,1000);
t2=linspace(pi+.0001,3*pi,1000); % pi + .0001 has been used to avoid unnecessary vertical line at point of discontinuity.
plot(t1,f(t1),t2,f(t2));
xlabel('t');ylabel('f(t)');
```

MATLAB Output:



Sample MATLAB Code for problems 30-32:

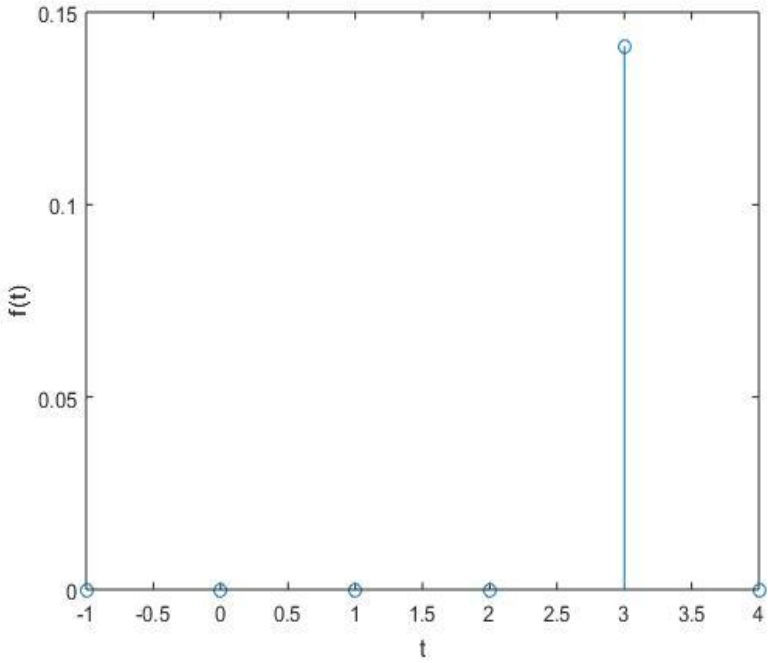
Question: Sketch the following functions and find their laplace transforms:

32. $f(t) = \sin t\, \delta(t-3)$.

MATLAB Code:

```
t = -1:4;
f= sin(t).*dirac(t-3); % defining Dirac's delta function
find = f == Inf; % find Infinity
f(find) = sin(3); % set Infinity to finite value
stem(t,f);
xlabel('t');
ylabel('f(t)');
```

MATLAB Output:



MATLAB Code:

```
syms t s;
f=sin(t)*dirac(t-3);
ans=laplace(f,t,s)
```

MATLAB Output

exp(-3 s) sin(3)

Problem Set 2.1 & 2.2

Sample MATLAB Code for problems 1-31:

4. $\boldsymbol{F(s) = \frac{2+4s}{s^2+25}}$

MATLAB Code:

```
syms t s;
F=(2+4*s)/(s^2+25);
ans=ilaplace(F,s,t)  % command to find inverse laplace  transform.
```

MATLAB Output

ans=

4*cos(5*t) + (2*sin(5*t))/5

10. $\boldsymbol{F(s) = \frac{s}{s^2+4s-9}}$.

MATLAB Code:

```
syms t s;
F=s/(s^2+4*s-9);
ans=ilaplace(F,s,t)
```

MATLAB Output:

exp(-2*t)*(cosh(13^(1/2)*t) - (2*13^(1/2)*sinh(13^(1/2)*t))/13)

19. $\boldsymbol{F(s) = \frac{s}{(s^2+4)\,(s-1)}}$.

MATLAB Code:

```
syms s t;
F=s/((s^2+4)*(s-1));
ans=ilaplace(F,s,t)
```

MATLAB Output:

```
ans =
(2*sin(2*t))/5 - cos(2*t)/5 + exp(t)/5
```

28. $\boldsymbol{F(s) = \frac{5\left(e^{-\pi s} + e^{-2\pi s}\right)}{s^2+25}}$

MATLAB Code:

```
syms s t;
F=5*(exp(-pi*s)+exp(-2*pi*s))/(s^2+25);
ans=ilaplace(F,s,t)
```

MATLAB Output:

```
ans =
sin(5*t)*heaviside(t - 2*pi) - sin(5*t)*heaviside(t - pi)
```

31. $\boldsymbol{f(t) = 25\,\delta(t-2)}$.

MATLAB Code:

```
syms s t;
F=25*exp(-2*s);
ans=ilaplace(F,s,t)
```

MATLAB Output:

```
25*dirac(t - 2)
```

Problem Set 3.1

Sample MATLAB code for problems 1-11

Question: Apply Laplace transform to solve the following ordinary differential equations and hence justify your ans:

8. $\ddot{y}(t) - 2\dot{y}(t) = \cos t\,;\;\; y(0) = 0, \dot{y}(0) = 1$

MATLAB Code: (Step 1)

```
syms y(t) equation(t) s;
equation(t)=diff(y(t),t,2)-2*diff(y(t),t,1)==cos(t);
```

% defining differential equation

```
ans1=laplace(equation(t),t,s)
```

% laplace transform of the ODE

MATLAB Output: (Step 1)

ans1=

2*y(0) - D(y)(0) - 2*s*laplace(y(t), t, s) - s*y(0) + s^2*laplace(y(t), t, s) == s/(s^2 + 1)

MATLAB Code: (Step 2)

```
syms lap1 y1 y2;
ans1=subs(ans1,{'laplace(y(t),t,s)','y(0)','D(y)(0)'},{lap1,y1,y2})
```

% code to substitute laplace(y(t),t,s), y(0) and D(y)(0) by lap1, y1 and y2 respectively in ans1.

MATLAB Ouput: (Step 2)

ans1 =

2*y1 - y2 - 2*lap1*s - s*y1 + lap1*s^2 == s/(s^2 + 1)

MATLAB Code: (Step 3)

```
ans2=solve(ans1,lap1)
```

% solving equation in ans1 for lap1 and storing it in ans2.

MATLAB Output: (Step 3)

ans2 =

-(y2 - 2*y1 + s*y1 + s/(s^2 + 1))/(- s^2 + 2*s)

MATLAB Code: (Step 4)

```
initial_cond=[y1 y2]; % recalling y1 and y2 for y(0) and ẏ(0)
initial_val=[0 1]; % setting initial value for y1=y(0) = 0 and y2=ẏ(0) = 1
ans3=subs(ans2,initial_cond,initial_val) % substituting initial value in last result ans2 and storing updated result in ans3.
```

MATLAB Output: (Step 4)

ans3 =

-(s/(s^2 + 1) + 1)/(- s^2 + 2*s)

MATLAB Code: (final step)

```
final_ans=ilaplace(ans3,s,t) % evaluating inverse laplace transform
```

MATLAB Output: (final step)

final_ans =

(7*exp(2*t))/10 - cos(t)/5 - (2*sin(t))/5 - 1/2

Justification of answer:

MATLAB Code:

```
if diff(final_ans,t,2)-2*diff(final_ans,t,1) = = cos(t)
   disp('Answer is correct')
end
```

MATLAB Output:

Answer is correct