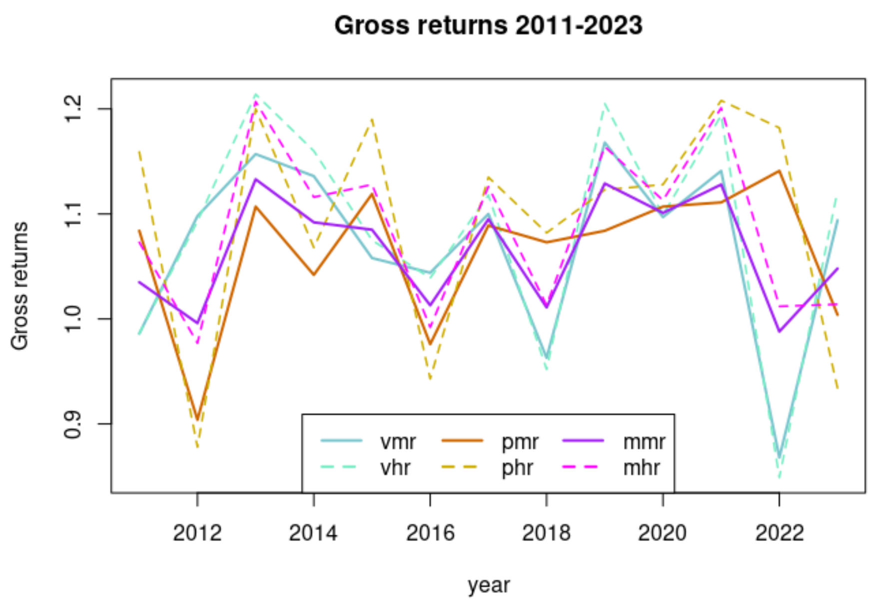
Pension returns analysis

15:48 29 April 2024

Fit log returns to F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated shape parameter (degrees of freedom).
`xi` is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.
`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.



Summary of gross returns

```
##       vmr              vhr              pmr              phr
##  Min.   :0.868   Min.   :0.849   Min.   :0.904   Min.   :0.878
```

```
##  1st Qu.:1.044   1st Qu.:1.039   1st Qu.:1.042   1st Qu.:1.068  
##  Median :1.097   Median :1.099   Median :1.084   Median :1.128  
##  Mean   :1.070   Mean   :1.085   Mean   :1.065   Mean   :1.095  
##  3rd Qu.:1.136   3rd Qu.:1.160   3rd Qu.:1.107   3rd Qu.:1.182  
##  Max.   :1.168   Max.   :1.214   Max.   :1.141   Max.   :1.208  
##       mmr             mhr            vm_ph_r          vh_pm_r      
##  Min.   :0.988   Min.   :0.977   Min.   :0.9791   Min.   :0.9666  
##  1st Qu.:1.013   1st Qu.:1.013   1st Qu.:1.0213   1st Qu.:1.0115  
##  Median :1.085   Median :1.113   Median :1.1024   Median :1.0938  
##  Mean   :1.066   Mean   :1.087   Mean   :1.0807   Mean   :1.0736  
##  3rd Qu.:1.101   3rd Qu.:1.128   3rd Qu.:1.1211   3rd Qu.:1.1065  
##  Max.   :1.133   Max.   :1.207   Max.   :1.1778   Max.   :1.1630
```

```
##       vmrl      
##  Min.   :0.801  
##  1st Qu.:1.013  
##  Median :1.085  
##  Mean   :1.061  
##  3rd Qu.:1.128  
##  Max.   :1.193
```

```
##            vmr   vhr   pmr   phr   mmr   mhr vm_ph_r vh_pm_r
## Min.   : 0.868 0.849 0.904 0.878 0.988 0.977  0.9791  0.9666
## 1st Qu.: 1.044 1.039 1.042 1.068 1.013 1.013  1.0213  1.0115
## Median : 1.097 1.099 1.084 1.128 1.085 1.113  1.1024  1.0938
## Mean   : 1.070 1.085 1.065 1.095 1.066 1.087  1.0807  1.0736
## 3rd Qu.: 1.136 1.160 1.107 1.182 1.101 1.128  1.1211  1.1065
## Max.   : 1.168 1.214 1.141 1.208 1.133 1.207  1.1778  1.1630
```

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.182	phr	1.214	vhr
0.979	vm_ph_r	1.044	vmr	1.113	mhr	1.087	mhr	1.160	vhr	1.208	phr
0.977	mhr	1.042	pmr	1.102	vm_ph_r	1.085	vhr	1.136	vmr	1.207	mhr
0.967	vh_pm_r	1.039	vhr	1.099	vhr	1.081	vm_ph_r	1.128	mhr	1.178	vm_ph_r
0.904	pmr	1.021	vm_ph_r	1.097	vmr	1.074	vh_pm_r	1.121	vm_ph_r	1.168	vmr
0.878	phr	1.013	mmr	1.094	vh_pm_r	1.070	vmr	1.107	pmr	1.163	vh_pm_r
0.868	vmr	1.013	mhr	1.085	mmr	1.066	mmr	1.106	vh_pm_r	1.141	pmr
0.849	vhr	1.012	vh_pm_r	1.084	pmr	1.065	pmr	1.101	mmr	1.133	mmr

Covariance and correlations

Covariances

```
##         vmr     vhr     pmr     phr
## vmr  0.0072  0.0087 -0.0011 -0.0008
## vhr  0.0087  0.0106 -0.0008 -0.0002
## pmr -0.0011 -0.0008  0.0043  0.0066
## phr -0.0008 -0.0002  0.0066  0.0111
```

Correlations

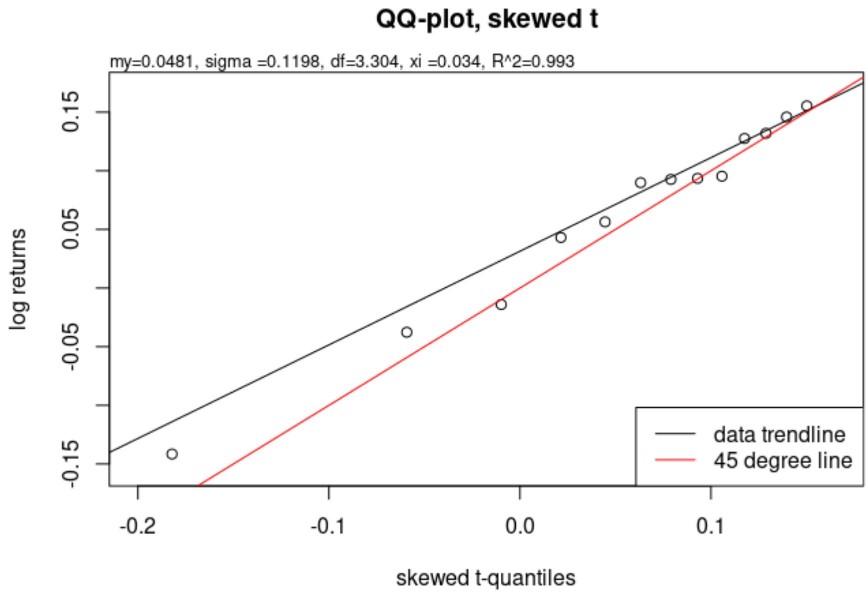
```
##         vmr     vhr     pmr     phr
## vmr  1.0000  0.9926 -0.1971 -0.0949
## vhr  0.9926  1.0000 -0.1186 -0.0159
## pmr -0.1971 -0.1186  1.0000  0.9569
## phr -0.0949 -0.0159  0.9569  1.0000
```

`vhr` and `phr` are clearly the least correlated.

Velliv medium risk, 2011 - 2023

```
##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 63.16667 percent
## What is the chance of gaining min 25 %? >= 49.33333 percent
## What is the chance of gaining min 50 %? >= 40.16667 percent
## What is the chance of gaining min 90 %? >= 32.66667 percent
## What is the chance of gaining min 99 %? >= 31.5 percent
```

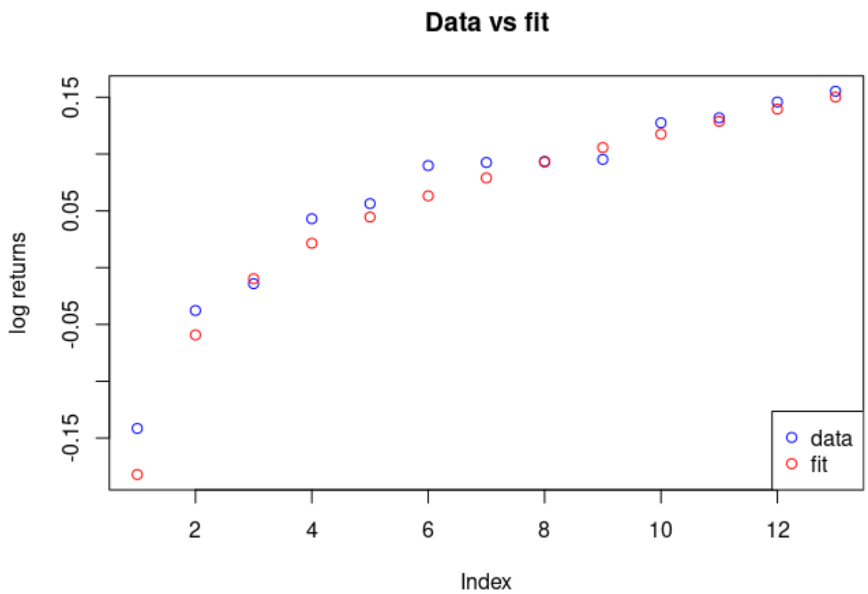
QQ Plot



The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

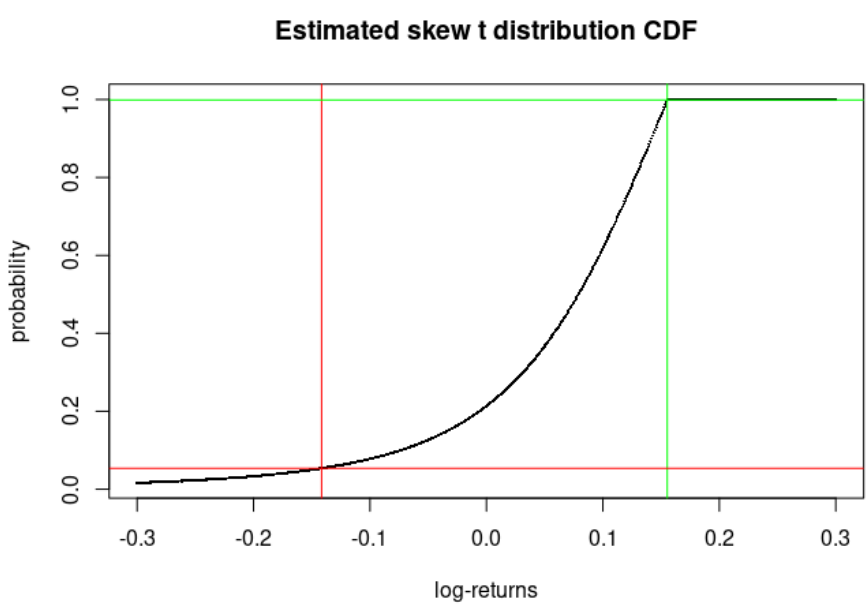
Data vs fit

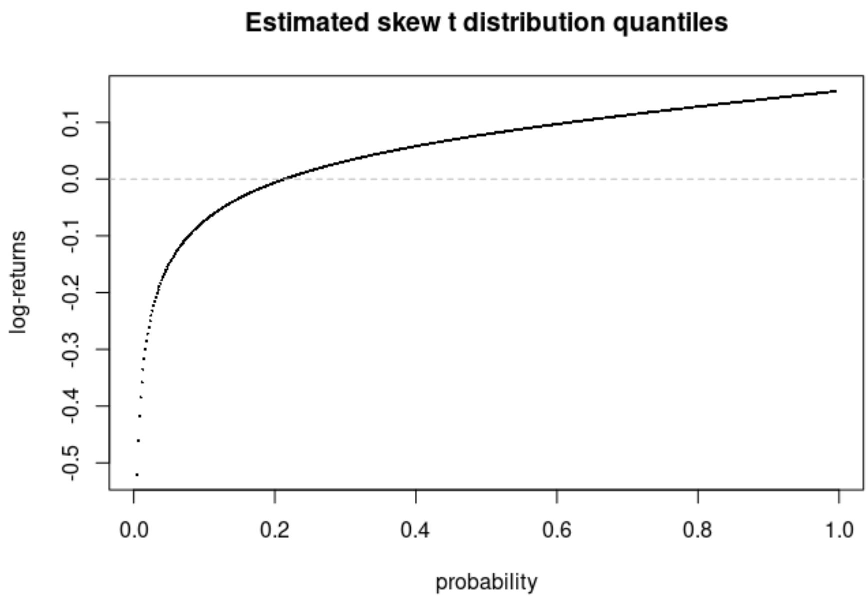
Let's plot the fit and the observed returns together.



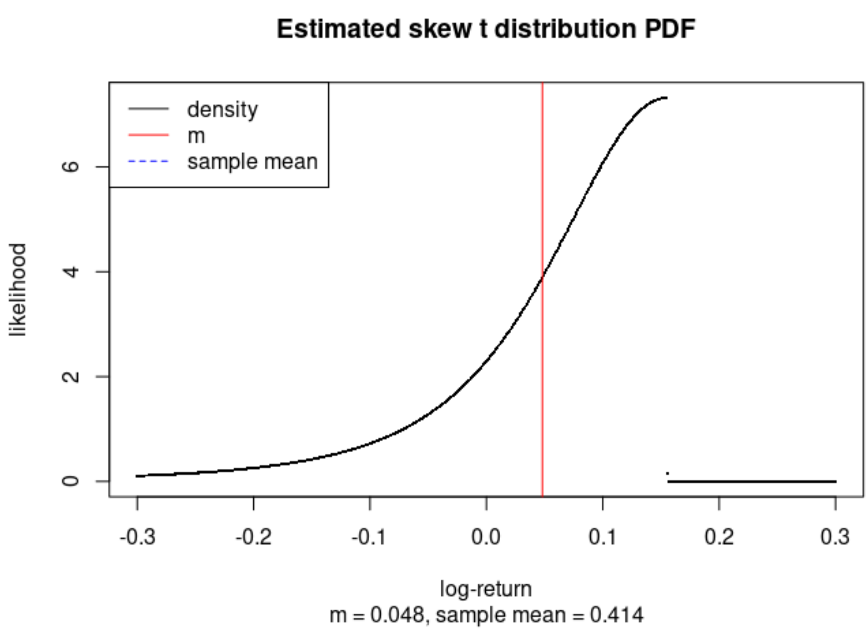
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

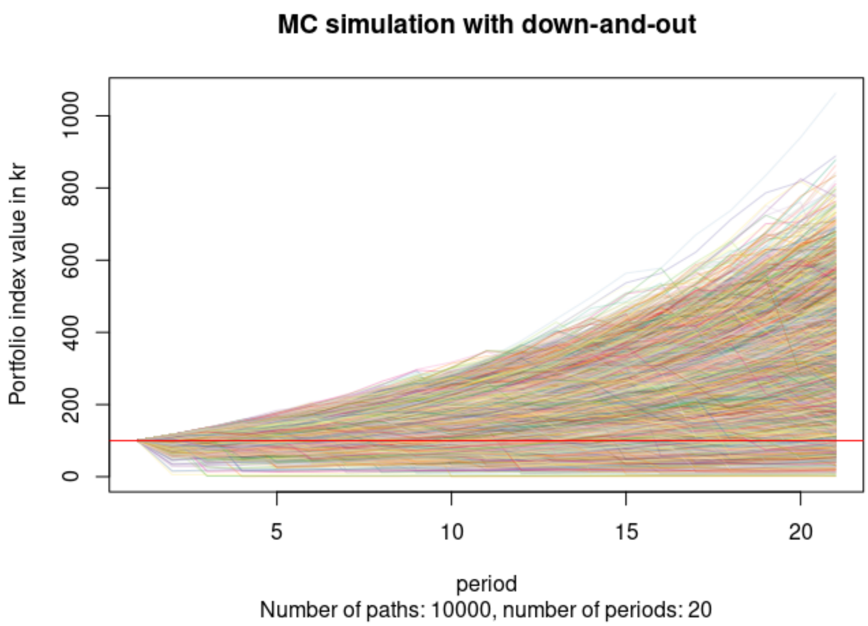


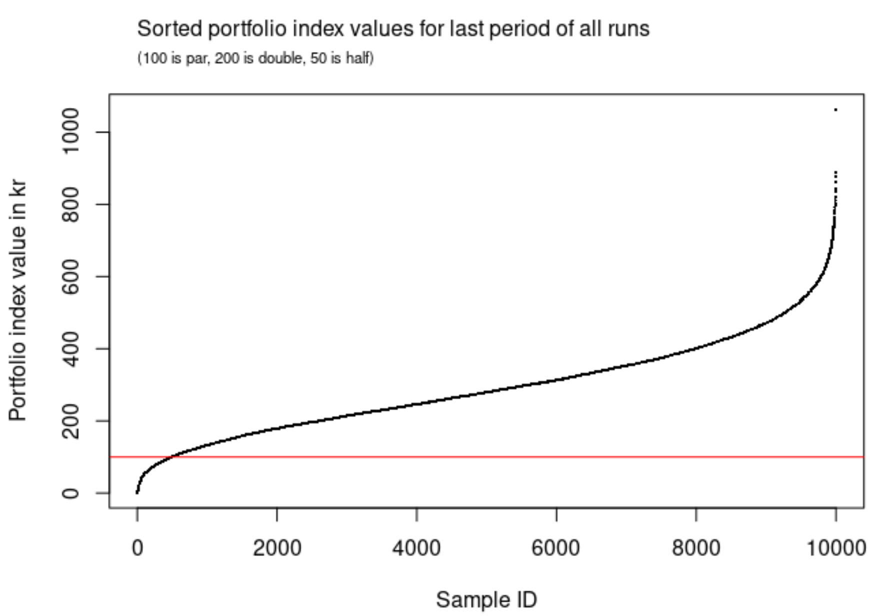


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.



Monte Carlo

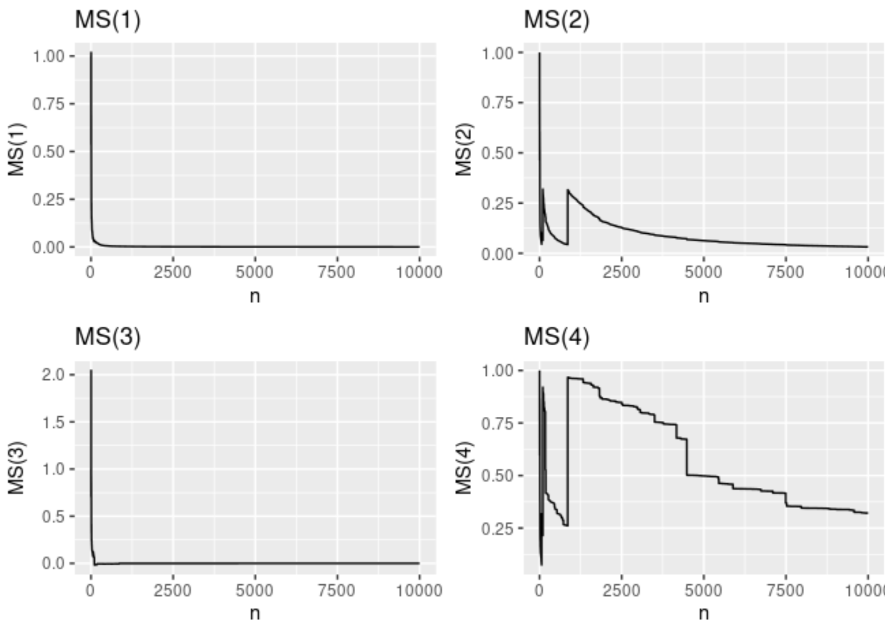

MC simulation with down-and-out
Portfolio index value in kr
0
200
400
600
800
1000
5
10
15
20
period
Number of paths: 10000, number of periods: 20


Sorted portfolio index values for last period of all runs
(100 is par, 200 is double, 50 is half)
Portfolio index value in kr
0
200
400
600
800
1000
0
2000
4000
6000
8000
10000
Sample ID

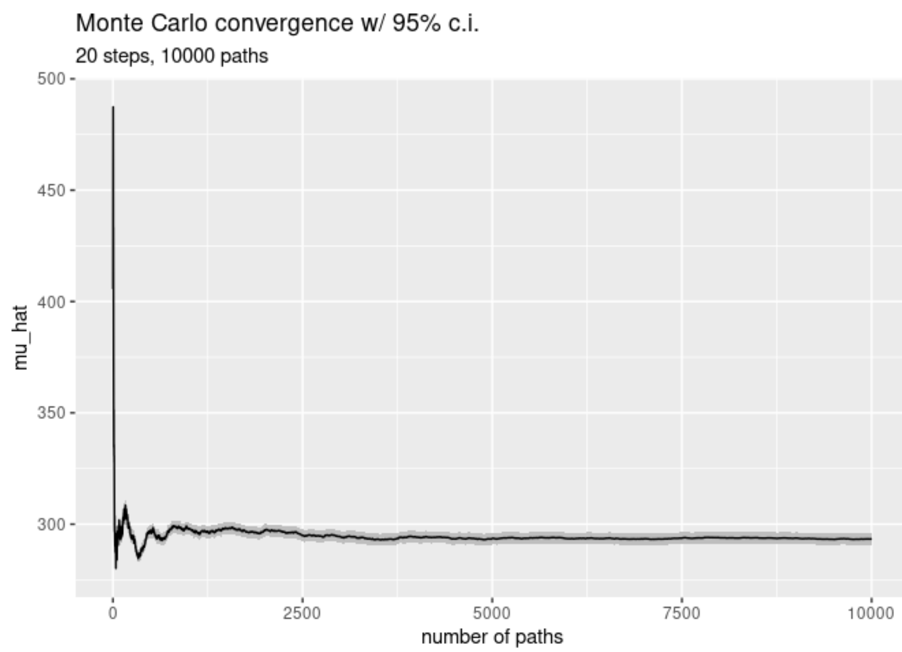
Convergence

Max vs sum

Max vs sum plots for the first four moments:



MC

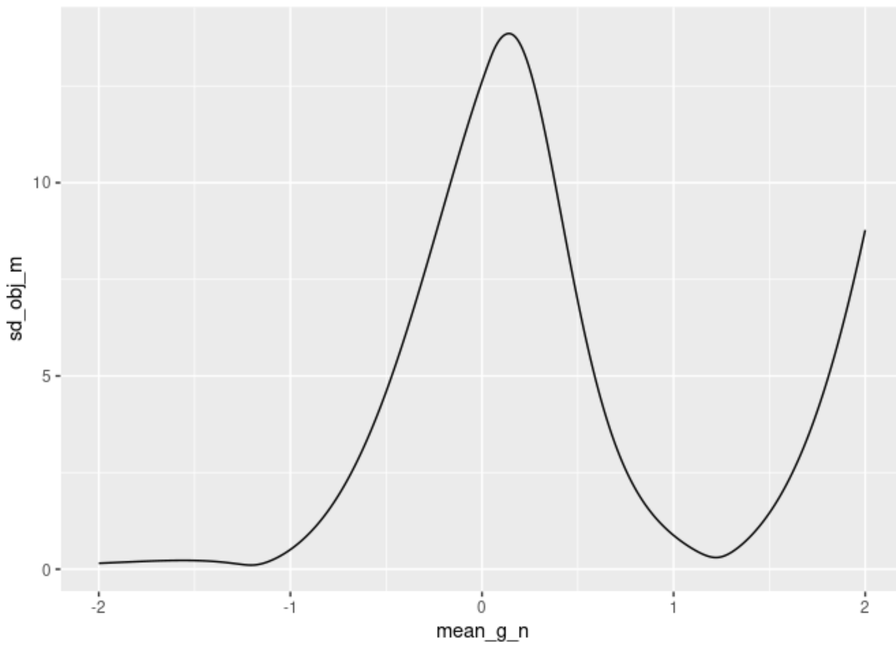


IS

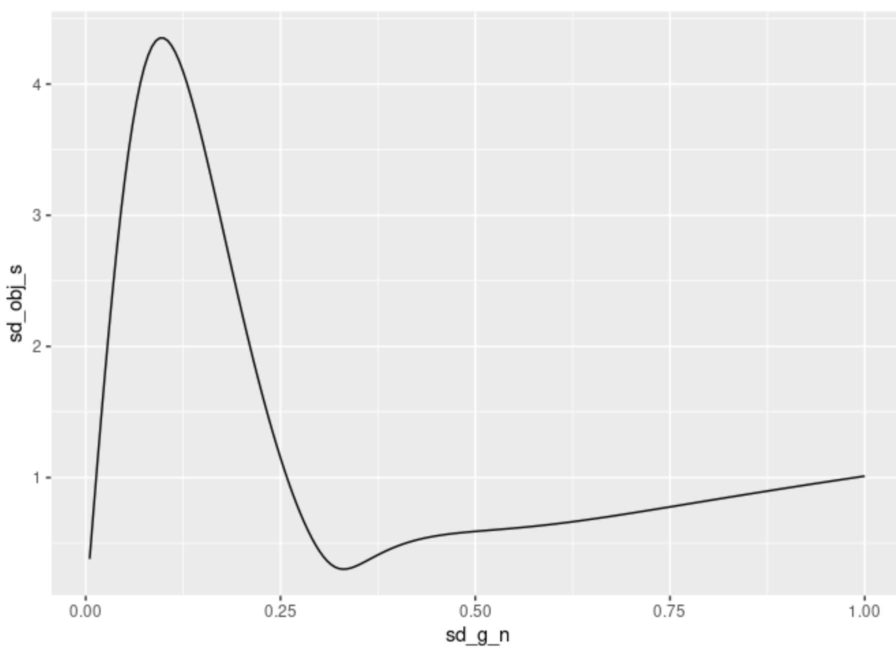
Parameters

```
## [1] 1.2209089 0.3309714
```

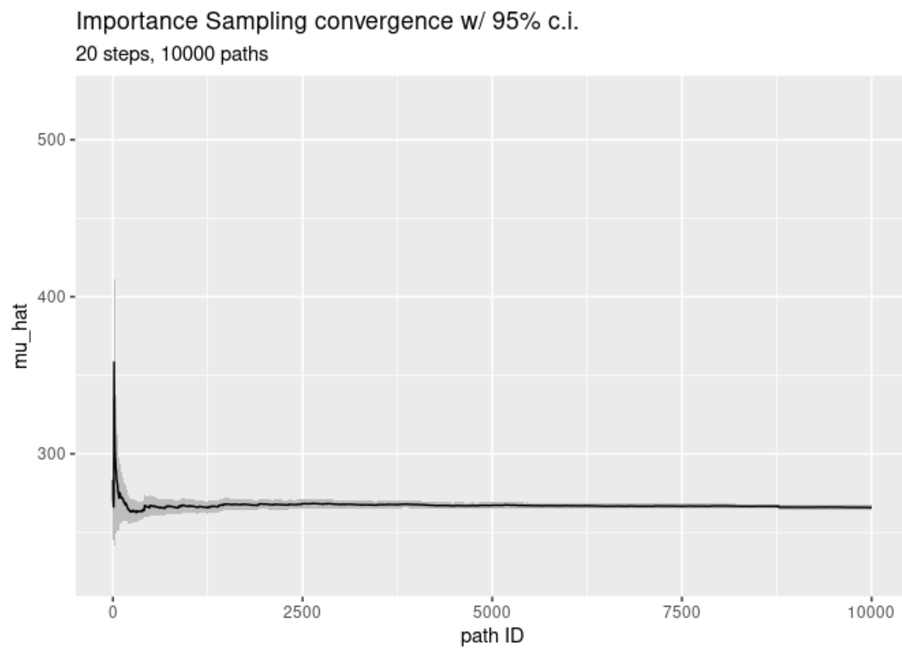
Objective function plots



sd_obj_m
10
5
0
-2
-1
0
1
2
mean_g_n



sd_obj_s
4
3
2
1
0.00
0.25
0.50
0.75
1.00
sd_g_n

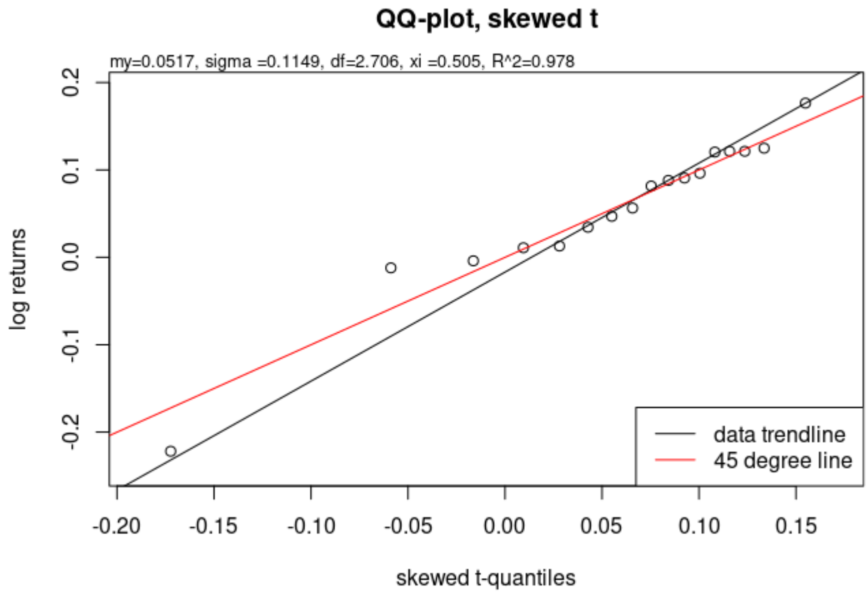


Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##
## AIC: -34.35752
## BIC: -31.02467
## m: 0.05171176
## s: 0.1149408
## nu (df): 2.706099
## xi: 0.5049945
## R^2: 0.978
##
## An R^2 of 0.978 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 58.66667 percent
## What is the chance of gaining min 25 %? >= 47.5 percent
## What is the chance of gaining min 50 %? >= 40.16667 percent
## What is the chance of gaining min 90 %? >= 34 percent
## What is the chance of gaining min 99 %? >= 33 percent
```

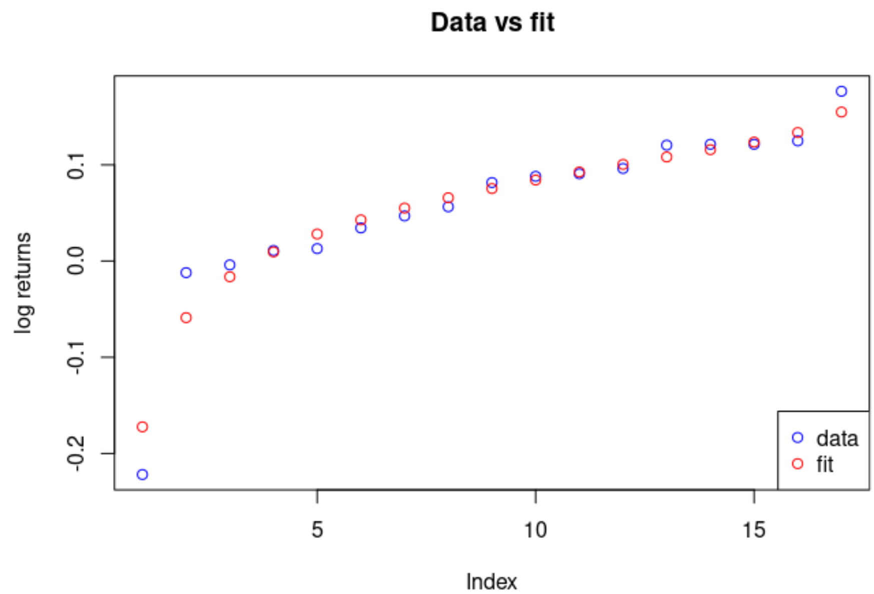
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

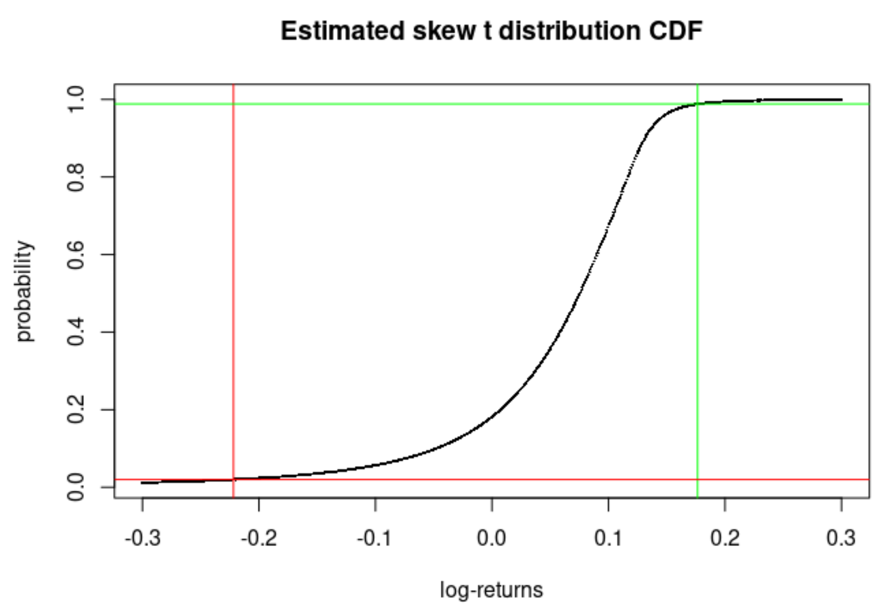
Data vs fit

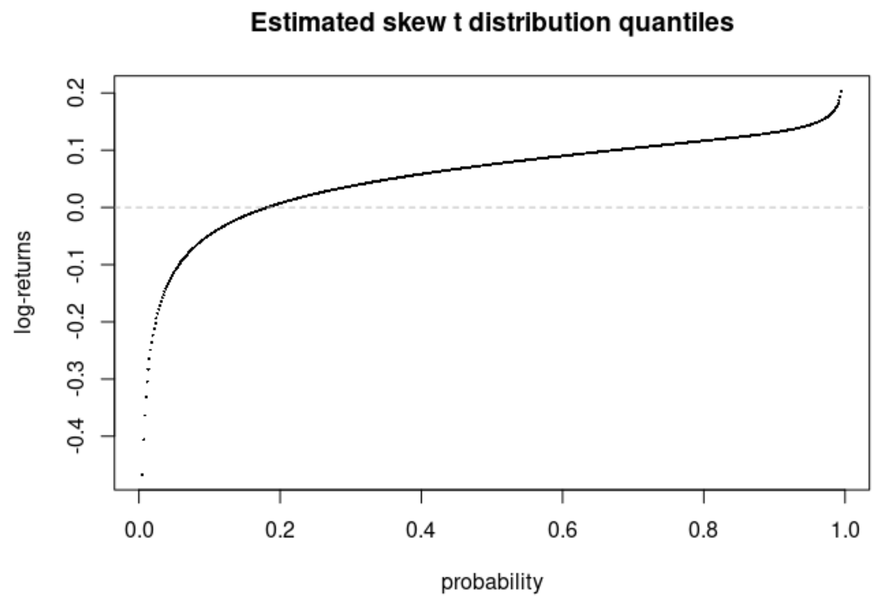
Let's plot the fit and the observed returns together.



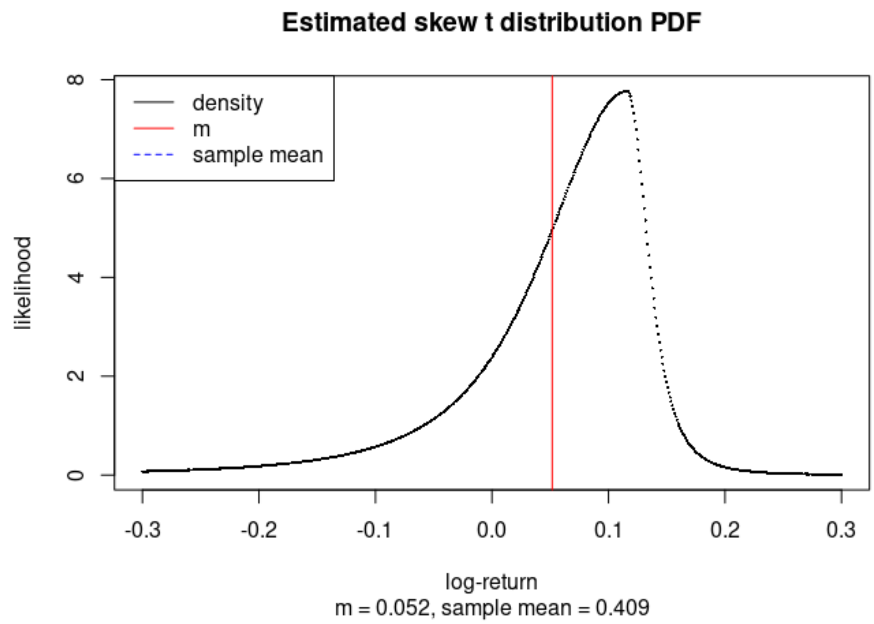
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

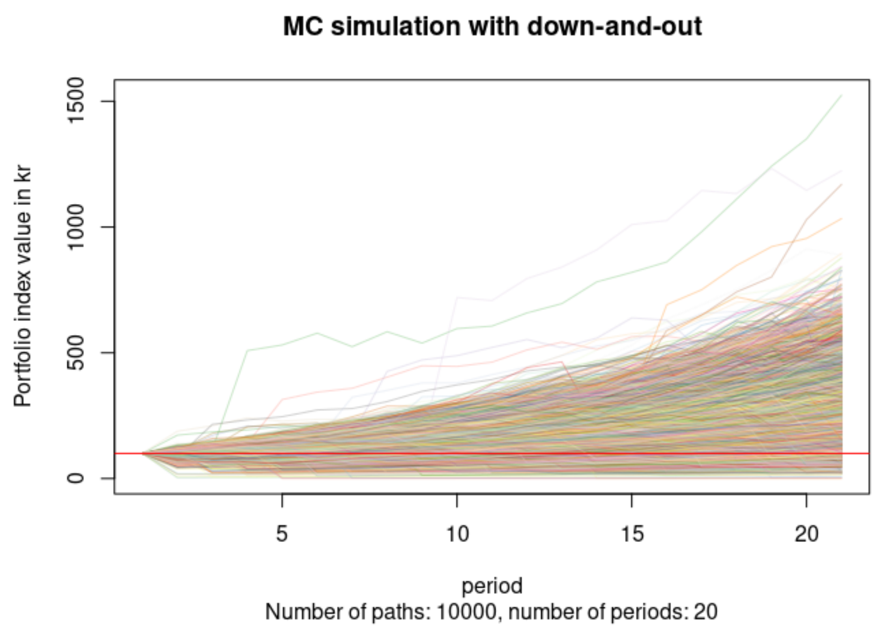


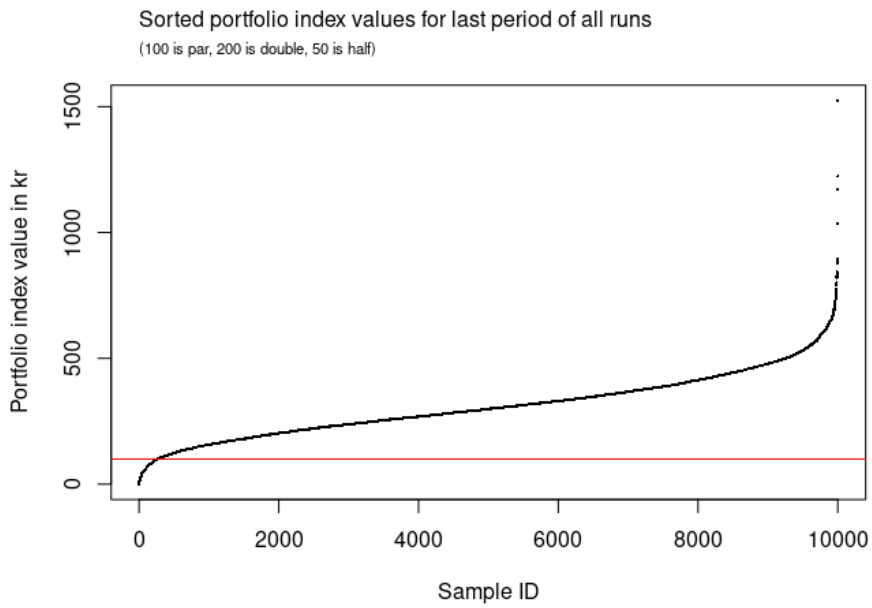


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!


Estimated skew t distribution PDF
density
m
sample mean
likelihood
0
2
4
6
8
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
log-return
m = 0.052, sample mean = 0.409

Monte Carlo

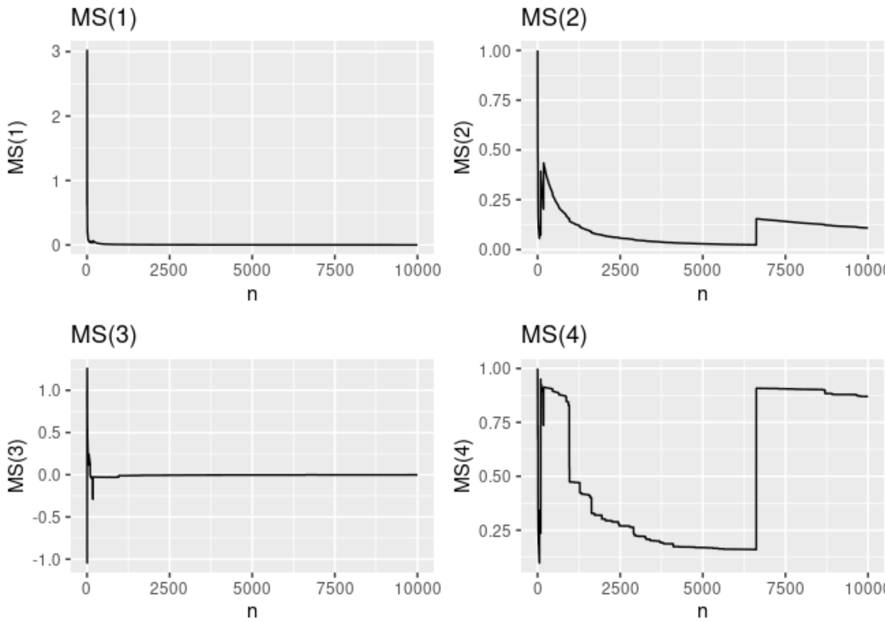

MC simulation with down-and-out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20



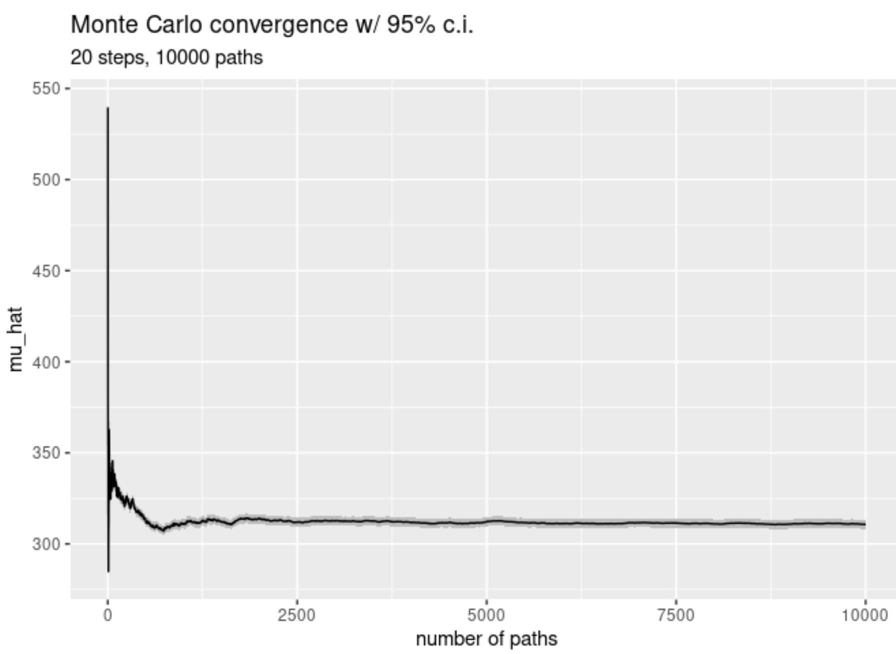
Convergence

Max vs sum

Max vs sum plots for the first four moments:


MS(1)
MS(2)
MS(3)
MS(4)
n

MC

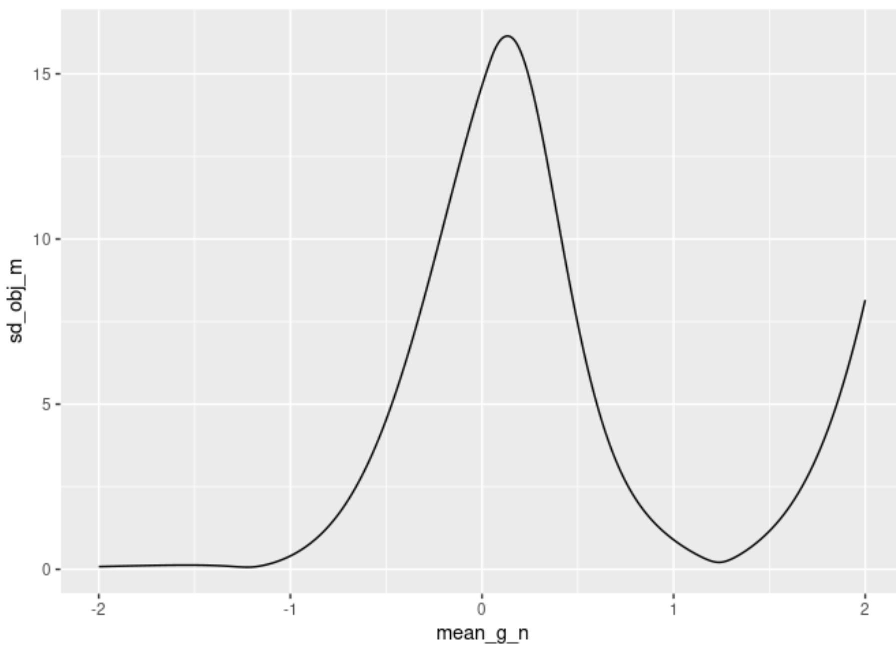

Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
number of paths

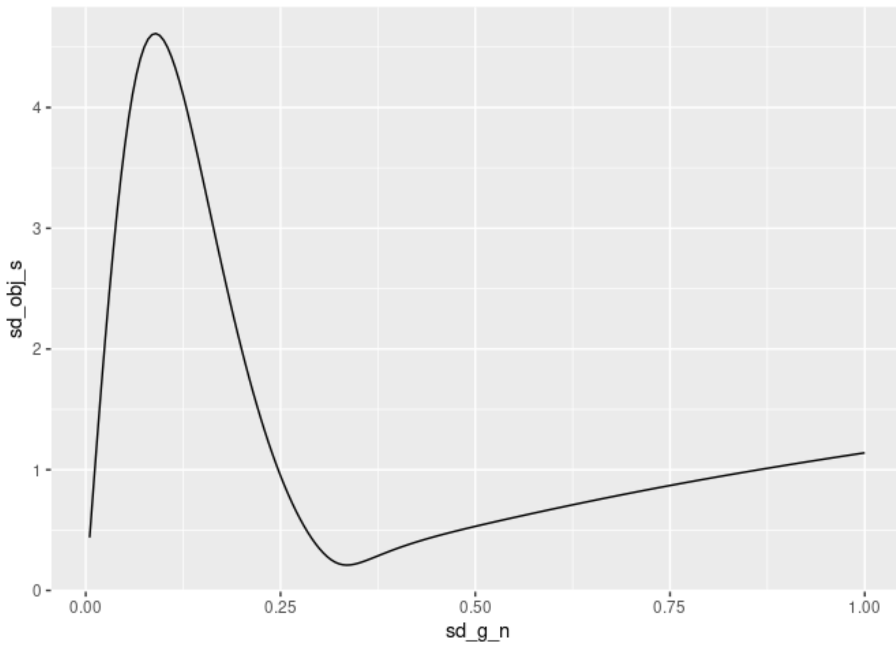
IS

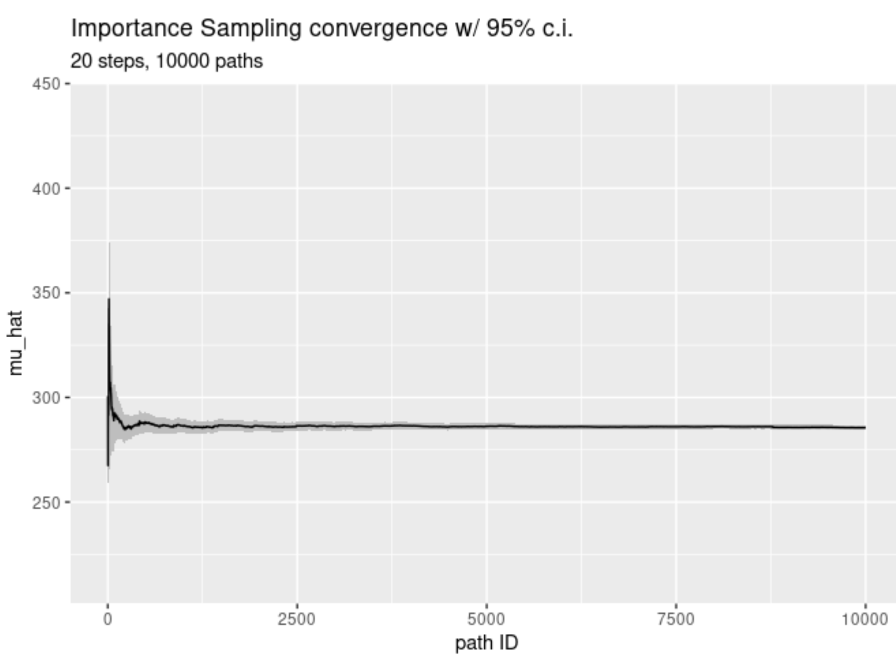
Parameters

```
## [1] 1.2367098 0.3352537
```

Objective function plots




sd_obj_s
sd_g_n
0
1
2
3
4
0.00
0.25
0.50
0.75
1.00

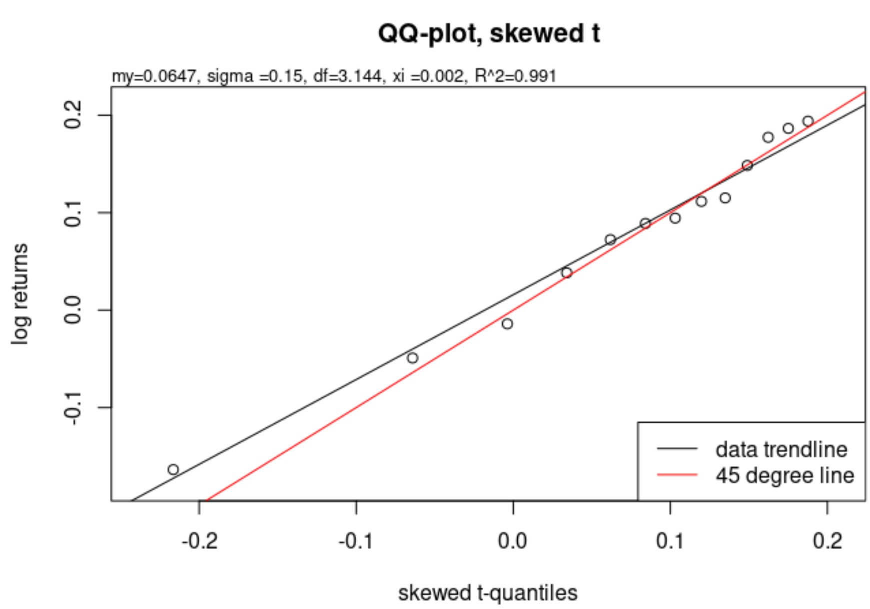

Importance Sampling convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
path ID
250
300
350
400
450
0
2500
5000
7500
10000

Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -21.42488
## BIC: -19.16508
## m: 0.06471454
## s: 0.1499924
## nu (df): 3.144355
## xi: 0.002367034
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 64.66667 percent
## What is the chance of gaining min 25 %? >= 47.83333 percent
## What is the chance of gaining min 50 %? >= 36.83333 percent
## What is the chance of gaining min 90 %? >= 28 percent
## What is the chance of gaining min 99 %? >= 26.5 percent
```

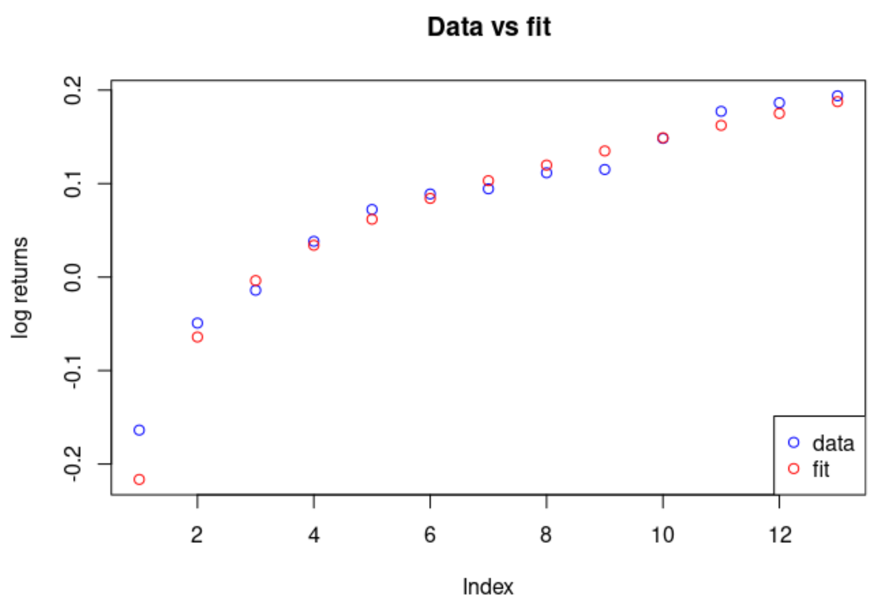
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

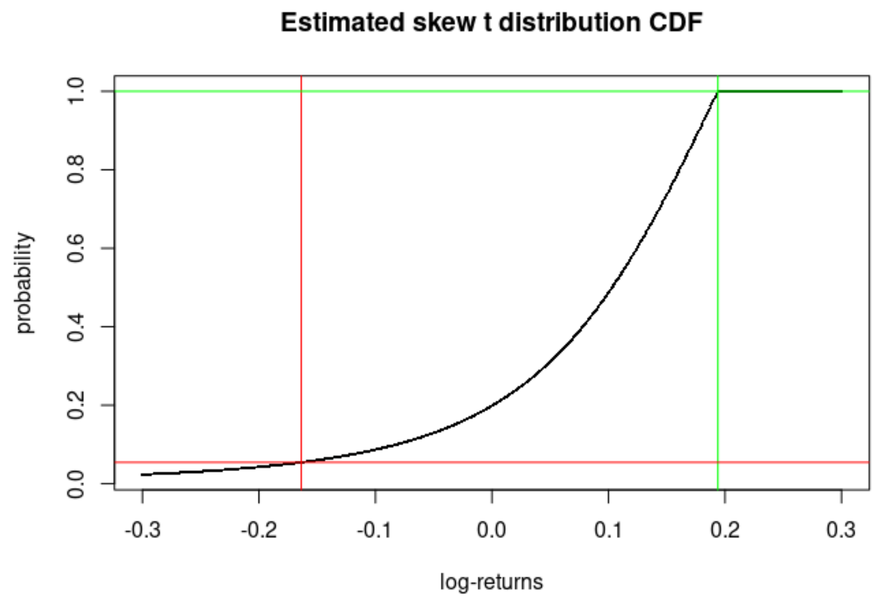
Data vs fit

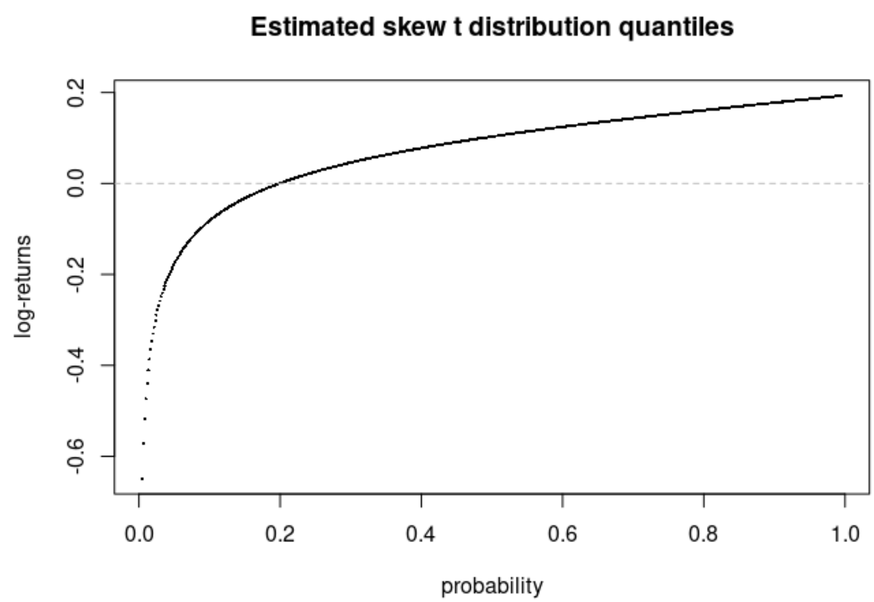
Let's plot the fit and the observed returns together.



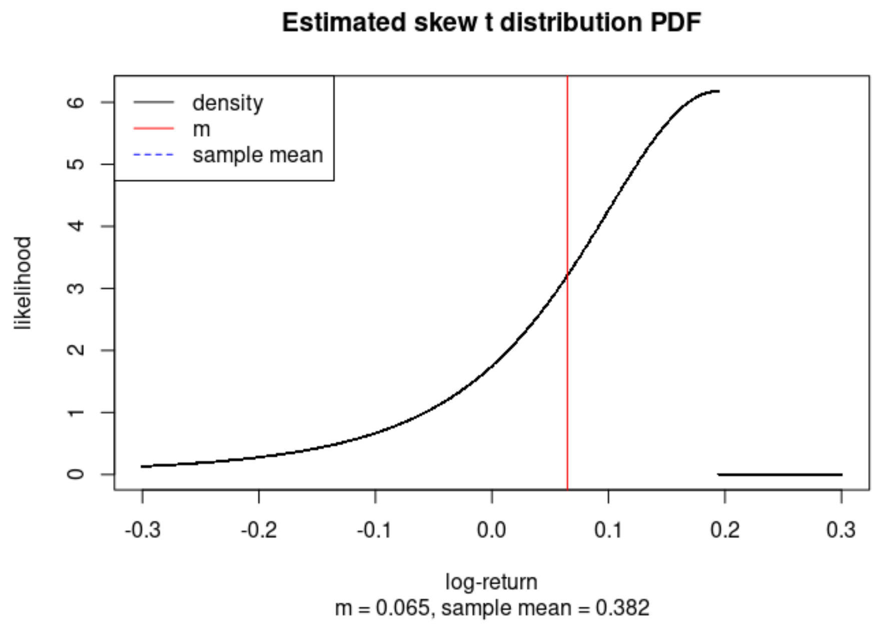
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

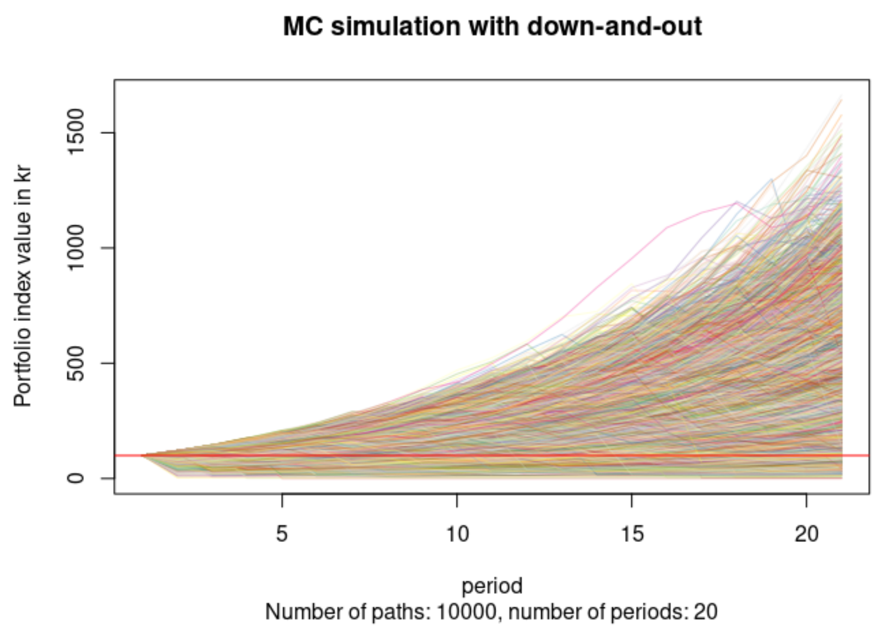


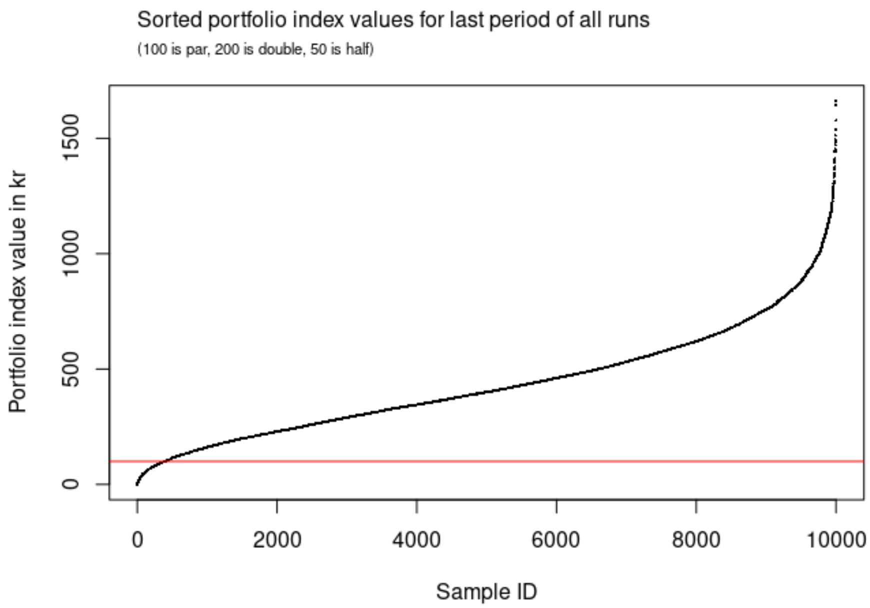


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.


Estimated skew t distribution PDF
density
m
sample mean
likelihood
0
1
2
3
4
5
6
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
log-return
m = 0.065, sample mean = 0.382

Monte Carlo

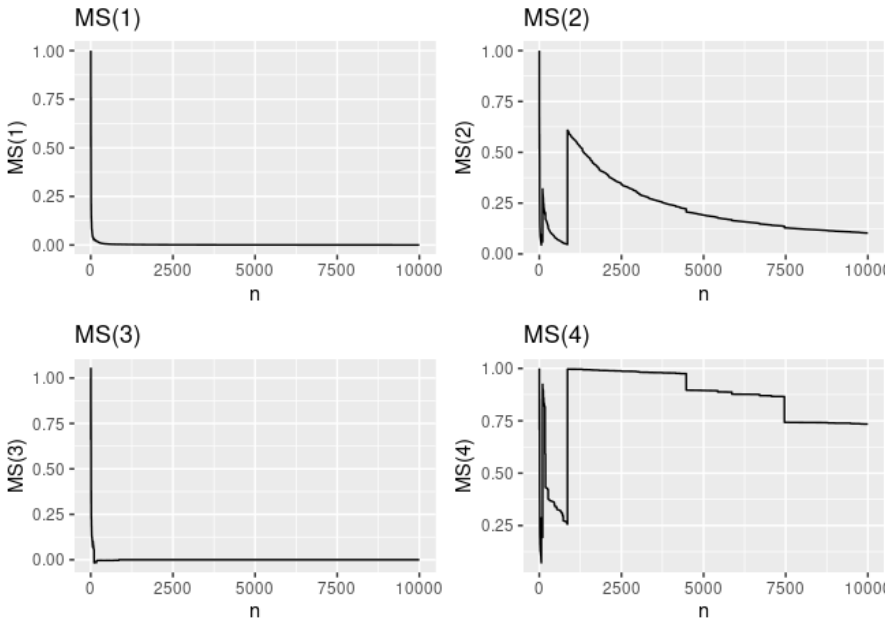

MC simulation with down-and-out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20



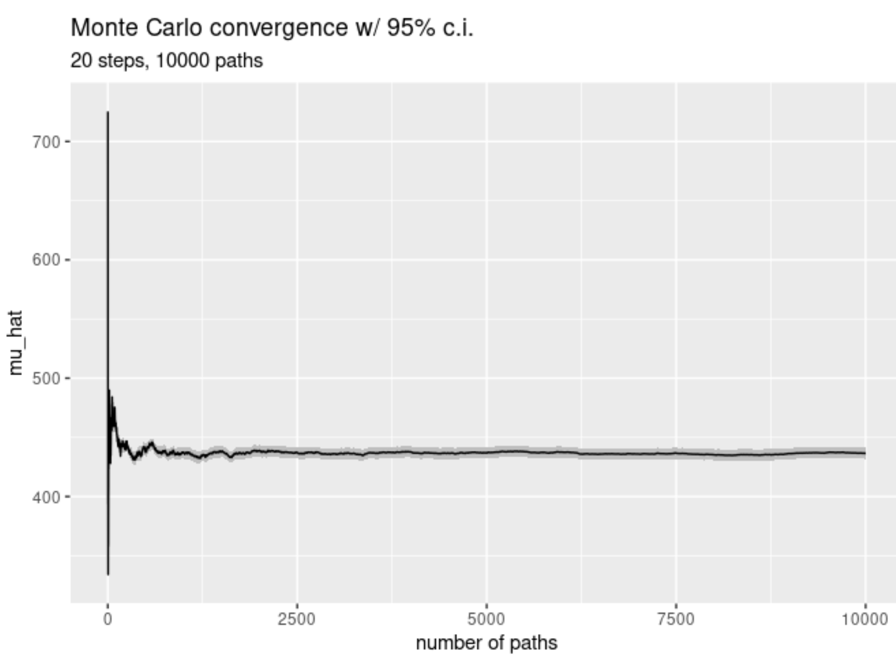
Convergence

Max vs sum

Max vs sum plots for the first four moments:


MS(1)
MS(2)
MS(3)
MS(4)
n
1.00
0.75
0.50
0.25
0.00
0
2500
5000
7500
10000

MC

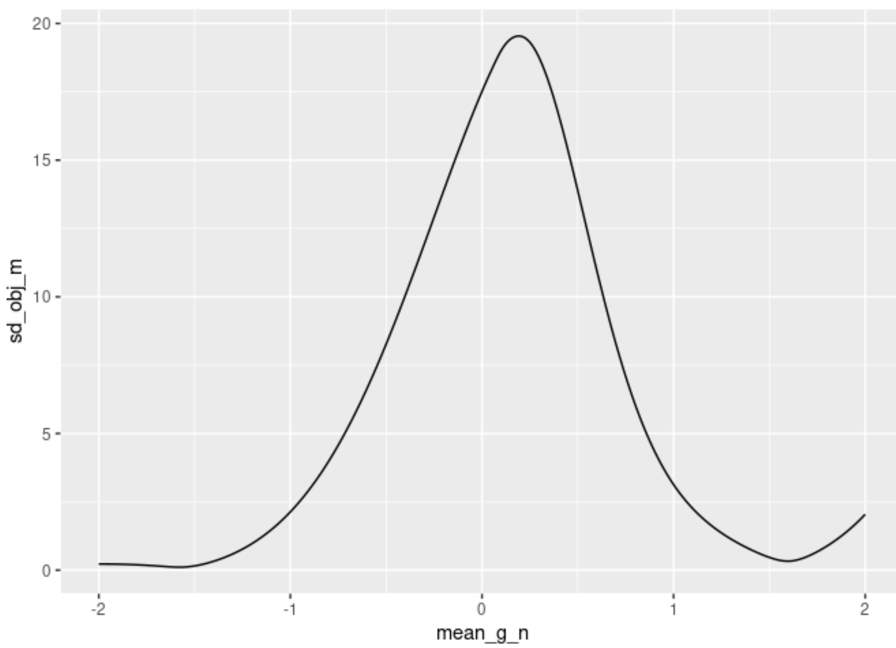

Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
700
600
500
400
0
2500
5000
7500
10000
number of paths

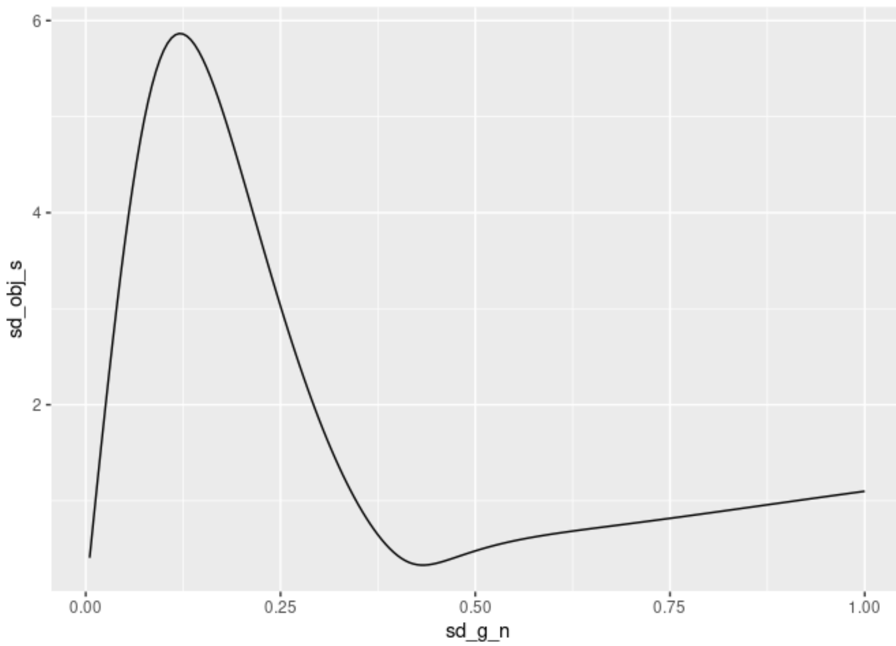
IS

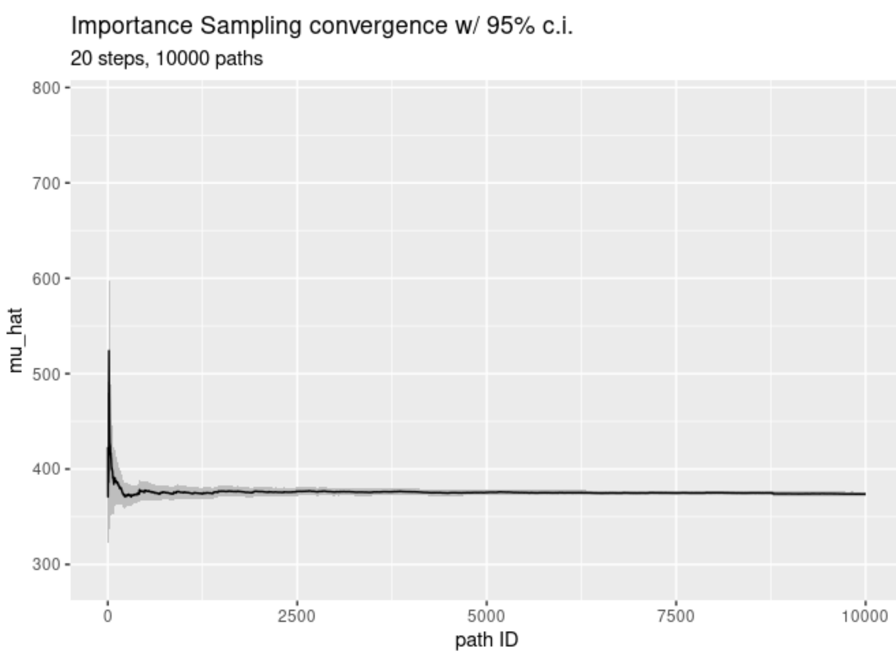
Parameters

```
## [1] 1.595924 0.432716
```

Objective function plots




6
4
2
sd_obj_s
0.00
0.25
0.50
0.75
1.00
sd_g_n

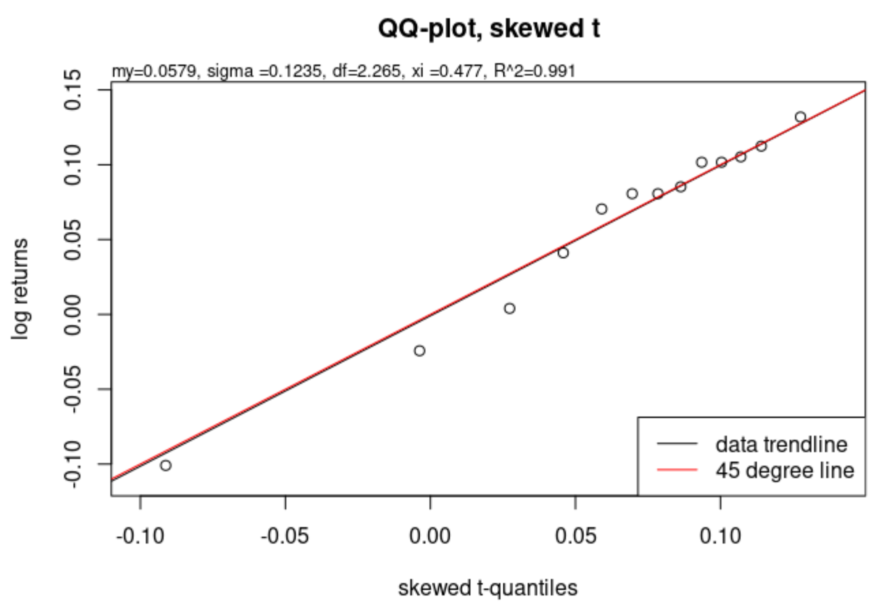

Importance Sampling convergence w/ 95% c.i.
20 steps, 10000 paths
800
700
600
500
400
300
mu_hat
0
2500
5000
7500
10000
path ID

PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -33.22998
## BIC: -30.97018
## m: 0.05789224
## s: 0.1234592
## nu (df): 2.265273
## xi: 0.477324
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 52.83333 percent
## What is the chance of gaining min 25 %? >= 44 percent
## What is the chance of gaining min 50 %? >= 38.83333 percent
## What is the chance of gaining min 90 %? >= 34.66667 percent
## What is the chance of gaining min 99 %? >= 34 percent
```

QQ Plot

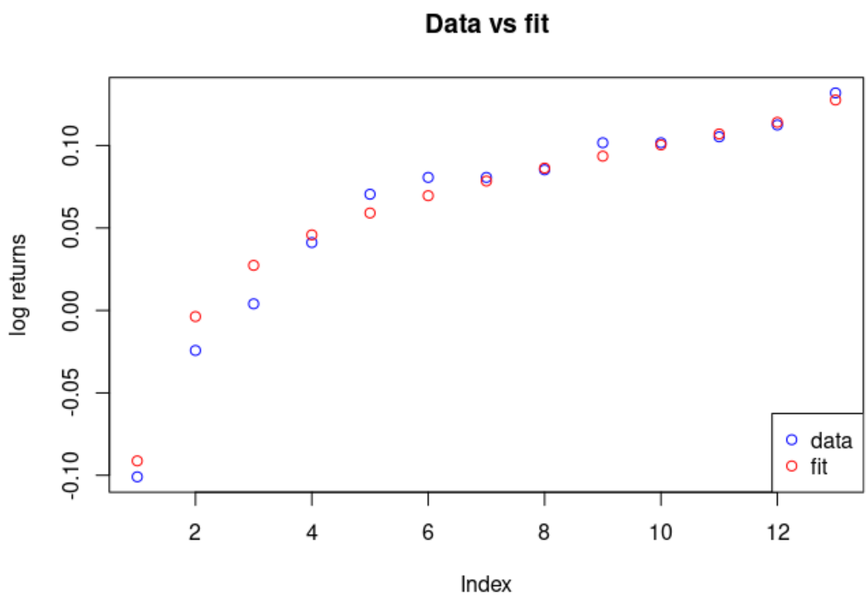


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
##  [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
##  [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11]  0.107018607  0.114081432  0.127604387
```

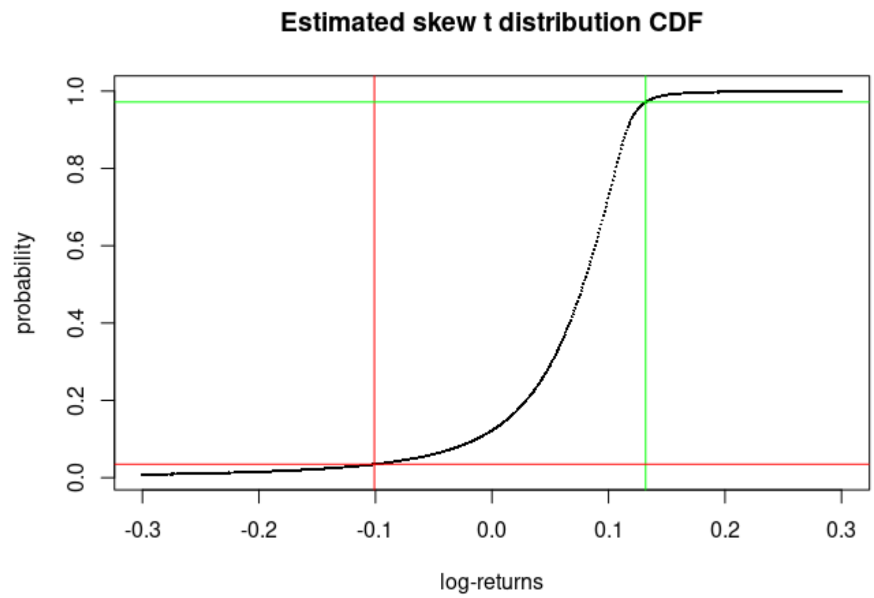
Data vs fit

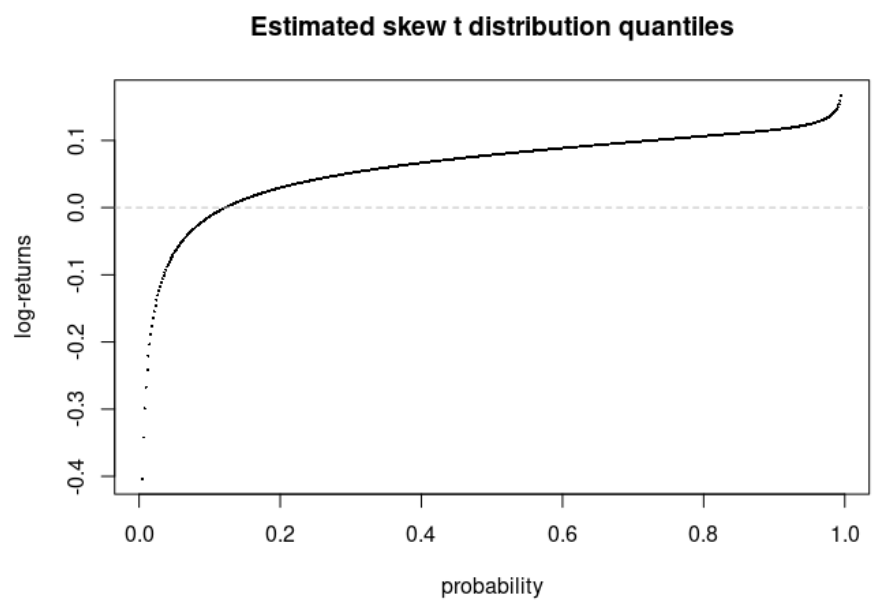
Let's plot the fit and the observed returns together.



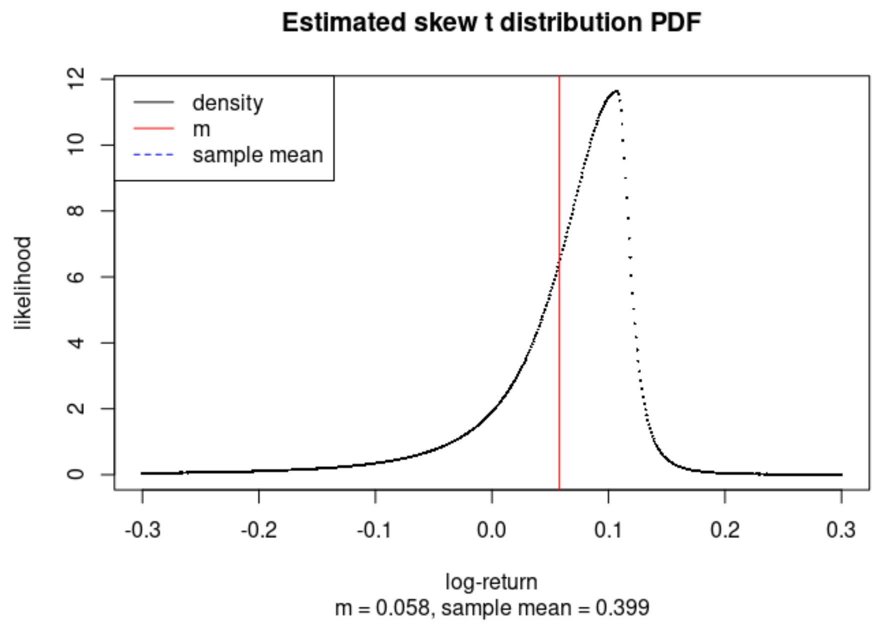
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

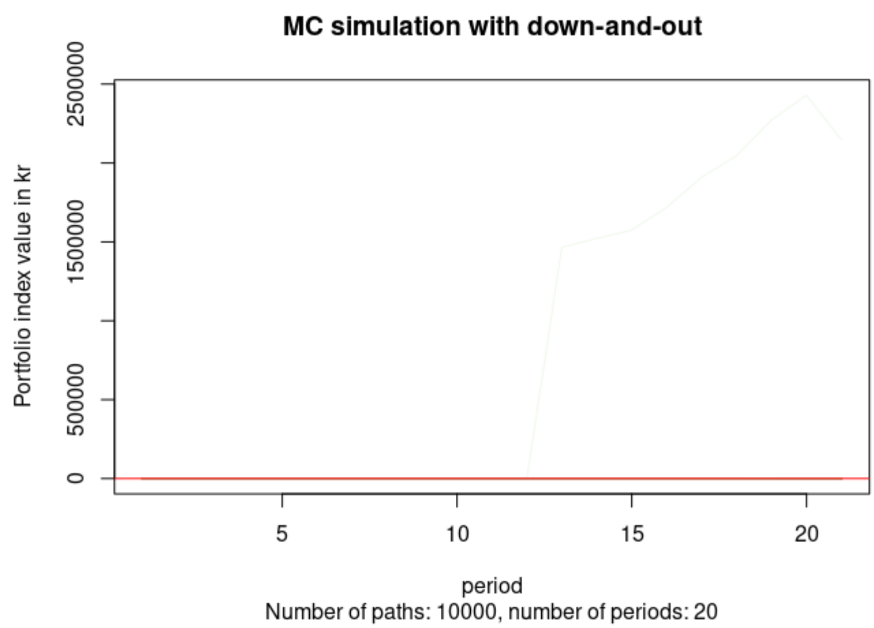


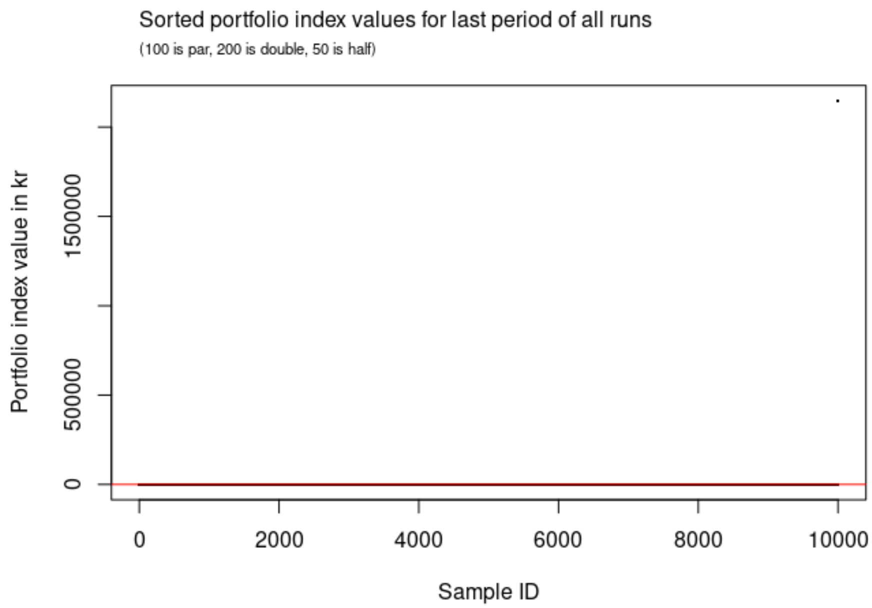


We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.


Estimated skew t distribution PDF
density
m
sample mean
likelihood
log-return
m = 0.058, sample mean = 0.399

Monte Carlo

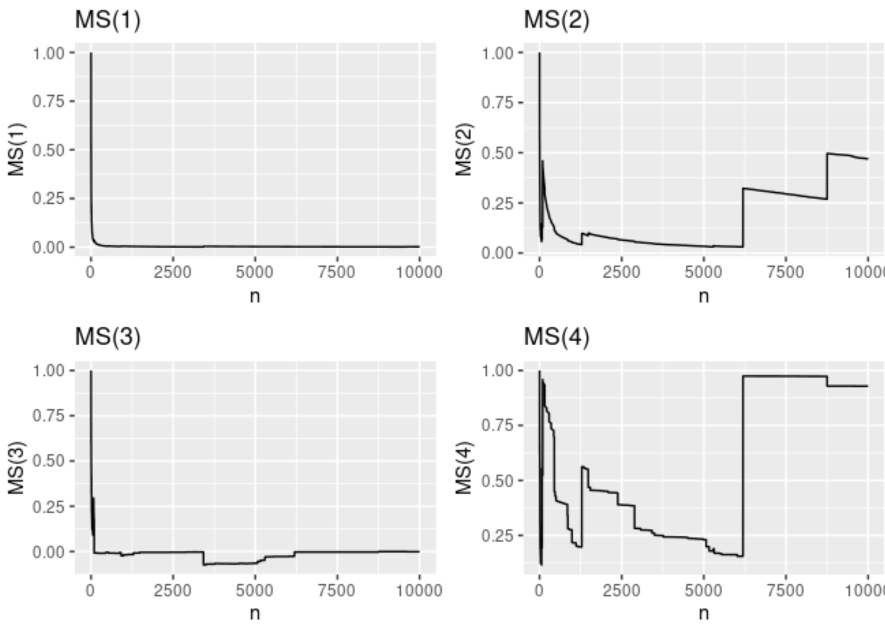

MC simulation with down-and-out
Portfolio index value in kr
period
Number of paths: 10000, number of periods: 20



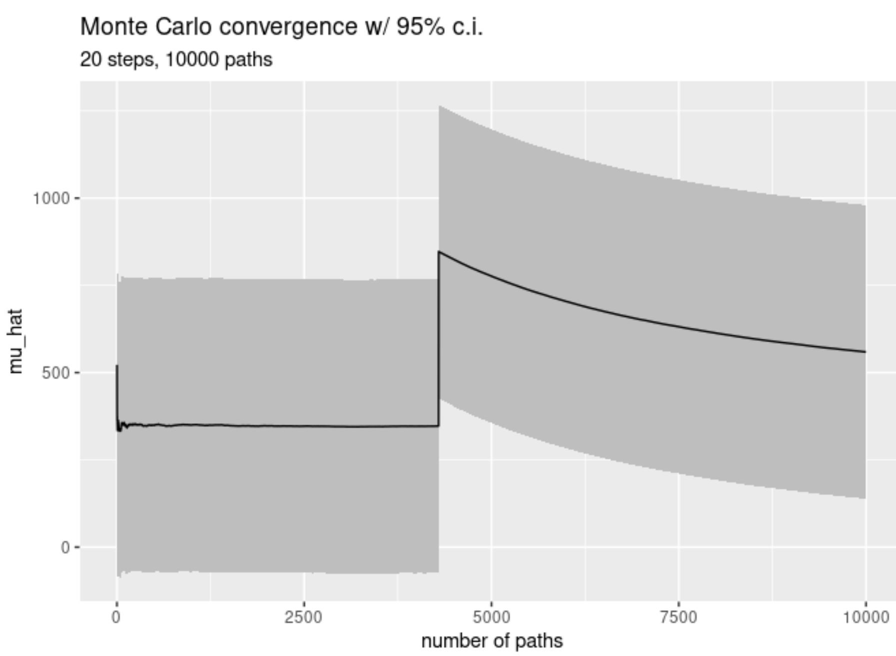
Convergence

Max vs sum

Max vs sum plots for the first four moments:


MS(1)
MS(2)
MS(3)
MS(4)
1.00
0.75
0.50
0.25
0.00
0
2500
5000
7500
10000
n

MC

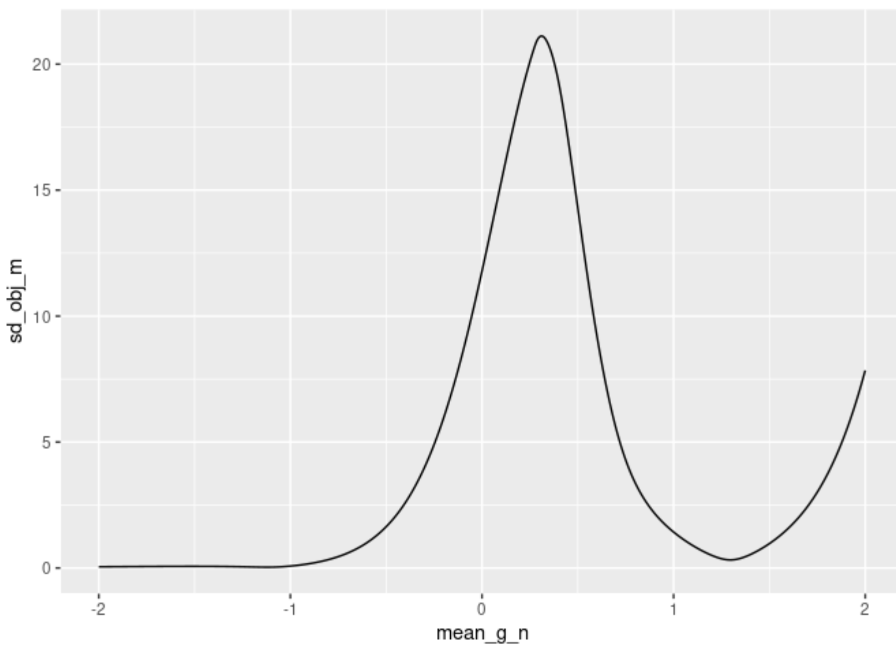

Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
1000
500
0
0
2500
5000
7500
10000
number of paths

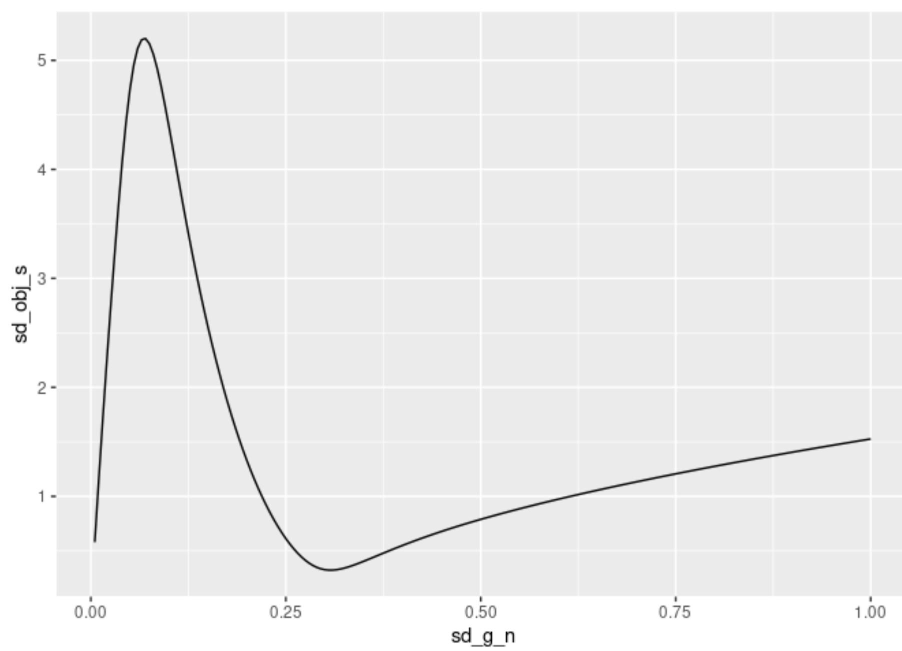
IS

Parameters

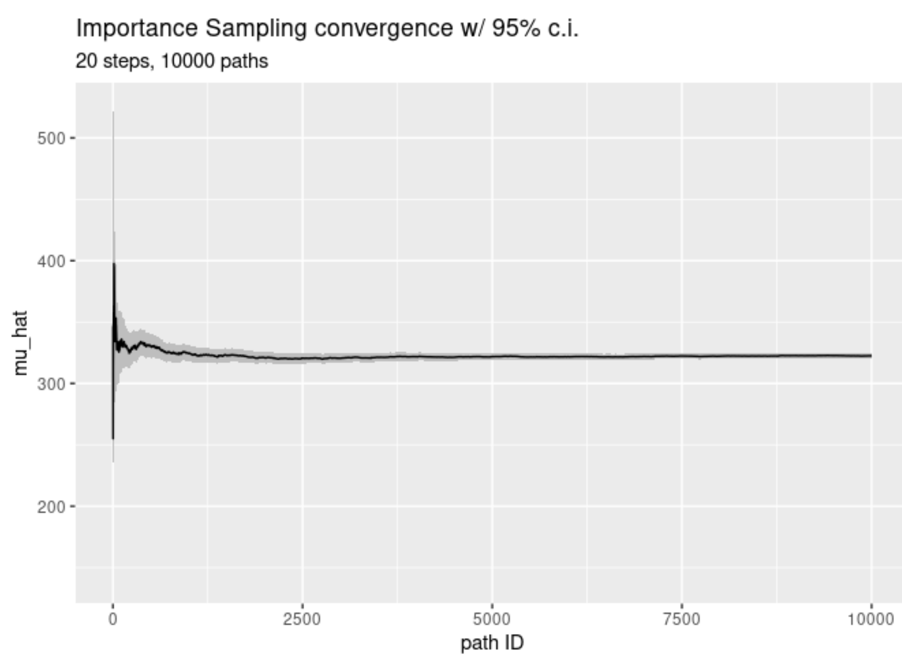
```
## [1] 1.2963050 0.3073762
```

Objective function plots





sd_obj_s
sd_g_n
5
4
3
2
1
0.00
0.25
0.50
0.75
1.00



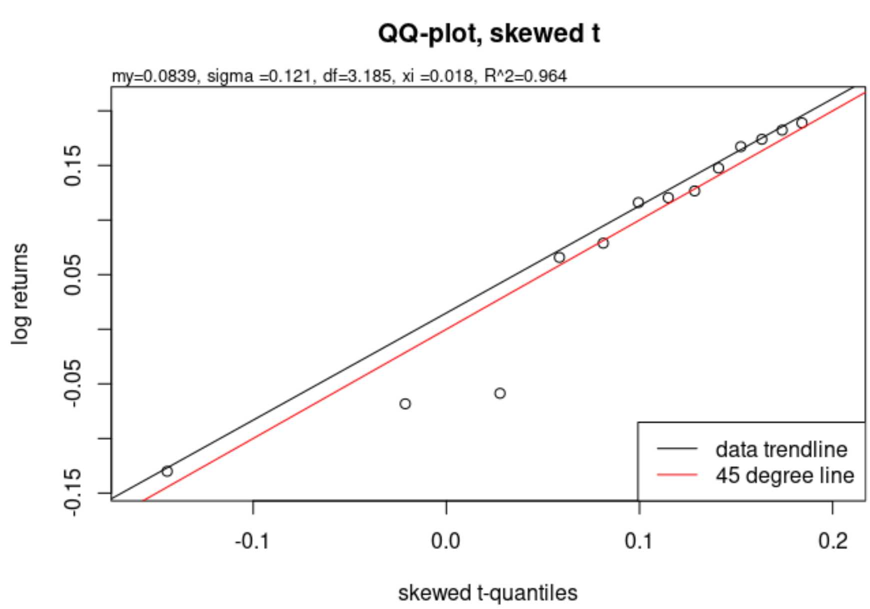
Importance Sampling convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
path ID
500
400
300
200
0
2500
5000
7500
10000

PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -23.72565
## BIC: -21.46585
## m: 0.08386034
## s: 0.1210107
## nu (df): 3.184569
## xi: 0.01790306
## R^2: 0.964
##
## An R^2 of 0.964 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 56.83333 percent
## What is the chance of gaining min 25 %? >= 43.16667 percent
## What is the chance of gaining min 50 %? >= 34.16667 percent
## What is the chance of gaining min 90 %? >= 26.83333 percent
## What is the chance of gaining min 99 %? >= 25.66667 percent
```

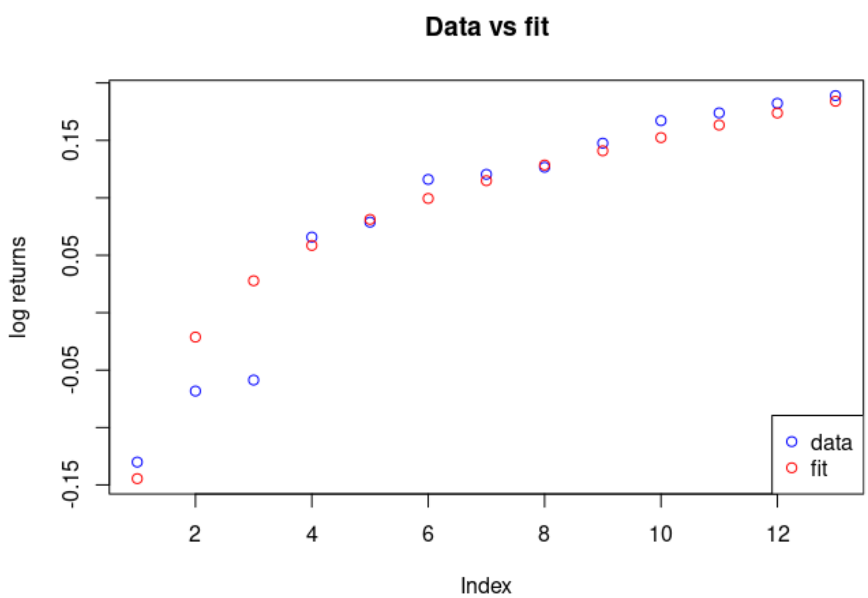
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

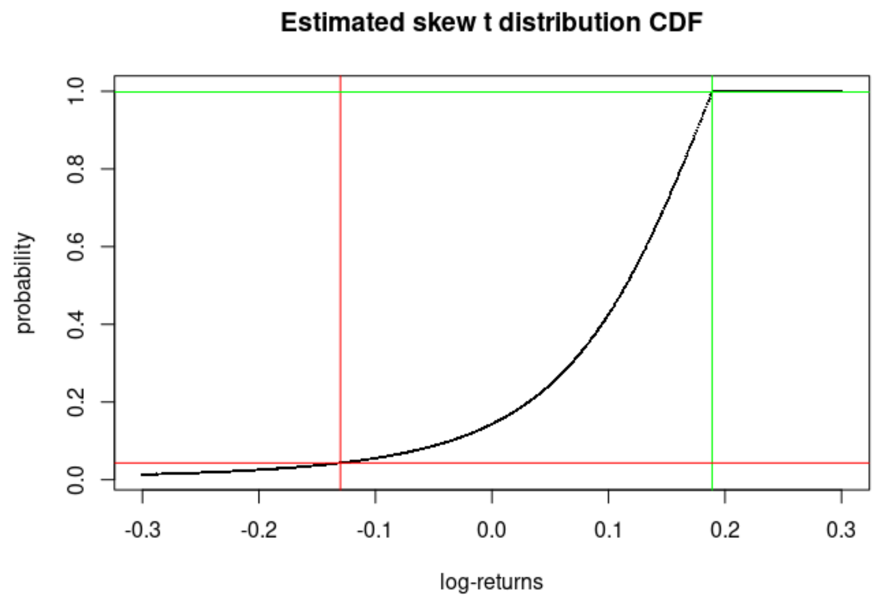
Data vs fit

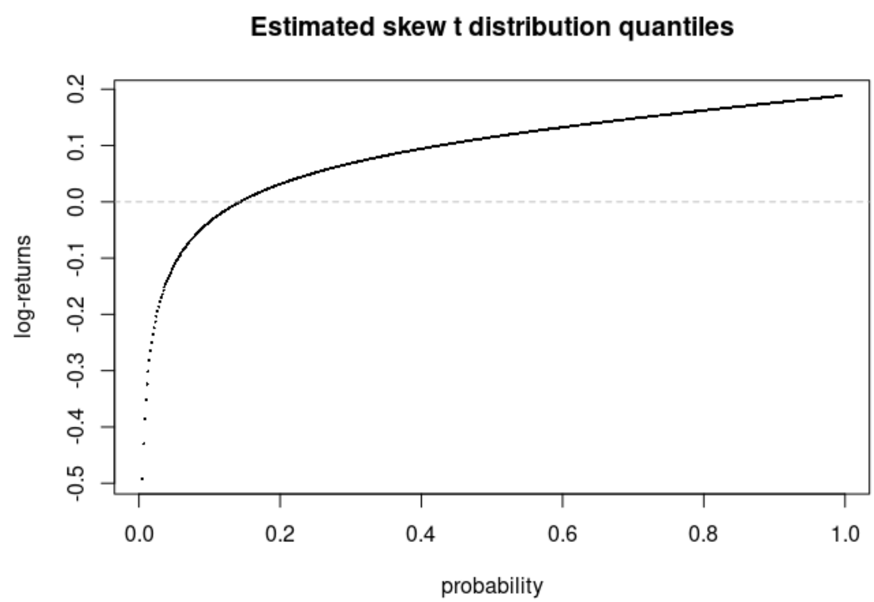
Let’s plot the fit and the observed returns together.



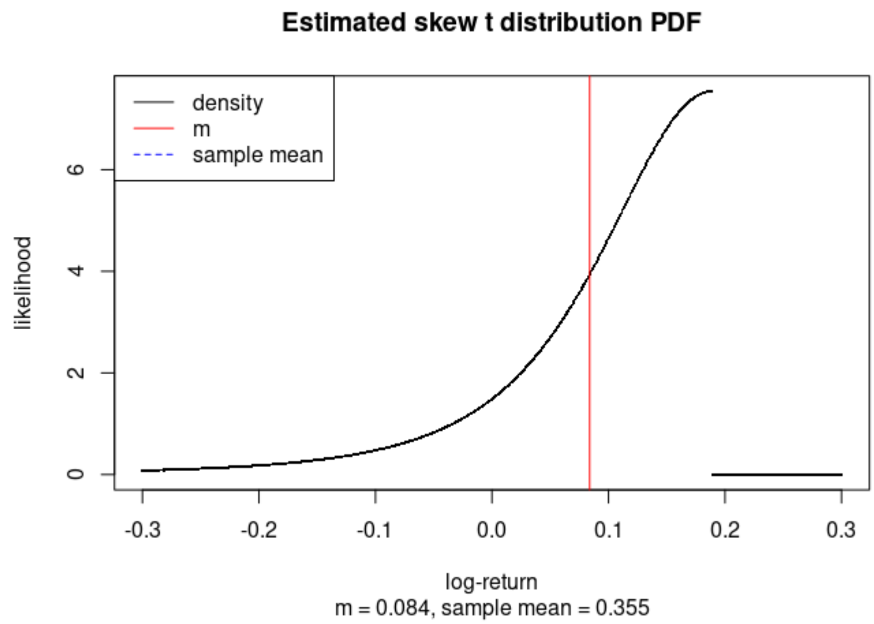
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



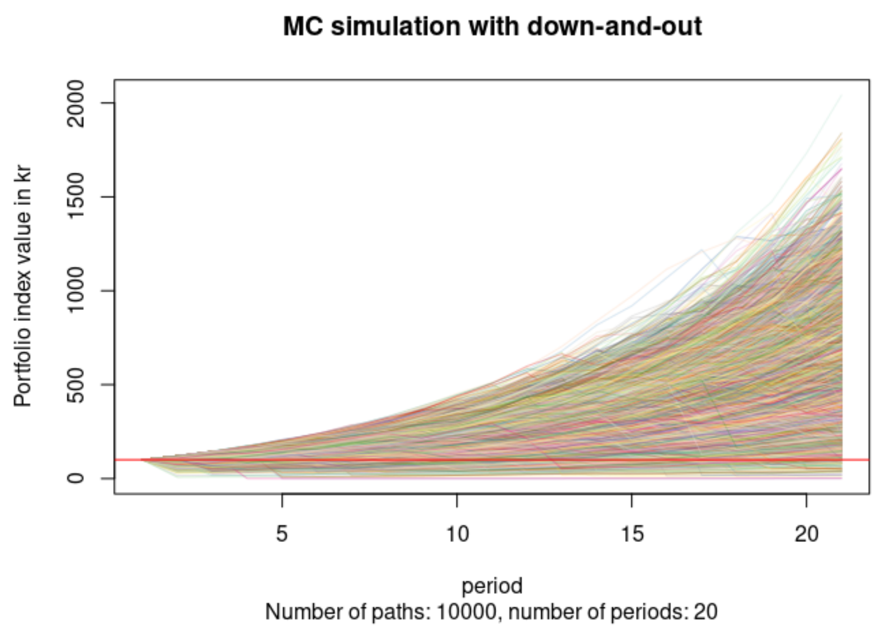


We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

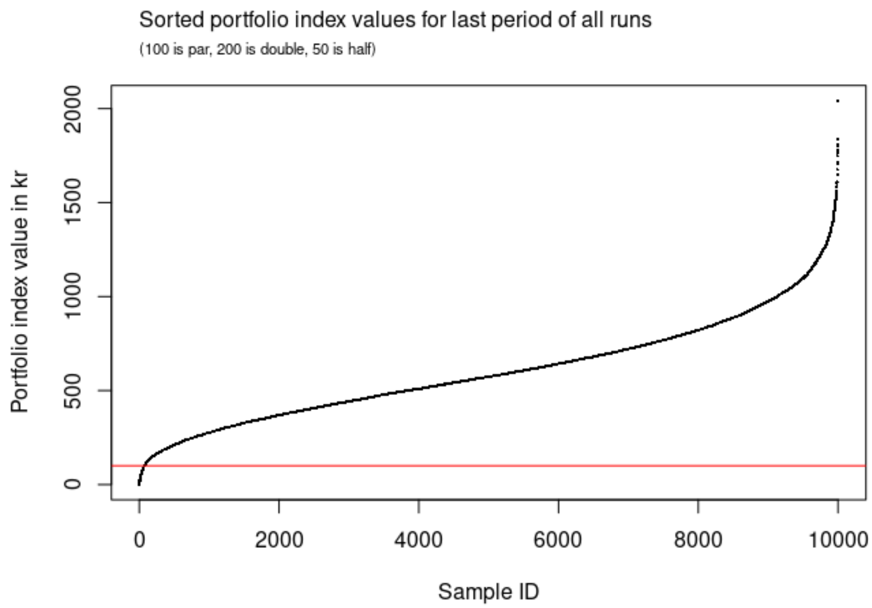


Estimated skew t distribution PDF
density
m
sample mean
likelihood
0
2
4
6
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
log-return
m = 0.084, sample mean = 0.355

Monte Carlo



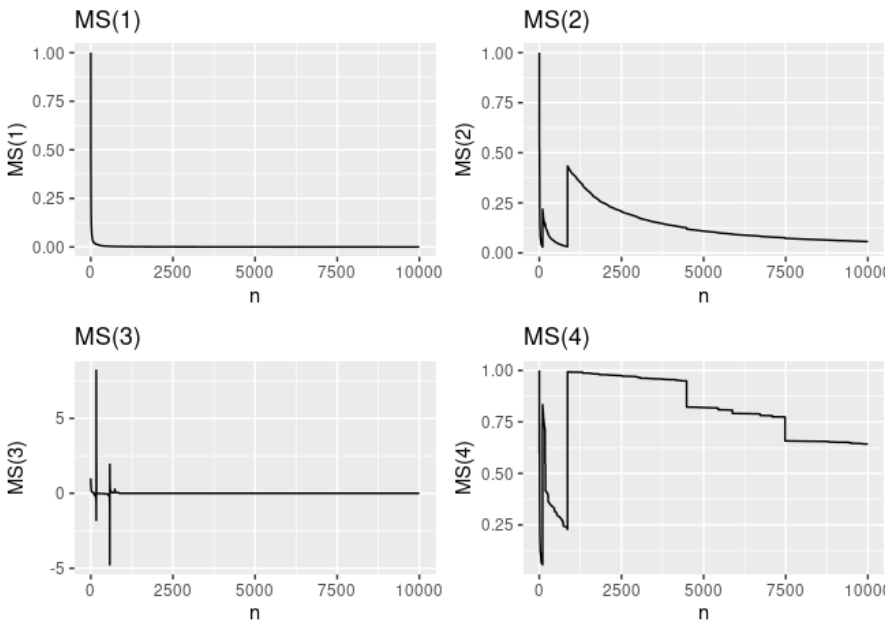
MC simulation with down-and-out
Portfolio index value in kr
0
500
1000
1500
2000
5
10
15
20
period
Number of paths: 10000, number of periods: 20



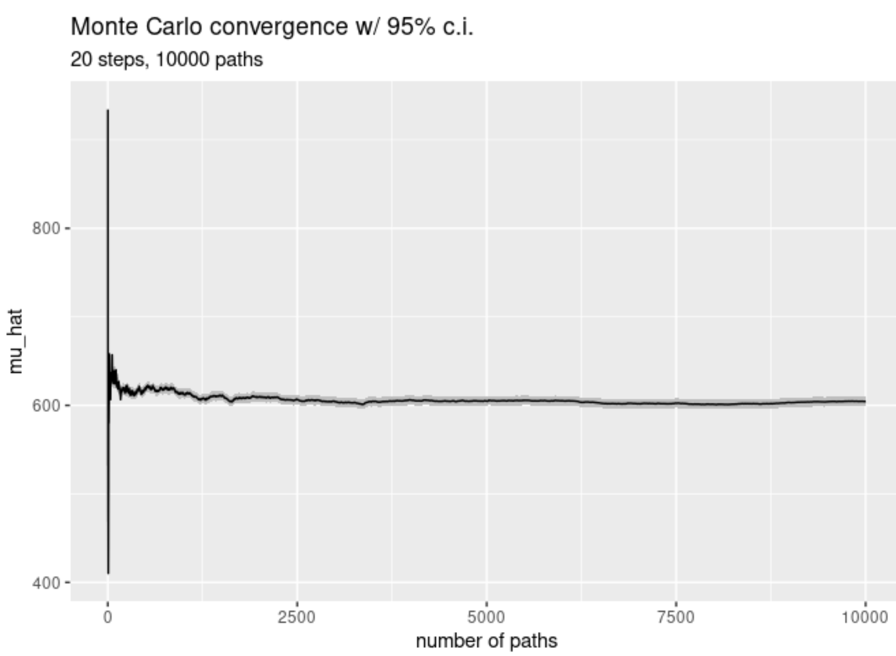
Convergence

Max vs sum

Max vs sum plots for the first four moments:


MS(1)
MS(2)
MS(3)
MS(4)
n
1.00
0.75
0.50
0.25
0.00
5
0
-5
0
2500
5000
7500
10000

MC

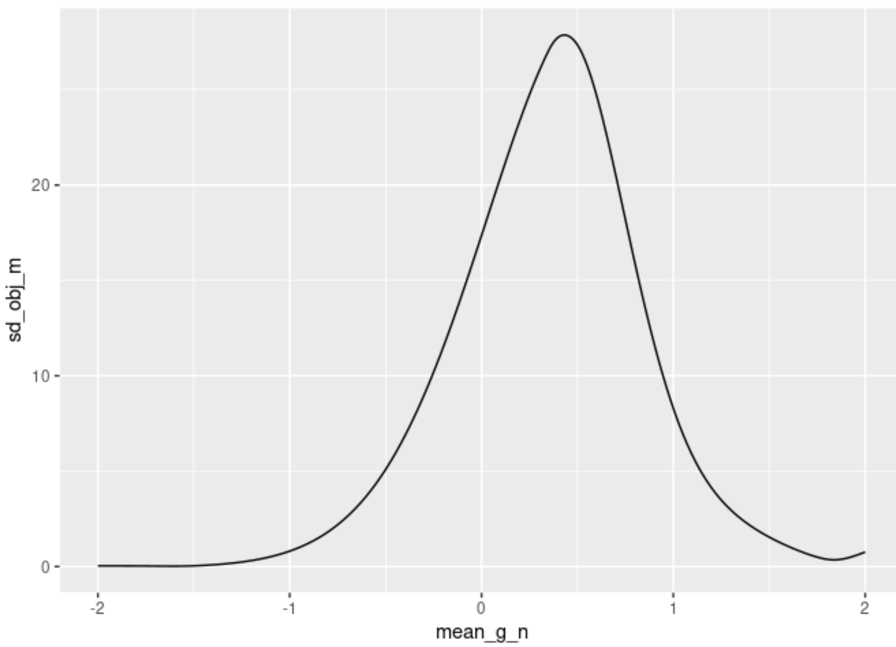

Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
800
600
400
0
2500
5000
7500
10000
number of paths

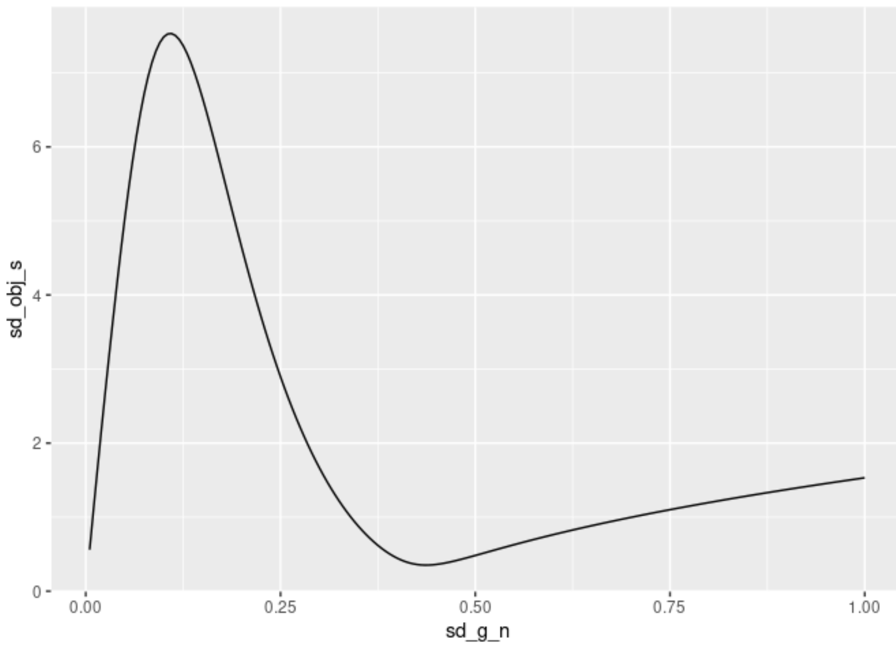
IS

Parameters

```
## [1] 1.8378581 0.4367791
```

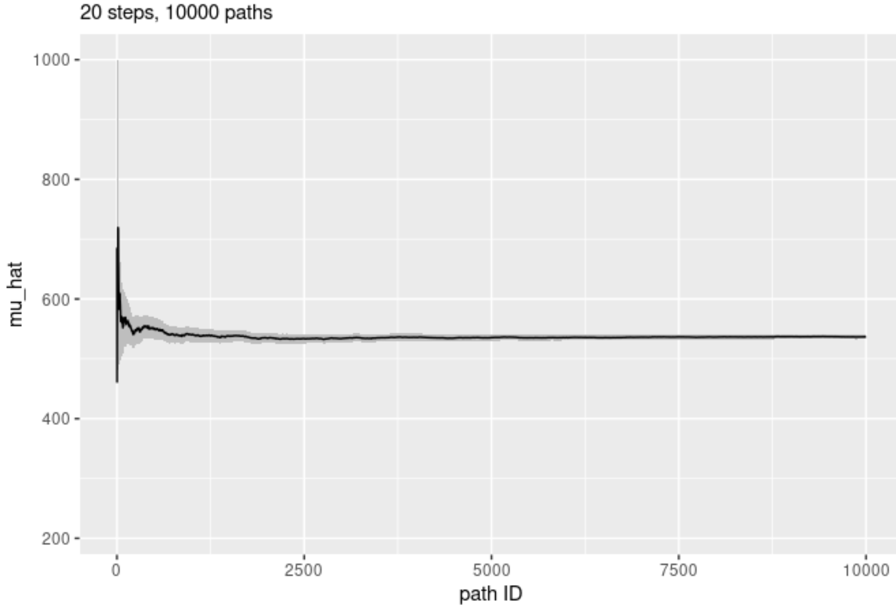
Objective function plots





Importance Sampling convergence w/ 95% c.i.

20 steps, 10000 paths

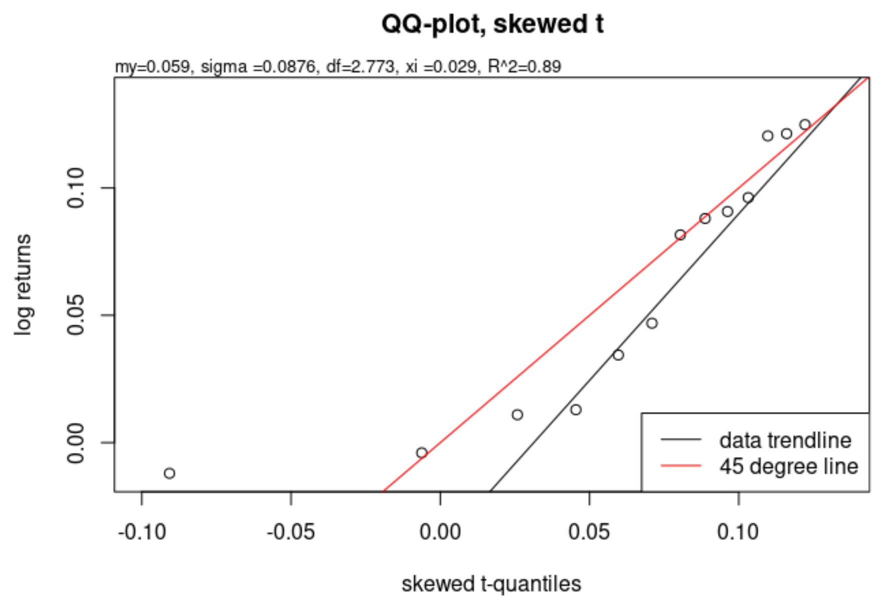


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -36.9603
## BIC: -34.7005
## m: 0.05902873
## s: 0.08757749
## nu (df): 2.772621
## xi: 0.02904471
## R^2: 0.89
##
## An R^2 of 0.89 suggests that the fit is not completely random.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 53.16667 percent
## What is the chance of gaining min 25 %? >= 44.16667 percent
## What is the chance of gaining min 50 %? >= 38.66667 percent
## What is the chance of gaining min 90 %? >= 34.16667 percent
## What is the chance of gaining min 99 %? >= 33.5 percent
```

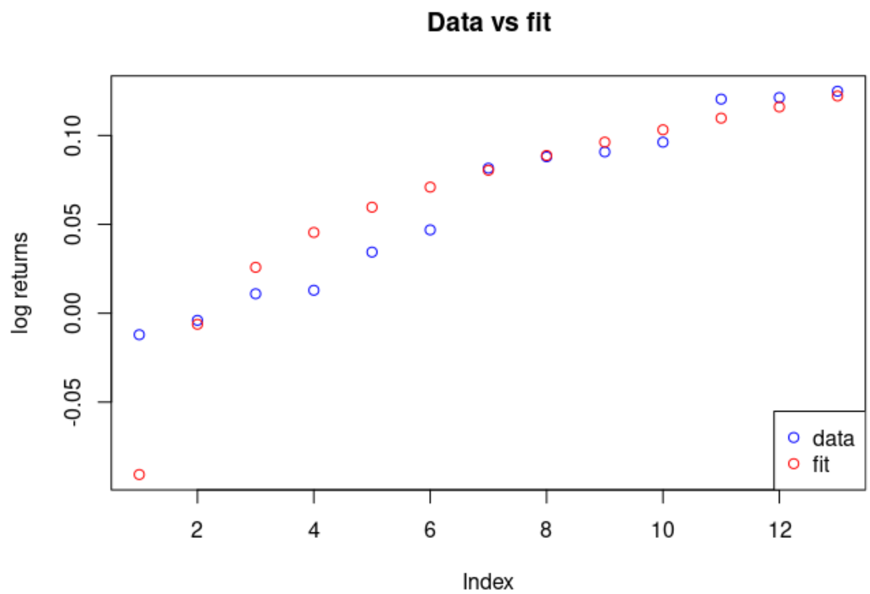
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

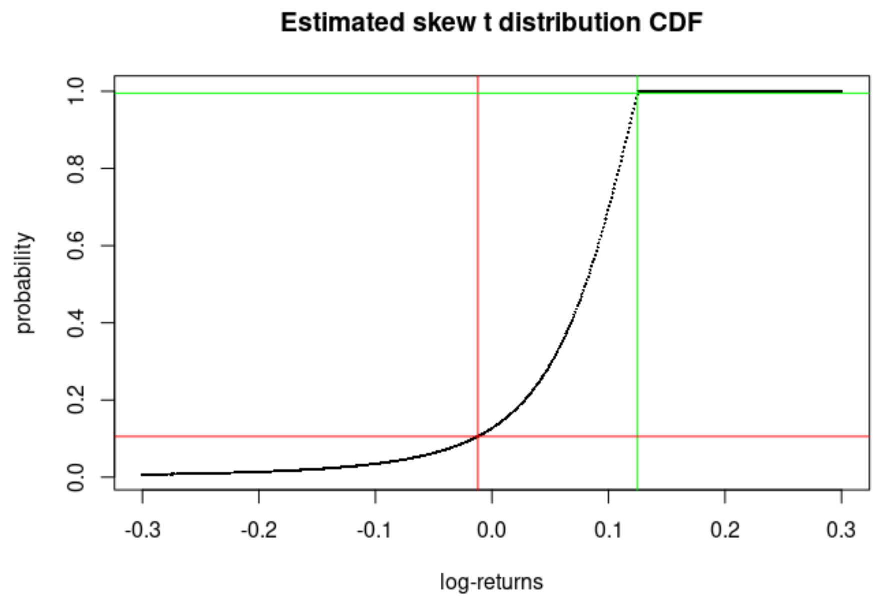
Let's plot the fit and the observed returns together.

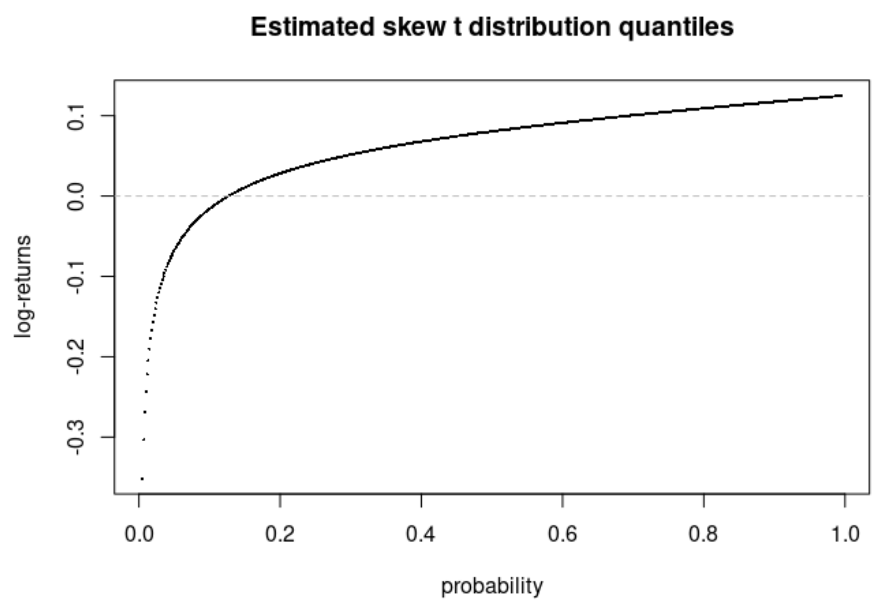


Interestingly, the fit predicts a much bigger "biggest loss" than the actual data. This is the main reason that R^2 is 0.90 and not higher.

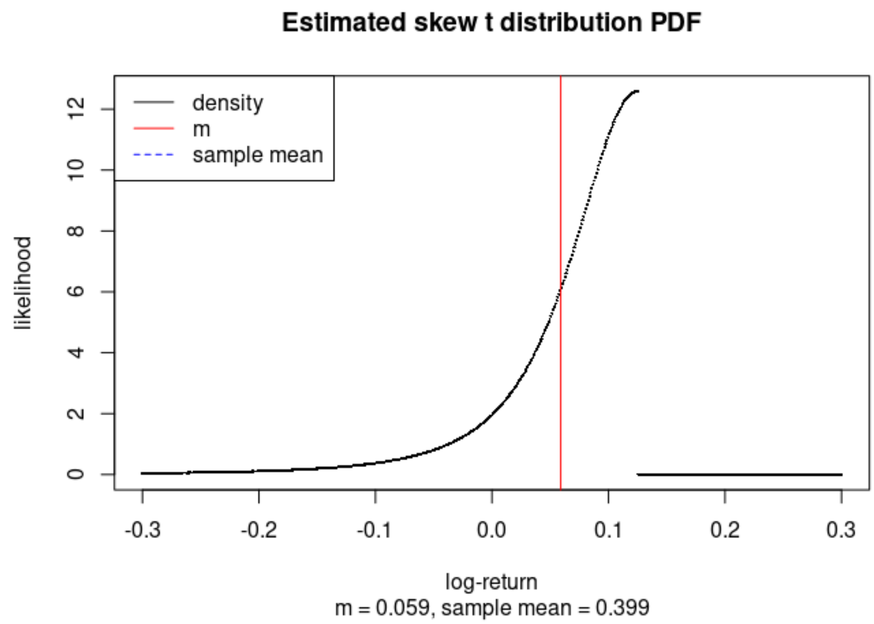
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



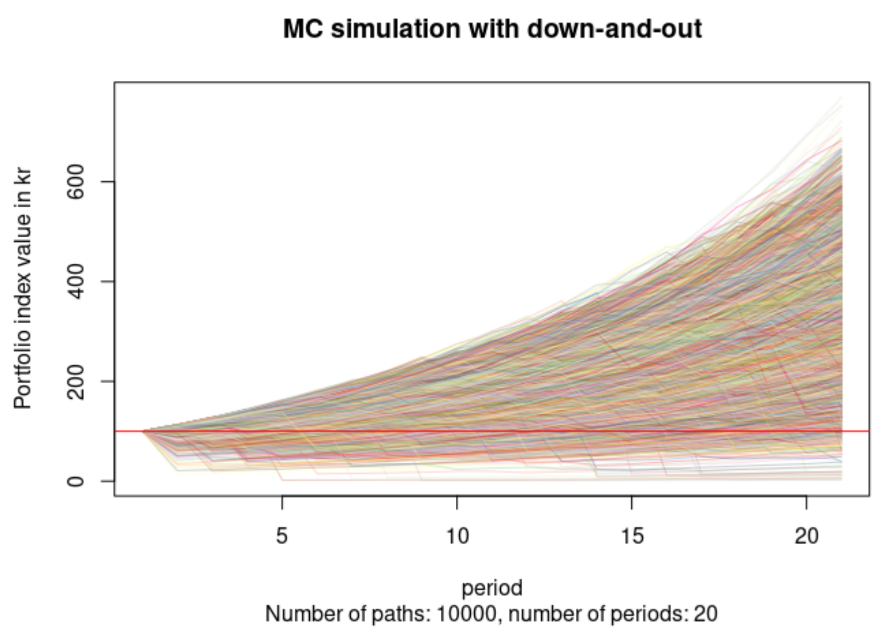


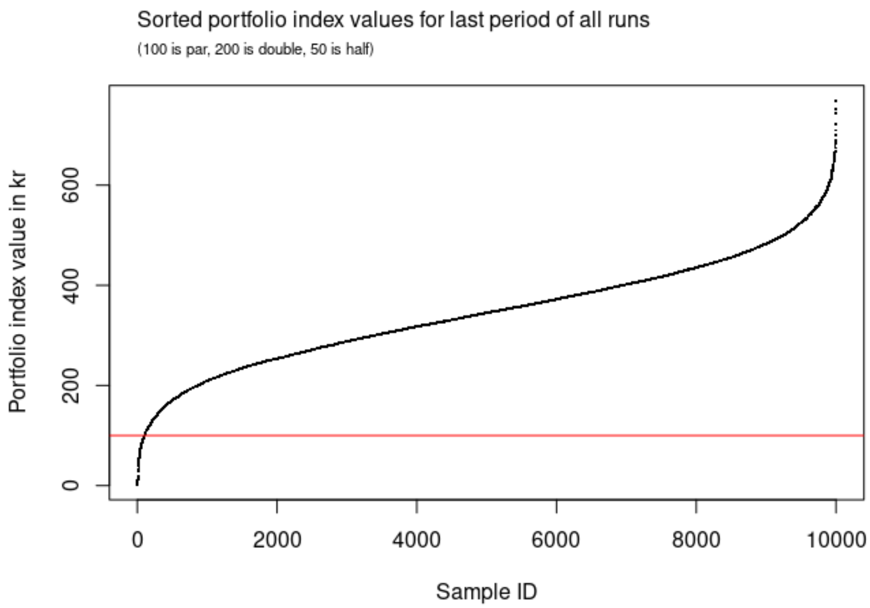
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.



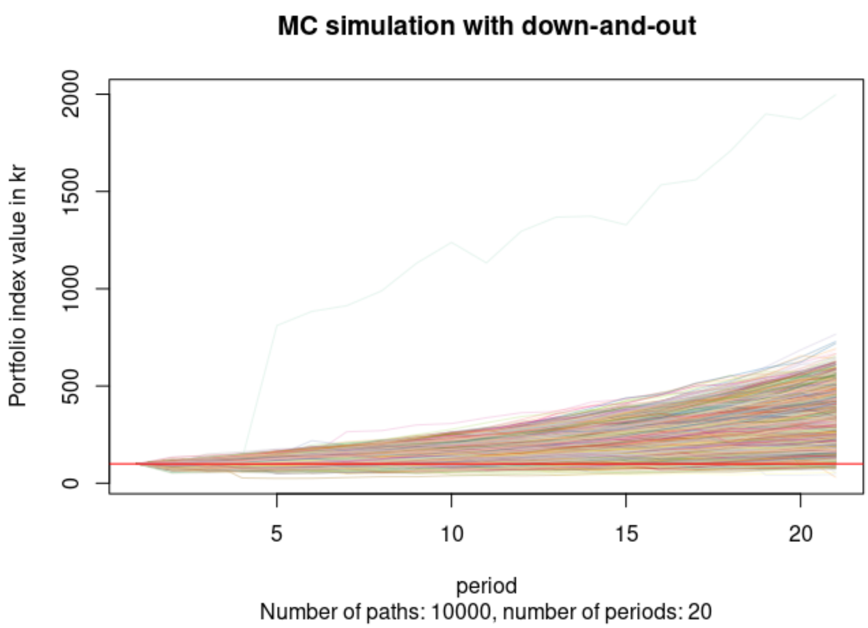
Monte Carlo

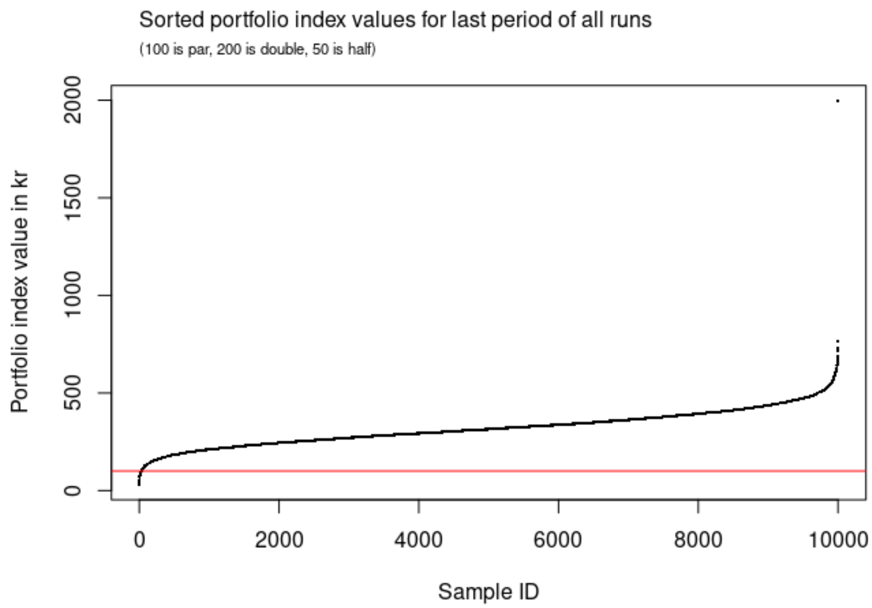
Version a: Simulation from estimated distribution of returns of mix.





Version b: Mix of simulations from estimated distribution of returns from individual funds.

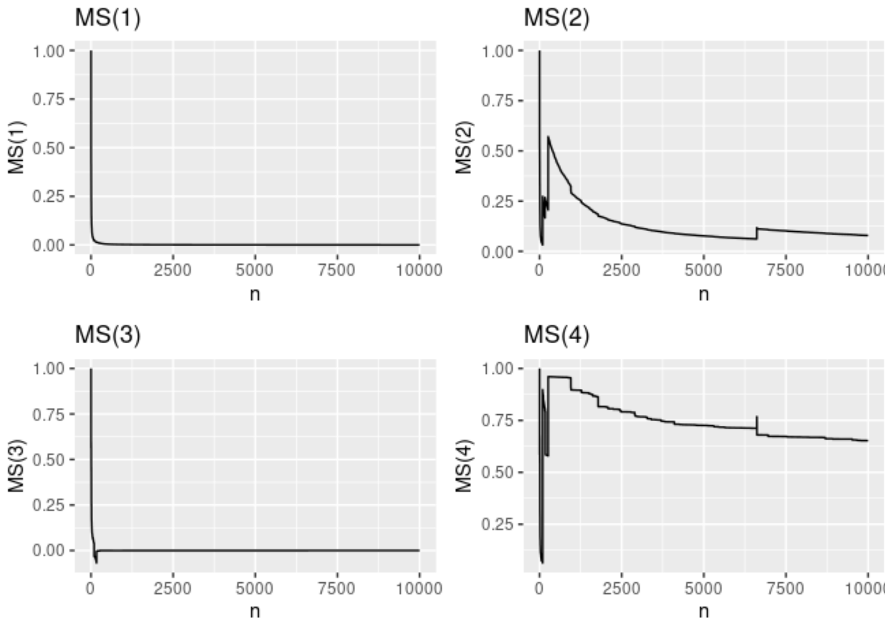




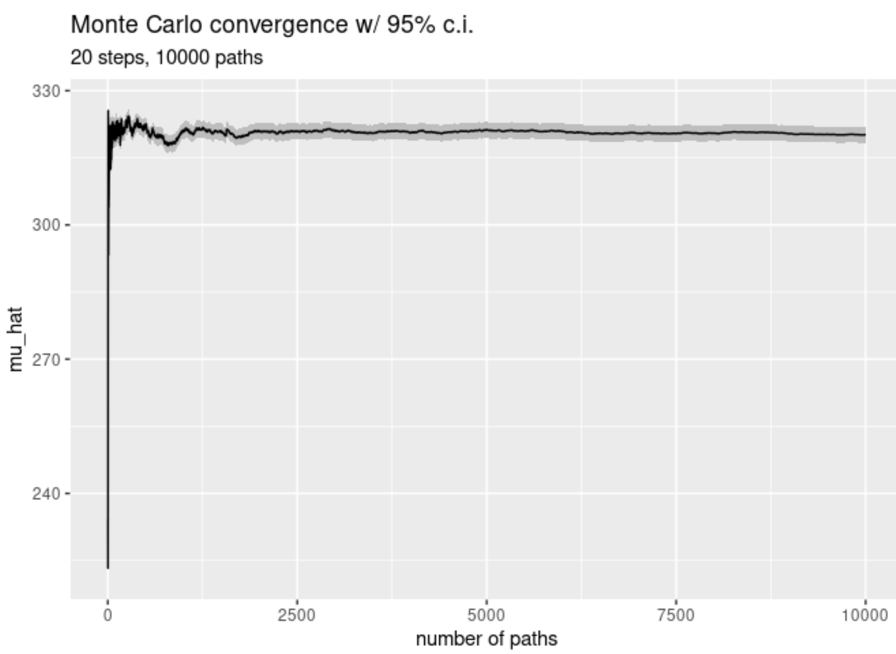
Convergence

Max vs sum

Max vs sum plots for the first four moments:


MS(1)
MS(2)
MS(3)
MS(4)
1.00
0.75
0.50
0.25
0.00
0
2500
5000
7500
10000
n

MC

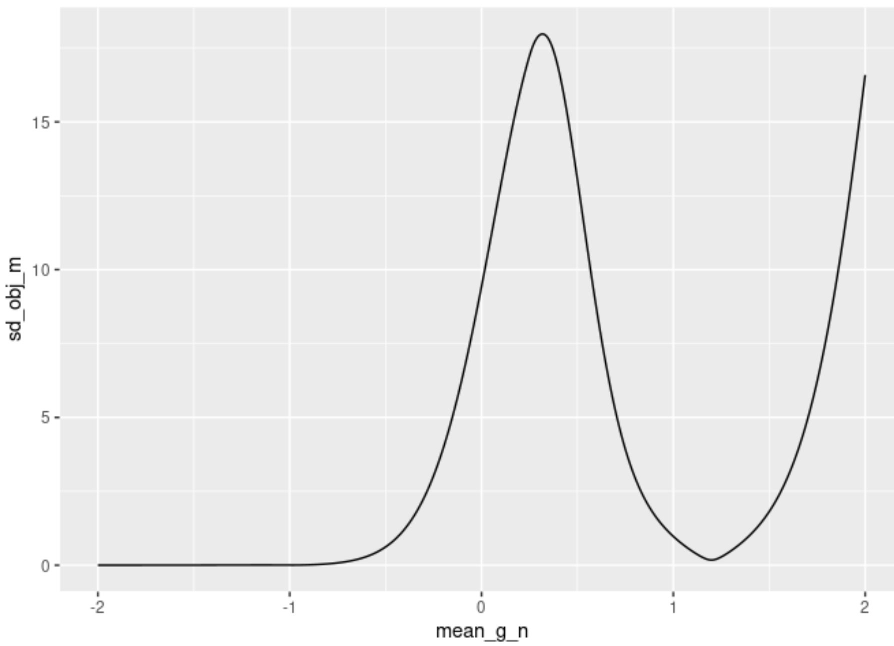

Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
330
300
270
240
mu_hat
0
2500
5000
7500
10000
number of paths

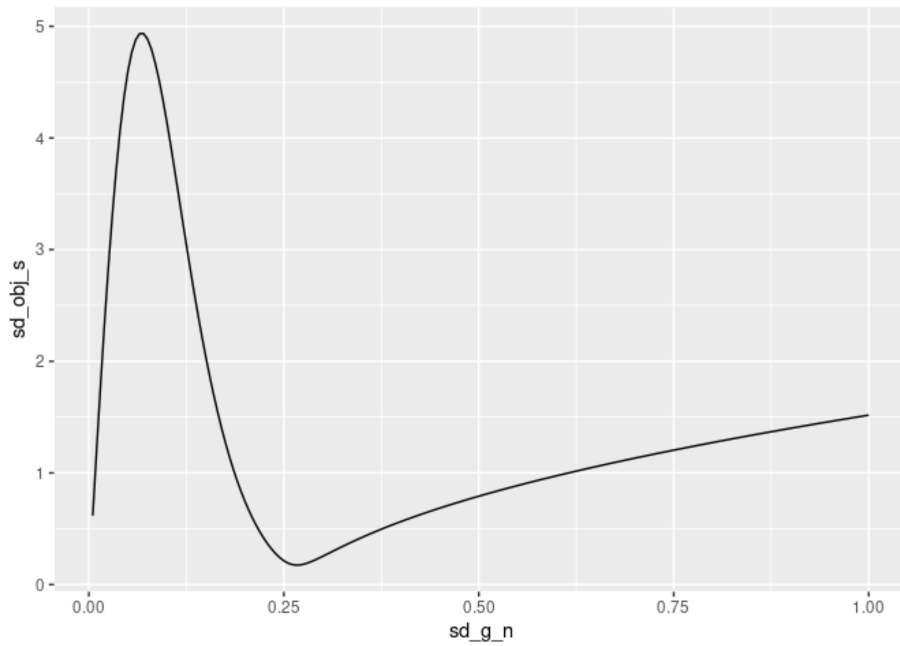
IS

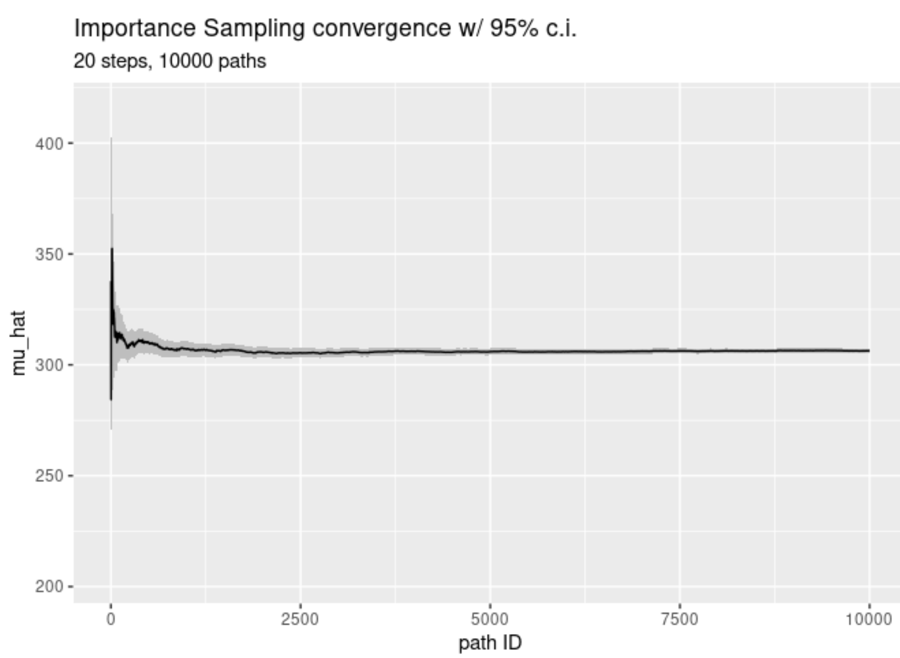
Parameters

```
## [1] 1.1990016 0.2668179
```

Objective function plots




sd_obj_s
sd_g_n
0
1
2
3
4
5
0.00
0.25
0.50
0.75
1.00

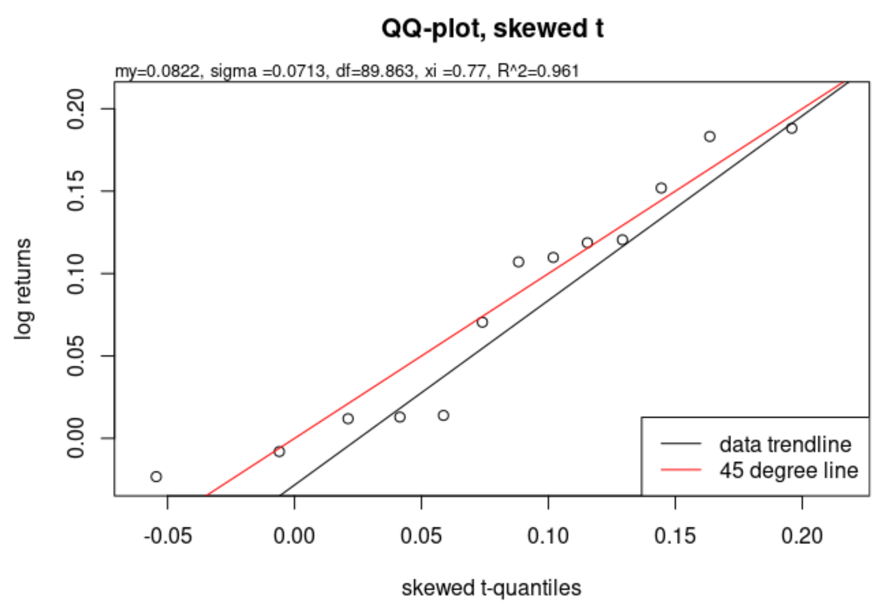

Importance Sampling convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
path ID
200
250
300
350
400
0
2500
5000
7500
10000

Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -24.26084
## BIC: -22.00104
## m: 0.0822419
## s: 0.07129843
## nu (df): 89.86289
## xi: 0.7697502
## R^2: 0.961
##
## An R^2 of 0.961 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 52.5 percent
## What is the chance of gaining min 25 %? >= 45 percent
## What is the chance of gaining min 50 %? >= 38.33333 percent
## What is the chance of gaining min 90 %? >= 31.16667 percent
## What is the chance of gaining min 99 %? >= 29.83333 percent
```

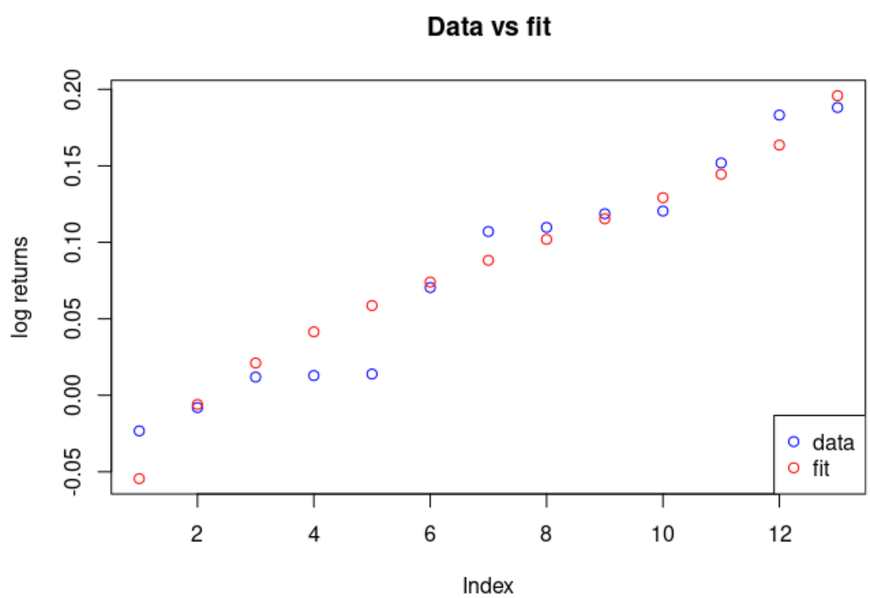
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

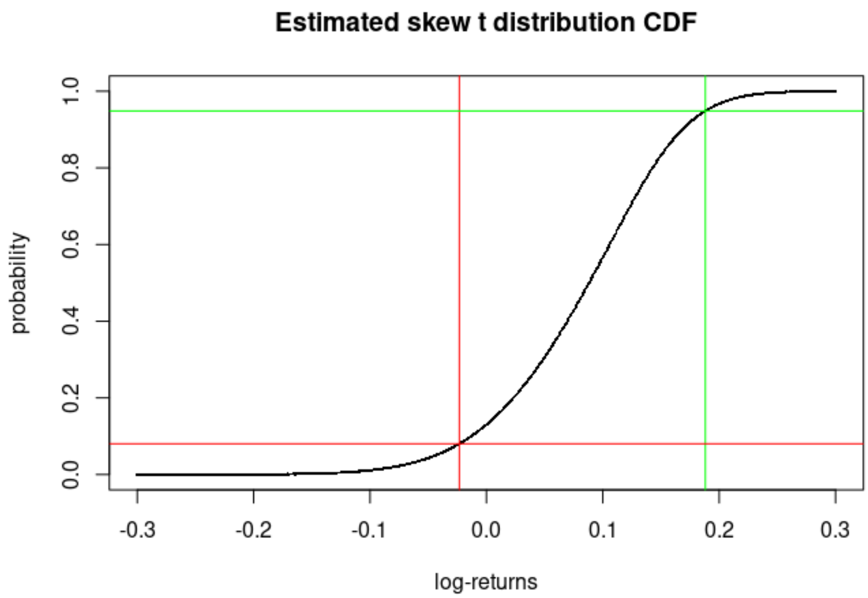
Data vs fit

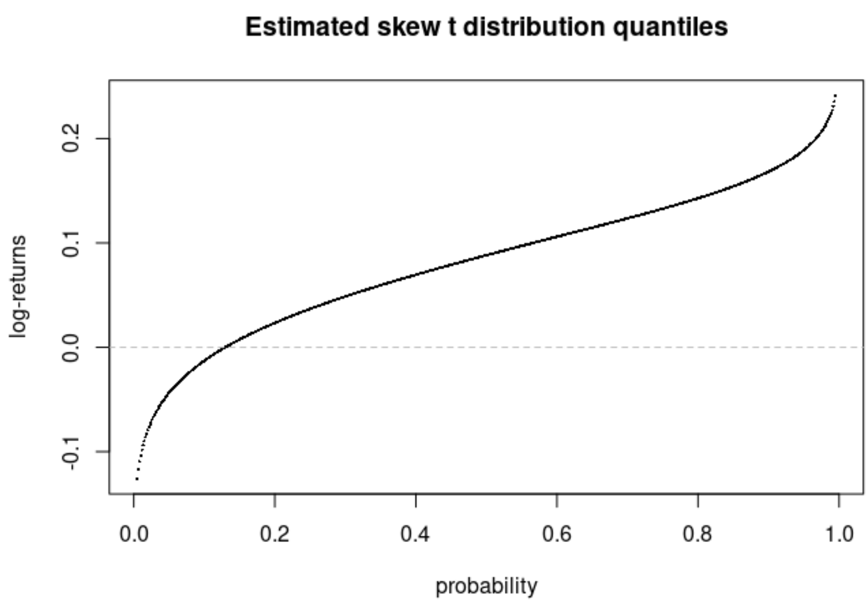
Let's plot the fit and the observed returns together.



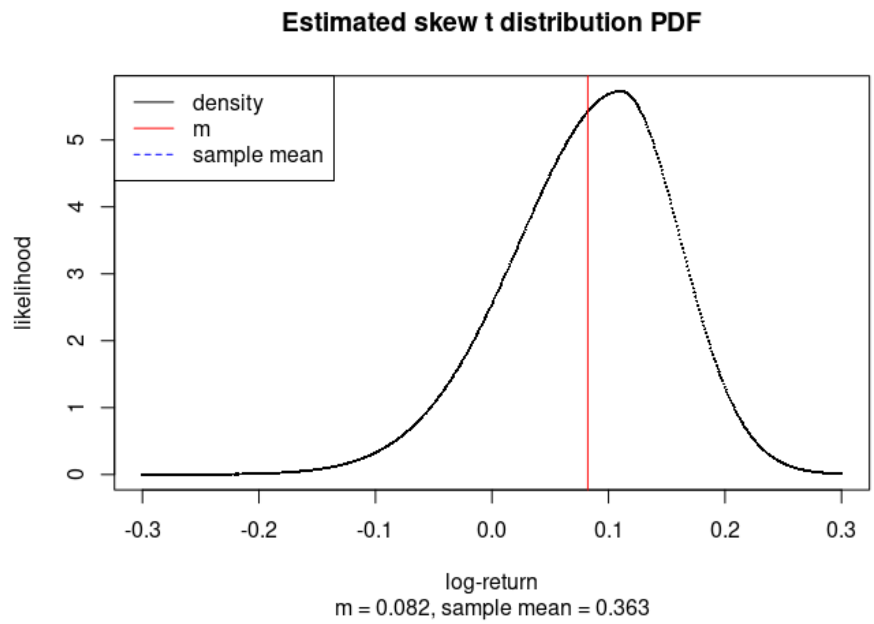
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



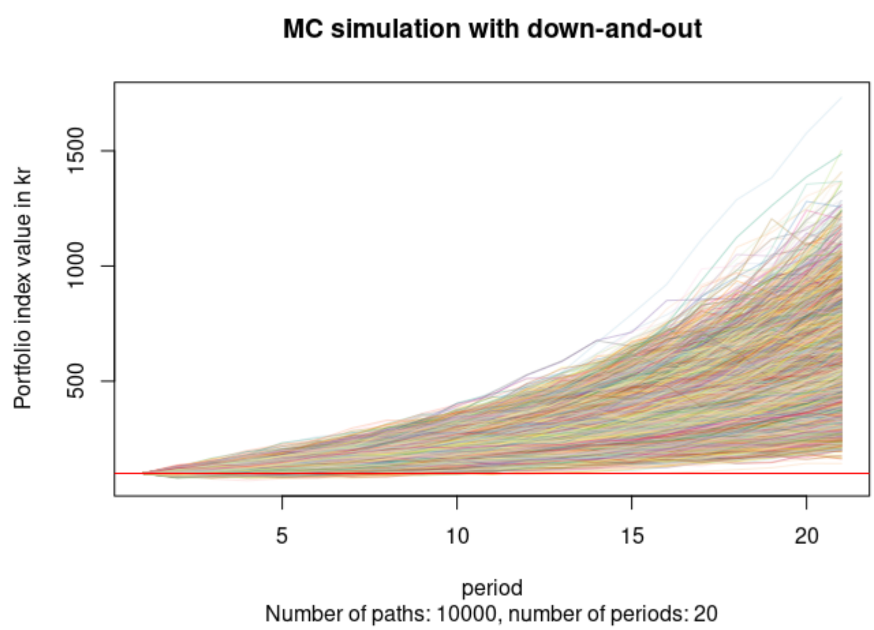


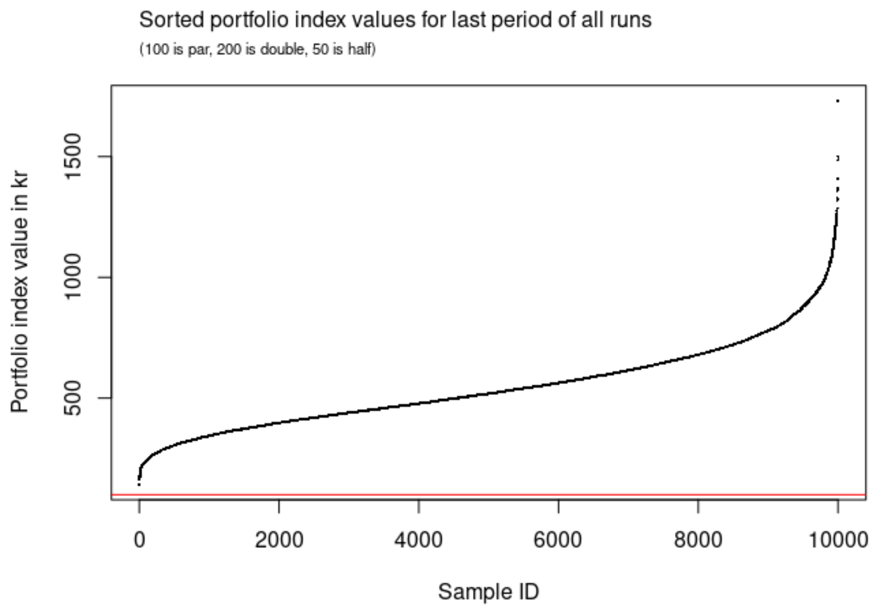
We see that the high risk mix provides a much better upside and smaller downside.



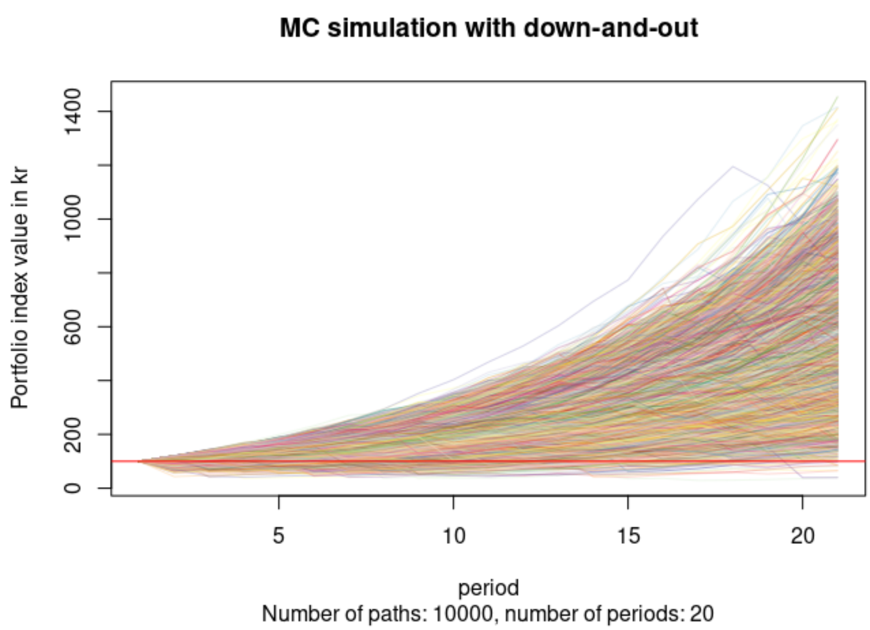
Monte Carlo

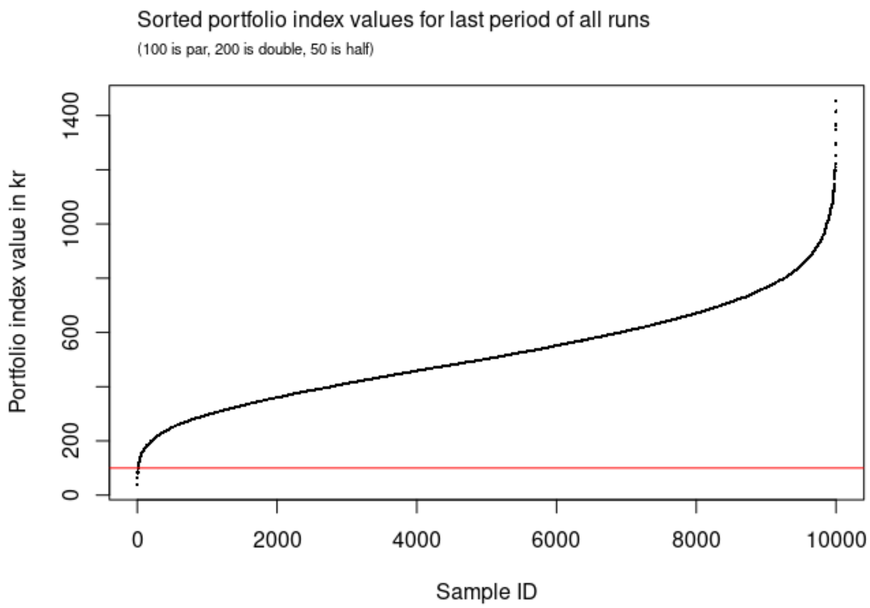
Version a: Simulation from estimated distribution of returns of mix.





Version b: Mix of simulations from estimated distribution of returns from individual funds.





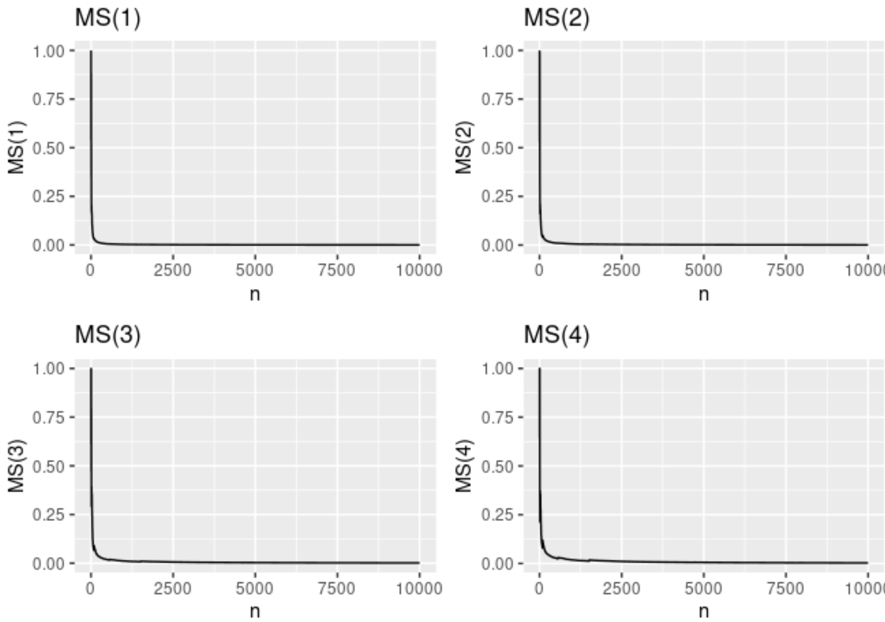
Many simulations 1e6 paths:

```
# Down-and-out simulation:
# Probability of down-and-out: 0 percent
#
# Mean portfolio index value after 20 years: 478.339 kr.
# SD of portfolio index value after 20 years: 163.093 kr.
# Min total portfolio index value after 20 years: 2.233 kr.
# Max total portfolio index value after 20 years: 1561.965 kr.
#
# Share of paths finishing below 100: 0.1181 percent
```

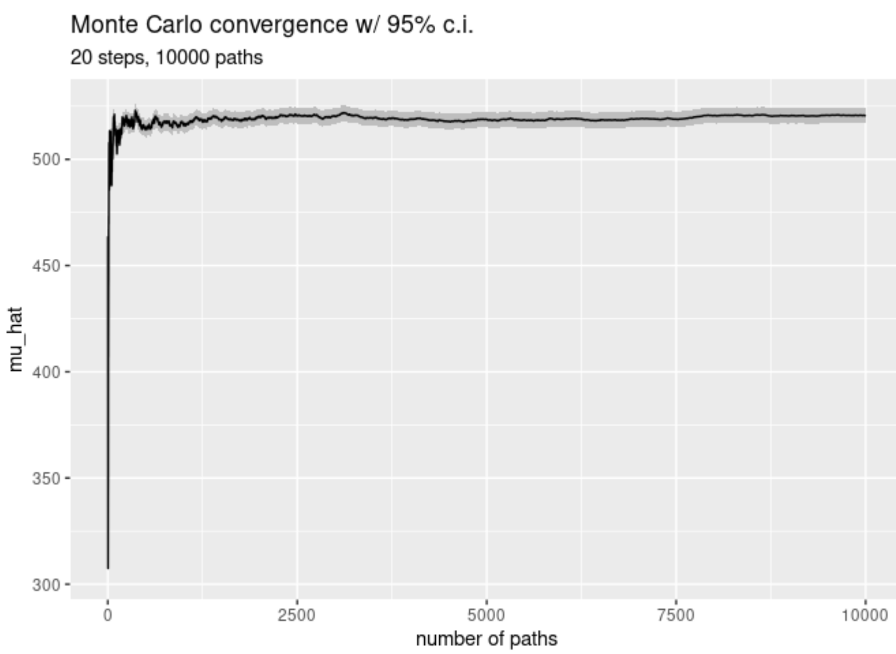
Convergence

Max vs sum

Max vs sum plots for the first four moments:


MS(1)
MS(2)
MS(3)
MS(4)
1.00
0.75
0.50
0.25
0.00
0
2500
5000
7500
10000
n

MC

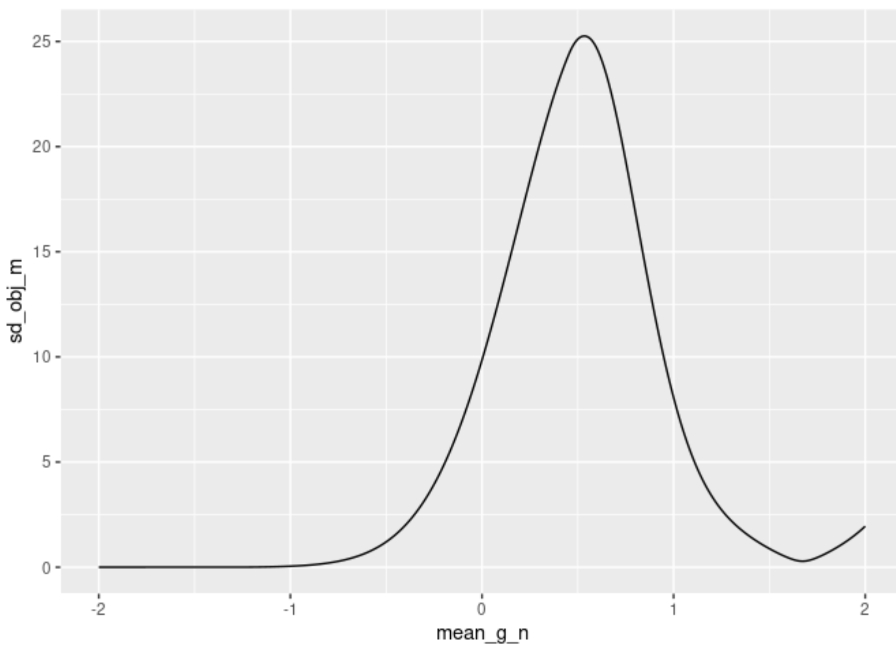

Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
500
450
400
350
300
0
2500
5000
7500
10000
number of paths

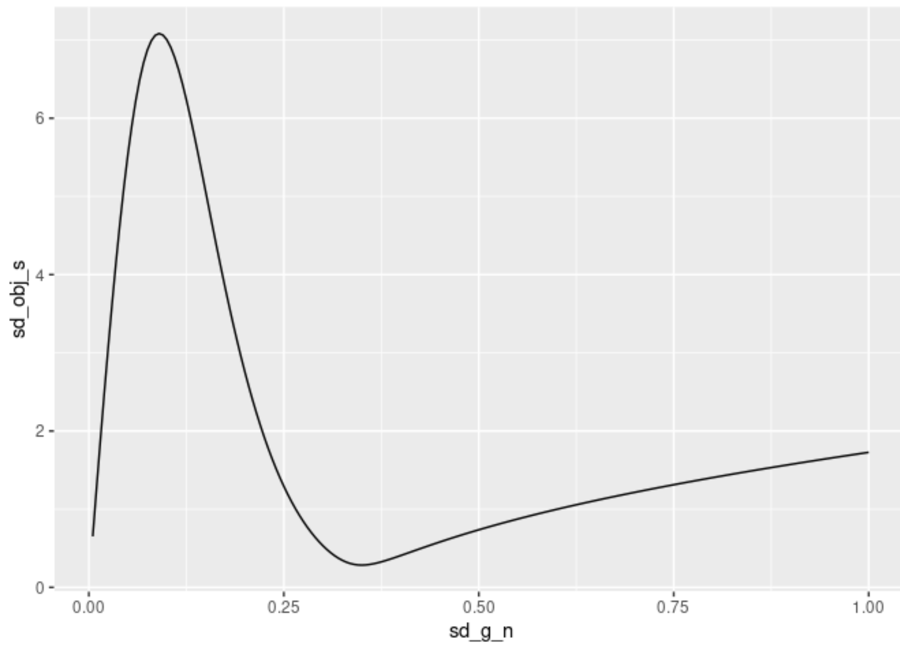
IS

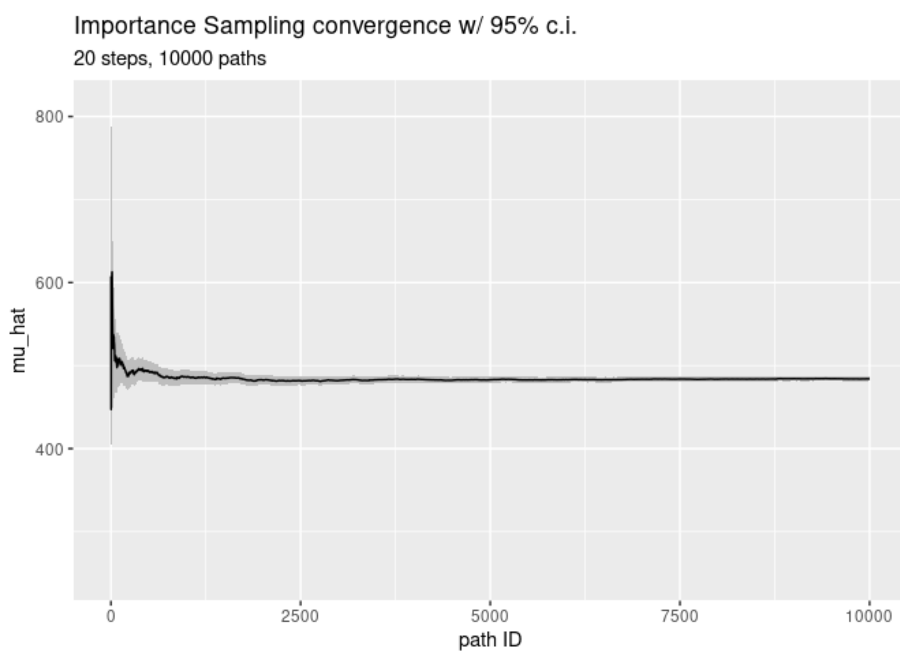
Parameters

```
## [1] 1.6731455 0.3497998
```

Objective function plots







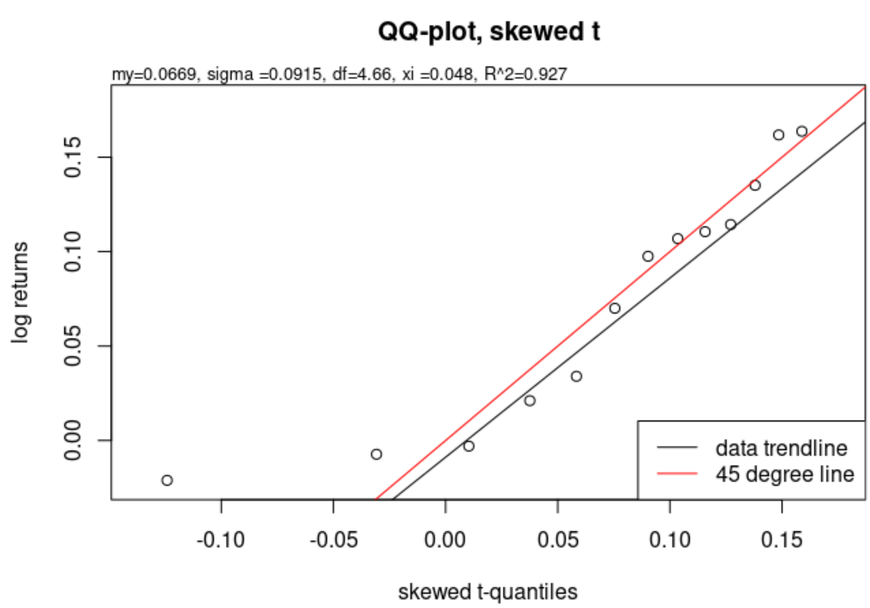
Mix vmr+phr, 2011 - 2023

Log-returns for mix of Velliv medium risk (vm) and PFA high risk (ph):

Fit to skew t distribution

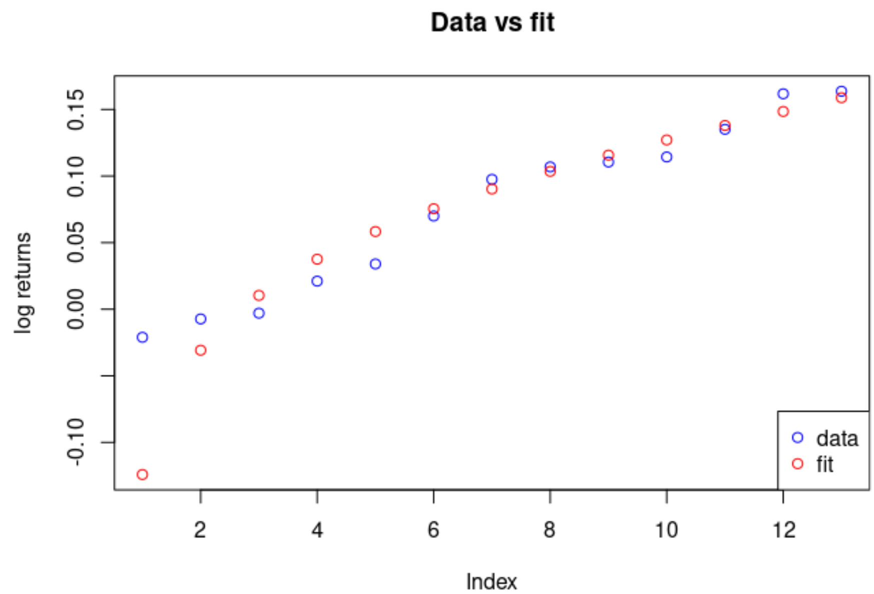
```
##
## AIC: -29.6509
## BIC: -27.3911
## m: 0.0668553
## s: 0.09147987
## nu (df): 4.659549
## xi: 0.04824493
## R^2: 0.927
##
## An R^2 of 0.927 suggests that the fit is good.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 57.66667 percent
## What is the chance of gaining min 25 %? >= 46.33333 percent
## What is the chance of gaining min 50 %? >= 38 percent
## What is the chance of gaining min 90 %? >= 31 percent
## What is the chance of gaining min 99 %? >= 29.83333 percent
```

QQ Plot



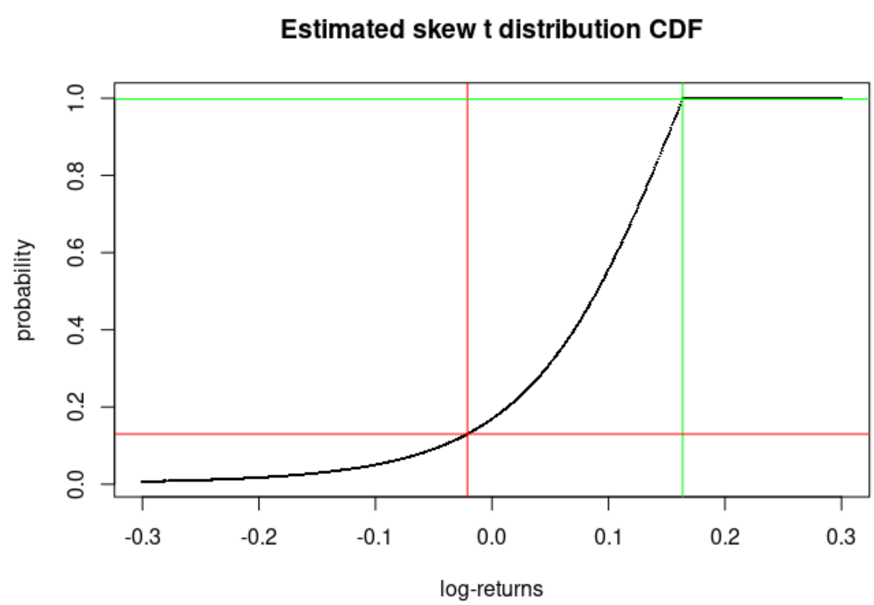
Data vs fit

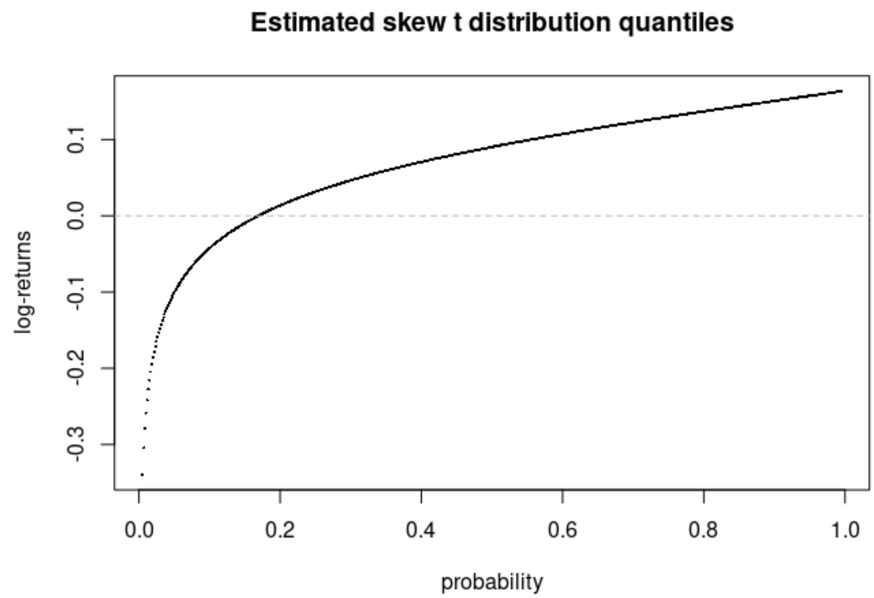
Let's plot the fit and the observed returns together.

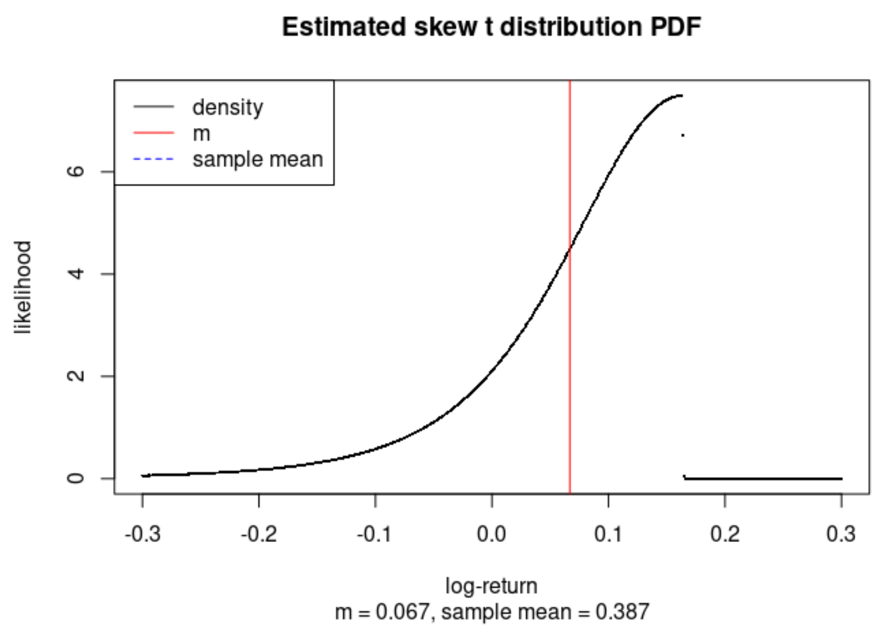


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

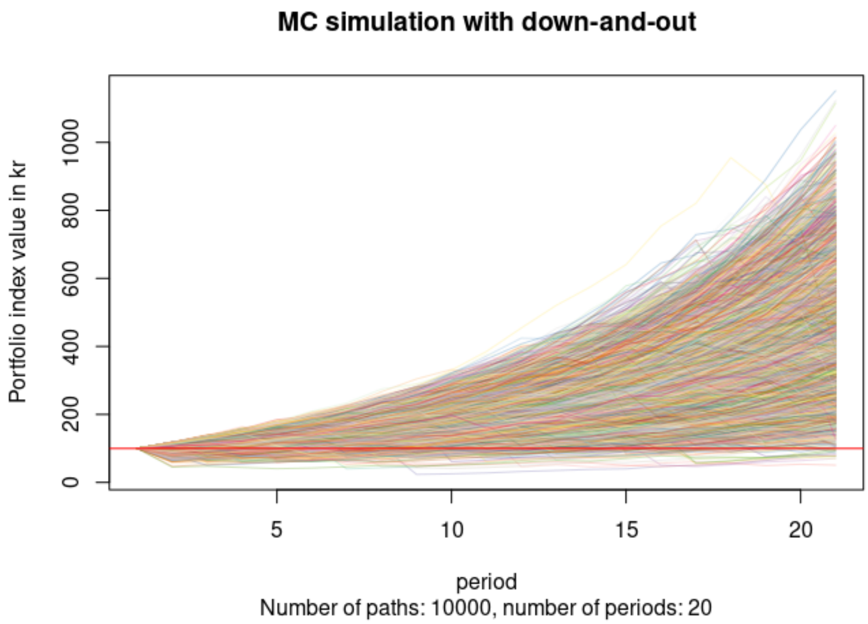


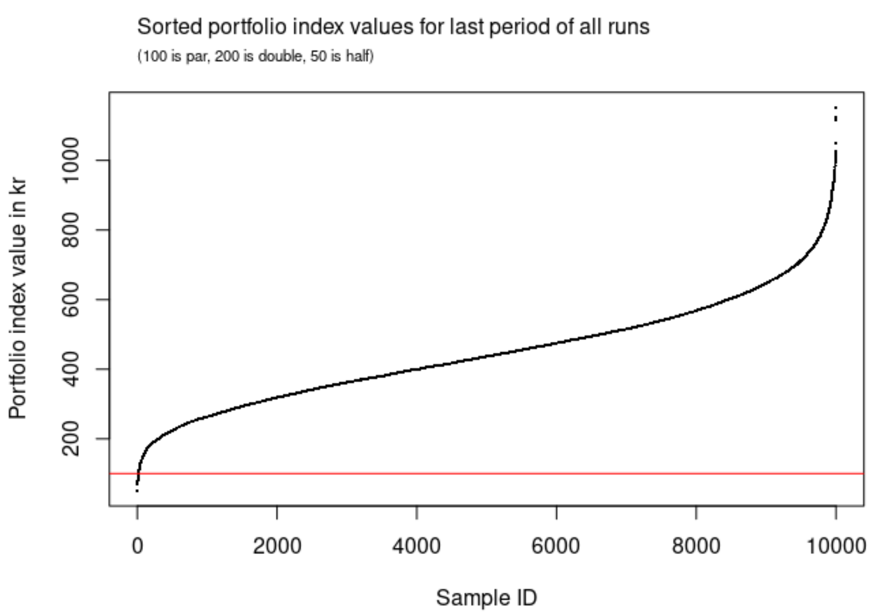

Estimated skew t distribution quantiles
log-returns
0.1
0.0
-0.1
-0.2
-0.3
0.0
0.2
0.4
0.6
0.8
1.0
probability


Estimated skew t distribution PDF
density
m
sample mean
likelihood
6
4
2
0
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
log-return
m = 0.067, sample mean = 0.387

Monte Carlo

Mix of simulations from estimated distribution of returns from individual funds.

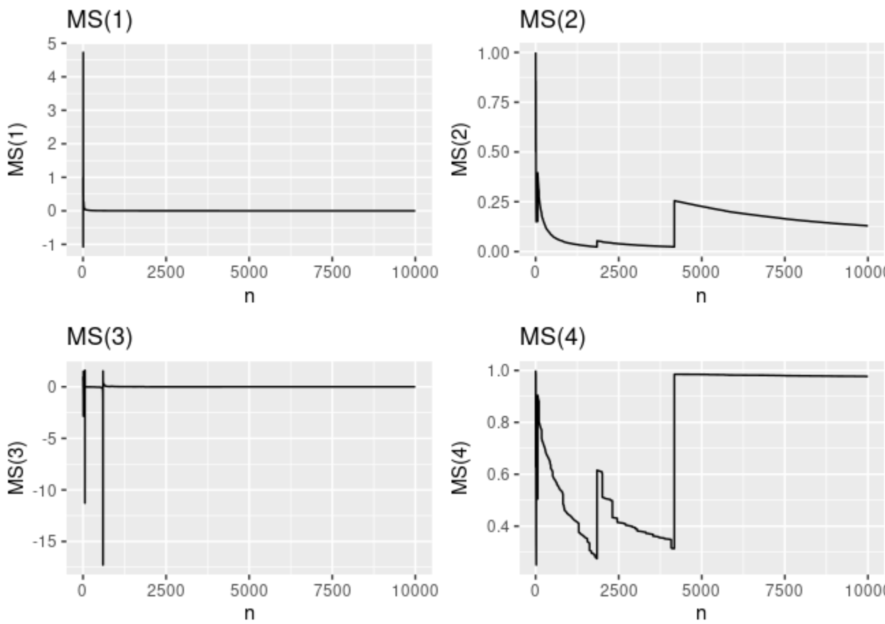




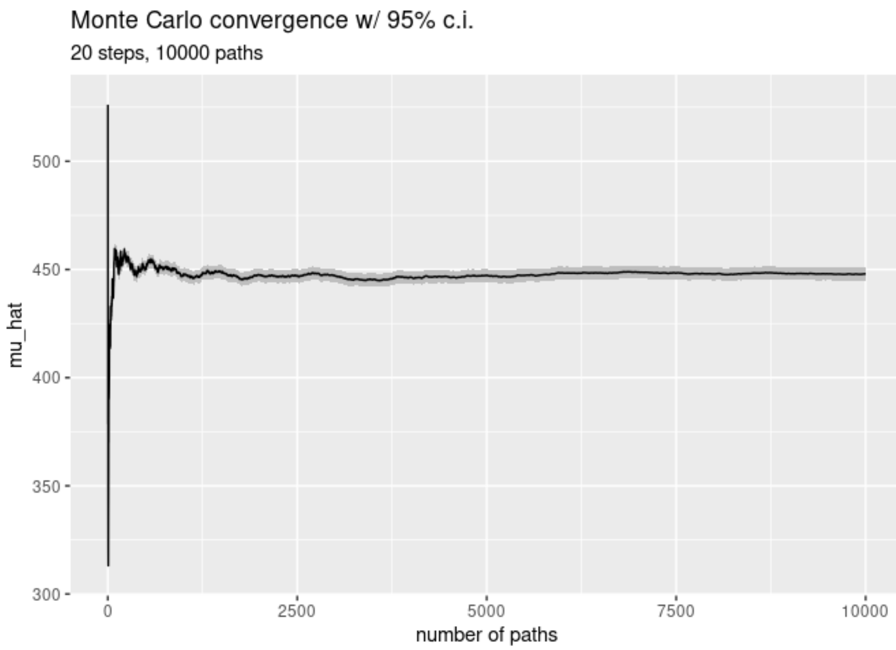
Convergence

Max vs sum

Max vs sum plots for the first four moments:



MC

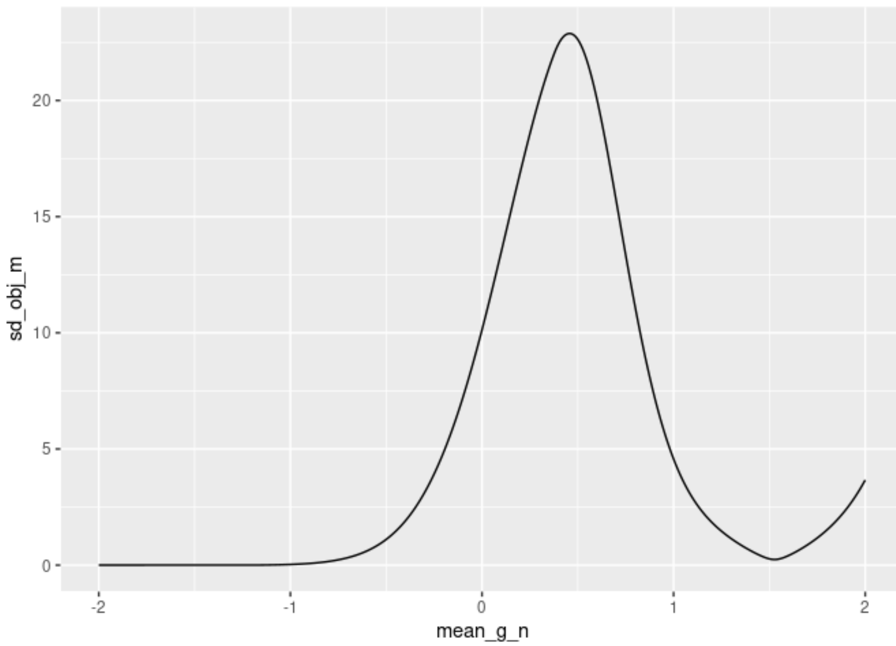


IS

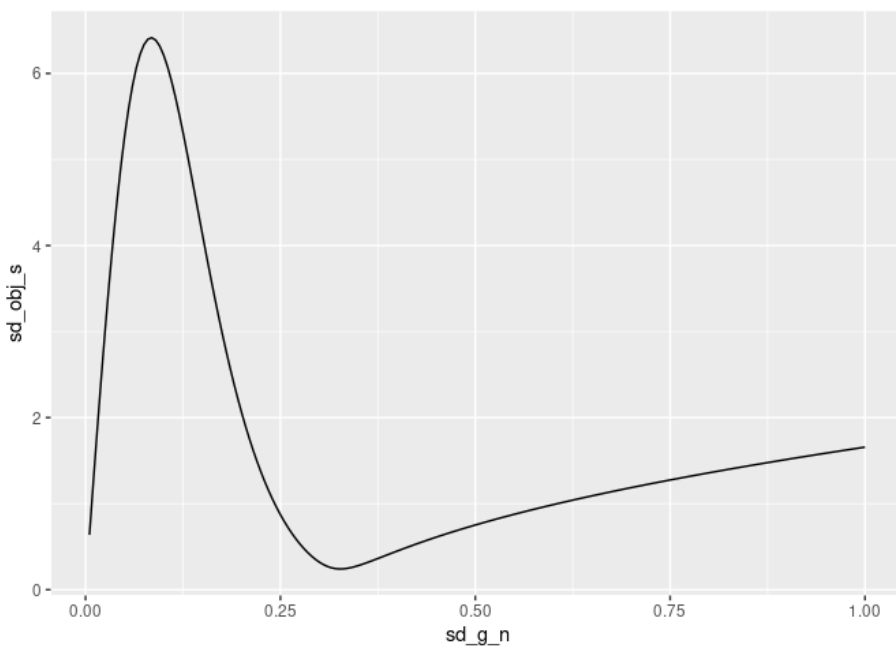
Parameters

```
## [1] 1.526841 0.326815
```

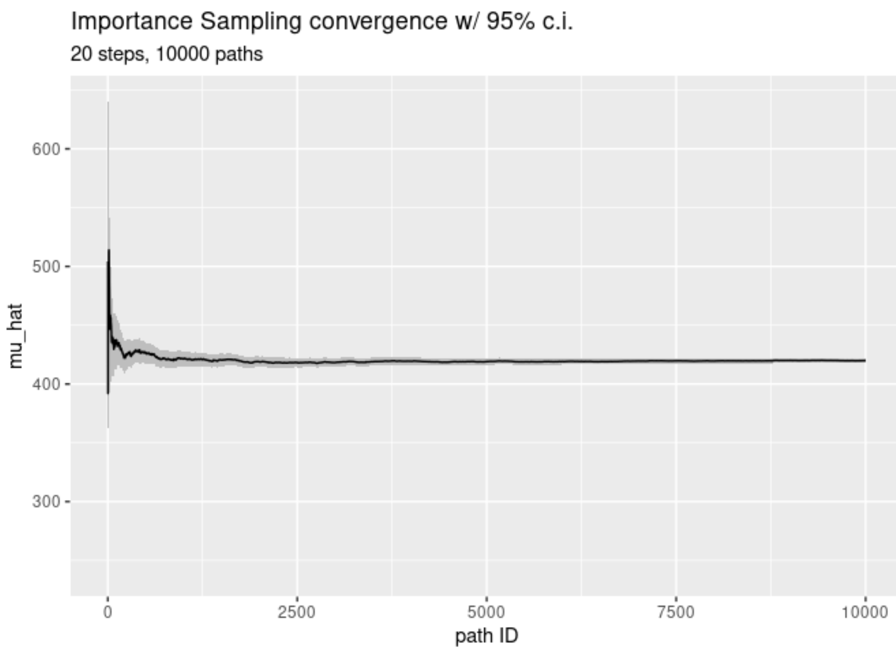
Objective function plots



20
15
10
5
0
sd_obj_m
-2
-1
0
1
2
mean_g_n



6
4
2
0
sd_obj_s
0.00
0.25
0.50
0.75
1.00
sd_g_n



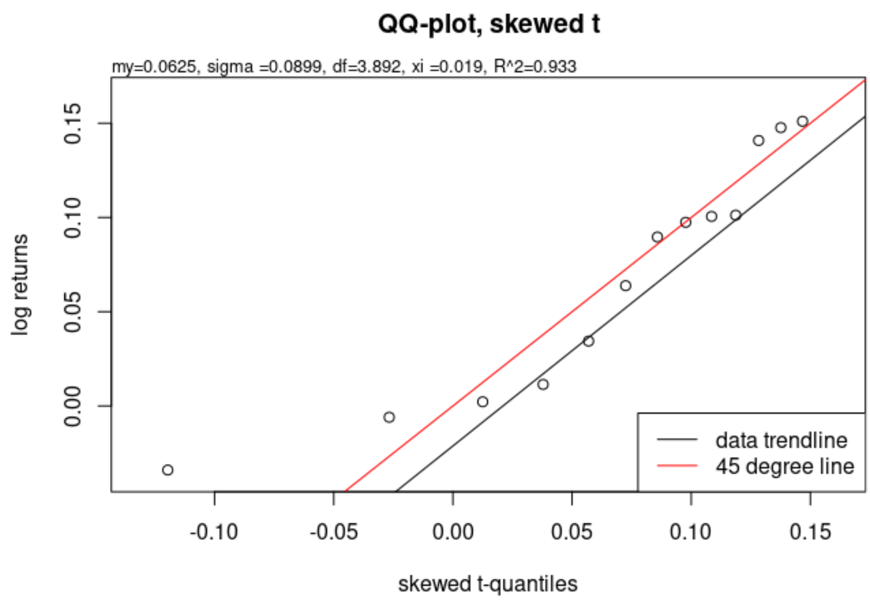
Mix vhr+pmr, 2011 - 2023

Log-returns for mix of Velliv high risk (vh) and PFA medium risk (pm):

Fit to skew t distribution

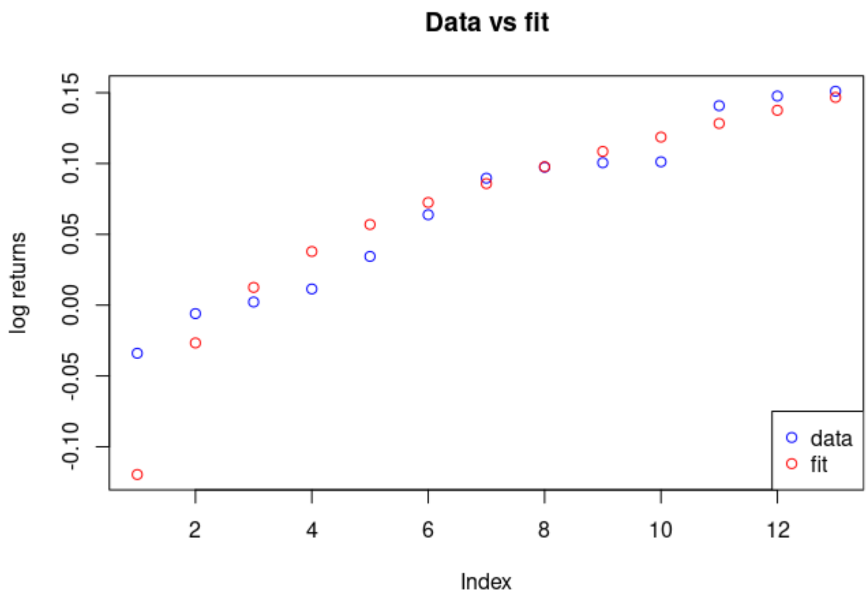
```
##
## AIC: -31.1004
## BIC: -28.84061
## m: 0.06249566
## s: 0.0898826
## nu (df): 3.89221
## xi: 0.01893003
## R^2: 0.933
##
## An R^2 of 0.933 suggests that the fit is good.
##
## What is the risk of losing max 10 %? =< 0 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 57 percent
## What is the chance of gaining min 25 %? >= 46 percent
## What is the chance of gaining min 50 %? >= 38.5 percent
## What is the chance of gaining min 90 %? >= 32.16667 percent
## What is the chance of gaining min 99 %? >= 31.16667 percent
```

QQ Plot



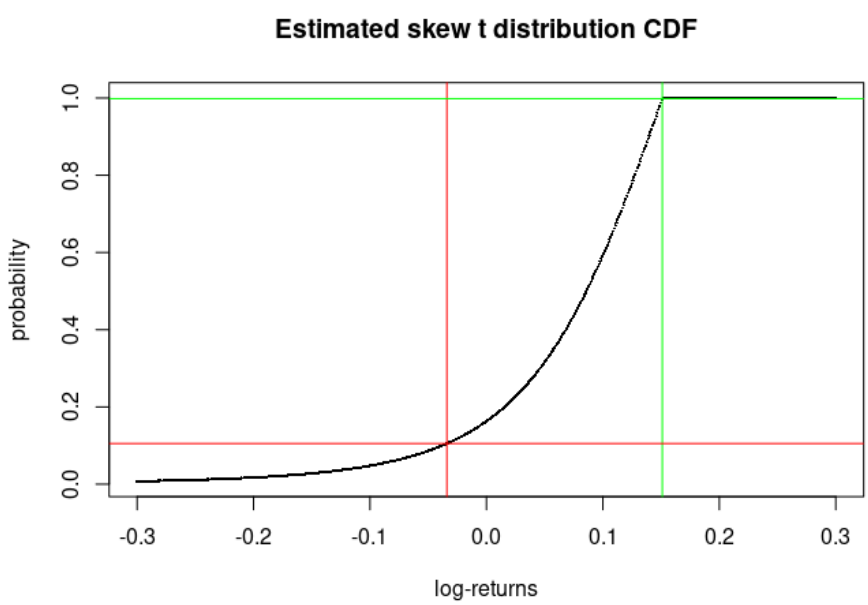
Data vs fit

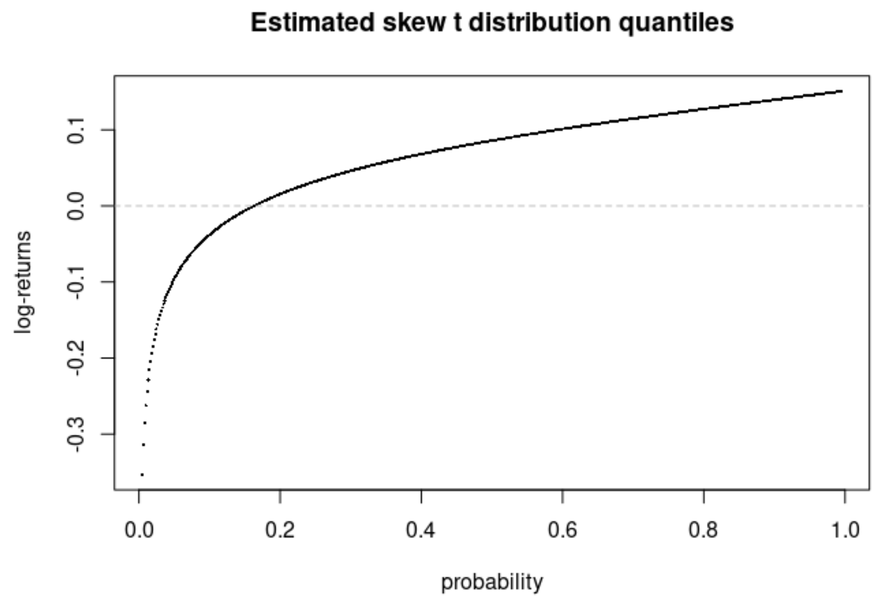
Let's plot the fit and the observed returns together.

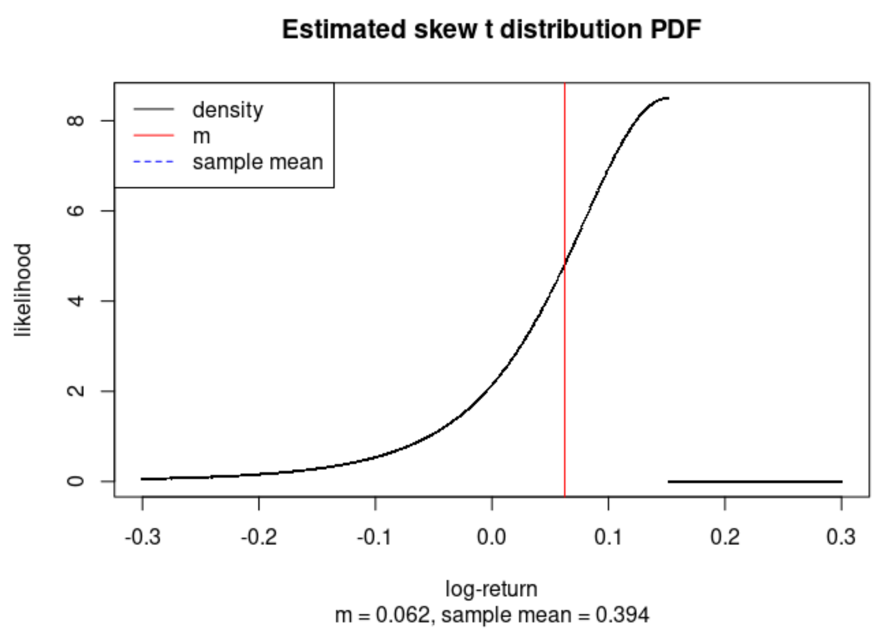


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

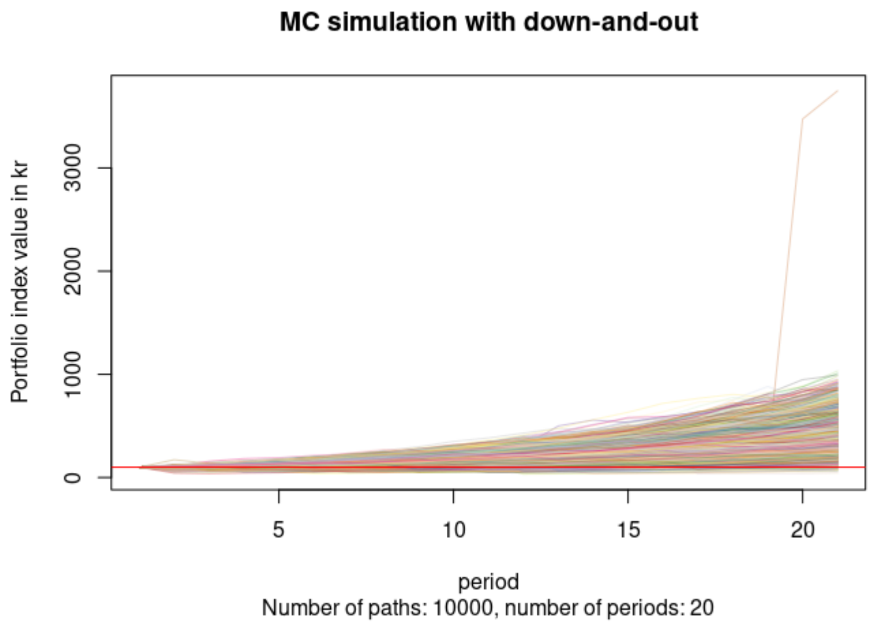


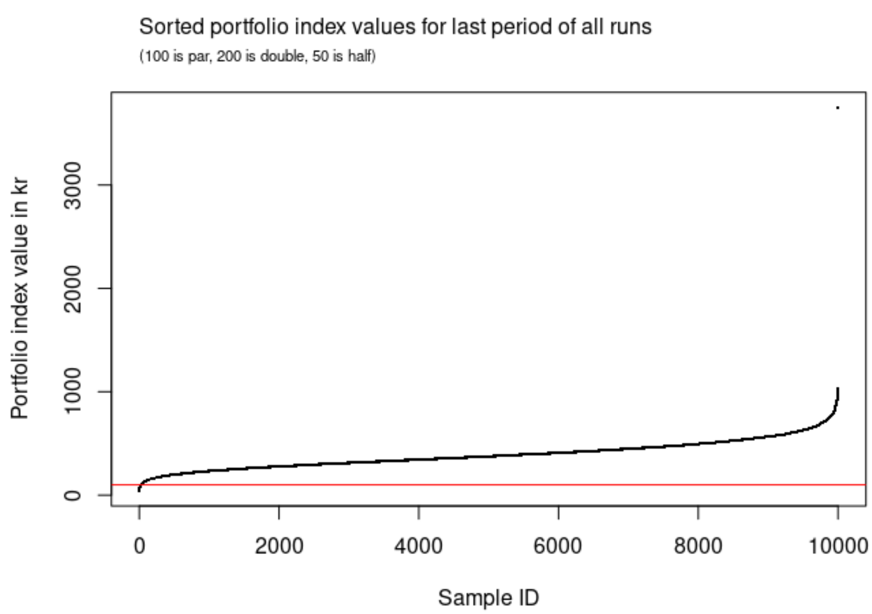

Estimated skew t distribution quantiles
log-returns
probability
0.1
0.0
-0.1
-0.2
-0.3
0.0
0.2
0.4
0.6
0.8
1.0


Estimated skew t distribution PDF
density
m
sample mean
likelihood
log-return
m = 0.062, sample mean = 0.394
8
6
4
2
0
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3

Monte Carlo

Mix of simulations from estimated distribution of returns from individual funds.

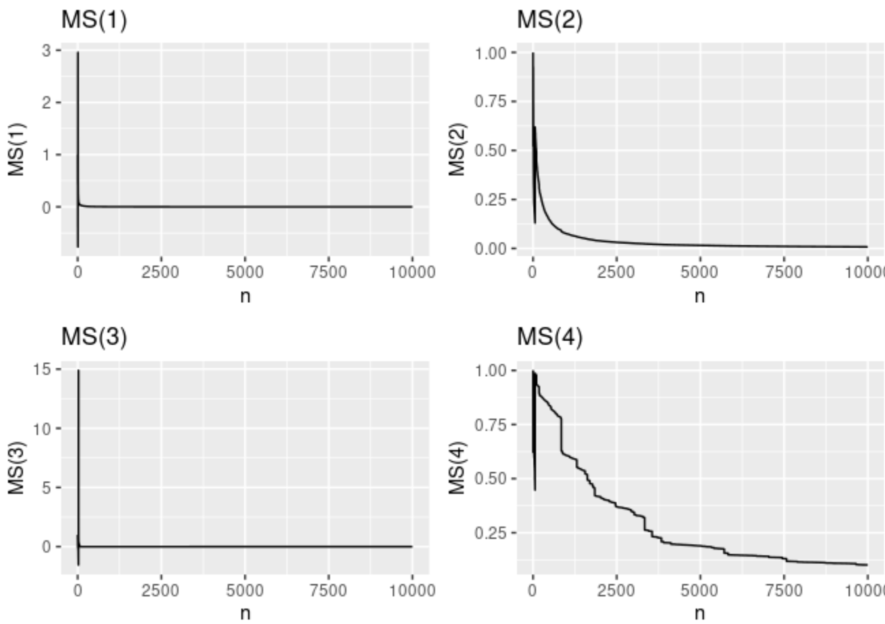




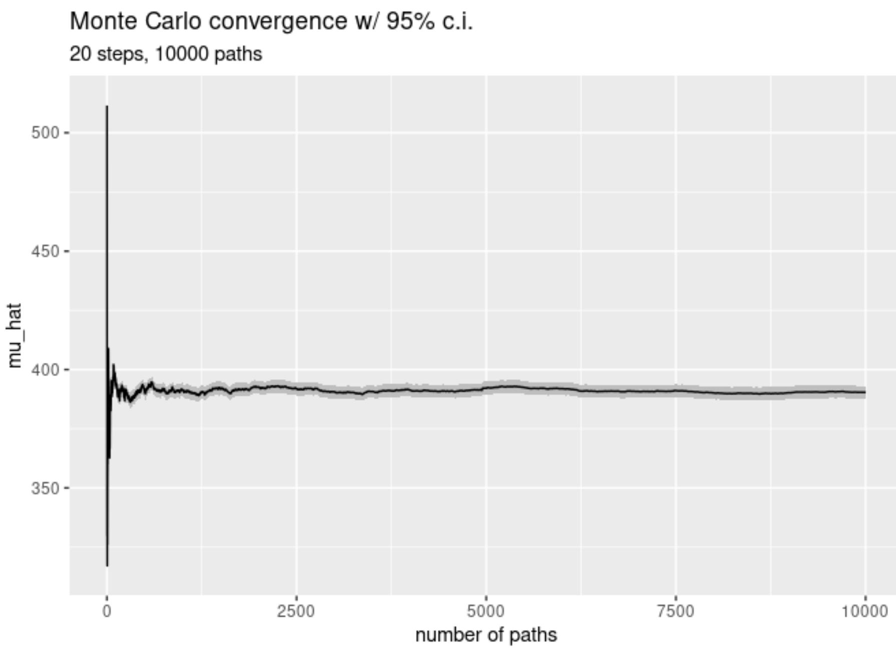
Convergence

Max vs sum

Max vs sum plots for the first four moments:



MC

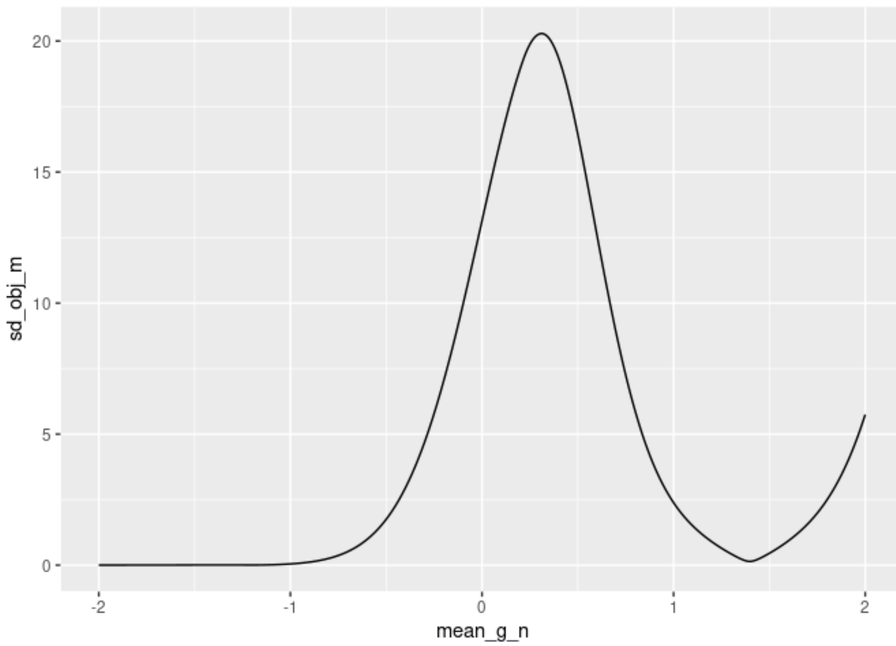


IS

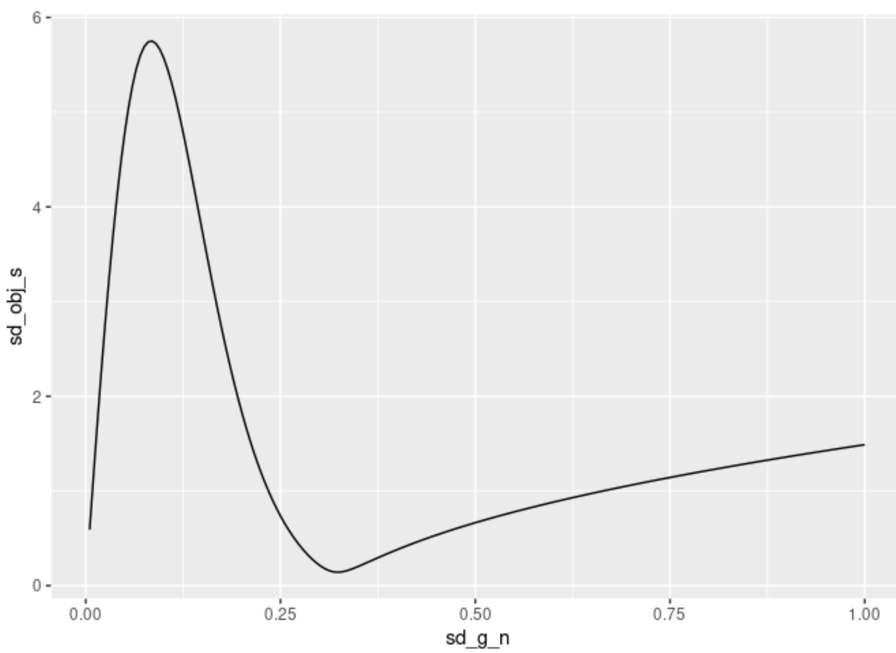
Parameters

```
## [1] 1.3976614 0.3233002
```

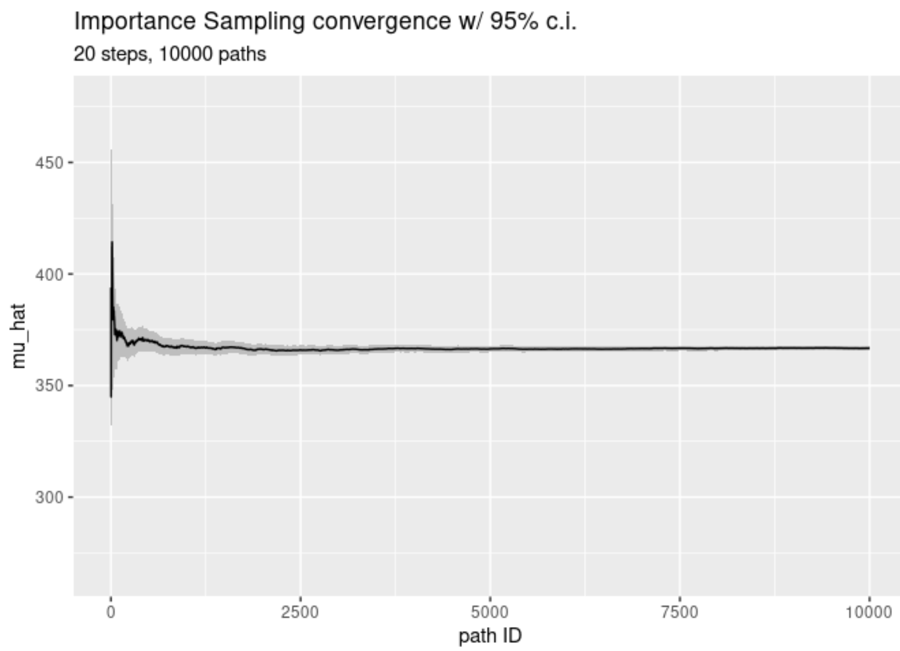
Objective function plots



20
15
10
5
0
-2
-1
0
1
2
sd_obj_m
mean_g_n



6
4
2
0
0.00
0.25
0.50
0.75
1.00
sd_obj_s
sd_g_n



Compare pension plans

Risk of max loss

Risk of max loss of x percent for a single period (year).
x values are row names.

	Vel_m	Vel_ml	Vel_h	PFA_m	PFA_h	mix_m	mix_h	vm_ph	vh_pm
0	21.167	17.833	19.667	11.833	14.000	12.333	12.667	16.667	16.000
5	12.167	9.333	12.500	5.667	8.333	5.833	3.833	8.667	8.167
10	7.000	5.000	8.000	3.000	5.000	2.833	0.500	4.333	4.167
25	1.333	0.833	2.167	0.500	1.000	0.333	0.000	0.333	0.333
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.167	Vel_m	12.500	Vel_h	8.000	Vel_h	2.167	Vel_h	0	Vel_m	0	Vel_m	0	Vel_m
19.667	Vel_h	12.167	Vel_m	7.000	Vel_m	1.333	Vel_m	0	Vel_ml	0	Vel_ml	0	Vel_ml
17.833	Vel_ml	9.333	Vel_ml	5.000	Vel_ml	1.000	PFA_h	0	Vel_h	0	Vel_h	0	Vel_h
16.667	vm_ph	8.667	vm_ph	5.000	PFA_h	0.833	Vel_ml	0	PFA_m	0	PFA_m	0	PFA_m
16.000	vh_pm	8.333	PFA_h	4.333	vm_ph	0.500	PFA_m	0	PFA_h	0	PFA_h	0	PFA_h
14.000	PFA_h	8.167	vh_pm	4.167	vh_pm	0.333	mix_m	0	mix_m	0	mix_m	0	mix_m
12.667	mix_h	5.833	mix_m	3.000	PFA_m	0.333	vm_ph	0	mix_h	0	mix_h	0	mix_h

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
12.333	mix_m	5.667	PFA_m	2.833	mix_m	0.333	vh_pm	0	vm_ph	0	vm_ph	0	vm_ph
11.833	PFA_m	3.833	mix_h	0.500	mix_h	0.000	mix_h	0	vh_pm	0	vh_pm	0	vh_pm

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h	vm_ph	vh_pm
0	78.833	82.167	80.333	88.167	86.000	87.667	87.333	83.333	84.000
5	63.833	65.000	69.333	71.667	76.000	71.667	70.167	69.333	69.000
10	40.833	36.000	53.333	32.500	59.667	35.500	46.000	47.167	43.833
25	0.000	0.000	0.000	0.000	0.000	0.000	0.833	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
88.167	PFA_m	76.000	PFA_h	59.667	PFA_h	0.833	mix_h	0	Velliv_m	0	Velliv_m
87.667	mix_m	71.667	PFA_m	53.333	Velliv_h	0.000	Velliv_m	0	Velliv_m_l	0	Velliv_m_l
87.333	mix_h	71.667	mix_m	47.167	vm_ph	0.000	Velliv_m_l	0	Velliv_h	0	Velliv_h
86.000	PFA_h	70.167	mix_h	46.000	mix_h	0.000	Velliv_h	0	PFA_m	0	PFA_m
84.000	vh_pm	69.333	Velliv_h	43.833	vh_pm	0.000	PFA_m	0	PFA_h	0	PFA_h
83.333	vm_ph	69.333	vm_ph	40.833	Velliv_m	0.000	PFA_h	0	mix_m	0	mix_m
82.167	Velliv_m_l	69.000	vh_pm	36.000	Velliv_m_l	0.000	mix_m	0	mix_h	0	mix_h
80.333	Velliv_h	65.000	Velliv_m_l	35.500	mix_m	0.000	vm_ph	0	vm_ph	0	vm_ph
78.833	Velliv_m	63.833	Velliv_m	32.500	PFA_m	0.000	vh_pm	0	vh_pm	0	vh_pm

MC risk percentiles

Risk of loss from first to last period.

`_m` is medium.
`_h` is high.

`a` is simulation from estimated distribution of returns of mix.
`b` is mix of simulations from estimated distribution of returns from individual funds.

`l` for “long”, going back to 2007.

	Vel_m	Vel_ml	Vel_h	PFA_m	PFA_h	mix_ma	mix_ha	mix_mb	mix_hb	vm_ph	vh_pm
0	5.33	2.89	4.17	1.95	0.82	1.11	0	0.32	0.13	0.18	0.28
5	4.71	2.48	3.75	1.82	0.71	0.99	0	0.26	0.11	0.12	0.19
10	4.14	2.16	3.34	1.67	0.67	0.86	0	0.24	0.07	0.11	0.18
25	2.61	1.48	2.37	1.17	0.48	0.55	0	0.07	0.04	0.04	0.09
50	0.91	0.62	1.08	0.56	0.24	0.21	0	0.01	0.02	0.00	0.02
90	0.08	0.06	0.08	0.15	0.02	0.04	0	0.00	0.00	0.00	0.00
99	0.00	0.02	0.02	0.06	0.01	0.00	0	0.00	0.00	0.00	0.00

1e6 simulation paths of `mhr_b`:

	0	5	10	25	50	90	99
prob_pct	0.118	0.095	0.076	0.036	0.008	0	0

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
5.33	Vel_m	4.71	Vel_m	4.14	Vel_m	2.61	Vel_m	1.08	Vel_h	0.15	PFA_m	0.06	PFA_m
4.17	Vel_h	3.75	Vel_h	3.34	Vel_h	2.37	Vel_h	0.91	Vel_m	0.08	Vel_m	0.02	Vel_ml
2.89	Vel_ml	2.48	Vel_ml	2.16	Vel_ml	1.48	Vel_ml	0.62	Vel_ml	0.08	Vel_h	0.02	Vel_h
1.95	PFA_m	1.82	PFA_m	1.67	PFA_m	1.17	PFA_m	0.56	PFA_m	0.06	Vel_ml	0.01	PFA_h
1.11	mix_ma	0.99	mix_ma	0.86	mix_ma	0.55	mix_ma	0.24	PFA_h	0.04	mix_ma	0.00	Vel_m
0.82	PFA_h	0.71	PFA_h	0.67	PFA_h	0.48	PFA_h	0.21	mix_ma	0.02	PFA_h	0.00	mix_ma
0.32	mix_mb	0.26	mix_mb	0.24	mix_mb	0.09	vh_pm	0.02	mix_hb	0.00	mix_ha	0.00	mix_ha
0.28	vh_pm	0.19	vh_pm	0.18	vh_pm	0.07	mix_mb	0.02	vh_pm	0.00	mix_mb	0.00	mix_mb
0.18	vm_ph	0.12	vm_ph	0.11	vm_ph	0.04	mix_hb	0.01	mix_mb	0.00	mix_hb	0.00	mix_hb
0.13	mix_hb	0.11	mix_hb	0.07	mix_hb	0.04	vm_ph	0.00	mix_ha	0.00	vm_ph	0.00	vm_ph
0.00	mix_ha	0.00	mix_ha	0.00	mix_ha	0.00	mix_ha	0.00	vm_ph	0.00	vh_pm	0.00	vh_pm

MC gains percentiles

Chance of gains from first to last period.
`_a` is simulation from estimated distribution of returns of mix.
`_b` is mix of simulations from estimated distribution of returns from individual funds.

	Vel_m	Vel_ml	Vel_h	PFA_m	PFA_h	mix_ma	mix_ha	mix_mb	mix_hb	vm_ph	vh_pm
0	94.67	97.11	95.83	98.05	99.18	98.89	100.00	99.68	99.87	99.82	99.72
5	93.99	96.69	95.40	97.89	99.08	98.70	100.00	99.64	99.85	99.78	99.68
10	93.30	96.24	94.95	97.66	98.92	98.47	100.00	99.55	99.81	99.72	99.62
25	90.57	94.47	93.46	96.83	98.59	97.79	100.00	99.08	99.66	99.56	99.30
50	85.82	90.51	90.59	94.79	97.66	96.11	99.99	97.70	99.30	99.00	98.36
100	71.75	78.71	83.25	88.35	94.78	90.17	99.68	89.91	97.33	96.13	93.80
200	39.43	44.18	65.33	59.73	85.79	59.98	93.09	48.93	86.46	79.55	67.91
300	15.96	17.44	45.13	22.39	71.81	22.13	71.60	11.68	66.16	51.65	34.81
400	5.03	4.93	29.01	4.32	54.54	3.91	44.58	1.33	42.05	24.74	13.33
500	1.14	1.07	17.58	0.54	38.04	0.16	23.53	0.11	22.26	9.12	4.01
1000	0.00	0.02	0.70	0.02	2.30	0.00	0.29	0.01	0.10	0.00	0.01

1e6 simulation paths of `mhr_b`:

	0	5	10	25	50	100	200	300	400	500	1000
prob	99.882	99.854	99.824	99.686	99.301	97.513	86.912	65.992	41.486	21.693	0.086

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mix_ha	100.00	mix_ha	100.00	mix_ha	100.00	mix_ha	99.99	mix_ha	99.68	mix_ha
99.87	mix_hb	99.85	mix_hb	99.81	mix_hb	99.66	mix_hb	99.30	mix_hb	97.33	mix_hb
99.82	vm_ph	99.78	vm_ph	99.72	vm_ph	99.56	vm_ph	99.00	vm_ph	96.13	vm_ph
99.72	vh_pm	99.68	vh_pm	99.62	vh_pm	99.30	vh_pm	98.36	vh_pm	94.78	PFA_h
99.68	mix_mb	99.64	mix_mb	99.55	mix_mb	99.08	mix_mb	97.70	mix_mb	93.80	vh_pm
99.18	PFA_h	99.08	PFA_h	98.92	PFA_h	98.59	PFA_h	97.66	PFA_h	90.17	mix_ma
98.89	mix_ma	98.70	mix_ma	98.47	mix_ma	97.79	mix_ma	96.11	mix_ma	89.91	mix_mb
98.05	PFA_m	97.89	PFA_m	97.66	PFA_m	96.83	PFA_m	94.79	PFA_m	88.35	PFA_m
97.11	Vel_ml	96.69	Vel_ml	96.24	Vel_ml	94.47	Vel_ml	90.59	Vel_h	83.25	Vel_h
95.83	Vel_h	95.40	Vel_h	94.95	Vel_h	93.46	Vel_h	90.51	Vel_ml	78.71	Vel_ml
94.67	Vel_m	93.99	Vel_m	93.30	Vel_m	90.57	Vel_m	85.82	Vel_m	71.75	Vel_m

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
93.09	mix_ha	71.81	PFA_h	54.54	PFA_h	38.04	PFA_h	2.30	PFA_h
86.46	mix_hb	71.60	mix_ha	44.58	mix_ha	23.53	mix_ha	0.70	Vel_h
85.79	PFA_h	66.16	mix_hb	42.05	mix_hb	22.26	mix_hb	0.29	mix_ha
79.55	vm_ph	51.65	vm_ph	29.01	Vel_h	17.58	Vel_h	0.10	mix_hb
67.91	vh_pm	45.13	Vel_h	24.74	vm_ph	9.12	vm_ph	0.02	Vel_ml
65.33	Vel_h	34.81	vh_pm	13.33	vh_pm	4.01	vh_pm	0.02	PFA_m
59.98	mix_ma	22.39	PFA_m	5.03	Vel_m	1.14	Vel_m	0.01	mix_mb
59.73	PFA_m	22.13	mix_ma	4.93	Vel_ml	1.07	Vel_ml	0.01	vh_pm
48.93	mix_mb	17.44	Vel_ml	4.32	PFA_m	0.54	PFA_m	0.00	Vel_m
44.18	Vel_ml	15.96	Vel_m	3.91	mix_ma	0.16	mix_ma	0.00	mix_ma
39.43	Vel_m	11.68	mix_mb	1.33	mix_mb	0.11	mix_mb	0.00	vm_ph

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated degrees of freedom, or shape parameter.
`xi` is the estimated skewness parameter.

	Vel_m	Vel_ml	Vel_h	PFA_m	PFA_h	mix_m	mix_h	vm_ph	vh_pm
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082	0.067	0.062
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071	0.091	0.090
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863	4.660	3.892
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770	0.048	0.019
R^2	0.993	0.978	0.991	0.991	0.964	0.890	0.961	0.927	0.933

Fit statistics ranking

m	ranking	s	ranking	R^2	ranking
0.084	PFA_h	0.071	mix_h	0.993	Vel_m
0.082	mix_h	0.088	mix_m	0.991	Vel_h
0.067	vm_ph	0.090	vh_pm	0.991	PFA_m

m	ranking	s	ranking	R^2	ranking
0.065	Vel_h	0.091	vm_ph	0.978	Vel_ml
0.062	vh_pm	0.115	Vel_ml	0.964	PFA_h
0.059	mix_m	0.120	Vel_m	0.961	mix_h
0.058	PFA_m	0.121	PFA_h	0.933	vh_pm
0.052	Vel_ml	0.123	PFA_m	0.927	vm_ph
0.048	Vel_m	0.150	Vel_h	0.890	mix_m

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).
Statistics are given for the final state of all paths.
Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.
0 is defined as any value below a threshold.
`dai_pct` (for down-and-in) is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

	Vel_m	Vel_ml	Vel_h	PFA_m	PFA_h	mix_ma	mix_mb	mix_ha	mix_hb	vm_ph	vh_pm
mc_m	293.36	310.72	436.49	559.20	604.48	345.46	320.16	545.49	520.51	447.96	390.33
mc_s	132.88	127.38	238.77	21464.17	272.58	107.65	90.70	177.24	184.77	149.96	137.05
mc_min	1.15	0.11	0.01	0.01	0.72	2.26	31.57	142.72	39.60	50.95	43.65
mc_max	1062.48	1524.06	1662.79	2146730.28	2041.09	768.50	1996.78	1730.75	1454.07	1151.68	3747.97
dao_pct	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dai_pct	4.89	2.64	3.99	1.91	0.73	1.04	0.30	0.00	0.10	0.16	0.24

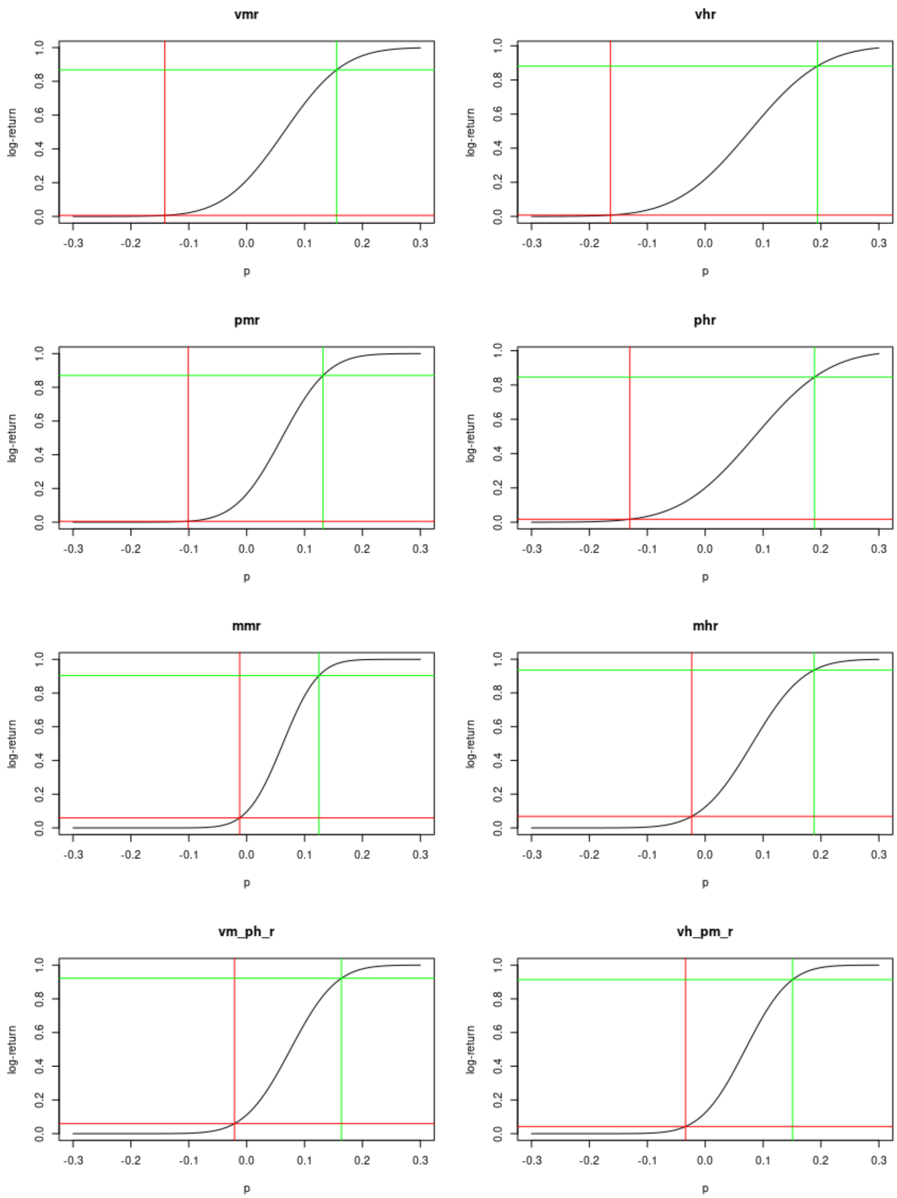
Ranking

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
604.48	PFA_h	90.70	mix_mb	142.72	mix_ha	2146730.28	PFA_m	0.00	Vel_m	0.00	mix_ha
559.20	PFA_m	107.65	mix_ma	50.95	vm_ph	3747.97	vh_pm	0.00	Vel_ml	0.10	mix_hb
545.49	mix_ha	127.38	Vel_ml	43.65	vh_pm	2041.09	PFA_h	0.00	PFA_h	0.16	vm_ph
520.51	mix_hb	132.88	Vel_m	39.60	mix_hb	1996.78	mix_mb	0.00	mix_ma	0.24	vh_pm
447.96	vm_ph	137.05	vh_pm	31.57	mix_mb	1730.75	mix_ha	0.00	mix_mb	0.30	mix_mb
436.49	Vel_h	149.96	vm_ph	2.26	mix_ma	1662.79	Vel_h	0.00	mix_ha	0.73	PFA_h
390.33	vh_pm	177.24	mix_ha	1.15	Vel_m	1524.06	Vel_ml	0.00	mix_hb	1.04	mix_ma
345.46	mix_ma	184.77	mix_hb	0.72	PFA_h	1454.07	mix_hb	0.00	vm_ph	1.91	PFA_m
320.16	mix_mb	238.77	Vel_h	0.11	Vel_ml	1151.68	vm_ph	0.00	vh_pm	2.64	Vel_ml
310.72	Vel_ml	272.58	PFA_h	0.01	Vel_h	1062.48	Vel_m	0.01	PFA_m	3.99	Vel_h
293.36	Vel_m	21464.17	PFA_m	0.01	PFA_m	768.50	mix_ma	0.02	Vel_h	4.89	Vel_m

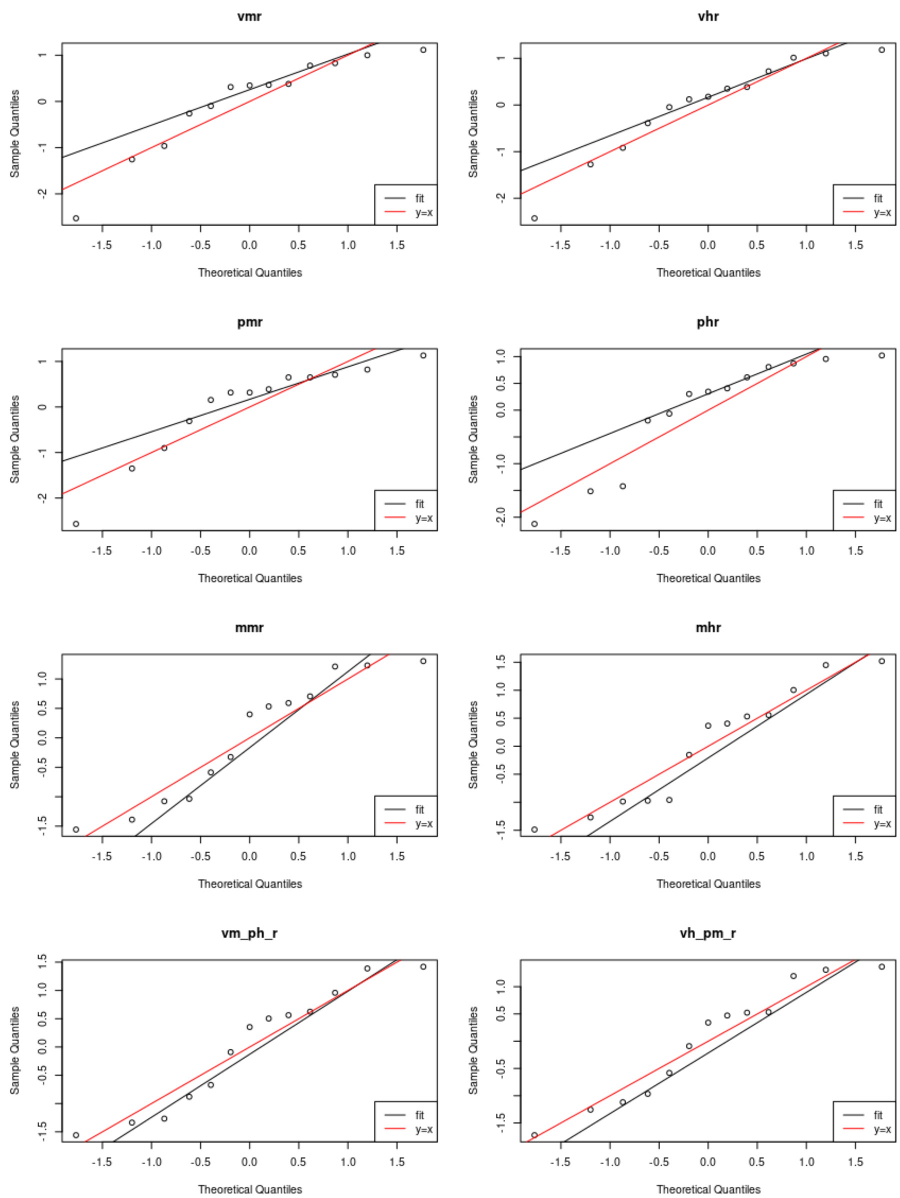
Compare Gaussian and skewed t-distribution fits

Gaussian fits

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
m	0.064	0.077	0.061	0.085	0.062	0.081	0.076	0.069
s	0.081	0.099	0.063	0.101	0.048	0.070	0.062	0.060



Gaussian QQ plots



Gaussian vs skewed t

Probability in percent that the smallest and largest (respectively) observed return for each fund was generated by a normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
P_norm(X_min)	0.571	0.758	0.511	1.676	5.971	6.842	5.945	4.228
P_norm(X_max)	13.230	11.876	12.922	15.359	9.628	6.429	7.796	8.592
P_t(X_min)	5.377	5.457	3.489	4.315	10.570	8.015	13.008	10.520
P_t(X_max)	0.118	0.001	2.825	0.188	0.488	5.141	0.229	0.175

Average number of years between min or max events (respectively):

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
norm: avg yrs btw min	175.248	131.911	195.568	59.669	16.748	14.616	16.820	23.650
norm: avg yrs btw max	7.559	8.420	7.739	6.511	10.386	15.556	12.827	11.639
t: avg yrs btw min	18.596	18.324	28.663	23.173	9.461	12.476	7.688	9.506
t: avg yrs btw max	848.548	178349.076	35.400	531.552	205.104	19.450	437.280	572.483

Comments

(Ignoring `mhr_a`…)

`mhr` has some nice properties:
- It has a relatively high `nu` value of 90, which means it is tending more towards exponential tails than polynomial tails. All other funds have `nu` values close to 3, except `phr` which is even worse at close to 2. (Note that for a Gaussian, `nu` is infinite.)
- It has the lowest losing percentage of all simulations, which is better than 1/6 that of `phr`.
- It has a DAO percentage of 0, which is the same as `mmr`, and less than `phr`.
- Only `phr` has a higher `mc_m`.
- It has a smaller `mc_s` than the individual components, `vhr` and `phr`.
- It has the highest `xi` of all fits, suggesting less left skewness. Density plots for `vmr`, `phr` and `mmr` have an extremely sharp drop, as if an upward limiter has been applied, which corresponds to extremely low `xi` values. The density plot for `mhr` is by far the most symmetrical of all the fits. As seen in the section "Compare Gaussian and skewed t-distribution fits", the other skewed t-distribution fits don't capture the max observed returns at all.
- Only `mmr` has as higher `mc_min`. However, that of `mmr` is 18 times higher with 62, so `mmr` is a clear winner here.
- Naturally, it has a `mc_max` smaller than the individual components, `vhr` and `phr`, but ca. 1.5 times higher then `mmr`.
- All the first 4 moments converge nicely. For all other fits, the 4th moment doesn't seem to converge.

Taleb, Statistical Consequences Of Fat Tails, p. 97:
"the variance of a finite variance random variable with tail exponent < 4 will be infinite".

And p. 363:
"The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, un- der a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance."

- Importance Sampling seems to converge to a lower level than Monte Carlo does. Is that because IS catches more observations in the lower tail? Supporting this thesis is that MC for `mhr` with `1e4` paths gives a mean of 520, while `1e6` paths gives a mean of 478 (see under "Many simulations").

- Note: QQ lines by design pass through 1st and 3rd quantiles. They are not trendlines in the sense of linear regression.

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2}$$
$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2}\right) \Big/ \left(\frac{x_{t-1} + y_{t-1}}{2}\right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns $=$ Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2\frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1} y_t = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

$$(x_{t-1} x_1 y_{t-1} + y_{t-1} x_t y_{t-1}) + (x_{t-1} x_{t-1} y_t + x_{t-1} y_{t-1} y_t) = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: `R = 1+r`

```
## Let x_0 be 100.

## Let y_0 be 200.

## So the initial value of the pf is 300 .

## Let R_x be 0.5.

## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.

## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): 0.001886388
## s(data_x): 0.3785598
## m(data_y): 10.49528
## s(data_y): 3.994056
##
## m(data_x + data_y): 5.248582
## s(data_x + data_y): 2.039439
```

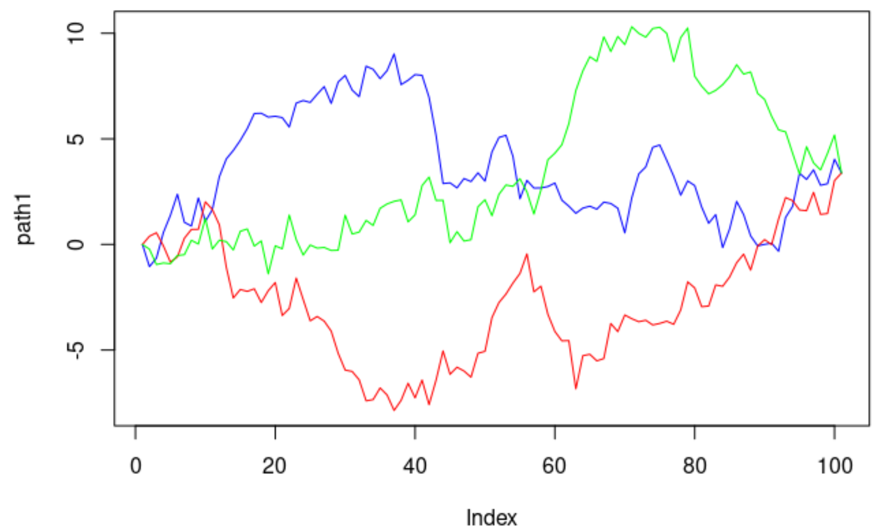
m and s of final state of all paths.
`_a` is mix of simulated returns.
`_b` is simulated mixed returns.

m_a	m_b	s_a	s_b
104.297	105.042	9.035	8.970
104.960	104.778	8.983	9.224
104.997	104.708	8.824	9.085
104.800	104.649	9.337	9.366
104.839	105.561	8.834	9.015
105.171	104.774	9.074	9.230
105.178	104.915	8.905	9.020
105.002	104.513	9.156	9.356
105.310	104.526	8.911	8.846
105.117	104.837	8.381	9.094

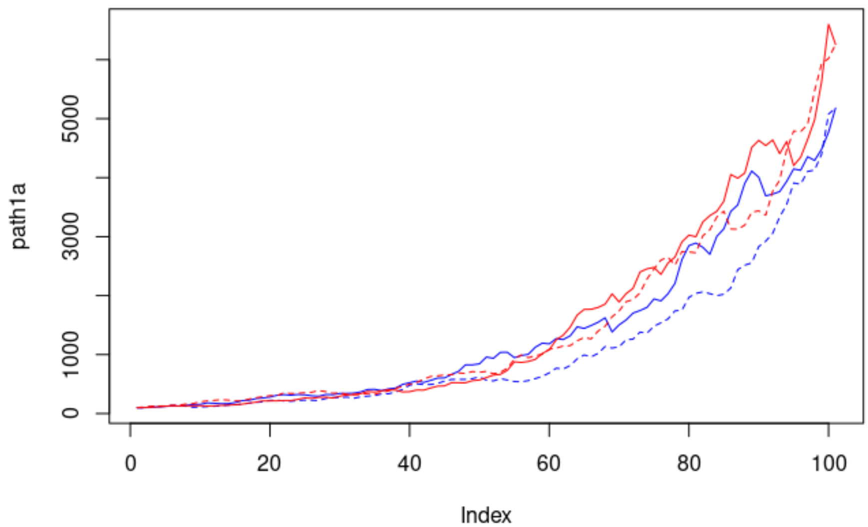
```
##       m_a             m_b             s_a             s_b
##  Min.   :104.3   Min.   :104.5   Min.   :8.381   Min.   :8.846
##  1st Qu.:104.9   1st Qu.:104.7   1st Qu.:8.852   1st Qu.:9.016
##  Median :105.0   Median :104.8   Median :8.947   Median :9.090
##  Mean   :105.0   Mean   :104.8   Mean   :8.944   Mean   :9.121
##  3rd Qu.:105.2   3rd Qu.:104.9   3rd Qu.:9.064   3rd Qu.:9.228
##  Max.   :105.3   Max.   :105.6   Max.   :9.337   Max.   :9.366
```

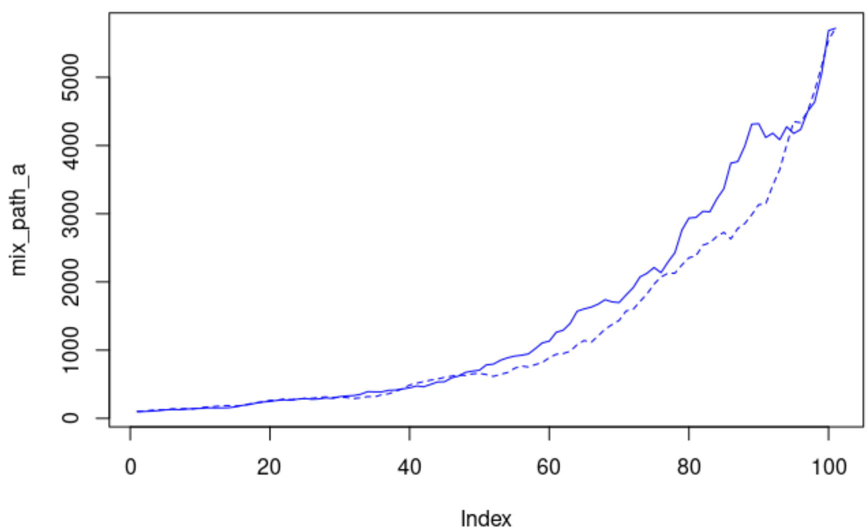
`_a` and `_b` are very close to equal.
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?





The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

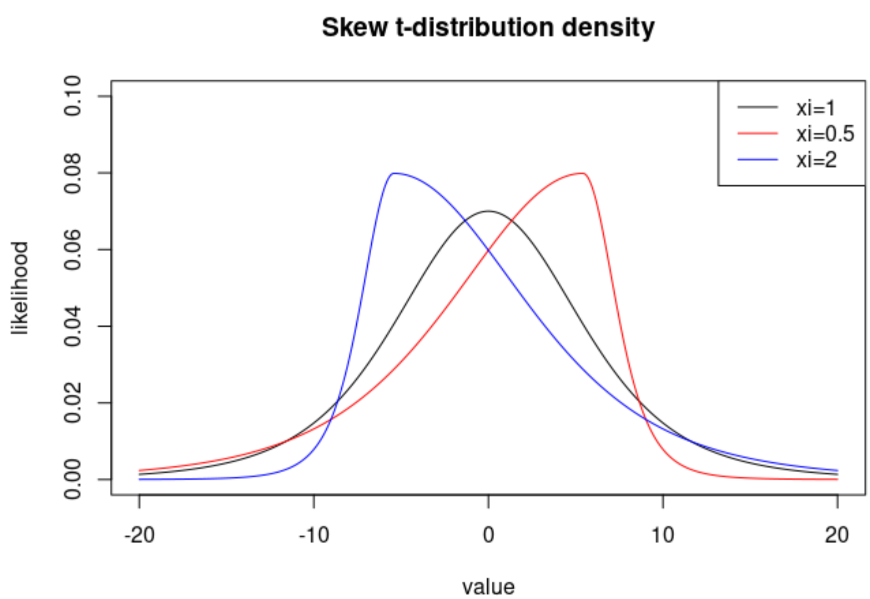
```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?
Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##        m                  s
##  Min.   :0.05926   Min.   :0.04394
##  1st Qu.:0.06719   1st Qu.:0.05978
##  Median :0.06962   Median :0.06679
##  Mean   :0.07039   Mean   :0.06693
##  3rd Qu.:0.07361   3rd Qu.:0.07466
##  Max.   :0.08310   Max.   :0.09379
```

The meaning of `xi`

The fit for `mhr` has the highest `xi` value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, ..., X^p)}{\sum_{i=1}^{n} X_i^p} \to 0$$

for $n \to \infty$.

If not, X doesn't have a p'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192