MATH 253X-UX1 Quiz 2 Key Name: ______________________

Fall 2021

Instructions. Either print this three-page file and fill in your answers or use exactly three blank pages and answer your response following this format (gradescope will not accept more or fewer number of pages). Then attach your work as a PDF. Show all your work in order to receive full credit. You have a total a 45 minutes from the time you enter the assignment to the time you submit it. No outside help allowed.

1. (2 Points) Find equations for the contour of $f(x, y) = x^2 + y^2$ and draw a contour diagram of f. Then draw the graph of f. The contour at height c is given by

$$f(x, y) = x^2 + y^2 = c.$$

This is a contour only for $c \geq 0$, for $c = 0$ it is a single point(the origin) and for $c > 0$ it is a circle of radius $\sqrt{c}$. Thus the contours at an elevation of $c = 1, 2, \ldots$ are all circles centered at the origin of radius $1, \sqrt{2}, \sqrt{3}, \ldots$. The contour diagram is shown in the following figure (left). The bowl-shaped graph of f is shown in the right.

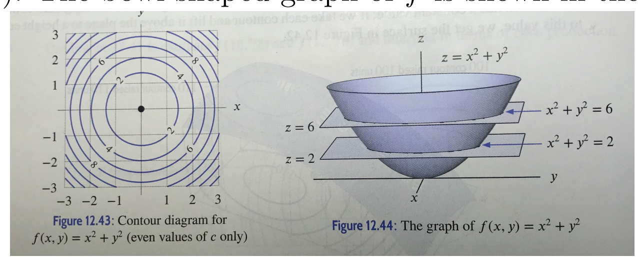

Figure 12.43: Contour diagram for $f(x, y) = x^2 + y^2$ (even values of c only)

Figure 12.44: The graph of $f(x, y) = x^2 + y^2$

2. (2 Points) Show that

$$f(x, y) = \frac{x^2}{x^2 + y}, x^2 + y \neq 0$$

does not have a limit as $(x, y) \to (0, 0)$.

First let's approach $(0, 0)$ along the $x-$ axis. Then $y = 0$ gives $f(x, 0) = \frac{x^2}{x^2} = 1$ for all $x \neq 0$, so $f(x, y) \to 1$ as $(x, y) \to (0, 0)$.

We now approach $(0, 0)$ along the curve $y = x^2$. Then $f(x, y^2) = \frac{x^2}{x^2+x^2} = \frac{1}{2}$ for all $x \neq 0$.

Since f has two different limits along two different lines, the given limit does not exist.

3. (2 Points) The Dubois formula relates a person's surface area, s, in m^2, to weight, w, in kg, and height, h, in cm, by

$$s = f(w, h) = 0.01w^{0.25}h^{0.75}.$$

Find $f(66, 165)$, $f_w(66, 165)$, and $f_h(66, 165)$. Interpret your answers in terms of surface area, height, and weight.

$f(66, 165) = 0.01(66)^{0.25}(165)^{0.75} = 1.31m^2$. The surface area of a person of weight 66 kg and height 166 cm is $1.31m^2$. $f_w(w, h) = 0.01(0.25)w^{-0.75}h^{0.75} \Rightarrow f_w(66, 165) = 0.01(0.25)(66)^{-0.75}(165)^{0.75} = 0.005m^2/kg$. If the height of a person is fixed at $165cm$ then surface area will increase at rate of $0.005m^2/kg$ if the weight is increased by 1 kg from 66 kg. $f_h(w, h) = 0.01(0.75)w^{.25}h^{-0.25} \Rightarrow f_w(66, 165) = 0.01(0.75)(66)^{.25}(165)^{-0.25} = 0.006m^2/cm$. If the weight of a person is fixed at $66kg$, then the surface area will increase at a rate of $0.006m^2/cm$ if the height is increased by 1 cm from 165 cm.

4. (2 Points) Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at the point $(3, 4)$. Then find the local linearization at the point $(3, 4)$. Estimate $f(2.9, 4.2)$ and $f(2, 2)$ using the linearization and compare your answer to the true values. Let $f(x, y) = x^2+y^2$, $f_x(x, y) = 2x$, $f_x(3, 4) = 6$, $f_y(x, y) = 2y$, $f_y(3, 4) = 8$. At $(3, 4)$, $z = 25$. Then the equation of the tangent plane is $z-25 = 6(x-3)+8(y-4)$ or $z = 6x+8y-25$. The local linearization of f near (x, y) is $L(x, y) = 6x + 8y - 25$ at $(3, 4)$ we have $L(3, 4) = 6(3) + 8(4) - 25 = 25$.
The linear approximation at $(2.9, 4.2)$ is $f(2.9, 4.2) \approx 6(2.9) + 8(4.2) - 25 = 17.4 + 33.6 - 25 = 26$. The true value is $f(2.9, 4.2) = 26.5$. The linear approximation at $(2, 2)$ is $f(2, 2) \approx 6(2) + 8(2) - 25 = 12 + 16 - 25 = 3$. The true value is $f(2, 2) = 8$.

5. (2 Points) Let $w = x^2e^y$, $x = 4u$, and $y = 3u^2 - 2v$. Compute $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.

$$
\begin{aligned}
w &= x^2e^y, \frac{\partial w}{\partial x} = 2xe^y, \frac{\partial w}{\partial y} = x^2e^y, \\
x &= 4u, \frac{\partial x}{\partial u} = 4, \frac{\partial x}{\partial v} = 0, \\
y &= 3u^2 - 2v, \frac{\partial y}{\partial u} = 6u, \frac{\partial y}{\partial v} = -2.
\end{aligned}
$$

Now

$$
\begin{aligned}
\frac{\partial w}{\partial u} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} \\
&= 2xe^y(4) + x^2e^y(6u) \\
&= 4(4u)e^{3u^2-2v}(2 + 96u^2) \\
&= 32ue^{3u^2-2v}(1 + 3u^2)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial w}{\partial v} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} \\
&= 2xe^y(0) + x^2e^y(-2) = -32u^2e^{3u^2-v}.
\end{aligned}
$$