MATH253X-UX1 Quiz 1 Key

Fall 2021

Name: ____________________

Instructions. Either print this two-page file and fill in your answers or use a separate piece of paper. Then attach your work as a PDF. Show all your work in order to receive full credit. You have a total a 45 minutes from the time you enter the assignment to the time you submit it. No outside help allowed.

1. An object is located at the point $P(2, 0, -1)$, but is constrained so that it can only move in the straight-line direction toward the point $Q(0, 1, 3)$.

(a) (1 Points) Give, in coordinate form, a vector $\mathbf{v}$ representing the direction in which the object can move.
Here $P(2, 0, -1)$ and $Q(0, 1, 3)$. So $\mathbf{v} = \langle -2, 1, 4\rangle$.

(b) (2 Points) Give, in coordinate form, a *unit* vector pointing in the direction that the object can move.
Here $\mathbf{v} = \langle -2, 1, 4\rangle$, so $|\mathbf{v}| = \sqrt{(-2)^2 + 1^2 + 4^2} = \sqrt{21}$. Hence $\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{21}}\langle -2, 1, 4\rangle$.

2. (2 Points) Find parametric equation of the line through $(-2, 2, 4)$ and perpendicular to the plane $2x - y + 5z = 12$.
A vector perpendicular to the plane $2x - y + 5z = 12$ is $\langle 2, -1, 5\rangle$. The parametric equation of the line through the point $(-2, 2, 4)$ and perpendicular to the plane $2x - y + 5z = 12$ are

$$\begin{aligned} x &= -2 + 2t \\ y &= 2 - t \\ z &= 4 + 5t. \end{aligned}$$

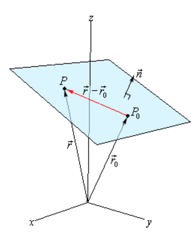
3. (1 Points) Find the angle between the vectors $\langle 8, -1, 4\rangle$ and $\langle 0, 4, 2\rangle$. Let $\mathbf{a} = \langle 8, -1, 4\rangle$ and $\mathbf{b} = \langle 0, 4, 2\rangle$. Then $|\mathbf{a}| = \sqrt{8^2 + (-1)^2 + 4^2} = \sqrt{81} = 9$ and $|\mathbf{b}| = \sqrt{0^2 + 4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$ and $\mathbf{a}\cdot\mathbf{b} = \langle 8, -1, 4\rangle \cdot \langle 0, 4, 2\rangle = 8(0) + (-1)(4) + 4(2) = 4$.
Thus the angle between the vectors is $\theta = \cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right) = \cos^{-1}\left(\frac{4}{18\sqrt{5}}\right) \approx 60.20^0$.

4. (2 Points) Find the scalar and vector projection of $\langle 3, -3, 1\rangle$ onto $\langle 2, 4, 1\rangle$. Let $\mathbf{a} = \langle 2, 4, 1\rangle$ and $\mathbf{b} = \langle 3, -3, 1\rangle$. Then the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is

$$\text{Comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \frac{\langle 3, -3, 1\rangle \cdot \langle 2, 4, 1\rangle}{|\langle 2, 4, 1\rangle|}$$
$$= \frac{6 - 12 + 1}{\sqrt{4 + 16 + 1}} = \frac{-5}{\sqrt{21}}.$$

$$\begin{aligned}\text{Pro}_{\mathbf{a}}\mathbf{b} &= \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\mathbf{a} \\ &= \frac{-5}{\sqrt{21}}\frac{\langle 2, 4, 1\rangle}{\sqrt{21}} \\ &= \frac{-5}{21}\langle 2, 4, 1\rangle.\end{aligned}$$

5. (2 Points) Find the equation of the plane through the point $(5, 3, 5)$ and normal to the vector $\langle 2, 1, -1\rangle$.



Let $P(x, y, z)$ be any point on the plane and $\mathbf{r}_0$ and $\mathbf{r}$ be the position vector of $(5, 3, 5)$ and $P(x, y, z)$. Then $\mathbf{r}_0 = \langle 5, 3, 5\rangle$ and $\mathbf{r} = \langle x, y, z\rangle$. Since $\mathbf{n} = \langle 2, 1, -1\rangle$ is the normal to the plane $\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}_0) = 0$. This gives us the equation of the plane $\langle 2, 1, -1\rangle\cdot\langle x-5, y-3, z-5\rangle 0$ or $2x + y - z = 8$.