AIUB COURSE SOLUTION

MATH-3

CHAPTER 1

AIUB COURSE SOLUTION

SOLVED BY ELME KHAN

AIUB COURSE SOLUTION

https://www.youtube.com/channel/UCC3KjA8kstFtM-2Cx

1.1

① $\mathcal{L}\{2e^{3t}-6\}^2$

$= \mathcal{L}\{4e^{6t} - 24e^{3t} + 36\}$

$= \mathcal{L}\{4e^{6t}\} - \mathcal{L}\{24e^{3t}\} + \mathcal{L}\{36\}$

$= \frac{4}{s-6} - \frac{24}{s-3} + \frac{36}{s}$ A

② $\mathcal{L}\{\sin^2 4t\}$

$= \mathcal{L}\{\frac{1}{2}(6 - \cos 8t)\}$

$= \frac{1}{2}\mathcal{L}\{6\} - \mathcal{L}\{6\cos 8t\}$

$= \frac{1}{2}\left\{\frac{6}{s} - \frac{6s}{s^2+64}\right\}$

$= \frac{3}{s} - \frac{3s}{s^2+64}$ A

(3)

(3) $\mathcal{L}\{t \sin 7t\}$

$$= -\frac{d}{dt}\left(\frac{7}{s^2+49}\right)$$

$$= \frac{14s}{(s^2+49)^2} \text{ Ans.}$$

(4) $\mathcal{L}\{t \cos 5t\}$

$$= -\frac{d}{dt}\left(\frac{s}{s^2+25}\right)$$

$$= \frac{s^2+25-2s.s}{(s^2+25)^2}$$

$$= \frac{s^2-25}{(s^2+25)^2} \text{ Ans.}$$

(4)

(5) $\mathcal{L}\left\{ \dfrac{\cos 3t}{e^{-5t}} \right\}$

$= \mathcal{L}\left\{ \cos 3t \; e^{5t} \right\}$

$= \dfrac{s-5}{(s-5)^2+3^2}$

$= \dfrac{s-5}{(s-5)^2+9}$ A.

(6) $\mathcal{L}\left\{ t^2 e^{-3t} \right\}$

$= \dfrac{2!}{(s+3)^3}$

$= \dfrac{2}{(s+3)^3}$ A

(5)

(7) $\mathcal{L}\{3t^5 e^{4t} + 4t\}$

$$= \frac{5!}{(s-4)^6} + \frac{4}{s^2} \quad A$$

(8) $\mathcal{L}\{t^3 e^{5t}\}$

$$= \frac{6}{(s-5)^4} \quad A$$

(9) $\mathcal{L}\{t \sinh at\}$

$$= -\frac{d}{ds} \cdot \frac{a}{s^2-a^2}$$

$$= \frac{2as}{(s^2-a^2)^2} \quad A$$

(10) $\mathcal{L}\{t \cos at\}$

$$= -\frac{d}{ds} \frac{s}{s^2+a^2}$$

$$= \frac{-s^2-a^2+2s^2}{(s^2+a^2)^2}$$

$$= \frac{s^2-a^2}{(s^2+a^2)^2}$$

(6)

11. $\mathcal{L}\{3\cos 4t + 4\sin 3t - t^3\}$

$$= \frac{3s}{s^2+16} + \frac{12}{s^2+9} - \frac{6}{s^4} \quad \text{Ans.}$$

(12) $\mathcal{L}\left\{\frac{1}{5}\sin 5t - 2\cos\left(\frac{t}{3}\right)\right\}$

$$= \frac{1}{s^2+25} - \frac{2s}{s^2+\frac{1}{9}}$$

$$= \frac{1}{s^2+25} - \frac{18s}{9s^2+1} \quad \text{Ans.}$$

(13) $\mathcal{L}\{e^{-t}\sin 4t - 3e^{-t}\cos 4t\}$

$$= \frac{4}{(s+1)^2+16} - \frac{3(s+1)}{(s+1)^2+16}$$

$$= \frac{4-3s-3}{(s+1)^2+16}$$

$$= \frac{1-3s}{(s+1)^2+16} \quad \text{Ans.}$$

(7)

(14) $\mathcal{L}\{e^{-t}\cos^2 t\}$

$= \mathcal{L}\{e^{-t}\frac{1}{2}(1+\cos 2t)\}$

$= \frac{1}{2}\mathcal{L}\{e^{-t} + e^{-t}\cos 2t\}$

$= \frac{1}{2}\left(\frac{1}{s+1} + \frac{s+1}{(s+1)^2+2}\right)$

$= \frac{1}{2s+2} + \frac{s+1}{2s^2+4s+10}$ A

(15) $\mathcal{L}\{\sin 3t + 2\cos t\}$

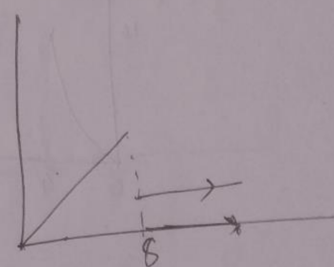
$= \mathcal{L}\{\sin 3t\} + 2\mathcal{L}\{\cos t\}$

$= \frac{3}{s^2+9} + \frac{2s}{s^2+1}$ A

(8)

Piecewise function

① $f(t) = \begin{cases} 4t, & 0 \le t \le 8 \\ 1, & t > 8 \end{cases}$



$$\mathcal{L}\{f(t)\} = \int_0^8 e^{-st}\, 4t\,dt + \int_8^\infty e^{-st}\, 1\, dt$$

$$= \left[\frac{-4t\, e^{-st}}{s} - \frac{4e^{-st}}{s^2}\right]_0^8 + \left[\frac{-e^{-st}}{s}\right]_8^\infty$$

$$= \frac{-32e^{-8s}}{s} - \frac{4e^{-8s}}{s^2} + \frac{4}{s^2} + \frac{e^{-8s}}{s}$$

$$= \frac{4}{s^2}\left(1 - e^{8s}\right) - \frac{31}{s} e^{-8s}$$

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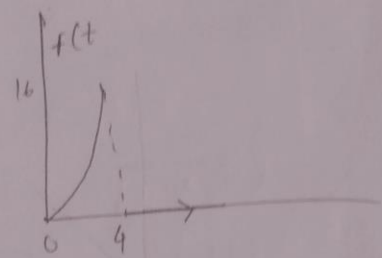
(5)

(2) sketch

$$f(t) = \begin{cases} t^2, & 0 \le t \le 4 \\ 0, & t > 4 \end{cases}$$



$$\mathcal{L}\{f(t)\} = \int_0^4 e^{-st}\, t^2\, dt + \int_4^\infty e^{-st}\, 0 \, dt$$

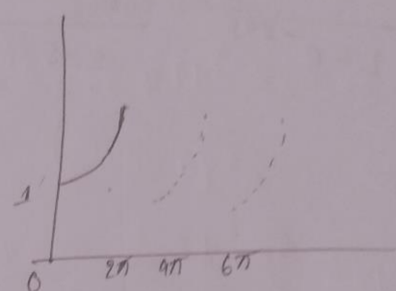
$$= \left[\frac{-t^2 e^{-st}}{s} - \frac{2t e^{-st}}{s^2} - \frac{2 e^{-st}}{s^3} \right]_0^4 + 0$$

$$= -\frac{16}{s} e^{-4s} - \frac{8}{s^2} e^{-4s} - \frac{2}{s^3} e^{-4s} + \frac{2}{s^3}$$

(10)

1.2

① $f(t) = e^{2t}, \quad 0 < t < 2\pi$, period $T = 2\pi$.



$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-st}} \int_0^{2\pi} e^{-st} e^{2t}\, dt$$

$$= \frac{1}{1-e^{-st}} \int_0^{2\pi} e^{-st+2t}\, dt$$

$$= \frac{1}{1-e^{-st}} \int_0^{2\pi} e^{(2-s)t}\, dt$$

$$= \frac{1}{1-e^{-2\pi s}} \left[\frac{e^{(2-s)t}}{2-s} \right]_0^{2\pi}$$

PTO.

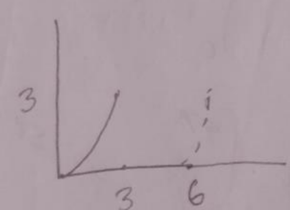
(1)

$$\Rightarrow \frac{1}{1-e^{-2\pi s}}\left(\frac{e^{(2-s)2\pi}}{2-s}-\frac{1}{2s}\right)$$

$$= \frac{1}{1-e^{-2\pi s}} \cdot \frac{e^{(2-s)2\pi}-1}{2-s}$$ Ans

(2) বাদ

(3) $f(t) = t^2,\ 0<t<3,\ f(t+3)=f(t)$



$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-3s}}\int_0^3 e^{-st}\, t^2\, dt$$

$$= \frac{1}{1-e^{-3s}}\left[-t^2\frac{e^{-st}}{s}-\frac{2te^{-st}}{s^2}-\frac{2e^{-st}}{s^3}\right]_0^3$$

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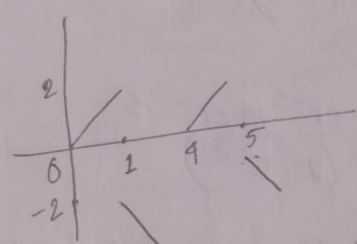
(12)

$$= \frac{s}{1-e^{-3s}}\left[\frac{9e^{-3s}}{s} - \frac{5e^{-3s}}{s^2} - \frac{2e^{-3s}}{s^3} + \frac{2}{s^3}\right] \text{ Ans.}$$

(4) $f(t) = \begin{cases} t, & 0 < t < 1 \\ -t, & 1 < t < 3 \end{cases}$ period $T = 3$.



$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-3s}}\int_0^1 e^{-st}\, t\, dt + \int_1^3 e^{-st}(-t)\,dt$$

$$= \frac{1}{1-e^{-3s}}\int_0^1 e^{-st}\, t\, dt - \int_1^3 e^{-st}\, t\, dt$$

$$\cancel{= \frac{1}{1-e^{-3s}}\left[-\frac{e^{-st}}{s}\right]_0^1 - \left[\right.}$$

$$= \frac{1}{1-e^{-3s}}\left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^1 - \left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_1^3$$

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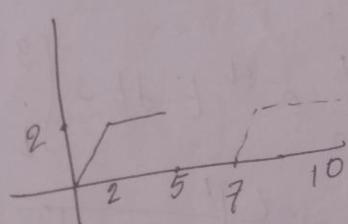
(13)

$$= \frac{1}{1-e^{-3s}}\left[\frac{-e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{1}{s^2}\right] - \left[\frac{-3e^{-3s}}{s} - \frac{e^{-3s}}{s^2} + \frac{e^{-s}}{s} + \frac{e^{-s}}{s^2}\right]$$

$$= \frac{-se^{-s} - e^{-s} + 1 + 3se^{-3s} + e^{-3s} + se^{-s} - e^{-s}}{s^2(1-e^{-3s})}$$

$$= \frac{1 - 2e^{-s} + e^{-3s} - 2e^{-s} + 3se^{-3s}}{s^2(1-e^{-3s})}$$ Ans.

(3) $f(t) = \begin{cases} t, & 0<t<2 \\ 2, & 2<t<5 \end{cases}, \quad f(t+5) = f(t).$



$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-st}} \int_0^2 e^{-st}\, t\, dt + \int_2^5 e^{-st}\, 2\, dt$$

(14)

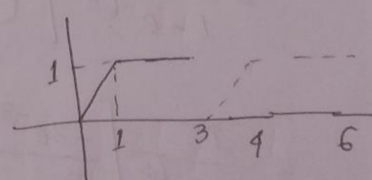
$$= \frac{1}{1-e^{-5s}} \left[\frac{-te^{-st}}{s} - \frac{e^{-st}}{s^2} \right]_0^2 + \left[\frac{-2e^{-st}}{s} \right]_2^5$$

$$= \frac{1}{1-e^{-5s}} \left[-\frac{2e^{-2s}}{s} - \frac{e^{-2s}}{s^2} + \frac{1}{s^2} - \frac{2e^{-5s}}{s} + \frac{2e^{-2s}}{s} \right]$$

$$= \frac{1 - e^{-2s} - \cancel{e}\, 2se^{-5s}}{s^2(1-e^{-5s})}$$ Ans.

(b) $f(t) = \begin{cases} t, & 0<t<1 \\ 0, & 1<t<3 \end{cases}, \quad f(t+3) = f(t)$

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-st}} \int_0^1 e^{-st}\, t\, dt + \int_1^3 e^{-st}\, 0\, dt$$

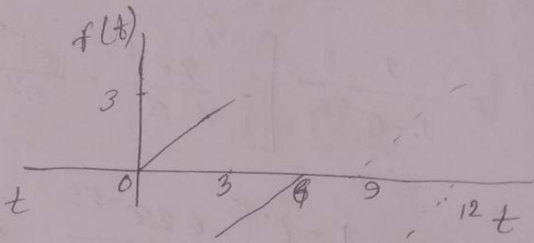


$$= \frac{1}{1-e^{-3s}} \left[\frac{-te^{-st}}{\cancel{0}\, s} - \frac{e^{-st}}{s^2} \right]_0^1 + [0]$$

$$= \frac{1 - e^{-s} - se^{-s}}{s^2(1-e^{-3s})}$$ A.

(15)

(7) $f(t) = \begin{cases} t, & 0 < t < 3 \\ 3-t & 3 < t < 6 \end{cases}$, period $T = 6$.



$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-6s}} \int_0^3 e^{-st}\, t\, dt + \int_3^6 e^{-st}(3-t)\, dt$$

$$= \frac{1}{1-e^{-6s}} \left[\left(\frac{-te^{-st}}{s} - \frac{e^{-st}}{s^2} \right)_0^3 + \left(\frac{-(3-t)e^{-st}}{s} + \frac{e^{-st}}{s^2} \right)_3^6 \right]$$

$$= \frac{1}{1-e^{-6s}} \left[\frac{-3e^{-3s}}{s} - \frac{e^{-3s}}{s^2} + \frac{1}{s^2} + \frac{3e^{-6s}}{s} + \frac{e^{-6s}}{s^2} - \frac{e^{-3s}}{s^2} \right]$$

$$= \frac{1}{1-e^{-6s}} \left[\frac{-3e^{-s}}{s} - \frac{2e^{-3s}}{s^2} + \frac{1}{s^2} + \frac{3e^{-6s}}{s} + \frac{e^{-6s}}{s^2} \right]$$

$$= \frac{1 - 2e^{-3s} - 3se^{-3s} + 3se^{-6s} + e^{-6s}}{\left(1 - e^{-6s}\right) s^2}$$ Ans.

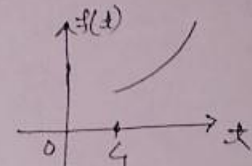
(1)

Exercise 1.3

সূত্র: (i) $\mathcal{L}\{u_a(t)\}$ OR $\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$

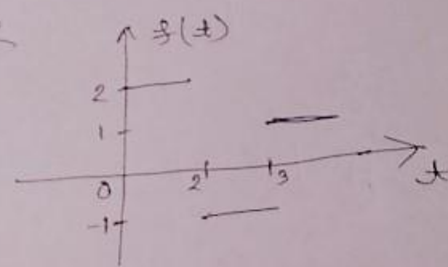
(ii) $\mathcal{L}\{f(t)\, u(t-a)\} = e^{-as}\, \mathcal{L}\, f(t+a)\}$

(4) $t^2 u_4(t)$



$$\mathcal{L}\, t^2 u_4(t) = e^{-4s} \mathcal{L}\{(t+4)^2\}$$
$$= e^{-4s} \mathcal{L}\{t^2 + 8t + 16\}$$
$$= e^{-4s} \left(\frac{2}{s^3} + \frac{8}{s^2} + \frac{16}{s}\right)$$

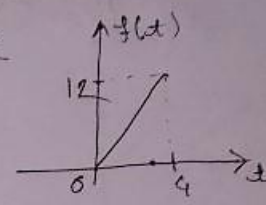
(3) $f(t) = \begin{cases} 2, & 0 < t < 2 \\ -1, & 2 \le t < 3 \\ 1, & t \ge 3 \end{cases}$



$$f(t) = 2[u(t) - u(t-2)] - 1[u(t-2) - u(t-3)] + u(t-3)$$
$$= 2u(t) - 2u(t-2) - u(t-2) + u(t-3) + u(t-3)$$
$$= 2u(t) - 3u(t-2) + 2u(t-3)$$
$$\therefore \mathcal{L}\{f(t)\} = \frac{2}{s} - \frac{3e^{-2s}}{s} + \frac{2e^{-3s}}{s}$$

Ans

② $f(t) = \begin{cases} 3t, & 0 < t < 4 \\ 0, & \text{otherwise} \end{cases}$



$$f(t) = 3t \{ u(t) - u(t-4) \}$$

$$= 3t\,u(t) - 3t\,u(t-4)$$

$$\mathcal{L}f(t) = e^{-0s} \mathcal{L}\,3t - e^{-4s}\mathcal{L}\{3(t+4)\}$$

$$= 1 \cdot 3\frac{1}{s^2} - e^{-4s}\mathcal{L}(3t+12)$$

$$= \frac{3}{s^2} - e^{-4s}\left(\frac{3}{s^2} + \frac{12}{s}\right)$$

$$= \frac{3}{s^2} - e^{-4s}\left(\frac{3+12s}{s^2}\right)$$

$$= \frac{3 - 3(4s+1)e^{-4s}}{s^2}$$

Ans.

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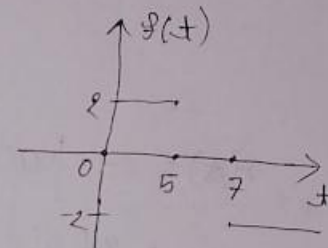
(3)

9. $$f(t) = \begin{cases} 2, & 0 \le t < 5 \\ 0, & 5 \le t < 7 \\ -2, & t \ge 7 \end{cases}$$

$$f(t) = 2\{u(t) - u(t-5)\} + 0 - 2\,u(t-7)$$



$$= 2u(t) - 2u(t-5) - 2u(t-7)$$

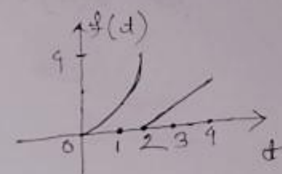
$$\therefore \mathcal{L}f(t) = 2\frac{e^{-0s}}{s} - 2\frac{e^{-5s}}{s} - \frac{2e^{-7s}}{s}$$

$$= \frac{2}{s}(1 - e^{-5s} - e^{-7s})$$

Ans.

(4)

(5) $f(t) = \begin{cases} t^2, & 0 \le t < 2 \\ t-2, & t \ge 2 \end{cases}$



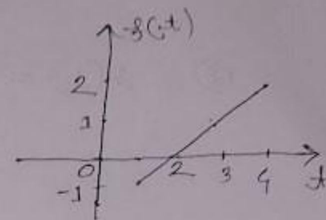
$$f(t) = t^2\{u(t) - u(t-2)\} + (t-2)\,u(t-2)$$
$$= t^2 u(t) - t^2 u(t-2) + (t-2)\,u(t-2)$$

$$\mathcal{L} f(t) = e^{-0s}\mathcal{L}\, t^2 - e^{-2s}\mathcal{L}(t+2)^2 + e^{-2s}\mathcal{L}(t-2+2)$$
$$= \frac{2}{s^3} - e^{-2s}\mathcal{L}\{t^2+4t+4\} + e^{-2s}\frac{1}{s^2}$$
$$= \frac{2}{s^3} - e^{-2s}\left\{\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right\} + e^{-2s}\frac{1}{s^2}$$
$$= \frac{2}{s^3} - \frac{2}{s^3}e^{-2s} - \frac{4}{s^2}e^{-2s} - \frac{4}{s}e^{-2s} + \frac{1}{s^2}e^{-2s}$$
$$= \frac{2}{s^3} - \frac{2}{s^3}e^{-2s} - \frac{3}{s^2}e^{-2s} - \frac{4}{s}e^{-2s}$$

Ans

(5)

⑥ $f(t) = \begin{cases} t-2, & 1<t<4 \\ 0, & \text{otherwise} \end{cases}$



$f(t) = (t-2)\{u(t-1) - u(t-4)\} + 0$

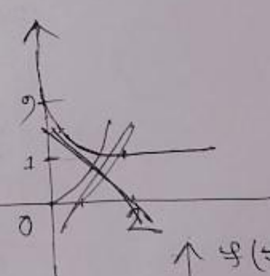
$= (t-2)\,u(t-1) - (t-2)\,u(t-4)$

$\mathcal{L} f(t) = e^{-s}\mathcal{L}\{(t-2+1)\} - e^{-4s}\mathcal{L}\{(t-2+4)\}$

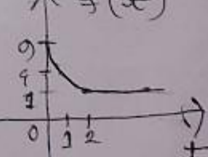
$= e^{-s}\left(\frac{1}{s^2} - \frac{1}{s}\right) - e^{-4s}\left(\frac{1}{s^2} + \frac{2}{s}\right)$

Ans

7. $f(t) = \begin{cases} (t-3)^2, & 0<t<2 \\ 1, & t>2 \end{cases}$



$f(t) = (t-3)^2\{u(t) - u(t-2)\} + u(t-2)$



$= (t-3)^2 u(t) - (t-3)^2 u(t-2) + u(t-2)$

$\mathcal{L} f(t) = e^{-0s}\mathcal{L}(t-3)^2 - e^{-2s}\mathcal{L}(t-3+2)^2 + \frac{e^{-2s}}{s}$

$= \mathcal{L}\{t^2 - 6t + 9\} - e^{-2s}\mathcal{L}\{t^2 - 2t + 1\} + \frac{e^{-2s}}{s}$

(6)

$$= \frac{2}{s^3} - \frac{6}{s^2} + \frac{9}{s} - e^{-2s}\left(\frac{2!}{s^3} - \frac{2}{s^2} + \frac{1}{s}\right) + \frac{e^{-2s}}{s}$$

$$= \frac{2}{s^3} - \frac{6}{s^2} + \frac{9}{s} - e^{-2s}\left(\frac{2}{s^3} - \frac{2}{s^2}\right)$$

Ans.