Demystifying Simpson's Paradox

The world is a perplexing place, where illusions are ubiquitous. We are constantly reminded that what we see or feel may not be the complete manifestation of the world. Change of perspective may often reveal the truth, but we must be careful in choosing the right one.

Let us take a look at the following table, which summarizes the scores of two students in two exams.

|  |  |  |
| --- | --- | --- |
| Studnet→  Exam ↓ | Sayem | Siyam |
| Exam-1 | 63/90=70% | 8/10=80% |
| Exam-2 | 4/10=40% | 45/90=50% |

Who do you think is the better student? The first look tells us that Siyam outscored Sayem in both exams. In the first exam, Siyam secured 8 out of 10, which is 80% of the total marks, and more than what Sayem scored (70%). Similarly, in the second exam, he again obtained a greater grade, attaining 50%, while Sayem achieved a score worth 40% of the total.

We, thus, are to conclude Siyam is the better student. Or is he?

Let us take a second look, after rearranging sequence of the exams.

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| --- | --- | --- |
| Studnet→  Exam ↓ | Sayem | Siyam |
| Exam-1 | 4/10=40% | 8/10=80% |
| Exam-2 | 63/90=70% | 45/90=50% |

Your perception may have changed now. In the first exam, Sayem achieved 4 out of 10 marks (40%), while Siyam got 8 (80%), being the clear winner. In the second exam, however, Sayem outsmarted him, secuirng 63 out 90 (70%), while Siyam scored just 45 (50%). Both have secured an exam. Is this a tie now?

Well, before drawing the final conslusion, let us take another look. As is already evident from the data, the two exams together constitute 100 marks. We would like to see how much they obtained in total.

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| --- | --- | --- |
| Studnet→  Exam ↓ | Sayem | Siyam |
| Exam-1 | 63/90=70% | 8/10=80% |
| Exam-2 | 4/10=40% | 45/90=50% |
| Total | 63+4=69% | 8+45 = 53% |

Sayem scored 69 out 100 (69%), and Siyam secured 53% marks. Sayem wins by a clear margin. Why is there a obvious difference now?

First of all, initially we compared two incomparable exams, one worth 10 marks and the other 90 marks. We should compare a 90-marks exam with another 90-mark exam.

Secondly, the exam won by Siyam is worth only 10 marks, while the exam in which Sayem came first is worth 90 marks. Securing good grade in a big exam is a sign of consistency.

Finally, percentages often hide the true picture, especially when the actual values are not mentioned or are very small.

In statistics, figures may seem to lie, but they do not. It is for us to correctly analyze and understand the data. The apparent misunderstanding created in the above example is called Simpson's paradox, named after Edward H. Simpson, who first illustrated the phenomenon.

References

Simpson, Edward H. (1951). "The Interpretation of Interaction in Contingency Tables". Journal of the Royal Statistical Society, Series B. 13: 238–241.

Clifford H. Wagner (February 1982). "Simpson's Paradox in Real Life". The American Statistician. 36 (1): 46–48.