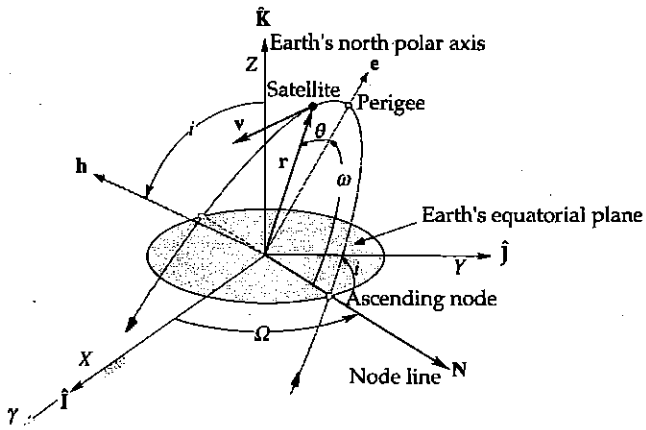
AAE 412

# SPACE SYSTEMS ENGINEERING

## Classical Orbital Elements




**The Orbital Elements:**

| | |
|---|---|
| $h$ | Specific angular momentum |
| $i$ | Inclination |
| $\Omega$ | Right Ascension of the ascending node |
| $e$ | Eccentricity |
| $\omega$ | Argument of perigee |
| $\theta$ | True anomaly |

Given the state vectors **r** and **v**: $r = (\mathbf{r}\cdot\mathbf{r})^{1/2}$ $\quad v = (\mathbf{v}\cdot\mathbf{v})^{1/2}$ $\quad v_r = \mathbf{r}\cdot\mathbf{v}/r$

1. **Specific angular momentum:** $\mathbf{h} = \mathbf{r} \times \mathbf{v}$ and $h = (\mathbf{h}\cdot\mathbf{h})^{1/2}$

2. **Inclination:** $i = \cos^{-1}(h_Z/h)$

3. **Right ascension of the ascending node:** $\mathbf{N} = \mathbf{K} \times \mathbf{h}$ and $N = (\mathbf{n}\cdot\mathbf{n})^{1/2}$

$$\Omega = \cos^{-1}(N_X/N) \qquad N_Y \geq 0$$

$$\Omega = 360° - \cos^{-1}(N_X/N) \qquad N_Y < 0$$

4. **Eccentricity:** $\mathbf{e} = [\,(v^2 - \mu/r)\,\mathbf{r} - r\ v_r\,\mathbf{v}]\,/\,\mu$ and $e = (\mathbf{e}\cdot\mathbf{e})^{1/2}$

5. **Argument of perigee:**

$$\omega = \cos^{-1}(\mathbf{N}\cdot\mathbf{e}/Ne) \qquad \mathbf{e}_Z \geq 0$$

$$\omega = 360° - \cos^{-1}(\mathbf{N}\cdot\mathbf{e}/Ne) \qquad \mathbf{e}_Z < 0$$

6. **True anomaly**

$$\theta = \cos^{-1}(\mathbf{e}\cdot\mathbf{r}/er) \qquad v_r \geq 0$$

$$\theta = 360° - \cos^{-1}(\mathbf{e}\cdot\mathbf{r}/er) \qquad v_r < 0$$

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**Orbital Elements → State Vector in GEF**

**Problem:** Given the orbital elements, find the state vectors $\mathbf{R_X}$ and $\mathbf{V_X}$ in the geocentric equatorial frame $X$.

From the orbital elements, the position and velocity vectors in perifocal frame $x$ are:

$$\mathbf{R}_x = (h^2/\mu)/(1 + e\cos(\theta))\,[\cos(\theta) \quad \sin(\theta) \quad 0]$$

$$\mathbf{V}_x = \mu/h\,[-\sin(\theta) \quad e + \cos(\theta) \quad 0]$$

Rotation of the geocentric equatorial frame $X$ to the perifocal frame $x$:

Rotation matrix:

$$[\mathbf{R}_3(\Omega)] = \begin{bmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\mathbf{R}_1(i)] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & \sin(i) \\ 0 & -\sin(i) & \cos(i) \end{bmatrix}$$

$$[\mathbf{R}_3(\omega)] \quad \begin{bmatrix} \cos(\omega) & \sin(\omega) & 0 \\ -\sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix: $[\mathbf{Q}_{Xx}] = [\mathbf{R}_3(\omega)]\,[\mathbf{R}_1(i)]\,[\mathbf{R}_3(\Omega)]$

$\mathbf{R}_x = [\mathbf{Q}_{Xx}]\,\mathbf{R}_X$ Transformation from geocentric equatorial frame to perifocal frame

$\mathbf{V}_x = [\mathbf{Q}_{Xx}]\,\mathbf{V}_X$

Find the state vectors in geocentric equatorial frame:

$\mathbf{R}_X = [\mathbf{Q}_{Xx}]^T\,\mathbf{R}_x$ Transformation from perifocal frame to geocentric equatorial frame

$\mathbf{V}_X = [\mathbf{Q}_{Xx}]^T\,\mathbf{V}_x$