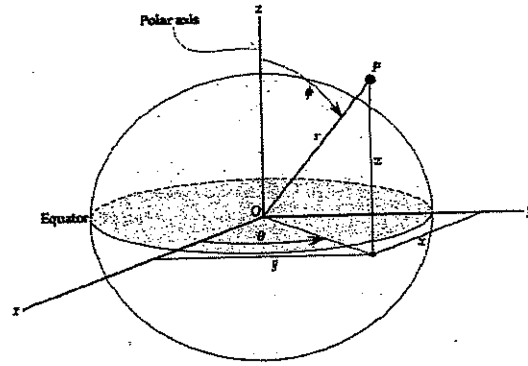
AAE 412

SPACE SYSTEMS ENGINEERING
Gravitational Perturbations

The spherical coordinate system:



Rotationally symmetric perturbation $\Phi(r,\phi)$ is given by:

$$\Phi(r,\phi) = \frac{\mu}{r}\sum_{k=2}^{\infty} J_k \left(\frac{R}{r}\right)^k P_k(\cos\phi)$$

where J_k are the zonal harmonics of the planet, R is the equatorial radius ($R/r < 1$) and P_k are Legendre polynomials of order k. For the earth, the six zonal harmonics are:

$J_2 = 0.00108263$ $\quad J_3 = -2.33936 \times 10^{-3} J_2$

$J_4 = -1.49601 \times 10^{-3} J_2$ $\quad J_5 = -0.20995 \times 10^{-3} J_2$ $\quad$ Earth zonal harmonics

$J_6 = 0.49941 \times 10^{-3} J_2$ $\quad J_7 = 0.32547 \times 10^{-3} J_2$

The perturbing acceleration $\mathbf{a}_P$ is given by $\mathbf{a}_P = -\nabla\Phi$

The gravitational acceleration perturbation due to J_2:

$$\mathbf{a}_P = \frac{3}{2}\frac{J_2\mu R^2}{r^4}\left[\frac{x}{r}\left(5\frac{z^2}{r^2}-1\right)\hat{\mathbf{i}} + \frac{y}{r}\left(5\frac{z^2}{r^2}-1\right)\hat{\mathbf{j}} + \frac{z}{r}\left(5\frac{z^2}{r^2}-3\right)\hat{\mathbf{k}}\right]$$

The gravitational acceleration perturbation due to J_3:

$$\mathbf{a}_P = \frac{1}{2}\frac{J_3\mu R^3}{r^5}\left[5\frac{x}{r}\left(7\frac{z^3}{r^3}-3\frac{z}{r}\right)\hat{\mathbf{i}} + 5\frac{y}{r}\left(7\frac{z^3}{r^3}-3\frac{z}{r}\right)\hat{\mathbf{j}} + \left(35\frac{z^4}{r^4}-30\frac{z^2}{r^2}+3\right)\hat{\mathbf{k}}\right]$$

Solution of the equation of motion yields the new, perturbed orbit: $\ddot{\mathbf{r}} = -\mu/r^3\,\mathbf{r} + \mathbf{a}_P$