New Uzbekistan University

Discrete Mathematics | MATH 221

Tutorial 1: Propositional Calculus

P1.1. Let p and q be the propositions

p: It is below freezing.

q: It is snowing.

Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing. $p \wedge q$

b) It is below freezing but not snowing. $p \wedge \neg q$, "but- "and"

c) It is not below freezing and it is not snowing. $\neg p \wedge \neg q$

d) It is either snowing or below freezing (or both). $p \vee q$

P1.1. Let p and q be the propositions

p: It is below freezing.

q: It is snowing.

Write these propositions using p and q and logical connectives (including negations).

e) If it is below freezing, it is also snowing. $p \rightarrow q$

f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

$(p \vee q) \wedge (p \rightarrow \neg q)$

g) That it is below freezing is necessary and sufficient for it to be snowing. $p \leftrightarrow q$

P1.2. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$.

b) If $1 + 1 = 3$, then $2 + 2 = 4$.

c) If $1 + 1 = 3$, then $2 + 2 = 5$.

d) If monkeys can fly, then $1 + 1 = 3$.

Answer: F - T - T - T

P1.3. For each of these sentences, determine whether an inclusive or, or an exclusive or is intended. Explain your answer.

a) Coffee or tea comes with dinner.

b) A password must have at least three digits or be at least eight characters long.

c) The prerequisite for the course is a course in number theory or a course in cryptography.

d) You can pay using U.S. dollars or euros.

Answer: exclusive - inclusive - inclusive - exclusive/but might be inclusive too.

P1.4. Write each of these statements in the form $p \rightarrow q$.

a) It snows whenever the wind blows from the northeast.

Answer: $p \rightarrow q$, where $p =$the wind blows from the northeast, $q =$it snows.

b) The apple trees will bloom if it stays warm for a week.

Answer: If it stays warm for a week, then the apple trees will bloom ($p \rightarrow q$).

c) That the Pistons win the championship implies that they beat the Lakers.

Answer: If the Pistons win the championship, then they beat the Lakers ($p \rightarrow q$).

d) It is necessary to walk 8 miles to get to the top of Long's Peak.

Answer: The necessary condition is the conclusion. If you get to the top of Long's Peak, then you must have walked eight miles. ($p \rightarrow q$).

e) To get tenure as a professor, it is sufficient to be world-famous.

Answer: If you are world famous, then you will get tenure as a professor ($p \rightarrow q$).

P1.4. Write each of these statements in the form $p \longrightarrow q$.

f) Your guarantee is good only if you bought your CD player less than 90 days ago.
Answer: If your guarantee is good, then you must have bought your CD player less than 90 days ago ($p \longrightarrow q$).

h) Jan will go swimming unless the water is too cold.
Answer: If the water is not too cold, then Jan will go swimming ($p \longrightarrow q$).

P1.5. Construct a truth table for $(p \vee q) \to (p \oplus q)$ and $(p \oplus q) \to (p \wedge q)$.

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \to (p \oplus q)$	$p \wedge q$	$(p \oplus q) \to (p \wedge q)$
1	1	1	0	0	1	1
1	0	1	1	1	0	0
0	1	1	1	1	0	0
0	0	0	0	1	0	1

P1.6. Construct a truth table for $(p \longleftrightarrow q) \longleftrightarrow (r \longleftrightarrow s)$.

p	q	r	s	$p \longleftrightarrow q$	$r \longleftrightarrow s$	$(p \longleftrightarrow q) \longleftrightarrow (r \longleftrightarrow s)$
1	1	1	1	1	1	1
1	1	1	0	1	0	0
1	1	0	1	1	0	0
1	1	0	0	1	1	1
1	0	1	1	0	1	0
1	0	1	0	0	0	1
1	0	0	1	0	0	1
1	0	0	0	0	1	0
0	1	1	1	0	1	0
0	1	1	0	0	0	1
0	1	0	1	0	0	1
0	1	0	0	0	1	0
0	0	1	1	1	1	1
0	0	1	0	1	0	0
0	0	0	1	1	0	0
0	0	0	0	1	1	1

P1.7. Find the bitwise OR, bitwise AND, and bitwise XOR of the bitstrings 1111 0000 and 1010 1010.

Solution: Bitwise logical operations by definitions are carried out "bitwise". Take first values of both bitstrings and perform required operation and do the same with the bits in the second position and etc:

$$11110000 \vee 10101010 = 11111010$$
$$11110000 \wedge 10101010 = 10100000$$
$$11110000 \oplus 10101010 = 01011010$$

Road to Russel's Paradox

P1.8*. Is the assertion $p =$"This statement is false" a proposition?

Solution: Assume that p is a proposition.

If it has truth value 1 then "This statement is false" is true and contradicts to itself.

If p has value 0, then "This statement is false" is true statement, which contradicts with p being false.

Hence, p is not a proposition.

P1.10. Are these system specifications ***consistent*** (all true at the same time)?
"The router can send packets to the edge system only if it supports the new address space. For the router to support the new address space it is necessary that the latest software release be installed. The router can send packets to the edge system if the latest software release is installed, The router does not support the new address space."

Let s = "The router can send packets to the edge system"
a = "The router supports the new address space",
r = "The latest software release is installed."
Then we are told $s \to a$, $a \to r$, $r \to s$ and $\neg a$.
Since a is false, the first conditional statement tells us that s must be false.
From that we deduce from the third conditional statement that r must be false. If indeed all three propositions are false, then all four specifications are true, so they are consistent.

P1.11. When three professors are seated in a restaurant, the hostess asks them: “Does everyone want coffee?” The first professor says: “I do not know.” The second professor then says: “I do not know.” Finally, the third professor says: “No, not every one wants coffee.” The hostess comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

Solution: The question was "Does everyone want coffee?"

If the first professor did not want coffee, then he would know that the answer to the hostess’s question was “no.” Therefore we and the hostess and the remaining professors know that the first professor does want coffee.

The same argument applies to the second professor, so she, too, must want coffee. The third professor can now answer the question. Because she said “no”, we conclude that she does not want coffee.

Therefore the hostess knows to bring coffee to the first two professors but not to the third.

P1.12. Construct the disjunctive normal form of the proposition $(p \rightarrow q) \wedge \neg r$.

Solution:

p	q	r	$p \rightarrow q$	$\neg r$	$(p \rightarrow q) \wedge \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	F	F
F	T	T	T	F	F
T	F	F	F	T	F
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

The disjunctive normal form will be a disjunction of three conjunctions, one for each row in the truth table that gives the truth value T for $(p \rightarrow q) \wedge \neg r$. These rows are rows 2, 6, and 8. The disjunctive normal form for $(p \rightarrow q) \wedge \neg r$ is then:

$$(p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r),$$