Spline Interpolation: Cubic Spline Interpolation (CSI)

Lecture-2

Objectives

- ❑ **Introduce MATLAB Spline Interpolation Functions**
- ❑ **Solve problems by using Cubic Spline Interpolation**
- ❑ **Draw curve by applying MATLAB Spline Interpolation Functions**

MATLAB Spline Interpolation Functions

- MATLAB function **spline**

yy=spline(x, Y, xx)   for **not-a-knot** cubic spline

x, Y are inputs and xx is expolant.

yy=spline(x, [dY0, Y, dYn], xx)   for clamped cubic spline

$$dY0 = Y'(x0)\text{and} \quad dYn = Y'(xn)$$

- **csape** spline interpolation with various end conditions

Syntax: sp=csape(X, Y, conds)

some of the conditions are

I. 'second' adjusted second derivatives if not mentioned it uses $[0, 0]$
II. 'clamped' adjusted first derivatives
III. 'not-a-knot' uses not-a-knot condition

Solve Problems

Example #1: A natural cubic spline is defined by

$$f(x) = \begin{cases} A(x+1)^3 + B(x+1)^2 + C(x+1) + 1, & -1 \le x < 1 \\ D(x-1)^3 + 6(x-1)^2 + E(x-1) - 1, & 1 \le x \le 2 \end{cases}$$

I. Use continuity and boundary conditions to estimate A, B, C, D and E.

II. Find the value of $f(1.4)$ from the spline curve'

III. Use MATLB function “**sp=csape(x, y, ‘conditions’)**” to construct natural cubic spline for the data set $(-1, 1), (1, -1)$ and $(2, 10)$.

Find $f(1.4)$ using “**fnval(**sp,**x)**”, and Plot the spline curve using “**fnplt(sp)**” along with the data points.

Solutions:

Let $$f_1(x) = A(x+1)^3 + B(x+1)^2 + C(x+1) + 1,$$
and $$f_2(x) = D(x-1)^3 + 6(x-1)^2 + E(x-1) - 1,$$
Then

$$f_1'(x) = 3A(x+1)^2 + 2B(x+1) + C$$
$$f_2'(x) = 3D(x-1)^2 + 12(x-1) + E$$

and

$$f_1''(x) = 6A(x+1) + 2B$$
$$f_2''(x) = 6D(x-1) + 12$$

i. Conditions at the interior point $x = 1$ give

$$f_1(1) = f_2(1) \quad \Longrightarrow \quad 8A + 4B + 2C + 1 = -1 \qquad (1)$$
$$f_1'(1) = f_2'(1) \quad \Longrightarrow \quad 12\,A + 4B + C = E \qquad (2)$$
$$f_1''(1) = f_2''(1) \quad \Longrightarrow \quad 12A + 2B = 12 \qquad (3)$$

For natural cubic spline the boundary conditions give

$$f_1''(-1) = 2B = 0 \qquad \text{or} \qquad B = 0$$

$$f_2''(2) = 6D + 12 = 0 \qquad \text{or} \qquad D = -2$$

From (3), $12A = 12$ or $A = 1$

From (1), $8(1)+2C + 1 = -1$ or $C = -5$

From (2), $12(1) + (-5) = E$ or $E = 7$

The natural cubic spline function is

$$f(x) = \begin{cases} (x+1)^3 - 5(x+1) + 1\,, & -1 \le x < 1 \\ -2(x-1)^3 + 6(x-1)^2 + 7(x-1) - 1, & 1 \le x \le 2 \end{cases}$$

(ii) $f(x) = -2(x-1)^3 + 6(x-1)^2 + 7(x-1) - 1, \ \&1 \le x \le 2$

$$f(1.4) = f_2(1.4) = -2(0.4)^3 + 6(0.4)^2 + 7(0.4)$$
$$= 2.632.$$

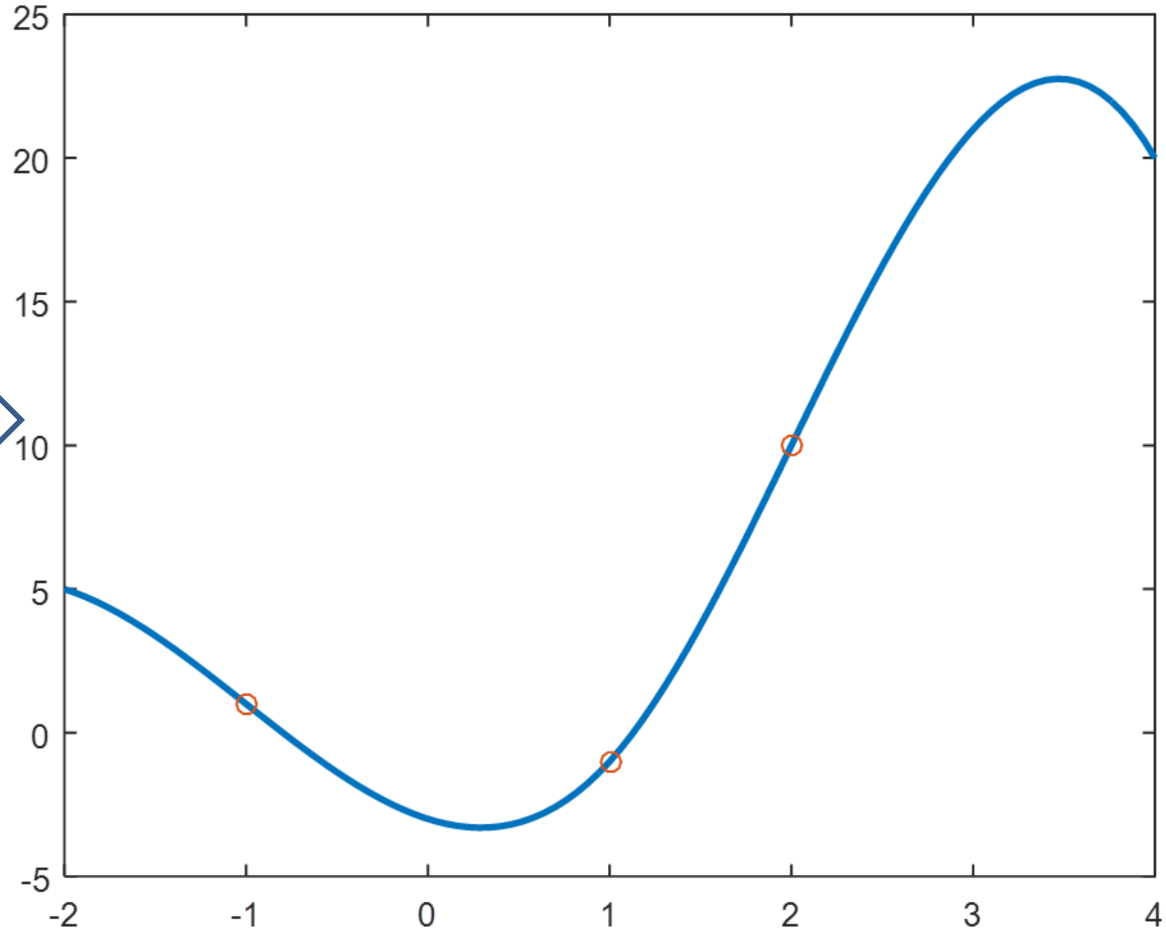
iii.

```
>> clear
>> x=[-1 1 2];
>> y=[1 -1 10];
>> sp=csape(x,y,'second');
>> y1=fnval(sp,1.4)

y1 =
   2.6320
```

```
>> fnplt(sp, 2, [-2, 4])
>> hold on                          % used to plot in the same figure
>> plot(x, y,'O')
>> hold off
```

Cubic spline curve



Outcomes

- **Functions can be derived by using Cubic Spline Interpolation (CSI) from given data sets.**
- **Functions can be plotted by using built in function in MATLAB for Cubic Spline Interpolation (CSI).**

Multiple questions:

S.No.	Questions
1	Which command can be used to find coefficients after constructing spline curve in MATHLAB by? (a) sp.coefs, (b) esape(x, y, ‘conditions’), (c) None of them
2	By using Cubic Spline Interpolation (CSI) from available data sets- (a) Functions can be derived, (b) Functions can not be derived, (c) None of them, (d) Both of them
3	By applying built in function in MATLAB for Cubic Spline Interpolation - (a) Functions can be plotted, (b) Functions can not be plotted , (c) Both of them
4	Which command can be used for to construct natural cubic spline for the given data set? (a) csape(x, y, ‘conditions’), (b) esape(x, y, ‘conditions’), (c) None of them

Try to do yourself

Exercise 1: A natural cubic spline $f(x)$ which interpolates the data points $(-1, 5), (2, 2), (4, 60)$ is defined by

$$f(x) = \begin{cases} A(x+1)^3 + B(x+1)^2 + C(x+1) + 5, & -1 \leq x < 2 \\ D(x-1)^3 + E(x-1)^2 + 17(x-1) + 2, & 2 \leq x \leq 4 \end{cases}$$

i. Use continuity and boundary conditions to find the values of *A, B, C, D* and *E*.

ii. Estimate $f(1)$ and $f(3)$ from .the spline curve.

Exercise 2: A clamped cubic spline function through (-1,1), (2,-2) (4,30) is defined by

$$f(x) = \begin{cases} A(x+1)^3 + 3(x+1)^2 + B(x+1) + 1, & -1 \le x < 2 \\ C(x-2)^3 + D(x-2)^2 + E(x-2) - 2, & 2 \le x \le 4 \end{cases}$$

i. Given that $f'(-1) = 1$ and $f'(4) = 30$, find the values of *A, B, C,D* and *E*.

ii. Estimate the value of $f(2.5)$

iii. Write a MATLAB code using function "**spline(x, y)**" to construct the spline curve and "**fnval(function,x)**" to estimate the values of $f(x)$ for $x = -0.5, 2.5$ and 3.8

References

[1] Applied Numerical Methods With Matlab for Engineers and Scientists (Steven C.Chapra).

[2] Applied Numerical Analysis – C.F.Gerald & P.O.Wheatley, 7th Edition, 2003, Pearson Education Limited, USA.

[3] Numerical Analysis & Computing – W. Cheney & D. Kincaid, 6th Edition, 2007, Cengage Learning, Inc, USA.

[4] Numerical Analysis – J. Douglas Faires , Annette Burden , Richard Burden, 10th Edition, 2015, Cengage Learning, Inc, USA.