Pension returns analysis

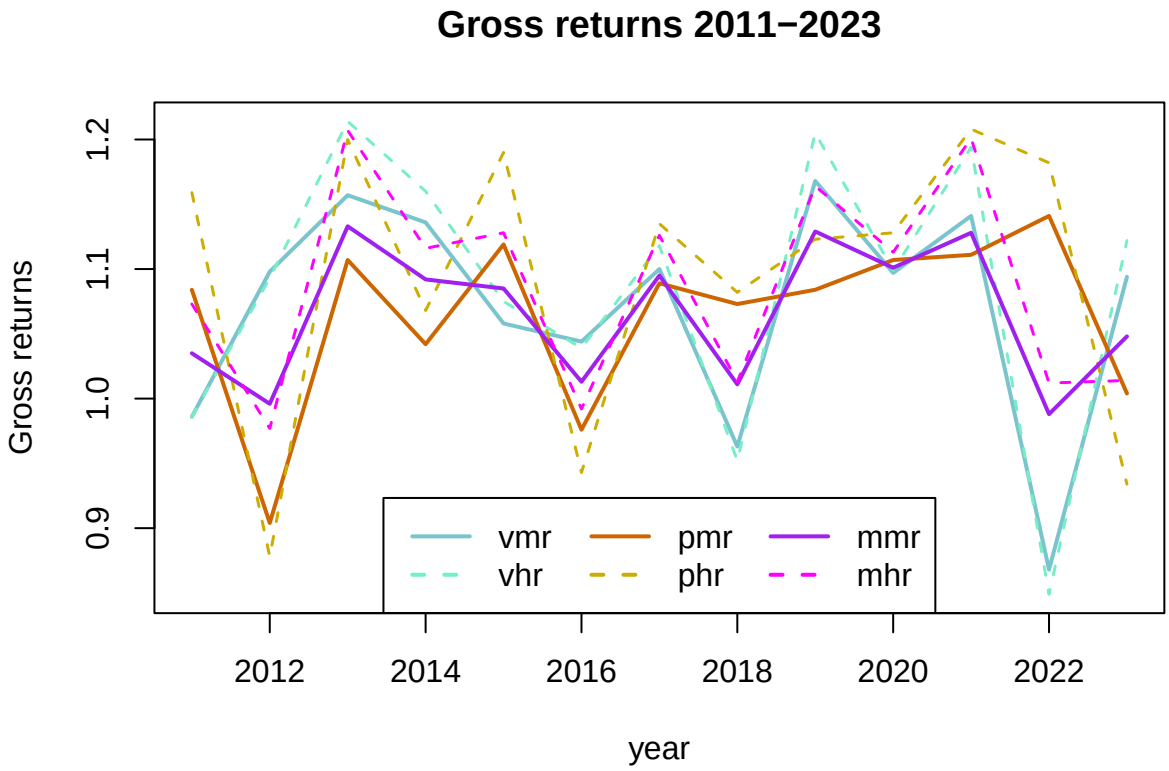
21:24 11 April 2024

Contents

Fit log returns to F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated shape parameter (degrees of freedom).
`xi` is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.
`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.



Summary of gross returns

```
##       vmr              pmr              mmr              vhr        
##  Min.   :0.868   Min.   :0.904   Min.   :0.988   Min.   :0.849  
##  1st Qu.:1.044   1st Qu.:1.042   1st Qu.:1.013   1st Qu.:1.039  
##  Median :1.097   Median :1.084   Median :1.085   Median :1.099  
##  Mean   :1.070   Mean   :1.065   Mean   :1.066   Mean   :1.085  
##  3rd Qu.:1.136   3rd Qu.:1.107   3rd Qu.:1.101   3rd Qu.:1.160  
##  Max.   :1.168   Max.   :1.141   Max.   :1.133   Max.   :1.214  
##       phr              mhr        
##  Min.   :0.878   Min.   :0.977  
##  1st Qu.:1.068   1st Qu.:1.013  
##  Median :1.128   Median :1.113  
##  Mean   :1.095   Mean   :1.087  
##  3rd Qu.:1.182   3rd Qu.:1.128  
##  Max.   :1.208   Max.   :1.207
```

```
##       vmrl        
##  Min.   :0.801  
##  1st Qu.:1.013  
##  Median :1.085  
##  Mean   :1.061  
##  3rd Qu.:1.128  
##  Max.   :1.193
```

```
##            vmr   pmr   mmr   vhr   phr   mhr
## Min.   : 0.868 0.904 0.988 0.849 0.878 0.977
## 1st Qu.: 1.044 1.042 1.013 1.039 1.068 1.013
## Median : 1.097 1.084 1.085 1.099 1.128 1.113
## Mean   : 1.070 1.065 1.066 1.085 1.095 1.087
## 3rd Qu.: 1.136 1.107 1.101 1.160 1.182 1.128
## Max.   : 1.168 1.141 1.133 1.214 1.208 1.207
```

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.136	vmr	1.168	vmr
0.977	mhr	1.044	vmr	1.113	mhr	1.087	mhr	1.107	pmr	1.141	pmr
0.904	pmr	1.042	pmr	1.099	vhr	1.085	vhr	1.101	mmr	1.133	mmr
0.878	phr	1.039	vhr	1.097	vmr	1.070	vmr	1.160	vhr	1.214	vhr
0.868	vmr	1.013	mmr	1.085	mmr	1.066	mmr	1.182	phr	1.208	phr
0.849	vhr	1.013	mhr	1.084	pmr	1.065	pmr	1.128	mhr	1.207	mhr

Covariance

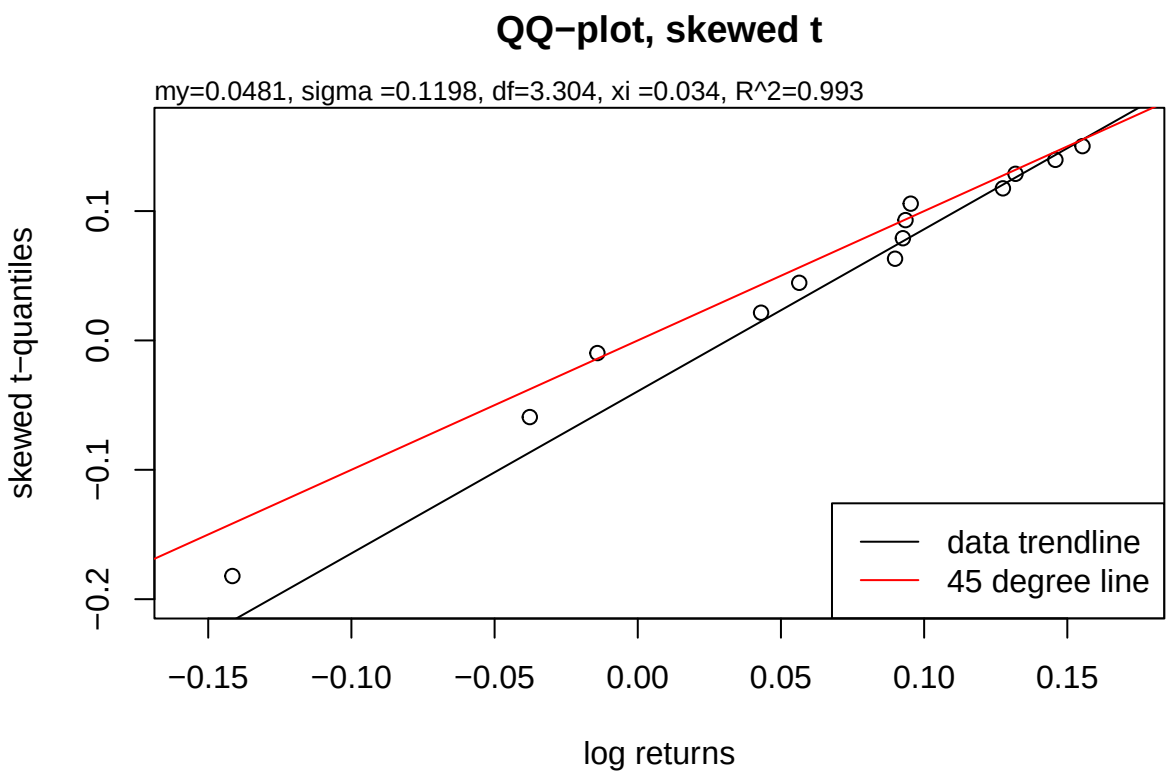
```
## cov(vmr, pmr) =  -0.001094875
```

```
## cov(vhr, phr) =  -0.0001730651
```

Velliv medium risk, 2011 - 2023

```
##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 7.4 percent
## What is the risk of losing max 25 %? =< 1.8 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 41 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

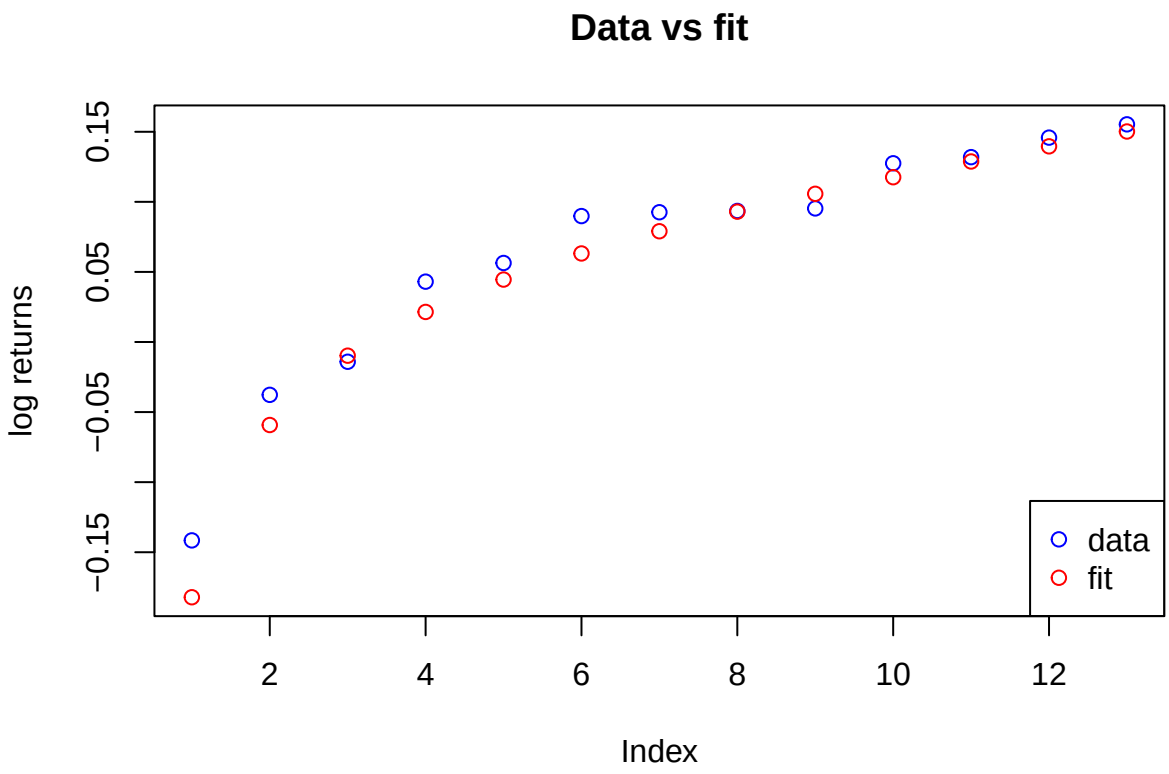
QQ Plot



The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

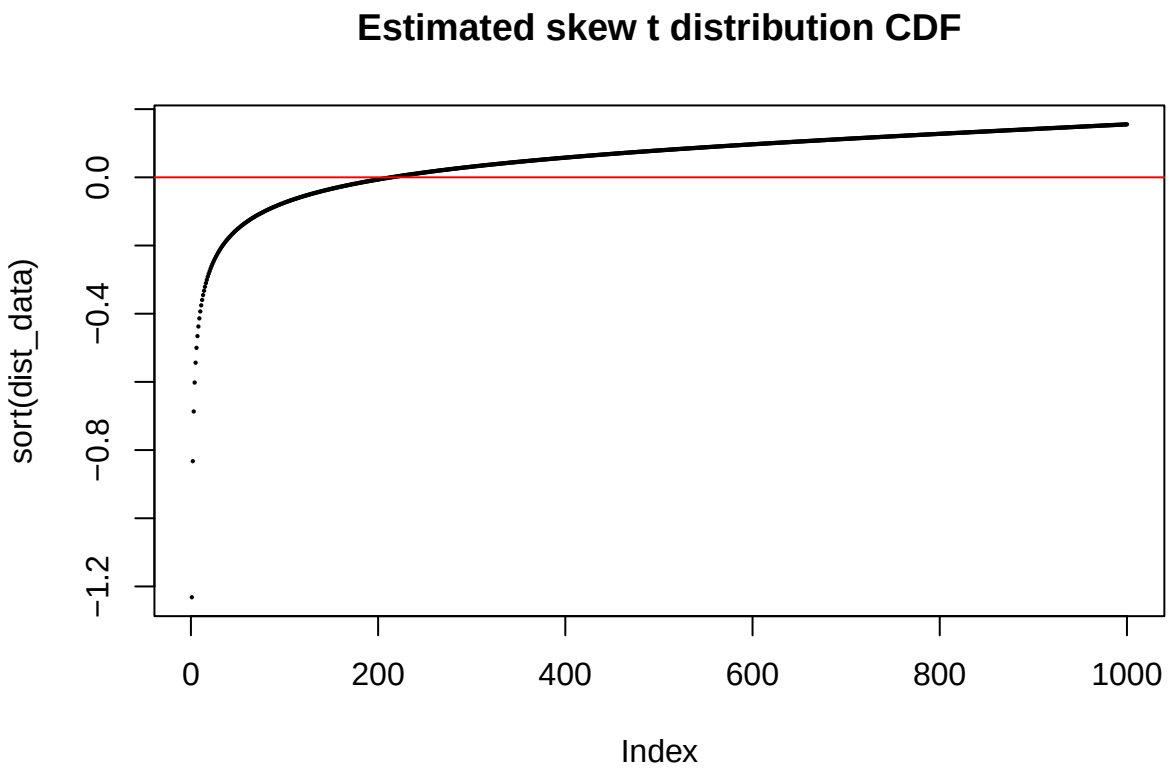
Data vs fit

Let's plot the fit and the observed returns together.

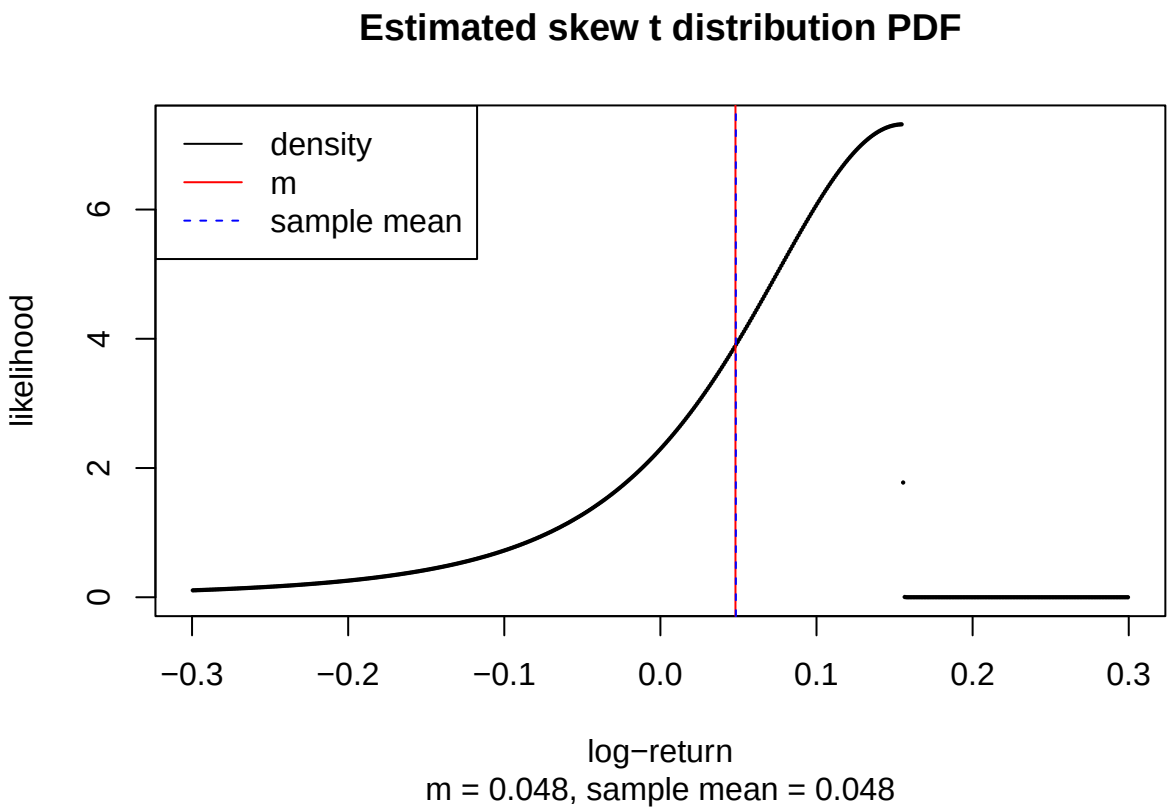


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

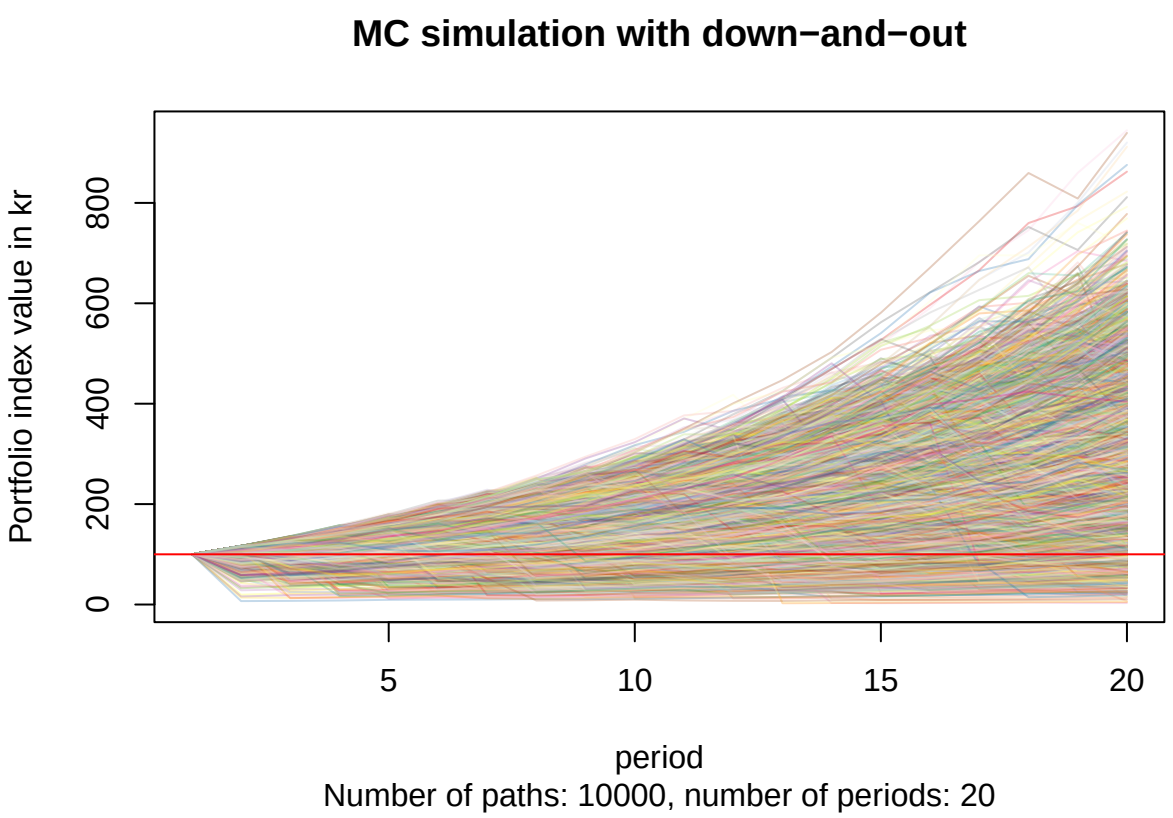


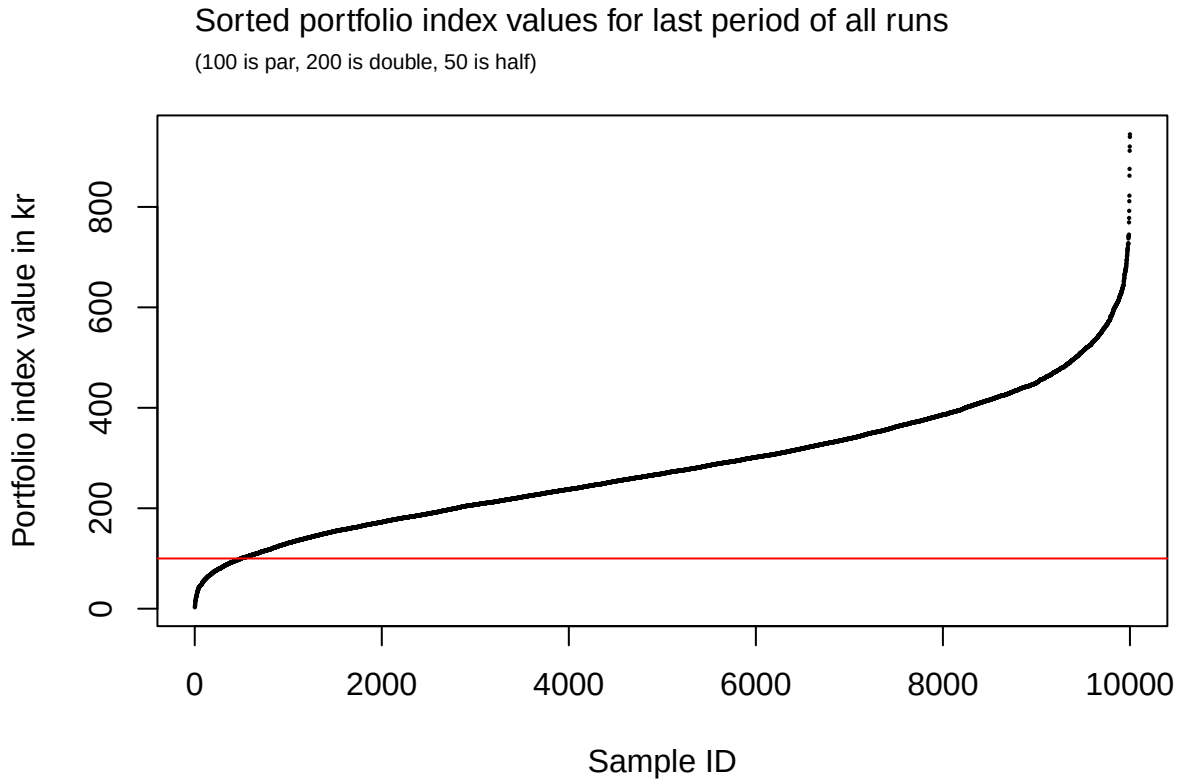
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.



Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 282.756 kr.
## SD of portfolio index value after 20 years: 126.515 kr.
## Min total portfolio index value after 20 years: 2.896 kr.
## Max total portfolio index value after 20 years: 944.49 kr.
##
## Share of paths finishing below 100: 4.97 percent
```

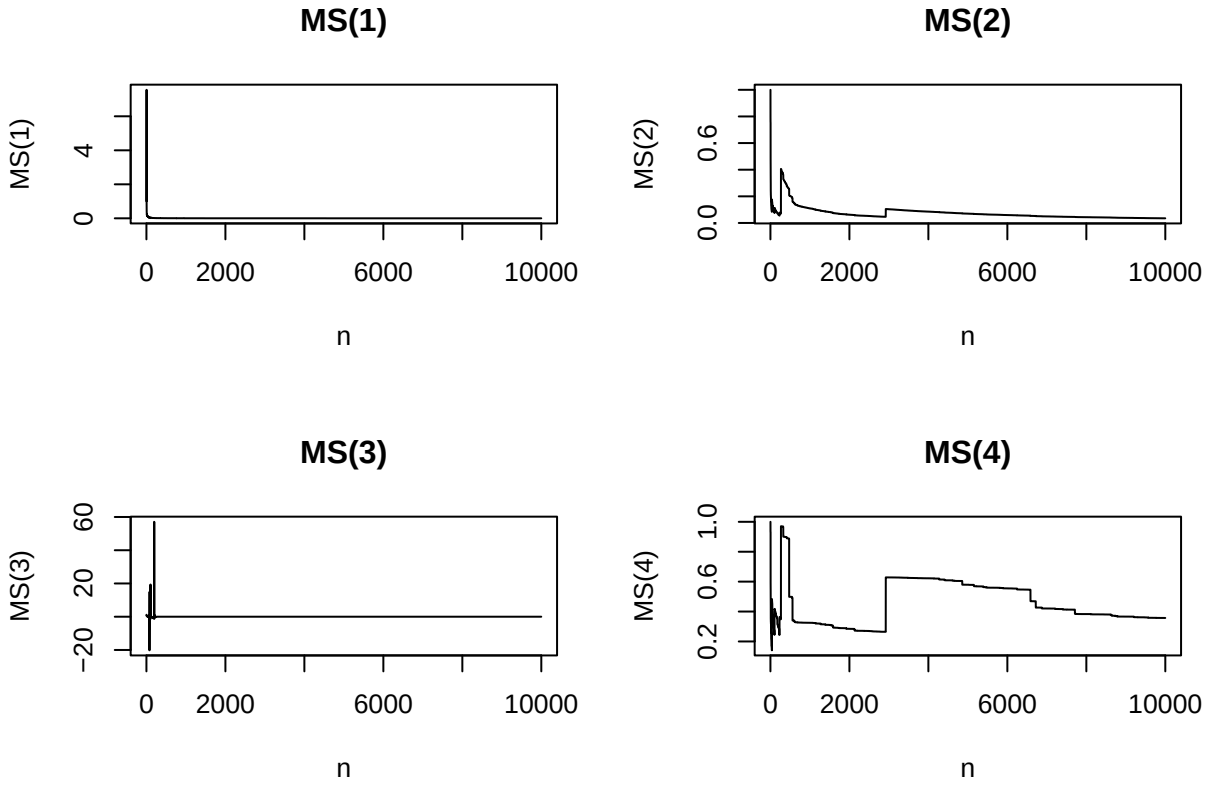

MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Importance sampling

Max vs sum plots

Max vs sum plots for the first four moments:

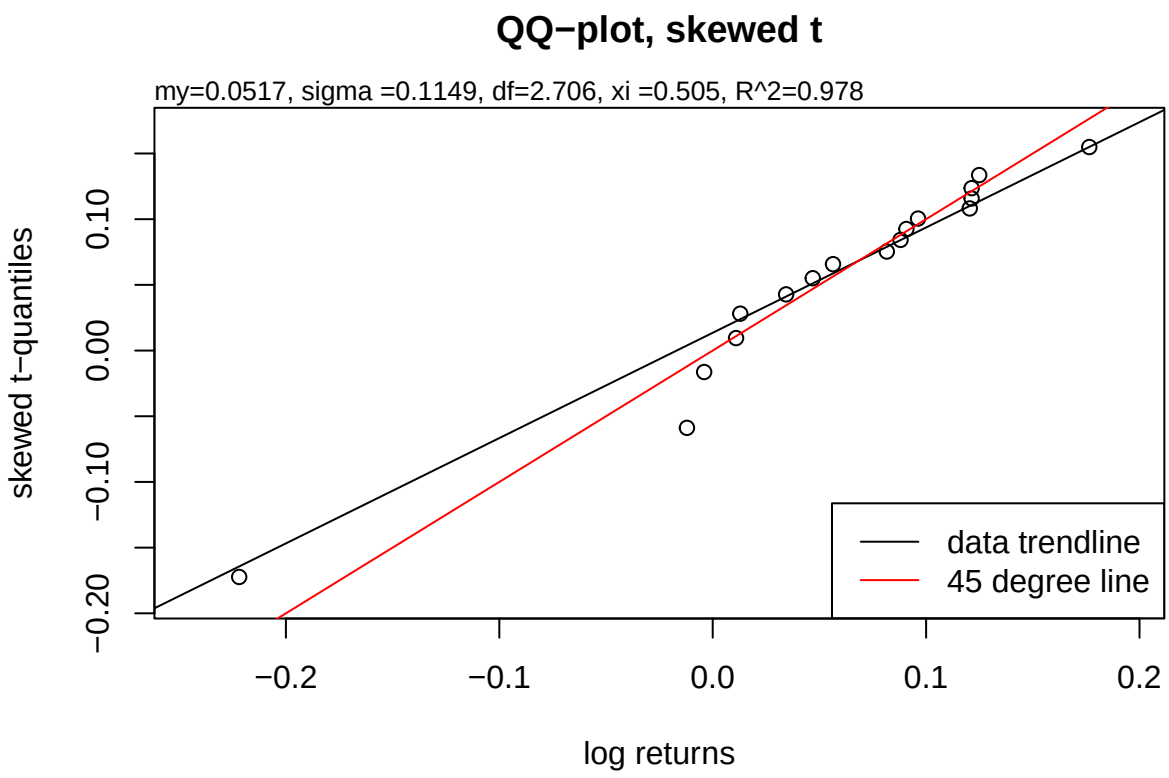


Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##
## AIC: -34.35752
## BIC: -31.02467
## m: 0.05171176
## s: 0.1149408
## nu (df): 2.706099
## xi: 0.5049945
## R^2: 0.978
##
## An R^2 of 0.978 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 5.4 percent
## What is the risk of losing max 25 %? =< 1.3 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 36.2 percent
## What is the chance of gaining min 25 %? >= 0.3 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

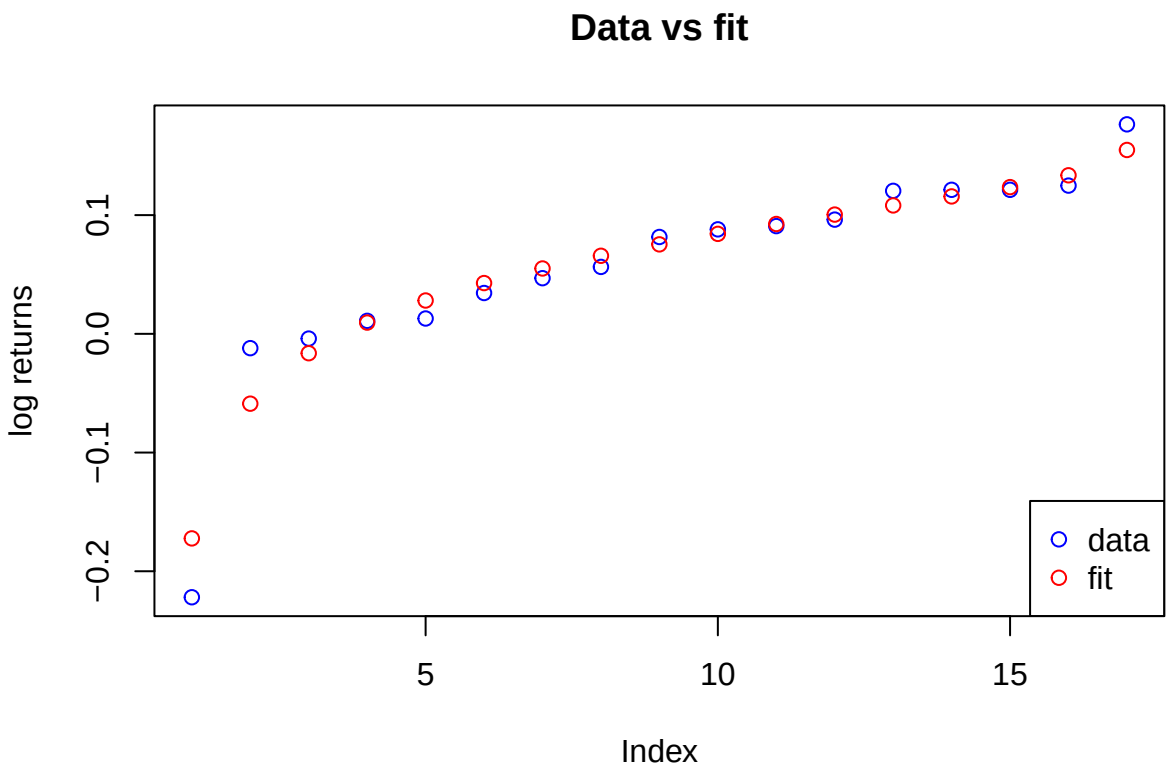
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

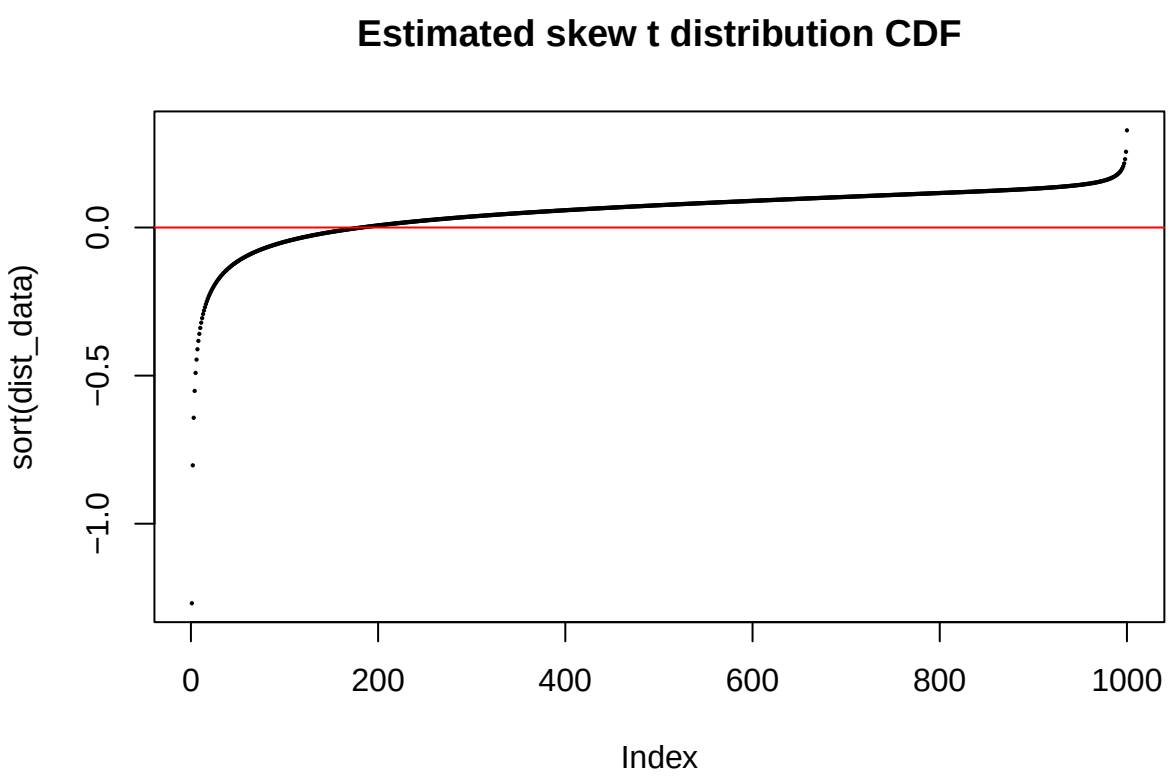
Data vs fit

Let's plot the fit and the observed returns together.

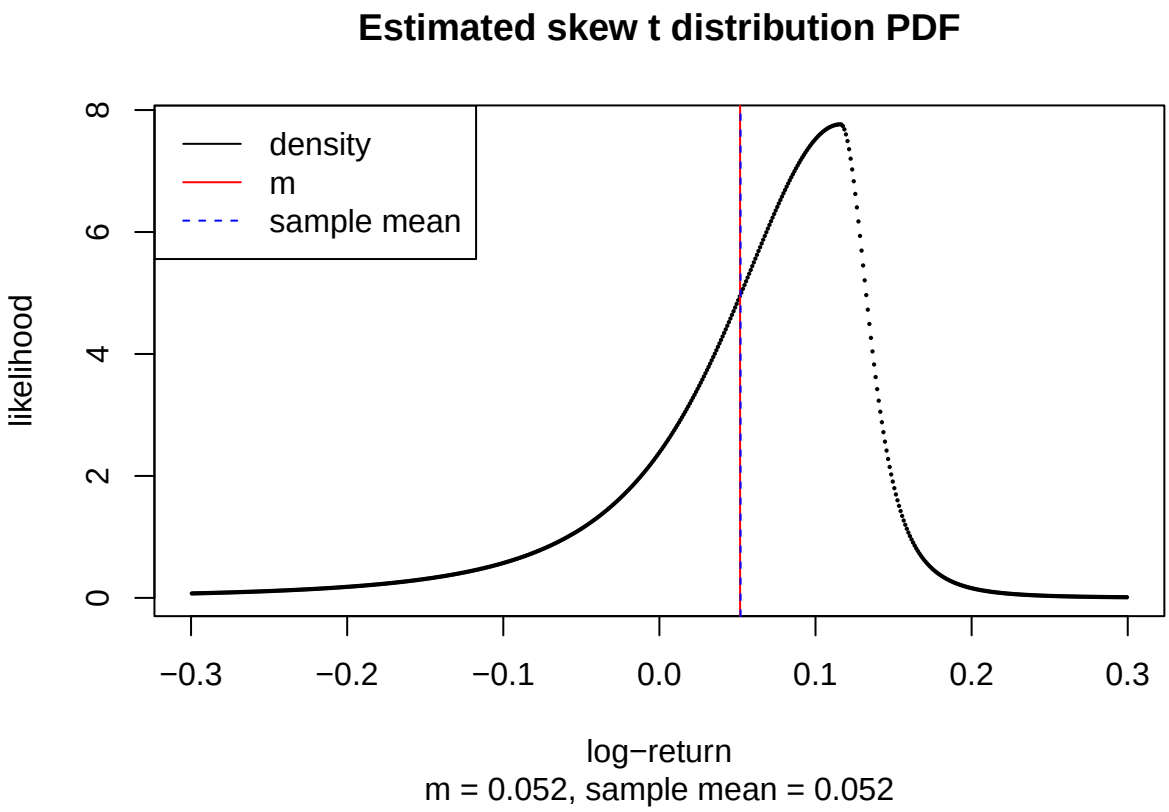


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

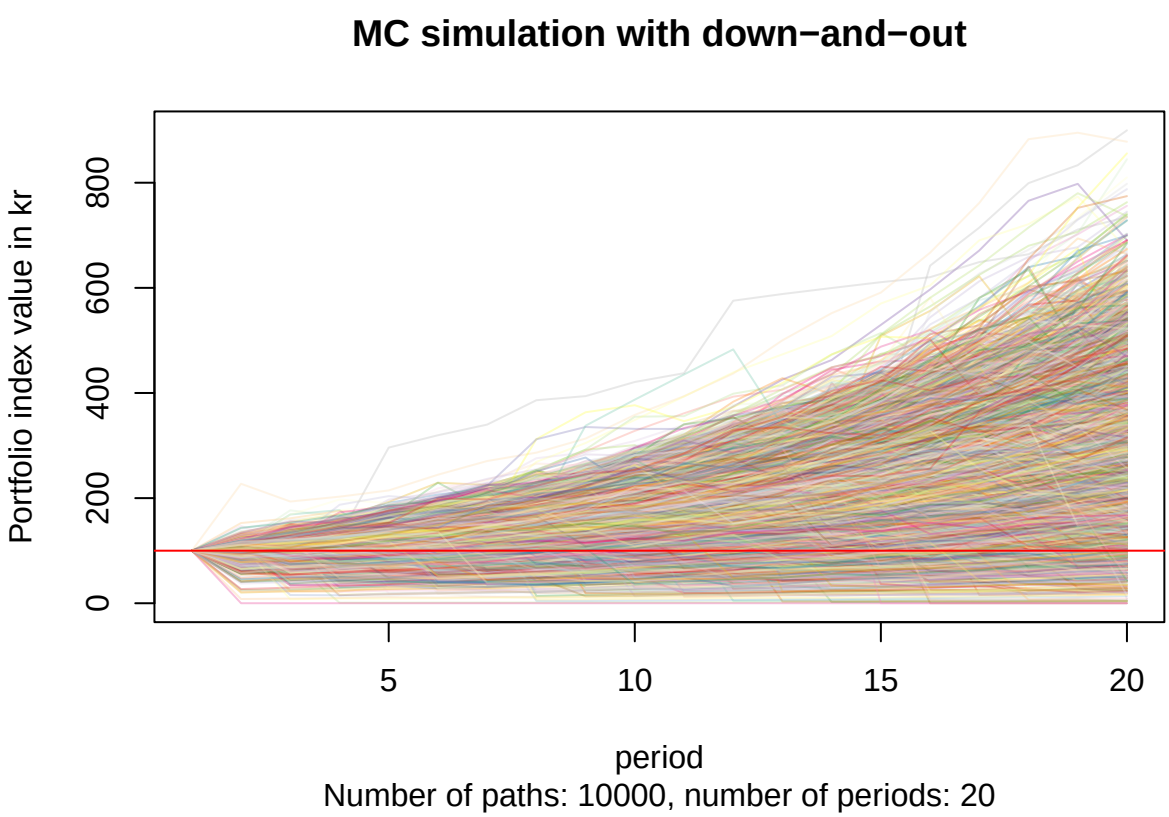


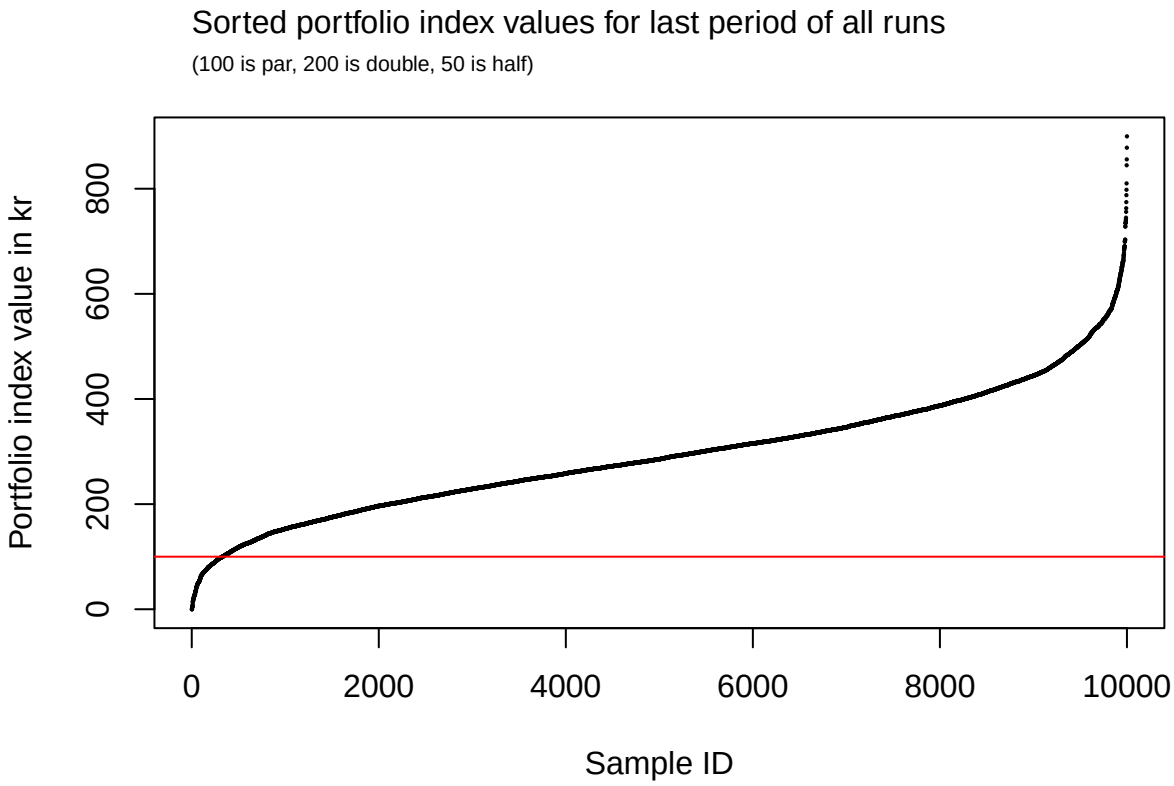
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!



Monte Carlo

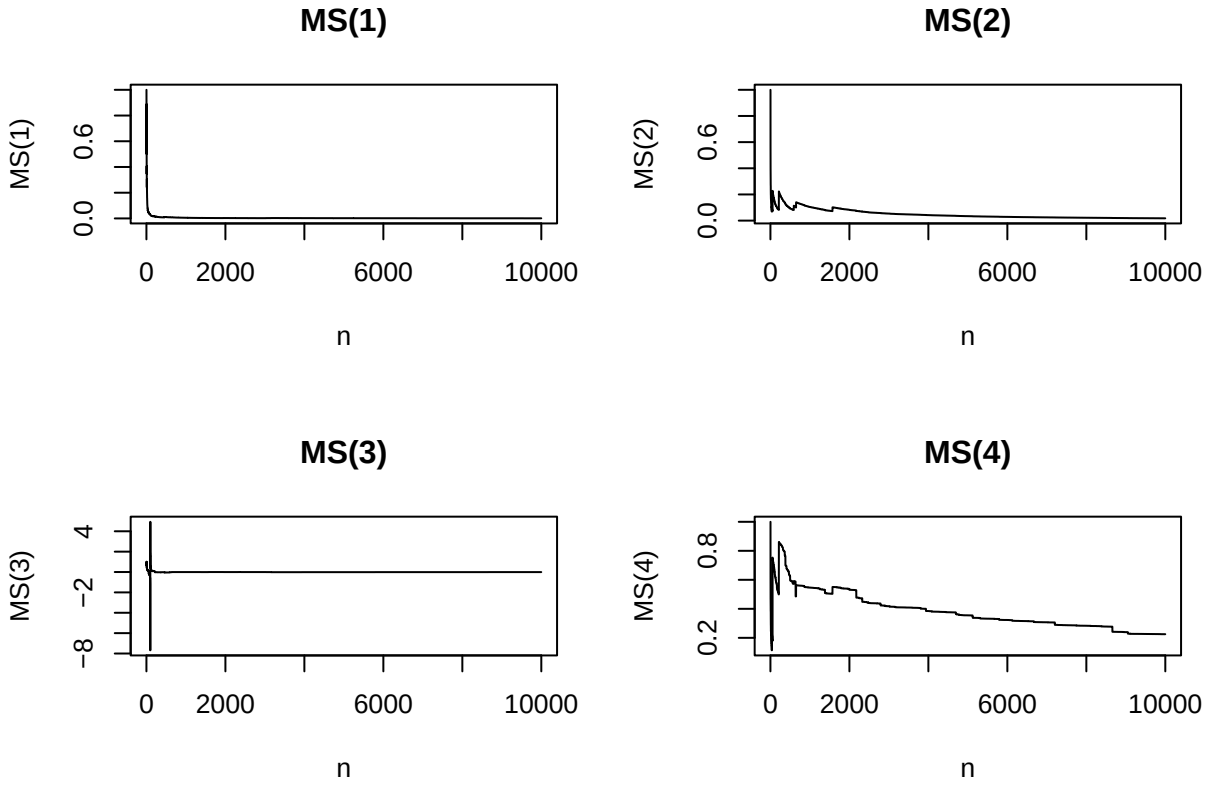
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 294.553 kr.
## SD of portfolio index value after 20 years: 116.778 kr.
## Min total portfolio index value after 20 years: 0.02 kr.
## Max total portfolio index value after 20 years: 899.641 kr.
##
## Share of paths finishing below 100: 3.29 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Max vs sum plots

Max vs sum plots for the first four moments:

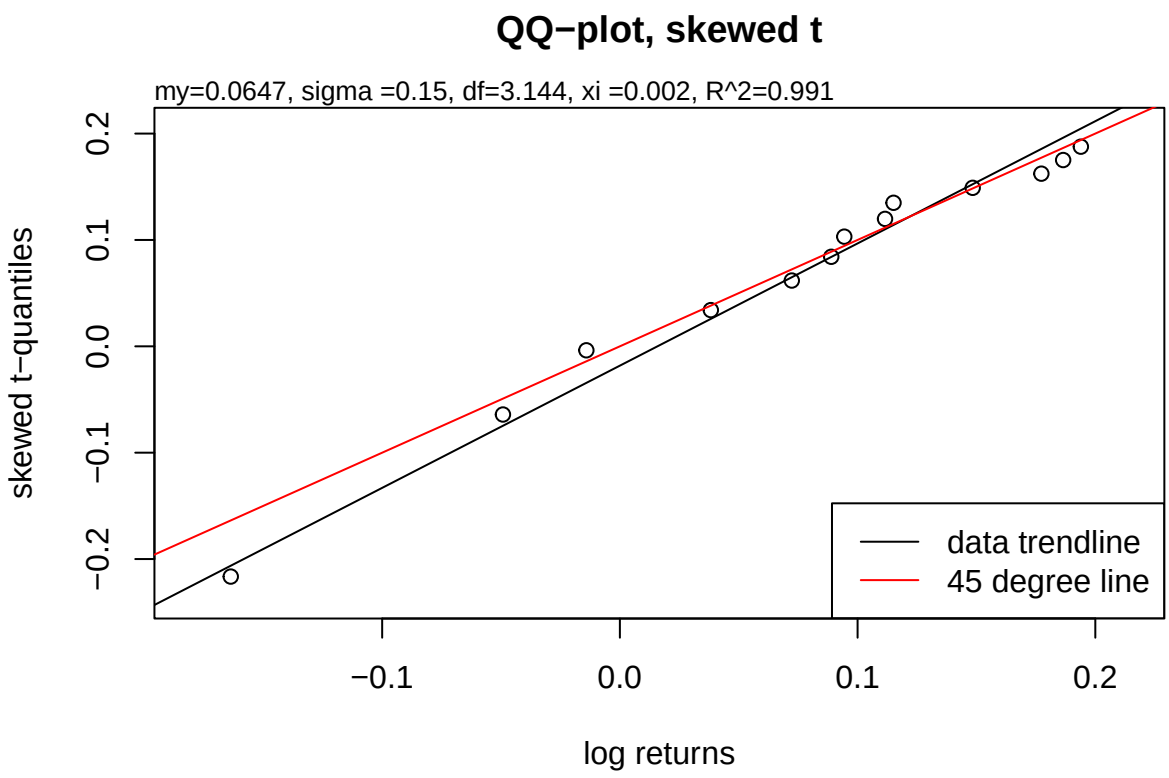


Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -21.42488
## BIC: -19.16508
## m: 0.06471454
## s: 0.1499924
## nu (df): 3.144355
## xi: 0.002367034
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 8.3 percent
## What is the risk of losing max 25 %? =< 2.5 percent
## What is the risk of losing max 50 %? =< 0.4 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 53.3 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

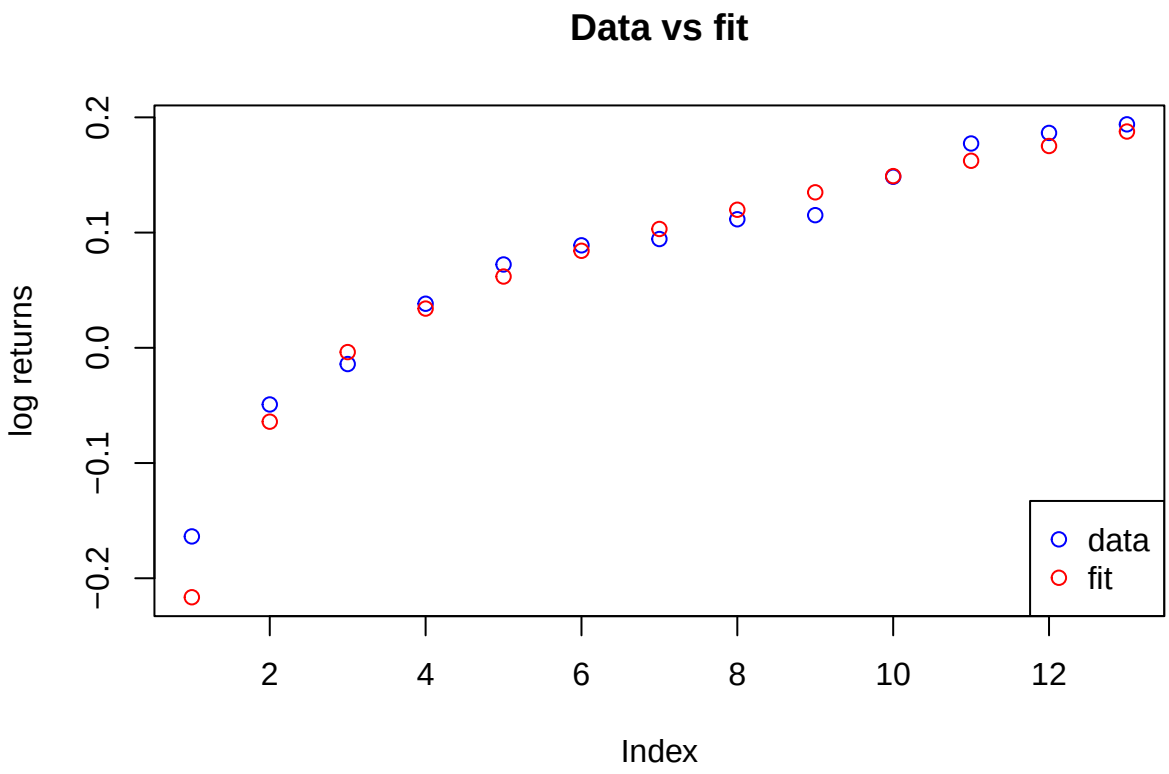
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

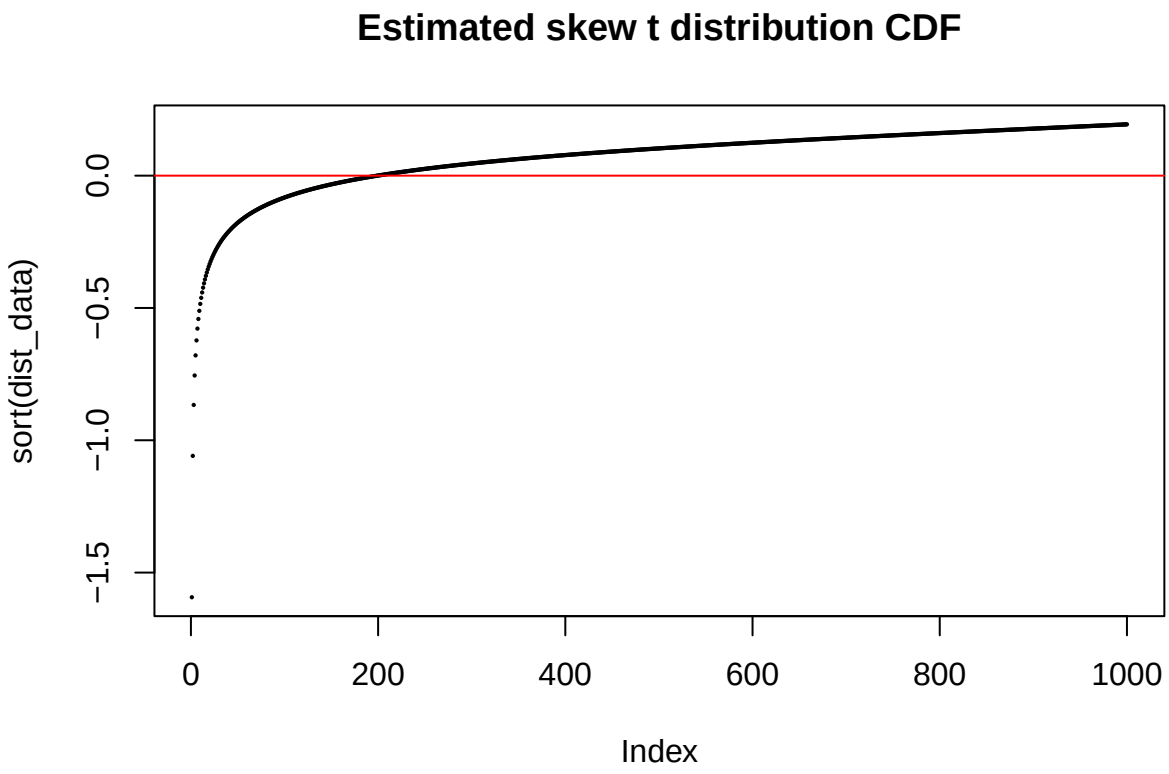
Data vs fit

Let's plot the fit and the observed returns together.

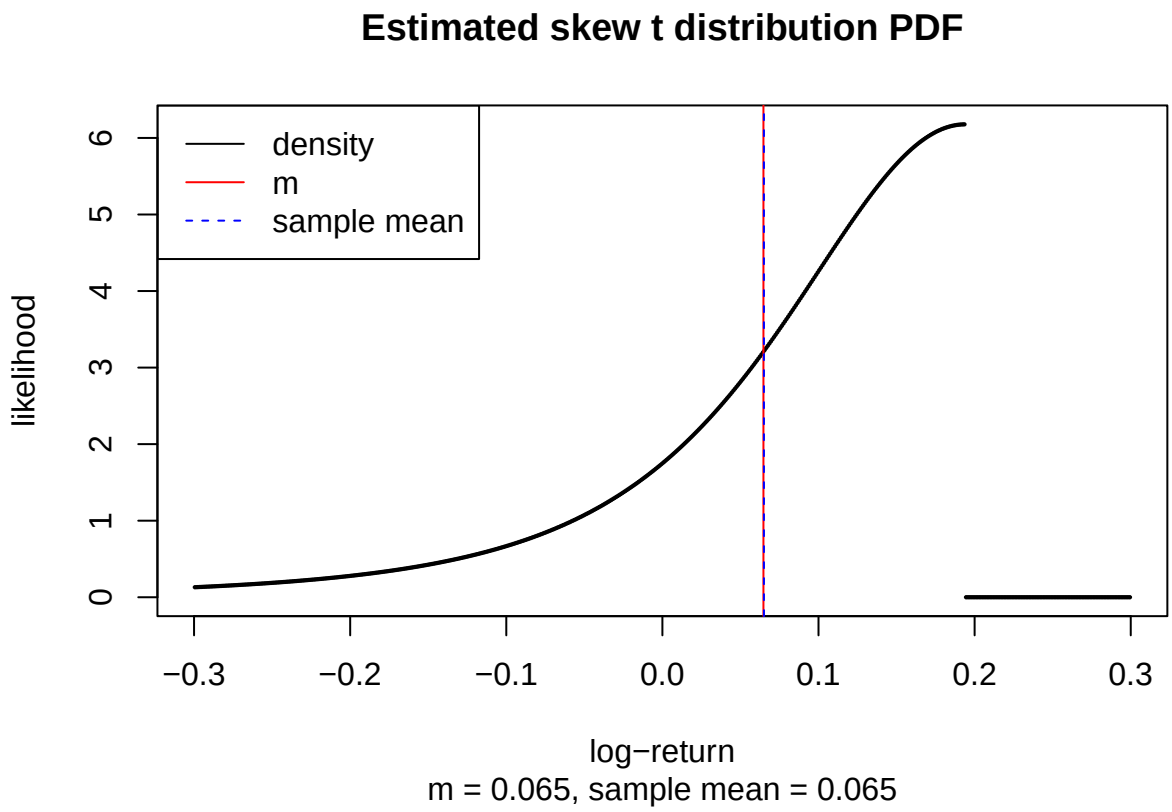


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

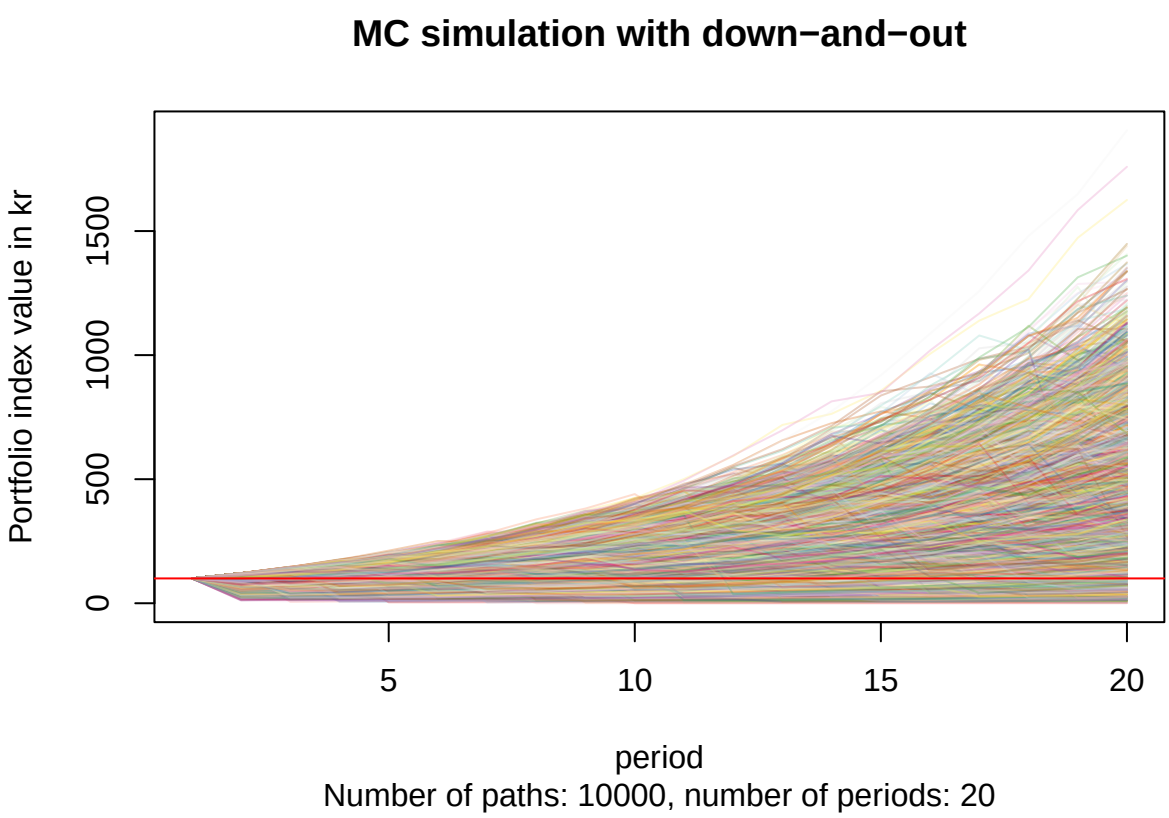


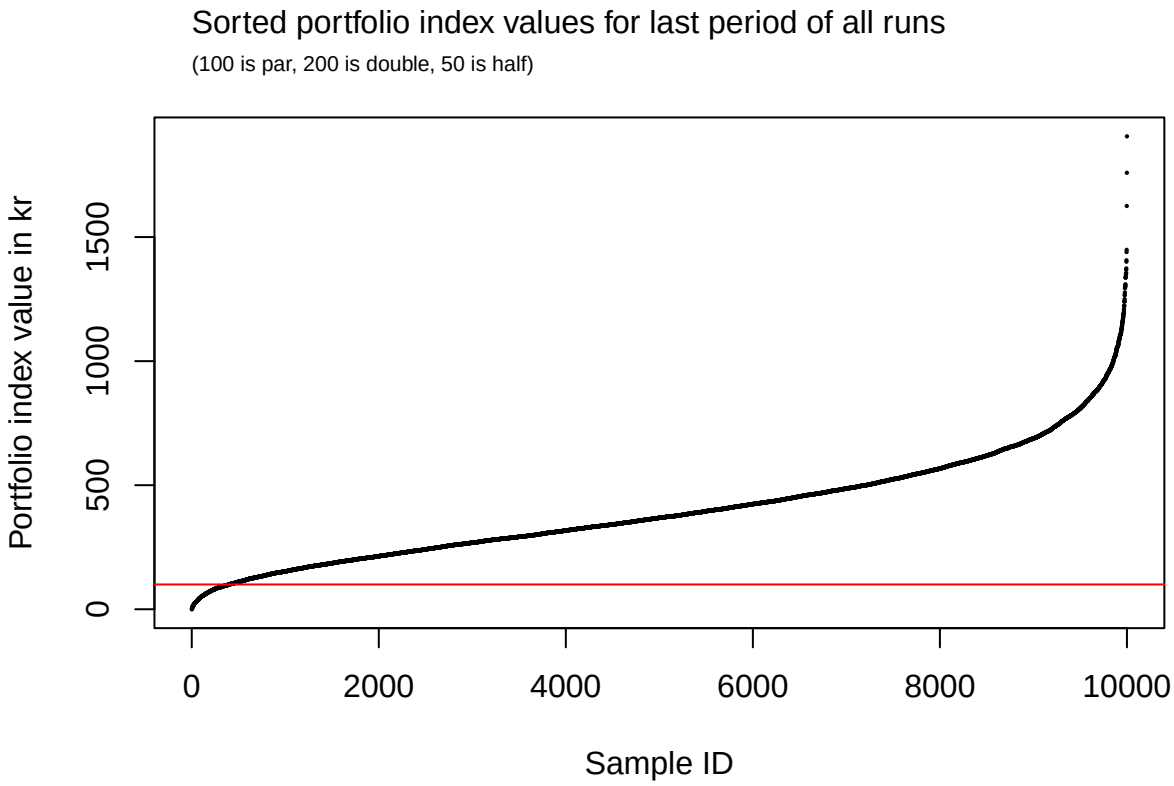
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.



Monte Carlo

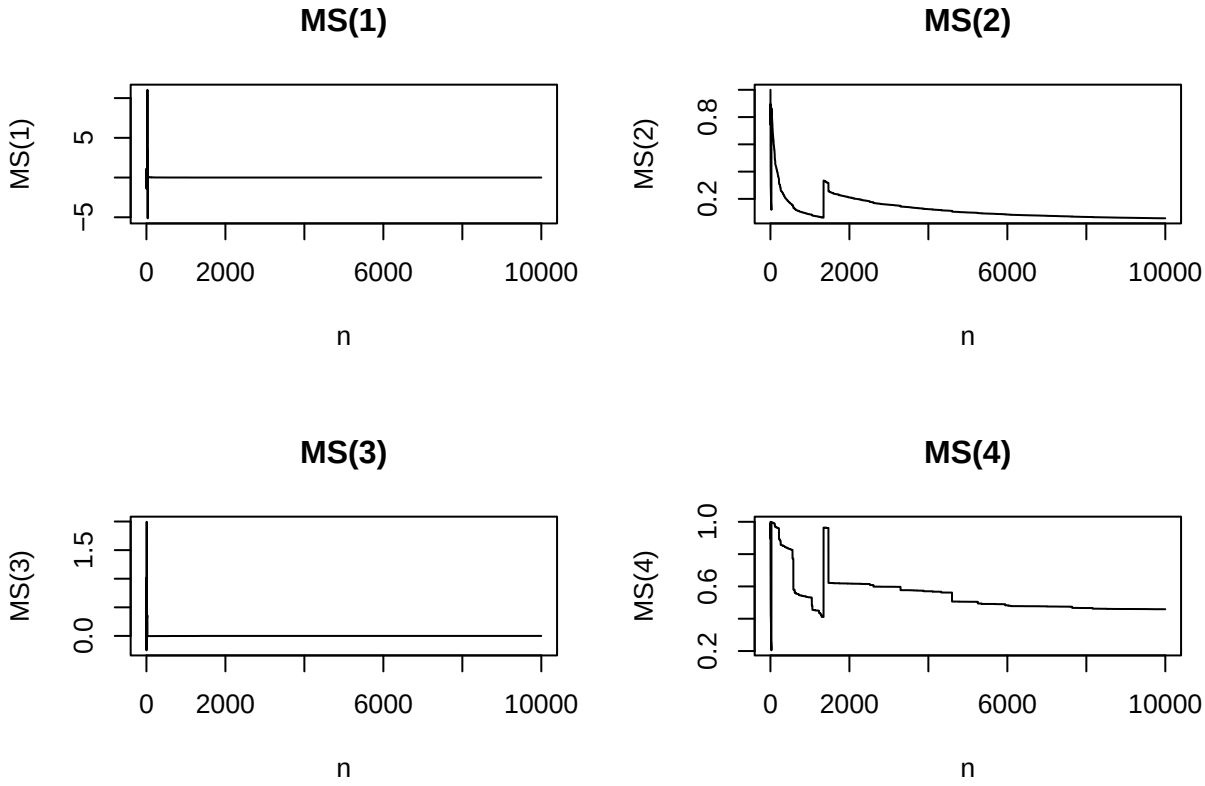
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 401.591 kr.
## SD of portfolio index value after 20 years: 218.086 kr.
## Min total portfolio index value after 20 years: 0.051 kr.
## Max total portfolio index value after 20 years: 1905.916 kr.
##
## Share of paths finishing below 100: 4.02 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Max vs sum plots

Max vs sum plots for the first four moments:

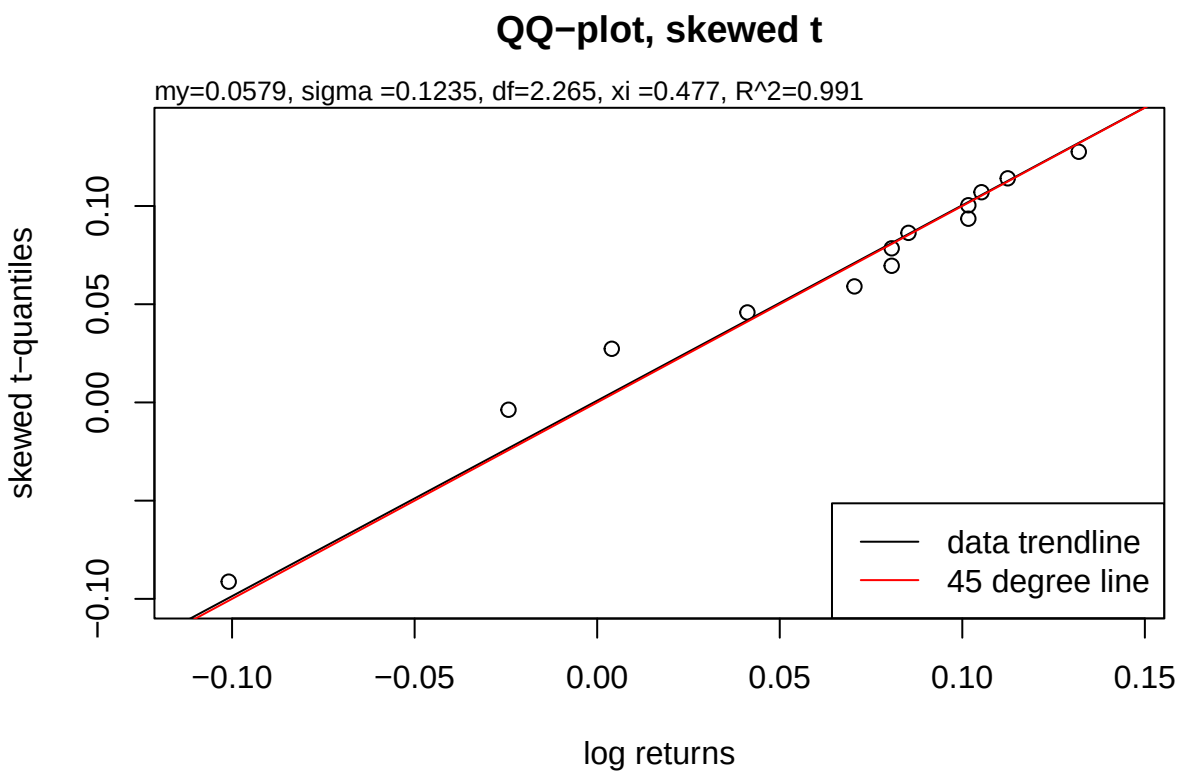


PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -33.22998
## BIC: -30.97018
## m: 0.05789224
## s: 0.1234592
## nu (df): 2.265273
## xi: 0.477324
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 3.3 percent
## What is the risk of losing max 25 %? =< 0.9 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 32.7 percent
## What is the chance of gaining min 25 %? >= 0.1 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

QQ Plot

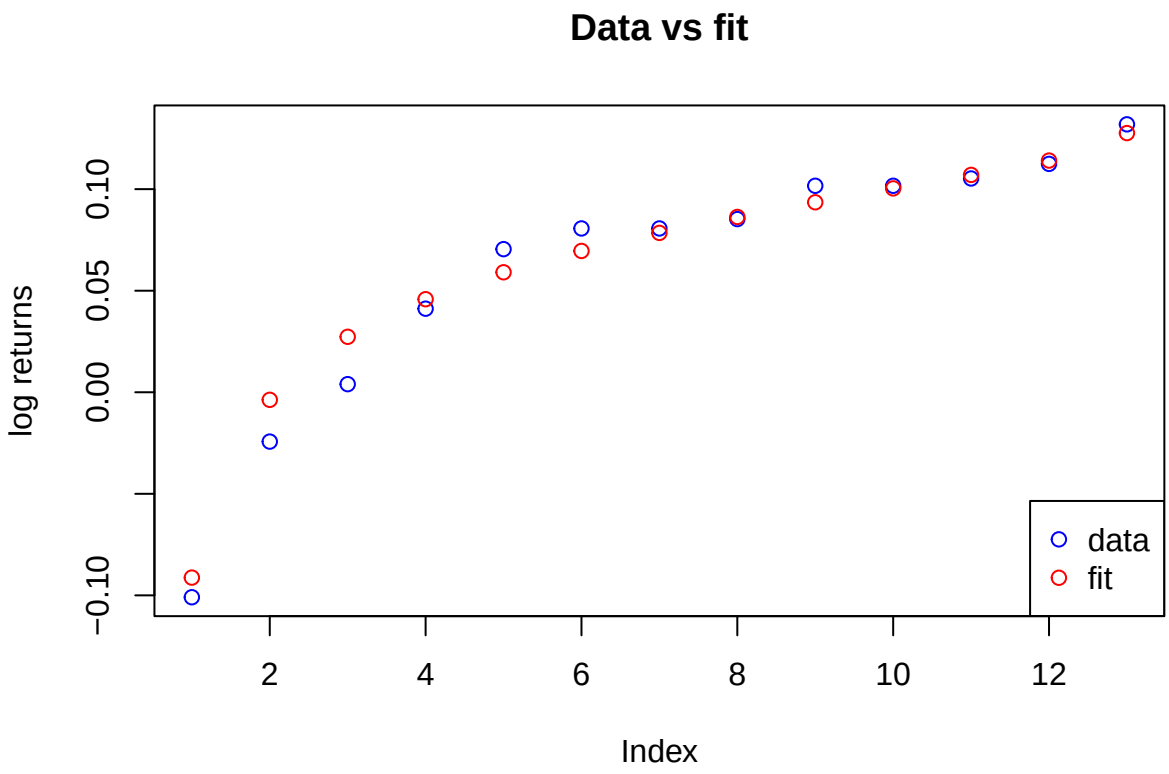


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
##  [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
##  [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11]  0.107018607  0.114081432  0.127604387
```

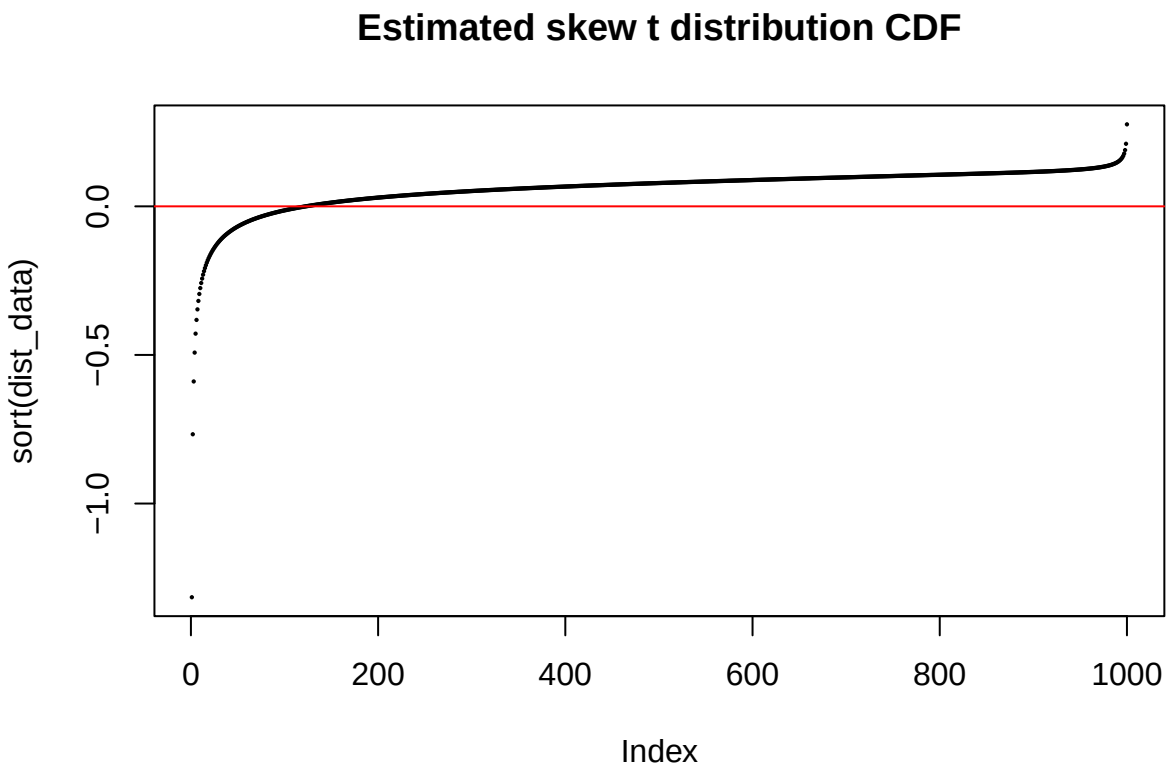
Data vs fit

Let's plot the fit and the observed returns together.

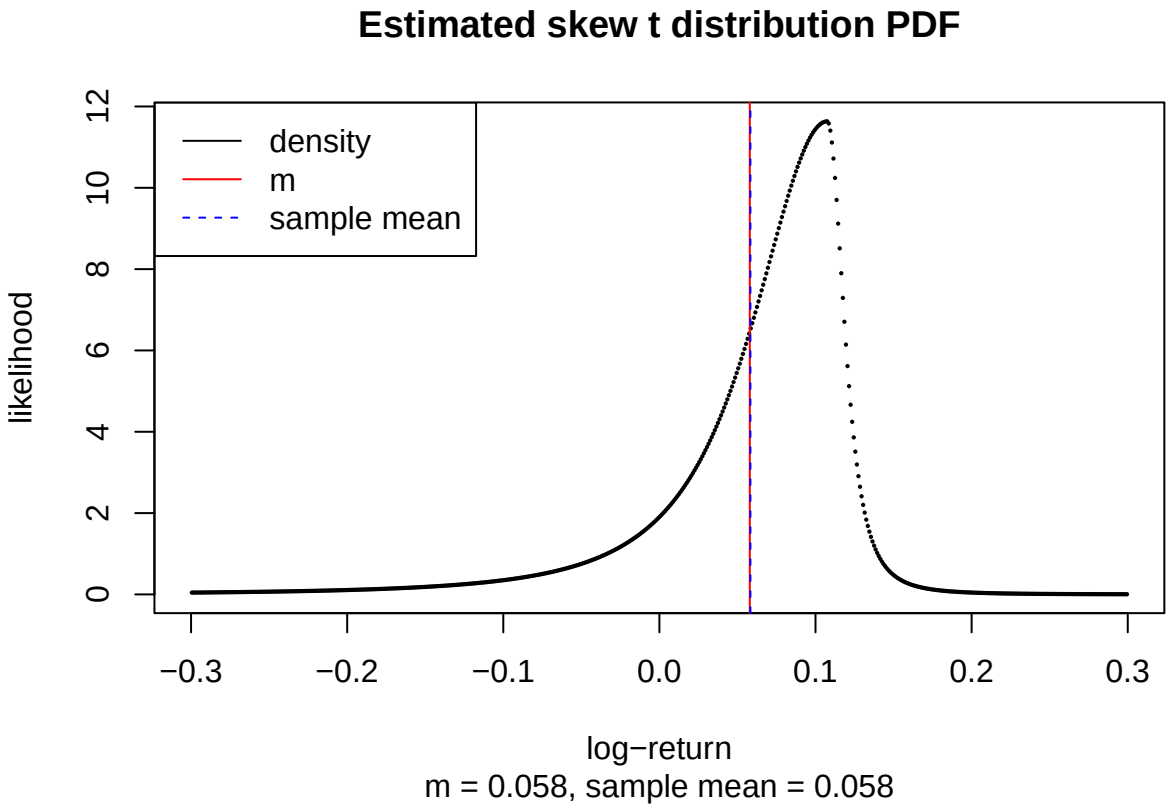


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

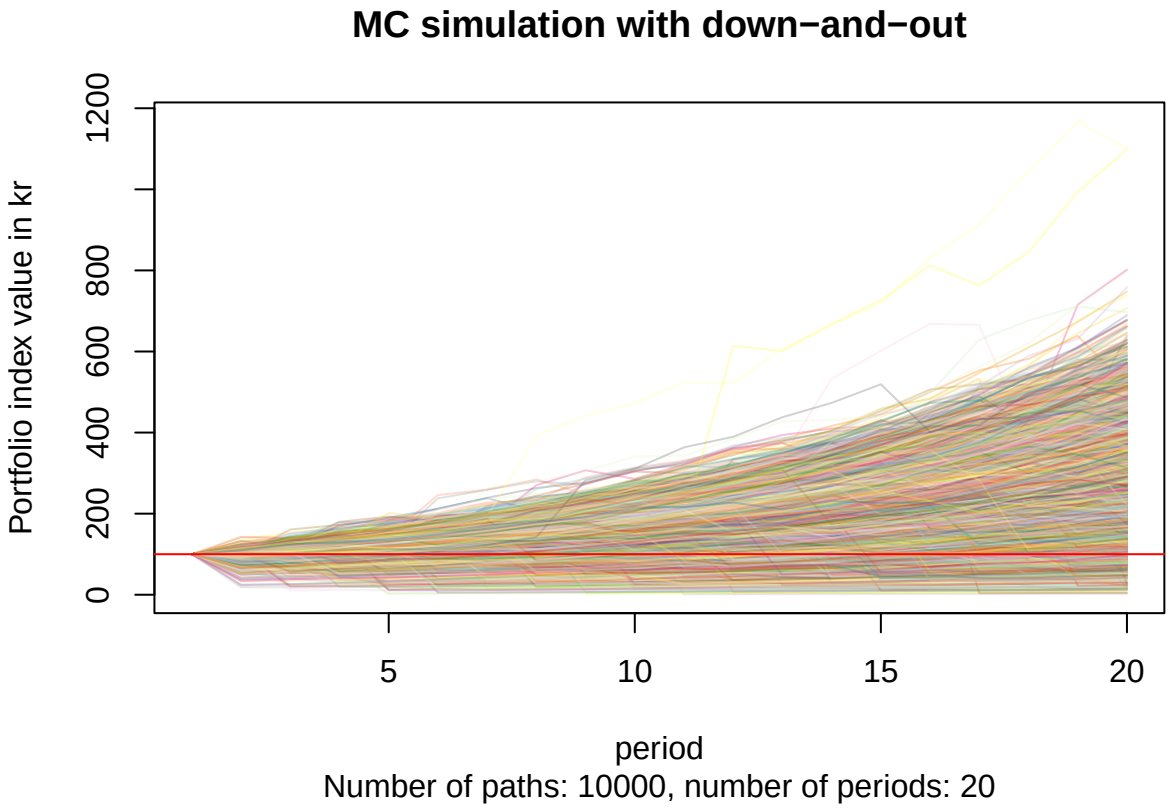


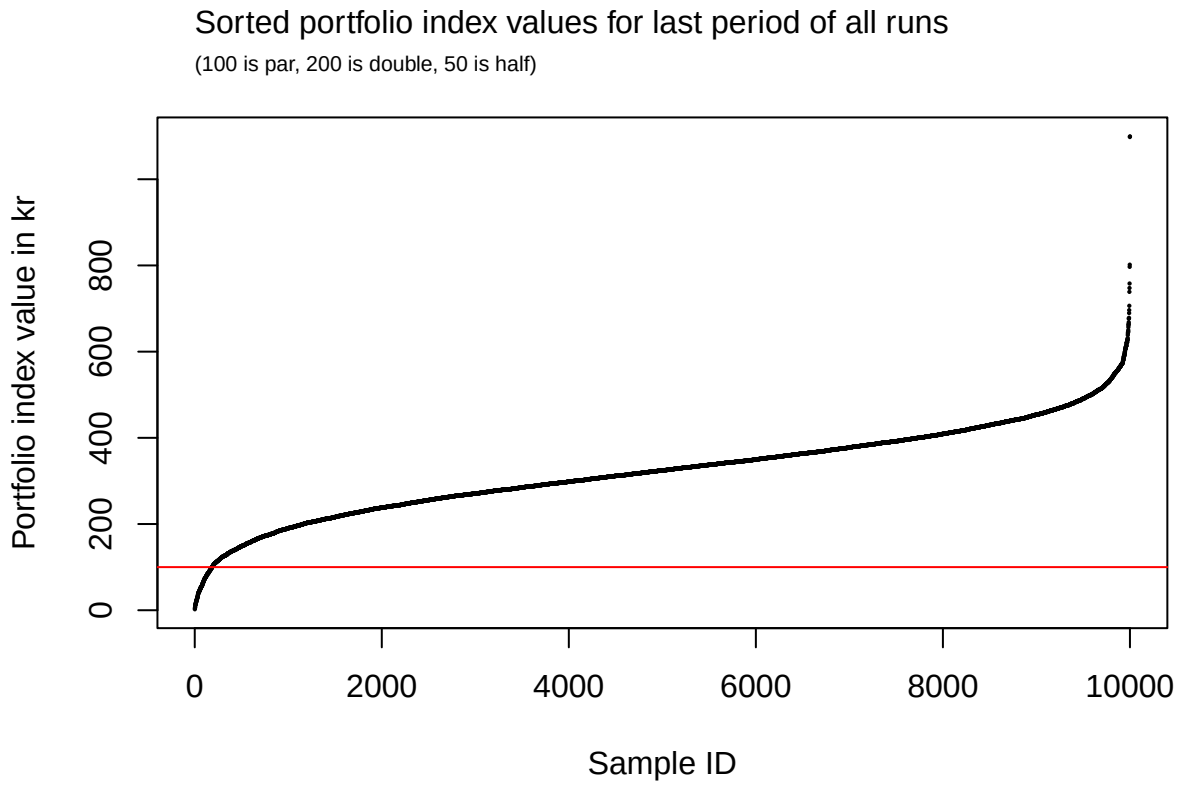
We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.



Monte Carlo

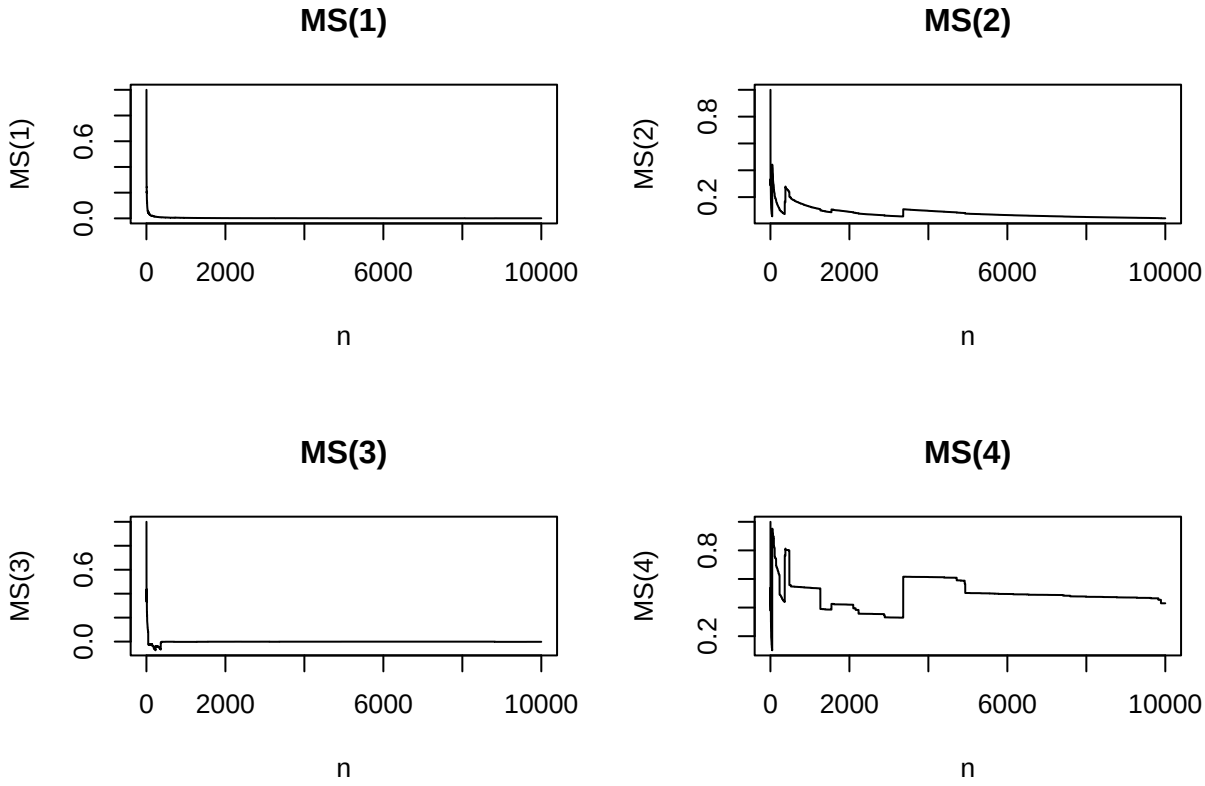
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 323.343 kr.
## SD of portfolio index value after 20 years: 104.601 kr.
## Min total portfolio index value after 20 years: 2.281 kr.
## Max total portfolio index value after 20 years: 1099.534 kr.
##
## Share of paths finishing below 100: 1.85 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
1200
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Max vs sum plots

Max vs sum plots for the first four moments:

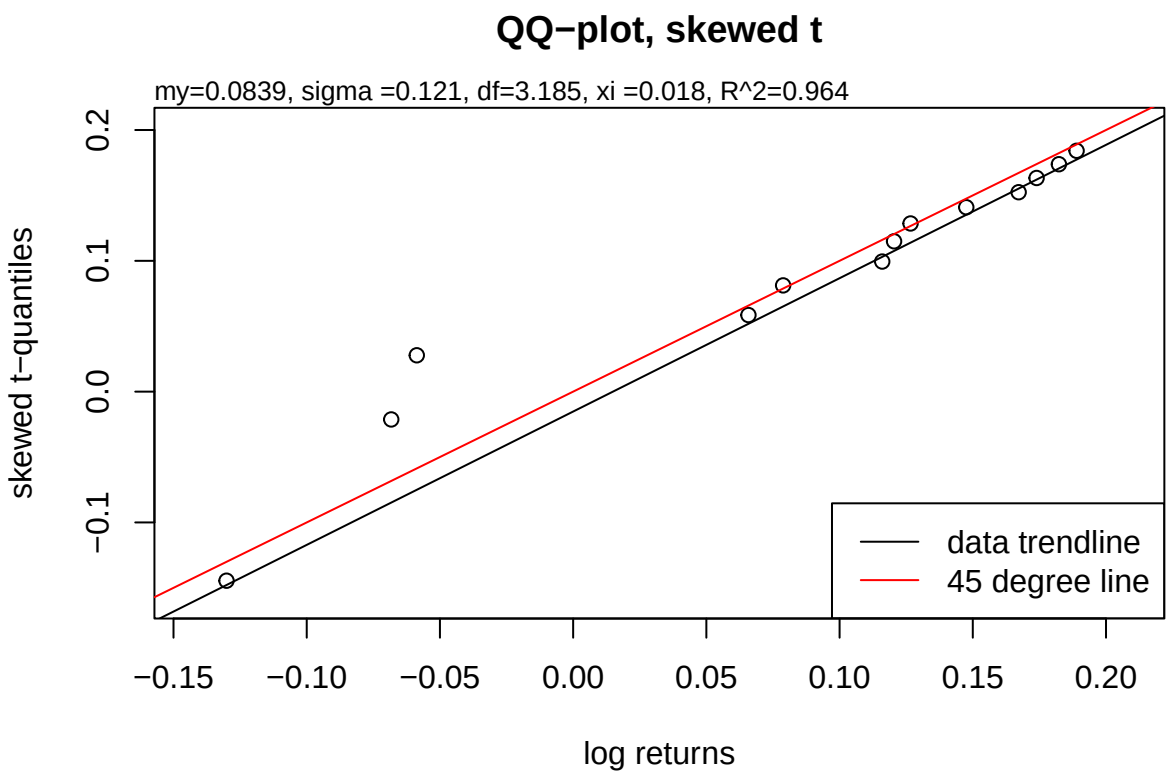


PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -23.72565
## BIC: -21.46585
## m: 0.08386034
## s: 0.1210107
## nu (df): 3.184569
## xi: 0.01790306
## R^2: 0.964
##
## An R^2 of 0.964 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 5.3 percent
## What is the risk of losing max 25 %? =< 1.4 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 59.6 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

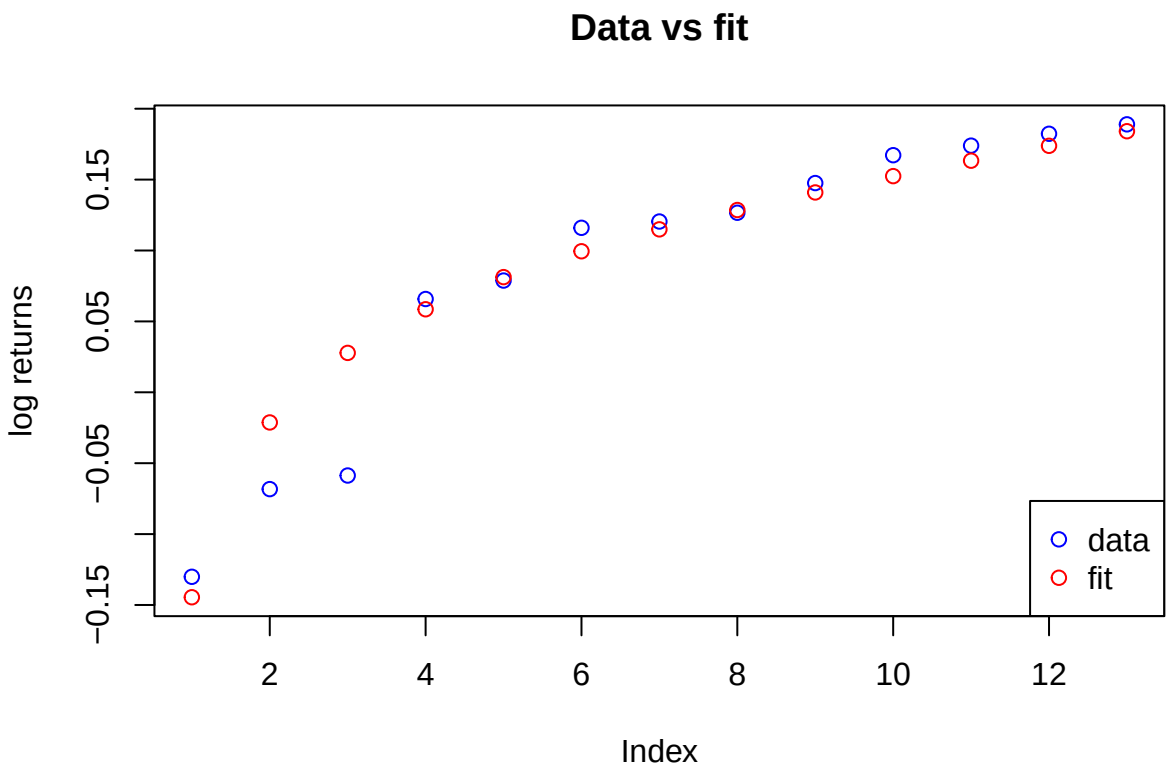
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

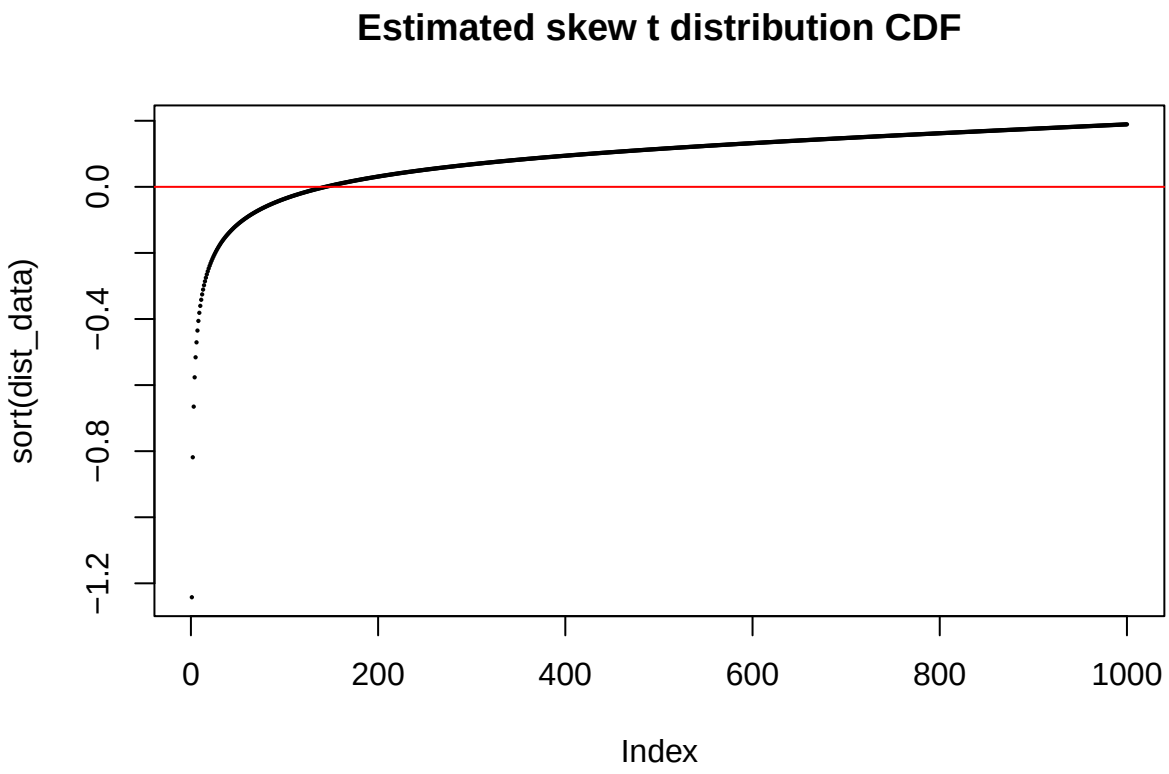
Data vs fit

Let's plot the fit and the observed returns together.

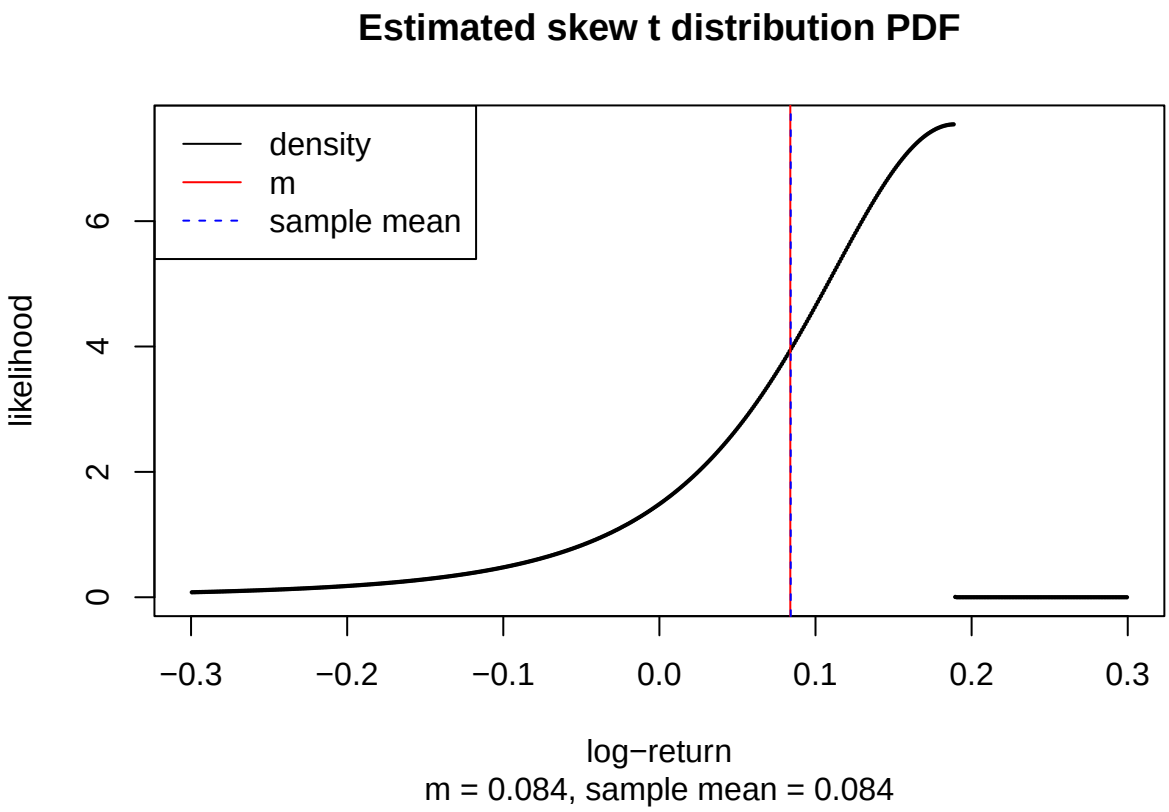


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

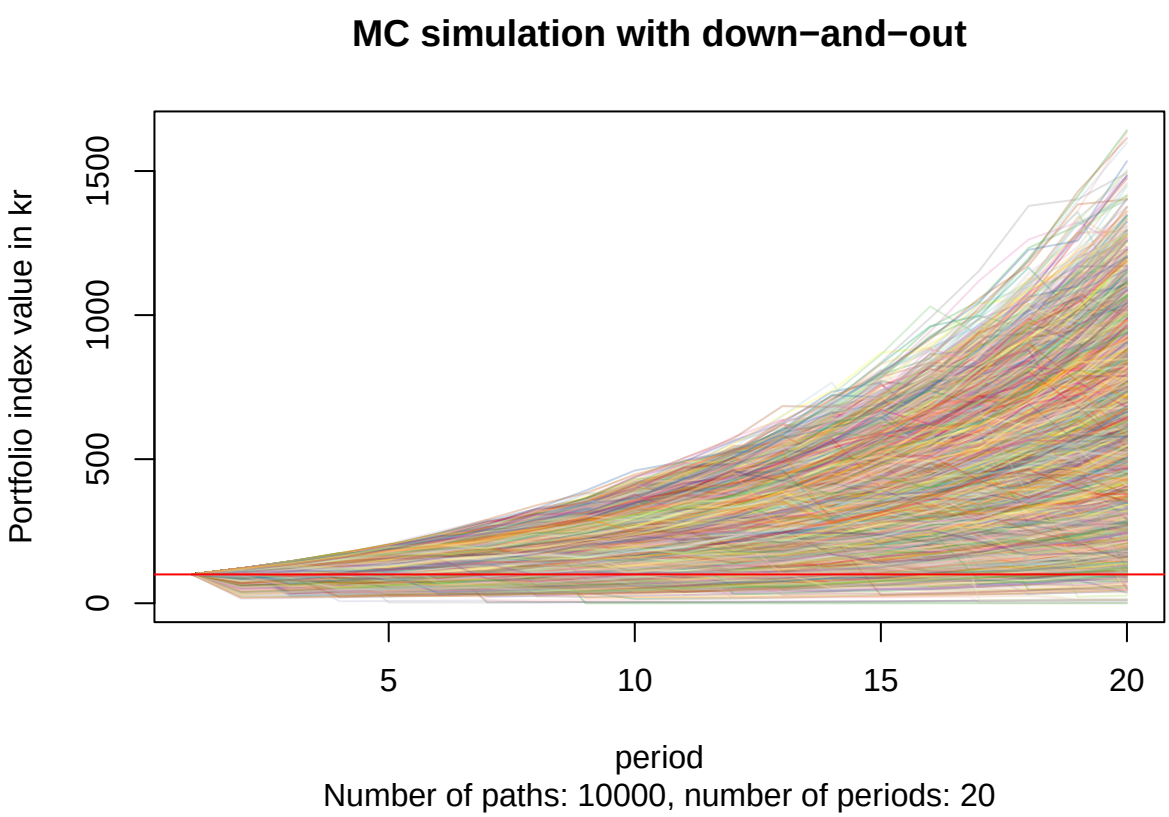


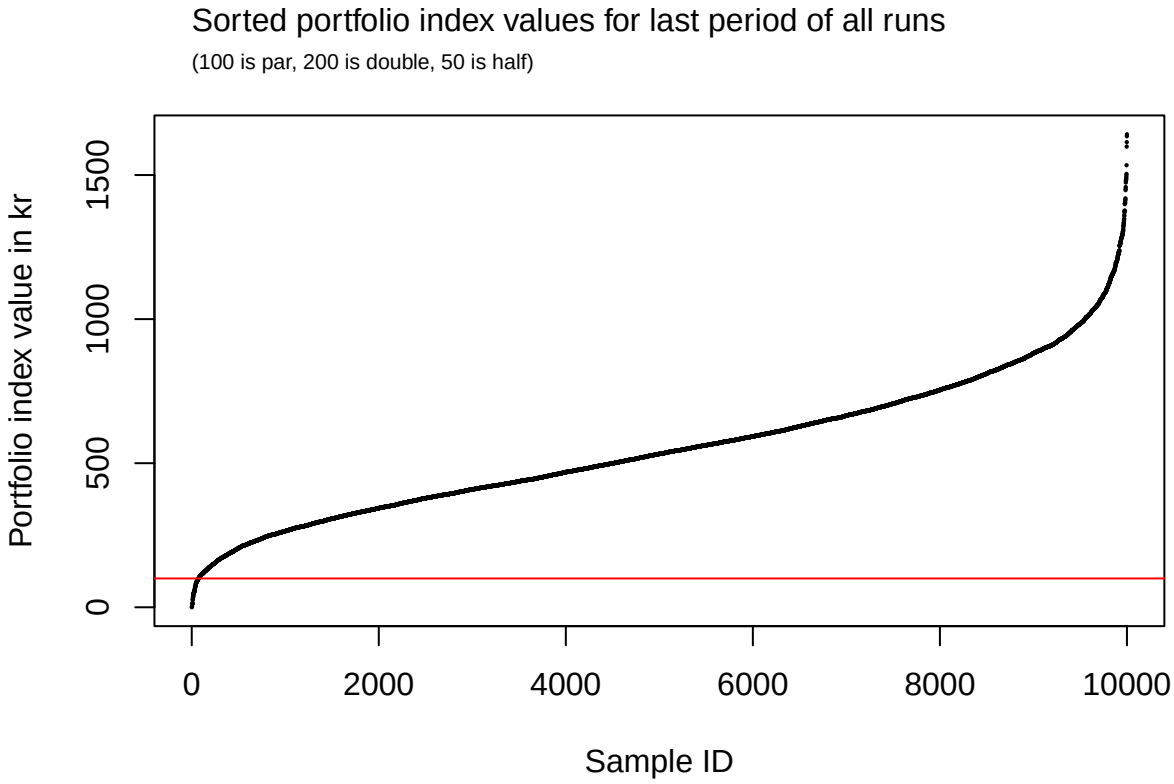
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.



Monte Carlo

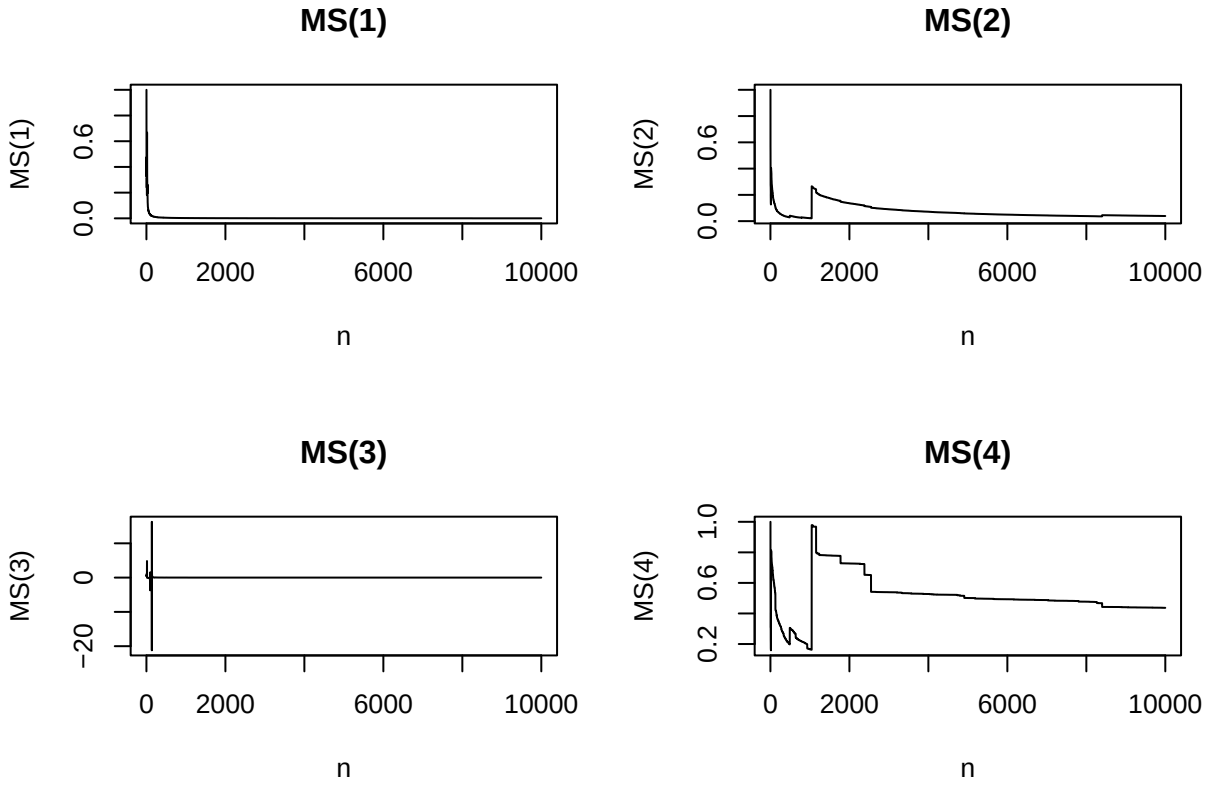
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 554.695 kr.
## SD of portfolio index value after 20 years: 241.376 kr.
## Min total portfolio index value after 20 years: 0.094 kr.
## Max total portfolio index value after 20 years: 1641.229 kr.
##
## Share of paths finishing below 100: 0.72 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Max vs sum plots

Max vs sum plots for the first four moments:

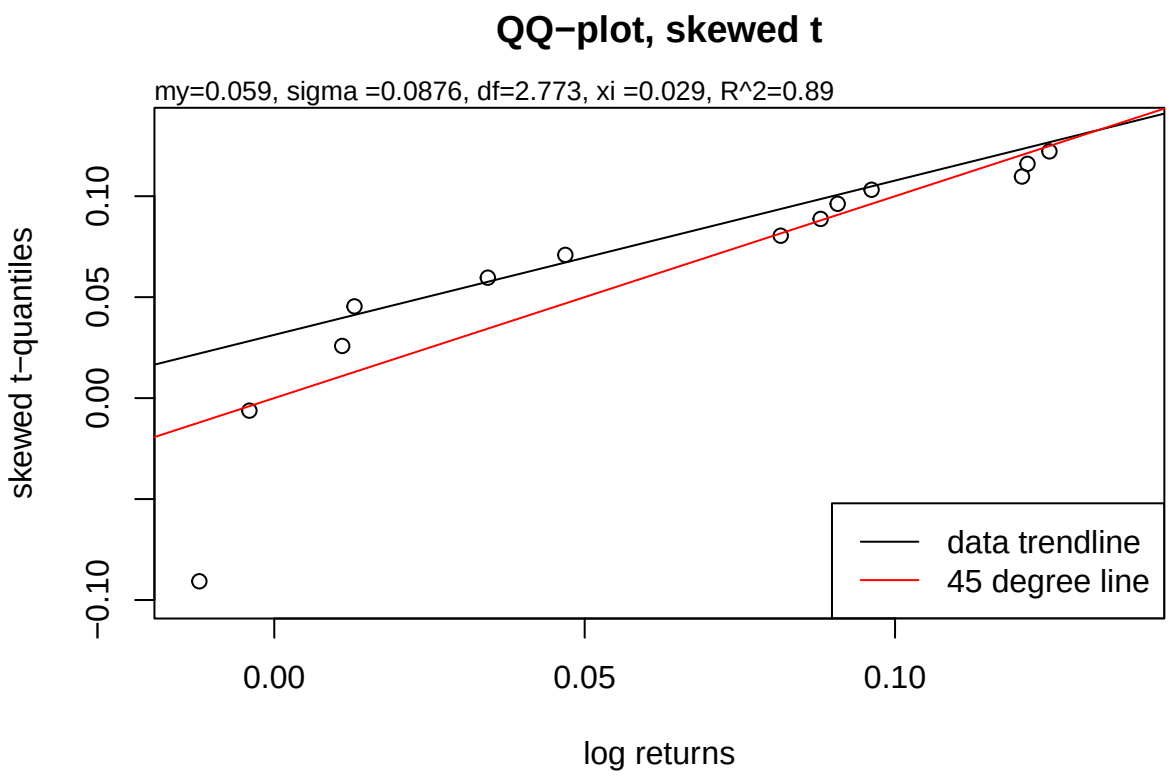


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -36.9603
## BIC: -34.7005
## m: 0.05902873
## s: 0.08757749
## nu (df): 2.772621
## xi: 0.02904471
## R^2: 0.89
##
## An R^2 of 0.89 suggests that the fit is not completely random.
##
## What is the risk of losing max 10 %? =< 3.3 percent
## What is the risk of losing max 25 %? =< 0.7 percent
## What is the risk of losing max 50 %? =< 0.1 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 35.6 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

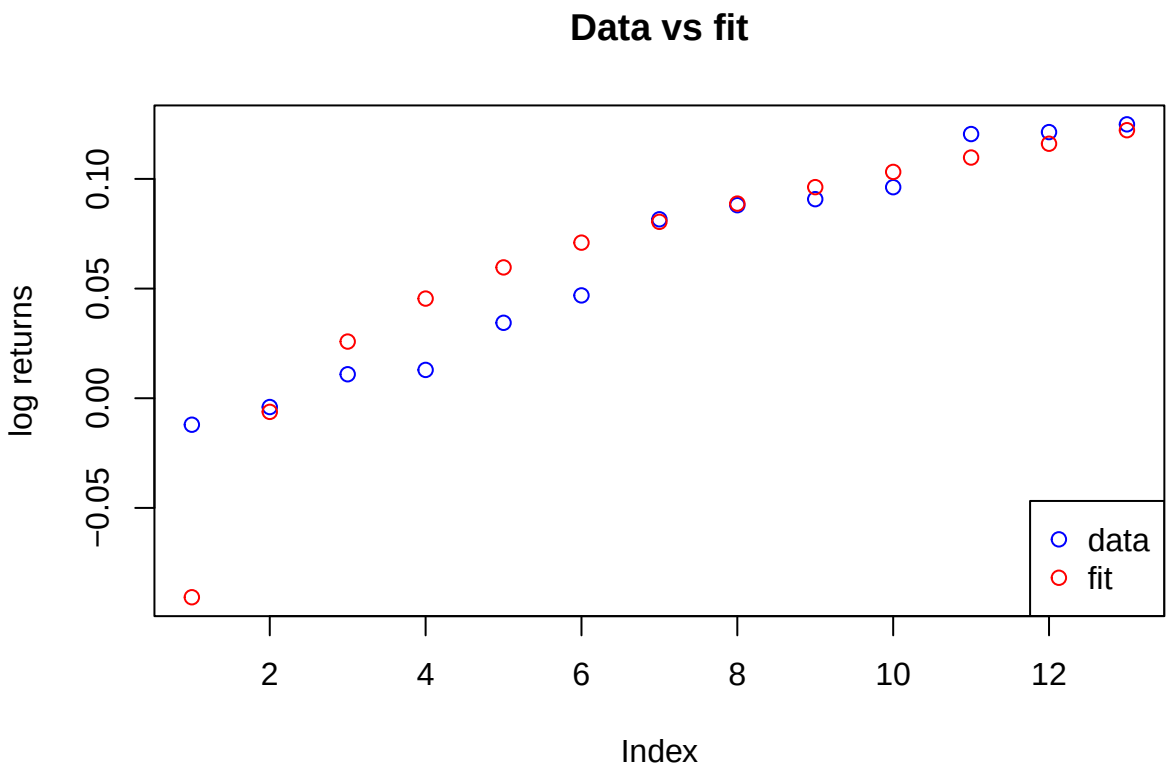
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

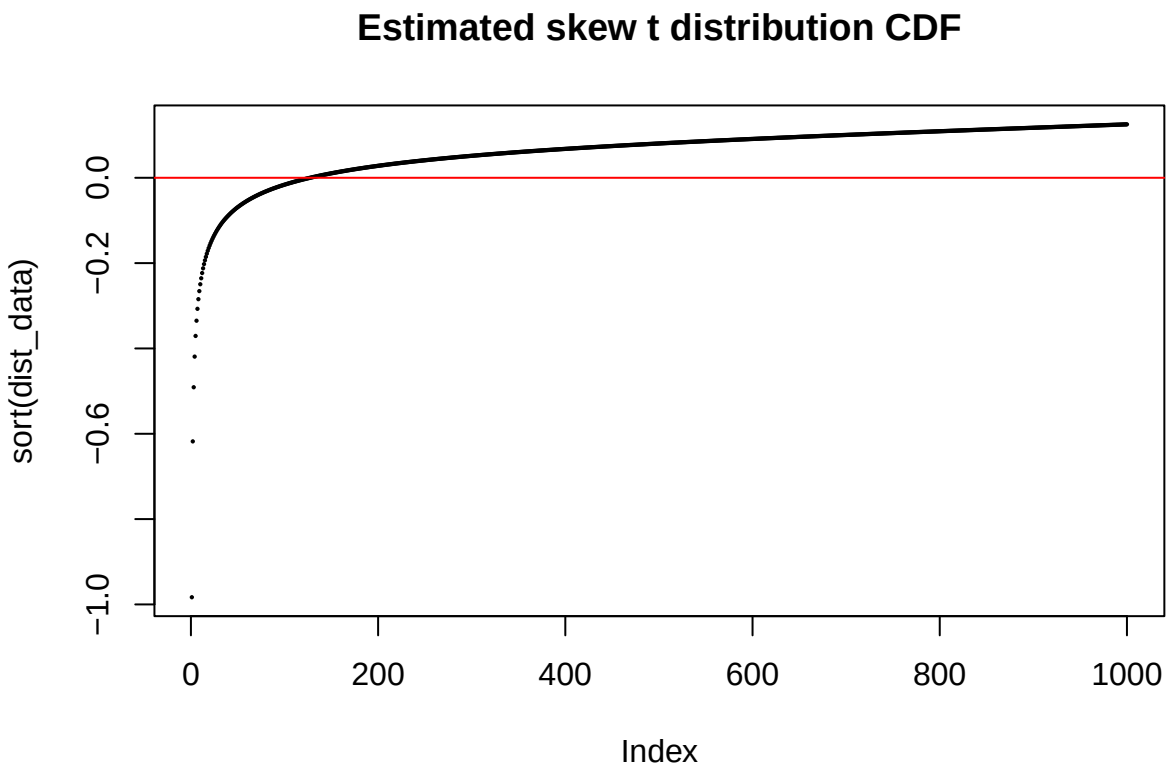
Let's plot the fit and the observed returns together.



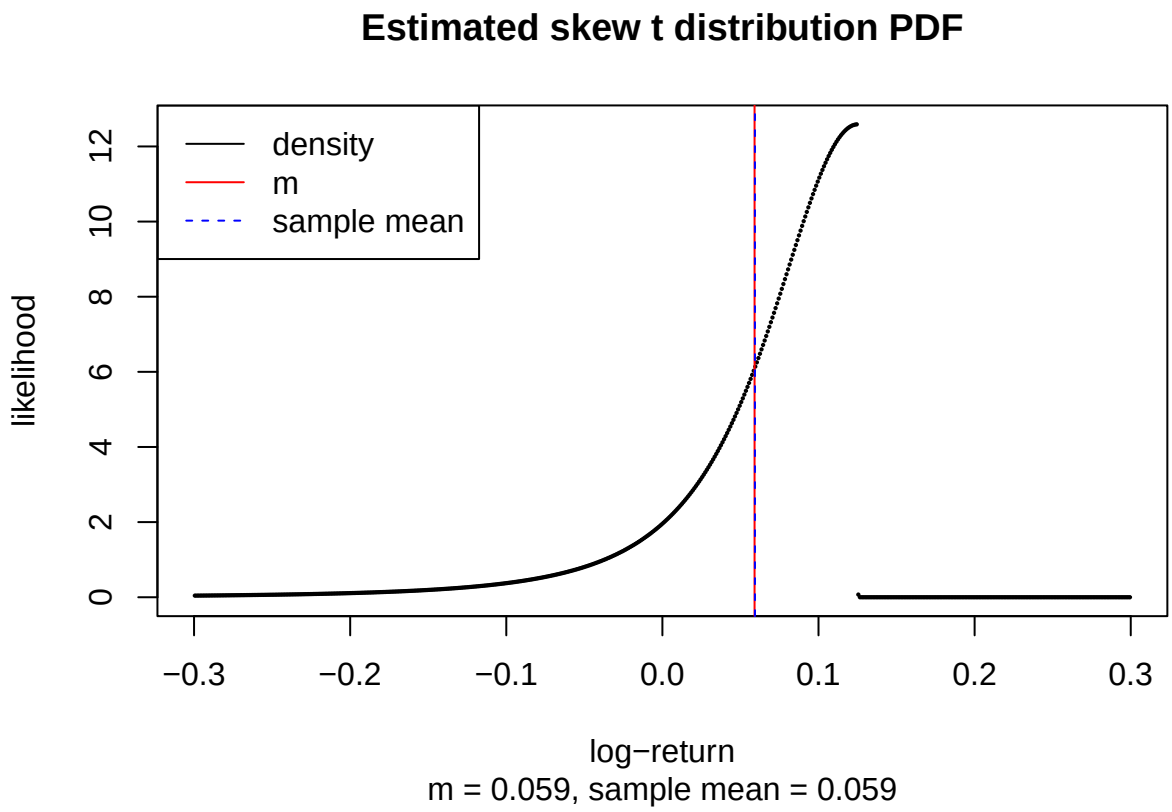
Interestingly, the fit predicts a much bigger “biggest loss” than the actual data. This is the main reason that R^2 is 0.90 and not higher.

Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



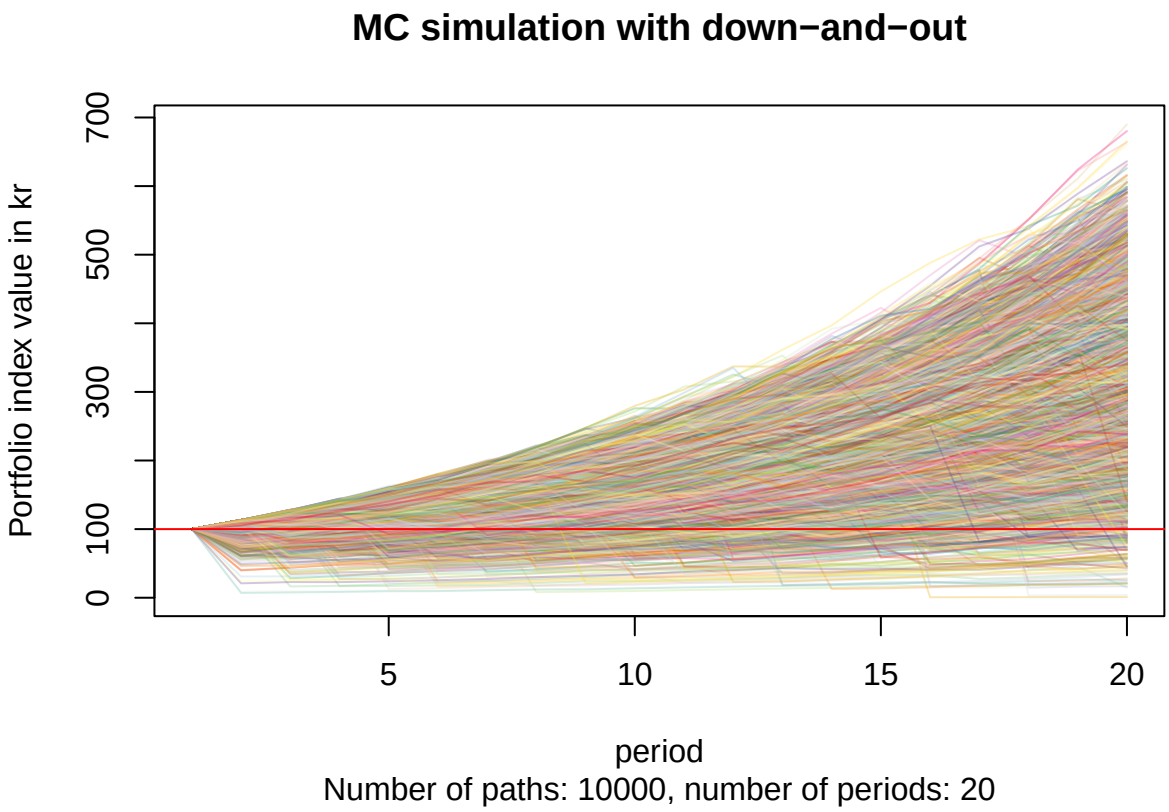
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

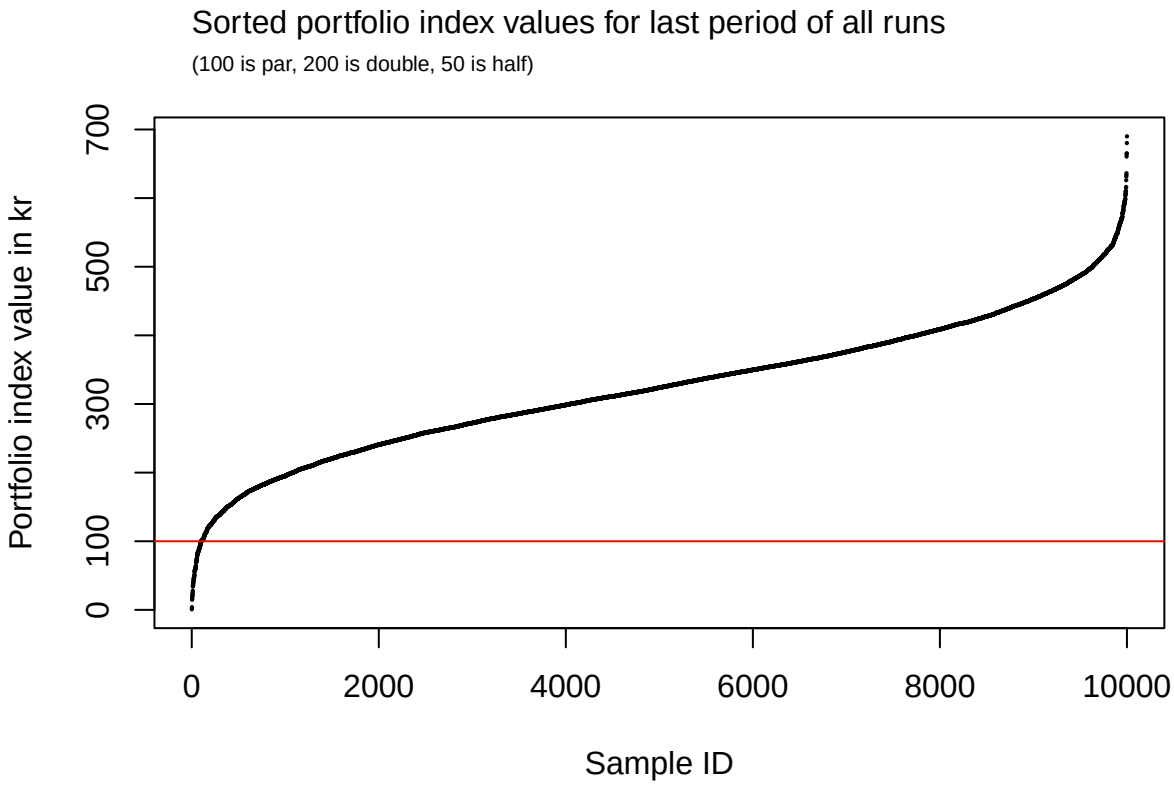


Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

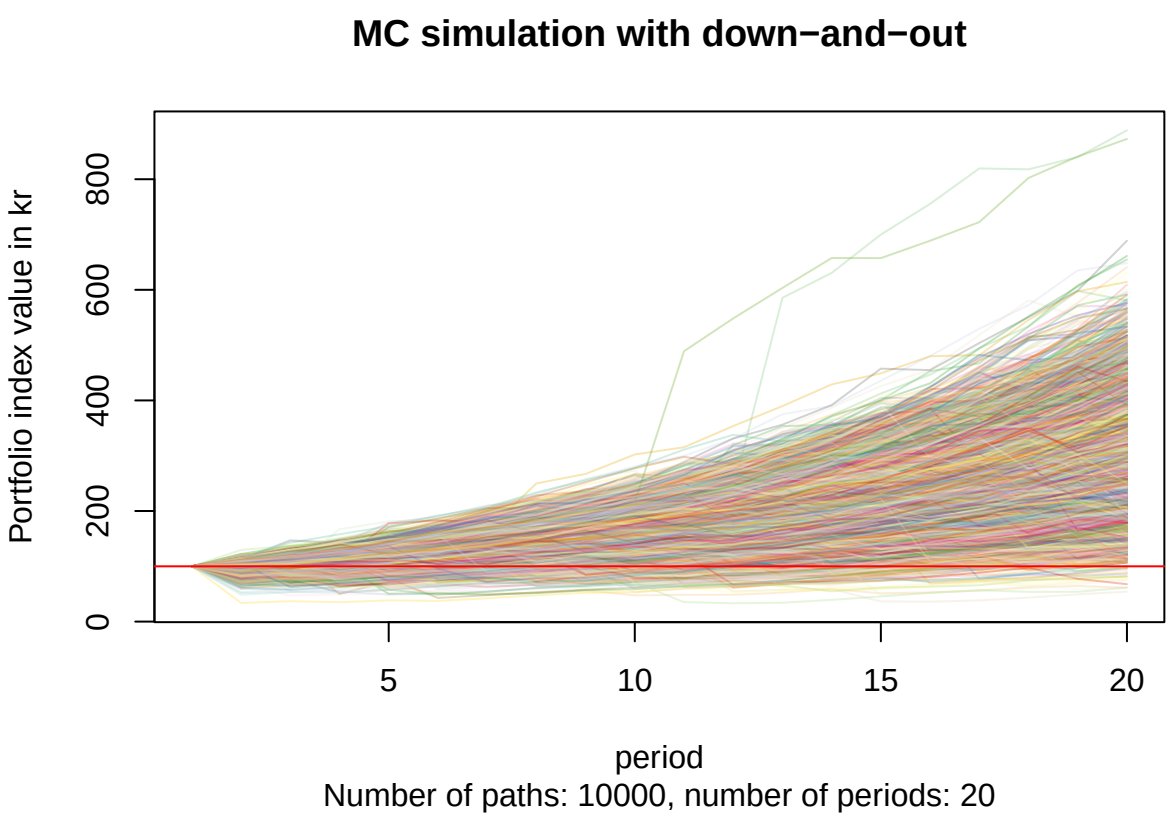
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 324.341 kr.
## SD of portfolio index value after 20 years: 98.828 kr.
## Min total portfolio index value after 20 years: 0.908 kr.
## Max total portfolio index value after 20 years: 689.992 kr.
##
## Share of paths finishing below 100: 1 percent
```

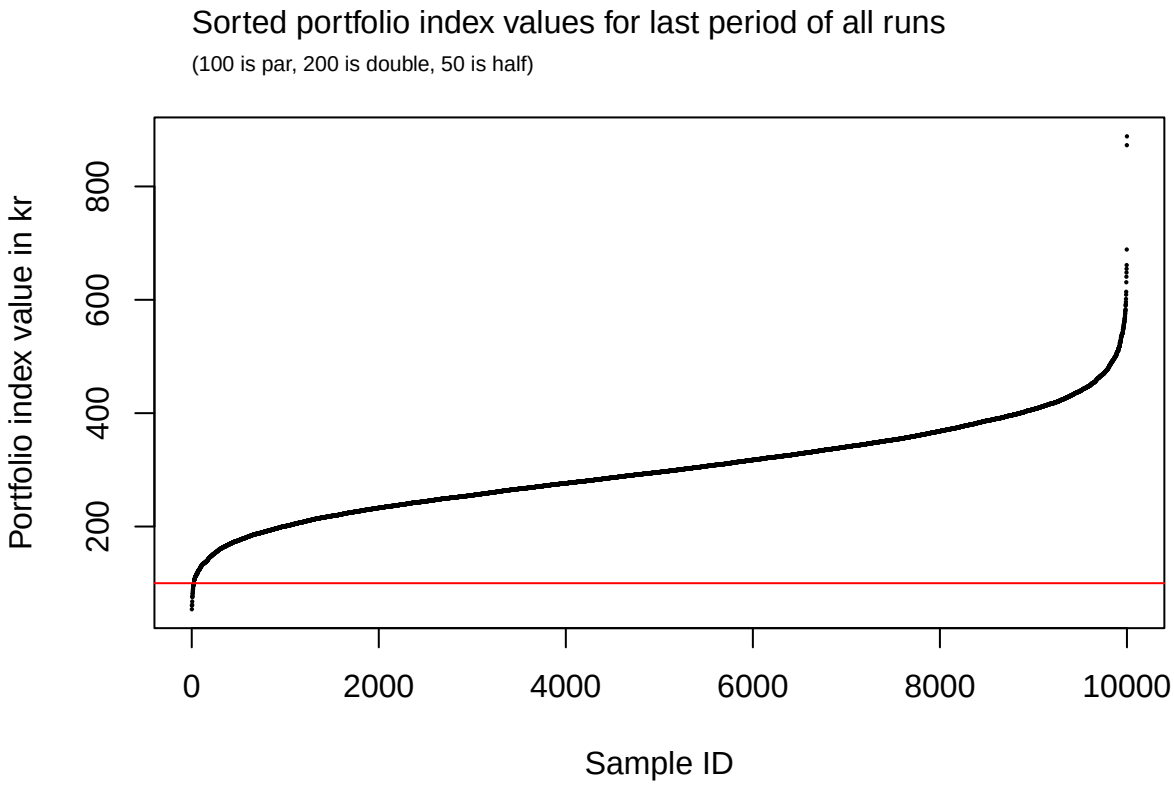

MC simulation with down−and−out
Portfolio index value in kr
0
100
300
500
700
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Version b: Mix of simulations from estimated distribution of returns from individual funds.

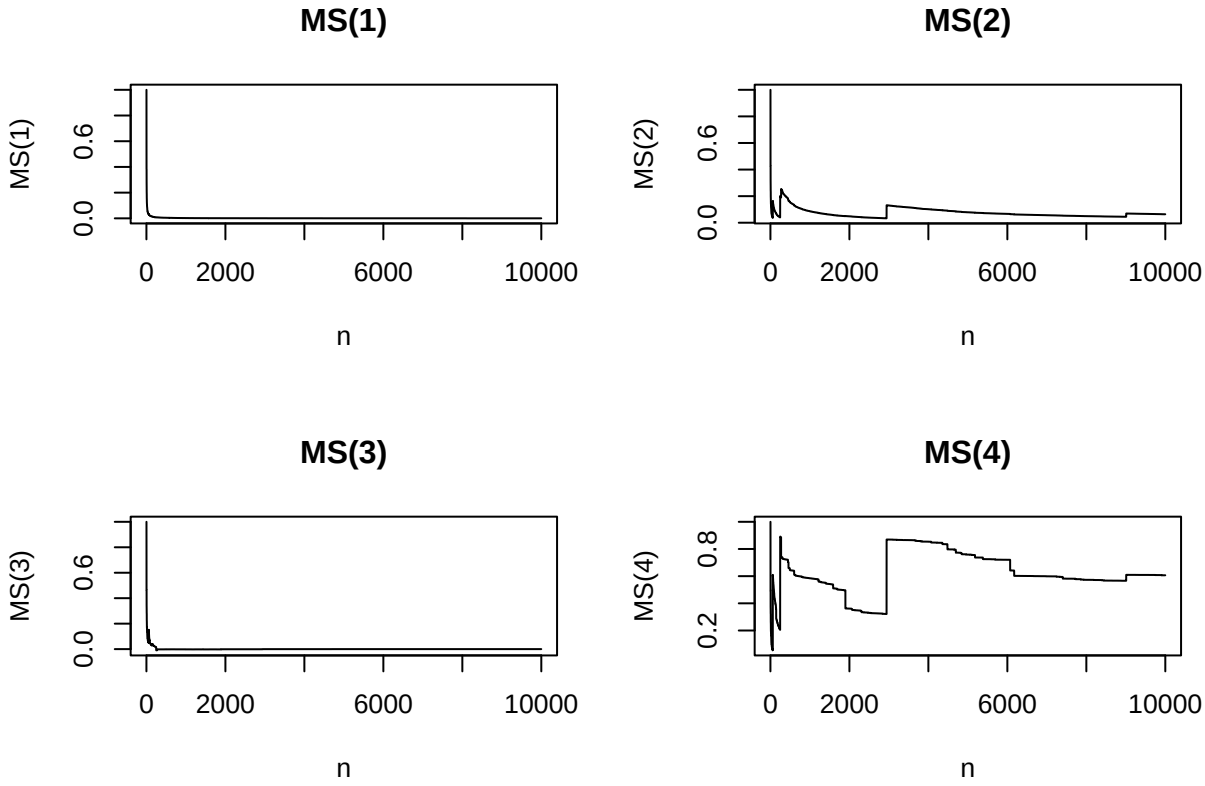
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 301.077 kr.
## SD of portfolio index value after 20 years: 81.417 kr.
## Min total portfolio index value after 20 years: 54.068 kr.
## Max total portfolio index value after 20 years: 888.384 kr.
##
## Share of paths finishing below 100: 0.23 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Max vs sum plots

Max vs sum plots for the first four moments:

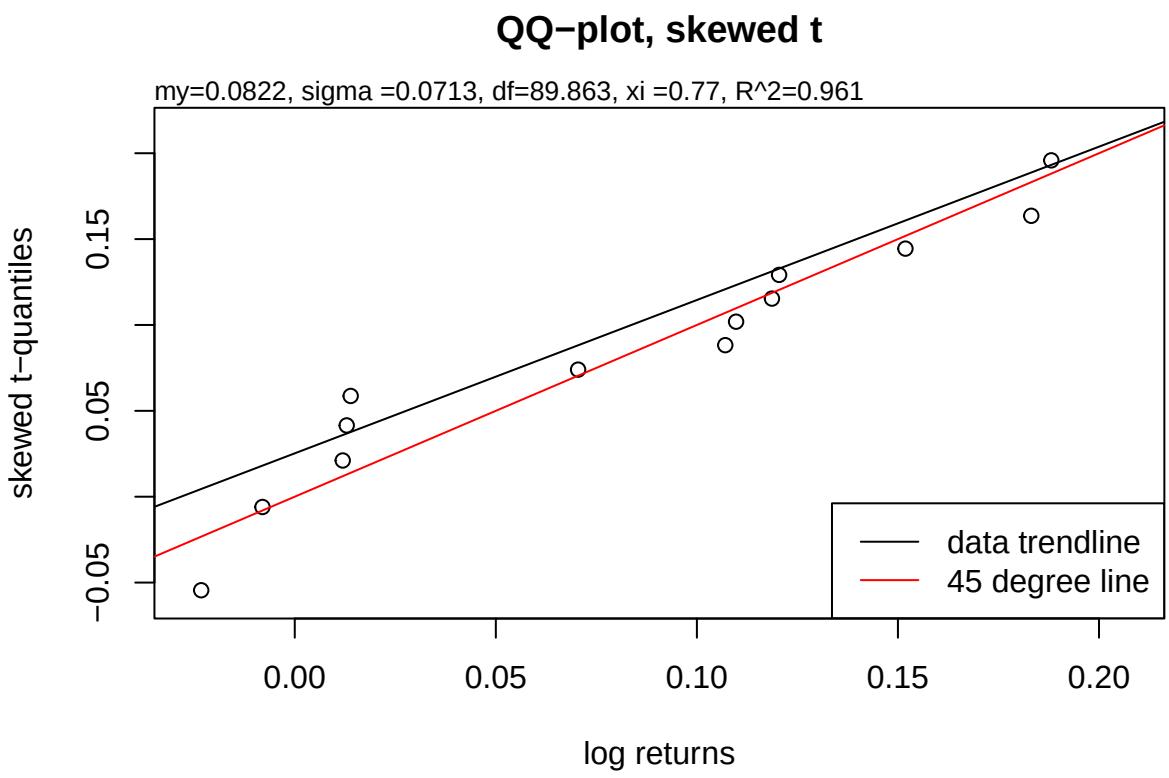


Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -24.26084
## BIC: -22.00104
## m: 0.0822419
## s: 0.07129843
## nu (df): 89.86289
## xi: 0.7697502
## R^2: 0.961
##
## An R^2 of 0.961 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 0.9 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 46.1 percent
## What is the chance of gaining min 25 %? >= 1.2 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

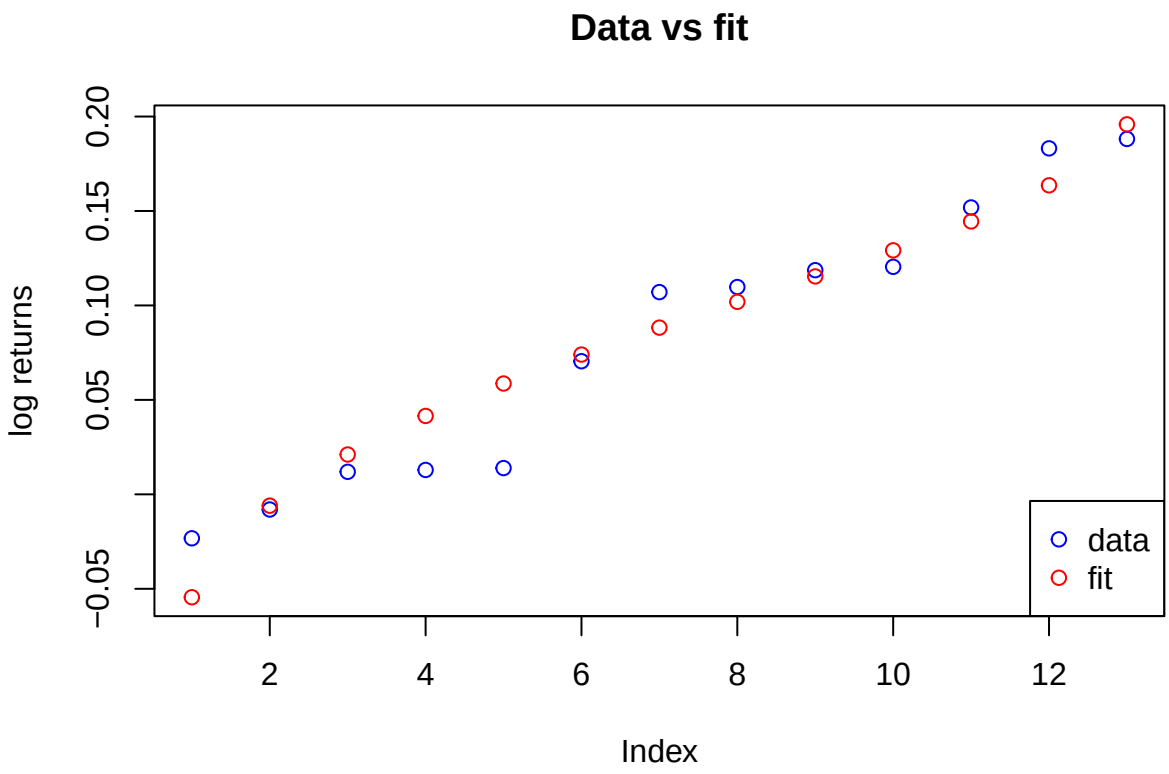
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

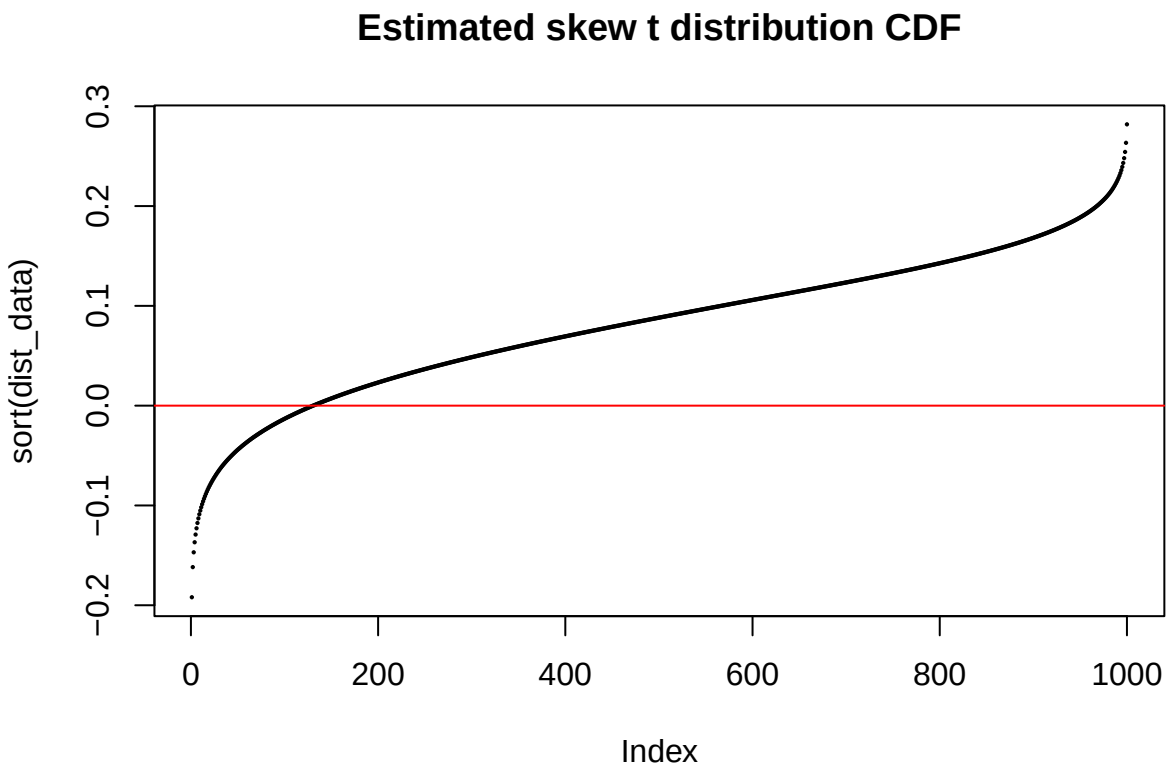
Data vs fit

Let's plot the fit and the observed returns together.

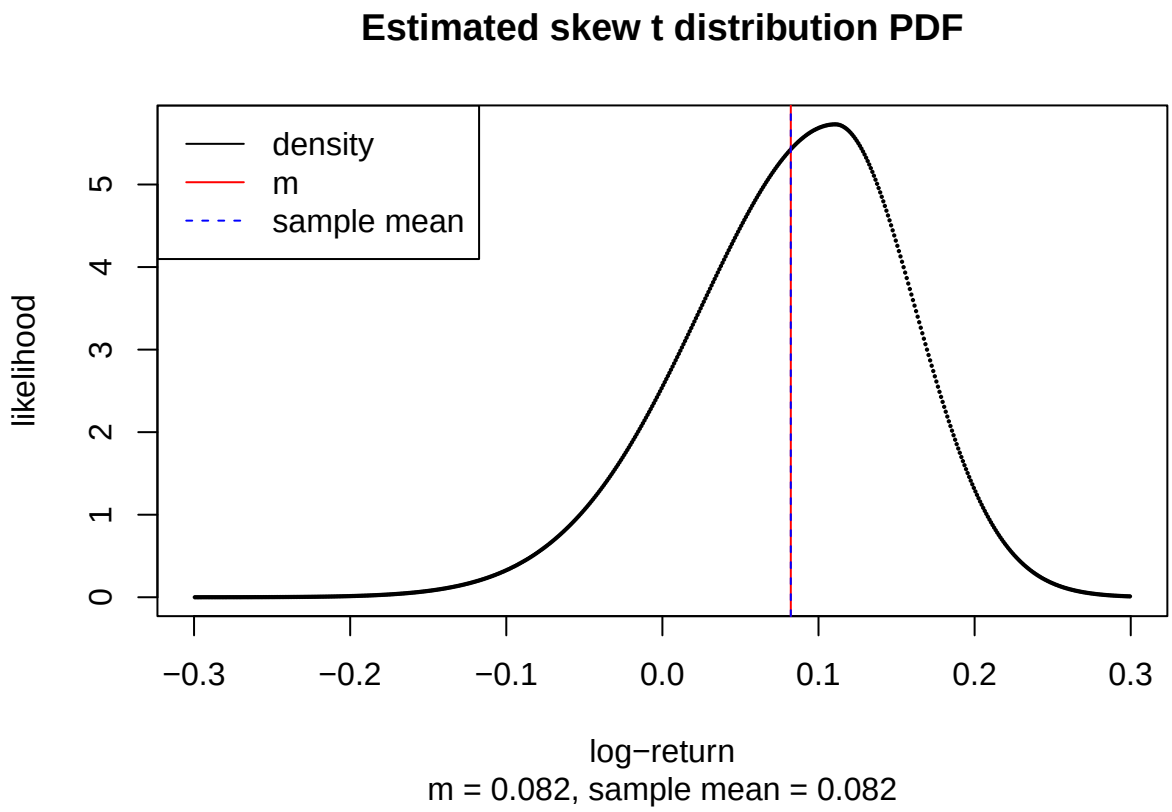


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



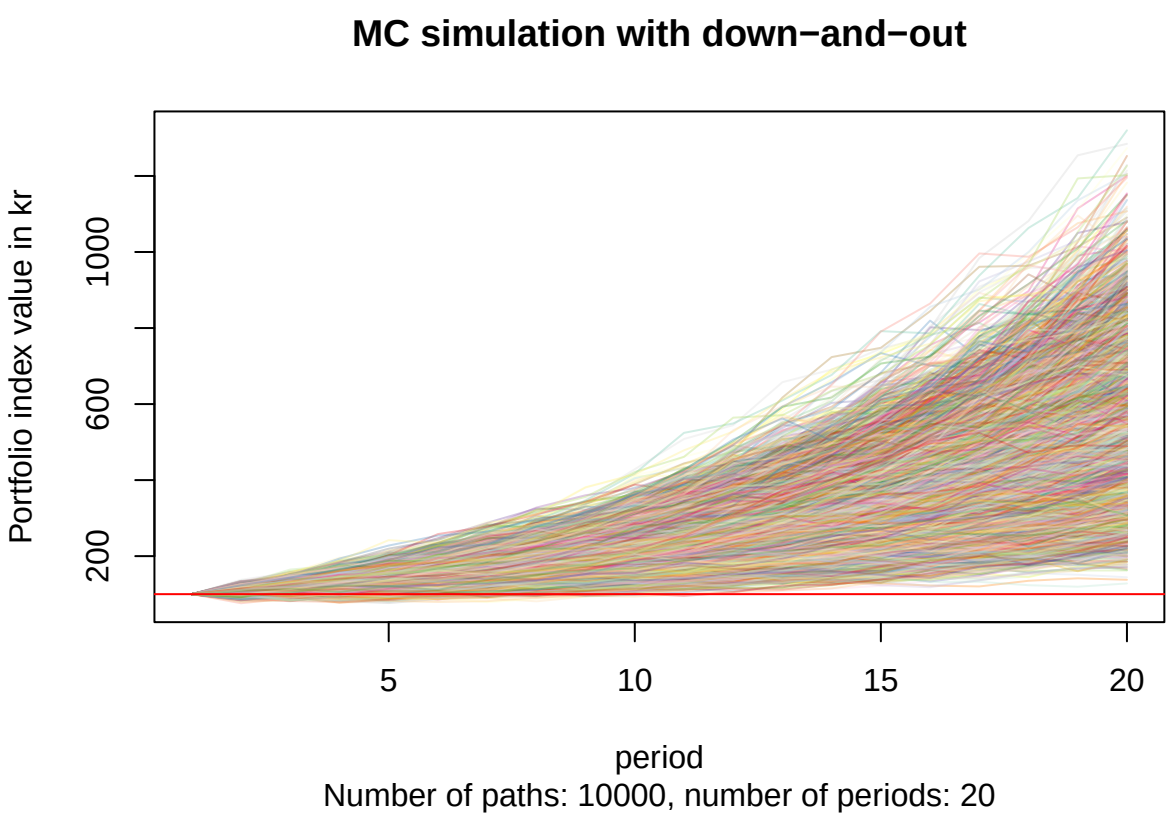
We see that the high risk mix provides a much better upside and smaller downside.

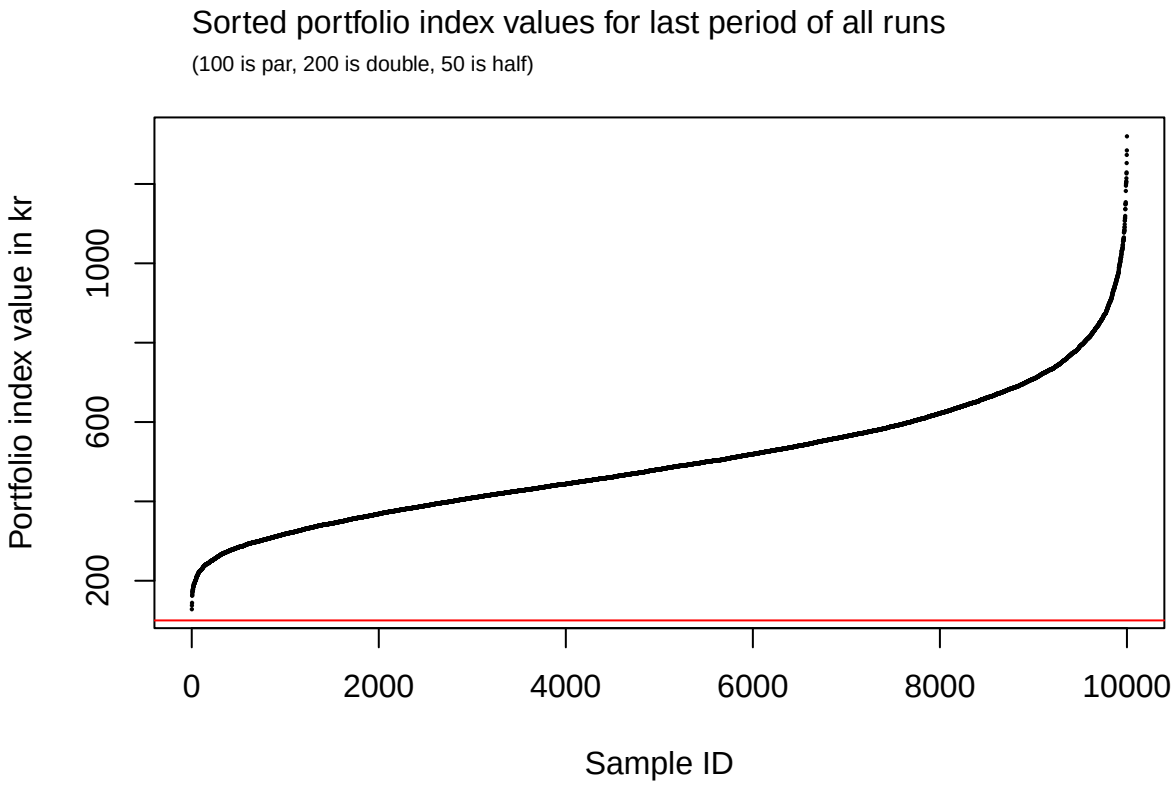


Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

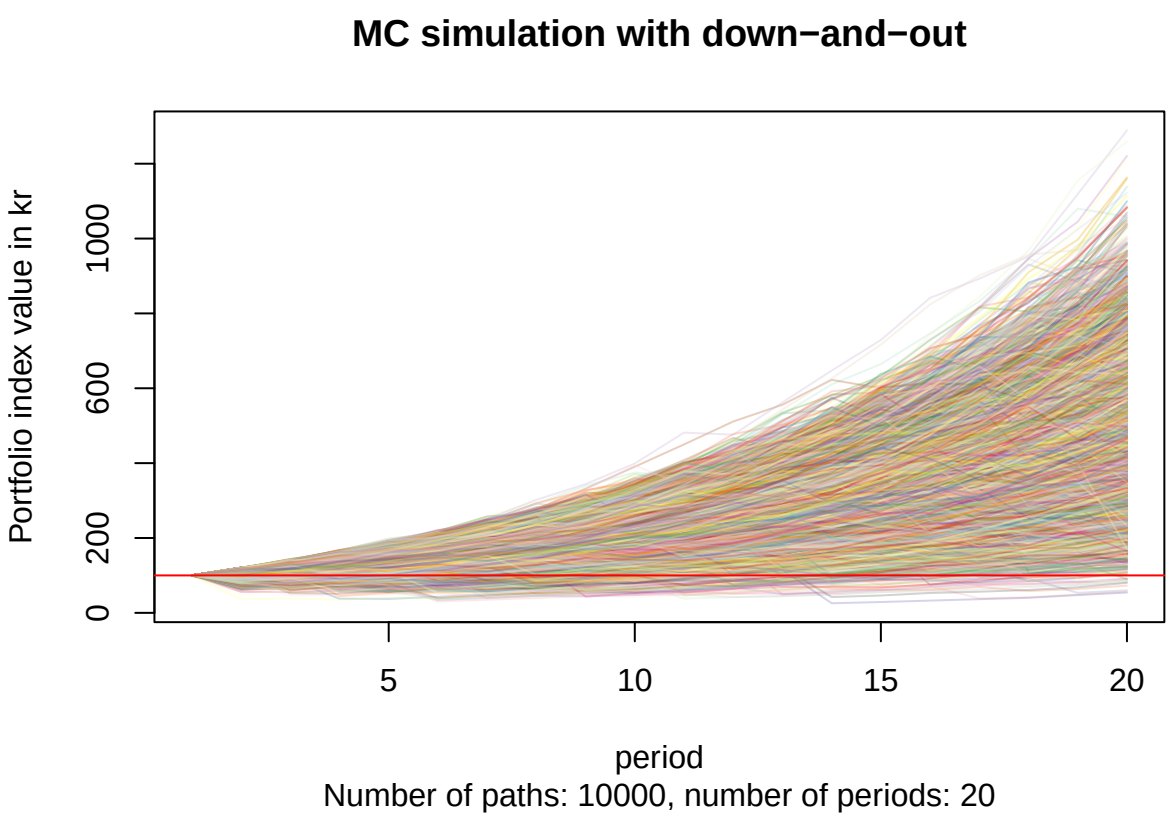
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 501.217 kr.
## SD of portfolio index value after 20 years: 157.181 kr.
## Min total portfolio index value after 20 years: 128.193 kr.
## Max total portfolio index value after 20 years: 1320.013 kr.
##
## Share of paths finishing below 100: 0 percent
```

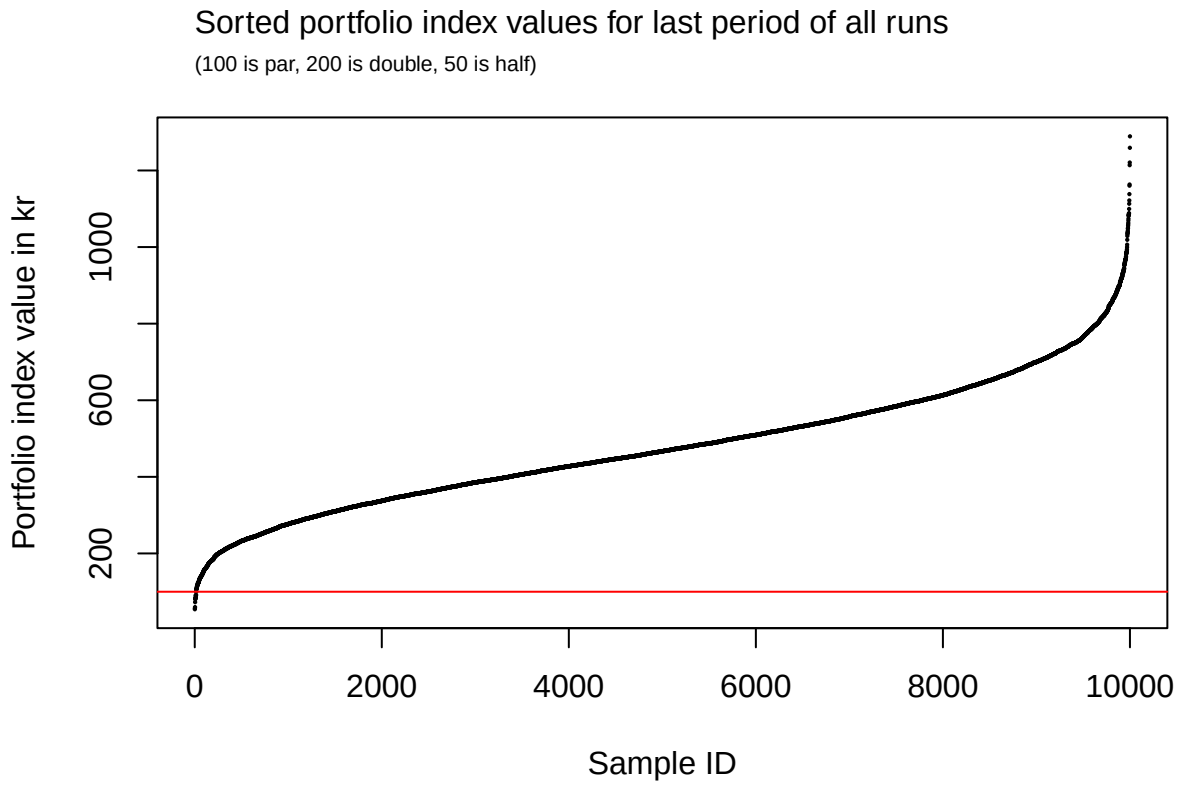

MC simulation with down−and−out
Portfolio index value in kr
200
600
1000
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Version b: Mix of simulations from estimated distribution of returns from individual funds.

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 479.705 kr.
## SD of portfolio index value after 20 years: 164.484 kr.
## Min total portfolio index value after 20 years: 54.305 kr.
## Max total portfolio index value after 20 years: 1289.077 kr.
##
## Share of paths finishing below 100: 0.14 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
600
1000
5
10
15
20
period
Number of paths: 10000, number of periods: 20

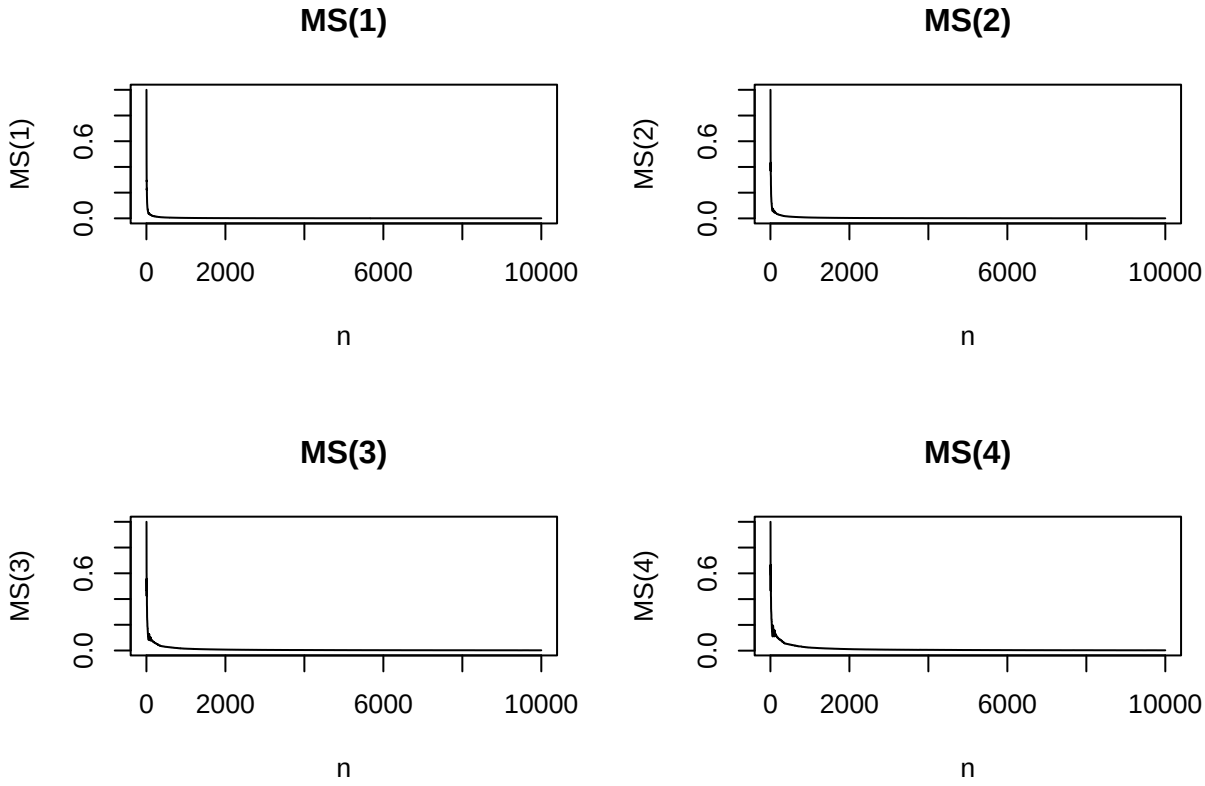


Many simulations 1e6 paths:

```
# Down-and-out simulation:
# Probability of down-and-out: 0 percent
#
# Mean portfolio index value after 20 years: 478.339 kr.
# SD of portfolio index value after 20 years: 163.093 kr.
# Min total portfolio index value after 20 years: 2.233 kr.
# Max total portfolio index value after 20 years: 1561.965 kr.
#
# Share of paths finishing below 100: 0.1181 percent
```

Max vs sum plots

Max vs sum plots for the first four moments:



Compare pension plans

Risk of max loss

Risk of max loss of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	21.3	18.2	19.9	12.2	14.3	12.7	13.0
5	12.5	9.6	12.8	6.0	8.6	6.2	4.2
10	7.4	5.4	8.3	3.3	5.3	3.3	0.9
25	1.8	1.3	2.5	0.9	1.4	0.7	0.0
50	0.2	0.2	0.4	0.2	0.2	0.1	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.3	Velliv_m	12.8	Velliv_h	8.3	Velliv_h	2.5	Velliv_h	0.4	Velliv_h	0	Velliv_m	0	Velliv_m
19.9	Velliv_h	12.5	Velliv_m	7.4	Velliv_m	1.8	Velliv_m	0.2	Velliv_m	0	Velliv_m_l	0	Velliv_m_l
18.2	Velliv_m_l	9.6	Velliv_m_l	5.4	Velliv_m_l	1.4	PFA_h	0.2	Velliv_m_l	0	Velliv_h	0	Velliv_h
14.3	PFA_h	8.6	PFA_h	5.3	PFA_h	1.3	Velliv_m_l	0.2	PFA_m	0	PFA_m	0	PFA_m
13.0	mix_h	6.2	mix_m	3.3	PFA_m	0.9	PFA_m	0.2	PFA_h	0	PFA_h	0	PFA_h
12.7	mix_m	6.0	PFA_m	3.3	mix_m	0.7	mix_m	0.1	mix_m	0	mix_m	0	mix_m
12.2	PFA_m	4.2	mix_h	0.9	mix_h	0.0	mix_h	0.0	mix_h	0	mix_h	0	mix_h

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	78.7	81.8	80.1	87.8	85.7	87.3	87.0
5	63.8	64.9	69.2	71.5	75.8	71.4	69.9
10	41.0	36.2	53.3	32.7	59.6	35.6	46.1
25	0.0	0.3	0.0	0.1	0.0	0.0	1.2
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
87.8	PFA_m	75.8	PFA_h	59.6	PFA_h	1.2	mix_h	0	Velliv_m	0	Velliv_m
87.3	mix_m	71.5	PFA_m	53.3	Velliv_h	0.3	Velliv_m_l	0	Velliv_m_l	0	Velliv_m_l
87.0	mix_h	71.4	mix_m	46.1	mix_h	0.1	PFA_m	0	Velliv_h	0	Velliv_h
85.7	PFA_h	69.9	mix_h	41.0	Velliv_m	0.0	Velliv_m	0	PFA_m	0	PFA_m
81.8	Velliv_m_l	69.2	Velliv_h	36.2	Velliv_m_l	0.0	Velliv_h	0	PFA_h	0	PFA_h
80.1	Velliv_h	64.9	Velliv_m_l	35.6	mix_m	0.0	PFA_h	0	mix_m	0	mix_m
78.7	Velliv_m	63.8	Velliv_m	32.7	PFA_m	0.0	mix_m	0	mix_h	0	mix_h

MC risk percentiles

Risk of loss from first to last period.

`_a` is simulation from estimated distribution of returns of mix.
`_b` is mix of simulations from estimated distribution of returns from individual funds.

`_m` is medium.
`_h` is high.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	4.97	3.29	4.02	1.85	0.72	1.00	0	0.23	0.14
5	4.31	2.80	3.62	1.70	0.65	0.92	0	0.16	0.14
10	3.66	2.48	3.16	1.55	0.54	0.83	0	0.13	0.10
25	2.20	1.51	2.09	1.11	0.40	0.55	0	0.05	0.04
50	0.78	0.67	1.00	0.61	0.21	0.27	0	0.00	0.00
90	0.04	0.10	0.09	0.04	0.04	0.02	0	0.00	0.00
99	0.00	0.03	0.02	0.00	0.01	0.01	0	0.00	0.00

1e6 simulation paths of `mhr_b`:

	0	5	10	25	50	90	99
prob_pct	0.118	0.095	0.076	0.036	0.008	0	0

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.97	Velliv_m	4.31	Velliv_m	3.66	Velliv_m	2.20	Velliv_m	1.00	Velliv_h	0.10	Velliv_m_l	0.03	Velliv_m_l
4.02	Velliv_h	3.62	Velliv_h	3.16	Velliv_h	2.09	Velliv_h	0.78	Velliv_m	0.09	Velliv_h	0.02	Velliv_h
3.29	Velliv_m_l	2.80	Velliv_m_l	2.48	Velliv_m_l	1.51	Velliv_m_l	0.67	Velliv_m_l	0.04	Velliv_m	0.01	PFA_h
1.85	PFA_m	1.70	PFA_m	1.55	PFA_m	1.11	PFA_m	0.61	PFA_m	0.04	PFA_m	0.01	mix_m_a
1.00	mix_m_a	0.92	mix_m_a	0.83	mix_m_a	0.55	mix_m_a	0.27	mix_m_a	0.04	PFA_h	0.00	Velliv_m
0.72	PFA_h	0.65	PFA_h	0.54	PFA_h	0.40	PFA_h	0.21	PFA_h	0.02	mix_m_a	0.00	PFA_m
0.23	mix_m_b	0.16	mix_m_b	0.13	mix_m_b	0.05	mix_m_b	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.14	mix_h_b	0.14	mix_h_b	0.10	mix_h_b	0.04	mix_h_b	0.00	mix_m_b	0.00	mix_m_b	0.00	mix_m_b
0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_b	0.00	mix_h_b	0.00	mix_h_b

MC gains percentiles

Chance of gains from first to last period.
_a is simulation from estimated distribution of returns of mix.
_b is mix of simulations from estimated distribution of returns from individual funds.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	95.03	96.71	95.98	98.15	99.28	99.00	100.00	99.77	99.86
5	94.23	96.23	95.50	98.00	99.18	98.72	100.00	99.75	99.84
10	93.38	95.78	95.05	97.79	99.05	98.61	100.00	99.66	99.78
25	90.97	94.14	93.57	96.95	98.58	97.90	100.00	99.16	99.59
50	86.02	90.63	90.40	94.83	97.68	96.22	99.97	97.76	99.11
100	72.09	78.91	82.57	88.39	95.27	89.20	99.63	90.22	97.47
200	40.49	45.31	63.21	59.34	85.82	59.52	92.76	47.97	86.61
300	17.57	17.36	44.11	22.60	71.41	22.55	72.15	11.46	66.38
400	5.79	5.26	27.89	4.16	54.92	3.67	44.94	1.24	42.24
500	1.63	1.16	16.78	0.53	38.90	0.18	23.13	0.11	22.16
1000	0.00	0.00	0.75	0.00	2.21	0.00	0.24	0.00	0.09

1e6 simulation paths of `mhr_b`:

	0	5	10	25	50	100	200	300	400	500	1000
prob	99.882	99.854	99.824	99.686	99.301	97.513	86.912	65.992	41.486	21.693	0.086

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	99.97	mix_h_a	99.63	mix_h_a
99.86	mix_h_b	99.84	mix_h_b	99.78	mix_h_b	99.59	mix_h_b	99.11	mix_h_b	97.47	mix_h_b
99.77	mix_m_b	99.75	mix_m_b	99.66	mix_m_b	99.16	mix_m_b	97.76	mix_m_b	95.27	PFA_h
99.28	PFA_h	99.18	PFA_h	99.05	PFA_h	98.58	PFA_h	97.68	PFA_h	90.22	mix_m_b
99.00	mix_m_a	98.72	mix_m_a	98.61	mix_m_a	97.90	mix_m_a	96.22	mix_m_a	89.20	mix_m_a
98.15	PFA_m	98.00	PFA_m	97.79	PFA_m	96.95	PFA_m	94.83	PFA_m	88.39	PFA_m
96.71	Velliv_m_l	96.23	Velliv_m_l	95.78	Velliv_m_l	94.14	Velliv_m_l	90.63	Velliv_m_l	82.57	Velliv_h
95.98	Velliv_h	95.50	Velliv_h	95.05	Velliv_h	93.57	Velliv_h	90.40	Velliv_h	78.91	Velliv_m_l
95.03	Velliv_m	94.23	Velliv_m	93.38	Velliv_m	90.97	Velliv_m	86.02	Velliv_m	72.09	Velliv_m

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
92.76	mix_h_a	72.15	mix_h_a	54.92	PFA_h	38.90	PFA_h	2.21	PFA_h
86.61	mix_h_b	71.41	PFA_h	44.94	mix_h_a	23.13	mix_h_a	0.75	Velliv_h
85.82	PFA_h	66.38	mix_h_b	42.24	mix_h_b	22.16	mix_h_b	0.24	mix_h_a
63.21	Velliv_h	44.11	Velliv_h	27.89	Velliv_h	16.78	Velliv_h	0.09	mix_h_b
59.52	mix_m_a	22.60	PFA_m	5.79	Velliv_m	1.63	Velliv_m	0.00	Velliv_m
59.34	PFA_m	22.55	mix_m_a	5.26	Velliv_m_l	1.16	Velliv_m_l	0.00	Velliv_m_l
47.97	mix_m_b	17.57	Velliv_m	4.16	PFA_m	0.53	PFA_m	0.00	PFA_m
45.31	Velliv_m_l	17.36	Velliv_m_l	3.67	mix_m_a	0.18	mix_m_a	0.00	mix_m_a
40.49	Velliv_m	11.46	mix_m_b	1.24	mix_m_b	0.11	mix_m_b	0.00	mix_m_b

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.
`m` is the location parameter.

`s` is the scale parameter.
`nu` is the estimated degrees of freedom, or shape parameter.
`xi` is the estimated skewness parameter.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770
R-squared	0.993	0.978	0.991	0.991	0.964	0.890	0.961

Fit statistics ranking

m	ranking	s	ranking	R-squared	ranking
0.084	PFA_high	0.071	mix_high	0.993	Velliv_medium
0.082	mix_high	0.088	mix_medium	0.991	Velliv_high
0.065	Velliv_high	0.115	Velliv_medium_long	0.991	PFA_medium
0.059	mix_medium	0.120	Velliv_medium	0.978	Velliv_medium_long
0.058	PFA_medium	0.121	PFA_high	0.964	PFA_high
0.052	Velliv_medium_long	0.123	PFA_medium	0.961	mix_high
0.048	Velliv_medium	0.150	Velliv_high	0.890	mix_medium

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).
Statistics are given for the final state of all paths.
Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.
0 is defined as any value below a threshold.
`losing_prob_pct` is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_m_b	mix_h_a	mix_h_b
mc_m	282.756	294.553	401.591	323.343	554.695	324.341	301.077	501.217	479.705
mc_s	126.515	116.778	218.086	104.601	241.376	98.828	81.417	157.181	164.484
mc_min	2.896	0.020	0.051	2.281	0.094	0.908	54.068	128.193	54.305
mc_max	944.490	899.641	1905.916	1099.534	1641.229	689.992	888.384	1320.013	1289.077
dao_pct	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
losing_pct	4.970	3.290	4.020	1.850	0.720	1.000	0.230	0.000	0.140

Ranking

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	losing_pct	ranking
554.695	PFA_h	81.417	mix_m_b	128.193	mix_h_a	1905.916	Velliv_h	0	Velliv_m	0.00	mix_h_a
501.217	mix_h_a	98.828	mix_m_a	54.305	mix_h_b	1641.229	PFA_h	0	Velliv_m_l	0.14	mix_h_b
479.705	mix_h_b	104.601	PFA_m	54.068	mix_m_b	1320.013	mix_h_a	0	Velliv_h	0.23	mix_m_b
401.591	Velliv_h	116.778	Velliv_m_l	2.896	Velliv_m	1289.077	mix_h_b	0	PFA_m	0.72	PFA_h
324.341	mix_m_a	126.515	Velliv_m	2.281	PFA_m	1099.534	PFA_m	0	PFA_h	1.00	mix_m_a
323.343	PFA_m	157.181	mix_h_a	0.908	mix_m_a	944.490	Velliv_m	0	mix_m_a	1.85	PFA_m
301.077	mix_m_b	164.484	mix_h_b	0.094	PFA_h	899.641	Velliv_m_l	0	mix_m_b	3.29	Velliv_m_l
294.553	Velliv_m_l	218.086	Velliv_h	0.051	Velliv_h	888.384	mix_m_b	0	mix_h_a	4.02	Velliv_h
282.756	Velliv_m	241.376	PFA_h	0.020	Velliv_m_l	689.992	mix_m_a	0	mix_h_b	4.97	Velliv_m

Comments

(Ignoring `mhr_a`…)

`mhr` has some nice properties:
- It has a relatively high `nu` value of 90, which means it is tending more towards exponential tails than polynomial tails. All other funds have `nu` values close to 3, except `phr` which is even worse at close to 2. (Note that for a Gaussian, `nu` is infinite.)
- It has the lowest losing percentage of all simulations, which is better than 1/6 that of `phr`.
- It has a DAO percentage of 0, which is the same as `mmr`, and less than `phr`.
- Only `phr` has a higher `mc_m`.
- It has a smaller `mc_s` than the individual components, `vhr` and `phr`.
- It has the highest `xi` of all fits, suggesting less left skewness. Density plots for `vmr`, `phr` and `mmr` have an extremely sharp drop, as if an upward limiter has been applied, which corresponds to extremely low `xi` values. The density plot for `mhr` is by far the most symmetrical of all the fits.
- Only `mmr` has as higher `mc_min`. However, that of `mmr` is 18 times higher with 62, so `mmr` is a clear winner here.
- Naturally, it has a `mc_max` smaller than the individual components, `vhr` and `phr`, but ca. 1.5 times higher then `mmr`.
- All the first 4 moments converge nicely. For all other fits, the 4th moment doesn't seem to converge.

Taleb, Statistical Consequences Of Fat Tails, p. 97:
"the variance of a finite variance random variable with tail exponent < 4 will be infinite".

And p. 363:
"The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, un- der a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance."

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2}\right) \Big/ \left(\frac{x_{t-1} + y_{t-1}}{2}\right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns $=$ Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2\frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1} y_t = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

$$(x_{t-1} x_1 y_{t-1} + y_{t-1} x_t y_{t-1}) + (x_{t-1} x_{t-1} y_t + x_{t-1} y_{t-1} y_t) = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: `R = 1+r`

```
## Let x_0 be 100.

## Let y_0 be 200.

## So the initial value of the pf is 300 .

## Let R_x be 0.5.

## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.
```

```
## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): 0.1531626
## s(data_x): 0.4450475
## m(data_y): 9.368863
## s(data_y): 2.966004
##
## m(data_x + data_y): 4.761013
## s(data_x + data_y): 1.47759
```

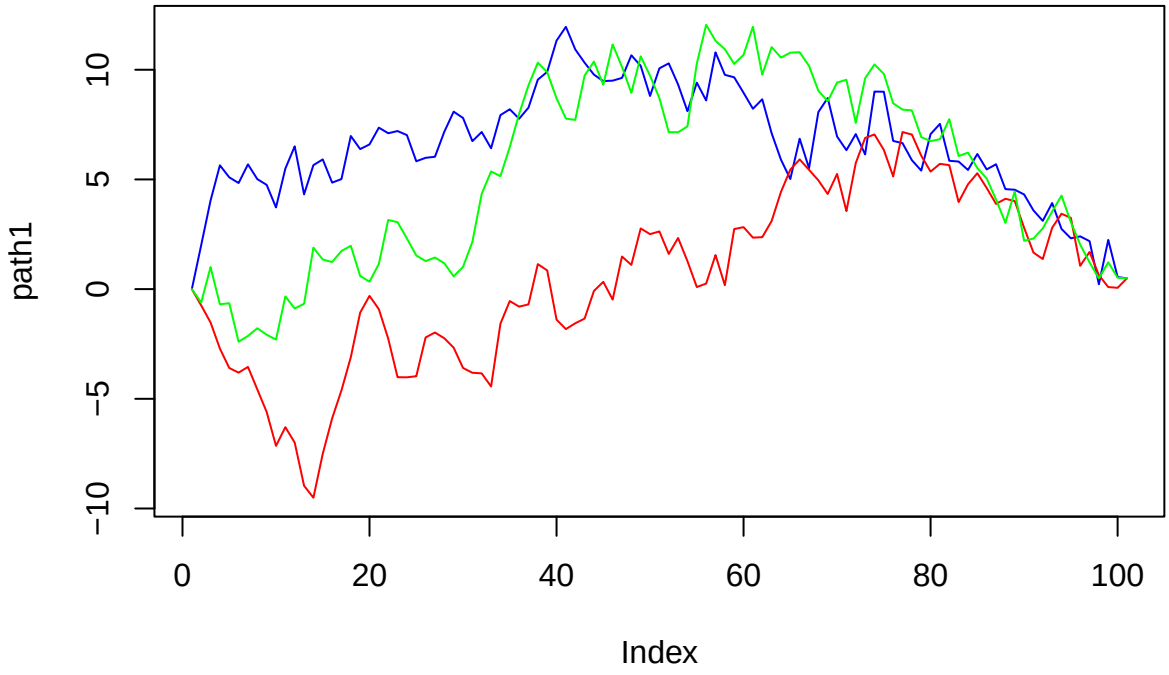
m and s of final state of all paths.
_a is mix of simulated returns.
_b is simulated mixed returns.

m_a	m_b	s_a	s_b
94.790	95.239	6.301	6.413
95.316	95.557	6.948	6.896
95.313	95.350	6.821	6.794
95.352	95.040	6.598	6.782
95.460	95.047	6.571	6.814
95.309	95.088	6.680	6.658
95.046	95.444	6.621	6.621
95.090	95.027	6.759	6.618
95.716	95.098	6.646	6.671
95.218	95.239	6.666	6.812

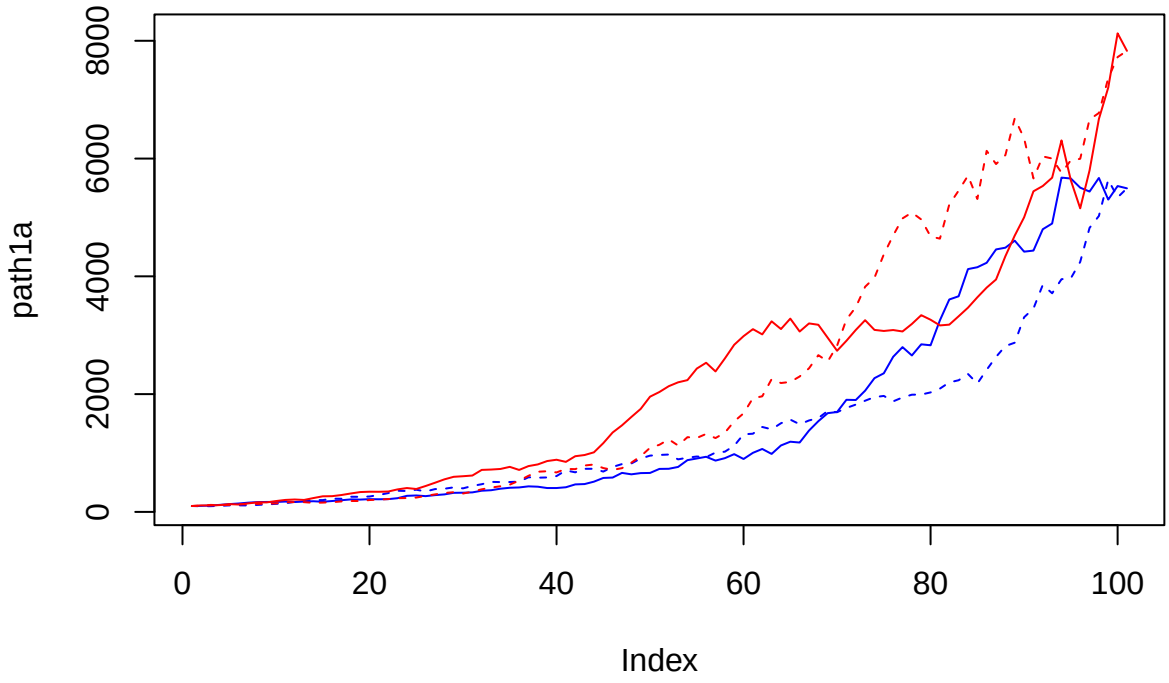
```
##       m_a              m_b              s_a              s_b        
##  Min.   :94.79   Min.   :95.03   Min.   :6.301   Min.   :6.413  
##  1st Qu.:95.12   1st Qu.:95.06   1st Qu.:6.604   1st Qu.:6.630  
##  Median :95.31   Median :95.17   Median :6.656   Median :6.726  
##  Mean   :95.26   Mean   :95.21   Mean   :6.661   Mean   :6.708  
##  3rd Qu.:95.34   3rd Qu.:95.32   3rd Qu.:6.739   3rd Qu.:6.808  
##  Max.   :95.72   Max.   :95.56   Max.   :6.948   Max.   :6.896
```

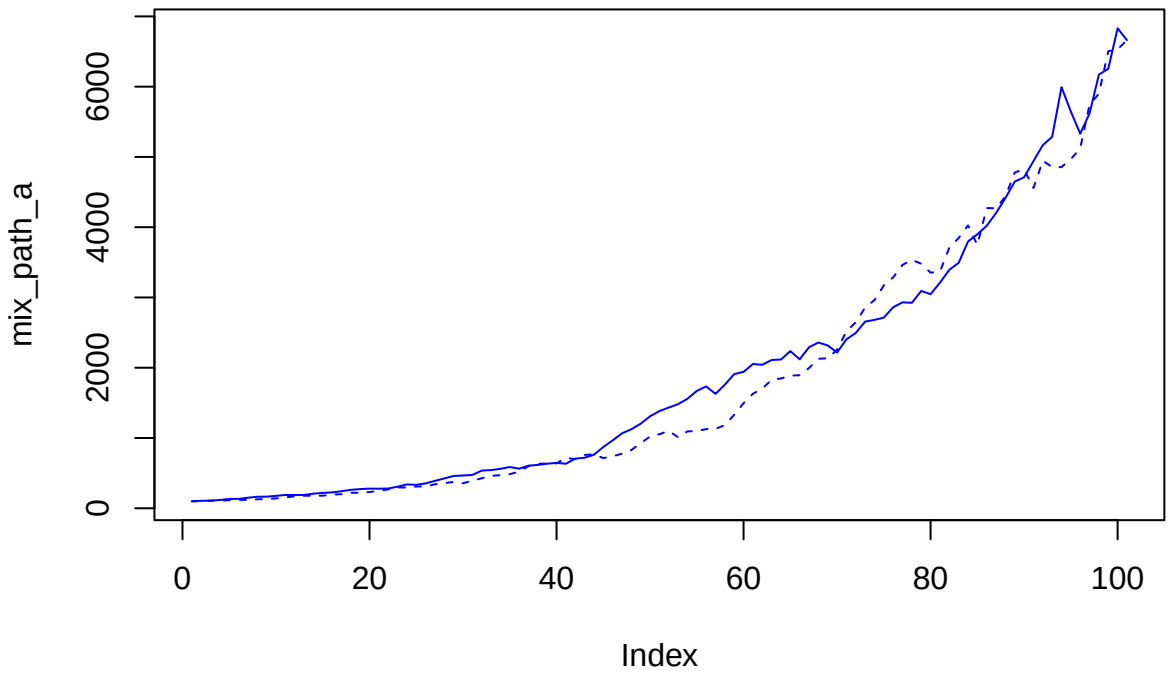
_a and _b are very close to equal.
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?


path1a
0
2000
4000
6000
8000
0
20
40
60
80
100
Index



The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

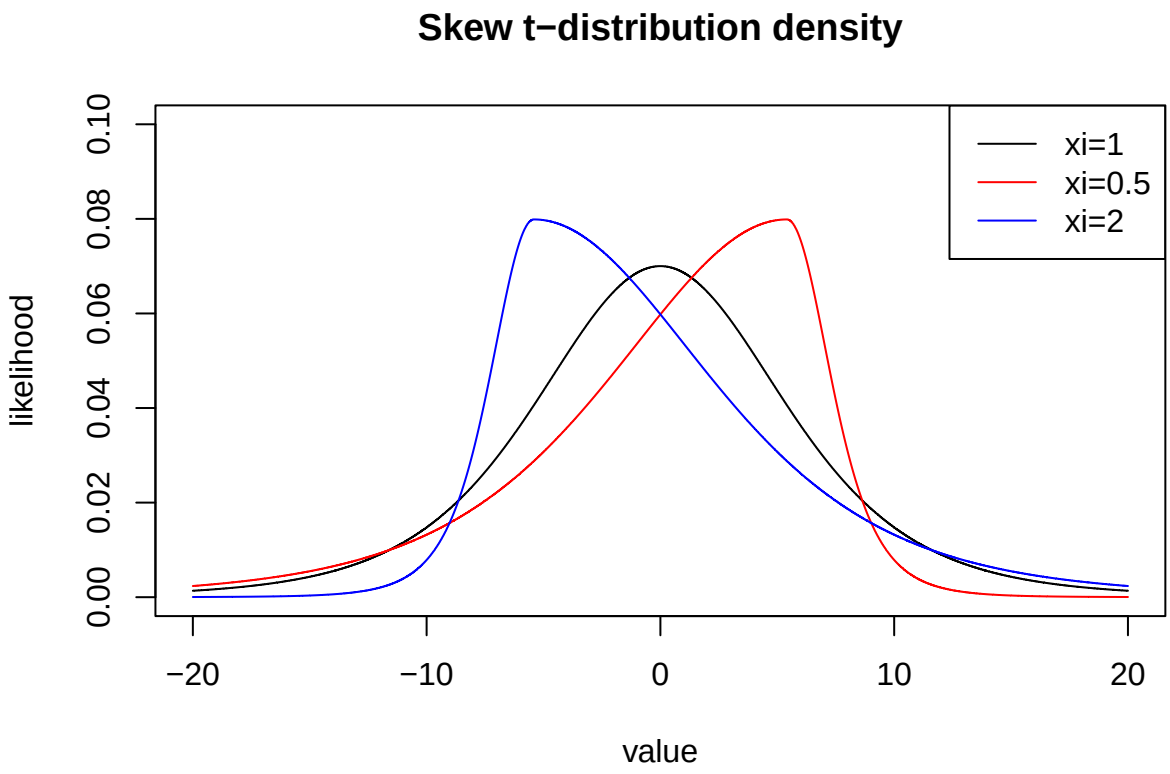
```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?
Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##        m                s          
##  Min.   :0.06152   Min.   :0.04551  
##  1st Qu.:0.06616   1st Qu.:0.05917  
##  Median :0.06907   Median :0.06482  
##  Mean   :0.07029   Mean   :0.06796  
##  3rd Qu.:0.07313   3rd Qu.:0.07759  
##  Max.   :0.08514   Max.   :0.09383
```

The meaning of `xi`

The fit for `mhr` has the highest `xi` value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, ..., X^p)}{\sum_{i=1}^{n} X_i^p} \to 0$$

for $n \to \infty$.

If not, X doesn't have a p'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192