Applied Numerical Methods

with MATLAB® for Engineers and Scientists

Third Edition

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PREFACE

This book is designed to support a one-semester course in numerical methods. It has been written for students who want to learn and apply numerical methods in order to solve problems in engineering and science. As such, the methods are motivated by problems rather than by mathematics. That said, sufficient theory is provided so that students come away with insight into the techniques and their shortcomings.

MATLAB® provides a great environment for such a course. Although other environments (e.g., Excel/VBA, Mathcad) or languages (e.g., Fortran 90, C++) could have been chosen, MATLAB presently offers a nice combination of handy programming features with powerful built-in numerical capabilities. On the one hand, its M-file programming environment allows students to implement moderately complicated algorithms in a structured and coherent fashion. On the other hand, its built-in, numerical capabilities empower students to solve more difficult problems without trying to "reinvent the wheel."

The basic content, organization, and pedagogy of the second edition are essentially preserved in the third edition. In particular, the conversational writing style is intentionally maintained in order to make the book easier to read. This book tries to speak directly to the reader and is designed in part to be a tool for self-teaching.

That said, this edition differs from the past edition in three major ways: (1) two new chapters, (2) several new sections, and (3) revised homework problems.

1. **New Chapters.** As shown in Fig. P.1, I have developed two new chapters for this edition. Their inclusion was primarily motivated by my classroom experience. That is, they are included because they work well in the undergraduate numerical methods course I teach at Tufts. The students in that class typically represent all areas of engineering and range from sophomores to seniors with the majority at the junior level. In addition, we typically draw a few math and science majors. The two new chapters are:
   - **Eigenvalues.** When I first developed this book, I considered that eigenvalues might be deemed an "advanced" topic. I therefore presented the material on this topic at the end of the semester and covered it in the book as an appendix. This sequencing had the ancillary advantage that the subject could be partly motivated by the role of eigenvalues in the solution of linear systems of ODEs. In recent years, I have begun

PART ONE Modeling, Computers, and Error Analysis	PART TWO Roots and Optimization	PART THREE Linear Systems	PART FOUR Curve Fitting	PART FIVE Integration and Differentiation	PART SIX Ordinary Differential Equations
CHAPTER 1 Mathematical Modeling, Numerical Methods, and Problem Solving	CHAPTER 5 Roots: Bracketing Methods	CHAPTER 8 Linear Algebraic Equations and Matrices	CHAPTER 14 Linear Regression	CHAPTER 19 Numerical Integration Formulas	CHAPTER 22 Initial-Value Problems
CHAPTER 2 MATLAB Fundamentals	CHAPTER 6 Roots: Open Methods	CHAPTER 9 Gauss Elimination	CHAPTER 15 General Linear Least-Squares and Nonlinear Regression	CHAPTER 20 Numerical Integration of Functions	CHAPTER 23 Adaptive Methods and Stiff Systems
CHAPTER 3 Programming with MATLAB	CHAPTER 7 Optimization	CHAPTER 10 *LU* Factorization	CHAPTER 16 Fourier Analysis	CHAPTER 21 Numerical Differentiation	CHAPTER 24 Boundary-Value Problems
CHAPTER 4 Roundoff and Truncation Errors		CHAPTER 11 Matrix Inverse and Condition	CHAPTER 17 Polynomial Interpolation		
		CHAPTER 12 Iterative Methods	CHAPTER 18 Splines and Piecewise Interpolation		
		CHAPTER 13 Eigenvalues			

FIGURE P.1
An outline of this edition. The shaded areas represent new material. In addition, several of the original chapters have been supplemented with new topics.

PART ONE

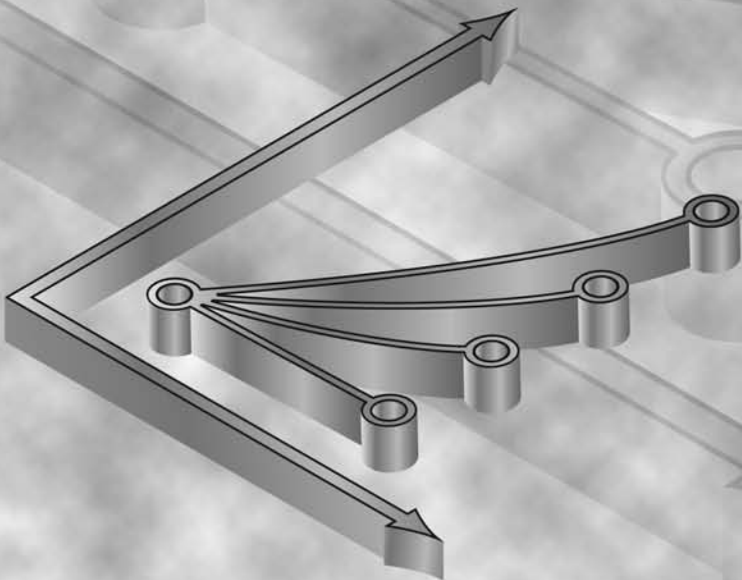
Modeling, Computers, and Error Analysis

1.1 MOTIVATION

What are numerical methods and why should you study them?

Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic and logical operations. Because digital computers excel at performing such operations, numerical methods are sometimes referred to as *computer mathematics*.

In the pre–computer era, the time and drudgery of implementing such calculations seriously limited their practical use. However, with the advent of fast, inexpensive digital computers, the role of numerical methods in engineering and scientific problem solving has exploded. Because they figure so prominently in much of our work, I believe that numerical methods should be a part of every engineer's and scientist's basic education. Just as we all must have solid foundations in the other areas of mathematics and science, we should also have a fundamental understanding of numerical methods. In particular, we should have a solid appreciation of both their capabilities and their limitations.



Beyond contributing to your overall education, there are several additional reasons why you should study numerical methods:

1. Numerical methods greatly expand the types of problems you can address. They are capable of handling large systems of equations, nonlinearities, and complicated geometries that are not uncommon in engineering and science and that are often impossible to solve analytically with standard calculus. As such, they greatly enhance your problem-solving skills.
2. Numerical methods allow you to use "canned" software with insight. During

your career, you will invariably have occasion to use commercially available prepackaged computer programs that involve numerical methods. The intelligent use of these programs is greatly enhanced by an understanding of the basic theory underlying the methods. In the absence of such understanding, you will be left to treat such packages as "black boxes" with little critical insight into their inner workings or the validity of the results they produce.

3. Many problems cannot be approached using canned programs. If you are conversant with numerical methods, and are adept at computer programming, you can design your own programs to solve problems without having to buy or commission expensive software.
4. Numerical methods are an efficient vehicle for learning to use computers. Because numerical methods are expressly designed for computer implementation, they are ideal for illustrating the computer's powers and limitations. When you successfully implement numerical methods on a computer, and then apply them to solve otherwise intractable problems, you will be provided with a dramatic demonstration of how computers can serve your professional development. At the same time, you will also learn to acknowledge and control the errors of approximation that are part and parcel of large-scale numerical calculations.
5. Numerical methods provide a vehicle for you to reinforce your understanding of mathematics. Because one function of numerical methods is to reduce higher mathematics to basic arithmetic operations, they get at the "nuts and bolts" of some otherwise obscure topics. Enhanced understanding and insight can result from this alternative perspective.

With these reasons as motivation, we can now set out to understand how numerical methods and digital computers work in tandem to generate reliable solutions to mathematical problems. The remainder of this book is devoted to this task.

1.2 PART ORGANIZATION

This book is divided into six parts. The latter five parts focus on the major areas of numerical methods. Although it might be tempting to jump right into this material, *Part One* consists of four chapters dealing with essential background material.

Chapter 1 provides a concrete example of how a numerical method can be employed to solve a real problem. To do this, we develop a *mathematical model* of a free-falling bungee jumper. The model, which is based on Newton's second law, results in an ordinary differential equation. After first using calculus to develop a closed-form solution, we then show how a comparable solution can be generated with a simple numerical method. We end the chapter with an overview of the major areas of numerical methods that we cover in Parts Two through Six.

Chapters 2 and 3 provide an introduction to the MATLAB® software environment. *Chapter 2* deals with the standard way of operating MATLAB by entering commands one at a time in the so-called *calculator,* or *command, mode*. This interactive mode provides a straightforward means to orient you to the environment and illustrates how it is used for common operations such as performing calculations and creating plots.