Maths1

Intro To Modular Arithmetic
Count Pair whose sum%m is 0
GCD
GCD(a, b)
Delete one to maximize GCD of all array elements

Intro To Modular Arithmetic

Modular Arithmetic

A % B = Remainder when A is divided by B.

[0, B-1] range.

why % ?

— Helps in limit the range of data.

Modulus Arithmetic Properties

1) $(a+b) \% m = (a \% m + b \% m) \% m$

2) $(a*b) \% m = (a \% m * b \% m) \% m$

3) $(a+m) \% m = (a \% m)$ ---- from (1)

4) $(a-b) \% m = (a \% m - b \% m) \% m$

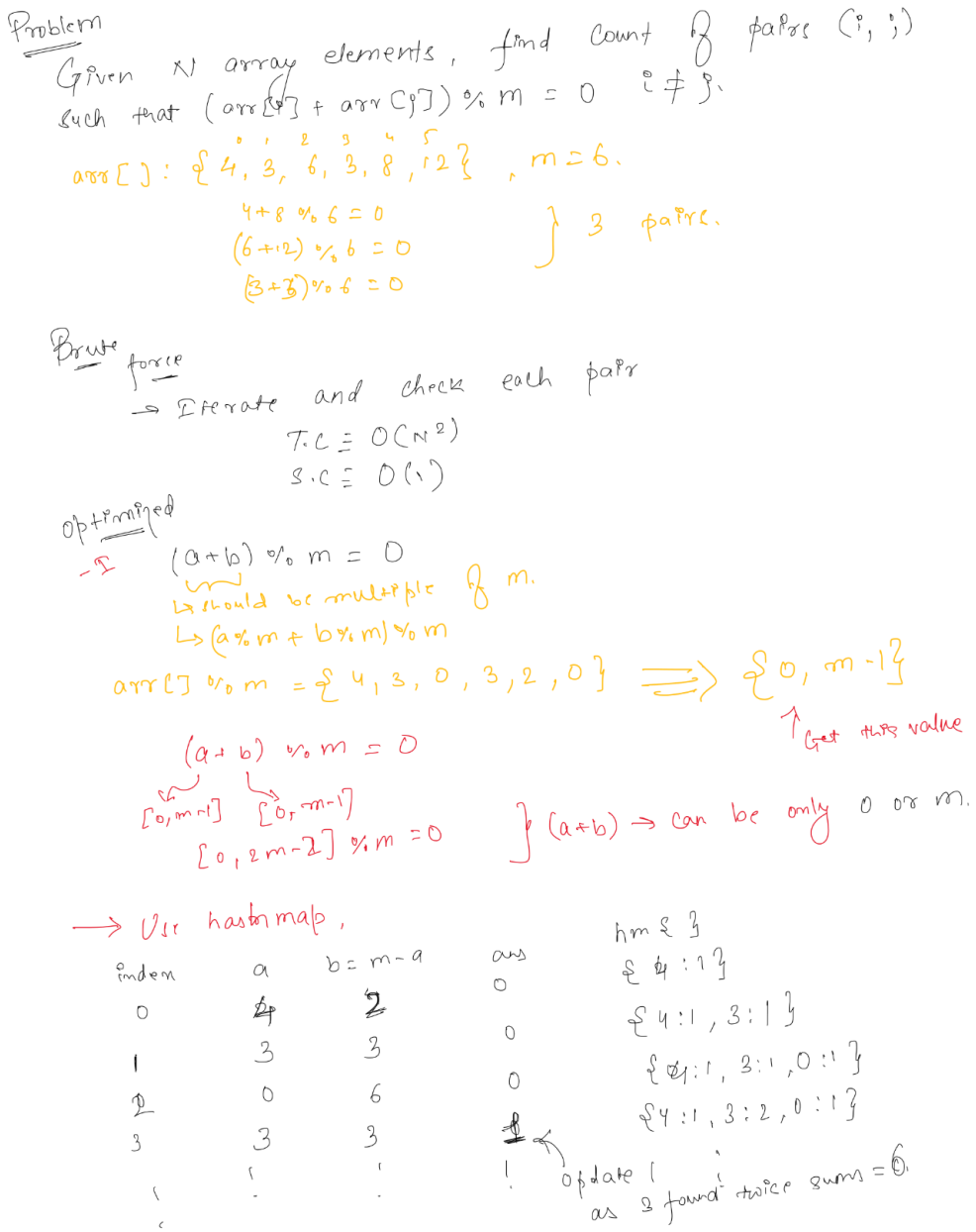
Possible it can be negative. So,

$= (a \% m - b \% m + m) \% m.$

5) $(a \% m) = (a \% m) \% m.$

6) $(a \wedge b) \% m = ((a \% m)^b) \% m.$

Count Pair whose sum%m is 0



Code :-

```
hash<int, int> hm;
for (int i=0; i<N; i++)
{ int a = arr[i] % m        //step 1
  if(a==0)                  //step 2
      b=0
  else
      b = m-a
  if (hm.contains(b)) {
        ans = ans + hm[b];
  if (hm.contains(a))
        hm[a]++;
  else
        hm.insert({a, 1});
}
```

T.C : O(N)

S.C : O(N)

GCD

GCD (Greatest Common divisor)

HCF (Highest Common factor) same as GCD.

GCD (a, b) = the greatest divisor / factor that divides both a & b.

if gcd (a, b) = x

a % x = 0

b % x = 0

gcd (15, 25) = 5

1 5

3 25

5

15

gcd (12, 30) = 6

2 2

3 3

6 5

12 30

6

15

gcd (10, -25) = 5

2 5

5 -5

0 % — = 0

gcd (0, 4) = 4

0 1

1 2

2 4

∞

gcd (0, -10) = 10

1 10

2 1

∞

gcd (0, 0) = ∞

Properties of GCD

1. gcd (a, b) = gcd (b, a)
2. gcd (a, b) = gcd (|a|, |b|) — |x| = absolute of |x|
3. gcd (0, a) = |a| $b \neq 0$
4. gcd (a, b, c) = gcd (a, gcd (b, c))
   = gcd (gcd (a, b), c)

GCD(a, b)

Q Write a function to find gcd (a, b)

Brute force :- for both a, b find all the factors & then find largest common between them.

```
a>0 , b>0
for (i= min(a,b) ; i>=1 ; i--
{   if (a%i==0 && b%i==0) {
        return i;
    }
}
```

Assume a > b ;

gcd(a, b) = gcd (a-b, b)

(1)

```
Using Recursion;
int gcd (a, b) {
    if (a<b)
        swap (a, b);
    if (b==0)
        return a;
    return gcd(a-b, a);
}
```

Say a > b ;

gcd (a, b) = gcd (a-b, b)
= gcd (b, a-b)
= gcd (b, a-2b)
= gcd (b, a-3b)
.
.
.
a- Maximul of b<=a

(2)

∴ gcd (a,b) = gcd (b, a % b)

Delete one to maximize GCD of all array elements

Q Given N array elements. We have to delete one element such that GCD of remaining element is maximized.

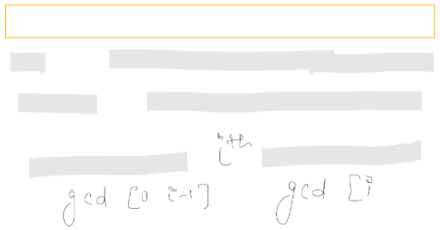
Eg: arr[] : {24, 16, 18, 30, 15}

0 1 2 3 4

After removing 16, GCD will be maximized.

Brute force

```
for (i in arr)
    for (i in arr)
        except i, gcd;
    }
    ans = max (
}
```



Maintain prefix of GCD.
Maintain suffix of GCD

```
P[i] = GCD from 0 to i;
P[0] = arr[0]
for(i=1; i<N; i++){
    P(i) = gcd (P[i-1], arr[i]);
}
S[i] = GCD from i to N-1
S[N-1] = arr[N-1]
for (i=N-2; i>=0; i--){
    S[i]=gcd(S[i+1], arr[i])
}
for (i=1; i<N-1; i++){
    x=gcd (P[i-1], S[i+1]);
    ans = max (ans, x)
}
return ans;
```