Exercise 1: Basic Matrix Operations

Define the following matrices and vectors.

$$A = \begin{pmatrix} -4 & 2 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}, C = \begin{pmatrix} 1 & -3 & 2 \\ 1 & -9 & -4 \end{pmatrix}, D = \begin{pmatrix} 1 & 2 \\ 1 & -3 \\ -2 & 3 \end{pmatrix},$$
$$\boldsymbol{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \boldsymbol{y} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \boldsymbol{z} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}.$$

Calculate the following:

- (a) A B,
- (b) 4x 2y,
- (c) A**y**,
- (d) $A(\boldsymbol{y} + \boldsymbol{x})$,
- (e) $D\boldsymbol{x}$,
- (f) $D\boldsymbol{y} + \boldsymbol{z}$,
- (g) AB,
- (h) BA,
- (i) AC,
- (j) CD.

Save the answers in ten separate .dat files, named **A1.dat**, **A2.dat**, ..., **A10.dat**. Are the answers in (g) and (h) the same? Why or why not?

Next, access and save the following elements:

- (k) The second column of C,
- (l) Both columns of the last two rows of D,
- (m) The first two columns of the second row of C.

Save the answers in three separate .dat files, named ${\bf A11.dat},\,{\bf A12.dat}$ and ${\bf A13.dat}.$

Exercise 2: Truncation Errors

The following four expressions are exactly equal to zero:

$$x_{1} = \left| 1000 - \sum_{k=1}^{10,000} 0.1 \right|, \quad x_{2} = \left| 1000 - \sum_{k=1}^{8,000} 0.125 \right|,$$

$$x_{3} = \left| 1000 - \sum_{k=1}^{5,000} 0.2 \right|, \quad x_{4} = \left| 1000 - \sum_{k=1}^{4,000} 0.25 \right|$$

However, computers store floating-point numbers with a binary representation, so most decimals representations have a small truncation error. This error accumulates in the summation of a series. As a result, x and y might not be numerically identical to zero. To verify this effect, use MATLAB to compute x and y. Save $\begin{bmatrix} x_1, x_2, x_3, x_4 \end{bmatrix}^T$ as a column vector in **A14.dat**. **Hint:** Use a **for** loop.

Can you explain the differences in these values?