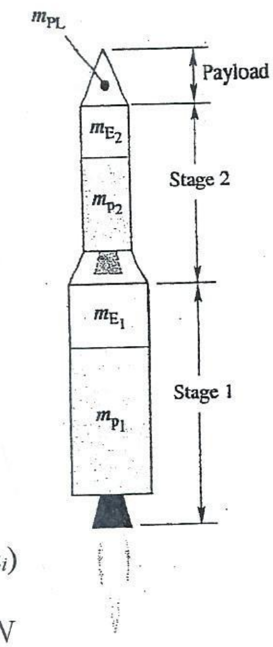
AAE 412

SPACE SYSTEMS ENGINEERING
Mass Minimization of an N-Stage Rocket

Problem: The following parameters are given for each stage *i* of an *N*-stage rocket:

The required burnout velocity $\mathcal{V}_{bo}$
Specific Impulse Isp_i
Structural mass ratio ε_i
Payload mass m_{PL}

Tandem 2-stage booster



Find the minimum propellant mass for each stage.

Let $c_i = Isp_i \, g_o$

Solve iteratively for the Lagrange multiplier λ :

$$\mathcal{V}_{bo} = \Sigma \, c_i \, ln\,(c_i\lambda - 1) \; - \; c_i \, ln\,(c_i\lambda\, \varepsilon_i)$$

Find the optimum mass ratio n_i for each stage: $n_i = (c_i\lambda - 1) \,/\, (c_i\lambda\,\varepsilon_i)$

Verify minimization : $\lambda c_i (\varepsilon_i \, n_i - 1)^2 + 2\,\varepsilon_i\, n_i - 1 > 0$ for $i = 1,2 \ldots N$

Then the minimized stage masses are:

$$m_N = m_{PL}\,(n_N - 1)\,/(1 - n_N\,\varepsilon_N)$$

$$m_{N\text{-}1} = (m_N + m_{PL})\,(n_{N\text{-}1} - 1)\,/(1 - n_{N\text{-}1}\,\varepsilon_{N\text{-}1})$$

$$m_{N\text{-}2} = (m_{N\text{-}1} + m_N + m_{PL})\,(n_{N\text{-}2} - 1)\,/(1 - n_{N\text{-}2}\,\varepsilon_{N\text{-}2})$$

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$$m_1 = (m_2 + m_3 + \ldots\, m_N + m_{PL})\,(n_1 - 1)\,/(1 - n_1\,\varepsilon_1)$$

Stage masses are $m_i = m_{Ei} + m_{pi}$

m_{Ei} is the structural mass for each stage: $m_{Ei} = \varepsilon_i \, m_i$

m_{pi} is the propellant mass for each stage: $m_{pi} = m_i - m_{Ei}$