## Exercise 1: Salmon Runs

Download the file **salmon_data.csv** included with the homework. This file contains the annual Chinook salmon counts taken at Bonneville on the Columbia river from the years 1938 to 2014 (*www.cbr.washington.edu*). You can load this file into MATLAB using `>>load salmon_data.csv`. **Do not upload this file to compsoftbook, your code will be tested using the counts for another species of salmon**.

a) You should begin by creating a time vector `>>t=(1:length(salmon_data)).'` and plotting the salmon counts against the year in which they were taken (plotting usually helps you get a better understanding of the data), but make sure to comment out the plot before submitting. Note that we have let 1938 be represented by year 1 (making 2014 year 77), continue with this convention in the rest of the problem. In the video lectures you learned that the following matrix equations could be used to determine the coefficients of a linear best-fit:

$$\underbrace{\begin{bmatrix} \sum_{k=1}^{N} t_k^2 & \sum_{k=1}^{N} t_k \\ \sum_{k=1}^{N} t_k & \sum_{k=1}^{N} 1 \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} A \\ B \end{bmatrix}}_{P} = \underbrace{\begin{bmatrix} \sum_{k=1}^{N} t_k y_k \\ \sum_{k=1}^{N} y_k \end{bmatrix}}_{R}$$

where $y_k$ is the $k$-th index of the salmon count data. The solution provides $A$ and $B$ such that $y = At + B$ is the RMS best-fit line. Compute the $Q$ , $R$ , and $P$ matrices and save them in `A1.dat`, `A2.dat`, and `A3.dat`. You can confirm that you $A$ and $B$ are correct by finding a first order fit using `polyfit`.

b) Use `polyfit` to find the best-fit polynomials of order 2, 5, and 8, and save these coefficients in `A4.dat`, `A5.dat`, and `A6.dat`, respectively. You should plot each of these best-fits, but make sure to comment out the plot before submitting.

c) Using each polynomial fit from part (b), predict the salmon counts in 2015. Save the predictions from each polynomial fit in a column vector with 3 elements in `A7.dat`. If you are curious, a year from now look at the website provided above to see if any of the predictions were accurate.

d) We will now use this data to study interpolations. Create coarse vectors of time and the salmon data which contain the first element and every fourth subsequent element. For example, the course time vector would begin with `[1 ; 5 ; 9 ; 13 ; ...` . Save the course salmon data as `A8.dat`.

e) Now, interpolate this coarse data onto the original time vector using each of the following methods: nearest neighbor, linear, cubic, and spline. Save the interpolated salmon count values for these methods as column vectors in `A9-A12.dat`.

f) For each of these interpolation methods, compute the RMS error between the interpolated values and the true values using

$$RMS = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(y_k - \hat{y}_k)^2}$$

where $\hat{y}$ represents the interpolated values. Save the RMS errors for each method in a single column vector with four elements in `A13.dat`

## Exercise 2: Electron orbital wave function for the hydrogen atom

The equation for the probability density function of the $n = 3, l = 1, m = 0$ electron orbital of the hydrogen atom is given as a function of the angle ($\theta$) and radius ($r$):

$$f(\theta, r) = \left| \frac{\sqrt{2}}{\sqrt{\pi}81} \left(6r - r^2\right) e^{-r/3} \cos(\theta) \right|^2$$

where $r$ is in units of the Bohr radius.

a) You will begin by making a 3d plot of the function. First, create a mesh grid with $\theta$ from 0 to $2\pi$ with $0.05\pi$ increments, and r from 0 to 20 with .5 increments (see `>>help meshgrid`). Assuming you name the resulting matrices `TH` and `R`, then use them to evaluate the above function to find `F=f(TH,R)` (make sure your function $f(\theta, r)$ can handle matrix inputs). To plot, first convert the polar-coordinates to cartesian, `[X,Y]= pol2cart(TH,R)`, and then make a 3d plot with `surf(X,Y,F)`. Make sure to comment out this plot before submission to comsoftbook. Save `X`, `Y`, and `F` as `A14-A16.dat`.

b) Looking at the plot in part a), you should notice that there are 4 maximums. You will now find the coordinates $(\theta, r)$ of these peaks.. Using `fminsearch` find the $(\theta, r)$ which maximizes $f(\theta, r)$ with each of the following initial guesses: $(0, 1)$ , $(0, 10)$ , $(\pi, 1)$, and $(\pi, 10)$. Save the resulting $(\theta, r)$ values as row vectors in `A17-A20.dat`. Notice how dependant `fminsearch` is on initial conditions.

## Problem 3: Maximizing Profits

An aircraft manufacturer can produce three different types of aircraft: small, medium, and large. Aircraft costs consist of a materials cost and a labor cost. The small aircraft sells for **$40 million** more than its materials cost, the medium aircraft sells for **$50 million** more than its materials cost, and the large aircraft sells for **$75 million** more than its materials cost. Labor cost for each aircraft consists of three parts: fuselage production at **$32/hour**, wing production at **$45/hour**, and assembly at **$30/hour**. Constructing 1 small aircraft requires **.1 million man hours** (mmh) of fuselage production, **.22 mmh** of wing production, and **.15 mmh** of assembly. Constructing 1 medium aircraft requires **.23 mmh** of fuselage production, **.25 mmh** of wing production, and **.17 mmh** of assembly. Constructing 1 large aircraft requires **.31 mmh** of fuselage production, **.38 mmh** of wing production, and **.27 mmh** of assembly. The following constraints are placed on production:

1. Fuselage production facility can accommodate a maximum of 5 mmh.
2. Wing production facility can accommodate a maximum of 7 mmh.
3. Assembly facility can accommodate a maximum of 9 mmh.
4. In order to receive a needed tax break, the fusel lodge production facility must have at least 4 mmh.
5. As they have an overlapping workforce, the combined wing production and assembly labor must not exceed 15 mmh.
6. Due to limited large scale facilities, labor spent on assembly of the large aircraft must not exceed 2 mmh.
7. To fulfill existing contracts, the number of small aircraft produced must be greater than 2.
8. To fulfill existing contracts, the number of medium aircraft produced must be greater than 3.
9. To fulfill existing contracts, the number of large aircraft produced must be greater than 1.

You will be trying to find numbers of small, medium, and large ($S, M$, and $L$) aircraft whose production maximizes profit.

a) Make a column matrix `f`, such that `f'*[S;M;L]` is the net profit in millions of dollars (note: this profit should be in millions of dollars, so if the net profit were $4 million, `f'*[S;M;L]=4`). Save as `A21.dat`

b) Make a matrix $A$ and a column vector $b$ such that

$$A \begin{bmatrix} S \\ M \\ L \end{bmatrix} \leq b$$

represents the constraints listed above. Each listed constraint should be represented by the same numbered row of $A$ and $b$. For example, you should have `A(1,:)= [.1 .23 .31]` and `B(1) = 5`. Save $A$ as `A22.dat`, and save $b$ as `A23.dat`.

c) Use `linprog` to find `[S;M;L]`. Round each value in the vector to the nearest whole number and save as `A24.dat`