Statistics Question Bank

Second Paper

Abdullah Al Mahmud

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Contents

Chapter 1

Probability

1.1 Creative Questions

1. **Events that do not depend on each other are called independent events, and events that cannot occurr simulataneously are called disjoint events.**

   (a) Provide an example of disjoint events, using the set theory. 1

   (b) Prove that $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ 2

   (c) If there are k mutually and exhaustive events, prove $\sum_{i=1}^{k} P(A_i) = 1$ 3

   (d) Prove that two events cannot be simulataneously independent and mutually exclusive. 4

2. **A quality control analyst in an industry tracks the no. of defective items produced per day. He observes 150 successive days and then prepares a table.**

No. of items	0	1	2	3	4
Frequency	30	32	40	28	20

   (a) What is the formula of classical probability? 1

   (b) Explain the difference between Priori Approach and Empirical Approach of probability. 2

   (c) What is the probability that less than 2 defective items would be produced on a particular day? 3

   (d) Explain the relationship between independency and mutual excluvity in the light of the stem. 4

3. **Ratul and Tomal both have an unbiased die. Both have randomly thrown their die once.**

   (a) What are equally likely events? 1

   (b) If a die is thrown once, what is the probability of getting a prime number? 2

   (c) From the stem, what is the probability that the sum of numbers appearing on the dice is greater than 6. 3

   (d) Examine: the probabilities of getting the sum less than 6 and greater than 6 are equal. 4

4. **It is observed that 50% of mails are spam. A software filters spam mail before reaching the inbox. Its accuracy for detecting a spam mail is 99% and chances of tagging a non-spam mail as spam mail is 5%.**

   (a) What is a disjoint event? 1

   (b) For two independent events, what does the Bayes' theorem reduce to? 2

   (c) What is the probability that a mail is tagged as spam? 3

(d) If a certain mail is tagged as spam, find the probability that it is not a spam mail. 4

5. **A company receives 60% of its job applications from applicants with the required qualifications. A hiring software screens applications for minimum qualifications. It correctly identifies qualified applications 97% of the time, but it also incorrectly marks 4% of unqualified applications as qualified.**

   (a) What is the probability that an application is marked as qualified? 3

   (b) If an application is marked as qualified, find the probability that it actually does not meet the required qualifications. 4

6. **In a survey of a town's population of 500 people, it was found that 150 people read the local newspaper daily, 200 people listen to the radio daily, and 80 people do both.**

   (a) What is the probability that a randomly selected person reads the newspaper given that they listen to the radio? 3

   (b) Calculate the probability that a randomly selected person neither reads the newspaper nor listens to the radio. 4

7. **In a school with 200 students, 60 students participate in the science club, 80 participate in the math club, and 30 participate in both.**

   (a) What is the probability that a randomly selected student participates in both clubs? 3

   (b) If a student is chosen at random, what is the probability that they are in exactly one of the clubs? 4

8. **In a community of 300 residents, it was found that 90 people use public transportation regularly, 120 use bicycles, and 40 use both.**

   (a) What is the probability that a randomly selected resident uses either public transportation or bicycles? 3

   (b) What is the conditional probability that a resident uses public transportation given that they use a bicycle? 4

9. **A dope test correctly identifies a drug user as positive 90% of the time, but incorrectly identifes 20% non-users as users. The probability of drug use is 0.05.**

   (a) Write down the formula of conditional probability. 1

   (b) Express $P(A|B)$ in terms of $P(B|A)$. 2

   (c) Find the probability of testing positive in the test. 3

   (d) If the test shows a user positive, what is the probability that the person is actually a user? 4

10. **A red and a blue dice are thrown once. The dice are absolutely neutral and independent.**

    (a) What is a simple event? 1

    (b) Give an example of a certain event using set theory. 2

    (c) Find the probability that the difference of two digits from two dices is less than 3. 3

    (d) Are the probabilities of getting greater digit from the blue die and that from the red die equal? Justify. 4

11. **An unbiased coin is tossed 10 times.**

    (a) If a coin is flung 3 times, how many outcomes are generated? 1

    (b) If a coin is flung n times, show how many outcomes are generated. 2

    (c) What is the probability of getting a) at least 3 heads, b) at most 3 heads? 3

(d) Are these probabilities equal? a) Getting at least 2 heads & b) Getting at least 2 tails. Also justify logically. 4

12. **It is observed that in a college, there are 100 students, of whom 30 play football, 40 play cricket, and 20 play both.**

    (a) What is the range of probability? 1

    (b) What is the relationship between independence and mutual excluvity? 2

    (c) Are the probabilities of playing cricket and that of football independent? Prove. 3

    (d) If a student is selected randomly, and if he does not play cricket, what is the probability that he plays football? 4

13. **A box contains four blue and 6 green balls. 3 balls are drawn randomly.**

    (a) What is the value of ${}^nC_r$? 1

    (b) Illustrate the difference between permutation and combination with an example. 2

    (c) What is the probability that all balls are green? 3

    (d) What is the probabilith that one ball has a different color? 4

14. **Sadman has an urn with 5 red and 4 white balls. He has randomly drawn two balls from the urn.**

    (a) What is the probability of an uncertain event? 1

    (b) Write the third axiom of probability. 2

    (c) What is the probability that both the balls drawn by Sadman are white? 3

    (d) Are the probabilities of both balls being same color and different color equal? Analyze. 4

15. **Two dice are thrown together. The dice are named A and B.**

    (a) What is P(A=7)? 1

    (b) Create the sample space. 2

    (c) What is the probability that the outcomes of A & B are different? 3

    (d) Determine the probability that the summation of outcome of two dice is a prime number. 4

16. **A magician draws two cards from a pack (i) with replacement and then (ii) without replacement. The cards were well-shuffled before drawing.**

    (a) What is the probability of an impossible event? 1

    (b) How to determine the probability of a joint event? 2

    (c) As per (i), what is the probability that the cards have different color? 3

    (d) As per (ii), what is the probability that the cardsare aces of same color? 4

17. $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}, P(B \cup A) = \frac{1}{2}$

    (a) What is an independent event? 1

    (b) What is the relationship between independency and mutual excluvity? 2

    (c) Find $P(A|B)$ and $P(B|A)$ 3

    (d) Verify the equality mathematically & empirically: $P(B) = P(A) \cdot P(B|A) + P(\bar{A}) \cdot P(B|\bar{A})$ 4

18. $P(A|B) = \frac{1}{8}, P(A) = \frac{1}{2}, P(B) = \frac{1}{5}$

    (a) Write down the range of probability. 1

    (b) Find $P(A \cap B)$. 2

    (c) Find $P(A|\bar{B})$. 3

    (d) Are the probabilities $P(A|B)$ and $P(B|A)$ equal? Justify 4

19. **Sakib has recently graduated from the University of Dhaka. he applies to two firms - EduCube & Digic- for a Data Analyst job. The probability of hiring by EduCube is 0.8 and by Digic is 0.4. The probability that none hires is 0.5.**

    (a) What is a sample space? 1

    (b) Explain how to find $P(\bar{A} \cap B)$ using Venn Diagram. 2

    (c) Find the probability of hiirng by by Digic but not by EduCube. 3

    (d) Find the probability that no firm will reject him. 4

20. **Recently there is an increase in the number of electronic medias in Bangladesh. A professor stated in the class room that very few people now resort to print media for news. A research indicates 70% people collect news from electronic media, 60% from print media, and 50% from both.**

    (a) What is an impossible event? 1

    (b) Write the event ”None of the two occurs” in two different notations. 2

    (c) What is the probability of getting news from at most one type of media? 3

    (d) Is the professor correct in his/her statement? Analyze. 4

1.2 Short Questions

1. Question 1
2. Question 2
3. Question 3
4. Question 4

Chapter 2

Random Variable and Probability Function

2.1 Creative Questions

1. **A deck of 52 card is well-shuffled and three cards are drawn from them at random. The number of kings obtained is denoted by x.**

   (a) What are equaly likely events? 1

   (b) Differentiate between with replacement and without replacement drawings. 2

   (c) Form the probability fucntion using the above information and then form the distribution. 3

   (d) Examine the statement: $P(1 \leq x \leq 3) = F(3) - F(1)$ 4

   (a) **The joint probability function of two random variables X and Y is given by:**

   $$P(X, Y) = \frac{x + 2y}{28}; \quad x = 0, 1; \quad y = 0, 1, 2, 3$$

   i. Write down the formula for conditional probability. 1

   ii. What is the relationship between marginal and joint probability? 2

   iii. Find $P(X)$. 3

   iv. Find $P(X|Y)$ and $P(X|Y = 0)$. 4

   (b) **The joint probability function of two random variables X and Y is described by:**

   $$P(X, Y) = \frac{2x + 3y}{45}; \quad x = 0, 1, 2; \quad y = 0, 1, 2$$

   i. Write down the formula for conditional probability. 1

   ii. What is the relationship between marginal and joint probability? 2

   iii. Find $P(X)$. 3

   iv. Find $P(X|Y)$ and $P(X|Y = 0)$. 4

2. **The joint probability function of two random variables X and Y is given by:**

   $$P(X, Y) = \frac{x + y + 1}{42}; \quad x = 0, 1, 2; \quad y = 0, 1, 2, 3$$

   (a) Calculate the marginal probability $P(Y)$. 3

   (b) Determine $P(Y|X = 1)$ and $P(Y|X = 0)$. 4

3. **The joint probability function of two random variables X and Y is described by:**

   $$P(X, Y) = \frac{2x + y + 1}{52}; \quad x = 1, 2; \quad y = 1, 2, 3, 4$$

   (a) Find the marginal distribution $P(X)$. 3

Table 2.1: **Distribution - A**

x	0	1	2	3	4	5	6
P(x)	0.20	0.10	0.08	w	0.02	0.10	0.30

Table 2.2: **Distribution - B**

x	0	1	2	3	4
P(x)	0.20	0.10	0.30	0.50	0.20

(b) Compute $P(Y|X)$ for $X = 2$. 4

4. **The probability distributions of a random variable X in two different cases are given below:**

   (a) What is a probability mass function? 1

   (b) Can we dtermine the probability of a certain value of a discrete random variable? 2

   (c) What is the value of w? 3

   (d) Which table is a proper probability distribution? Justify with mathematical reasoning. 4

5. **A continuos random variable X follows the following probability density function (pdf).**

$$f(x) = 6x(1 - x); 0 \leq x \leq 1$$

   (a) Give an example of a continous random variable. 1

   (b) Examine whether the given function is a pdf. 2

   (c) If $P(X > a) = P(X < a)$, find the value of a. 3

   (d) Should $P(0.5 \leq X \leq 1)$ be equal to 0.5? 4

6. **The probability mass function (pmf) of a football striker scoring no. of hattricks during the course of a league season is given below**

$$P(x) = \frac{|2 - x|}{k}; x = 0, 1, 2, 3, 4, 5$$

   (a) What is a random variable? 1

   (b) Is probability a discrete variable? Explain in brief. 2

   (c) Find the value of k. 3

   (d) Find the probability that the no. of hattricks would be less than the expectation. 4

7. **A fair coin is tossed five times. Number of heads appearing are noted, considering it a discrete random variable.**

   (a) Give a real life example of a discrete random variable. 1

   (b) Can discrete variable have infinite number of possible outcomes? 2

   (c) Find the probability distribution from the stem. 3

   (d) Construct thedistribution function and hence find $F(X \leq 3)$. 4

8. **The probability density function of a continuous random variable is**

$$f(x) = \begin{cases} k(x + 1), & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

(a) What is a random variable? 1

(b) Find the value of k 2

(c) Find the probability that the values of x would lie between 0 and 0.5. 3

(d) What is the probability that X is greater than 0.8? 4

9. **The probability density function of a continuous random variable is**

$$f(x) = \begin{cases} kx(x-1), & 1 \leq x \leq 4 \\ 0, & otherwise \end{cases}$$

(a) What is the range of probability? 1

(b) Find the value of k 2

(c) Justify the pdf property of the fucntion. 3

(d) What is the probability that X is greater than 3? 4

10. **The probability distribution of a discrete random variable X is given below:**

x	-2	-1	0	1	3	4
P(x)	0.1	k	2k	3k	4k	0.2

(a) What is $\Sigma P(x)$? 1

(b) Find the value of k. 2

(c) Find $P(X \geq 0)$ and $P(X < 1)$. 3

(d) Find the cumulative distribution function, F(X) and F(2) and explain. 4

11. **The joint probability function of two random variables X & Y is given below:**

$$P(x, y) = \frac{1}{21}(x + y); x = 1, 2, 3 \ \& \ y = 1, 2$$

(a) What is a probability density function (pdf)? 1

(b) What is P(X=a) in a pdf, where a is an aribitrary number? 2

(c) Find the marginal probabilities. 3

(d) Find $P(x|y), P(x|1)$ and $P(y|4)$ 4

12. **The probability density function of a continuos random variable X is given as:**

$$f(x) = \frac{1}{b-a}; a \leq x \leq b$$

(a) In this distribution, what is P(a)? 1

(b) What is the shape of the distribution? 2

(c) Find $P(a \leq x \leq b)$. 3

(d) Find and explain the median of the distribution. 4

13. **The probability density function of a continuous random variable is**

$$f(x) = \begin{cases} kx^2 + kx + \frac{1}{8}, & 0 \leq x \leq 2 \\ 0, & otherwise \end{cases}$$

(a) What is a continuous random variable? 1

(b) Find the value of k 2

(c) Find the probability that the values of x would lie between1 and 3. 3

(d) Find the 40th percentile of the distribution and explain. 4

2.2 Short Questions

1. What is a continuous random variable? 1
2. Question 1
3. Question 1
4. Question 1

Chapter 3

Mathematical Expectation

3.1 Creative Questions

1. **The probability distribution of a random X is provided below:**

X	-1	0	1	2	3
P(x)	$\frac{3}{20}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{20}$

   (a) What is the expectation of a constant m? 1
   (b) Find $E(X)$. 2
   (c) Find $E(Y)$, where $Y = \frac{X}{2}$ 3
   (d) Find Variance of (2X+3). 4

2. **A random variable is distributed as below:**

$$P(X) = \frac{3-|4-x|}{k}; x = 2, 3, 4, 5, 6$$

   (a) What is the Expectation equivalent to? 1
   (b) Find the value of k. 2
   (c) Determine the value of the expectation. 3
   (d) Find $V(2X - 1)$ 4

3. **The probability distributions of demand of mobile phones of two operating systems (OS) Android (X) and iPhone OS (iOS) (Y) are:**

Demand	100	200	300	400	500
P(X)	0.1	0.4	m	0.15	0.1
P(Y)	0.09	0.45	0.32	0.11	0.03

   (a) What is Expectation? 1
   (b) Can Expectation be negative? 2
   (c) Find m from the table. 3
   (d) Which OS has higher demand? Analyze. 4

4. **An umbrella seller earns a revenue of BDT. 5000 if it rains. If it does not rain, he loses BDT. 1000. The probability that it rains on a given day is 0.04.**

   (a) Write down the formula of Expectation for a continuous random variable. 1
   (b) Can the value of Expectation be zero? 2
   (c) What is the umbrella seller's expected revenue? 3

(d) What should be the minimum probability of raining for him to achieve revenue greater than zero? 4

5. **A box contains 5 red and 6 white balls. 3 balls are drawn at random. X is the number of white balls drawn.**

(a) What does variance measure? 1

(b) Can the variance be smaller than standard deviation? 2

(c) Find the E(X) from the stem. 3

(d) Find the variance from the stem assuming X is the number of red balls drawn. 4

6. **A professor showed a probability distribution in a class:**

x	1	2	3	4	5
p(x)	0.1	a	0.3	b	0.2

The value of the arithmetic mean of the distribution is 3.

(a) What is the formula of expectation? 1

(b) What is the variance of a constant? Explain logically. 2

(c) What are the values of a & b? 3

(d) Find and explain the variance of the distribution. 4

7. **X is a random variable having the below functional form:**

$$P(X) = \frac{6-|7-x|}{k}; x = 1, 2, \cdots, 10$$

Y is another variable having the relationship y = 3x+5

(a) What is joint probability? 1

(b) What is the minimum possible value of variance? Why? 2

(c) Find the value of k. 3

(d) Find E(X) and E(Y). Why are they different? 4

8. **Various sales and their probabilities of a grocery store is given below**

Sales	200	250	275	310	350
Probability	0.10	0.20	0.40	0.25	0.05

(a) Can the expectation of a random variable be negative? 1

(b) Find the expected sales of the store on a given day. 2

(c) Compute the dispersion of sales f the store. 3

(d) To make the expected sale 280, what sale does the store need in place of 200? 4

9. **A survey of Television (TV) users at Gulshan in Dhaka was conducted to find how many sets each family use. The following data were obtained:**

No of TV set	0	1	2	3
No of family	10	75	10	5

(a) What is Expectation equivalent to? 1

(b) Can Variance be negative? Why or why not? 2

(c) Find the variance of the number of TV sets. 3

(d) Find and compoare between arithmetic mean and expectation. 4

3.2 Short Questions

Chapter 4

Binomial Distribution

4.1 Creative Questions

1. **A farmer selected a paddy field for seed collection. He found out that 10 out of each 25 paddies are damaged. He collected a sample of 15 paddies.**
   (a) What is a Bernoulli trial? 1
   (b) IF a Bernoulli trial is repeated n times, in how many ways are outcomes generated? Explain. 2
   (c) Find the probability that at least one paddy is damaged? 3
   (d) Comment on the skewness of the data. 4
   [Hint: For a binomial distribution, $\gamma_1 = \frac{q-p}{\sqrt{npq}}$]

2. **A farmer plans to store rice seeds for future use. It was found that 8 out of 20 seeds are rotten. He then collected a sample of 15 seeds.**
   (a) What is Bernoulli trial? 1
   (b) How are Bernoulli and Binomial distributions related? 2
   (c) What is the probability that at least one seed is rotten out of 15? 3
   (d) What is the probability that the number of rotten seeds is greater than the arithmetic mean? 4

3. **The number of defective pen produced by a company follows a binomial distribution with expectation 1.5 and variance 1.125.**.
   (a) What is the mean of binomial distribution 1
   (b) Can variance be greater than mean in binomial distribution? 2
   (c) Determine the probability function of the number of defective items produced by the company. 3
   (d) What is the probability that the number of defective items is no less than 3? 4

4.2 Short Questions

Chapter 5

Poisson Distribution

5.1 Creative Questions

1. **Between 1000hrs and 1700 hrs, the average number of phonce calls per minute received by a power distribution company is 2.5.**

   (a) Give an example where Poisson distribution is applicable. 1

   (b) Find the relationship between expectationa and standard deviation of Poisson distribution. 2

   (c) Find the probability that the number of calls is between 1 and 3 (inclusive). 3

   (d) What is the probability that the number of calls received is greater than the average? 4

2. **The frequency distribution of defective items in packets of key rings is given below.**

Number of defective items	0	1	2	3	4	5
Number of packets	76	74	29	17	3	1

   (a) What is another way to write $P(X \geq 1)$? 1

   (b) Can the mean of Poisson distribution be negative? 2

   (c) From the given stem, find mean and variance. 3

   (d) Find the expected frequencies and comment. 4

3. **A can manufacturer observes that 0.1% of the produced cans are faulty. The cans are packaged in carton boxes, with 500 cans in each box. A wholeseller purchases 100 boxes from the manufacturer.**

   (a) What is shape of Poisson distribution? 1

   (b) For a Poisson distribution, P(2) = P (3). What is P(2)? 2

   (c) Find the probability of exactly one defective can. 3

   (d) Find the expected number of boxes with no defective cans. 4

4. **In winter, the probability that it rains on a particular day is 0.015. An analyst observes 100 winter days.**

   (a) What is an experiment? 1

   (b) When can the Poisson distribution be approximated by the Binomial distribution? 2

   (c) Find, using Binomial distribution, the probability that it would not rain at all on the observed days. 3

   (d) Find the probability in 3(c) using Poisson distribution. 4

5. **BTCL receives 2.5 telephone calls on average from 4 pm to 6 pm. The number of calls received is a random variable.**

(a) When is Poisson variate applicable? 1

(b) Show conversion criteria and method from Binomial to Poisson distribution. 2

(c) Find the probability of receiving no more than 3 calls. 3

(d) Find the pattern of calls and show on graph paper. 4
Hint: Find probabilities: P(0), P(1), $\cdots$

6. **The number of customers coming at a shop per minute follows a Poisson distribution, whose mean is 3.**

   (a) What is a Poisson variate? 1

   (b) Can the mean of Poisson distribution be negative? 2

   (c) Find the probability that the number of customers coming is between 1 and 2. 3

   (d) Are the probabilities of coming to 2 and 3 customers equal? 4

7. **A random variable is distributed as follows:**

Value	0	1	2	3	4	5
Frequency	70	73	27	15	4	1

   (a) What is the mean of Poisson distribution? 1

   (b) What is the relationship between mean and standard deviation of Poisson distribution? 2

   (c) Find the mean and variance of the given distribution. 3

   (d) Compare the observed and expected frequencies, assuming a Possion distribution. 4

5.2 Short Questions

Chapter 6

Normal Distribution

6.1 Creative Questions

6.2 Short Questions

Chapter 7

Index Number

7.1 Creative Questions

7.2 Short Questions

Chapter 8

Sampling

8.1 Creative Questions

8.2 Short Questions

Chapter 9

Vital Statistics

9.1 Creative Questions

1. **A reseracher uses the following data to know about some demographic characterisics.**

   (a) What is General Fertility Rate? 1

   (b) What is the difference between GRR and NRR? 2

   (c) Compute the population density. 3

   (d) Are TFR and GRR same for this data? 4

2. **For projection of population in a future time period, demographers use simple, geometric or exponential growth technique. Each method has its advantages and disadvantages.**

   (a) What is geometric growth? 1

   (b) In geometric growth method, obtain the formula for time required for the population to get doubled [denote rate as r]. 2

   (c) In exponential method, how much unit of time is required for the population to get tripled? 3

   (d) For projecting (predicting future values), is geometric growth method better than the exponential method? Justify. 4

3. **Population of Dhaka and Sylhet by different age groups and areas are given below:**

Division	Age			Area (km^2)
	0-14	15-64	65+	
Dhaka	10,000,00	5,00,000	5,80,000	1,880
Sylhet	7,00,000	2,70,000	4,70,000	2,319

   (a) Write down the formula of dependency ratio. 1

   (b) What is meant by NRR = 0.983? 2

   (c) Find and compare between the dependency ratios of the cities. 3

   (d) Based on data, which city is more comfortable for living? 4

4. **As part of an analysis, a researcher collected data on women and live births.**

Age	15-19	20-24	25-29	30-34	35-39	40-44	45-49
No. of Women	540	760	530	495	450	505	430
No. of live births	109	198	86	90	65	76	60

(a) What is the formula of death rate? 1

(b) Write down the uses of vital statistics. 2

(c) Find teh Age Specific Birth Rates (ASFR). 3

(d) Find the GFR and compare its concept and value with ASFRs. 4

9.2 Short Questions

Conclusion

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