

[N.B. – The figures of the right margin indicate full marks. Read the stems carefully and answer the associated questions. Answer any **FIVE** questions taking at least two questions from each group]

Group - A

1. In a survey of 150 people, it is found that 70 own a car, 90 own a house, and 50 own both.
- (a) What is a simple event? 1
  - (b) How many different ways can a team of 4 players be selected from a group of 12 players? 2
  - (c) What is the probability that a randomly selected person has only one item? 3
  - (d) If a person is selected randomly, and if they do not own a house, what is the probability that they own a car? 4
2. Two dice are thrown together. The dice are named A and B.
- (a) If two events  $A$  and  $A^c$  are complementary, what is the relationship between them? 1
  - (b) What is the sample space for the possible outcomes of the two dice? 2
  - (c) What is the probability that the absolute difference between the two dice is exactly 1? 3
  - (d) What is the probability that the sum of the dice is a prime number and the outcome of die A is greater than that of die B? 4
3. A deck of 52 card is well-shuffled and three cards are drawn from them at random. The number of kings obtained is denoted by  $x$ .
- (a) What is the formula of cumulative distribution function for a continuous variable 1
  - (b) Differentiate between with replacement and without replacement drawings. 2
  - (c) Form the probability fucntion using the above information and then form the distribution. 3
  - (d) Examine the statement:  $P(1 \leq x \leq 3) = F(3) - F(1)$  4
4. A random variable is distributed as below:
- $$P(X) = \frac{7-|5-x|}{k}; x = 1, 2, 3, 4, 5, 6$$
- (a) If  $Z = 2Y + 7$  and  $E(Y) = 10$ , what is  $E\left(\frac{Z}{2} - 3\right)$ ? 1
  - (b) Find the value of  $k$ . 2
  - (c) Determine the value of the expectation. 3
  - (d) Find  $V(4X - 3)$  4

Group - B

5. A smartphone company finds that 9% of its phones have minor defects. Each shipment contains 35 phones. A retailer purchases 500 shipments.
- (a) How many outcomes are there in a Bernoulli trial? 1
  - (b) How does  $n$ ,  $p$ , and  $q$  affect the shape of binomial distribution? 2
  - (c) What is the probability that a randomly selected shipment has at most 3 defective phones? 3
  - (d) In how many shipments can we expect to find between 4 and 7 defective phones (inclusive)? 4
6. An emergency room receives an average of 3.8 patient arrivals per hour during nighttime (2200 hrs to 0600 hrs).
- (a) If the mean of a Poisson distribution is 7, what is its standard deviation? 1
  - (b) If a Poisson distribution has  $P(1) = P(2)$ , what is its variance? 2
  - (c) Find the probability that the number of patient arrivals in an hour is between 2 and 4 (inclusive). 3
  - (d) What is the probability that the number of arrivals exceeds 2? 4
7. The number of machine failures in a factory follows a Poisson distribution with a standard deviation of 2.5.
- (a) How is Poisson distribution skewed? 1
  - (b) In a Poisson distribution,  $P(a) = P(a+1)$ . What is the value of  $a$  if parameter is 3? 2
  - (c) Find  $P(X \geq 4)$ . 3
  - (d) If  $P(a) = P(a + 2)$ , what is the value of  $a$ ? Does it hold for Poisson distribution? 4

8. A study was conducted to analyze the relationship between women’s age groups and live births. The following table presents the number of women surveyed in different age groups along with the corresponding number of live births.

Age Group	No. of Women	No. of Live Births
15-19	540	109
20-24	760	198
25-29	530	86
30-34	495	90
35-39	450	65
40-44	505	76
45-49	430	60

- (a) Why is ASFR calculated for specific age groups instead of the entire population?1
- (b) Distinguish between TFR and GFR.2
- (c) Calculate the Age Specific Birth Rates (ASFR) for each age group.3
- (d) Compute the General Fertility Rate (GFR) and compare its value and concept with ASFR.4

“It is easy to lie with statistics; it is easier to lie without them.” - Frederick Mosteller