

Pension returns analysis

12:02 18 April 2024

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Fit log returns to F-S skew standardized Student-t distribution.

m is the location parameter.

s is the scale parameter.

ν is the estimated shape parameter (degrees of freedom).

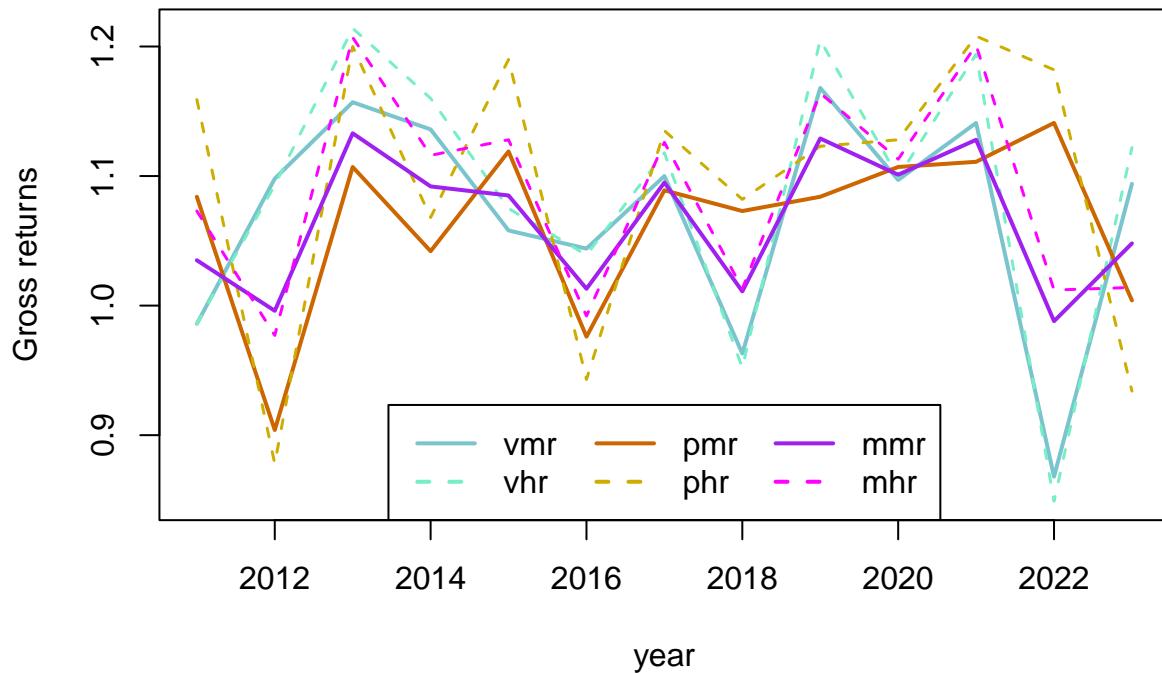
ξ is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.

`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.

Gross returns 2011–2023



Summary of gross returns

```

##      vmr        pmr        mmr        vhr
## Min.  :0.868  Min.  :0.904  Min.  :0.988  Min.  :0.849
## 1st Qu.:1.044  1st Qu.:1.042  1st Qu.:1.013  1st Qu.:1.039
## Median :1.097  Median :1.084  Median :1.085  Median :1.099
## Mean   :1.070  Mean   :1.065  Mean   :1.066  Mean   :1.085
## 3rd Qu.:1.136  3rd Qu.:1.107  3rd Qu.:1.101  3rd Qu.:1.160
## Max.   :1.168  Max.   :1.141  Max.   :1.133  Max.   :1.214
##      phr        mhr
## Min.  :0.878  Min.  :0.977
## 1st Qu.:1.068  1st Qu.:1.013
## Median :1.128  Median :1.113
## Mean   :1.095  Mean   :1.087
## 3rd Qu.:1.182  3rd Qu.:1.128
## Max.   :1.208  Max.   :1.207

##      vmrl
## Min.  :0.801
## 1st Qu.:1.013
## Median :1.085
## Mean   :1.061
## 3rd Qu.:1.128
## Max.   :1.193

##      vmr    pmr    mmr    vhr    phr    mhr
## Min.  : 0.868 0.904 0.988 0.849 0.878 0.977
## 1st Qu.: 1.044 1.042 1.013 1.039 1.068 1.013
## Median : 1.097 1.084 1.085 1.099 1.128 1.113
## Mean   : 1.070 1.065 1.066 1.085 1.095 1.087
## 3rd Qu.: 1.136 1.107 1.101 1.160 1.182 1.128
## Max.   : 1.168 1.141 1.133 1.214 1.208 1.207

```

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.136	vmr	1.168	vmr
0.977	mhr	1.044	vmr	1.113	mhr	1.087	mhr	1.107	pmr	1.141	pmr
0.904	pmr	1.042	pmr	1.099	vhr	1.085	vhr	1.101	mmr	1.133	mmr
0.878	phr	1.039	vhr	1.097	vmr	1.070	vmr	1.160	vhr	1.214	vhr
0.868	vmr	1.013	mmr	1.085	mmr	1.066	mmr	1.182	phr	1.208	phr
0.849	vhr	1.013	mhr	1.084	pmr	1.065	pmr	1.128	mhr	1.207	mhr

Covariance

```
## cov(vmr, pmr) = -0.001094875
```

```
## cov(vhr, phr) = -0.0001730651
```

Gaussian fits

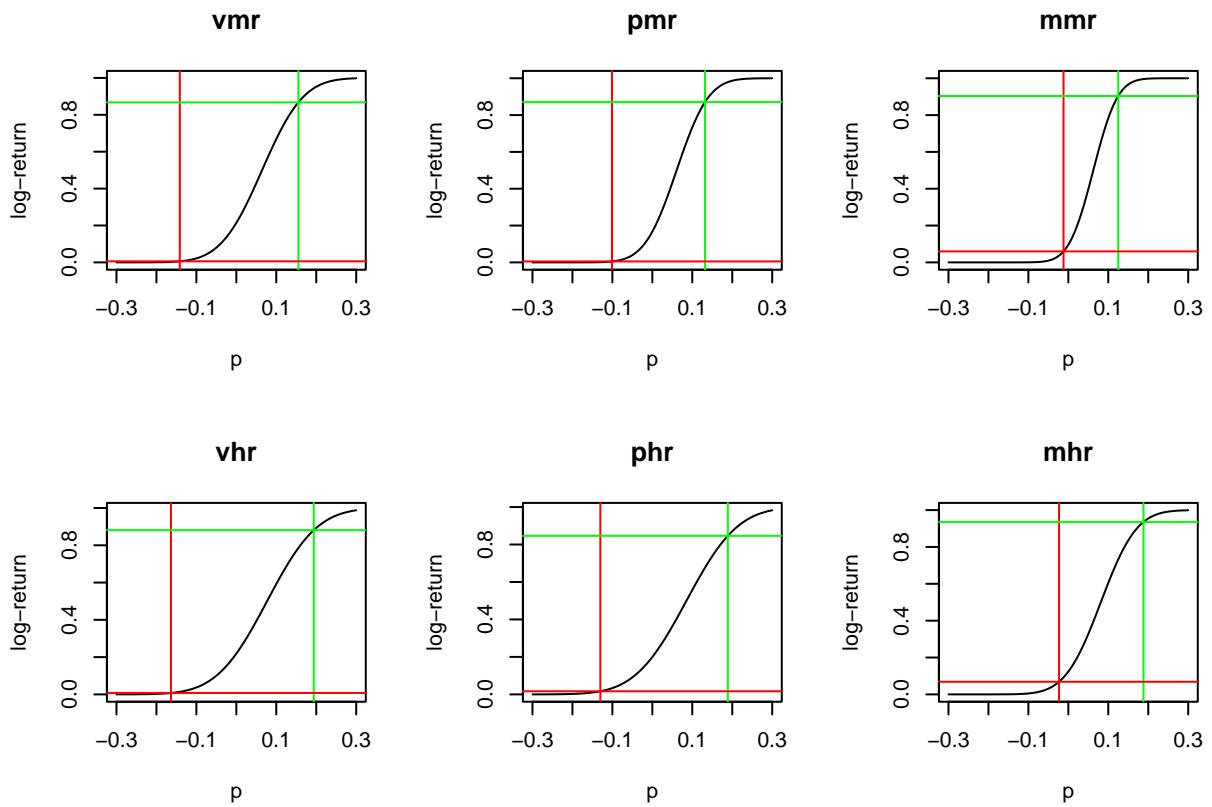
	vmr	pmr	mmr	vhr	phr	mhr
m	0.064	0.061	0.062	0.077	0.085	0.081
s	0.081	0.063	0.048	0.099	0.101	0.070

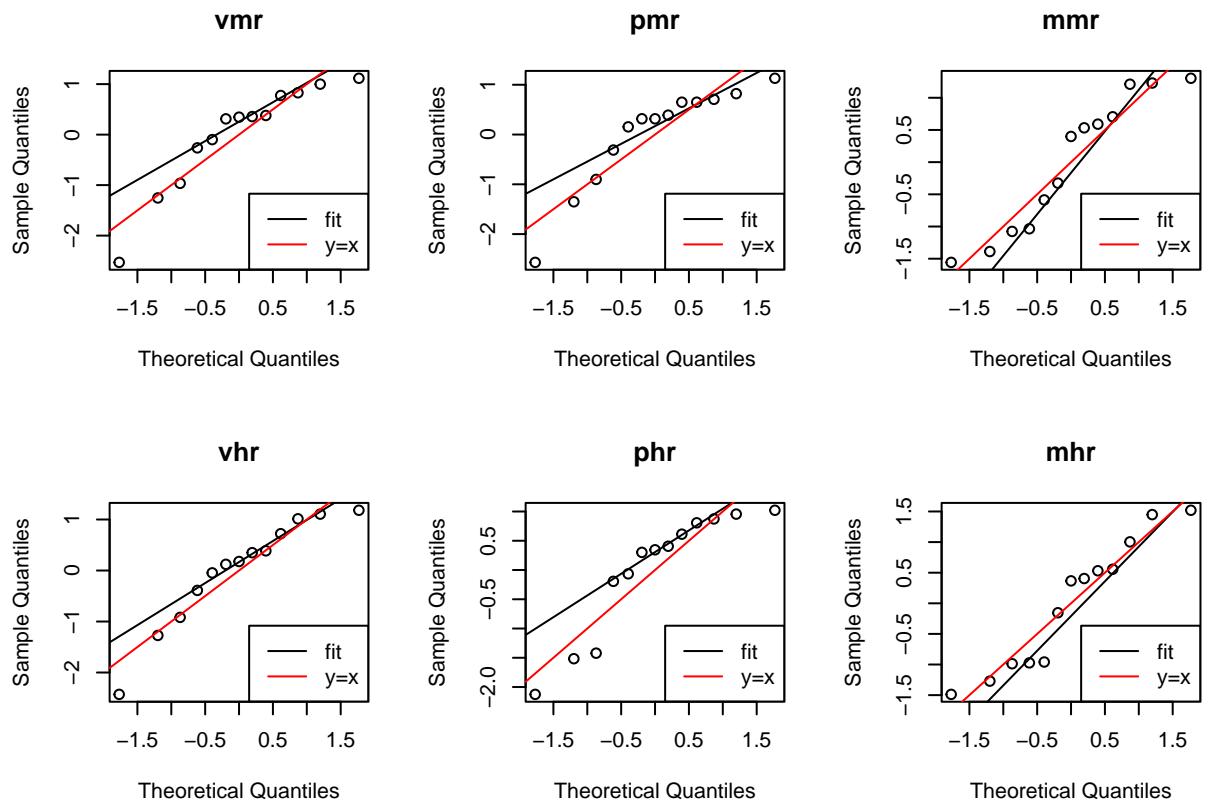
Probability in percent that the smallest and largest (respectively) observed return for each fund was generated by a normal distribution:

	vmr	pmr	mmr	vhr	phr	mhr
P(X_min)	0.754	0.68	6.729	0.982	2.055	7.645
P(X_max)	14.190	13.88	10.528	12.822	16.327	7.212

Average number of years between min or max events (respectively):

	vmr	pmr	mmr	vhr	phr	mhr
avg yrs btw min	132.699	146.958	14.860	101.874	48.659	13.080
avg yrs btw max	7.047	7.205	9.499	7.799	6.125	13.866



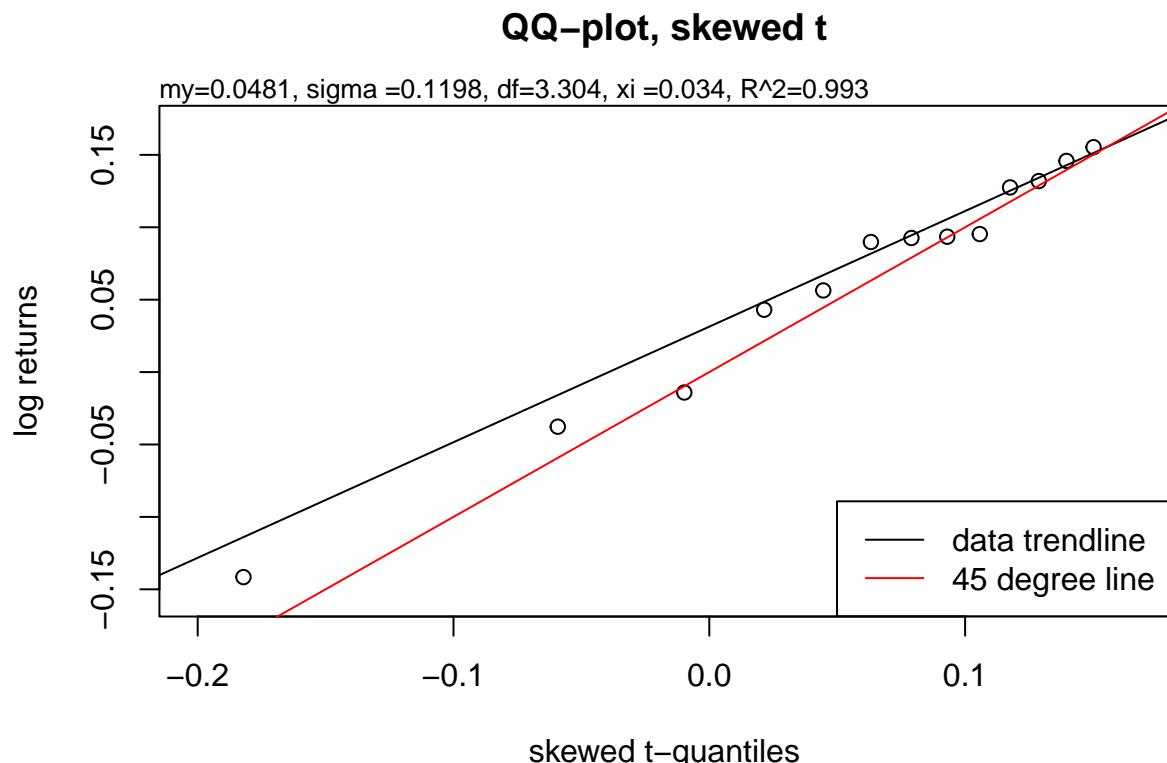


Gaussian QQ plots

Velliv medium risk, 2011 - 2023

```
##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 7.4 percent
## What is the risk of losing max 25 %? =< 1.8 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 41 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

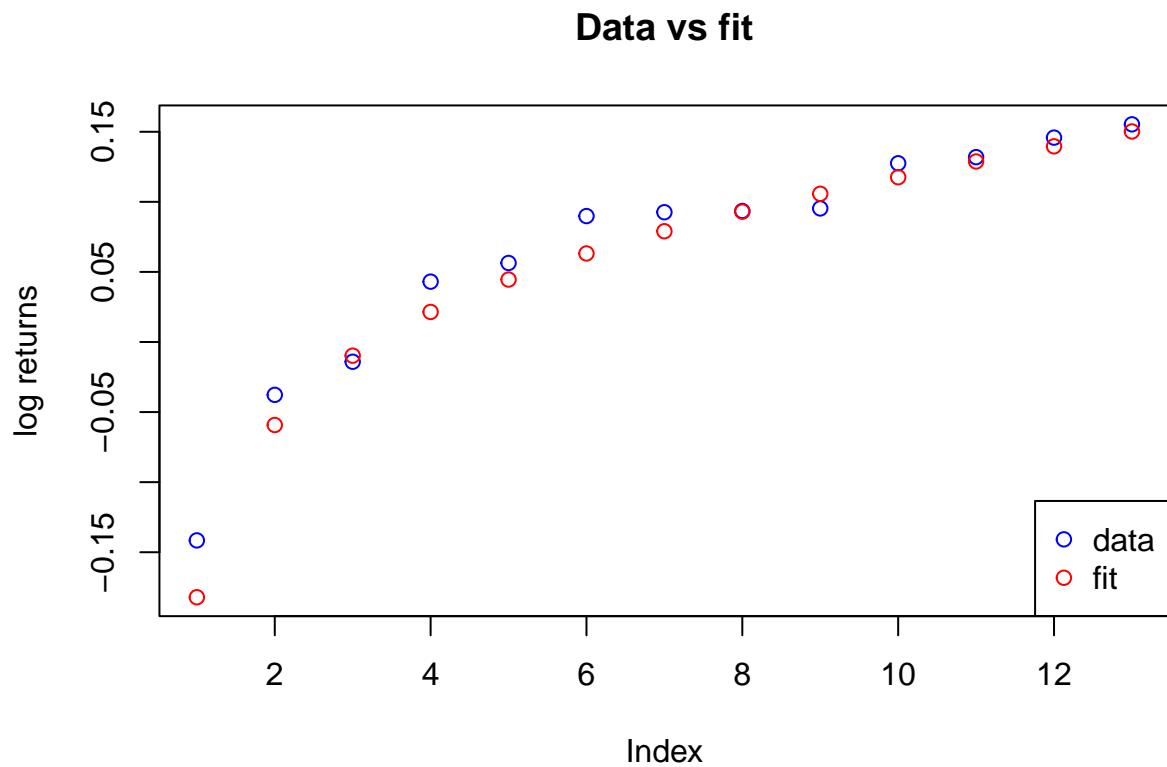
QQ Plot



The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

Data vs fit

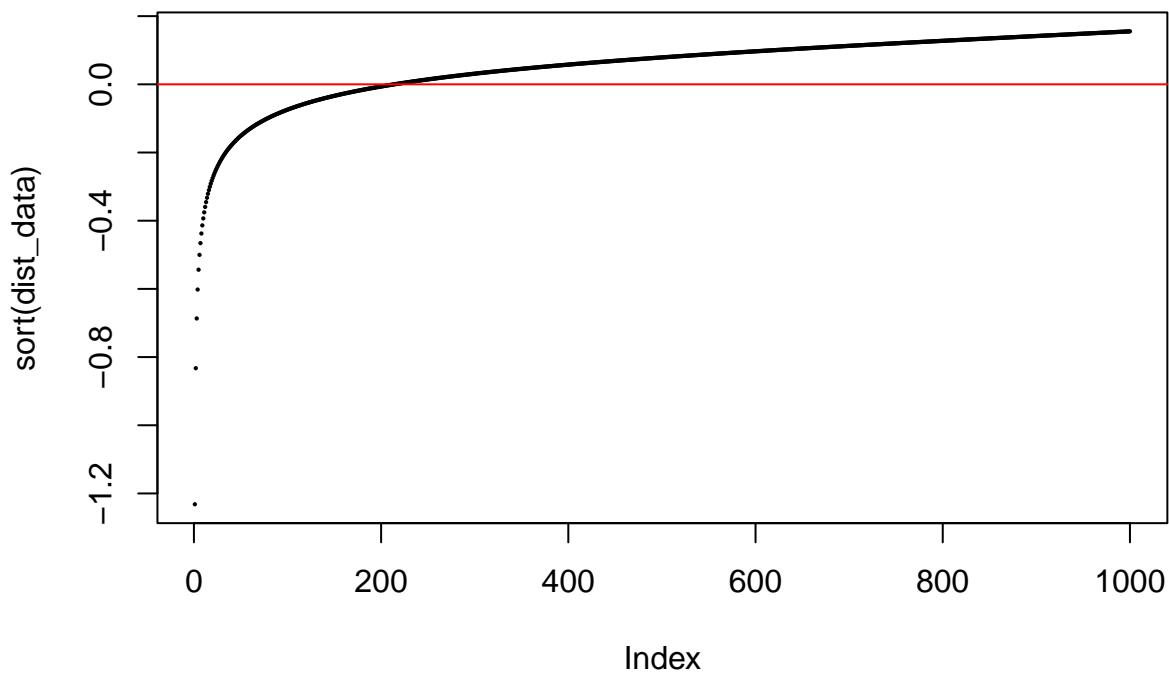
Let's plot the fit and the observed returns together.



Estimated distribution

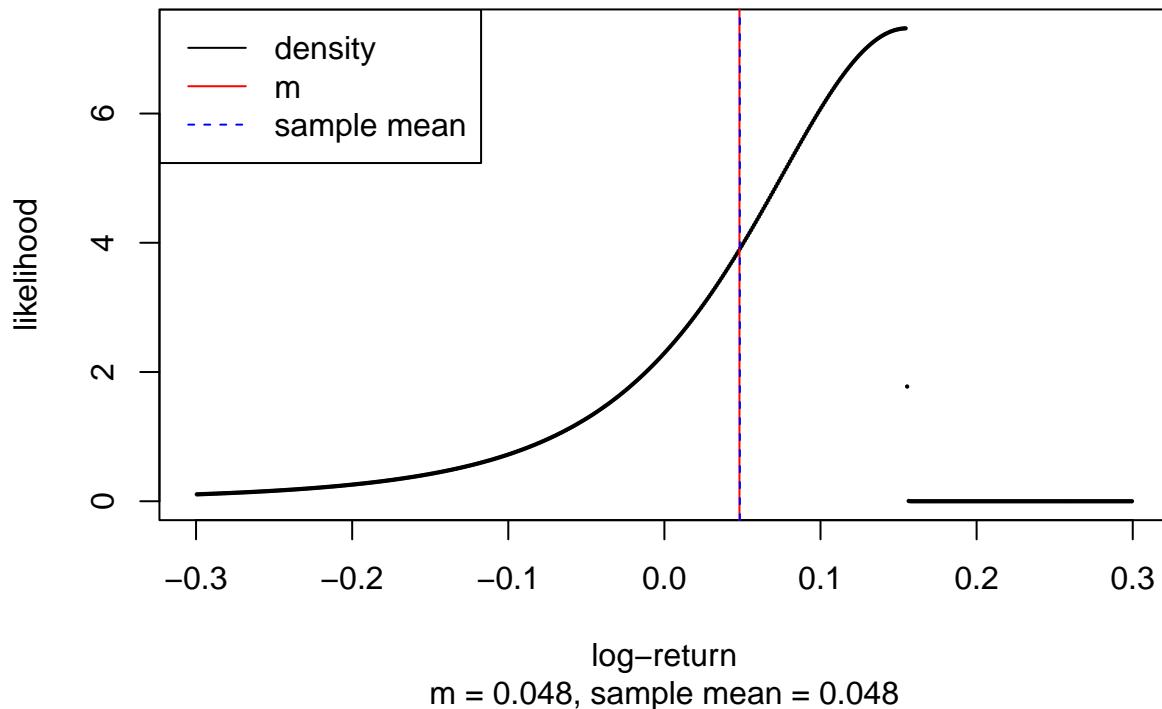
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

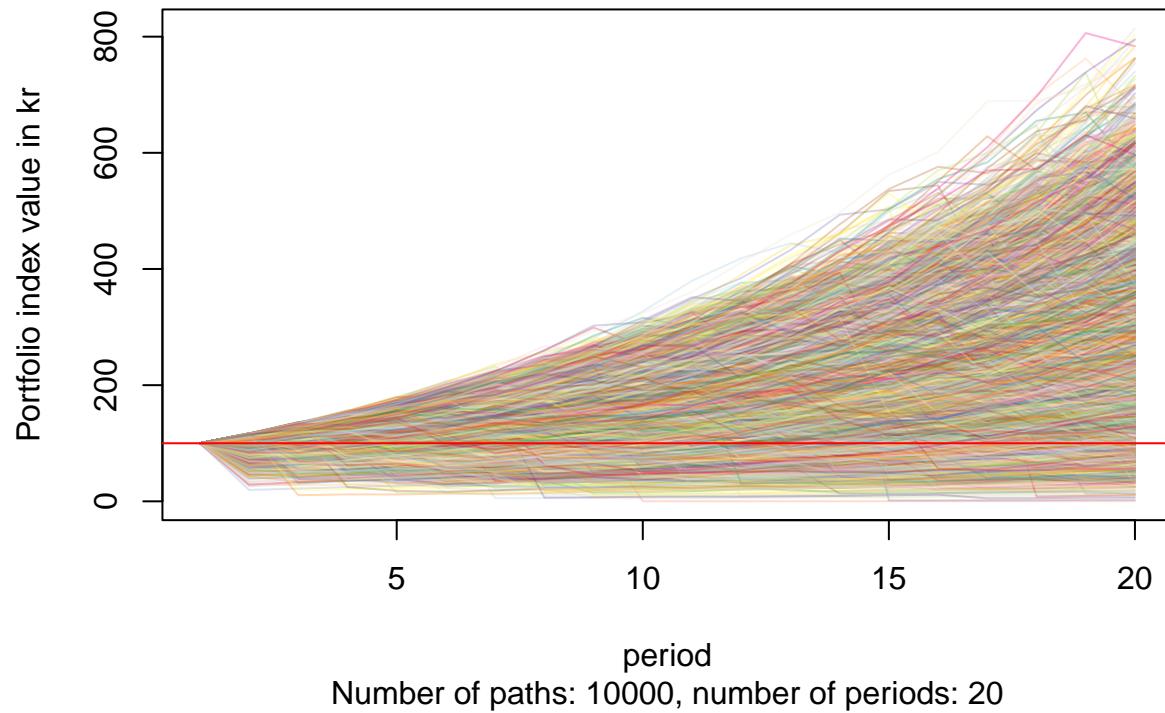
Estimated skew t distribution PDF



Monte Carlo

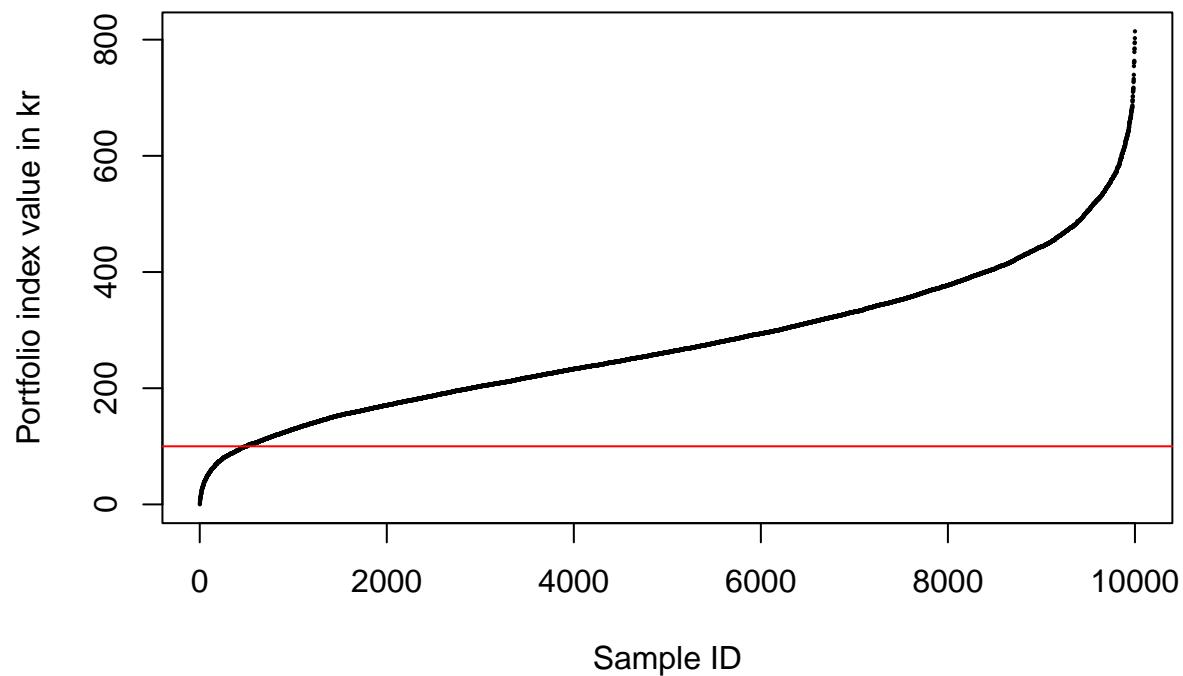
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 277.26 kr.  
## SD of portfolio index value after 20 years: 123.712 kr.  
## Min total portfolio index value after 20 years: 0.283 kr.  
## Max total portfolio index value after 20 years: 814.308 kr.  
##  
## Share of paths finishing below 100: 4.97 percent
```

MC simulation with down-and-out



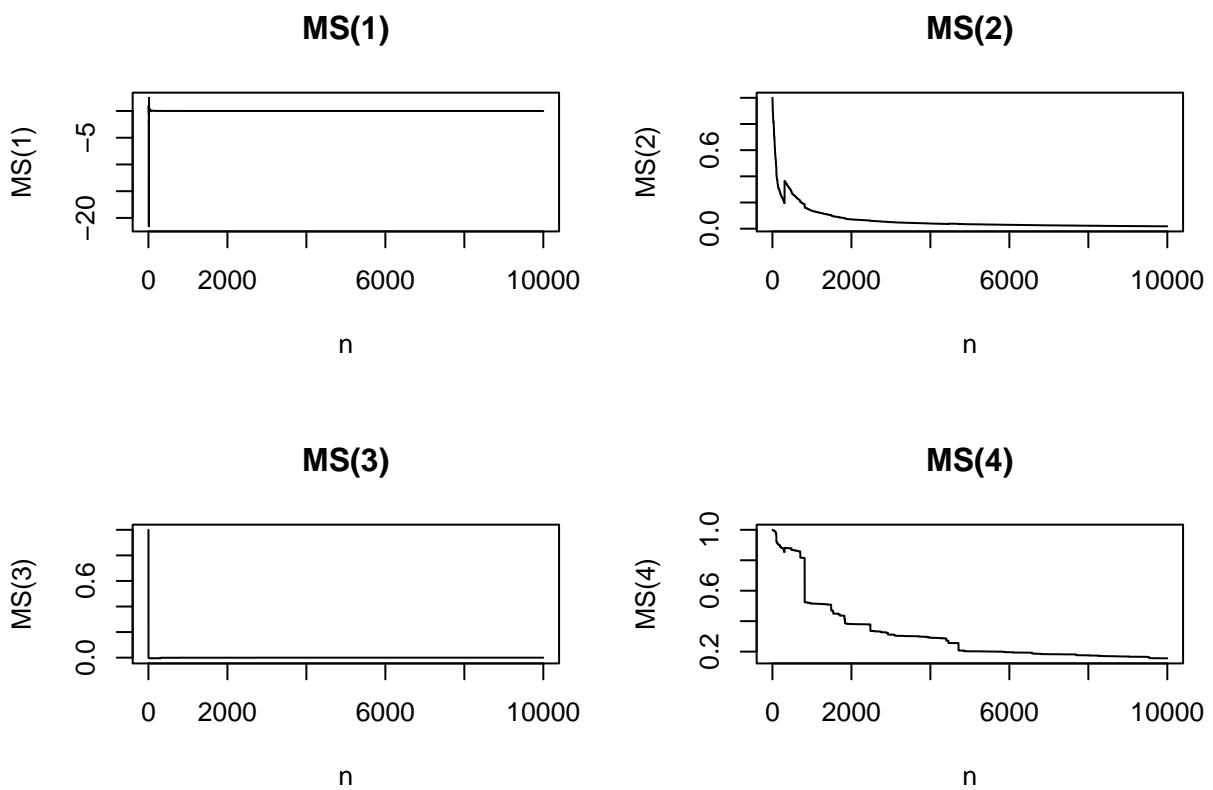
Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)



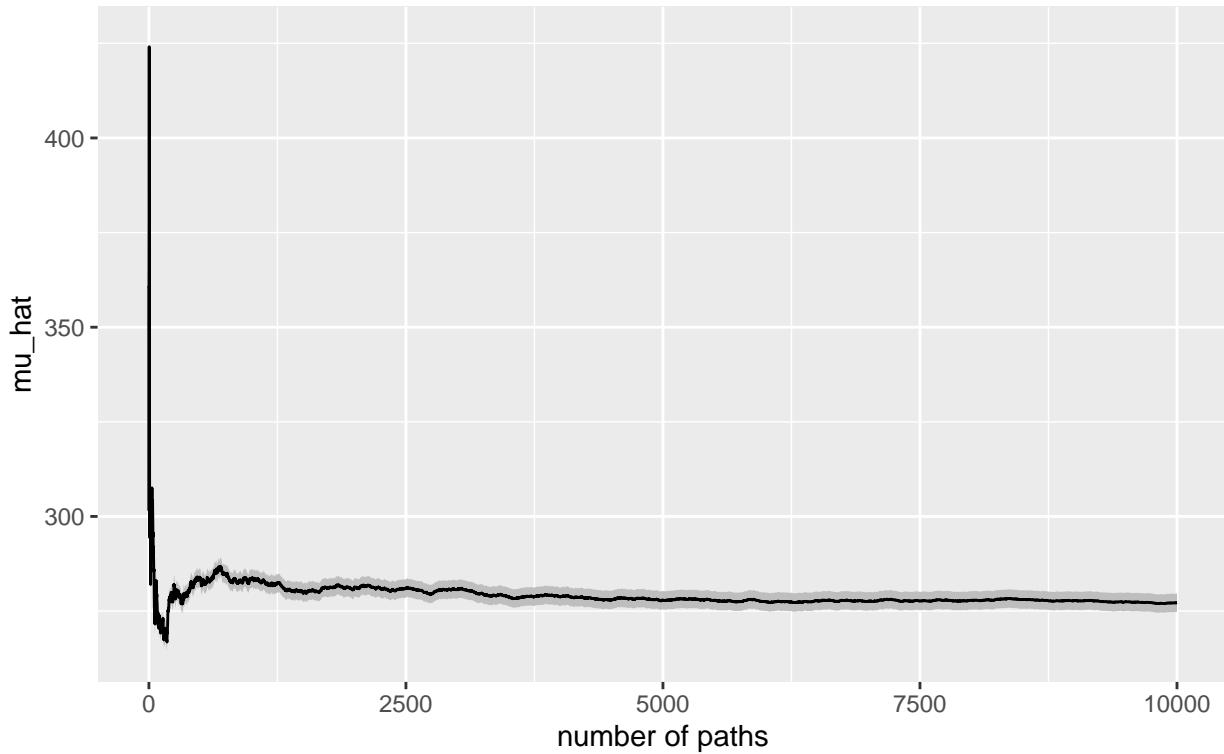
Convergence

Max vs sum plots for the first four moments:



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

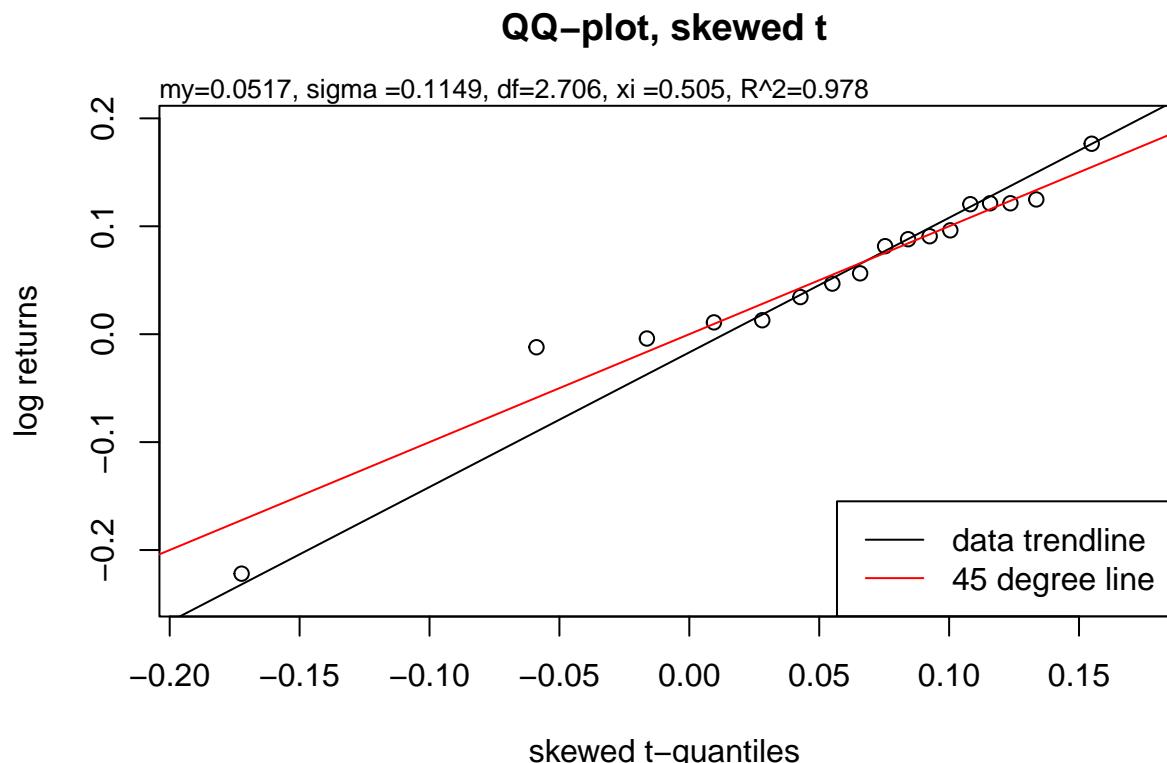


Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##  
## AIC: -34.35752  
## BIC: -31.02467  
## m: 0.05171176  
## s: 0.1149408  
## nu (df): 2.706099  
## xi: 0.5049945  
## R^2: 0.978  
##  
## An R^2 of 0.978 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 5.4 percent  
## What is the risk of losing max 25 %? <= 1.3 percent  
## What is the risk of losing max 50 %? <= 0.2 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 36.2 percent  
## What is the chance of gaining min 25 %? >= 0.3 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

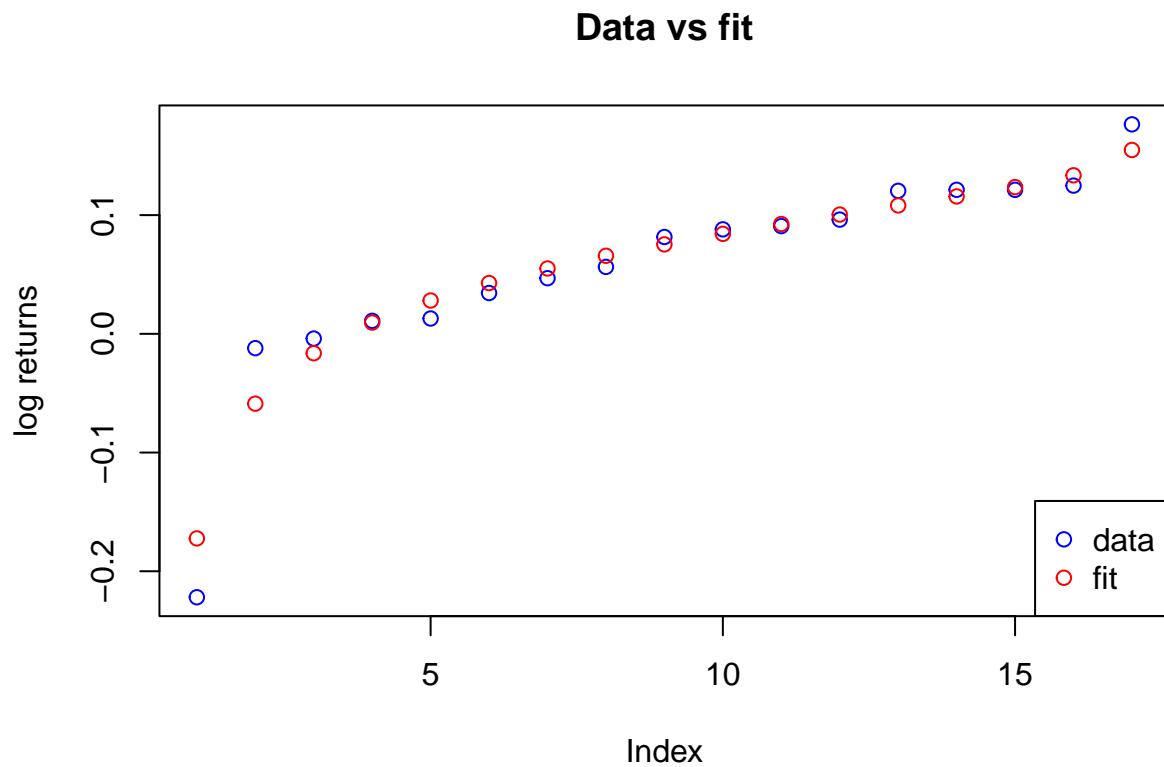
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

Data vs fit

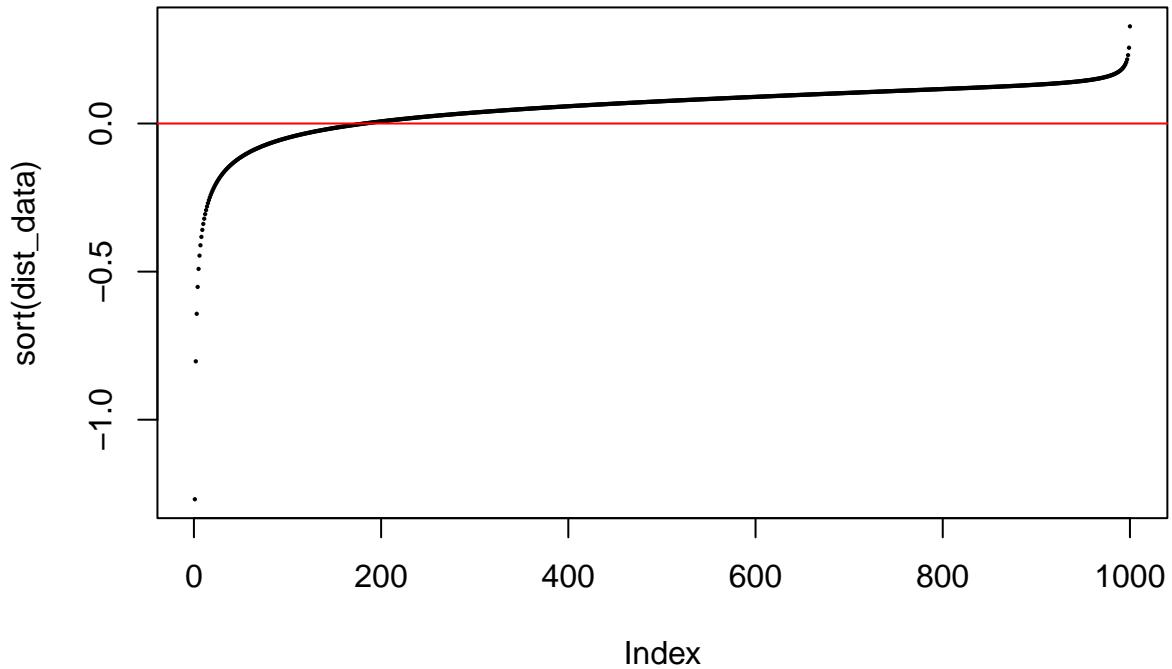
Let's plot the fit and the observed returns together.



Estimated distribution

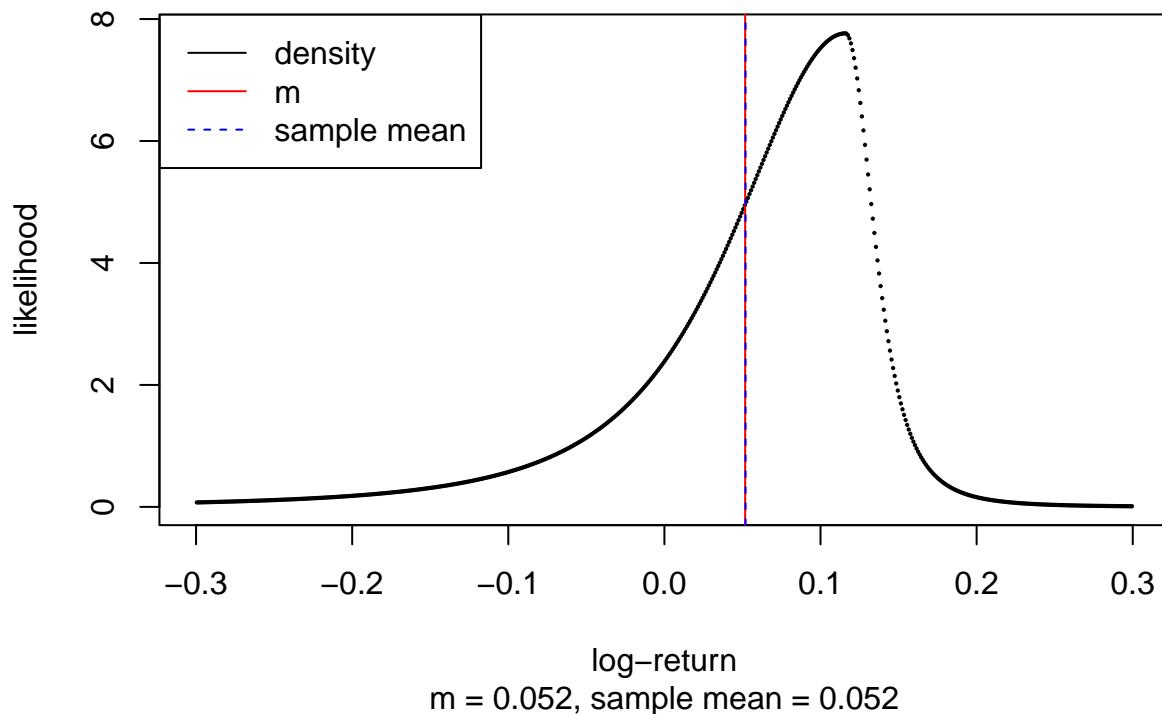
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!

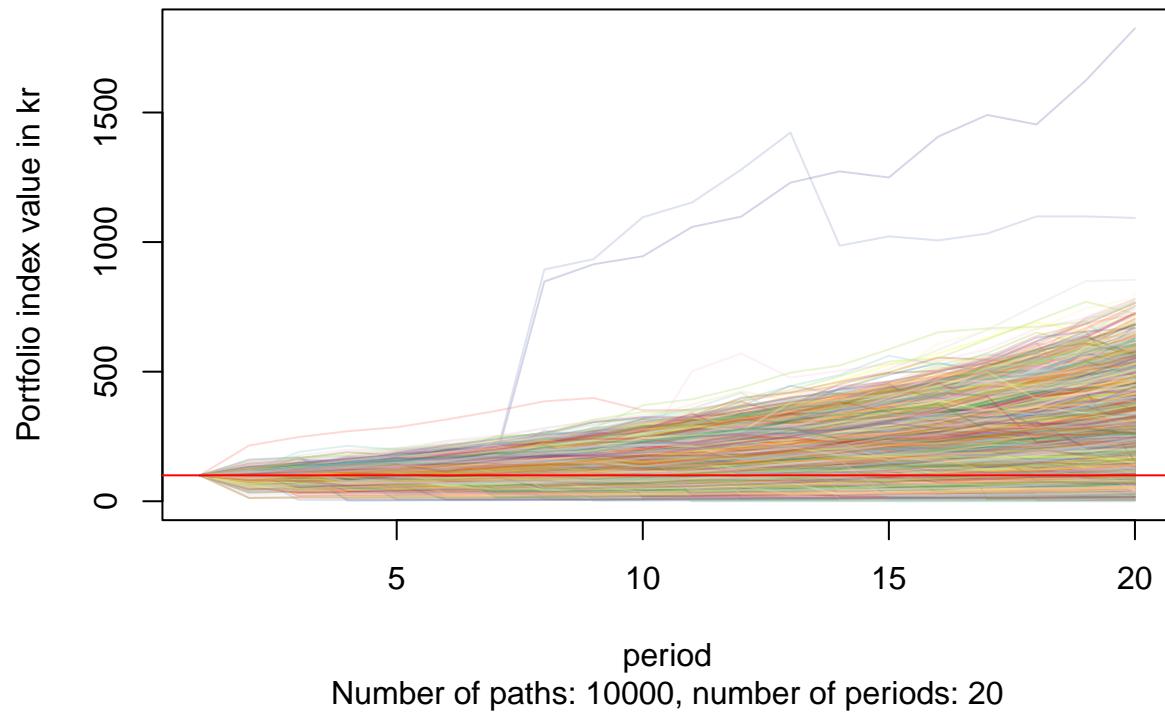
Estimated skew t distribution PDF



Monte Carlo

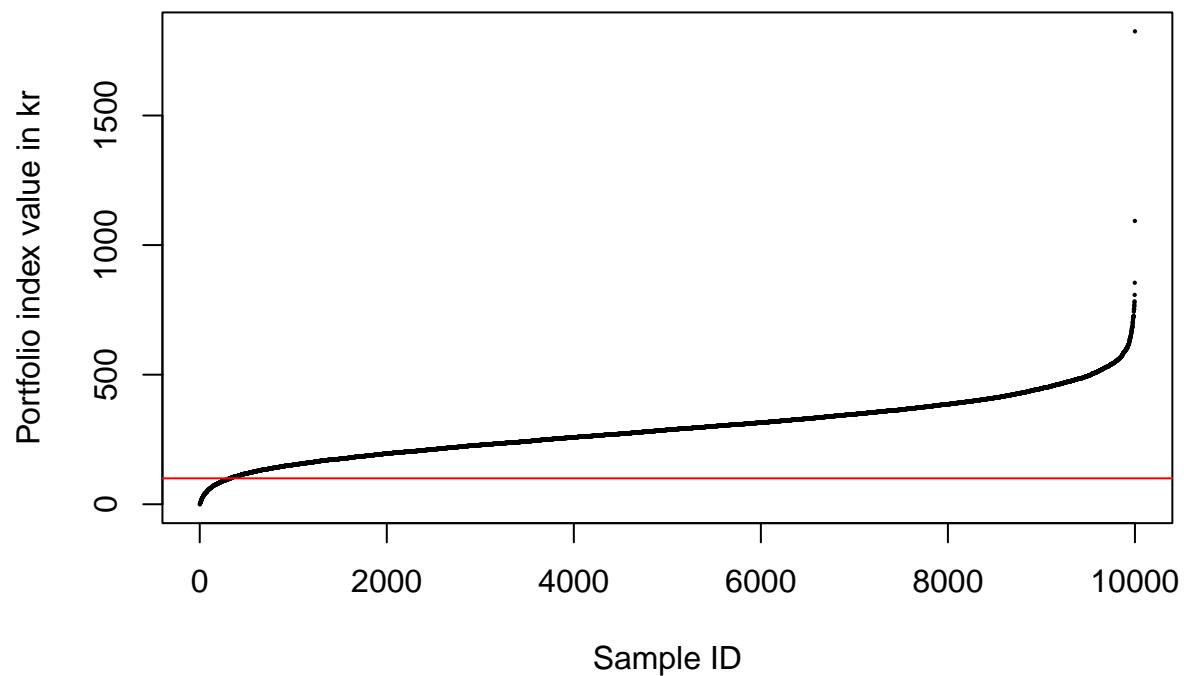
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 293.822 kr.  
## SD of portfolio index value after 20 years: 116.562 kr.  
## Min total portfolio index value after 20 years: 0.05 kr.  
## Max total portfolio index value after 20 years: 1824.687 kr.  
##  
## Share of paths finishing below 100: 3.25 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

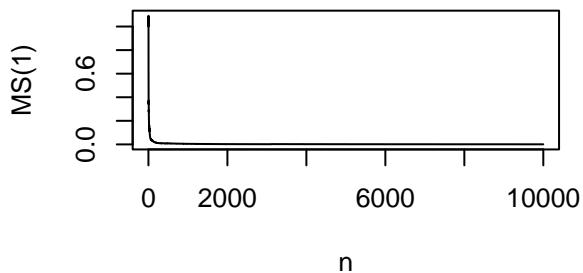
(100 is par, 200 is double, 50 is half)



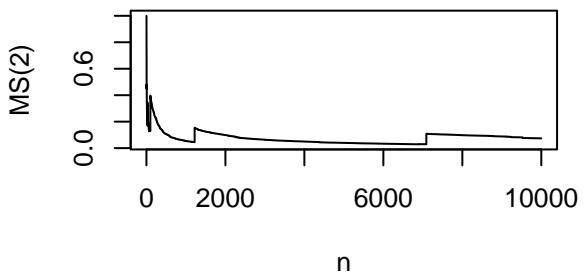
Convergence

Max vs sum plots for the first four moments:

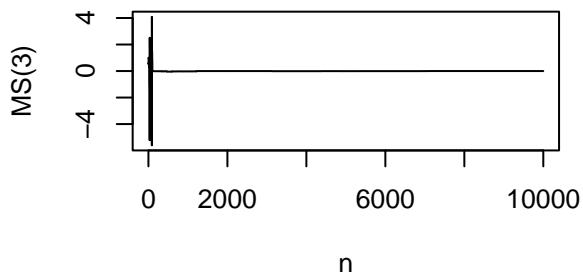
MS(1)



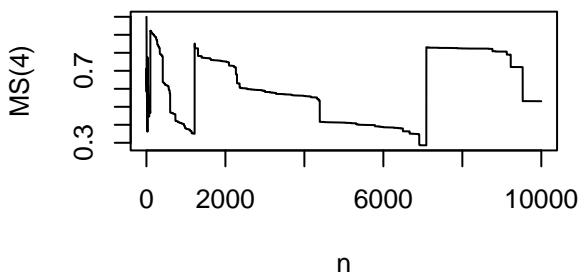
MS(2)



MS(3)

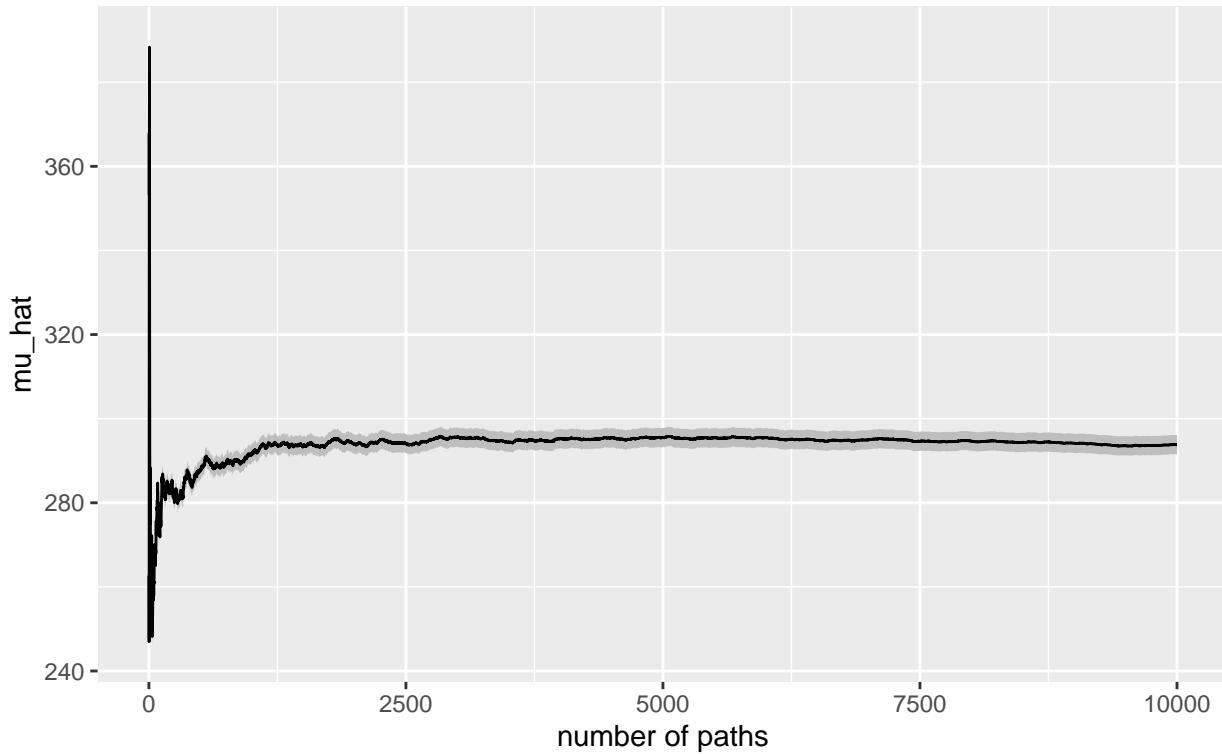


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

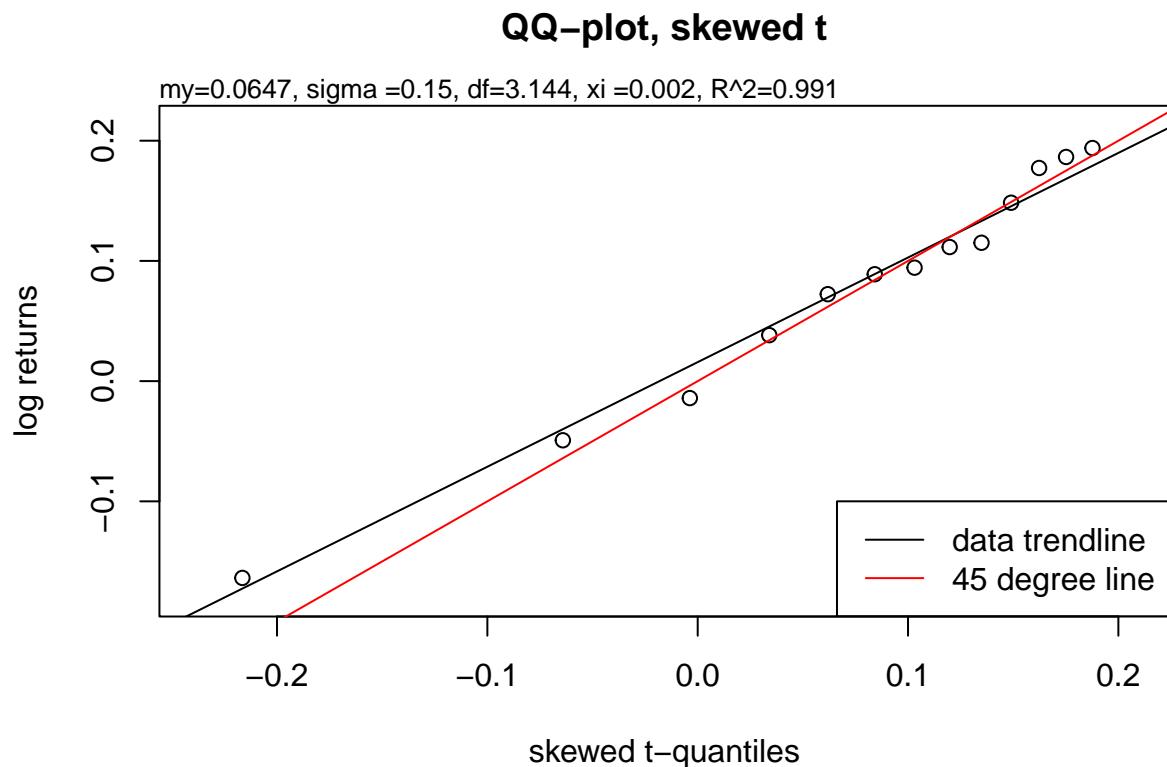


Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -21.42488  
## BIC: -19.16508  
## m: 0.06471454  
## s: 0.1499924  
## nu (df): 3.144355  
## xi: 0.002367034  
## R^2: 0.991  
##  
## An R^2 of 0.991 suggests that the fit is extremely good.  
##  
## What is the risk of losing max 10 %? <= 8.3 percent  
## What is the risk of losing max 25 %? <= 2.5 percent  
## What is the risk of losing max 50 %? <= 0.4 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 53.3 percent  
## What is the chance of gaining min 25 %? >= 0 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

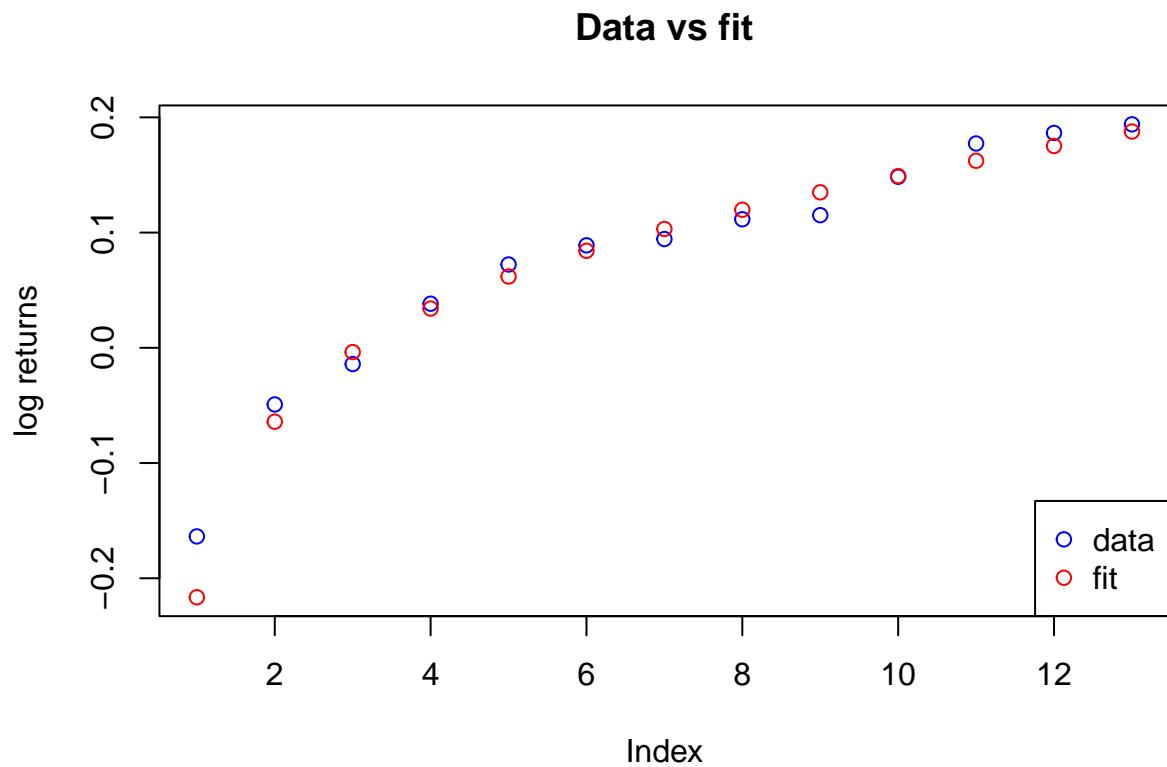
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

Data vs fit

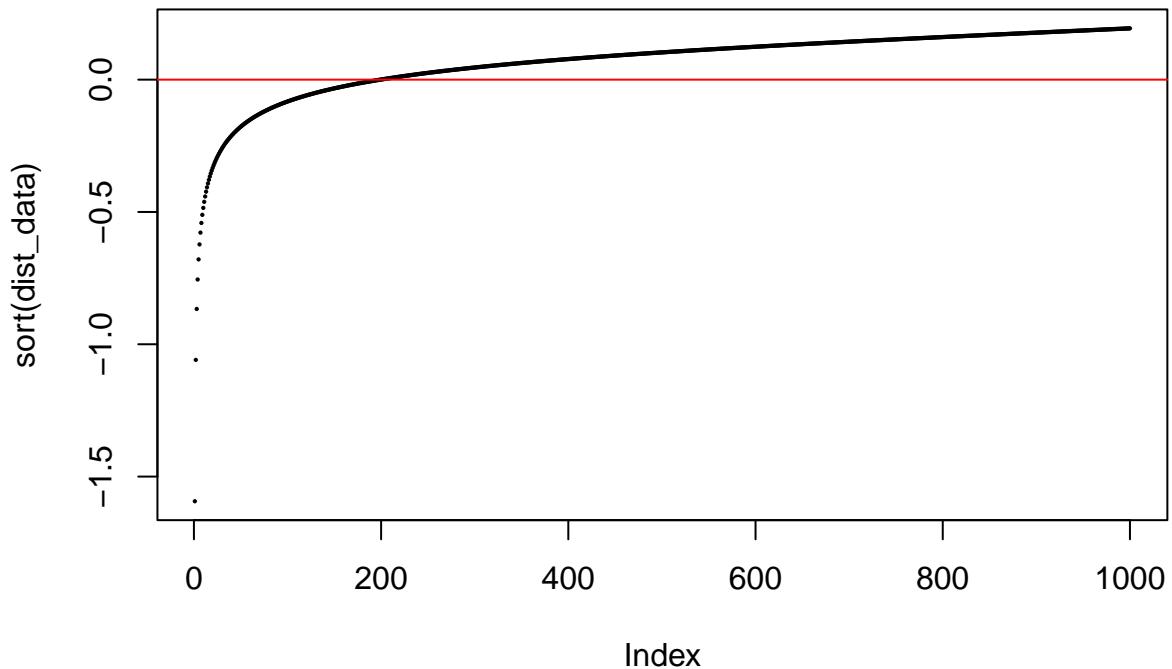
Let's plot the fit and the observed returns together.



Estimated distribution

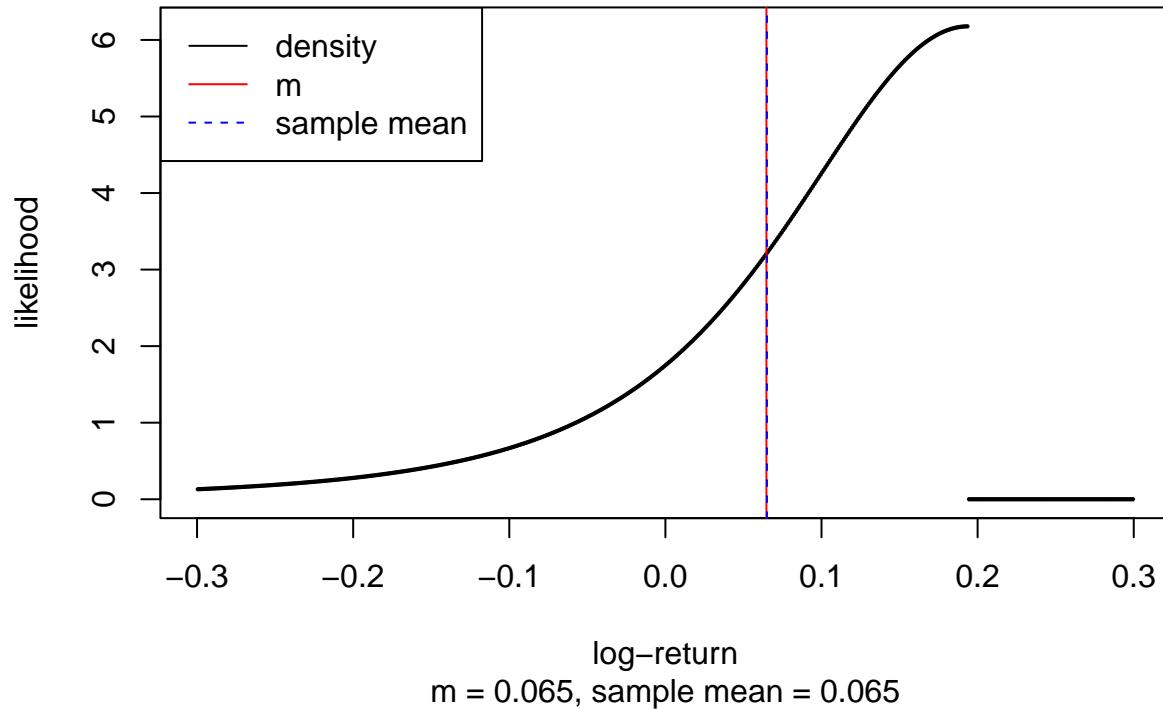
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

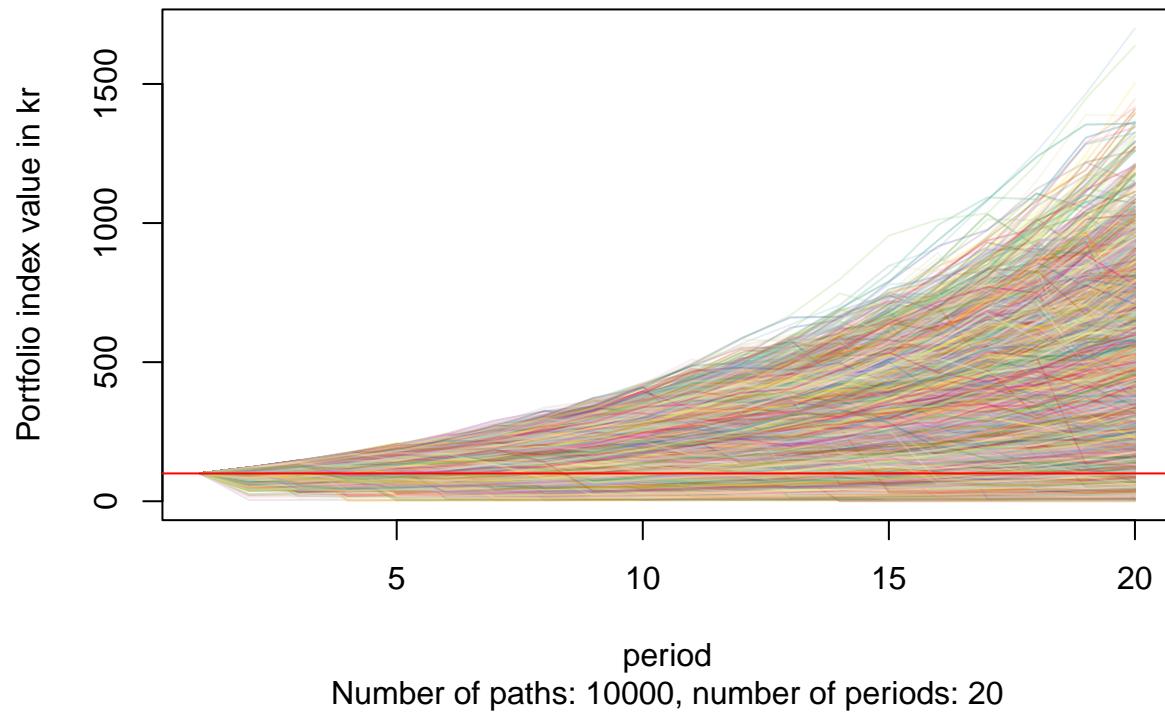
Estimated skew t distribution PDF



Monte Carlo

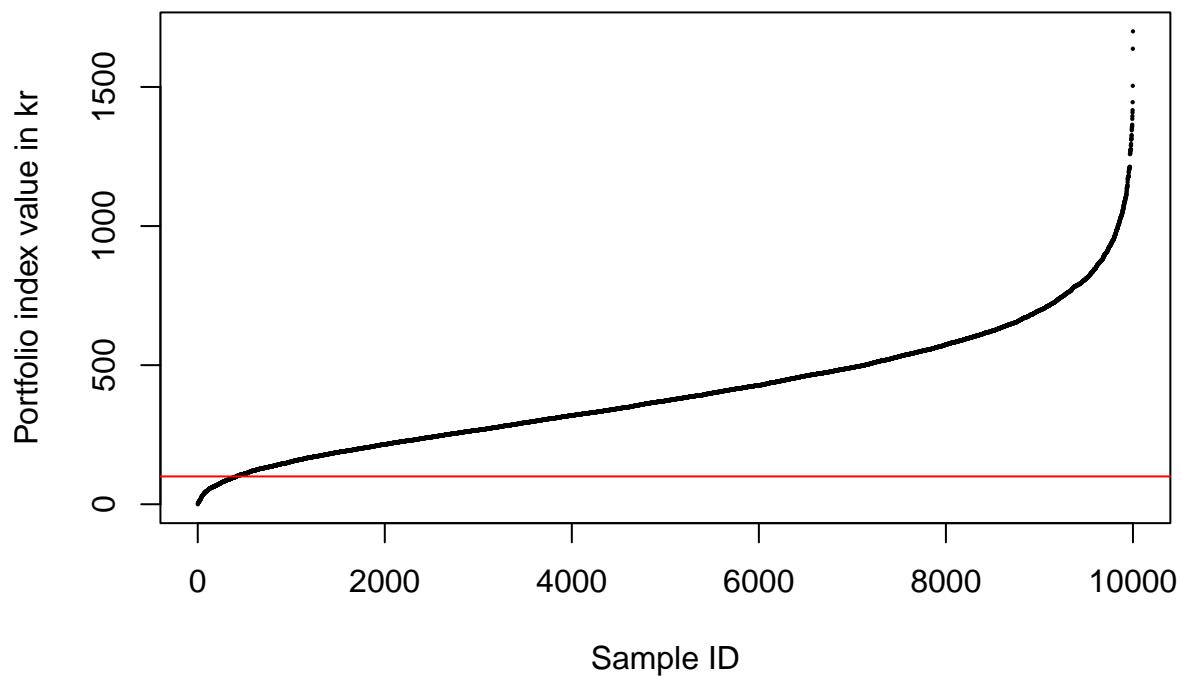
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.01 percent  
##  
## Mean portfolio index value after 20 years: 404.547 kr.  
## SD of portfolio index value after 20 years: 220.357 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 1699.981 kr.  
##  
## Share of paths finishing below 100: 4.15 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

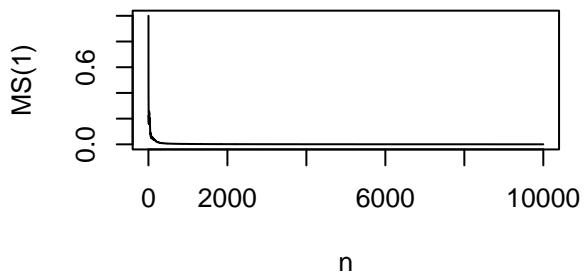
(100 is par, 200 is double, 50 is half)



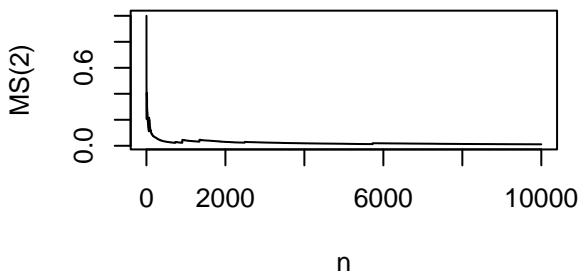
Convergence

Max vs sum plots for the first four moments:

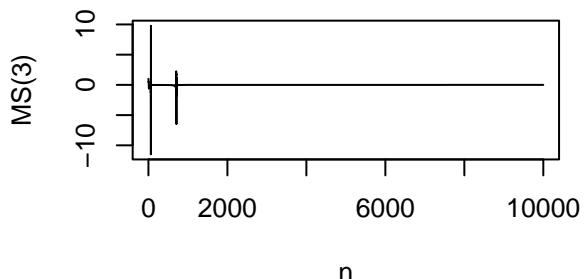
MS(1)



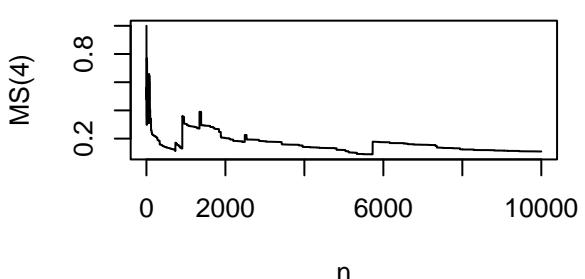
MS(2)



MS(3)

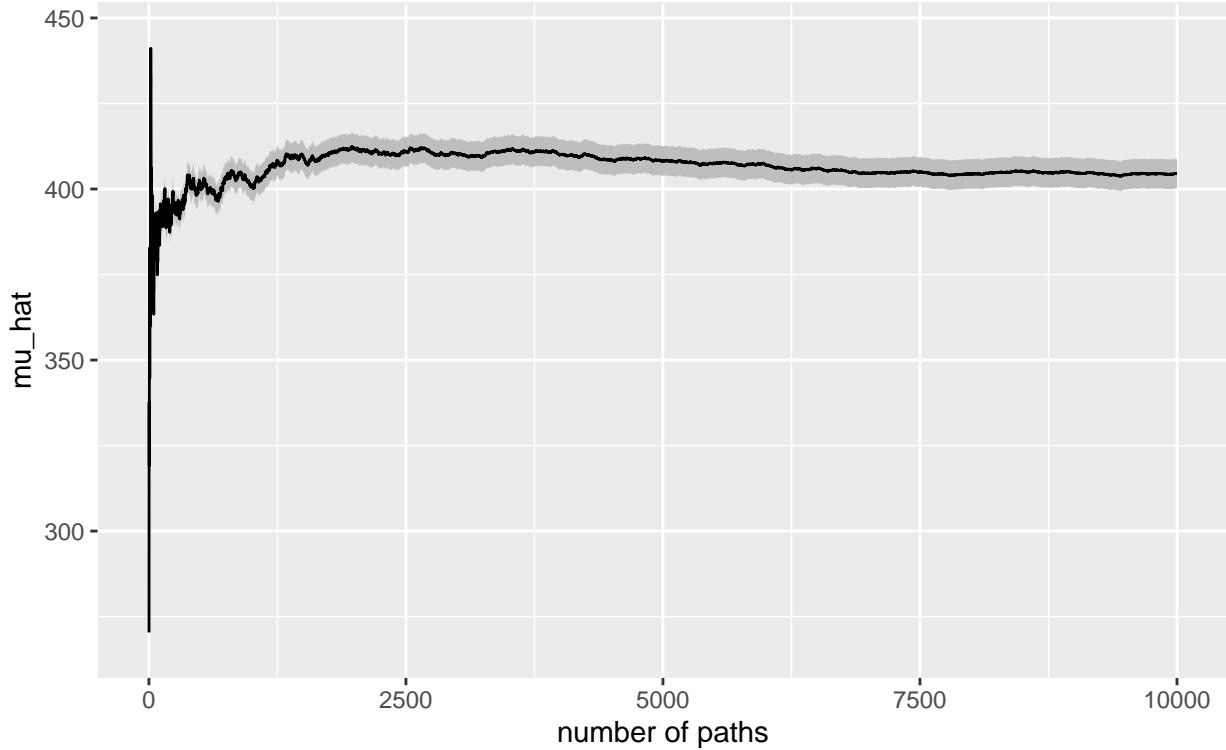


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

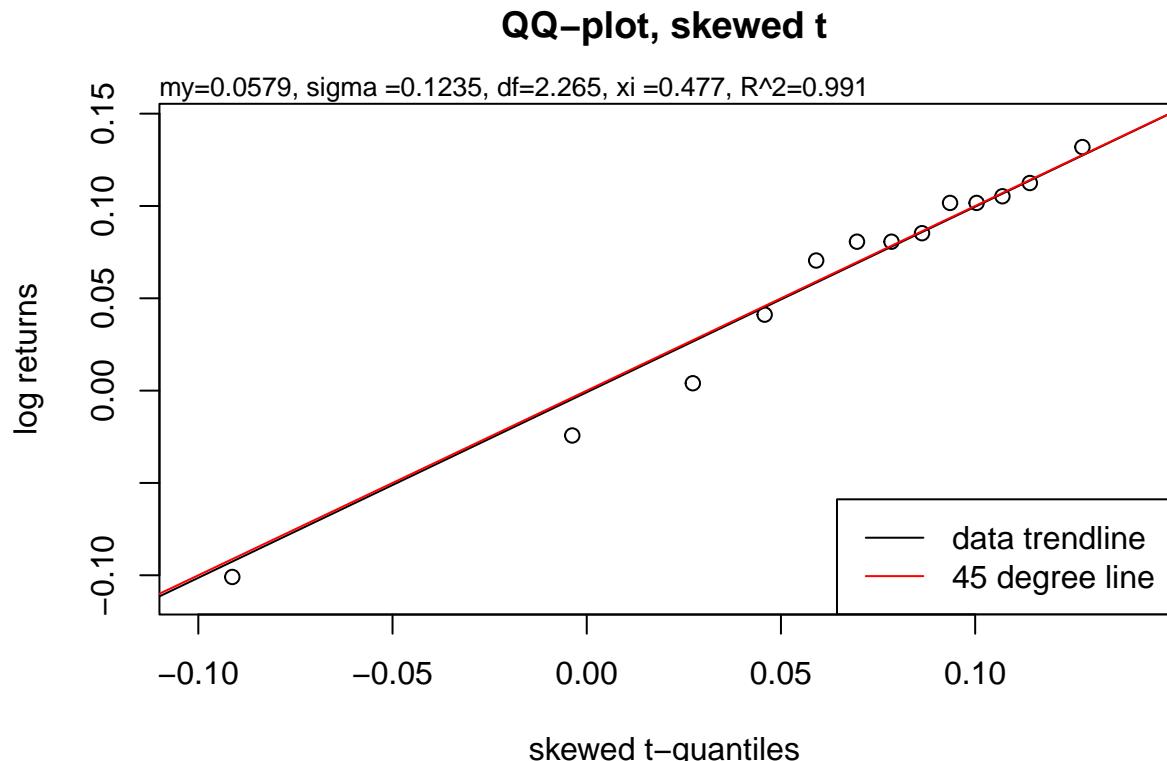


PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -33.22998  
## BIC: -30.97018  
## m: 0.05789224  
## s: 0.1234592  
## nu (df): 2.265273  
## xi: 0.477324  
## R^2: 0.991  
##  
## An R^2 of 0.991 suggests that the fit is extremely good.  
##  
## What is the risk of losing max 10 %? <= 3.3 percent  
## What is the risk of losing max 25 %? <= 0.9 percent  
## What is the risk of losing max 50 %? <= 0.2 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 32.7 percent  
## What is the chance of gaining min 25 %? >= 0.1 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

QQ Plot

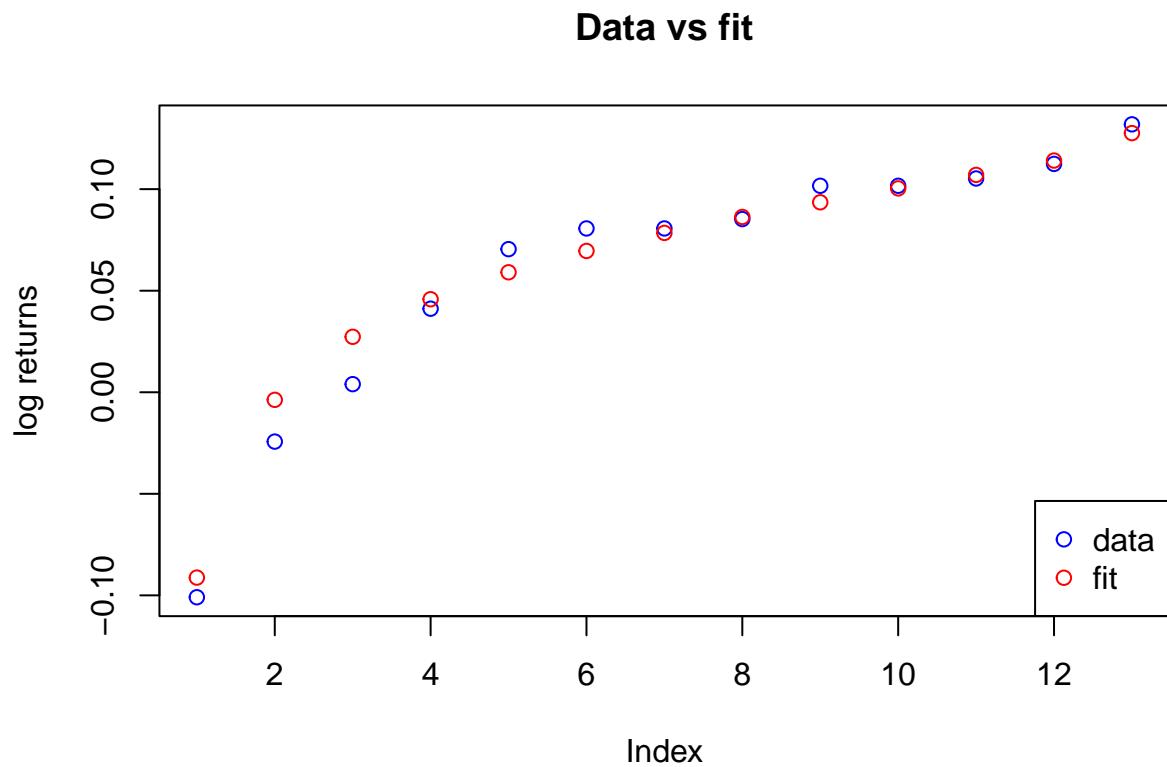


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
## [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
## [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11] 0.107018607  0.114081432  0.127604387
```

Data vs fit

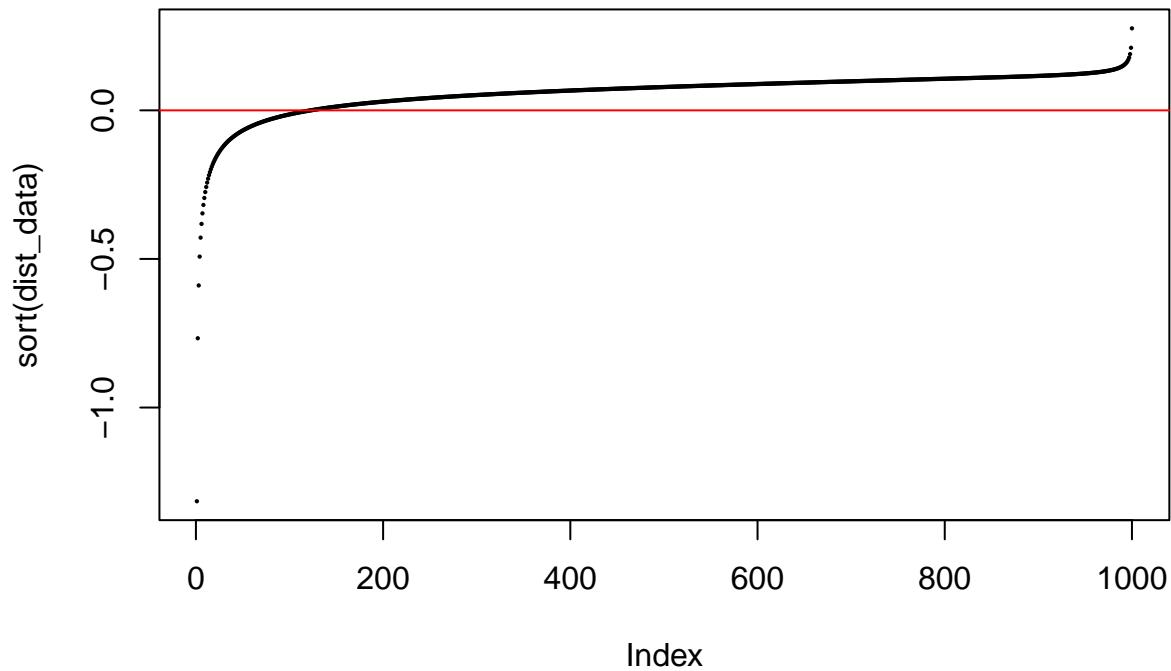
Let's plot the fit and the observed returns together.



Estimated distribution

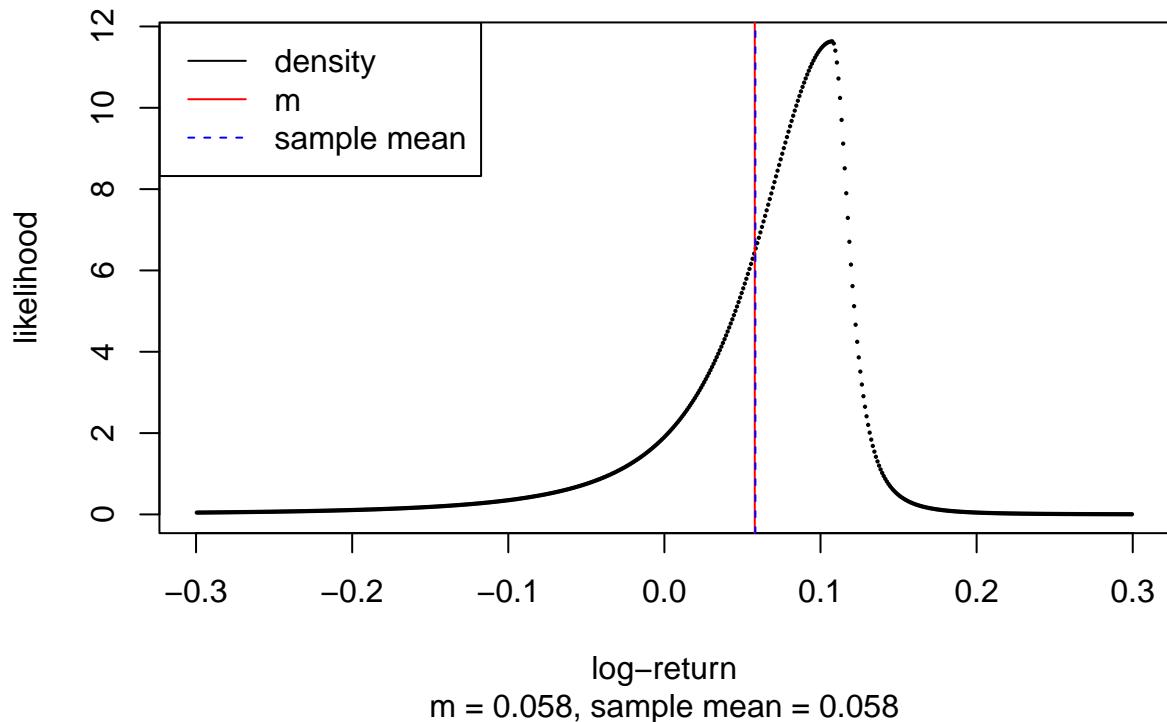
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.

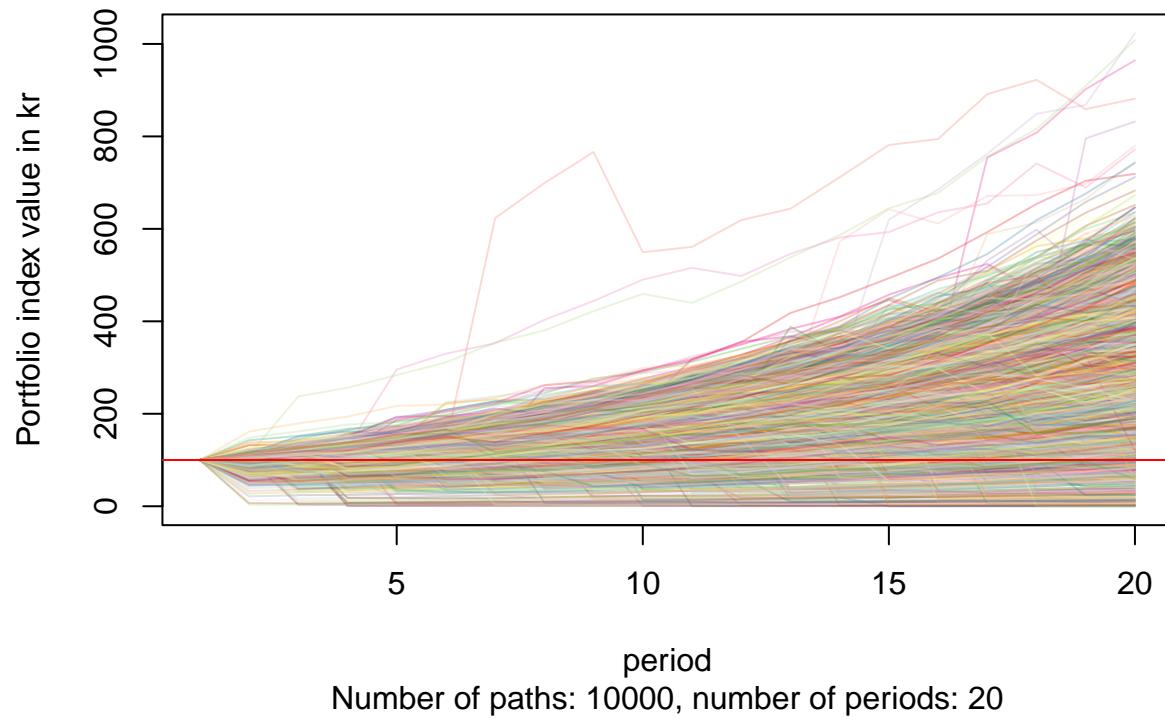
Estimated skew t distribution PDF



Monte Carlo

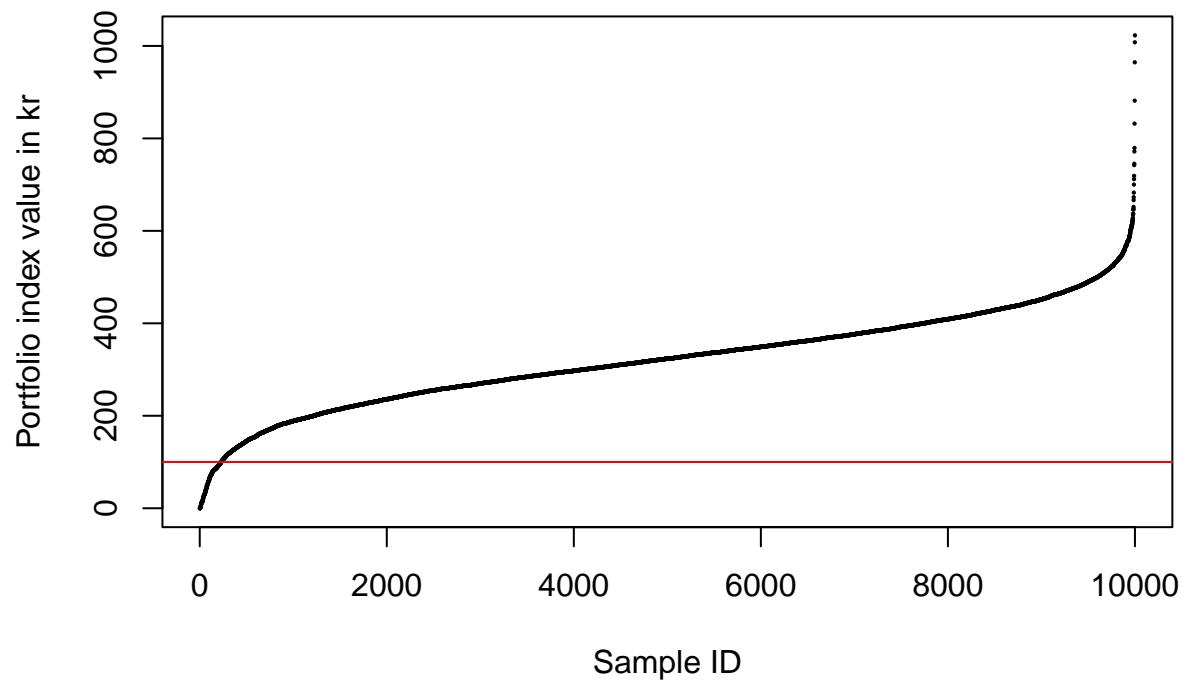
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.03 percent  
##  
## Mean portfolio index value after 20 years: 321.903 kr.  
## SD of portfolio index value after 20 years: 105.482 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 1022.902 kr.  
##  
## Share of paths finishing below 100: 2.3 percent
```

MC simulation with down-and-out



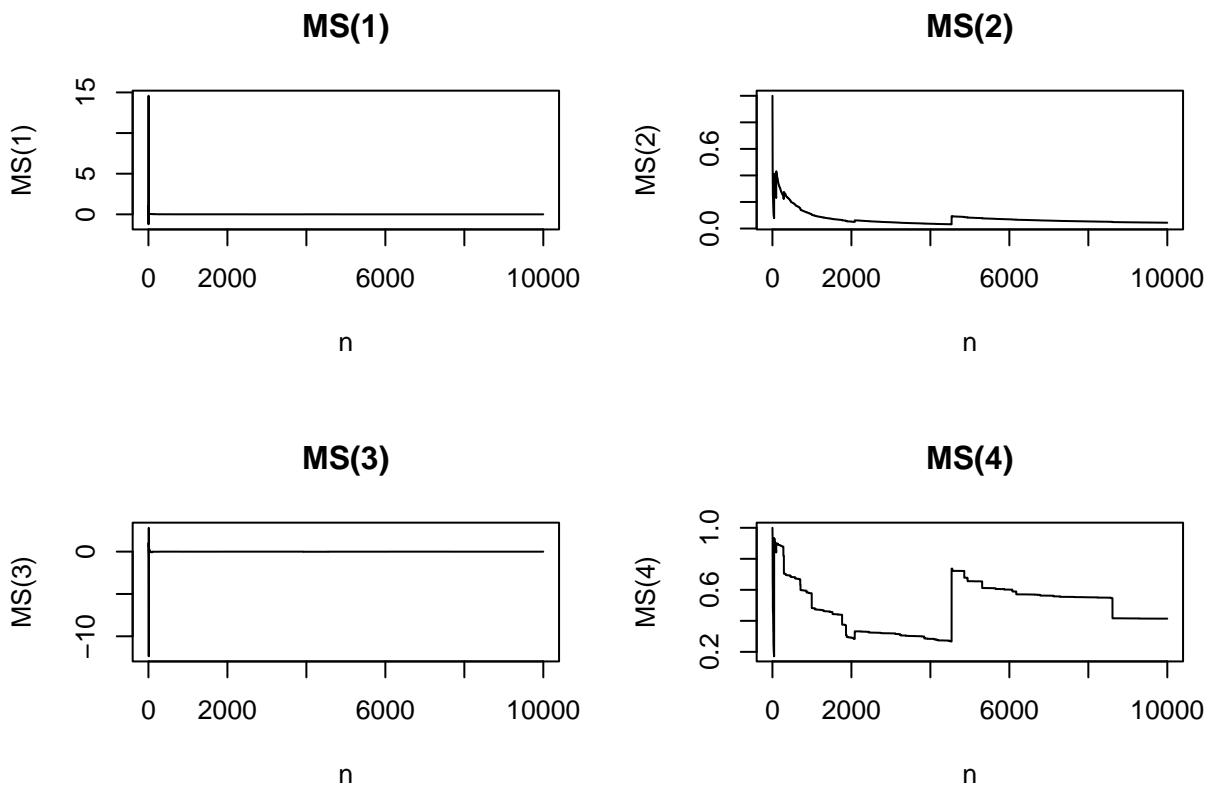
Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)



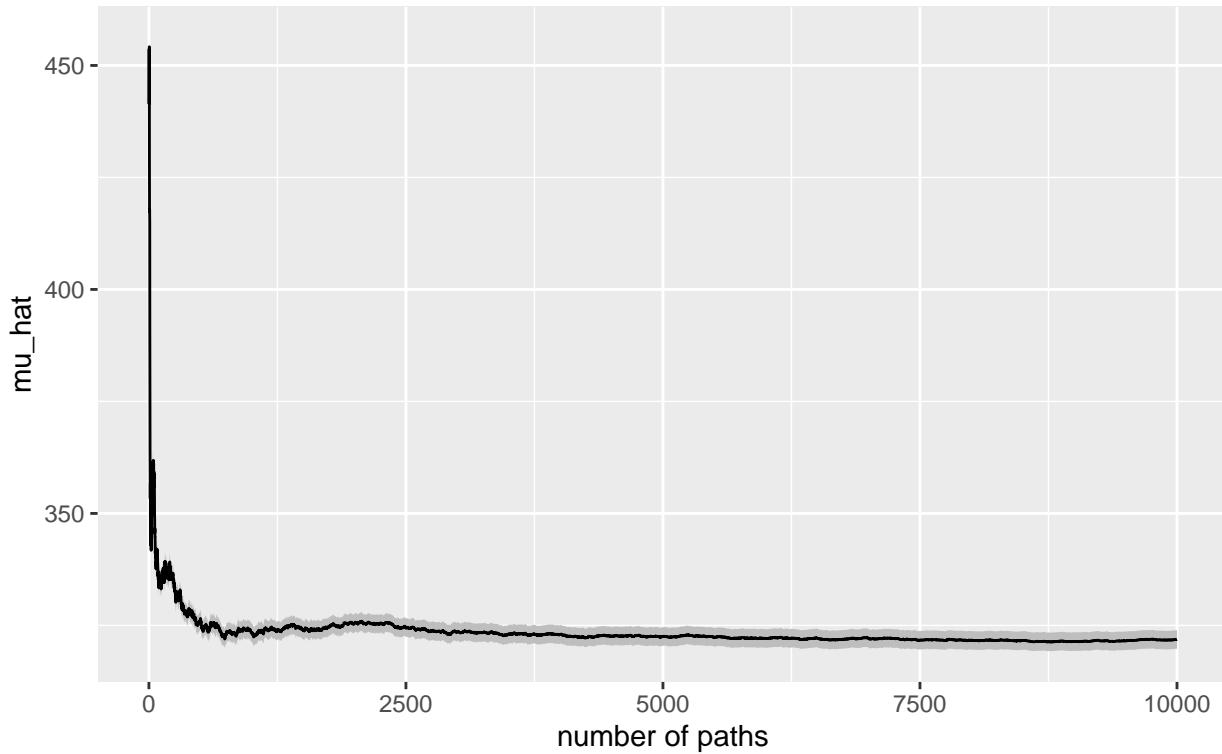
Convergence

Max vs sum plots for the first four moments:



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

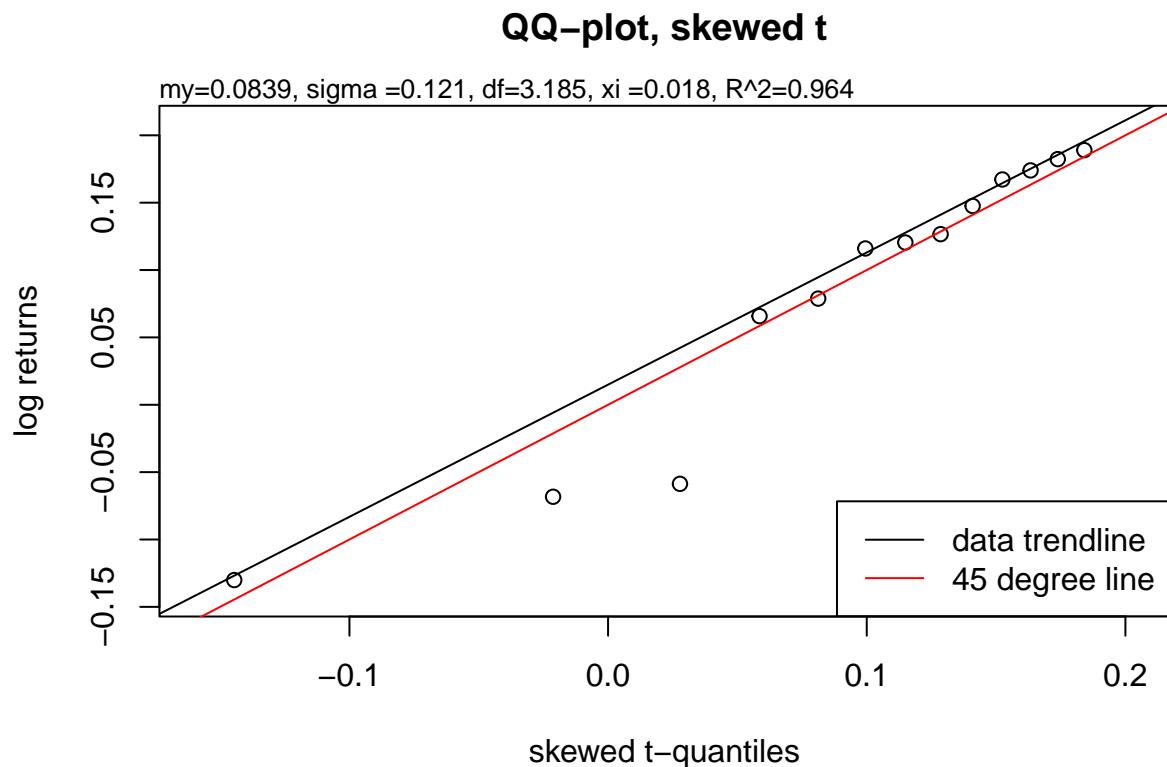


PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -23.72565  
## BIC: -21.46585  
## m: 0.08386034  
## s: 0.1210107  
## nu (df): 3.184569  
## xi: 0.01790306  
## R^2: 0.964  
##  
## An R^2 of 0.964 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 5.3 percent  
## What is the risk of losing max 25 %? <= 1.4 percent  
## What is the risk of losing max 50 %? <= 0.2 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 59.6 percent  
## What is the chance of gaining min 25 %? >= 0 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

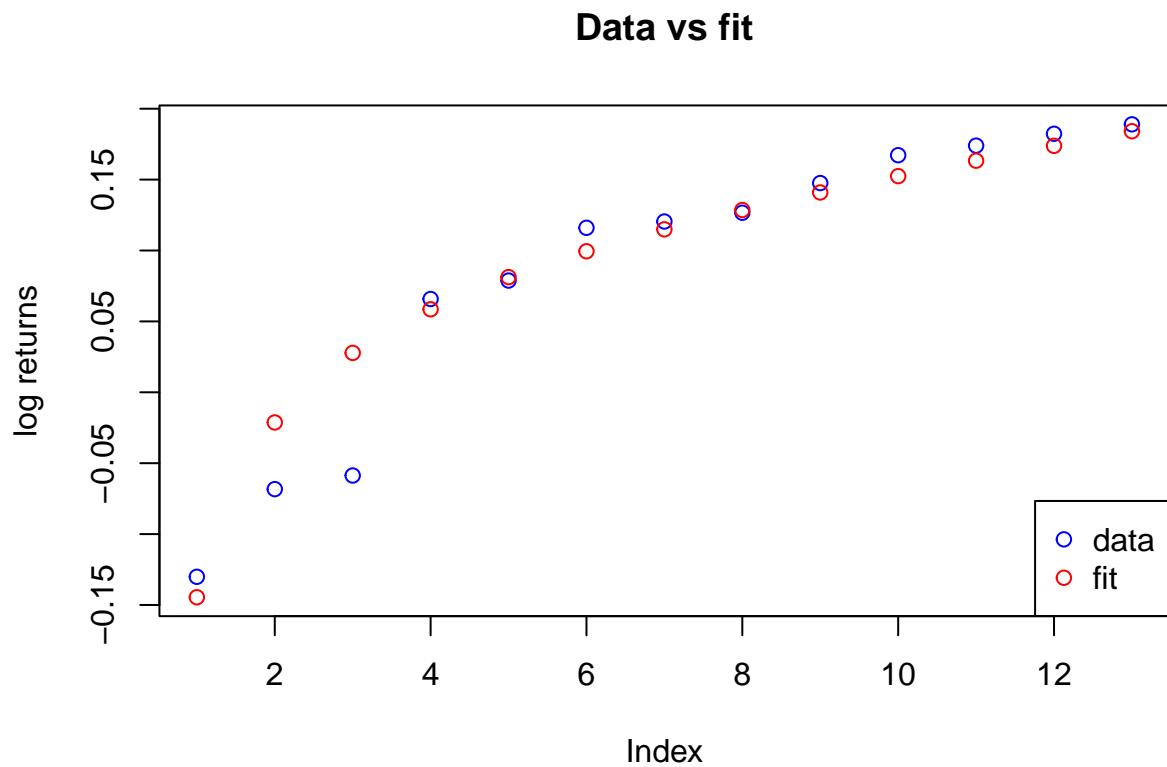
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

Data vs fit

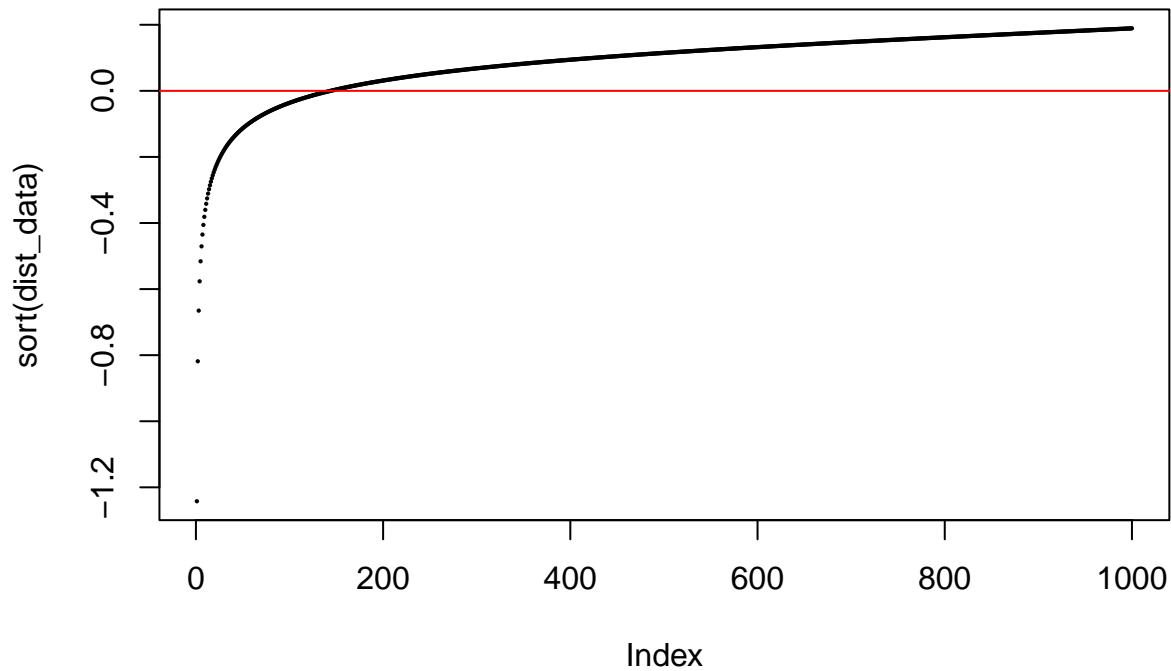
Let's plot the fit and the observed returns together.



Estimated distribution

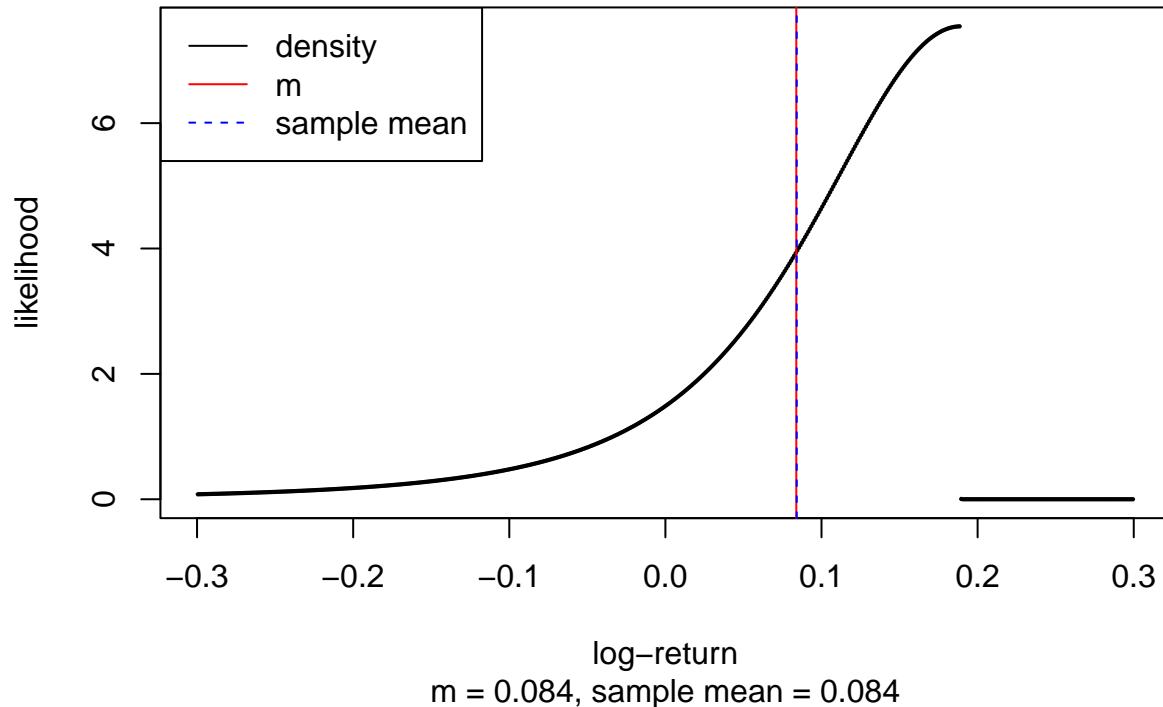
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

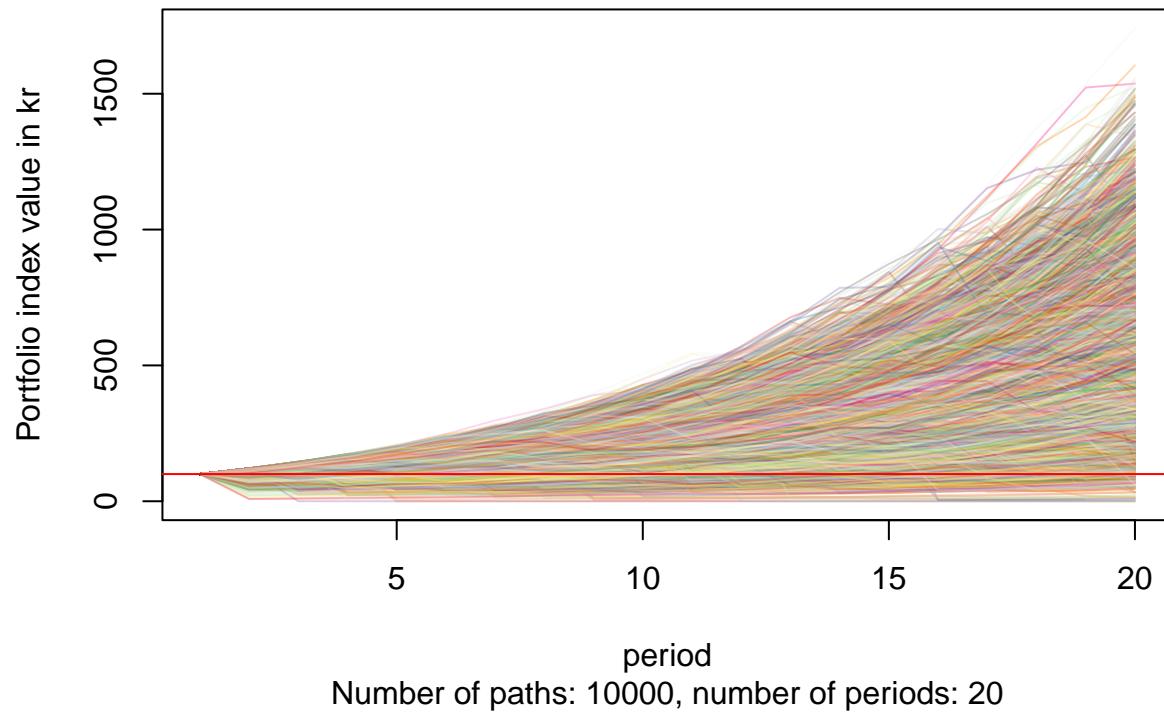
Estimated skew t distribution PDF



Monte Carlo

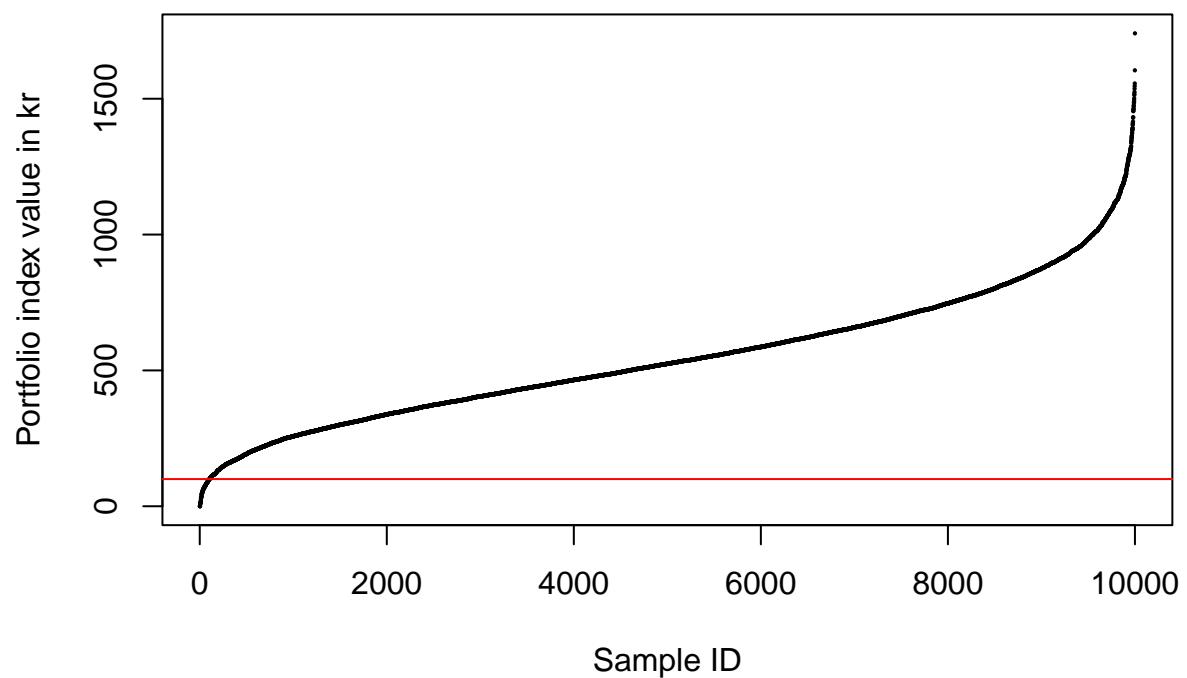
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.01 percent  
##  
## Mean portfolio index value after 20 years: 548.853 kr.  
## SD of portfolio index value after 20 years: 243.592 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 1740.595 kr.  
##  
## Share of paths finishing below 100: 1.03 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

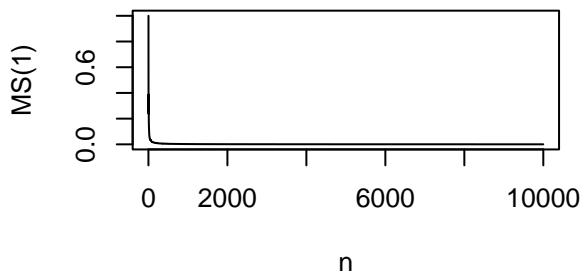
(100 is par, 200 is double, 50 is half)



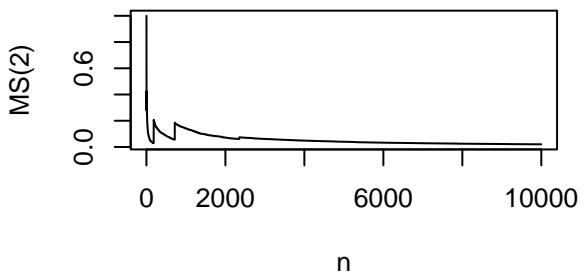
Convergence

Max vs sum plots for the first four moments:

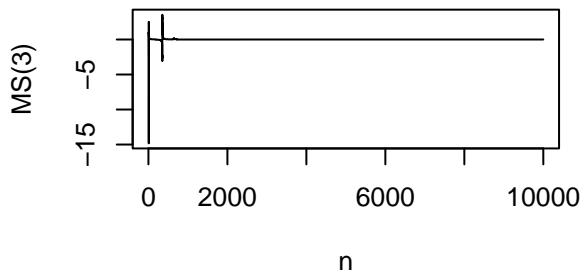
MS(1)



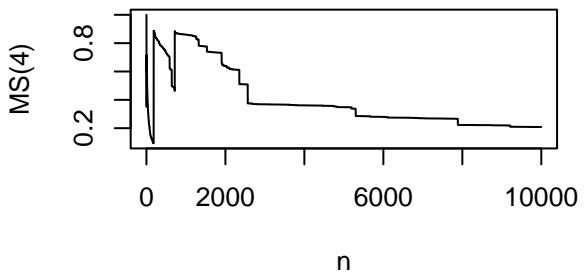
MS(2)



MS(3)

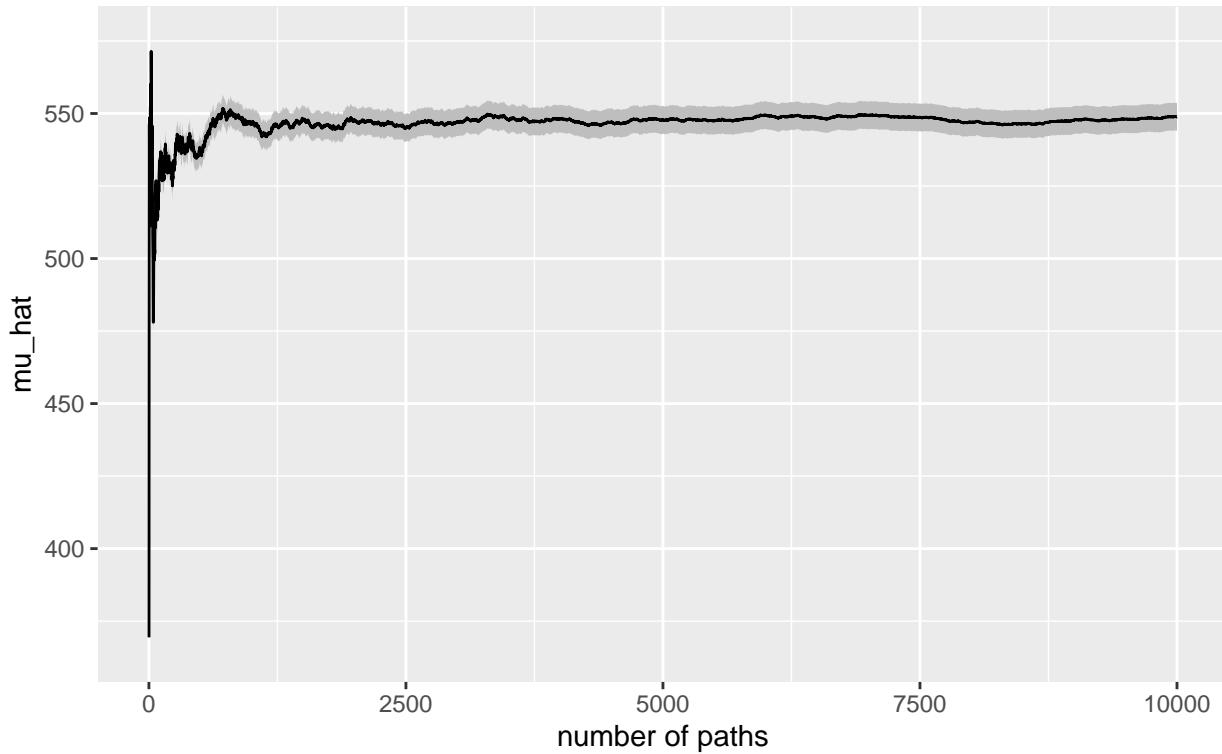


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

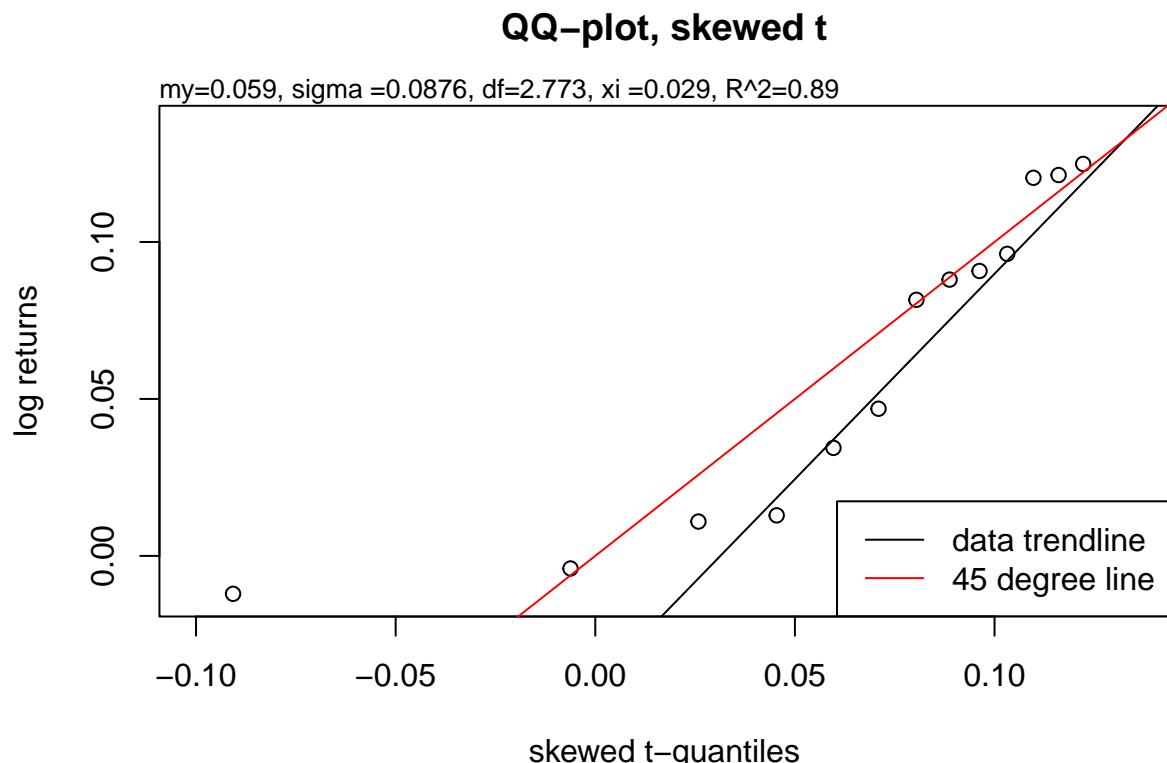


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -36.9603  
## BIC: -34.7005  
## m: 0.05902873  
## s: 0.08757749  
## nu (df): 2.772621  
## xi: 0.02904471  
## R^2: 0.89  
##  
## An R^2 of 0.89 suggests that the fit is not completely random.  
##  
## What is the risk of losing max 10 %? <= 3.3 percent  
## What is the risk of losing max 25 %? <= 0.7 percent  
## What is the risk of losing max 50 %? <= 0.1 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 35.6 percent  
## What is the chance of gaining min 25 %? >= 0 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

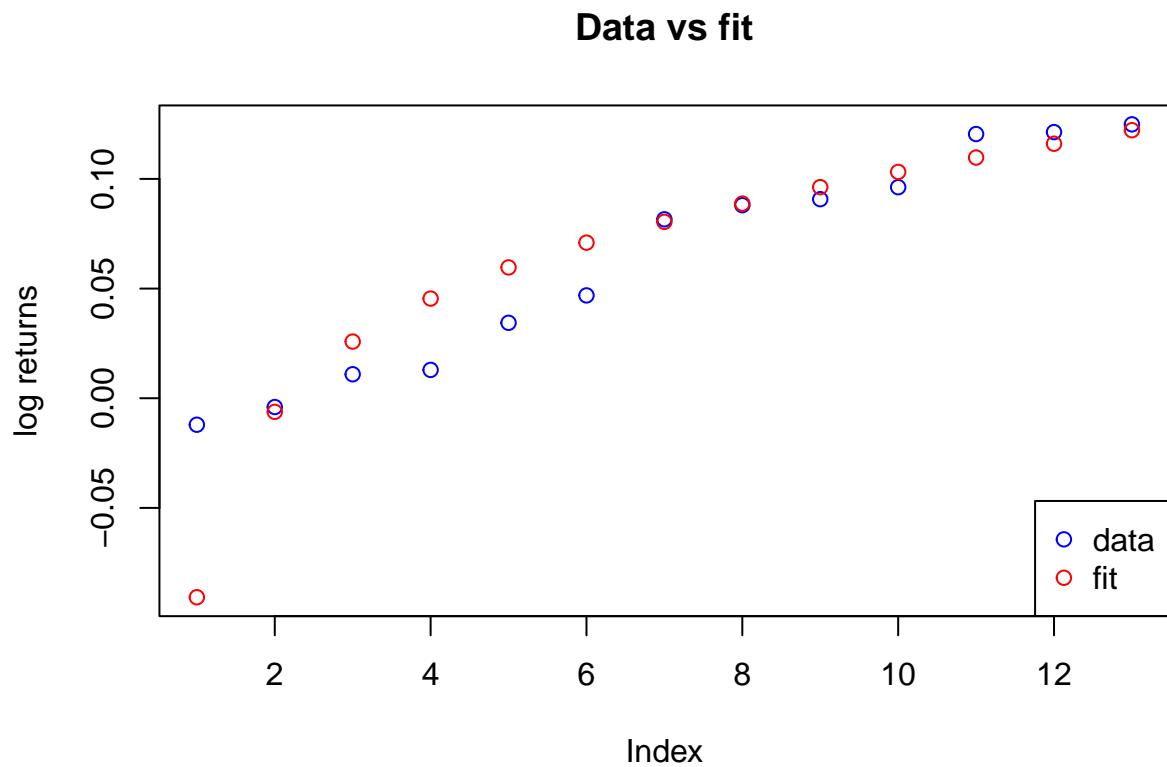
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

Let's plot the fit and the observed returns together.

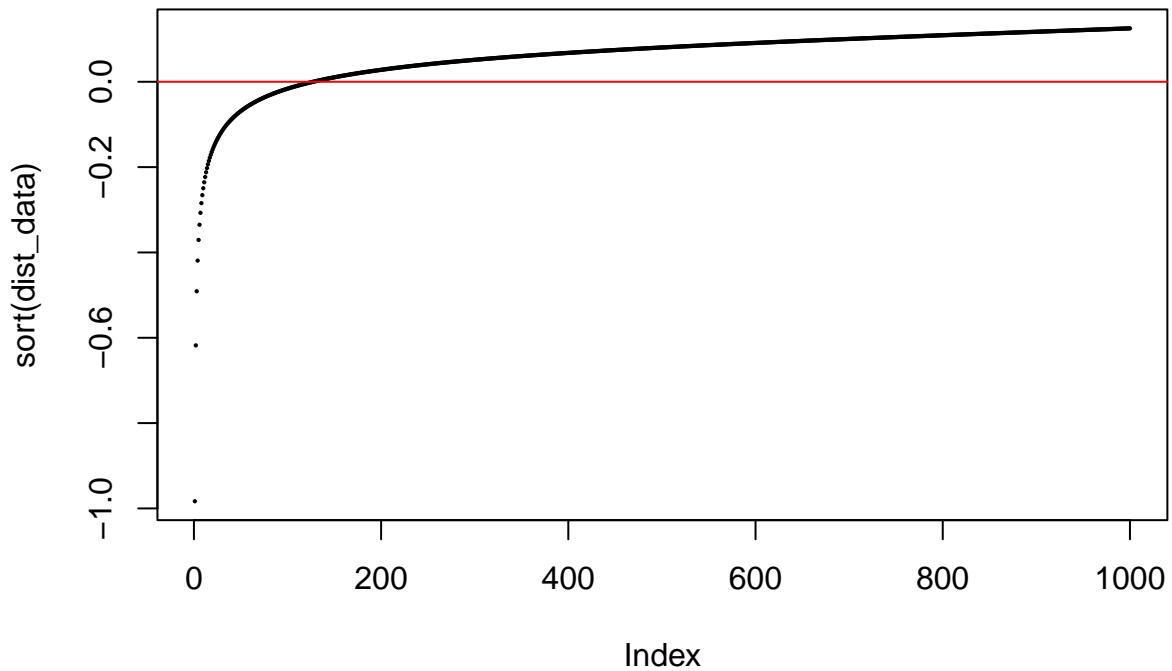


Interestingly, the fit predicts a much bigger “biggest loss” than the actual data. This is the main reason that R^2 is 0.90 and not higher.

Estimated distribution

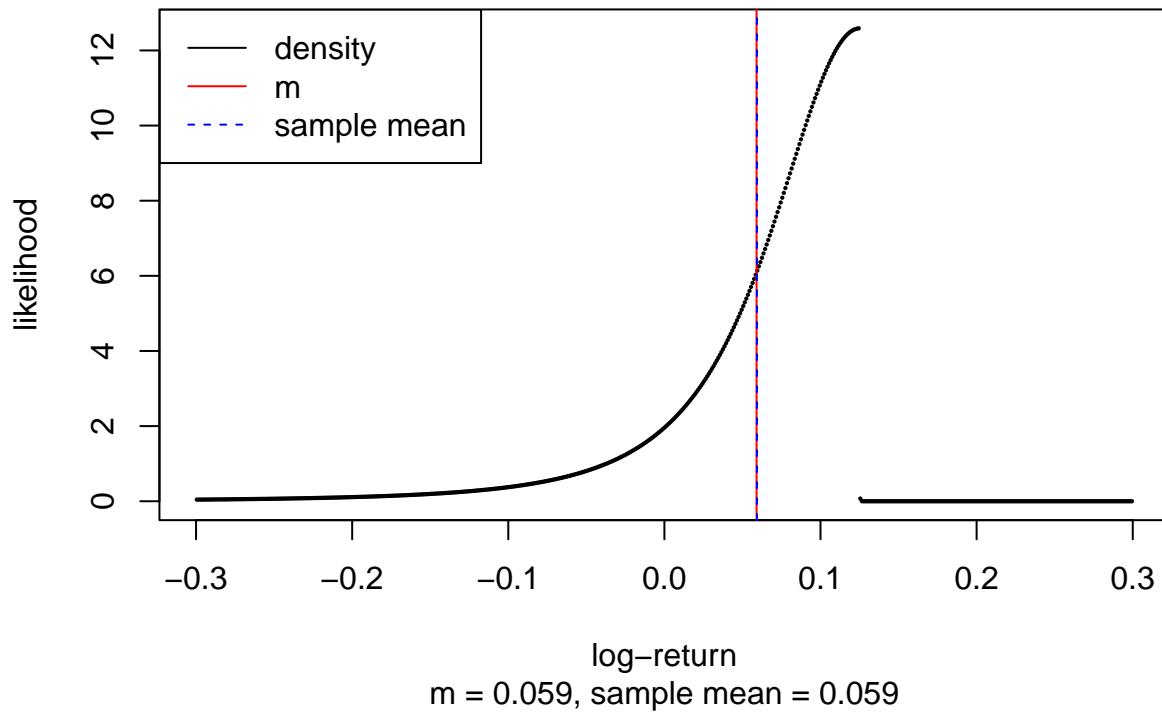
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Estimated skew t distribution PDF

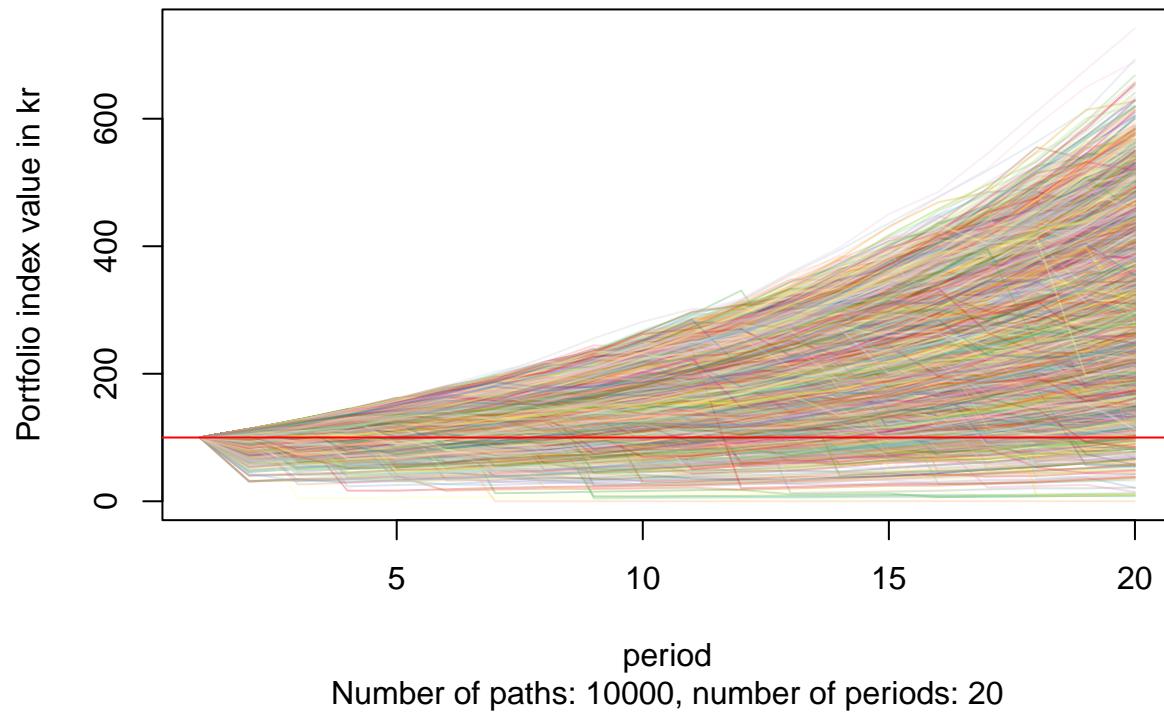


Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

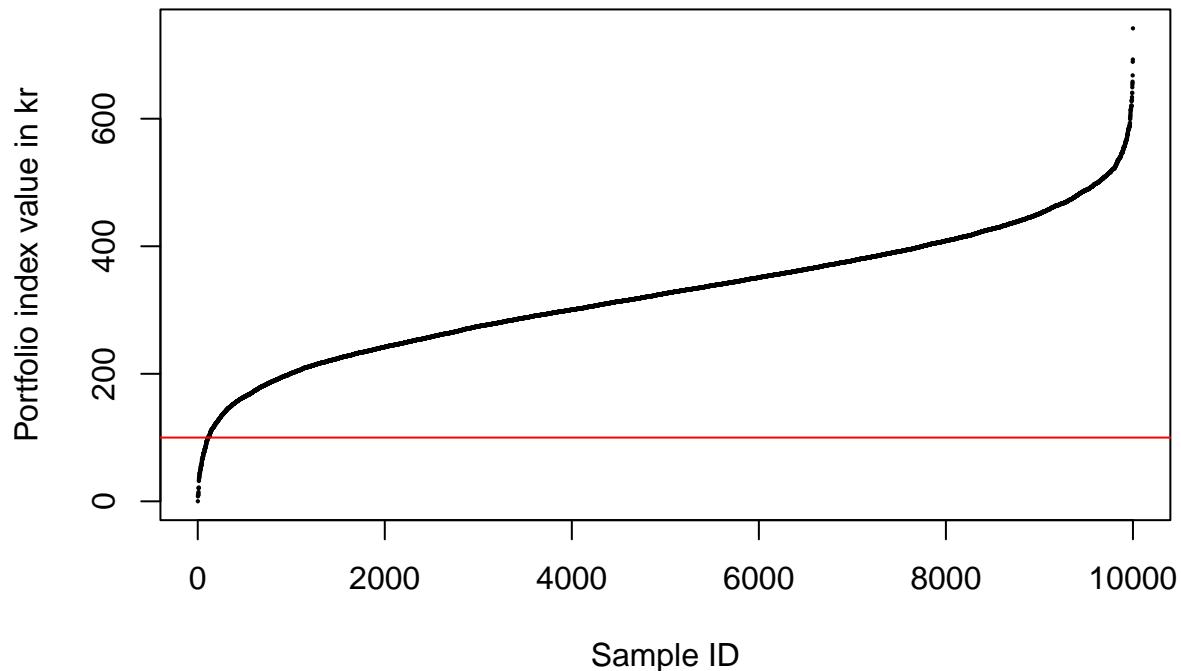
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 325.625 kr.  
## SD of portfolio index value after 20 years: 98.291 kr.  
## Min total portfolio index value after 20 years: 0.155 kr.  
## Max total portfolio index value after 20 years: 741.723 kr.  
##  
## Share of paths finishing below 100: 1.1 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

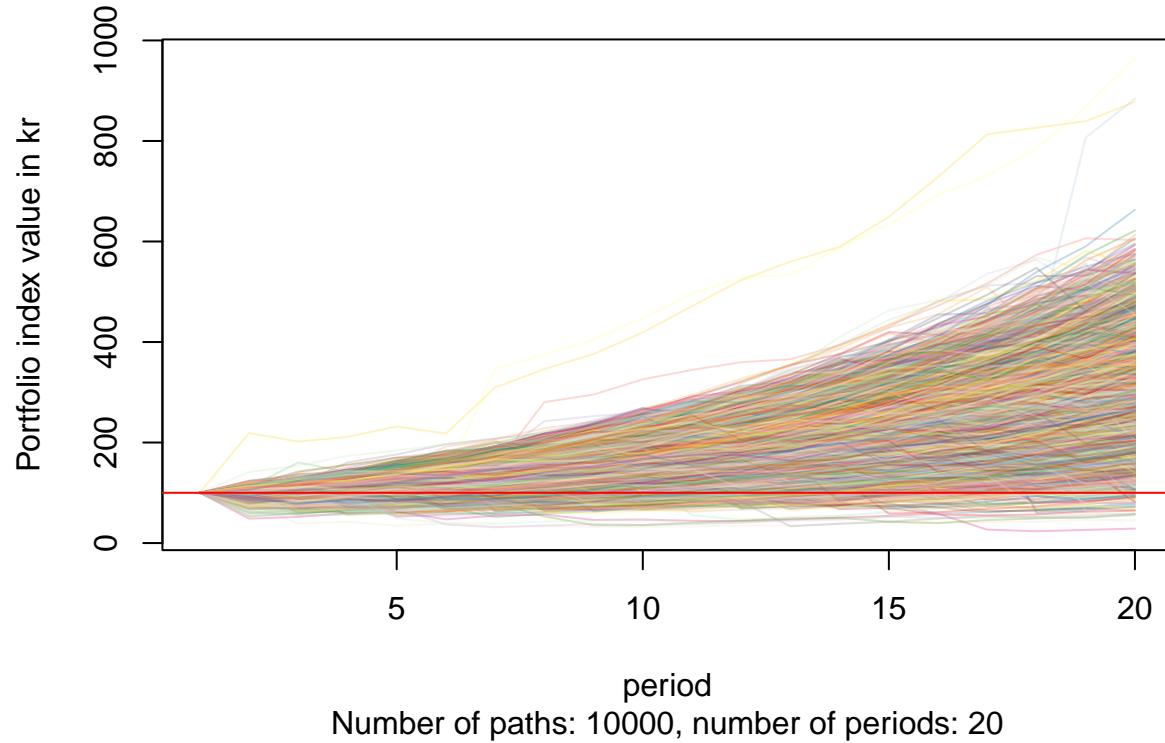
(100 is par, 200 is double, 50 is half)



Version b: Mix of simulations from estimated distribution of returns from individual funds.

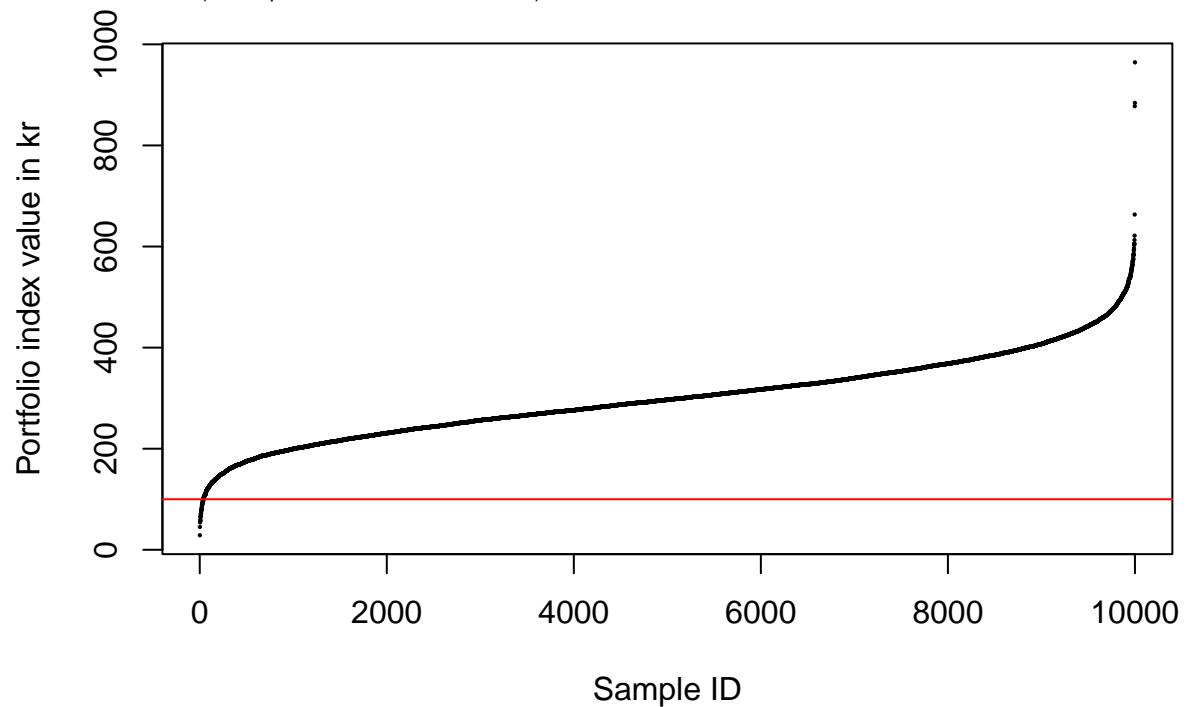
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 300.962 kr.  
## SD of portfolio index value after 20 years: 82.667 kr.  
## Min total portfolio index value after 20 years: 28.79 kr.  
## Max total portfolio index value after 20 years: 964.382 kr.  
##  
## Share of paths finishing below 100: 0.38 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

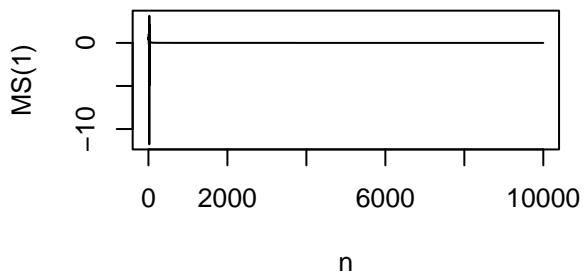
(100 is par, 200 is double, 50 is half)



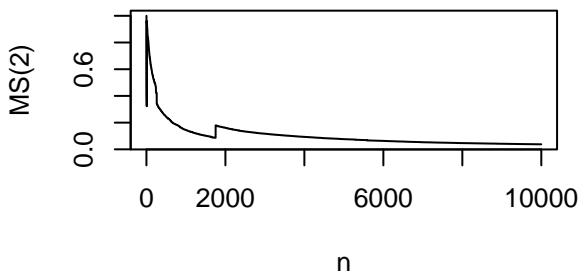
Convergence

Max vs sum plots for the first four moments:

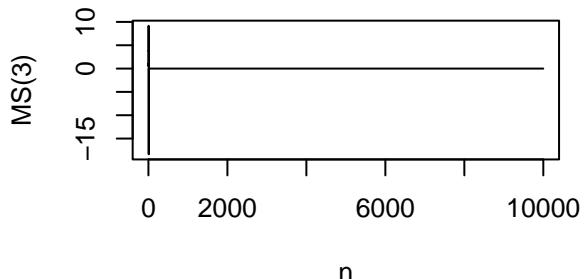
MS(1)



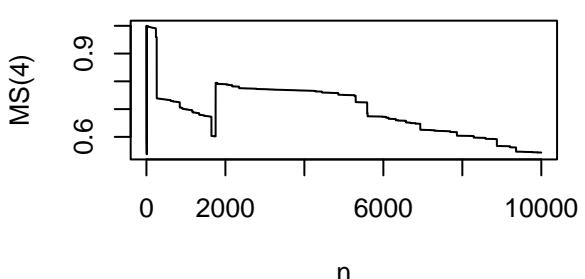
MS(2)



MS(3)

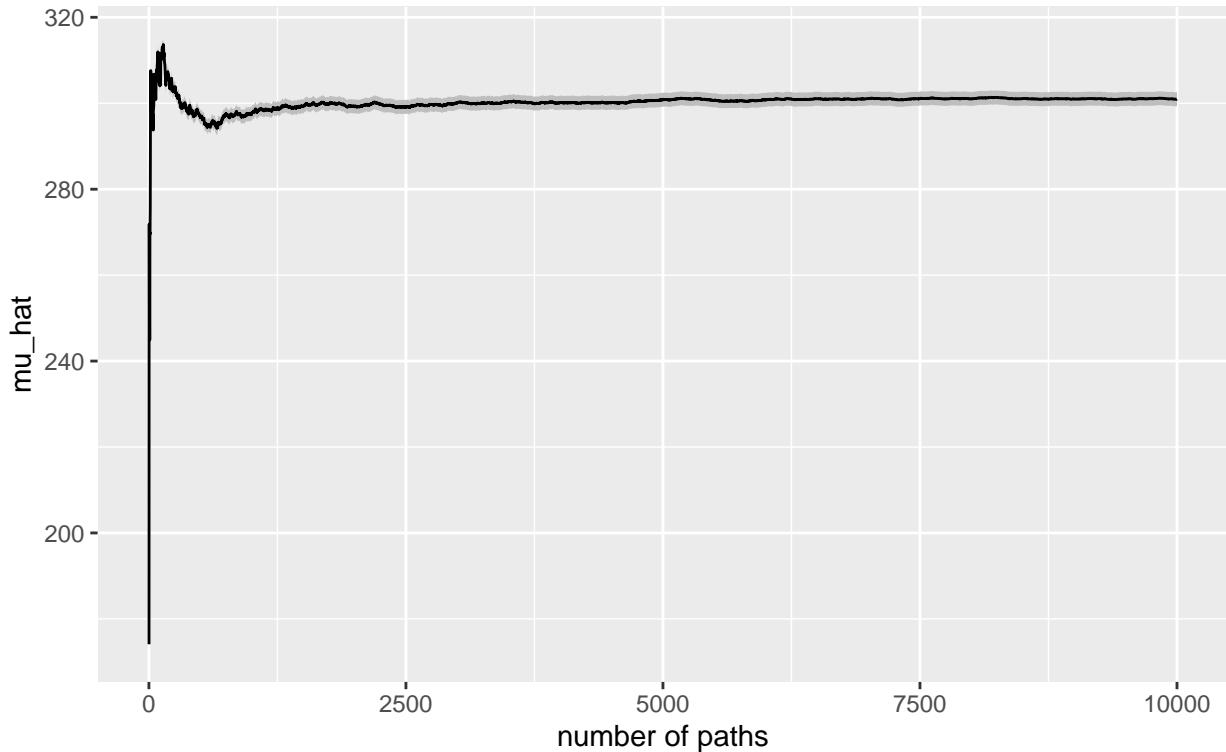


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

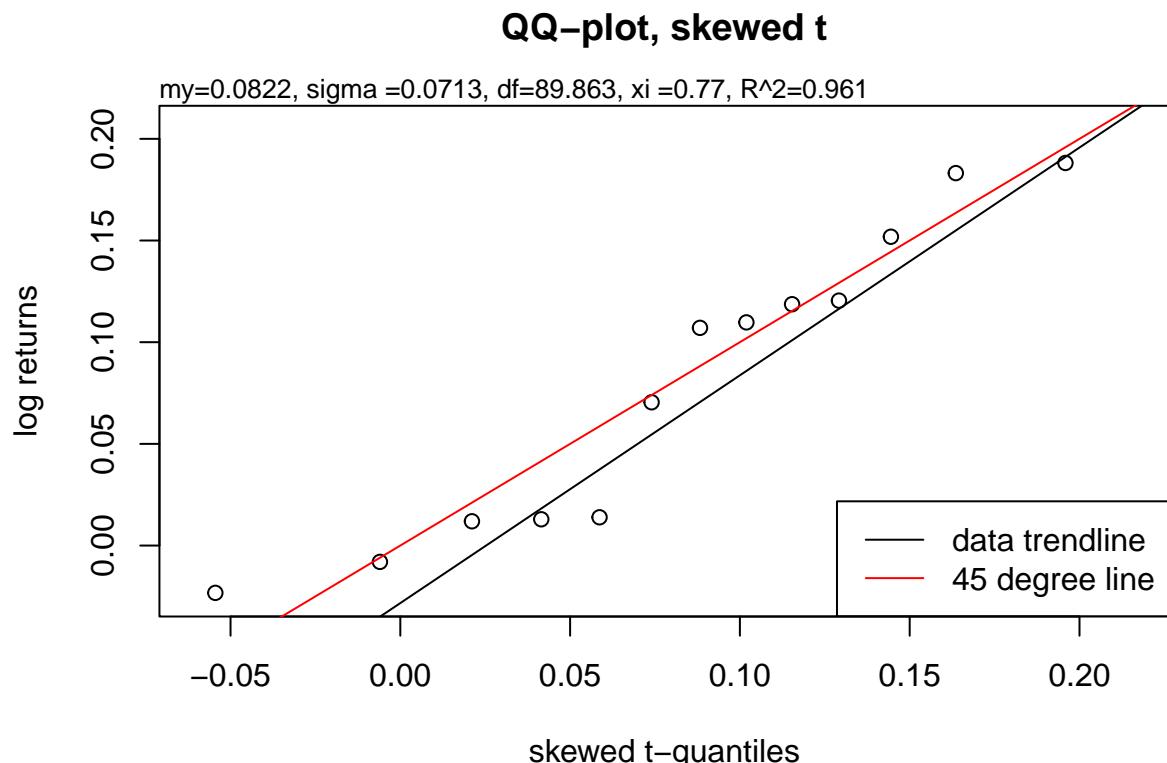


Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -24.26084  
## BIC: -22.00104  
## m: 0.0822419  
## s: 0.07129843  
## nu (df): 89.86289  
## xi: 0.7697502  
## R^2: 0.961  
##  
## An R^2 of 0.961 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 0.9 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 46.1 percent  
## What is the chance of gaining min 25 %? >= 1.2 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

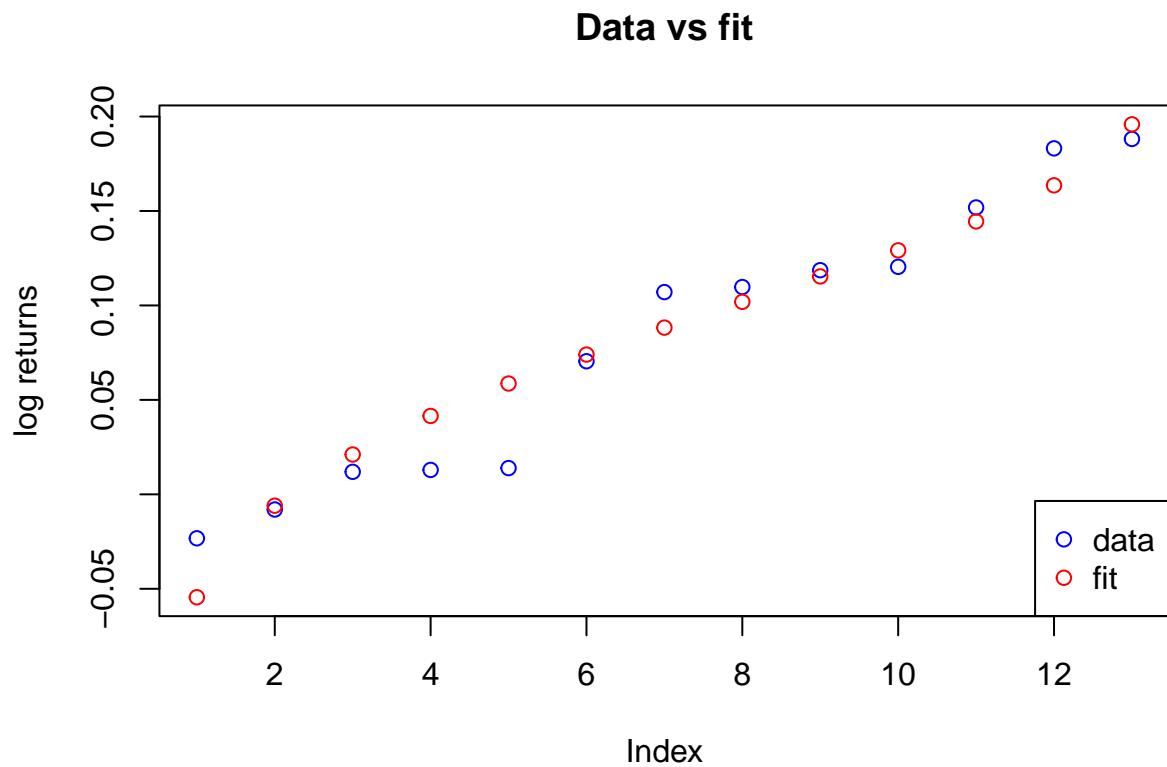
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

Data vs fit

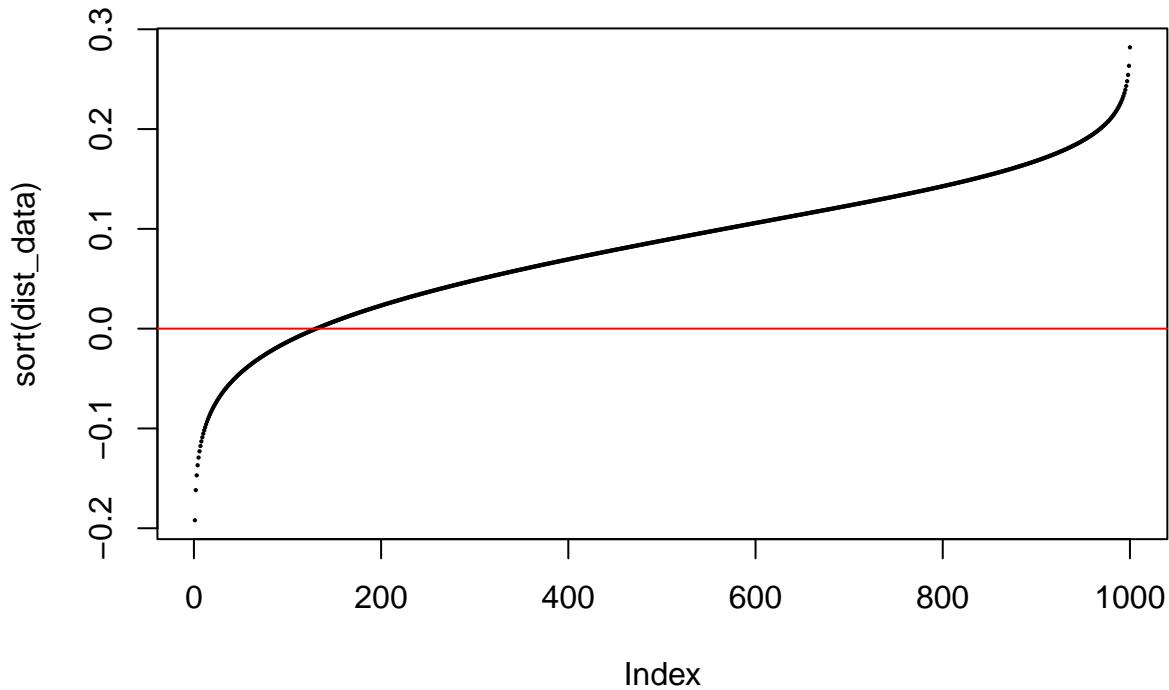
Let's plot the fit and the observed returns together.



Estimated distribution

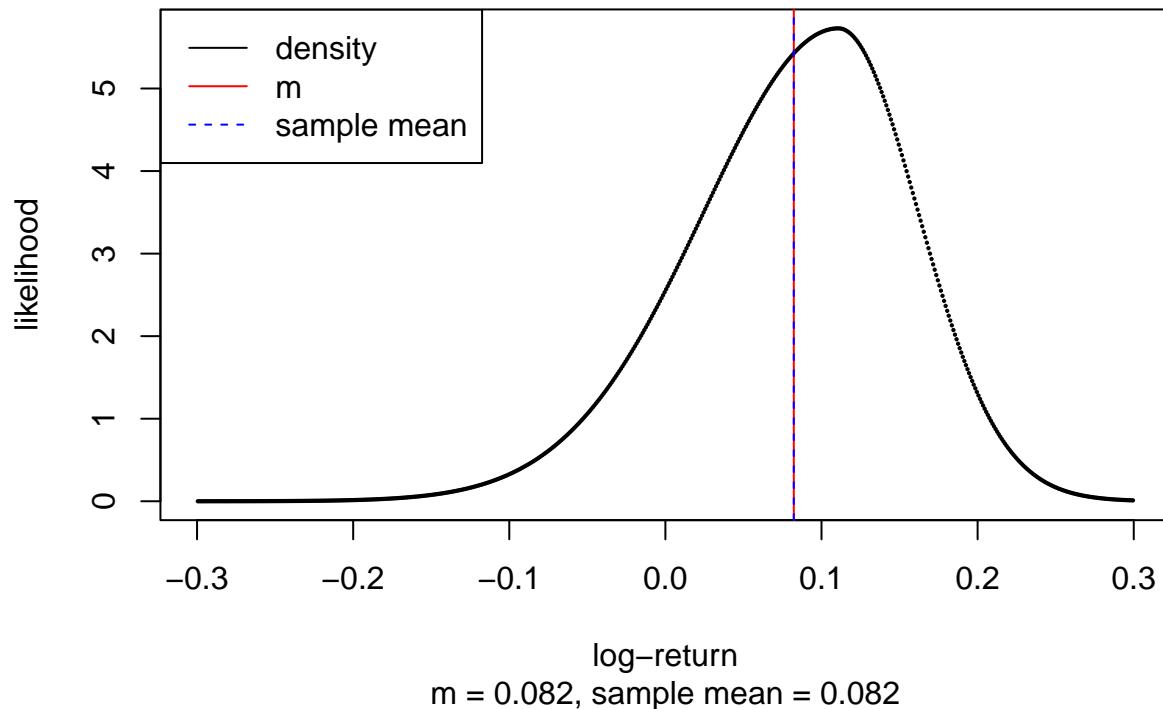
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that the high risk mix provides a much better upside and smaller downside.

Estimated skew t distribution PDF

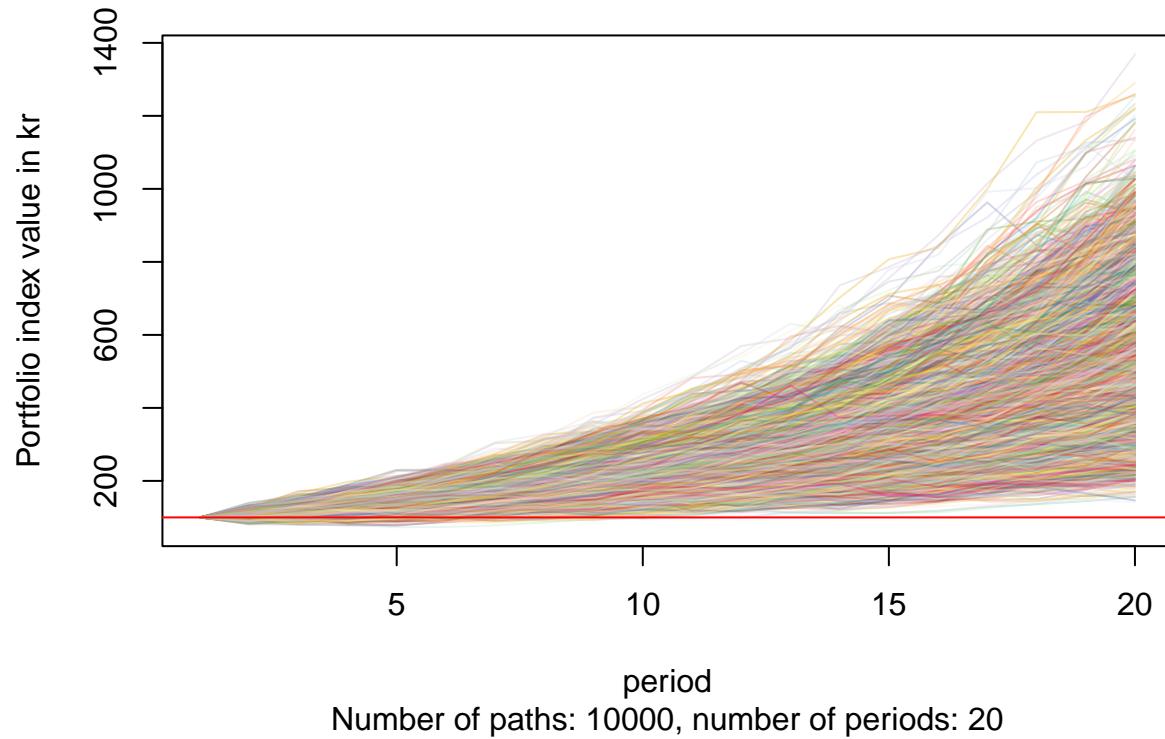


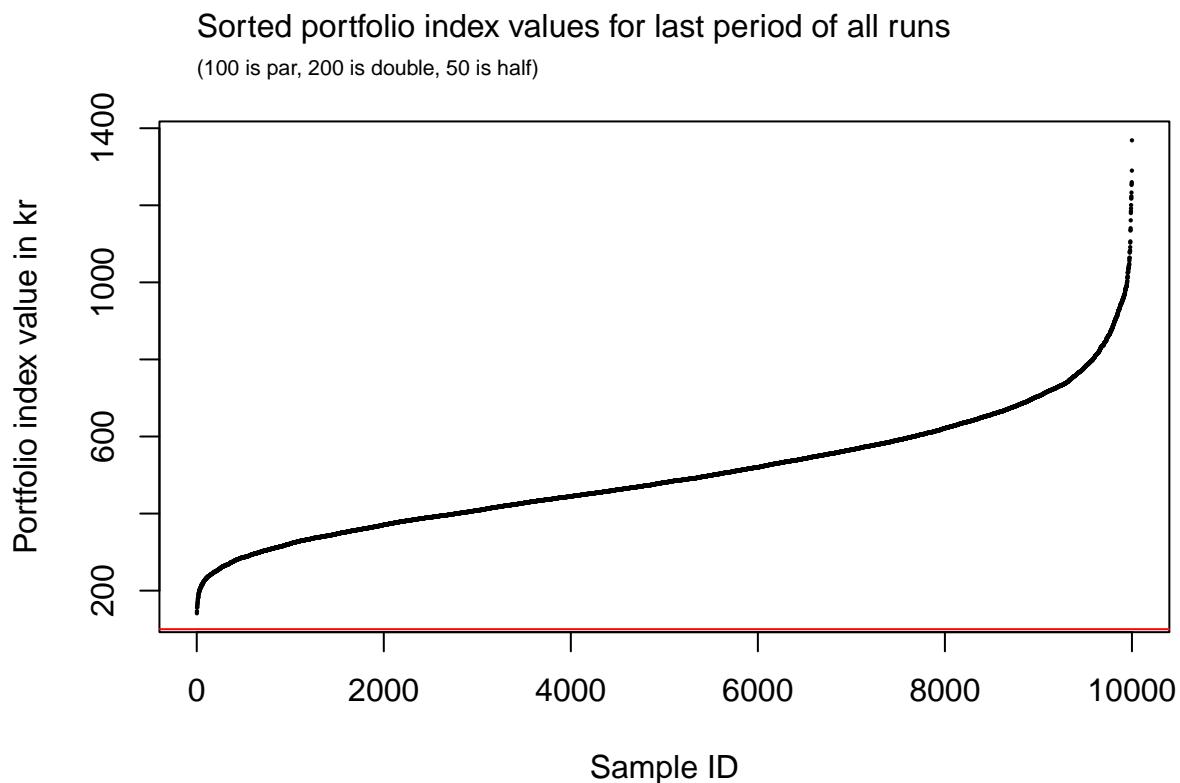
Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 501.08 kr.  
## SD of portfolio index value after 20 years: 154.282 kr.  
## Min total portfolio index value after 20 years: 141.372 kr.  
## Max total portfolio index value after 20 years: 1368.541 kr.  
##  
## Share of paths finishing below 100: 0 percent
```

MC simulation with down-and-out





Version b: Mix of simulations from estimated distribution of returns from individual funds.

```
## Down-and-out simulation:  

## Probability of down-and-out: 0 percent  

##  

## Mean portfolio index value after 20 years: 477.697 kr.  

## SD of portfolio index value after 20 years: 160.897 kr.  

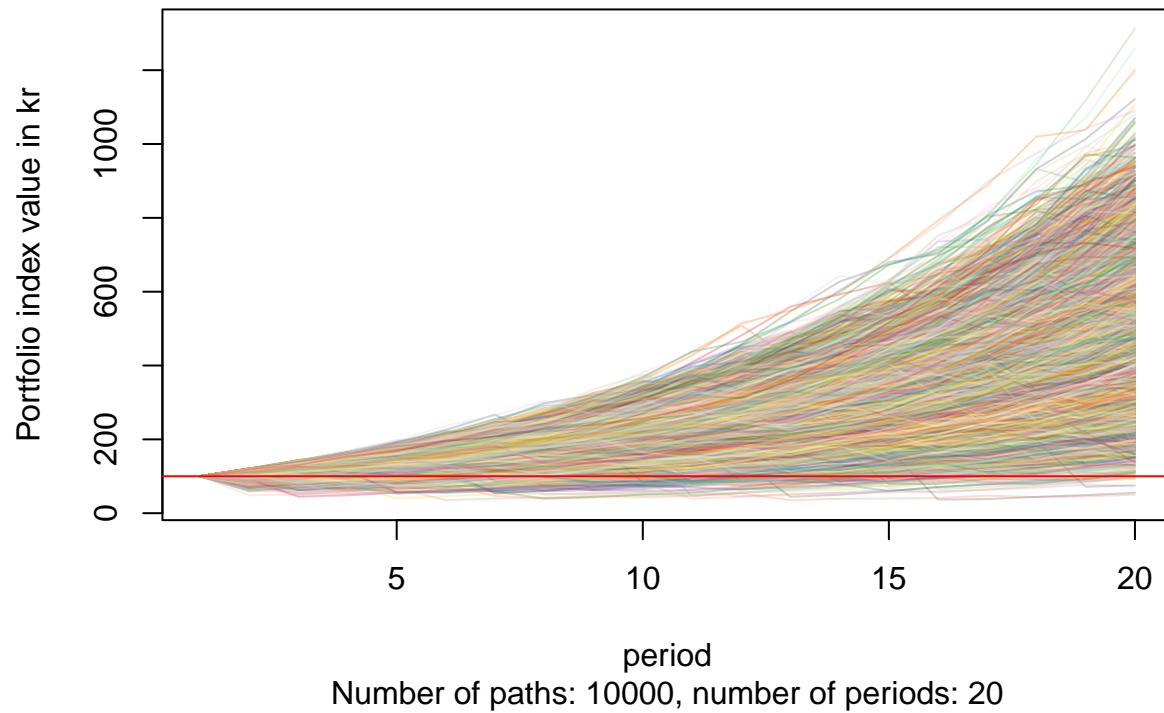
## Min total portfolio index value after 20 years: 50.137 kr.  

## Max total portfolio index value after 20 years: 1313.325 kr.  

##  

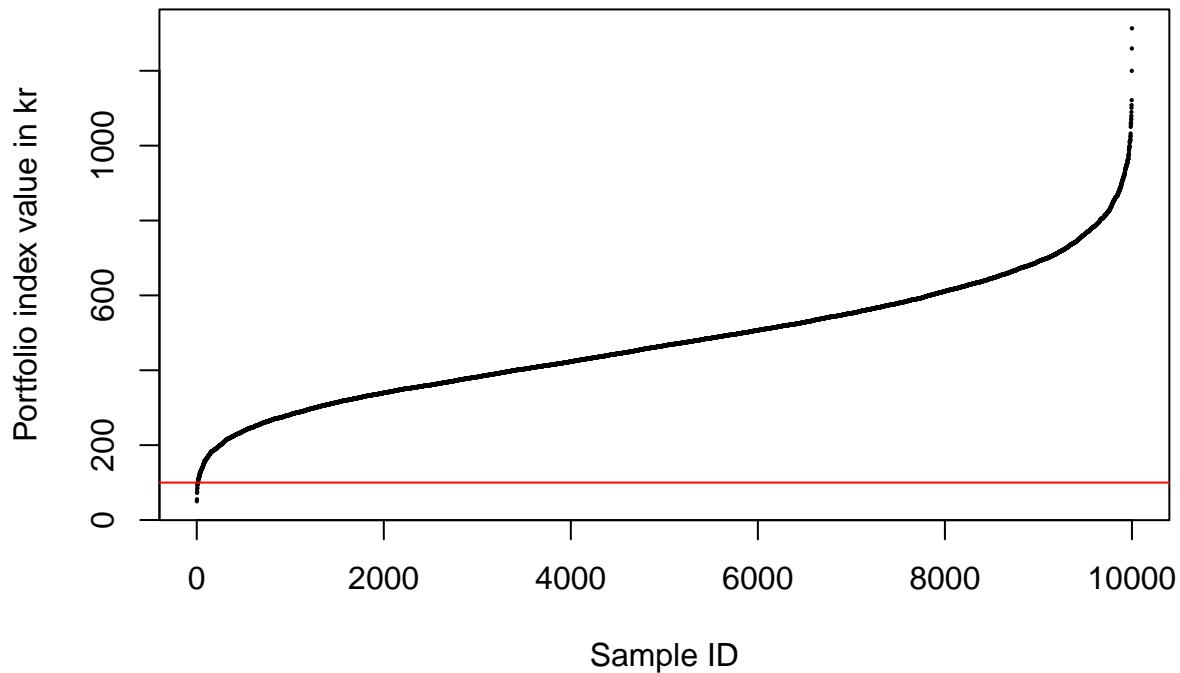
## Share of paths finishing below 100: 0.13 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)



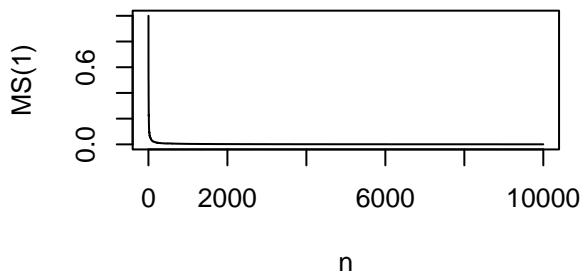
Many simulations 1e6 paths:

```
# Down-and-out simulation:  
# Probability of down-and-out: 0 percent  
#  
# Mean portfolio index value after 20 years: 478.339 kr.  
# SD of portfolio index value after 20 years: 163.093 kr.  
# Min total portfolio index value after 20 years: 2.233 kr.  
# Max total portfolio index value after 20 years: 1561.965 kr.  
#  
# Share of paths finishing below 100: 0.1181 percent
```

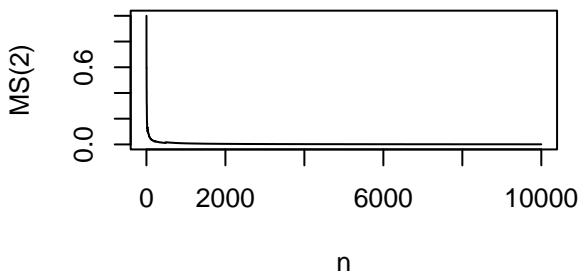
Convergence

Max vs sum plots for the first four moments:

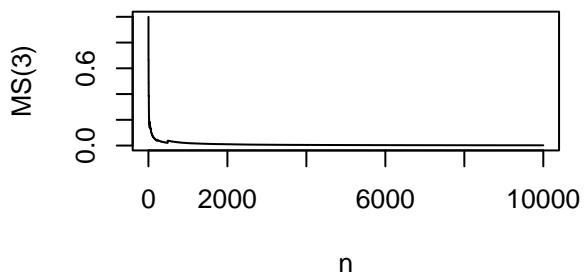
MS(1)



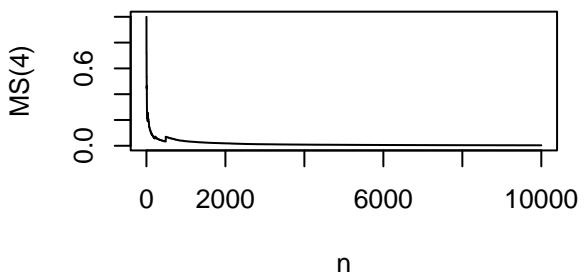
MS(2)



MS(3)

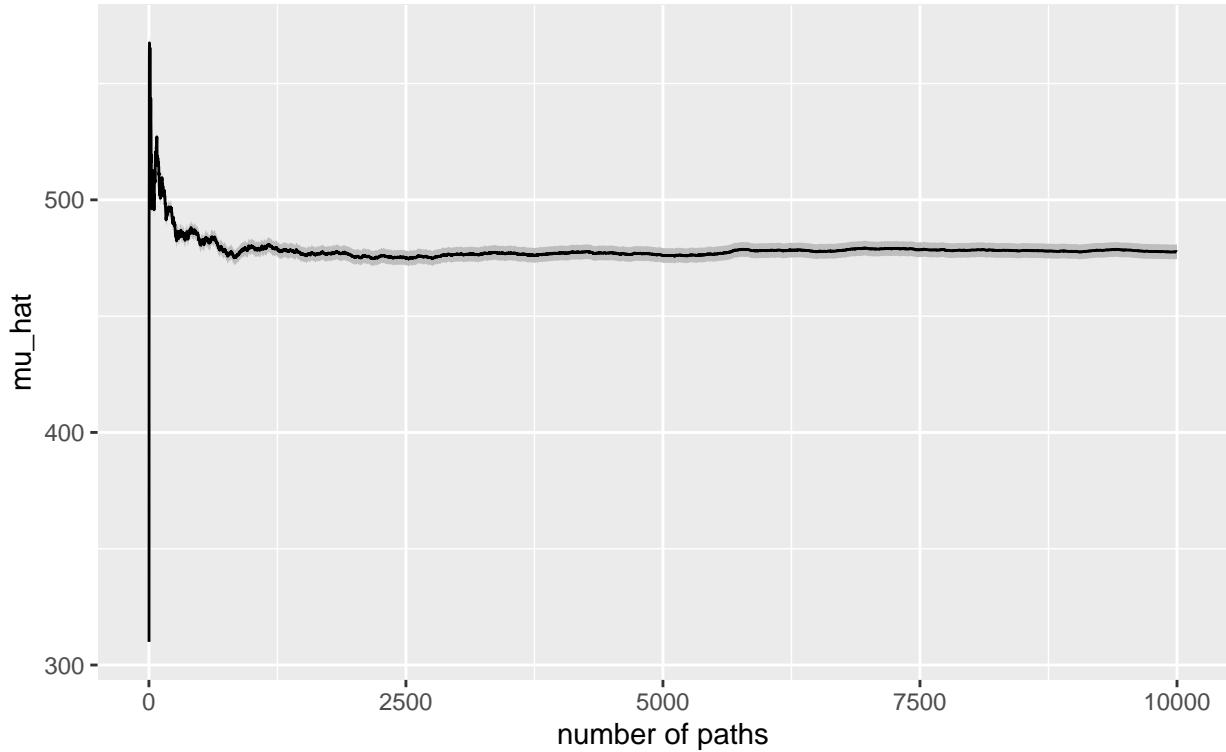


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths



Compare pension plans

Risk of max loss

Risk of max loss of x percent for a single period (year).

x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	21.3	18.2	19.9	12.2	14.3	12.7	13.0
5	12.5	9.6	12.8	6.0	8.6	6.2	4.2
10	7.4	5.4	8.3	3.3	5.3	3.3	0.9
25	1.8	1.3	2.5	0.9	1.4	0.7	0.0
50	0.2	0.2	0.4	0.2	0.2	0.1	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.3	Velliv_m	12.8	Velliv_h	8.3	Velliv_h	2.5	Velliv_h	0.4	Velliv_h	0	Velliv_m	0	Velliv_m
19.9	Velliv_h	12.5	Velliv_m	7.4	Velliv_m	1.8	Velliv_m	0.2	Velliv_m	0	Velliv_m_l	0	Velliv_m_l
18.2	Velliv_m_l	9.6	Velliv_m_l	5.4	Velliv_m_l	1.4	PFA_h	0.2	Velliv_m_l	0	Velliv_h	0	Velliv_h
14.3	PFA_h	8.6	PFA_h	5.3	PFA_h	1.3	Velliv_m_l	0.2	PFA_m	0	PFA_m	0	PFA_m
13.0	mix_h	6.2	mix_m	3.3	PFA_m	0.9	PFA_m	0.2	PFA_h	0	PFA_h	0	PFA_h
12.7	mix_m	6.0	PFA_m	3.3	mix_m	0.7	mix_m	0.1	mix_m	0	mix_m	0	mix_m
12.2	PFA_m	4.2	mix_h	0.9	mix_h	0.0	mix_h	0.0	mix_h	0	mix_h	0	mix_h

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	78.7	81.8	80.1	87.8	85.7	87.3	87.0
5	63.8	64.9	69.2	71.5	75.8	71.4	69.9
10	41.0	36.2	53.3	32.7	59.6	35.6	46.1
25	0.0	0.3	0.0	0.1	0.0	0.0	1.2
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
87.8	PFA_m	75.8	PFA_h	59.6	PFA_h	1.2	mix_h	0	Velliv_m	0	Velliv_m
87.3	mix_m	71.5	PFA_m	53.3	Velliv_h	0.3	Velliv_m_l	0	Velliv_m_l	0	Velliv_m_l
87.0	mix_h	71.4	mix_m	46.1	mix_h	0.1	PFA_m	0	Velliv_h	0	Velliv_h
85.7	PFA_h	69.9	mix_h	41.0	Velliv_m	0.0	Velliv_m	0	PFA_m	0	PFA_m
81.8	Velliv_m_l	69.2	Velliv_h	36.2	Velliv_m_l	0.0	Velliv_h	0	PFA_h	0	PFA_h
80.1	Velliv_h	64.9	Velliv_m_l	35.6	mix_m	0.0	PFA_h	0	mix_m	0	mix_m
78.7	Velliv_m	63.8	Velliv_m	32.7	PFA_m	0.0	mix_m	0	mix_h	0	mix_h

MC risk percentiles

Risk of loss from first to last period.

_a is simulation from estimated distribution of mix.

_b is mix of simulations from estimated distribution of returns from individual funds.

_m is medium.

_h is high.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	4.97	3.25	4.15	2.30	1.03	1.10	0	0.38	0.13
5	4.27	2.94	3.74	2.10	0.89	0.98	0	0.32	0.09
10	3.69	2.52	3.42	1.91	0.84	0.91	0	0.26	0.06
25	2.18	1.64	2.41	1.27	0.58	0.62	0	0.14	0.03
50	0.82	0.79	1.05	0.80	0.26	0.29	0	0.02	0.00
90	0.05	0.11	0.15	0.17	0.06	0.04	0	0.00	0.00
99	0.01	0.02	0.01	0.04	0.03	0.01	0	0.00	0.00

1e6 simulation paths of mhr_b:

	0	5	10	25	50	90	99
prob_pct	0.118	0.095	0.076	0.036	0.008	0	0

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.97	Velliv_m	4.27	Velliv_m	3.69	Velliv_m	2.41	Velliv_h	1.05	Velliv_h	0.17	PFA_m	0.04	PFA_m
4.15	Velliv_h	3.74	Velliv_h	3.42	Velliv_h	2.18	Velliv_m	0.82	Velliv_m	0.15	Velliv_h	0.03	PFA_h
3.25	Velliv_m_l	2.94	Velliv_m_l	2.52	Velliv_m_l	1.64	Velliv_m_l	0.80	PFA_m	0.11	Velliv_m_l	0.02	Velliv_m_l
2.30	PFA_m	2.10	PFA_m	1.91	PFA_m	1.27	PFA_m	0.79	Velliv_m_l	0.06	PFA_h	0.01	Velliv_m
1.10	mix_m_a	0.98	mix_m_a	0.91	mix_m_a	0.62	mix_m_a	0.29	mix_m_a	0.05	Velliv_m	0.01	Velliv_h
1.03	PFA_h	0.89	PFA_h	0.84	PFA_h	0.58	PFA_h	0.26	PFA_h	0.04	mix_m_a	0.01	mix_m_a
0.38	mix_m_b	0.32	mix_m_b	0.26	mix_m_b	0.14	mix_m_b	0.02	mix_m_b	0.00	mix_h_a	0.00	mix_h_a

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.13	mix_h_b	0.09	mix_h_b	0.06	mix_h_b	0.03	mix_h_b	0.00	mix_h_a	0.00	mix_m_b	0.00	mix_m_b
0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_b	0.00	mix_h_b	0.00	mix_h_b

MC gains percentiles

Chance of gains from first to last period.

_a is simulation from estimated distribution of returns of mix.

_b is mix of simulations from estimated distribution of returns from individual funds.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	95.03	96.75	95.85	97.70	98.97	98.90	100.00	99.62	99.87
5	94.25	96.40	95.52	97.53	98.88	98.71	100.00	99.56	99.85
10	93.37	95.91	94.93	97.31	98.75	98.60	100.00	99.39	99.81
25	90.80	94.26	93.60	96.49	98.23	97.92	100.00	99.00	99.66
50	85.85	90.45	90.35	94.60	97.36	96.37	99.98	97.64	99.26
100	71.03	78.66	82.62	87.77	94.60	90.13	99.71	89.89	97.49
200	38.08	45.37	63.67	58.91	84.97	60.19	93.24	48.29	87.34
300	16.01	17.01	44.94	22.57	70.68	22.46	72.27	11.50	65.97
400	5.34	4.70	28.71	3.99	53.99	3.75	44.85	1.43	41.51
500	1.43	0.88	17.25	0.49	37.99	0.31	23.32	0.10	21.69
1000	0.00	0.01	0.82	0.00	2.38	0.00	0.19	0.00	0.06

1e6 simulation paths of mhr_b:

	0	5	10	25	50	100	200	300	400	500	1000
prob	99.882	99.854	99.824	99.686	99.301	97.513	86.912	65.992	41.486	21.693	0.086

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	99.98	mix_h_a	99.71	mix_h_a
99.87	mix_h_b	99.85	mix_h_b	99.81	mix_h_b	99.66	mix_h_b	99.26	mix_h_b	97.49	mix_h_b
99.62	mix_m_b	99.56	mix_m_b	99.39	mix_m_b	99.00	mix_m_b	97.64	mix_m_b	94.60	PFA_h
98.97	PFA_h	98.88	PFA_h	98.75	PFA_h	98.23	PFA_h	97.36	PFA_h	90.13	mix_m_a
98.90	mix_m_a	98.71	mix_m_a	98.60	mix_m_a	97.92	mix_m_a	96.37	mix_m_a	89.89	mix_m_b
97.70	PFA_m	97.53	PFA_m	97.31	PFA_m	96.49	PFA_m	94.60	PFA_m	87.77	PFA_m
96.75	Velliv_m_l	96.40	Velliv_m_l	95.91	Velliv_m_l	94.26	Velliv_m_l	90.45	Velliv_m_l	82.62	Velliv_h
95.85	Velliv_h	95.52	Velliv_h	94.93	Velliv_h	93.60	Velliv_h	90.35	Velliv_h	78.66	Velliv_m_l
95.03	Velliv_m	94.25	Velliv_m	93.37	Velliv_m	90.80	Velliv_m	85.85	Velliv_m	71.03	Velliv_m

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
93.24	mix_h_a	72.27	mix_h_a	53.99	PFA_h	37.99	PFA_h	2.38	PFA_h
87.34	mix_h_b	70.68	PFA_h	44.85	mix_h_a	23.32	mix_h_a	0.82	Velliv_h
84.97	PFA_h	65.97	mix_h_b	41.51	mix_h_b	21.69	mix_h_b	0.19	mix_h_a
63.67	Velliv_h	44.94	Velliv_h	28.71	Velliv_h	17.25	Velliv_h	0.06	mix_h_b
60.19	mix_m_a	22.57	PFA_m	5.34	Velliv_m	1.43	Velliv_m	0.01	Velliv_m_l
58.91	PFA_m	22.46	mix_m_a	4.70	Velliv_m_l	0.88	Velliv_m_l	0.00	Velliv_m
48.29	mix_m_b	17.01	Velliv_m_l	3.99	PFA_m	0.49	PFA_m	0.00	PFA_m
45.37	Velliv_m_l	16.01	Velliv_m	3.75	mix_m_a	0.31	mix_m_a	0.00	mix_m_a
38.08	Velliv_m	11.50	mix_m_b	1.43	mix_m_b	0.10	mix_m_b	0.00	mix_m_b

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.

m is the location parameter.

s is the scale parameter.

nu is the estimated degrees of freedom, or shape parameter.

xi is the estimated skewness parameter.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770
R-squared	0.993	0.978	0.991	0.991	0.964	0.890	0.961

Fit statistics ranking

m	ranking	s	ranking	R-squared	ranking
0.084	PFA_high	0.071	mix_high	0.993	Velliv_medium
0.082	mix_high	0.088	mix_medium	0.991	Velliv_high
0.065	Velliv_high	0.115	Velliv_medium_long	0.991	PFA_medium
0.059	mix_medium	0.120	Velliv_medium	0.978	Velliv_medium_long
0.058	PFA_medium	0.121	PFA_high	0.964	PFA_high
0.052	Velliv_medium_long	0.123	PFA_medium	0.961	mix_high
0.048	Velliv_medium	0.150	Velliv_high	0.890	mix_medium

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).

Statistics are given for the final state of all paths.

Probability of down_and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.

0 is defined as any value below a threshold.

losing_prob_pct is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_m_b	mix_h_a	mix_h_b
mc_m	277.260	293.822	404.547	321.903	548.853	325.625	300.962	501.080	477.697
mc_s	123.712	116.562	220.357	105.482	243.592	98.291	82.667	154.282	160.897
mc_min	0.283	0.050	0.000	0.000	0.000	0.155	28.790	141.372	50.137
mc_max	814.308	1824.687	1699.981	1022.902	1740.595	741.723	964.382	1368.541	1313.325
dao_pct	0.000	0.000	0.010	0.030	0.010	0.000	0.000	0.000	0.000
losing_pct	4.970	3.250	4.150	2.300	1.030	1.100	0.380	0.000	0.130

Ranking

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	losing_pct	ranking
548.853	PFA_h	82.667	mix_m_b	141.372	mix_h_a	1824.687	Velliv_m_l	0.00	Velliv_m	0.00	mix_h_a
501.080	mix_h_a	98.291	mix_m_a	50.137	mix_h_b	1740.595	PFA_h	0.00	Velliv_m_l	0.13	mix_h_b
477.697	mix_h_b	105.482	PFA_m	28.790	mix_m_b	1699.981	Velliv_h	0.00	mix_m_a	0.38	mix_m_b
404.547	Velliv_h	116.562	Velliv_m_l	0.283	Velliv_m	1368.541	mix_h_a	0.00	mix_m_b	1.03	PFA_h
325.625	mix_m_a	123.712	Velliv_m	0.155	mix_m_a	1313.325	mix_h_b	0.00	mix_h_a	1.10	mix_m_a
321.903	PFA_m	154.282	mix_h_a	0.050	Velliv_m_l	1022.902	PFA_m	0.00	mix_h_b	2.30	PFA_m
300.962	mix_m_b	160.897	mix_h_b	0.000	Velliv_h	964.382	mix_m_b	0.01	Velliv_h	3.25	Velliv_m_l
293.822	Velliv_m_l	220.357	Velliv_h	0.000	PFA_m	814.308	Velliv_m	0.01	PFA_h	4.15	Velliv_h
277.260	Velliv_m	243.592	PFA_h	0.000	PFA_h	741.723	mix_m_a	0.03	PFA_m	4.97	Velliv_m

Comments

(Ignoring `mhr_a...`)

`mhr` has some nice properties:

- It has a relatively high `nu` value of 90, which means it is tending more towards exponential tails than polynomial tails. All other funds have `nu` values close to 3, except `phr` which is even worse at close to 2. (Note that for a Gaussian, `nu` is infinite.)
- It has the lowest losing percentage of all simulations, which is better than 1/6 that of `phr`.
- It has a DAO percentage of 0, which is the same as `mmr`, and less than `phr`.
- Only `phr` has a higher `mc_m`.
- It has a smaller `mc_s` than the individual components, `vhr` and `phr`.
- It has the highest `xi` of all fits, suggesting less left skewness. Density plots for `vmr`, `phr` and `mmr` have an extremely sharp drop, as if an upward limiter has been applied, which corresponds to extremely low `xi` values. The density plot for `mhr` is by far the most symmetrical of all the fits.
- Only `mmr` has a higher `mc_min`. However, that of `mmr` is 18 times higher with 62, so `mmr` is a clear winner here.
- Naturally, it has a `mc_max` smaller than the individual components, `vhr` and `phr`, but ca. 1.5 times higher than `mmr`.
- All the first 4 moments converge nicely. For all other fits, the 4th moment doesn't seem to converge.

Taleb, Statistical Consequences Of Fat Tails, p. 97:

"the variance of a finite variance random variable with tail exponent < 4 will be infinite".

And p. 363:

"The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, under a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance."

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} \right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2} \right) / \left(\frac{x_{t-1} + y_{t-1}}{2} \right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns = Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} \right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2 \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1} y_t = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

$$(x_{t-1} x_t y_{t-1} + y_{t-1} x_t y_{t-1}) + (x_{t-1} x_{t-1} y_t + x_{t-1} y_{t-1} y_t) = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: $R = 1+r$

```
## Let x_0 be 100.
```

```
## Let y_0 be 200.
```

```
## So the initial value of the pf is 300 .
```

```
## Let R_x be 0.5.
```

```
## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.
```

```
## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): 0.03592915
## s(data_x): 0.458387
## m(data_y): 10.70493
## s(data_y): 2.297038
##
## m(data_x + data_y): 5.370428
## s(data_x + data_y): 1.22428
```

m and s of final state of all paths.

_a is mix of simulated returns.

_b is simulated mixed returns.

m_a	m_b	s_a	s_b
107.246	107.369	5.527	5.564
107.127	107.522	5.203	5.634
107.411	107.560	5.352	5.370
107.558	107.676	5.196	5.541
107.513	107.301	5.401	5.415
107.395	107.311	5.169	5.276
107.596	107.280	5.261	5.378
107.482	107.407	5.137	5.698
107.489	107.306	5.296	5.497
107.404	107.587	5.068	5.135

```

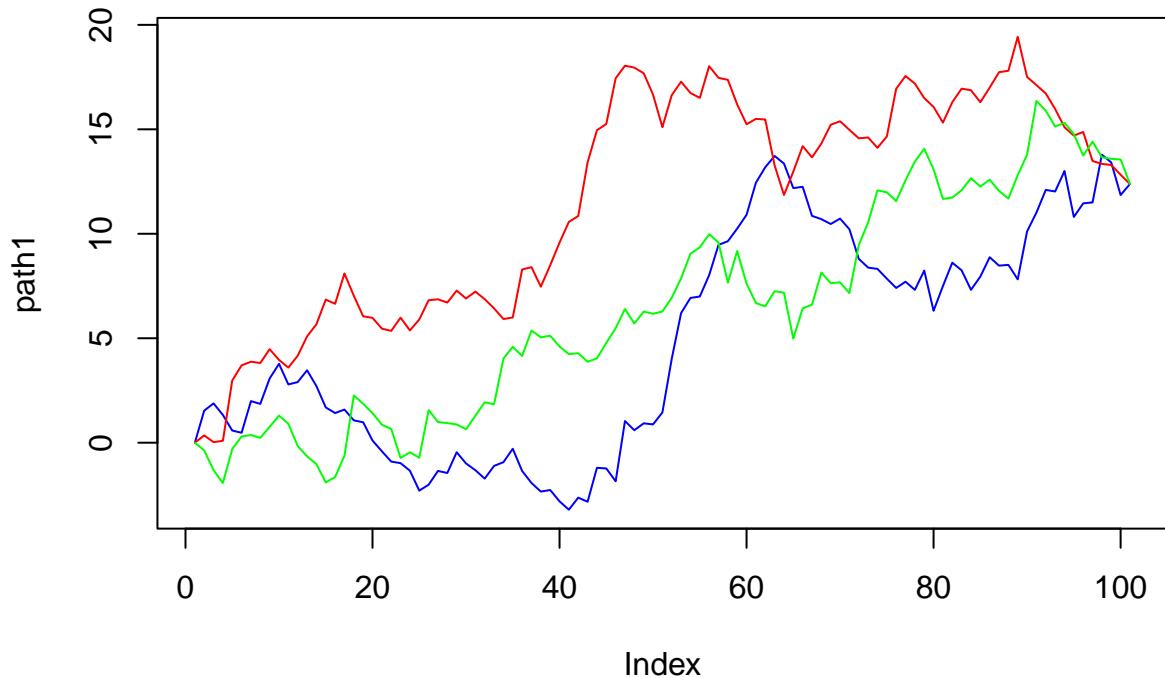
##      m_a      m_b      s_a      s_b
## Min.  :107.1  Min.  :107.3  Min.  :5.068  Min.  :5.135
## 1st Qu.:107.4 1st Qu.:107.3 1st Qu.:5.176 1st Qu.:5.372
## Median :107.4 Median :107.4 Median :5.232 Median :5.456
## Mean   :107.4 Mean   :107.4 Mean   :5.261 Mean   :5.451
## 3rd Qu.:107.5 3rd Qu.:107.6 3rd Qu.:5.338 3rd Qu.:5.558
## Max.   :107.6 Max.   :107.7 Max.   :5.527 Max.   :5.698

```

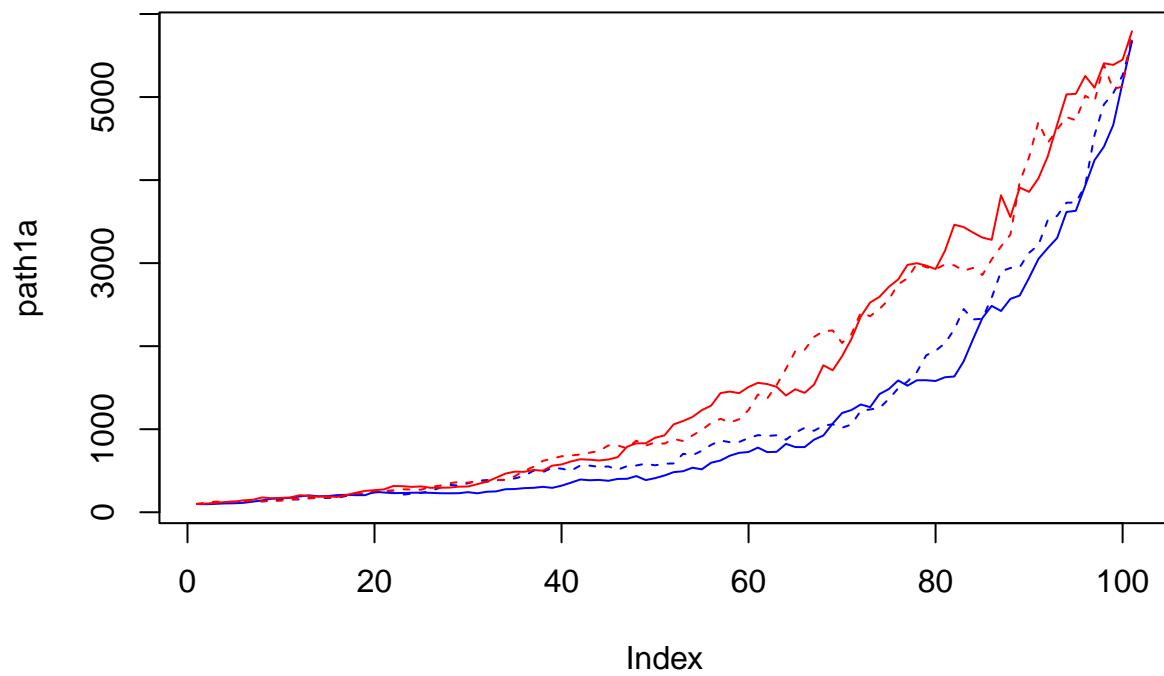
_a and _b are very close to equal.

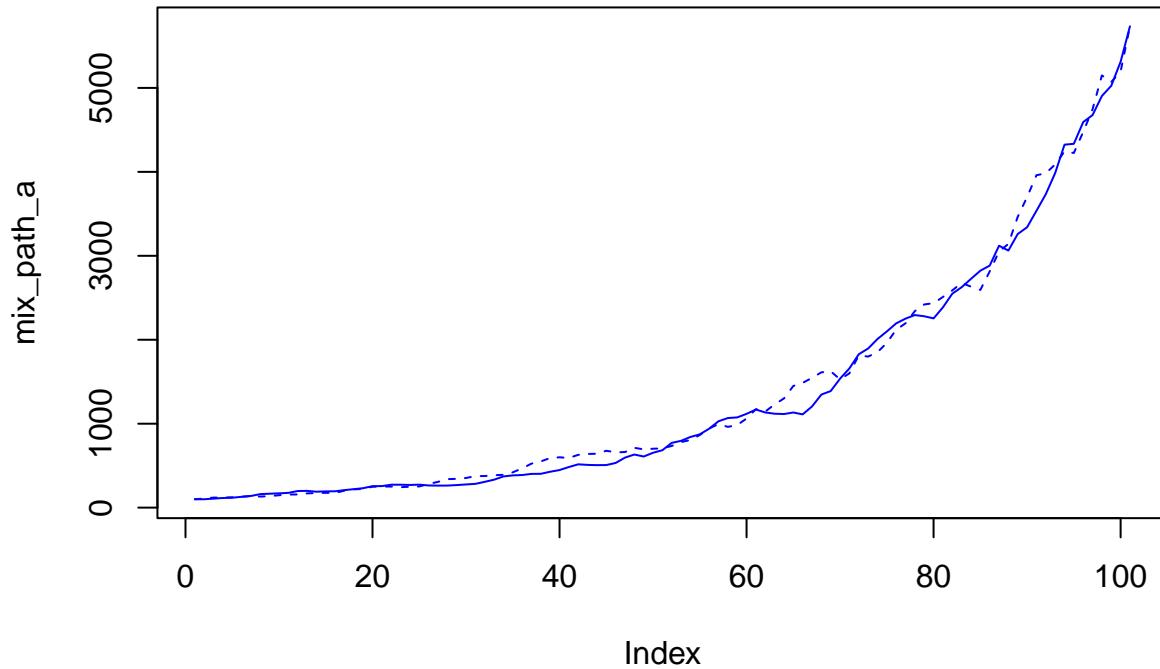
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?





The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?

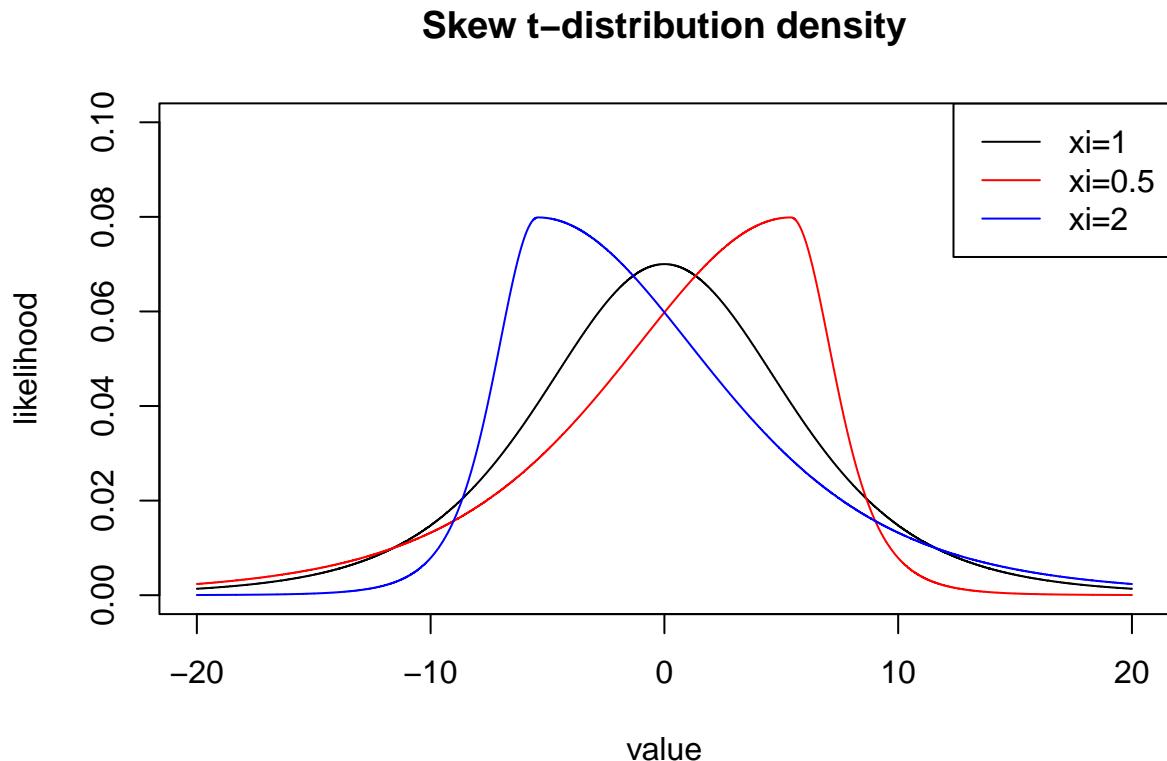
Let's try to include the Velliv data from 2007-2010.

We do this by sampling 13 observations from `vmrl1`.

```
##          m             s
##  Min. :0.05947  Min.  :0.03993
##  1st Qu.:0.06518  1st Qu.:0.05850
##  Median :0.06904  Median :0.06598
##  Mean   :0.06981  Mean   :0.06748
##  3rd Qu.:0.07246  3rd Qu.:0.07696
##  Max.   :0.08317  Max.   :0.09592
```

The meaning of ξ_i

The fit for `mhr` has the highest ξ_i value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, \dots, X_n^p)}{\sum_{i=1}^n X_i^p} \rightarrow 0$$

for $n \rightarrow \infty$.

If not, X doesn't have a p 'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192