

10 Sorting

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Agenda :

1. Introduction
2. Problems
3. Basic Sorting Algorithms

1. Introduction

- Sorting is an arrangement of data in particular order on the basis of some parameter

Example :

$A[] = \{ 2, 3, 9, 12, 17, 19 \}$

Above Array is sorted in ascending order.

2. Problems

Problem 1 : Minimize the cost to empty an array.

Given an array of n integers, minimize the cost to empty given array where cost of removing an element is equal to sum of all elements left in an array.

Example :

$A[] = \{ 2, 1, 4 \}$

Ans = 11

After removing 4 cost = $4+2+1 = 7$

After removing 2 cost = $2+1 = 3$

After removing 1 cost = $1 = 1$

Comment

Suggest edit

Total cost = 11

Approach :

Start Removing from the largest element. i.e removing largest element sequentially will result in lowest cost.

[a b c d]

Remove a

$a + b + c + d$

Remove b

+

$b + c + d$

Remove c

+

$c + d$

Remove d

+

d

$a + 2b + 3c + 4d$



$a > b > c > d$

- Sort the data in descending order.
- Initialise the ans equal to 0.
- Run a loop for i from 0 to $n - 1$, where n is the size of the array.
- For every element add $\text{arr}[i] * i$ to the ans.
 - TC - $O(n \log n)$
 - SC - $O(n)$

Problem 2 : Find Count of Noble Integers

Given an array of distinct elements of size n , find the count of noble integers.

Note: $\text{arr}[i]$ is noble if count of elements smaller than $\text{arr}[i]$ is equal to $\text{arr}[i]$ where $\text{arr}[i]$ is element at index i .

Example :

$A[] = \{ 1, -5, 3, 5, -10, 4 \}$

Ans = 3

Explanation

For $\text{arr}[2]$ there are three elements less than 3 that is 1, -5 and -10. So $\text{arr}[2]$ is noble integer.

For $\text{arr}[3]$ there are five elements less than 5 that is 1, 3, 4, -5 and -10. So $\text{arr}[3]$ is noble integer.

For $\text{arr}[5]$ there are four elements less than 4 that is 1, 3, -5 and -10. So $\text{arr}[5]$ is noble integer.

In total there are 3 noble elements.

BruteForce Approach :

Iterate through every element in the array, for every element count the number of smaller elements.

– TC - $O(N^2)$

– SC - $O(1)$

Optimized Approach :

Sort the elements and then for each element compare total elements in left side with the current element value.

```
int find_nobel_integers(int arr[], int n) {
    sort(arr);
    int ans = 0;
    for (int i = 0; i < n; i++) {
        if (arr[i] == i) {
            ans = ans + 1;
        }
    }
    return ans;
}
```

– TC - $O(n \log n)$

– SC - $O(1)$

Problem 3 : Find count of noble integers (Not Distinct)

Note: Same as previous question, but all elements need not to be distinct

Brute Force Approach :

Iterate and count all smaller elements for current element.

Optimized Approach :

- If the current element is same as previous element then the total number of smaller elements will be same as previous element.
- If current element is not equal to previous element then the total number of smaller elements is equal to its index.

```
int find_nobel_integers(int arr[], int n) {
    sort(arr);
    int count = 0, ans = 0;
    if (arr[0] == 0) ans++;

    for (int i = 1; i < n; i++) {
        if (arr[i] != arr[i - 1])
            count = i;

        if (count == arr[i])
            ans++;
    }
    return ans;
}
```

– TC - $O(n \log n)$

– SC - $O(1)$

Problem 4 : Sort an array in ascending order of count of factors if count of factors are same then sort on base of magnitude.

Q3) Sort an array in ascending order of count of factors
if count of factors are equal, then sort based on
magnitude. (asc)

$\begin{array}{ccccc} 9 & 3 & 10 & 6 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3 & 2 & 4 & 4 & 3 \end{array} \Rightarrow \begin{array}{ccccc} 3 & 4 & 3 & 6 & 10 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 3 & 3 & 4 & 4 \end{array}$

Sort (P₁, P₂, P₃) By default, sort based on magnitude
 [10, 1, 2] → [1, 2, 10]
 → P₁: input
 → P₂: order: 'Asc', 'Des'
 → P₃: Comparator. Define your own custom sort function

```
Comparator(x, y) {
    if CountFactor(x) < CountFactor(y)
        return True
    else
        return False
}
```

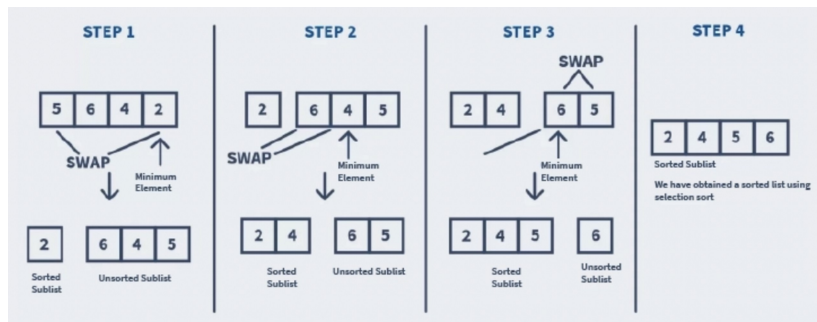
```
bool comp(int x, int y) {
    int cntx = CountFactor(x)
    int cnty = CountFactor(y)
    if (cntx < cnty)
        return true
    else if (cntx > cnty)
        return false
    else { // cntx == cnty
        if (x < y)
            return True
        else
            return false
    }
}
```

Sort(arr, ..., comparator=comp)

3. Basic Sorting Algorithms

A. Selection Sort

- To begin with, place all the students in the unarranged queue.
- From this unarranged queue, search for the shortest student and place him/her in the list of arranged students.
- Again, from the unarranged queue, select the second-shortest student. Place this student in the arranged queue, just after the smallest student.
- Repeat the above-given steps until all the students are placed into the arranged queue.



```
void selectionSort(int arr[], int size) {
    int i, j, minIndex;
    for (i = 0; i < size - 1; i++) {
        // set minIndex equal to the first unsorted element
        minIndex = i;
        //iterate over unsorted sublist and find the minimum element
        for (j = i + 1; j < size; j++) {
            if (arr[j] < arr[minIndex]) {
                minIndex = j;
            }
        }
        // swapping the minimum element with the element at minIndex to place it at its correct position
        swap(arr[minIndex], arr[i]);
    }
}
```

Time Complexity : $O(N^2)$ since we have to iterate entire list to search for a minimum element every time.
Space Complexity : $O(1)$

B. Insertion Sort

Approach :

Line 2: We don't process the first element, as it has nothing to compare against.

Line 3: Loop from $i=1$ till the end, to process each element.

Line 4: Extract the element at position i i.e. $arr[i]$. Let it be called E .

Line 5: To compare E with its left elements, loop j from $i-1$ to 0

Line 6, 7: Compare E with the left element, if E is lesser, then move $arr[j]$ to right by 1.

Line 8: Once we have found the position for E , place it there.

```
void insertionSort(int arr[], int n) {
    for (int i = 1; i < n; i++) { // Start from 1 as arr[0] is always sorted
        int currentElement = arr[i];
        int j = i - 1;
        // Move elements of arr[0..i-1], that are greater than key,
        // to one position ahead of their current position
        while (j >= 0 && arr[j] > currentElement) {
            arr[j + 1] = arr[j];
            j = j - 1;
        }
        // Finally place the Current element at its correct position.
        arr[j + 1] = currentElement;
    }
}
```

Time Complexity:

Worst Case: $O(N^2)$, when the array is sorted in reverse order.

Best Case: $O(N)$, when the data is already sorted in desired order, in that case there will be no swap.

Space Complexity: $O(1)$

Note:

Both Selection & Insertion are in-place sorting algorithms, means they don't need extra space.

Since the time complexity of both can go to $O(N^2)$, it is only useful when we have a lesser number of elements to sort in an array.