

Pension returns analysis

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Fit log returns to F-S skew standardized Student-t distribution.

\bar{m} is the location parameter.

s is the scale parameter.

ν is the estimated shape parameter (degrees of freedom).

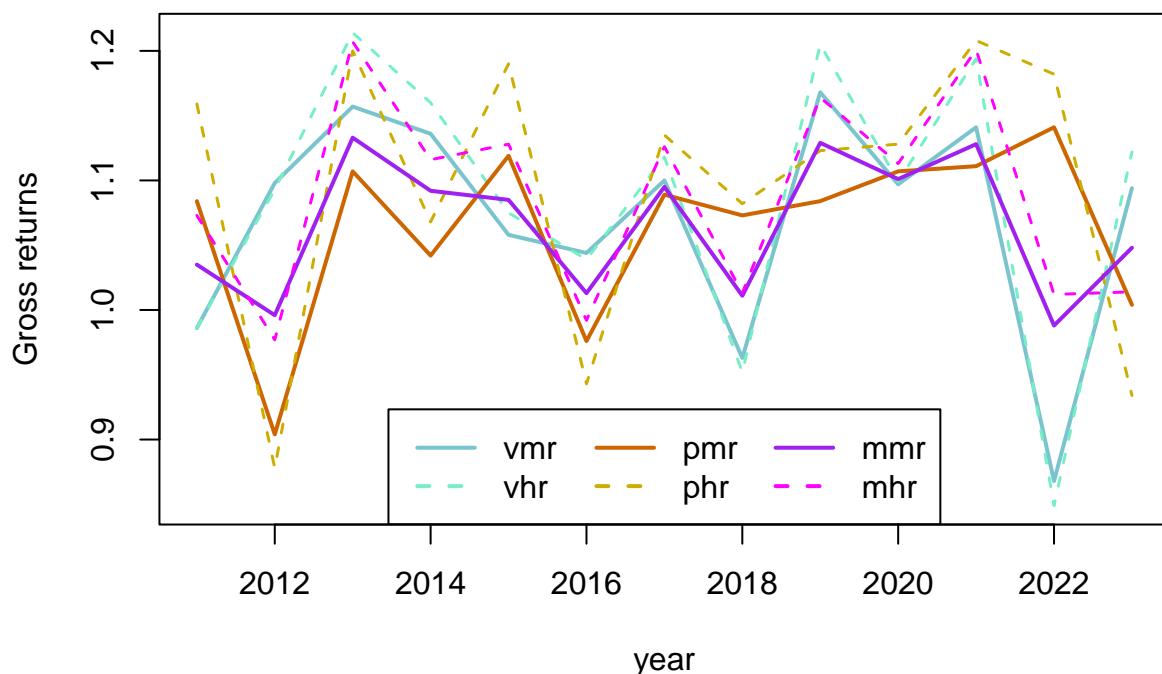
ξ is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.

`vmr1` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.

Gross returns 2011–2023



Summary of gross returns

```

##      vmr      pmr      mmr      vhr
## Min. :0.868  Min. :0.904  Min. :0.988  Min. :0.849
## 1st Qu.:1.044 1st Qu.:1.042 1st Qu.:1.013 1st Qu.:1.039
## Median :1.097 Median :1.084 Median :1.085 Median :1.099
## Mean   :1.070 Mean   :1.065 Mean   :1.066 Mean   :1.085
## 3rd Qu.:1.136 3rd Qu.:1.107 3rd Qu.:1.101 3rd Qu.:1.160
## Max.   :1.168 Max.   :1.141 Max.   :1.133 Max.   :1.214
##      phr      mhr
## Min. :0.878  Min. :0.977
## 1st Qu.:1.068 1st Qu.:1.013
## Median :1.128 Median :1.113
## Mean   :1.095 Mean   :1.087
## 3rd Qu.:1.182 3rd Qu.:1.128
## Max.   :1.208 Max.   :1.207

##      vmrl
## Min. :0.801
## 1st Qu.:1.013
## Median :1.085
## Mean   :1.061
## 3rd Qu.:1.128
## Max.   :1.193

##      vmr pmr mmr vhr phr mhr
## Min. : 0.868 0.904 0.988 0.849 0.878 0.977
## 1st Qu.: 1.044 1.042 1.013 1.039 1.068 1.013
## Median : 1.097 1.084 1.085 1.099 1.128 1.113
## Mean   : 1.070 1.065 1.066 1.085 1.095 1.087
## 3rd Qu.: 1.136 1.107 1.101 1.160 1.182 1.128
## Max.   : 1.168 1.141 1.133 1.214 1.208 1.207

```

Ranking

Min.:	ranking	1st Qu.:	ranking	Median:	ranking	Mean:	ranking	3rd Qu.:	ranking	Max.:	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.136	vmr	1.168	vmr
0.977	mhr	1.044	vmr	1.113	mhr	1.087	mhr	1.107	pmr	1.141	pmr
0.904	pmr	1.042	pmr	1.099	vhr	1.085	vhr	1.101	mmr	1.133	mmr
0.878	phr	1.039	vhr	1.097	vmr	1.070	vmr	1.160	vhr	1.214	vhr
0.868	vmr	1.013	mmr	1.085	mmr	1.066	mmr	1.182	phr	1.208	phr
0.849	vhr	1.013	mhr	1.084	pmr	1.065	pmr	1.128	mhr	1.207	mhr

Covariance

```

## cov(vmr, pmr) = -0.001094875

## cov(vhr, phr) = -0.0001730651

```

Velliv medium risk, 2011 - 2023

```

##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 7.4 percent

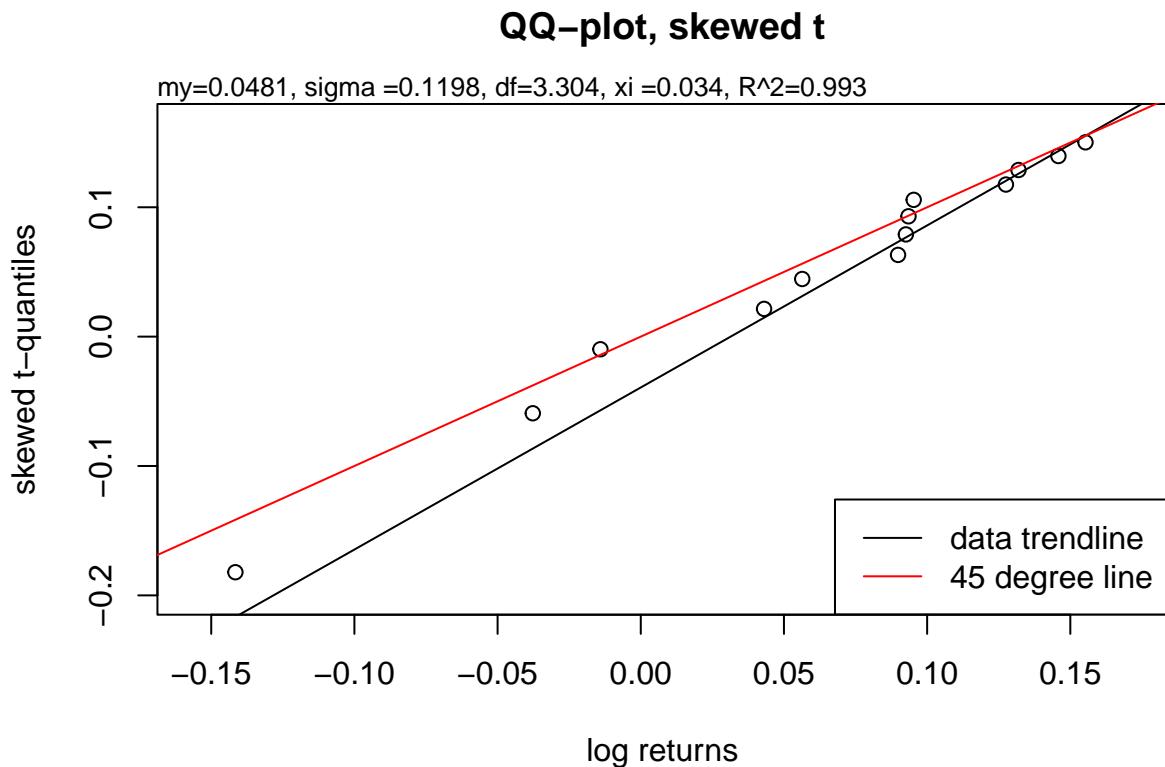
```

```

## What is the risk of losing max 25 %? =< 1.8 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 41 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent

```

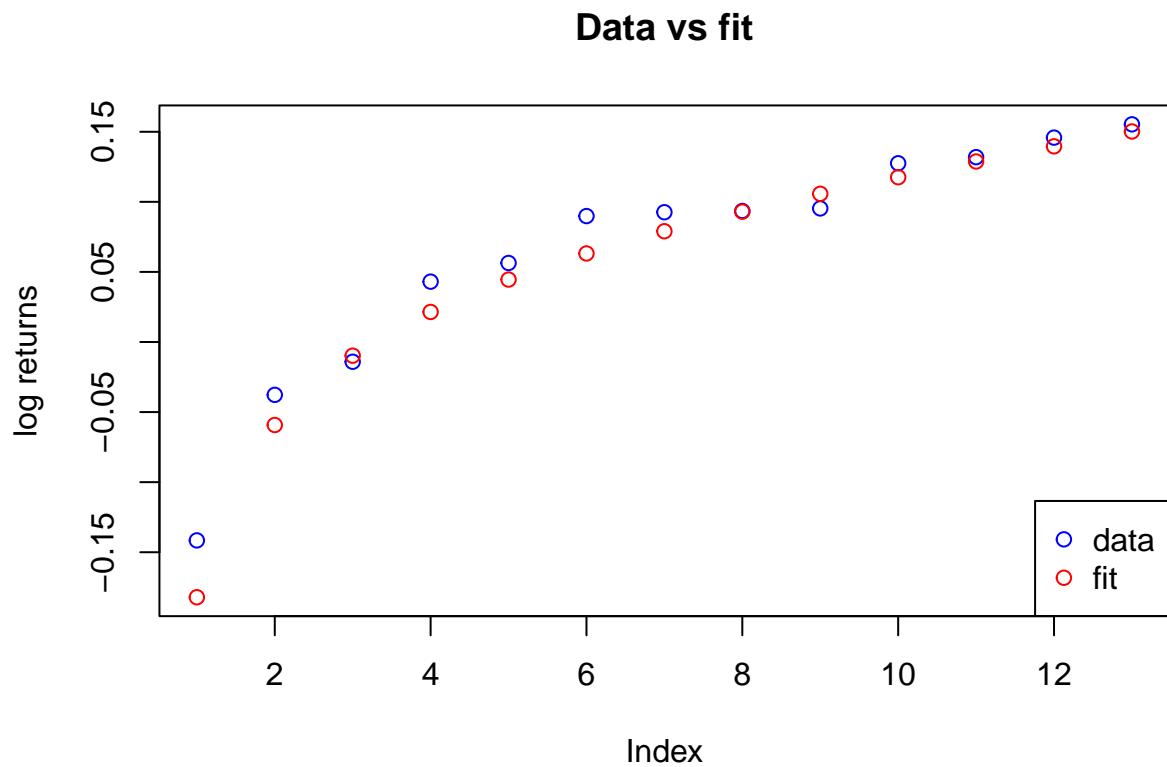
QQ Plot



The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

Data vs fit

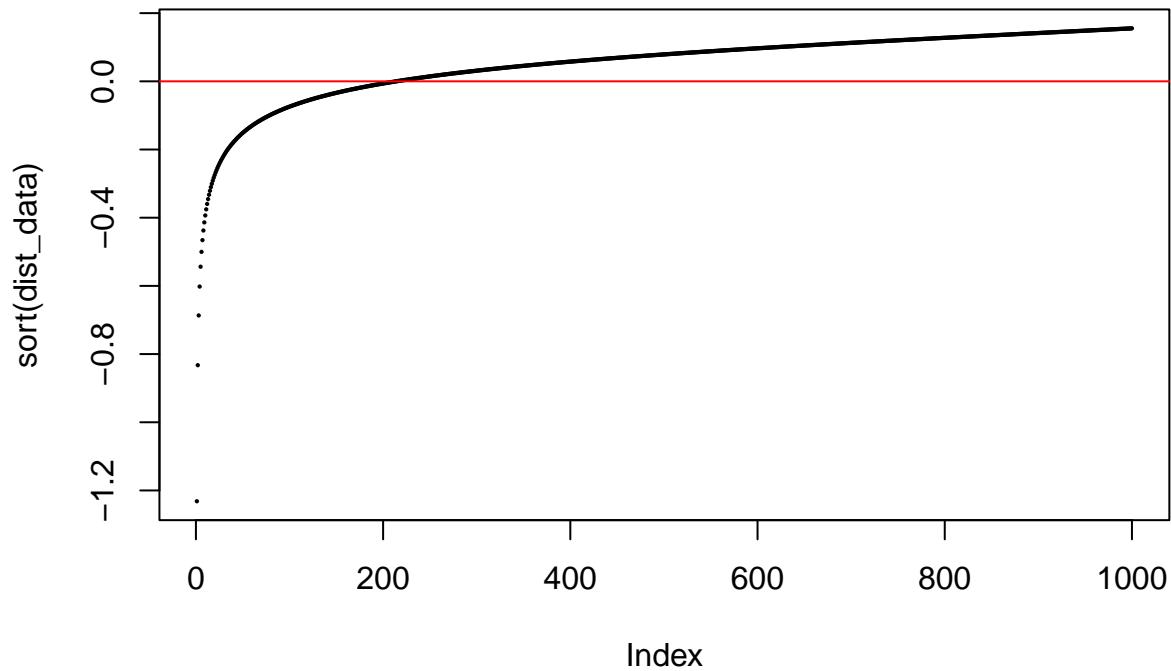
Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

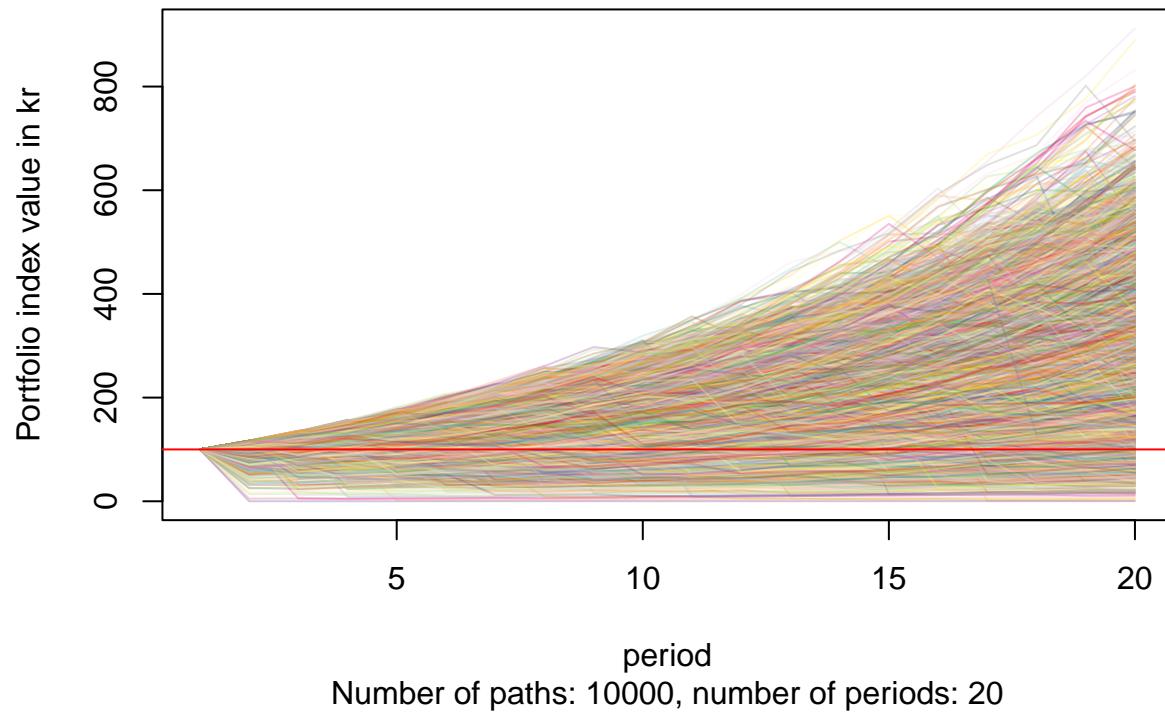


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

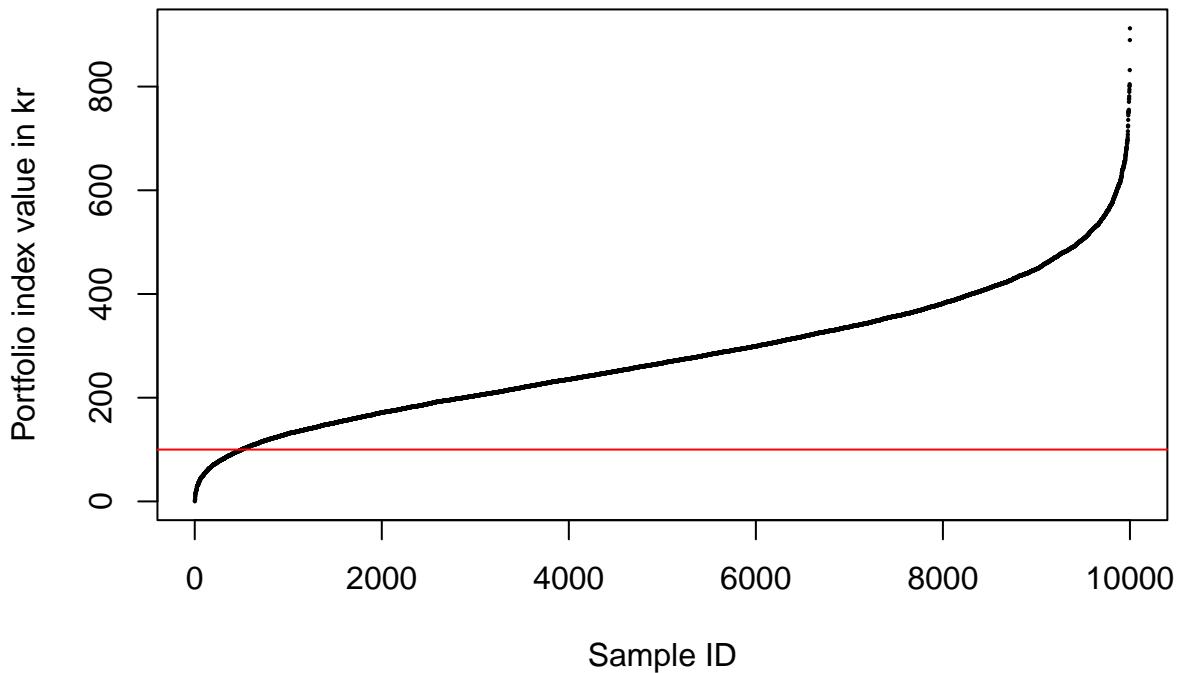
Monte Carlo

```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 280.176 kr.  
## SD of portfolio index value after 20 years: 125.216 kr.  
## Min total portfolio index value after 20 years: 0.344 kr.  
## Max total portfolio index value after 20 years: 912.296 kr.  
##  
## Share of paths finishing below 100: 5.05 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs
 (100 is par, 200 is double, 50 is half)



Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##  

## AIC: -34.35752  

## BIC: -31.02467  

## m: 0.05171176  

## s: 0.1149408  

## nu (df): 2.706099  

## xi: 0.5049945  

## R^2: 0.978  

##  

## An R^2 of 0.978 suggests that the fit is very good.  

##  

## What is the risk of losing max 10 %? =< 5.4 percent  

## What is the risk of losing max 25 %? =< 1.3 percent  

## What is the risk of losing max 50 %? =< 0.2 percent  

## What is the risk of losing max 90 %? =< 0 percent  

## What is the risk of losing max 99 %? =< 0 percent  

##  

## What is the chance of gaining min 10 %? >= 36.2 percent  

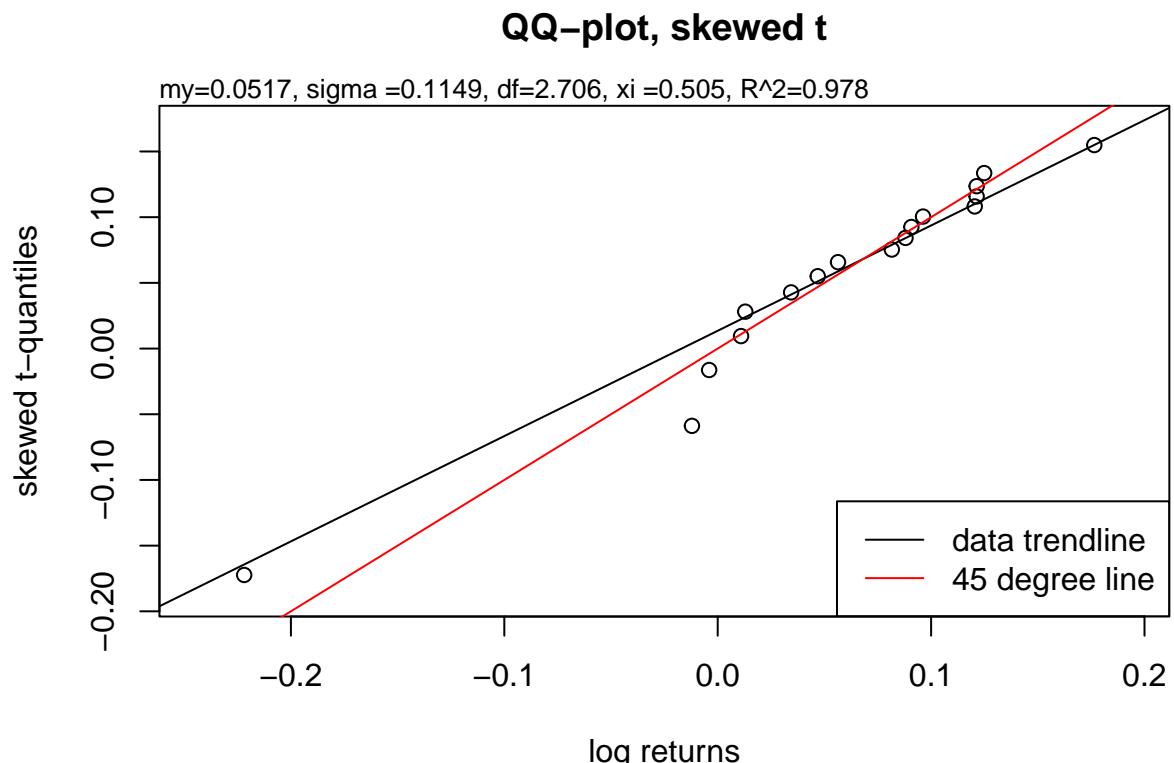
## What is the chance of gaining min 25 %? >= 0.3 percent  

## What is the chance of gaining min 50 %? >= 0 percent  

## What is the chance of gaining min 90 %? >= 0 percent  

## What is the chance of gaining min 99 %? >= 0 percent
```

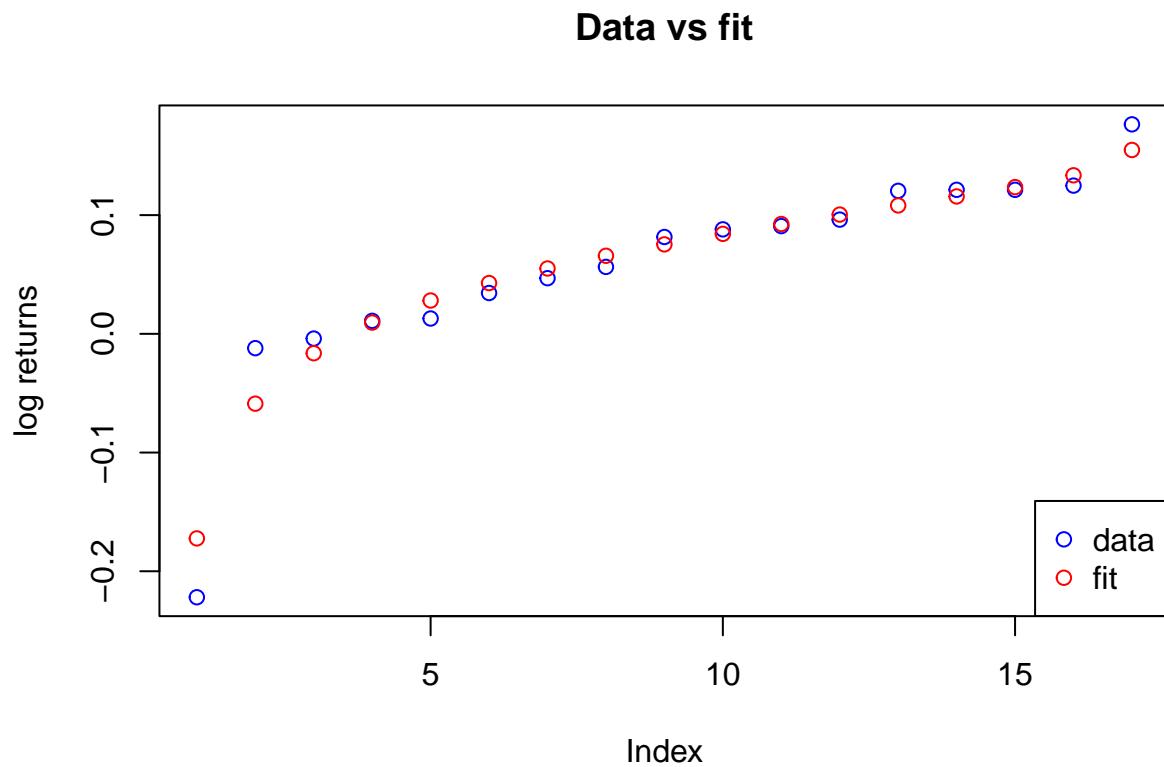
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

Data vs fit

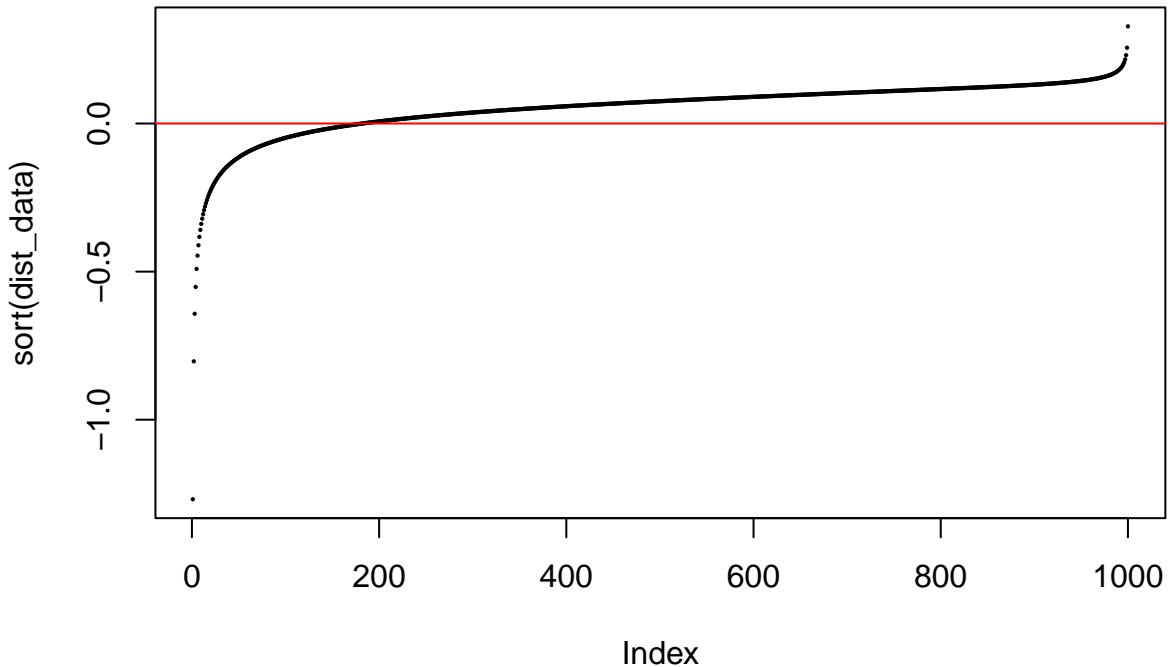
Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

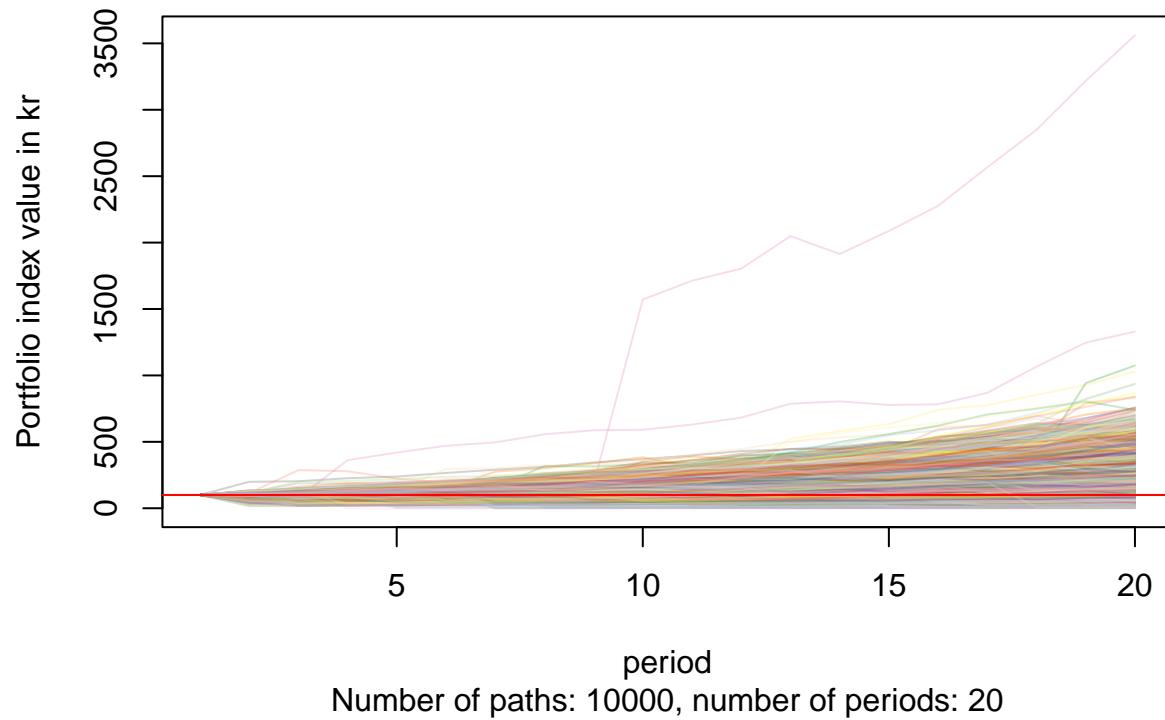


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!

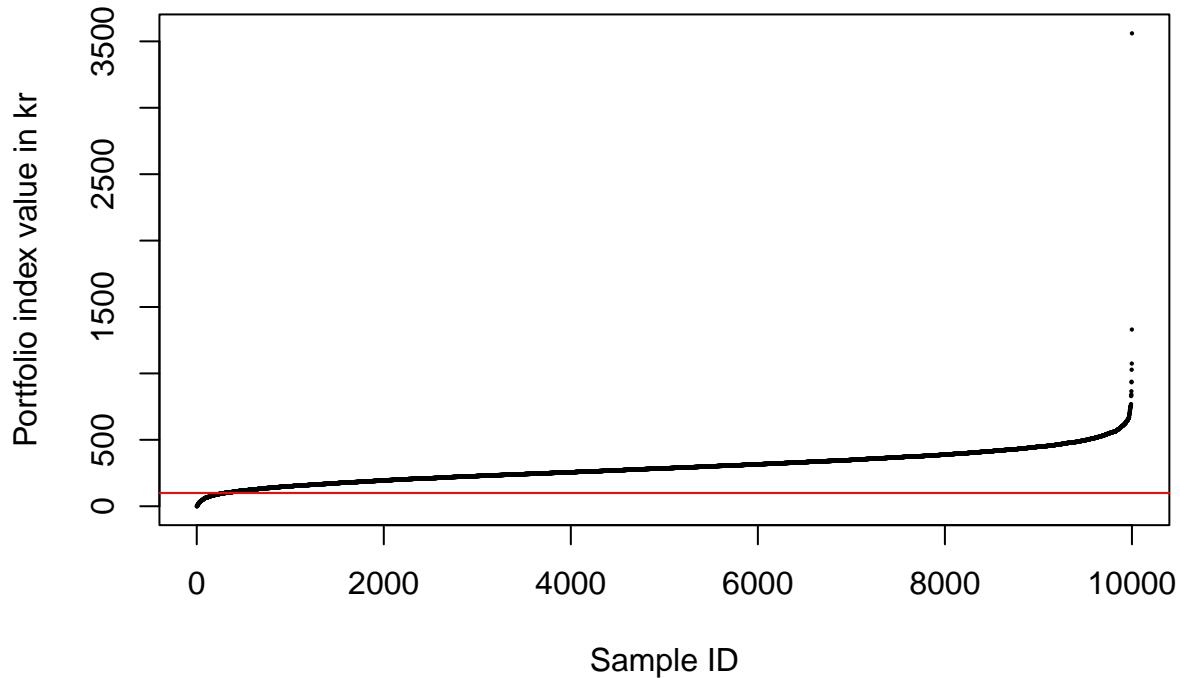
Monte Carlo

```
## Down-and-out simulation:  
## Probability of down-and-out: 0.01 percent  
##  
## Mean portfolio index value after 20 years: 294.583 kr.  
## SD of portfolio index value after 20 years: 121.955 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 3560.339 kr.  
##  
## Share of paths finishing below 100: 3.03 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs
 (100 is par, 200 is double, 50 is half)



Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##  

## AIC: -21.42488  

## BIC: -19.16508  

## m: 0.06471454  

## s: 0.1499924  

## nu (df): 3.144355  

## xi: 0.002367034  

## R^2: 0.991  

##  

## An R^2 of 0.991 suggests that the fit is extremely good.  

##  

## What is the risk of losing max 10 %? =< 8.3 percent  

## What is the risk of losing max 25 %? =< 2.5 percent  

## What is the risk of losing max 50 %? =< 0.4 percent  

## What is the risk of losing max 90 %? =< 0 percent  

## What is the risk of losing max 99 %? =< 0 percent  

##  

## What is the chance of gaining min 10 %? >= 53.3 percent  

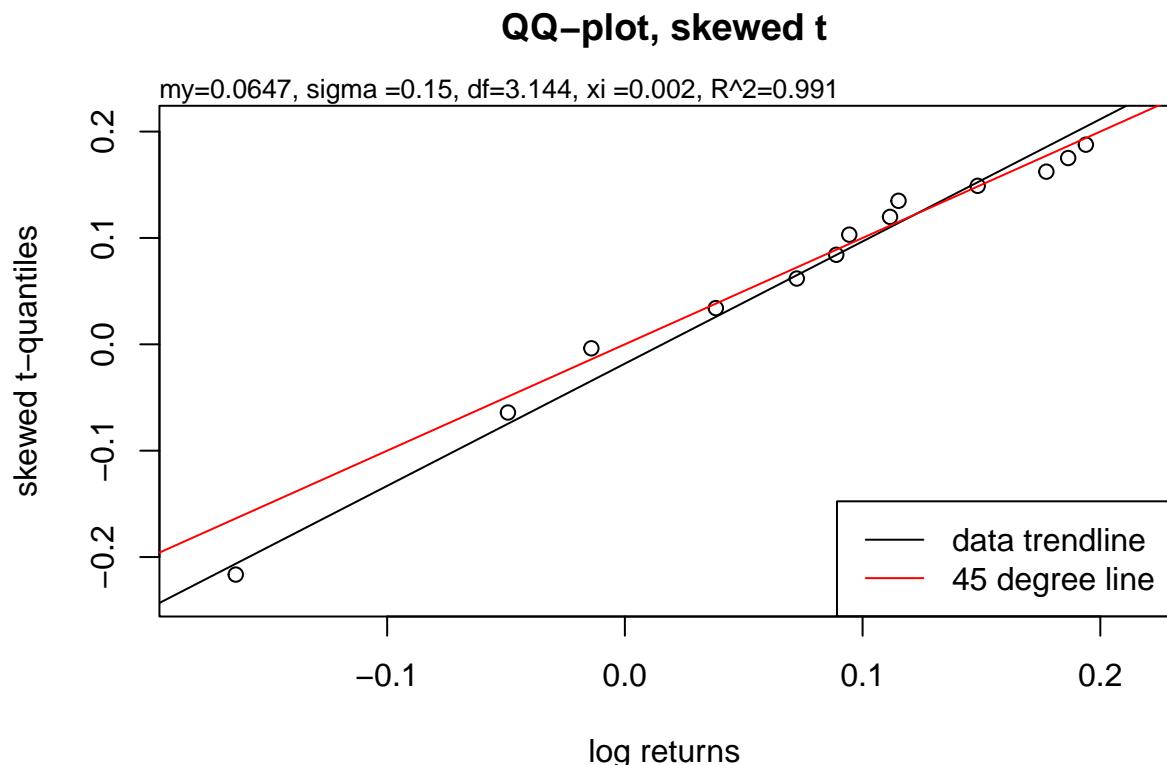
## What is the chance of gaining min 25 %? >= 0 percent  

## What is the chance of gaining min 50 %? >= 0 percent  

## What is the chance of gaining min 90 %? >= 0 percent  

## What is the chance of gaining min 99 %? >= 0 percent
```

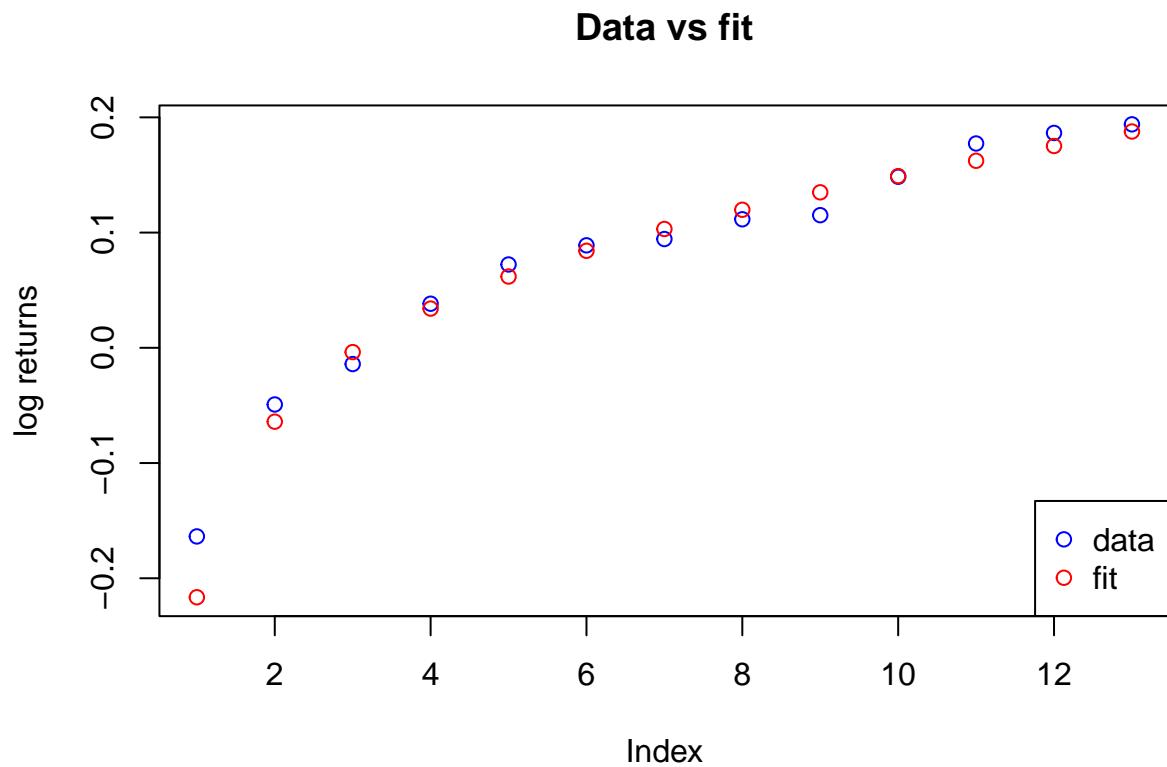
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

Data vs fit

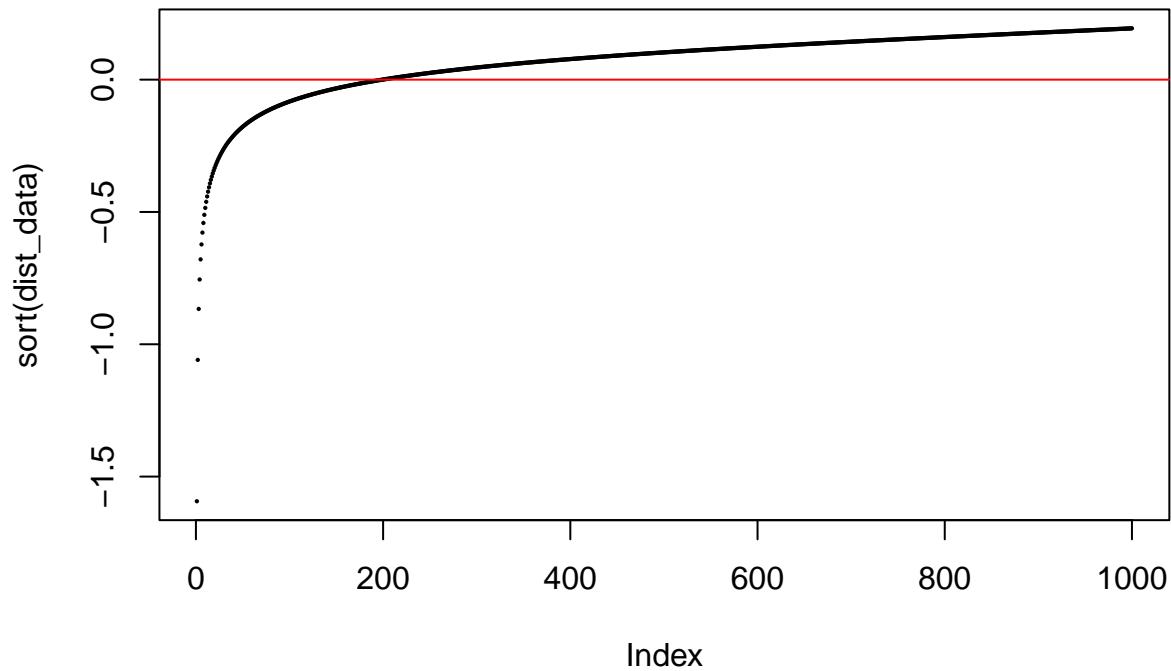
Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

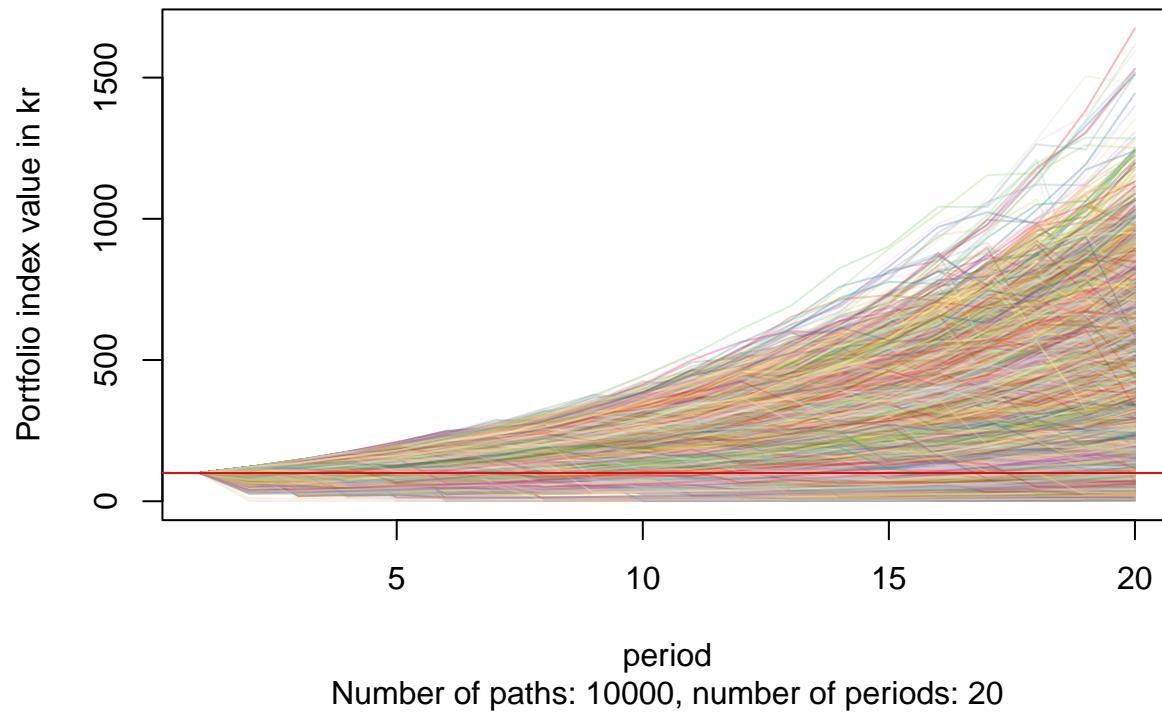


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

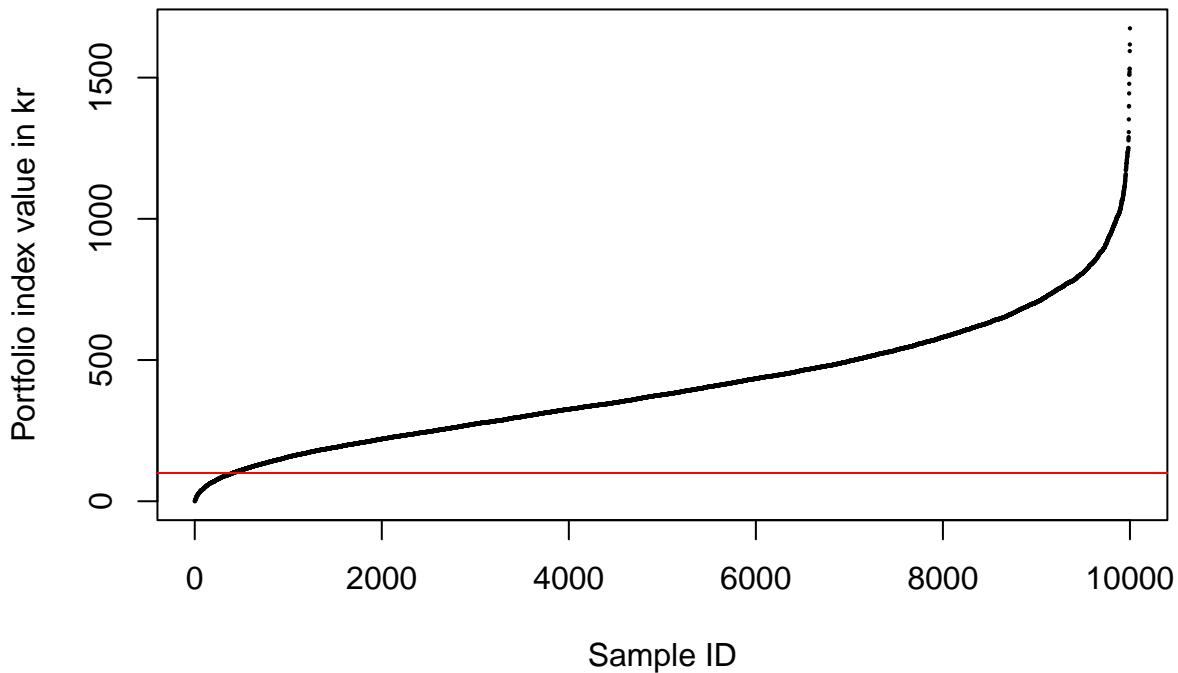
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.01 percent  
##  
## Mean portfolio index value after 20 years: 408.863 kr.  
## SD of portfolio index value after 20 years: 219.108 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 1674.477 kr.  
##  
## Share of paths finishing below 100: 4.07 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)

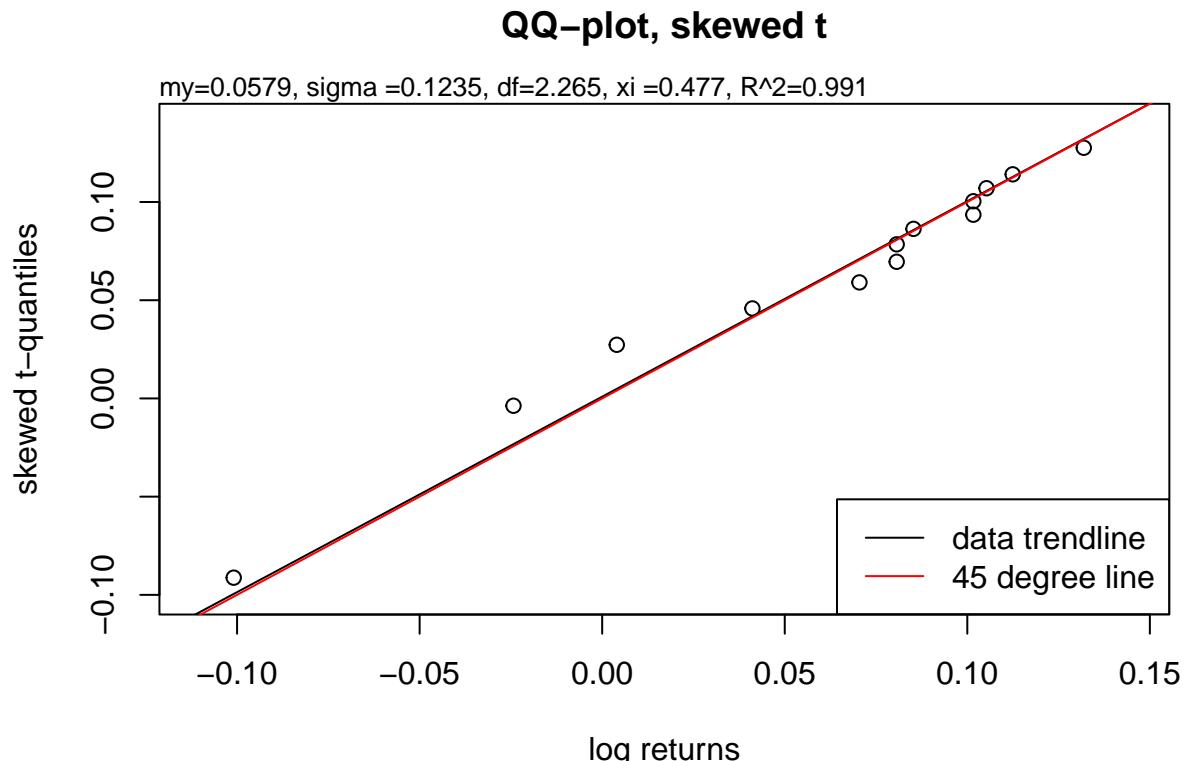


PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -33.22998  
## BIC: -30.97018  
## m: 0.05789224  
## s: 0.1234592  
## nu (df): 2.265273  
## xi: 0.477324  
## R^2: 0.991  
##  
## An R^2 of 0.991 suggests that the fit is extremely good.  
##  
## What is the risk of losing max 10 %? <= 3.3 percent  
## What is the risk of losing max 25 %? <= 0.9 percent  
## What is the risk of losing max 50 %? <= 0.2 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 32.7 percent  
## What is the chance of gaining min 25 %? >= 0.1 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

QQ Plot

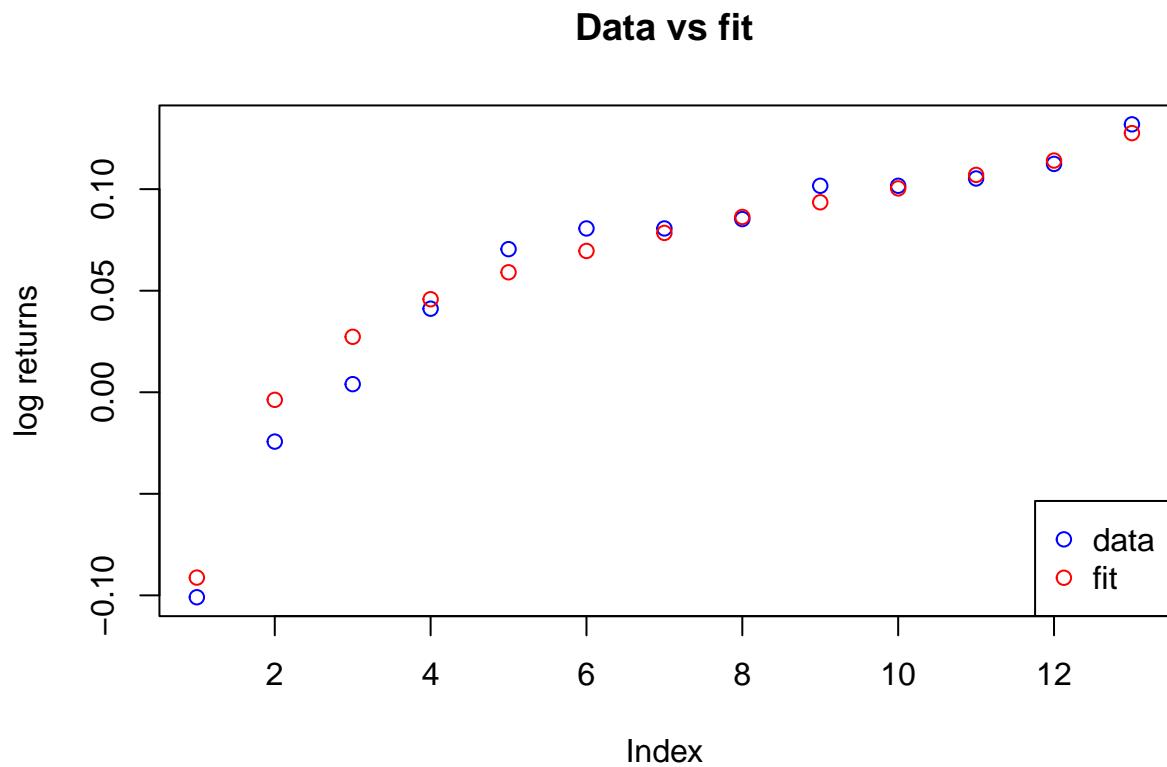


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
## [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
## [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11]  0.107018607  0.114081432  0.127604387
```

Data vs fit

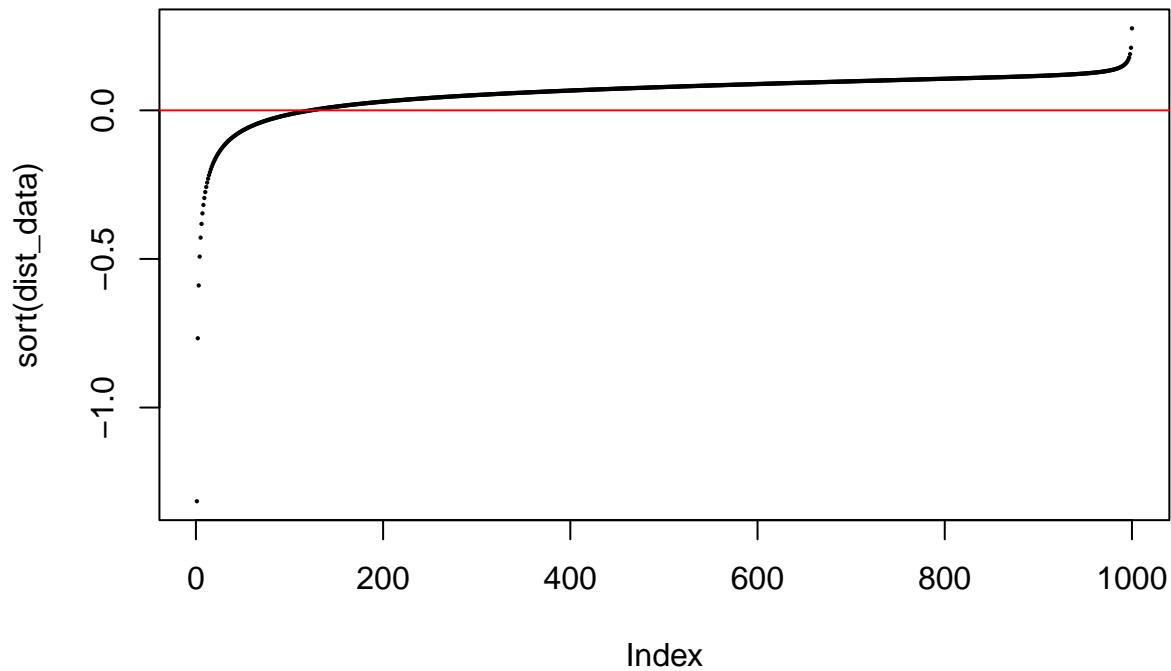
Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

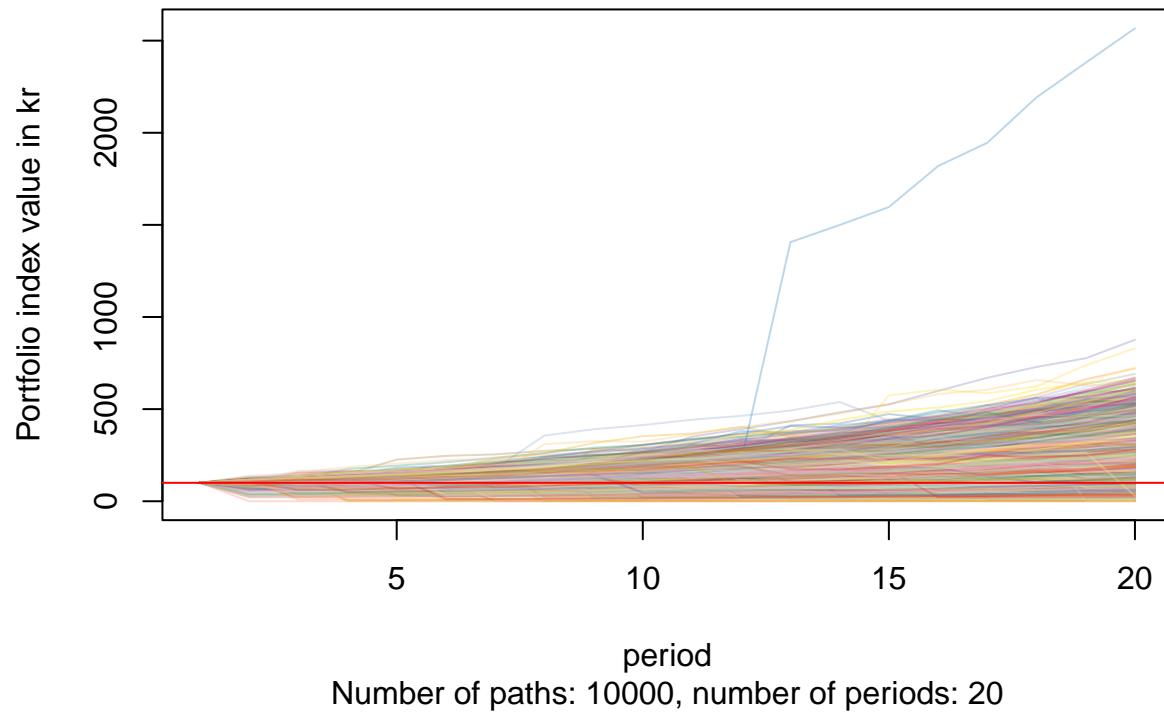


We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.

Monte Carlo

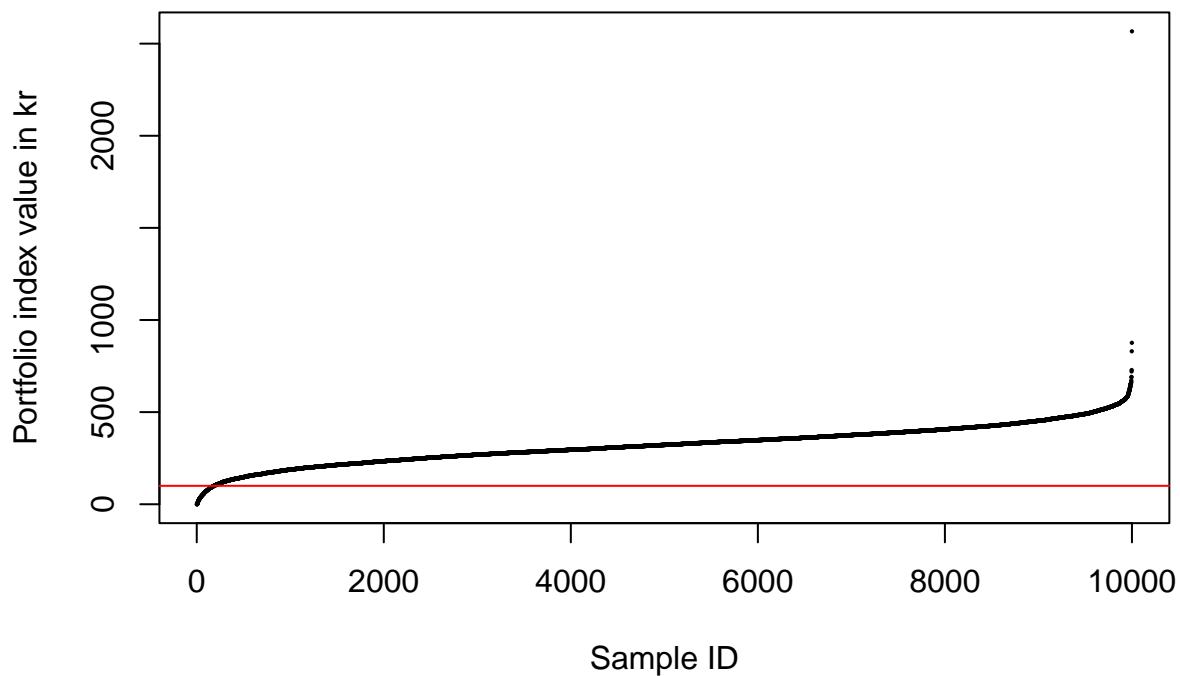
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.02 percent  
##  
## Mean portfolio index value after 20 years: 321.123 kr.  
## SD of portfolio index value after 20 years: 106.423 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 2566.719 kr.  
##  
## Share of paths finishing below 100: 1.85 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)

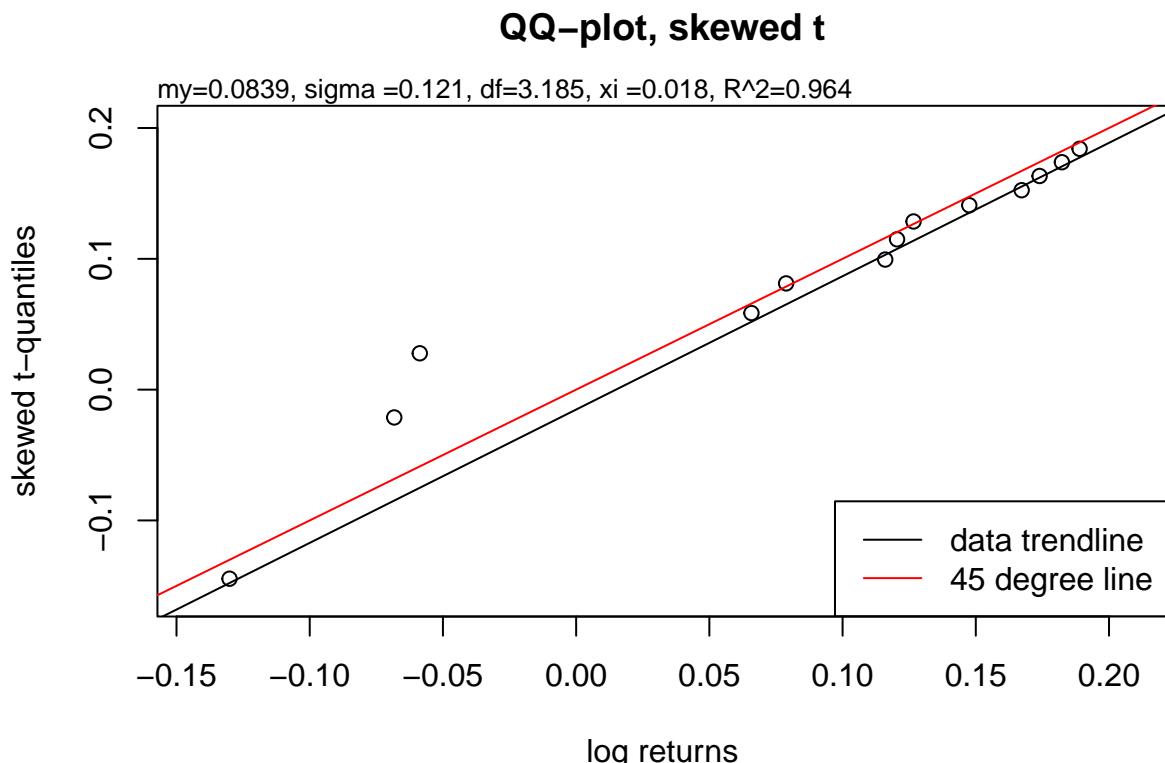


PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -23.72565  
## BIC: -21.46585  
## m: 0.08386034  
## s: 0.1210107  
## nu (df): 3.184569  
## xi: 0.01790306  
## R^2: 0.964  
##  
## An R^2 of 0.964 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 5.3 percent  
## What is the risk of losing max 25 %? <= 1.4 percent  
## What is the risk of losing max 50 %? <= 0.2 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 59.6 percent  
## What is the chance of gaining min 25 %? >= 0 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

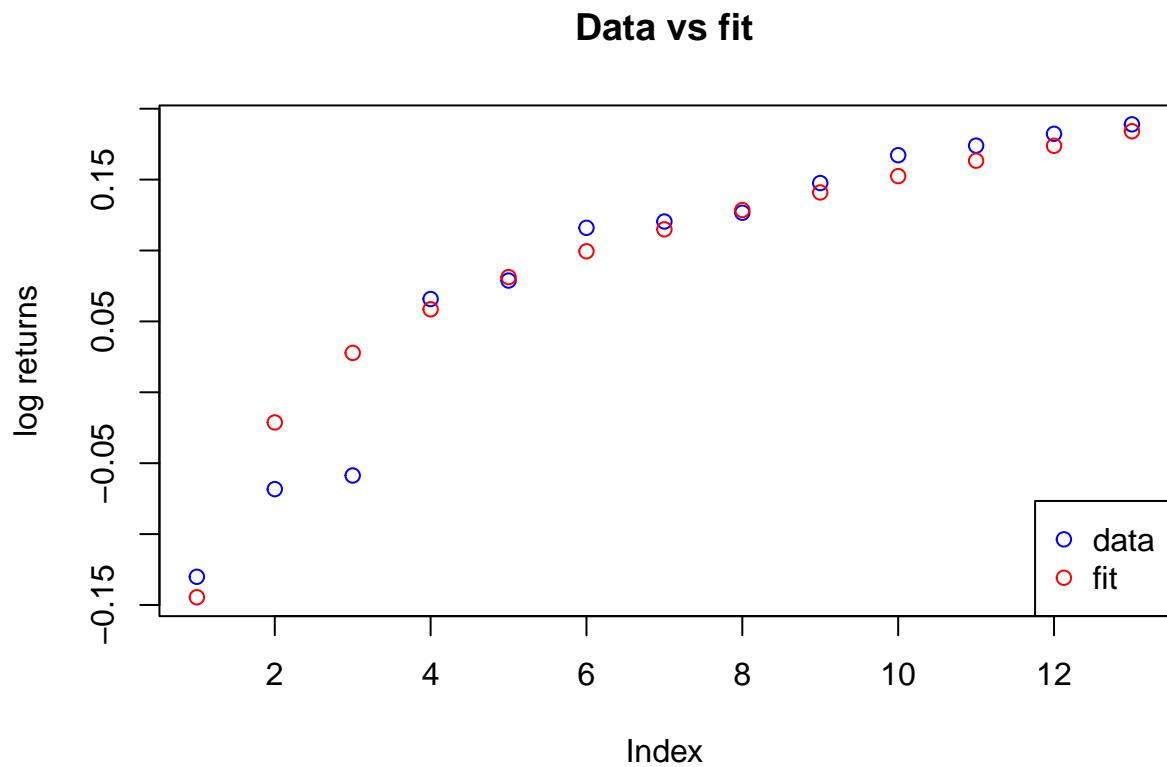
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

Data vs fit

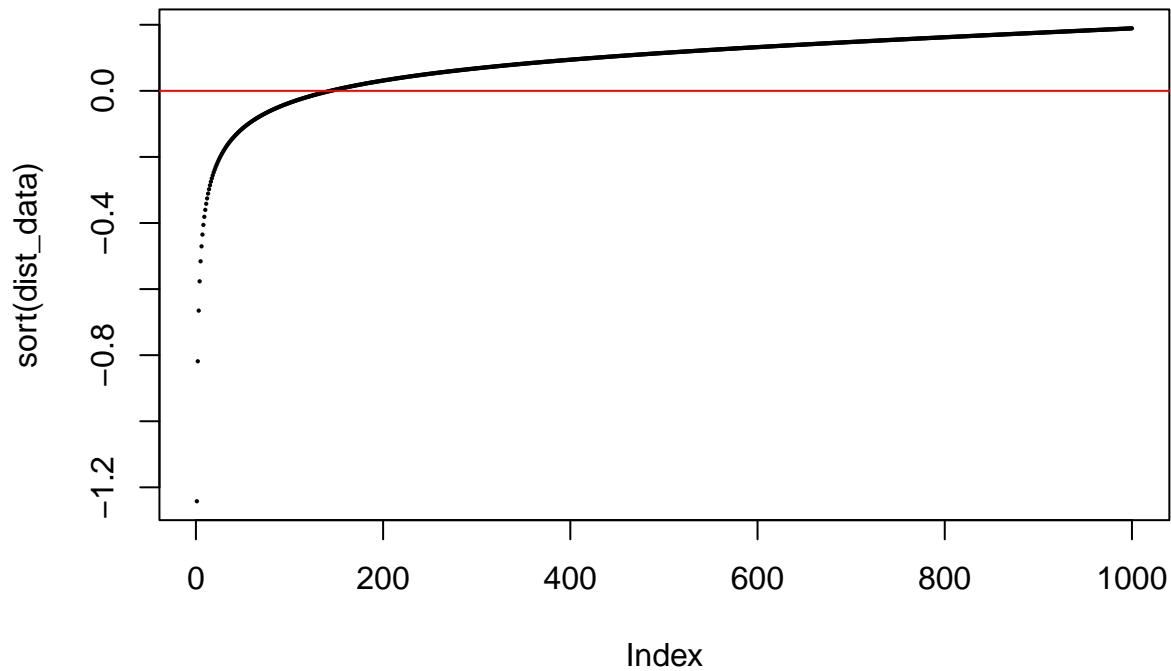
Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

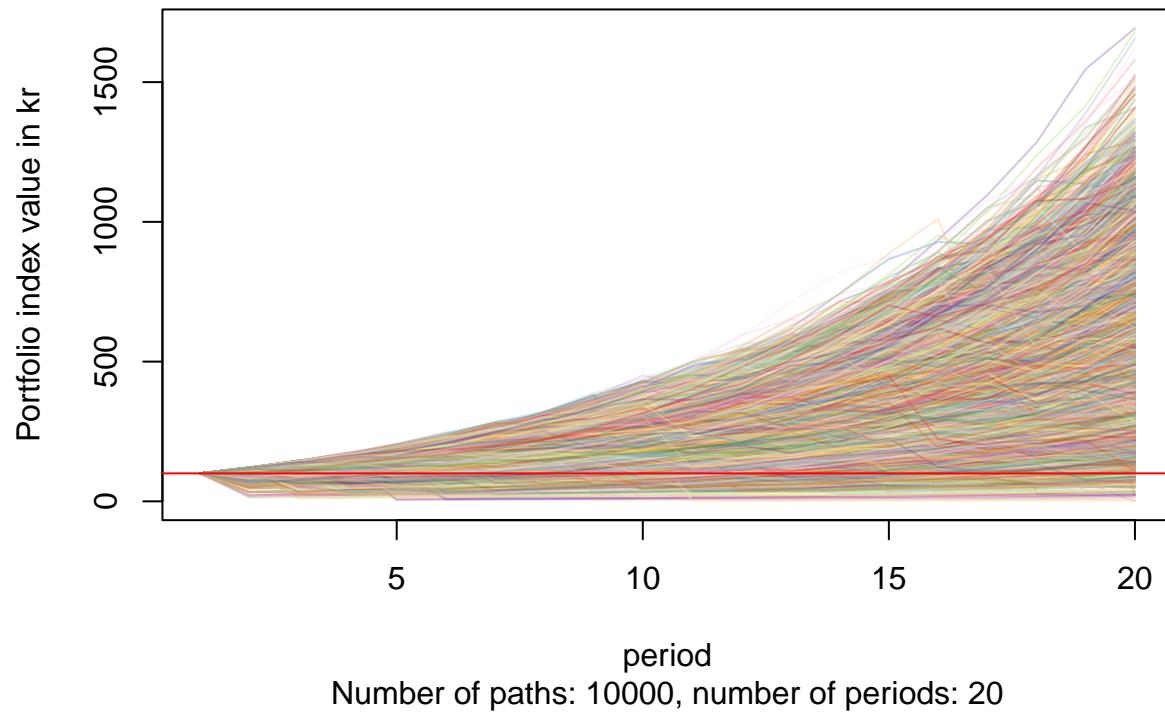


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

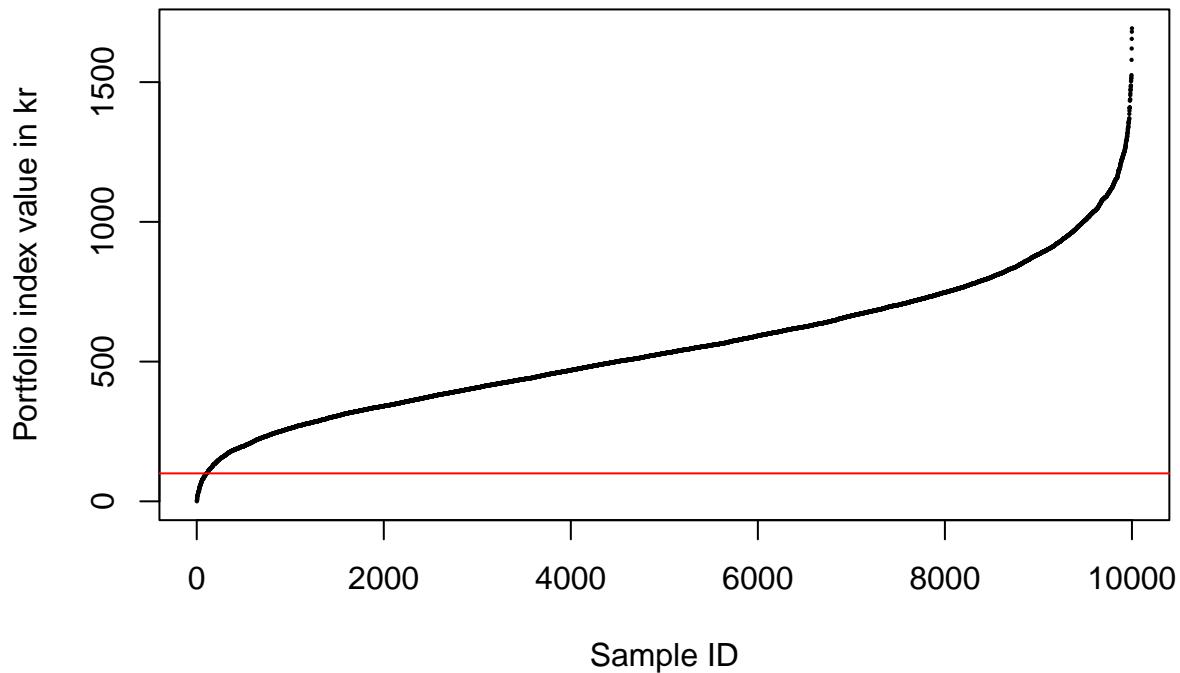
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 553.794 kr.  
## SD of portfolio index value after 20 years: 245.211 kr.  
## Min total portfolio index value after 20 years: 0.303 kr.  
## Max total portfolio index value after 20 years: 1692.399 kr.  
##  
## Share of paths finishing below 100: 1.07 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)

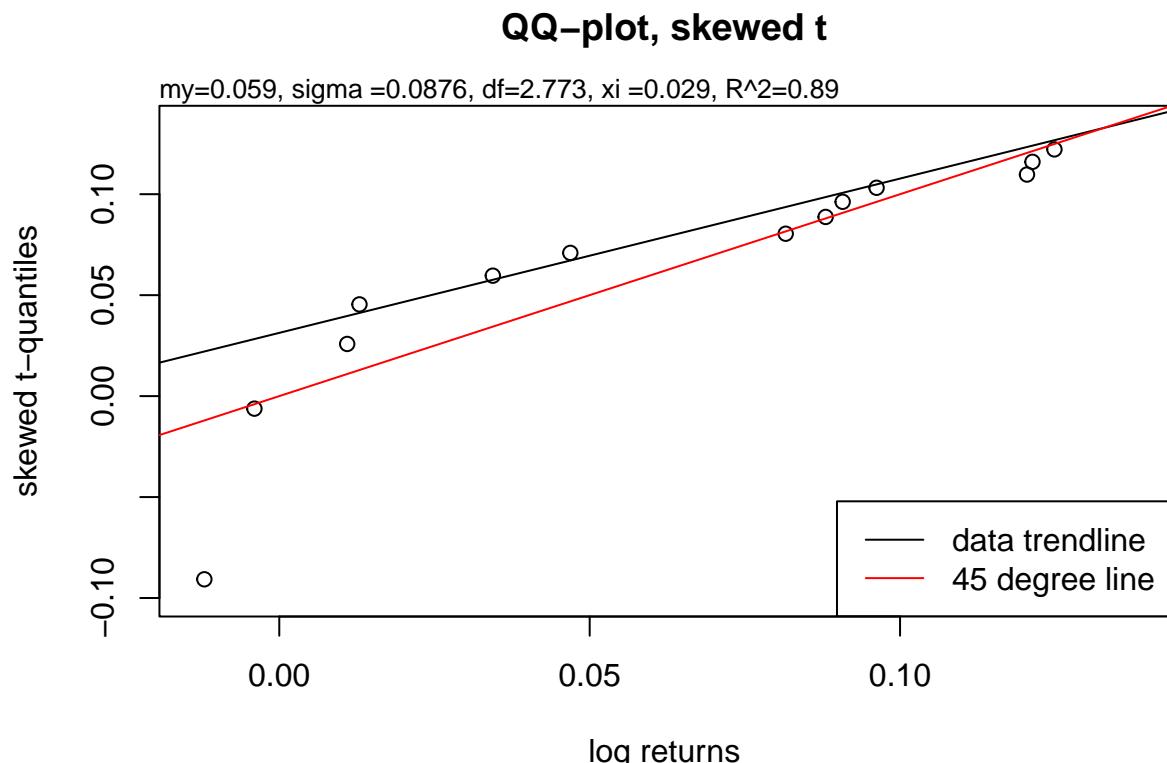


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -36.9603  
## BIC: -34.7005  
## m: 0.05902873  
## s: 0.08757749  
## nu (df): 2.772621  
## xi: 0.02904471  
## R^2: 0.89  
##  
## An R^2 of 0.89 suggests that the fit is not completely random.  
##  
## What is the risk of losing max 10 %? <= 3.3 percent  
## What is the risk of losing max 25 %? <= 0.7 percent  
## What is the risk of losing max 50 %? <= 0.1 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 35.6 percent  
## What is the chance of gaining min 25 %? >= 0 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

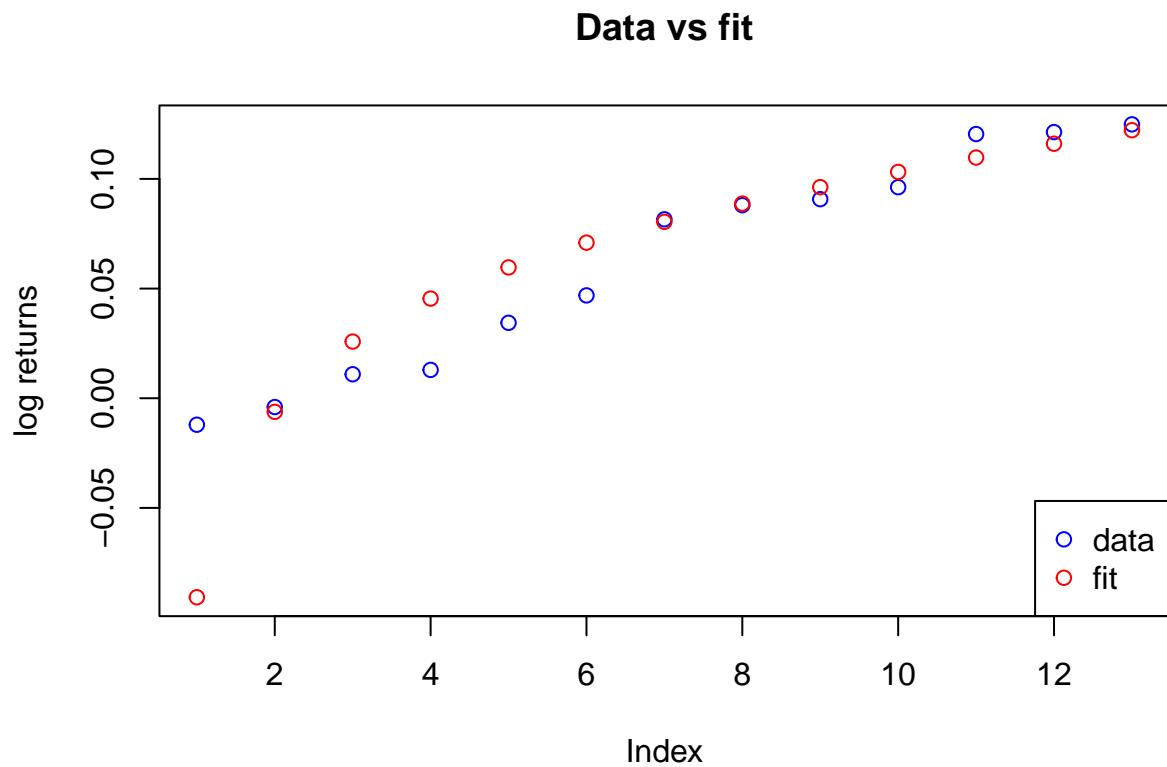
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

Let's plot the fit and the observed returns together.

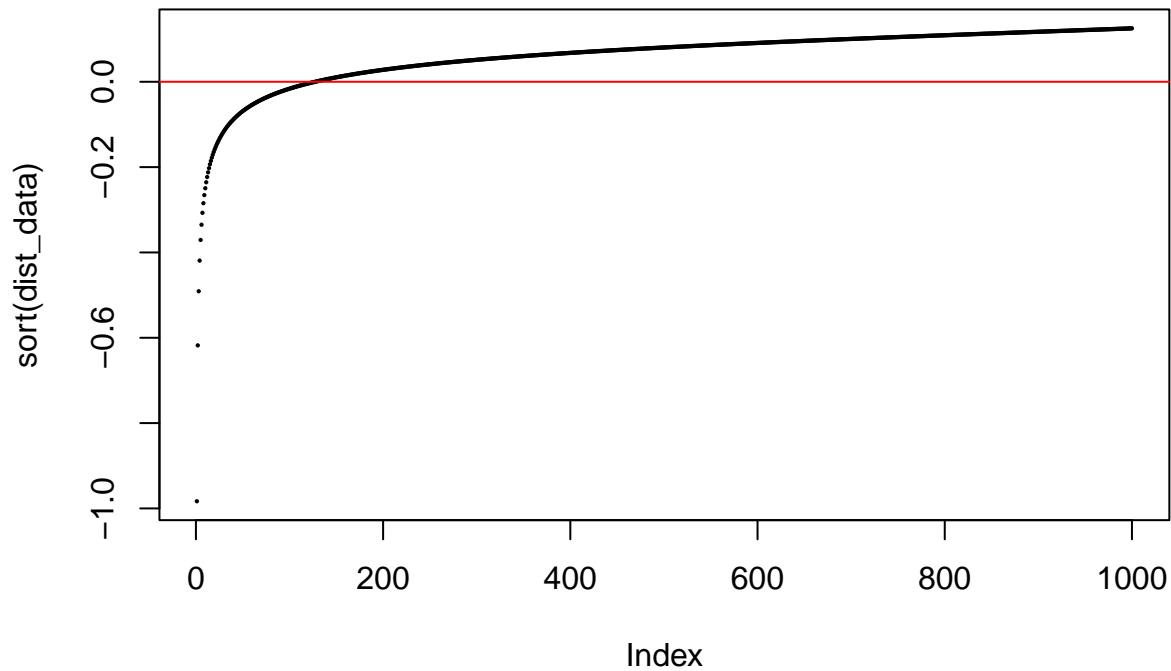


Interestingly, the fit predicts a much bigger “biggest loss” than the actual data. This is the main reason that R^2 is 0.90 and not higher.

Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



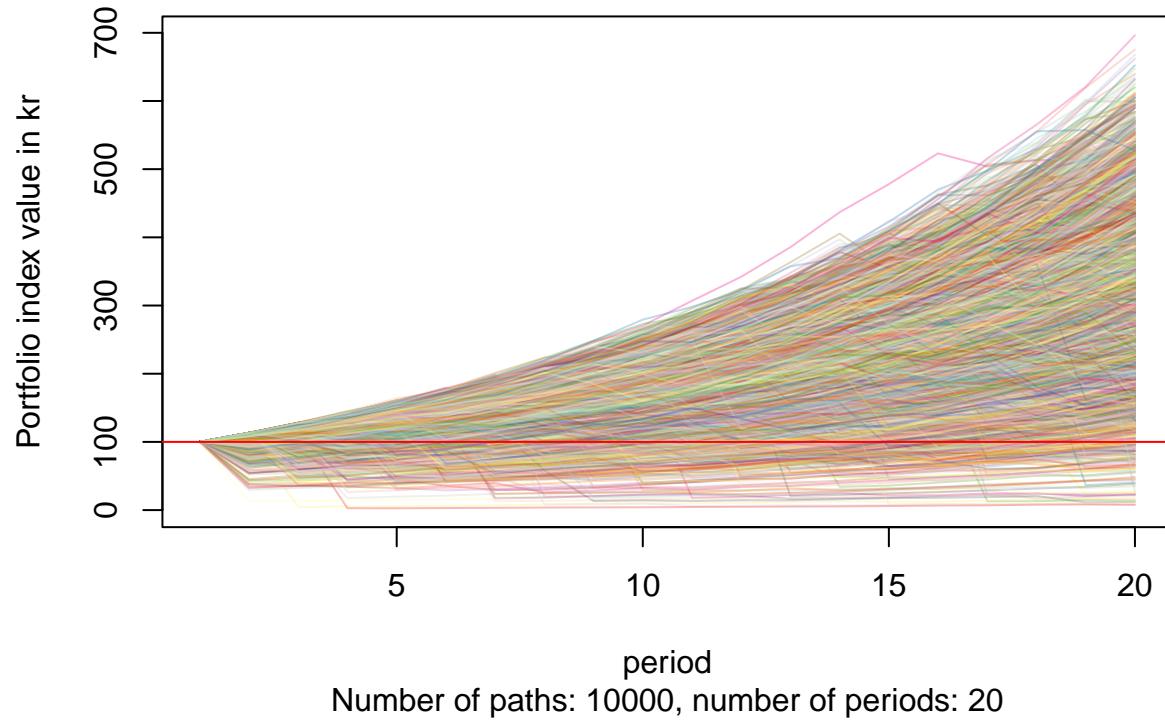
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

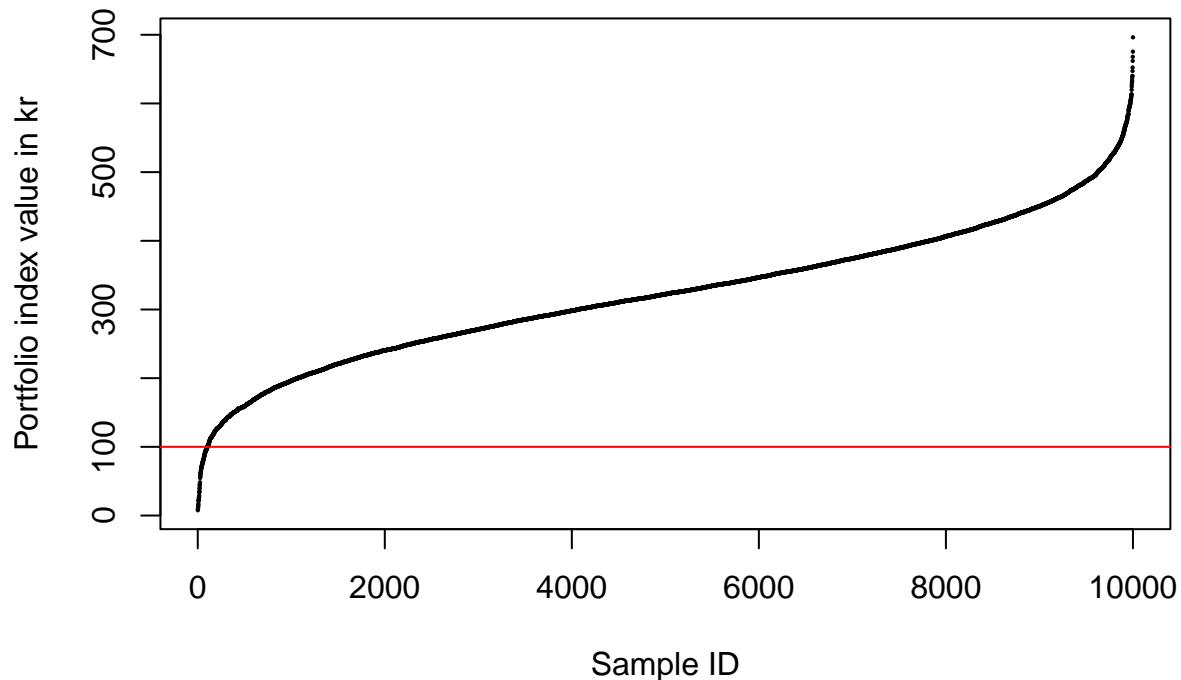
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 323.244 kr.  
## SD of portfolio index value after 20 years: 98.709 kr.  
## Min total portfolio index value after 20 years: 7.697 kr.  
## Max total portfolio index value after 20 years: 696.259 kr.  
##  
## Share of paths finishing below 100: 1.02 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

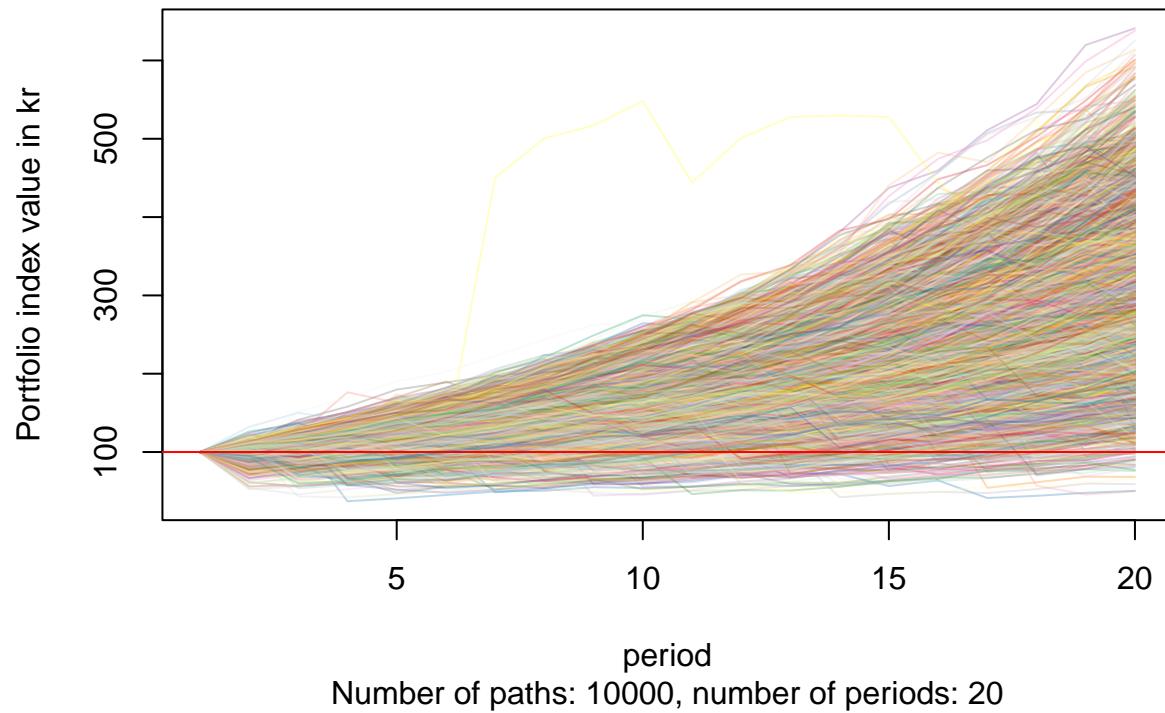
(100 is par, 200 is double, 50 is half)



Version b: Mix of simulations from estimated distribution of returns from individual funds.

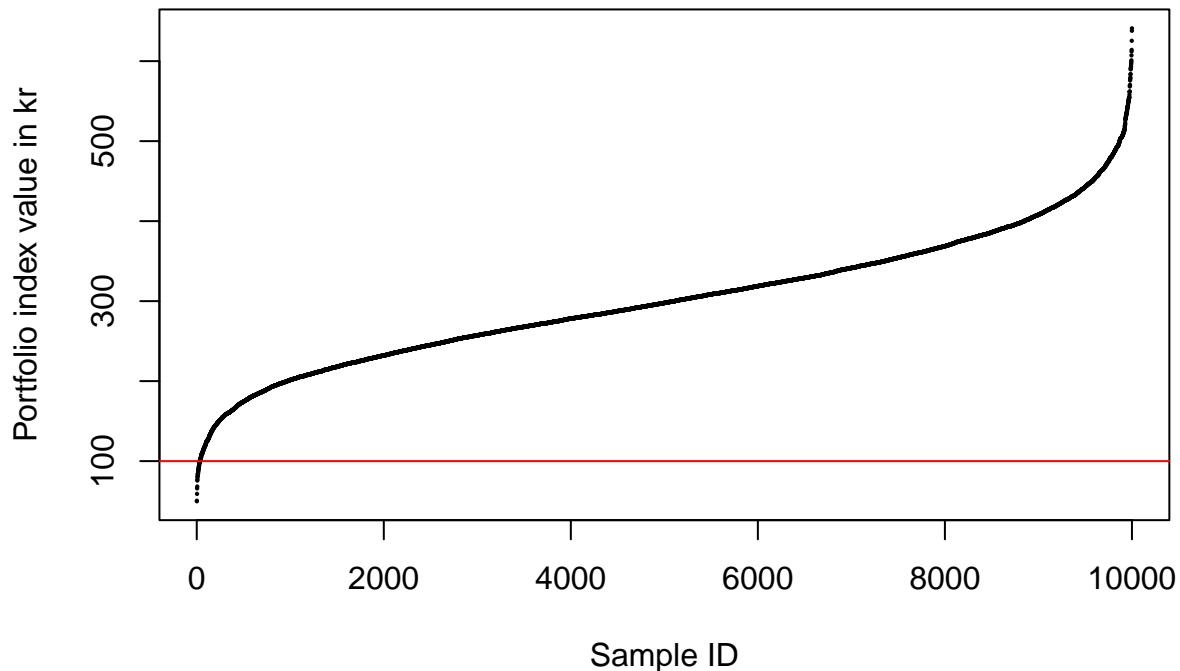
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 301.846 kr.  
## SD of portfolio index value after 20 years: 81.884 kr.  
## Min total portfolio index value after 20 years: 49.846 kr.  
## Max total portfolio index value after 20 years: 641.004 kr.  
##  
## Share of paths finishing below 100: 0.33 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)

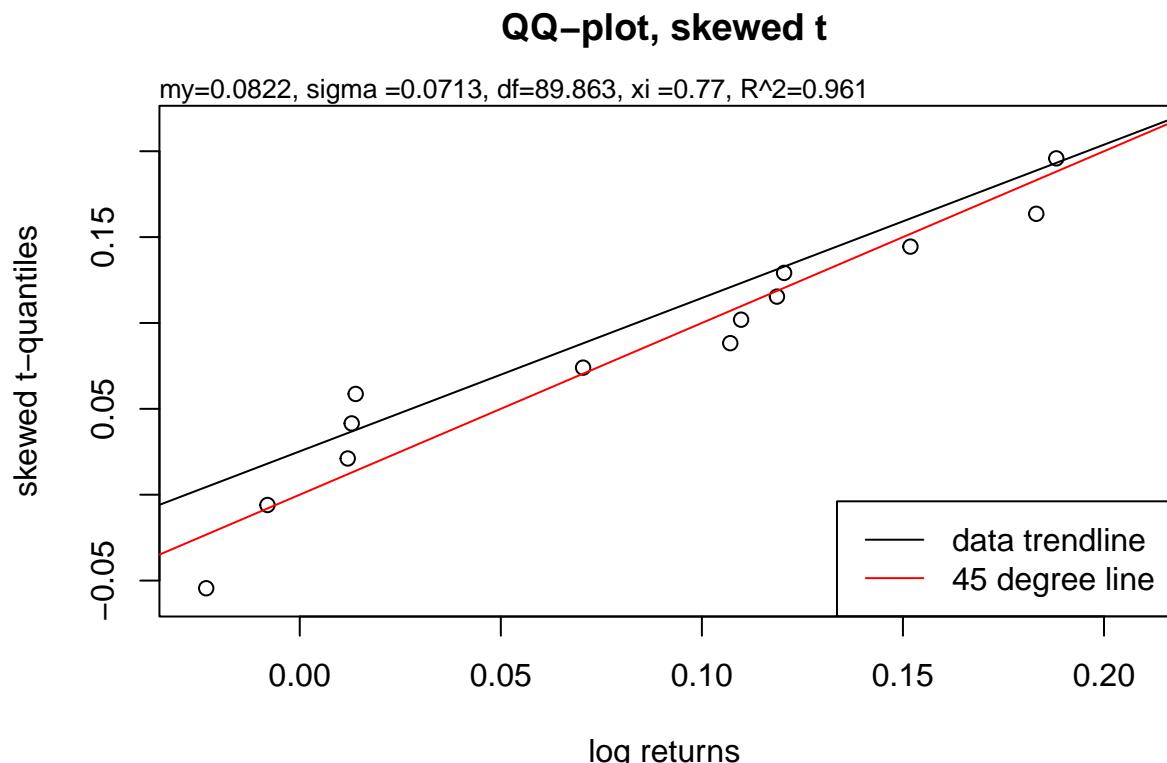


Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -24.26084  
## BIC: -22.00104  
## m: 0.0822419  
## s: 0.07129843  
## nu (df): 89.86289  
## xi: 0.7697502  
## R^2: 0.961  
##  
## An R^2 of 0.961 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 0.9 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 46.1 percent  
## What is the chance of gaining min 25 %? >= 1.2 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

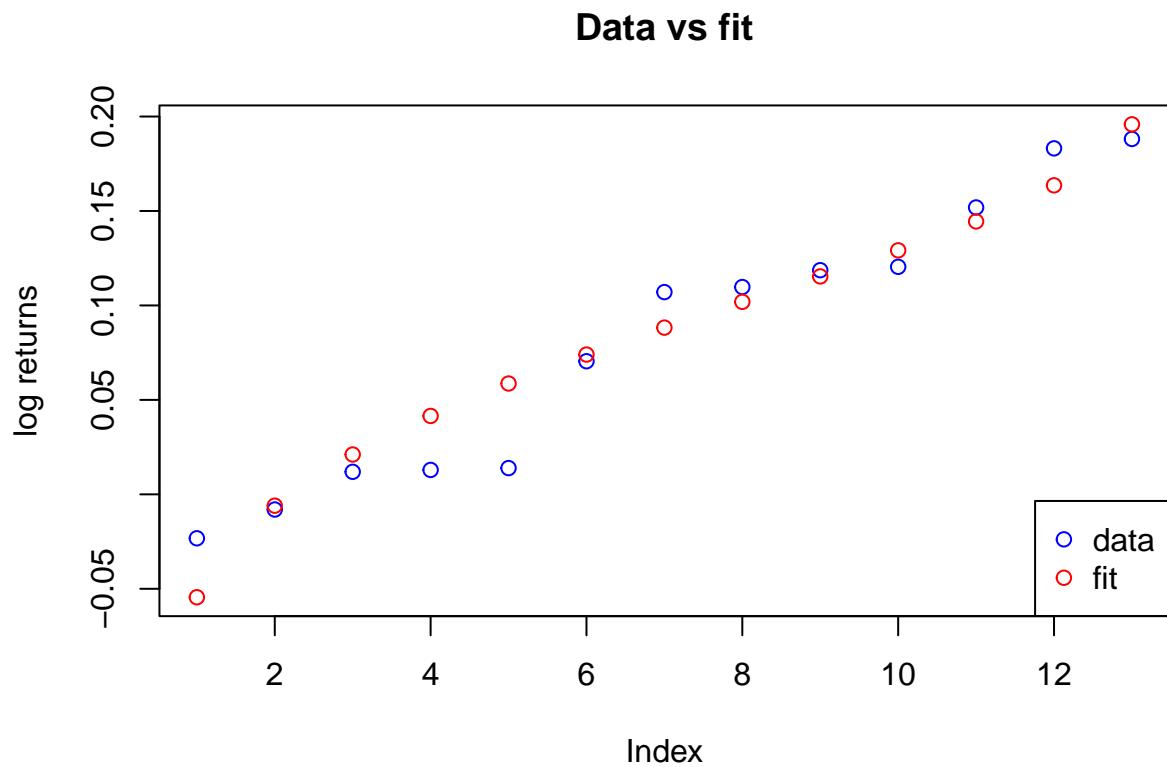
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

Data vs fit

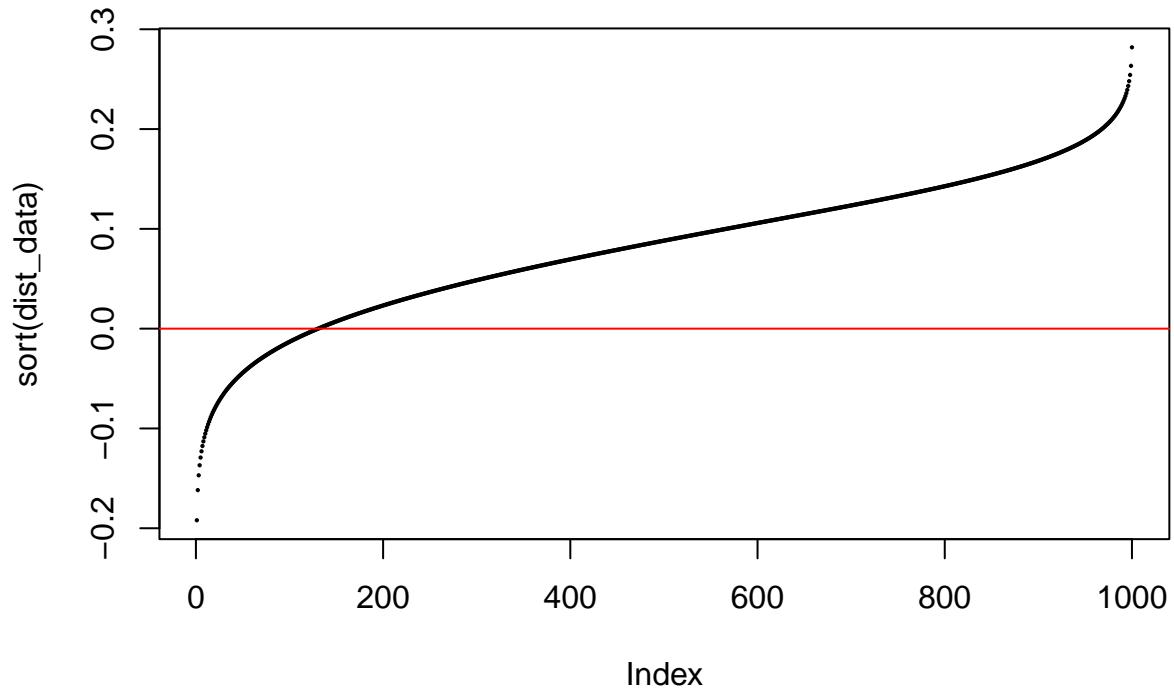
Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



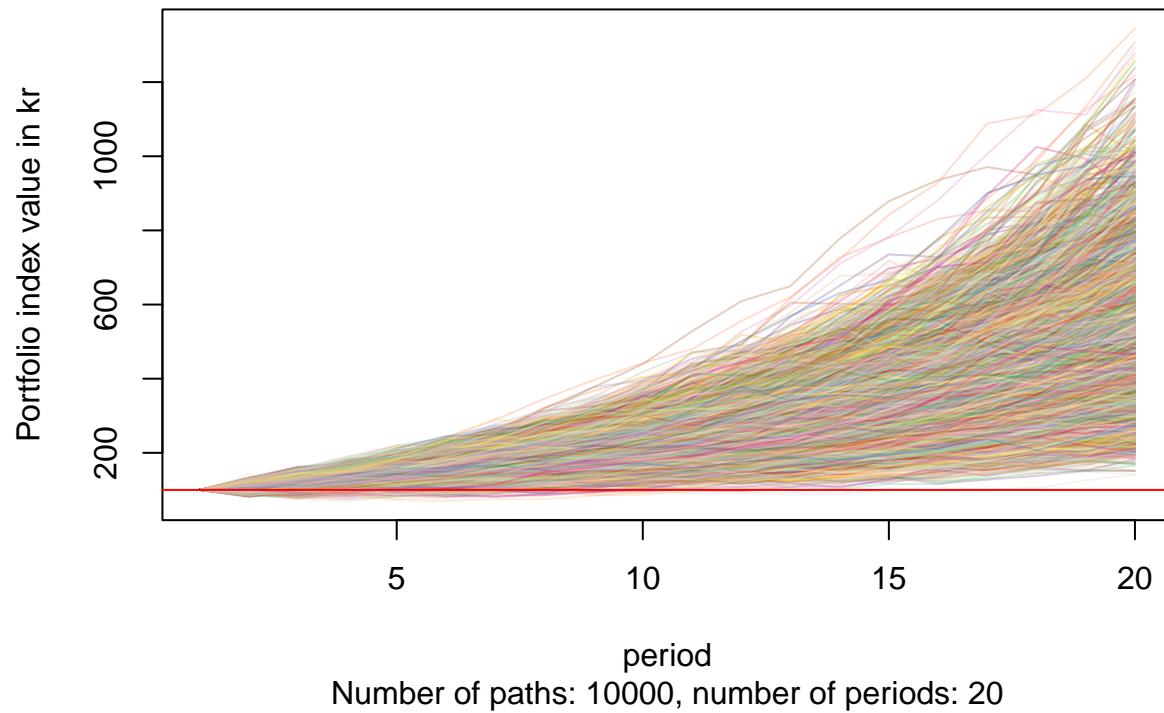
We see that the high risk mix provides a much better upside and smaller downside.

Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

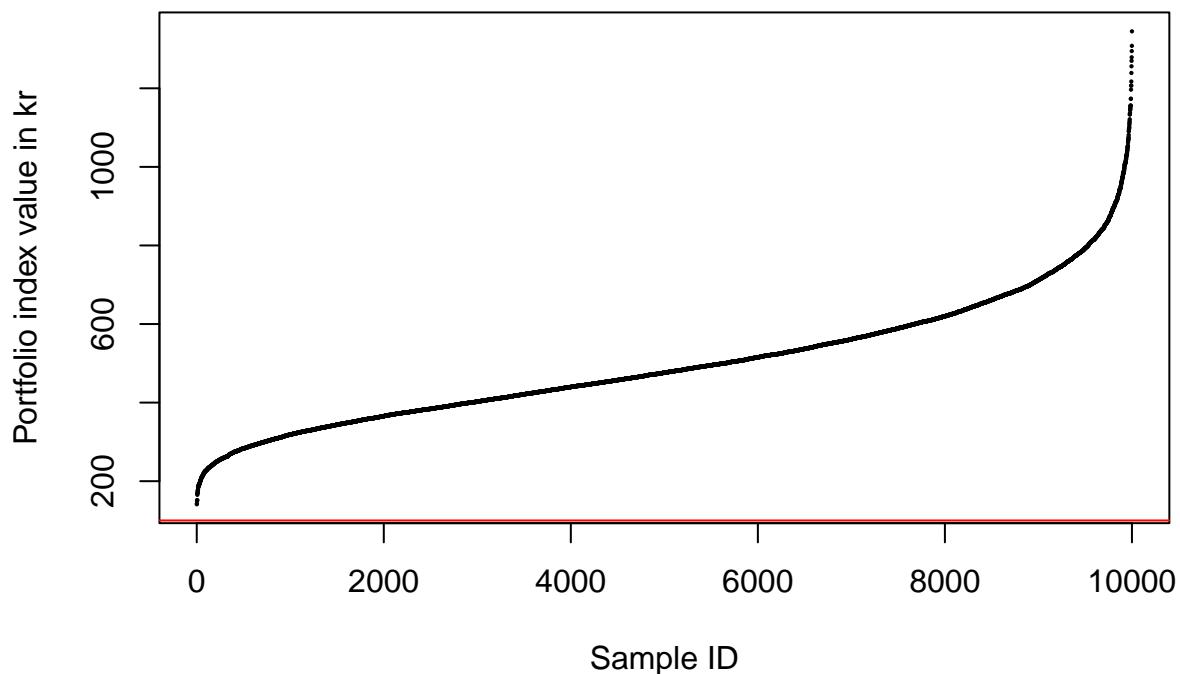
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 499.077 kr.  
## SD of portfolio index value after 20 years: 158.895 kr.  
## Min total portfolio index value after 20 years: 141.306 kr.  
## Max total portfolio index value after 20 years: 1344.969 kr.  
##  
## Share of paths finishing below 100: 0 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

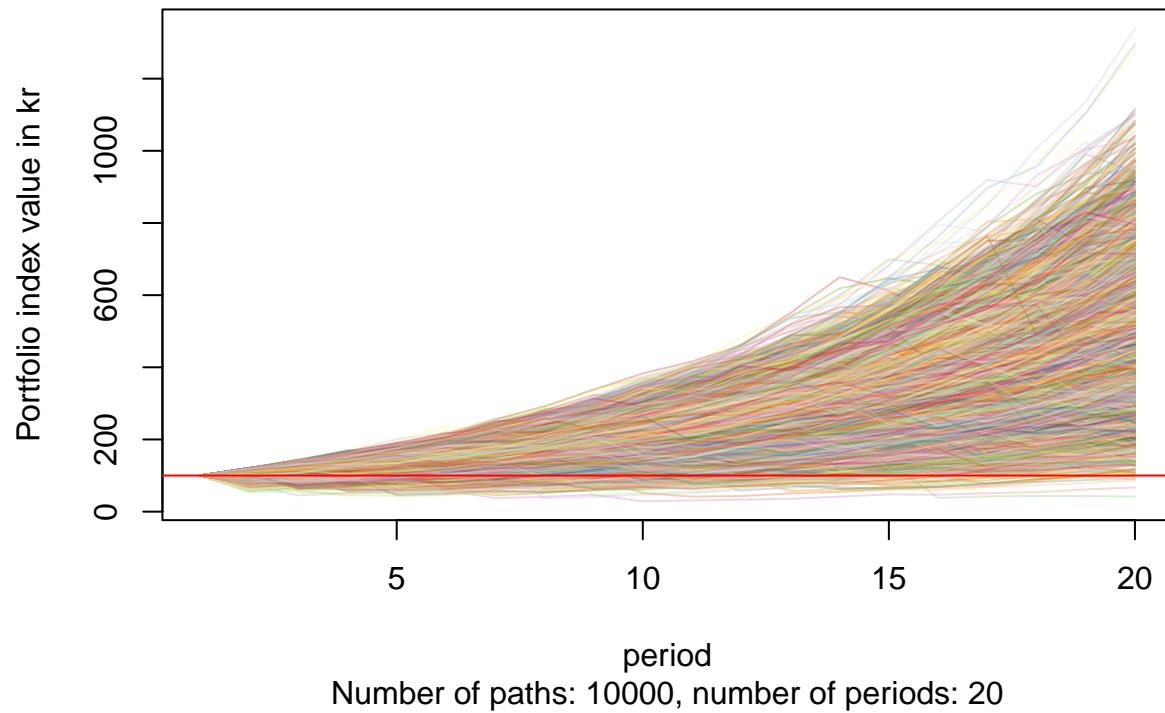
(100 is par, 200 is double, 50 is half)



Version b: Mix of simulations from estimated distribution of returns from individual funds.

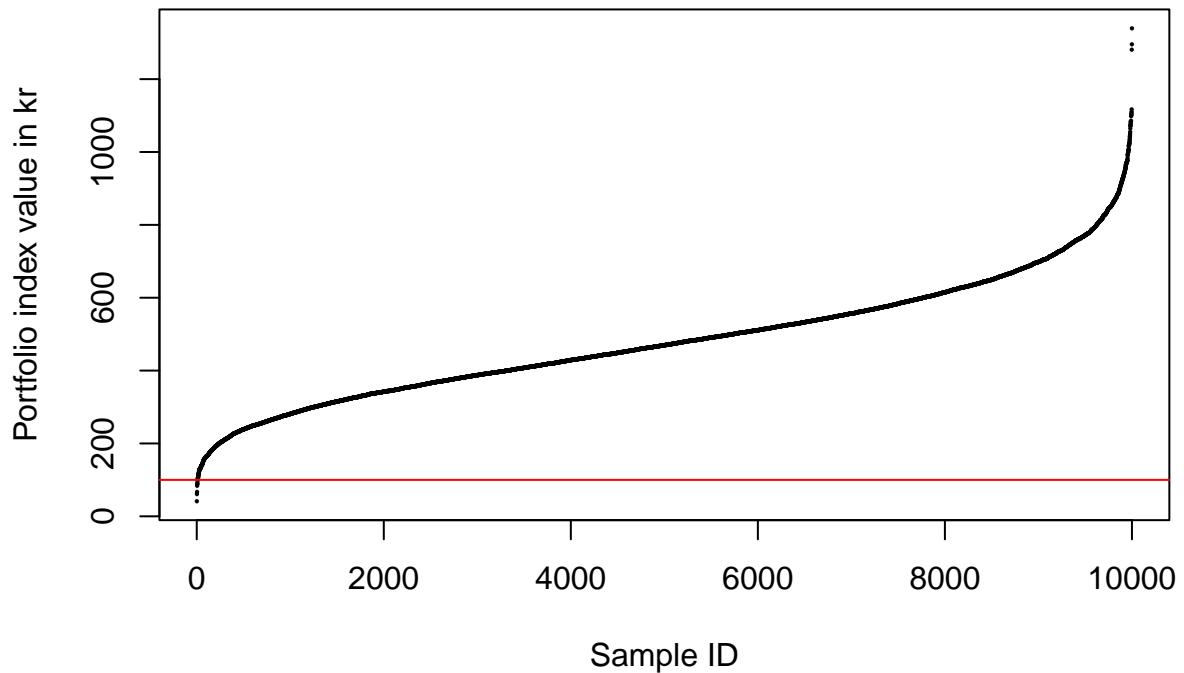
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 482.246 kr.  
## SD of portfolio index value after 20 years: 163.777 kr.  
## Min total portfolio index value after 20 years: 41.382 kr.  
## Max total portfolio index value after 20 years: 1339.403 kr.  
##  
## Share of paths finishing below 100: 0.09 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)



Compare pension plans

Risk of max loss

Risk of max loss of x percent for a single period (year).

x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	21.3	18.2	19.9	12.2	14.3	12.7	13.0
5	12.5	9.6	12.8	6.0	8.6	6.2	4.2
10	7.4	5.4	8.3	3.3	5.3	3.3	0.9
25	1.8	1.3	2.5	0.9	1.4	0.7	0.0
50	0.2	0.2	0.4	0.2	0.2	0.1	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.3	Velliv_m	12.8	Velliv_h	8.3	Velliv_h	2.5	Velliv_h	0.4	Velliv_h	0	Velliv_m	0	Velliv_m
19.9	Velliv_h	12.5	Velliv_m	7.4	Velliv_m	1.8	Velliv_m	0.2	Velliv_m	0	Velliv_m_l	0	Velliv_m_l
18.2	Velliv_m_l	9.6	Velliv_m_l	5.4	Velliv_m_l	1.4	PFA_h	0.2	Velliv_m_l	0	Velliv_h	0	Velliv_h
14.3	PFA_h	8.6	PFA_h	5.3	PFA_h	1.3	Velliv_m_l	0.2	PFA_m	0	PFA_m	0	PFA_m
13.0	mix_h	6.2	mix_m	3.3	PFA_m	0.9	PFA_m	0.2	PFA_h	0	PFA_h	0	PFA_h
12.7	mix_m	6.0	PFA_m	3.3	mix_m	0.7	mix_m	0.1	mix_m	0	mix_m	0	mix_m
12.2	PFA_m	4.2	mix_h	0.9	mix_h	0.0	mix_h	0.0	mix_h	0	mix_h	0	mix_h

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	78.7	81.8	80.1	87.8	85.7	87.3	87.0
5	63.8	64.9	69.2	71.5	75.8	71.4	69.9
10	41.0	36.2	53.3	32.7	59.6	35.6	46.1
25	0.0	0.3	0.0	0.1	0.0	0.0	1.2
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
87.8	PFA_m	75.8	PFA_h	59.6	PFA_h	1.2	mix_h	0	Velliv_m	0	Velliv_m
87.3	mix_m	71.5	PFA_m	53.3	Velliv_h	0.3	Velliv_m_l	0	Velliv_m_l	0	Velliv_m_l
87.0	mix_h	71.4	mix_m	46.1	mix_h	0.1	PFA_m	0	Velliv_h	0	Velliv_h
85.7	PFA_h	69.9	mix_h	41.0	Velliv_m	0.0	Velliv_m	0	PFA_m	0	PFA_m
81.8	Velliv_m_l	69.2	Velliv_h	36.2	Velliv_m_l	0.0	Velliv_h	0	PFA_h	0	PFA_h
80.1	Velliv_h	64.9	Velliv_m_l	35.6	mix_m	0.0	PFA_h	0	mix_m	0	mix_m
78.7	Velliv_m	63.8	Velliv_m	32.7	PFA_m	0.0	mix_m	0	mix_h	0	mix_h

MC risk percentiles

Risk of loss from first to last period.

_a is simulation from estimated distribution of mix.

_b is mix of simulations from estimated distribution of returns from individual funds.

_m is medium.

_h is high.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	5.05	3.03	4.07	1.85	1.07	1.02	0	0.33	0.09
5	4.41	2.60	3.74	1.66	0.96	0.87	0	0.25	0.07
10	3.86	2.22	3.24	1.47	0.83	0.75	0	0.18	0.06
25	2.39	1.43	2.29	1.09	0.55	0.48	0	0.05	0.03
50	0.91	0.66	1.10	0.58	0.31	0.24	0	0.01	0.01
90	0.05	0.08	0.13	0.10	0.05	0.02	0	0.00	0.00
99	0.01	0.03	0.01	0.03	0.01	0.00	0	0.00	0.00

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
5.05	Velliv_m	4.41	Velliv_m	3.86	Velliv_m	2.39	Velliv_m	1.10	Velliv_h	0.13	Velliv_h	0.03	Velliv_m_l
4.07	Velliv_h	3.74	Velliv_h	3.24	Velliv_h	2.29	Velliv_h	0.91	Velliv_m	0.10	PFA_m	0.03	PFA_m
3.03	Velliv_m_l	2.60	Velliv_m_l	2.22	Velliv_m_l	1.43	Velliv_m_l	0.66	Velliv_m_l	0.08	Velliv_m_l	0.01	Velliv_m
1.85	PFA_m	1.66	PFA_m	1.47	PFA_m	1.09	PFA_m	0.58	PFA_m	0.05	Velliv_m	0.01	Velliv_h
1.07	PFA_h	0.96	PFA_h	0.83	PFA_h	0.55	PFA_h	0.31	PFA_h	0.05	PFA_h	0.01	PFA_h
1.02	mix_m_a	0.87	mix_m_a	0.75	mix_m_a	0.48	mix_m_a	0.24	mix_m_a	0.02	mix_m_a	0.00	mix_m_a
0.33	mix_m_b	0.25	mix_m_b	0.18	mix_m_b	0.05	mix_m_b	0.01	mix_m_b	0.00	mix_h_a	0.00	mix_h_a
0.09	mix_h_b	0.07	mix_h_b	0.06	mix_h_b	0.03	mix_h_b	0.01	mix_h_b	0.00	mix_m_b	0.00	mix_m_b
0.00	mix_h_a	0.00	mix_h_b	0.00	mix_h_b								

MC gains percentiles

Chance of gains from first to last period.

_a is simulation from estimated distribution of returns of mix.

_b is mix of simulations from estimated distribution of returns from individual funds.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	94.95	96.97	95.93	98.15	98.93	98.98	100.00	99.67	99.91
5	94.31	96.45	95.53	97.96	98.81	98.82	100.00	99.55	99.89
10	93.62	95.94	95.06	97.71	98.71	98.71	100.00	99.41	99.85
25	91.07	94.14	93.61	96.91	98.33	98.03	100.00	98.88	99.76
50	85.40	90.12	90.71	94.80	97.58	96.17	99.98	97.64	99.34
100	71.24	78.45	83.54	87.90	94.83	89.32	99.63	90.34	97.62
200	39.74	45.14	64.90	57.98	85.66	59.35	92.72	48.91	87.48
300	16.95	17.62	45.79	21.88	71.14	21.74	70.75	11.50	66.75
400	5.44	4.84	29.47	4.18	55.10	3.78	43.57	1.33	42.69
500	1.41	1.10	18.12	0.38	38.82	0.29	23.23	0.07	22.31
1000	0.00	0.02	0.65	0.01	2.57	0.00	0.30	0.00	0.09

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	99.98	mix_h_a	99.63	mix_h_a
99.91	mix_h_b	99.89	mix_h_b	99.85	mix_h_b	99.76	mix_h_b	99.34	mix_h_b	97.62	mix_h_b
99.67	mix_m_b	99.55	mix_m_b	99.41	mix_m_b	98.88	mix_m_b	97.64	mix_m_b	94.83	PFA_h
98.98	mix_m_a	98.82	mix_m_a	98.71	PFA_h	98.33	PFA_h	97.58	PFA_h	90.34	mix_m_b
98.93	PFA_h	98.81	PFA_h	98.71	mix_m_a	98.03	mix_m_a	96.17	mix_m_a	89.32	mix_m_a
98.15	PFA_m	97.96	PFA_m	97.71	PFA_m	96.91	PFA_m	94.80	PFA_m	87.90	PFA_m
96.97	Velliv_m_l	96.45	Velliv_m_l	95.94	Velliv_m_l	94.14	Velliv_m_l	90.71	Velliv_h	83.54	Velliv_h
95.93	Velliv_h	95.53	Velliv_h	95.06	Velliv_h	93.61	Velliv_h	90.12	Velliv_m_l	78.45	Velliv_m_l
94.95	Velliv_m	94.31	Velliv_m	93.62	Velliv_m	91.07	Velliv_m	85.40	Velliv_m	71.24	Velliv_m

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
92.72	mix_h_a	71.14	PFA_h	55.10	PFA_h	38.82	PFA_h	2.57	PFA_h
87.48	mix_h_b	70.75	mix_h_a	43.57	mix_h_a	23.23	mix_h_a	0.65	Velliv_h
85.66	PFA_h	66.75	mix_h_b	42.69	mix_h_b	22.31	mix_h_b	0.30	mix_h_a
64.90	Velliv_h	45.79	Velliv_h	29.47	Velliv_h	18.12	Velliv_h	0.09	mix_h_b
59.35	mix_m_a	21.88	PFA_m	5.44	Velliv_m	1.41	Velliv_m	0.02	Velliv_m_l
57.98	PFA_m	21.74	mix_m_a	4.84	Velliv_m_l	1.10	Velliv_m_l	0.01	PFA_m
48.91	mix_m_b	17.62	Velliv_m_l	4.18	PFA_m	0.38	PFA_m	0.00	Velliv_m
45.14	Velliv_m_l	16.95	Velliv_m	3.78	mix_m_a	0.29	mix_m_a	0.00	mix_m_a
39.74	Velliv_m	11.50	mix_m_b	1.33	mix_m_b	0.07	mix_m_b	0.00	mix_m_b

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.

m is the location parameter.

s is the scale parameter.

nu is the estimated degrees of freedom, or shape parameter.

xi is the estimated skewness parameter.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770
R-squared	0.993	0.978	0.991	0.991	0.964	0.890	0.961

Fit statistics ranking

m	ranking	s	ranking	R-squared	ranking
0.084	PFA_high	0.071	mix_high	0.993	Velliv_medium
0.082	mix_high	0.088	mix_medium	0.991	Velliv_high
0.065	Velliv_high	0.115	Velliv_medium_long	0.991	PFA_medium
0.059	mix_medium	0.120	Velliv_medium	0.978	Velliv_medium_long
0.058	PFA_medium	0.121	PFA_high	0.964	PFA_high
0.052	Velliv_medium_long	0.123	PFA_medium	0.961	mix_high
0.048	Velliv_medium	0.150	Velliv_high	0.890	mix_medium

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).

Statistics are given for the final state of all paths.

Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.

0 is defined as any value below a threshold.

losing_prob_pct is the probability of losing money. This is calculated as the share of paths finishing below index 100.

Number of paths: 10000

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_m_b	mix_h_a	mix_h_b
mc_m	280.176	294.583	408.863	321.123	553.794	323.244	301.846	499.077	482.246
mc_s	125.216	121.955	219.108	106.423	245.211	98.709	81.884	158.895	163.777
mc_min	0.344	0.000	0.000	0.000	0.303	7.697	49.846	141.306	41.382
mc_max	912.296	3560.339	1674.477	2566.719	1692.399	696.259	641.004	1344.969	1339.403
dao_pct	0.000	0.010	0.010	0.020	0.000	0.000	0.000	0.000	0.000
losing_pct	5.050	3.030	4.070	1.850	1.070	1.020	0.330	0.000	0.090

Ranking

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	losing_pct	ranking
553.794	PFA_h	81.884	mix_m_b	141.306	mix_h_a	3560.339	Velliv_m_l	0.00	Velliv_m	0.00	mix_h_a
499.077	mix_h_a	98.709	mix_m_a	49.846	mix_m_b	2566.719	PFA_m	0.00	PFA_h	0.09	mix_h_b
482.246	mix_h_b	106.423	PFA_m	41.382	mix_h_b	1692.399	PFA_h	0.00	mix_m_a	0.33	mix_m_b
408.863	Velliv_h	121.955	Velliv_m_l	7.697	mix_m_a	1674.477	Velliv_h	0.00	mix_m_b	1.02	mix_m_a
323.244	mix_m_a	125.216	Velliv_m	0.344	Velliv_m	1344.969	mix_h_a	0.00	mix_h_a	1.07	PFA_h
321.123	PFA_m	158.895	mix_h_a	0.303	PFA_h	1339.403	mix_h_b	0.00	mix_h_b	1.85	PFA_m
301.846	mix_m_b	163.777	mix_h_b	0.000	Velliv_m_l	912.296	Velliv_m	0.01	Velliv_m_l	3.03	Velliv_m_l
294.583	Velliv_m_l	219.108	Velliv_h	0.000	Velliv_h	696.259	mix_m_a	0.01	Velliv_h	4.07	Velliv_h
280.176	Velliv_m	245.211	PFA_h	0.000	PFA_m	641.004	mix_m_b	0.02	PFA_m	5.05	Velliv_m

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} \right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2} \right) / \left(\frac{x_{t-1} + y_{t-1}}{2} \right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns = Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} \right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2 \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$c\{x_t + y_t\}\{x_{t-1} + y_{t-1}\}$

$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1}y_t = 2(x_{t-1}y_{t-1}x_t + x_{t-1}y_{t-1}y_t) \$\$$

$$(x_{t-1}x_1y_{t-1} + y_{t-1}x_ty_{t-1}) + (x_{t-1}x_ty_{t-1}y_t + x_{t-1}y_{t-1}y_t) = 2(x_{t-1}y_{t-1}x_t + x_{t-1}y_{t-1}y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: $R = 1+r$

Let x_0 be 100.

Let y_0 be 200.

So the initial value of the pf is 300 .

Let R_x be 0.5.

Let R_y be 1.5.

Then,

x_1 is $R_x * x_0 = 50$.

y_1 is $R_y * y_0 = 300$.

Average of returns:

$0.5 * (R_x + R_y) = 1$

So here the value of the pf at t=1 should be unchanged from t=0:

$(x_0 + y_0) * 0.5 * (R_x + R_y) = 300$

But this is clearly not the case:

$0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175$

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

$0.5 * (\log(R_x) + \log(R_y)) = -0.143841$

We now get:

$(x_0 + y_0) * \exp(0.5 * (\log(R_x) + \log(R_y))) = 259.8076$

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): -0.1138774
## s(data_x): 0.3973582
## m(data_y): 10.17686
## s(data_y): 3.685686
##
## m(data_x + data_y): 5.03149
## s(data_x + data_y): 1.833164
```

m and s of final state of all paths.

_a is mix of simulated returns.

_b is simulated mixed returns.

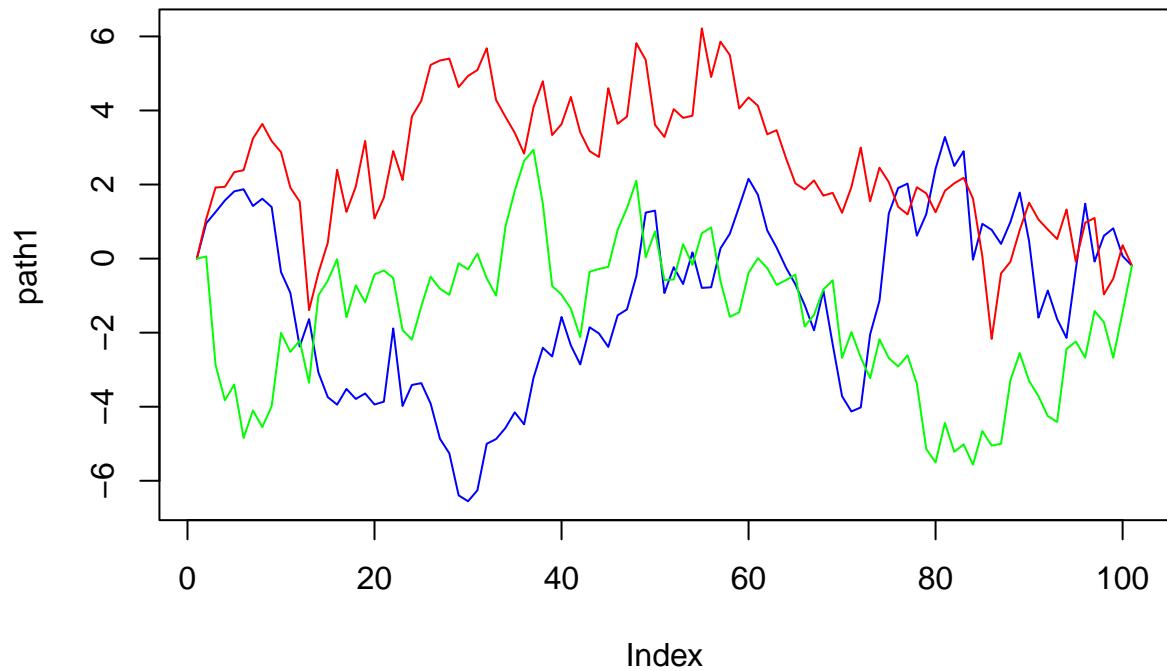
	m_a	m_b	s_a	s_b
100.304	100.357	8.610	8.345	
100.476	100.604	8.587	7.884	
100.453	100.713	8.280	8.120	
100.715	100.249	8.424	8.187	
100.185	100.683	7.899	7.893	
100.925	100.201	8.531	7.888	
100.933	100.416	8.174	8.296	
100.560	101.101	8.277	8.071	
100.517	100.642	8.539	8.141	
100.320	100.887	8.293	8.141	

```
##      m_a          m_b          s_a          s_b
## Min. :100.2  Min. :100.2  Min. :7.899  Min. :7.884
## 1st Qu.:100.4  1st Qu.:100.4  1st Qu.:8.278  1st Qu.:7.938
## Median :100.5  Median :100.6  Median :8.359  Median :8.131
## Mean   :100.5  Mean   :100.6  Mean   :8.362  Mean   :8.097
## 3rd Qu.:100.7  3rd Qu.:100.7  3rd Qu.:8.537  3rd Qu.:8.176
## Max.  :100.9  Max.  :101.1  Max.  :8.610  Max.  :8.345
```

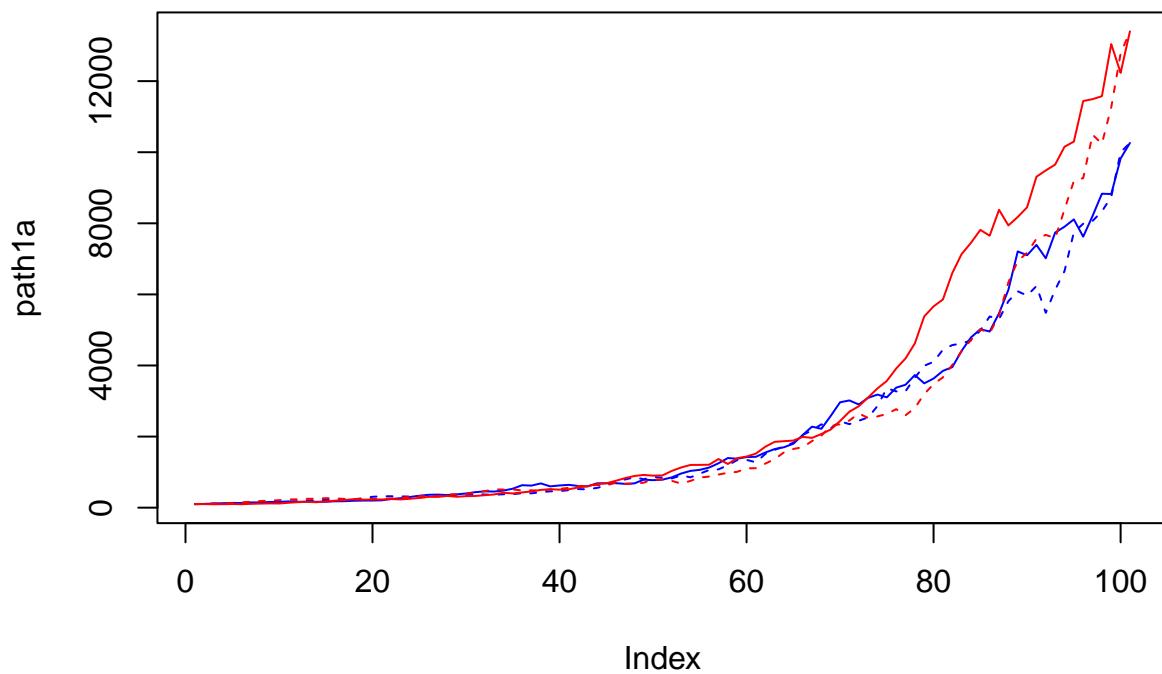
_a and _b are very close to equal.

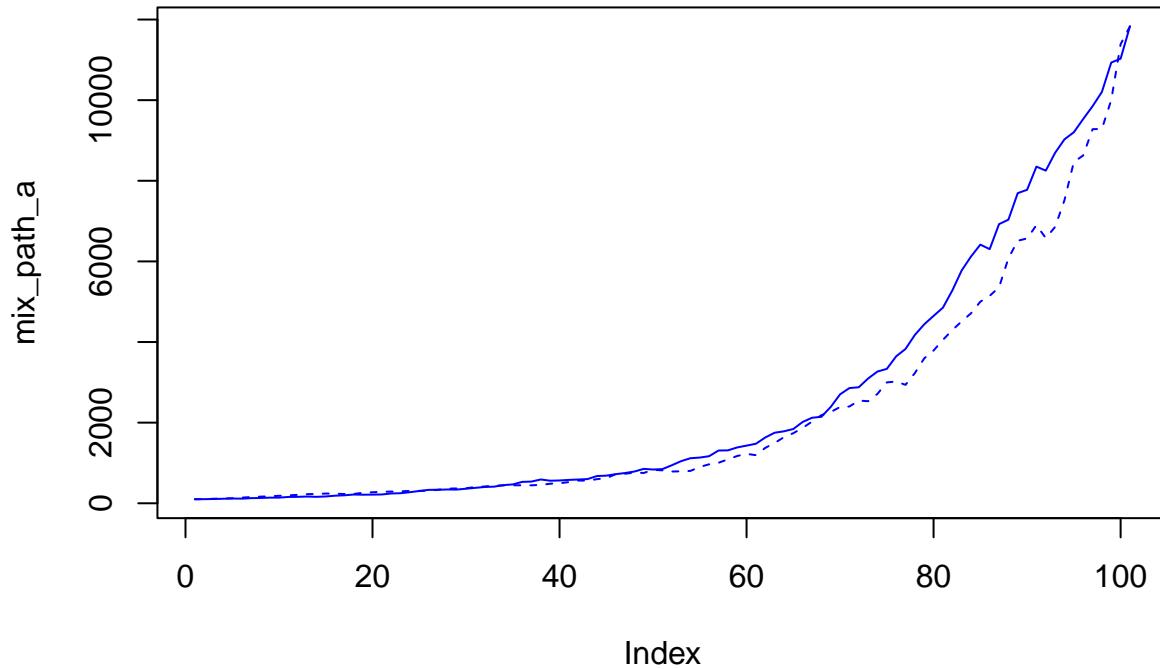
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?





The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?

Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##          m              s
##  Min. :0.06079  Min.  :0.04880
##  1st Qu.:0.06610  1st Qu.:0.06051
##  Median :0.07000  Median :0.06707
##  Mean   :0.07110  Mean   :0.07048
##  3rd Qu.:0.07486  3rd Qu.:0.08125
##  Max.   :0.08461  Max.   :0.09220
```