

Pension returns analysis

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Fit log returns to F-S skew standardized Student-t distribution.

m is the location parameter.

s is the scale parameter.

ν is the estimated shape parameter (degrees of freedom).

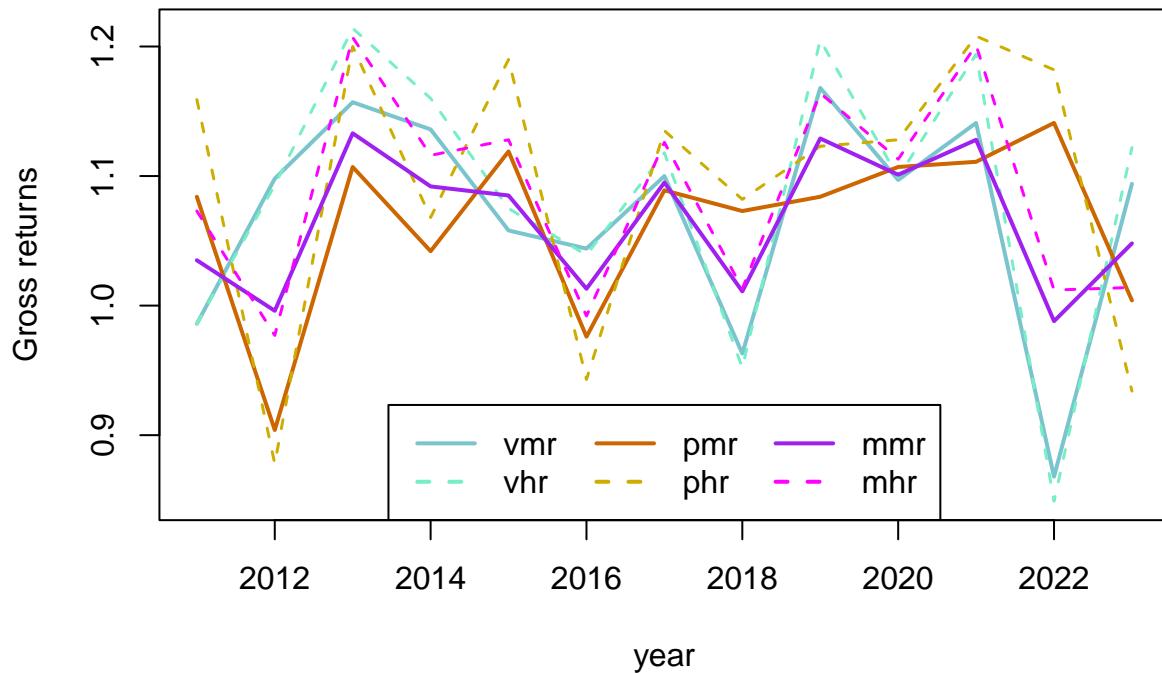
ξ_i is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.

`vmrl1` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.

Gross returns 2011–2023



Summary of gross returns

```

##      vmr        pmr        mmr        vhr
## Min.  :0.868  Min.  :0.904  Min.  :0.988  Min.  :0.849
## 1st Qu.:1.044  1st Qu.:1.042  1st Qu.:1.013  1st Qu.:1.039
## Median :1.097  Median :1.084  Median :1.085  Median :1.099
## Mean   :1.070  Mean   :1.065  Mean   :1.066  Mean   :1.085
## 3rd Qu.:1.136  3rd Qu.:1.107  3rd Qu.:1.101  3rd Qu.:1.160
## Max.   :1.168  Max.   :1.141  Max.   :1.133  Max.   :1.214
##      phr        mhr
## Min.  :0.878  Min.  :0.977
## 1st Qu.:1.068  1st Qu.:1.013
## Median :1.128  Median :1.113
## Mean   :1.095  Mean   :1.087
## 3rd Qu.:1.182  3rd Qu.:1.128
## Max.   :1.208  Max.   :1.207

##      vmrl
## Min.  :0.801
## 1st Qu.:1.013
## Median :1.085
## Mean   :1.061
## 3rd Qu.:1.128
## Max.   :1.193

##      vmr    pmr    mmr    vhr    phr    mhr
## Min.  : 0.868 0.904 0.988 0.849 0.878 0.977
## 1st Qu.: 1.044 1.042 1.013 1.039 1.068 1.013
## Median : 1.097 1.084 1.085 1.099 1.128 1.113
## Mean   : 1.070 1.065 1.066 1.085 1.095 1.087
## 3rd Qu.: 1.136 1.107 1.101 1.160 1.182 1.128
## Max.   : 1.168 1.141 1.133 1.214 1.208 1.207

```

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.136	vmr	1.168	vmr
0.977	mhr	1.044	vmr	1.113	mhr	1.087	mhr	1.107	pmr	1.141	pmr
0.904	pmr	1.042	pmr	1.099	vhr	1.085	vhr	1.101	mmr	1.133	mmr
0.878	phr	1.039	vhr	1.097	vmr	1.070	vmr	1.160	vhr	1.214	vhr
0.868	vmr	1.013	mmr	1.085	mmr	1.066	mmr	1.182	phr	1.208	phr
0.849	vhr	1.013	mhr	1.084	pmr	1.065	pmr	1.128	mhr	1.207	mhr

Covariance

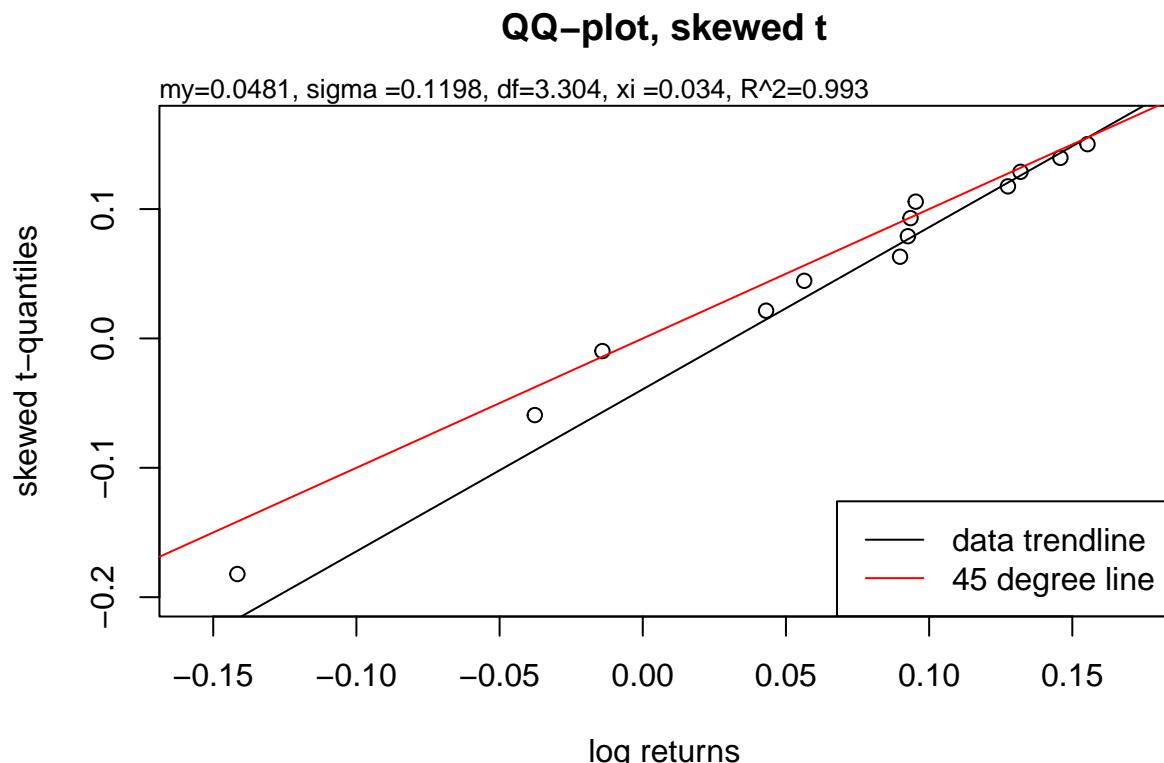
```
## cov(vmr, pmr) = -0.001094875
```

```
## cov(vhr, phr) = -0.0001730651
```

Velliv medium risk, 2011 - 2023

```
##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 7.4 percent
## What is the risk of losing max 25 %? =< 1.8 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 41 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

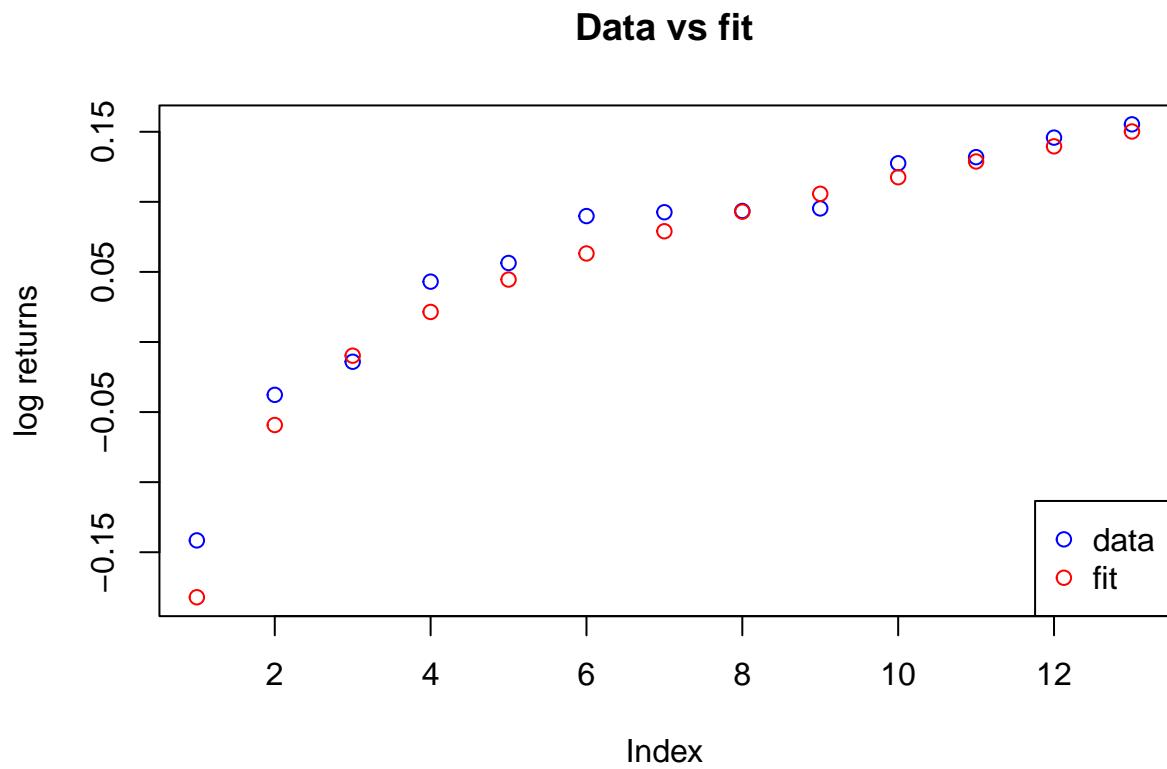
QQ Plot



The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

Data vs fit

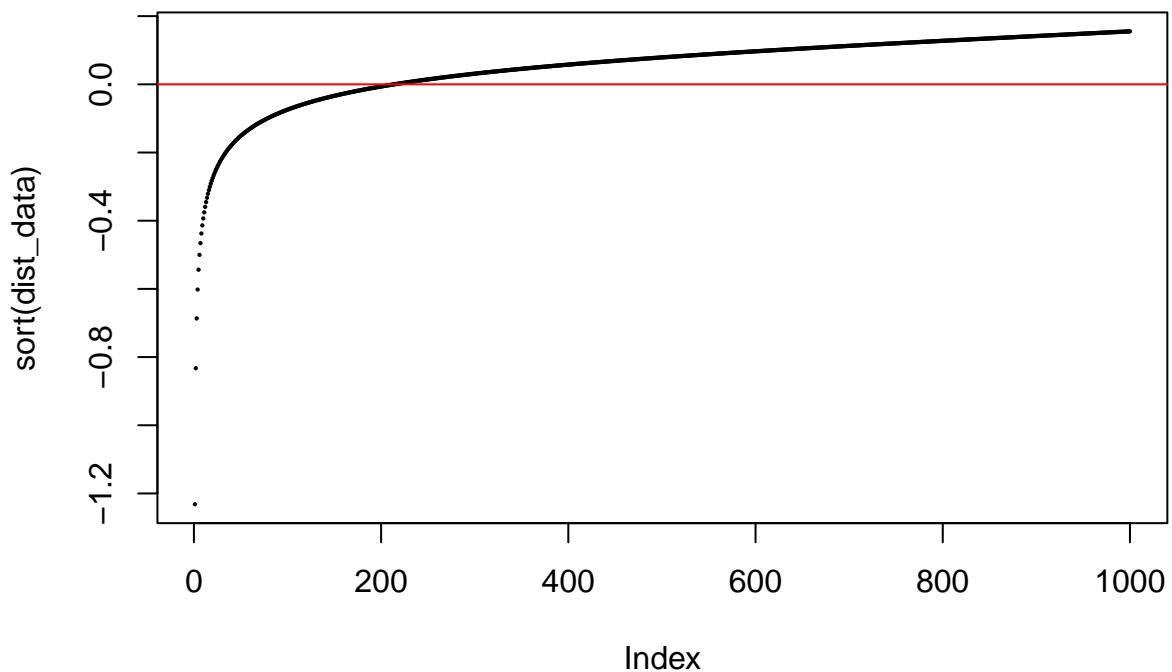
Let's plot the fit and the observed returns together.



Estimated distribution

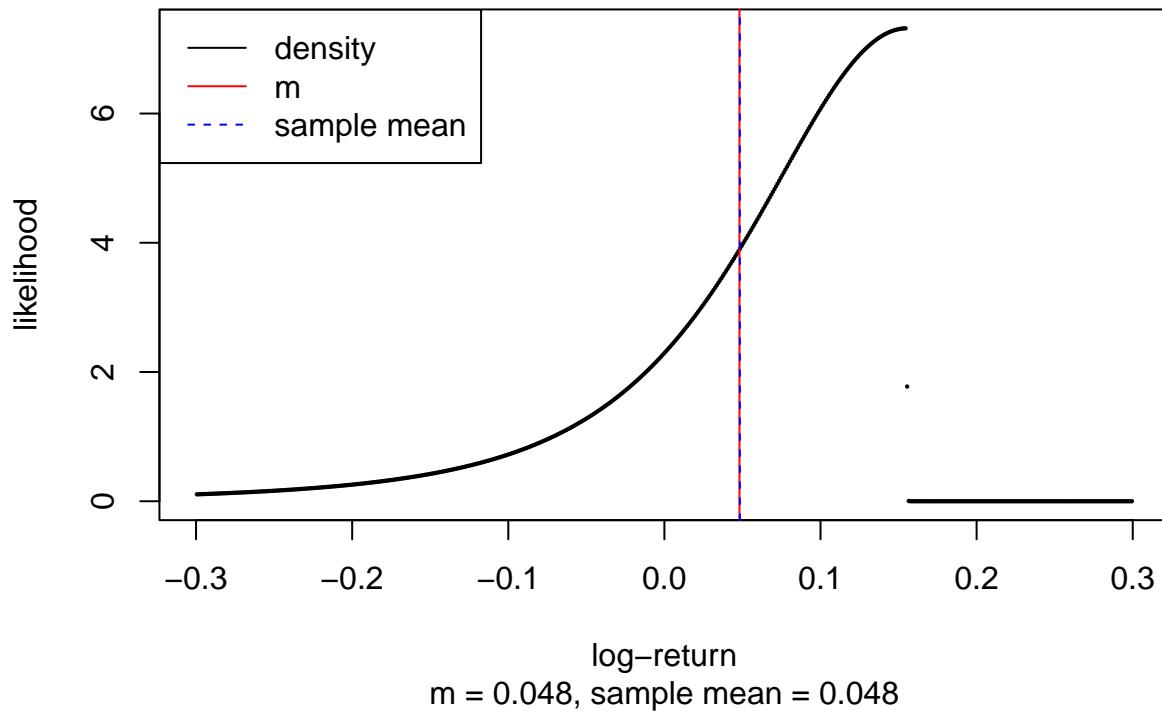
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

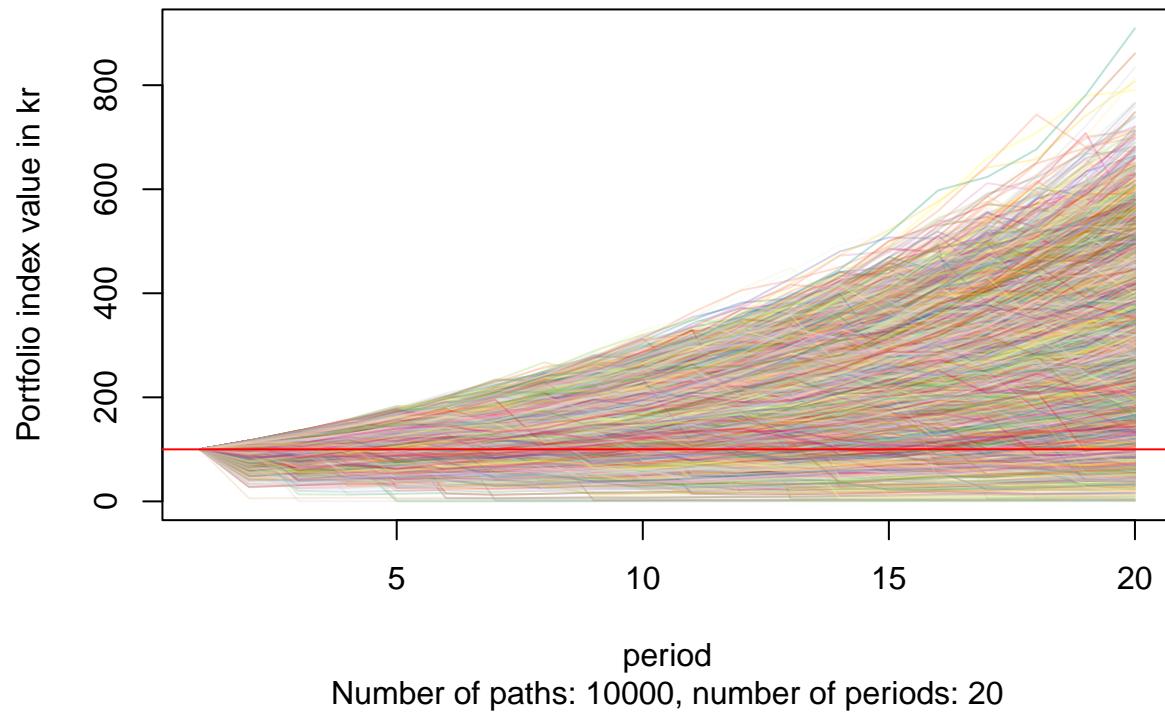
Estimated skew t distribution PDF



Monte Carlo

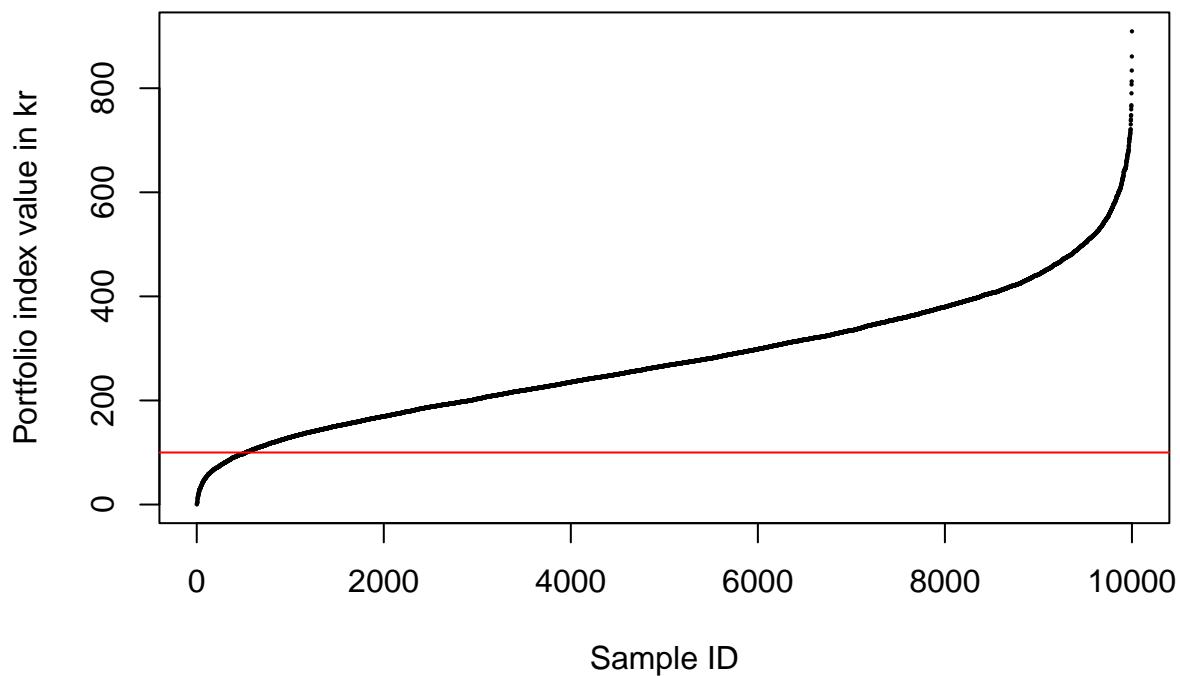
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 278.527 kr.  
## SD of portfolio index value after 20 years: 124.32 kr.  
## Min total portfolio index value after 20 years: 0.567 kr.  
## Max total portfolio index value after 20 years: 909.277 kr.  
##  
## Share of paths finishing below 100: 5.24 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

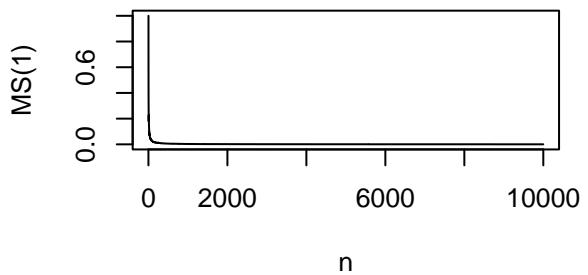
(100 is par, 200 is double, 50 is half)



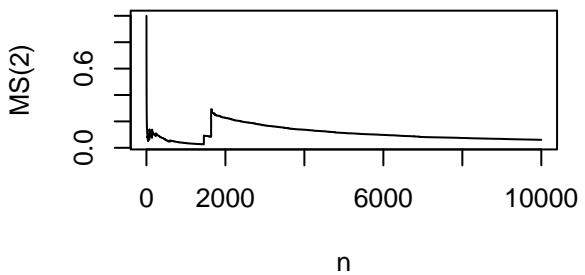
Convergence

Max vs sum plots for the first four moments:

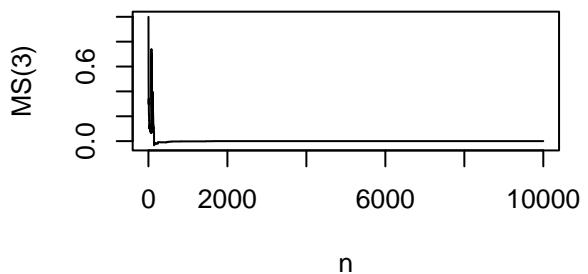
MS(1)



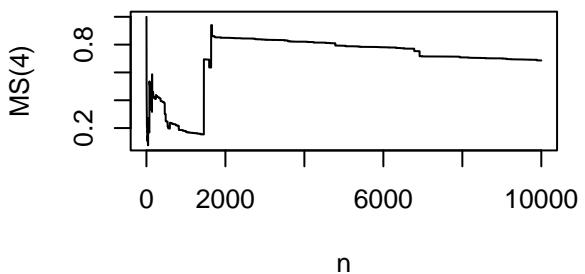
MS(2)



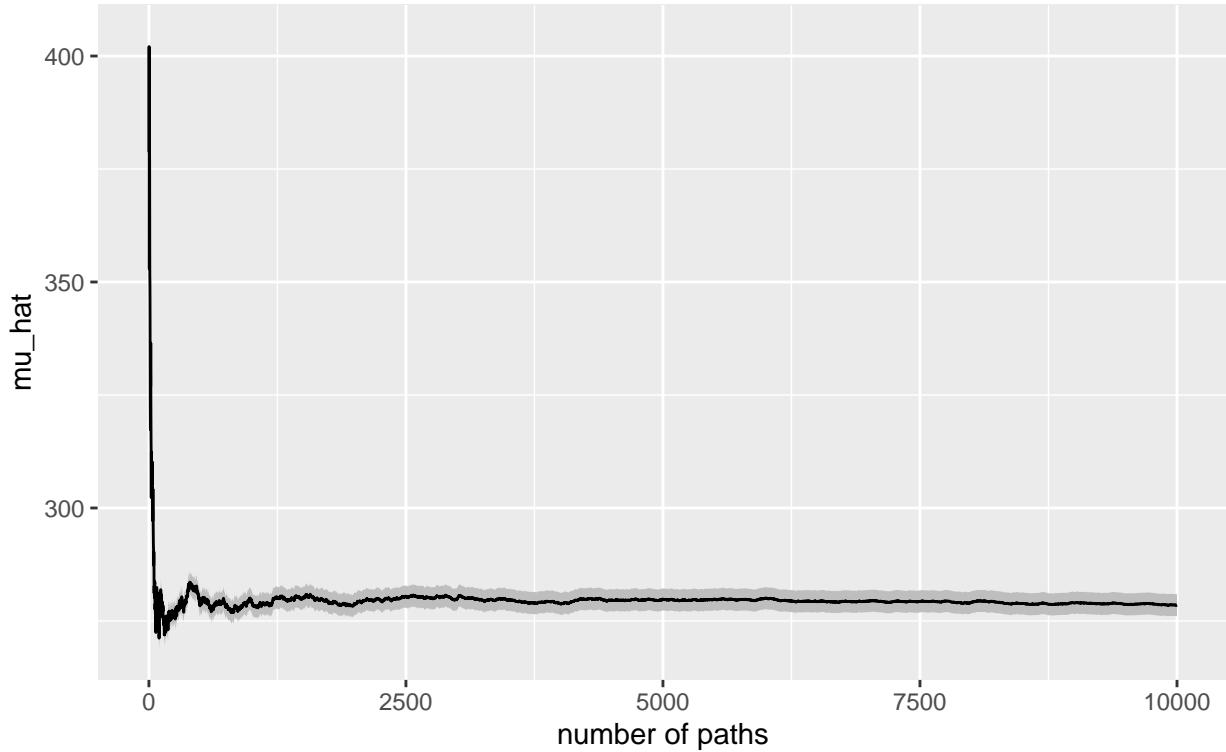
MS(3)



MS(4)



Monte Carlo
20 steps, 10000 paths

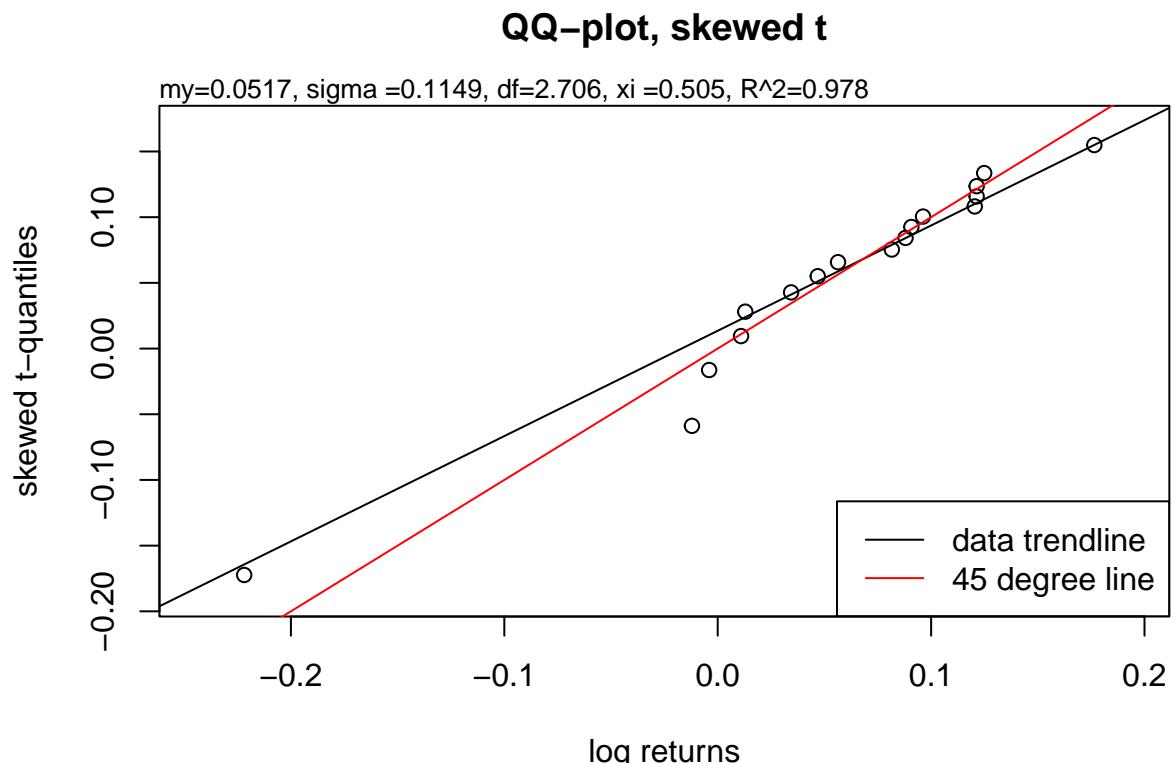


Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##  
## AIC: -34.35752  
## BIC: -31.02467  
## m: 0.05171176  
## s: 0.1149408  
## nu (df): 2.706099  
## xi: 0.5049945  
## R^2: 0.978  
##  
## An R^2 of 0.978 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 5.4 percent  
## What is the risk of losing max 25 %? <= 1.3 percent  
## What is the risk of losing max 50 %? <= 0.2 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 36.2 percent  
## What is the chance of gaining min 25 %? >= 0.3 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

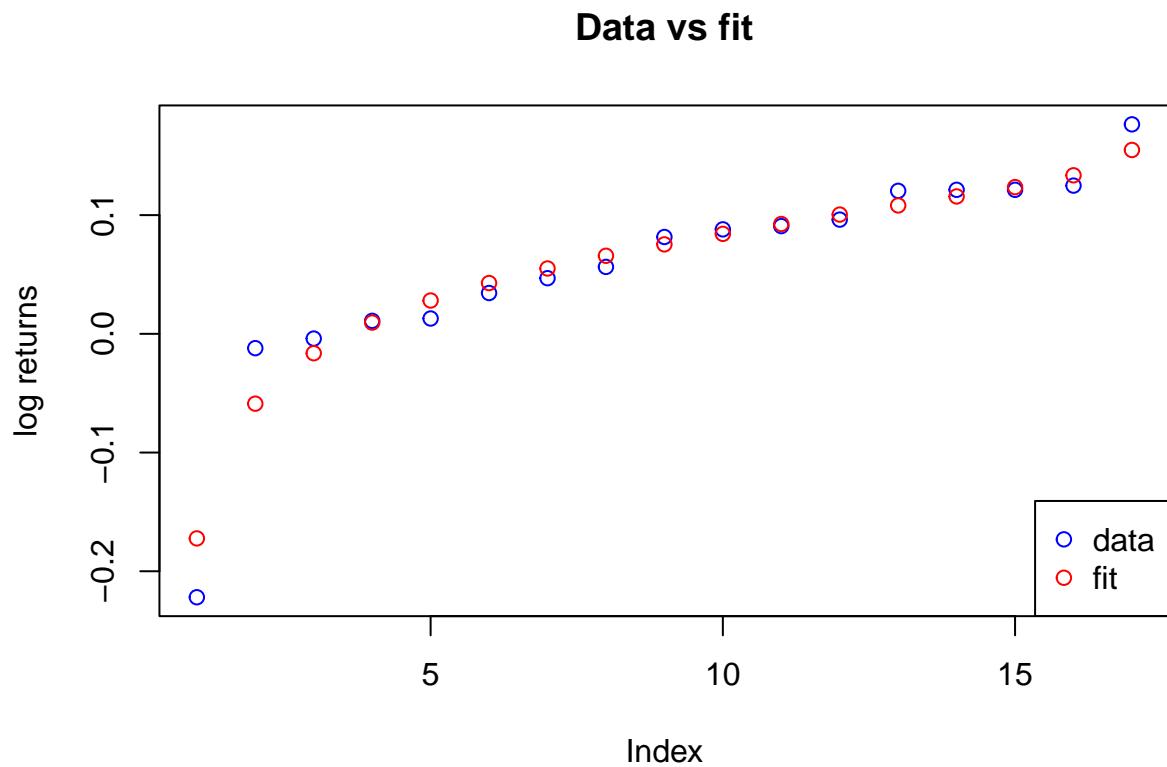
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

Data vs fit

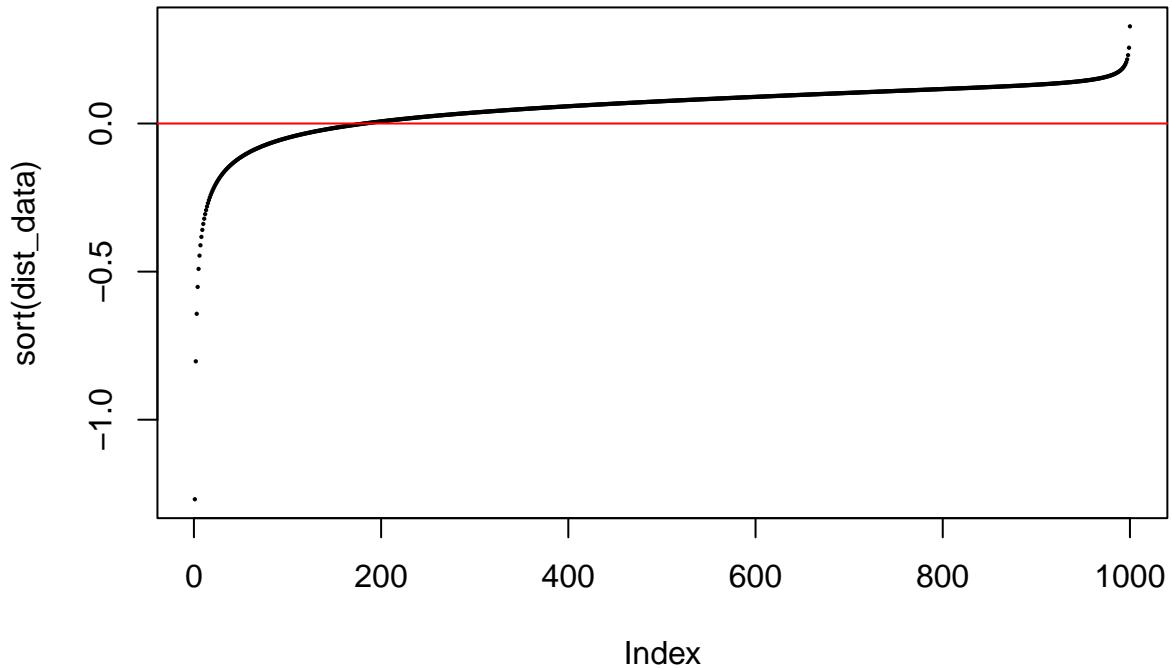
Let's plot the fit and the observed returns together.



Estimated distribution

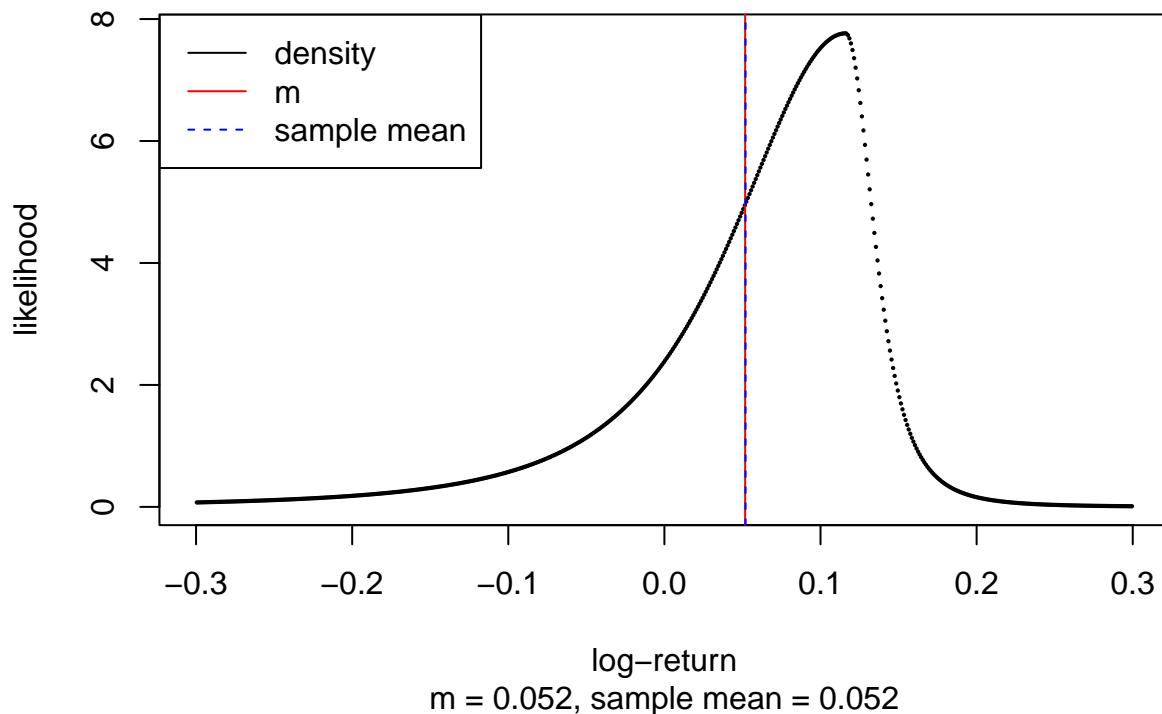
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!

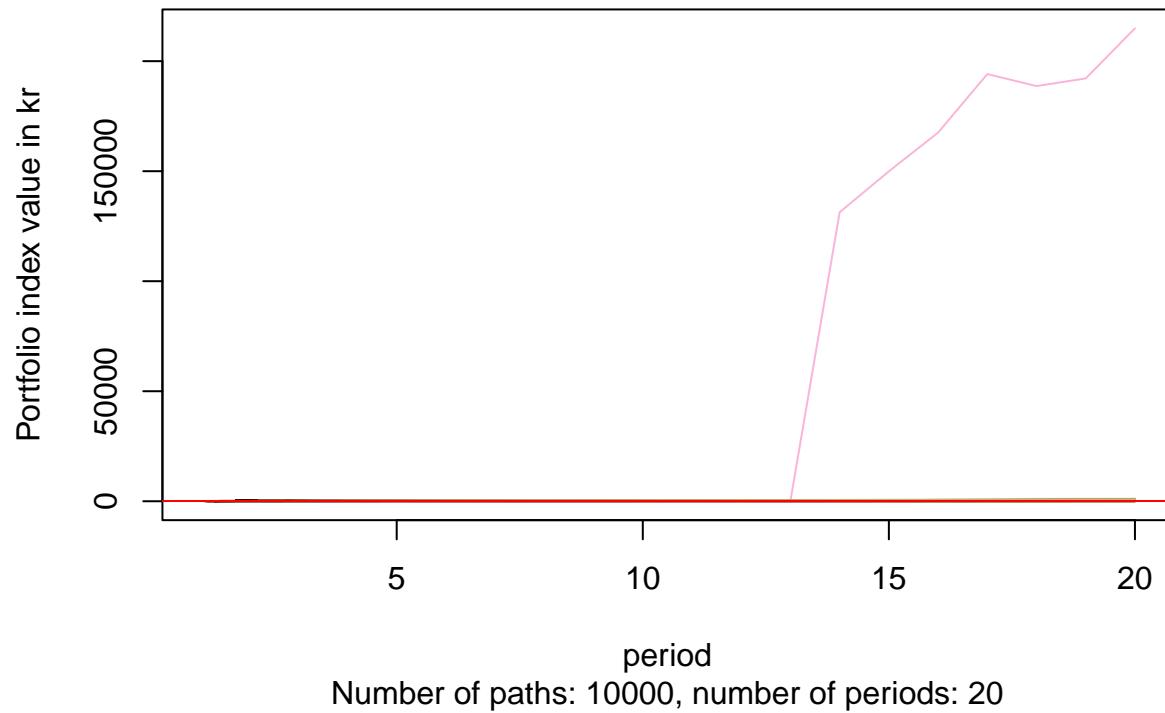
Estimated skew t distribution PDF



Monte Carlo

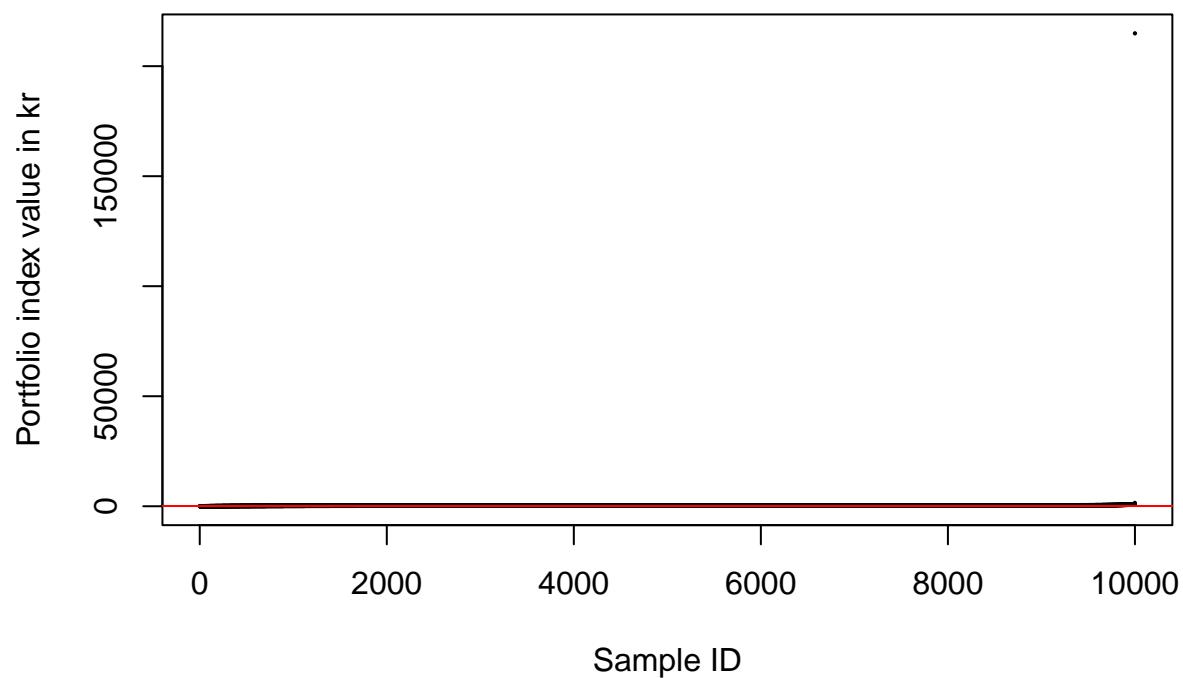
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.02 percent  
##  
## Mean portfolio index value after 20 years: 316.525 kr.  
## SD of portfolio index value after 20 years: 2149.399 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 214912.1 kr.  
##  
## Share of paths finishing below 100: 3.23 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

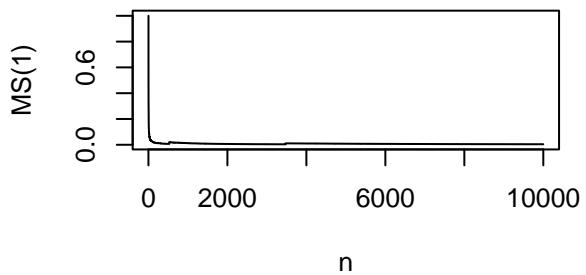
(100 is par, 200 is double, 50 is half)



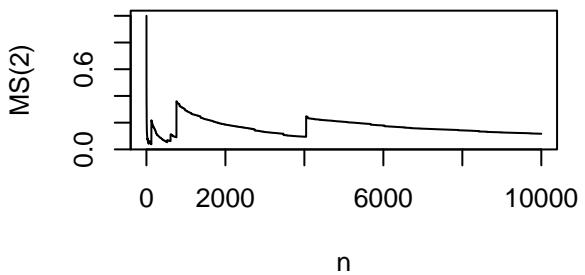
Convergence

Max vs sum plots for the first four moments:

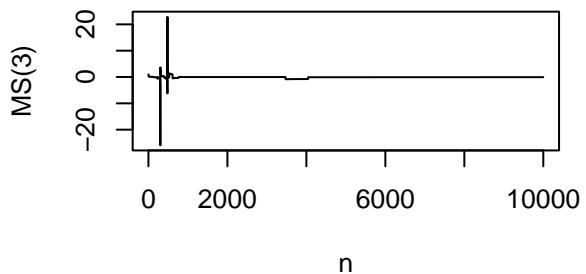
MS(1)



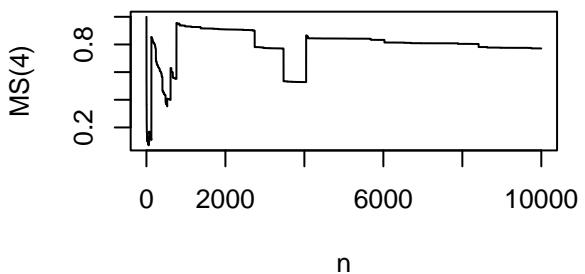
MS(2)



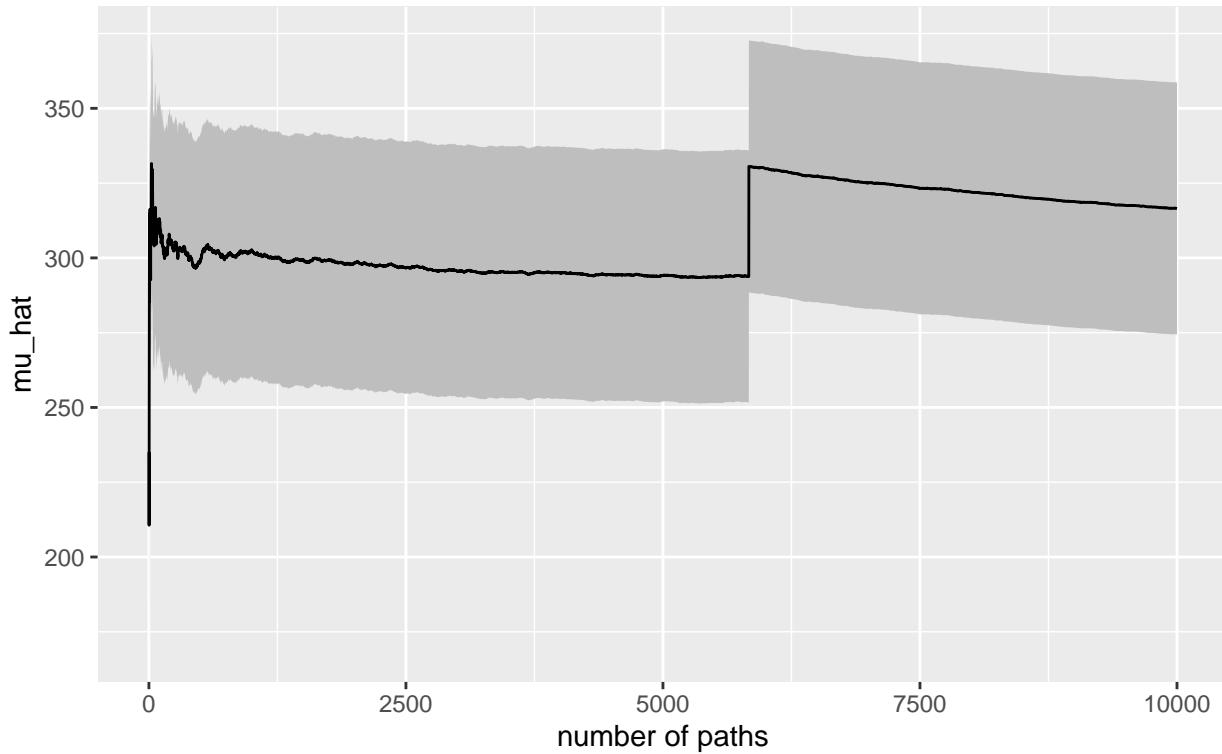
MS(3)



MS(4)



Monte Carlo
20 steps, 10000 paths

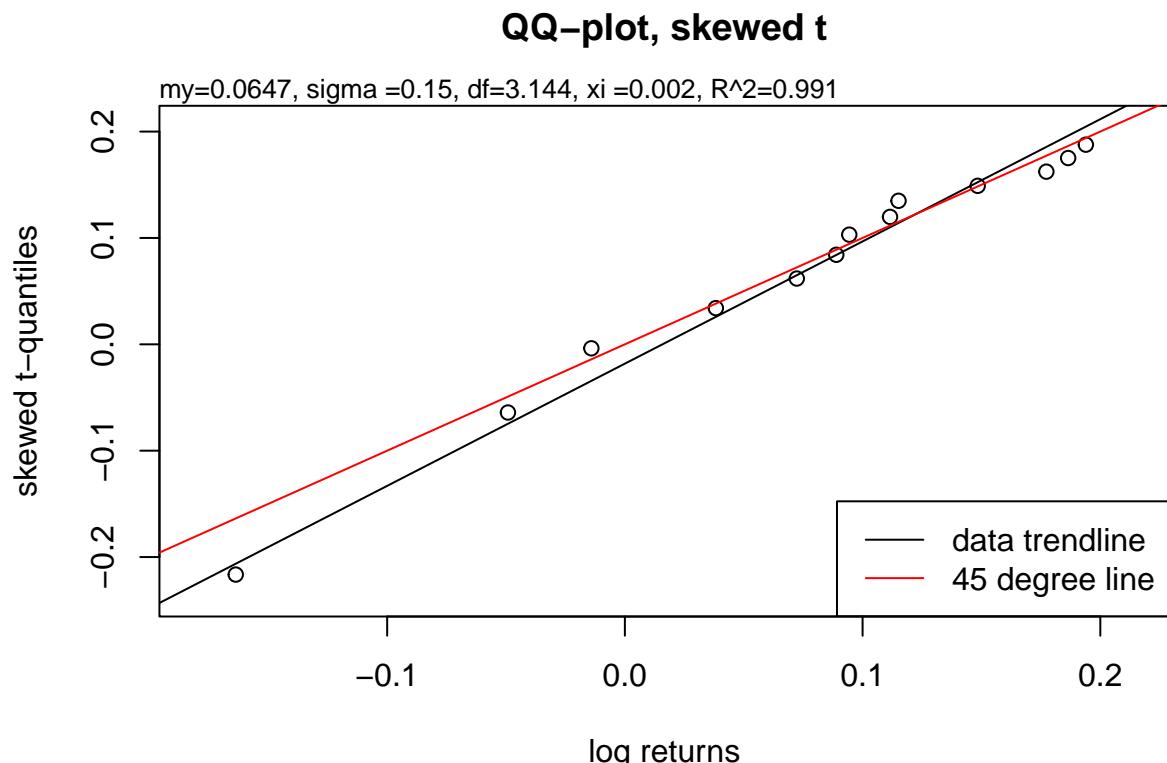


Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -21.42488  
## BIC: -19.16508  
## m: 0.06471454  
## s: 0.1499924  
## nu (df): 3.144355  
## xi: 0.002367034  
## R^2: 0.991  
##  
## An R^2 of 0.991 suggests that the fit is extremely good.  
##  
## What is the risk of losing max 10 %? <= 8.3 percent  
## What is the risk of losing max 25 %? <= 2.5 percent  
## What is the risk of losing max 50 %? <= 0.4 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 53.3 percent  
## What is the chance of gaining min 25 %? >= 0 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

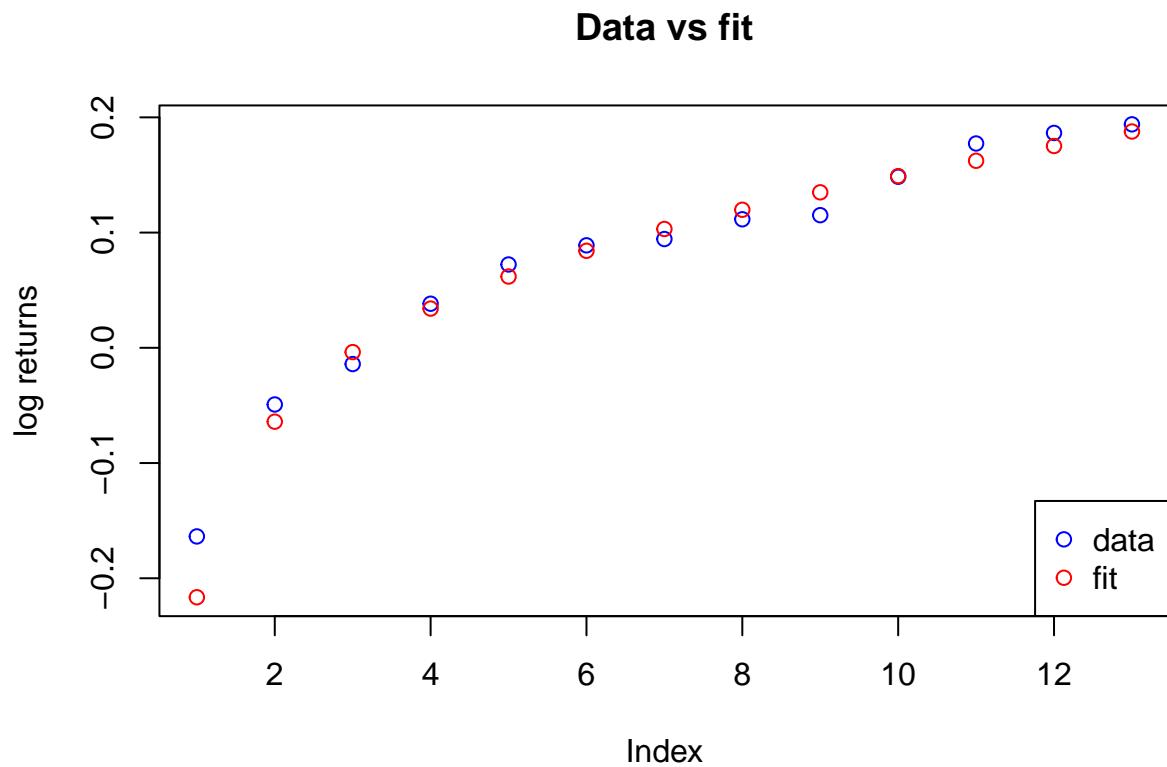
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

Data vs fit

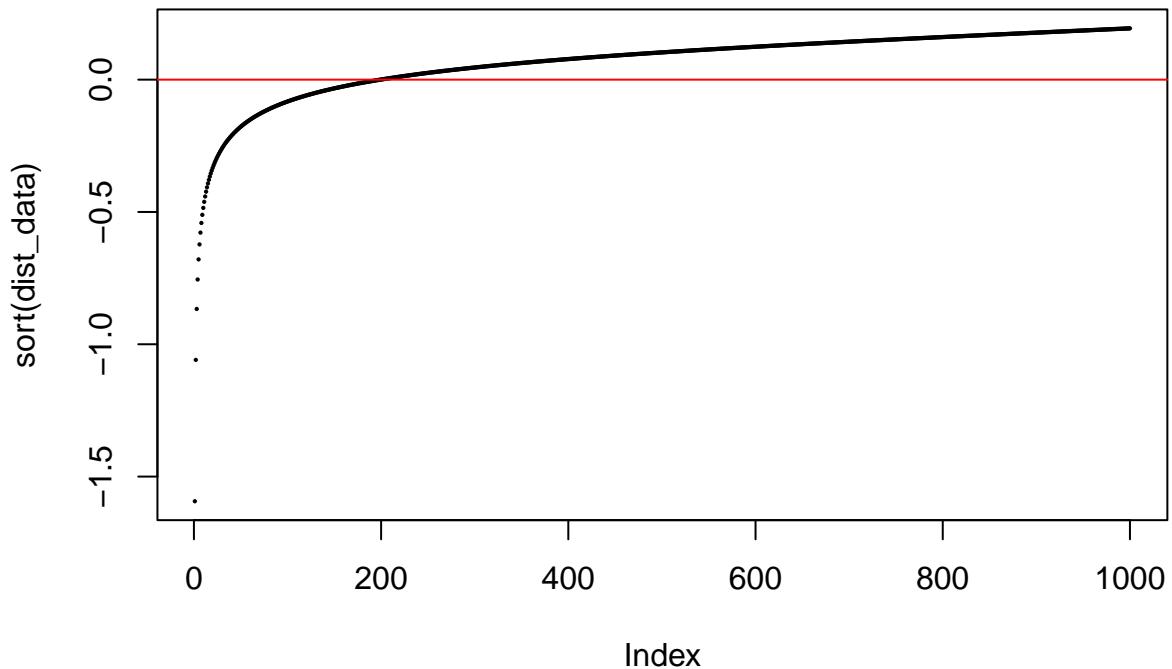
Let's plot the fit and the observed returns together.



Estimated distribution

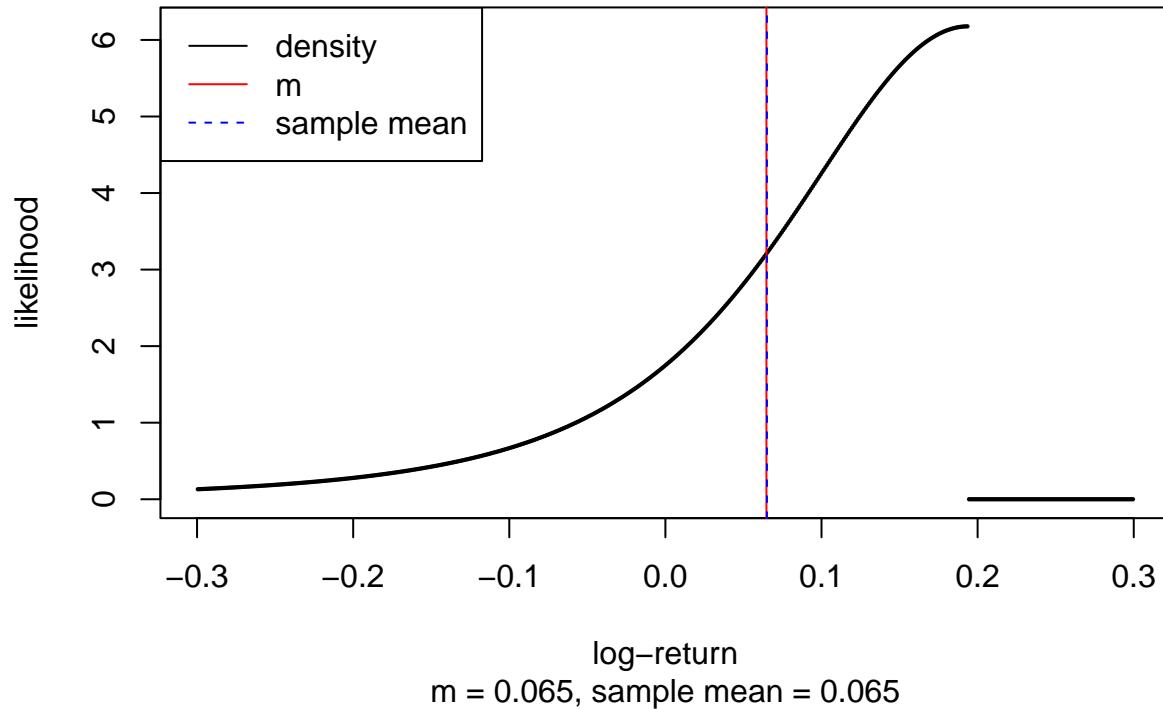
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

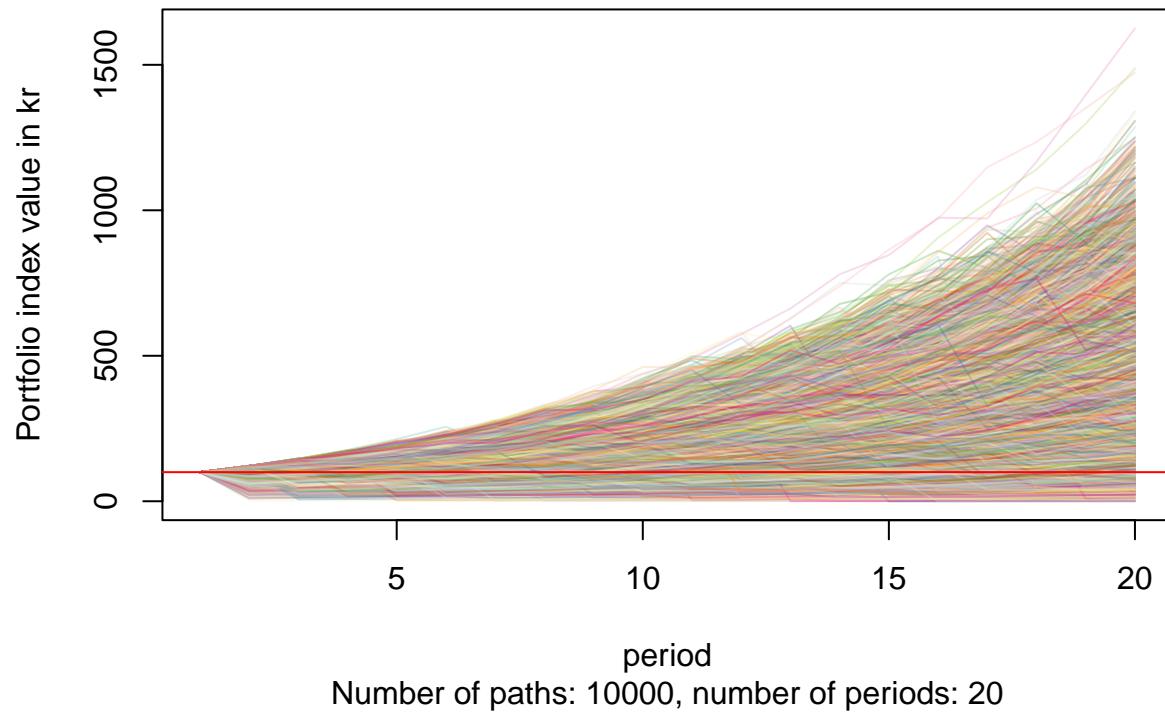
Estimated skew t distribution PDF



Monte Carlo

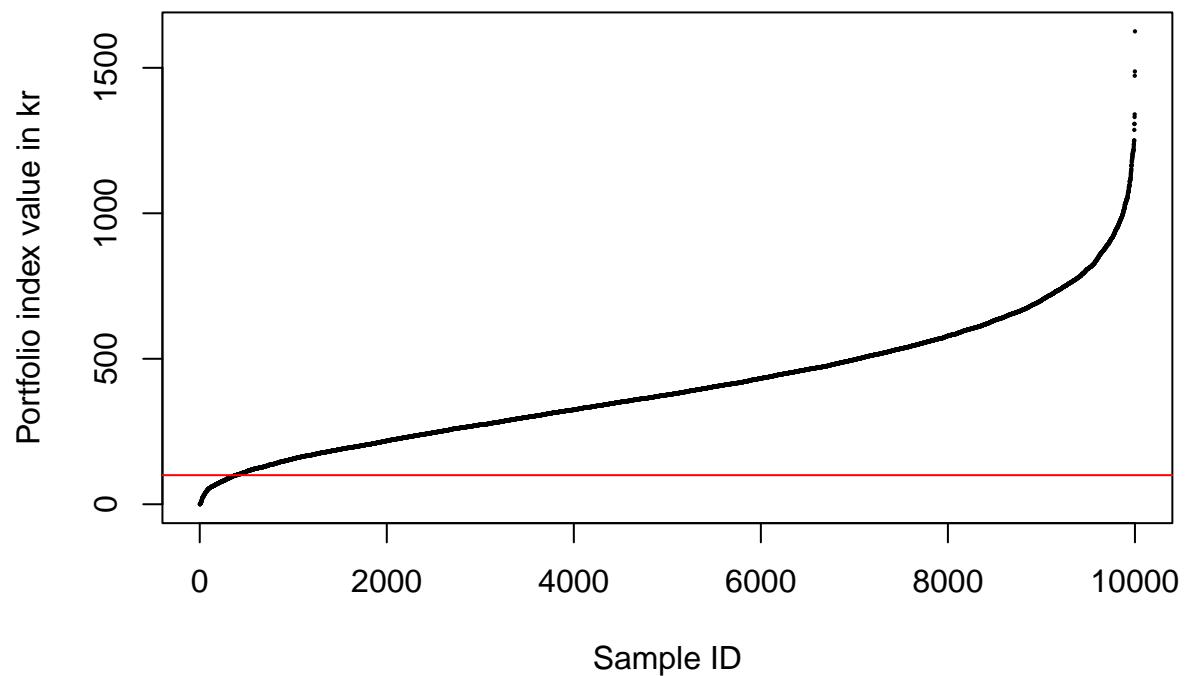
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.01 percent  
##  
## Mean portfolio index value after 20 years: 407.438 kr.  
## SD of portfolio index value after 20 years: 216.472 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 1625.391 kr.  
##  
## Share of paths finishing below 100: 3.89 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

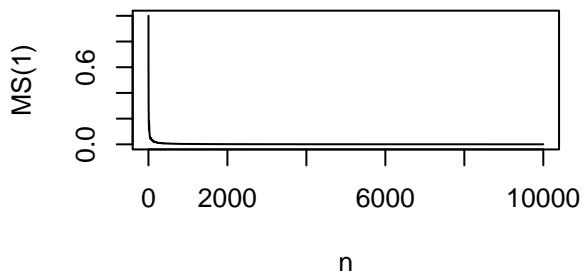
(100 is par, 200 is double, 50 is half)



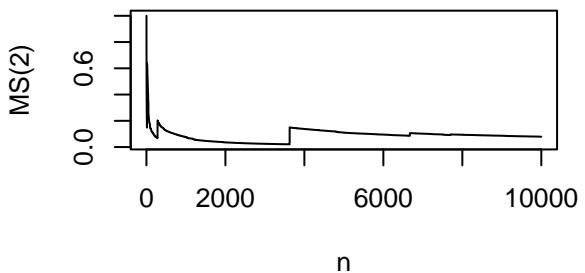
Convergence

Max vs sum plots for the first four moments:

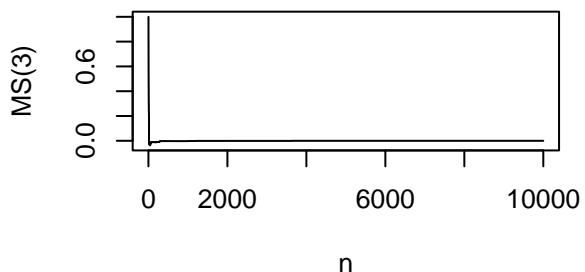
MS(1)



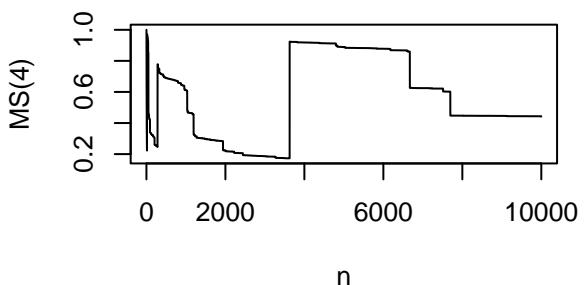
MS(2)



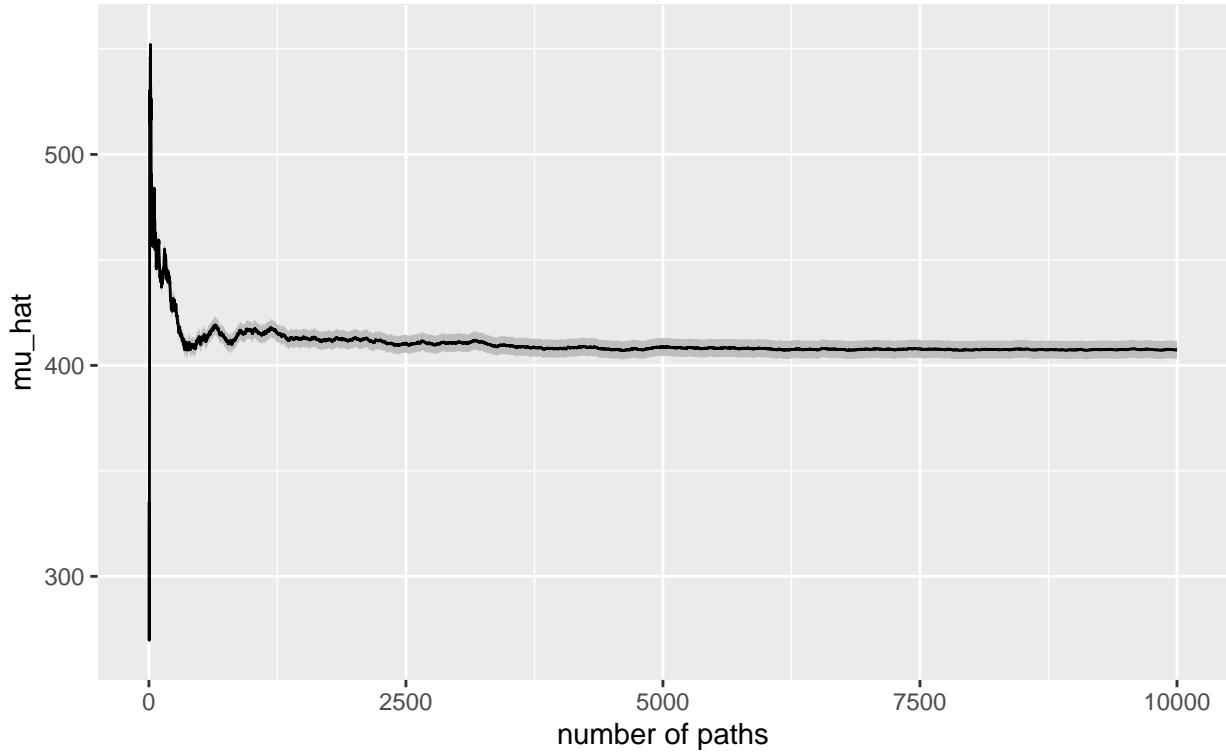
MS(3)



MS(4)



Monte Carlo
20 steps, 10000 paths

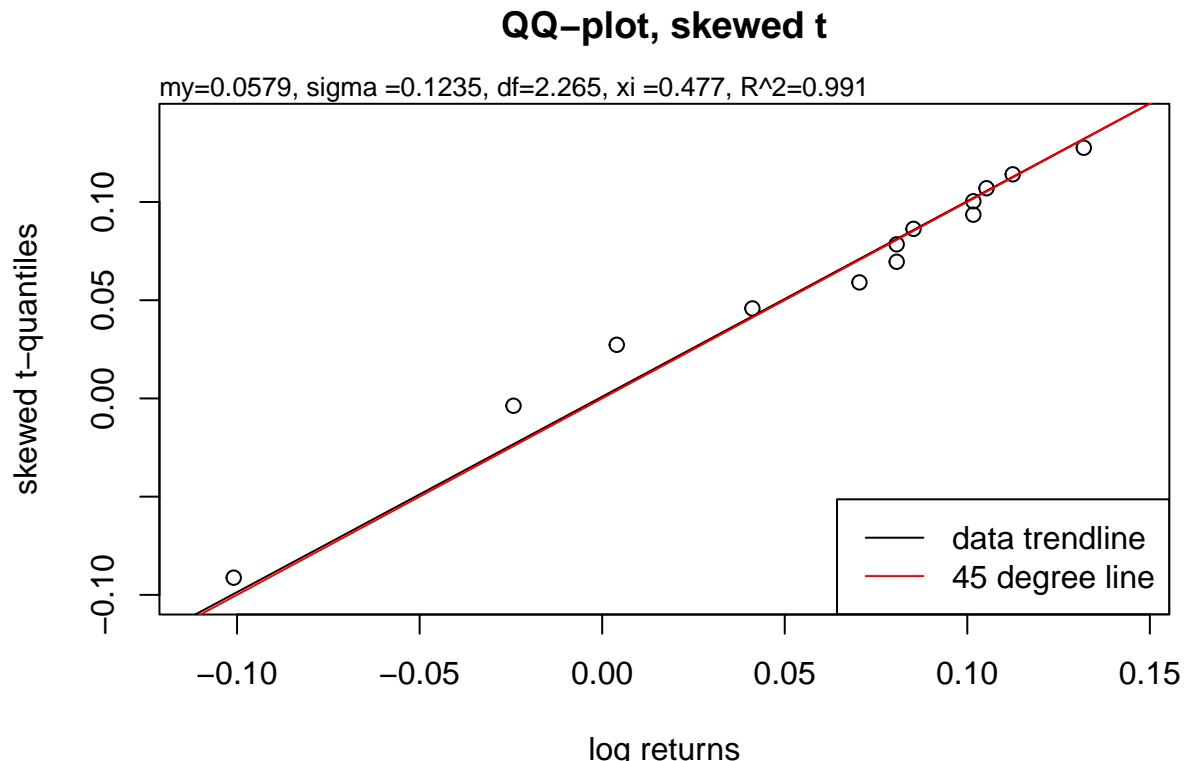


PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -33.22998  
## BIC: -30.97018  
## m: 0.05789224  
## s: 0.1234592  
## nu (df): 2.265273  
## xi: 0.477324  
## R^2: 0.991  
##  
## An R^2 of 0.991 suggests that the fit is extremely good.  
##  
## What is the risk of losing max 10 %? <= 3.3 percent  
## What is the risk of losing max 25 %? <= 0.9 percent  
## What is the risk of losing max 50 %? <= 0.2 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 32.7 percent  
## What is the chance of gaining min 25 %? >= 0.1 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

QQ Plot

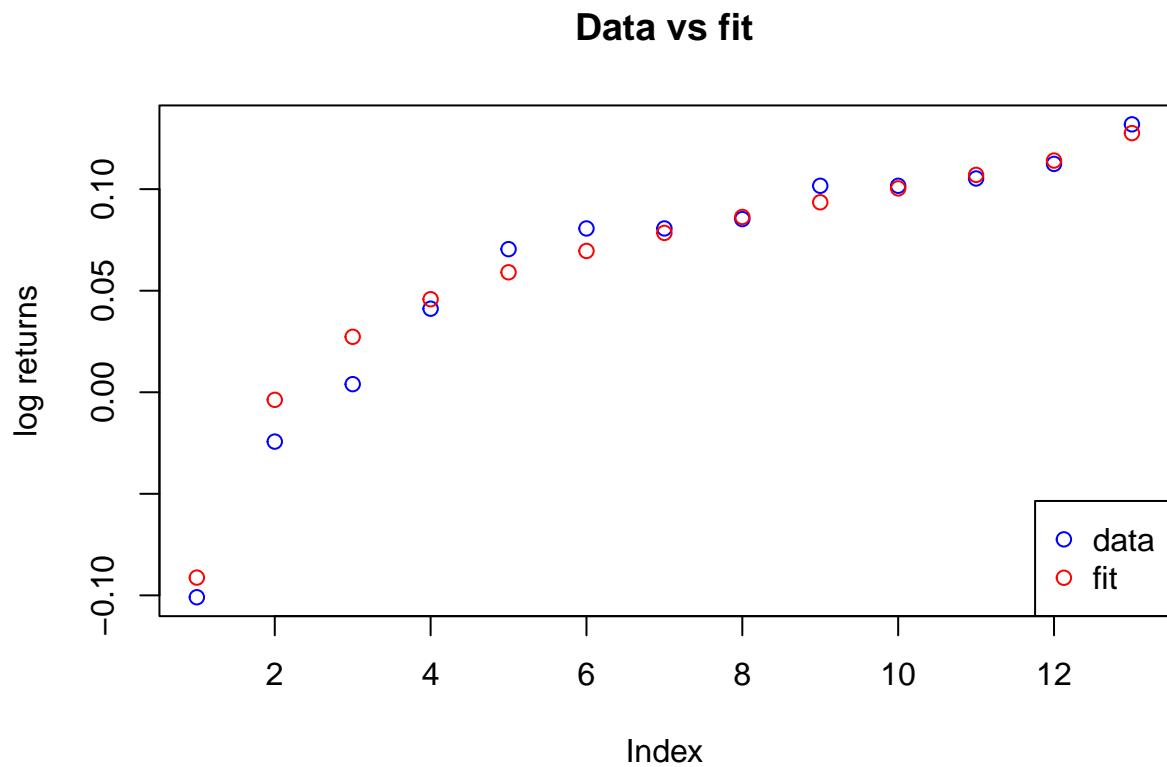


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
## [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
## [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11]  0.107018607  0.114081432  0.127604387
```

Data vs fit

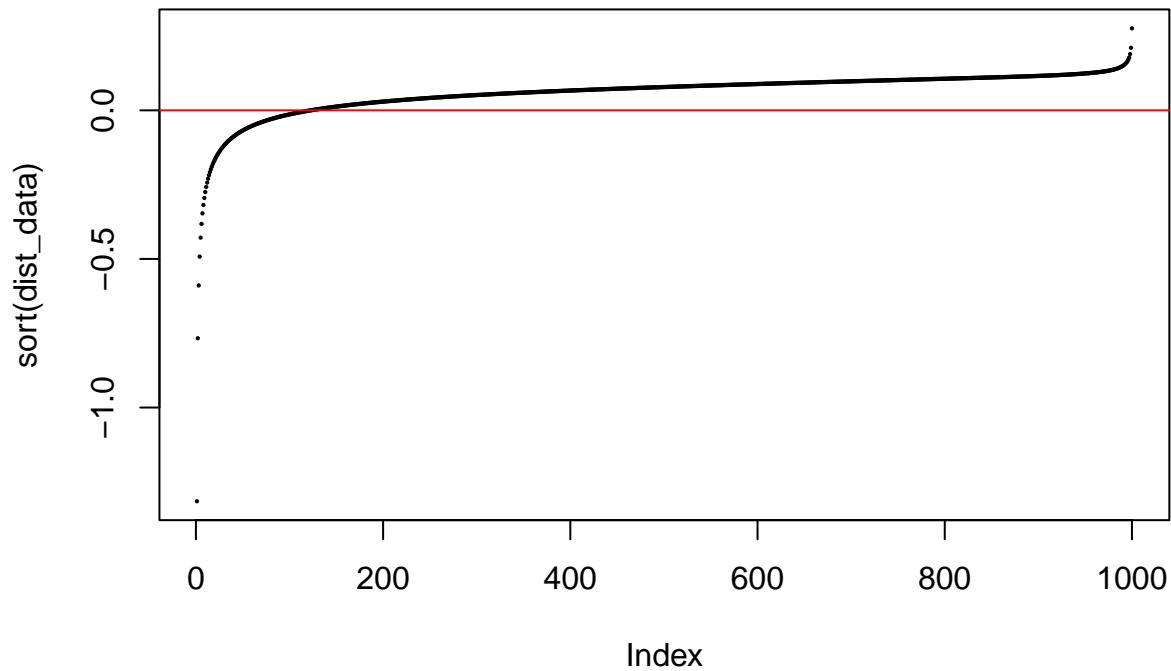
Let's plot the fit and the observed returns together.



Estimated distribution

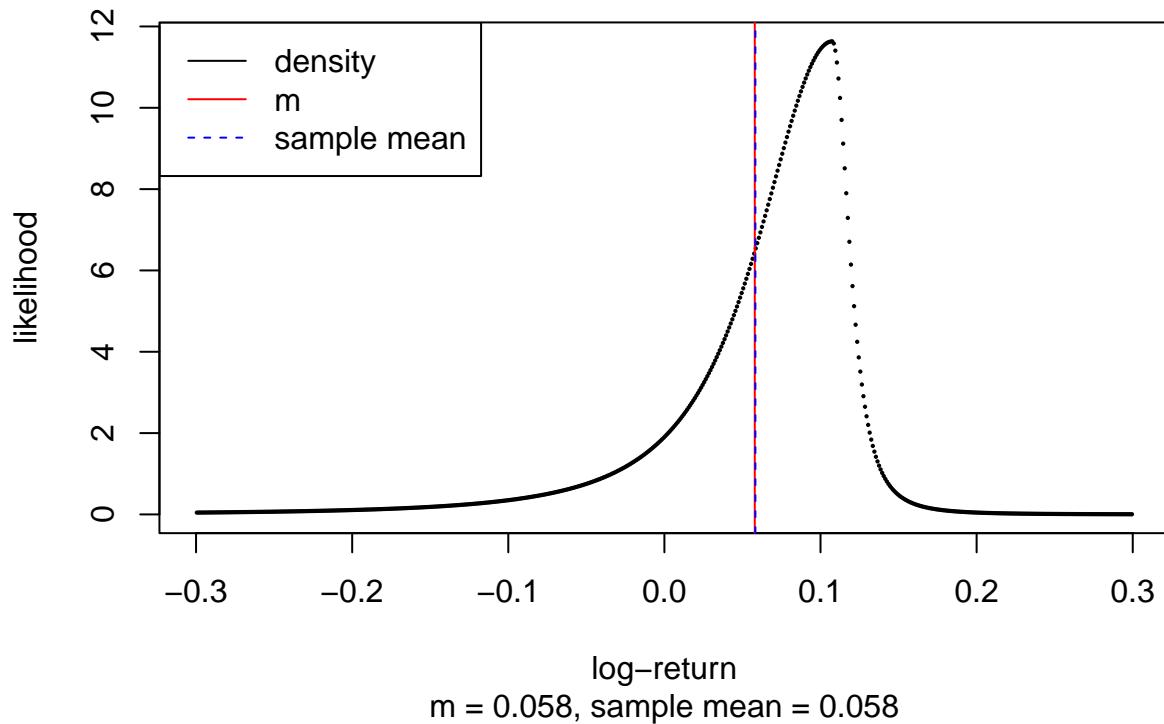
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.

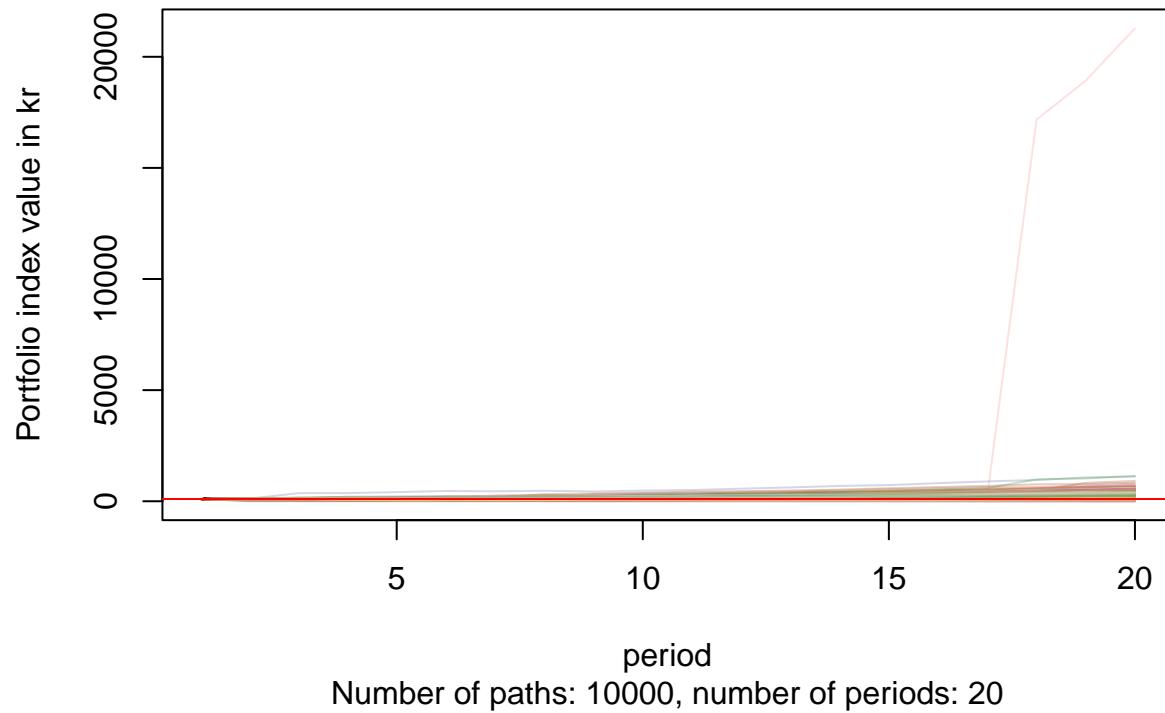
Estimated skew t distribution PDF



Monte Carlo

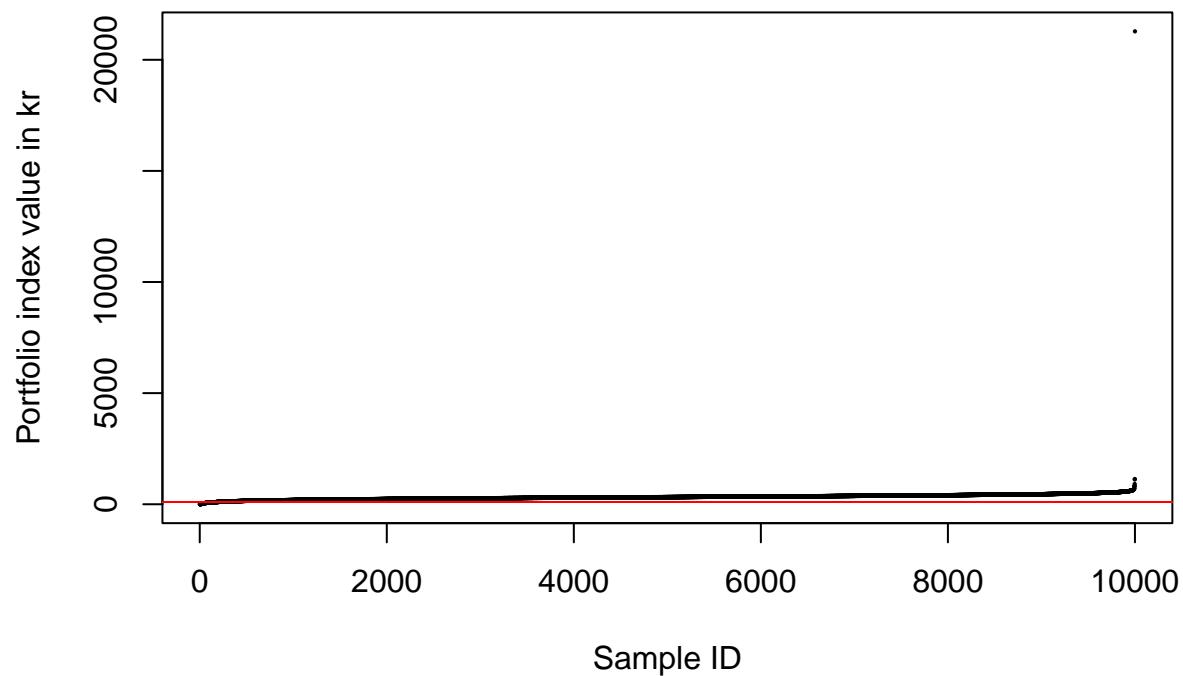
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.01 percent  
##  
## Mean portfolio index value after 20 years: 324.155 kr.  
## SD of portfolio index value after 20 years: 234.349 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 21280.18 kr.  
##  
## Share of paths finishing below 100: 1.83 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

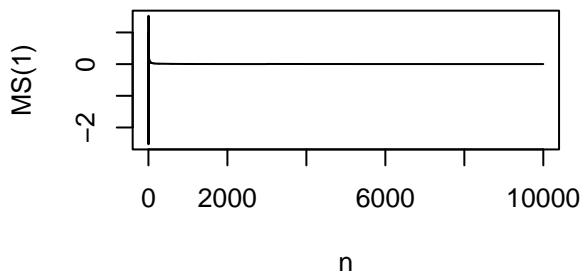
(100 is par, 200 is double, 50 is half)



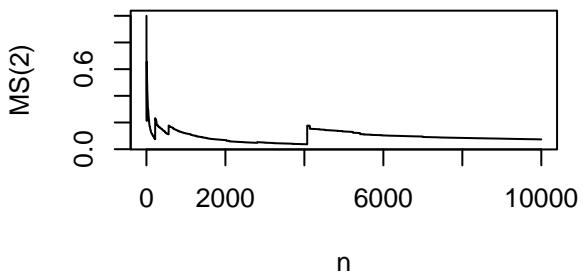
Convergence

Max vs sum plots for the first four moments:

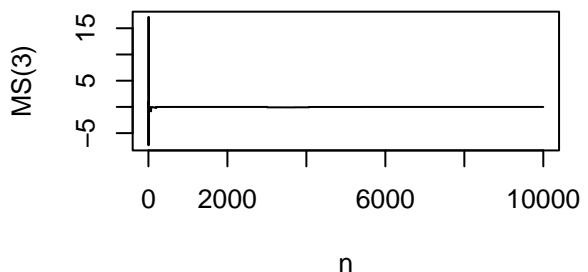
MS(1)



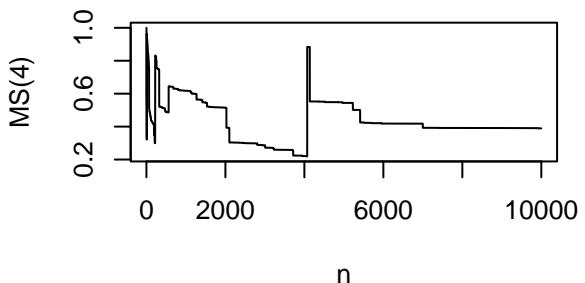
MS(2)



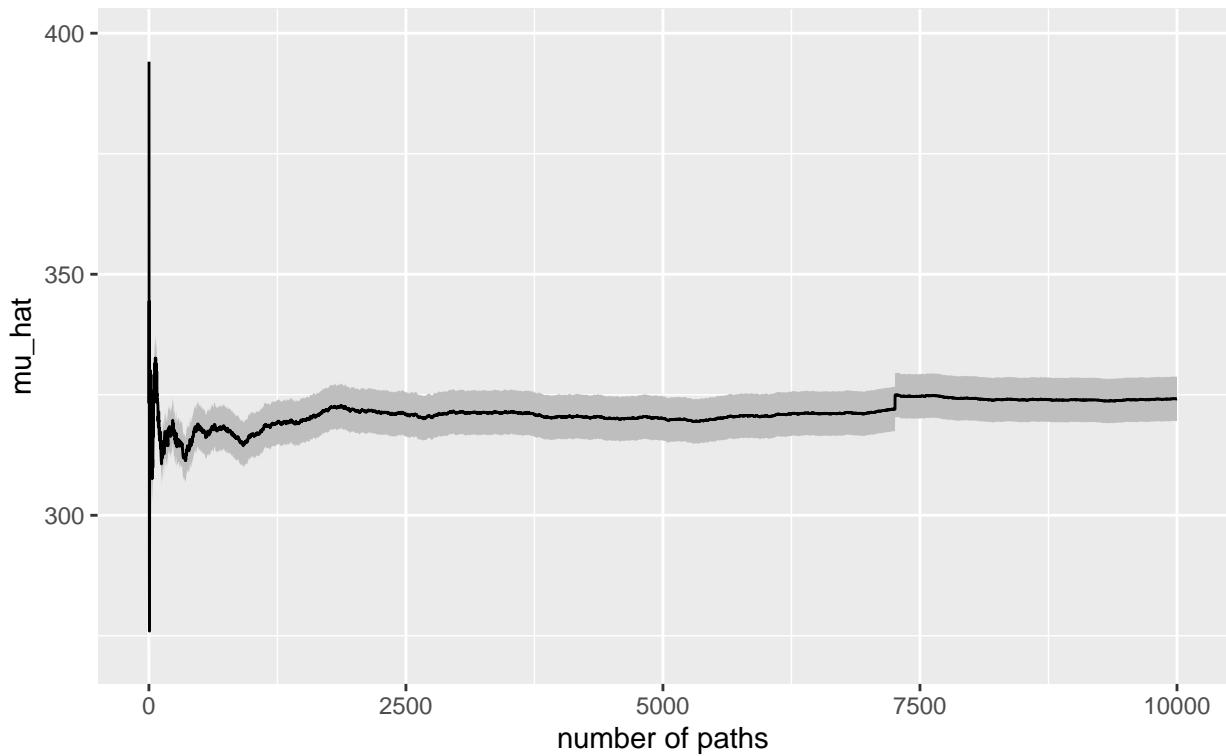
MS(3)



MS(4)



Monte Carlo
20 steps, 10000 paths

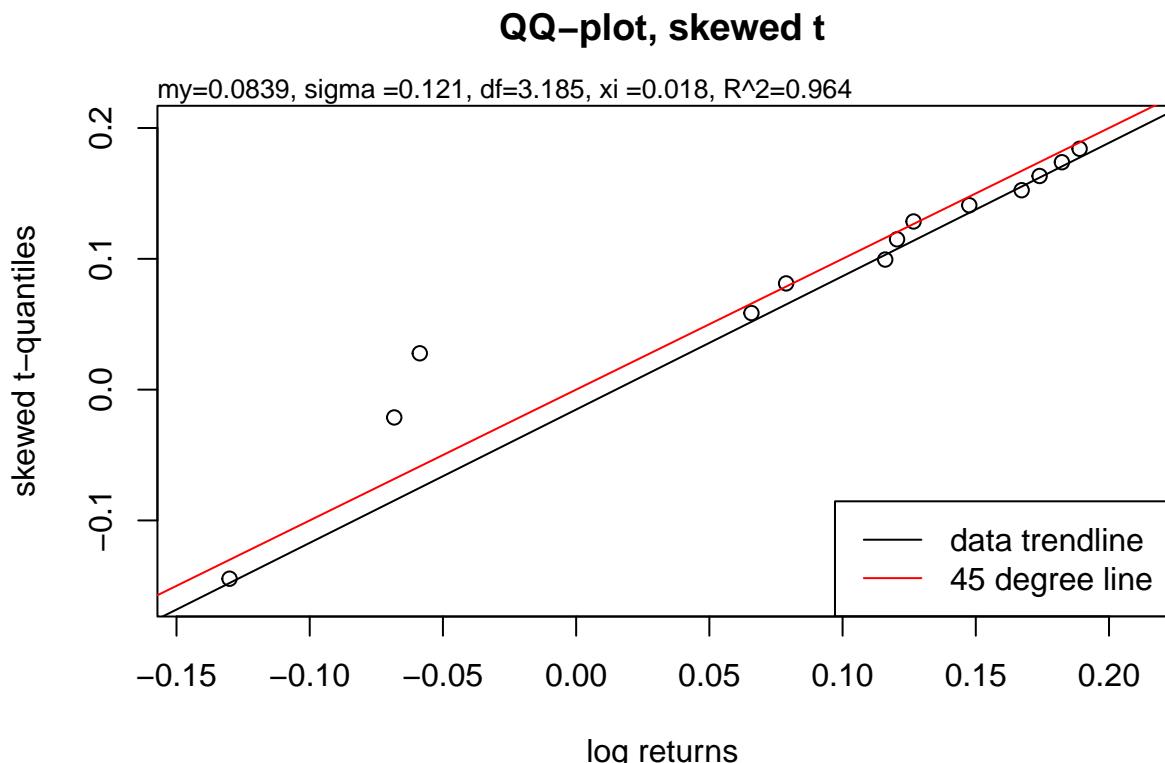


PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -23.72565  
## BIC: -21.46585  
## m: 0.08386034  
## s: 0.1210107  
## nu (df): 3.184569  
## xi: 0.01790306  
## R^2: 0.964  
##  
## An R^2 of 0.964 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 5.3 percent  
## What is the risk of losing max 25 %? <= 1.4 percent  
## What is the risk of losing max 50 %? <= 0.2 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 59.6 percent  
## What is the chance of gaining min 25 %? >= 0 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

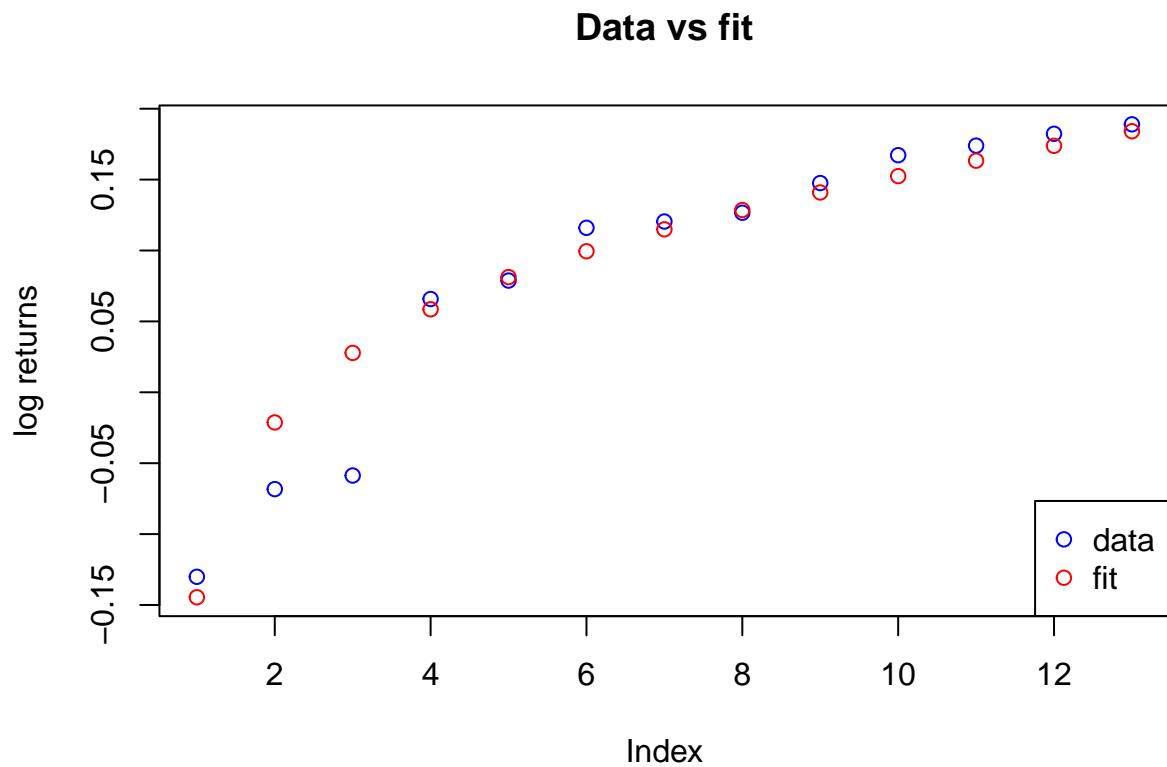
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

Data vs fit

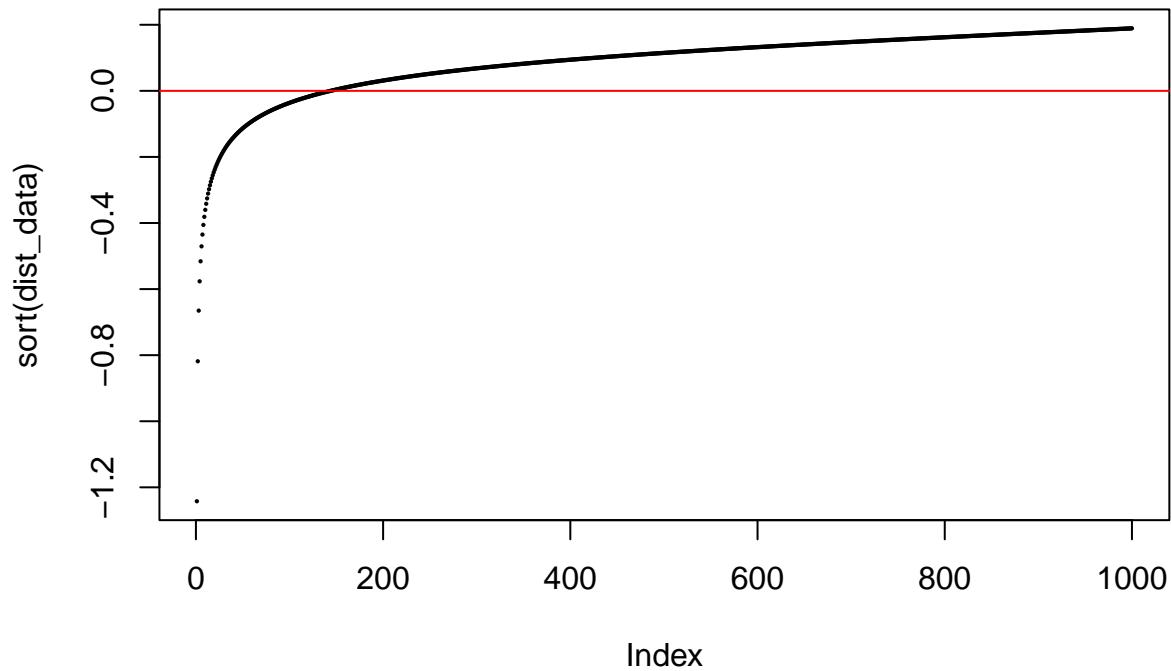
Let's plot the fit and the observed returns together.



Estimated distribution

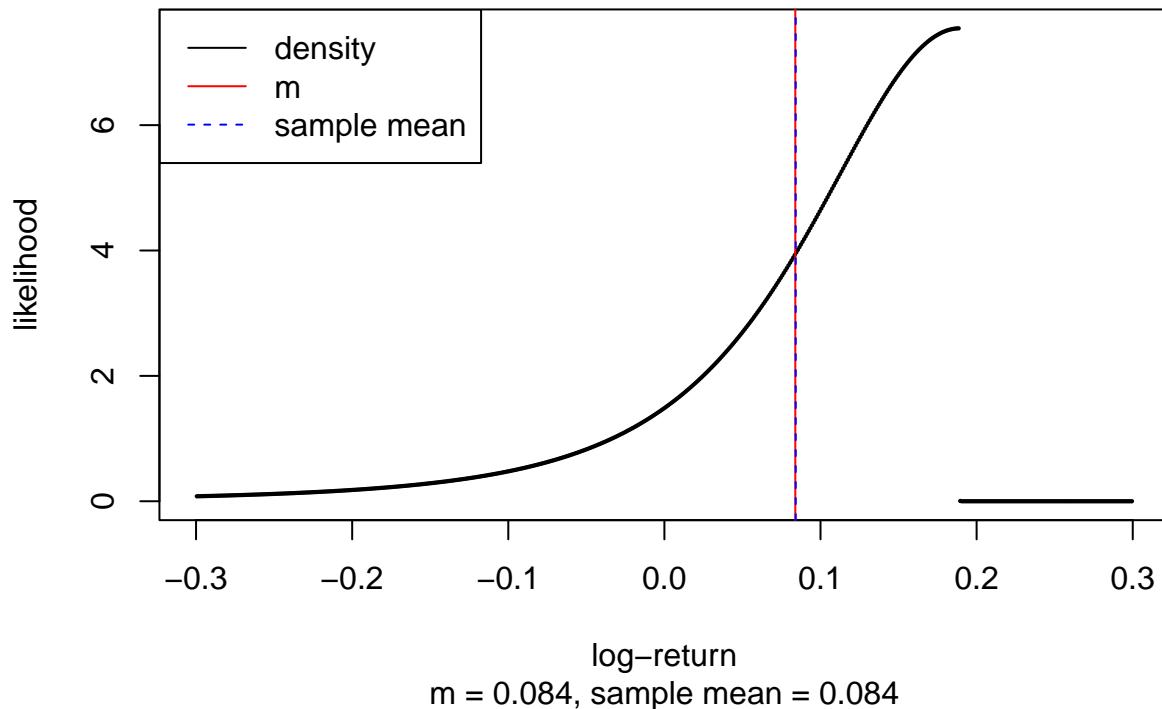
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

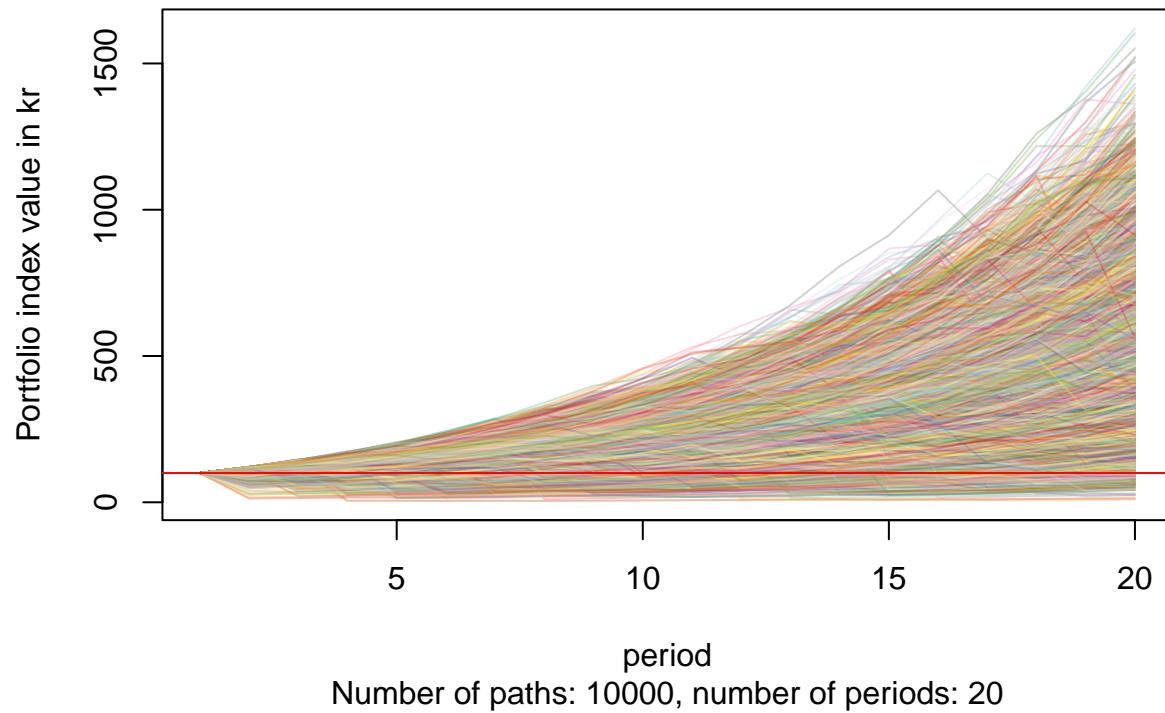
Estimated skew t distribution PDF



Monte Carlo

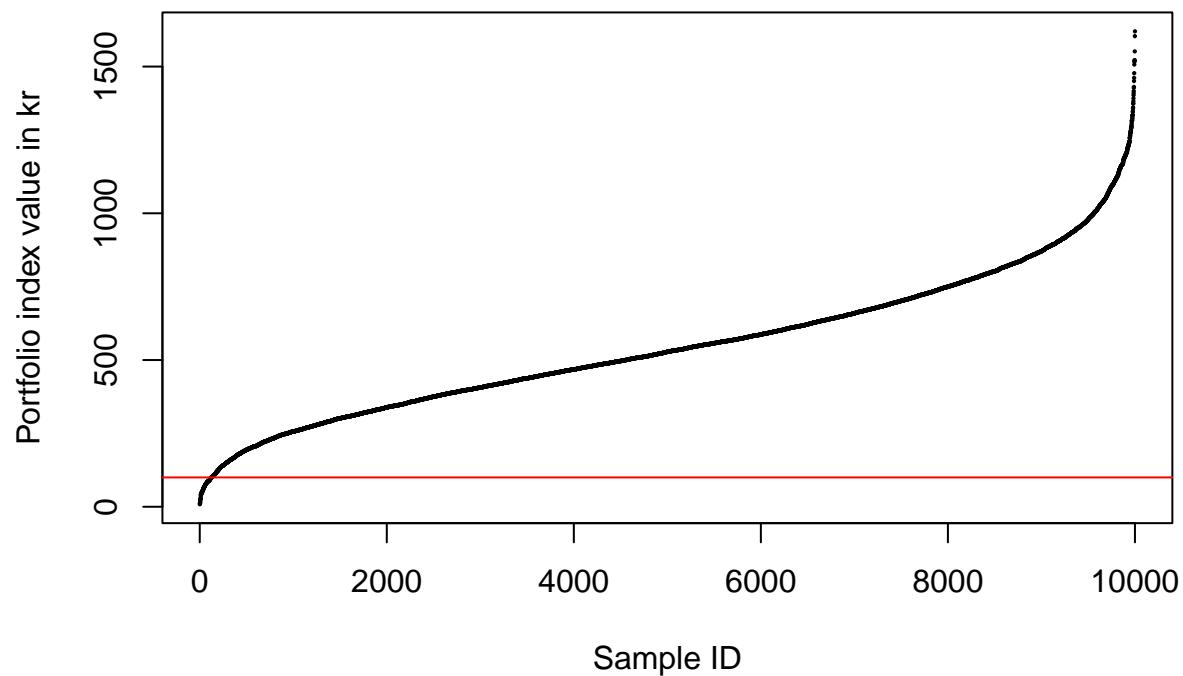
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 549.462 kr.  
## SD of portfolio index value after 20 years: 241.696 kr.  
## Min total portfolio index value after 20 years: 8.581 kr.  
## Max total portfolio index value after 20 years: 1620.013 kr.  
##  
## Share of paths finishing below 100: 1.24 percent
```

MC simulation with down-and-out



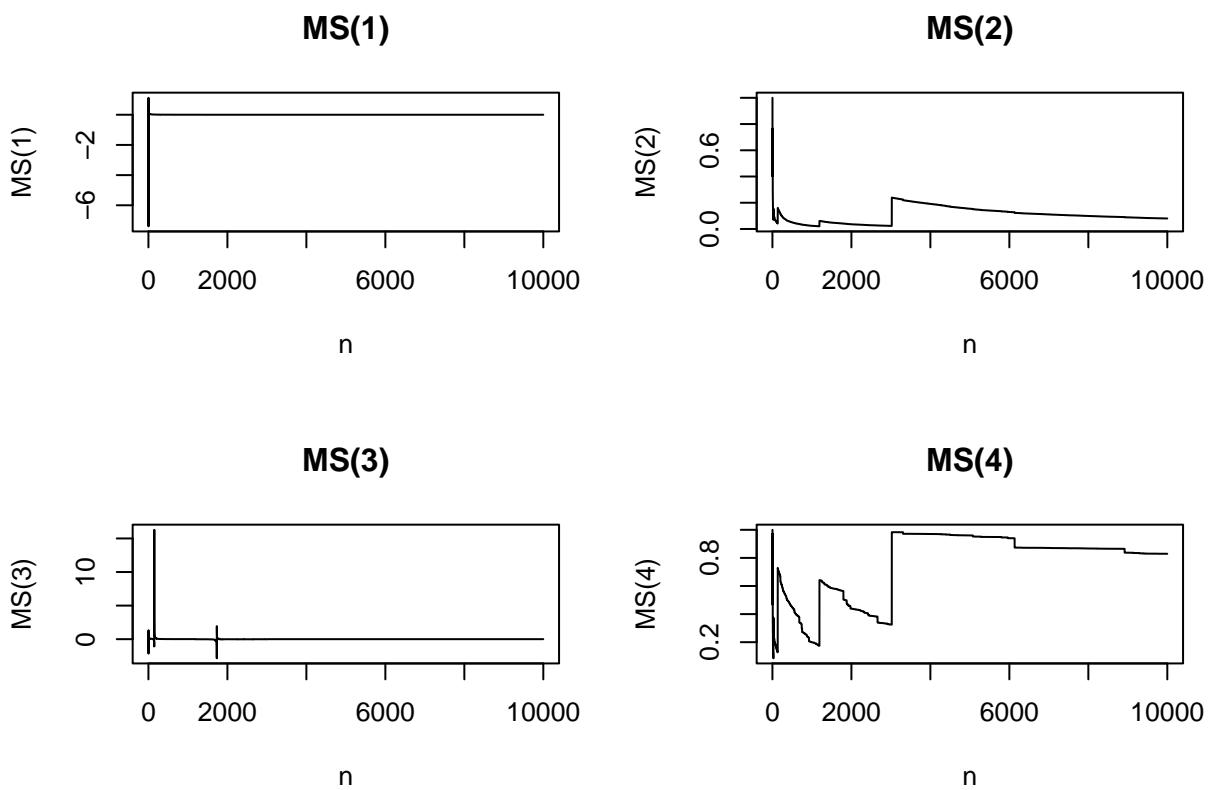
Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)

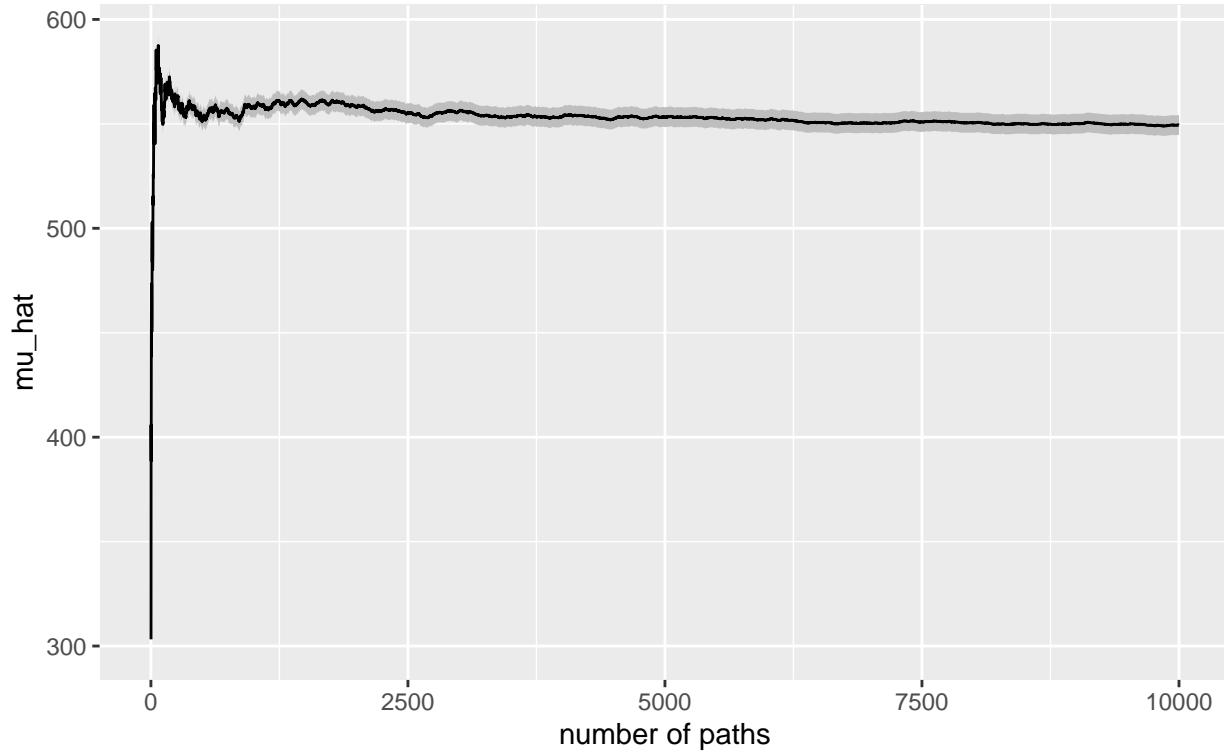


Convergence

Max vs sum plots for the first four moments:



Monte Carlo
20 steps, 10000 paths

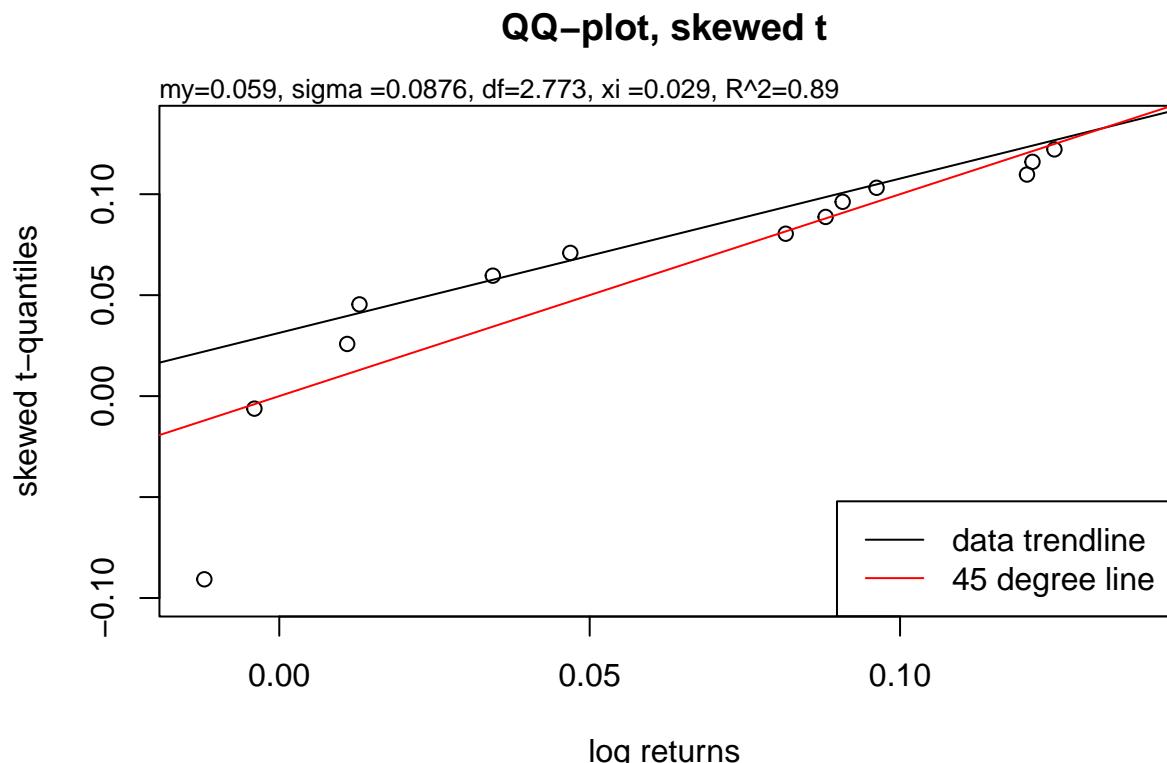


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -36.9603  
## BIC: -34.7005  
## m: 0.05902873  
## s: 0.08757749  
## nu (df): 2.772621  
## xi: 0.02904471  
## R^2: 0.89  
##  
## An R^2 of 0.89 suggests that the fit is not completely random.  
##  
## What is the risk of losing max 10 %? <= 3.3 percent  
## What is the risk of losing max 25 %? <= 0.7 percent  
## What is the risk of losing max 50 %? <= 0.1 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 35.6 percent  
## What is the chance of gaining min 25 %? >= 0 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

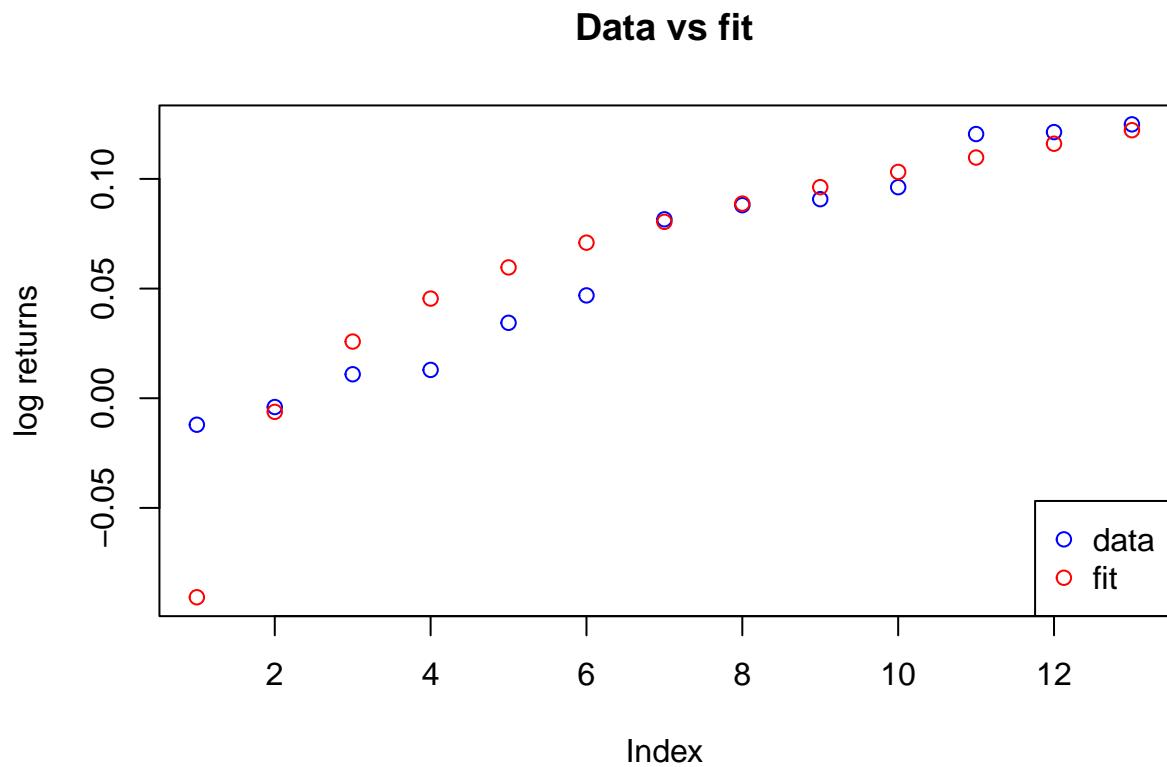
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

Let's plot the fit and the observed returns together.

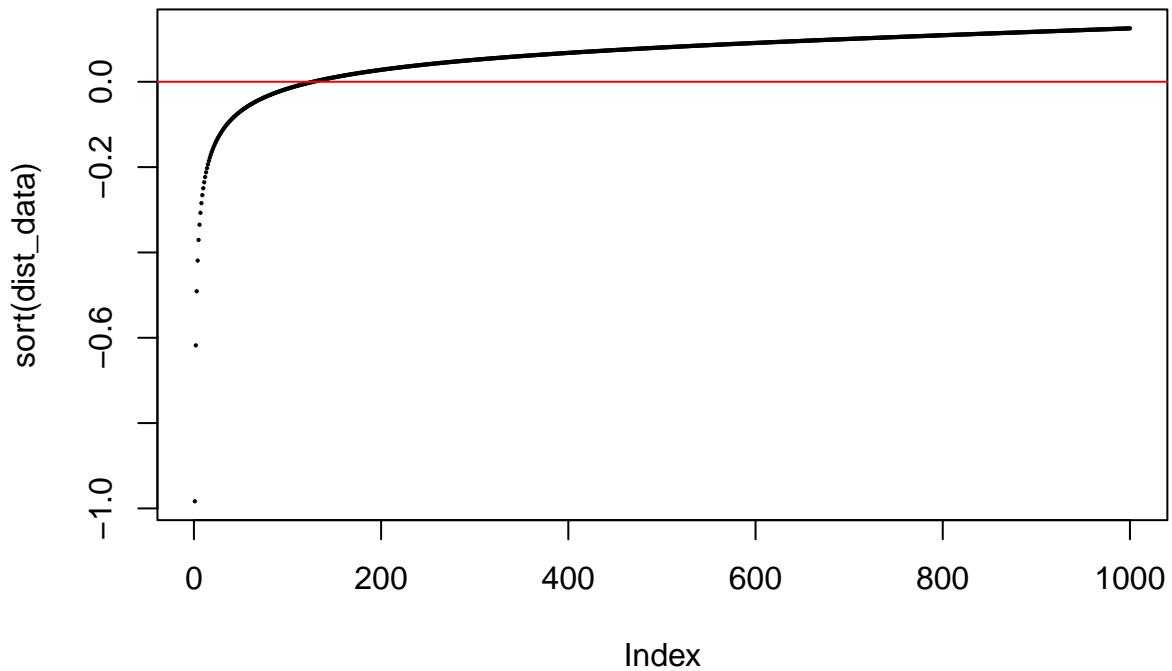


Interestingly, the fit predicts a much bigger “biggest loss” than the actual data. This is the main reason that R^2 is 0.90 and not higher.

Estimated distribution

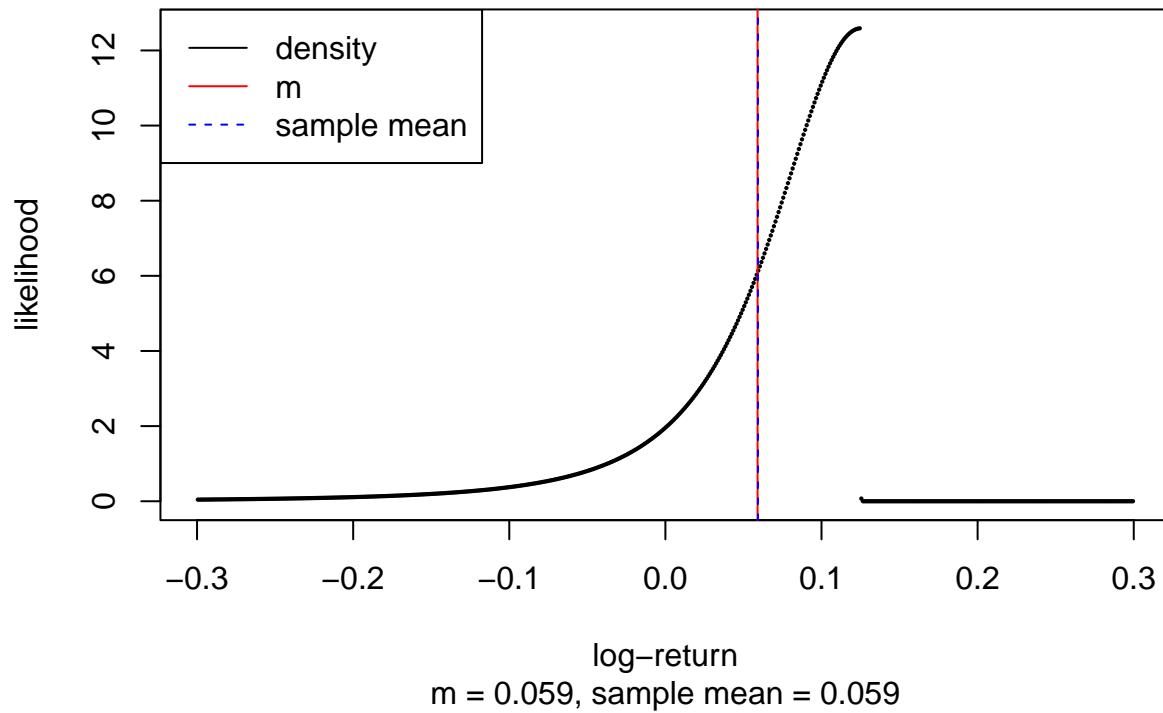
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Estimated skew t distribution PDF

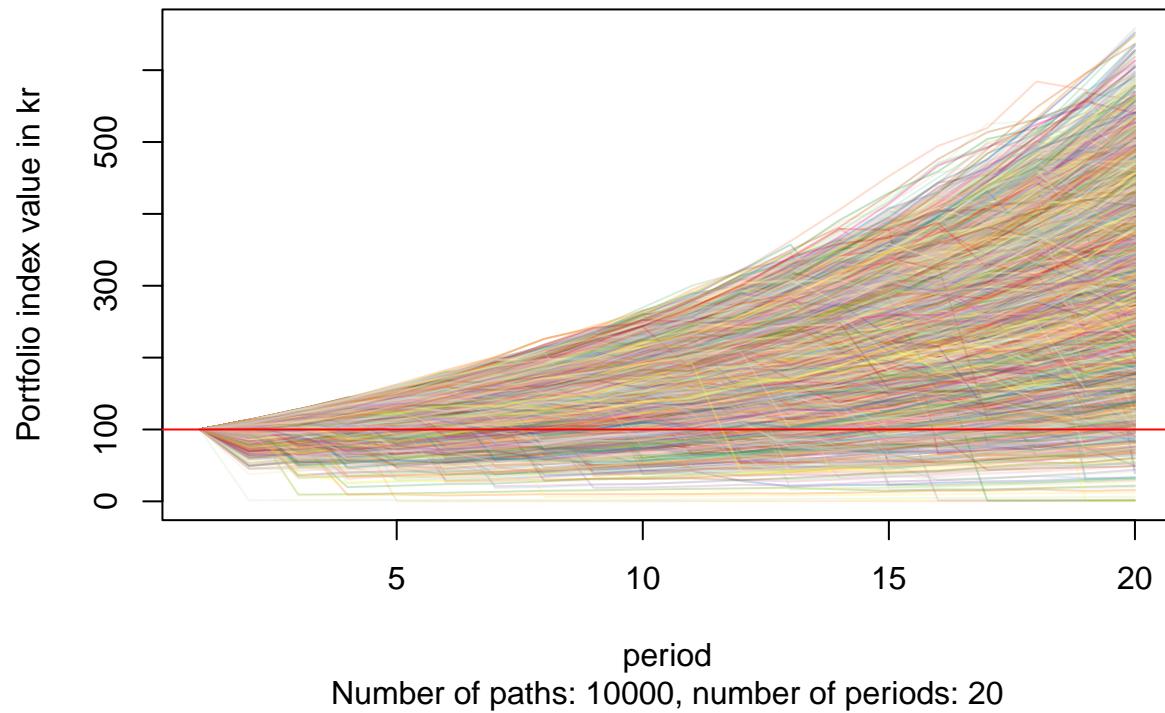


Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

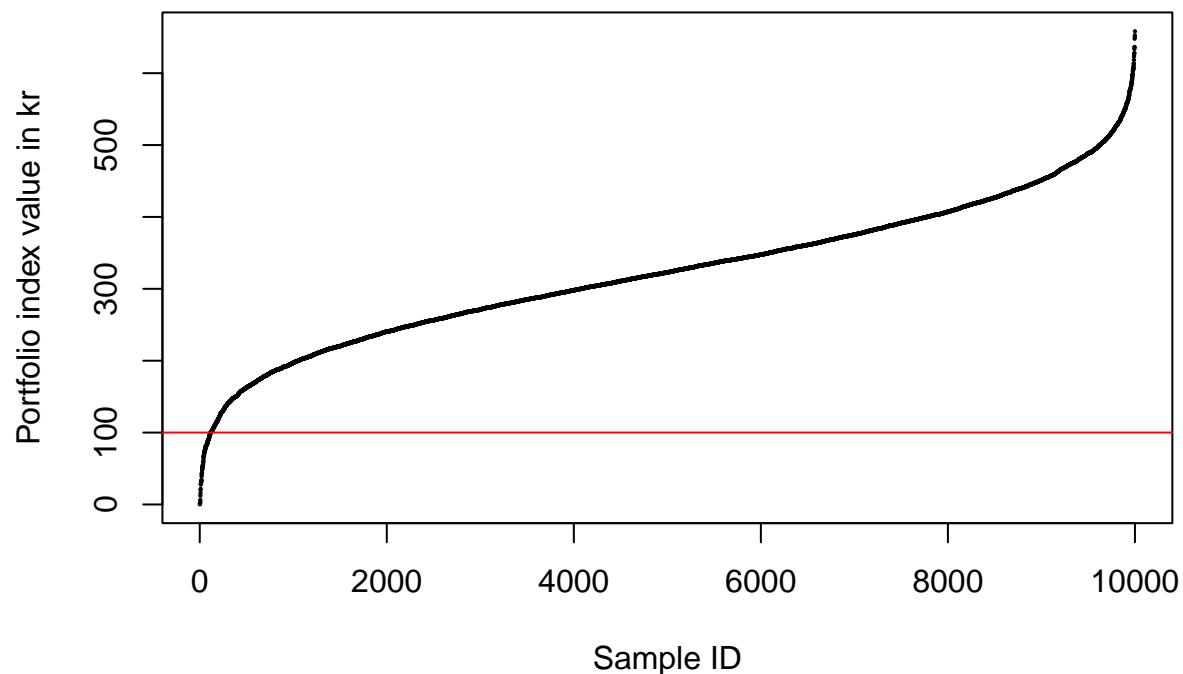
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 323.72 kr.  
## SD of portfolio index value after 20 years: 98.91 kr.  
## Min total portfolio index value after 20 years: 0.304 kr.  
## Max total portfolio index value after 20 years: 658.188 kr.  
##  
## Share of paths finishing below 100: 1.17 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

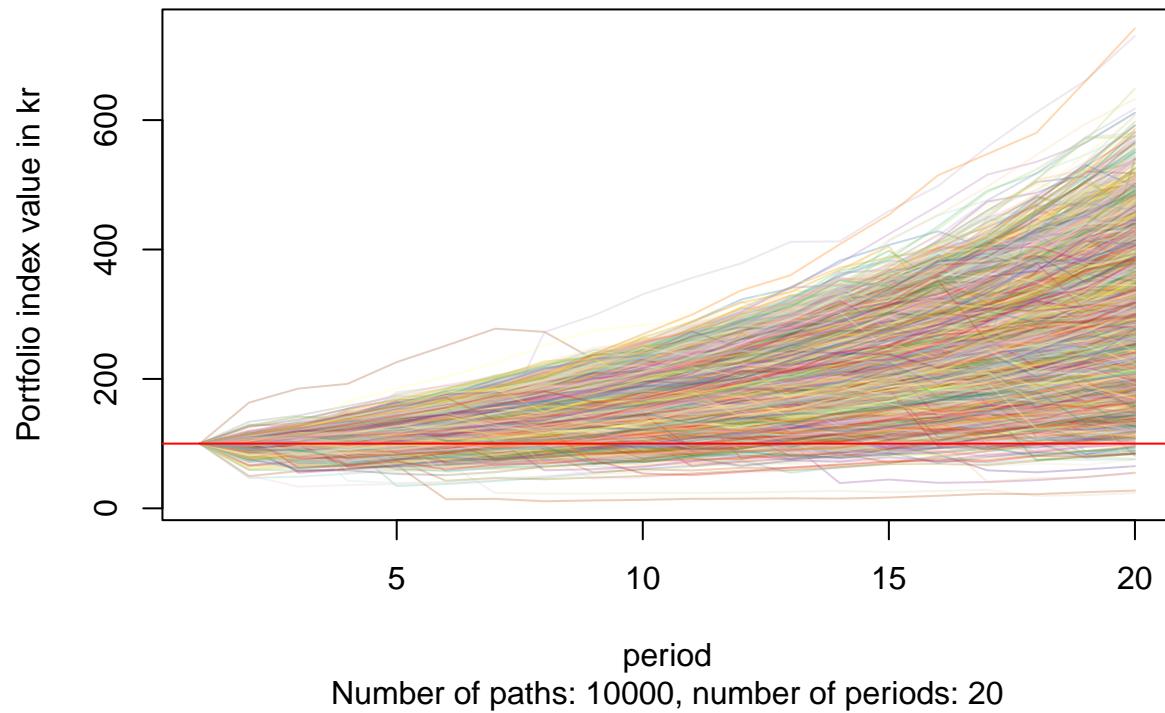
(100 is par, 200 is double, 50 is half)



Version b: Mix of simulations from estimated distribution of returns from individual funds.

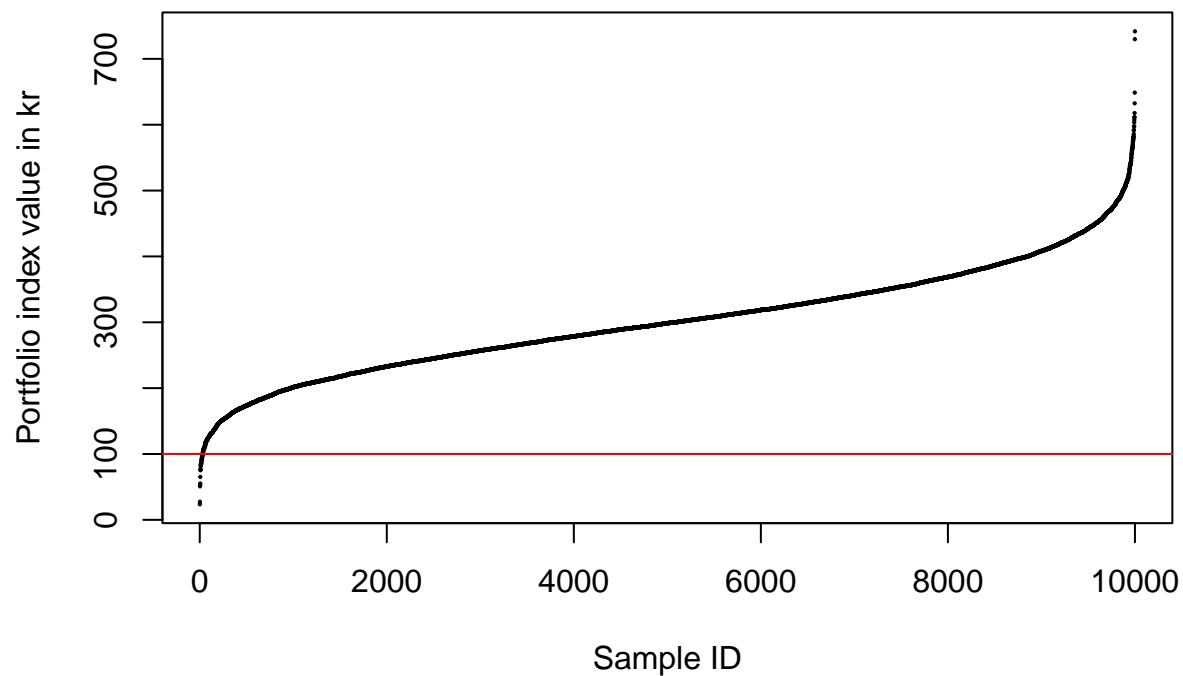
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 301.909 kr.  
## SD of portfolio index value after 20 years: 81.816 kr.  
## Min total portfolio index value after 20 years: 23.718 kr.  
## Max total portfolio index value after 20 years: 741.78 kr.  
##  
## Share of paths finishing below 100: 0.31 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

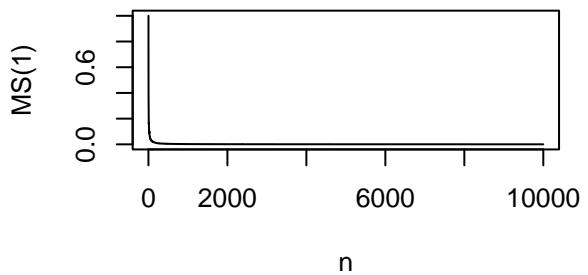
(100 is par, 200 is double, 50 is half)



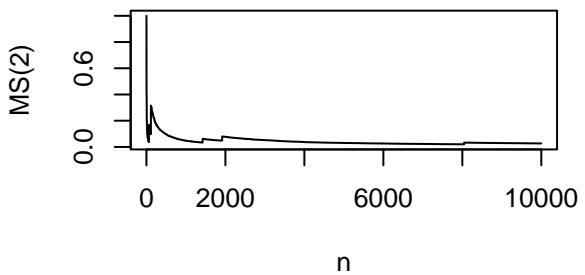
Convergence

Max vs sum plots for the first four moments:

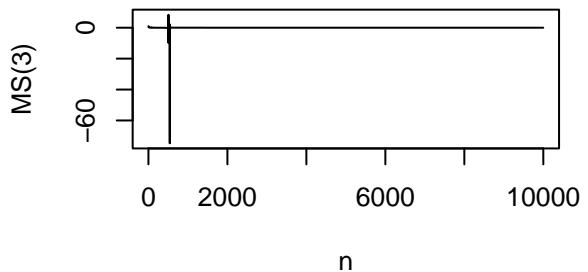
MS(1)



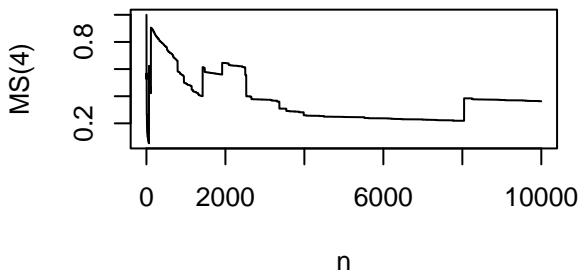
MS(2)



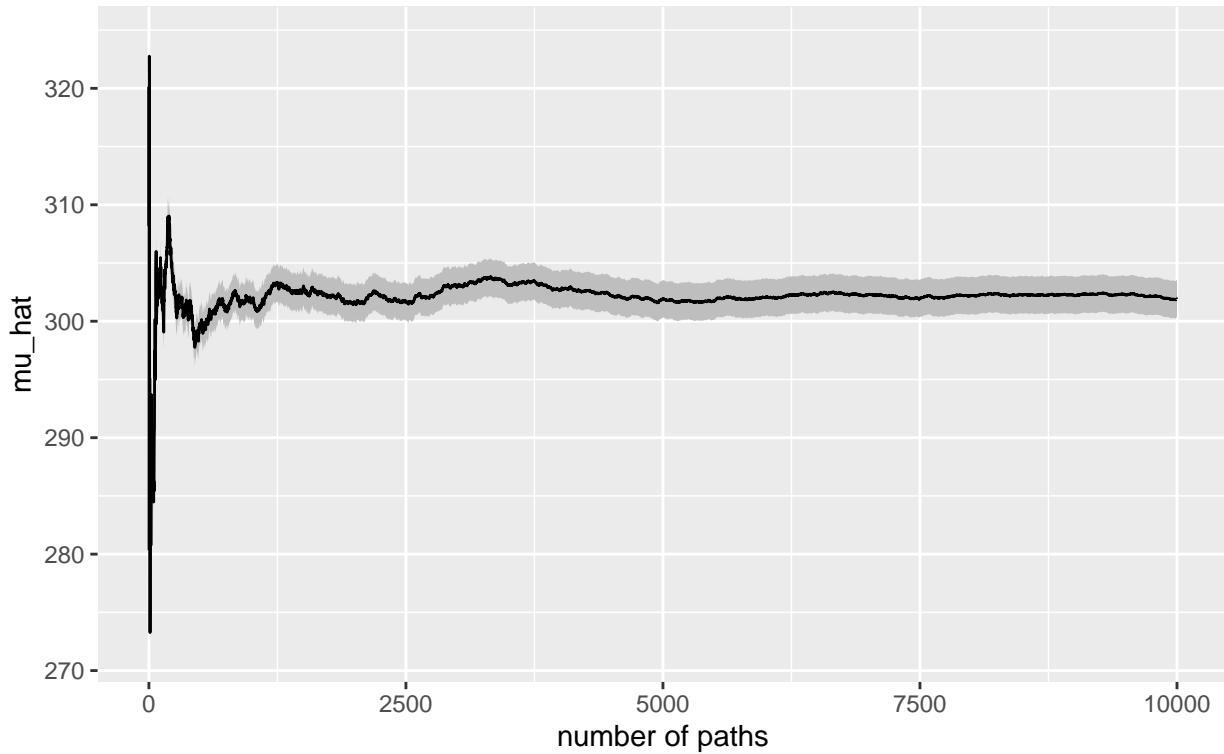
MS(3)



MS(4)



Monte Carlo
20 steps, 10000 paths

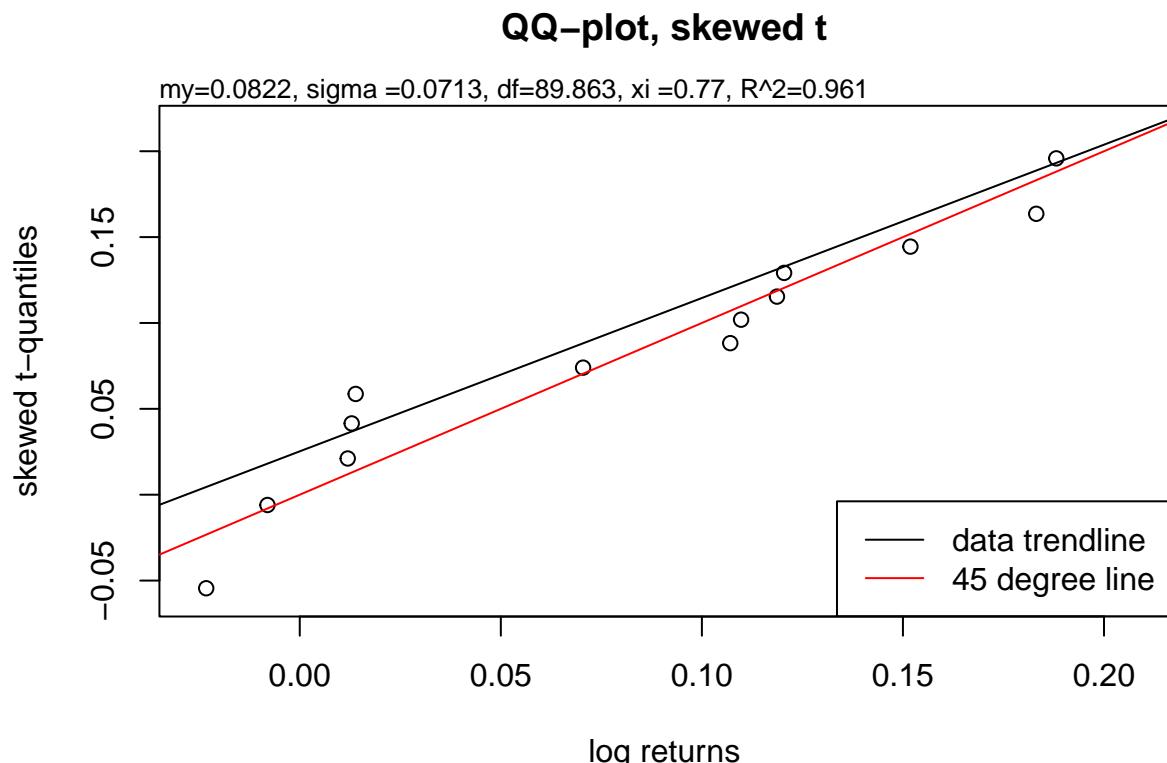


Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -24.26084  
## BIC: -22.00104  
## m: 0.0822419  
## s: 0.07129843  
## nu (df): 89.86289  
## xi: 0.7697502  
## R^2: 0.961  
##  
## An R^2 of 0.961 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 0.9 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 46.1 percent  
## What is the chance of gaining min 25 %? >= 1.2 percent  
## What is the chance of gaining min 50 %? >= 0 percent  
## What is the chance of gaining min 90 %? >= 0 percent  
## What is the chance of gaining min 99 %? >= 0 percent
```

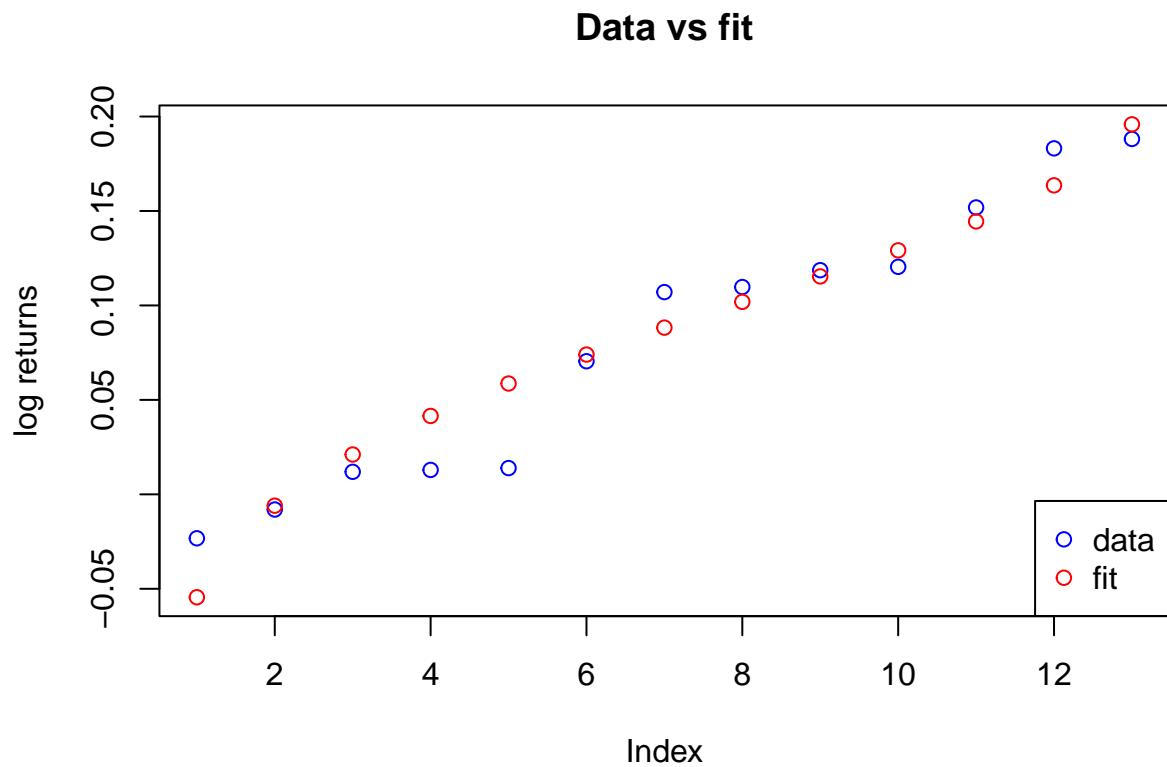
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

Data vs fit

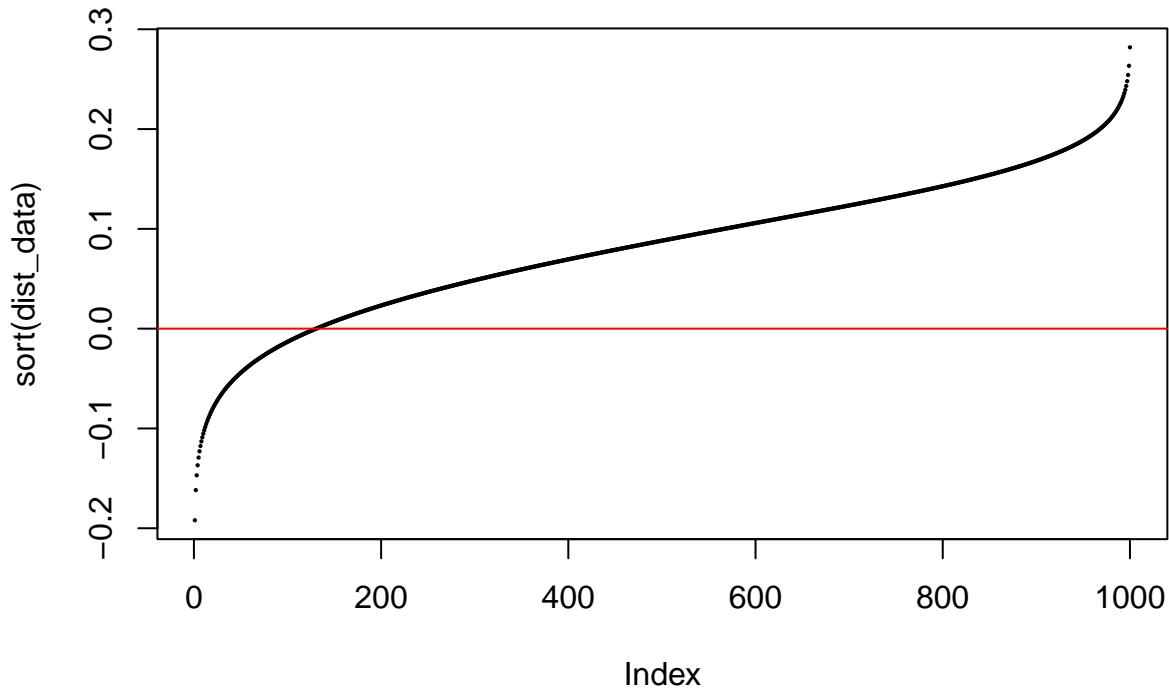
Let's plot the fit and the observed returns together.



Estimated distribution

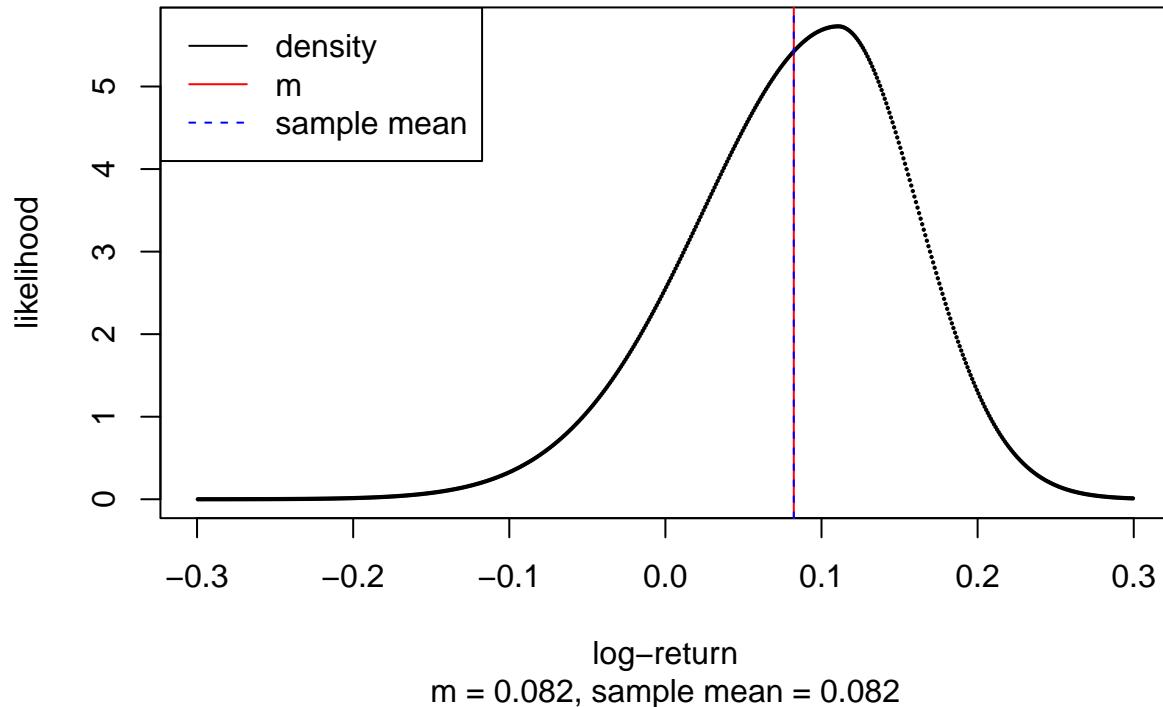
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF



We see that the high risk mix provides a much better upside and smaller downside.

Estimated skew t distribution PDF

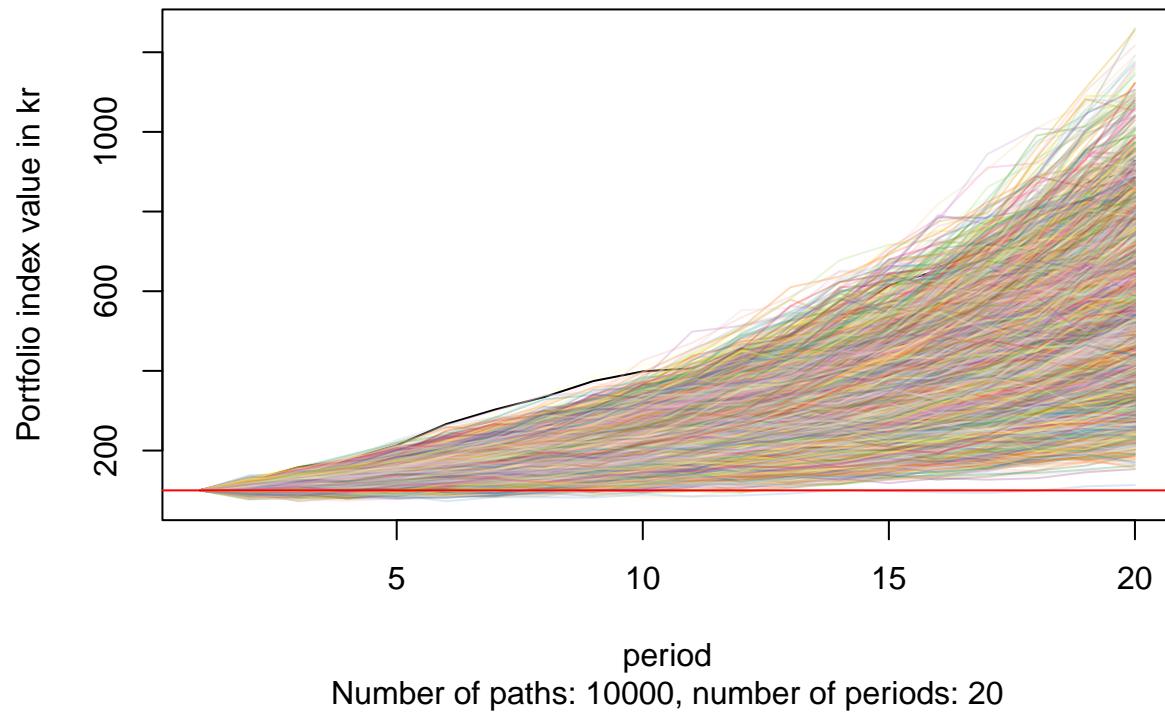


Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

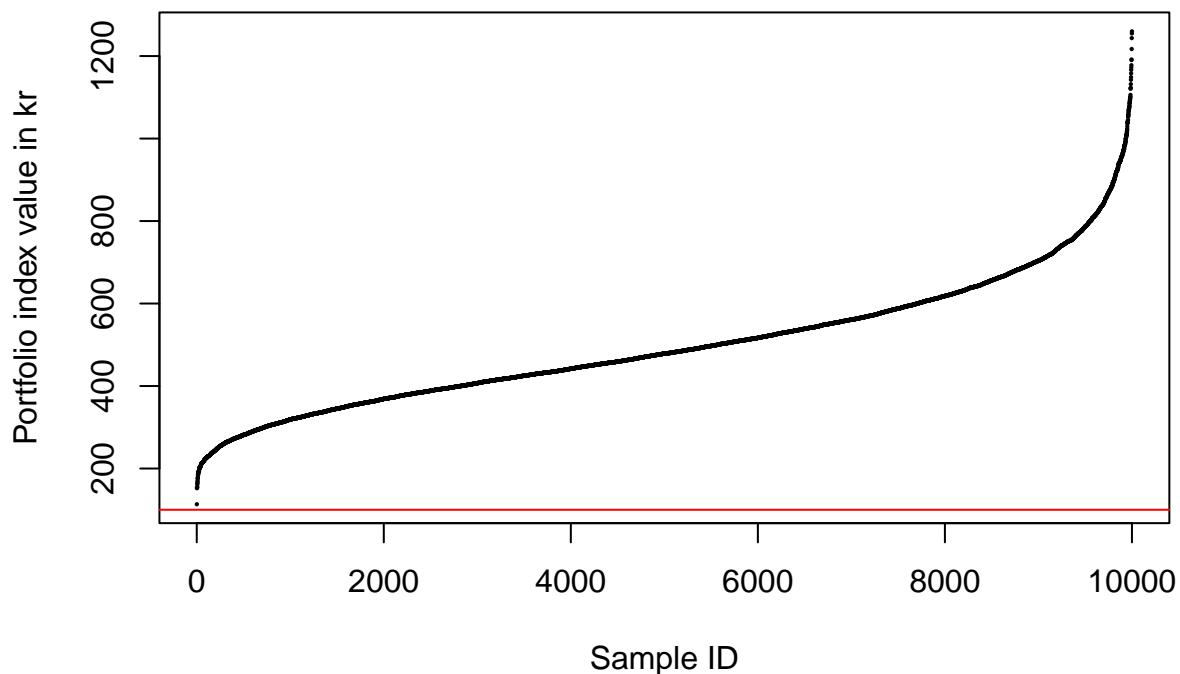
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 499.107 kr.  
## SD of portfolio index value after 20 years: 155.986 kr.  
## Min total portfolio index value after 20 years: 113.398 kr.  
## Max total portfolio index value after 20 years: 1259.941 kr.  
##  
## Share of paths finishing below 100: 0 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

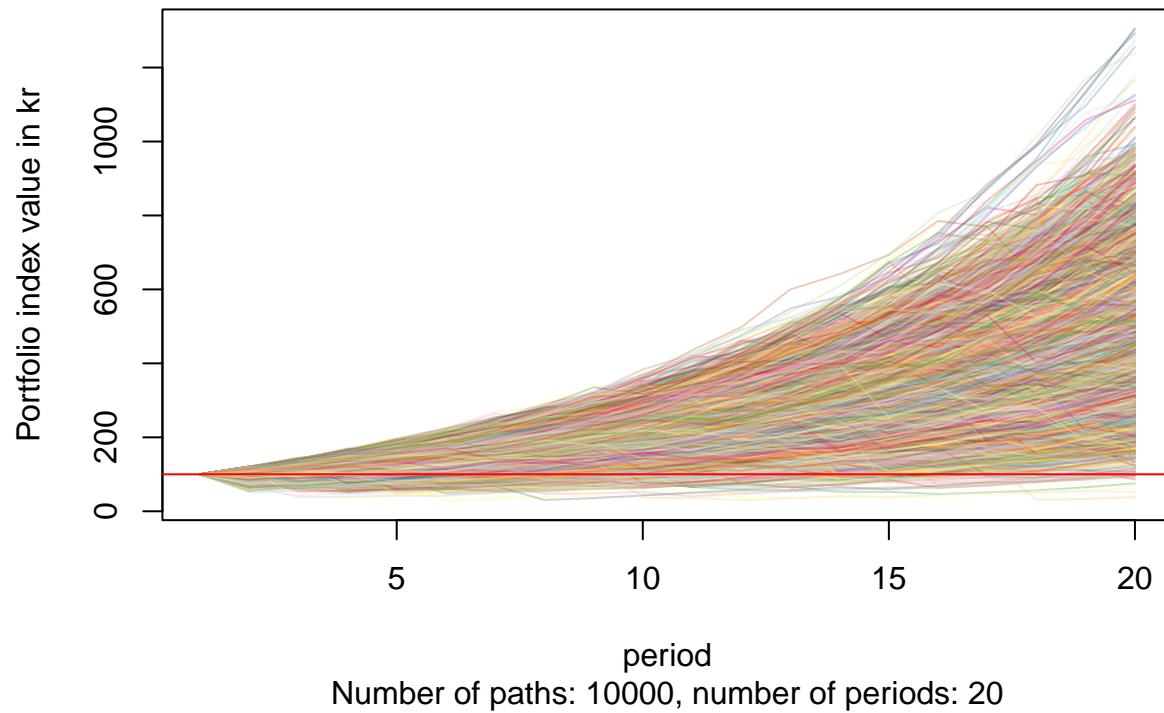
(100 is par, 200 is double, 50 is half)



Version b: Mix of simulations from estimated distribution of returns from individual funds.

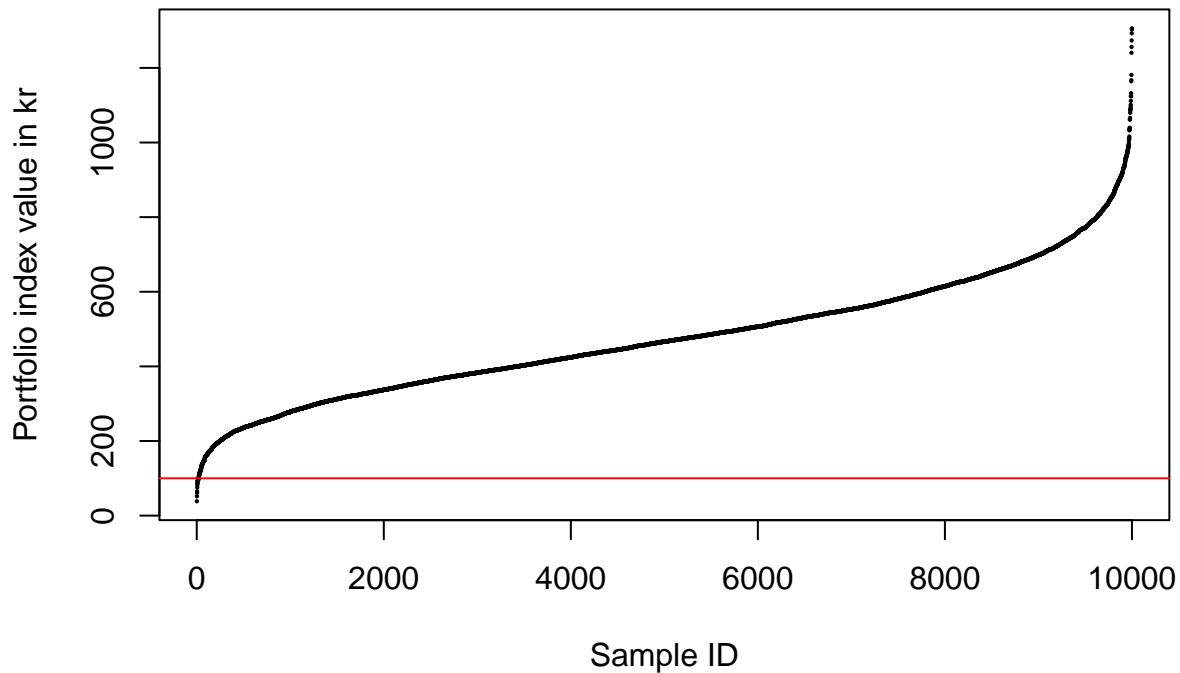
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 479.48 kr.  
## SD of portfolio index value after 20 years: 165.357 kr.  
## Min total portfolio index value after 20 years: 38.572 kr.  
## Max total portfolio index value after 20 years: 1306.009 kr.  
##  
## Share of paths finishing below 100: 0.18 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)



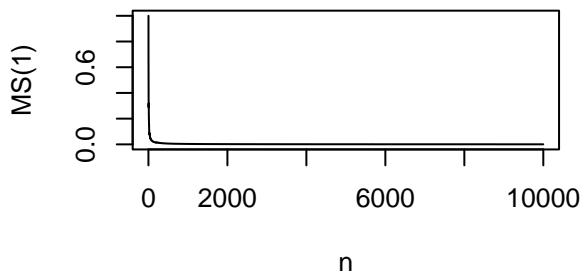
Many simulations 1e6 paths:

```
# Down-and-out simulation:  
# Probability of down-and-out: 0 percent  
#  
# Mean portfolio index value after 20 years: 478.339 kr.  
# SD of portfolio index value after 20 years: 163.093 kr.  
# Min total portfolio index value after 20 years: 2.233 kr.  
# Max total portfolio index value after 20 years: 1561.965 kr.  
#  
# Share of paths finishing below 100: 0.1181 percent
```

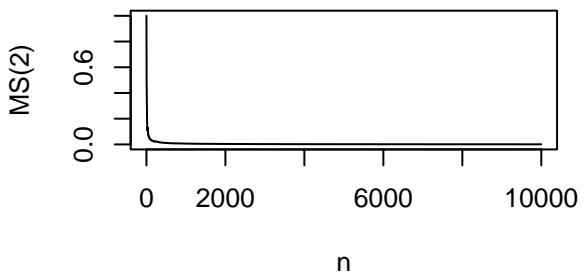
Convergence

Max vs sum plots for the first four moments:

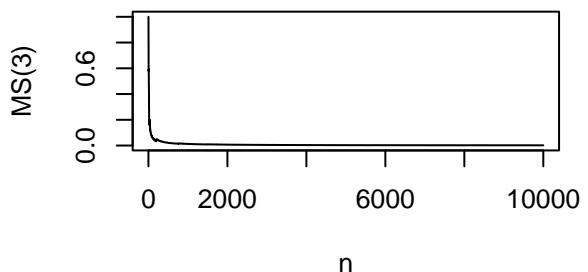
MS(1)



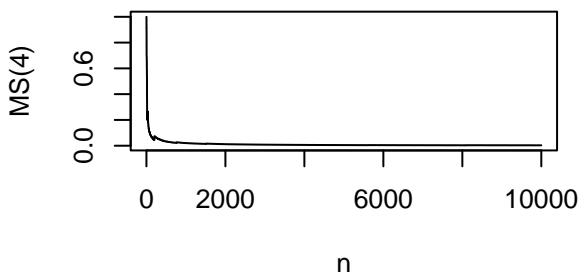
MS(2)



MS(3)

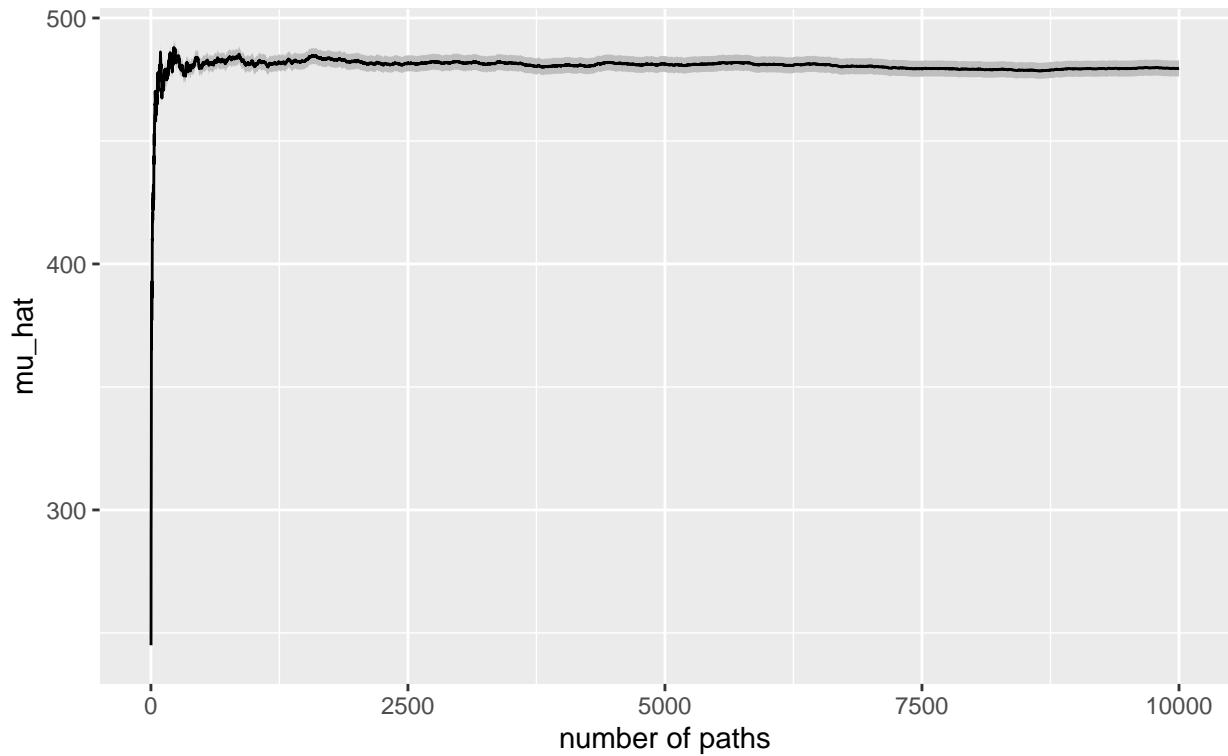


MS(4)



Monte Carlo

20 steps, 10000 paths



Compare pension plans

Risk of max loss

Risk of max loss of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	21.3	18.2	19.9	12.2	14.3	12.7	13.0
5	12.5	9.6	12.8	6.0	8.6	6.2	4.2
10	7.4	5.4	8.3	3.3	5.3	3.3	0.9
25	1.8	1.3	2.5	0.9	1.4	0.7	0.0
50	0.2	0.2	0.4	0.2	0.2	0.1	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.3	Velliv_m	12.8	Velliv_h	8.3	Velliv_h	2.5	Velliv_h	0.4	Velliv_h	0	Velliv_m	0	Velliv_m
19.9	Velliv_h	12.5	Velliv_m	7.4	Velliv_m	1.8	Velliv_m	0.2	Velliv_m	0	Velliv_m_l	0	Velliv_m_l
18.2	Velliv_m_l	9.6	Velliv_m_l	5.4	Velliv_m_l	1.4	PFA_h	0.2	Velliv_m_l	0	Velliv_h	0	Velliv_h
14.3	PFA_h	8.6	PFA_h	5.3	PFA_h	1.3	Velliv_m_l	0.2	PFA_m	0	PFA_m	0	PFA_m
13.0	mix_h	6.2	mix_m	3.3	PFA_m	0.9	PFA_m	0.2	PFA_h	0	PFA_h	0	PFA_h
12.7	mix_m	6.0	PFA_m	3.3	mix_m	0.7	mix_m	0.1	mix_m	0	mix_m	0	mix_m
12.2	PFA_m	4.2	mix_h	0.9	mix_h	0.0	mix_h	0.0	mix_h	0	mix_h	0	mix_h

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	78.7	81.8	80.1	87.8	85.7	87.3	87.0
5	63.8	64.9	69.2	71.5	75.8	71.4	69.9
10	41.0	36.2	53.3	32.7	59.6	35.6	46.1
25	0.0	0.3	0.0	0.1	0.0	0.0	1.2
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
87.8	PFA_m	75.8	PFA_h	59.6	PFA_h	1.2	mix_h	0	Velliv_m	0	Velliv_m
87.3	mix_m	71.5	PFA_m	53.3	Velliv_h	0.3	Velliv_m_l	0	Velliv_m_l	0	Velliv_m_l
87.0	mix_h	71.4	mix_m	46.1	mix_h	0.1	PFA_m	0	Velliv_h	0	Velliv_h
85.7	PFA_h	69.9	mix_h	41.0	Velliv_m	0.0	Velliv_m	0	PFA_m	0	PFA_m
81.8	Velliv_m_l	69.2	Velliv_h	36.2	Velliv_m_l	0.0	Velliv_h	0	PFA_h	0	PFA_h
80.1	Velliv_h	64.9	Velliv_m_l	35.6	mix_m	0.0	PFA_h	0	mix_m	0	mix_m
78.7	Velliv_m	63.8	Velliv_m	32.7	PFA_m	0.0	mix_m	0	mix_h	0	mix_h

MC risk percentiles

Risk of loss from first to last period.

_a is simulation from estimated distribution of mix.

_b is mix of simulations from estimated distribution of returns from individual funds.

_m is medium.

_h is high.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	5.24	3.23	3.89	1.83	1.24	1.17	0	0.31	0.18
5	4.50	2.93	3.44	1.71	1.13	1.03	0	0.23	0.13
10	3.86	2.66	3.16	1.49	1.02	0.92	0	0.19	0.08
25	2.48	1.64	2.17	0.97	0.61	0.54	0	0.06	0.04
50	0.88	0.72	0.82	0.55	0.25	0.28	0	0.02	0.01
90	0.08	0.12	0.18	0.16	0.01	0.06	0	0.00	0.00
99	0.02	0.03	0.06	0.02	0.00	0.02	0	0.00	0.00

1e6 simulation paths of mhr_b:

	0	5	10	25	50	90	99
prob_pct	0.118	0.095	0.076	0.036	0.008	0	0

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
5.24	Velliv_m	4.50	Velliv_m	3.86	Velliv_m	2.48	Velliv_m	0.88	Velliv_m	0.18	Velliv_h	0.06	Velliv_h
3.89	Velliv_h	3.44	Velliv_h	3.16	Velliv_h	2.17	Velliv_h	0.82	Velliv_h	0.16	PFA_m	0.03	Velliv_m_l
3.23	Velliv_m_l	2.93	Velliv_m_l	2.66	Velliv_m_l	1.64	Velliv_m_l	0.72	Velliv_m_l	0.12	Velliv_m_l	0.02	Velliv_m
1.83	PFA_m	1.71	PFA_m	1.49	PFA_m	0.97	PFA_m	0.55	PFA_m	0.08	Velliv_m	0.02	PFA_m
1.24	PFA_h	1.13	PFA_h	1.02	PFA_h	0.61	PFA_h	0.28	mix_m_a	0.06	mix_m_a	0.02	mix_m_a
1.17	mix_m_a	1.03	mix_m_a	0.92	mix_m_a	0.54	mix_m_a	0.25	PFA_h	0.01	PFA_h	0.00	PFA_h
0.31	mix_m_b	0.23	mix_m_b	0.19	mix_m_b	0.06	mix_m_b	0.02	mix_m_b	0.00	mix_h_a	0.00	mix_h_a

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.18	mix_h_b	0.13	mix_h_b	0.08	mix_h_b	0.04	mix_h_b	0.01	mix_h_b	0.00	mix_m_b	0.00	mix_m_b
0.00	mix_h_a	0.00	mix_h_b	0.00	mix_h_b								

MC gains percentiles

Chance of gains from first to last period.

_a is simulation from estimated distribution of returns of mix.

_b is mix of simulations from estimated distribution of returns from individual funds.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	94.76	96.77	96.11	98.17	98.76	98.83	100.00	99.69	99.82
5	94.09	96.36	95.61	97.97	98.59	98.60	100.00	99.62	99.78
10	93.34	95.80	95.15	97.74	98.44	98.39	100.00	99.51	99.73
25	90.80	94.26	93.69	97.05	98.01	97.81	99.99	99.07	99.53
50	85.28	90.61	90.73	95.23	97.14	96.15	99.99	97.73	99.17
100	70.68	78.83	82.94	87.98	94.58	89.49	99.71	90.25	97.55
200	39.59	45.60	64.84	58.76	85.24	59.26	92.76	49.14	87.04
300	16.16	17.66	45.63	22.13	71.04	22.24	71.79	11.49	65.74
400	5.17	4.88	29.62	4.44	54.44	3.76	44.33	1.28	41.50
500	1.43	1.23	17.89	0.51	38.08	0.23	22.95	0.09	22.10
1000	0.00	0.02	0.57	0.03	2.31	0.00	0.19	0.00	0.14

1e6 simulation paths of mhr_b:

	0	5	10	25	50	100	200	300	400	500	1000
prob	99.882	99.854	99.824	99.686	99.301	97.513	86.912	65.992	41.486	21.693	0.086

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	99.99	mix_h_a	99.99	mix_h_a	99.71	mix_h_a
99.82	mix_h_b	99.78	mix_h_b	99.73	mix_h_b	99.53	mix_h_b	99.17	mix_h_b	97.55	mix_h_b
99.69	mix_m_b	99.62	mix_m_b	99.51	mix_m_b	99.07	mix_m_b	97.73	mix_m_b	94.58	PFA_h
98.83	mix_m_a	98.60	mix_m_a	98.44	PFA_h	98.01	PFA_h	97.14	PFA_h	90.25	mix_m_b
98.76	PFA_h	98.59	PFA_h	98.39	mix_m_a	97.81	mix_m_a	96.15	mix_m_a	89.49	mix_m_a
98.17	PFA_m	97.97	PFA_m	97.74	PFA_m	97.05	PFA_m	95.23	PFA_m	87.98	PFA_m
96.77	Velliv_m_l	96.36	Velliv_m_l	95.80	Velliv_m_l	94.26	Velliv_m_l	90.73	Velliv_h	82.94	Velliv_h
96.11	Velliv_h	95.61	Velliv_h	95.15	Velliv_h	93.69	Velliv_h	90.61	Velliv_m_l	78.83	Velliv_m_l
94.76	Velliv_m	94.09	Velliv_m	93.34	Velliv_m	90.80	Velliv_m	85.28	Velliv_m	70.68	Velliv_m

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
92.76	mix_h_a	71.79	mix_h_a	54.44	PFA_h	38.08	PFA_h	2.31	PFA_h
87.04	mix_h_b	71.04	PFA_h	44.33	mix_h_a	22.95	mix_h_a	0.57	Velliv_h
85.24	PFA_h	65.74	mix_h_b	41.50	mix_h_b	22.10	mix_h_b	0.19	mix_h_a
64.84	Velliv_h	45.63	Velliv_h	29.62	Velliv_h	17.89	Velliv_h	0.14	mix_h_b
59.26	mix_m_a	22.24	mix_m_a	5.17	Velliv_m	1.43	Velliv_m	0.03	PFA_m
58.76	PFA_m	22.13	PFA_m	4.88	Velliv_m_l	1.23	Velliv_m_l	0.02	Velliv_m_l
49.14	mix_m_b	17.66	Velliv_m_l	4.44	PFA_m	0.51	PFA_m	0.00	Velliv_m
45.60	Velliv_m_l	16.16	Velliv_m	3.76	mix_m_a	0.23	mix_m_a	0.00	mix_m_a
39.59	Velliv_m	11.49	mix_m_b	1.28	mix_m_b	0.09	mix_m_b	0.00	mix_m_b

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.

m is the location parameter.

s is the scale parameter.

nu is the estimated degrees of freedom, or shape parameter.

xi is the estimated skewness parameter.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770
R-squared	0.993	0.978	0.991	0.991	0.964	0.890	0.961

Fit statistics ranking

m	ranking	s	ranking	R-squared	ranking
0.084	PFA_high	0.071	mix_high	0.993	Velliv_medium
0.082	mix_high	0.088	mix_medium	0.991	Velliv_high
0.065	Velliv_high	0.115	Velliv_medium_long	0.991	PFA_medium
0.059	mix_medium	0.120	Velliv_medium	0.978	Velliv_medium_long
0.058	PFA_medium	0.121	PFA_high	0.964	PFA_high
0.052	Velliv_medium_long	0.123	PFA_medium	0.961	mix_high
0.048	Velliv_medium	0.150	Velliv_high	0.890	mix_medium

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).

Statistics are given for the final state of all paths.

Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.

0 is defined as any value below a threshold.

losing_prob_pct is the probability of losing money. This is calculated as the share of paths finishing below index 100.

Number of paths: 10000

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_m_b	mix_h_a	mix_h_b
mc_m	278.527	316.525	407.438	324.155	549.462	323.720	301.909	499.107	479.480
mc_s	124.320	2149.399	216.472	234.349	241.696	98.910	81.816	155.986	165.357
mc_min	0.567	0.000	0.000	0.000	8.581	0.304	23.718	113.398	38.572
mc_max	909.277	214912.130	1625.391	21280.182	1620.013	658.188	741.780	1259.941	1306.009
dao_pct	0.000	0.020	0.010	0.010	0.000	0.000	0.000	0.000	0.000
losing_pct	5.240	3.230	3.890	1.830	1.240	1.170	0.310	0.000	0.180

Ranking

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	losing_pct	ranking
549.462	PFA_h	81.816	mix_m_b	113.398	mix_h_a	214912.130	Velliv_m_l	0.00	Velliv_m	0.00	mix_h_a
499.107	mix_h_a	98.910	mix_m_a	38.572	mix_h_b	21280.182	PFA_m	0.00	PFA_h	0.18	mix_h_b
479.480	mix_h_b	124.320	Velliv_m	23.718	mix_m_b	1625.391	Velliv_h	0.00	mix_m_a	0.31	mix_m_b
407.438	Velliv_h	155.986	mix_h_a	8.581	PFA_h	1620.013	PFA_h	0.00	mix_m_b	1.17	mix_m_a
324.155	PFA_m	165.357	mix_h_b	0.567	Velliv_m	1306.009	mix_h_b	0.00	mix_h_a	1.24	PFA_h
323.720	mix_m_a	216.472	Velliv_h	0.304	mix_m_a	1259.941	mix_h_a	0.00	mix_h_b	1.83	PFA_m
316.525	Velliv_m_l	234.349	PFA_m	0.000	Velliv_m_l	909.277	Velliv_m	0.01	Velliv_h	3.23	Velliv_m_l
301.909	mix_m_b	241.696	PFA_h	0.000	Velliv_h	741.780	mix_m_b	0.01	PFA_m	3.89	Velliv_h
278.527	Velliv_m	2149.399	Velliv_m_l	0.000	PFA_m	658.188	mix_m_a	0.02	Velliv_m_l	5.24	Velliv_m

Comments

(Ignoring `mhr_a...`)

`mhr` has some nice properties:

- It has a relatively high `nu` value of 90, which means it is tending more towards exponential tails than polynomial tails. All other funds have `nu` values close to 3, except `phr` which is even worse at close to 2. (Note that for a Gaussian, `nu` is infinite.)
- It has the lowest losing percentage of all simulations, which is better than 1/6 that of `phr`.
- It has a DAO percentage of 0, which is the same as `mmr`, and less than `phr`.
- Only `phr` has a higher `mc_m`.
- It has a smaller `mc_s` than the individual components, `vhr` and `phr`.
- It has the highest `xi` of all fits, suggesting less left skewness. Density plots for `vmr`, `phr` and `mmr` have an extremely sharp drop, as if an upward limiter has been applied, which corresponds to extremely low `xi` values. The density plot for `mhr` is by far the most symmetrical of all the fits.
- Only `mmr` has a higher `mc_min`. However, that of `mmr` is 18 times higher with 62, so `mmr` is a clear winner here.
- Naturally, it has a `mc_max` smaller than the individual components, `vhr` and `phr`, but ca. 1.5 times higher than `mmr`.
- All the first 4 moments converge nicely. For all other fits, the 4th moment doesn't seem to converge.

Taleb, Statistical Consequences Of Fat Tails, p. 97:

"the variance of a finite variance random variable with tail exponent < 4 will be infinite".

And p. 363:

"The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, under a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance."

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} \right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2} \right) / \left(\frac{x_{t-1} + y_{t-1}}{2} \right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns = Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} \right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2 \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1} y_t = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

$$(x_{t-1} x_t y_{t-1} + y_{t-1} x_t y_{t-1}) + (x_{t-1} x_{t-1} y_t + x_{t-1} y_{t-1} y_t) = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: $R = 1+r$

```
## Let x_0 be 100.
```

```
## Let y_0 be 200.
```

```
## So the initial value of the pf is 300 .
```

```
## Let R_x be 0.5.
```

```
## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.
```

```
## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): 0.04653069
## s(data_x): 0.4480524
## m(data_y): 9.971466
## s(data_y): 3.113235
##
## m(data_x + data_y): 5.008998
## s(data_x + data_y): 1.561021
```

m and s of final state of all paths.

_a is mix of simulated returns.

_b is simulated mixed returns.

m_a	m_b	s_a	s_b
100.331	100.296	7.329	6.828
100.105	100.248	7.185	6.828
99.938	100.075	7.085	6.838
100.886	100.061	6.753	7.192
100.142	99.967	6.894	6.990
100.395	100.413	7.102	6.633
100.085	100.053	6.838	6.995
100.057	100.121	6.914	7.256
100.184	100.260	7.156	7.106
100.074	99.854	6.985	7.036

```

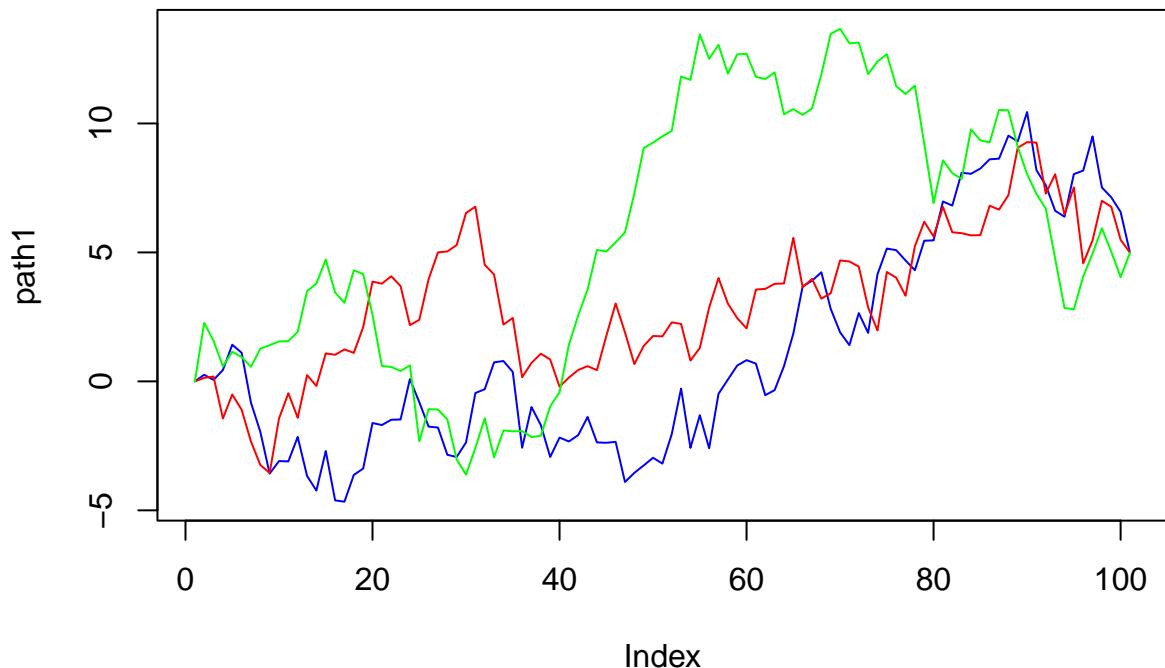
##      m_a          m_b          s_a          s_b
## Min.   :99.94   Min.   :99.85   Min.   :6.753   Min.   :6.633
## 1st Qu.:100.08  1st Qu.:100.06  1st Qu.:6.899   1st Qu.:6.831
## Median :100.12  Median :100.10  Median :7.035   Median :6.993
## Mean    :100.22  Mean    :100.13  Mean    :7.024   Mean    :6.970
## 3rd Qu.:100.29  3rd Qu.:100.26  3rd Qu.:7.142   3rd Qu.:7.088
## Max.    :100.89  Max.    :100.41  Max.    :7.329   Max.    :7.256

```

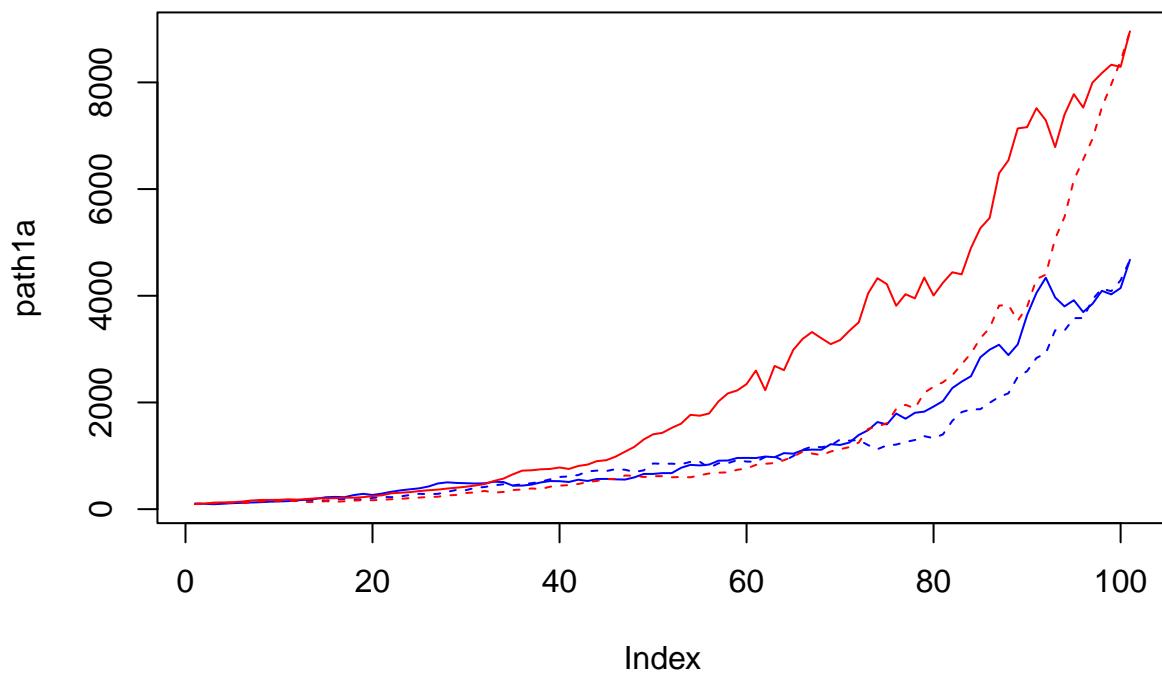
_a and _b are very close to equal.

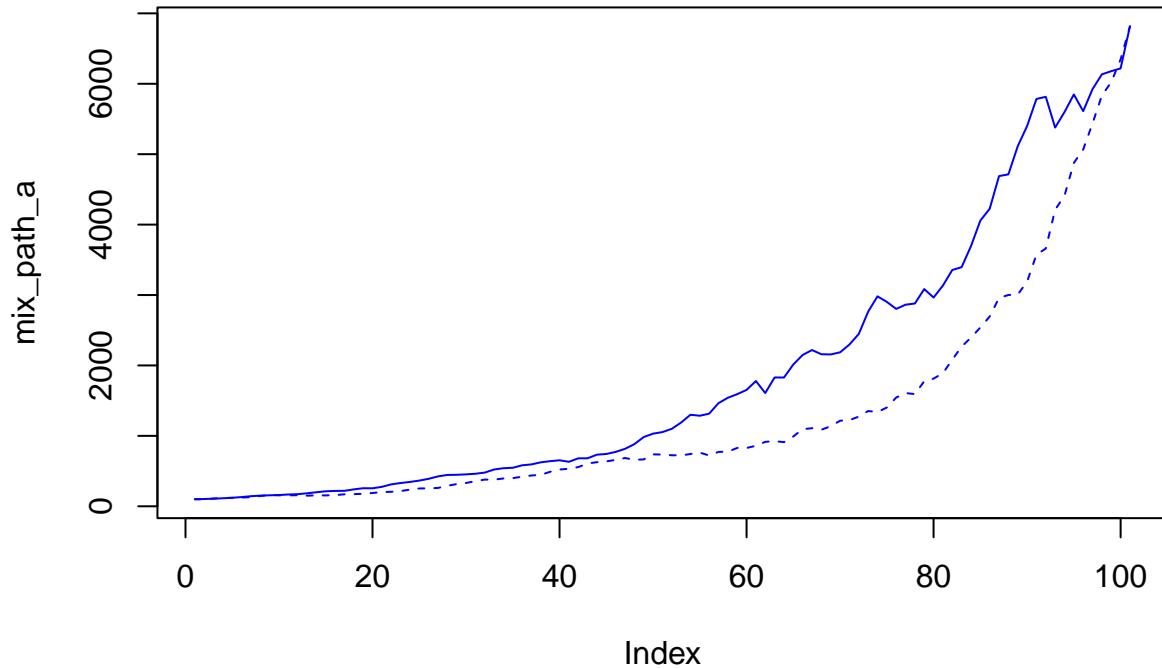
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?





The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?

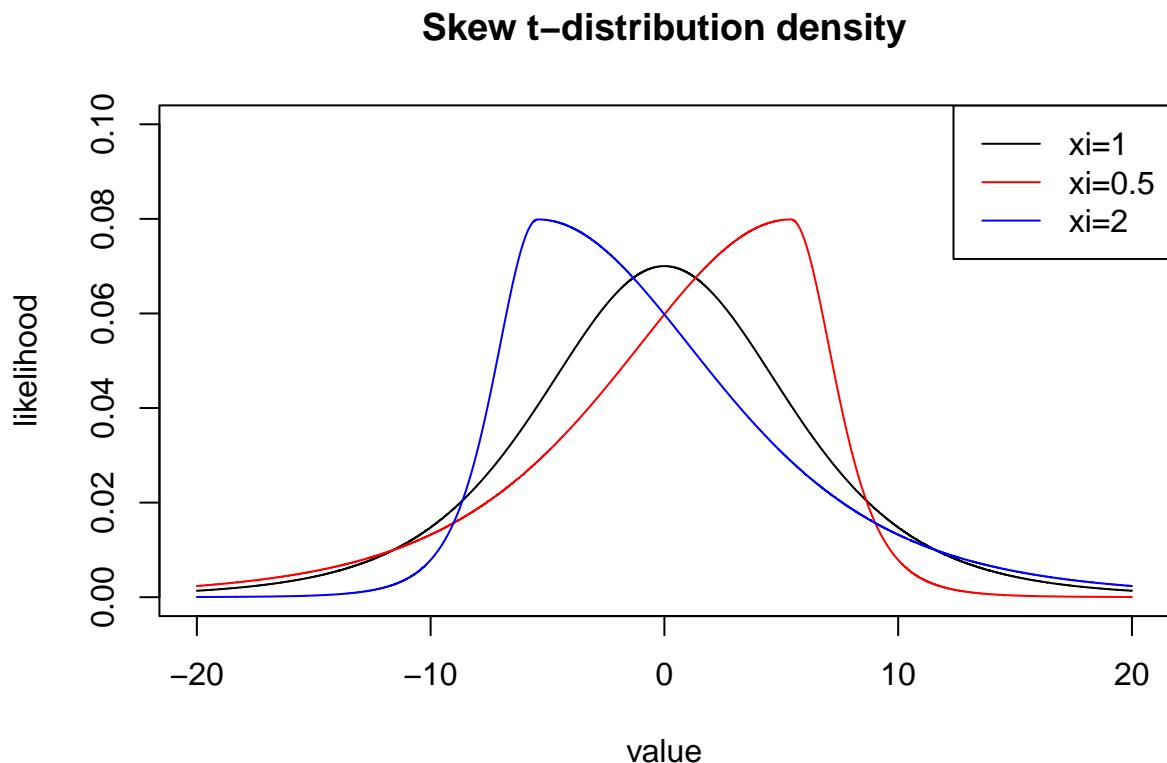
Let's try to include the Velliv data from 2007-2010.

We do this by sampling 13 observations from `vmrl`.

```
##          m             s
##  Min. :0.05943  Min.  :0.05402
##  1st Qu.:0.06561  1st Qu.:0.06095
##  Median :0.06771  Median :0.06697
##  Mean   :0.06876  Mean   :0.06841
##  3rd Qu.:0.07031  3rd Qu.:0.07555
##  Max.   :0.08406  Max.   :0.08943
```

The meaning of ξ_i

The fit for `mhr` has the highest ξ_i value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, \dots, X_n^p)}{\sum_{i=1}^n X_i^p} \rightarrow 0$$

for $n \rightarrow \infty$.

If not, X doesn't have a p 'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192