Statistics Notes (II)

Contents

1 Probbility

1.1 Important Concepts

1.1.1 Terms

Trial A single performance of well-defined experiment

Experiment a scientific test in which you perform a series of actions and carefully observe their effects in order to learn about something. or *An act that can be repeated under some specific condition.*

Random variable A variable whose values are associated with probability.

Sample space Set of all possible outcomes of a random experiment

Sample point Each outcome of a sample space

Event Any subset of a sample space

Simple event An event having a single outcome

Compound/Composite event An event having more than one outcome

Impossible event An event which cannot happen (If P(A) = 0, then A is an impossible event)

Certain event An event which surely will or will not happen. (P(A) = 0 or 1)

Uncertain event An event which may or may not happen ($0 < P(A) < 1$)

1.1.2 Set Theory

1.1.3 Permutaion

Permutaion is all about arranging items, while combination is used to find the ways to to select items.

If we have 3 items A, B, and C; we can arrange them in the following way.

- ABC
- ACB
- BAC
- BCA
- CAB
- CBA

There are 6 possible ways to arrange 3 items in 3 positions.

Thinking another way, there are 3 positions and 3 items. The first position can be filled up in 3 ways (A or B or C), the second in 2 ways (after one item is fixed in the first position, be it A or B, or C), and the third in 1 way.

Mathematically, this is also written as 3! (3 factorial), which is nothing but $3 \times 2 \times 1 = 6$

Position	1	2	3
Possible options	3	2	1

Similarly, if we have 4 items to arrange in 4 places, we can write:

Position	1	2	3	4
Possible options	4	3	2	1

Thus we can arrange this in $4 \times 3 \times 2 \times 1 = 4! = 24$ ways.

Not Using All Items

Now, what if want to arrange 2 items out of 4 items. In this case we have 2 places, but 4 items.

Position	1	2
Possible options	4	3

We get $4 \times 3 = 12$ ways.

This is also written as ${}^4P_2 = 12$ (shown below)

$$^nP_r = \frac{n!}{(n-r)!}$$

Repeating Items

In the above examples, items cannot be repeated in places. In some scenario, this is absurd; after all, one person cannot sit on 2 chairs.

However, consider using digits to make up telephone numbers.

Position	1	2	3	4	5	6	7
Possible options	10	10	10	10	10	10	10

All 10 digits can be used in each position.

Hence, for 7-digit telephone numbers, we can have telephone number in 10^7 ways.

Think

- What is the general formula of the above case?[1]
- What if the first digit is always zero (0)?
- What if not all 7 digits can be same?
- What if some particular digit cannot be repeated, or can be repeated only twice?
- How many license plates can make using 5 letters, 2 digits and 3 letters, or 1 letter and 3 digits, where items can be repeated?

1.1.4 Combination

Combination is used when we are concerned with selecting items or individuals.

Example: How can we select 2 items out of 3 (A, B, and C)?

AB, AC, BC (AB = BA, AC = CA, BC = CB)

In permutation, we had 6 ways. The reason is obvious.

Not Using All Items

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Think

- In how many ways can a coach select 11 footballers from a squad of 15?
- What if s/he must keep 5 specific players?
- In how many ways the players can be placed in the field?

Think More

- How many 5-digit numbers can be made using the digits 4, 5, 2, 1, 0?
- How many are odd and even?
- How many end with zero?

[1] n^r, where n = no. of items and r = no. of places

1.2 Three Definitions of Probability

Classical

$$P(A) = \frac{n(A)}{n(S)}$$

Relative frequency

$$\lim_{n(S)\to\infty} \frac{n(A)}{n(S)}$$

Axiomatic

Three axioms

Say, S is sample space and A_i is an event

- $0 \leq P(A) \leq 1$ (NOT $P(A) \geq 0$)
- At least one of S will occur. P (S) = 1; Certain event.
- $P(A_1UA_2U...UA_n) = P(A_1) + P(A_2) + ... + P(A_n)$ or
- $$P\left(\cup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

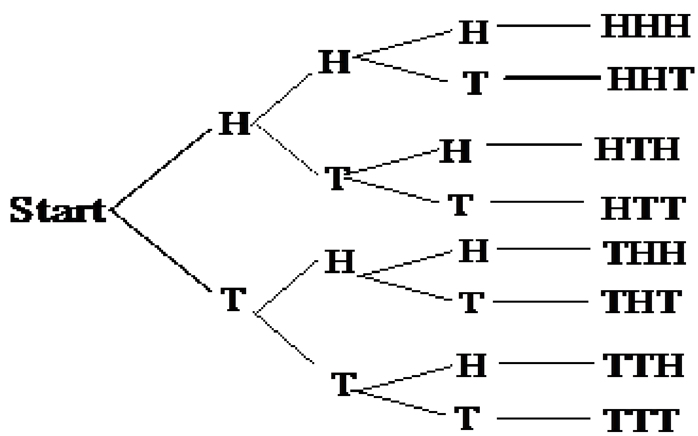
1.3 Probability Theorems

- $P(A) + P(\bar{A}) = 1$ (prove)
- $\sum_{i=1}^{k} P(A_i) = 1$
- $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$ (Venn)
- If A & B are independent, are $\bar{A}$ & $\bar{B}$ independent? (Prove by an example)

1.4 Coin and Die Problems

1.4.1 Tree Method

The sample space of a coin tossed thrice (or 3 coins tossed together)



What is the general formula? [2]

1.4.2 Table of Sample Space

A coin is tossed twice

	First Toss →	H	T
Second Toss ↓	H	HH	HT
	T	TH	TT

A coin is tossed thrice

	First 2 Toss →	HH	HT	TH	TT
Third Toss ↓	H	HHH	HHT	HTH	HTT
	T	THH	THT	TTH	TTT

[2] 2^n for a coin and 6^r for a die. What would be a more general formula?

Tables of higher order are similarly created using the combination of smaller tables. For example, the outcome of five tosses can be obtained by combining outcome of 3 tosses and that of 2 tosses.

DIY

Using tree and table, make a sample space of

- four coins tossed at once.
- 2 dice rolled together
- a coin and die thrown together
- 2 coins and a die

1.4.3 Problems

1. A coin is flipped thrice. What is the probability that

   a. the first toss gives Head
   b. The last two toss give Tails
   c. there exactly one H
   d. there are less than 3 heads

2. A coin and a die are thrown together. Find the probabilities that:

   a. the sample has an even digit
   b. the sample has a prime number

3. Two unbiased dice are rolled at once. Find the probabilities that:

   a. sum of the numbers is 7
   b. sum is less than 4
   c. both numbers are greater than 3
   d. the number are equal
   e. the numbers are different
   f. sum is a prime number

Solution without creating sample space

A fair coin is tossed 10 times.

Find the probability that a. there are exactly 3 heads b. at least 2 heads c. more than 8 heads d. at best 1 tail

Solution

$$n(S) = 2^10$$

a. there ${}^{10}C_3$ ways to select 3 items out of 10 items (heads)

For example

H T H T T T T H T T

or

T T H T T T T H T H

If there 3 coins, then 1 head can appear in the following ways.

HTT THT TTH (${}^{3}C_1 = 3$)

1.5 Set Theory Problems

1.6 Problems: Drawing Items at Once

1.7 Problems: Drawing Items One by One

1.8 Addition vs Multiplication

1.9 Playing card Concept and Problems

1.10 Condional Probability Theory

1.11 Condional Probability Problems

1.12 Digit Problems