Statistics Question Bank

Second Paper

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Contents

Chapter 1

Probability

1.1 Creative Questions

1. **Events that do not depend on each other are called independent events, and events that cannot occurr simulataneously are called disjoint events.**

   (a) Provide an example of disjoint events, using the set theory. 1

   (b) Prove that $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ 2

   (c) If there are k mutually and exhaustive events, prove $\sum_{i=1}^{k} P(A_i) = 1$ 3

   (d) Prove that two events cannot be simulataneously independent and mutually exclusive. 4

2. **A quality control analyst in an industry tracks the no. of defective items produced per day. He observes 150 successive days and then prepares a table.**

No. of items	0	1	2	3	4
Frequency	30	32	40	28	20

   (a) What is the formula of classical probability? 1

   (b) Explain the difference between Priori Approach and Empirical Approach of probability. 2

   (c) What is the probability that less than 2 defective items would be produced on a particular day? 3

   (d) Explain the relationship between independency and mutual excluvity in the light of the stem. 4

3. **Ratul and Tomal both have an unbiased die. Both have randomly thrown their die once.**

   (a) What are equally likely events? 1

   (b) If a die is thrown once, what is the probability of getting a prime number? 2

   (c) From the stem, what is the probability that the sum of numbers appearing on the dice is greater than 6. 3

   (d) Examine: the probabilities of getting the sum less than 6 and greater than 6 are equal. 4

4. **It is observed that 50% of mails are spam. A software filters spam mail before reaching the inbox. Its accuracy for detecting a spam mail is 99% and chances of tagging a non-spam mail as spam mail is 5%.**

   (a) What is a disjoint event? 1

   (b) For two independent events, what does the Bayes' theorem reduce to? 2

   (c) What is the probability that a mail is tagged as spam? 3

(d) If a certain mail is tagged as spam, find the probability that it is not a spam mail. 4

5. **A company receives 60% of its job applications from applicants with the required qualifications. A hiring software screens applications for minimum qualifications. It correctly identifies qualified applications 97% of the time, but it also incorrectly marks 4% of unqualified applications as qualified.**

   (a) What is the probability that an application is marked as qualified? 3

   (b) If an application is marked as qualified, find the probability that it actually does not meet the required qualifications. 4

6. **A university reports that 70% of its students pass a certain entrance exam. A new AI tool is implemented to predict if a student will pass, with a 92% accuracy rate for students who actually pass, and a 10% chance of falsely predicting that a failing student will pass.**

   (a) What is the probability that a student is predicted to pass the exam? 3

   (b) If a student is predicted to pass, what is the probability that the prediction is incorrect? 4

7. **A large retailer finds that 30% of the products returned by customers are actually defective. Its quality control system identifies defective products correctly 85% of the time, but it also incorrectly flags 8% of non-defective products as defective.**

   (a) What is the probability that a returned product is flagged as defective? 3

   (b) If a product is flagged as defective, determine the likelihood that it is indeed defective. 4

8. **A dope test correctly identifies a drug user as positive 90% of the time, but incorrectly identifes 20% non-users as users. The probability of drug use is 0.05.**

   (a) Write down the formula of conditional probability. 1

   (b) Express $P(A|B)$ in terms of $P(B|A)$. 2

   (c) Find the probability of testing positive in the test. 3

   (d) If the test shows a user positive, what is the probability that the person is actually a user? 4

9. **A red and a blue dice are thrown once. The dice are absolutely neutral and independent.**

   (a) What is a simple event? 1

   (b) Give an example of a certain event using set theory. 2

   (c) Find the probability that the difference of two digits from two dices is less than 3. 3

   (d) Are the probabilities of getting greater digit from the blue die and that from the red die equal? Justify. 4

10. **An unbiased coin is tossed 10 times.**

   (a) If a coin is flung 3 times, how many outcomes are generated? 1

   (b) If a coin is flung n times, show how many outcomes are generated. 2

   (c) What is the probability of getting a) at least 3 heads, b) at most 3 heads? 3

   (d) Are these probabilities equal? a) Getting at least 2 heads & b) Getting at least 2 tails. Also justify logically. 4

11. **It is observed that in a college, there are 100 students, of whom 30 play football, 40 play cricket, and 20 play both.**

   (a) What is the range of probability? 1

   (b) What is the relationship between independence and mutual excluvity? 2

   (c) Are the probabilities of playing cricket and that of football independent? Prove. 3

(d) If a student is selected randomly, and if he does not play cricket, what is the probability that he plays football? 4

12. **In a survey of a town's population of 500 people, it was found that 150 people read the local newspaper daily, 200 people listen to the radio daily, and 80 people do both.**

    (a) Are the events of reading the newspaper and listening to the radio independent? Prove. 3

    (b) Calculate the probability that a randomly selected person neither reads the newspaper nor listens to the radio. 4

13. **In a school with 200 students, 60 students participate in the science club, 80 participate in the math club, and 30 participate in both.**

    (a) Are the events of participating in the science club and the math club independent? Justify your answer. 3

    (b) If a student is chosen at random, what is the probability that they are in exactly one of the clubs? 4

14. **In a community of 300 residents, it was found that 90 people use public transportation regularly, 120 use bicycles, and 40 use both.**

    (a) Are the events of using public transportation and using bicycles independent? Show your work. 3

    (b) What is the conditional probability that a resident uses public transportation given that they use a bicycle? 4

15. **A box contains four blue and 6 green balls. 3 balls are drawn randomly.**

    (a) What is the value of ${}^nC_r$? 1

    (b) Illustrate the difference between permutation and combination with an example. 2

    (c) What is the probability that all balls are green? 3

    (d) What is the probabilith that one ball has a different color? 4

16. **Sadman has an urn with 5 red and 4 white balls. He has randomly drawn two balls from the urn.**

    (a) What is the probability of an uncertain event? 1

    (b) Write the third axiom of probability. 2

    (c) What is the probability that both the balls drawn by Sadman are white? 3

    (d) Are the probabilities of both balls being same color and different color equal? Analyze. 4

17. **Two dice are thrown together. The dice are named A and B.**

    (a) What is P(A=7)? 1

    (b) Create the sample space. 2

    (c) What is the probability that the outcomes of A & B are different? 3

    (d) Determine the probability that the summation of outcome of two dice is a prime number. 4

18. **A magician draws two cards from a pack (i) with replacement and then (ii) without replacement. The cards were well-shuffled before drawing.**

    (a) What is the probability of an impossible event? 1

    (b) How to determine the probability of a joint event? 2

    (c) As per (i), what is the probability that the cards have different color? 3

    (d) As per (ii), what is the probability that the cardsare aces of same color? 4

19. $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}, P(B \cup A) = \frac{1}{2}$

(a) What is an independent event? 1

(b) What is the relationship between independency and mutual excluvity? 2

(c) Find $P(A|B)$ and $P(B|A)$ 3

(d) Verify the equality mathematically & empirically: $P(B) = P(A) \cdot P(B|A) + P(\bar{A}) \cdot P(B|\bar{A})$ 4

20. $P(A|B) = \frac{1}{8}, P(A) = \frac{1}{2}, P(B) = \frac{1}{5}$

(a) Write down the range of probability. 1

(b) Find $P(A \cap B)$. 2

(c) Find $P(A|\bar{B})$. 3

(d) Are the probabilities $P(A|B)$ and $P(B|A)$ equal? Justify 4

21. **Sakib has recently graduated from the University of Dhaka. he applies to two firms - EduCube & Digic- for a Data Analyst job. The probability of hiring by EduCube is 0.8 and by Digic is 0.4. The probability that none hires is 0.5.**

(a) What is a sample space? 1

(b) Explain how to find $P(\bar{A} \cap B)$ using Venn Diagram. 2

(c) Find the probability of hiirng by by Digic but not by EduCube. 3

(d) Find the probability that no firm will reject him. 4

22. **Recently there is an increase in the number of electronic medias in Bangladesh. A professor stated in the class room that very few people now resort to print media for news. A research indicates 70% people collect news from electronic media, 60% from print media, and 50% from both.**

(a) What is an impossible event? 1

(b) Write the event ”None of the two occurs” in two different notations. 2

(c) What is the probability of getting news from at most one type of media? 3

(d) Is the professor correct in his/her statement? Analyze. 4

1.2 Short Questions

1. Question 1
2. Question 2
3. Question 3
4. Question 4

Chapter 2

Random Variable and Probability Function

2.1 Creative Questions

1. **A deck of 52 card is well-shuffled and three cards are drawn from them at random. The number of kings obtained is denoted by x.**

   (a) What are equaly likely events? 1

   (b) Differentiate between with replacement and without replacement drawings. 2

   (c) Form the probability fucntion using the above information and then form the distribution. 3

   (d) Examine the statement: $P(1 \leq x \leq 3) = F(3) - F(1)$ 4

2. **The joint probability function of two random variables X and Y is given below:**

$$P(X, Y) = \frac{x + 2y}{16}; x = 0, 1; y = 0, 1, 2, 3$$

   (a) Write down the formula of conditional proibability. 1

   (b) What is the relationship between marginal and joint probability? 2

   (c) Find P(X). 3

   (d) Find $P(X|Y)$ and $P(X|0)$. 4

3. **The joint probability function of two random variables X and Y is given by:**

$$P(X, Y) = \frac{3x + y}{30}; \quad x = 1, 2; \quad y = 0, 1, 2, 3$$

   (a) Find $P(X)$. 3

   (b) Calculate $P(X|Y)$ and $P(X|1)$. 4

4. **The joint probability function of two random variables X and Y is defined as:**

$$P(X, Y) = \frac{2x + 3y}{50}; \quad x = 0, 1; \quad y = 1, 2, 3, 4$$

   (a) Determine $P(Y)$. 3

   (b) Find $P(X|Y)$ and $P(X|2)$. 4

5. **The joint probability function of two random variables X and Y is given by:**

$$P(X, Y) = \frac{x + y + 1}{42}; \quad x = 0, 1, 2; \quad y = 0, 1, 2, 3$$

(a) Calculate the marginal probability $P(Y)$. 3

(b) Determine $P(Y|X=1)$ and $P(Y|X=0)$. 4

6. **The joint probability function of two random variables X and Y is described by:**

$$P(X,Y) = \frac{2x+y+1}{52}; \quad x = 1,2; \quad y = 1,2,3,4$$

(a) Find the marginal distribution $P(X)$. 3

(b) Compute $P(Y|X)$ for $X = 2$. 4

7. **The probability distributions of a random variable X in two different cases are given below:**

Table 2.1: **Distribution - A**

x	0	1	2	3	4	5	6
P(x)	0.20	0.10	0.08	w	0.02	0.10	0.30

Table 2.2: **Distribution - B**

x	0	1	2	3	4
P(x)	0.20	0.10	0.30	0.50	0.20

(a) What is a probability mass function? 1

(b) Can we dtermine the probability of a certain value of a discrete random variable? 2

(c) What is the value of w? 3

(d) Which table is a proper probability distribution? Justify with mathematical reasoning. 4

8. **A continuos random variable X follows the following probability density function (pdf).**

$$f(x) = 6x(1-x); 0 \leq x \leq 1$$

(a) Give an example of a continous random variable. 1

(b) Examine whether the given function is a pdf. 2

(c) If $P(X > a) = P(X < a)$, find the value of a. 3

(d) Should $P(0.5 \leq X \leq 1)$ be equal to 0.5? 4

9. **The probability mass function (pmf) of a football striker scoring no. of hattricks during the course of a league season is given below**

$$P(x) = \frac{|2-x|}{k}; x = 0,1,2,3,4,5$$

(a) What is a random variable? 1

(b) Is probability a discrete variable? Explain in brief. 2

(c) Find the value of k. 3

(d) Find the probability that the no. of hattricks would be less than the expectation. 4

10. **A fair coin is tossed five times. Number of heads appearing are noted, considering it a discrete random variable.**

(a) Give a real life example of a discrete random variable. 1

(b) Can discrete variable have infinite number of possible outcomes? 2

(c) Find the probability distribution from the stem. 3

(d) Construct thedistribution function and hence find $F(X \leq 3)$. 4

11. **The probability density function of a continuous random variable is**

$$f(x) = \begin{cases} k(x+1), & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

(a) What is a random variable? 1

(b) Find the value of k 2

(c) Find the probability that the values of x would lie between 0 and 0.5. 3

(d) What is the probability that X is greater than 0.8? 4

12. **The probability density function of a continuous random variable is**

$$f(x) = \begin{cases} kx(x-1), & 1 \leq x \leq 4 \\ 0, & otherwise \end{cases}$$

(a) What is the range of probability? 1

(b) Find the value of k 2

(c) Justify the pdf property of the fucntion. 3

(d) What is the probability that X is greater than 3? 4

13. **The probability distribution of a discrete random variable X is given below:**

x	-2	-1	0	1	3	4
P(x)	0.1	k	2k	3k	4k	0.2

(a) What is $\Sigma P(x)$? 1

(b) Find the value of k. 2

(c) Find $P(X \geq 0)$ and $P(X < 1)$. 3

(d) Find the cumulative distribution function, F(X) and F(2) and explain. 4

14. **The joint probability function of two random variables X & Y is given below:**

$$P(x, y) = \tfrac{1}{21}(x+y); x = 1, 2, 3 \ \& \ y = 1, 2$$

(a) What is a probability density function (pdf)? 1

(b) What is P(X=a) in a pdf, where a is an aribitrary number? 2

(c) Find the marginal probabilities. 3

(d) Find $P(x|y), P(x|1)$ and $P(y|4)$ 4

15. **The probability density function of a continuos random variable X is given as:**

$$f(x) = \frac{1}{b-a}; a \leq x \leq b$$

(a) In this distribution, what is P(a)? 1

(b) What is the shape of the distribution? 2

(c) Find $P(a \leq x \leq b)$. 3

(d) Find and explain the median of the distribution. 4

16. **The probability density function of a continuous random variable is**

$$f(x) = \begin{cases} kx^2 + kx + \frac{1}{8}, & 0 \leq x \leq 2 \\ 0, & otherwise \end{cases}$$

   (a) What is a continuous random variable? 1
   (b) Find the value of k 2
   (c) Find the probability that the values of x would lie between1 and 3. 3
   (d) Find the 40th percentile of the distribution and explain. 4

2.2 Short Questions

1. What is a continuous random variable? 1
2. Question 1
3. Question 1
4. Question 1

Chapter 3

Mathematical Expectation

3.1 Creative Questions

1. **The probability distribution of a random X is provided below:**

X	-1	0	1	2	3
P(x)	$\frac{3}{20}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{20}$

   (a) What is the expectation of a constant m? 1

   (b) Find $E(X)$. 2

   (c) Find $E(Y)$, where $Y = \frac{X}{2}$ 3

   (d) Find Variance of (2X+3). 4

2. **A random variable is distributed as below:**

$$P(X) = \frac{3-|4-x|}{k}; x = 2, 3, 4, 5, 6$$

   (a) What is the Expectation equivalent to? 1

   (b) Find the value of k. 2

   (c) Determine the value of the expectation. 3

   (d) Find $V(2X - 1)$ 4

3. **The probability distributions of demand of mobile phones of two operating systems (OS) Android (X) and iPhone OS (iOS) (Y) are:**

Demand	100	200	300	400	500
P(X)	0.1	0.4	m	0.15	0.1
P(Y)	0.09	0.45	0.32	0.11	0.03

   (a) What is Expectation? 1

   (b) Can Expectation be negative? 2

   (c) Find m from the table. 3

   (d) Which OS has higher demand? Analyze. 4

4. **An umbrella seller earns a revenue of BDT. 5000 if it rains. If it does not rain, he loses BDT. 1000. The probability that it rains on a given day is 0.04.**

   (a) Write down the formula of Expectation for a continuous random variable. 1

   (b) Can the value of Expectation be zero? 2

   (c) What is the umbrella seller's expected revenue? 3

(d) What should be the minimum probability of raining for him to achieve revenue greater than zero? 4

5. **A box contains 5 red and 6 white balls. 3 balls are drawn at random. X is the number of white balls drawn.**

(a) What does variance measure? 1

(b) Can the variance be smaller than standard deviation? 2

(c) Find the E(X) from the stem. 3

(d) Find the variance from the stem assuming X is the number of red balls drawn. 4

6. **A professor showed a probability distribution in a class:**

x	1	2	3	4	5
p(x)	0.1	a	0.3	b	0.2

The value of the arithmetic mean of the distribution is 3.

(a) What is the formula of expectation? 1

(b) What is the variance of a constant? Explain logically. 2

(c) What are the values of a & b? 3

(d) Find and explain the variance of the distribution. 4

7. **X is a random variable having the below functional form:**

$$P(X) = \frac{6-|7-x|}{k}; x = 1, 2, \cdots, 10$$

Y is another variable having the relationship y = 3x+5

(a) What is joint probability? 1

(b) What is the minimum possible value of variance? Why? 2

(c) Find the value of k. 3

(d) Find E(X) and E(Y). Why are they different? 4

8. **Various sales and their probabilities of a grocery store is given below**

Sales	200	250	275	310	350
Probability	0.10	0.20	0.40	0.25	0.05

(a) Can the expectation of a random variable be negative? 1

(b) Find the expected sales of the store on a given day. 2

(c) Compute the dispersion of sales f the store. 3

(d) To make the expected sale 280, what sale does the store need in place of 200? 4

9. **A survey of Television (TV) users at Gulshan in Dhaka was conducted to find how many sets each family use. The following data were obtained:**

No of TV set	0	1	2	3
No of family	10	75	10	5

(a) What is Expectation equivalent to? 1

(b) Can Variance be negative? Why or why not? 2

(c) Find the variance of the number of TV sets. 3

(d) Find and compoare between arithmetic mean and expectation. 4

3.2 Short Questions

Chapter 4

Binomial Distribution

4.1 Creative Questions

1. **A farmer selected a paddy field for seed collection. He found out that 10 out of each 25 paddies are damaged. He collected a sample of 15 paddies.**
   (a) What is a Bernoulli trial? 1
   (b) IF a Bernoulli trial is repeated n times, in how many ways are outcomes generated? Explain. 2
   (c) Find the probability that at least one paddy is damaged? 3
   (d) Comment on the skewness of the data. 4
   [Hint: For a binomial distribution, $\gamma_1 = \frac{q-p}{\sqrt{npq}}$]

2. **A farmer plans to store rice seeds for future use. It was found that 8 out of 20 seeds are rotten. He then collected a sample of 15 seeds.**
   (a) What is Bernoulli trial? 1
   (b) How are Bernoulli and Binomial distributions related? 2
   (c) What is the probability that at least one seed is rotten out of 15? 3
   (d) What is the probability that the number of rotten seeds is greater than the arithmetic mean? 4

3. **The number of defective pen produced by a company follows a binomial distribution with expectation 1.5 and variance 1.125.**.
   (a) What is the mean of binomial distribution 1
   (b) Can variance be greater than mean in binomial distribution? 2
   (c) Determine the probability function of the number of defective items produced by the company. 3
   (d) What is the probability that the number of defective items is no less than 3? 4

4.2 Short Questions

Chapter 5

Poisson Distribution

5.1 Creative Questions

1. **Between 1000hrs and 1700 hrs, the average number of phonce calls per minute received by a power distribution company is 2.5.**

   (a) Give an example where Poisson distribution is applicable. 1

   (b) Find the relationship between expectationa and standard deviation of Poisson distribution. 2

   (c) Find the probability that the number of calls is between 1 and 3 (inclusive). 3

   (d) What is the probability that the number of calls received is greater than the average? 4

2. **The frequency distribution of defective items in packets of key rings is given below.**

Number of defective items	0	1	2	3	4	5
Number of packets	76	74	29	17	3	1

   (a) What is another way to write $P(X \geq 1)$? 1

   (b) Can the mean of Poisson distribution be negative? 2

   (c) From the given stem, find mean and variance. 3

   (d) Find the expected frequencies and comment. 4

3. **A can manufacturer observes that 0.1% of the produced cans are faulty. The cans are packaged in carton boxes, with 500 cans in each box. A wholeseller purchases 100 boxes from the manufacturer.**

   (a) What is shape of Poisson distribution? 1

   (b) For a Poisson distribution, P(2) = P (3). What is P(2)? 2

   (c) Find the probability of exactly one defective can. 3

   (d) Find the expected number of boxes with no defective cans. 4

4. **In winter, the probability that it rains on a particular day is 0.015. An analyst observes 100 winter days.**

   (a) What is an experiment? 1

   (b) When can the Poisson distribution be approximated by the Binomial distribution? 2

   (c) Find, using Binomial distribution, the probability that it would not rain at all on the observed days. 3

   (d) Find the probability in 3(c) using Poisson distribution. 4

5. **BTCL receives 2.5 telephone calls on average from 4 pm to 6 pm. The number of calls received is a random variable.**

(a) When is Poisson variate applicable? 1

(b) Show conversion criteria and method from Binomial to Poisson distribution. 2

(c) Find the probability of receiving no more than 3 calls. 3

(d) Find the pattern of calls and show on graph paper. 4
Hint: Find probabilities: P(0), P(1), $\cdots$

6. **The number of customers coming at a shop per minute follows a Poisson distribution, whose mean is 3.**

(a) What is a Poisson variate? 1

(b) Can the mean of Poisson distribution be negative? 2

(c) Find the probability that the number of customers coming is between 1 and 2. 3

(d) Are the probabilities of coming to 2 and 3 customers equal? 4

7. **A random variable is distributed as follows:**

Value	0	1	2	3	4	5
Frequency	70	73	27	15	4	1

(a) What is the mean of Poisson distribution? 1

(b) What is the relationship between mean and standard deviation of Poisson distribution? 2

(c) Find the mean and variance of the given distribution. 3

(d) Compare the observed and expected frequencies, assuming a Possion distribution. 4

5.2 Short Questions

Chapter 6

Normal Distribution

6.1 Creative Questions

6.2 Short Questions

Chapter 7

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Chapter 8

Sampling

8.1 Creative Questions

8.2 Short Questions

Chapter 9

Vital Statistics

9.1 Creative Questions

1. **A reseracher uses the following data to know about some demographic characterisics.**

   (a) What is General Fertility Rate? 1

   (b) What is the difference between GRR and NRR? 2

   (c) Compute the population density. 3

   (d) Are TFR and GRR same for this data? 4

2. **For projection of population in a future time period, demographers use simple, geometric or exponential growth technique. Each method has its advantages and disadvantages.**

   (a) What is geometric growth? 1

   (b) In geometric growth method, obtain the formula for time required for the population to get doubled [denote rate as r]. 2

   (c) In exponential method, how much unit of time is required for the population to get tripled? 3

   (d) For projecting (predicting future values), is geometric growth method better than the exponential method? Justify. 4

3. **Population of Dhaka and Sylhet by different age groups and areas are given below:**

Division	Age			Area (km^2)
	0-14	15-64	65+	
Dhaka	10,000,00	5,00,000	5,80,000	1,880
Sylhet	7,00,000	2,70,000	4,70,000	2,319

   (a) Write down the formula of dependency ratio. 1

   (b) What is meant by NRR = 0.983? 2

   (c) Find and compare between the dependency ratios of the cities. 3

   (d) Based on data, which city is more comfortable for living? 4

4. **As part of an analysis, a researcher collected data on women and live births.**

Age	15-19	20-24	25-29	30-34	35-39	40-44	45-49
No. of Women	540	760	530	495	450	505	430
No. of live births	109	198	86	90	65	76	60

(a) What is the formula of death rate? 1

(b) Write down the uses of vital statistics. 2

(c) Find teh Age Specific Birth Rates (ASFR). 3

(d) Find the GFR and compare its concept and value with ASFRs. 4

9.2 Short Questions

Conclusion

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

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