Demystifying Simpson's paradox

The world is a perplexing place, wherein illusions are ubiquitous. We are constantly reminded that what we see or feel may not be the complete manifestation of the world. Change of perspective may often reveal the truth, but we must be careful in choosing the right one.

Let us take a look at the following table, which summarizes the scores of two students in two exams.

|  |  |  |
| --- | --- | --- |
| Studnet→  Exam ↓ | Sayem | Siyam |
| Exam-1 | 63/90=70% | 8/10=80% |
| Exam-2 | 4/10=40% | 45/90=50% |

Who do you think is the better student? The first look tells us that Siyam outscored Sayem in both exams. In the first exam, Siyam secured 8 out of 10, which is 80% of the total marks, and more than what Sayem scored (70%). Similarly, in the second exam, he again obtained a greater grade, attaining 50%, while Sayem achieved a score worth 40% of the total.

We, thus, are to conclude Siyam is the better student. Or is he?

Let us take a second look, after rearranging sequence of the exams.

|  |  |  |
| --- | --- | --- |
| Studnet→  Exam ↓ | Sayem | Siyam |
| Exam-1 | 4/10=40% | 8/10=80% |
| Exam-2 | 63/90=70% | 45/90=50% |

Your perception may have changed now. In the first exam, Sayem achieved 4 out of 10 marks (40%), while Siyam got 8 (80%), being the clear winner. In the second exam, however, Sayem outsmarted him, secuirng 63 out 90 (70%), while Siyam scored just 45 (50%). Both have secured an exam. Is this a tie now?

Well, before drawing the final conslusion, let us take another look. As is already evident from the data, Two exams together constitute 100 marks. We would like to see how much they obtained in total.

|  |  |  |
| --- | --- | --- |
| Studnet→  Exam ↓ | Sayem | Siyam |
| Exam-1 | 63/90=70% | 8/10=80% |
| Exam-2 | 4/10=40% | 45/90=50% |
| Total | 63+4=69% | 8+45 = 53% |

Sayem scored 69 out 100 (69%), and Siyam secured 53% marks. Sayem wins by a clear margin. Why is there a obvious difference now?

First of all, initially we compared two uncoparebale exams, one worth 10 marks and the other 90 marks. We should compare 90-marks exam with another 90-mark exam.

Secondly, the exam won by Siyam is worth only 10 marks, while the exam in which Sayem came first is worth 90 marks. Securing good grade in a big exam is a sign of consistency.

Finally, percentages often hide the true picture, especially when the actual values are not mentioned.

This apparent misunderstanding may arise from multiple reasons. One of the ways it can occur, as explained before, is when we compare the figures unfairly. This can occur due to the presence of a hidden causes as well, however.

A study of gender bias among graduate school admissions to University of California, Berkeley, seemed to revealed men applying were more likely than women to be admitted, the difference being too significant to be due to chance.

The figures are shown below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | All | | Men | | Women | |
|  | Applicants | Admitted | Applicants | Admitted | Applicants | Admitted |
| Total | 12,763 | 41% | 8,442 | 44% | 4,321 | 35% |

Source: stat.uiowa.edu

It seemed there was a discrimination aginst the women. However, further insight unfolded that the percentages obtained belonged to different departments, and women usually applied to more competitive departments with lower rates of admission. When the complete data were observed, with rates of admission to each department, it was astonishingly noticed that there was a small but statistically significant bias in favor of women.

In statistics, figures may seem to lie, but they do not. It is for us to correctly analyze and understand the data. The apparent misunderstanding created in the above example is called Simpson's paradox, anmed after Edward H. Simpson, who first illustrated the phenomenon.

References

Simpson, Edward H. (1951). "The Interpretation of Interaction in Contingency Tables". Journal of the Royal Statistical Society, Series B. 13: 238–241.

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