AAE 412 **SPACE SYSTEMS ENGINEERING**

Satellite Stability: Eulerian Angles

Given the unit vectors of a body frame in geocentric equatorial coordinates at a given instant:

$$\mathbf{i} = i_x\,\mathbf{I} + i_y\,\mathbf{J} + i_z\,\mathbf{K}$$
$$\mathbf{j} = j_x\,\mathbf{I} + j_y\,\mathbf{J} + j_z\,\mathbf{K}$$
$$\mathbf{k} = k_x\,\mathbf{I} + k_y\,\mathbf{J} + k_z\,\mathbf{K}$$

and the angular velocity $\quad \boldsymbol{\Omega} = \Omega_x\,\mathbf{I} + \Omega_y\,\mathbf{J} + \Omega_z\,\mathbf{K}$

find the rates of precession, nutation and spin at that instant.

$$[\mathbf{Q}]_{\mathbf{Xx}} = \begin{vmatrix} i_x & i_y & i_z \\ j_x & j_y & j_z \\ k_x & k_y & k_z \end{vmatrix}$$

Rotation matrix

Precession angle $\phi = \tan^{-1}(\mathbf{Q}_{31} / -\mathbf{Q}_{32})$ $\qquad 0 \le \phi \le 360°$

Nutation angle $\theta = \cos^{-1}(\mathbf{Q}_{33})$ $\qquad 0 \le \theta \le 180°$

Spin angle $\psi = \tan^{-1}(Q_{13} / Q_{23})$ $\qquad 0 \le \psi \le 360°$

The angular velocity is rotated into the body frame: $\boldsymbol{\omega} = [\mathbf{Q}]_{\mathbf{Xx}}\,\Omega$

Precession rate $\quad \dot{\phi} = (\omega_x \sin\psi + \omega_y \cos\psi) / \sin\theta$

Nutation rate $\quad \dot{\theta} = \omega_x \cos\psi - \omega_y \sin\psi$

Spin rate $\quad \dot{\phi} = -(\omega_x \sin\psi + \omega_y \cos\psi) / \tan\theta + \omega_z$