Pension returns analysis

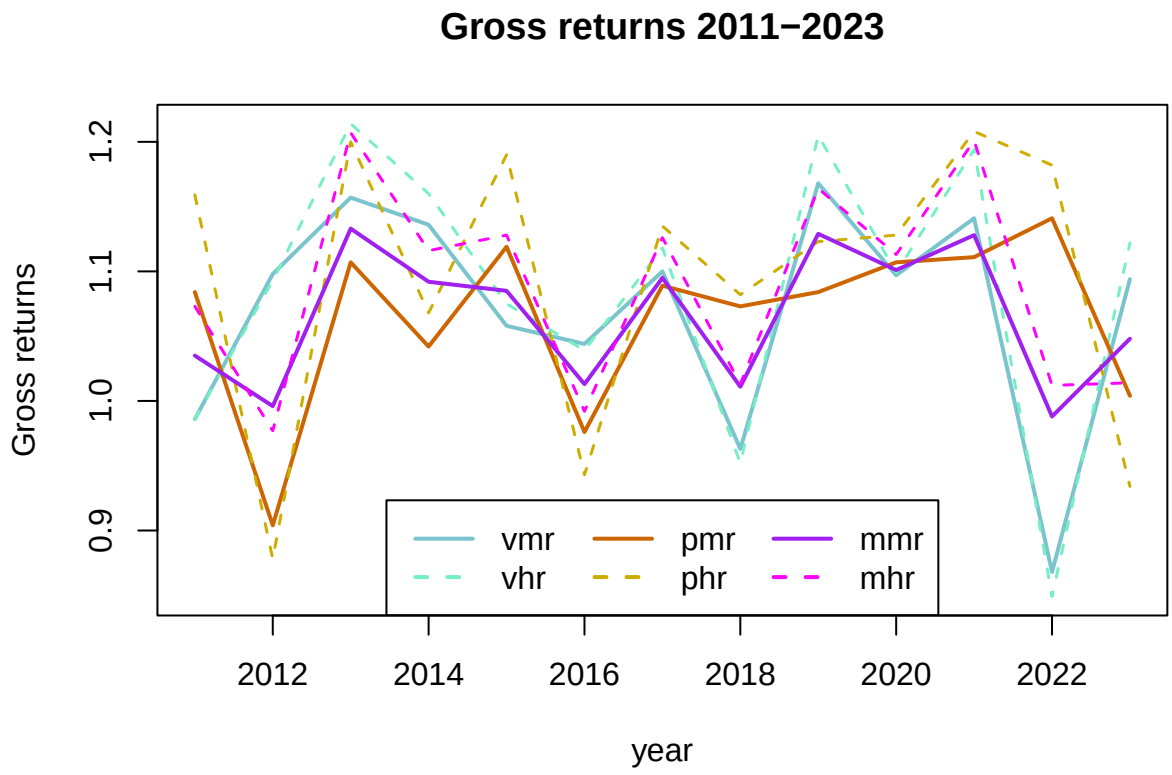
17:12 05 April 2024

Contents

Fit log returns to F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated shape parameter (degrees of freedom).
`xi` is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.
`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.



Summary of gross returns

```
##       vmr              pmr              mmr              vhr        
##  Min.   :0.868   Min.   :0.904   Min.   :0.988   Min.   :0.849  
##  1st Qu.:1.044   1st Qu.:1.042   1st Qu.:1.013   1st Qu.:1.039  
##  Median :1.097   Median :1.084   Median :1.085   Median :1.099  
##  Mean   :1.070   Mean   :1.065   Mean   :1.066   Mean   :1.085  
##  3rd Qu.:1.136   3rd Qu.:1.107   3rd Qu.:1.101   3rd Qu.:1.160  
##  Max.   :1.168   Max.   :1.141   Max.   :1.133   Max.   :1.214  
##       phr              mhr
```

```
##  Min.   :0.878   Min.   :0.977
##  1st Qu.:1.068   1st Qu.:1.013
##  Median :1.128   Median :1.113
##  Mean   :1.095   Mean   :1.087
##  3rd Qu.:1.182   3rd Qu.:1.128
##  Max.   :1.208   Max.   :1.207
```

```
##       vmrl
##  Min.   :0.801
##  1st Qu.:1.013
##  Median :1.085
##  Mean   :1.061
##  3rd Qu.:1.128
##  Max.   :1.193
```

```
##             vmr   pmr   mmr   vhr   phr   mhr
## Min.   : 0.868 0.904 0.988 0.849 0.878 0.977
## 1st Qu.: 1.044 1.042 1.013 1.039 1.068 1.013
## Median : 1.097 1.084 1.085 1.099 1.128 1.113
## Mean   : 1.070 1.065 1.066 1.085 1.095 1.087
## 3rd Qu.: 1.136 1.107 1.101 1.160 1.182 1.128
## Max.   : 1.168 1.141 1.133 1.214 1.208 1.207
```

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.136	vmr	1.168	vmr
0.977	mhr	1.044	vmr	1.113	mhr	1.087	mhr	1.107	pmr	1.141	pmr
0.904	pmr	1.042	pmr	1.099	vhr	1.085	vhr	1.101	mmr	1.133	mmr
0.878	phr	1.039	vhr	1.097	vmr	1.070	vmr	1.160	vhr	1.214	vhr
0.868	vmr	1.013	mmr	1.085	mmr	1.066	mmr	1.182	phr	1.208	phr
0.849	vhr	1.013	mhr	1.084	pmr	1.065	pmr	1.128	mhr	1.207	mhr

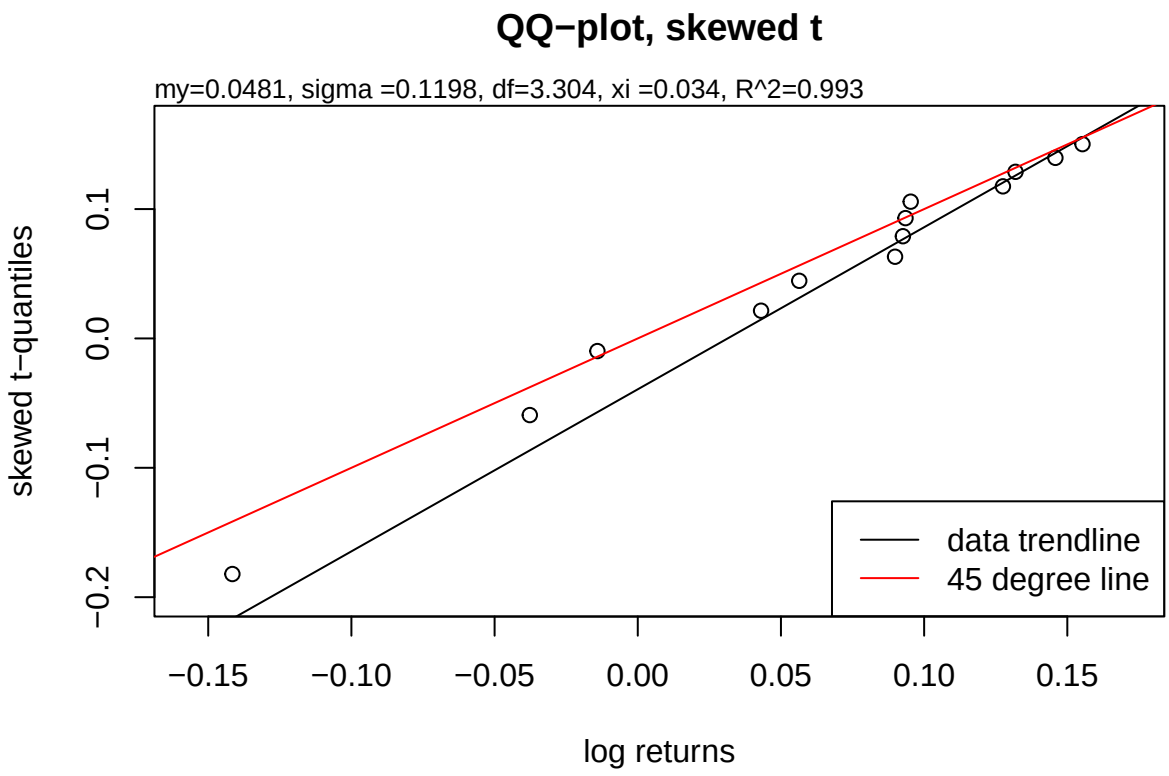
```
## cov(vmr, pmr) =  -0.001094875
```

```
## cov(vhr, phr) =  -0.0001730651
```

Velliv medium risk, 2011 - 2023

```
##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 7.4 percent
## What is the risk of losing max 25 %? =< 1.8 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 41 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

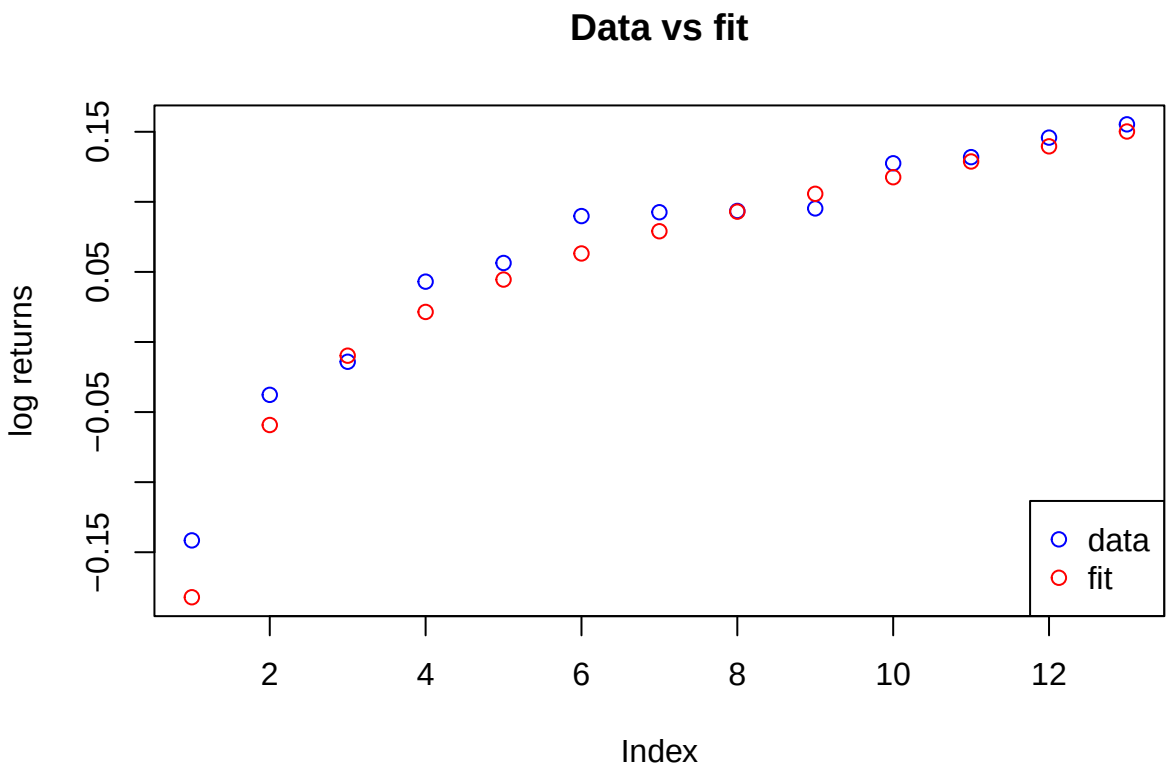
QQ Plot



The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

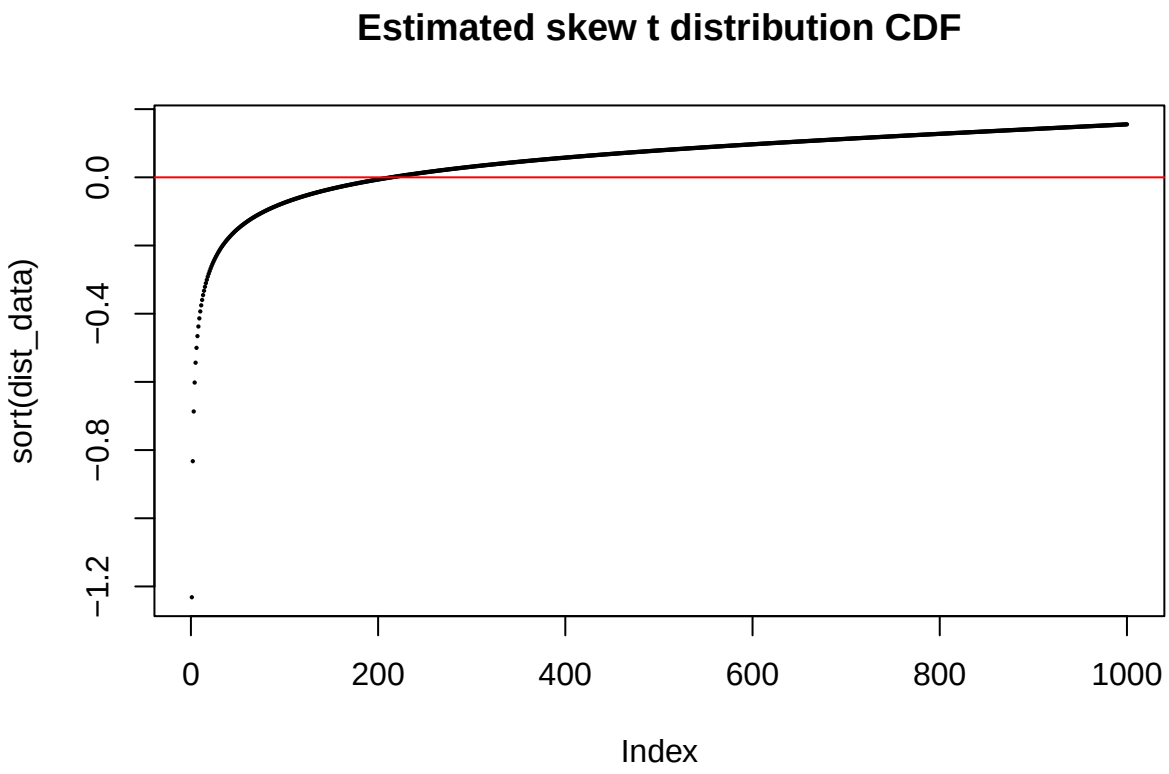
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

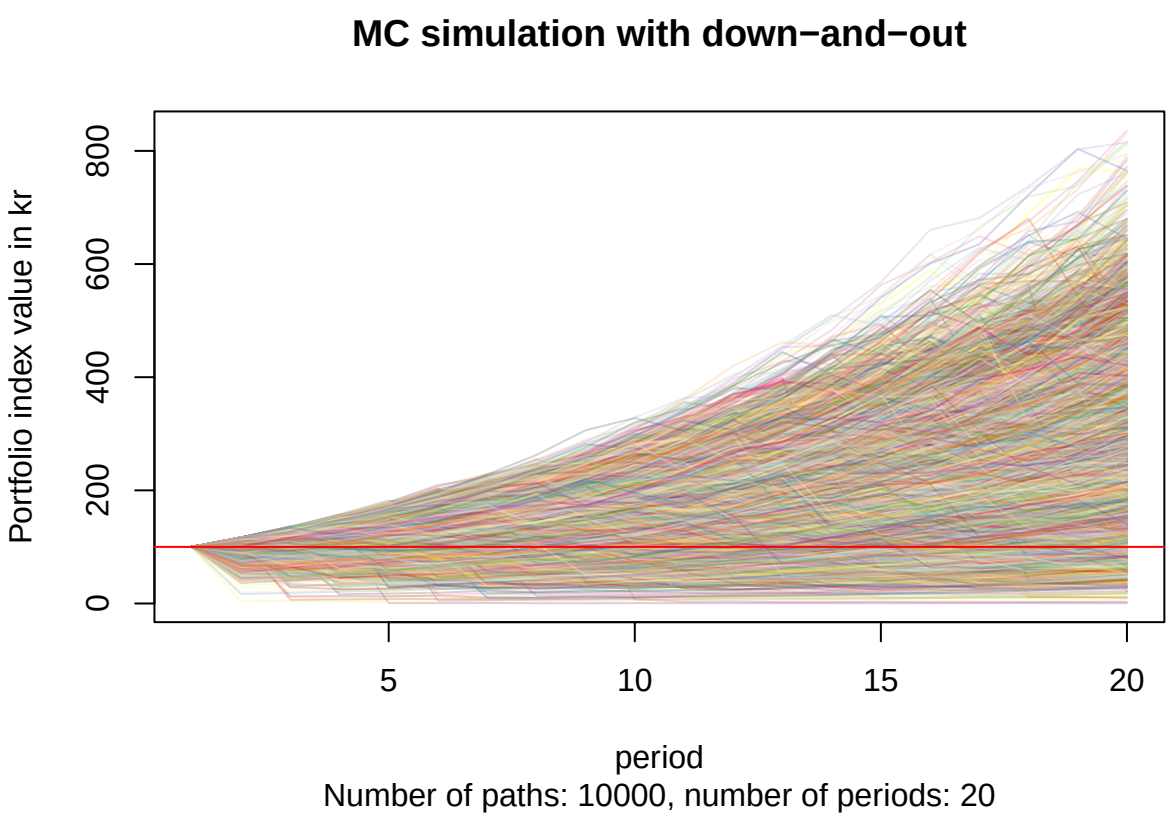
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

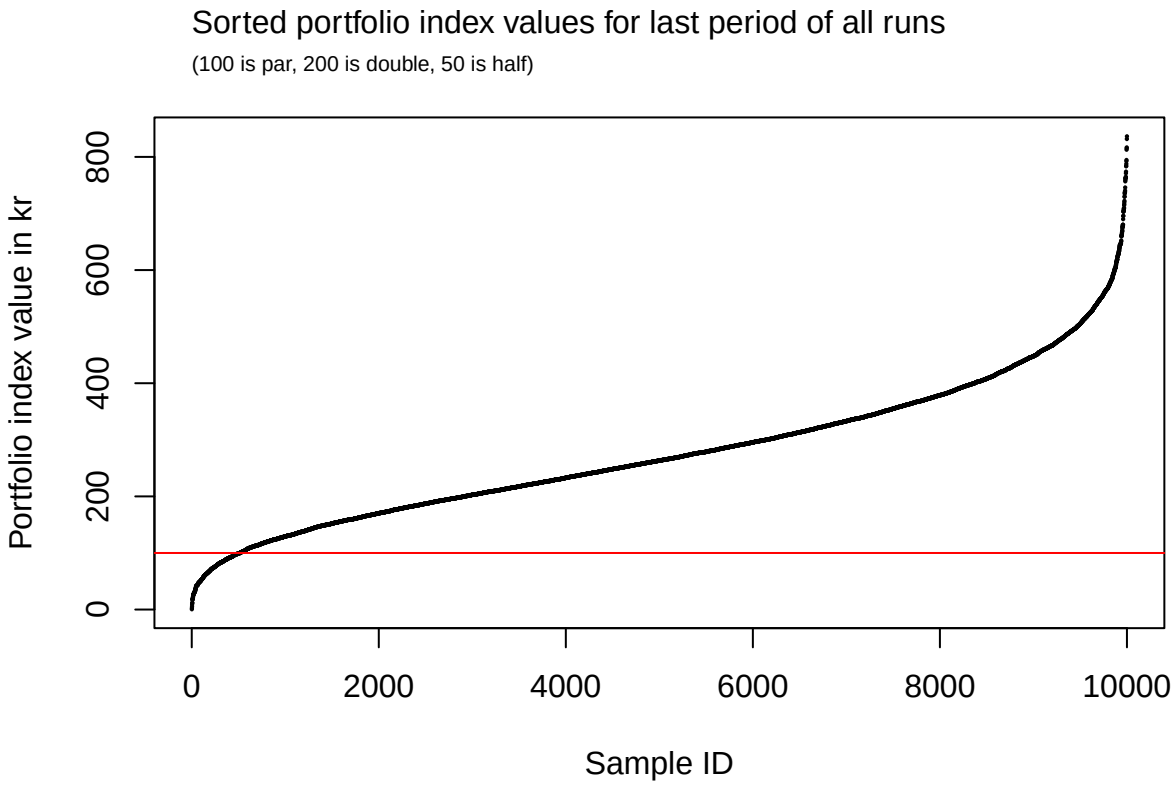


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 278.171 kr.
## SD of portfolio index value after 20 years: 125.008 kr.
## Min total portfolio index value after 20 years: 0.552 kr.
## Max total portfolio index value after 20 years: 836.325 kr.
##
## Share of paths finishing below 100: 5.15 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period
Number of paths: 10000, number of periods: 20

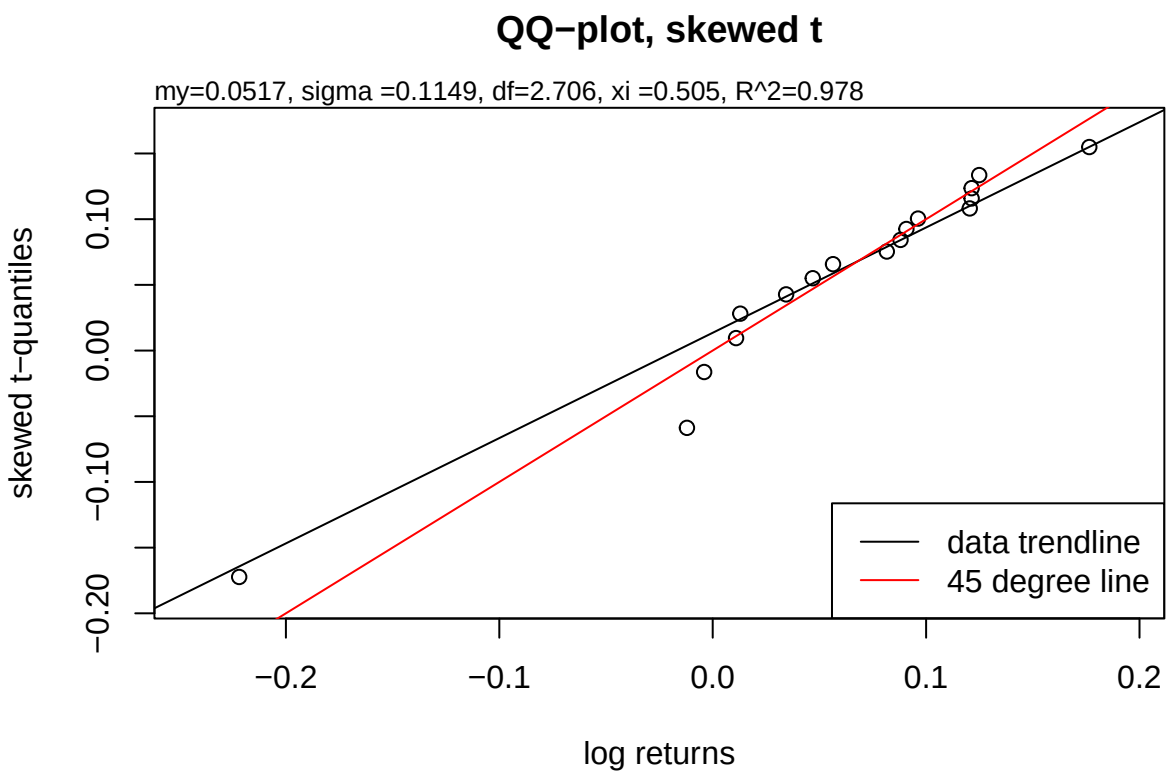


Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##
## AIC: -34.35752
## BIC: -31.02467
## m: 0.05171176
## s: 0.1149408
## nu (df): 2.706099
## xi: 0.5049945
## R^2: 0.978
##
## An R^2 of 0.978 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 5.4 percent
## What is the risk of losing max 25 %? =< 1.3 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 36.2 percent
## What is the chance of gaining min 25 %? >= 0.3 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

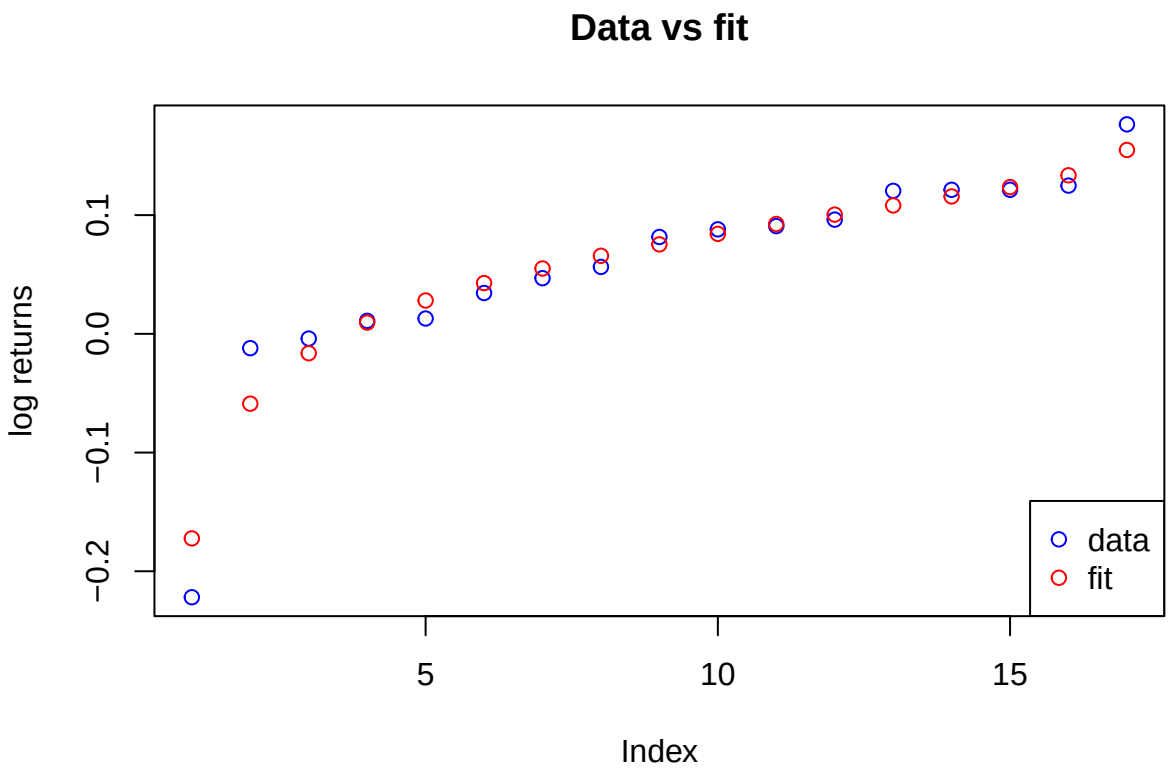
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

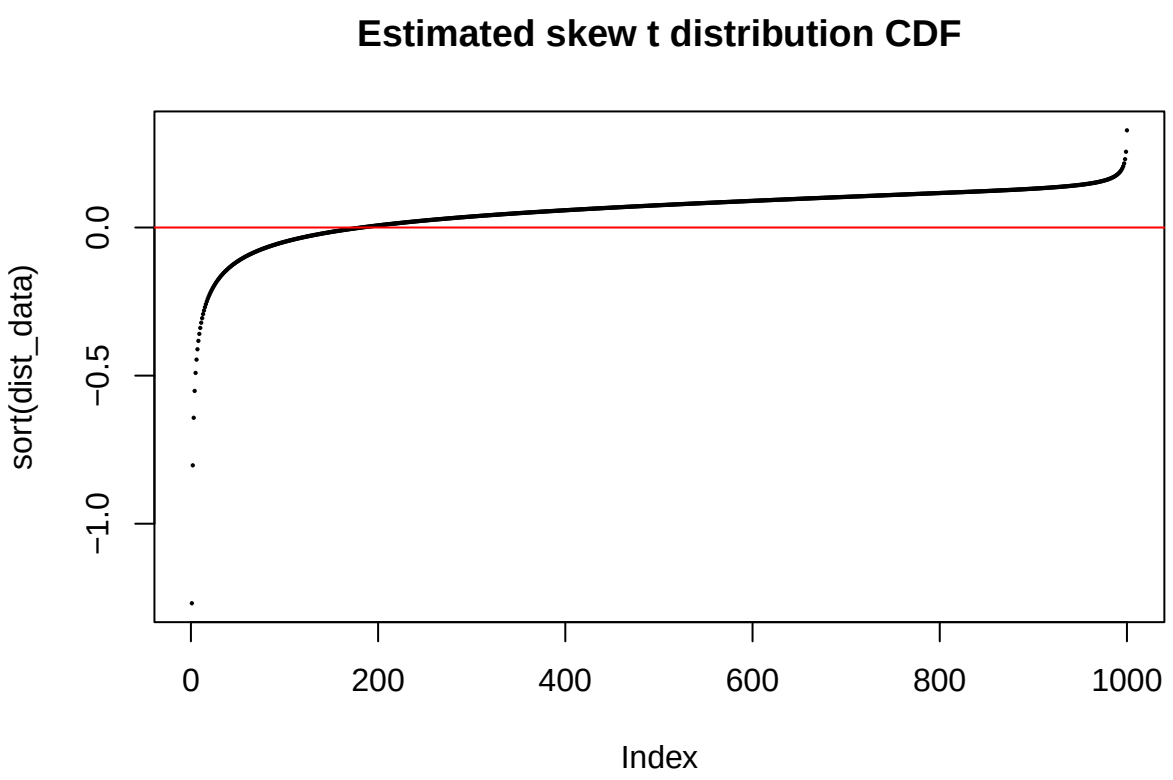
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

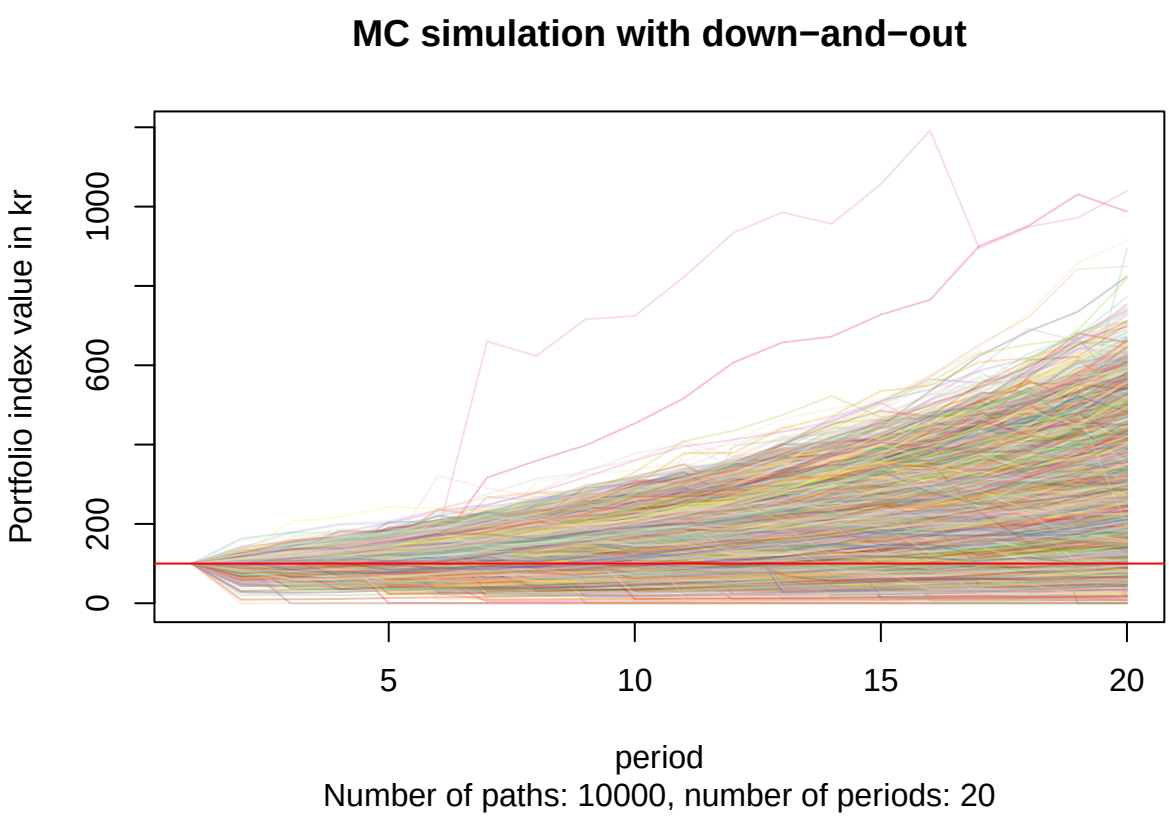
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

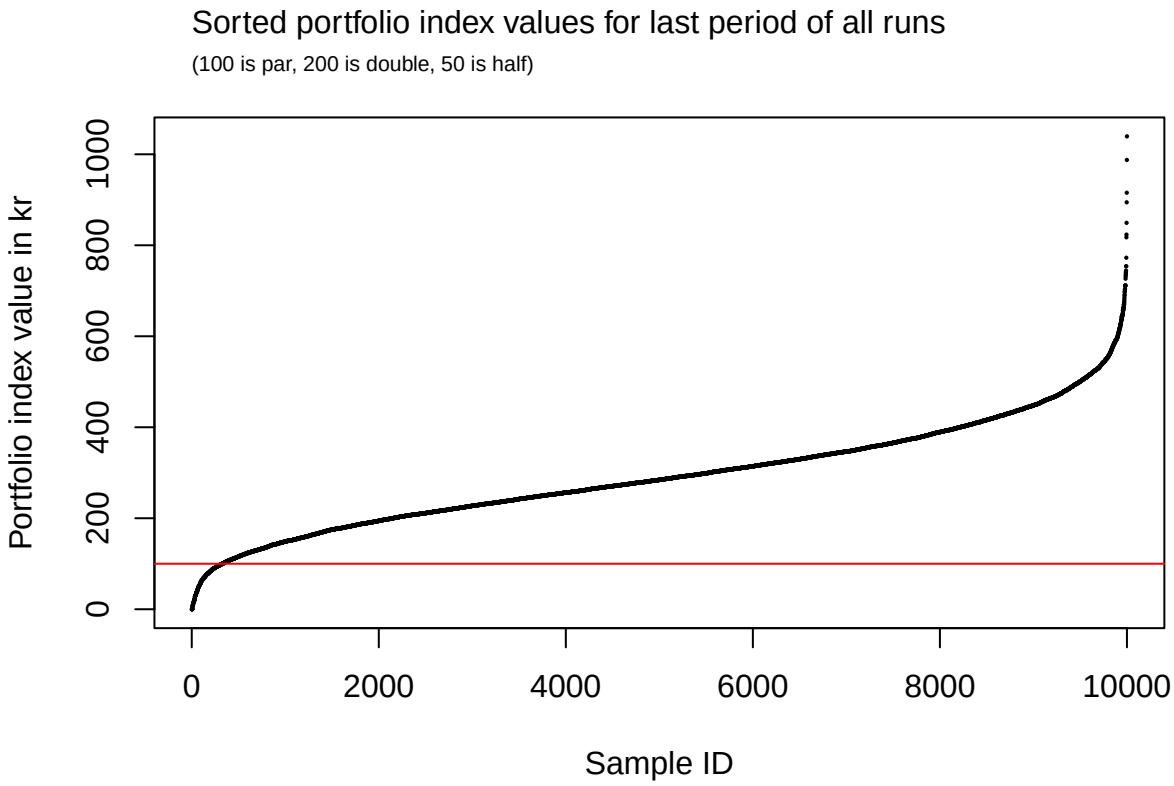


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0.02 percent
##
## Mean portfolio index value after 20 years: 293.566 kr.
## SD of portfolio index value after 20 years: 117.469 kr.
## Min total portfolio index value after 20 years: 0 kr.
## Max total portfolio index value after 20 years: 1039.413 kr.
##
## Share of paths finishing below 100: 3.24 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
600
1000
5
10
15
20
period
Number of paths: 10000, number of periods: 20

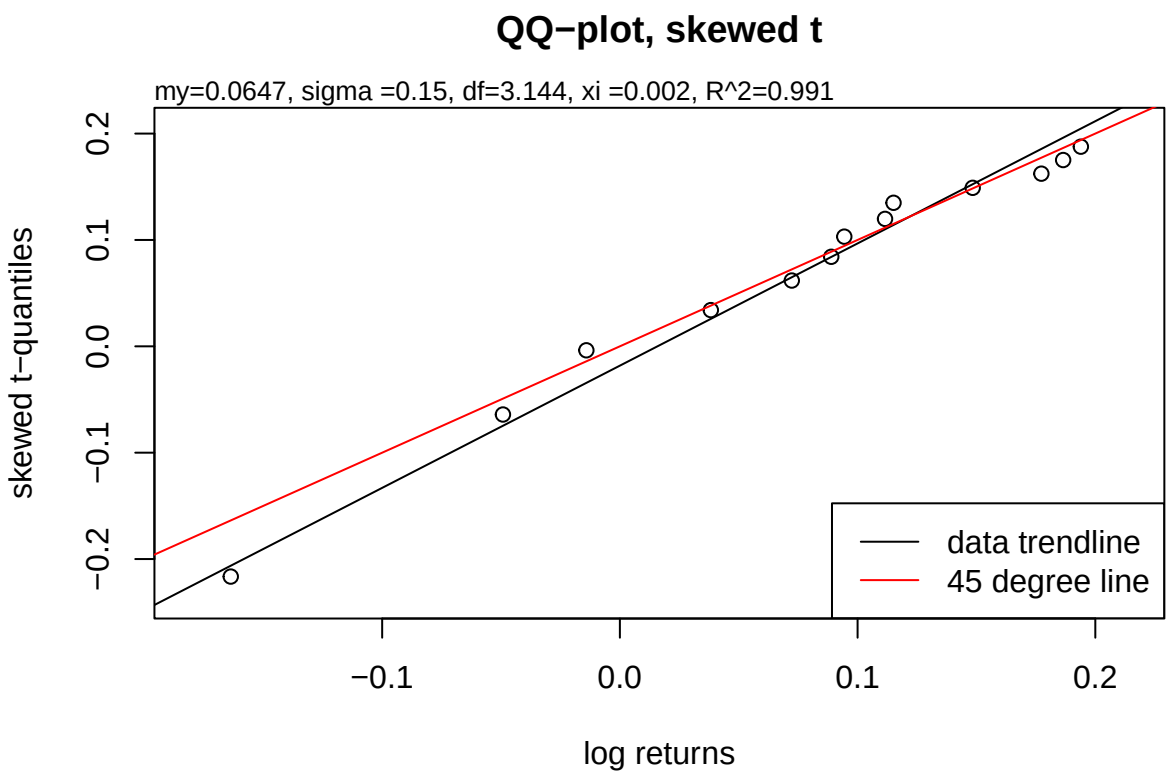


Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -21.42488
## BIC: -19.16508
## m: 0.06471454
## s: 0.1499924
## nu (df): 3.144355
## xi: 0.002367034
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 8.3 percent
## What is the risk of losing max 25 %? =< 2.5 percent
## What is the risk of losing max 50 %? =< 0.4 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 53.3 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

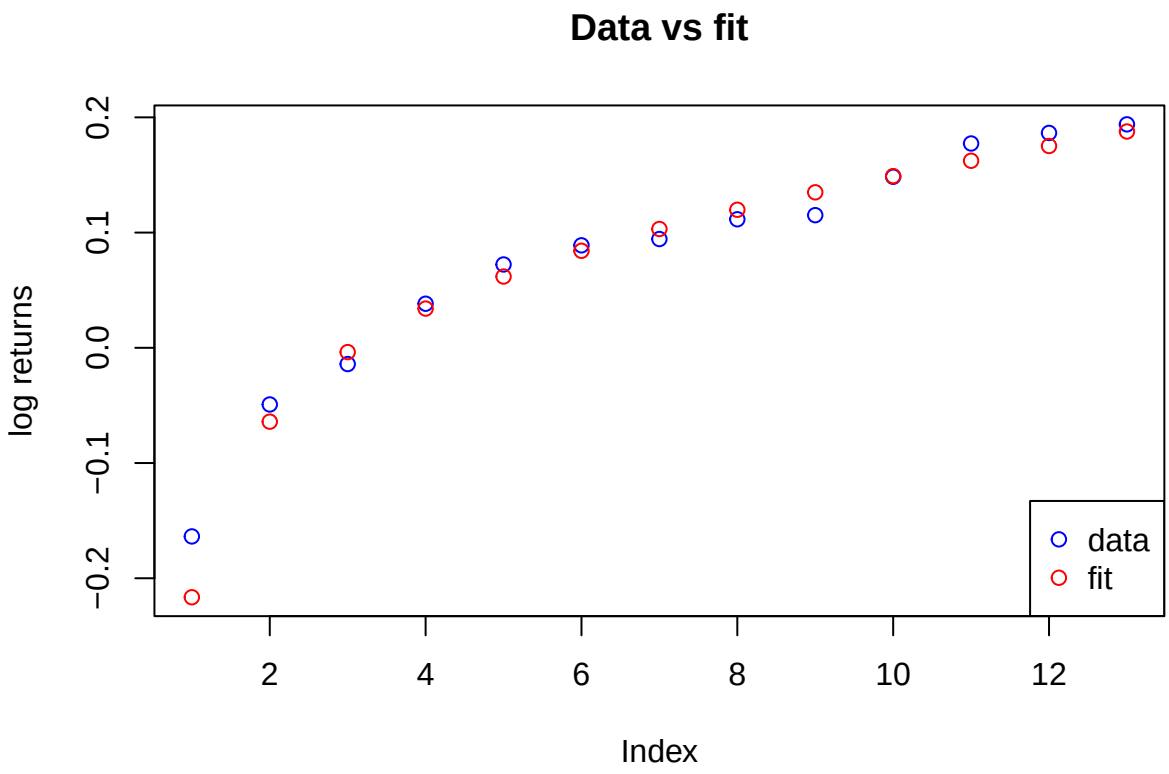
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

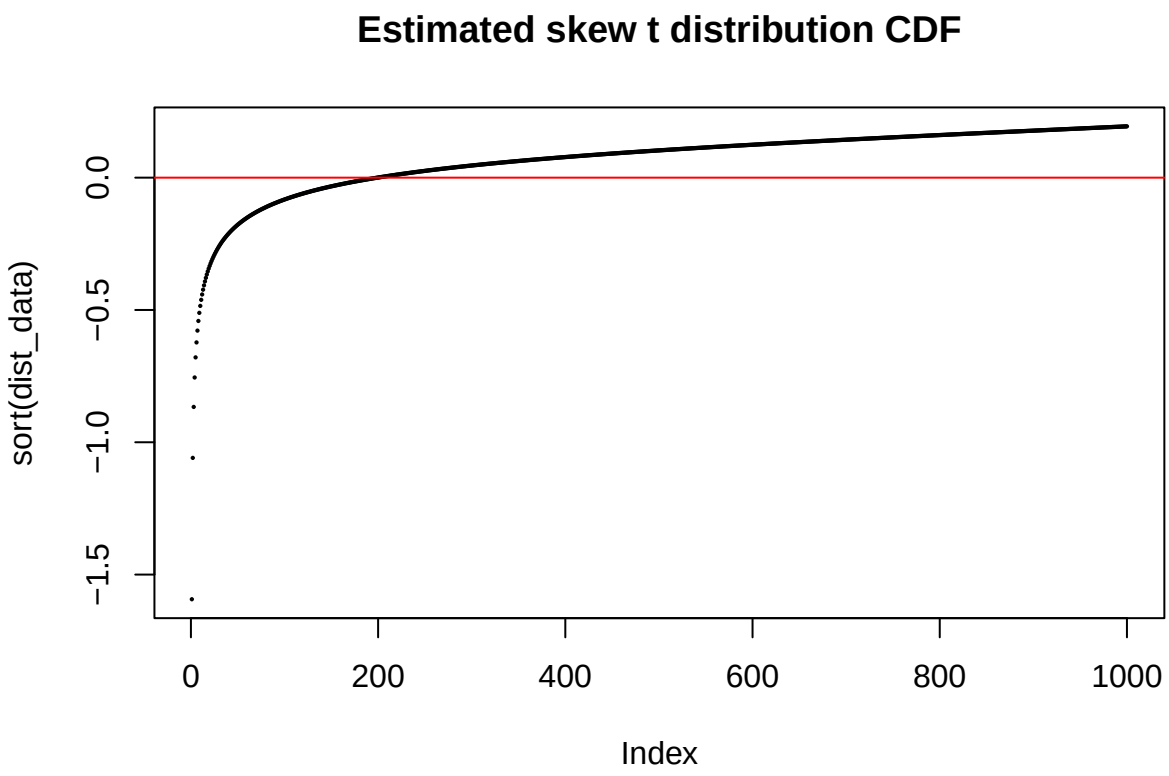
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

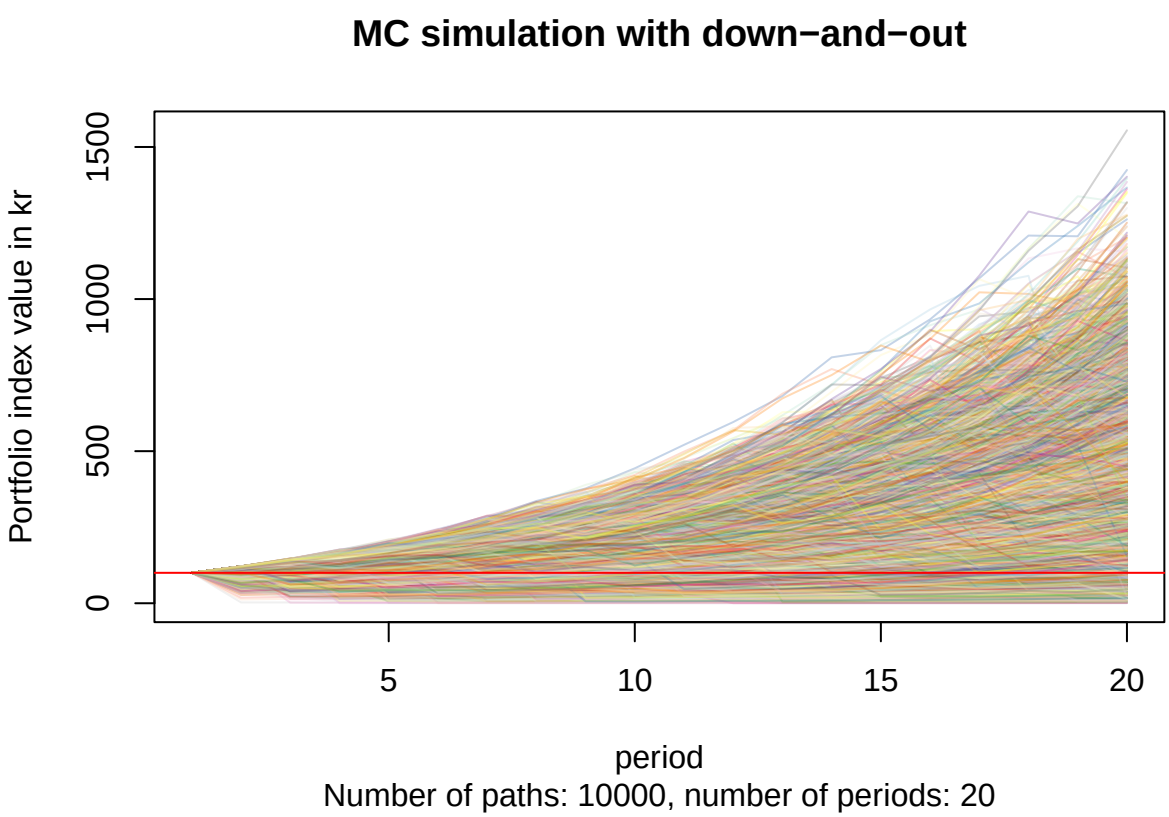
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

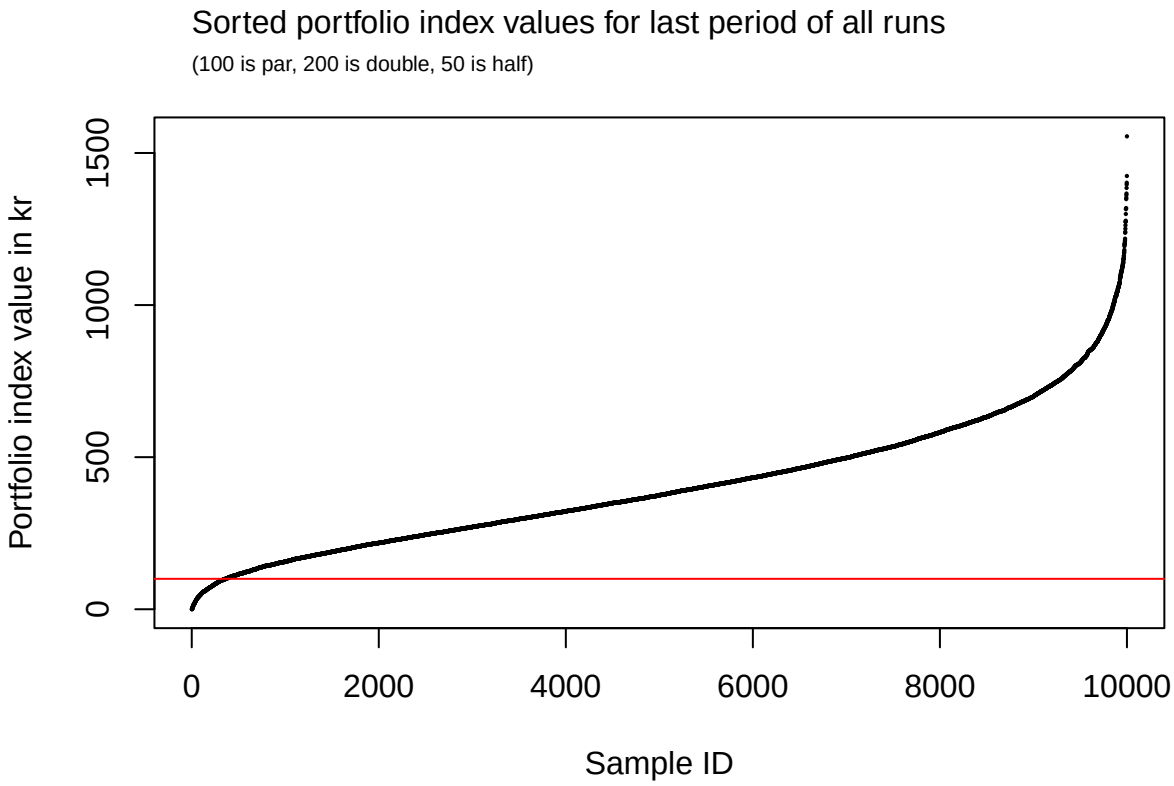


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0.01 percent
##
## Mean portfolio index value after 20 years: 407.755 kr.
## SD of portfolio index value after 20 years: 217.848 kr.
## Min total portfolio index value after 20 years: 0 kr.
## Max total portfolio index value after 20 years: 1554.709 kr.
##
## Share of paths finishing below 100: 3.62 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

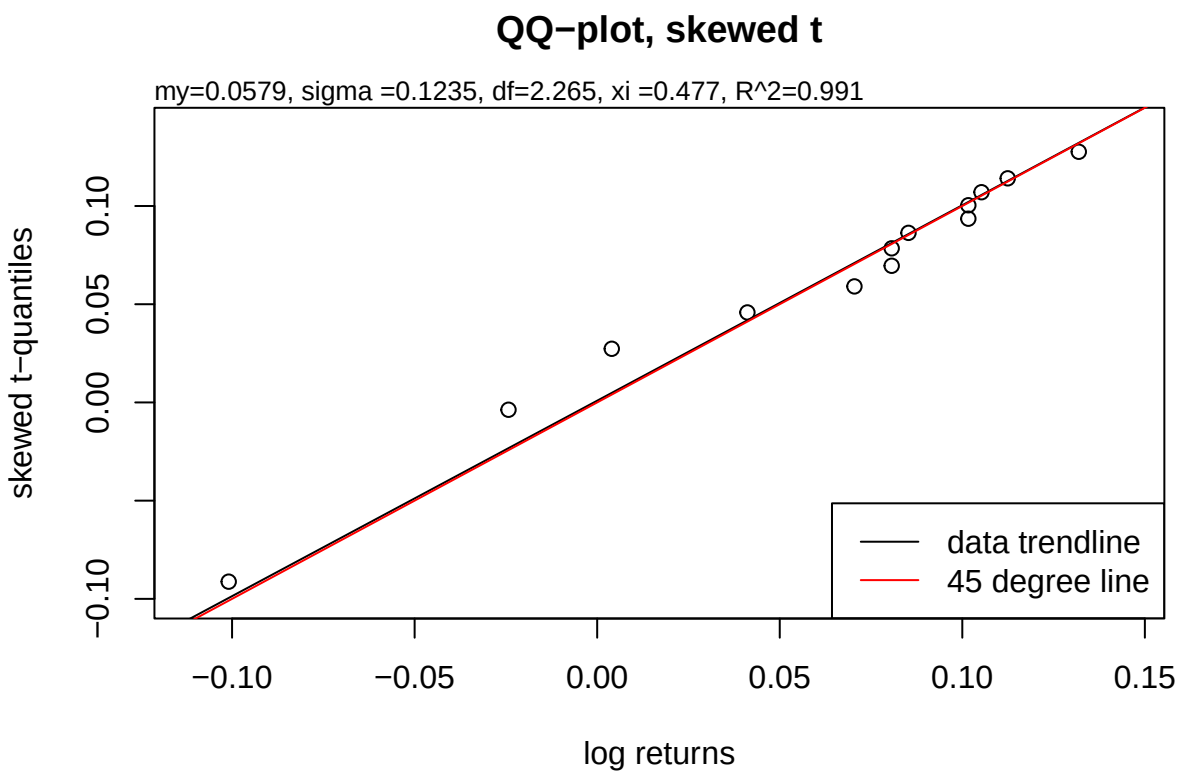


PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -33.22998
## BIC: -30.97018
## m: 0.05789224
## s: 0.1234592
## nu (df): 2.265273
## xi: 0.477324
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 3.3 percent
## What is the risk of losing max 25 %? =< 0.9 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 32.7 percent
## What is the chance of gaining min 25 %? >= 0.1 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

QQ Plot

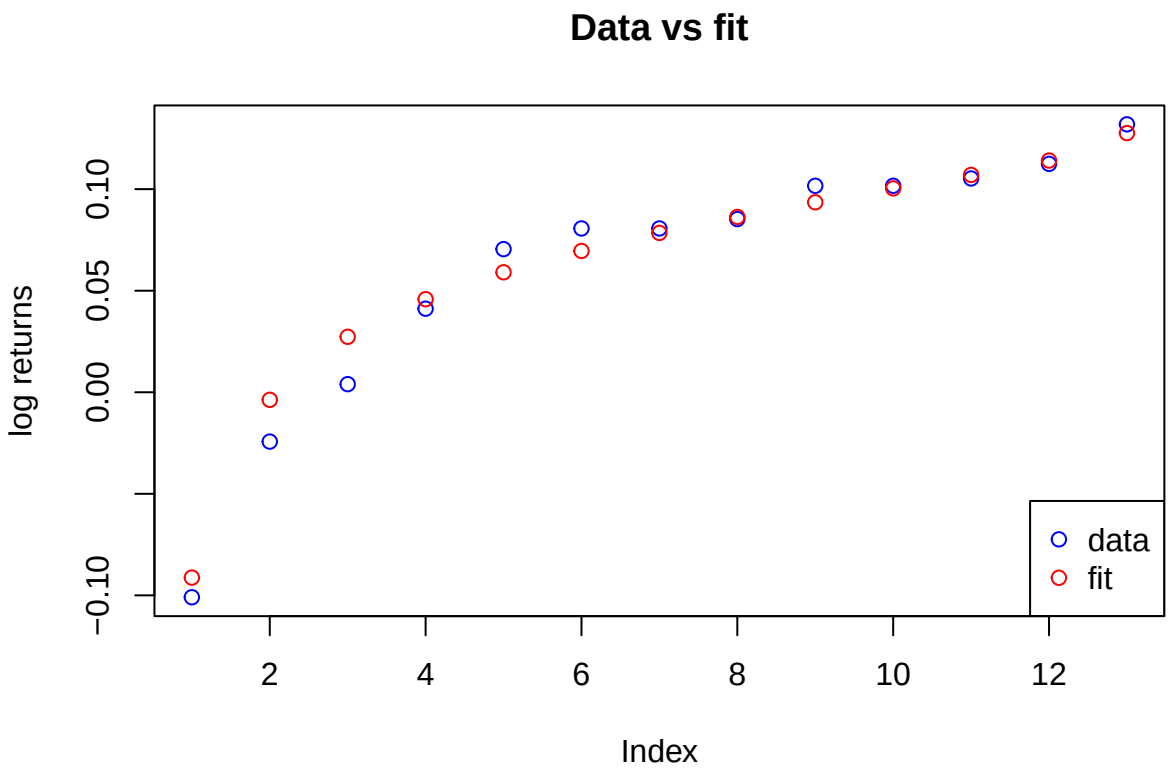


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
##  [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
##  [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11]  0.107018607  0.114081432  0.127604387
```

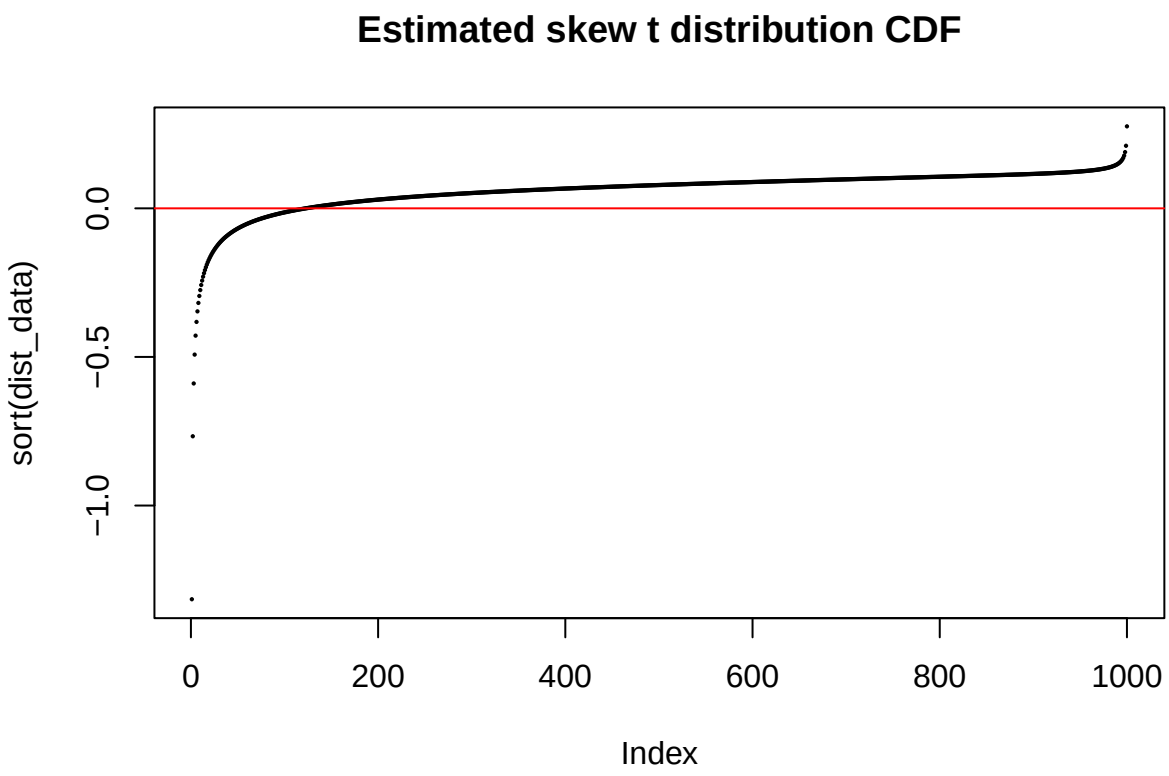
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

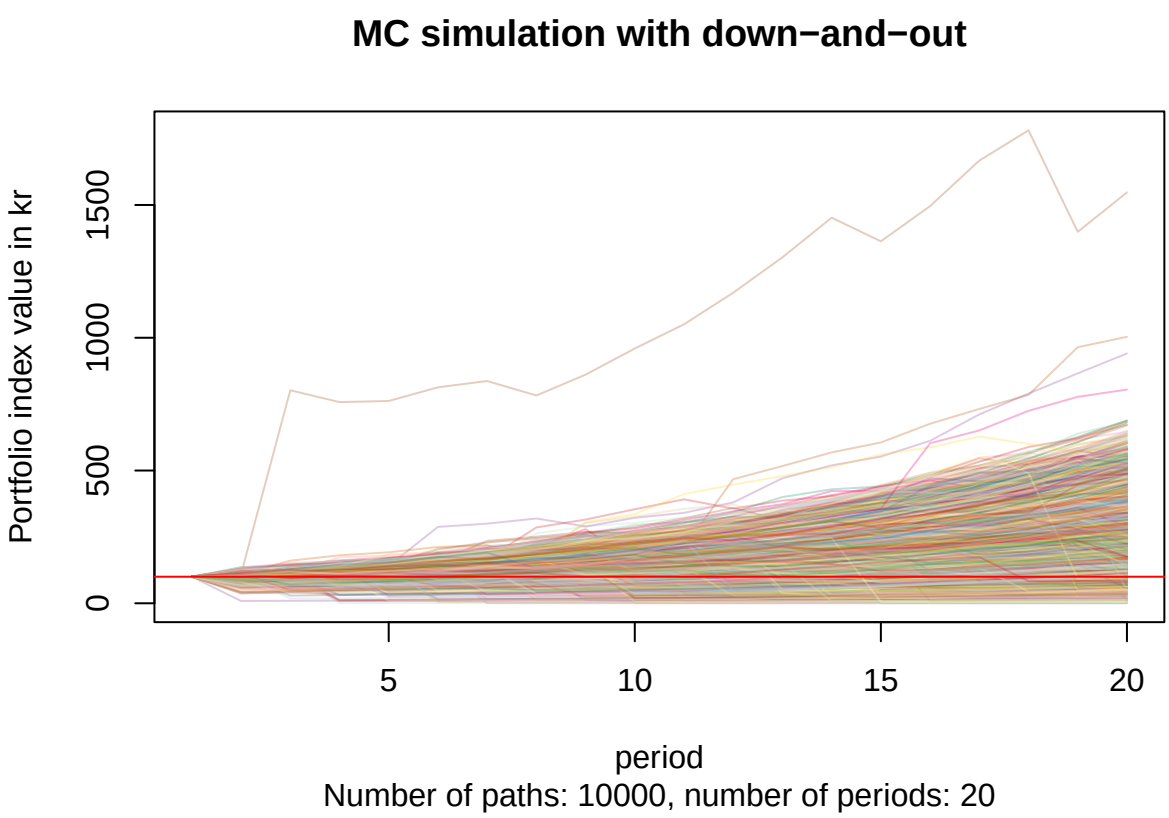
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

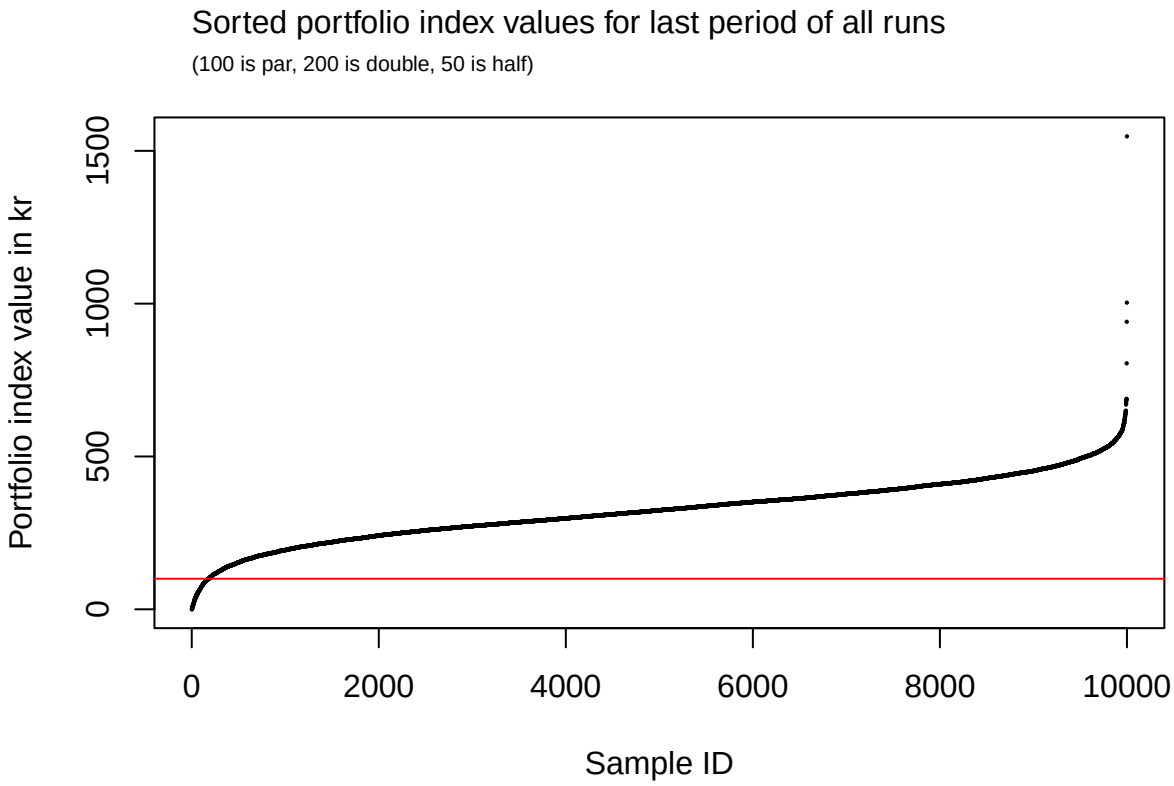


We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 324.14 kr.
## SD of portfolio index value after 20 years: 103.447 kr.
## Min total portfolio index value after 20 years: 0.199 kr.
## Max total portfolio index value after 20 years: 1547.412 kr.
##
## Share of paths finishing below 100: 1.83 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

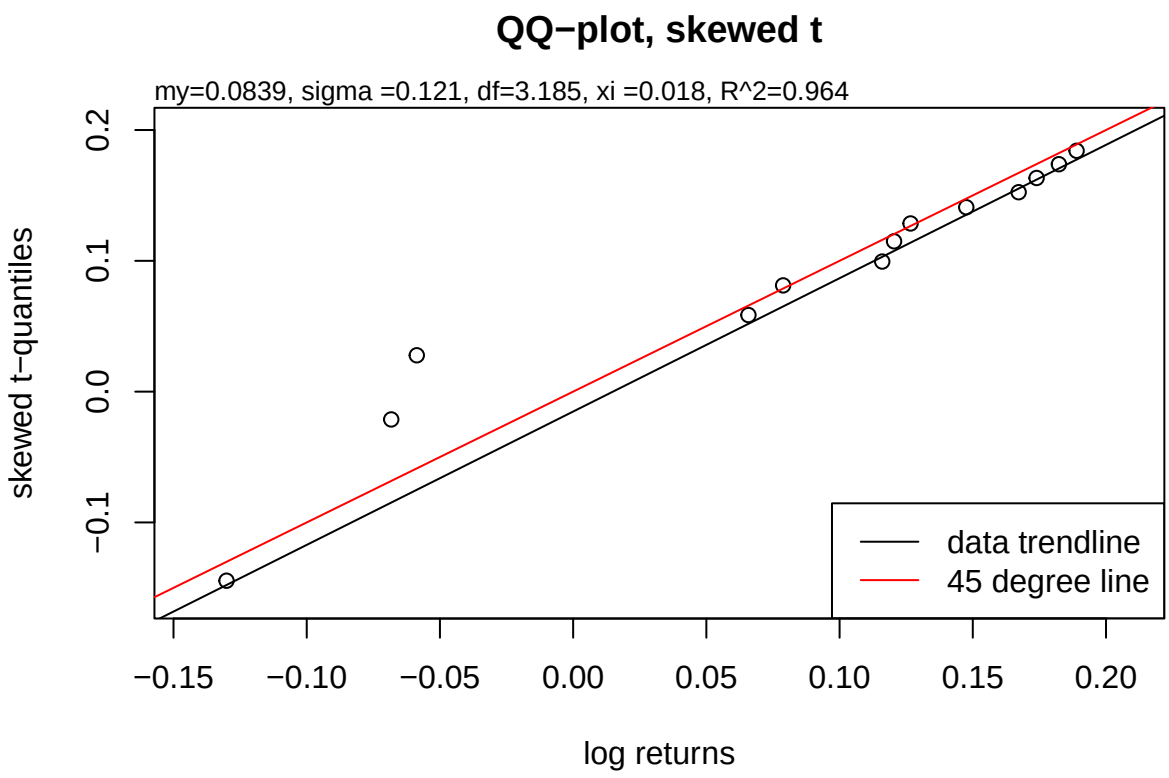


PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -23.72565
## BIC: -21.46585
## m: 0.08386034
## s: 0.1210107
## nu (df): 3.184569
## xi: 0.01790306
## R^2: 0.964
##
## An R^2 of 0.964 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 5.3 percent
## What is the risk of losing max 25 %? =< 1.4 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 59.6 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

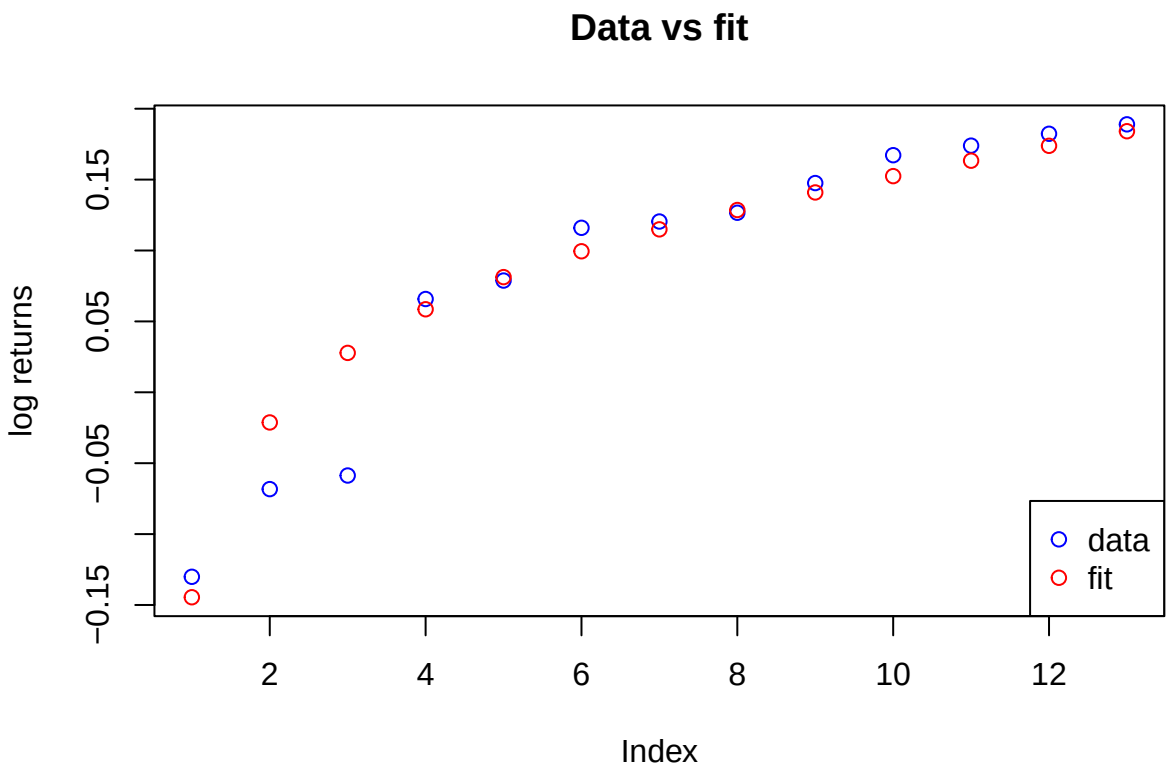
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

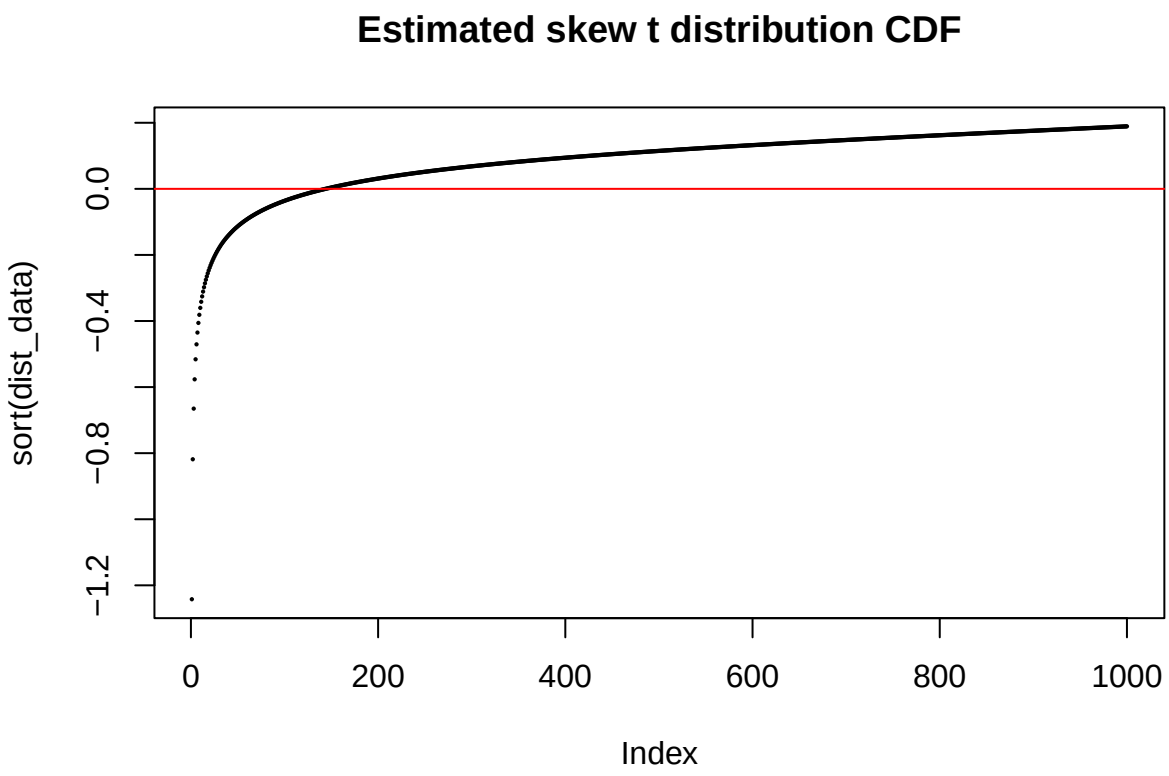
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

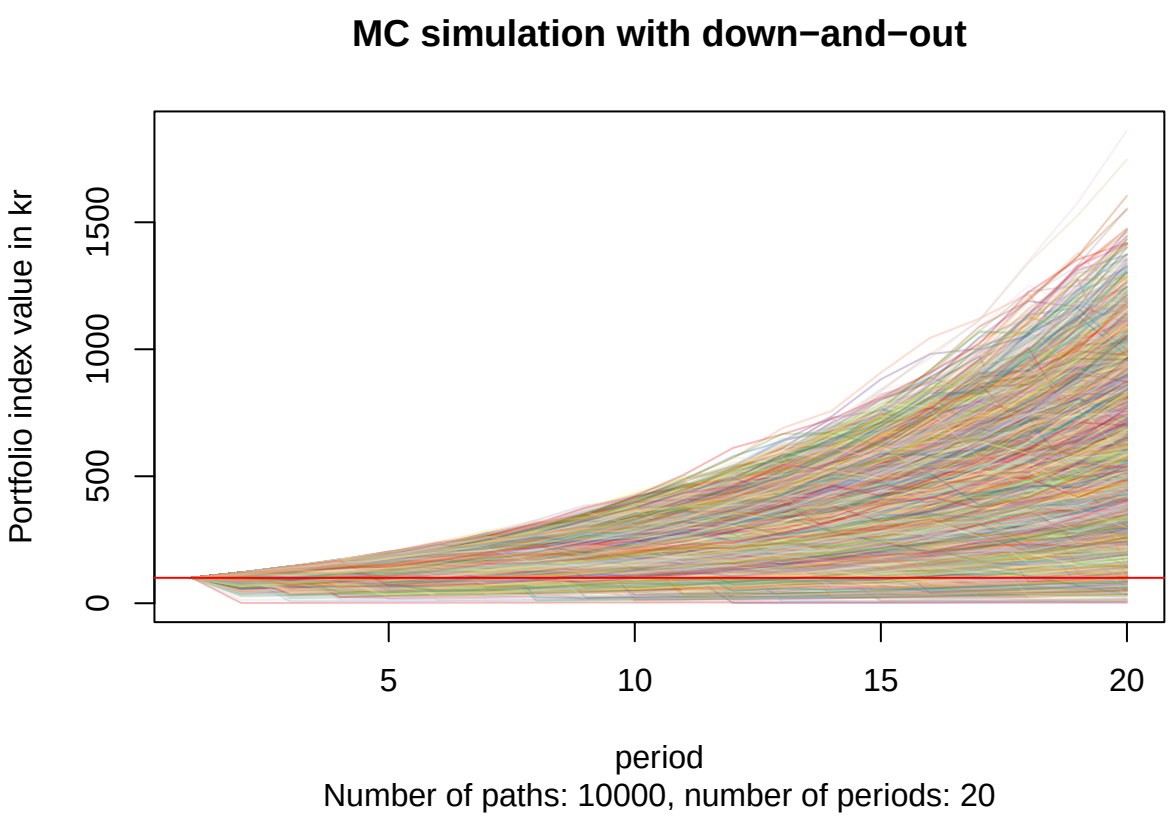
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

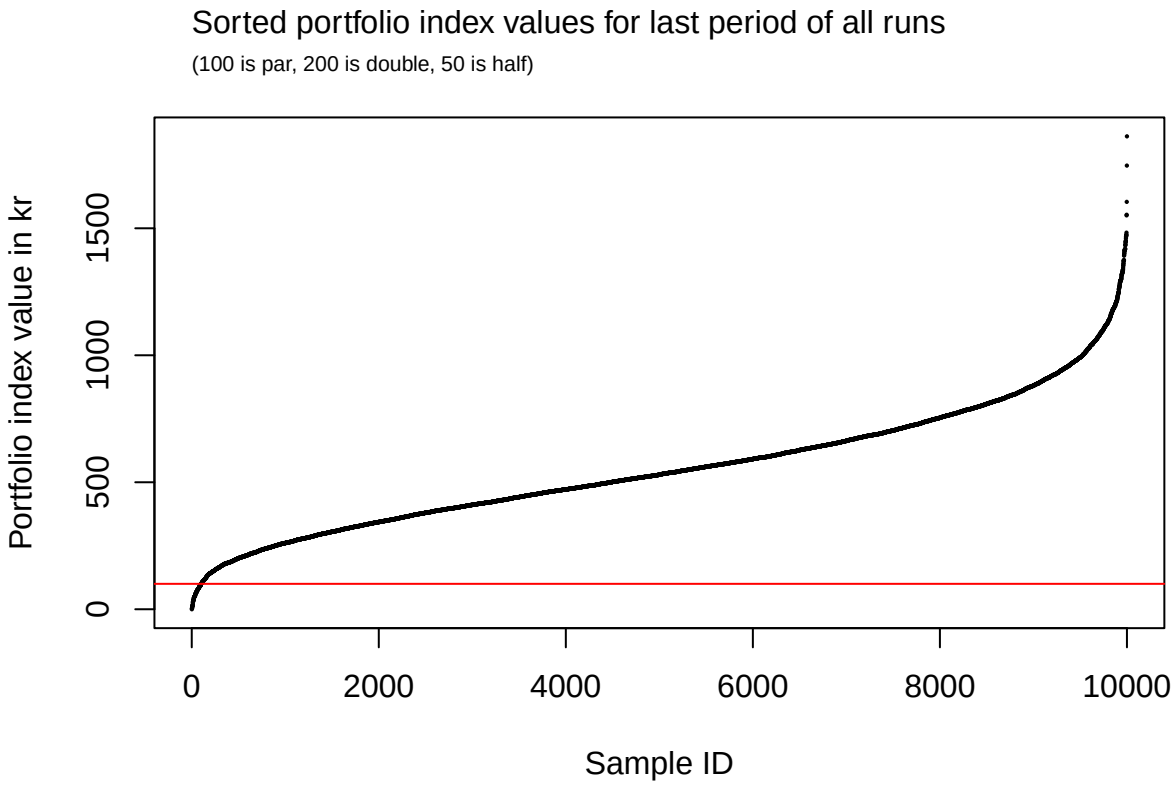


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 555.228 kr.
## SD of portfolio index value after 20 years: 244.452 kr.
## Min total portfolio index value after 20 years: 0.004 kr.
## Max total portfolio index value after 20 years: 1861.959 kr.
##
## Share of paths finishing below 100: 1.01 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

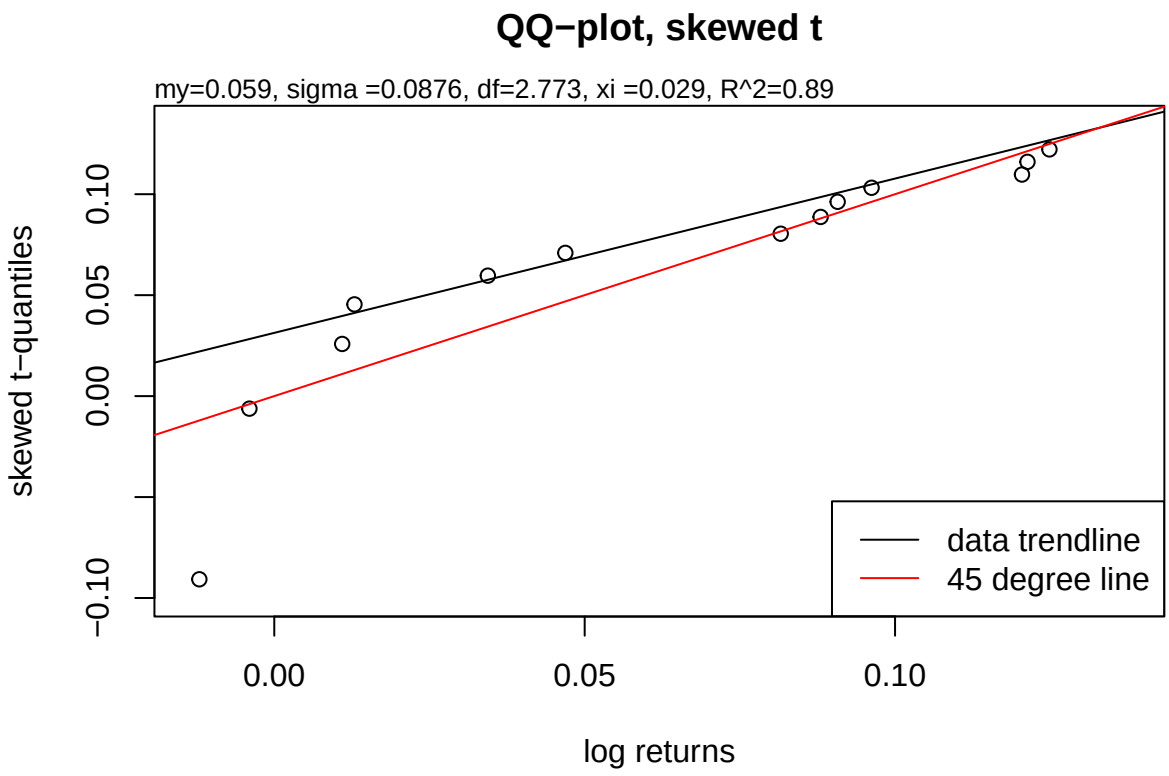


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -36.9603
## BIC: -34.7005
## m: 0.05902873
## s: 0.08757749
## nu (df): 2.772621
## xi: 0.02904471
## R^2: 0.89
##
## An R^2 of 0.89 suggests that the fit is not completely random.
##
## What is the risk of losing max 10 %? =< 3.3 percent
## What is the risk of losing max 25 %? =< 0.7 percent
## What is the risk of losing max 50 %? =< 0.1 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 35.6 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

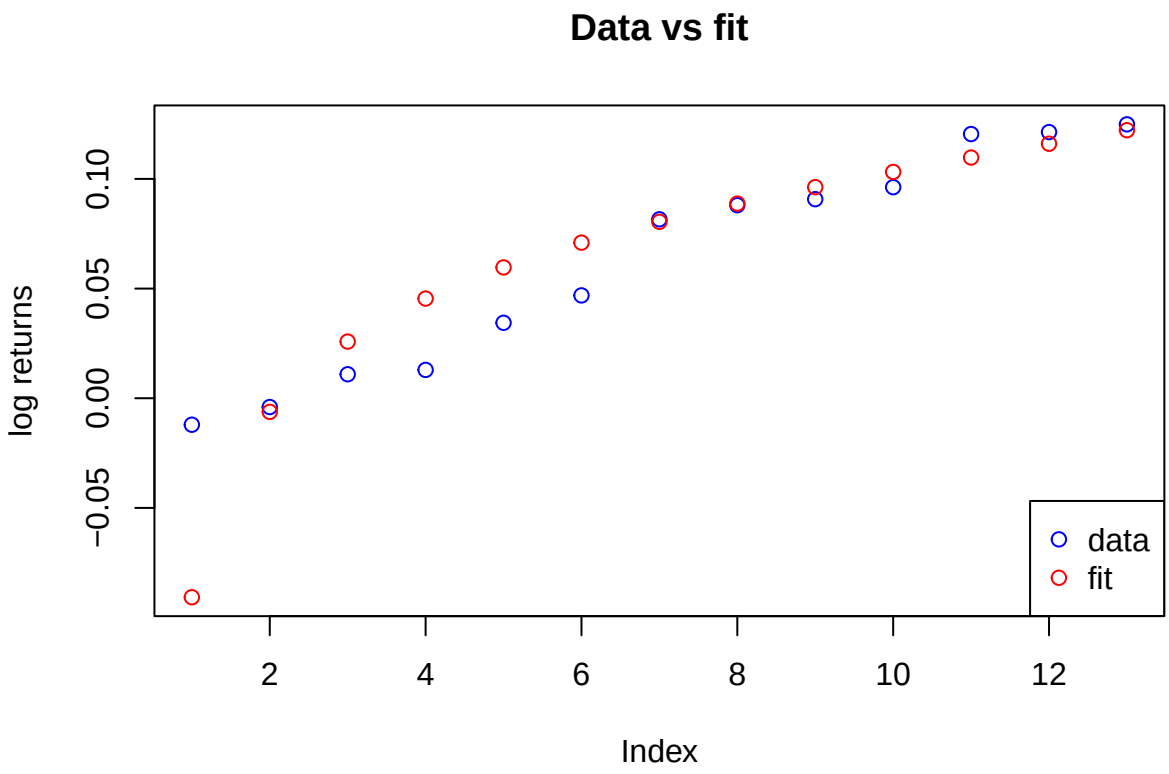
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

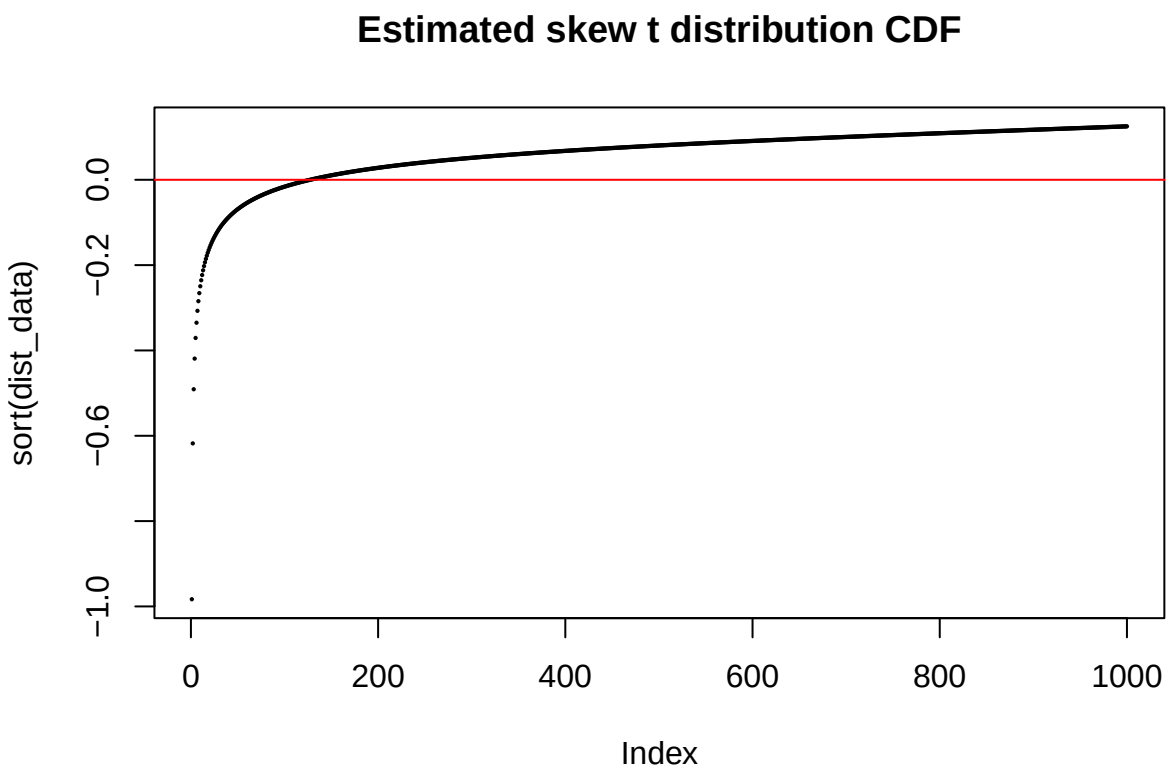
Let's plot the fit and the observed returns together.



Interestingly, the fit predicts a much bigger “biggest loss” than the actual data. This is the main reason that R^2 is 0.90 and not higher.

Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

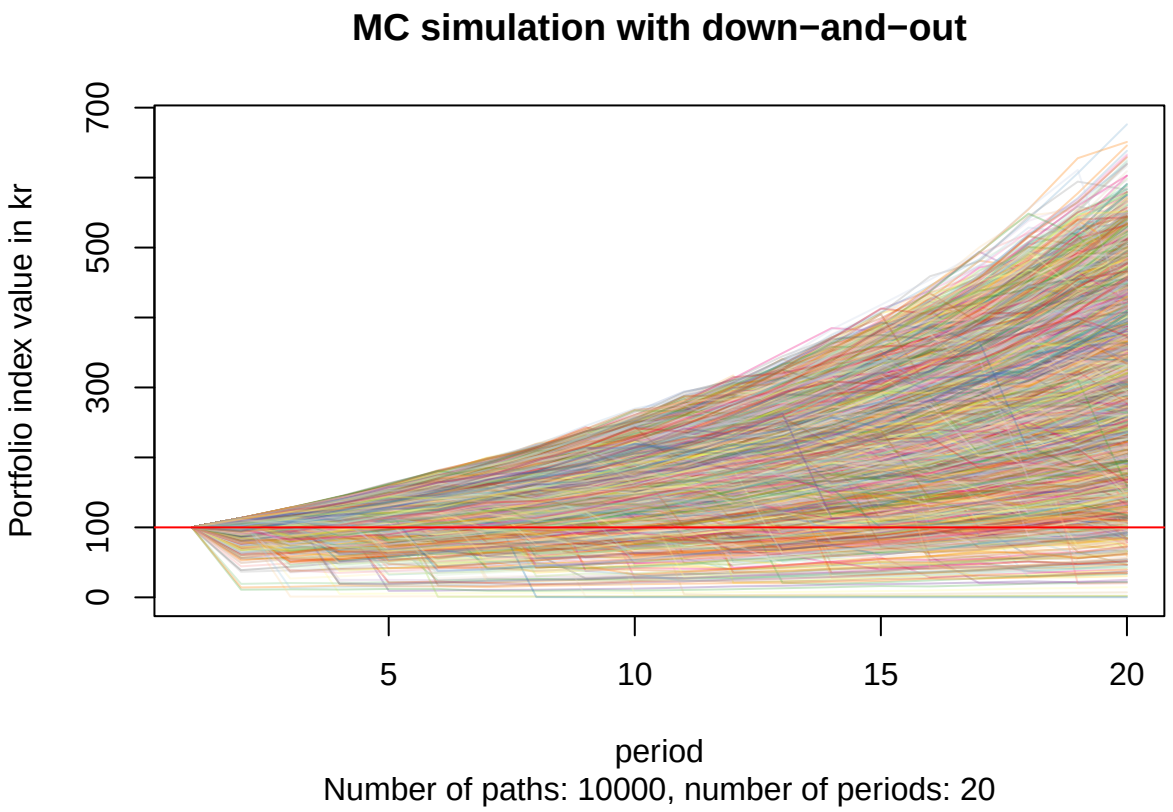


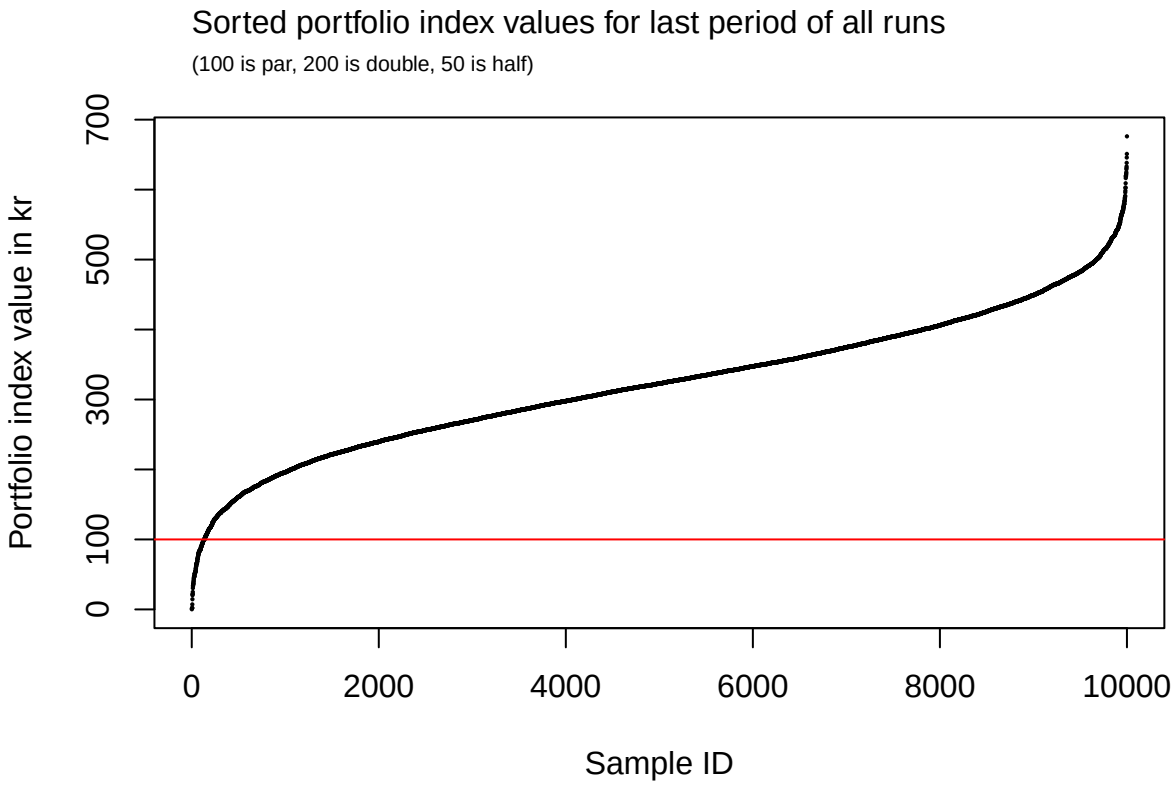
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

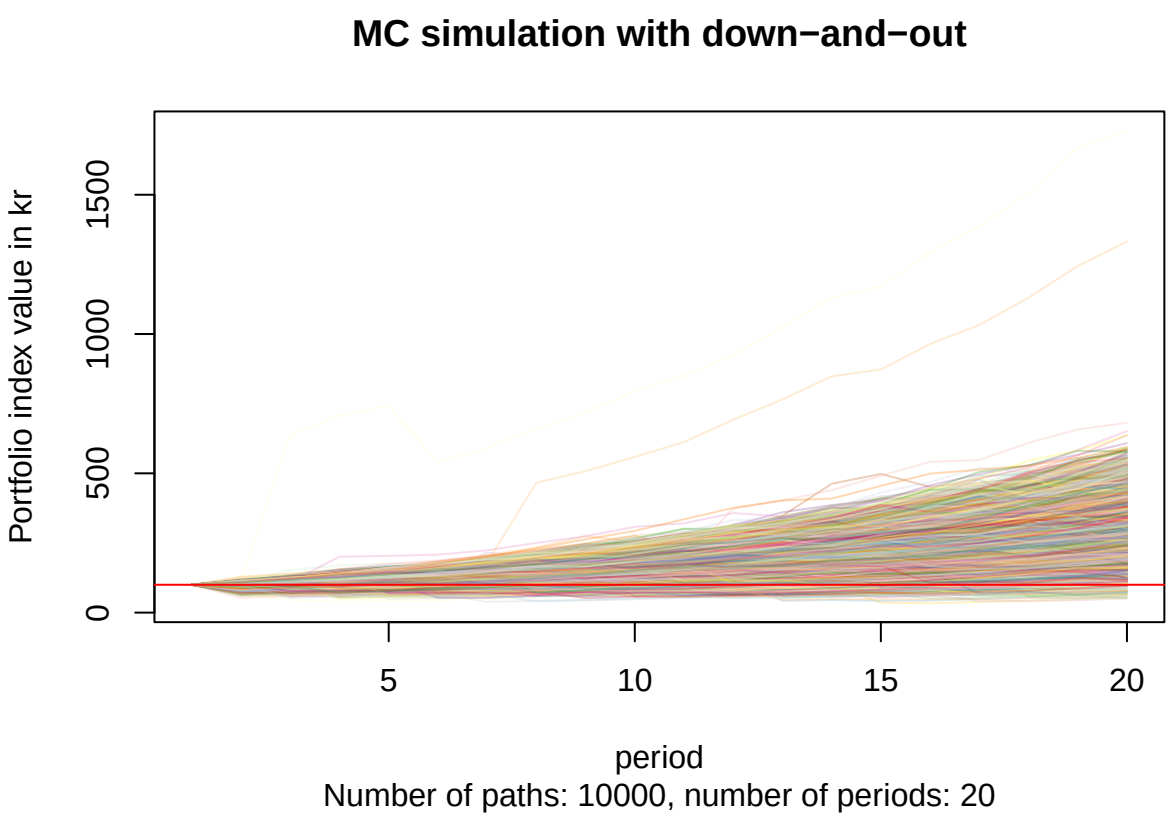
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 322.516 kr.
## SD of portfolio index value after 20 years: 98.127 kr.
## Min total portfolio index value after 20 years: 0.228 kr.
## Max total portfolio index value after 20 years: 676.128 kr.
##
## Share of paths finishing below 100: 1.35 percent
```

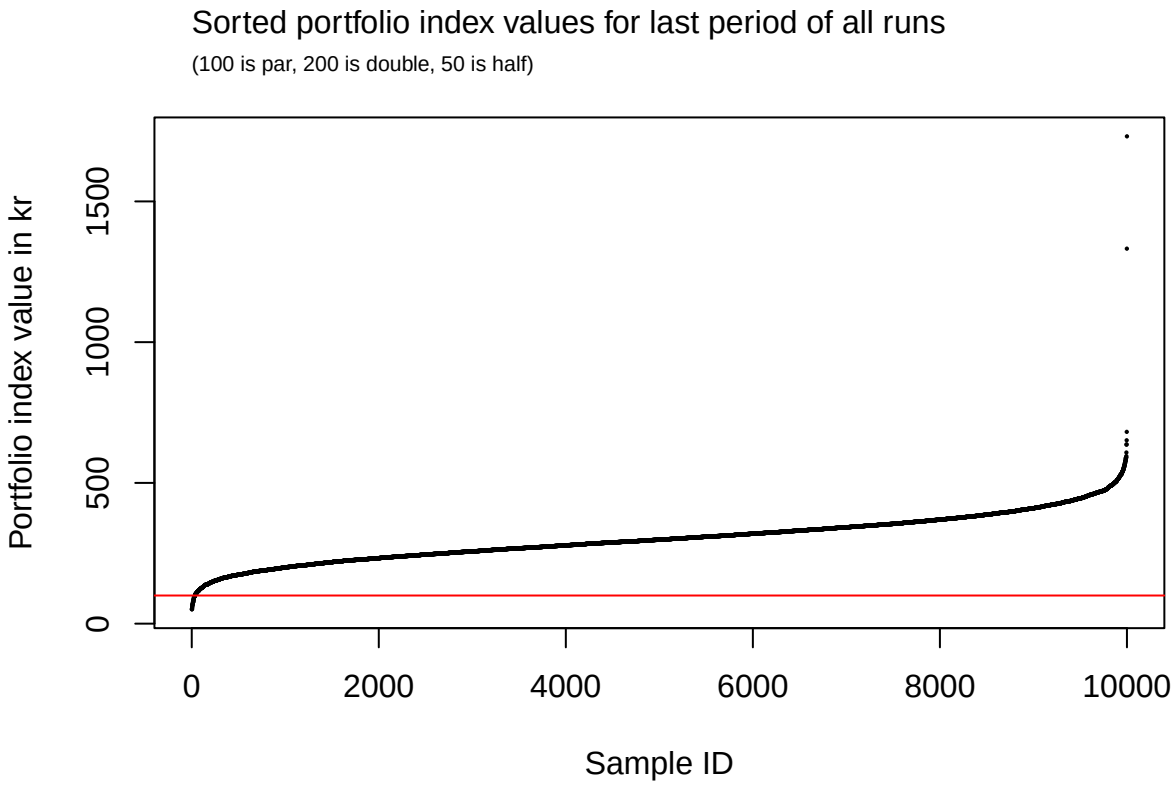

MC simulation with down−and−out
Portfolio index value in kr
0
100
300
500
700
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Version b: Mix of simulations from estimated distribution of returns from individual funds.

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 302.597 kr.
## SD of portfolio index value after 20 years: 84.175 kr.
## Min total portfolio index value after 20 years: 51.214 kr.
## Max total portfolio index value after 20 years: 1731.134 kr.
##
## Share of paths finishing below 100: 0.34 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

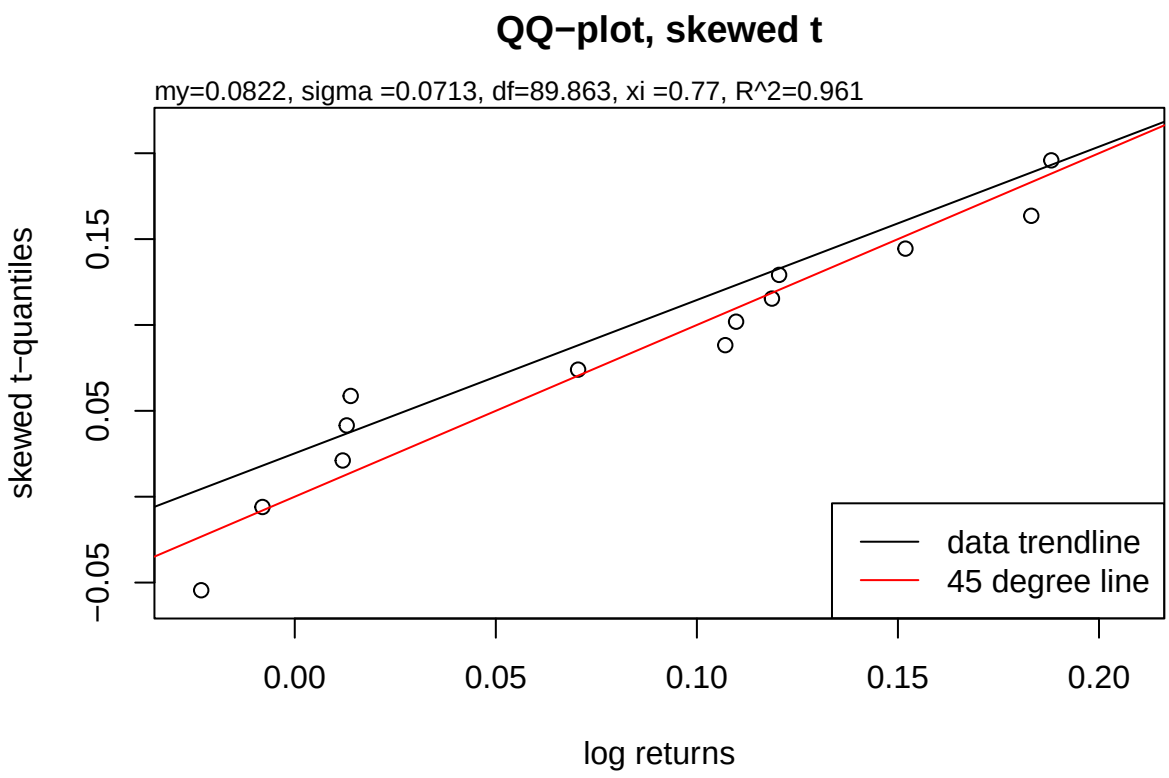


Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -24.26084
## BIC: -22.00104
## m: 0.0822419
## s: 0.07129843
## nu (df): 89.86289
## xi: 0.7697502
## R^2: 0.961
##
## An R^2 of 0.961 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 0.9 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 46.1 percent
## What is the chance of gaining min 25 %? >= 1.2 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

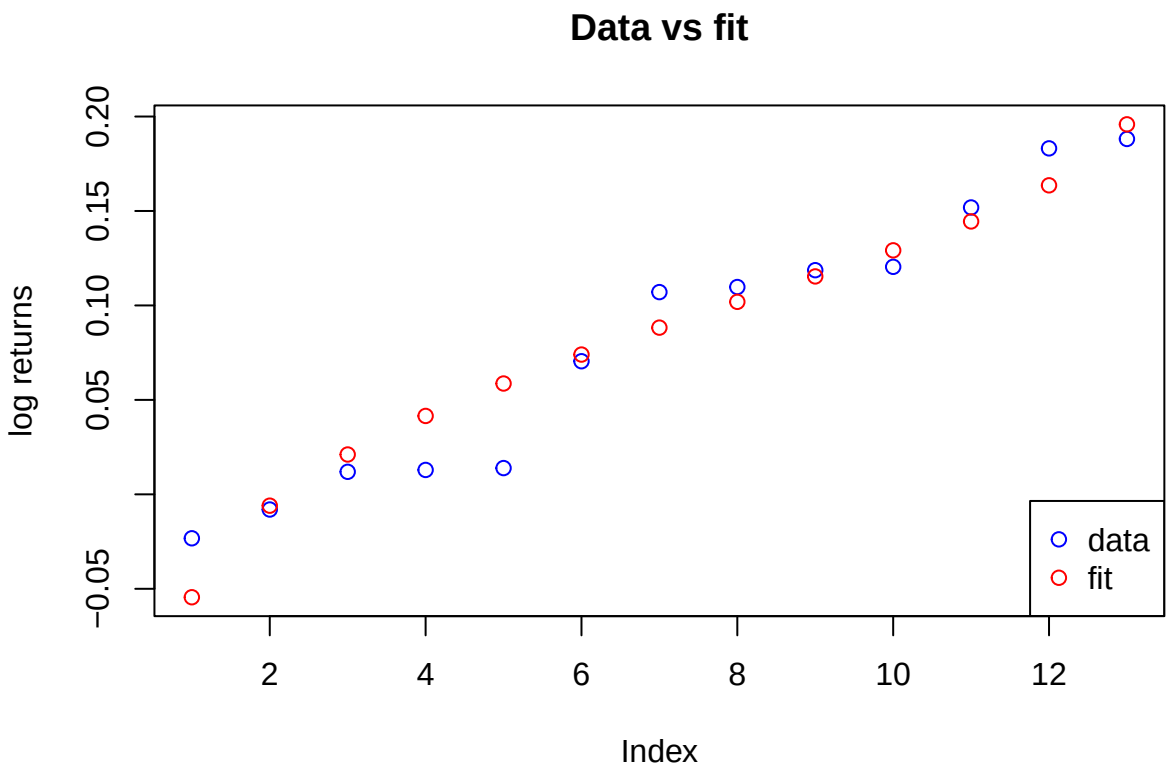
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

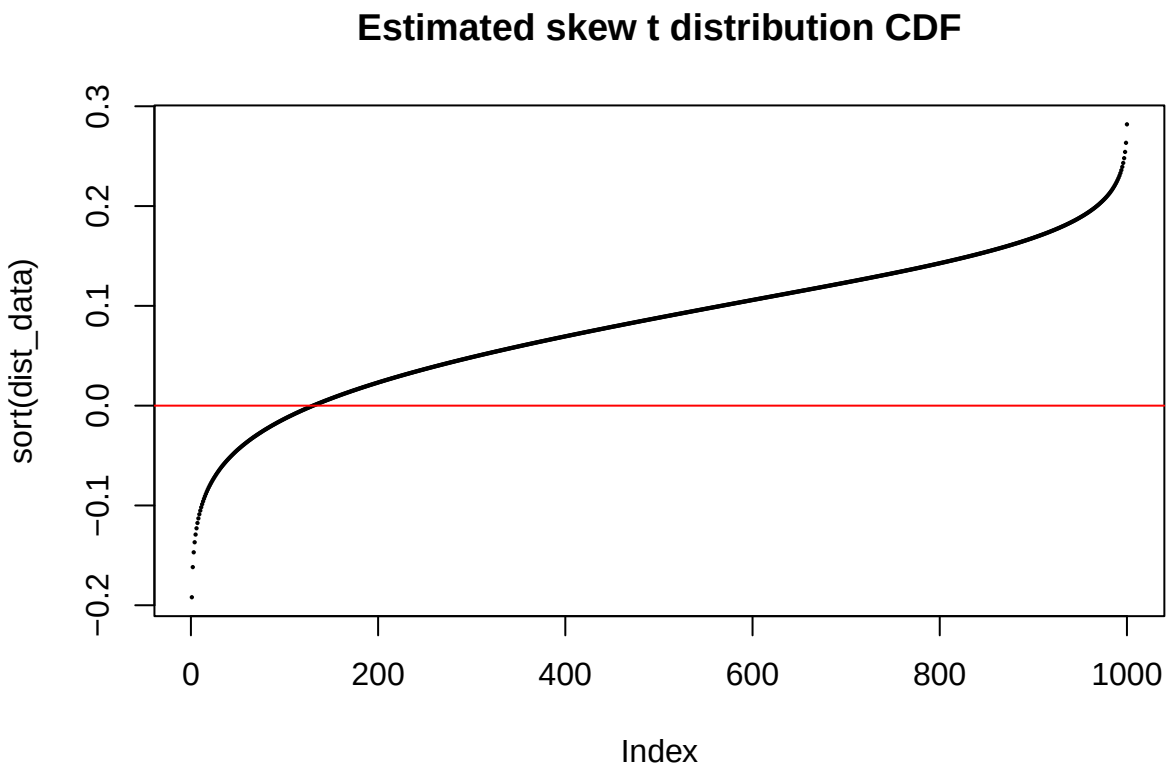
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

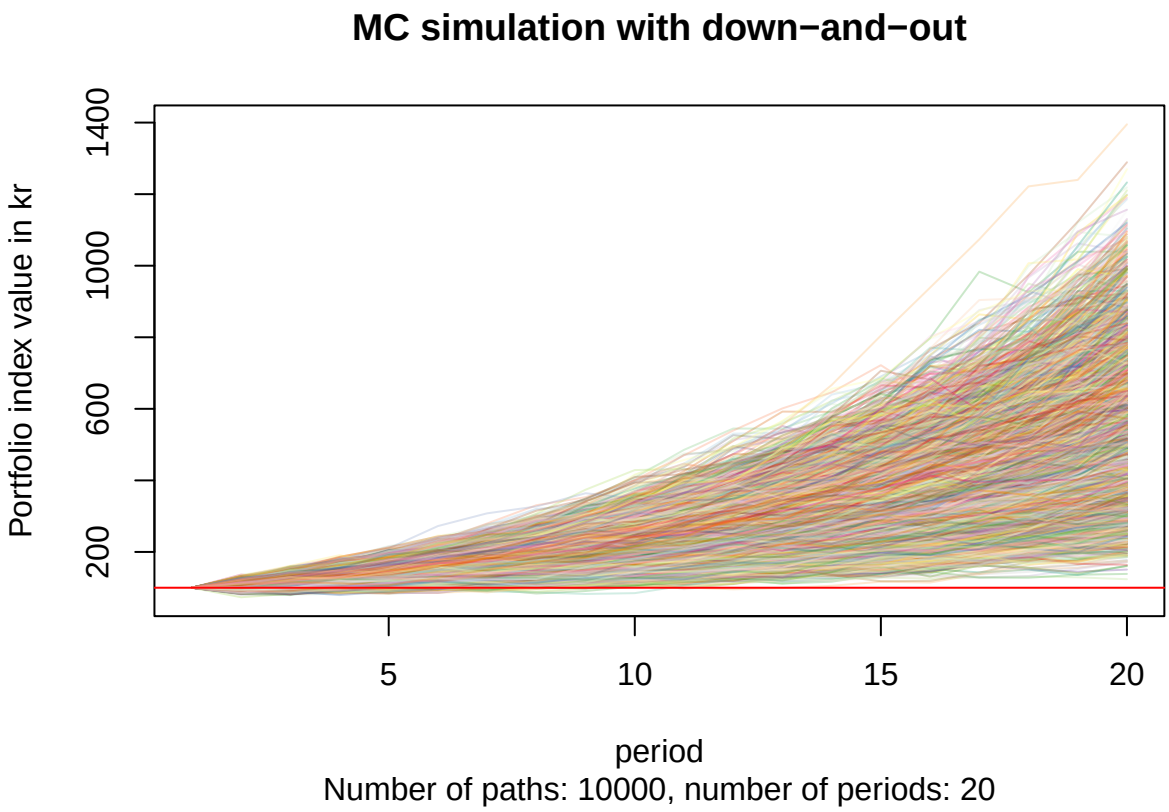


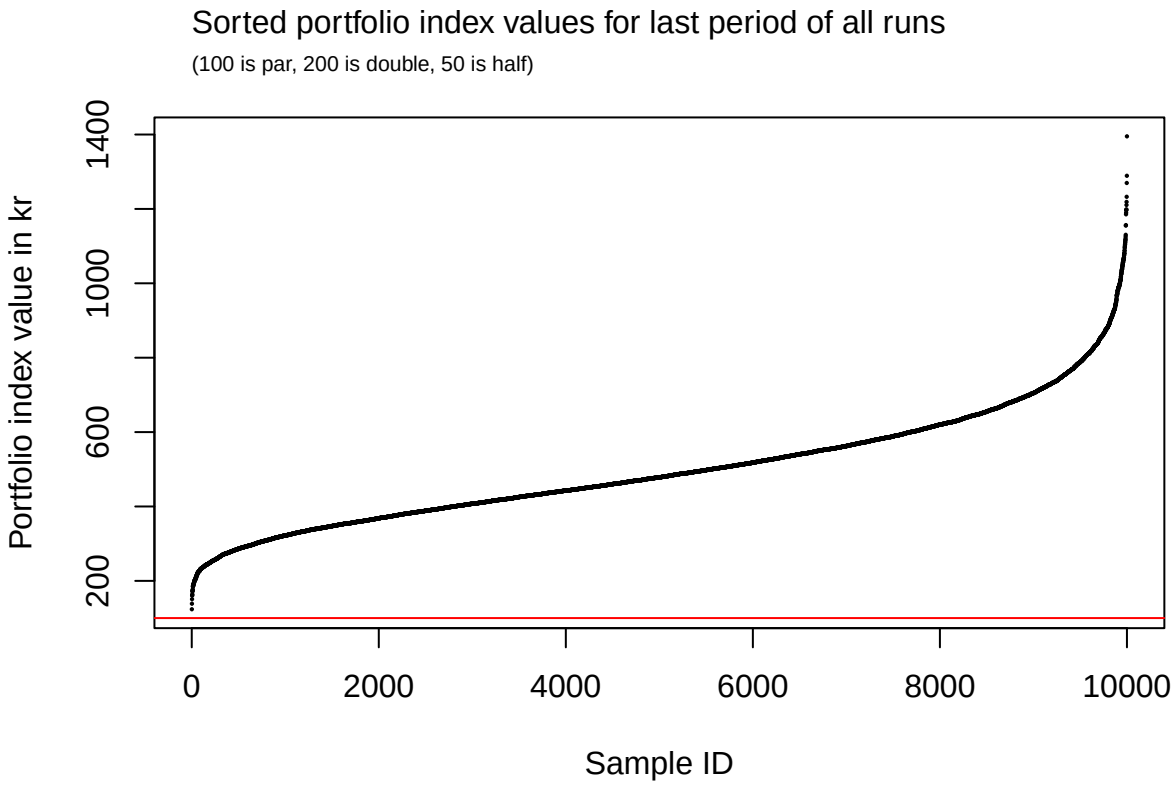
We see that the high risk mix provides a much better upside and smaller downside.

Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

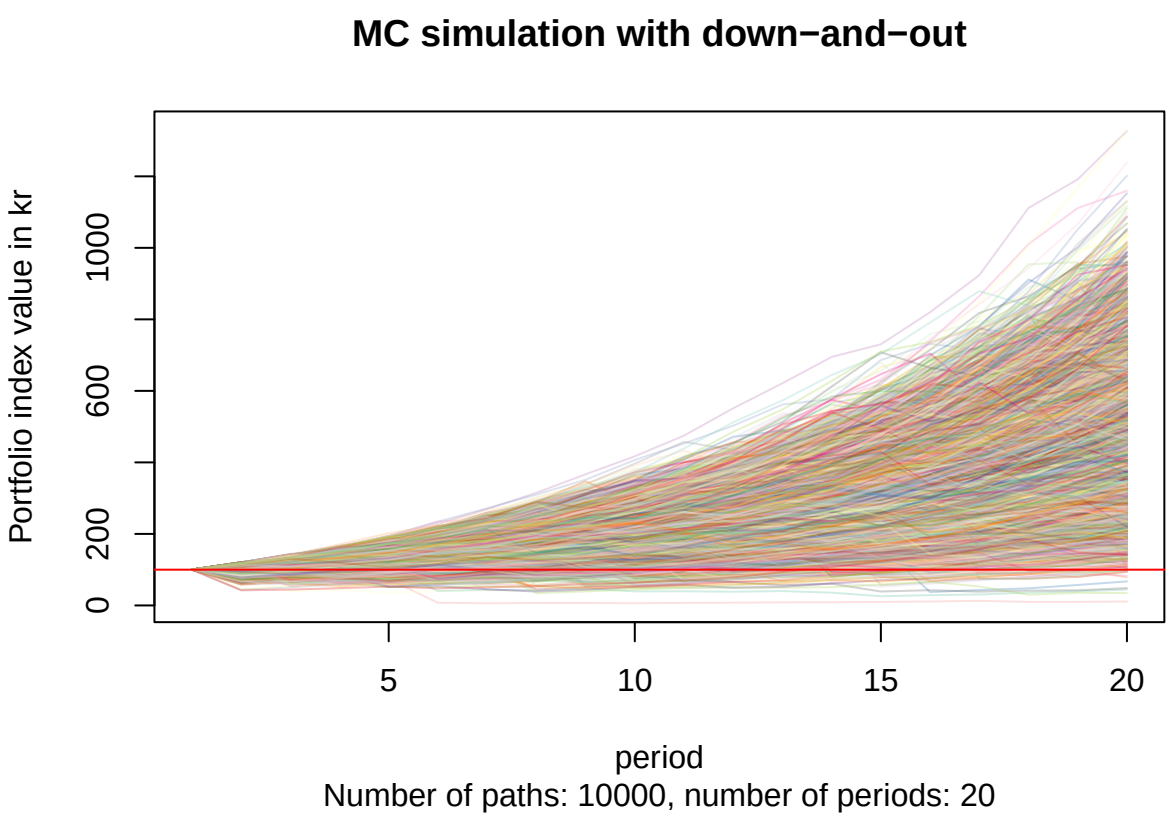
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 500.356 kr.
## SD of portfolio index value after 20 years: 155.754 kr.
## Min total portfolio index value after 20 years: 123.743 kr.
## Max total portfolio index value after 20 years: 1395.09 kr.
##
## Share of paths finishing below 100: 0 percent
```

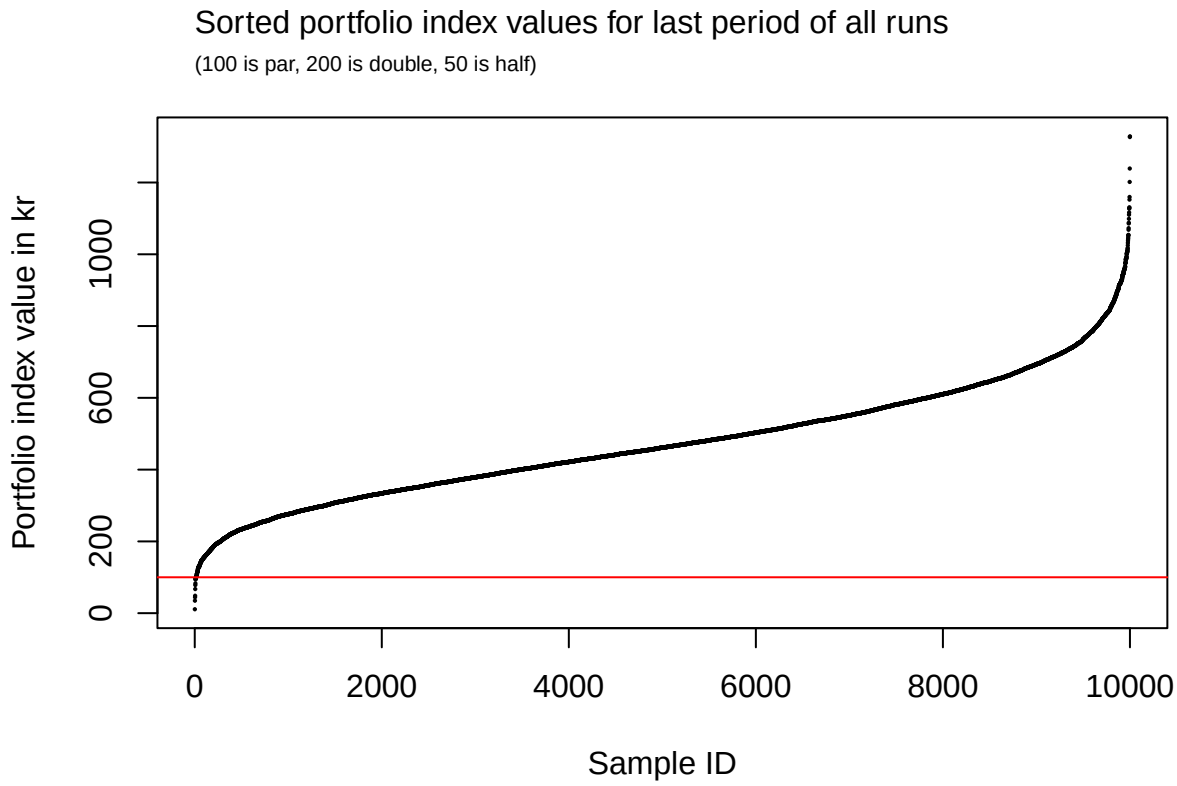

MC simulation with down−and−out
Portfolio index value in kr
200
600
1000
1400
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Version b: Mix of simulations from estimated distribution of returns from individual funds.

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 475.469 kr.
## SD of portfolio index value after 20 years: 164.292 kr.
## Min total portfolio index value after 20 years: 10.963 kr.
## Max total portfolio index value after 20 years: 1328.593 kr.
##
## Share of paths finishing below 100: 0.13 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
600
1000
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Compare pension plans

Risk of max loss of x percent for a single period (year).
x values are row names.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
0	21.3	18.2	19.9	12.2	14.3	12.7	13.0
5	12.5	9.6	12.8	6.0	8.6	6.2	4.2
10	7.4	5.4	8.3	3.3	5.3	3.3	0.9
25	1.8	1.3	2.5	0.9	1.4	0.7	0.0
50	0.2	0.2	0.4	0.2	0.2	0.1	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
0	78.7	81.8	80.1	87.8	85.7	87.3	87.0
5	63.8	64.9	69.2	71.5	75.8	71.4	69.9
10	41.0	36.2	53.3	32.7	59.6	35.6	46.1
25	0.0	0.3	0.0	0.1	0.0	0.0	1.2
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0

MC risk percentiles: Risk of loss from first to last period.
_a is simulation from estimated distribution of returns of mix.
_b is mix of simulations from estimated distribution of returns from individual funds.

_m is medium.
_h is high.

	Velliv_m	Velliv_m_long	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	5.15	3.24	3.62	1.83	1.01	1.35	0	0.34	0.13
5	4.44	2.82	3.24	1.61	0.96	1.16	0	0.26	0.08
10	3.81	2.41	2.92	1.42	0.87	1.02	0	0.22	0.07
25	2.42	1.53	2.13	1.06	0.59	0.66	0	0.12	0.05
50	0.92	0.76	0.99	0.59	0.29	0.30	0	0.00	0.04
90	0.04	0.13	0.14	0.10	0.04	0.06	0	0.00	0.00
99	0.01	0.07	0.04	0.03	0.01	0.01	0	0.00	0.00

MC gains percentiles: Chance of gains from first to last period.
_a is simulation from estimated distribution of returns of mix.
_b is mix of simulations from estimated distribution of returns from individual funds.

	Velliv_m	Velliv_m_long	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	94.85	96.76	96.38	98.17	98.99	98.65	100.00	99.66	99.87
5	94.24	96.26	96.01	98.02	98.90	98.51	100.00	99.59	99.80
10	93.68	95.67	95.53	97.81	98.83	98.32	100.00	99.48	99.76
25	90.86	93.80	93.90	97.05	98.48	97.73	99.99	99.02	99.64
50	85.56	89.76	90.94	95.30	97.69	95.92	99.98	97.70	99.18
100	70.82	78.53	83.10	89.06	95.06	89.40	99.72	90.01	97.27
200	38.60	44.68	64.23	59.02	85.60	59.05	93.21	49.11	86.04
300	16.35	17.87	45.67	22.65	71.67	21.84	72.00	11.94	65.20
400	5.36	5.06	29.58	4.37	55.14	3.26	44.44	1.36	40.58
500	1.33	0.99	18.10	0.41	38.46	0.17	23.15	0.07	21.67
1000	0.00	0.00	0.67	0.01	2.56	0.00	0.23	0.02	0.11

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.
m is the location parameter.
s is the scale parameter.
nu is the estimated degrees of freedom, or shape parameter.
xi is the estimated skewness parameter.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770
R-squared	0.993	0.978	0.991	0.991	0.964	0.890	0.961

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).
Statistics are given for the final state of all paths.
Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.
0 is defined as any value below a threshold.
losing_prob_pct is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

	Velliv_m	Velliv_m_long	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_m_b	mix_h_a	mix_h_b
mc_m	278.171	293.566	407.755	324.140	555.228	322.516	302.597	500.356	475.469
mc_s	125.008	117.469	217.848	103.447	244.452	98.127	84.175	155.754	164.292
mc_min	0.552	0.000	0.000	0.199	0.004	0.228	51.214	123.743	10.963
mc_max	836.325	1039.413	1554.709	1547.412	1861.959	676.128	1731.134	1395.090	1328.593
dao_prob_pct	0.000	0.020	0.010	0.000	0.000	0.000	0.000	0.000	0.000
losing_prob_pct	5.150	3.240	3.620	1.830	1.010	1.350	0.340	0.000	0.130

Ranking:

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_prob_pct	ranking	losing_prob_pct	ranking
555.228	PFA_h	84.175	mix_m_b	123.743	mix_h_a	1861.959	PFA_h	0.00	Velliv_m	0.00	mix_h_a
500.356	mix_h_a	98.127	mix_m_a	51.214	mix_m_b	1731.134	mix_m_b	0.00	PFA_m	0.13	mix_h_b
475.469	mix_h_b	103.447	PFA_m	10.963	mix_h_b	1554.709	Velliv_h	0.00	PFA_h	0.34	mix_m_b
407.755	Velliv_h	117.469	Velliv_m_long	0.552	Velliv_m	1547.412	PFA_m	0.00	mix_m_a	1.01	PFA_h
324.140	PFA_m	125.008	Velliv_m	0.228	mix_m_a	1395.090	mix_h_a	0.00	mix_m_b	1.35	mix_m_a
322.516	mix_m_a	155.754	mix_h_a	0.199	PFA_m	1328.593	mix_h_b	0.00	mix_h_a	1.83	PFA_m
302.597	mix_m_b	164.292	mix_h_b	0.004	PFA_h	1039.413	Velliv_m_long	0.00	mix_h_b	3.24	Velliv_m_long
293.566	Velliv_m_long	217.848	Velliv_h	0.000	Velliv_m_long	836.325	Velliv_m	0.01	Velliv_h	3.62	Velliv_h
278.171	Velliv_m	244.452	PFA_h	0.000	Velliv_h	676.128	mix_m_a	0.02	Velliv_m_long	5.15	Velliv_m

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2}\right) \Big/ \left(\frac{x_{t-1} + y_{t-1}}{2}\right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns $=$ Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2\frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

c{ x_t + y_t }{ x_{t-1} + y_{t-1}}

(x_{t-1} + y_{t-1}) x_t y_{t-1} + (x_{t-1} + y_{t-1}) x_{t-1} y_t = 2 (x_{t-1}y_{t-1}x_t + x_{t-1}y_{t-1}y_t)$$

$$(x_{t-1}x_1y_{t-1} + y_{t-1}x_ty_{t-1}) + (x_{t-1}x_{t-1}y_t + x_{t-1}y_{t-1}y_t) = 2(x_{t-1}y_{t-1}x_t + x_{t-1}y_{t-1}y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: `R = 1+r`

```
## Let x_0 be 100.

## Let y_0 be 200.

## So the initial value of the pf is 300 .
```

```
## Let R_x be 0.5.

## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.

## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): -0.06458577
## s(data_x): 0.3424259
## m(data_y): 9.768225
## s(data_y): 3.067499
##
## m(data_x + data_y): 4.85182
## s(data_x + data_y): 1.528284
```

m and s of final state of all paths.
_a is mix of simulated returns.
_b is simulated mixed returns.

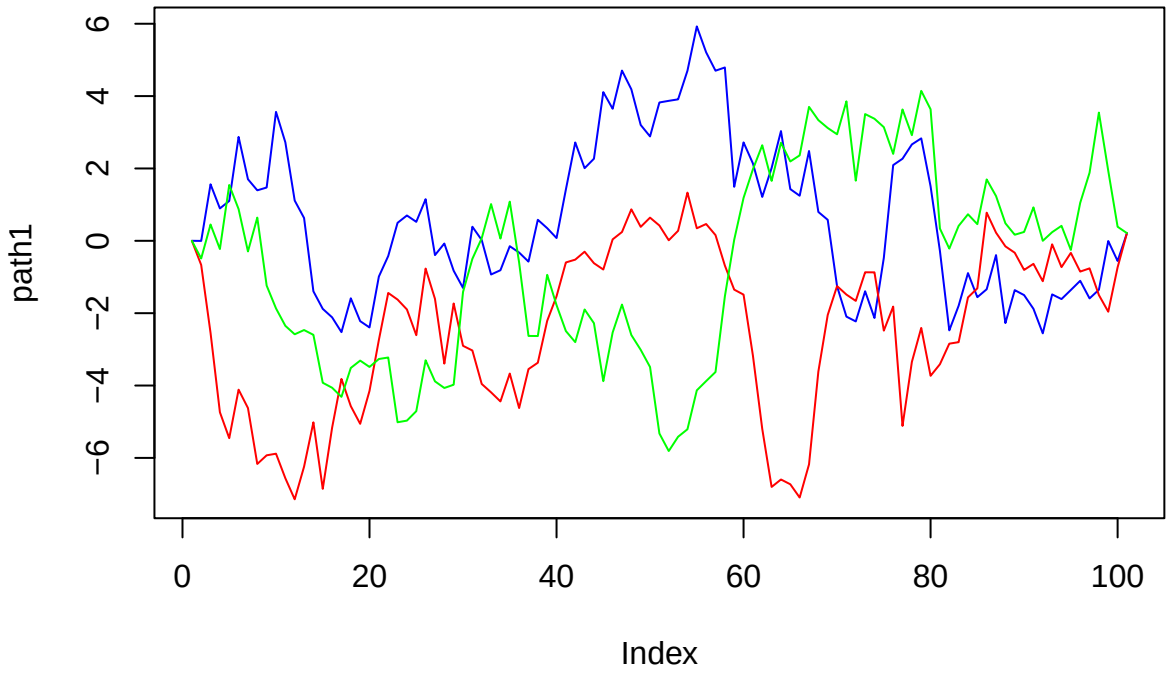
m_a	m_b	s_a	s_b
96.886	97.513	6.593	6.508
97.317	97.174	7.016	6.995
96.784	96.922	6.919	6.839
97.225	96.921	6.757	6.686
97.242	96.820	6.794	6.867
97.027	96.829	6.990	6.758
96.926	96.547	6.745	6.911

m_a	m_b	s_a	s_b
96.935	97.356	6.690	7.075
96.809	96.915	6.757	7.090
96.707	97.022	7.009	6.942

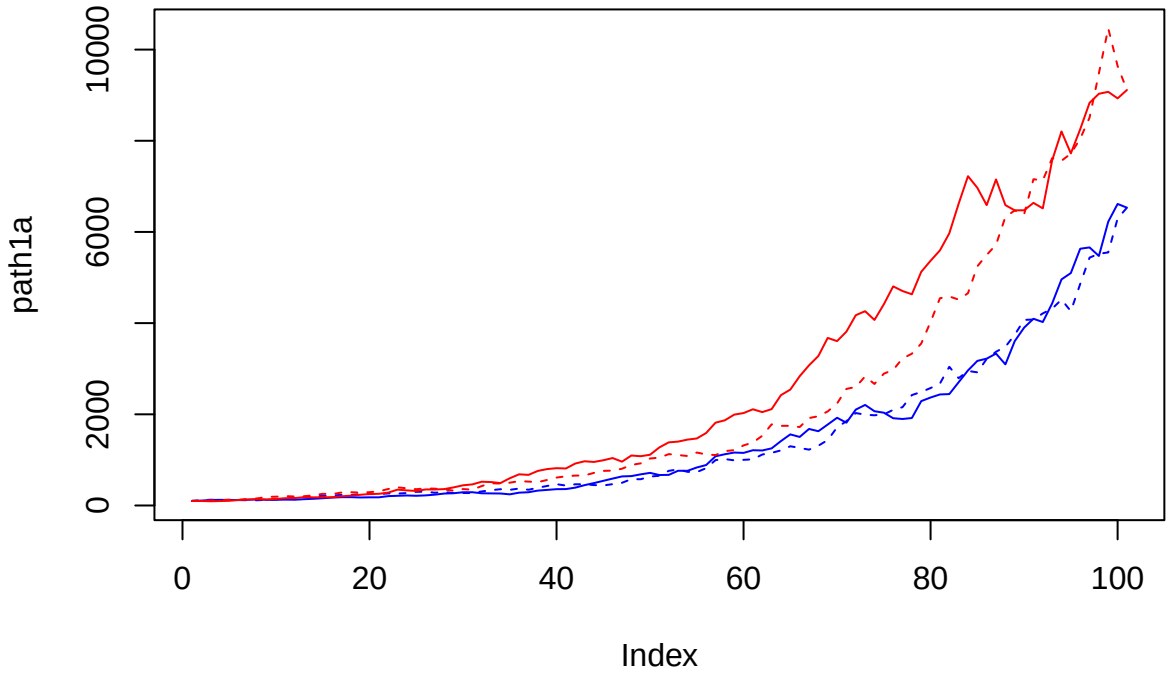
```
##       m_a              m_b              s_a              s_b        
##  Min.   :96.71   Min.   :96.55   Min.   :6.593   Min.   :6.508  
##  1st Qu.:96.83   1st Qu.:96.85   1st Qu.:6.748   1st Qu.:6.778  
##  Median :96.93   Median :96.92   Median :6.776   Median :6.889  
##  Mean   :96.99   Mean   :97.00   Mean   :6.827   Mean   :6.867  
##  3rd Qu.:97.18   3rd Qu.:97.14   3rd Qu.:6.972   3rd Qu.:6.982  
##  Max.   :97.32   Max.   :97.51   Max.   :7.016   Max.   :7.090
```

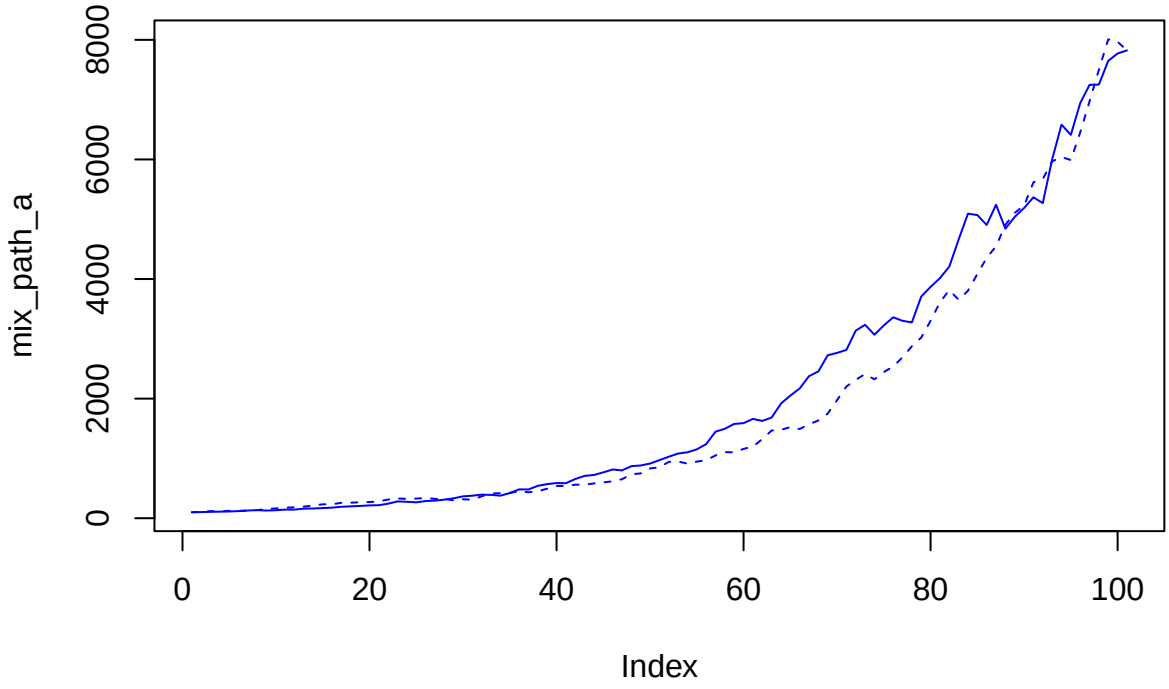
_a and _b are very close to equal.
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?


path1a
0
2000
6000
10000
0
20
40
60
80
100
Index



The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?
Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##        m                s
##  Min.   :0.05965   Min.   :0.04799
##  1st Qu.:0.06594   1st Qu.:0.06116
##  Median :0.06948   Median :0.06663
##  Mean   :0.07093   Mean   :0.06737
##  3rd Qu.:0.07576   3rd Qu.:0.07114
##  Max.   :0.08770   Max.   :0.09486
```