Pension returns analysis

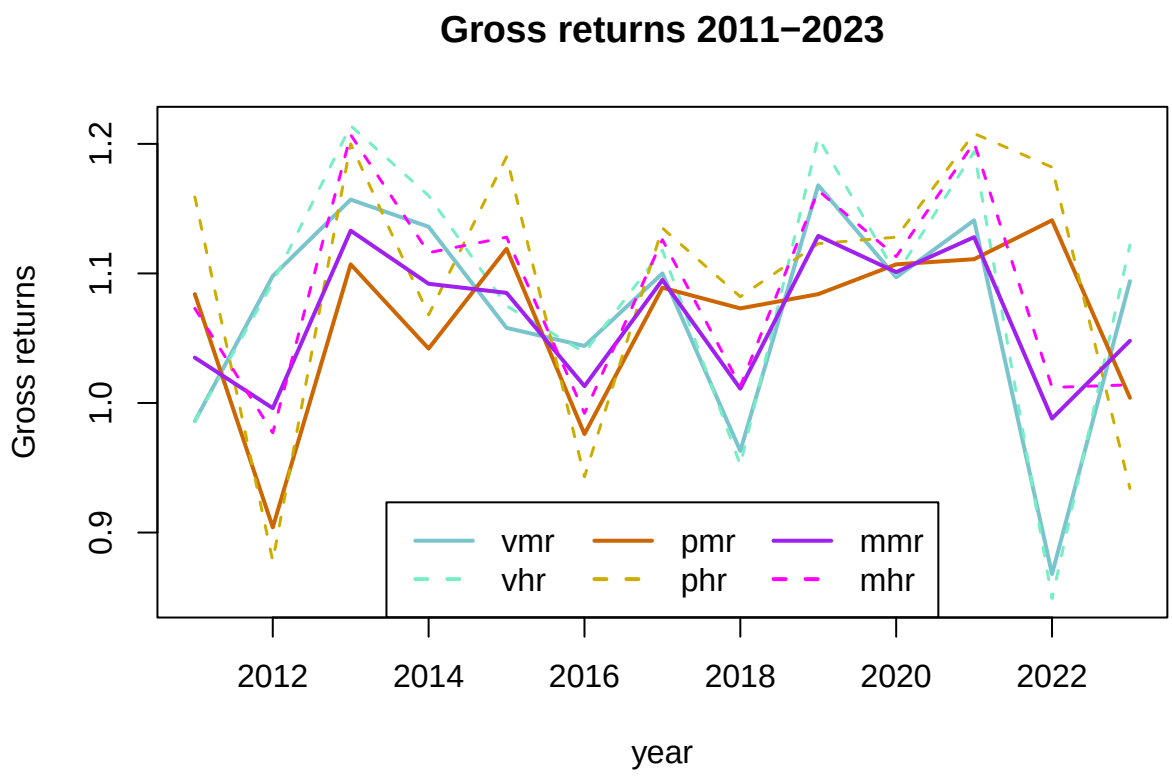
18:42 05 April 2024

Contents

Fit log returns to F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated shape parameter (degrees of freedom).
`xi` is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.
`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.



Summary of gross returns

```
##       vmr              pmr              mmr              vhr        
##  Min.   :0.868   Min.   :0.904   Min.   :0.988   Min.   :0.849  
##  1st Qu.:1.044   1st Qu.:1.042   1st Qu.:1.013   1st Qu.:1.039  
##  Median :1.097   Median :1.084   Median :1.085   Median :1.099  
##  Mean   :1.070   Mean   :1.065   Mean   :1.066   Mean   :1.085  
##  3rd Qu.:1.136   3rd Qu.:1.107   3rd Qu.:1.101   3rd Qu.:1.160  
##  Max.   :1.168   Max.   :1.141   Max.   :1.133   Max.   :1.214  
##       phr              mhr        
##  Min.   :0.878   Min.   :0.977  
##  1st Qu.:1.068   1st Qu.:1.013  
##  Median :1.128   Median :1.113  
##  Mean   :1.095   Mean   :1.087  
##  3rd Qu.:1.182   3rd Qu.:1.128  
##  Max.   :1.208   Max.   :1.207
```

```
##       vmrl        
##  Min.   :0.801  
##  1st Qu.:1.013  
##  Median :1.085  
##  Mean   :1.061  
##  3rd Qu.:1.128  
##  Max.   :1.193
```

```
##             vmr   pmr   mmr   vhr   phr   mhr
## Min.   : 0.868 0.904 0.988 0.849 0.878 0.977
## 1st Qu.: 1.044 1.042 1.013 1.039 1.068 1.013
## Median : 1.097 1.084 1.085 1.099 1.128 1.113
## Mean   : 1.070 1.065 1.066 1.085 1.095 1.087
## 3rd Qu.: 1.136 1.107 1.101 1.160 1.182 1.128
## Max.   : 1.168 1.141 1.133 1.214 1.208 1.207
```

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.136	vmr	1.168	vmr
0.977	mhr	1.044	vmr	1.113	mhr	1.087	mhr	1.107	pmr	1.141	pmr
0.904	pmr	1.042	pmr	1.099	vhr	1.085	vhr	1.101	mmr	1.133	mmr
0.878	phr	1.039	vhr	1.097	vmr	1.070	vmr	1.160	vhr	1.214	vhr
0.868	vmr	1.013	mmr	1.085	mmr	1.066	mmr	1.182	phr	1.208	phr
0.849	vhr	1.013	mhr	1.084	pmr	1.065	pmr	1.128	mhr	1.207	mhr

Covariance

```
## cov(vmr, pmr) =  -0.001094875
```

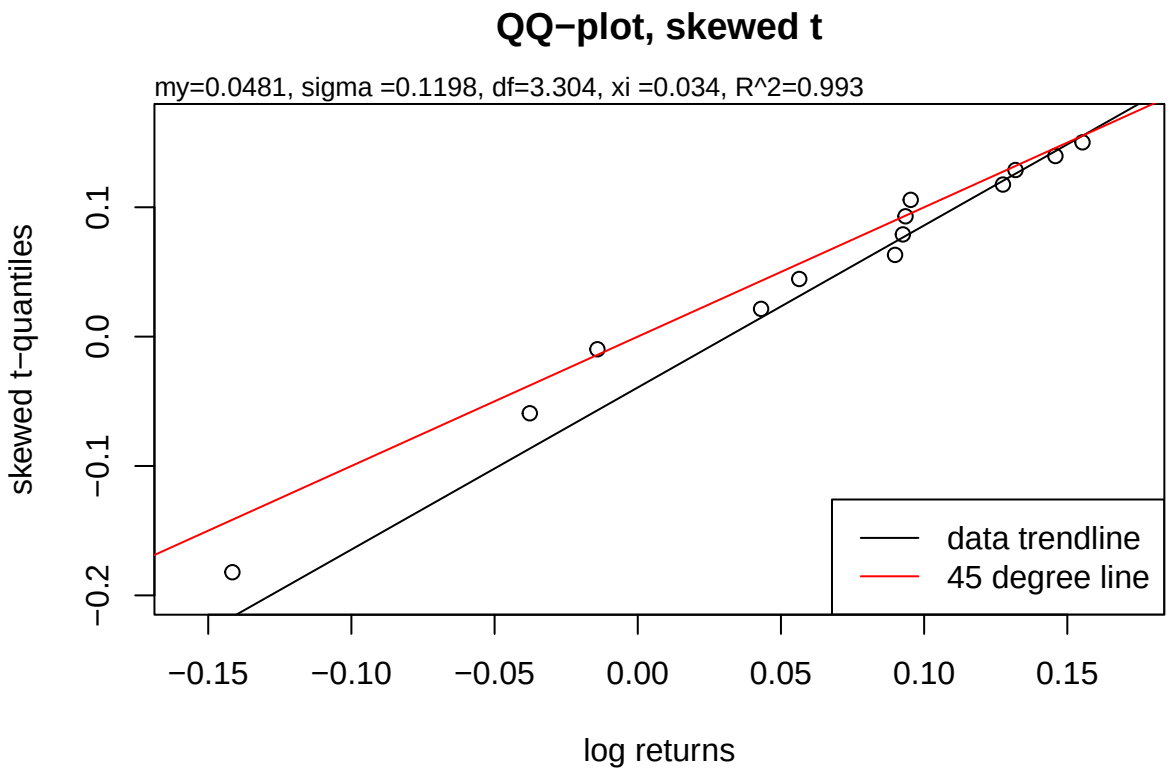
```
## cov(vhr, phr) =  -0.0001730651
```

Velliv medium risk, 2011 - 2023

```
##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 7.4 percent
```

```
## What is the risk of losing max 25 %? =< 1.8 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 41 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

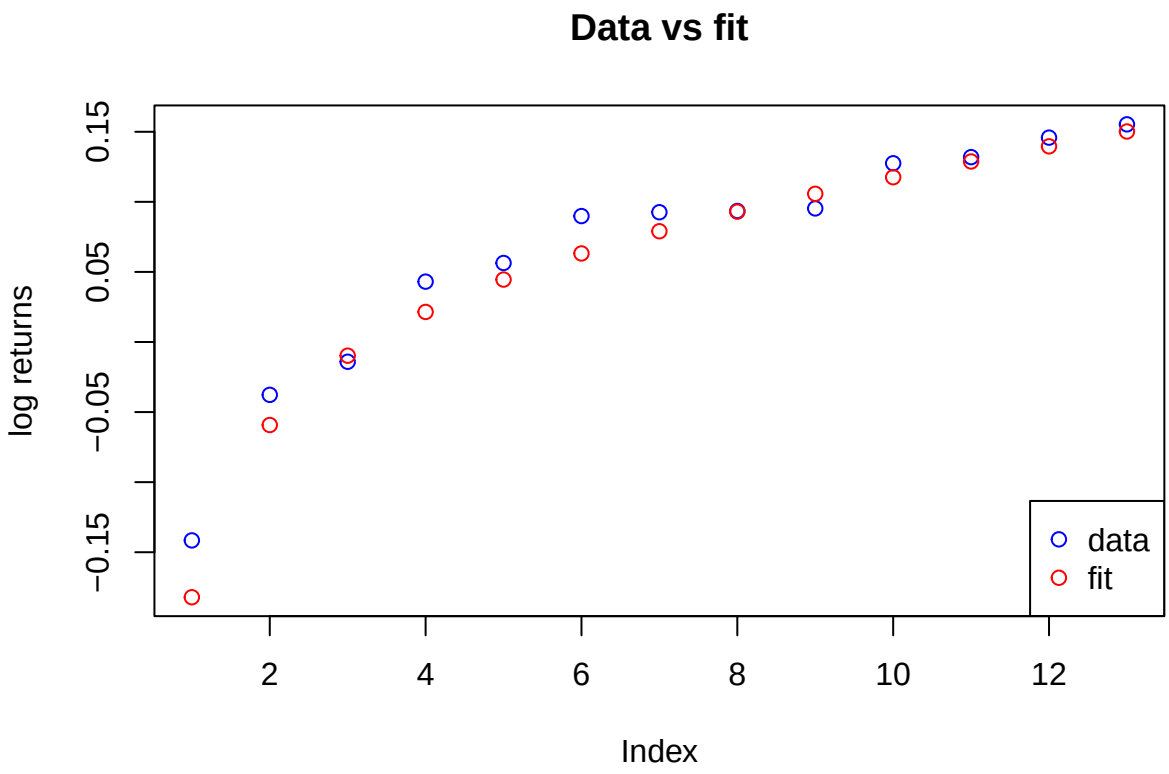
QQ Plot



The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

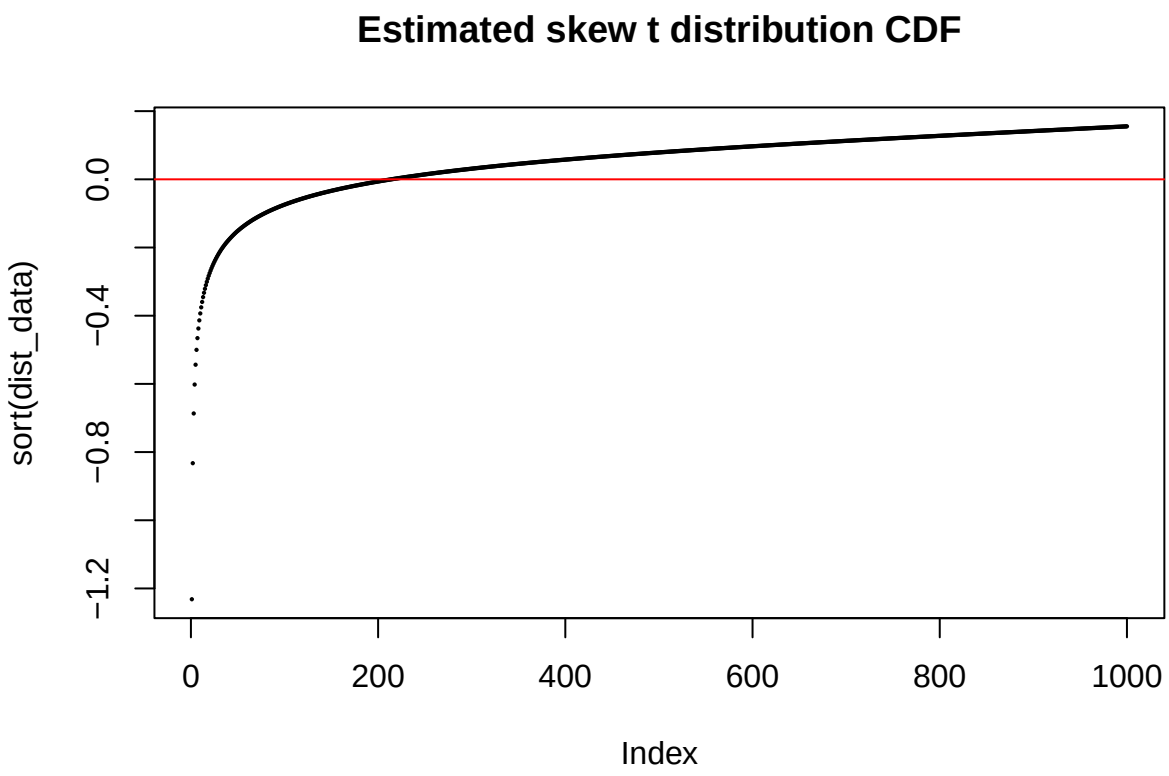
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

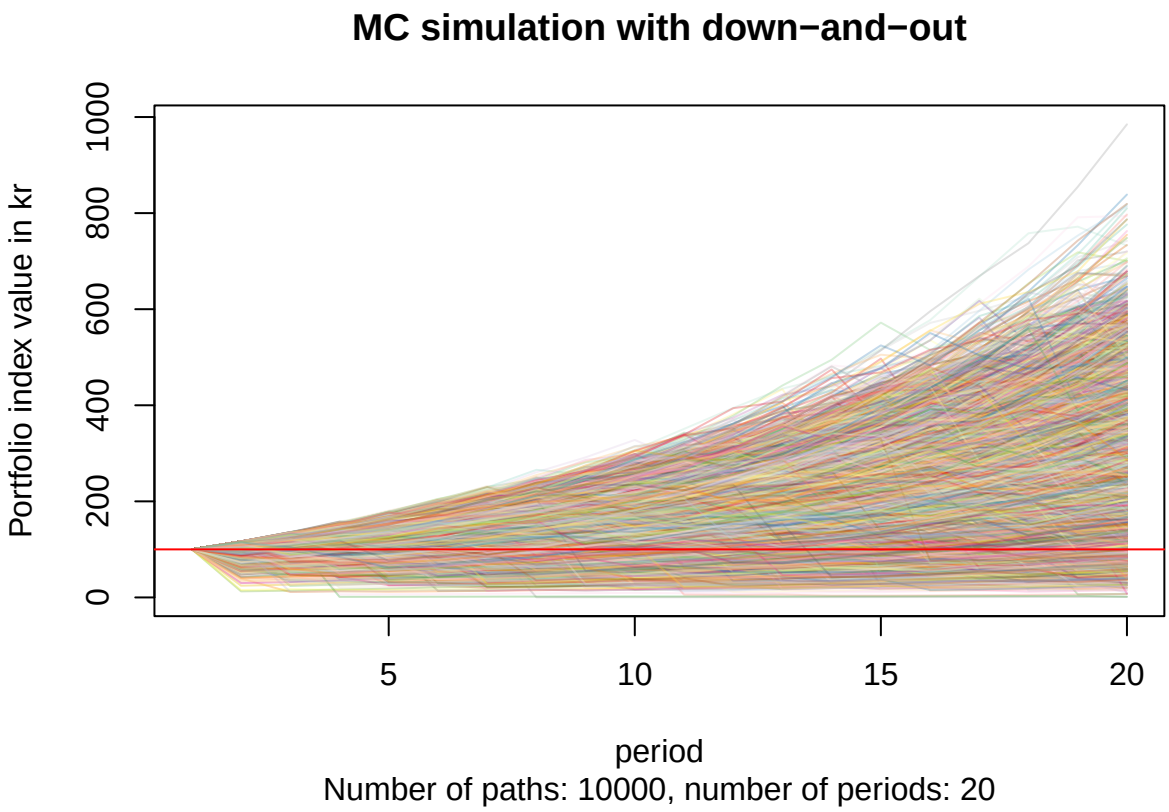
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

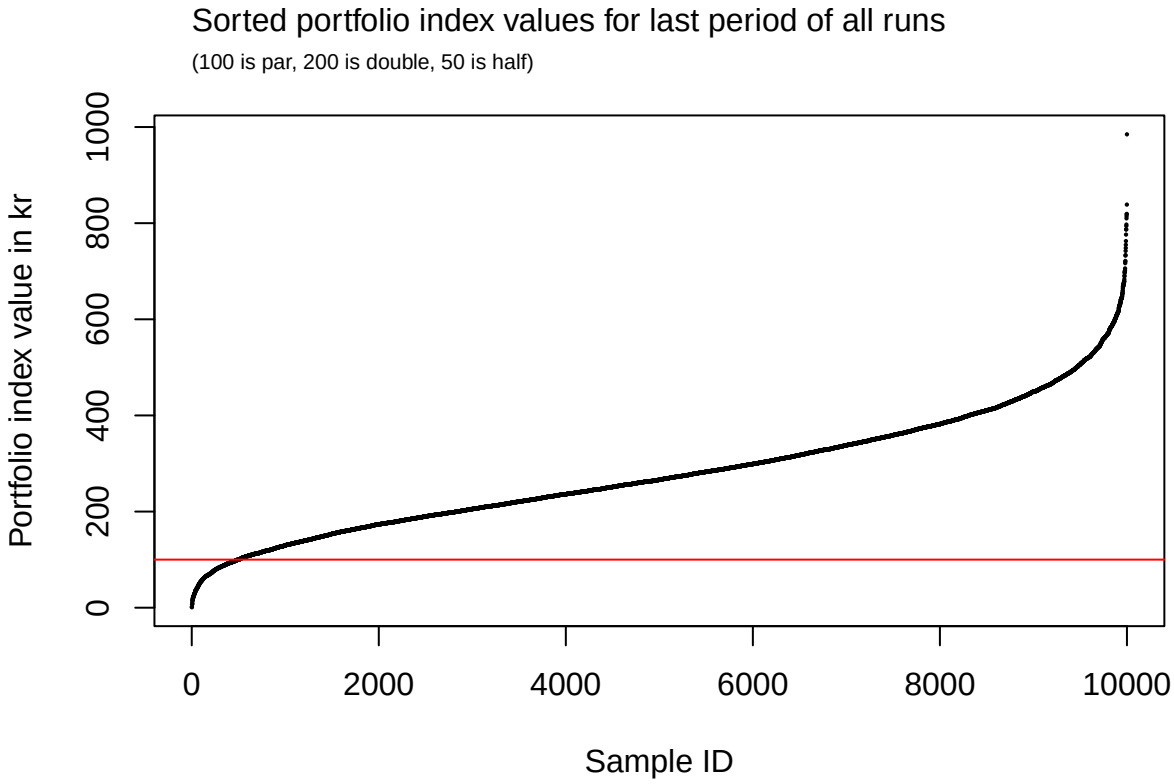


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 280.45 kr.
## SD of portfolio index value after 20 years: 124.455 kr.
## Min total portfolio index value after 20 years: 0.945 kr.
## Max total portfolio index value after 20 years: 984.799 kr.
##
## Share of paths finishing below 100: 4.94 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
1000
5
10
15
20
period
Number of paths: 10000, number of periods: 20

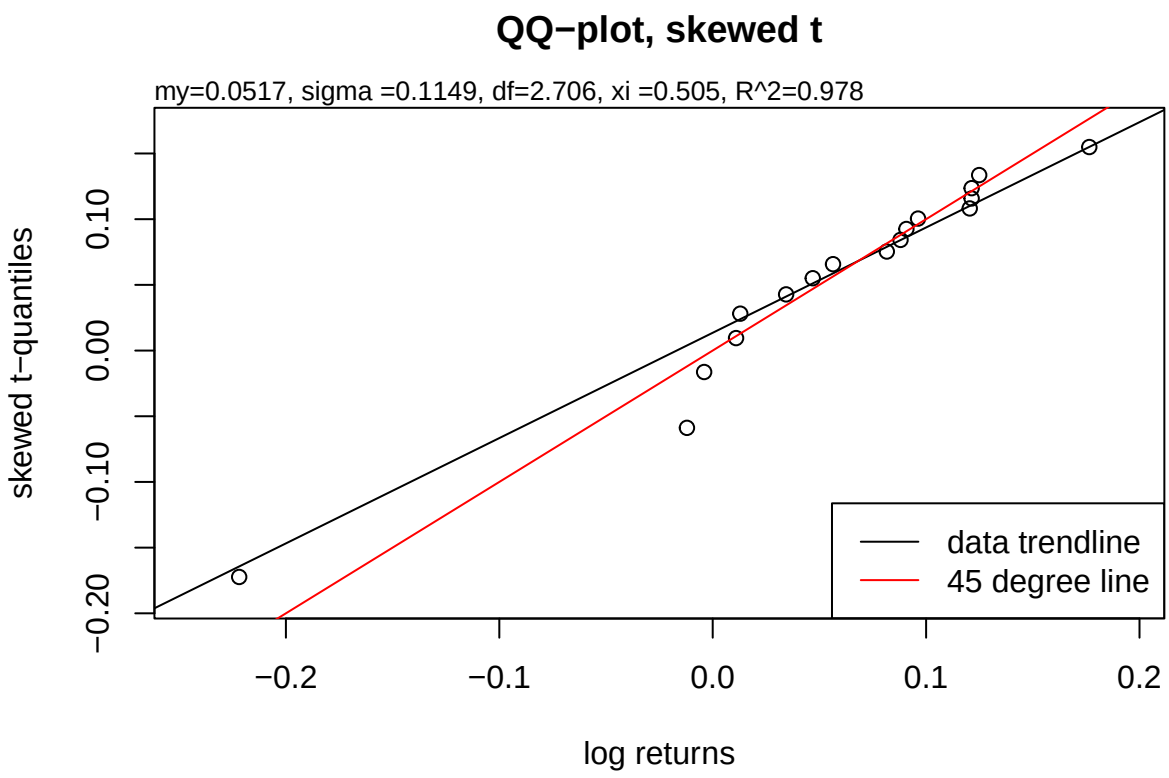


Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##
## AIC: -34.35752
## BIC: -31.02467
## m: 0.05171176
## s: 0.1149408
## nu (df): 2.706099
## xi: 0.5049945
## R^2: 0.978
##
## An R^2 of 0.978 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 5.4 percent
## What is the risk of losing max 25 %? =< 1.3 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 36.2 percent
## What is the chance of gaining min 25 %? >= 0.3 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

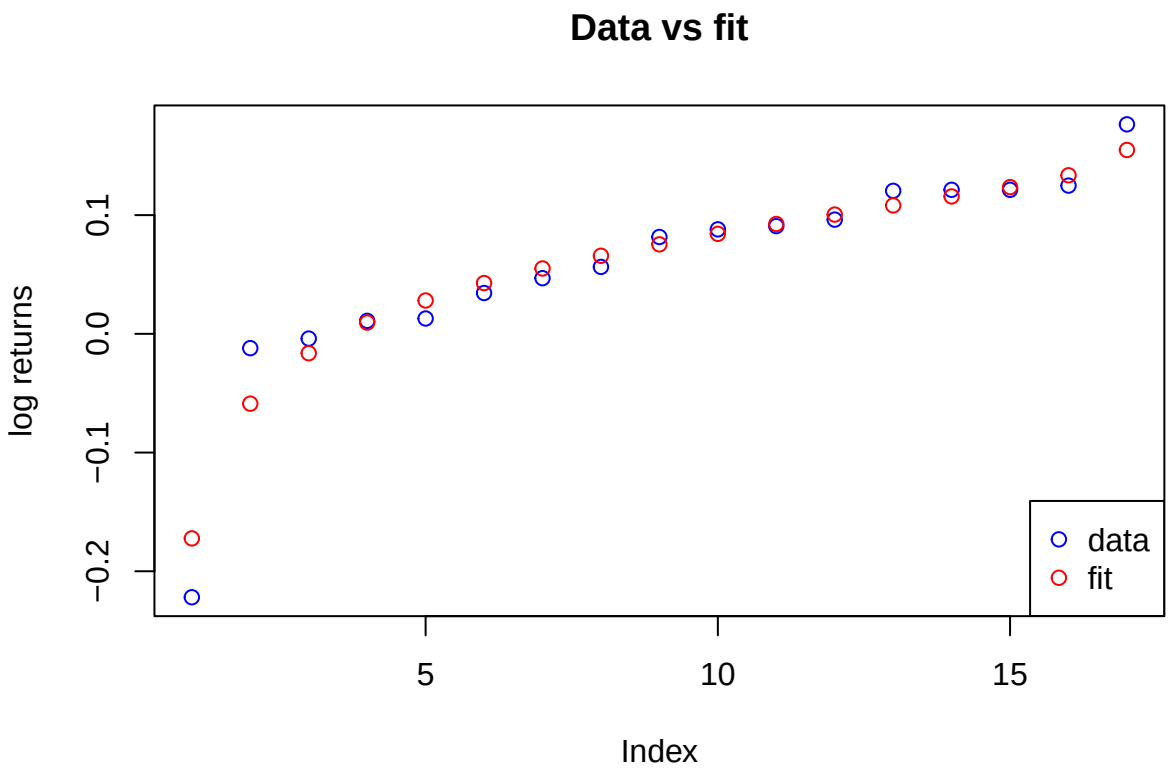
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

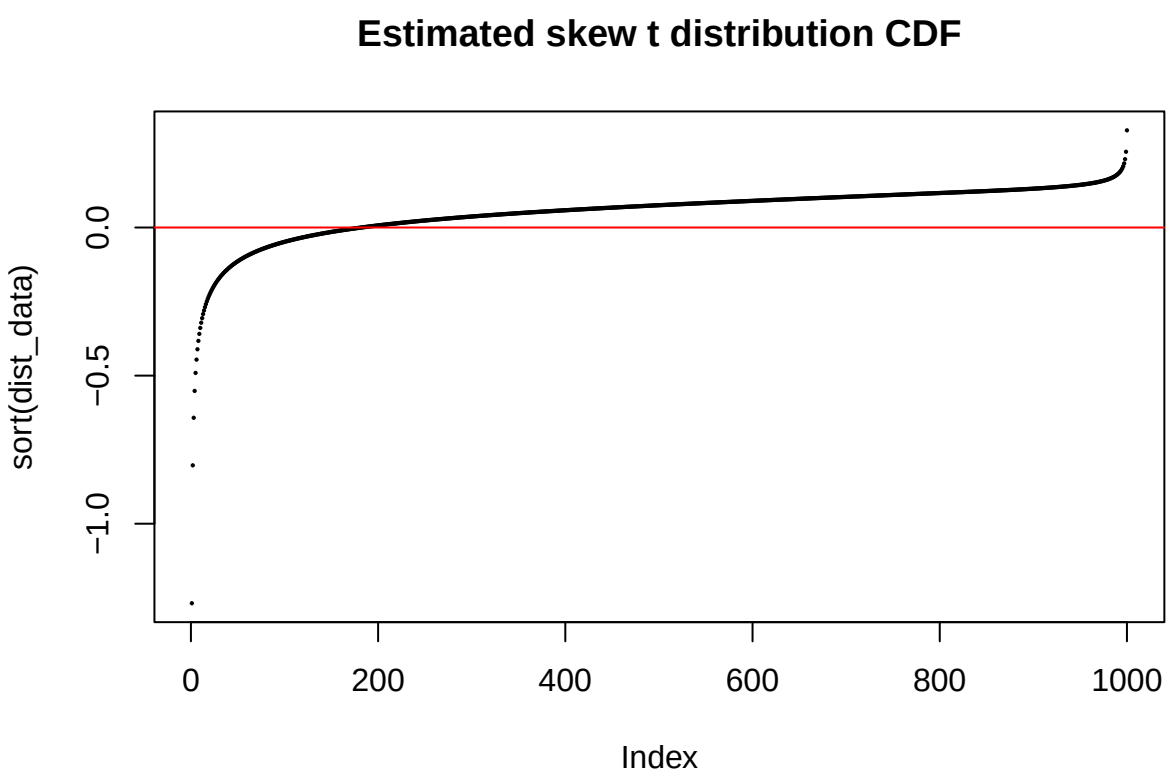
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

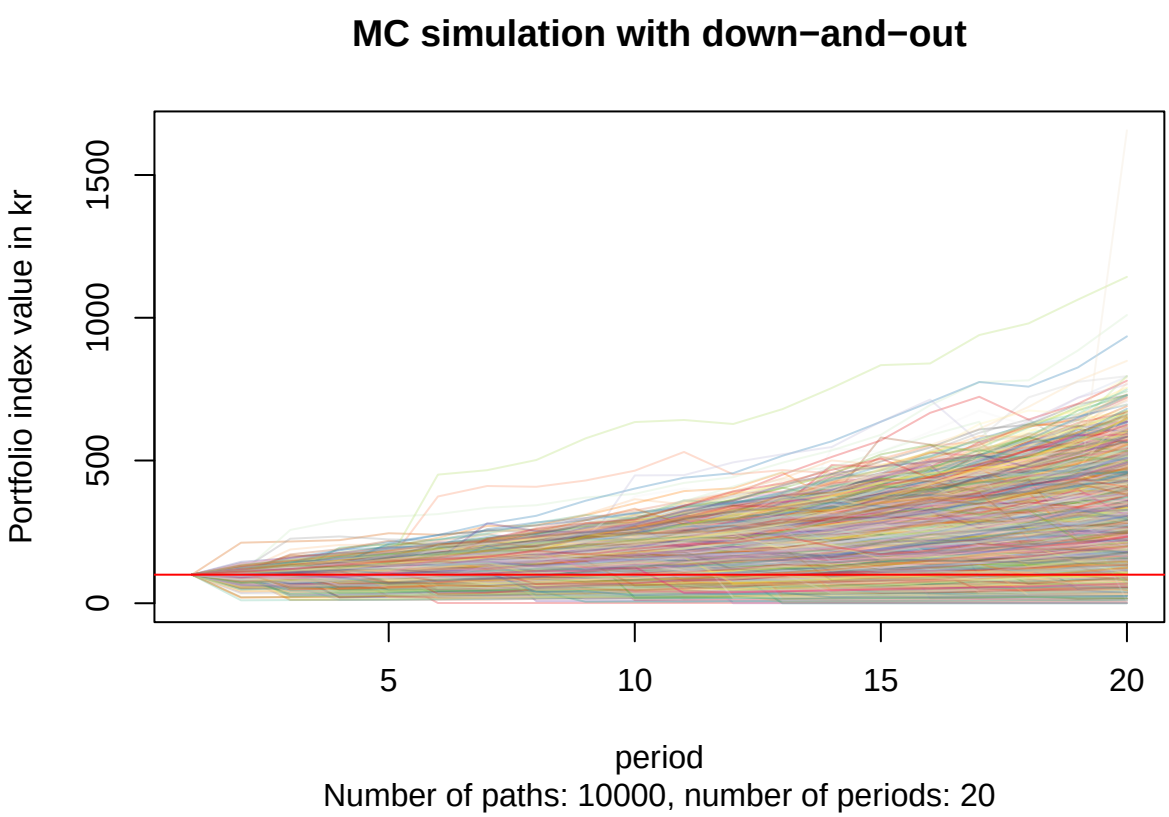
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

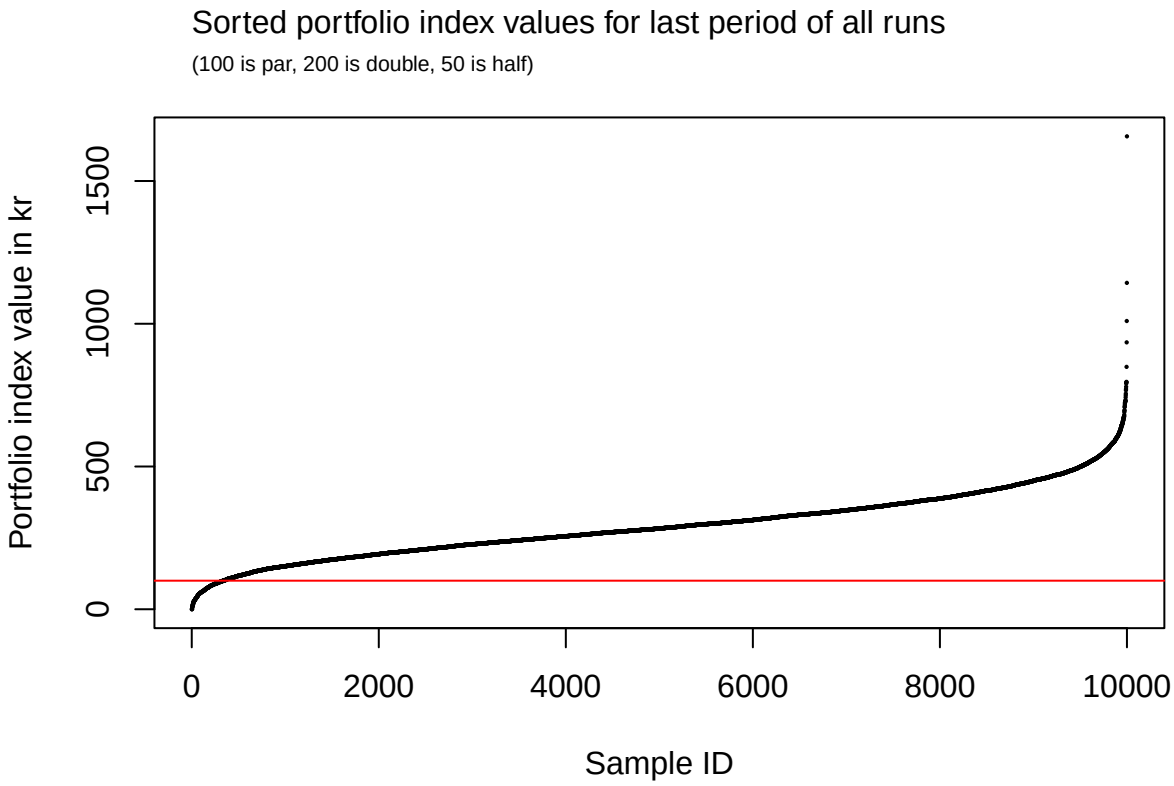


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0.01 percent
##
## Mean portfolio index value after 20 years: 293.509 kr.
## SD of portfolio index value after 20 years: 118.513 kr.
## Min total portfolio index value after 20 years: 0 kr.
## Max total portfolio index value after 20 years: 1656.391 kr.
##
## Share of paths finishing below 100: 3.37 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

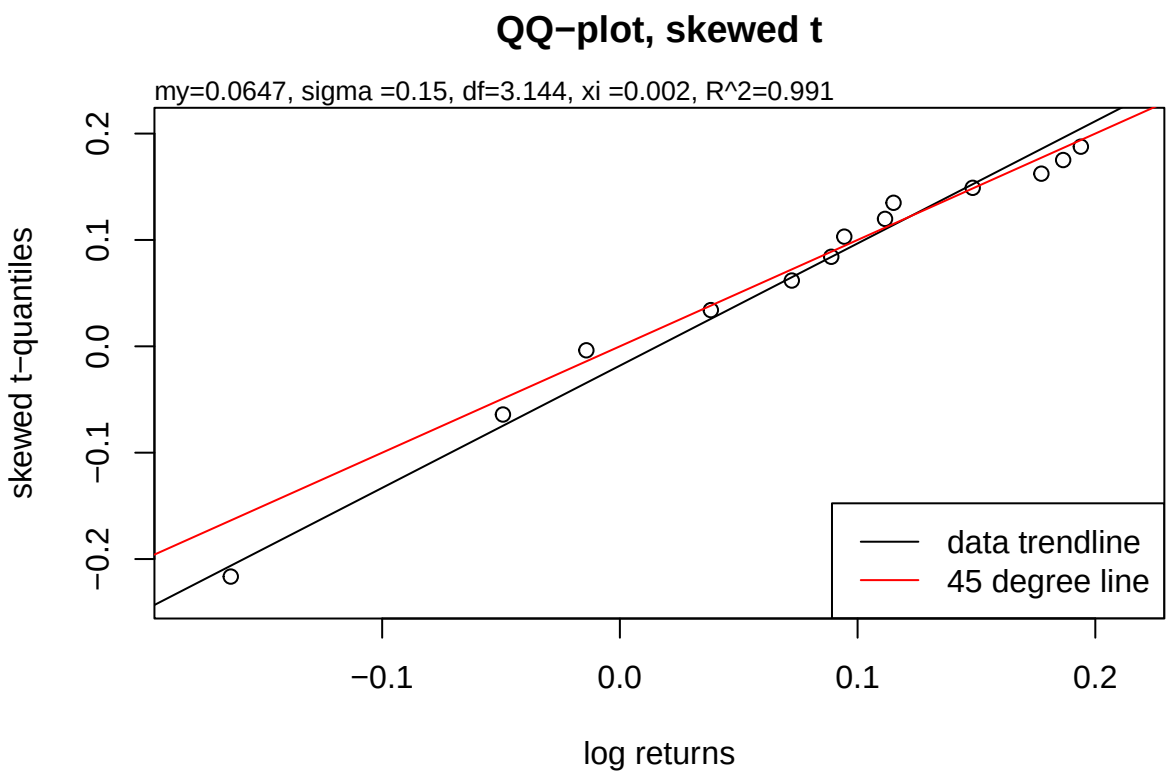


Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -21.42488
## BIC: -19.16508
## m: 0.06471454
## s: 0.1499924
## nu (df): 3.144355
## xi: 0.002367034
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 8.3 percent
## What is the risk of losing max 25 %? =< 2.5 percent
## What is the risk of losing max 50 %? =< 0.4 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 53.3 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

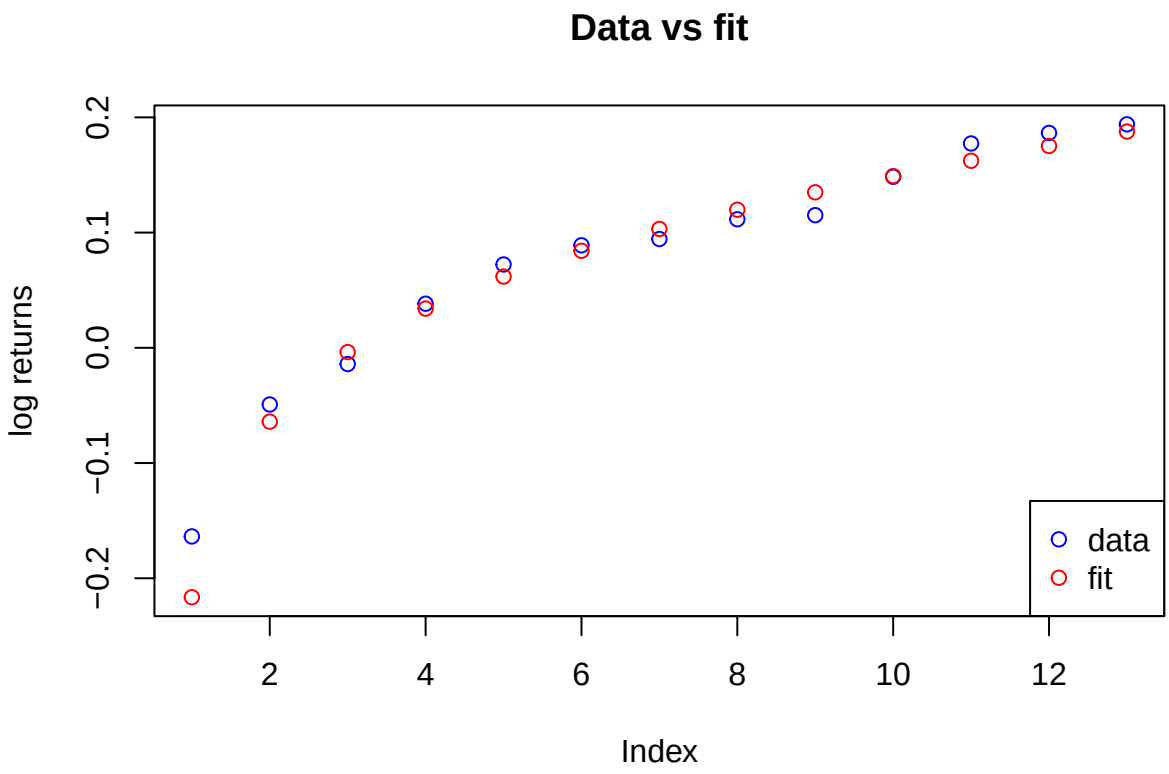
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

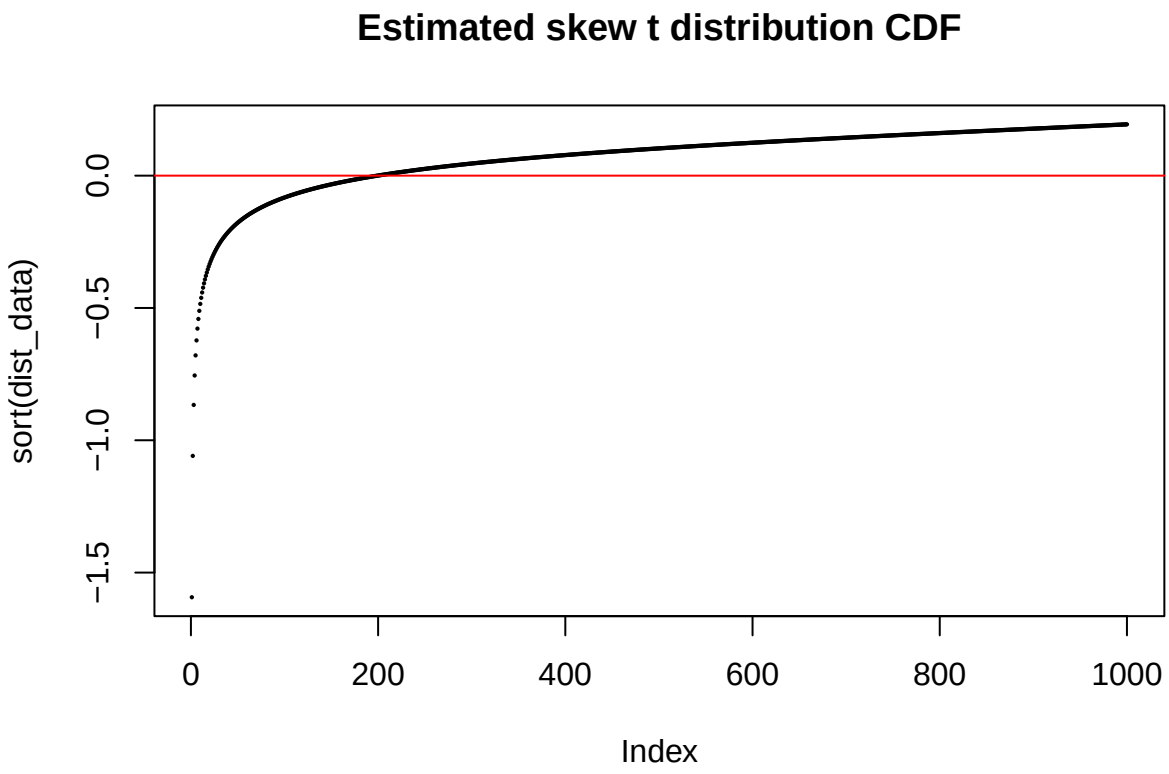
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

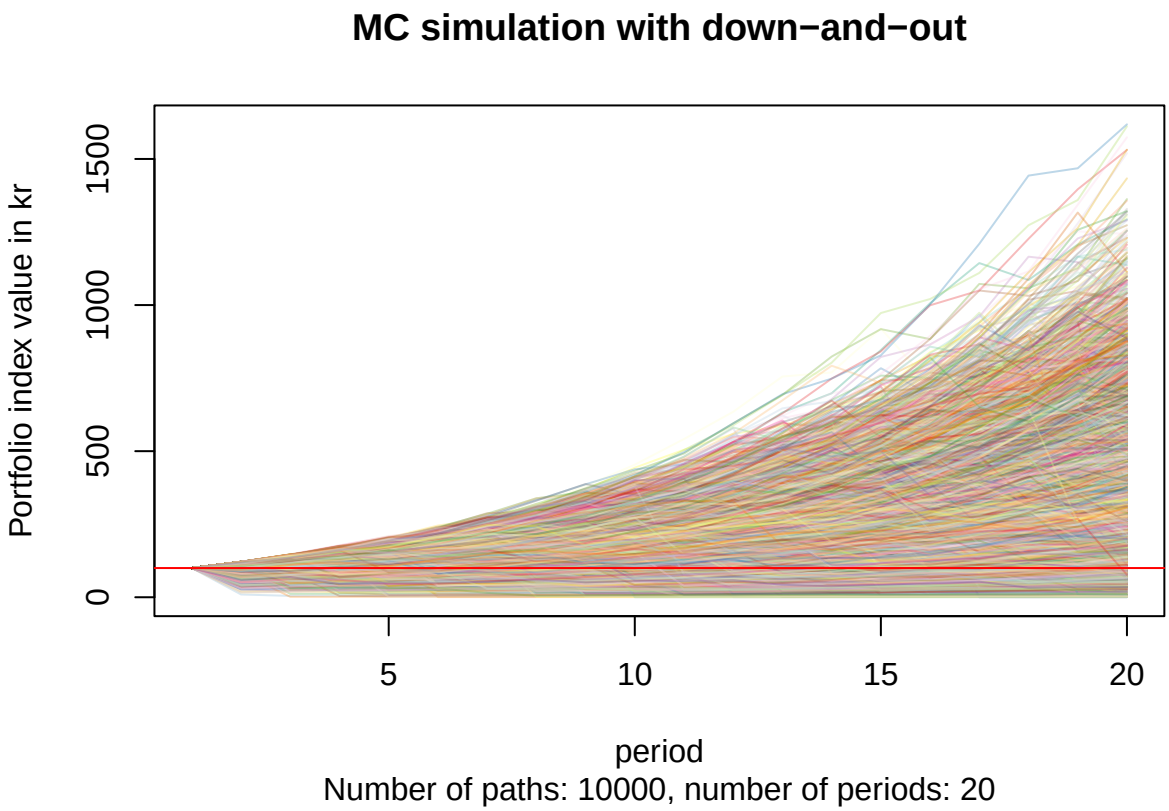
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

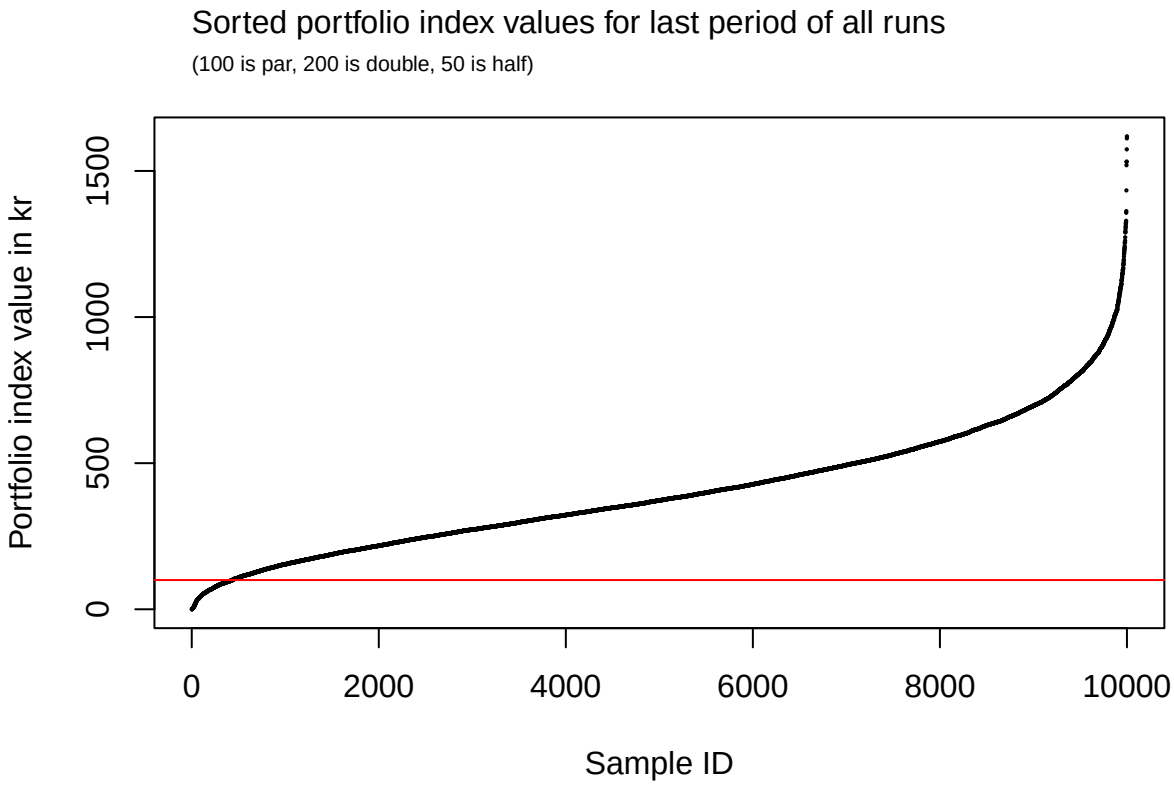


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 405.01 kr.
## SD of portfolio index value after 20 years: 217.541 kr.
## Min total portfolio index value after 20 years: 0.133 kr.
## Max total portfolio index value after 20 years: 1618.466 kr.
##
## Share of paths finishing below 100: 4.34 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

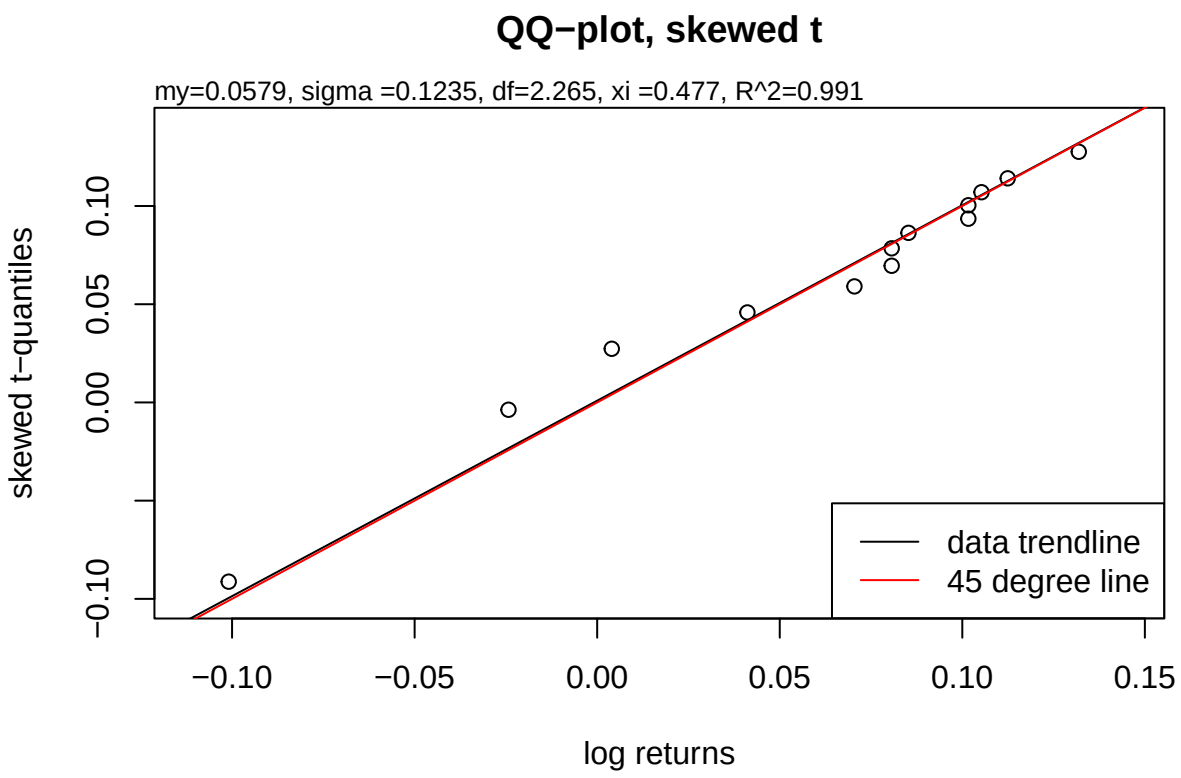


PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -33.22998
## BIC: -30.97018
## m: 0.05789224
## s: 0.1234592
## nu (df): 2.265273
## xi: 0.477324
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 3.3 percent
## What is the risk of losing max 25 %? =< 0.9 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 32.7 percent
## What is the chance of gaining min 25 %? >= 0.1 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

QQ Plot

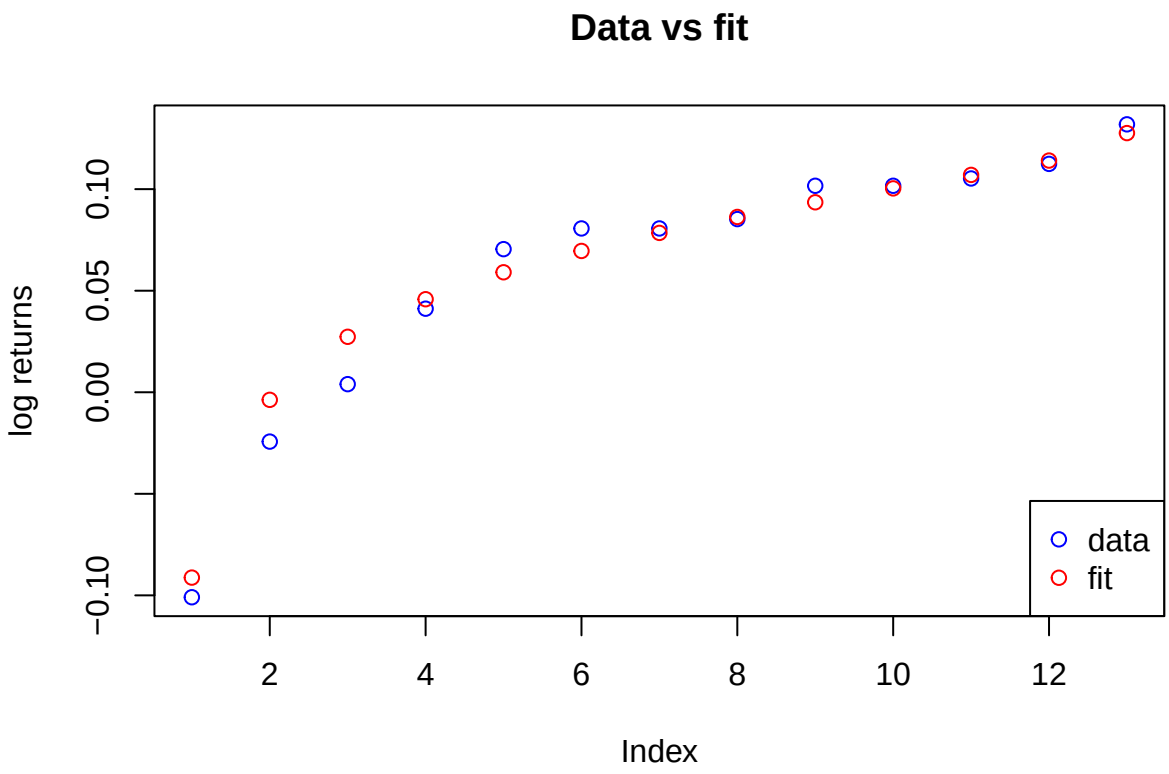


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
##  [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
##  [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11]  0.107018607  0.114081432  0.127604387
```

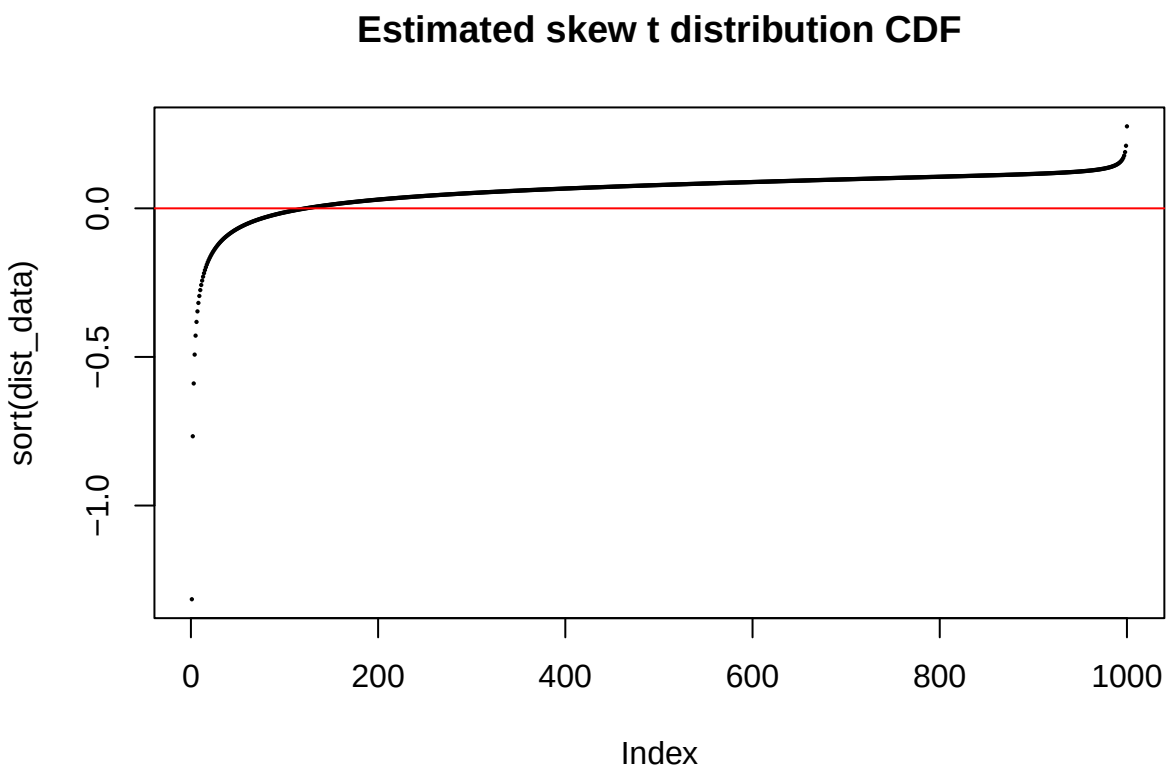
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

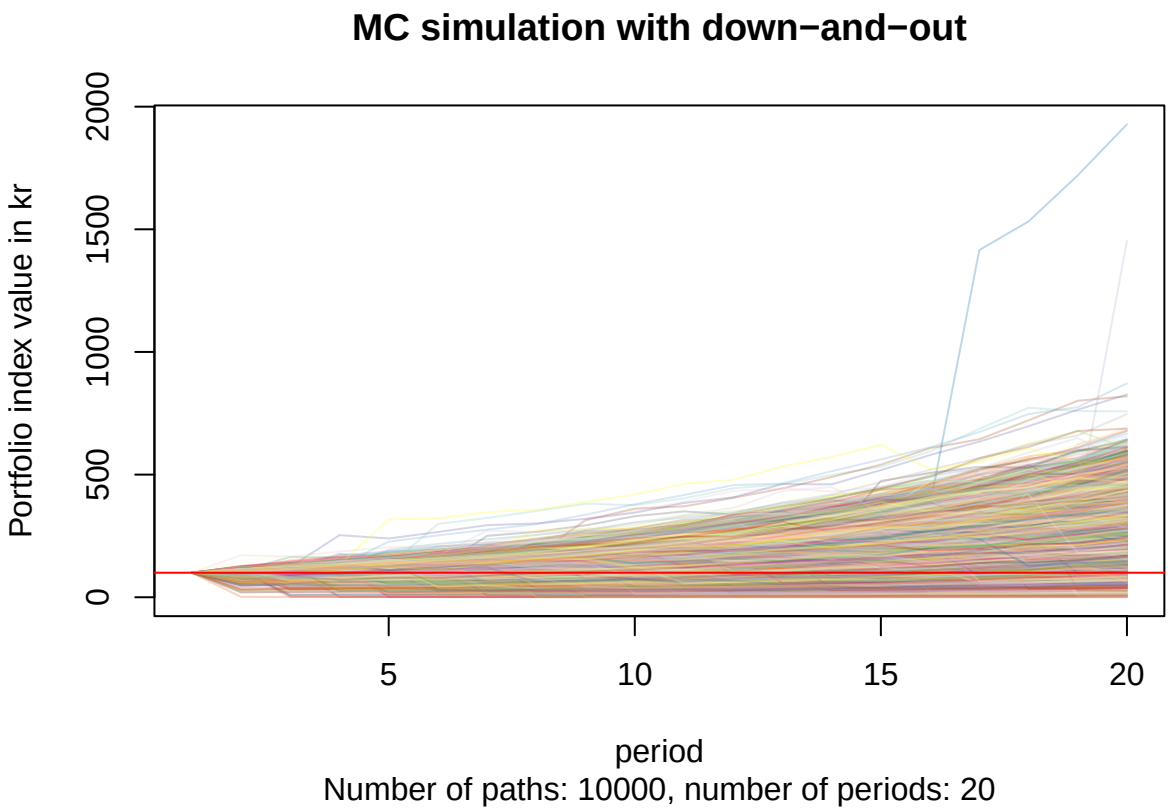
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

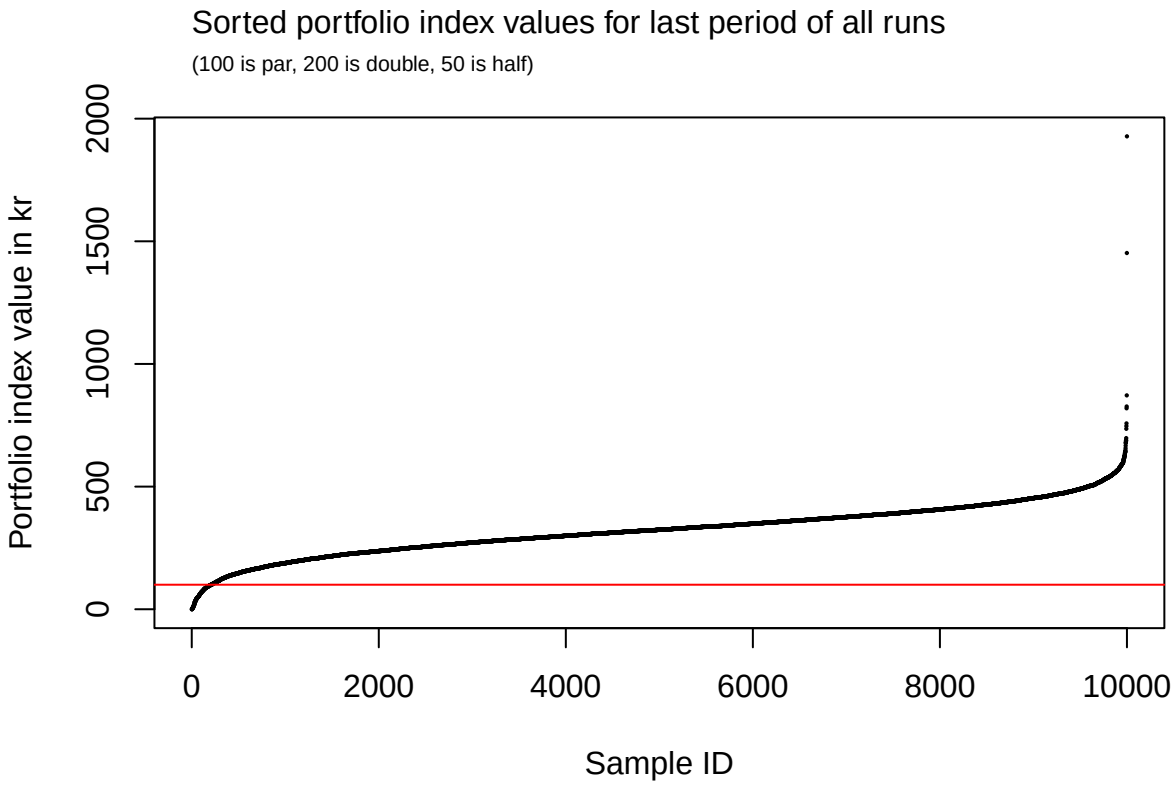


We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0.01 percent
##
## Mean portfolio index value after 20 years: 322.656 kr.
## SD of portfolio index value after 20 years: 105.871 kr.
## Min total portfolio index value after 20 years: 0 kr.
## Max total portfolio index value after 20 years: 1927.894 kr.
##
## Share of paths finishing below 100: 2.09 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
2000
5
10
15
20
period
Number of paths: 10000, number of periods: 20

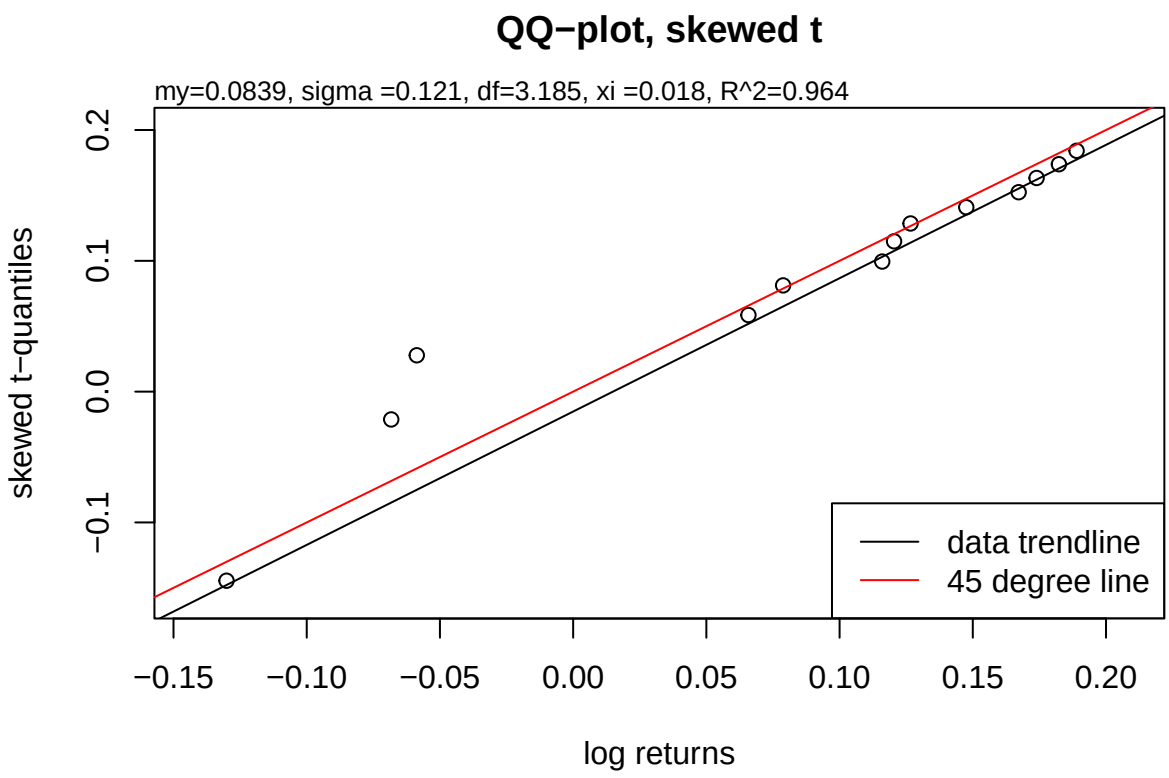


PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -23.72565
## BIC: -21.46585
## m: 0.08386034
## s: 0.1210107
## nu (df): 3.184569
## xi: 0.01790306
## R^2: 0.964
##
## An R^2 of 0.964 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 5.3 percent
## What is the risk of losing max 25 %? =< 1.4 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 59.6 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

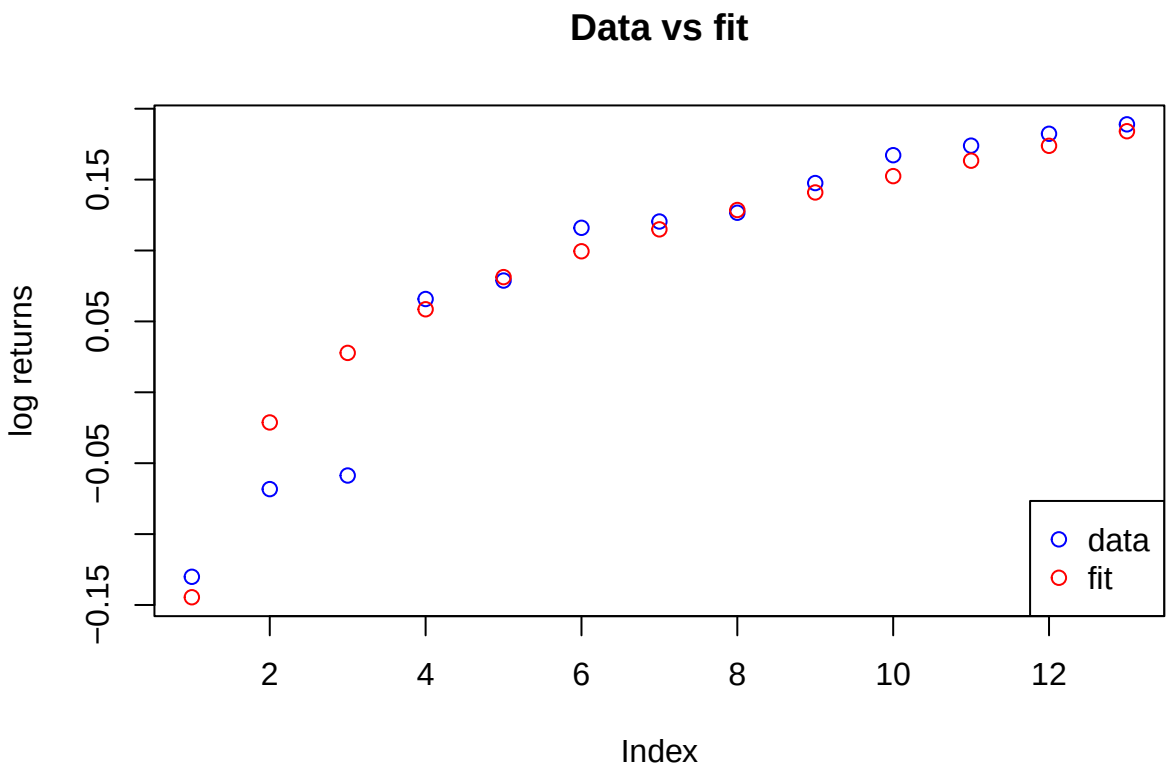
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

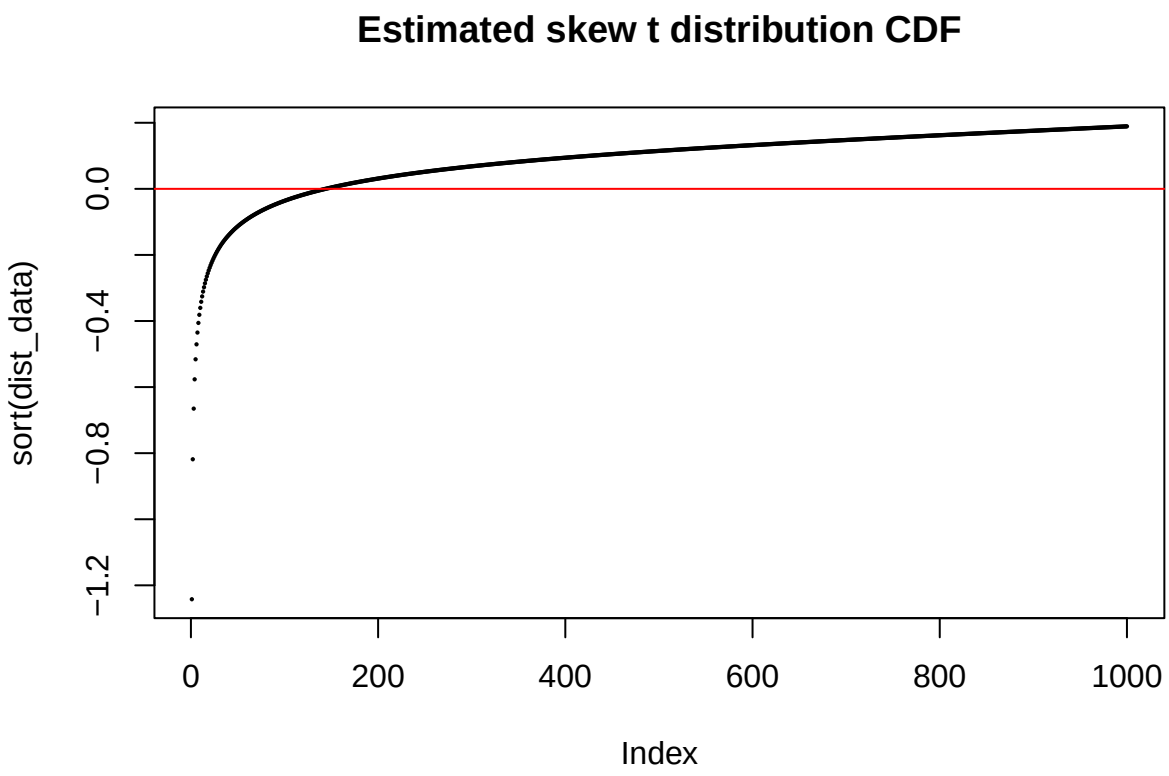
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

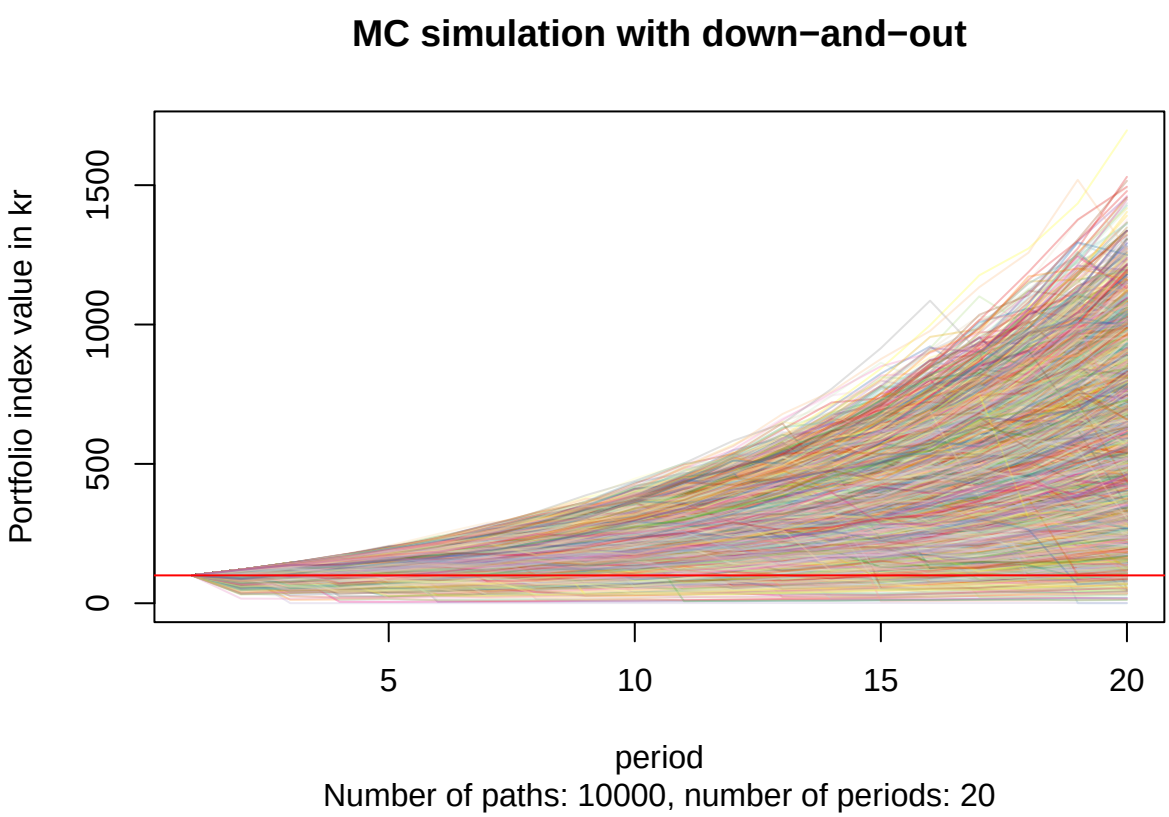
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

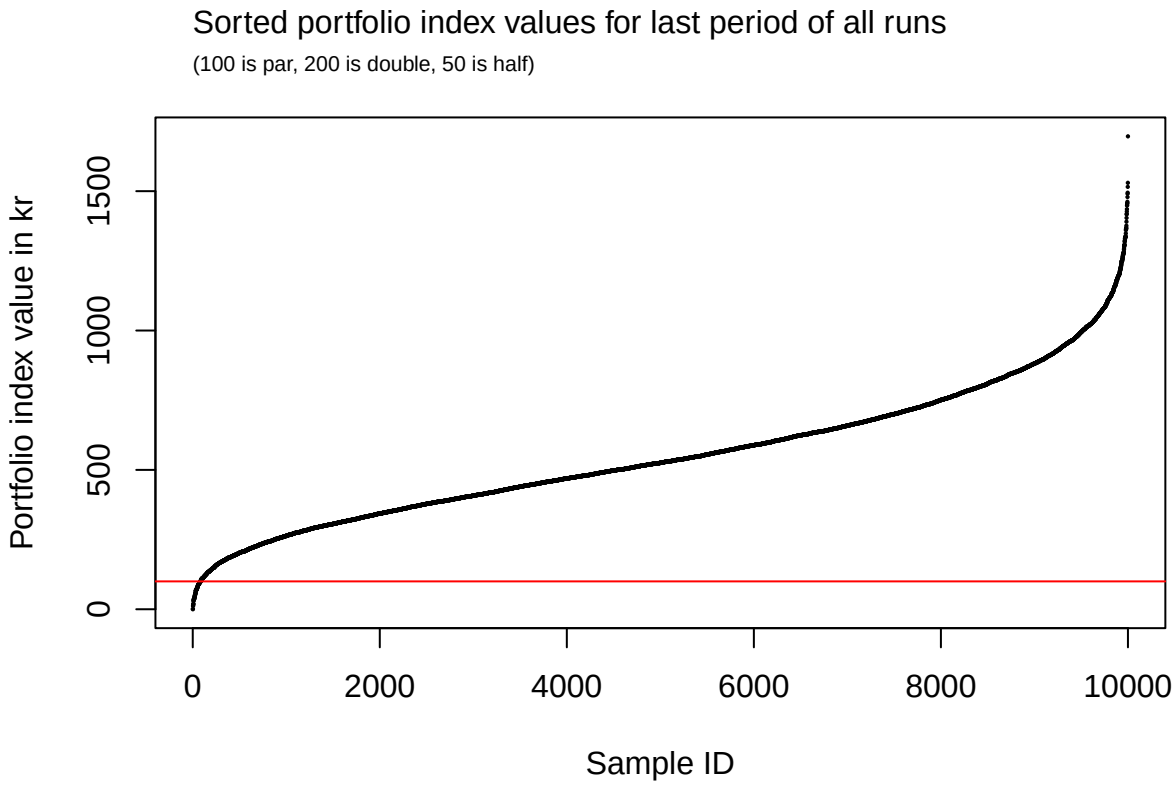


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 552.598 kr.
## SD of portfolio index value after 20 years: 240.837 kr.
## Min total portfolio index value after 20 years: 0.061 kr.
## Max total portfolio index value after 20 years: 1696.696 kr.
##
## Share of paths finishing below 100: 0.83 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

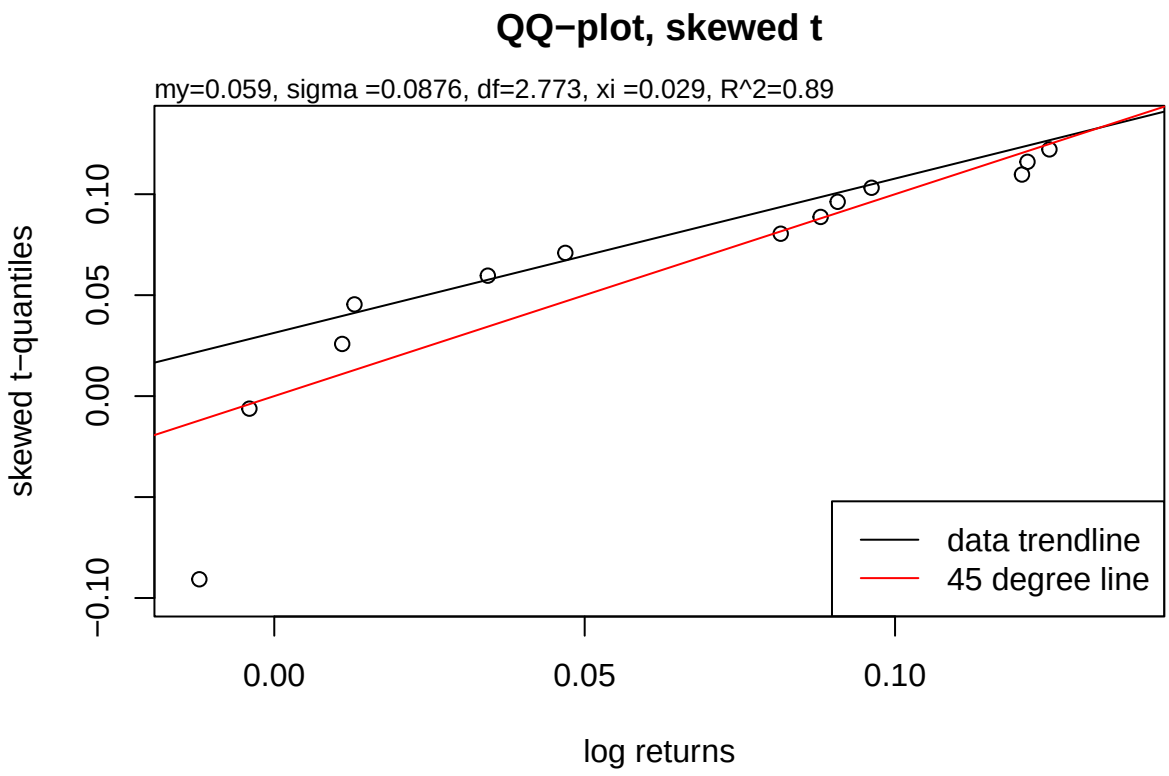


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -36.9603
## BIC: -34.7005
## m: 0.05902873
## s: 0.08757749
## nu (df): 2.772621
## xi: 0.02904471
## R^2: 0.89
##
## An R^2 of 0.89 suggests that the fit is not completely random.
##
## What is the risk of losing max 10 %? =< 3.3 percent
## What is the risk of losing max 25 %? =< 0.7 percent
## What is the risk of losing max 50 %? =< 0.1 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 35.6 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

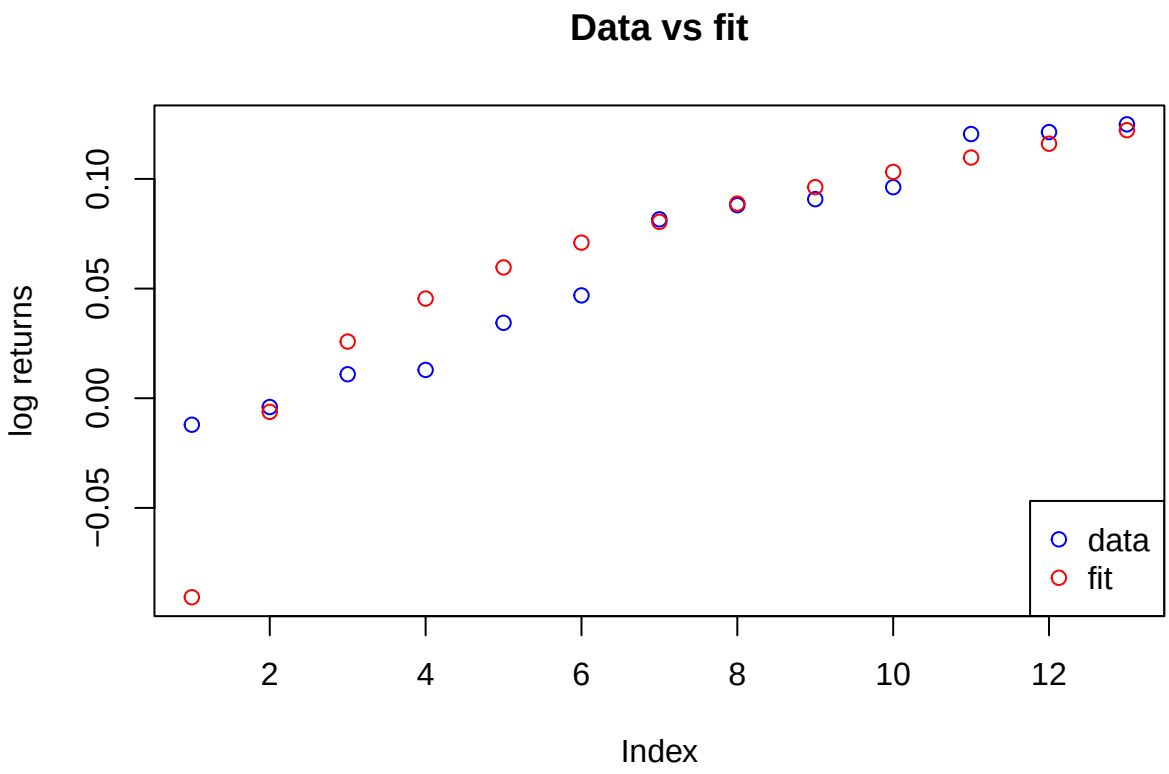
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

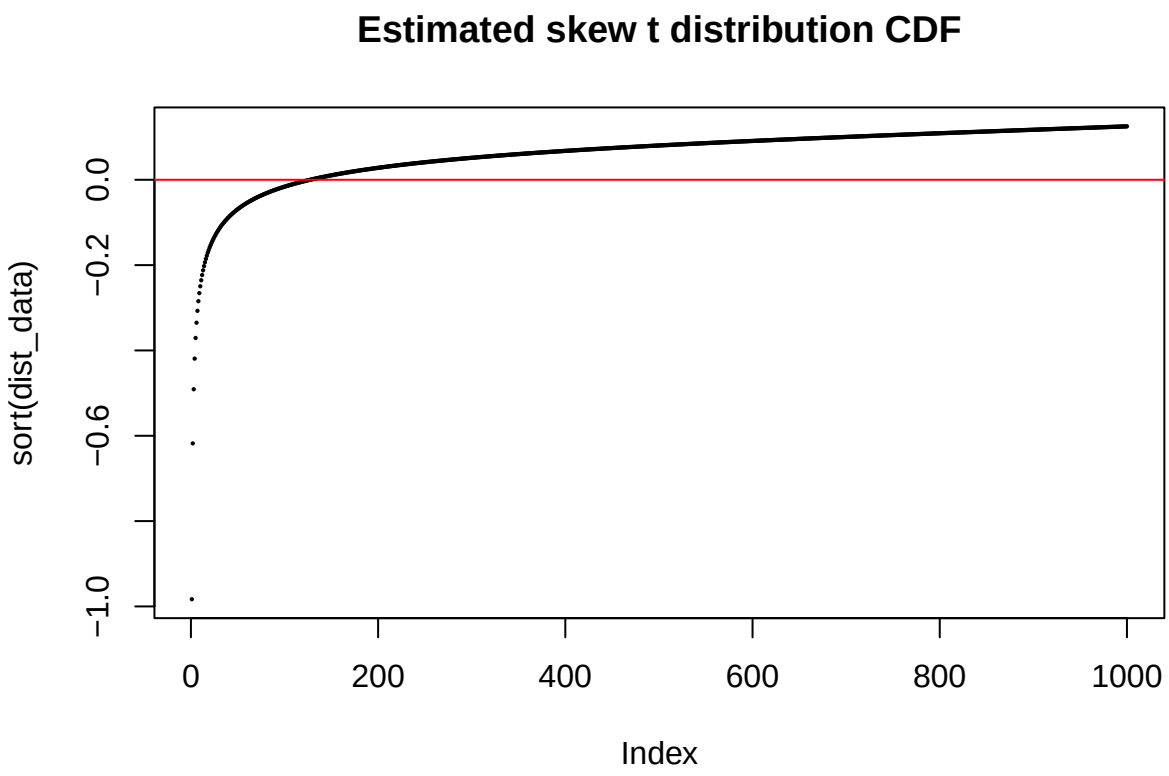
Let's plot the fit and the observed returns together.



Interestingly, the fit predicts a much bigger “biggest loss” than the actual data. This is the main reason that R^2 is 0.90 and not higher.

Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

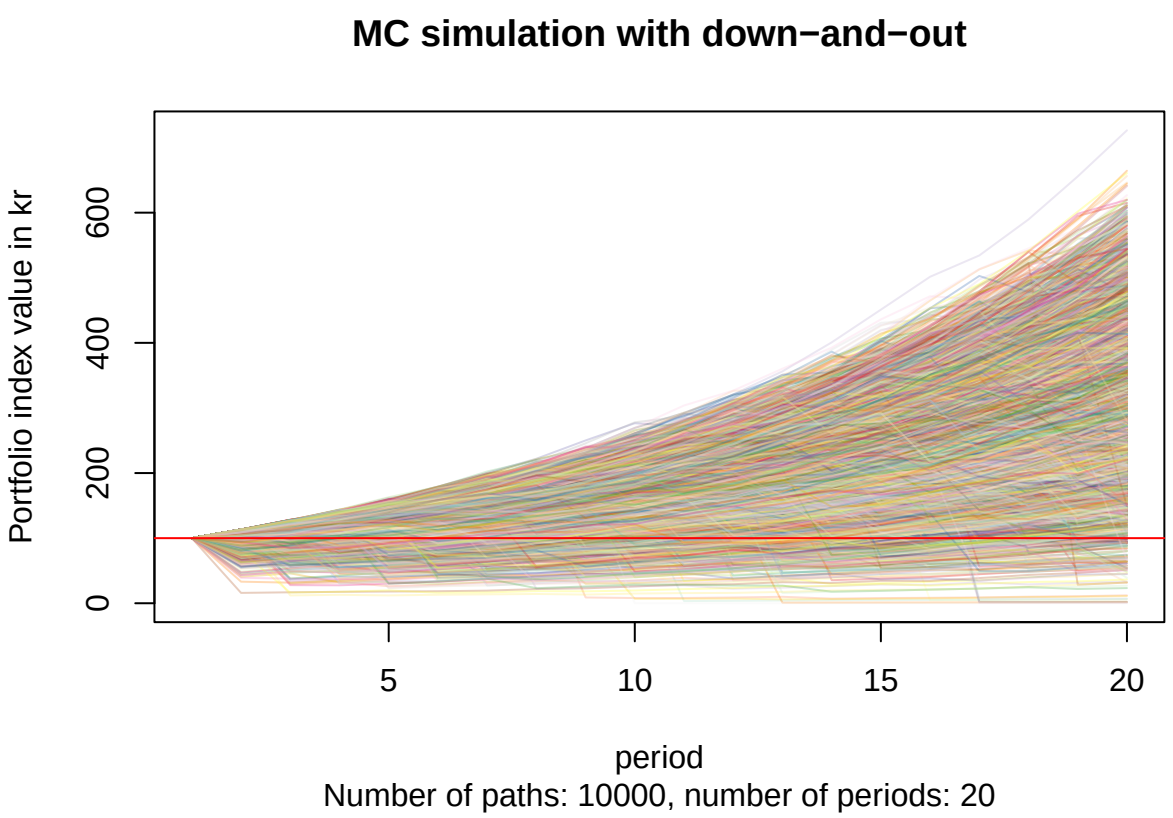


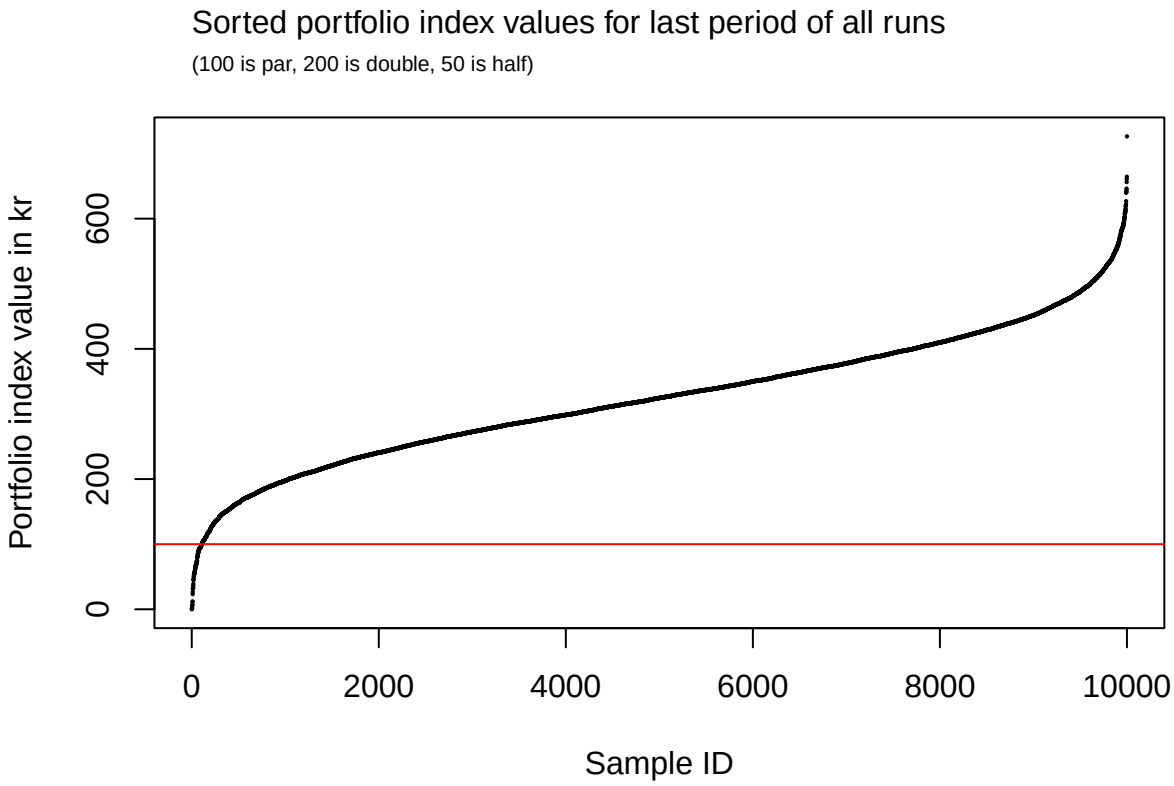
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

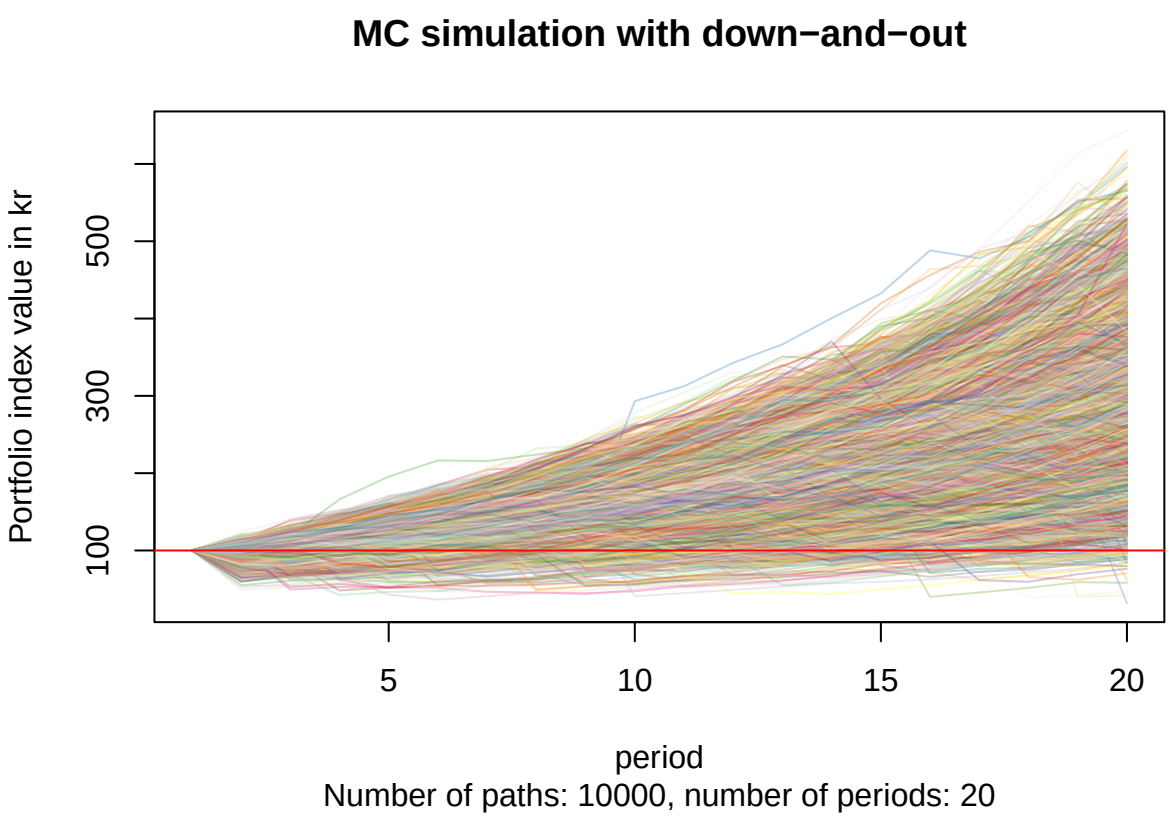
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 325.175 kr.
## SD of portfolio index value after 20 years: 99.272 kr.
## Min total portfolio index value after 20 years: 0.05 kr.
## Max total portfolio index value after 20 years: 726.437 kr.
##
## Share of paths finishing below 100: 1.08 percent
```

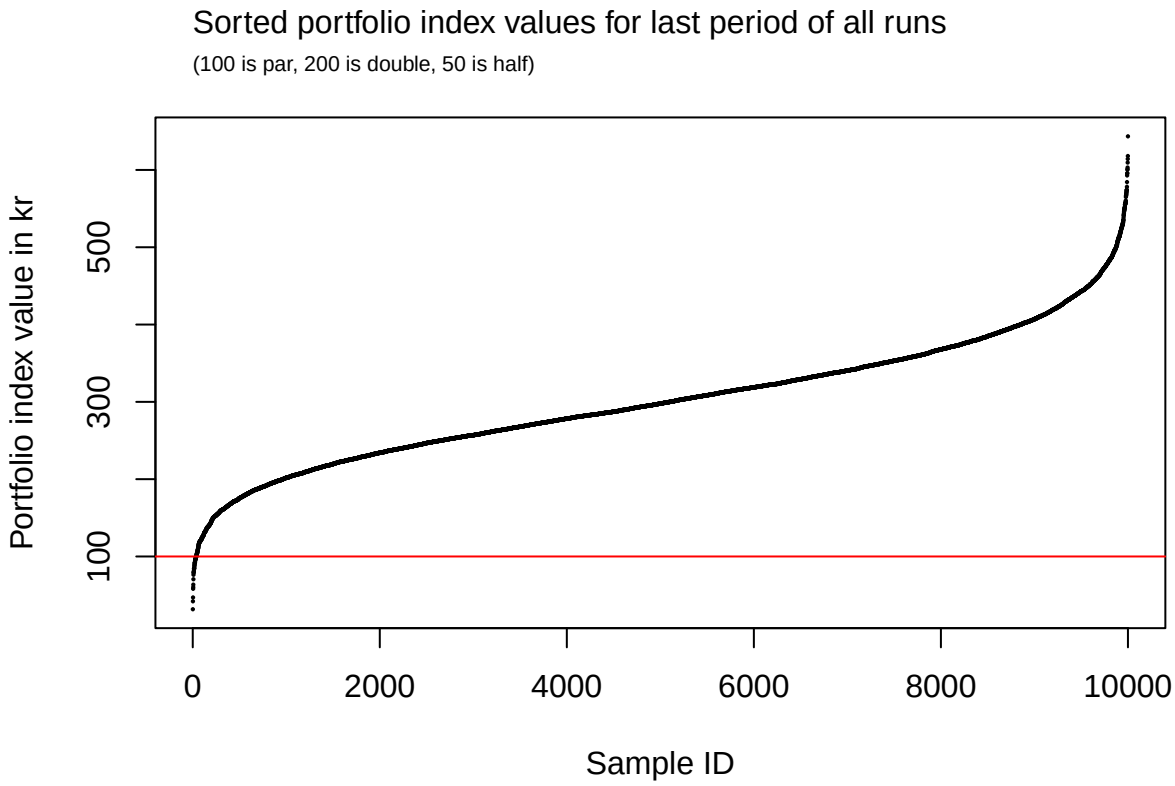

MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Version b: Mix of simulations from estimated distribution of returns from individual funds.

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 301.907 kr.
## SD of portfolio index value after 20 years: 81.167 kr.
## Min total portfolio index value after 20 years: 31.759 kr.
## Max total portfolio index value after 20 years: 643.45 kr.
##
## Share of paths finishing below 100: 0.35 percent
```


MC simulation with down−and−out
Portfolio index value in kr
100
300
500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

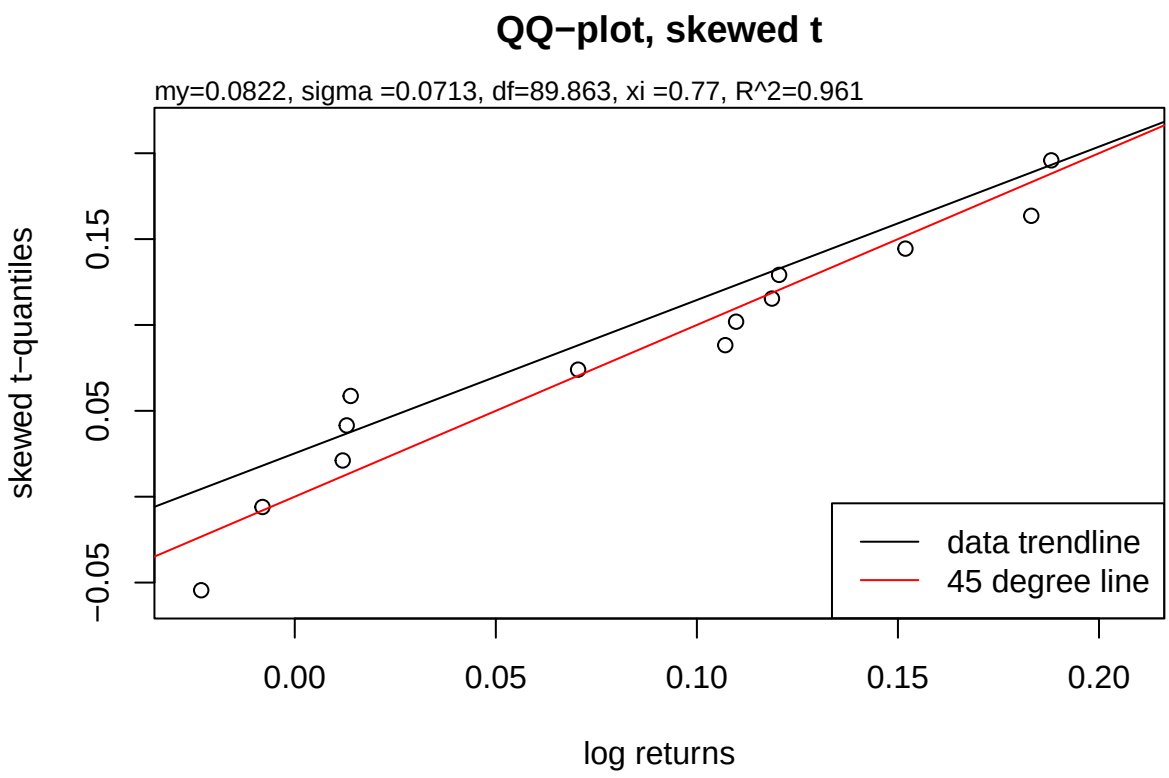


Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -24.26084
## BIC: -22.00104
## m: 0.0822419
## s: 0.07129843
## nu (df): 89.86289
## xi: 0.7697502
## R^2: 0.961
##
## An R^2 of 0.961 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 0.9 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 46.1 percent
## What is the chance of gaining min 25 %? >= 1.2 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

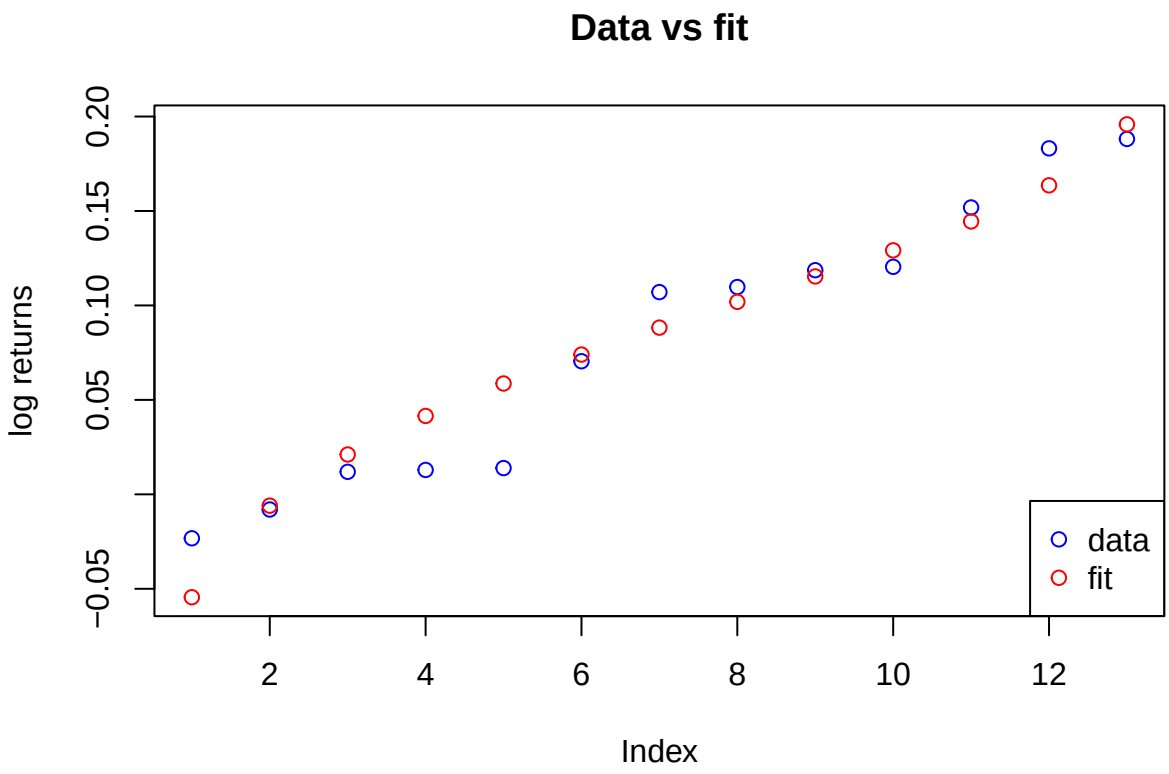
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

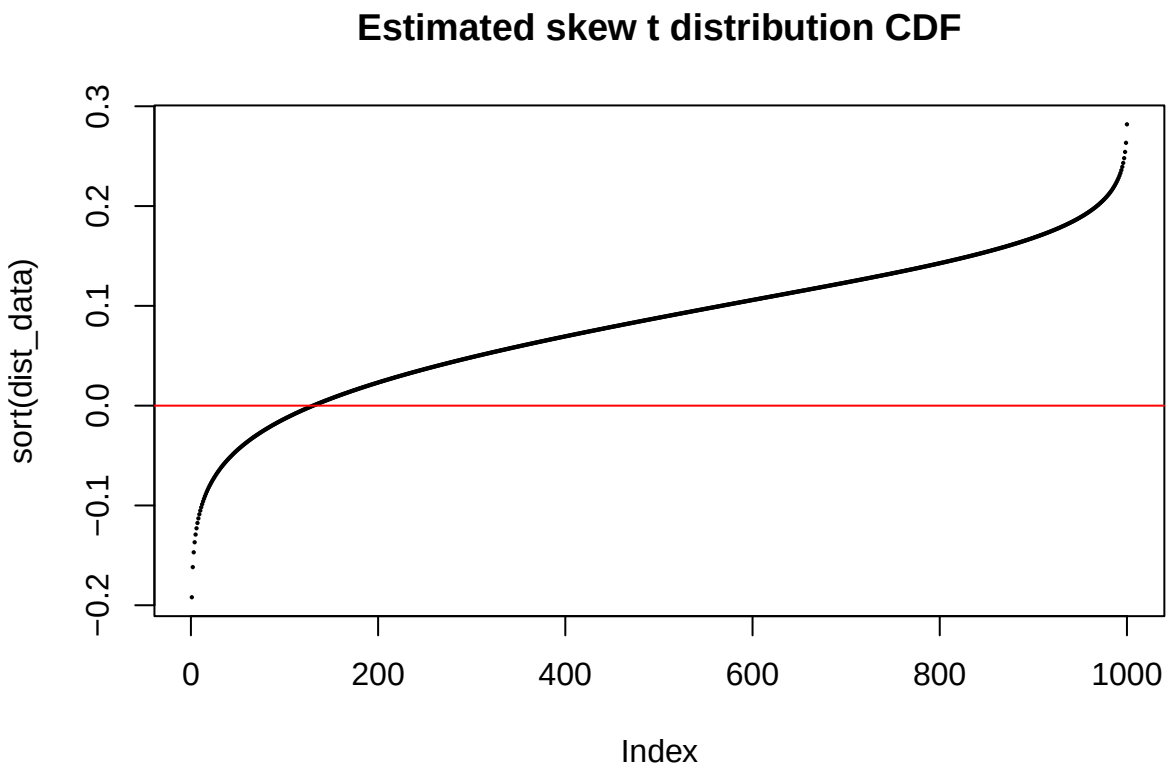
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

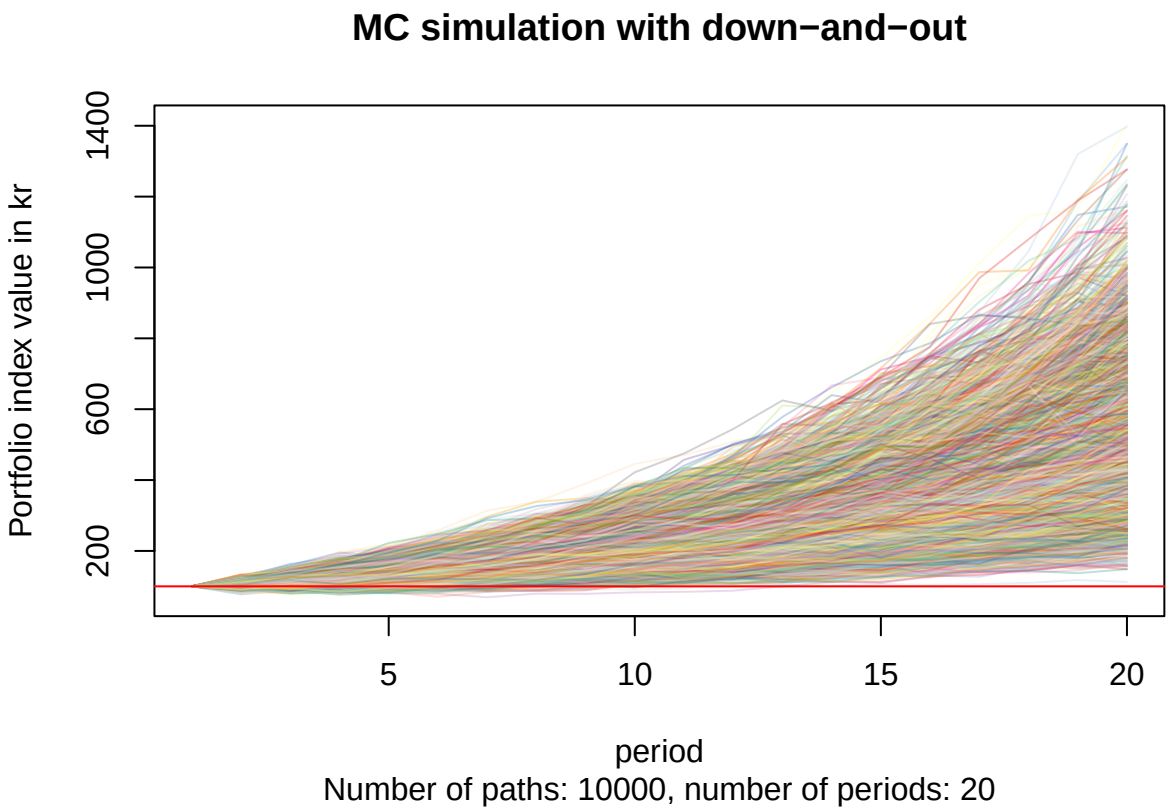


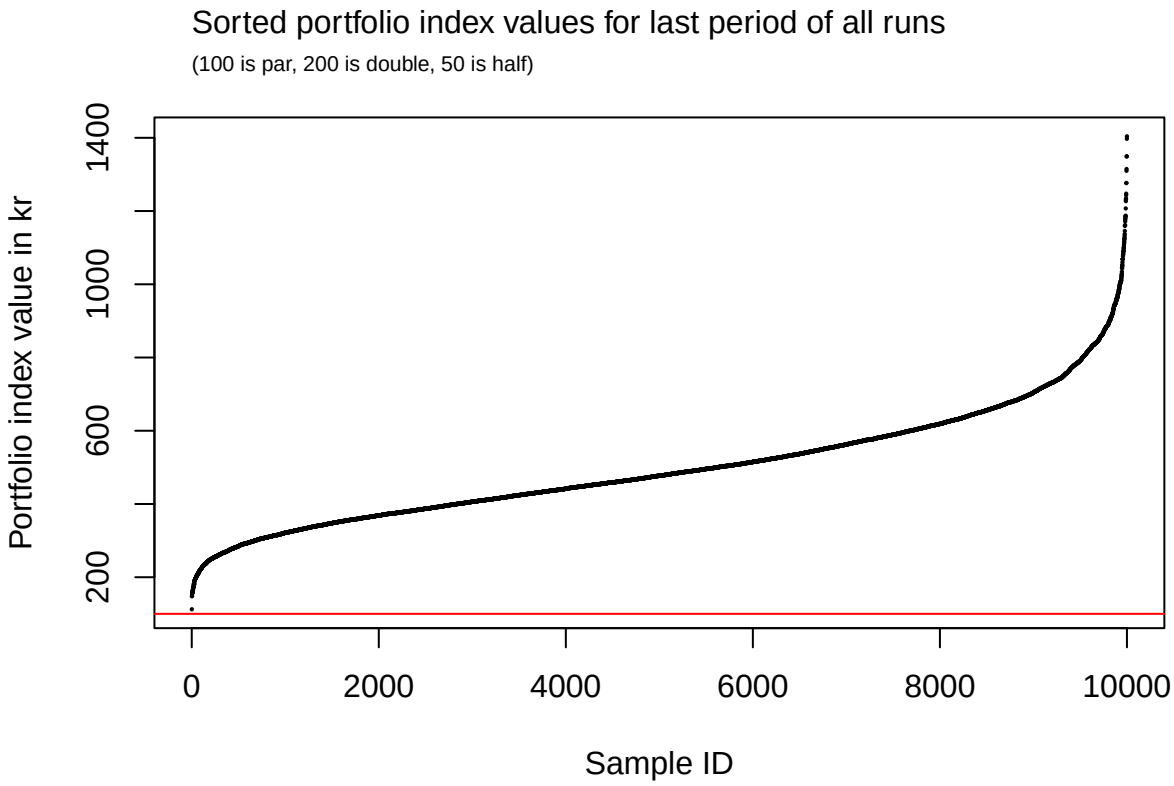
We see that the high risk mix provides a much better upside and smaller downside.

Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

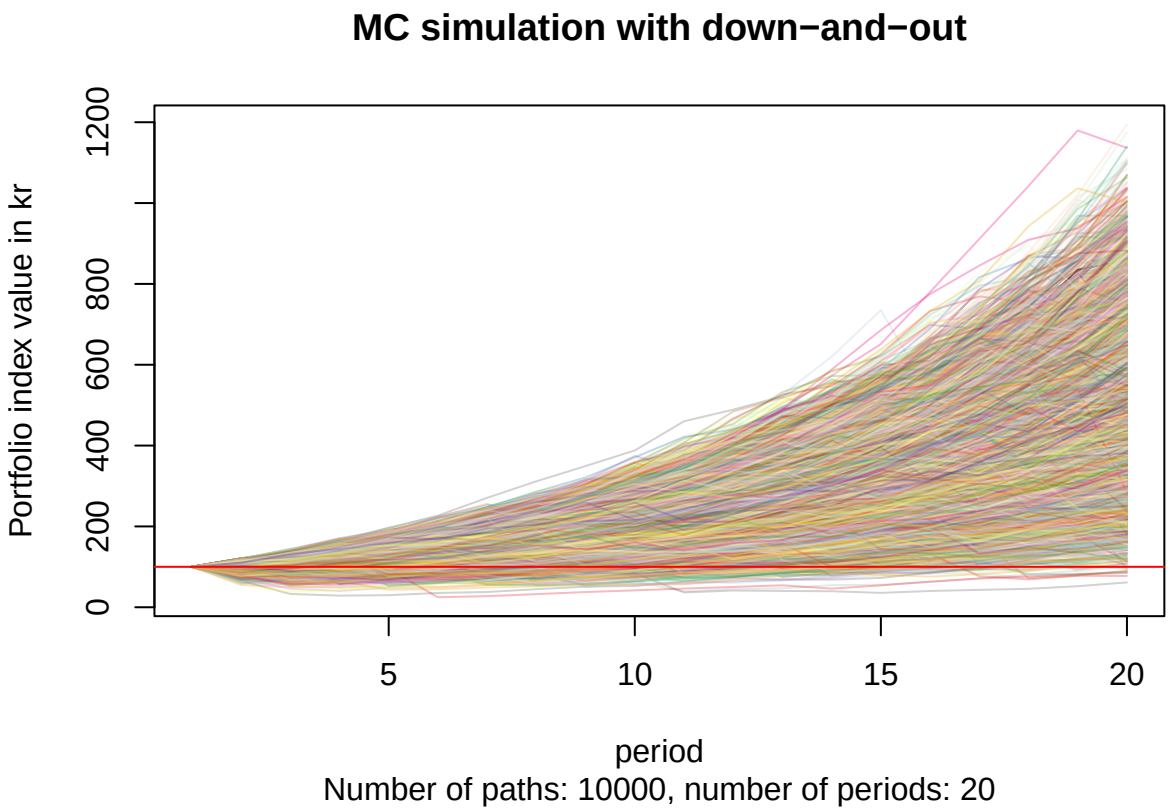
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 499.741 kr.
## SD of portfolio index value after 20 years: 157.751 kr.
## Min total portfolio index value after 20 years: 112.574 kr.
## Max total portfolio index value after 20 years: 1403.982 kr.
##
## Share of paths finishing below 100: 0 percent
```

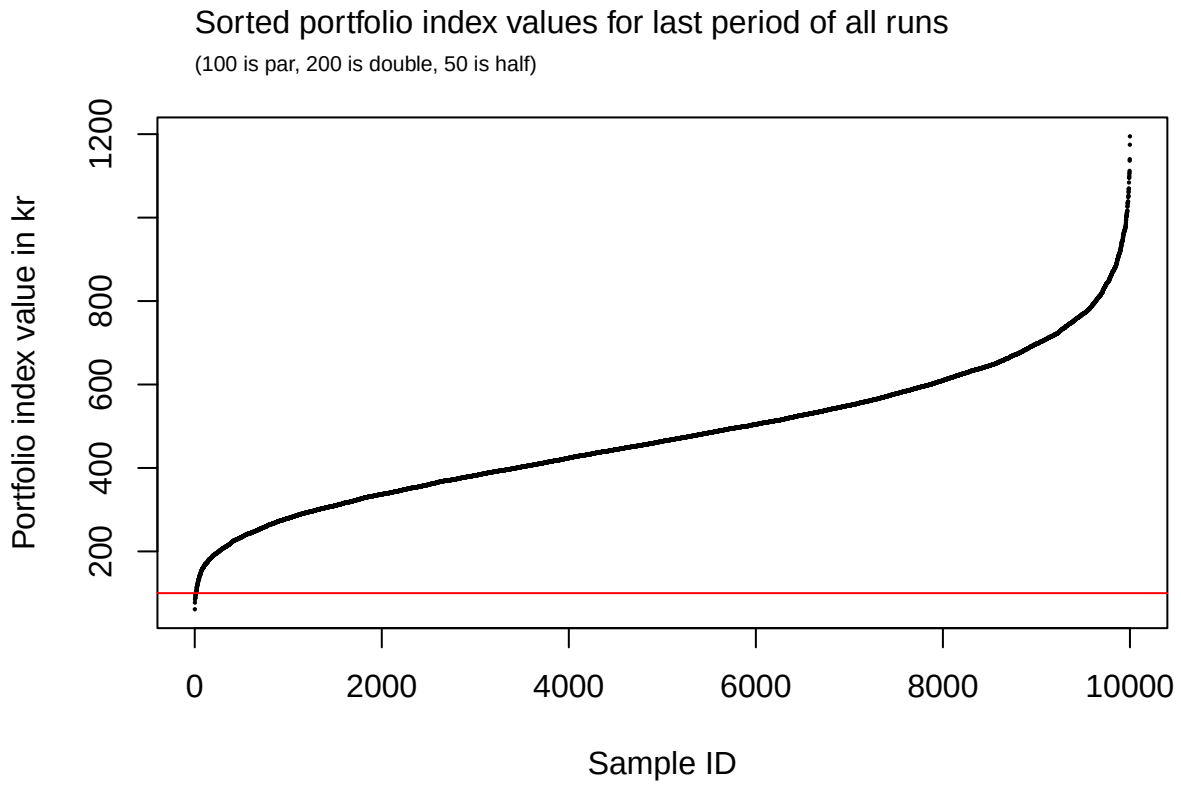

MC simulation with down−and−out
Portfolio index value in kr
200
600
1000
1400
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Version b: Mix of simulations from estimated distribution of returns from individual funds.

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 477.534 kr.
## SD of portfolio index value after 20 years: 163.648 kr.
## Min total portfolio index value after 20 years: 61.42 kr.
## Max total portfolio index value after 20 years: 1194.745 kr.
##
## Share of paths finishing below 100: 0.12 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
1200
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Compare pension plans

Risk of max loss

Risk of max loss of x percent for a single period (year).
x values are row names.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
0	21.3	18.2	19.9	12.2	14.3	12.7	13.0
5	12.5	9.6	12.8	6.0	8.6	6.2	4.2
10	7.4	5.4	8.3	3.3	5.3	3.3	0.9
25	1.8	1.3	2.5	0.9	1.4	0.7	0.0
50	0.2	0.2	0.4	0.2	0.2	0.1	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.3	Velliv_medium	12.8	Velliv_high	8.3	Velliv_high	2.5	Velliv_high	0.4	Velliv_high	0	Velliv_medium	0	Velliv_medium
19.9	Velliv_high	12.5	Velliv_medium	7.4	Velliv_medium	1.8	Velliv_medium	0.2	Velliv_medium	0	Velliv_medium_long	0	Velliv_medium_long
18.2	Velliv_medium_long	9.6	Velliv_medium_long	5.4	Velliv_medium_long	1.4	PFA_high	0.2	Velliv_medium_long	0	Velliv_high	0	Velliv_high
14.3	PFA_high	8.6	PFA_high	5.3	PFA_high	1.3	Velliv_medium_long	0.2	PFA_medium	0	PFA_medium	0	PFA_medium
13.0	mix_high	6.2	mix_medium	3.3	PFA_medium	0.9	PFA_medium	0.2	PFA_high	0	PFA_high	0	PFA_high
12.7	mix_medium	6.0	PFA_medium	3.3	mix_medium	0.7	mix_medium	0.1	mix_medium	0	mix_medium	0	mix_medium
12.2	PFA_medium	4.2	mix_high	0.9	mix_high	0.0	mix_high	0.0	mix_high	0	mix_high	0	mix_high

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
0	78.7	81.8	80.1	87.8	85.7	87.3	87.0
5	63.8	64.9	69.2	71.5	75.8	71.4	69.9
10	41.0	36.2	53.3	32.7	59.6	35.6	46.1
25	0.0	0.3	0.0	0.1	0.0	0.0	1.2
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
87.8	PFA_medium	75.8	PFA_high	59.6	PFA_high	1.2	mix_high	0	Velliv_medium	0	Velliv_medium
87.3	mix_medium	71.5	PFA_medium	53.3	Velliv_high	0.3	Velliv_medium_long	0	Velliv_medium_long	0	Velliv_medium_long
87.0	mix_high	71.4	mix_medium	46.1	mix_high	0.1	PFA_medium	0	Velliv_high	0	Velliv_high
85.7	PFA_high	69.9	mix_high	41.0	Velliv_medium	0.0	Velliv_medium	0	PFA_medium	0	PFA_medium
81.8	Velliv_medium_long	69.2	Velliv_high	36.2	Velliv_medium_long	0.0	Velliv_high	0	PFA_high	0	PFA_high
80.1	Velliv_high	64.9	Velliv_medium_long	35.6	mix_medium	0.0	PFA_high	0	mix_medium	0	mix_medium
78.7	Velliv_medium	63.8	Velliv_medium	32.7	PFA_medium	0.0	mix_medium	0	mix_high	0	mix_high

MC risk percentiles

Risk of loss from first to last period.

`_a` is simulation from estimated distribution of returns of mix.
`_b` is mix of simulations from estimated distribution of returns from individual funds.

`_m` is medium.
`_h` is high.

	Velliv_m	Velliv_m_long	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	4.94	3.37	4.34	2.09	0.83	1.08	0	0.35	0.12
5	4.33	2.97	3.90	1.84	0.72	0.94	0	0.25	0.08
10	3.69	2.56	3.51	1.61	0.64	0.74	0	0.19	0.06
25	2.25	1.68	2.43	1.17	0.44	0.57	0	0.07	0.01
50	0.83	0.67	1.11	0.65	0.24	0.21	0	0.03	0.00
90	0.05	0.07	0.25	0.16	0.02	0.07	0	0.00	0.00
99	0.01	0.03	0.05	0.04	0.01	0.02	0	0.00	0.00

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.94	Velliv_m	4.33	Velliv_m	3.69	Velliv_m	2.43	Velliv_h	1.11	Velliv_h	0.25	Velliv_h	0.05	Velliv_h
4.34	Velliv_h	3.90	Velliv_h	3.51	Velliv_h	2.25	Velliv_m	0.83	Velliv_m	0.16	PFA_m	0.04	PFA_m
3.37	Velliv_m_long	2.97	Velliv_m_long	2.56	Velliv_m_long	1.68	Velliv_m_long	0.67	Velliv_m_long	0.07	Velliv_m_long	0.03	Velliv_m_long
2.09	PFA_m	1.84	PFA_m	1.61	PFA_m	1.17	PFA_m	0.65	PFA_m	0.07	mix_m_a	0.02	mix_m_a
1.08	mix_m_a	0.94	mix_m_a	0.74	mix_m_a	0.57	mix_m_a	0.24	PFA_h	0.05	Velliv_m	0.01	Velliv_m
0.83	PFA_h	0.72	PFA_h	0.64	PFA_h	0.44	PFA_h	0.21	mix_m_a	0.02	PFA_h	0.01	PFA_h
0.35	mix_m_b	0.25	mix_m_b	0.19	mix_m_b	0.07	mix_m_b	0.03	mix_m_b	0.00	mix_h_a	0.00	mix_h_a
0.12	mix_h_b	0.08	mix_h_b	0.06	mix_h_b	0.01	mix_h_b	0.00	mix_h_a	0.00	mix_m_b	0.00	mix_m_b
0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_b	0.00	mix_h_b	0.00	mix_h_b

MC gains percentiles

Chance of gains from first to last period.
_a is simulation from estimated distribution of returns of mix.
_b is mix of simulations from estimated distribution of returns from individual funds.

	Velliv_m	Velliv_m_long	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	95.06	96.63	95.66	97.91	99.17	98.92	100.00	99.65	99.88
5	94.41	96.26	95.34	97.68	99.11	98.77	100.00	99.54	99.83
10	93.61	95.76	94.91	97.46	99.03	98.56	100.00	99.44	99.80
25	90.86	94.05	93.32	96.73	98.59	97.95	99.99	98.97	99.67
50	85.71	90.21	90.60	94.81	97.69	96.35	99.97	97.81	99.34
100	71.49	78.02	83.11	88.10	95.17	89.62	99.54	90.35	97.42
200	39.74	44.26	64.55	59.72	85.87	59.35	93.25	48.99	86.83
300	16.91	17.63	44.89	22.08	71.27	22.85	71.61	11.44	65.59
400	5.38	4.92	29.06	4.20	54.62	3.92	43.79	1.28	41.03
500	1.29	1.13	17.39	0.41	38.15	0.28	23.17	0.07	21.32
1000	0.00	0.02	0.68	0.02	2.23	0.00	0.34	0.00	0.07

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking	200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	99.99	mix_h_a	99.97	mix_h_a	99.54	mix_h_a	93.25	mix_h_a	71.61	mix_h_a	54.62	PFA_h	38.15	PFA_h	2.23	PFA_h
99.88	mix_h_b	99.83	mix_h_b	99.80	mix_h_b	99.67	mix_h_b	99.34	mix_h_b	97.42	mix_h_b	86.83	mix_h_b	71.27	PFA_h	43.79	mix_h_a	23.17	mix_h_a	0.68	Velliv_h
99.65	mix_m_b	99.54	mix_m_b	99.44	mix_m_b	98.97	mix_m_b	97.81	mix_m_b	95.17	PFA_h	85.87	PFA_h	65.59	mix_h_b	41.03	mix_h_b	21.32	mix_h_b	0.34	mix_h_a
99.17	PFA_h	99.11	PFA_h	99.03	PFA_h	98.59	PFA_h	97.69	PFA_h	90.35	mix_m_b	64.55	Velliv_h	44.89	Velliv_h	29.06	Velliv_h	17.39	Velliv_h	0.07	mix_h_b
98.92	mix_m_a	98.77	mix_m_a	98.56	mix_m_a	97.95	mix_m_a	96.35	mix_m_a	89.62	mix_m_a	59.72	PFA_m	22.85	mix_m_a	5.38	Velliv_m	1.29	Velliv_m	0.02	Velliv_m_long
97.91	PFA_m	97.68	PFA_m	97.46	PFA_m	96.73	PFA_m	94.81	PFA_m	88.10	PFA_m	59.35	mix_m_a	22.08	PFA_m	4.92	Velliv_m_long	1.13	Velliv_m_long	0.02	PFA_m
96.63	Velliv_m_long	96.26	Velliv_m_long	95.76	Velliv_m_long	94.05	Velliv_m_long	91.60	Velliv_h	83.11	Velliv_h	48.99	mix_m_b	17.63	Velliv_m_long	4.20	PFA_m	0.41	PFA_m	0.00	Velliv_m
95.66	Velliv_h	95.34	Velliv_h	94.91	Velliv_h	93.32	Velliv_h	90.21	Velliv_m_long	78.02	Velliv_m_long	44.26	Velliv_m_long	16.91	Velliv_m	3.92	mix_m_a	0.28	mix_m_a	0.00	mix_m_a
95.06	Velliv_m	94.41	Velliv_m	93.61	Velliv_m	90.86	Velliv_m	85.71	Velliv_m	71.49	Velliv_m	39.74	Velliv_m	11.44	mix_m_b	1.28	mix_m_b	0.07	mix_m_b	0.00	mix_m_b

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated degrees of freedom, or shape parameter.
`xi` is the estimated skewness parameter.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770
R-squared	0.993	0.978	0.991	0.991	0.964	0.890	0.961

Fit statistics ranking

m	ranking	s	ranking	R-squared	ranking
0.084	PFA_high	0.071	mix_high	0.993	Velliv_medium
0.082	mix_high	0.088	mix_medium	0.991	Velliv_high
0.065	Velliv_high	0.115	Velliv_medium_long	0.991	PFA_medium
0.059	mix_medium	0.120	Velliv_medium	0.978	Velliv_medium_long
0.058	PFA_medium	0.121	PFA_high	0.964	PFA_high
0.052	Velliv_medium_long	0.123	PFA_medium	0.961	mix_high
0.048	Velliv_medium	0.150	Velliv_high	0.890	mix_medium

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).
Statistics are given for the final state of all paths.
Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.
0 is defined as any value below a threshold.
`losing_prob_pct` is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_m_b	mix_h_a	mix_h_b
mc_m	280.450	293.509	405.010	322.656	552.598	325.175	301.907	499.741	477.534
mc_s	124.455	118.513	217.541	105.871	240.837	99.272	81.167	157.751	163.648
mc_min	0.945	0.000	0.133	0.000	0.061	0.050	31.759	112.574	61.420
mc_max	984.799	1656.391	1618.466	1927.894	1696.696	726.437	643.450	1403.982	1194.745
dao_pct	0.000	0.010	0.000	0.010	0.000	0.000	0.000	0.000	0.000
losing_pct	4.940	3.370	4.340	2.090	0.830	1.080	0.350	0.000	0.120

Ranking

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	losing_pct	ranking
552.598	PFA_h	81.167	mix_m_b	112.574	mix_h_a	1927.894	PFA_m	0.00	Velliv_m	0.00	mix_h_a
499.741	mix_h_a	99.272	mix_m_a	61.420	mix_h_b	1696.696	PFA_h	0.00	Velliv_h	0.12	mix_h_b
477.534	mix_h_b	105.871	PFA_m	31.759	mix_m_b	1656.391	Velliv_m_l	0.00	PFA_h	0.35	mix_m_b
405.010	Velliv_h	118.513	Velliv_m_l	0.945	Velliv_m	1618.466	Velliv_h	0.00	mix_m_a	0.83	PFA_h
325.175	mix_m_a	124.455	Velliv_m	0.133	Velliv_h	1403.982	mix_h_a	0.00	mix_m_b	1.08	mix_m_a
322.656	PFA_m	157.751	mix_h_a	0.061	PFA_h	1194.745	mix_h_b	0.00	mix_h_a	2.09	PFA_m
301.907	mix_m_b	163.648	mix_h_b	0.050	mix_m_a	984.799	Velliv_m	0.00	mix_h_b	3.37	Velliv_m_l
293.509	Velliv_m_l	217.541	Velliv_h	0.000	Velliv_m_l	726.437	mix_m_a	0.01	Velliv_m_l	4.34	Velliv_h
280.450	Velliv_m	240.837	PFA_h	0.000	PFA_m	643.450	mix_m_b	0.01	PFA_m	4.94	Velliv_m

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2}\right) \Big/ \left(\frac{x_{t-1} + y_{t-1}}{2}\right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns $=$ Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2\frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

c{ x_t + y_t }{ x_{t-1} + y_{t-1}}

(x_{t-1} + y_{t-1}) x_t y_{t-1} + (x_{t-1} + y_{t-1}) x_{t-1} y_t = 2 (x_{t-1}y_{t-1}x_t + x_{t-1}y_{t-1}y_t)$$

$$(x_{t-1}x_1y_{t-1} + y_{t-1}x_ty_{t-1}) + (x_{t-1}x_{t-1}y_t + x_{t-1}y_{t-1}y_t) = 2(x_{t-1}y_{t-1}x_t + x_{t-1}y_{t-1}y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: `R = 1+r`

```
## Let x_0 be 100.

## Let y_0 be 200.

## So the initial value of the pf is 300 .

## Let R_x be 0.5.

## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.

## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): 0.06566729
## s(data_x): 0.3822689
## m(data_y): 10.38592
## s(data_y): 2.038271
##
## m(data_x + data_y): 5.225793
## s(data_x + data_y): 0.9665913
```

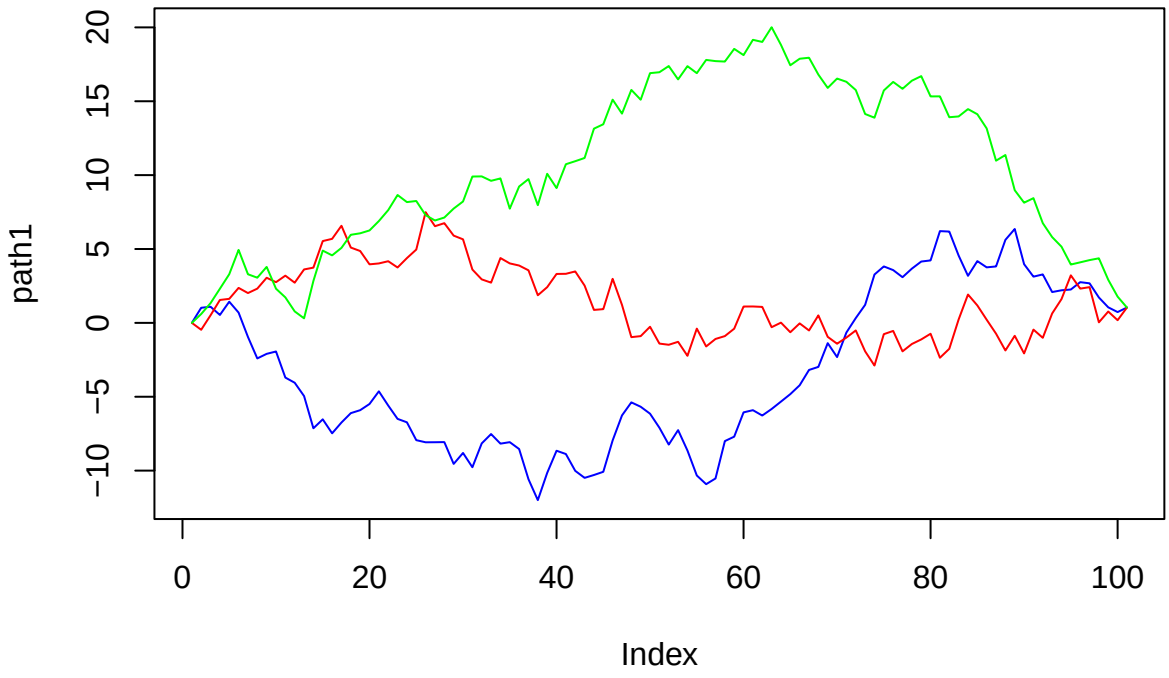
m and s of final state of all paths.
_a is mix of simulated returns.
_b is simulated mixed returns.

m_a	m_b	s_a	s_b
104.382	104.185	4.593	4.341
104.340	104.512	4.643	4.271
104.443	104.594	4.591	4.335
104.565	104.667	4.634	4.371
104.472	104.489	4.643	4.399
104.391	104.413	4.726	4.296
104.422	104.555	4.401	4.341
104.429	104.663	4.547	4.441
104.339	104.559	4.464	4.462
104.446	104.557	4.674	4.460

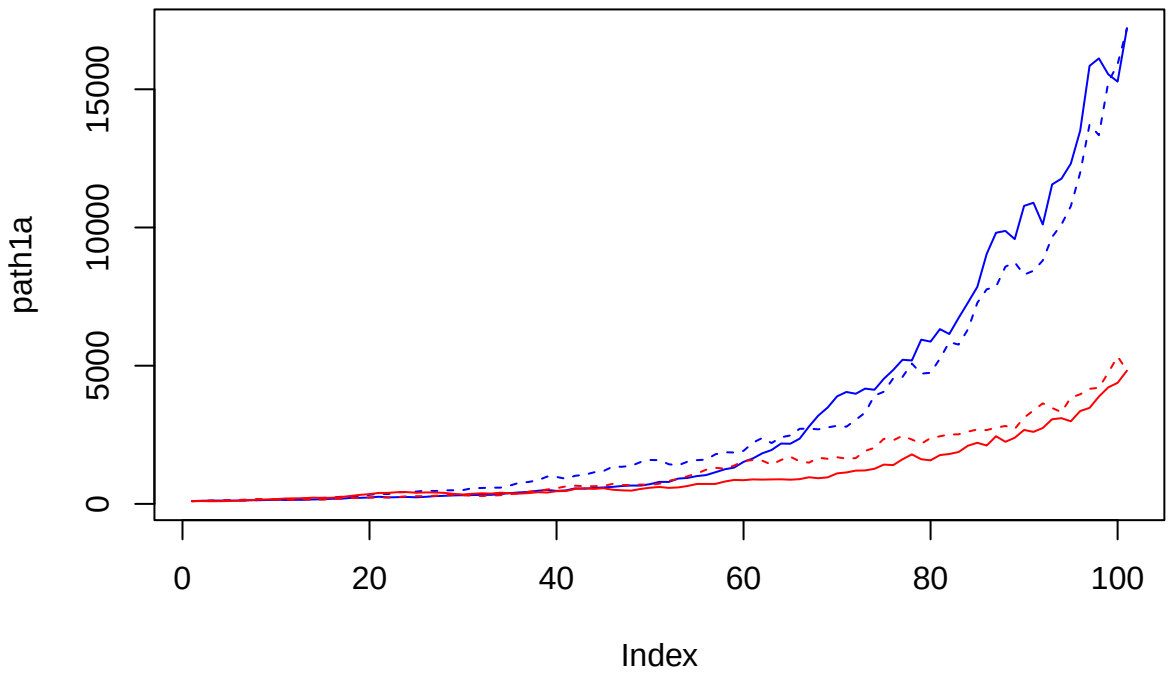
```
##       m_a             m_b             s_a             s_b       
##  Min.   :104.3   Min.   :104.2   Min.   :4.401   Min.   :4.271  
##  1st Qu.:104.4   1st Qu.:104.5   1st Qu.:4.558   1st Qu.:4.336  
##  Median :104.4   Median :104.6   Median :4.613   Median :4.356  
##  Mean   :104.4   Mean   :104.5   Mean   :4.591   Mean   :4.372  
##  3rd Qu.:104.4   3rd Qu.:104.6   3rd Qu.:4.643   3rd Qu.:4.430  
##  Max.   :104.6   Max.   :104.7   Max.   :4.726   Max.   :4.462
```

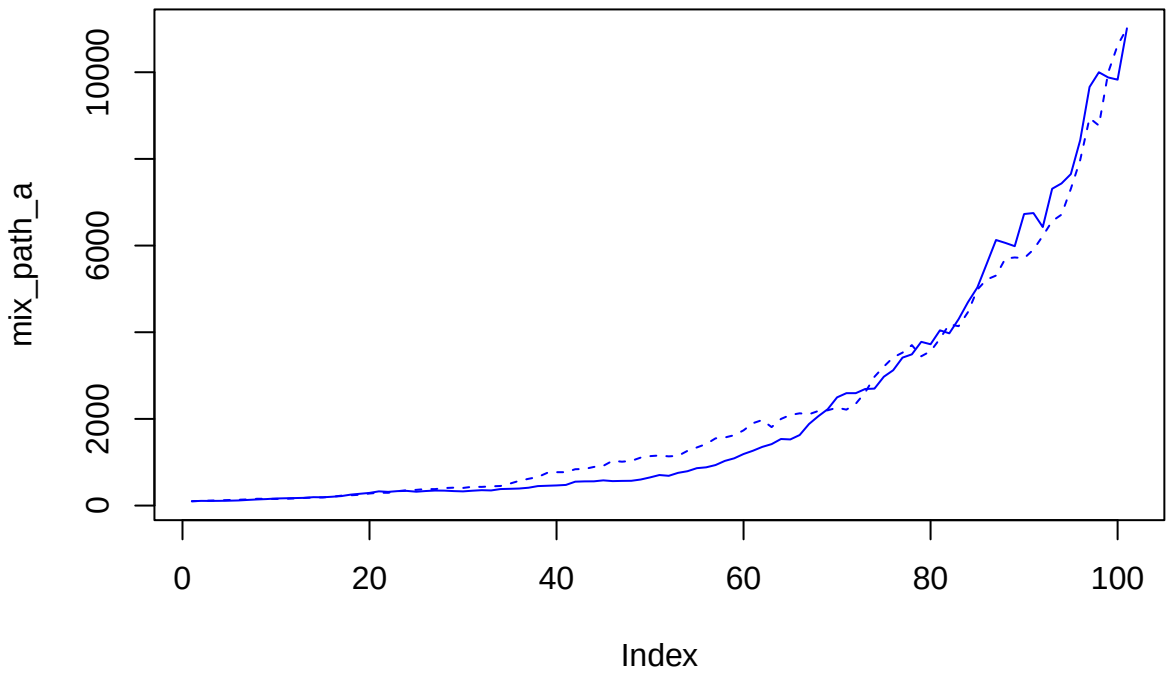
_a and _b are very close to equal.
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?


path1a
0
5000
10000
15000
0
20
40
60
80
100
Index



The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?
Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##        m                s
##  Min.   :0.05909   Min.   :0.04658
##  1st Qu.:0.06582   1st Qu.:0.06315
##  Median :0.06918   Median :0.06951
##  Mean   :0.07039   Mean   :0.06916
##  3rd Qu.:0.07404   3rd Qu.:0.07504
##  Max.   :0.08385   Max.   :0.09024
```