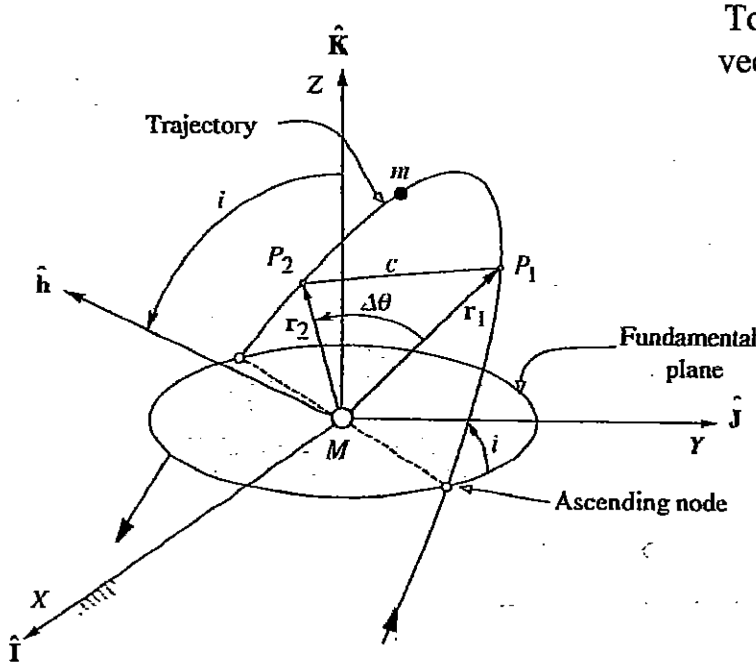
AAE 412 SPACE SYSTEMS ENGINEERING

Orbital Determination from 2 Position Vectors and Δt

The Lambert Problem:

Given 2 position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ in the geocentric equatorial frame and the time difference Δt in the siting of the 2 positions, find the orbital elements.



To determine the orbital elements, the state vector $\mathbf{r}$ and $\mathbf{v}$ at any one position is required:

$$\mathbf{v}_1 = \frac{1}{g}(\mathbf{r}_2 - f\mathbf{r}_1)$$

$$\mathbf{v}_2 = \frac{1}{g}(\dot{g}\mathbf{r}_2 - \mathbf{r}_1)$$

Where the universal anomaly parameter z is found from the roots of

$$\sqrt{\mu}\Delta t = \left[\frac{y(z)}{C(z)}\right]^{\frac{3}{2}} S(z) + A\sqrt{y(z)}$$

where

$$y(z) = r_1 + r_2 + A\frac{zS(z) - 1}{\sqrt{C(z)}} \qquad A = \sin\Delta\theta\sqrt{\frac{r_1 r_2}{1 - \cos\Delta\theta}}$$

$$\Delta\theta = \begin{cases} \cos^{-1}\left(\dfrac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) & \text{if } (\mathbf{r}_1 \times \mathbf{r}_2)_Z \geq 0 \\ 360^\circ - \cos^{-1}\left(\dfrac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) & \text{if } (\mathbf{r}_1 \times \mathbf{r}_2)_Z < 0 \end{cases} \quad \text{prograde trajectory}$$

$$\Delta\theta = \begin{cases} \cos^{-1}\left(\dfrac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) & \text{if } (\mathbf{r}_1 \times \mathbf{r}_2)_Z < 0 \\ 360^\circ - \cos^{-1}\left(\dfrac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) & \text{if } (\mathbf{r}_1 \times \mathbf{r}_2)_Z \geq 0 \end{cases} \quad \text{retrograde trajectory}$$

cont.

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The Lagrange coefficients as function of the universal anomaly parameter z:

$$f = 1 - \frac{\left[\sqrt{\frac{y(z)}{C(z)}}\right]^2}{r_1} C(z) = 1 - \frac{y(z)}{r_1}$$

$$g = \frac{1}{\sqrt{\mu}}\left\{\left[\frac{y(z)}{C(z)}\right]^{\frac{3}{2}} S(z) + A\sqrt{y(z)}\right\} - \frac{1}{\sqrt{\mu}}\left[\frac{y(z)}{C(z)}\right]^{\frac{3}{2}} S(z) = A\sqrt{\frac{y(z)}{\mu}}$$

$$\dot{f} = \frac{\sqrt{\mu}}{r_1 r_2}\sqrt{\frac{y(z)}{C(z)}}[zS(z) - 1]$$

$$\dot{g} = 1 - \frac{\left[\sqrt{\frac{y(z)}{C(z)}}\right]^2}{r_2} C(z) = 1 - \frac{y(z)}{r_2}$$

where C(z) and S(z) are the Stumpf functions. Note that S must be positive.

Note that once $\mathbf{r}_1$ and $\mathbf{v}_1$ (or $\mathbf{r}_2$ and $\mathbf{v}_2$) are known, the orbital elements can be calculated.