AAE 412 **SPACE SYSTEMS ENGINEERING**

The Two-Body Problem: Perifocal Frame

Problem: If the initial position vector $\mathbf{r}_o$ and velocity vector $\mathbf{v}_o$ of an orbiting body are known at a given instant, find the position $\mathbf{r}$ and velocity $\mathbf{v}$ vectors after the true anomaly changes by $\Delta\theta$:

$$\mathbf{r} = f\,\mathbf{r}_o + g\,\mathbf{v}_o$$

$$\mathbf{v} = \dot{f}\,\mathbf{r}_o + \dot{g}\,\mathbf{v}_o$$

where the Lagrange coefficients in terms of the change in true anomaly are:

$$f = 1 - \frac{\mu r}{h^2}(1 - \cos\Delta\theta)$$

$$g = \frac{r r_0}{h}\sin\Delta\theta$$

$$\dot{f} = \frac{\mu}{h}\,\frac{1-\cos\Delta\theta}{\sin\Delta\theta}\left[\frac{\mu}{h^2}(1-\cos\Delta\theta) - \frac{1}{r_0} - \frac{1}{r}\right]$$

$$\dot{g} = 1 - \frac{\mu r_0}{h^2}(1 - \cos\Delta\theta)$$

and the following scalars are defined by:

$r_o = [\mathbf{r}_o \cdot \mathbf{r}_o]^{1/2}$ is the magnitude of the $\mathbf{r}_o$ vector

$v_o = [\mathbf{v}_o \cdot \mathbf{v}_o]^{1/2}$ is magnitude of the $\mathbf{v}_o$ vector

$v_{ro} = \mathbf{r}_o \cdot \mathbf{v}_o / r_o$ is the radial component of $\mathbf{v}_o$

$h = r_o\,(v_o^2 - v_{ro}^2)^{1/2}$ is the specific angular momentum

New radial distance from the following form of the orbit equation:

$$r = \frac{h^2}{\mu}\,\frac{1}{1 + \left(\frac{h^2}{\mu r_0} - 1\right)\cos\Delta\theta - \frac{h v_{r0}}{\mu}\sin\Delta\theta}$$