# Statistics Question Bank

Second Paper

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### Contents

Chapter	1 Probability
1.1	Creative Questions
	1.1.1 Applied Set Theory
	1.1.2 Selection of Items
	1.1.3 Coin - Die
1.2	Short Questions
Chapter	2 Random Variable and Probability Function 9
2.1	Creative Questions
2.2	Short Questions
Chapter	3 Mathematical Expectation 13
3.1	Creative Questions
3.2	Short Questions
Chapter	4 Binomial Distribution 17
4.1	Creative Questions
4.2	Short Questions
Chapter	5 Poisson Distribution 20
5.1	Creative Questions
5.2	Short Questions
Chapter	6 Normal Distribution 23
6.1	Creative Questions
6.2	Short Questions
Chapter	7 Index Number 24
7.1	Creative Questions
7.2	Short Questions
Chapter	1 0
8.1	Creative Questions
8.2	Short Questions
Chapter	9 Vital Statistics 20
9.1	Creative Questions
9.2	Short Questions
Conclus	ion <b>2</b> 9

### Probability

#### 1.1 Creative Questions

- 1. Events that do not depend on each other are called independent events, and events that cannot occurr simulataneously are called disjoint events.
  - (a) Provide an example of disjoint events, using the set theory.
  - (b) Prove that  $P(A \cap \bar{B}) = P(A) P(A \cap B)$
  - (c) If there are k mutually and exhaustive events, prove  $\sum_{i=1}^{k} P(A_i) = 1$  3
  - (d) Prove that two events cannot be simulataneously independent and mutually exclusive.
- 2. A quality control analyst in an industry tracks the no. of defective items produced per day. He observes 150 successive days and then prepares a table.

No. of items	0	1	2	3	4
Frequency	30	32	40	28	20

- (a) What is the formula of classical probability?
- (b) Explain the difference between Priori Approach and Empirical Approach of probability. 2

1

- (c) What is the probability that less than 2 defective items would be produced on a particular day?
- (d) Explain the relationship between independency and mutual excluvity in the light of the stem. 4
- 3. Ratul and Tomal both have an unbiased die. Both have randomly thrown their die once.
  - (a) What are equally likely events?
  - (b) If a die is thrown once, what is the probability of getting a prime number?
  - (c) From the stem, what is the probability that the sum of numbers appearing on the dice is greater than 6.
    (d) Examine: the probabilities of getting the sum less than 6 and greater than 6 are equal.
  - (d) Examine: the probabilities of getting the sum less than 6 and greater than 6 are equal.
- 4. A red and a blue dice are thrown once. The dice are absolutely neutral and independent.
  - (a) What is a simple event?
  - (b) Give an example of a certain event using set theory.
  - (c) Find the probability that the difference of two digits from two dices is less than 3.

	(d) Are the probabilities of getting greater digit from the blue die and that from the red die equality.	ıal? 4
5.	An unbiased coin is tossed 10 times.	
	(a) If a coin is flung 3 times, how many outcomes are generated?	1
	(b) If a coin is flung n times, show how many outcomes are generated.	2
	(c) What is the probability of getting a) at least 3 heads, b) at most 3 heads?	3
	(d) Are these probabilities equal? a) Getting at least 2 heads & b) Getting at least 2 tails.  Also justify logically.	4
	Also justify logically.	4
	1.1.1 Applied Set Theory	
6.	It is observed that $50\%$ of mails are spam. A software filters spam mail before reach the inbox. Its accuracy for detecting a spam mail is $99\%$ and chances of taggin non-spam mail as spam mail is $5\%$ .	_
	(a) What is a disjoint event?	1
	(b) For two independent events, what does the Bayes' theorem reduce to?	2
	(c) What is the probability that a mail is tagged as spam?	3
	(d) If a certain mail is tagged as spam, find the probability that it is not a spam mail.	4
7.	A company receives $60\%$ of its job applications from applicants with the requiqualifications. A hiring software screens applications for minimum qualifications correctly identifies qualified applications $97\%$ of the time, but it also incorrectly ma $4\%$ of unqualified applications as qualified.	It
	(a) What is the probability that an application is marked as qualified?	3
	(b) If an application is marked as qualified, find the probability that it actually does not not the required qualifications.	neet
8.	In a survey of a town's population of 500 people, it was found that 150 people rethe local newspaper daily, 200 people listen to the radio daily, and 80 people do be	
	(a) What is the probability that a randomly selected person reads the newspaper given that t listen to the radio?	hey 3
	(b) Calculate the probability that a randomly selected person neither reads the newspaper listens to the radio.	nor
9.	In a school with $200$ students, $60$ students participate in the science club, $80$ participate in the math club, and $30$ participate in both.	ate
	(a) What is the probability that a randomly selected student participates in both clubs?	3
	(b) If a student is chosen at random, what is the probability that they are in exactly one of clubs?	the
10.	In a community of 300 residents, it was found that 90 people use public transportative regularly, 120 use bicycles, and 40 use both.	ion
	(a) What is the probability that a randomly selected resident uses either public transportation bicycles?	n or
	(b) What is the conditional probability that a resident uses public transportation given that t use a bicycle?	hey 4
11.	A dope test correctly identifies a drug user as positive 90% of the time, but incorrec	ctly

identifes 20% non-users as users. The probability of drug use is 0.05.

	(a) Write down the formula of conditional probability.	
	(b) Express $P(A B)$ in terms of $P(B A)$ .	
	(c) Find the probability of testing positive in the test.	
	(d) If the test shows a user positive, what is the probability that the person is actually a user? 4	t
12.	It is observed that in a college, there are 100 students, of whom 30 play football, 40 play cricket, and 20 play both.	1
	(a) What is the range of probability?	-
	(b) What is the relationship between independence and mutual excluvity?	)
	(c) Are the probabilities of playing cricket and that of football independent? Prove.	
	(d) If a student is selected randomly, and if he does not play cricket, what is the probability that he plays football?	
13.	In a school, there are 120 students, of whom 50 are in the drama club, 60 are in the music club, and 30 are in both.	,
	(a) Are the events of being in the drama club and the music club independent? Prove by calculating the conditional probability.	
	(b) If a student is selected randomly, and if they do not belong to the music club, what is the probability that they are a member of the drama club?	
14.	In a survey of 150 people, it is found that 70 own a car, 90 own a house, and 50 own both.	L
	(a) Are the events of owning a car and owning a house independent?	;
	(b) If a person is selected randomly, and if they do not own a house, what is the probability that they own a car?	
15.	In a survey of $150$ people, it is found that $60$ like basketball, $80$ like soccer, and $50$ like both.	)
	(a) What is the probability that a randomly selected person likes only one sport (either basketball or soccer, but not both)?	
	(b) If a person is selected randomly, and if they like soccer, what is the probability that they like basketball?	
	1.1.2 Selection of Items	
16.	A box contains four blue and 6 green balls. 3 balls are drawn randomly.	
	(a) What is the value of ${}^{n}C_{r}$ ?	_
	(b) Illustrate the difference between permutation and combination with an example.	,
	(c) What is the probability that all balls are green?	;
	(d) What is the probabilith that one ball has a different color?	ļ
17.	A jar contains 5 red marbles and 7 yellow marbles. Three marbles are drawn at random.	;
	(a) What is the probability that all marbles are yellow?	;
	(b) What is the probability that a marble has a different color?	Ŀ
18.	Sadman has an urn with 5 red and 4 white balls. He has randomly drawn two balls from the urn.	;
	(a) What is the probability of an uncertain event?	-
	(b) Write the third axiom of probability.	)
	(c) What is the probability that both the balls drawn by Sadman are white?	į
	(d) Are the probabilities of both balls being same color and different color equal? Analyze. 4	1

#### 1.1.3 Coin - Die

27.  $P(X|Y) = \frac{1}{6}, P(X \cap Y) = \frac{1}{10}, P(Y) = \frac{3}{4}$ 

19.	Two dice are thrown together. The dice are named A and B.	
	(a) What is the sample space for the sum of the two dice?	2
	(b) What is the probability that the difference between the outcomes of die A and die B is exact 3?	ctly 3
	(c) What is the probability that the sum of the dice is divisible by 3 and greater than 7?	4
20.	Two dice are thrown together. The dice are named A and B.	
	(a) What is the sample space for the possible outcomes of the two dice?	2
	(b) What is the probability that the absolute difference between the two dice is exactly 1?	3
	(c) What is the probability that the sum of the dice is a prime number and the outcome of dice is greater than that of die B?	e A 4
21.	Two dice are thrown together. The dice are named A and B.	
	(a) What is $P(A=7)$ ?	1
	(b) Create the sample space.	2
	(c) What is the probability that the outcomes of A & B are different?	3
	(d) Determine the probability that the summation of outcome of two dice is a prime number.	. 4
22.	A magician draws two cards from a pack (i) with replacement and then (ii) with replacement. The cards were well-shuffled before drawing.	out
	(a) What is the probability of an impossible event?	1
	(b) How to determine the probability of a joint event?	2
	(c) As per (i), what is the probability that the cards have different color?	3
	(d) As per (ii), what is the probability that the cardsare aces of same color?	4
23.	$P(A) = \frac{3}{10}, P(B) = \frac{2}{5}, P(B \cup A) = \frac{1}{2}$	
	(a) Find $P(A B)$ and $P(B A)$	3
	(b) Verify the equality mathematically & empirically: $P(B) = P(A) \cdot P(B A) + P(\bar{A}) \cdot P(B \bar{A})$	Ā) 4
24.	$P(A B) = \frac{1}{8}, P(A) = \frac{1}{2}, P(B) = \frac{1}{5}$	
	(a)	
	(b) Find $P(A \cap B)$ .	2
	(c) Find $P(A \bar{B})$ .	3
	(d) Are the probabilities $P(A B)$ and $P(B A)$ equal? Justify	4
25.	$P(B A) = \frac{1}{4}, P(B) = \frac{1}{2}, P(A) = \frac{1}{3}$	
	(a) Find $P(B \bar{A})$ .	3
	(b) Find $P(\bar{A} \cap \bar{B})$	4
26.	$P(C D) = \frac{2}{5}, P(C) = \frac{3}{4}, P(D) = \frac{1}{2}$	
	(a) Find $P(C \bar{D})$ .	3
	(b) Examine the following statements:	
	<ul><li>i) A and B are independent and</li><li>ii) A and B are mutually exclusive.</li></ul>	4

JΗA	APTER I. PROBABILITY	5
	(a) Find $P(X \bar{Y})$ .	3
	(b) Are $P(\bar{X})$ and $P(Y)$ independent?	4
28.	$P(M N) = \frac{2}{9}, P(M \cup N) = \frac{5}{7}, P(N) = \frac{4}{7}$	
	(a) Calculate $P(M \cap \bar{N})$ .	3
	(b) Examine whether	4
	i. $P(M N) = P(N M)$ ii. $P(M \cap \bar{N}) = P(\bar{M} \cap N)$	
29.	$P(G H) = \frac{3}{8}, P(G \cap H) = \frac{1}{6}, P(H) = \frac{2}{5}$	
	(a) Find $P(G \cup \bar{H})$ .	3
	(b) Verify the equality mathematically & empirically: $P(G) = P(H) \cdot P(G H) + P(\bar{H}) \cdot P(G \bar{H})$	4
30.	$P(T U) = \frac{5}{12}, P(T \cup U) = \frac{7}{10}, P(U) = \frac{1}{2}$	
	(a) Determine $P(T \cap \bar{U})$ .	3
	(b) What conditions must hold for $T$ and $U$ to be mutually exclusive?	4
31.	Sakib has recently graduated from the University of Dhaka. he applies to tw EduCube & Digic- for a Data Analyst job. The probability of hiring by Edu 0.8 and by Digic is 0.4. The probability that none hires is 0.5.	
	(a) What is a sample space?	1
	(b) Explain how to find $P(\bar{A} \cap B)$ using Venn Diagram.	2
	(c) Find the probability of hiring by by Digic but not by EduCube.	3
	(d) Find the probability that no firm will reject him.	4
32.	Recently there is an increase in the number of electronic medias in Bangla professor stated in the class room that very few people now resort to print mews. A research indicates 70% people collect news from electronic media, 6 print media, and 50% from both.	edia for
	(a) What is an impossible event?	1
	(b) Write the event "None of the two occurs" in two different notations.	2
	(c) What is the probability of getting news from at most one type of media?	3
	(d) Is the professor correct in his/her statement? Analyze.	4
<b>l.2</b>	Short Questions	
1.	What is a trial in the context of probability?	1
2.	What is an experiment in probability.	1
3.	What is a sample space?	1
4.	What is a sample point in probability?	1
5.	Explain what an event is in probability.	1
6.	What is a simple event?	1
7.	Define a compound event.	1
8.	What is an impossible event?	1

9. What is a certain event?

10.	Describe an uncertain event in probability.	1
11.	What does it mean when events are mutually exclusive?	2
12.	What is a complementary event?	1
13.	What are equally likely events.	1
14.	What is the difference between a permutation and a combination?	2
15.	In how many different ways can 5 books be arranged on a shelf?	2
16.	In how many different ways can 6 people be seated around a circular table?	2
17.	In how many ways can 3 books be chosen from a set of 8 books?	2
18.	How many different ways can a team of 4 players be selected from a group of 12 players?	2
19.	How many different ways can a committee of $5$ members be selected from a group of $15$ people?	2
20.	In how many ways can $3$ students be chosen from a class of $20$ students to represent the class at competition?	a 2
21.	How many different ways can 2 representatives be selected from each of 4 departments in a compangiven that there are 6 employees in each department?	ıy, 2
22.	In how many different ways can 4 red balls and 3 blue balls be arranged in a row?	2
23.	In how many ways can a committee of 3 people be selected from a group of 8 people?	2
24.	If there are 4 different letters, how many unique 2-letter permutations can be formed?	1
25.	Calculate the number of 3-letter combinations that can be formed from 7 different letters.	2
26.	In how many ways can a president and a vice president be chosen from a group of $10$ candidates $2$	s?
27.	How many different 4-digit passwords can be created using the digits 1 to 9 if repetition is no allowed?	$\frac{\text{ot}}{2}$
28.	What is the number of ways to arrange the letters in the word "APPLE"?	1
29.	If 10 people are at a meeting, how many ways can 2 people be chosen to speak?	2
30.	How many different teams of 4 players can be formed from a group of 12 players?	2
31.	What is the formula for calculating the number of permutations of n objects taken r at a time?	1
32.	How many different 3-digit combinations can be formed using the digits $2, 4, 6, 8$ , and $9$ if repetition is allowed?	on 2
33.	In how many ways can 6 people be seated in a row?	2
34.	If a deck of cards is shuffled, in how many ways can 5 cards be selected from the deck?	2
35.	What is the value of 5!?	1
36.	Expand $^{n}P_{r}$	
37.	Expand ${}^{n}C_{r}$	
38.	What is the classical definition of probability?	1
39.	What is the range of probability?	1
40.	Briefly explain empirical probability with an example.	2
41.	How does the classical definition of probability differ from the empirical definition?	2

42. Which definition of probability does this formula belong to:

$$P(E) = \lim_{n \to \infty} \frac{\mathrm{n(A)}}{n(S)}$$

- 43. What are the three axioms of probability in the axiomatic approach?
- 44. In the axiomatic approach, if P(S) = 1, where S is the sample space, what does this imply?
- 45. How does the axiomatic approach define the probability of the union of two mutually exclusive events A and B?
- 46. What is the third axiom of probability?
- 47. In the third axiom, what is the value of  $\sum_{i=1}^{n} P(A_i i)$
- 48. What does it mean when P(A) = 0 in probability theory?
- 49. When P(A) = 1, what does this signify about the event A?
- 50. What is the formula for conditional probability P(A|B)?
- 51. What is the value of  $P(A \cap B)$  if two events A and B are independent?
- 52. If events A and B are independent, what is the value of P(A|B)?
- 53. What is an independent event?
- 54. What is the additive law of probability for two events A and B?
- 55. How does the additive law of probability apply when events A and B are mutually exclusive? 2
- 56. What is the additive law of probability for n events, and how is it expressed mathematically? 2
- 57. What is the multiplicative law of probability for two events A and B?
- 58. How does the multiplicative law of probability apply when events A and B are independent? 2
- 59. If two events A and  $A^c$ ) are complementary, what is the relationship between them?
- 60. Prove using Venn diagram  $P(A \cap \bar{B}) = P(A) P(A \cap B)$
- 61. What is the relationship between independency and mutual exclusity?
- 62. What is the range of values that probability can take for any event?
- 63. Why is the probability of any event always between 0 and 1, inclusive?
- 64. Can the value of probability be 1.2?
- 65. Can the value of probability be -0.2?
- 66. How can the expression  $P(A \cap B)$  be expanded in terms of conditional probability?
- 67. Expand  $P(A' \cap B)$
- 68. Expand  $P(A \cap B')$
- 69. Can two events be independent and mutually exclusive at once?
- 70. What is the probability of getting a head on a fair coin toss?
- 71. If a fair die is rolled, what is the probability of getting a number greater than 4?
- 72. If a coin is tossed 4 times, how many outcomes are generated?
- 73. If a die is thrown 3 times, how many outcomes are generated?

CHAPTER 1. PROBABILITY	8
74. If 2 coins and a die are thrown together, how many outcomes are generated?	1
75. Is there any difference between tossin a coin thrice and tossing 3 coins together	2
76. Write down the formula of $P(\bar{A} \bar{B})$	1
77. Write down and expand the formula of $P(\bar{A} B)$	2

# Random Variable and Probability Function

#### 2.1 Creative Questions

- 1. A deck of 52 card is well-shuffled and three cards are drawn from them at random. The number of kings obtained is denoted by x.
  - (a) What are equaly likely events?
  - (b) Differentiate between with replacement and without replacement drawings.
  - (c) Form the probability fucntion using the above information and then form the distribution. 3
  - (d) Examine the statement:  $P(1 \le x \le 3) = F(3) F(1)$
  - (a) The joint probability function of two random variables X and Y is given by:

$$P(X,Y) = \frac{x+2y}{28}; \quad x = 0,1; \quad y = 0,1,2,3$$

- i. Write down the formula for conditional probability.
- ii. What is the relationship between marginal and joint probability?
- iii. Find P(X).
- iv. Find P(X|Y) and P(X|Y=0).
- (b) The joint probability function of two random variables X and Y is described by:

$$P(X,Y) = \frac{2x+3y}{45}; \quad x = 0,1,2; \quad y = 0,1,2$$

- i. Write down the formula for conditional probability.
- ii. What is the relationship between marginal and joint probability?
- iii. Find P(X).
- iv. Find P(X|Y) and P(X|Y=0).
- 2. The joint probability function of two random variables X and Y is given by:

$$P(X,Y) = \frac{x+y+1}{42}; \quad x = 0, 1, 2; \quad y = 0, 1, 2, 3$$

- (a) Calculate the marginal probability P(Y).
- (b) Determine P(Y|X=1) and P(Y|X=0).
- 3. The joint probability function of two random variables X and Y is described by:

$$P(X,Y) = \frac{2x+y+1}{52}; \quad x = 1,2; \quad y = 1,2,3,4$$

(a) Find the marginal distribution P(X).

(b) Compute 
$$P(Y|X)$$
 for  $X=2$ .

4. The joint probability function of two random variables X and Y is given by:

$$P(X,Y) = \frac{3x+y}{48}; \quad x = 1,2; \quad y = 0,1,2,3$$

- (a) Find P(X).
- (b) Calculate P(X|Y) and P(X|Y=1).
- 5. The probability distributions of a random variable X in two different cases are given below:

Table 2.1: **Distribution - A** 

Table 2.2: Distribution - B

- (a) What is a probability mass function?
- (b) Can we dtermine the probability of a certain value of a discrete random variable?
- (c) What is the value of w?
- (d) Which table is a proper probability distribution? Justify with mathematical reasoning. 4
- 6. A continuou random variable X follows the following probability density function (pdf).

$$f(x) = 6x(1-x); 0 < x < 1$$

- (a) Give an example of a continuous random variable.
- (b) Examine whether the given function is a pdf.
- (c) If P(X > a) = P(X < a), find the value of a.
- (d) Should  $P(0.5 \le X \le 1)$  be equal to 0.5?
- 7. The probability mass function (pmf) of a football striker scoring no. of hattricks during the course of a league season is given below

$$P(x) = \frac{|2-x|}{k}; x = 0, 1, 2, 3, 4, 5$$

- (a) What is a random variable?
- (b) Is probability a discrete variable? Explain in brief.
- (c) Find the value of k. 3
- (d) Find the probability that the no. of hattricks would be less than the expectation.
- 8. A fair coin is tossed five times. Number of heads appearing are noted, considering it a discrete random variable.
  - (a) Give a real life example of a discrete random variable.
  - (b) Can discrete variable have infinite number of possible outcomes?

(c) Find the probability distribution from the stem.	3
(d) Construct the distribution function and hence find $F(X \le 3)$ .	4
9. The probability density function of a continuous random variable is	
$f(x) = \begin{cases} k(x+1), & 0 \le x \le 1\\ 0, & otherwise \end{cases}$	
(a) What is a random variable?	1
(b) Find the value of k	2
(c) Find the probability that the values of x would lie between 0 and 0.5.	3
(d) What is the probability that X is greater than 0.8?	4
10. The probability density function of a continuous random variable is	
$f(x) = \begin{cases} kx(x-1), & 1 \le x \le 4\\ 0, & otherwise \end{cases}$	
$f(x) = \begin{cases} 0, & otherwise \end{cases}$	
(a) What is the range of probability?	1
(b) Find the value of k	2
(c) Justify the pdf property of the fucntion.	3
(d) What is the probability that X is greater than 3?	4
11. The probability distribution of a discrete random variable ${\bf X}$ is given	below:
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
$P(x) \mid 0.1  k  2k  3k  4k  0.2$	
(a) What is $\Sigma P(x)$ ?	1
(b) Find the value of k.	2
<ul> <li>(c) Find P(X ≥ 0) and P(X &lt; 1).</li> <li>(d) Find the cumulative distribution function, F(X) and F(2) and explain.</li> </ul>	3
	4
12. The joint probability function of two random variables X & Y is given	n below:
$P(x,y) = \frac{1}{21}(x+y); x = 1, 2, 3 \& y = 1, 2$	
(a) What is a probability density function (pdf)?	1
(b) What is P(X=a) in a pdf, where a is an aribitrary number?	2
(c) Find the marginal probabilities.	3
(d) Find $P(x y)$ , $P(x 1)$ and $P(y 4)$	4
13. The probability density function of a continuos random variable ${\bf X}$ is	given as:
$f(x) = \frac{1}{b-a}; a \le x \le b$	
(a) In this distribution, what is P(a)?	1
(b) What is the shape of the distribution?	2
(c) Find $P(a \le x \le b)$ .	3
(d) Find and explain the median of the distribution.	4

#### 14. The probability density function of a continuous random variable is

$$f(x) = \begin{cases} kx^2 + kx + \frac{1}{8}, & 0 \le x \le 2\\ 0, & otherwise \end{cases}$$

1 (a) What is a continuous random variable? 2 (b) Find the value of k (c) Find the probability that the values of x would lie between 1 and 3. 3 (d) Find the 40th percentile of the distribution and explain. 4 2.2Short Questions 1. What is a discrete random variable? 1 2. What is a continuous random variable? 1 3. Give an example of a continuous random variable 1 4. Is the number of cars passing through a toll booth in an hour an example of a discrete or continuous random variable? 5. Is the temperature in a city measured every hour an example of a discrete or continuous random 6. Is the number of students in a classroom an example of a discrete or continuous random variable? 7. Is the amount of time it takes for a light bulb to burn out an example of a discrete or continuous random variable? 8. Is the number of emails received in a day an example of a discrete or continuous random variable? 9. Is the weight of a person an example of a discrete or continuous random variable? 1 10. What is the integral of  $x^n$ , where  $n \neq -1$ ? 1 11. Compute the integral of  $x^3$  with respect to x. 1 12. Find the integral of  $x^5$  with respect to x. 1 13. Compute the definite integral of  $x^2$  from 0 to 3. 2 14. Find the value of the definite integral  $\int_1^4 x^4 dx$ . 2 15. What is the property of a probability distribution regarding the sum of all probabilities? 1 16. What are the required properties of a probability distribution? 2 17. What is the formula of cumulative distribution function for a discrete variable? 1 18. What is the formula of cumulative distribution function for a continuous variable 1 19. If a < b, F(b) - F(a) = ?, where F is cumulative distribution function? 2 20. How can you find f(x) from F(x) for a continuous distribution? 1 21. How can you find f(x) from F(x) for a discrete distribution? 1 22. How can calculate P(X > 3) using the concept of complementary probability? 1

### Mathematical Expectation

#### 3.1 Creative Questions

1. The probability distribution of a random X is provided below:

(a) What is the expectation of a constant m?

1

2 (b) Find E(X). (c) Find E(Y), where  $Y = \frac{X}{2}$ 3 (d) Find Variance of (2X+3). 4 2. A random variable is distributed as below:  $P(X) = \frac{3-|4-x|}{k}$ ; x = 2, 3, 4, 5, 61 (a) What is the Expectation equivalent to? (b) Find the value of k. 2 (c) Determine the value of the expectation. 3 (d) Find V(2X-1)4 3. A random variable is distributed as below:  $P(X) = \frac{5-|6-x|}{h}$ ; x = 1, 2, 3, 4, 5

$$P(X) = \frac{3-|6-x|}{k}; x = 1, 2, 3, 4, 5$$

2 (a) Find the value of k.

(c) Find V(3X+2)4

4. A random variable is distributed as below:

$$P(X) = \frac{7-|5-x|}{k}; x = 1, 2, 3, 4, 5, 6$$

(a) Find the value of k. 2

(c) Find V(4X-3)4

5. The probability distributions of demand of mobile phones of two operating systems (OS) Android (X) and iPhone OS (iOS) (Y) are:

(a) What is Expectation? 1

Demand	100	200	300	400	500
P(X)	0.1	0.4	m	0.15	0.1
P(Y)	0.09	0.45	0.32	0.11	0.03

(b) Can Expectation be negative?

2

(c) Find m from the table.

3

(d) Which OS has higher demand? Analyze.

4

6. The probability distributions of daily sales of two popular coffee brands, Brand A (X) and Brand B (Y), are:

Sales (cups)	50	100	150	200	250
P(X)	0.05	0.3	p	0.25	0.1
P(Y)	0.1	0.35	0.3	0.2	0.05

(a) Find p from the table.

3

(b) Which brand has a more consistent daily sales distribution? Justify your answer.

4

7. The probability distributions of sales for two different products, Product A (X) and Product B (Y), are:

Sales (units)	100	200	300	400	500
P(X)	0.1	0.4	p	0.2	0.1
P(Y)	0.2	0.3	0.3	0.1	0.1

(a) Find p from the table.

3

(b) Which product has a more concentrated sales distribution? Justify your answer.

4

- 8. An umbrella seller earns a revenue of BDT. 5000 if it rains. If it does not rain, he loses BDT. 1000. The probability that it rains on a given day is 0.04.
  - (a) Write down the formula of Expectation for a continuous random variable.

1 2

(b) Can the value of Expectation be zero?

3

(c) What is the umbrella seller's expected revenue?

- (d) What should be the minimum probability of raining for him to achieve revenue greater than zero?
- 9. A small business owner earns a profit of BDT. 3000 if a customer purchases a product. If no customer makes a purchase, he incurs a loss of BDT. 1500. The probability of a customer making a purchase on a given day is 0.10.
  - (a) What is the business owner's expected profit?

3

- (b) What should be the minimum probability of a customer making a purchase for the owner to have a profit greater than zero?
- 10. A factory produces gadgets. If the demand is high, the factory earns BDT. 12000; if the demand is low, it loses BDT. 3000. The probability of high demand is 0.15.
  - (a) What is the expected revenue for the factory?

3

- (b) What is the minimum probability of high demand required for the factory to make a profit greater than 5,000?
- 11. A box contains 5 red and 6 white balls. 3 balls are drawn at random. X is the number of white balls drawn.

(a) What does variance measure?	1
(b) Can the variance be smaller than standard deviation?	2
(c) Find the E(X) from the stem.	3
(d) Find the variance from the stem assuming X is the number of red balls draw	wn. 4
2. A professor showed a probability distribution in a class:	
$oxdot{x}  1  2  3  4  5$	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
The value of the arithmetic mean of the distribution is 3.	
(a) What is the formula of expectation?	1
(b) What is the variance of a constant? Explain logically.	2
(c) What are the values of a & b?	3
(d) Find and explain the variance of the distribution.	4
.,	I
3. X is a random variable having the below functional form:	
$P(X) = \frac{6 -  7 - x }{k}; x = 1, 2, \dots, 10$	
Y is another variable having the relationship $y = 3x+5$	
(a) What is joint probability?	1
(b) What is the minimum possible value of variance? Why?	2
(c) Find the value of k.	3
(d) Find E(X) and E(Y). Why are they different?	4
4. Various sales and their probabilities of a grocery store is given below	
Sales 200 250 275 310 350	
Probability 0.10 0.20 0.40 0.25 0.05	
(a) Can the expectation of a random variable be negative?	1
(b) Find the expected sales of the store on a given day.	2
(c) Compute the dispersion of sales f the store.	3
(d) To make the expected sale 280, what sale does the store need in place of 200	0? 4
5. A survey of Television (TV) users at Gulshan in Dhaka was conducted	ed to find how
many sets each family use. The following data were obtained:	
$\frac{\text{No of TV set}  0  1  2  3}{\text{No of family}  10  75  10  5}$	
100 of failing 10 75 10 5	
(a) What is Expectation equivalent to?	1
(b) Can Variance be negative? Why or why not?	2
(c) Find the variance of the number of TV sets.	3
(d) Find and compoare between arithmetic mean and expectation.	4

#### 3.2 Short Questions

- 1. What is the formula for the expectation E(X) of a discrete random variable X
- 2. What is  $E(X^2)$  equal to?
- 3. What is the relationship between expectation and arithmetic mean?
- 4. Derive Expectation from arithmetic mean.
- 5. Rewrite E(2X).
- 6. Rewrite E(4X+c).
- 7. Rewrite E(4X + 7).
- 8. Rewrite  $E(\frac{Y}{3} 3)$ .
- 9. How can you expand E(x+y)?
- 10. Between  $\{E(x)\}^2$  and  $E(x^2)$ , which one is greater?
- 11. How can you expand E(xy)?
- 12. Between  $E\left(\frac{1}{x}\right)$  and  $\frac{1}{E(x)}$ , which one is greater?
- 13. Determine the formula of variance in terms of expectation.
- 14. How is  $E(x^2)$  found using V(x) and E(x)?
- 15. What is the formula for the variance of a discrete random variable X?
- 16. How is the variance V(X) related to the expectation E(X)?
- 17. What is the variance of a constant value?
- 18. How is the variance of the sum of two independent random variables calculated?
- 19. What is the relationship between variance and standard deviation?
- 20. V(a) = ?, where a is an arbitrary constant?
- 21. V(x-a) = ?
- 22.  $V(\frac{x}{3}) = ?$
- 23.  $V(\frac{x}{4}+2)=?$
- 24. If V(x) = 1, what is the value of standard deviation?
- 25. If V(x) = -9, what is the value of standard deviation?
- 26. If V(x) = 10, what is the value of standard deviation?
- 27. If V(x) = 1, what is the value of standard deviation?
- 28. Can variance be smaller than standard deviation?
- 29. When can variance and standard deviation be equal?
- 30. V(x+y) = ?
- 31. V(x-y) = ?
- 32. What covariance?
- 33. Write down the formula of covariance.

### **Binomial Distribution**

#### 4.1 Creative Questions

- 1. A farmer selected a paddy field for seed collection. He found out that 10 out of each 25 paddies are damaged. He collected a sample of 15 paddies.
  - (a) What is a Bernoulli trial?
  - (b) IF a Bernoulli trial is repeated n times, in how many ways are outcomes generated? Explain.  $2\,$
  - (c) Find the probability that at least one paddy is damaged.
  - (d) Comment on the skewness of the data. [Hint: For a binomial distribution,  $\gamma_1 = \frac{q-p}{\sqrt{npq}}$ ]
- 2. A biologist is studying a group of plants and notes that 8 out of every 20 plants are infected with a certain disease. She collects a sample of 12 plants.
  - (a) Find the probability that at least one plant is infected.
  - (b) Determine the median of the distribution and explain its significance.
- 3. The electric kettles produced by a certain manufacturer are 12% defective on average. The company supplies 20 kettles in a packet. A retailer bought 1000 packets.
  - (a) What is the probability that no. of defective kettles is at most 2?
  - (b) In how many packtes, there are exactly 3 defective kettles?
- 4. A smartphone company finds that 9% of its phones have minor defects. Each shipment contains 35 phones. A retailer purchases 500 shipments.
  - (a) What is the probability that a randomly selected shipment has at most 3 defective phones? 3
  - (b) In how many shipments can we expect to find between 4 and 7 defective phones (inclusive)? 5
- 5. A bottling factory reports that 6% of its bottles have minor defects. Each batch contains 45 bottles. A supplier purchases 700 batches.
  - (a) What is the probability that a randomly selected batch contains at most 4 defective bottles? 3
  - (b) How many batches are expected to have at least 6 defective bottles?
- 6. A company produces smartphones, and it is known that 5% of the smartphones have a manufacturing defect on average. The company ships 15 smartphones in each box, and a retailer purchases 500 boxes.
  - (a) What is the probability that the number of defective smartphones in a box is at least 1?

	(b) How many boxes are expected to contain exactly 2 defective smartphones?	4
7.	A farmer plans to store rice seeds for future use. It was found that 8 out of 20 are rotten. He then collected a sample of 15 seeds.	$\mathbf{seeds}$
	(a) What is Bernoulli trial?	1
	(b) How are Bernoulli and Binomial distributions related?	2
	(c) What is the probability that at least one seed is rotten out of 15?	3
	(d) What is the probability that the number of rotten seeds is greater than the arithmetic 4	nean?
8.	The number of defective pen produced by a company follows a binomial distribution with expectation 1.5 and variance 1.125	ution
	(a) What is the mean of binomial distribution	1
	(b) Can variance be greater than mean in binomial distribution?	2
	(c) Determine the probability function of the number of defective items produced by the con $3$	ıpany.
	(d) What is the probability that the number of defective items is no less than 3?	4
9.	The number of faulty light bulbs produced by a factory follows a binomial distribution and a variance of 1.6.	ution
	(a) Determine the probability function for the number of faulty light bulbs produced lactory.	by the
	(b) What is the probability that the number of faulty light bulbs is at least 4?	4
4.2	Short Questions	
1.	What is $p$ in binomial distribution?	1
2.	What is a Bernoulli trial?	1
3.	How many outcomes are there in a Bernoulli trial?	1
4.	What is the probability function of Bernoulli distribution?	1
5.	Is Bernoulli distribution discrete or continuous?	1
6.	What is the value of n (number of trial) in a Bernoulli distribution?	1
7.	What is relationship between Bernoulli and Binomial distribution?	1
8.	How many parameters does the Binomial distribution have?	1
9.	What are the paremeters of Binomial distribution?	2
10.	What happens if $n = 1$ in Binomial distribution?	2
11.	What is denoted by $X$ in Binomial distribution?	1
12.	What is the mean of a binomial distribution with parameters $n$ and $p$ ?	1
13.	How is the variance of a binomial distribution with parameters $n$ and $p$ calculated?	1
14.	What is the skewness of a binomial distribution $p$ ?	1
15.	What is the kurtosis of a binomial distribution?	1
16.	When is the binomial distribution symmetric?	1
17.	How does the probability mass function (PMF) of a binomial distribution change with incr $n$ ?	easing

18.	What is the relationship between the mean, variance, and skewness in a binomial distribution?	2
19.	Is $E(X) < V(X)$ possible in binomial distribution?	1
20.	Can $V(X)$ be equal to $E(X)$ in binomial distribution? Examine.	2
21.	In a symmetric binomial distribution, what is the functional form of probability?	1

### Poisson Distribution

5.1	Creative	Questi	ions			
1.	The standard	deviation	of a Poisson	distribution	is	2.

	(a) Find $P(X \ge 2)$	3
	(b) If $P(a) = P(a+1)$ , what is the value of a?	4
2.	The number of machine failures in a factory follows a Poisson distribution with standard deviation of 2.5.	ıa
	(a) Find $P(X \ge 4)$ .	3
	(b) If $P(a) = P(a+2)$ , what is the value of a? Does it hold for Poisson distribution?	4
3.	A call center records the number of customer complaints per day, which follows Poisson distribution with a standard deviation of 3.	3 a
	(a) Find $P(X \ge 3)$ .	3
	(b) If $P(a) = P(a+1)$ , what is the value of $a$ ?	4
4.	Between 1000hrs and 1700 hrs, the average number of phonce calls per minute receive by a power distribution company is 2.5.	ed
	(a) Give an example where Poisson distribution is applicable.	1
	(b) Find the relationship between expectationa and standard deviation of Poisson distribution	. 2
	(c) Find the probability that the number of calls is between 1 and 3 (inclusive).	3
	(d) What is the probability that the number of calls received is greater than the average?	4
5.	A customer service center receives an average of 4.2 calls per minute during per hours (1200 hrs to 1800 hrs).	ak
	(a) Find the probability that the number of calls received in a minute is between 2 and 5 (inc sive).	lu-
	(b) What is the probability that the number of calls received is at least twice the average?	4
6.	An emergency room receives an average of 3.8 patient arrivals per hour during nightime (2200 hrs to 0600 hrs).	ht-
	(a) Find the probability that the number of patient arrivals in an hour is between 2 and 4 (inc sive).	lu-
	(b) What is the probability that the number of arrivals exceeds 2?	4

7. The frequency distribution of defective items in packets of key rings is given below.

Number of defective items	0	1	2	3	4	5
Number of packets	76	74	29	17	3	1

	(a) Wł	nat is another way to write $P(X \ge 1)$ ?	1
	(b) Ca	n the mean of Poisson distribution be negative?	2
	(c) Fro	om the given stem, find mean and variance.	3
	(d) Fir	nd the expected frequencies and comment.	4
8.	packag	manufacturer observes that $0.1\%$ of the produced cans are faulty. The can ed in carton boxes, with 500 cans in each box. A wholeseller purchases from the manufacturer.	
	(a) Wł	nat is shape of Poisson distribution?	1
	(b) For	r a Poisson distribution, $P(2) = P(3)$ . What is $P(2)$ ?	2
	(c) Fir	nd the probability of exactly one defective can.	3
	(d) Fir	nd the expected number of boxes with no defective cans.	4
9.		ter, the probability that it rains on a particular day is $0.015$ . An analyst observer days.	erves
	(a) Wł	nat is an experiment?	1
	(b) Wł	nen can the Poisson distribution be approximated by the Binomial distribution?	2
	` '	nd, using Binomial distribution, the probability that it would not rain at all on the served days.	3
	(d) Fir	nd the probability in $3(c)$ using Poisson distribution.	4
10.		receives $2.5$ telephone calls on average from $4$ pm to $6$ pm. The number of d is a random variable.	calls
	(a) Wł	nen is Poisson variate applicable?	1
	(b) Sho	ow conversion criteria and method from Binomial to Poisson distribution.	2
	(c) Fir	nd the probability of receiving no more than 3 calls.	3
		and the pattern of calls and show on graph paper. In the Find probabilities: $P(0)$ , $P(1)$ ,	4
11.	The numer mean 3	umber of customers coming at supershop follows a Poisson distribution 3.	with
	(a) De	termine the probability that in a particular minute, at least 1 customer arrives.	3
	(b) If <i>i</i>	P(X = a) = P(X = b), find the value of a. What pattern do you get?	4
12.		umber of cars passing through a toll booth follows a Poisson distribution of $5$ cars per minute.	with
	(a) De	termine the probability that exactly 3 cars pass through the toll booth in a minute.	3
	(b) If <i>I</i>	P(X = a) = P(X = b), find the value of a and b. What pattern do you observe?	4
13.	_	umber of customers coming at a shop per minute follows a Poisson distribumean is 3.	ition,
	(a) Wł	nat is a Poisson variate?	1
	. ,	n the mean of Poisson distribution be negative?	2
	(c) Fir	nd the probability that the number of customers coming is between 1 and 2.	3

(d) Are the probabilities of coming to 2 and 3 customers equal?

#### 14. A random variable is distributed as follows:

(a) What is the mean of Poisson distribution?

1

2

3

- (b) What is the relationship between mean and standard deviation of Poisson distribution?
- (c) Find the mean and variance of the given distribution.
- (d) Compare the observed and expected frequencies, assuming a Possion distribution. 4

#### 15. A random variable is distributed as follows:

- (a) Find the mean and standard deviation of the given distribution.
- (b) Compare the observed and expected frequencies, assuming a Poisson. 4

#### 5.2 Short Questions

- 1. What is a Poisson variate?
- 2. What is a Poisson process?
- 3. What is the mean of a Poisson distribution with parameter  $\lambda$ ?
- 4. What is the variance of a Poisson distribution with parameter  $\lambda$ ?
- 5. How is the probability mass function (PMF) of a Poisson distribution expressed?
- 6. When can the Poisson distribution be used as an approximation to the binomial distribution? 1
- 7. What is e in Poisson distribution?
- 8. What is the mean of Poisson distribution?
- 9. What is the variance of Poisson distribution?
- 10. What is the relationship between mean and variance of Poisson distribution?
- 11. Is Poisson distribution discrete or continuous?
- 12. What is the moment generating function of Poisson distribution?
- 13. If  $x_1 \sim Poisson(m_1)$  and  $x_2 \sim Poisson(m_2)$ ,  $x_1 + x_2 \sim$ ?
- 14. How is Poisson distribution skewed?
- 15. What is the kurtosis of Poisson distribution?
- 16. If a Poisson distribution is  $P(x) = \frac{e^{-m}m^x}{x!}$ , P(x+1) = ? Derive in terms of P(x).

17. Prove 
$$\sum_{i=1}^{\infty} P(x) = \sum_{i=1}^{\infty} \frac{e^{-m} m^x}{x!} = 1$$

# Normal Distribution

- 6.1 Creative Questions
- 6.2 Short Questions

### Index Number

- 7.1 Creative Questions
- 7.2 Short Questions

# Sampling

- 8.1 Creative Questions
- 8.2 Short Questions

### Vital Statistics

#### Creative Questions

(b) What is meant by NRR = 0.983?

.1 (	Creative Quest	ions				
1. <b>A</b> 1	reseracher uses the fe	ollowing da	ta to knov	w about s	ome demographic character	risics.
(a	) What is General Fert	ility Rate?				1
(b	) What is the difference	e between GI	RR and NR	R?		2
(c	) Compute the populat	ion density.				3
(d	) Are TFR and GRR sa	ame for this	data?			4
geo				- /	demographers use simple, thod has its advantages an	d
(a	) What is geometric gro	owth?				1
(b)	) In geometric growth a doubled [denote rate a		in the form	nula for tim	e required for the population t	o get 2
(c)	) In exponential metho 3	d, how much	unit of tir	ne is requir	ed for the population to get tr	ipled?
(d	) For projecting (prediction exponential method?	_	ralues), is g	eometric gr	rowth method better than the	4
3. <b>Po</b>	pulation of Dhaka aı	nd Sylhet b	y differen	t age grou	ıps and areas are given bel	ow:
	Division	0-14	Age 15-64	65+	Area $(km^2)$	
	Dhaka	10,000,00	5,00,000	5,80,000	1,880	
	Sylhet	7,00,000	2,70,000	4,70,000	2,319	
(a	) Write down the form	ıla of depend	ency ratio.			1
	•	-	-			

	· /	
	(c) Find and compare between the dependency ratios of the cities.	3
	(d) Based on data, which city is more comfortable for living?	4
1	Population of New York Los Angeles and Chicago by different age groups and are	295

2

- 4. Population of New York, Los Angeles, and Chicago by different age groups and areas are given below:
  - (a) Find and compare the dependency ratios of New York and Chicago. 3
  - (b) Based on the data, which city is more comfortable for living? Justify your choice. 4
- 5. As part of an analysis, a researcher collected data on women and live births.

3

3

3

4

1

1

City		Age		Area (sq. km)
	0-14	15-64	65+	
New York	1,200,000	5,000,000	700,000	789
Los Angeles	1,000,000	4,500,000	500,000	1,302
Chicago	900,000	3,800,000	600,000	606

Age	15-19	20-24	25 - 29	30-34	35-39	40-44	45-49	
No. of Women	540	760	530	495	450	505	430	
No. of live births	109	198	86	90	65	76	60	

- (a) What is the formula of death rate?
- (b) Write down the uses of vital statistics.
- (c) Find teh Age Specific Birth Rates (ASFR).
- (d) Find the GFR and compare its concept and value with ASFRs.
- 6. A study was conducted to analyze the relationship between women's age groups and live births. The following table presents the number of women surveyed in different age groups along with the corresponding number of live births.

Age Group	No. of Women	No. of Live Births
15-19	540	109
20-24	760	198
25-29	530	86
30-34	495	90
35-39	450	65
40-44	505	76
45-49	430	60

- (a) Calculate the Age Specific Birth Rates (ASFR) for each age group.
- (b) Compute the General Fertility Rate (GFR) and compare its value and concept with ASFR. 4
- 7. The following dataset records the number of women in different age groups and their respective live births as part of a demographic study.

Age Group	No. of Women	No. of Live Births
15-19	540	109
20-24	760	198
25-29	530	86
30-34	495	90
35-39	450	65
40-44	505	76
45-49	430	60

- (a) Determine the Age Specific Birth Rates (ASFR) for the given age groups.
- (b) Find the General Fertility Rate (GFR) and explain its significance in relation to ASFR.

#### 9.2 Short Questions

4. How is dependency ratio calculated?

- 1. What does vital statistics deal with?
- 2. Mention 4 sources of vital statistics.
- 3. What are the sources of vital statistics.

- 5. What is the formula of sex ratio? 1 6. What does Age Specific Fertility Rate (ASFR) measure? 1 7. Why is ASFR calculated for specific age groups instead of the entire population? 1 8. Write down the formula of population density? 1 9. What is the formula of crude birth rate 1 10. How is General Fertility Rate calculated? 1 2 11. Distinguish between TFR and GFR. 12. What is the purpose of Age-Specific Fertility Rate? 1 13. Two dependency ratios are  $d_1 = 98\%$  and  $d_2 = 104\%$ . In which case are there more dependent per 1000 individuals? 14. Ques
- 15. Ques

### Conclusion

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

# Contents