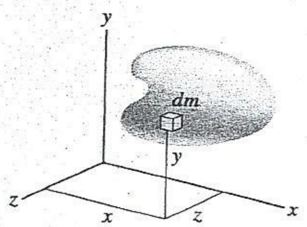
AAE 412 **SPACE SYSTEMS ENGINEERING**

# PROPERTIES OF VOLUMES AND HOMOGENEOUS OBJECTS

## Mass Moment of Inertia




$$I_{(x\text{ axis})} = I_{xx} = \int_m (y^2 + z^2)\, dm,$$

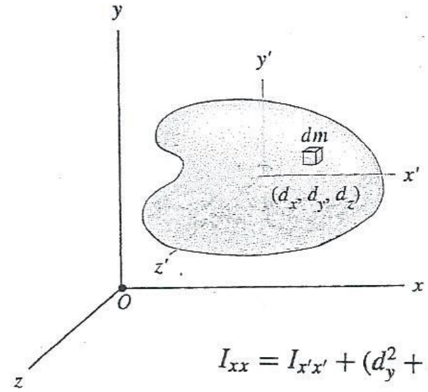
$$I_{(y\text{ axis})} = I_{yy} = \int_m (x^2 + z^2)\, dm,$$

$$I_{(z\text{ axis})} = I_{zz} = \int_m (x^2 + y^2)\, dm,$$

$$I_{xy} = \int_m xy\, dm, \qquad I_{yz} = \int_m yz\, dm,$$

$$I_{zx} = \int_m zx\, dm.$$

## Parallel Axis Theorems




$$I_{xx} = I_{x'x'} + (d_y^2 + d_z^2)m,$$

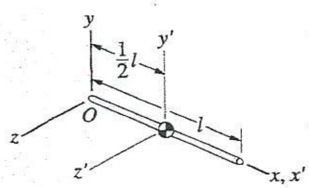
$$I_{yy} = I_{y'y'} + (d_x^2 + d_z^2)m,$$

$$I_{zz} = I_{z'z'} + (d_x^2 + d_y^2)m,$$

$$I_{xy} = I_{x'y'} + d_x d_y m,$$

$$I_{yz} = I_{y'z'} + d_y d_z m,$$

$$I_{zx} = I_{z'x'} + d_z d_x m.$$



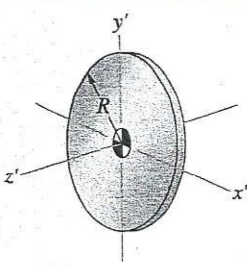

Slender Bar

$$I_{(x\text{ axis})} = 0, \qquad I_{(y\text{ axis})} = I_{(z\text{ axis})} = \frac{1}{3}ml^2,$$

$$I_{xy} = I_{yz} = I_{zx} = 0.$$

$$I_{(x'\text{ axis})} = 0, \qquad I_{(y'\text{ axis})} = I_{(z'\text{ axis})} = \frac{1}{12}ml^2,$$

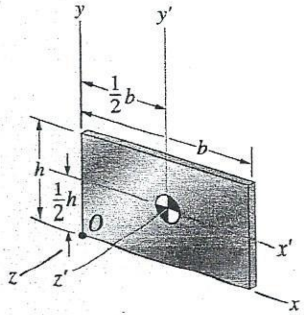
$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0.$$




Thin Circular Plate

$$I_{(x'\text{ axis})} = I_{(y'\text{ axis})} = \frac{1}{4}mR^2, \qquad I_{(z'\text{ axis})} = \frac{1}{2}mR^2,$$

$$I_{xy} = I_{yz} = I_{zx} = 0.$$




Thin Rectangular Plate

$$I_{(x\text{ axis})} = \frac{1}{3}mh^2, \qquad I_{(y\text{ axis})} = \frac{1}{3}mb^2, \qquad I_{(z\text{ axis})} = \frac{1}{3}m(b^2 + h^2),$$

$$I_{xy} = \frac{1}{4}mbh, \qquad I_{yz} = I_{zx} = 0.$$

$$I_{(x'\text{ axis})} = \frac{1}{12}mh^2, \qquad I_{(y'\text{ axis})} = \frac{1}{12}mb^2, \qquad I_{(z'\text{ axis})} = \frac{1}{12}m(b^2 + h^2),$$

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0.$$

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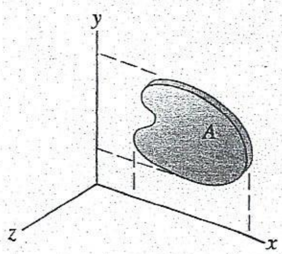
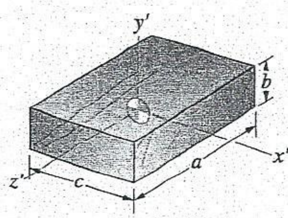


Thin Plate

$$I_{(x\text{ axis})} = \frac{m}{A} I_x^A, \qquad I_{(y\text{ axis})} = \frac{m}{A} I_y^A, \qquad I_{(z\text{ axis})} = I_{(x\text{ axis})} + I_{(y\text{ axis})},$$

$$I_{xy} = \frac{m}{A} I_{xy}^A, \qquad I_{yz} = I_{zx} = 0.$$

(The superscripts $A$ denote moments of inertia of the plate's cross-sectional area $A$.)

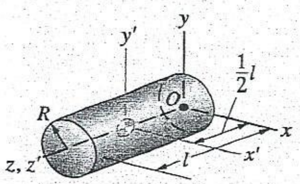


Rectangular Prism

Volume $= abc.$

$$I_{(x'\text{ axis})} = \frac{1}{12} m(a^2 + b^2), \qquad I_{(y'\text{ axis})} = \frac{1}{12} m(a^2 + c^2),$$

$$I_{(z'\text{ axis})} = \frac{1}{12} m(b^2 + c^2), \qquad I_{x'y'} = I_{y'z'} = I_{z'x'} = 0.$$



Circular Cylinder

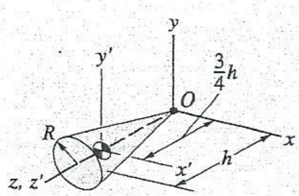
Volume $= \pi R^2 l.$

$$I_{(x\text{ axis})} = I_{(y\text{ axis})} = m\left(\frac{1}{3} l^2 + \frac{1}{4} R^2\right), \qquad I_{(z\text{ axis})} = \frac{1}{2} m R^2,$$

$$I_{xy} = I_{yz} = I_{zx} = 0.$$

$$I_{(x'\text{ axis})} = I_{(y'\text{ axis})} = m\left(\frac{1}{12} l^2 + \frac{1}{4} R^2\right), \qquad I_{(z'\ axis)} = \frac{1}{2} m R^2,$$

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0.$$



Circular Cone

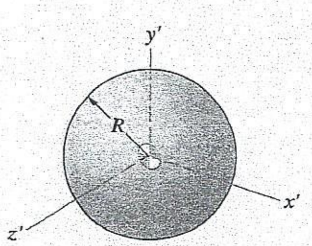
Volume $= \frac{1}{3} \pi R^2 h.$

$$I_{(x\text{ axis})} = I_{(y\text{ axis})} = m\left(\frac{3}{5} h^2 + \frac{3}{20} R^2\right), \qquad I_{(z\text{ axis})} = \frac{3}{10} m R^2,$$

$$I_{xy} = I_{yz} = I_{zx} = 0.$$

$$I_{(x'\text{ axis})} = I_{(y'\text{ axis})} = m\left(\frac{3}{80} h^2 + \frac{3}{20} R^2\right), \qquad I_{(z'\text{ axis})} = \frac{3}{10} m R^2.$$

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0.$$



Sphere

Volume $= \frac{4}{3} \pi R^3.$

$$I_{(x'\text{ axis})} = I_{(y'\text{ axis})} = I_{(z'\text{ axis})} = \frac{2}{5} m R^2,$$

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0.$$