

Pension returns analysis

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Fit log returns to F-S skew standardized Student-t distribution.

\bar{m} is the location parameter.

s is the scale parameter.

ν is the estimated shape parameter (degrees of freedom).

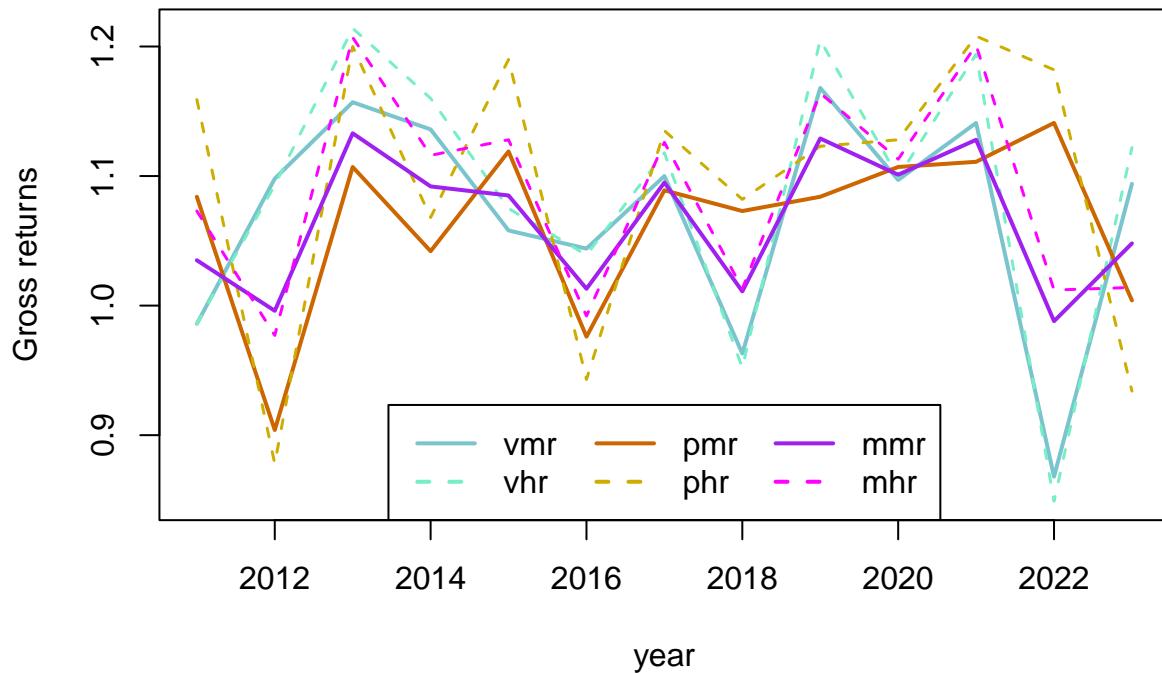
ξ is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.

`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.

Gross returns 2011–2023



Summary of gross returns

```

##      vmr        vhr        pmr        phr
## Min. :0.868  Min. :0.849  Min. :0.904  Min. :0.878
## 1st Qu.:1.044  1st Qu.:1.039  1st Qu.:1.042  1st Qu.:1.068
## Median :1.097  Median :1.099  Median :1.084  Median :1.128
## Mean   :1.070  Mean   :1.085  Mean   :1.065  Mean   :1.095
## 3rd Qu.:1.136  3rd Qu.:1.160  3rd Qu.:1.107  3rd Qu.:1.182
## Max.  :1.168  Max.  :1.214  Max.  :1.141  Max.  :1.208
##
##      mmr        mhr
## Min. :0.988  Min. :0.977
## 1st Qu.:1.013  1st Qu.:1.013
## Median :1.085  Median :1.113
## Mean   :1.066  Mean   :1.087
## 3rd Qu.:1.101  3rd Qu.:1.128
## Max.  :1.133  Max.  :1.207

##      vmrl
## Min. :0.801
## 1st Qu.:1.013
## Median :1.085
## Mean   :1.061
## 3rd Qu.:1.128
## Max.  :1.193

##      vmr    vhr    pmr    phr    mmr    mhr
## Min. : 0.868 0.849 0.904 0.878 0.988 0.977
## 1st Qu.: 1.044 1.039 1.042 1.068 1.013 1.013
## Median : 1.097 1.099 1.084 1.128 1.085 1.113
## Mean   : 1.070 1.085 1.065 1.095 1.066 1.087
## 3rd Qu.: 1.136 1.160 1.107 1.182 1.101 1.128
## Max.  : 1.168 1.214 1.141 1.208 1.133 1.207

```

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.182	phr	1.214	vhr
0.977	mhr	1.044	vmr	1.113	mhr	1.087	mhr	1.160	vhr	1.208	phr
0.904	pmr	1.042	pmr	1.099	vhr	1.085	vhr	1.136	vmr	1.207	mhr
0.878	phr	1.039	vhr	1.097	vmr	1.070	vmr	1.128	mhr	1.168	vmr
0.868	vmr	1.013	mmr	1.085	mmr	1.066	mmr	1.107	pmr	1.141	pmr
0.849	vhr	1.013	mhr	1.084	pmr	1.065	pmr	1.101	mmr	1.133	mmr

Covariance

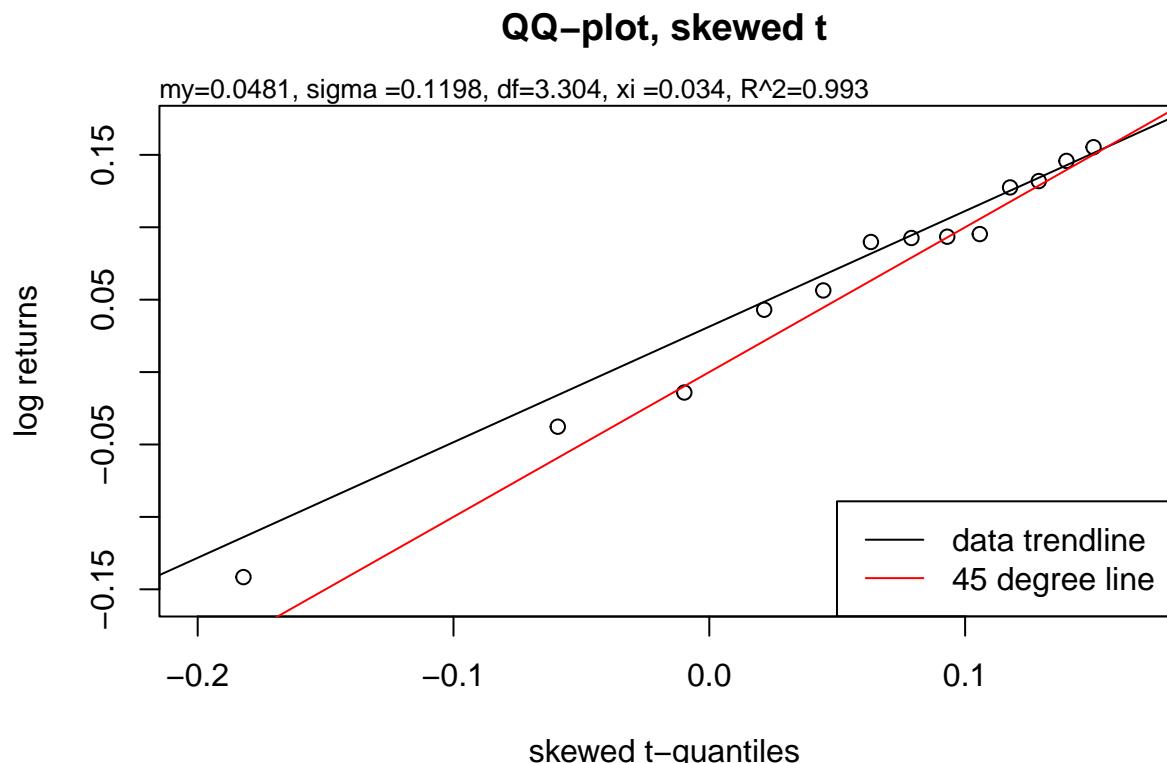
```
## cov(vmr, pmr) = -0.001094875
```

```
## cov(vhr, phr) = -0.0001730651
```

Velliv medium risk, 2011 - 2023

```
##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? <= 0 percent
## What is the risk of losing max 25 %? <= 0 percent
## What is the risk of losing max 50 %? <= 0 percent
## What is the risk of losing max 90 %? <= 0 percent
## What is the risk of losing max 99 %? <= 0 percent
##
## What is the chance of gaining min 10 %? >= 63.16667 percent
## What is the chance of gaining min 25 %? >= 49.33333 percent
## What is the chance of gaining min 50 %? >= 40.16667 percent
## What is the chance of gaining min 90 %? >= 32.66667 percent
## What is the chance of gaining min 99 %? >= 31.5 percent
```

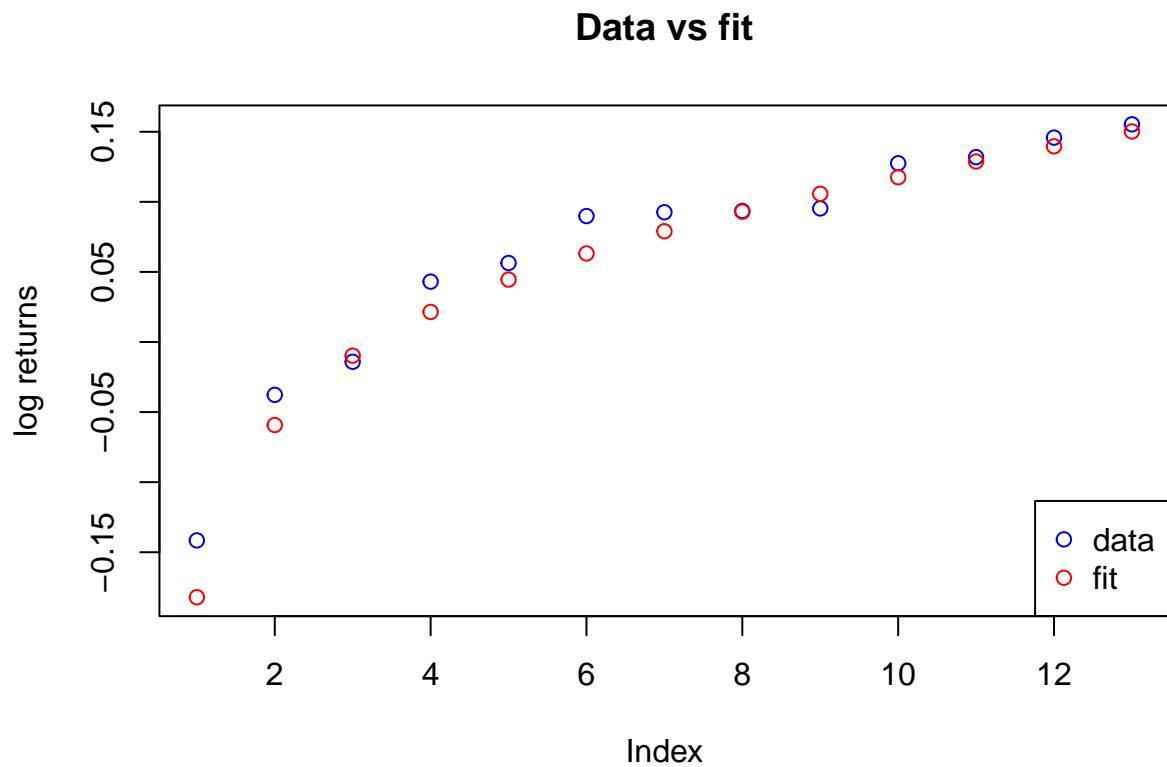
QQ Plot



The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

Data vs fit

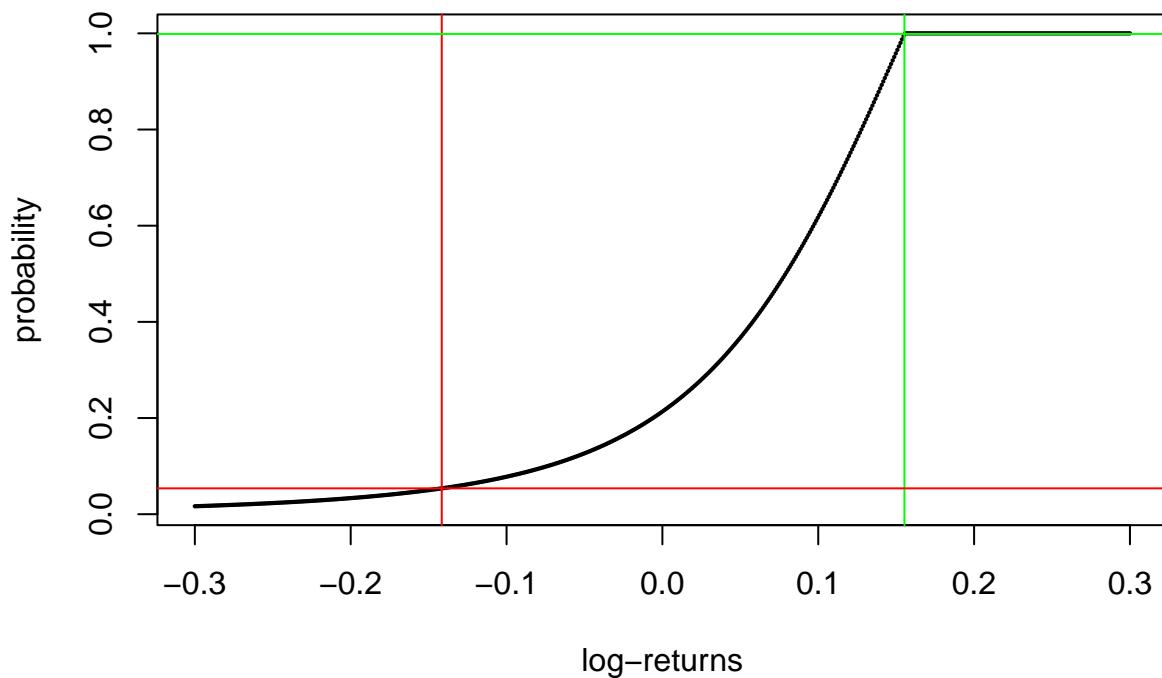
Let's plot the fit and the observed returns together.



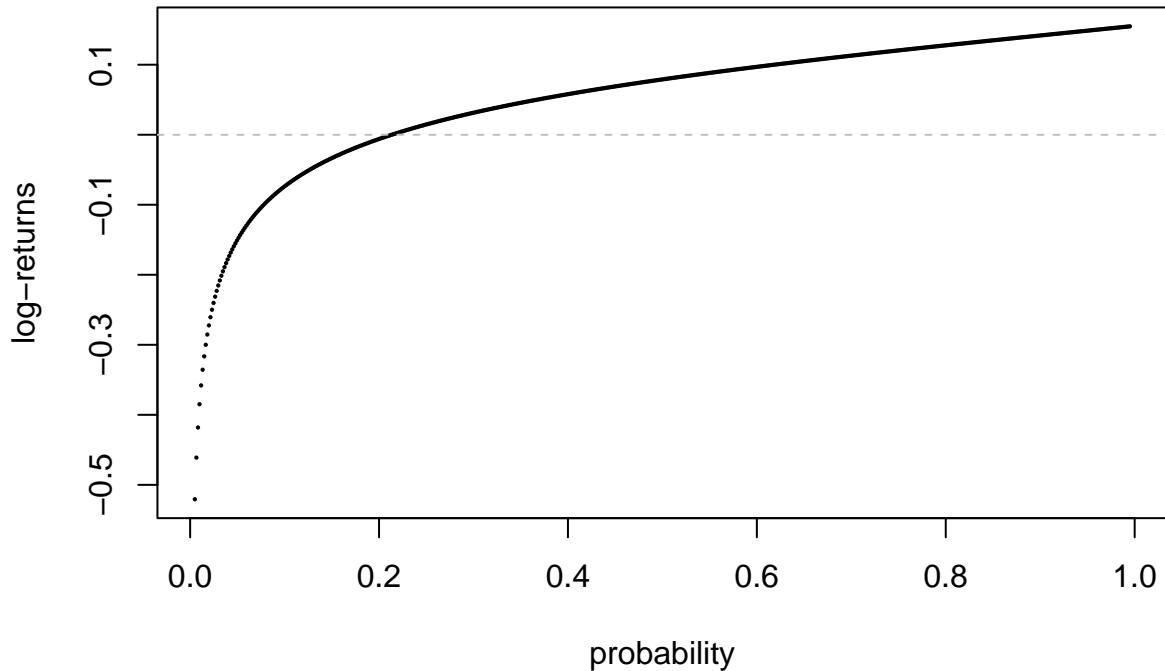
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

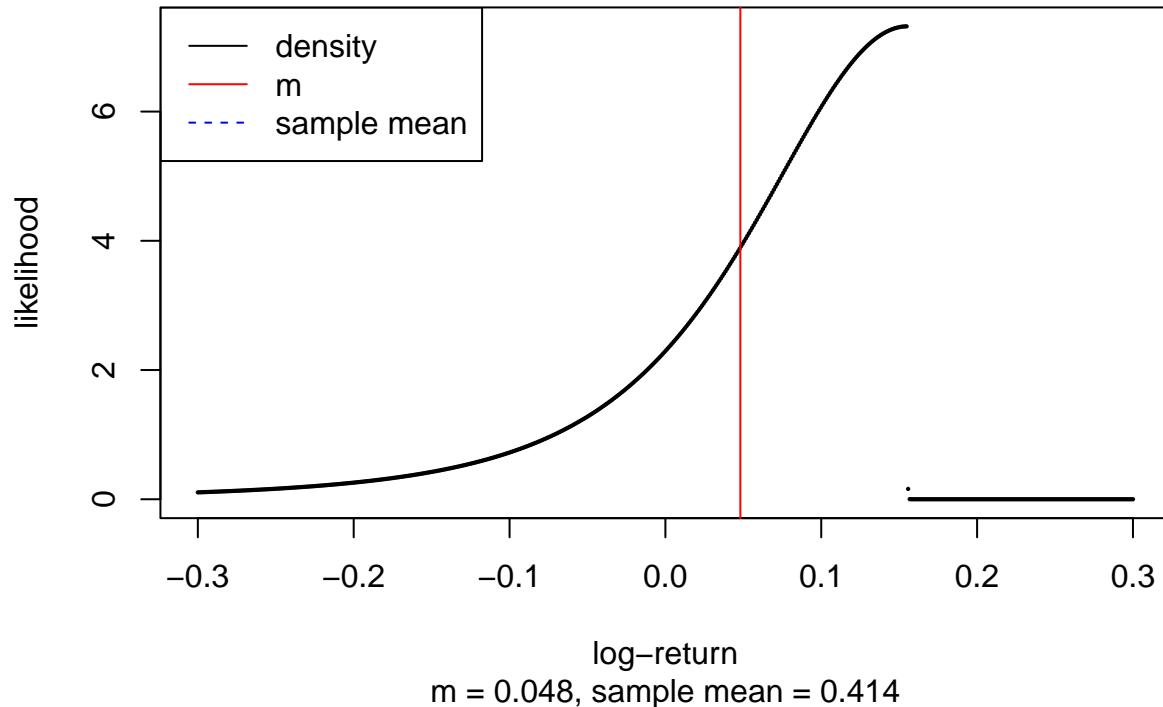


Estimated skew t distribution quantiles



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

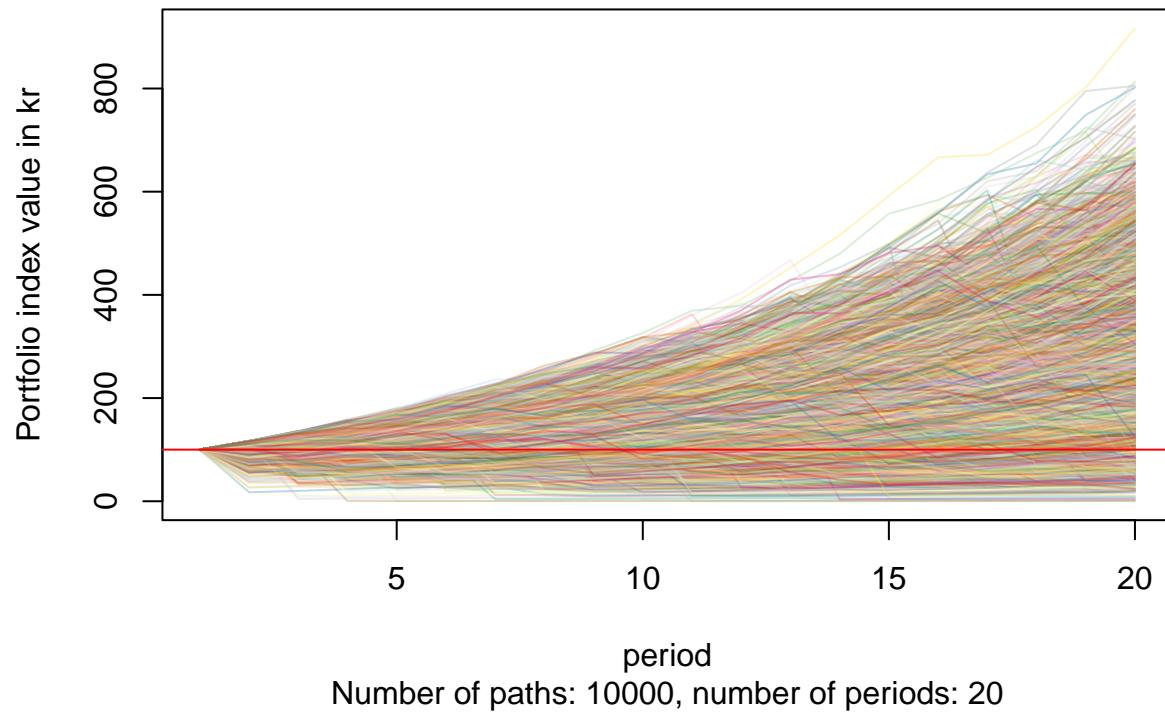
Estimated skew t distribution PDF



Monte Carlo

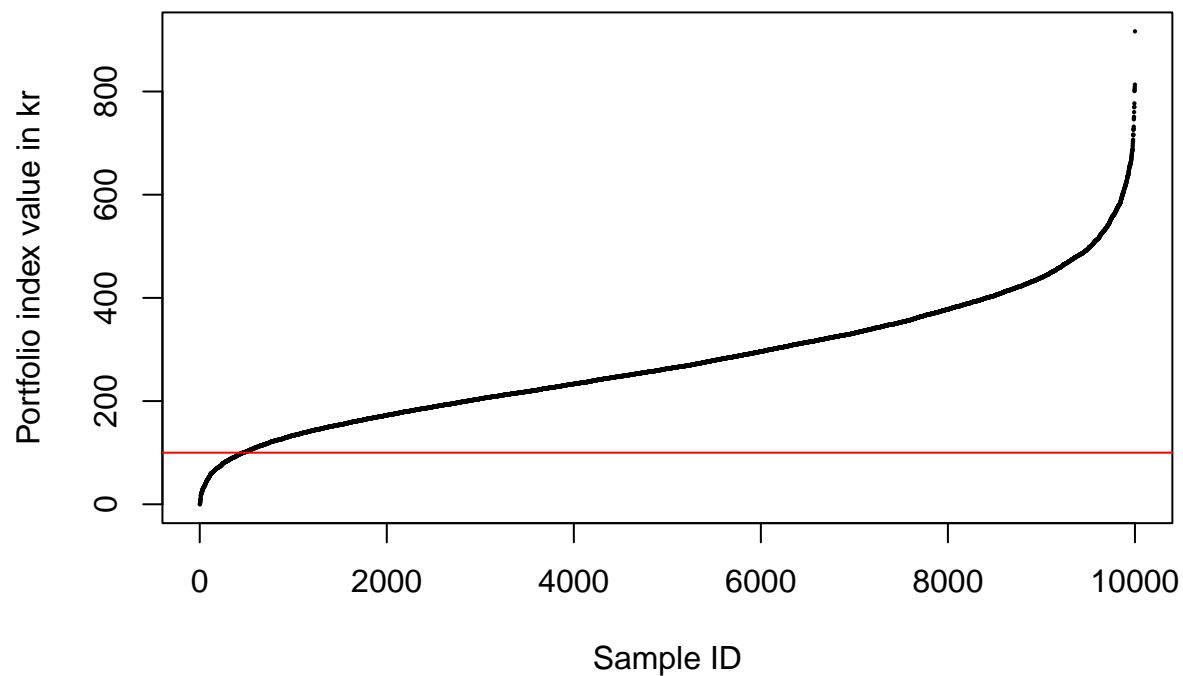
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 277.65 kr.  
## SD of portfolio index value after 20 years: 122.041 kr.  
## Min total portfolio index value after 20 years: 0.086 kr.  
## Max total portfolio index value after 20 years: 916.564 kr.  
##  
## Share of paths finishing below 100: 4.65 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

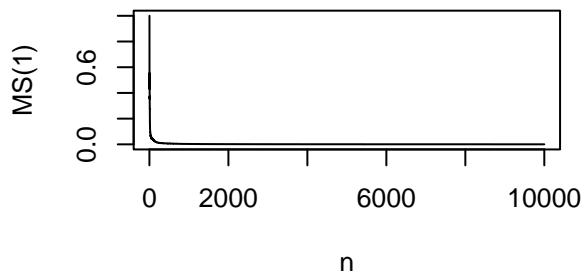
(100 is par, 200 is double, 50 is half)



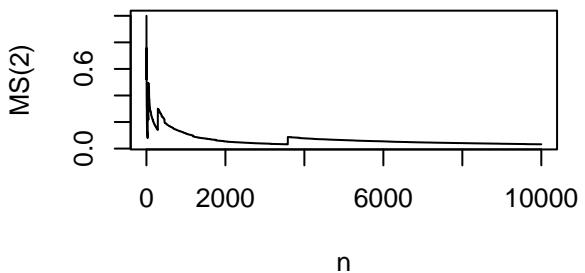
Convergence

Max vs sum plots for the first four moments:

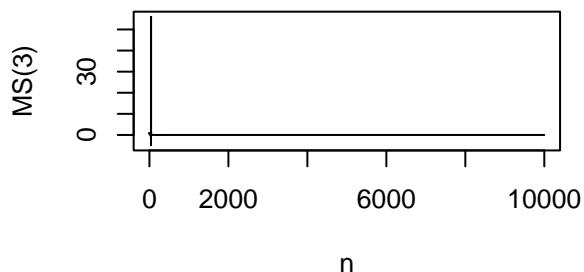
MS(1)



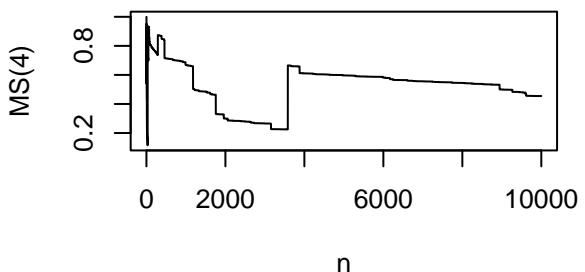
MS(2)



MS(3)

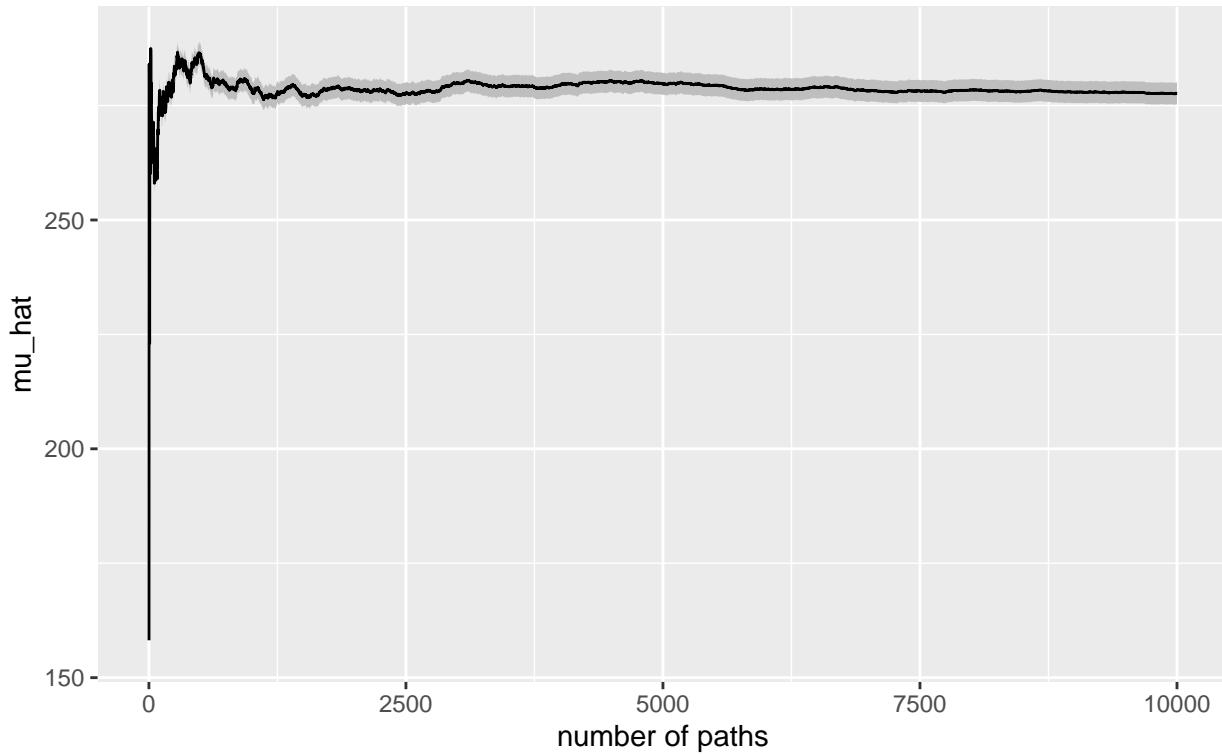


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

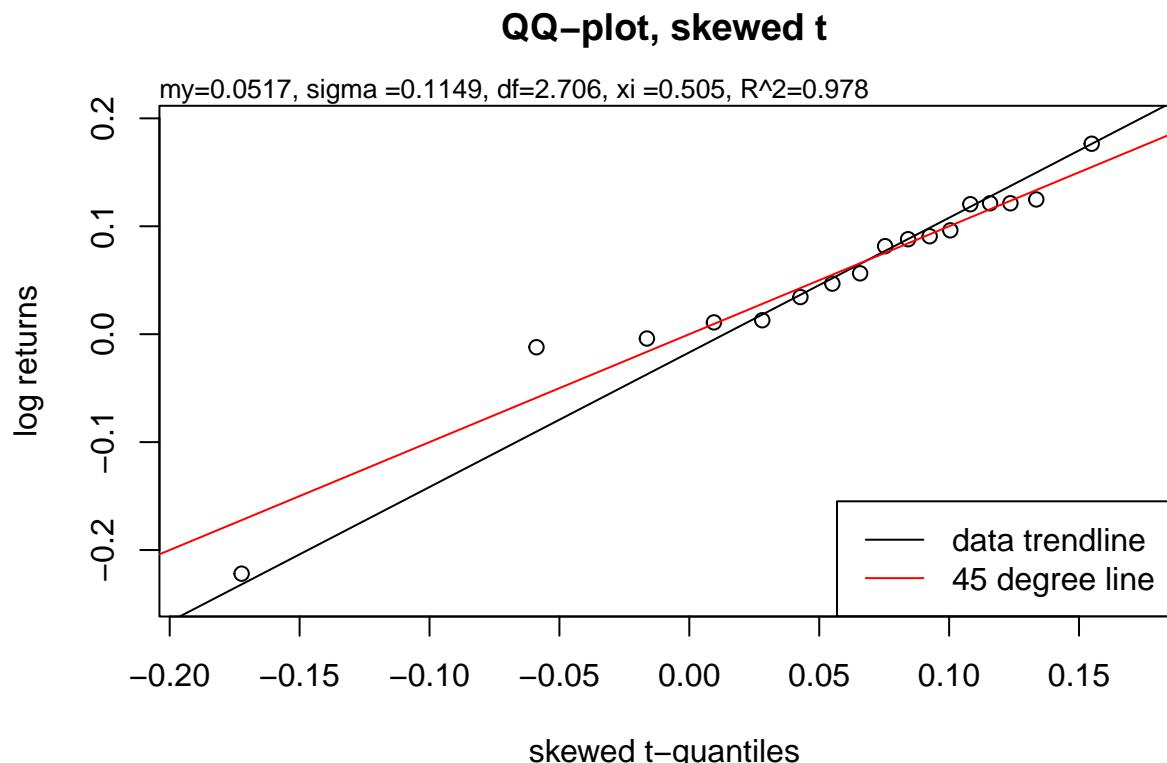


Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##  
## AIC: -34.35752  
## BIC: -31.02467  
## m: 0.05171176  
## s: 0.1149408  
## nu (df): 2.706099  
## xi: 0.5049945  
## R^2: 0.978  
##  
## An R^2 of 0.978 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 0 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 58.66667 percent  
## What is the chance of gaining min 25 %? >= 47.5 percent  
## What is the chance of gaining min 50 %? >= 40.16667 percent  
## What is the chance of gaining min 90 %? >= 34 percent  
## What is the chance of gaining min 99 %? >= 33 percent
```

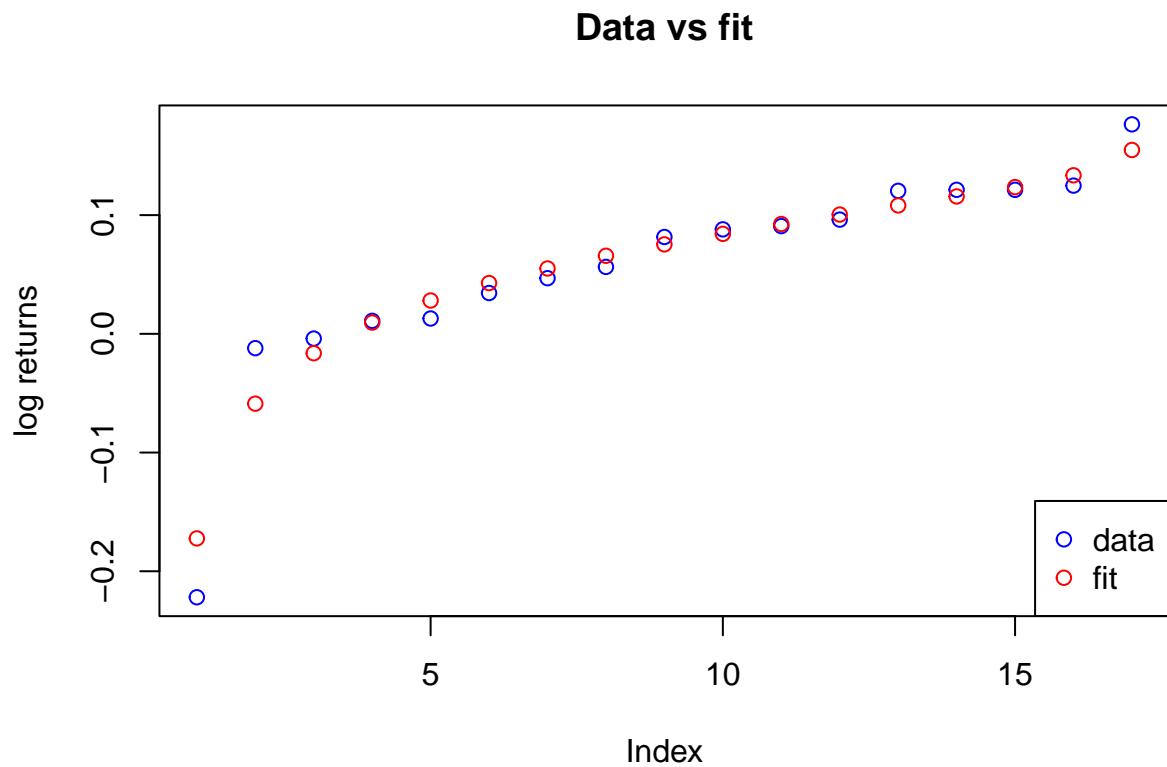
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

Data vs fit

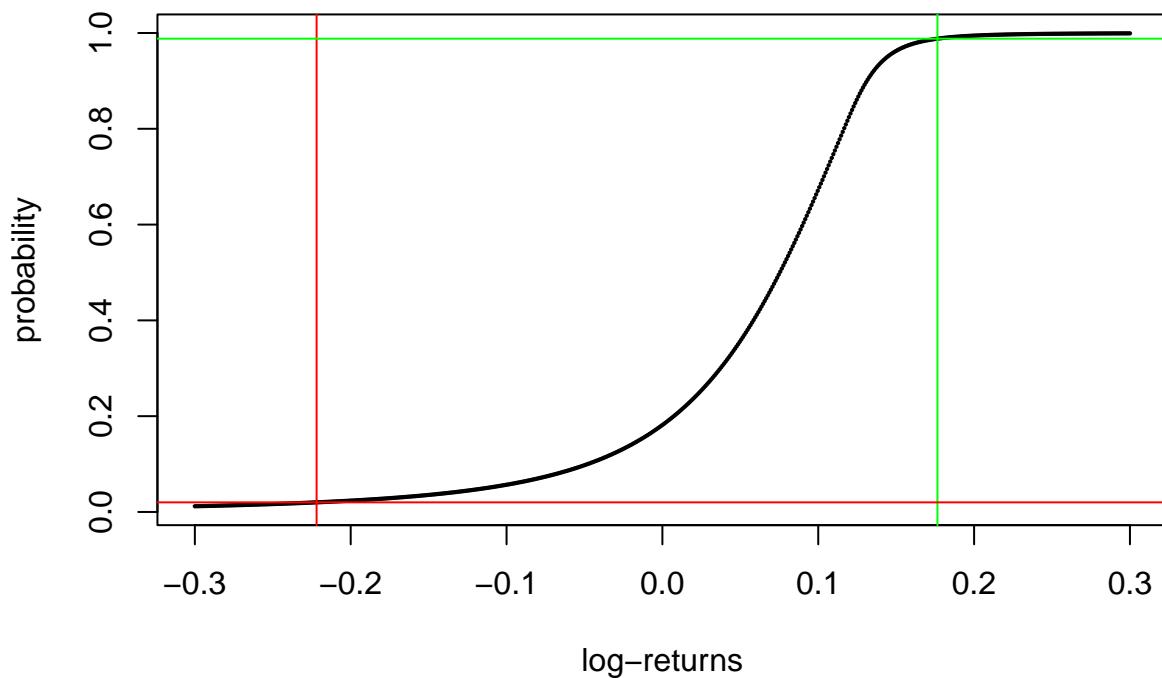
Let's plot the fit and the observed returns together.



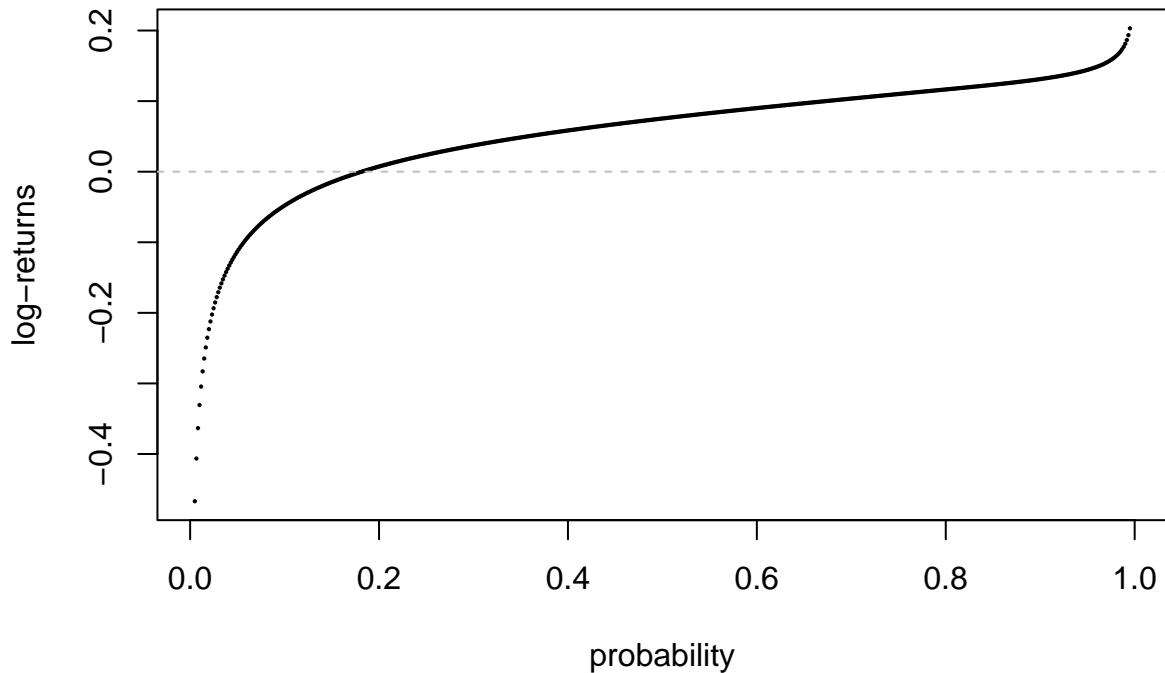
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

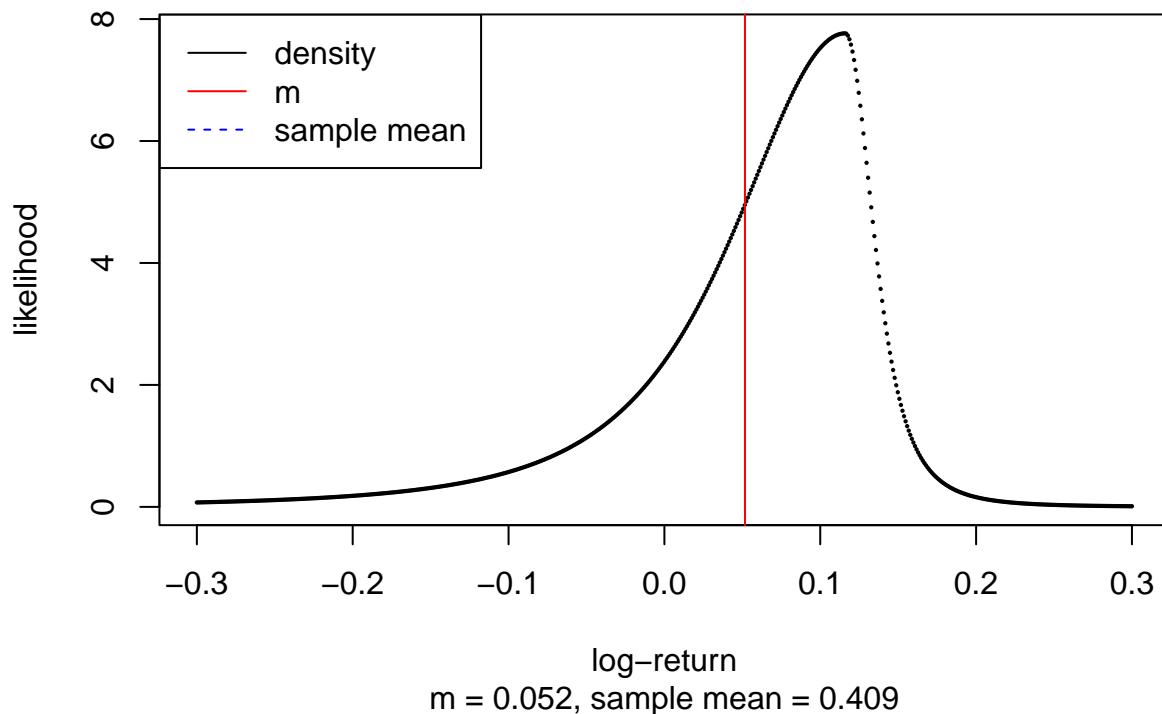


Estimated skew t distribution quantiles



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!

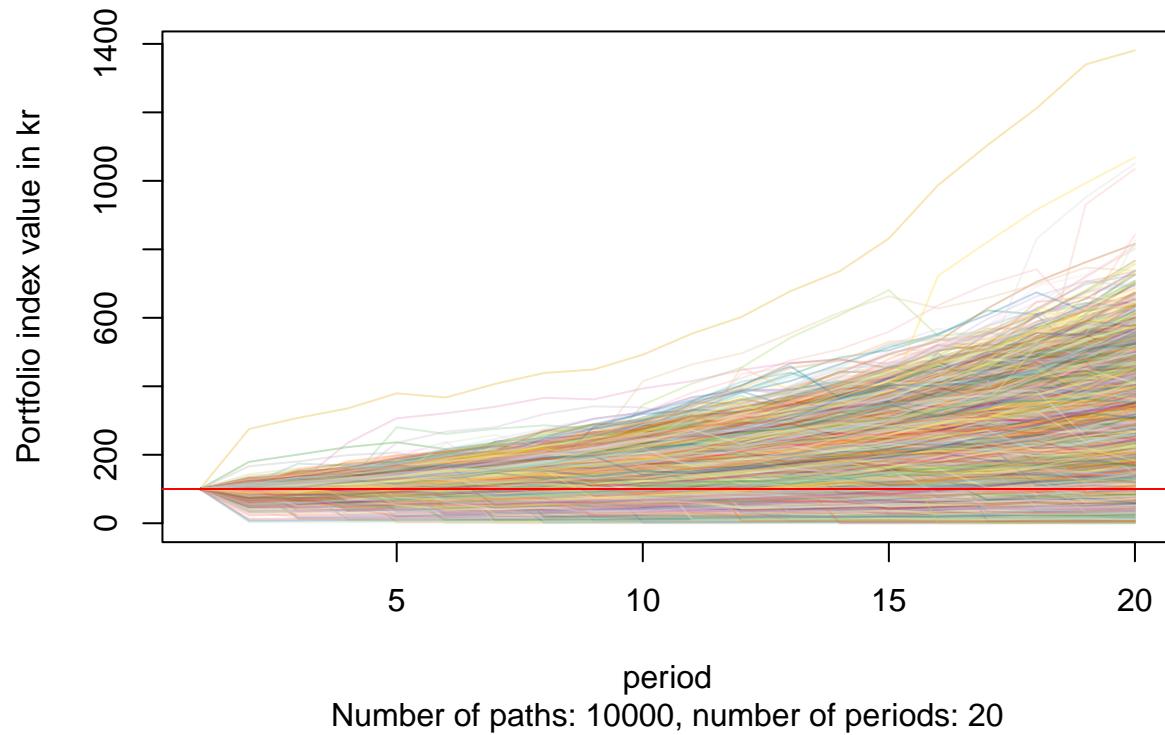
Estimated skew t distribution PDF



Monte Carlo

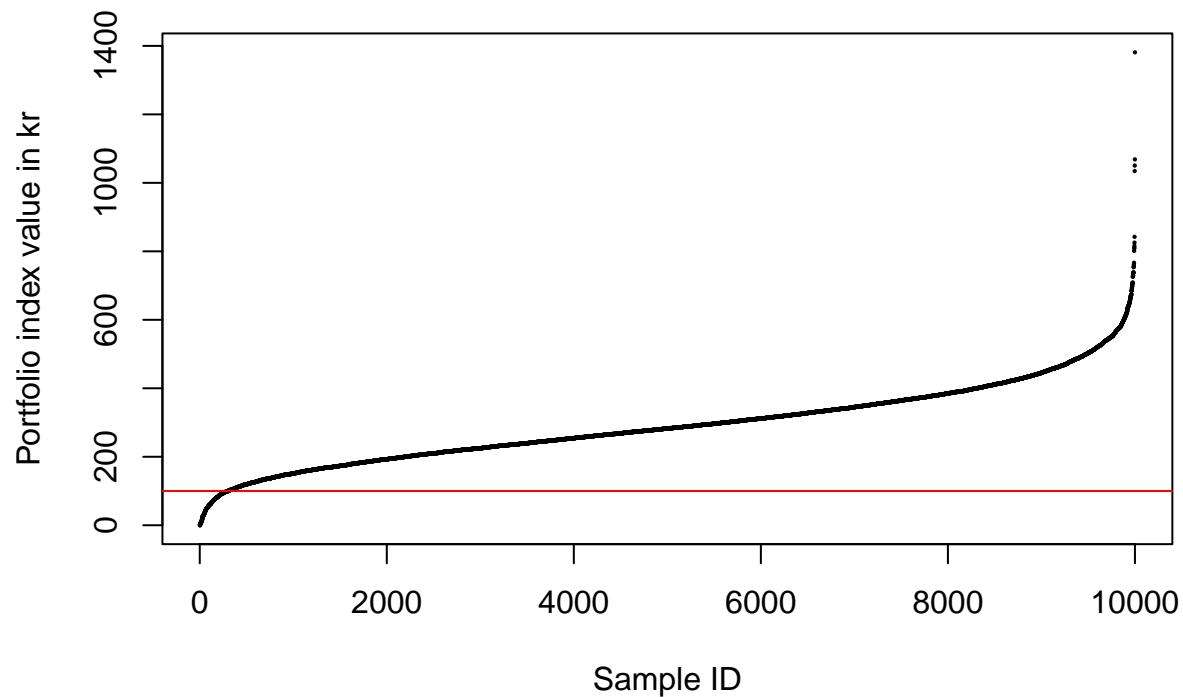
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.01 percent  
##  
## Mean portfolio index value after 20 years: 292.628 kr.  
## SD of portfolio index value after 20 years: 118.206 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 1381.145 kr.  
##  
## Share of paths finishing below 100: 3.02 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

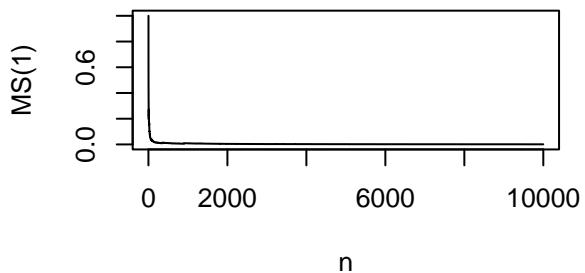
(100 is par, 200 is double, 50 is half)



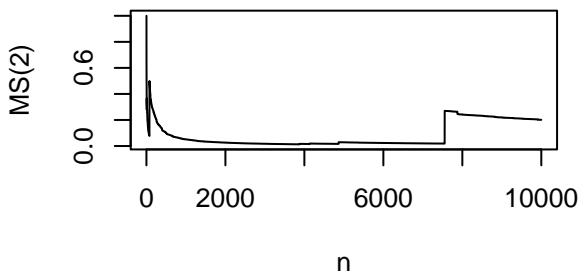
Convergence

Max vs sum plots for the first four moments:

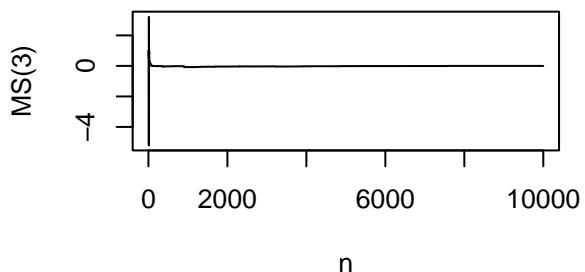
MS(1)



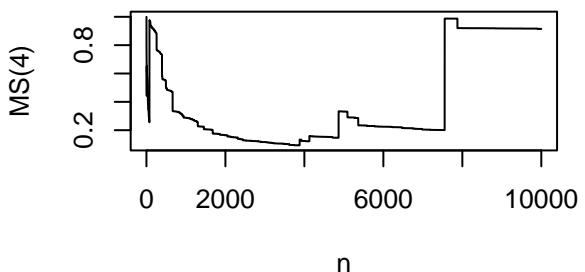
MS(2)



MS(3)

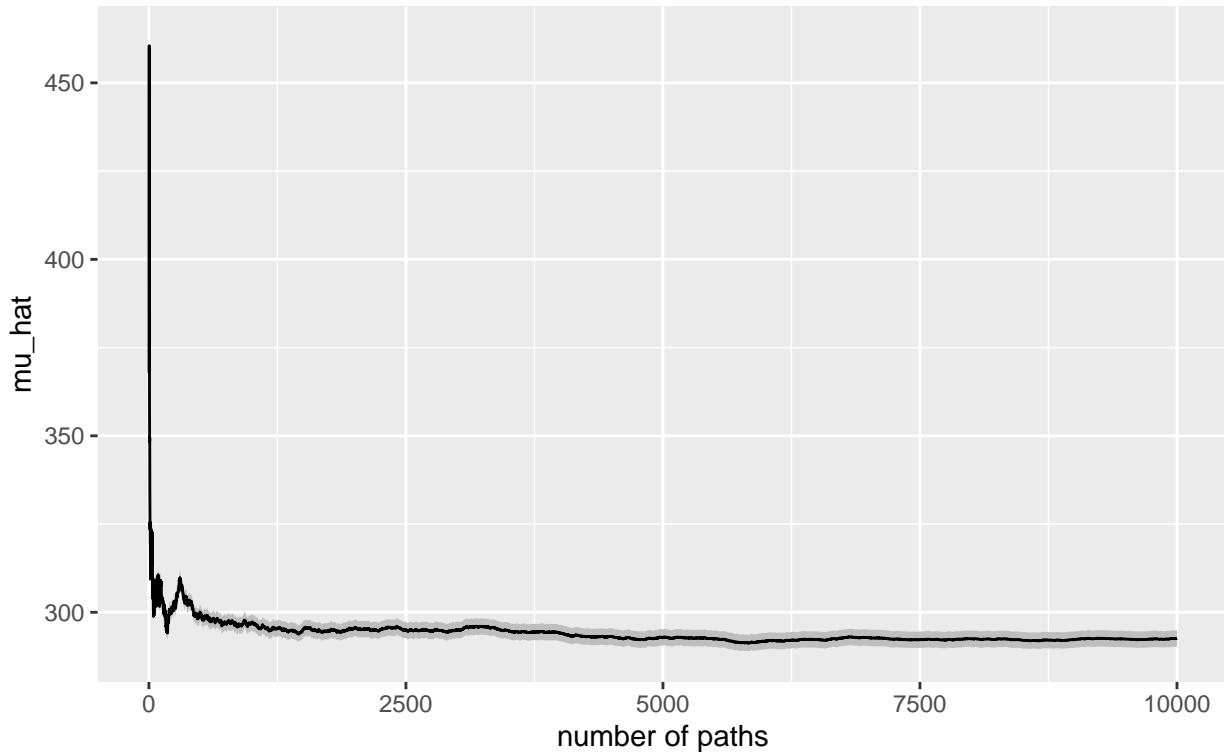


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

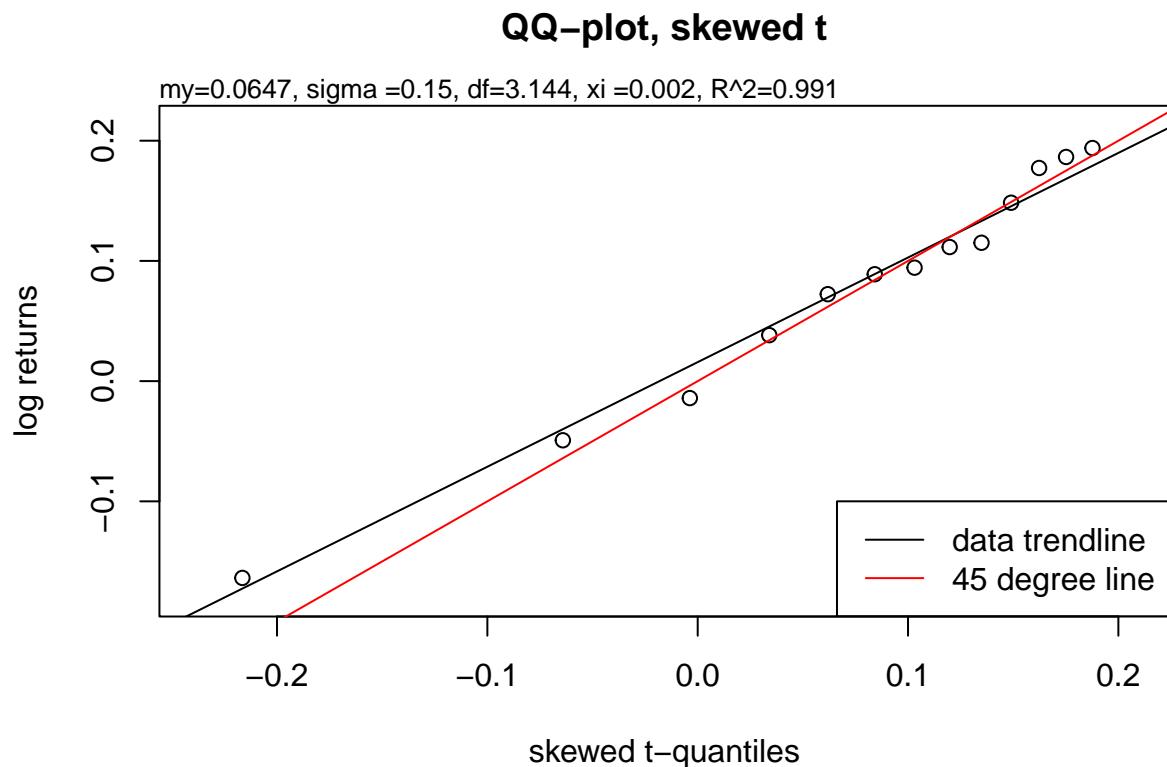


Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -21.42488  
## BIC: -19.16508  
## m: 0.06471454  
## s: 0.1499924  
## nu (df): 3.144355  
## xi: 0.002367034  
## R^2: 0.991  
##  
## An R^2 of 0.991 suggests that the fit is extremely good.  
##  
## What is the risk of losing max 10 %? <= 0 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 64.66667 percent  
## What is the chance of gaining min 25 %? >= 47.83333 percent  
## What is the chance of gaining min 50 %? >= 36.83333 percent  
## What is the chance of gaining min 90 %? >= 28 percent  
## What is the chance of gaining min 99 %? >= 26.5 percent
```

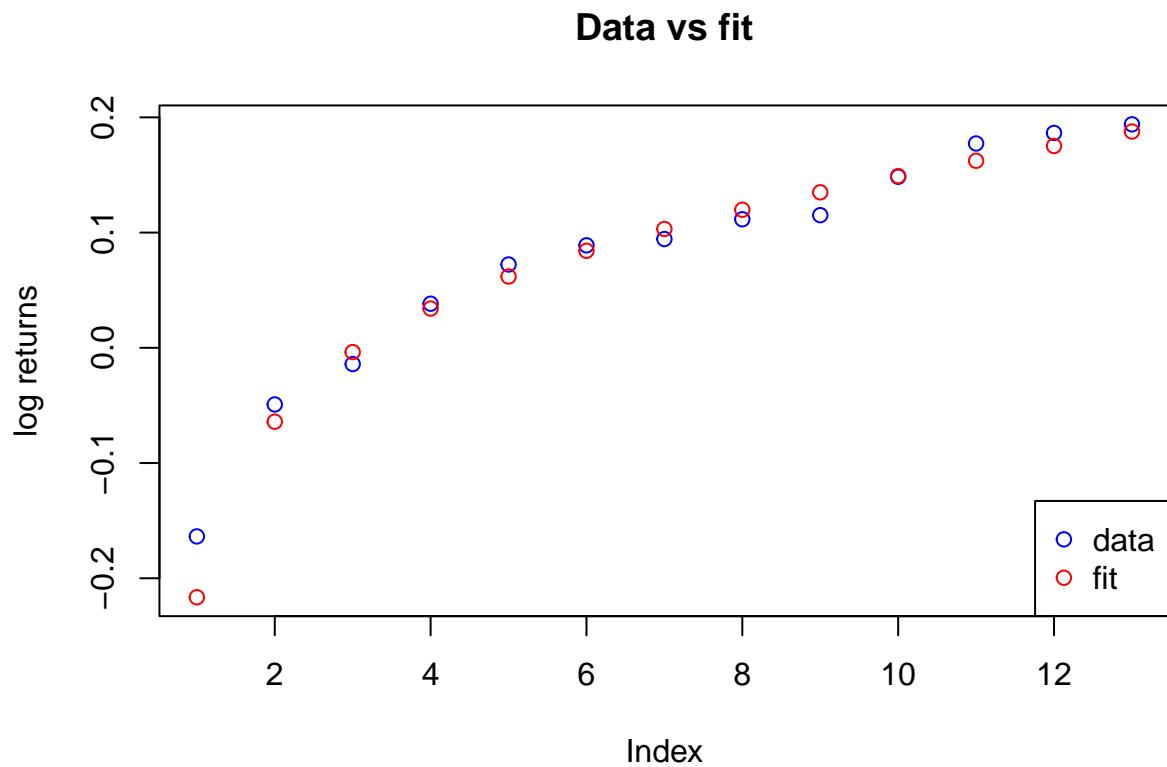
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

Data vs fit

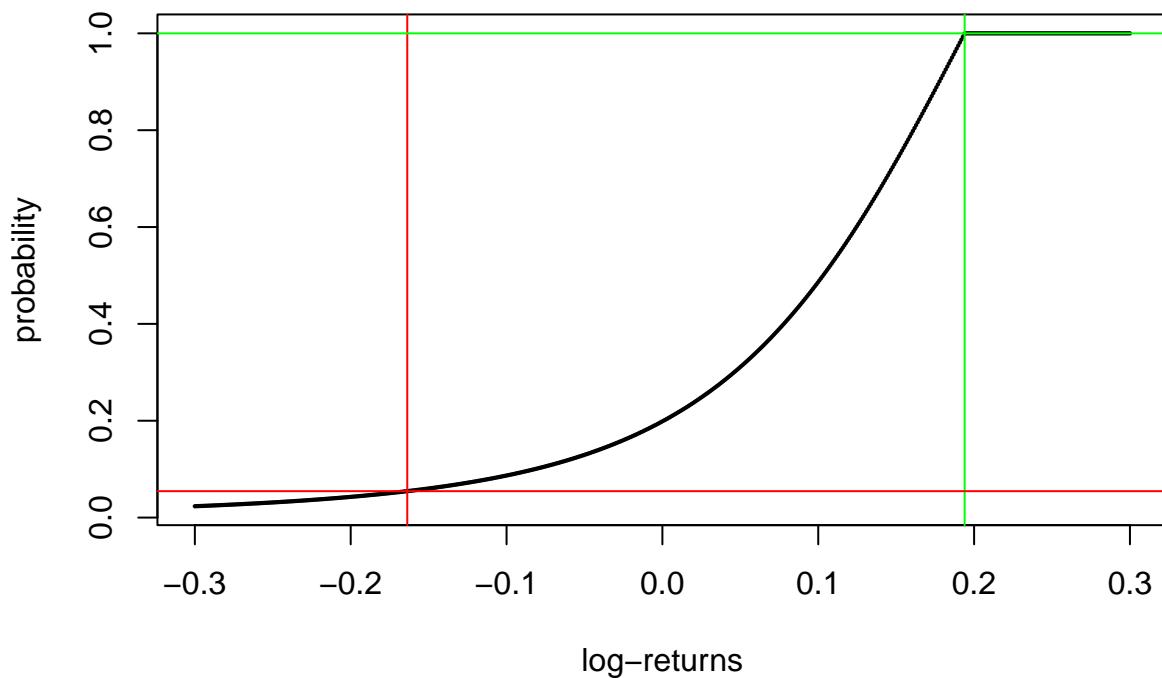
Let's plot the fit and the observed returns together.



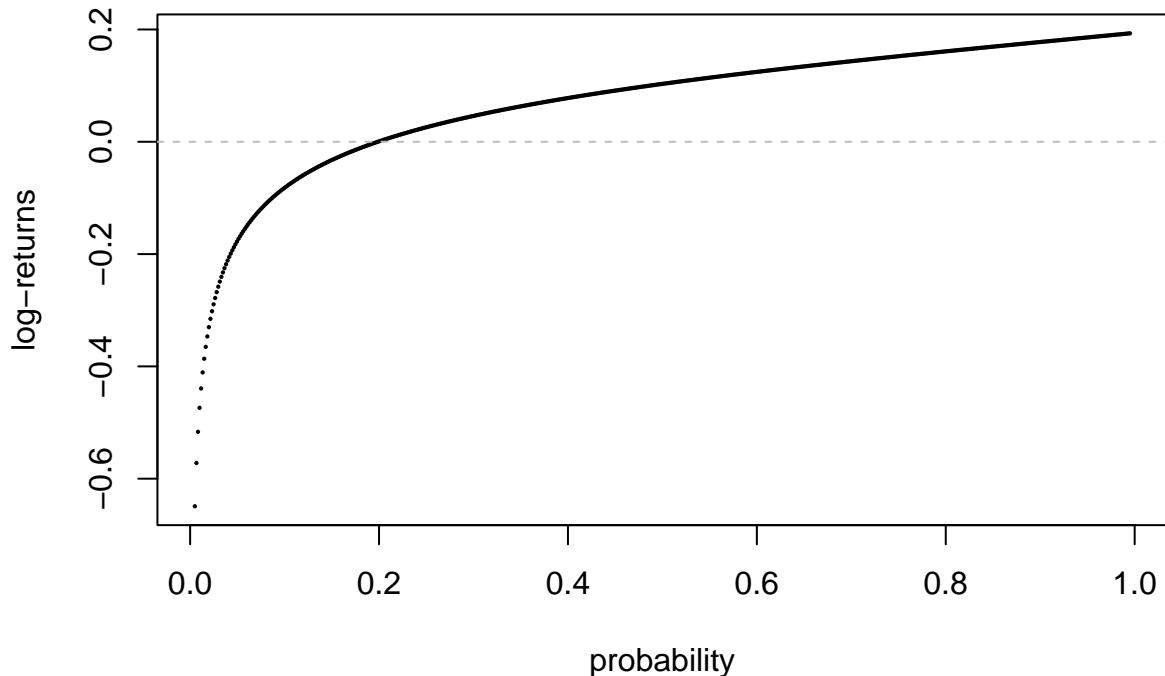
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

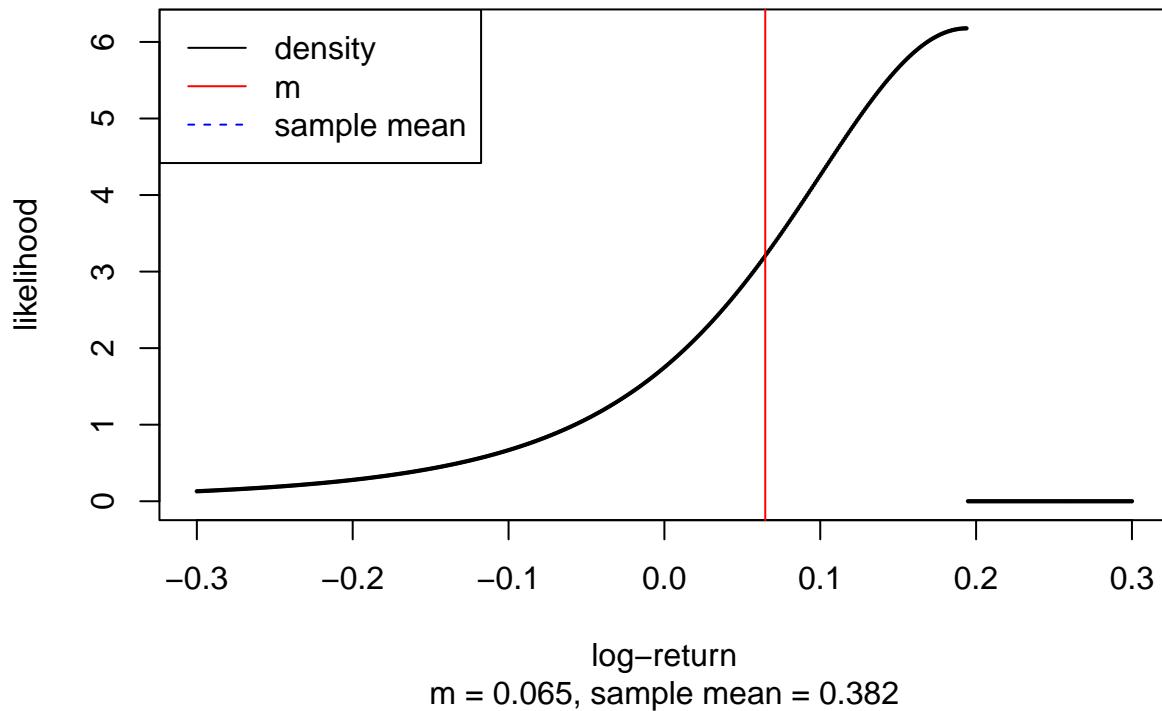


Estimated skew t distribution quantiles



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

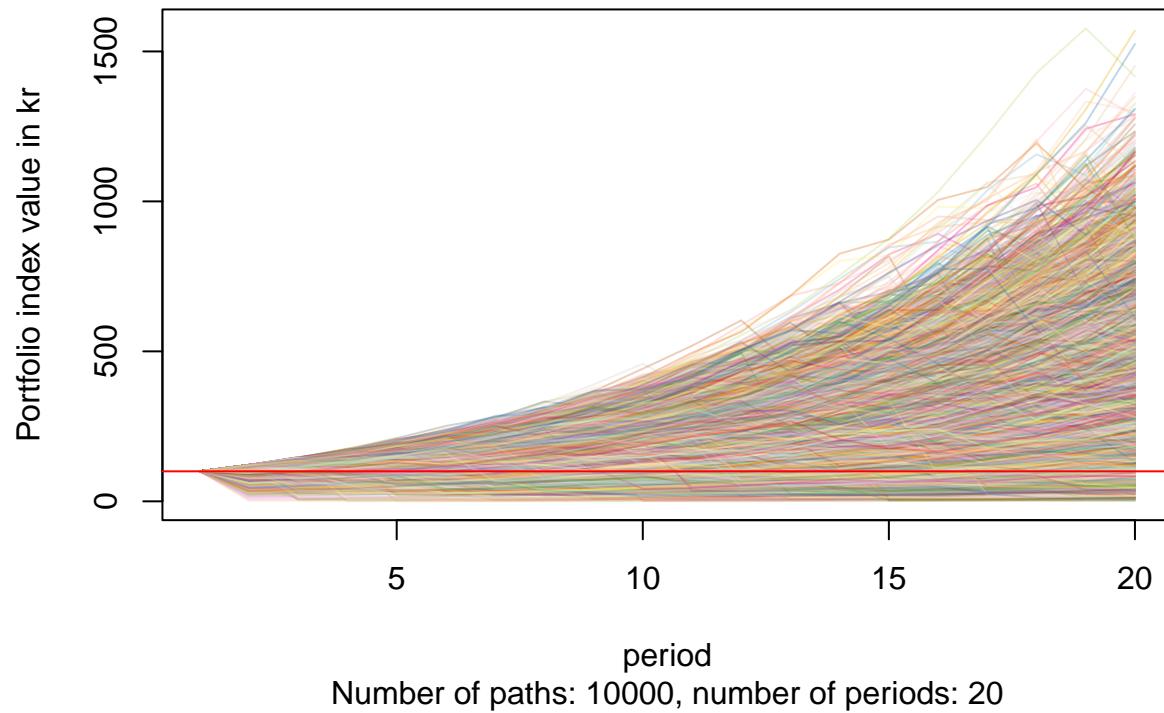
Estimated skew t distribution PDF



Monte Carlo

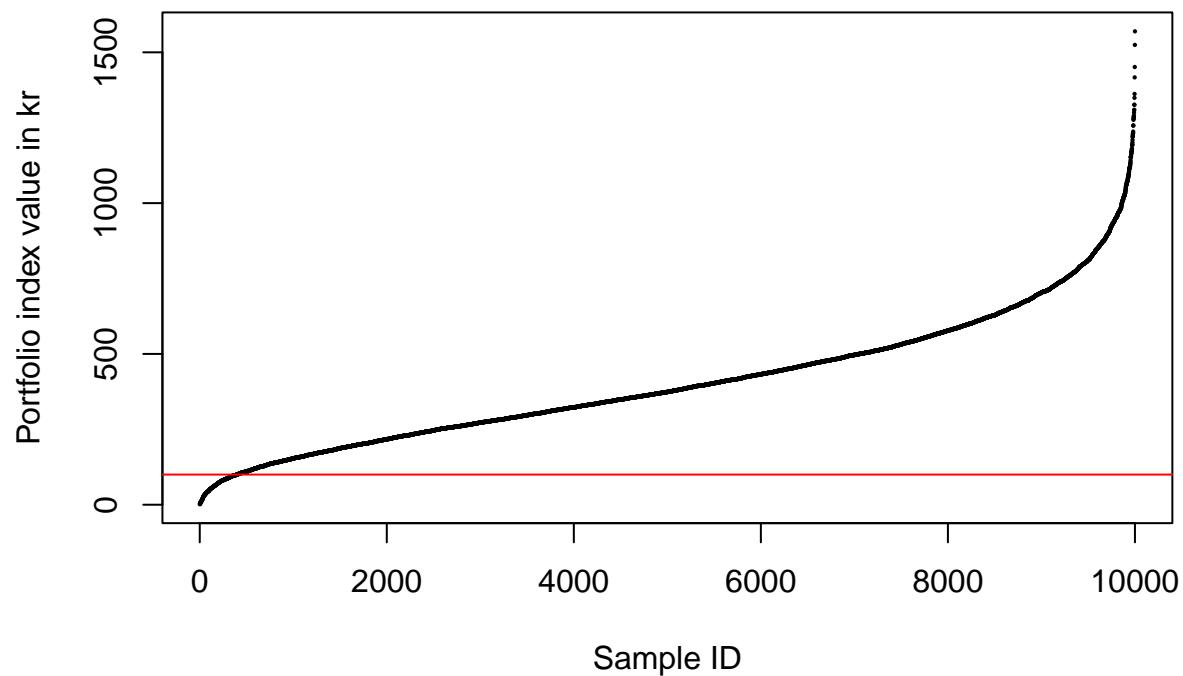
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 406.903 kr.  
## SD of portfolio index value after 20 years: 218.183 kr.  
## Min total portfolio index value after 20 years: 1.606 kr.  
## Max total portfolio index value after 20 years: 1569.591 kr.  
##  
## Share of paths finishing below 100: 3.99 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

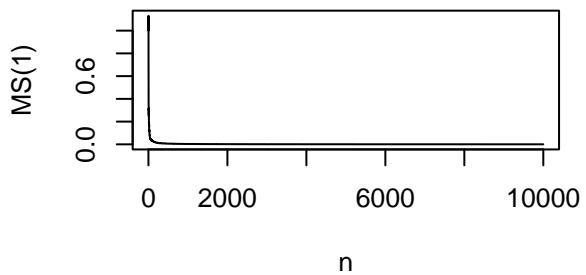
(100 is par, 200 is double, 50 is half)



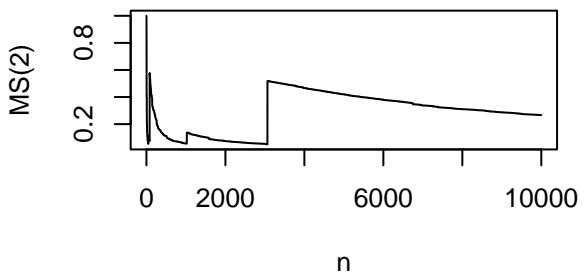
Convergence

Max vs sum plots for the first four moments:

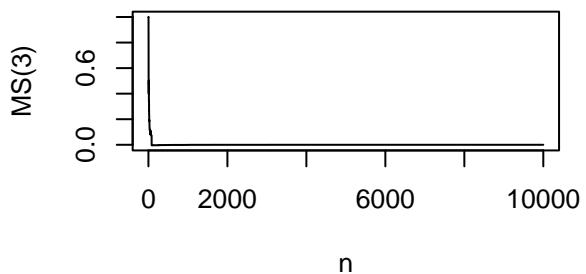
MS(1)



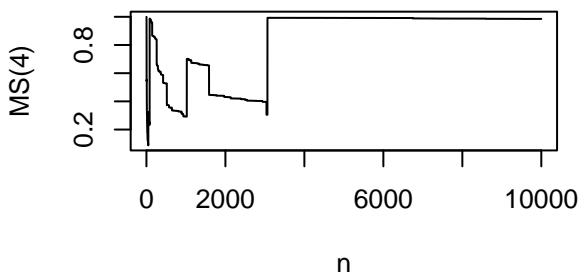
MS(2)



MS(3)

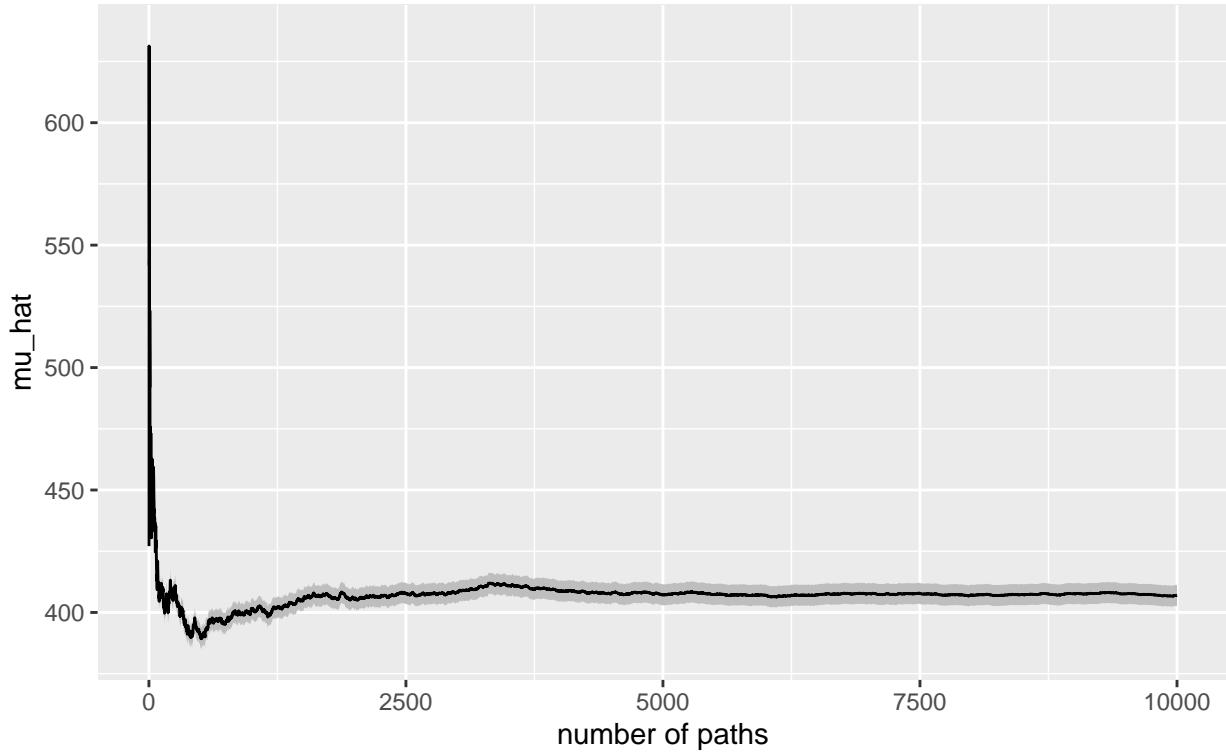


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

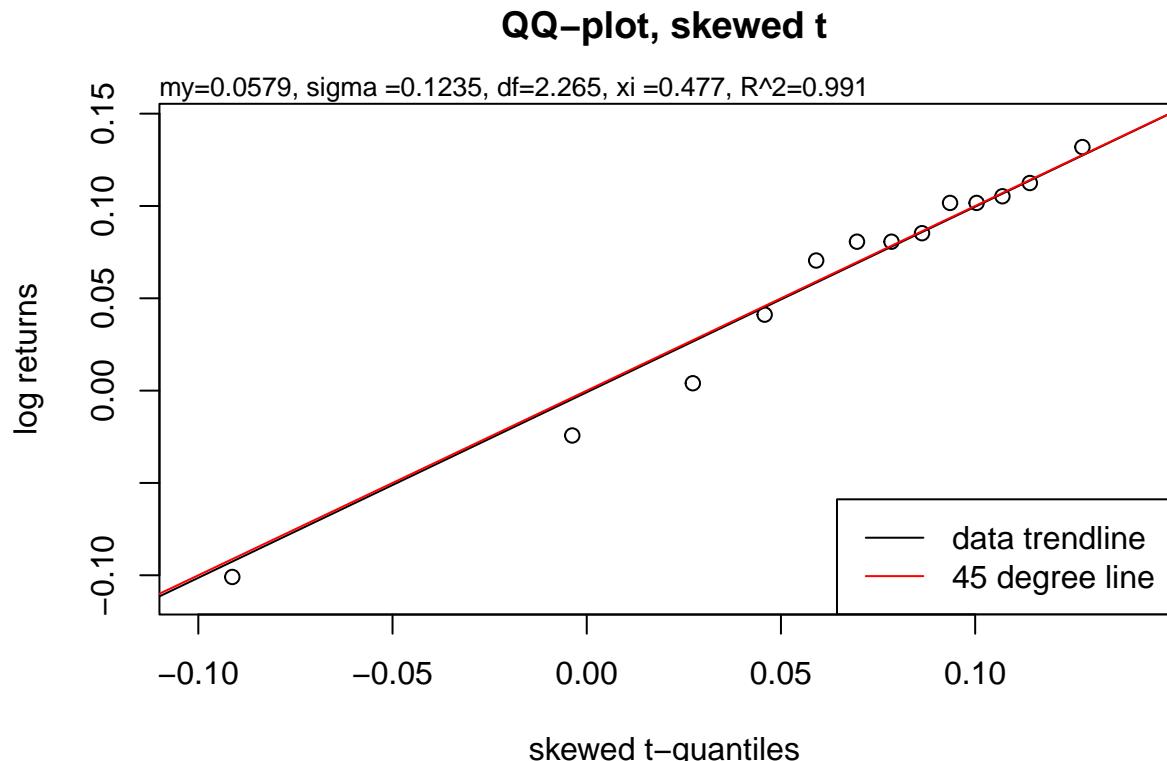


PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -33.22998  
## BIC: -30.97018  
## m: 0.05789224  
## s: 0.1234592  
## nu (df): 2.265273  
## xi: 0.477324  
## R^2: 0.991  
##  
## An R^2 of 0.991 suggests that the fit is extremely good.  
##  
## What is the risk of losing max 10 %? <= 0 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 52.83333 percent  
## What is the chance of gaining min 25 %? >= 44 percent  
## What is the chance of gaining min 50 %? >= 38.83333 percent  
## What is the chance of gaining min 90 %? >= 34.66667 percent  
## What is the chance of gaining min 99 %? >= 34 percent
```

QQ Plot

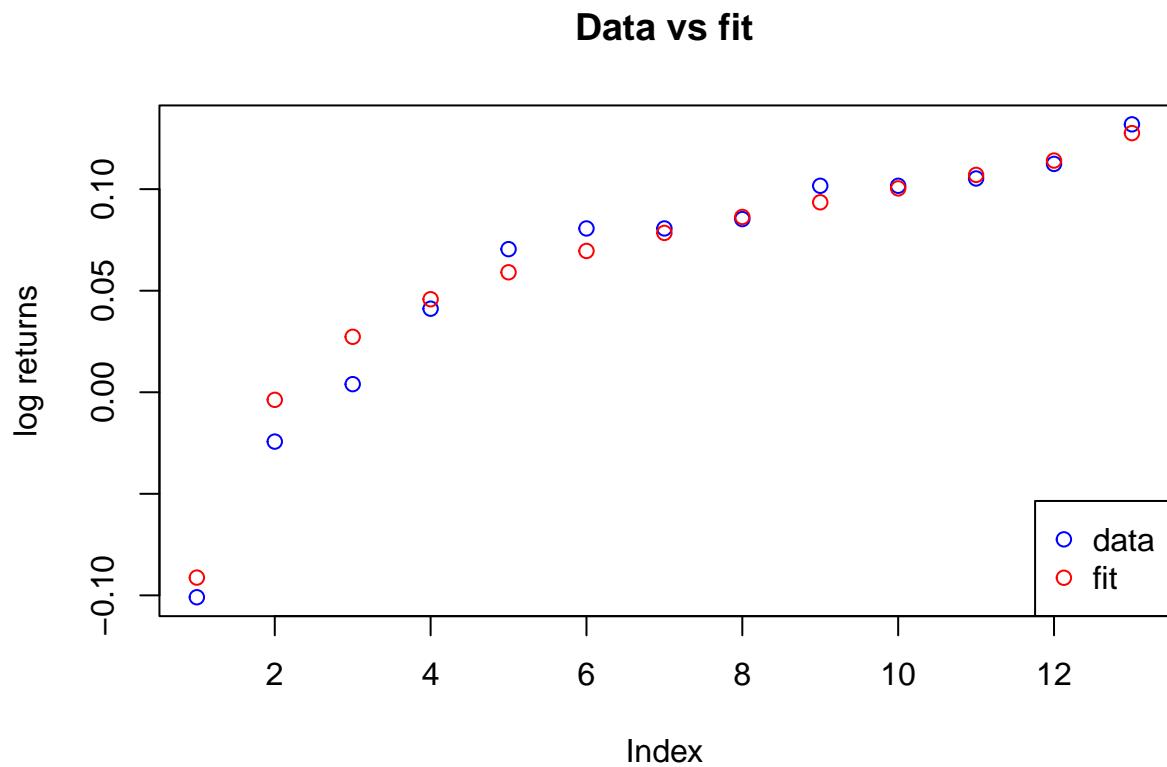


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
## [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
## [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11] 0.107018607  0.114081432  0.127604387
```

Data vs fit

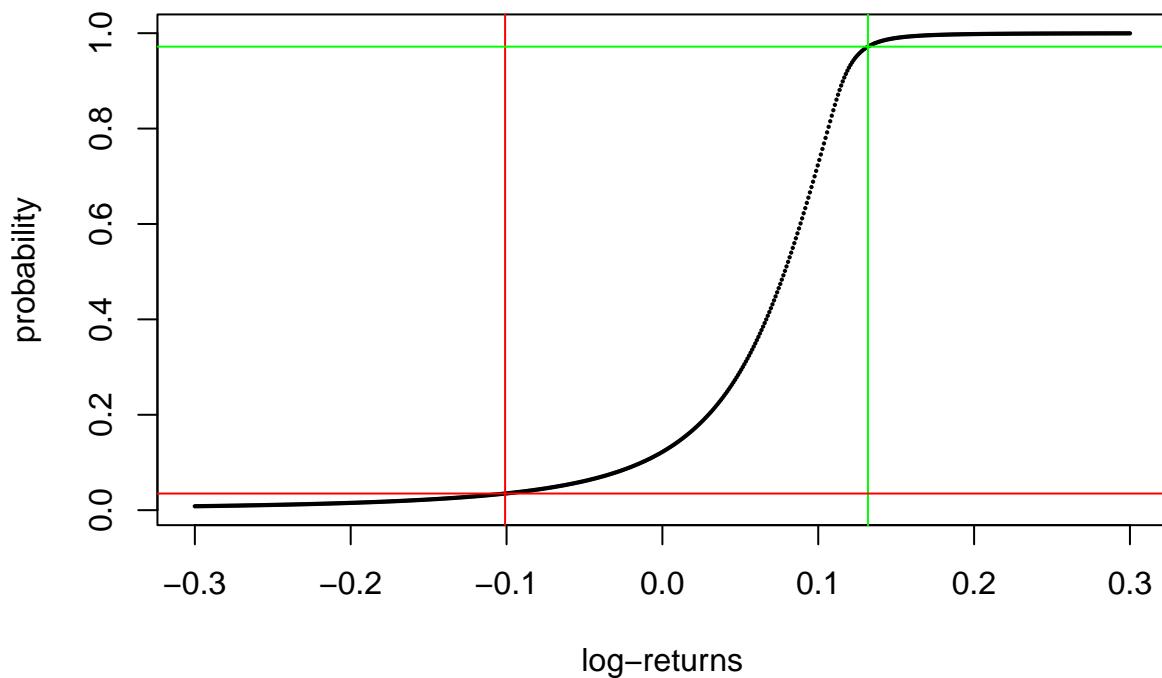
Let's plot the fit and the observed returns together.



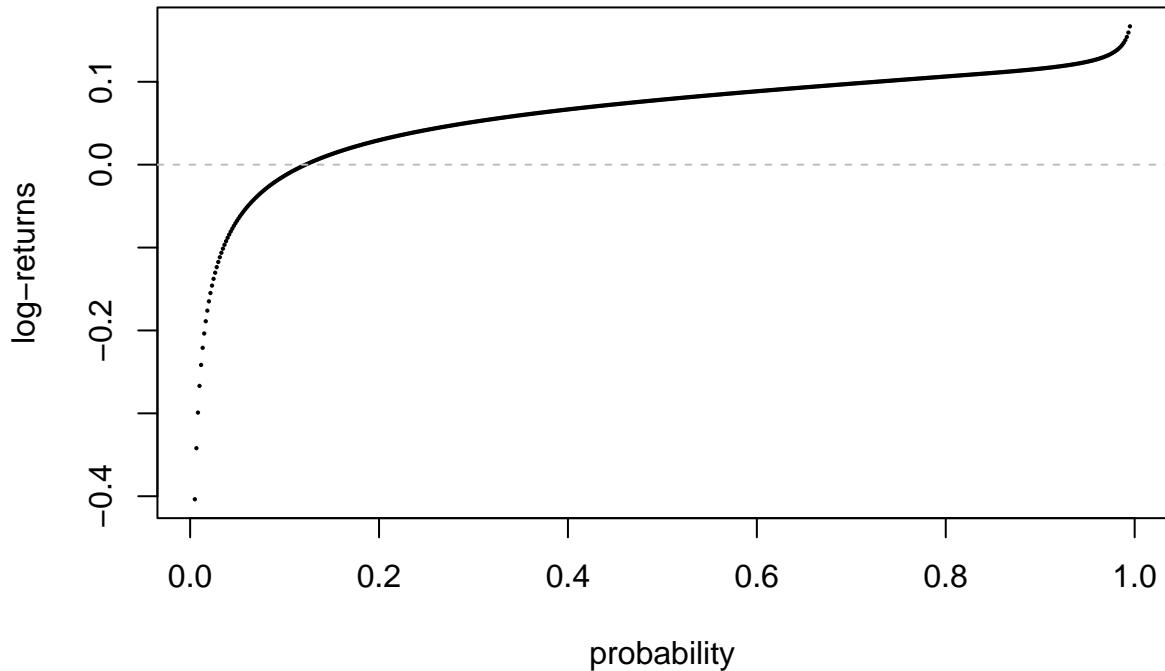
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

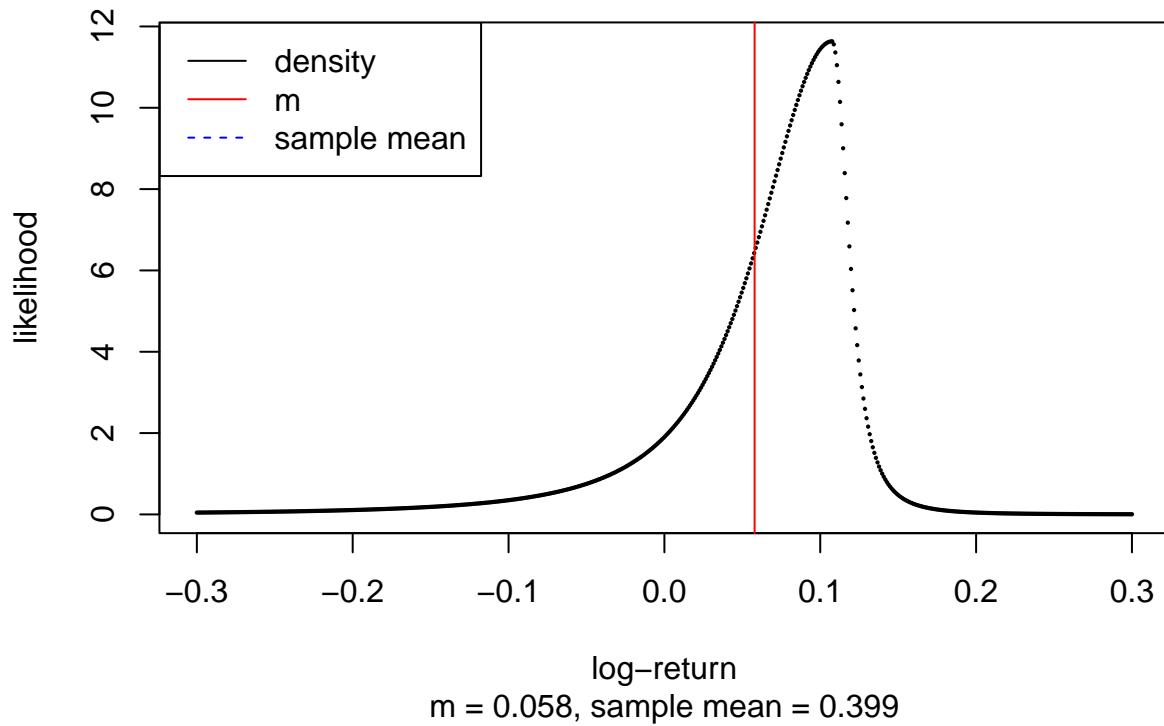


Estimated skew t distribution quantiles



We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.

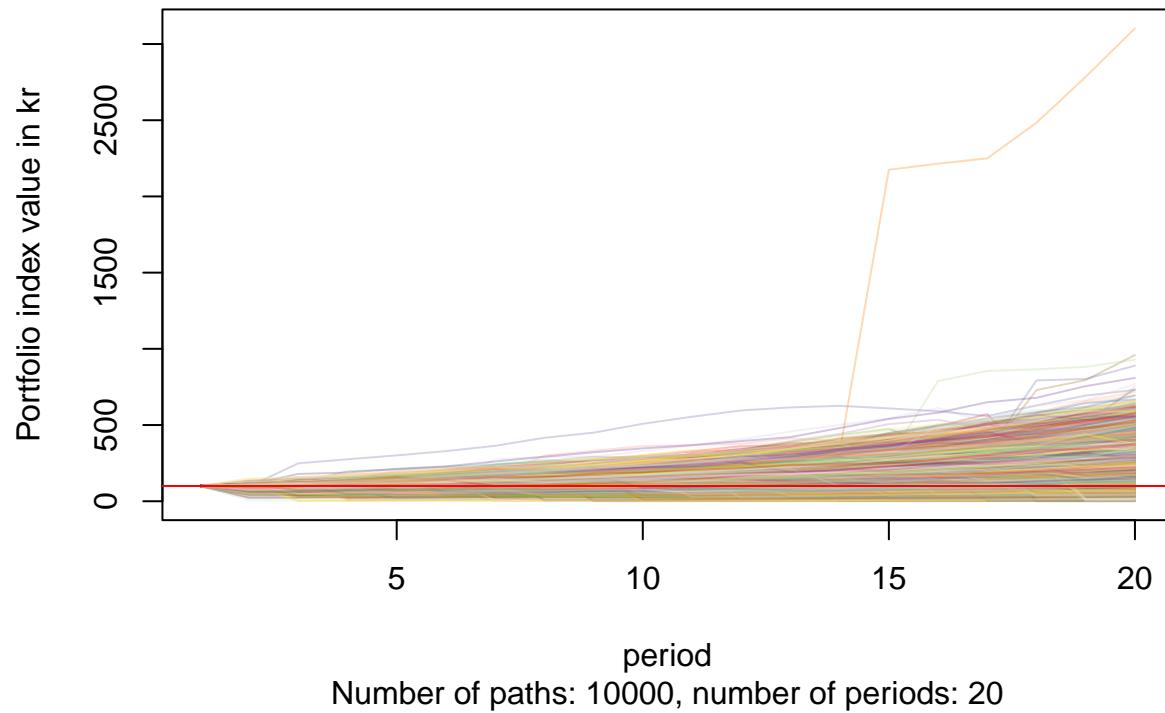
Estimated skew t distribution PDF



Monte Carlo

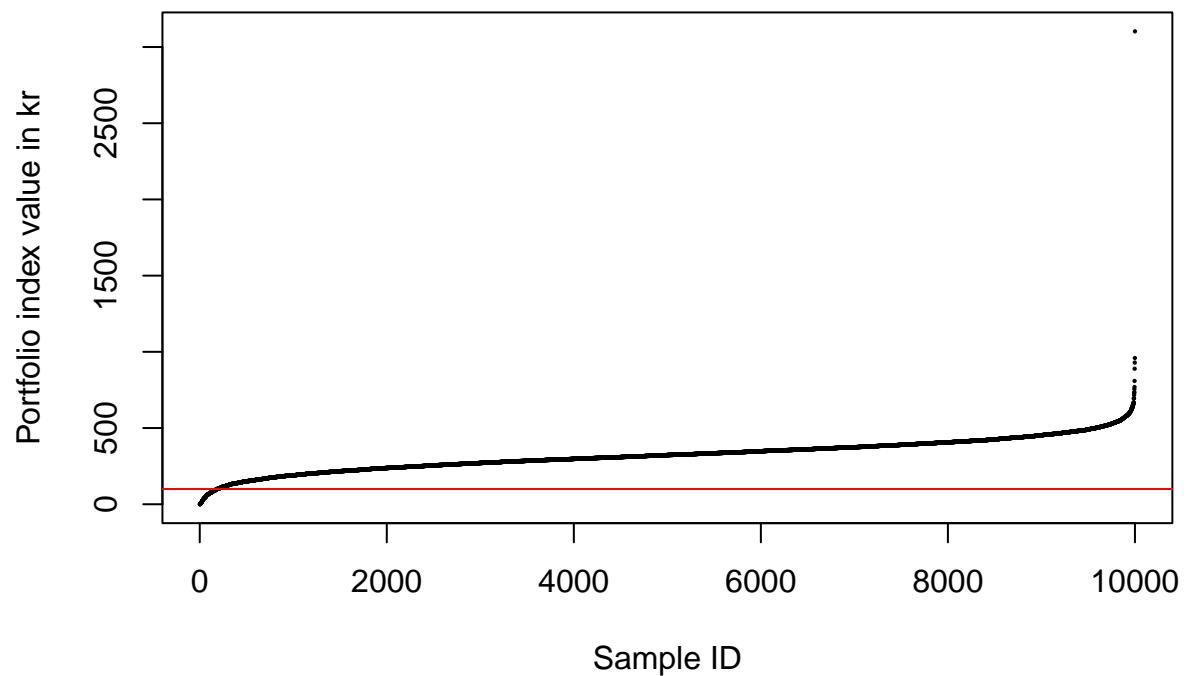
```
## Down-and-out simulation:  
## Probability of down-and-out: 0.01 percent  
##  
## Mean portfolio index value after 20 years: 322.484 kr.  
## SD of portfolio index value after 20 years: 107.876 kr.  
## Min total portfolio index value after 20 years: 0 kr.  
## Max total portfolio index value after 20 years: 3102.847 kr.  
##  
## Share of paths finishing below 100: 1.88 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

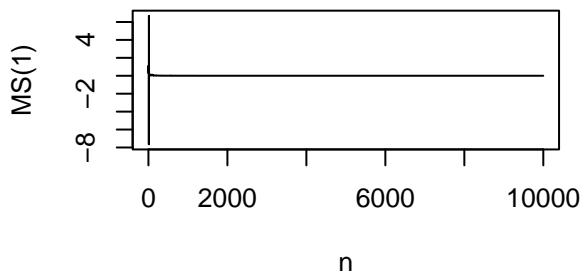
(100 is par, 200 is double, 50 is half)



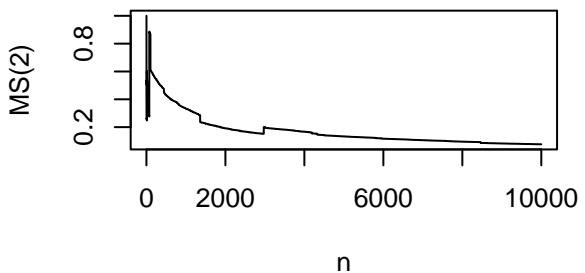
Convergence

Max vs sum plots for the first four moments:

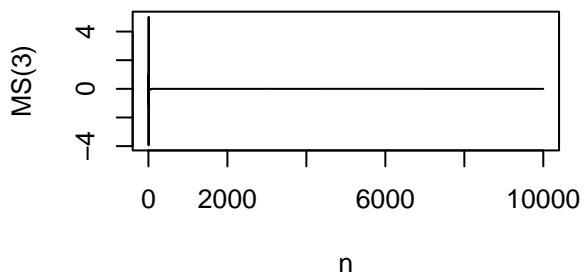
MS(1)



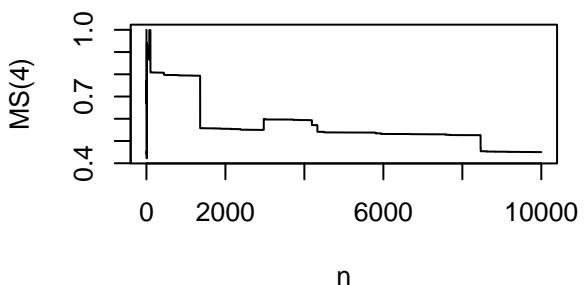
MS(2)



MS(3)

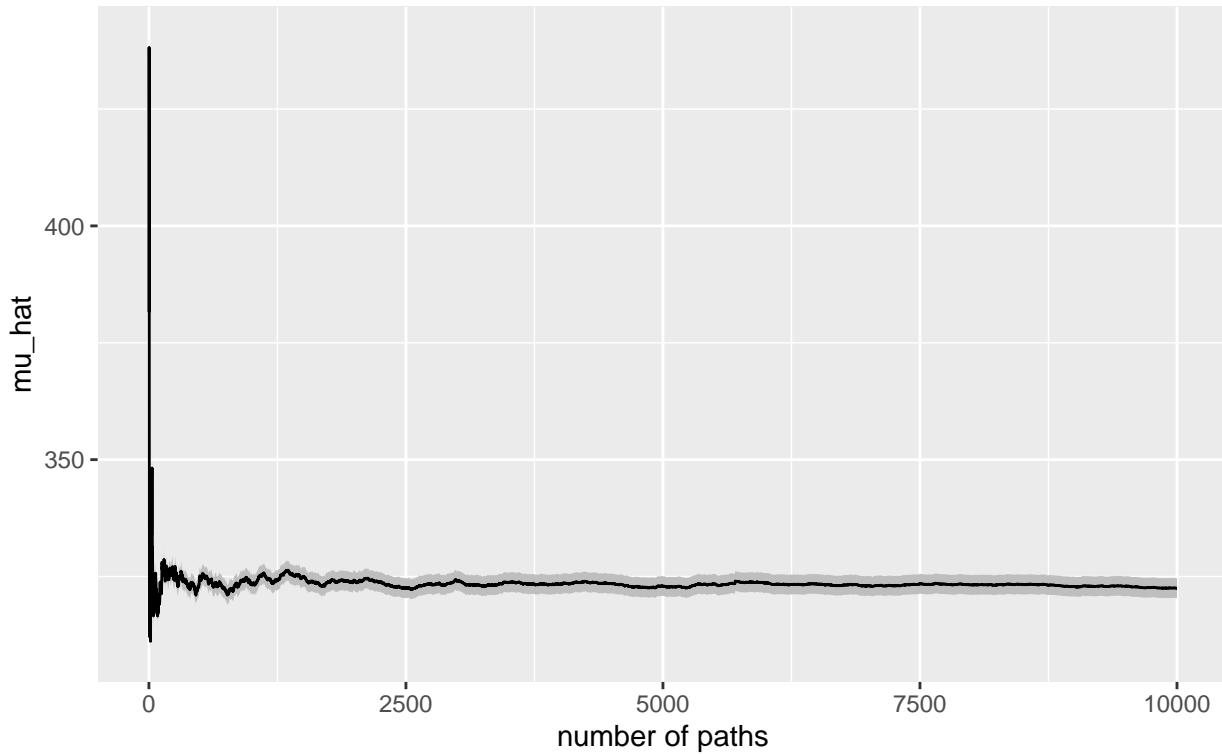


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

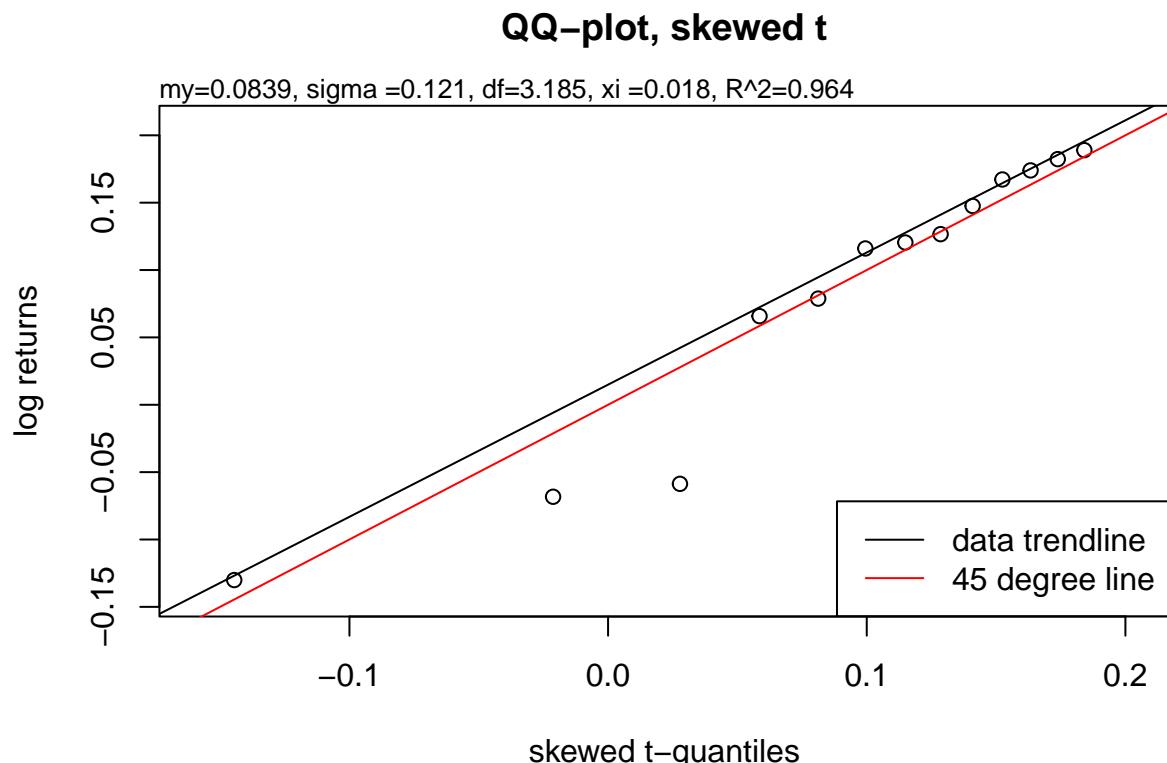


PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -23.72565  
## BIC: -21.46585  
## m: 0.08386034  
## s: 0.1210107  
## nu (df): 3.184569  
## xi: 0.01790306  
## R^2: 0.964  
##  
## An R^2 of 0.964 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 0 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 56.83333 percent  
## What is the chance of gaining min 25 %? >= 43.16667 percent  
## What is the chance of gaining min 50 %? >= 34.16667 percent  
## What is the chance of gaining min 90 %? >= 26.83333 percent  
## What is the chance of gaining min 99 %? >= 25.66667 percent
```

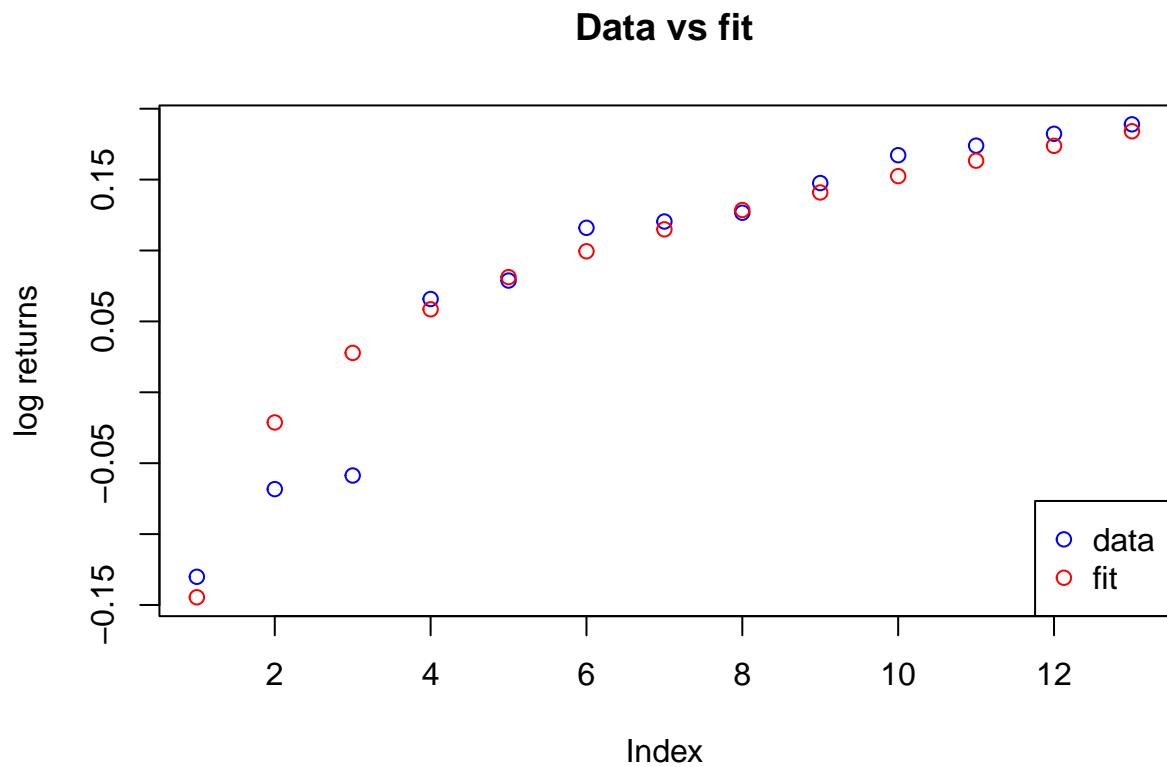
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

Data vs fit

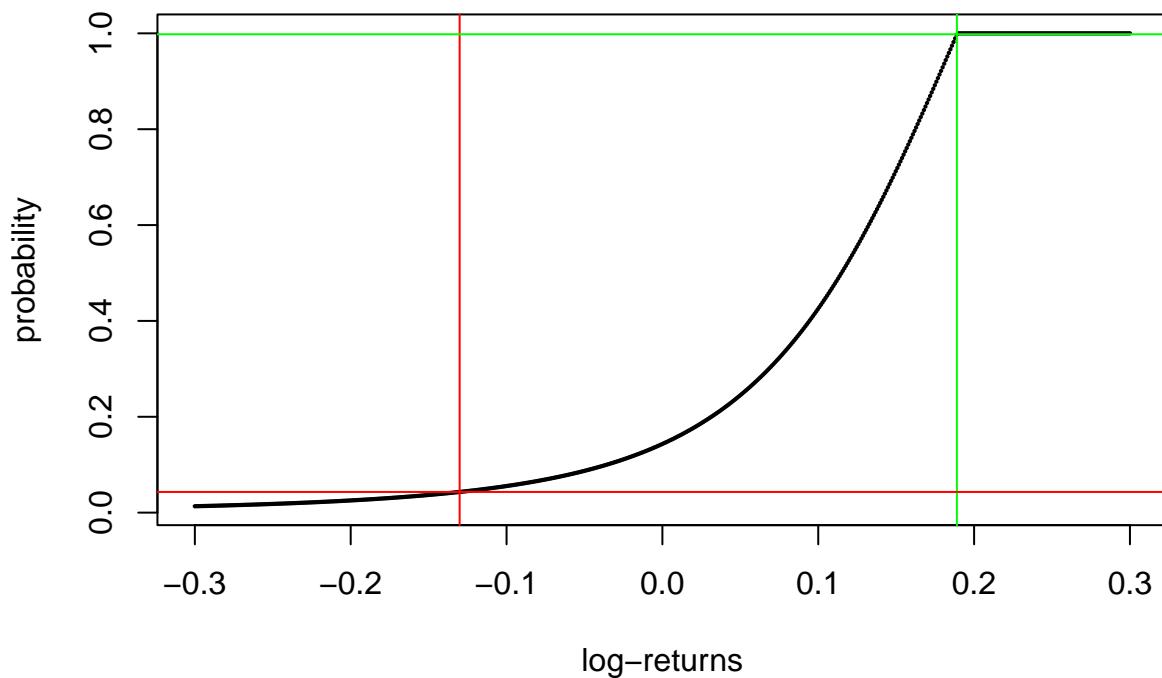
Let's plot the fit and the observed returns together.



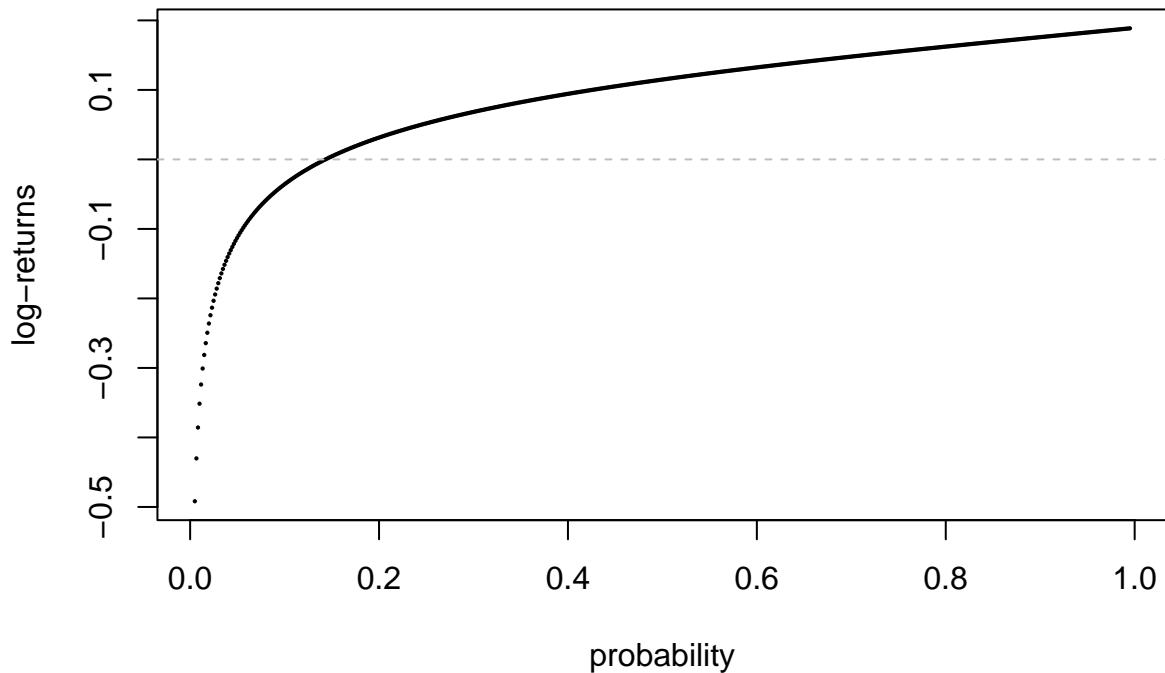
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

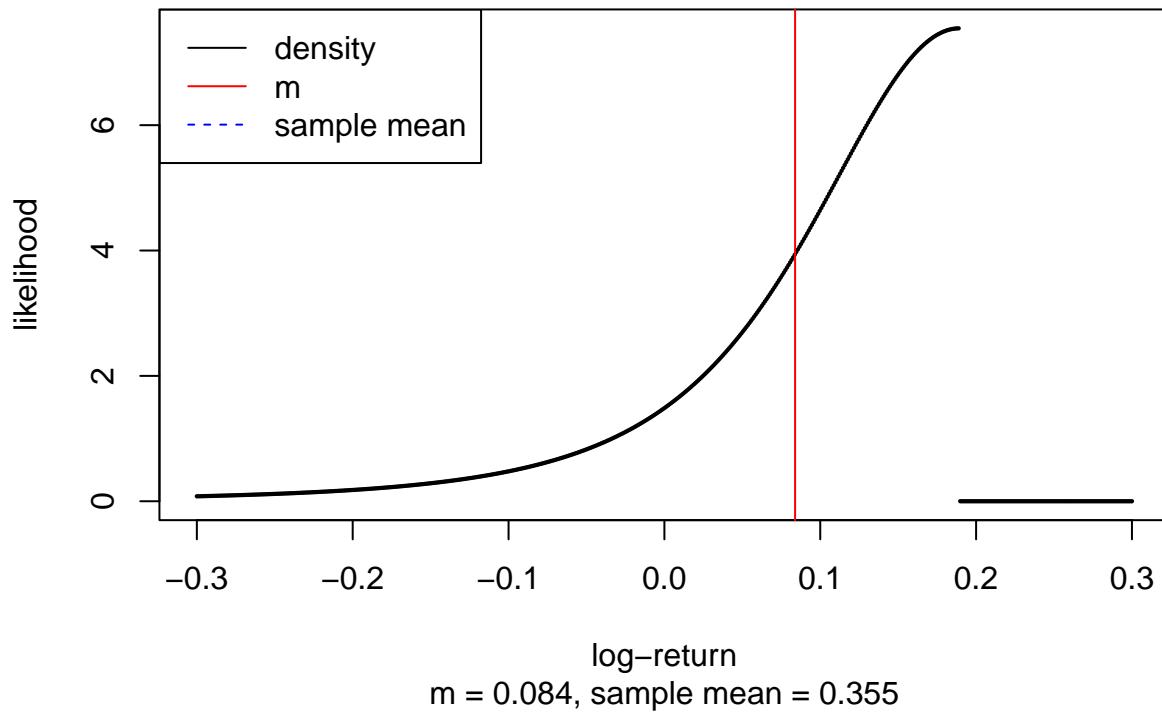


Estimated skew t distribution quantiles



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

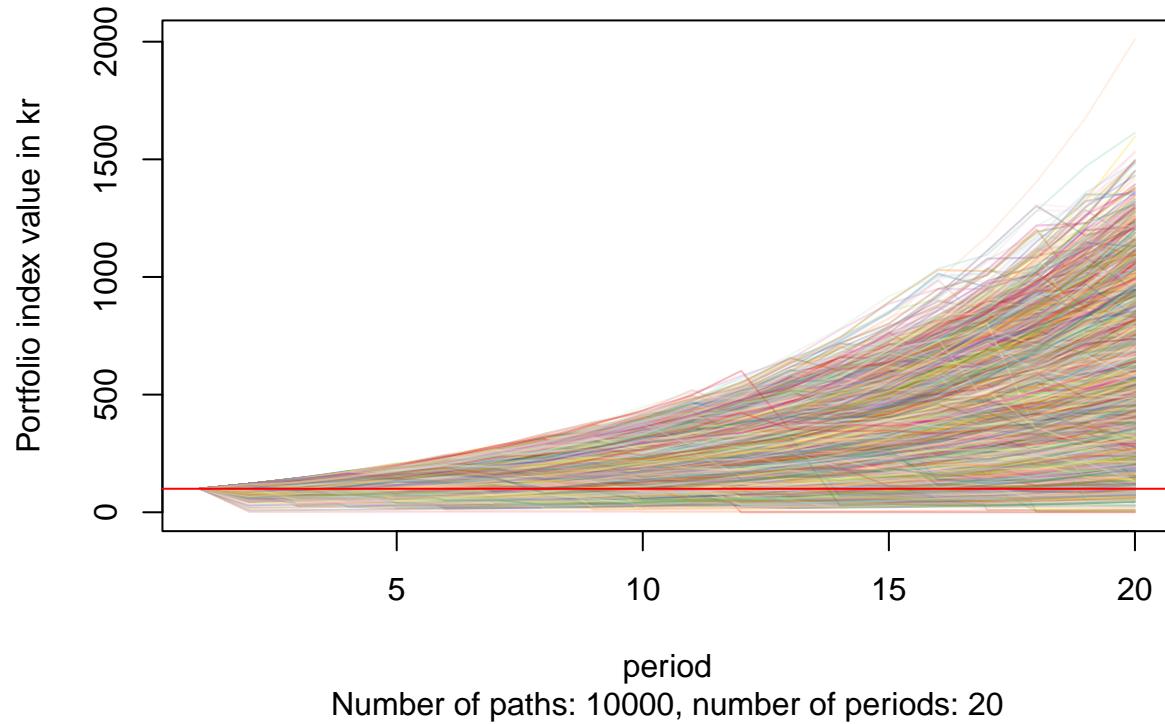
Estimated skew t distribution PDF



Monte Carlo

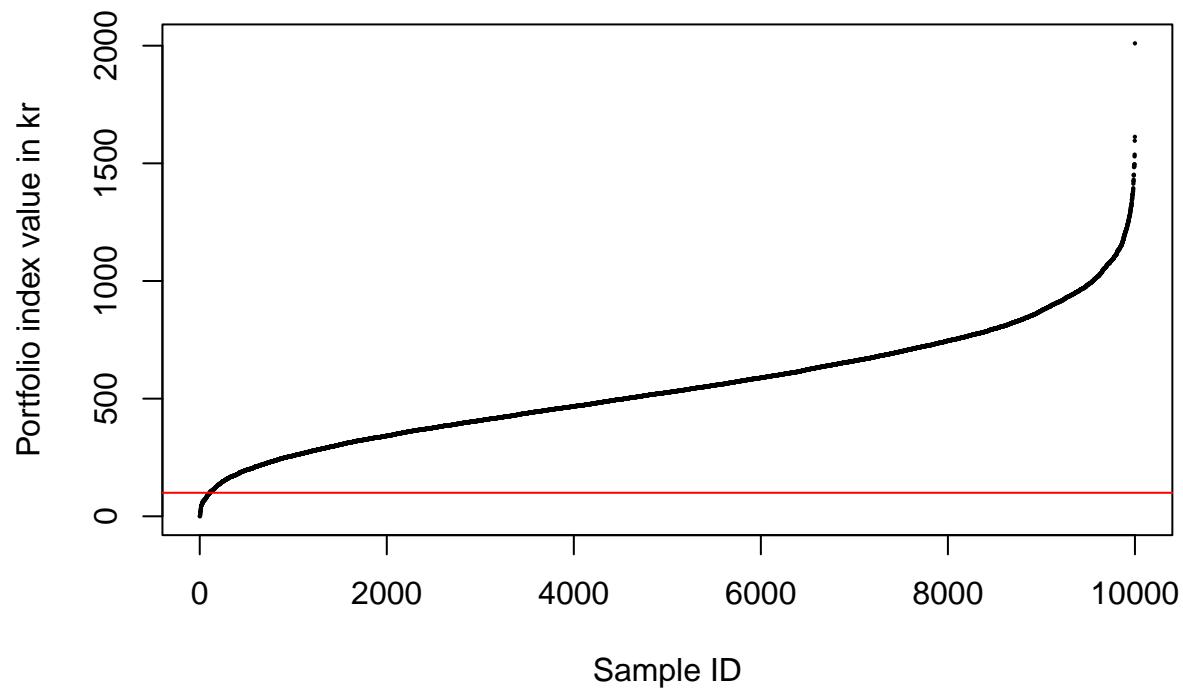
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 550.414 kr.  
## SD of portfolio index value after 20 years: 241.439 kr.  
## Min total portfolio index value after 20 years: 0.298 kr.  
## Max total portfolio index value after 20 years: 2009.894 kr.  
##  
## Share of paths finishing below 100: 1.06 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

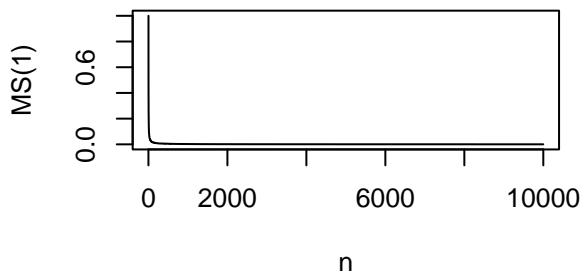
(100 is par, 200 is double, 50 is half)



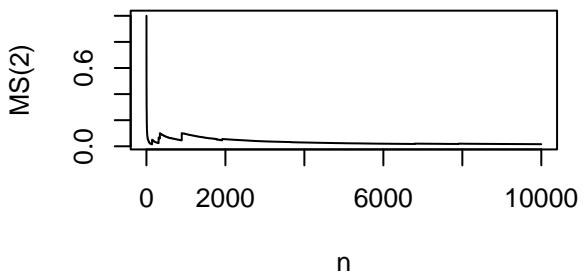
Convergence

Max vs sum plots for the first four moments:

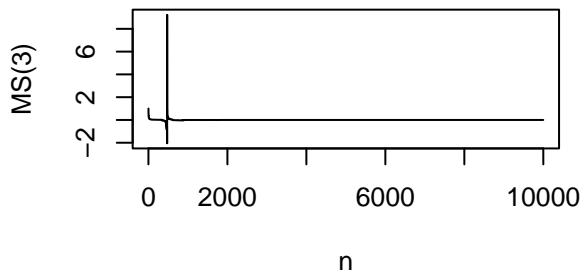
MS(1)



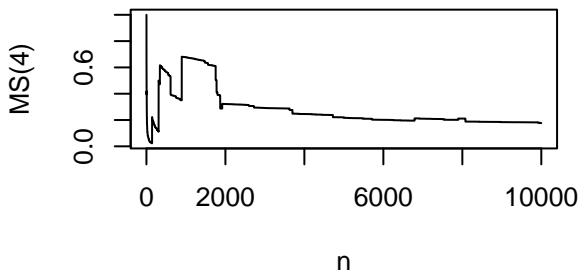
MS(2)



MS(3)

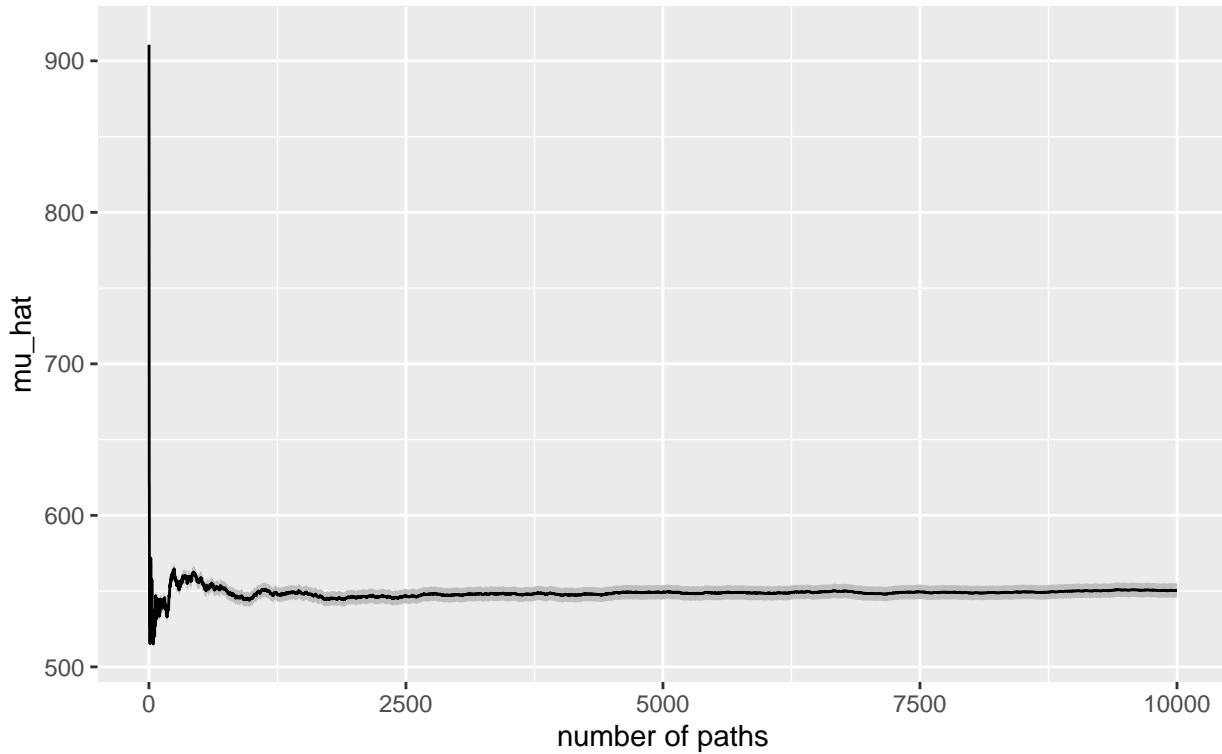


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

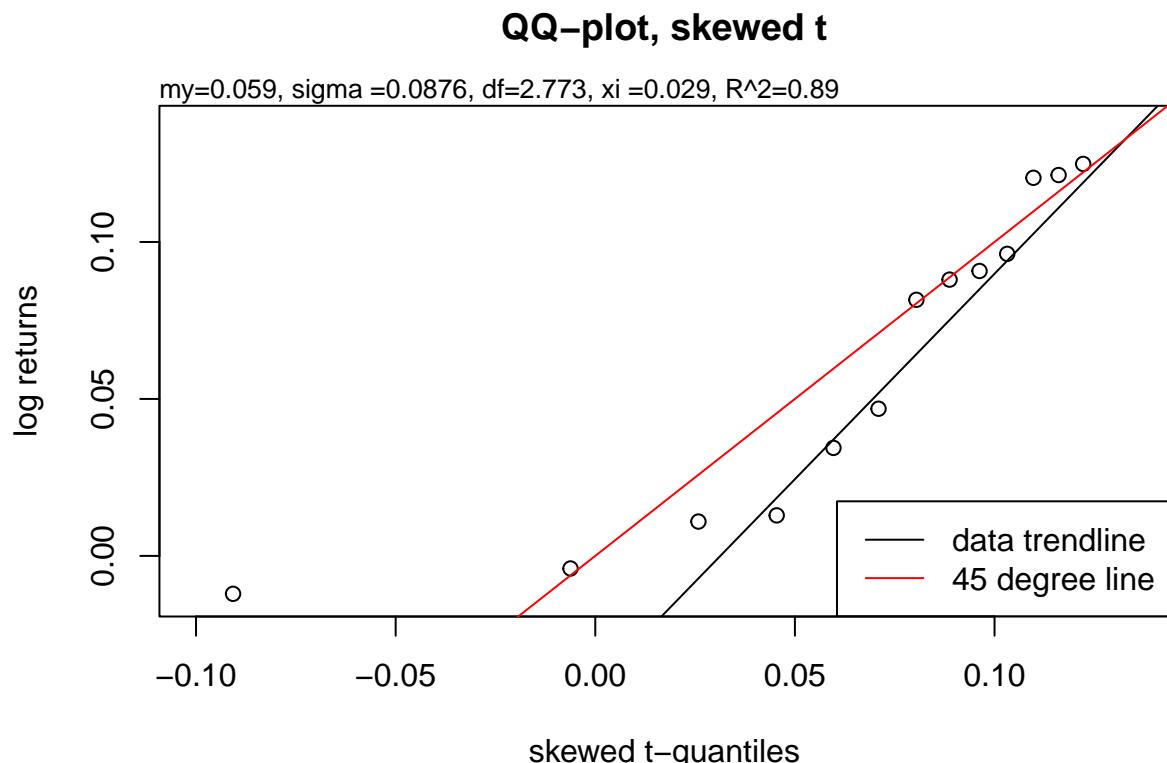


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -36.9603  
## BIC: -34.7005  
## m: 0.05902873  
## s: 0.08757749  
## nu (df): 2.772621  
## xi: 0.02904471  
## R^2: 0.89  
##  
## An R^2 of 0.89 suggests that the fit is not completely random.  
##  
## What is the risk of losing max 10 %? <= 0 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 53.16667 percent  
## What is the chance of gaining min 25 %? >= 44.16667 percent  
## What is the chance of gaining min 50 %? >= 38.66667 percent  
## What is the chance of gaining min 90 %? >= 34.16667 percent  
## What is the chance of gaining min 99 %? >= 33.5 percent
```

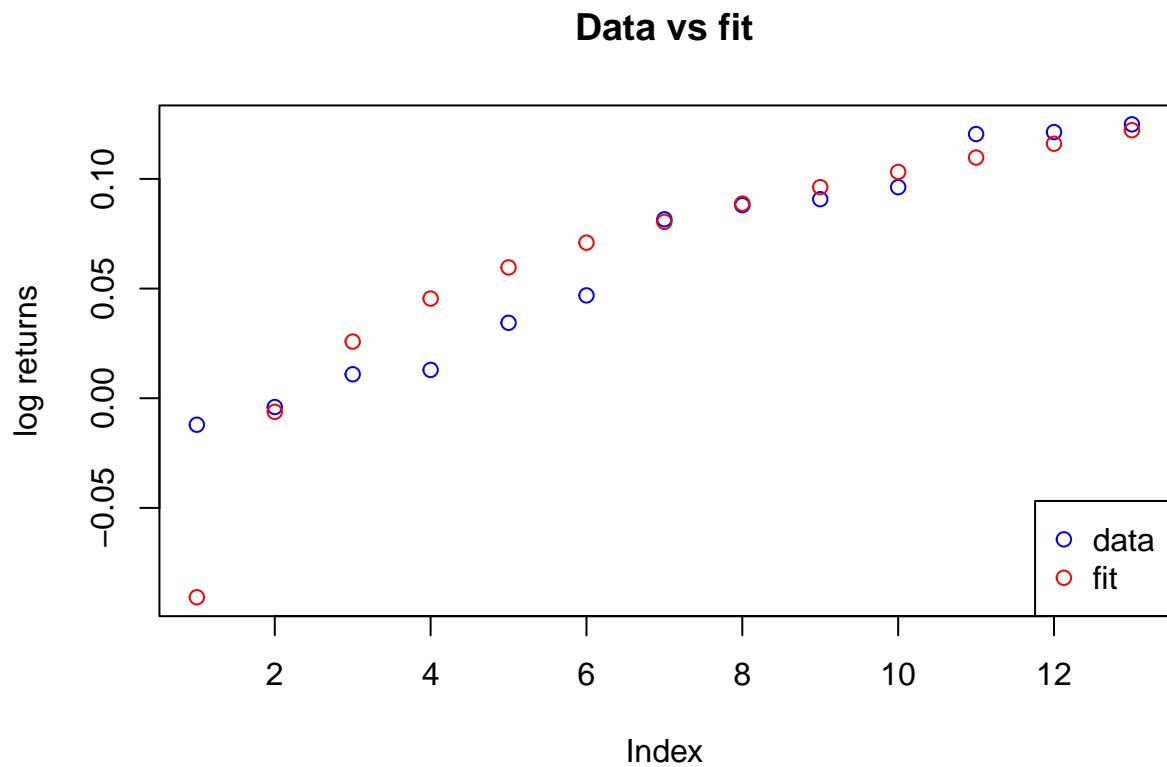
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

Let's plot the fit and the observed returns together.

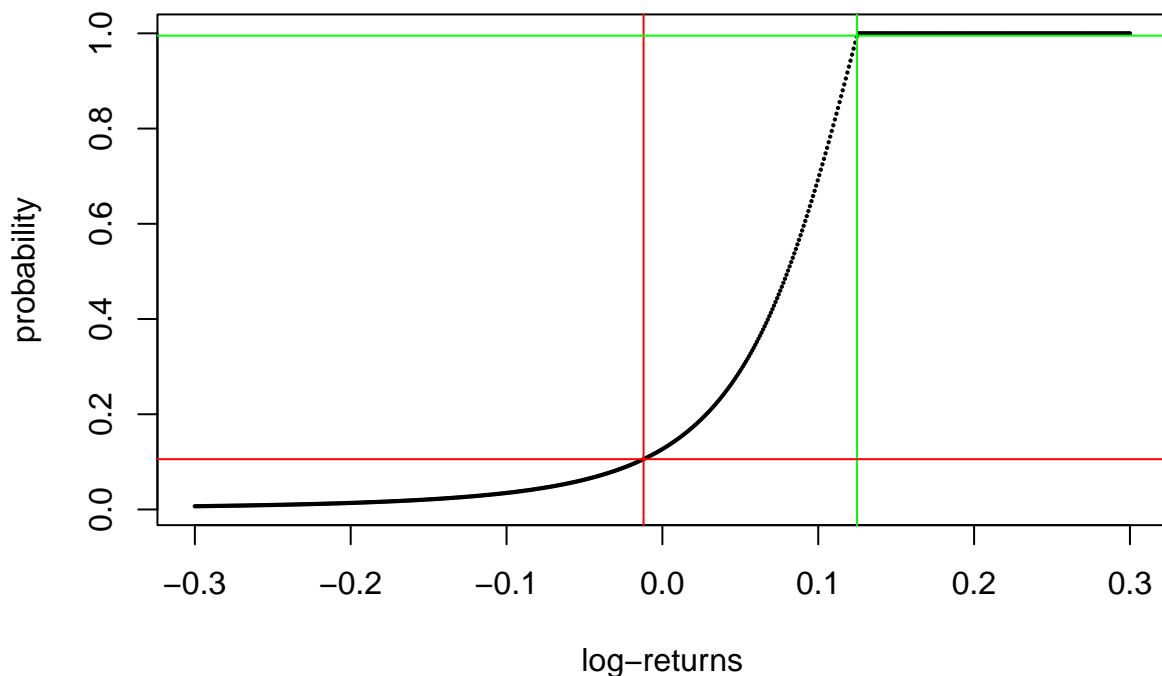


Interestingly, the fit predicts a much bigger “biggest loss” than the actual data. This is the main reason that R^2 is 0.90 and not higher.

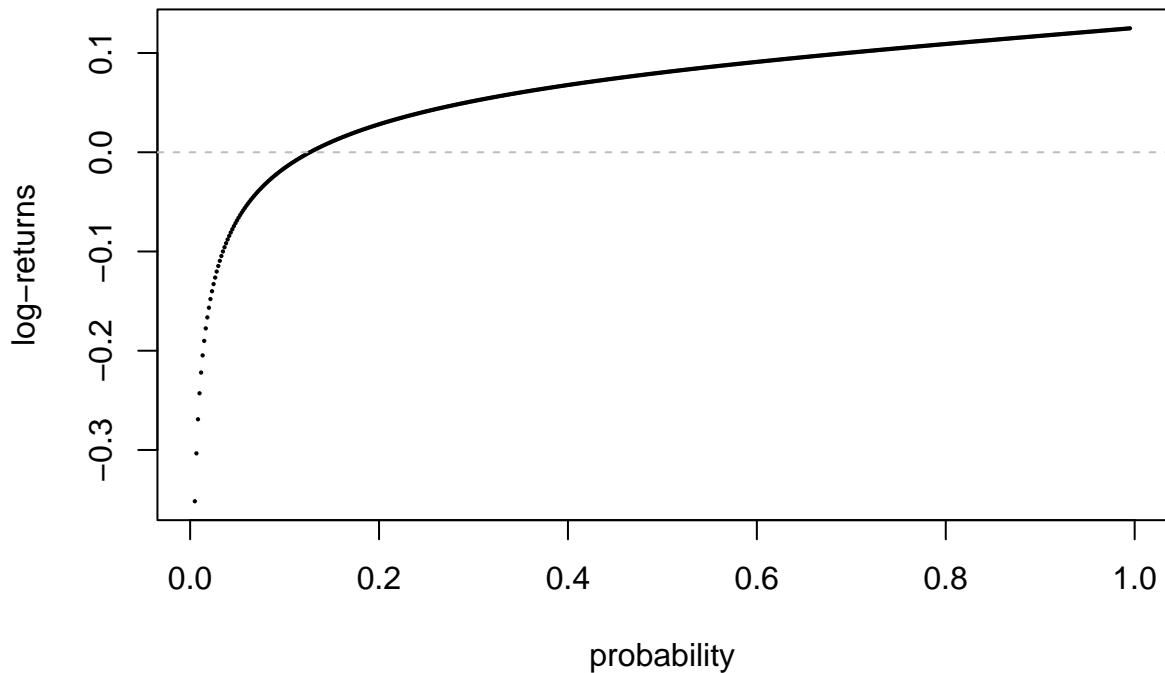
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

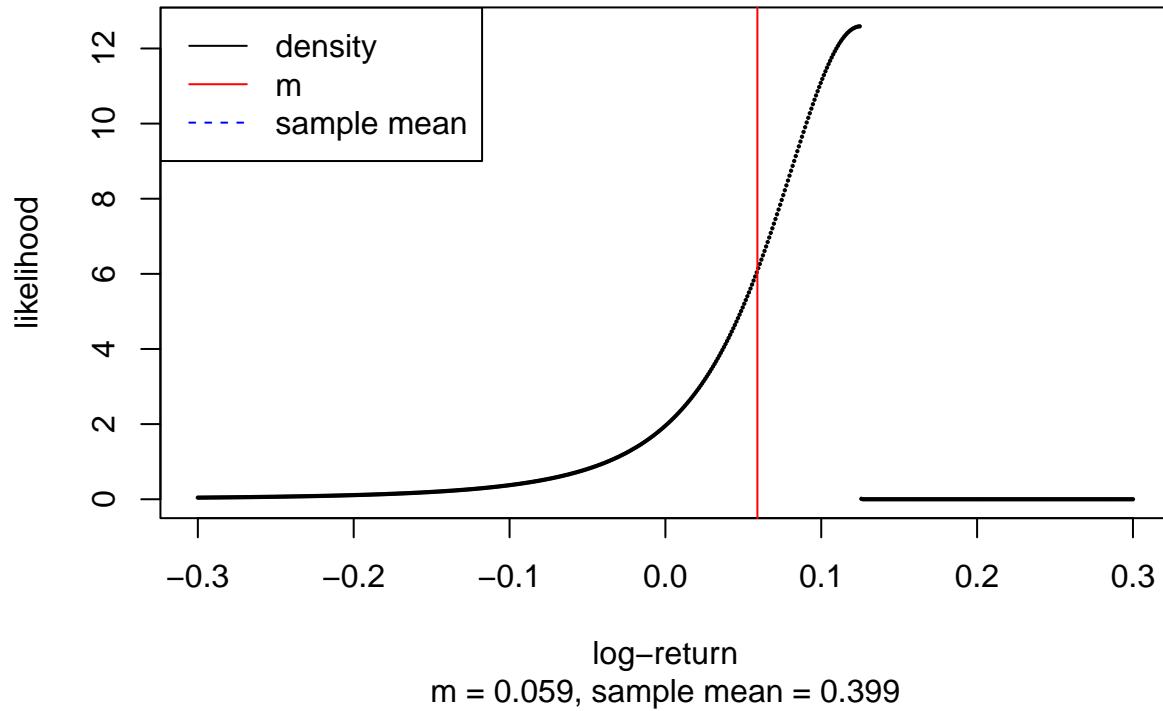


Estimated skew t distribution quantiles



We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

Estimated skew t distribution PDF

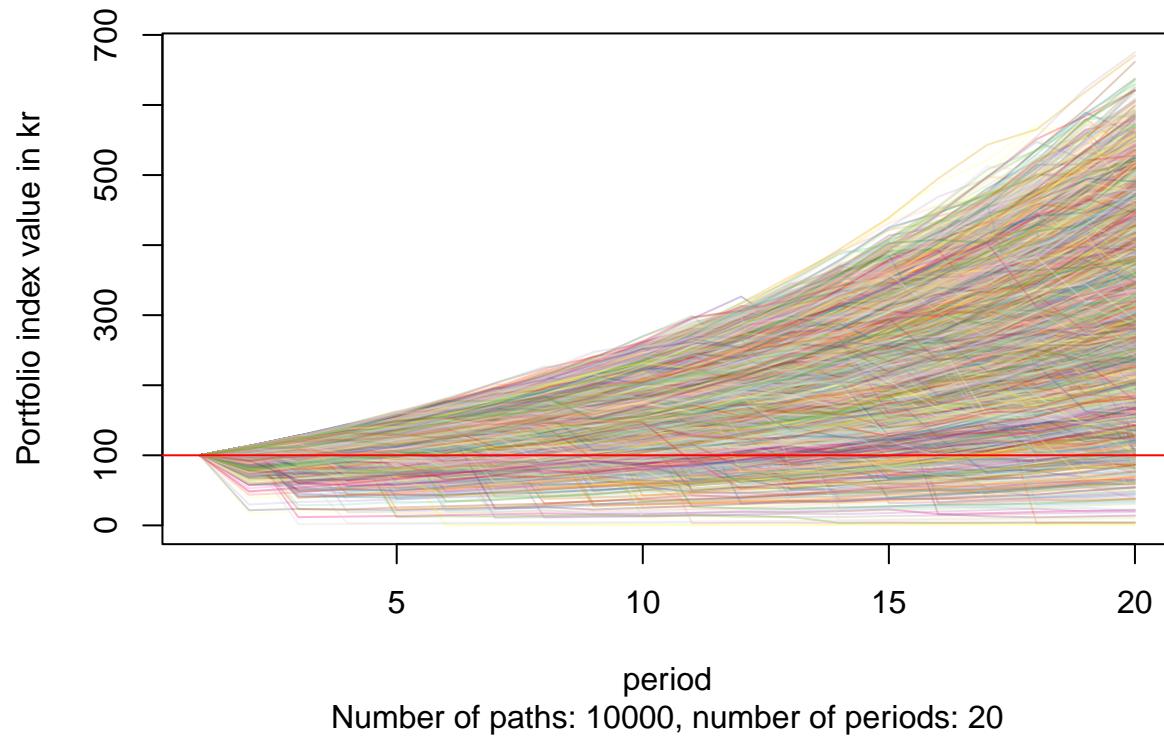


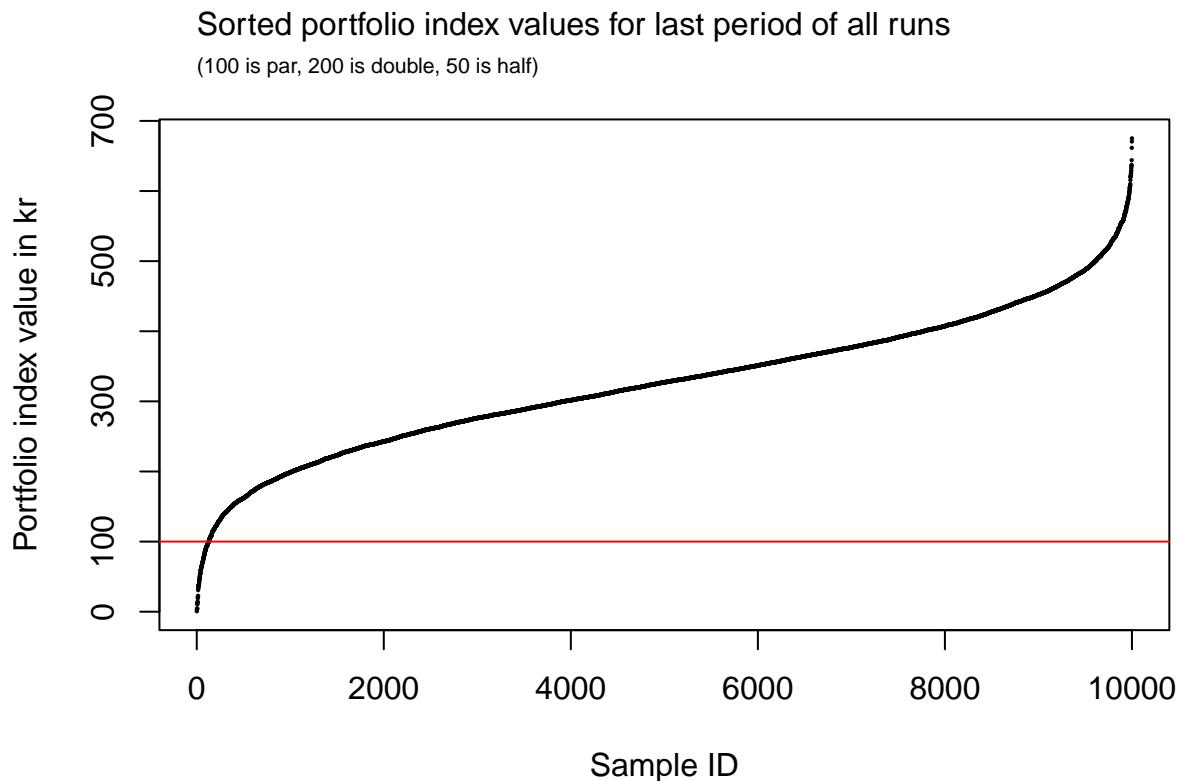
Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 325.984 kr.  
## SD of portfolio index value after 20 years: 98.788 kr.  
## Min total portfolio index value after 20 years: 0.667 kr.  
## Max total portfolio index value after 20 years: 675.13 kr.  
##  
## Share of paths finishing below 100: 1.28 percent
```

MC simulation with down-and-out





Version b: Mix of simulations from estimated distribution of returns from individual funds.

```
## Down-and-out simulation:  

## Probability of down-and-out: 0 percent  

##  

## Mean portfolio index value after 20 years: 301.707 kr.  

## SD of portfolio index value after 20 years: 85.601 kr.  

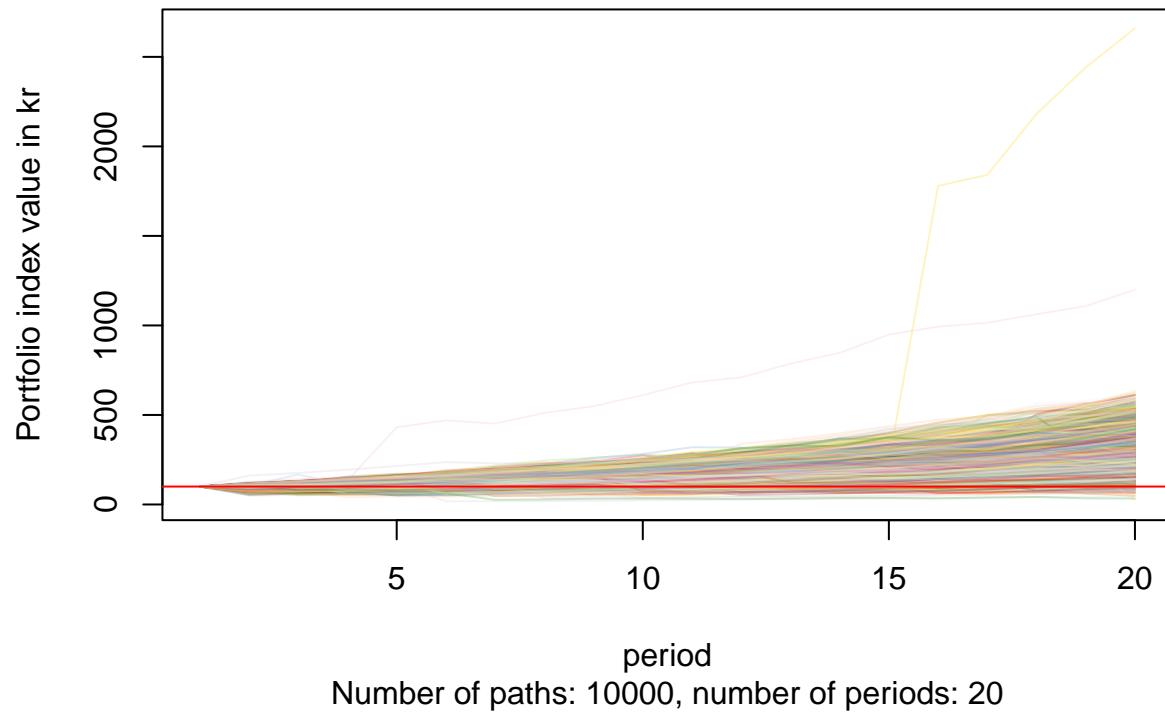
## Min total portfolio index value after 20 years: 25.402 kr.  

## Max total portfolio index value after 20 years: 2659.343 kr.  

##  

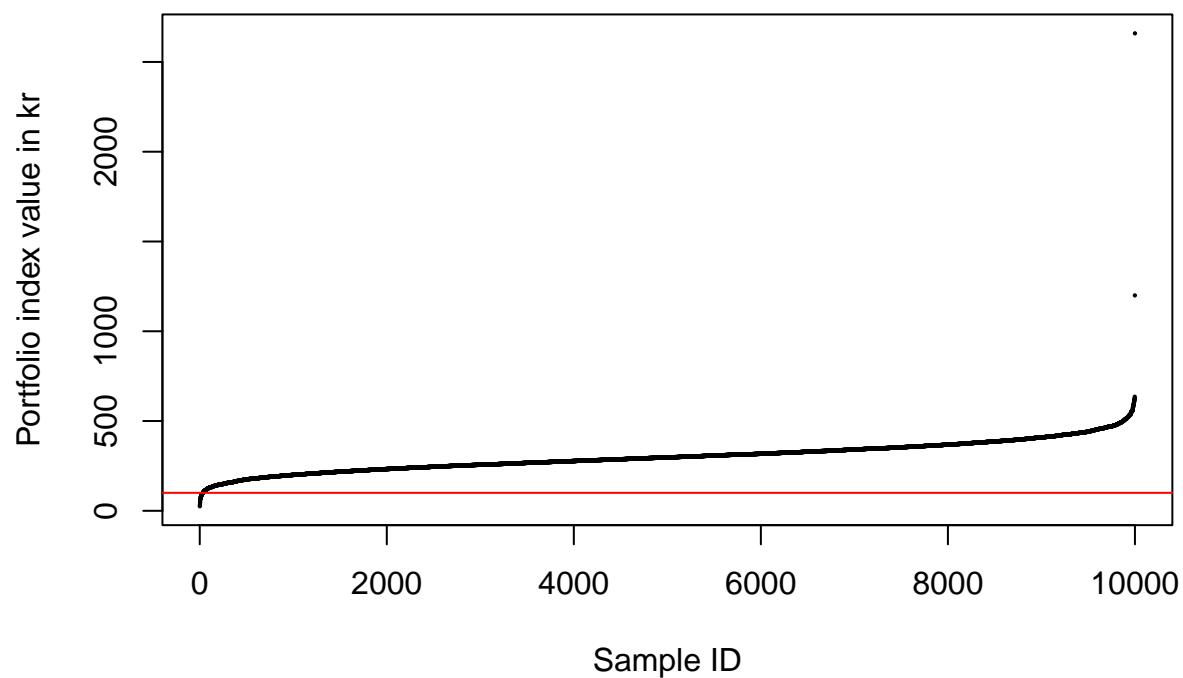
## Share of paths finishing below 100: 0.34 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

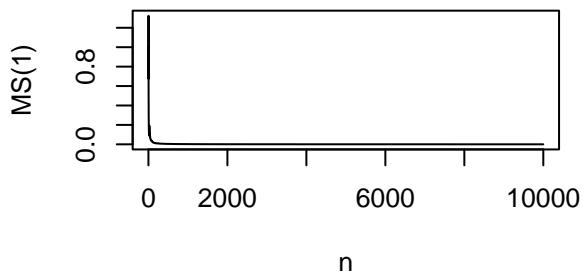
(100 is par, 200 is double, 50 is half)



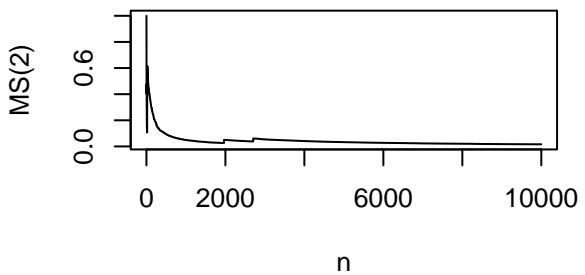
Convergence

Max vs sum plots for the first four moments:

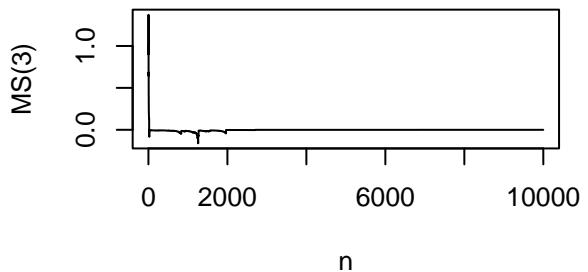
MS(1)



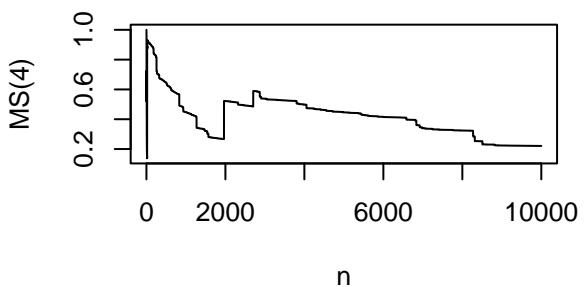
MS(2)



MS(3)

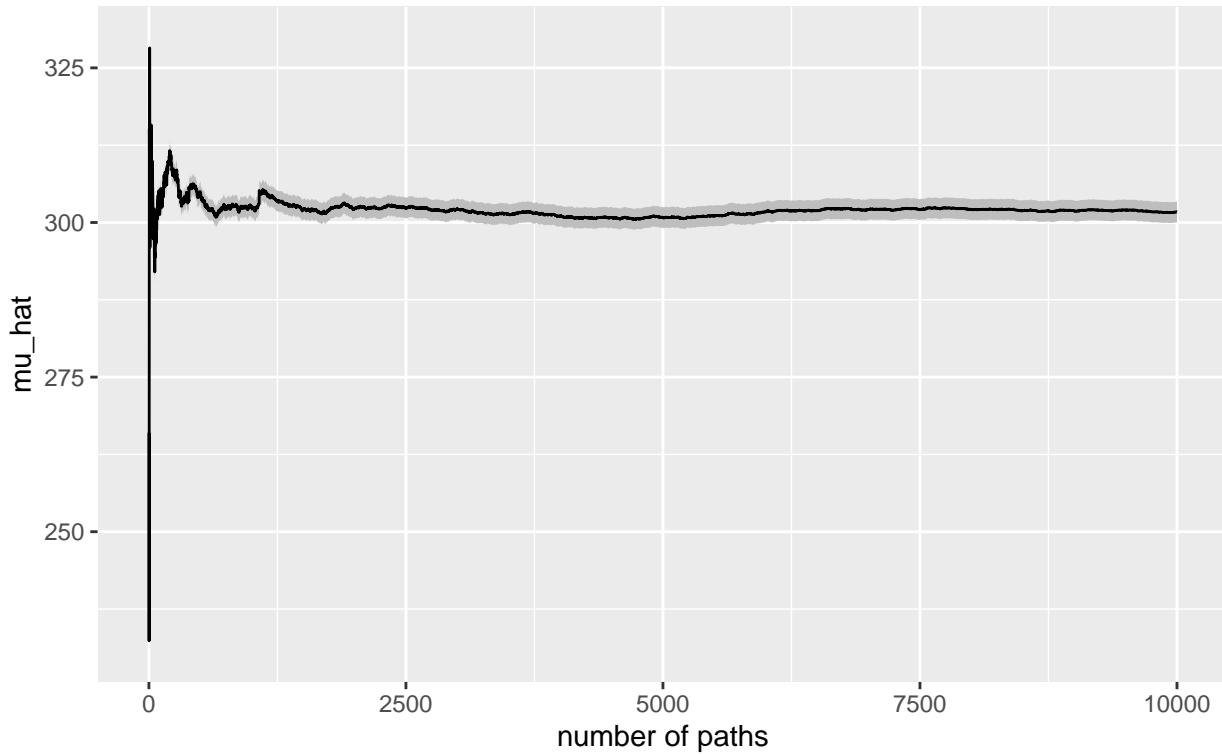


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths

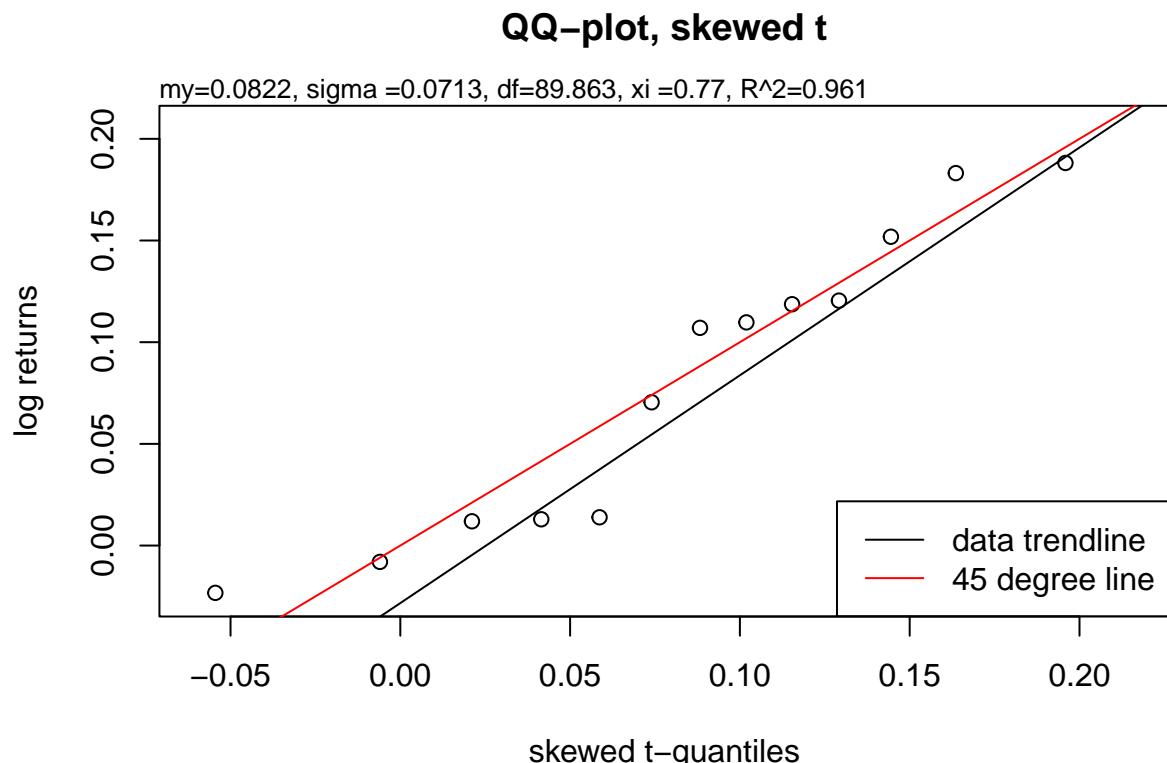


Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##  
## AIC: -24.26084  
## BIC: -22.00104  
## m: 0.0822419  
## s: 0.07129843  
## nu (df): 89.86289  
## xi: 0.7697502  
## R^2: 0.961  
##  
## An R^2 of 0.961 suggests that the fit is very good.  
##  
## What is the risk of losing max 10 %? <= 0 percent  
## What is the risk of losing max 25 %? <= 0 percent  
## What is the risk of losing max 50 %? <= 0 percent  
## What is the risk of losing max 90 %? <= 0 percent  
## What is the risk of losing max 99 %? <= 0 percent  
##  
## What is the chance of gaining min 10 %? >= 52.5 percent  
## What is the chance of gaining min 25 %? >= 45 percent  
## What is the chance of gaining min 50 %? >= 38.33333 percent  
## What is the chance of gaining min 90 %? >= 31.16667 percent  
## What is the chance of gaining min 99 %? >= 29.83333 percent
```

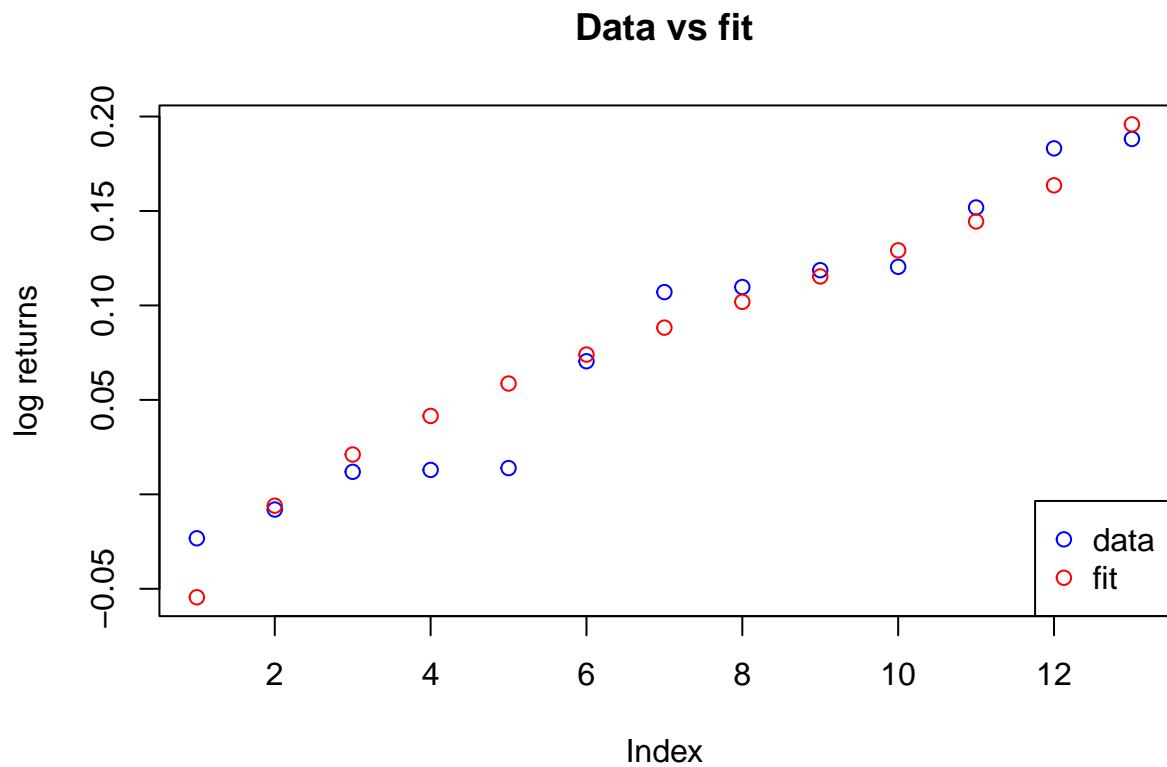
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

Data vs fit

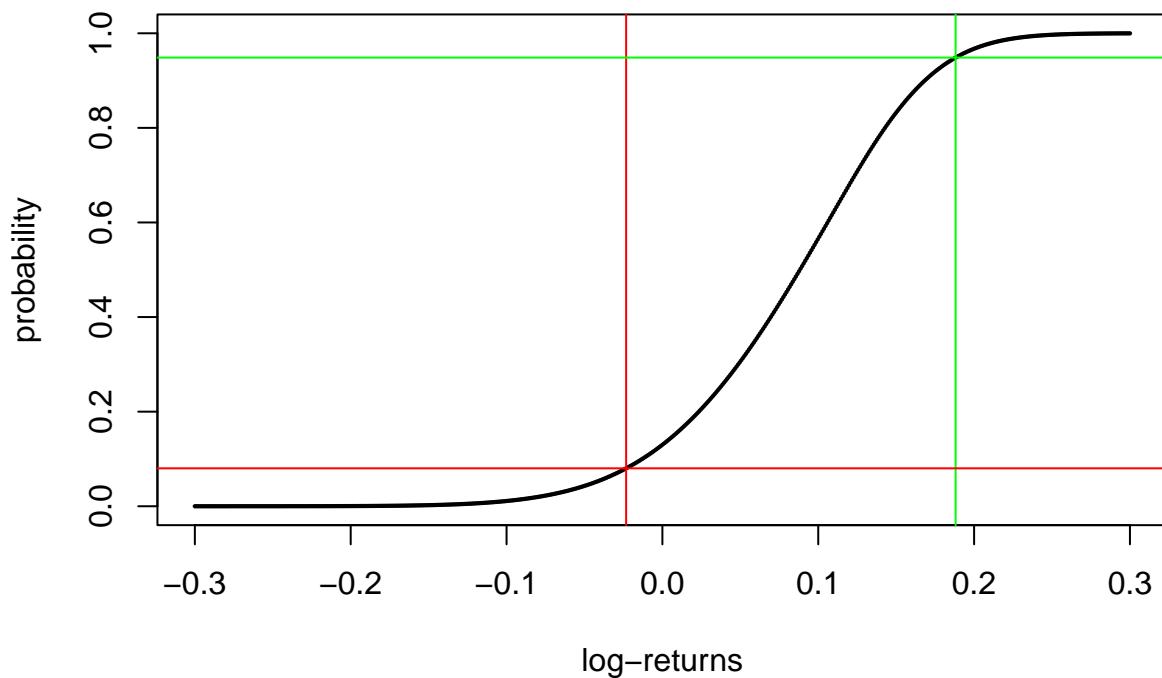
Let's plot the fit and the observed returns together.



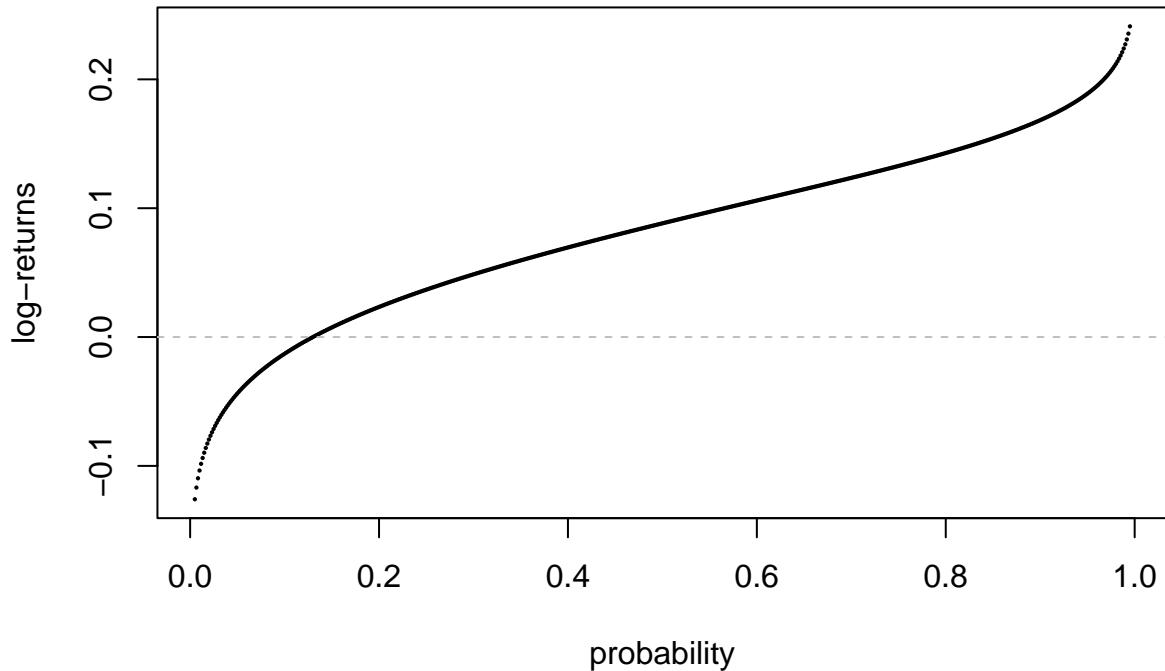
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

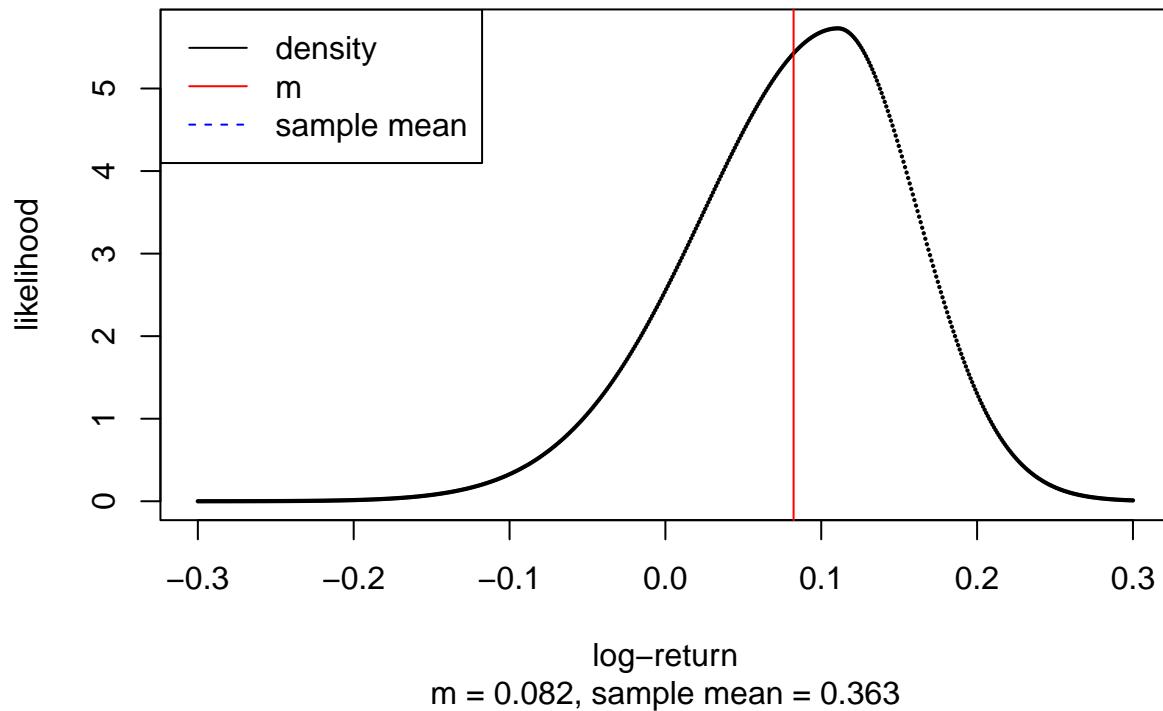


Estimated skew t distribution quantiles



We see that the high risk mix provides a much better upside and smaller downside.

Estimated skew t distribution PDF

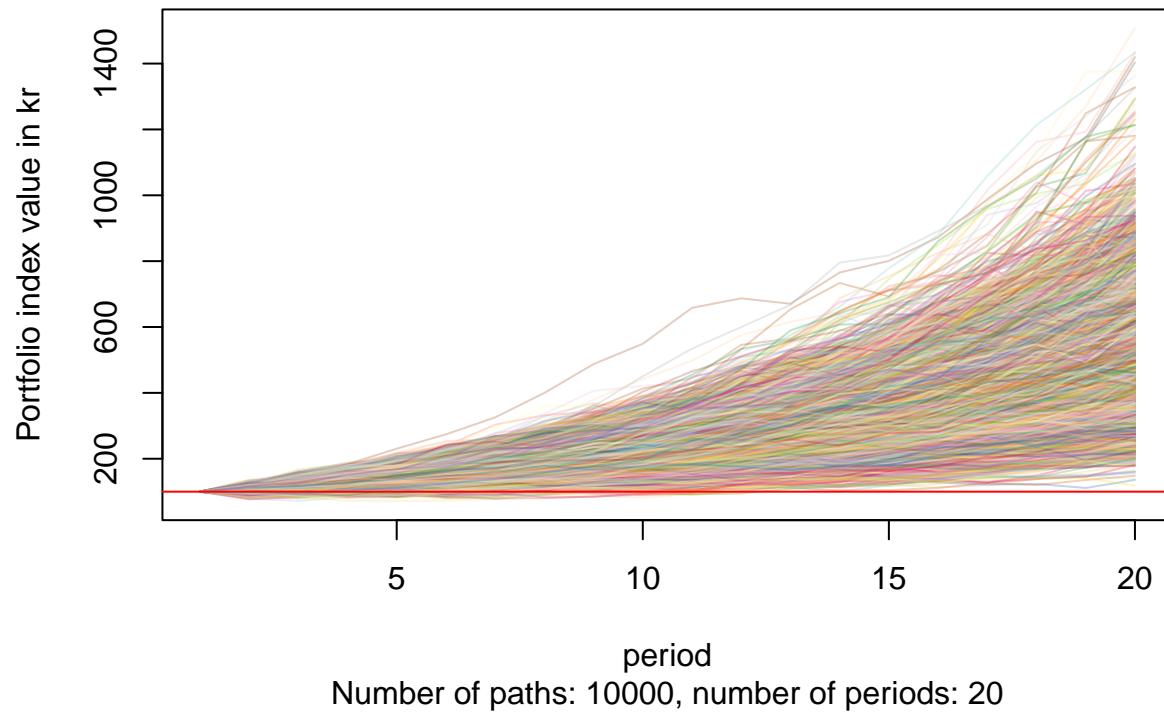


Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

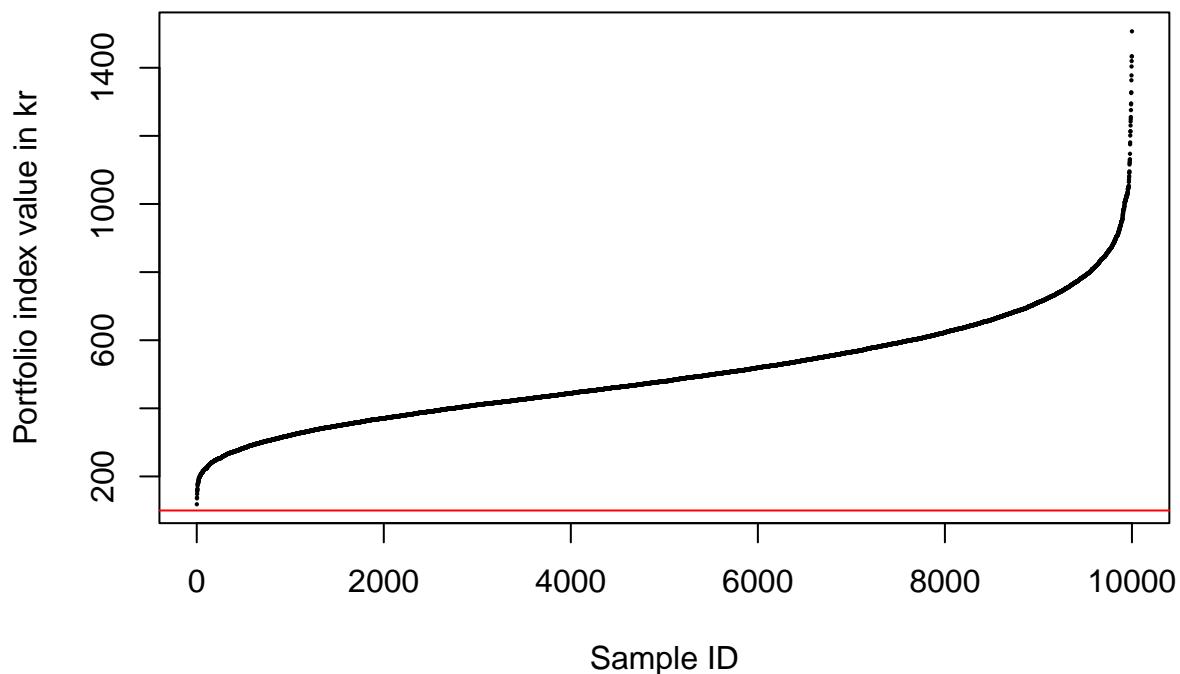
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 502.114 kr.  
## SD of portfolio index value after 20 years: 157.784 kr.  
## Min total portfolio index value after 20 years: 118.371 kr.  
## Max total portfolio index value after 20 years: 1506.946 kr.  
##  
## Share of paths finishing below 100: 0 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

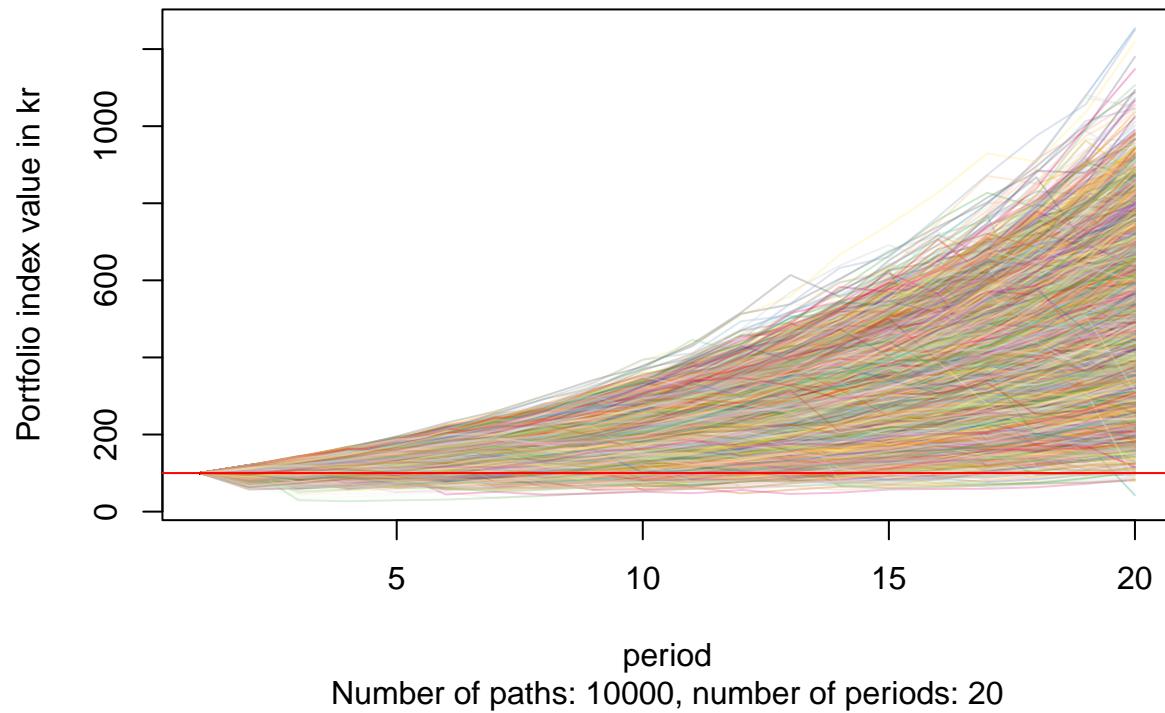
(100 is par, 200 is double, 50 is half)



Version b: Mix of simulations from estimated distribution of returns from individual funds.

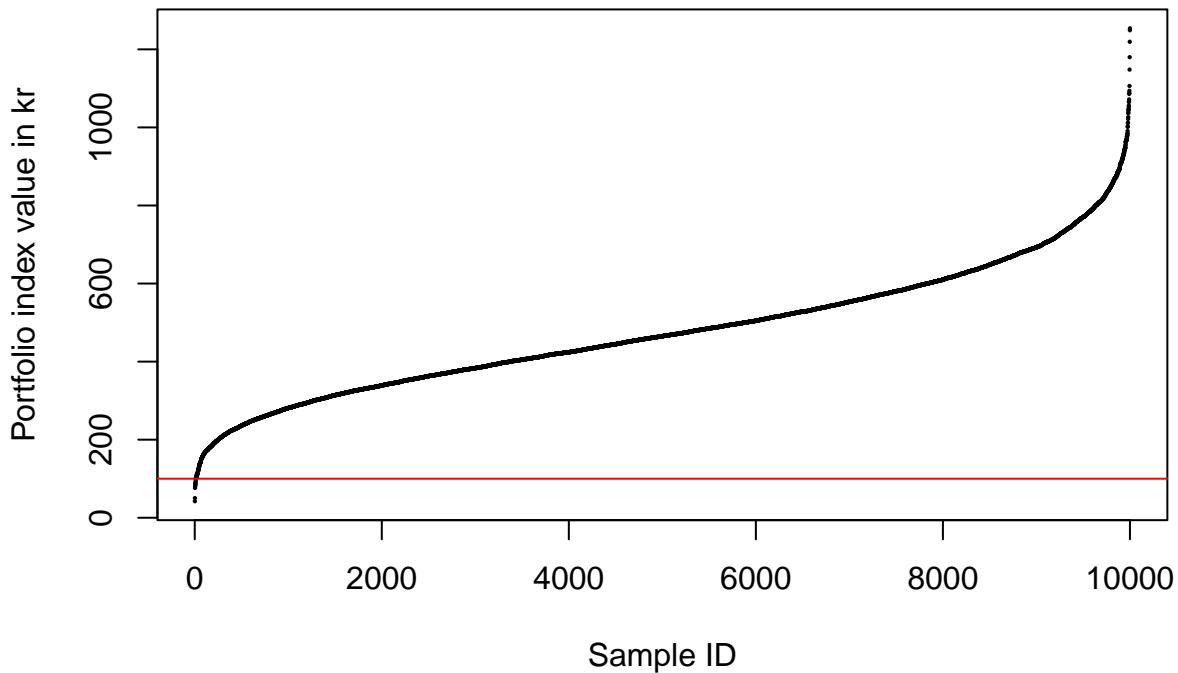
```
## Down-and-out simulation:  
## Probability of down-and-out: 0 percent  
##  
## Mean portfolio index value after 20 years: 478.402 kr.  
## SD of portfolio index value after 20 years: 162.03 kr.  
## Min total portfolio index value after 20 years: 42.309 kr.  
## Max total portfolio index value after 20 years: 1253.803 kr.  
##  
## Share of paths finishing below 100: 0.1 percent
```

MC simulation with down-and-out



Sorted portfolio index values for last period of all runs

(100 is par, 200 is double, 50 is half)



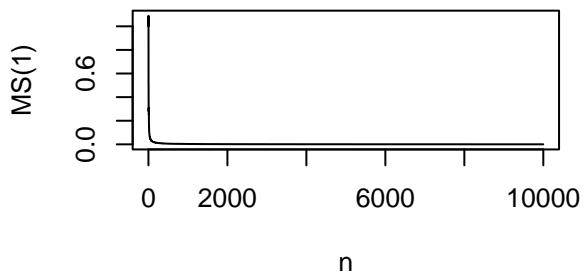
Many simulations 1e6 paths:

```
# Down-and-out simulation:  
# Probability of down-and-out: 0 percent  
#  
# Mean portfolio index value after 20 years: 478.339 kr.  
# SD of portfolio index value after 20 years: 163.093 kr.  
# Min total portfolio index value after 20 years: 2.233 kr.  
# Max total portfolio index value after 20 years: 1561.965 kr.  
#  
# Share of paths finishing below 100: 0.1181 percent
```

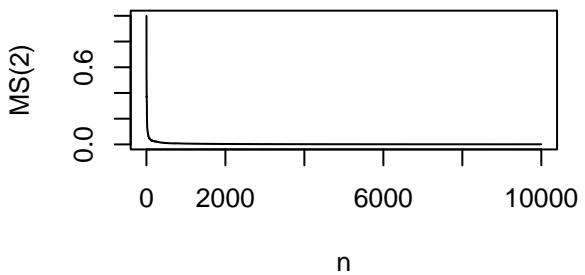
Convergence

Max vs sum plots for the first four moments:

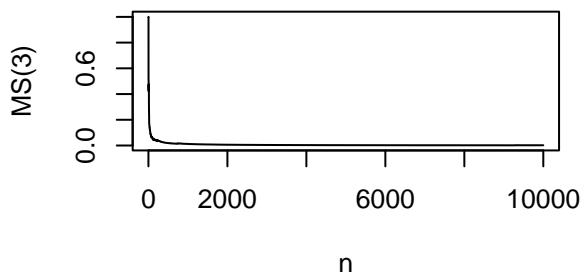
MS(1)



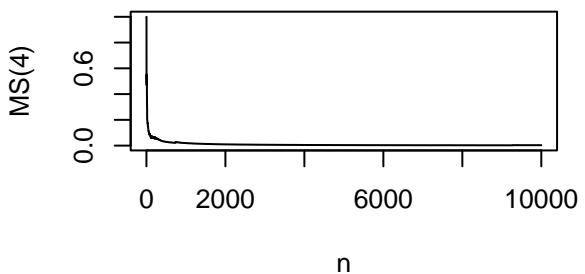
MS(2)



MS(3)

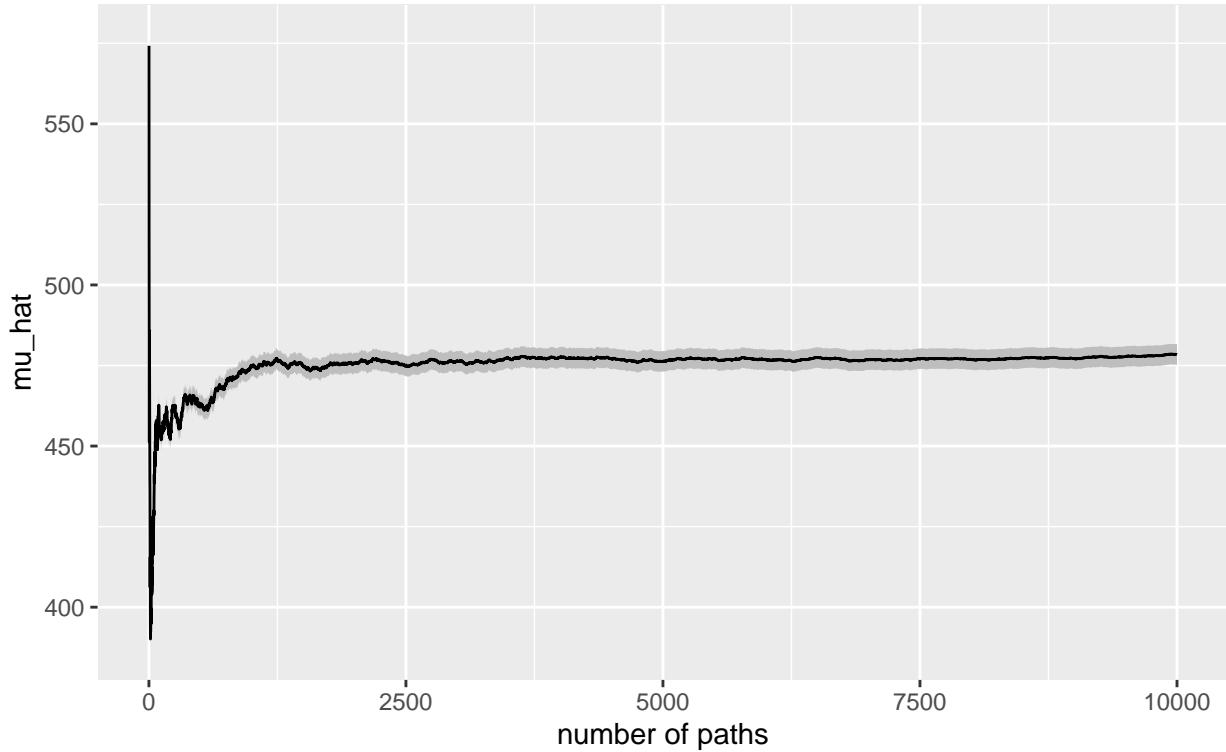


MS(4)



Monte Carlo convergence w/ 95% c.i.

20 steps, 10000 paths



Compare pension plans

Risk of max loss

Risk of max loss of x percent for a single period (year).

x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	21.167	17.833	19.667	11.833	14.000	12.333	12.667
5	12.167	9.333	12.500	5.667	8.333	5.833	3.833
10	7.000	5.000	8.000	3.000	5.000	2.833	0.500
25	1.333	0.833	2.167	0.500	1.000	0.333	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.167	Velliv_m	12.500	Velliv_h	8.000	Velliv_h	2.167	Velliv_h	0	Velliv_m	0	Velliv_m	0	Velliv_m
19.667	Velliv_h	12.167	Velliv_m	7.000	Velliv_m	1.333	Velliv_m	0	Velliv_m_l	0	Velliv_m_l	0	Velliv_m_l
17.833	Velliv_m_l	9.333	Velliv_m_l	5.000	Velliv_m_l	1.000	PFA_h	0	Velliv_h	0	Velliv_h	0	Velliv_h
14.000	PFA_h	8.333	PFA_h	5.000	PFA_h	0.833	Velliv_m_l	0	PFA_m	0	PFA_m	0	PFA_m
12.667	mix_h	5.833	mix_m	3.000	PFA_m	0.500	PFA_m	0	PFA_h	0	PFA_h	0	PFA_h
12.333	mix_m	5.667	PFA_m	2.833	mix_m	0.333	mix_m	0	mix_m	0	mix_m	0	mix_m
11.833	PFA_m	3.833	mix_h	0.500	mix_h	0.000	mix_h	0	mix_h	0	mix_h	0	mix_h

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	78.833	82.167	80.333	88.167	86.000	87.667	87.333
5	63.833	65.000	69.333	71.667	76.000	71.667	70.167
10	40.833	36.000	53.333	32.500	59.667	35.500	46.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.833
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
88.167	PFA_m	76.000	PFA_h	59.667	PFA_h	0.833	mix_h	0	Velliv_m	0	Velliv_m
87.667	mix_m	71.667	PFA_m	53.333	Velliv_h	0.000	Velliv_m	0	Velliv_m_l	0	Velliv_m_l
87.333	mix_h	71.667	mix_m	46.000	mix_h	0.000	Velliv_m_l	0	Velliv_h	0	Velliv_h
86.000	PFA_h	70.167	mix_h	40.833	Velliv_m	0.000	Velliv_h	0	PFA_m	0	PFA_m
82.167	Velliv_m_l	69.333	Velliv_h	36.000	Velliv_m_l	0.000	PFA_m	0	PFA_h	0	PFA_h
80.333	Velliv_h	65.000	Velliv_m_l	35.500	mix_m	0.000	PFA_h	0	mix_m	0	mix_m
78.833	Velliv_m	63.833	Velliv_m	32.500	PFA_m	0.000	mix_m	0	mix_h	0	mix_h

MC risk percentiles

Risk of loss from first to last period.

_a is simulation from estimated distribution of mix.

_b is mix of simulations from estimated distribution of returns from individual funds.

_m is medium.

_h is high.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	4.65	3.02	3.99	1.88	1.06	1.28	0	0.34	0.10
5	4.03	2.56	3.46	1.70	1.00	1.13	0	0.29	0.09
10	3.54	2.25	3.17	1.61	0.87	0.97	0	0.21	0.08
25	2.26	1.55	2.13	1.14	0.61	0.69	0	0.10	0.02
50	0.90	0.79	1.06	0.63	0.23	0.32	0	0.04	0.01
90	0.08	0.17	0.14	0.15	0.04	0.05	0	0.00	0.00
99	0.02	0.03	0.00	0.03	0.01	0.01	0	0.00	0.00

1e6 simulation paths of mhr_b:

	0	5	10	25	50	90	99
prob_pct	0.118	0.095	0.076	0.036	0.008	0	0

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.65	Velliv_m	4.03	Velliv_m	3.54	Velliv_m	2.26	Velliv_m	1.06	Velliv_h	0.17	Velliv_m_l	0.03	Velliv_m_l
3.99	Velliv_h	3.46	Velliv_h	3.17	Velliv_h	2.13	Velliv_h	0.90	Velliv_m	0.15	PFA_m	0.03	PFA_m
3.02	Velliv_m_l	2.56	Velliv_m_l	2.25	Velliv_m_l	1.55	Velliv_m_l	0.79	Velliv_m_l	0.14	Velliv_h	0.02	Velliv_m
1.88	PFA_m	1.70	PFA_m	1.61	PFA_m	1.14	PFA_m	0.63	PFA_m	0.08	Velliv_m	0.01	PFA_h
1.28	mix_m_a	1.13	mix_m_a	0.97	mix_m_a	0.69	mix_m_a	0.32	mix_m_a	0.05	mix_m_a	0.01	mix_m_a
1.06	PFA_h	1.00	PFA_h	0.87	PFA_h	0.61	PFA_h	0.23	PFA_h	0.04	PFA_h	0.00	Velliv_h
0.34	mix_m_b	0.29	mix_m_b	0.21	mix_m_b	0.10	mix_m_b	0.04	mix_m_b	0.00	mix_h_a	0.00	mix_h_a

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.10	mix_h_b	0.09	mix_h_b	0.08	mix_h_b	0.02	mix_h_b	0.01	mix_h_b	0.00	mix_m_b	0.00	mix_m_b
0.00	mix_h_a	0.00	mix_h_b	0.00	mix_h_b								

MC gains percentiles

Chance of gains from first to last period.

_a is simulation from estimated distribution of returns of mix.

_b is mix of simulations from estimated distribution of returns from individual funds.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	95.35	96.98	96.01	98.12	98.94	98.72	100.00	99.66	99.90
5	94.66	96.48	95.61	97.91	98.83	98.61	100.00	99.62	99.82
10	94.02	96.07	95.05	97.70	98.65	98.41	100.00	99.55	99.78
25	91.57	94.31	93.63	97.00	98.24	97.82	99.99	99.12	99.59
50	86.19	90.21	90.47	95.03	97.43	96.27	99.96	97.46	99.31
100	71.45	78.02	82.90	88.41	94.77	89.77	99.67	89.99	97.51
200	38.94	43.79	64.46	58.95	85.51	60.76	93.07	48.68	87.27
300	15.88	16.91	45.49	21.85	71.27	22.26	72.59	11.73	66.30
400	4.76	5.18	29.46	4.22	54.52	3.95	44.72	1.24	41.21
500	1.34	1.19	17.71	0.57	38.20	0.27	23.64	0.12	21.74
1000	0.00	0.01	0.65	0.01	2.24	0.00	0.27	0.02	0.06

1e6 simulation paths of mhr_b:

	0	5	10	25	50	100	200	300	400	500	1000
prob	99.882	99.854	99.824	99.686	99.301	97.513	86.912	65.992	41.486	21.693	0.086

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	99.99	mix_h_a	99.96	mix_h_a	99.67	mix_h_a
99.90	mix_h_b	99.82	mix_h_b	99.78	mix_h_b	99.59	mix_h_b	99.31	mix_h_b	97.51	mix_h_b
99.66	mix_m_b	99.62	mix_m_b	99.55	mix_m_b	99.12	mix_m_b	97.46	mix_m_b	94.77	PFA_h
98.94	PFA_h	98.83	PFA_h	98.65	PFA_h	98.24	PFA_h	97.43	PFA_h	89.99	mix_m_b
98.72	mix_m_a	98.61	mix_m_a	98.41	mix_m_a	97.82	mix_m_a	96.27	mix_m_a	89.77	mix_m_a
98.12	PFA_m	97.91	PFA_m	97.70	PFA_m	97.00	PFA_m	95.03	PFA_m	88.41	PFA_m
96.98	Velliv_m_l	96.48	Velliv_m_l	96.07	Velliv_m_l	94.31	Velliv_m_l	90.47	Velliv_h	82.90	Velliv_h
96.01	Velliv_h	95.61	Velliv_h	95.05	Velliv_h	93.63	Velliv_h	90.21	Velliv_m_l	78.02	Velliv_m_l
95.35	Velliv_m	94.66	Velliv_m	94.02	Velliv_m	91.57	Velliv_m	86.19	Velliv_m	71.45	Velliv_m

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
93.07	mix_h_a	72.59	mix_h_a	54.52	PFA_h	38.20	PFA_h	2.24	PFA_h
87.27	mix_h_b	71.27	PFA_h	44.72	mix_h_a	23.64	mix_h_a	0.65	Velliv_h
85.51	PFA_h	66.30	mix_h_b	41.21	mix_h_b	21.74	mix_h_b	0.27	mix_h_a
64.46	Velliv_h	45.49	Velliv_h	29.46	Velliv_h	17.71	Velliv_h	0.06	mix_h_b
60.76	mix_m_a	22.26	mix_m_a	5.18	Velliv_m_l	1.34	Velliv_m	0.02	mix_m_b
58.95	PFA_m	21.85	PFA_m	4.76	Velliv_m	1.19	Velliv_m_l	0.01	Velliv_m_l
48.68	mix_m_b	16.91	Velliv_m_l	4.22	PFA_m	0.57	PFA_m	0.01	PFA_m
43.79	Velliv_m_l	15.88	Velliv_m	3.95	mix_m_a	0.27	mix_m_a	0.00	Velliv_m
38.94	Velliv_m	11.73	mix_m_b	1.24	mix_m_b	0.12	mix_m_b	0.00	mix_m_a

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.

m is the location parameter.

s is the scale parameter.

nu is the estimated degrees of freedom, or shape parameter.

xi is the estimated skewness parameter.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770
R-squared	0.993	0.978	0.991	0.991	0.964	0.890	0.961

Fit statistics ranking

m	ranking	s	ranking	R-squared	ranking
0.084	PFA_high	0.071	mix_high	0.993	Velliv_medium
0.082	mix_high	0.088	mix_medium	0.991	Velliv_high
0.065	Velliv_high	0.115	Velliv_medium_long	0.991	PFA_medium
0.059	mix_medium	0.120	Velliv_medium	0.978	Velliv_medium_long
0.058	PFA_medium	0.121	PFA_high	0.964	PFA_high
0.052	Velliv_medium_long	0.123	PFA_medium	0.961	mix_high
0.048	Velliv_medium	0.150	Velliv_high	0.890	mix_medium

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).

Statistics are given for the final state of all paths.

Probability of down_and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.

0 is defined as any value below a threshold.

losing_prob_pct is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_m_b	mix_h_a	mix_h_b
mc_m	277.650	292.628	406.903	322.484	550.414	325.984	301.707	502.114	478.402
mc_s	122.041	118.206	218.183	107.876	241.439	98.788	85.601	157.784	162.030
mc_min	0.086	0.000	1.606	0.000	0.298	0.667	25.402	118.371	42.309
mc_max	916.564	1381.145	1569.591	3102.847	2009.894	675.130	2659.343	1506.946	1253.803
dao_pct	0.000	0.010	0.000	0.010	0.000	0.000	0.000	0.000	0.000
losing_pct	4.650	3.020	3.990	1.880	1.060	1.280	0.340	0.000	0.100

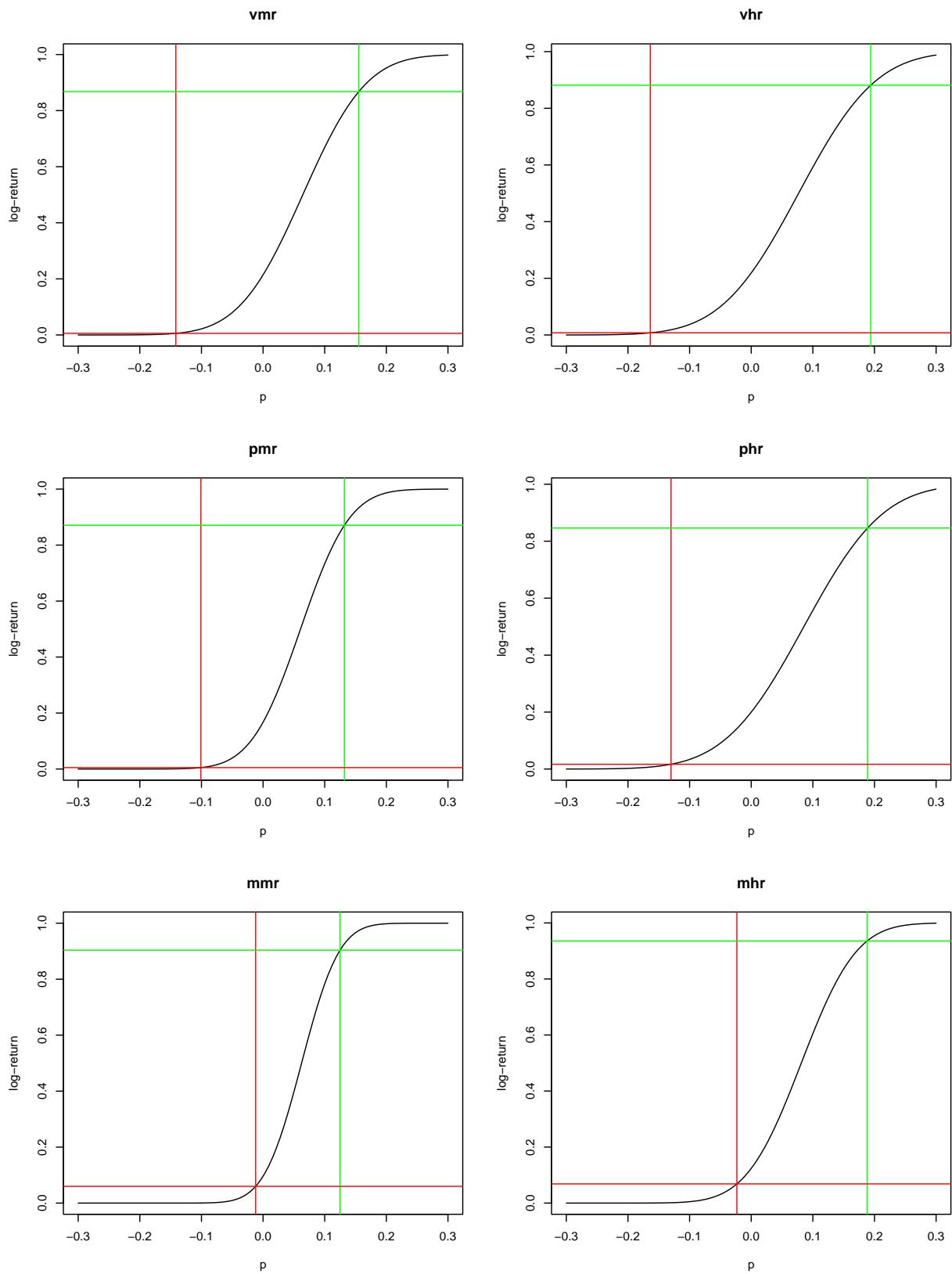
Ranking

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	losing_pct	ranking
550.414	PFA_h	85.601	mix_m_b	118.371	mix_h_a	3102.847	PFA_m	0.00	Velliv_m	0.00	mix_h_a
502.114	mix_h_a	98.788	mix_m_a	42.309	mix_h_b	2659.343	mix_m_b	0.00	Velliv_h	0.10	mix_h_b
478.402	mix_h_b	107.876	PFA_m	25.402	mix_m_b	2009.894	PFA_h	0.00	PFA_h	0.34	mix_m_b
406.903	Velliv_h	118.206	Velliv_m_l	1.606	Velliv_h	1569.591	Velliv_h	0.00	mix_m_a	1.06	PFA_h
325.984	mix_m_a	122.041	Velliv_m	0.667	mix_m_a	1506.946	mix_h_a	0.00	mix_m_b	1.28	mix_m_a
322.484	PFA_m	157.784	mix_h_a	0.298	PFA_h	1381.145	Velliv_m_l	0.00	mix_h_a	1.88	PFA_m
301.707	mix_m_b	162.030	mix_h_b	0.086	Velliv_m	1253.803	mix_h_b	0.00	mix_h_b	3.02	Velliv_m_l
292.628	Velliv_m_l	218.183	Velliv_h	0.000	Velliv_m_l	916.564	Velliv_m	0.01	Velliv_m_l	3.99	Velliv_h
277.650	Velliv_m	241.439	PFA_h	0.000	PFA_m	675.130	mix_m_a	0.01	PFA_m	4.65	Velliv_m

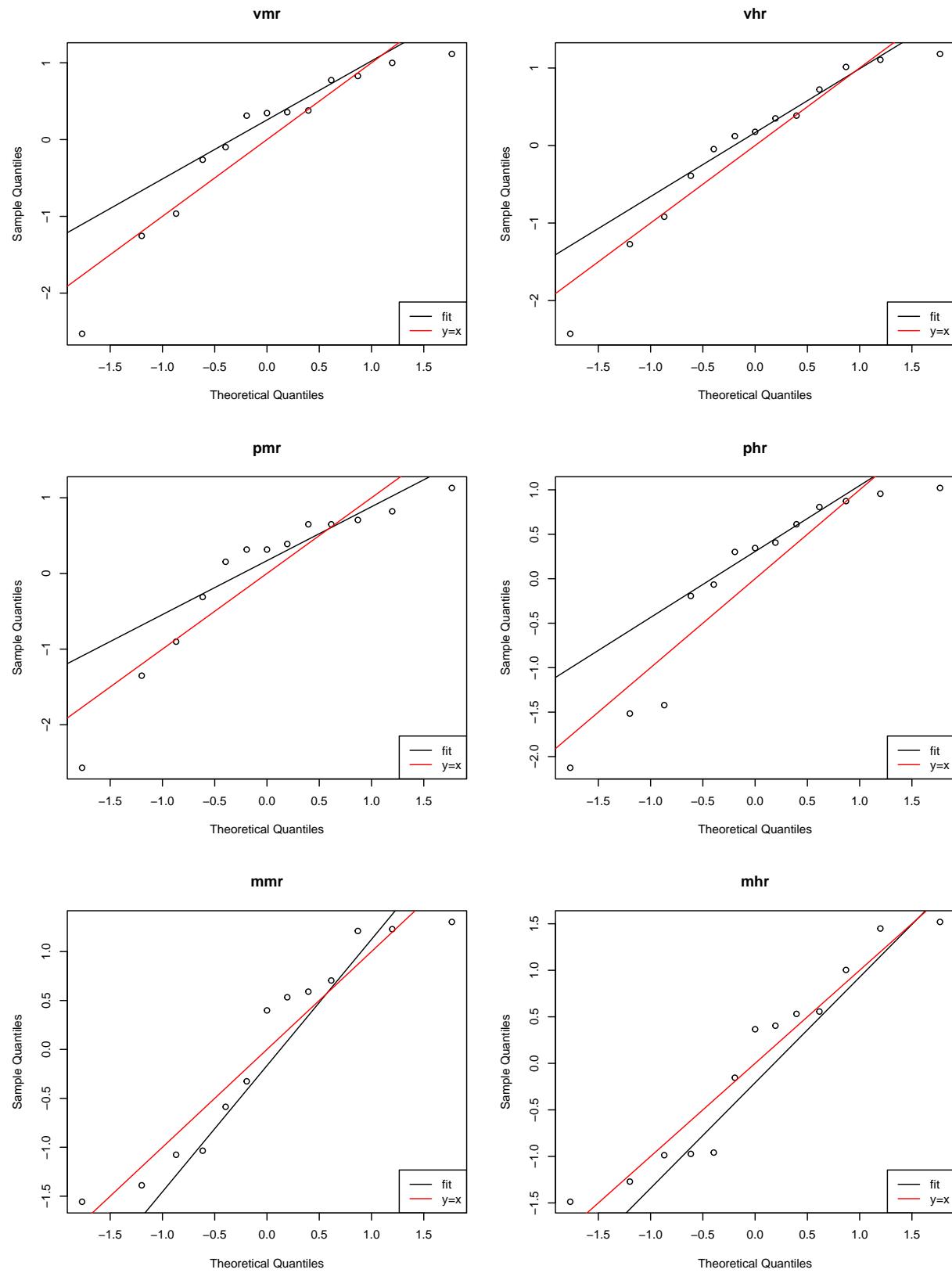
Compare Gaussian and skewed t-distribution fits

Gaussian fits

	vmr	vhr	pmr	phr	mmr	mhr
m	0.064	0.077	0.061	0.085	0.062	0.081
s	0.081	0.099	0.063	0.101	0.048	0.070



Gaussian QQ plots



Gaussian vs skewed t

Probability in percent that the smallest and largest (respectively) observed return for each fund was generated by a normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr
P_norm(X_min)	0.571	0.758	0.511	1.676	5.971	6.842
P_norm(X_max)	13.230	11.876	12.922	15.359	9.628	6.429
P_t(X_min)	5.377	5.457	3.489	4.315	10.570	8.015
P_t(X_max)	0.118	0.001	2.825	0.188	0.488	5.141

Average number of years between min or max events (respectively):

	vmr	vhr	pmr	phr	mmr	mhr
norm: avg yrs btw min	175.248	131.911	195.568	59.669	16.748	14.616
norm: avg yrs btw max	7.559	8.420	7.739	6.511	10.386	15.556
t: avg yrs btw min	18.596	18.324	28.663	23.173	9.461	12.476
t: avg yrs btw max	848.548	178349.076	35.400	531.552	205.104	19.450

Comments

(Ignoring mhr_a...)

mhr has some nice properties:

- It has a relatively high nu value of 90, which means it is tending more towards exponential tails than polynomial tails. All other funds have nu values close to 3, except phr which is even worse at close to 2. (Note that for a Gaussian, nu is infinite.)
- It has the lowest losing percentage of all simulations, which is better than 1/6 that of phr.
- It has a DAO percentage of 0, which is the same as mmr, and less than phr.
- Only phr has a higher mc_m.
- It has a smaller mc_s than the individual components, vhr and phr.
- It has the highest xi of all fits, suggesting less left skewness. Density plots for vmr, phr and mmr have an extremely sharp drop, as if an upward limiter has been applied, which corresponds to extremely low xi values. The density plot for mhr is by far the most symmetrical of all the fits. As seen in the section “Compare Gaussian and skewed t-distribution fits”, the other skewed t-distribution fits don’t capture the max observed returns at all.
- Only mmr has a higher mc_min. However, that of mmr is 18 times higher with 62, so mmr is a clear winner here.
- Naturally, it has a mc_max smaller than the individual components, vhr and phr, but ca. 1.5 times higher than mmr.
- All the first 4 moments converge nicely. For all other fits, the 4th moment doesn’t seem to converge.

Taleb, Statistical Consequences Of Fat Tails, p. 97:

“the variance of a finite variance random variable with tail exponent < 4 will be infinite”.

And p. 363:

“The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, under a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance.”

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} \right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2} \right) / \left(\frac{x_{t-1} + y_{t-1}}{2} \right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns = Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} \right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2 \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1} y_t = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

$$(x_{t-1} x_1 y_{t-1} + y_{t-1} x_t y_{t-1}) + (x_{t-1} x_{t-1} y_t + x_{t-1} y_{t-1} y_t) = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: $R = 1+r$

```
## Let x_0 be 100.

## Let y_0 be 200.

## So the initial value of the pf is 300 .

## Let R_x be 0.5.

## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.

## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): -0.02204096
## s(data_x): 0.354636
## m(data_y): 10.15401
## s(data_y): 3.524218
##
## m(data_x + data_y): 5.065983
## s(data_x + data_y): 1.80421
```

m and s of final state of all paths.

_a is mix of simulated returns.

_b is simulated mixed returns.

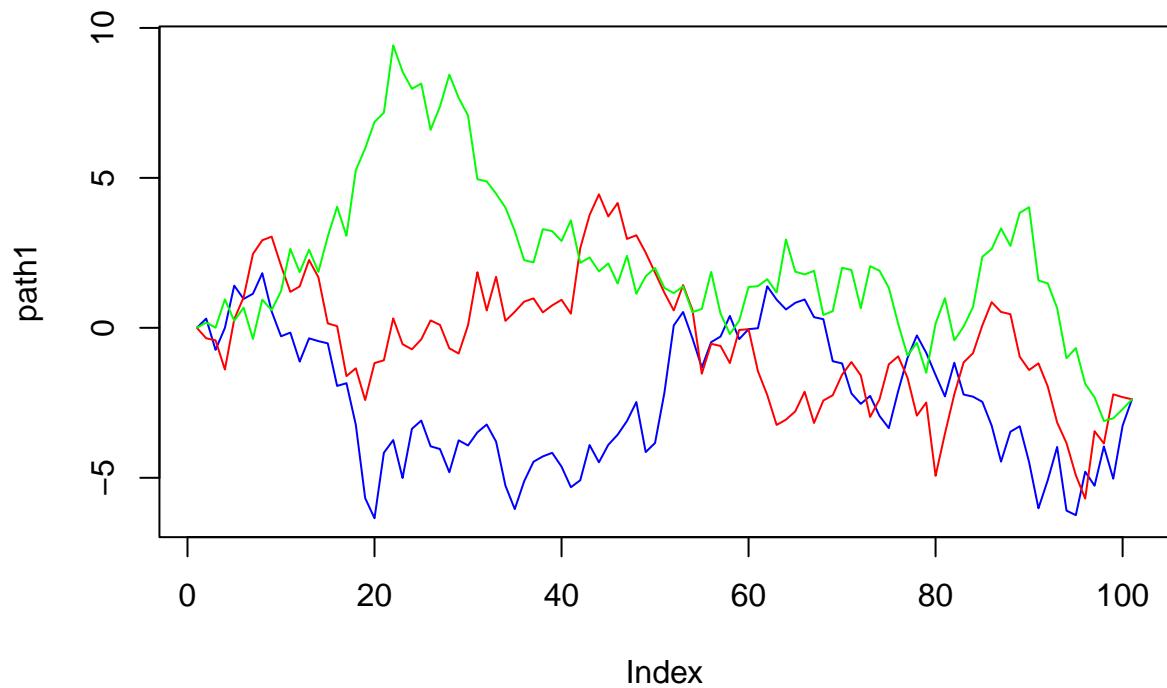
	m_a	m_b	s_a	s_b
101.190	101.214	7.096	8.063	
101.172	101.173	7.569	8.128	
101.290	101.154	7.830	7.687	
101.612	101.125	7.977	8.074	
101.177	101.694	7.973	7.946	
101.796	101.355	7.891	8.118	
101.471	101.033	7.741	7.968	
101.135	101.302	8.117	8.077	
100.830	101.028	7.870	8.528	
101.626	101.118	7.984	8.020	

```
##      m_a          m_b          s_a          s_b
## Min. :100.8  Min. :101.0  Min. :7.096  Min. :7.687
## 1st Qu.:101.2  1st Qu.:101.1  1st Qu.:7.763  1st Qu.:7.981
## Median :101.2  Median :101.2  Median :7.881  Median :8.069
## Mean   :101.3  Mean   :101.2  Mean   :7.805  Mean   :8.061
## 3rd Qu.:101.6  3rd Qu.:101.3  3rd Qu.:7.976  3rd Qu.:8.108
## Max.  :101.8  Max.  :101.7  Max.  :8.117  Max.  :8.528
```

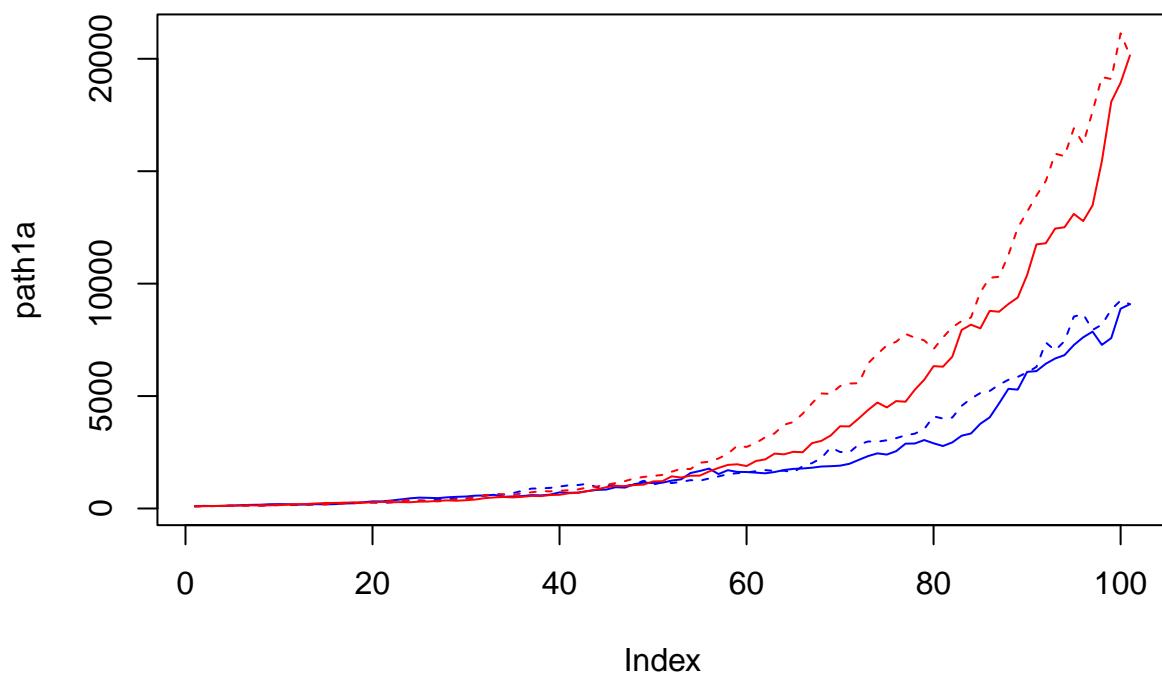
_a and _b are very close to equal.

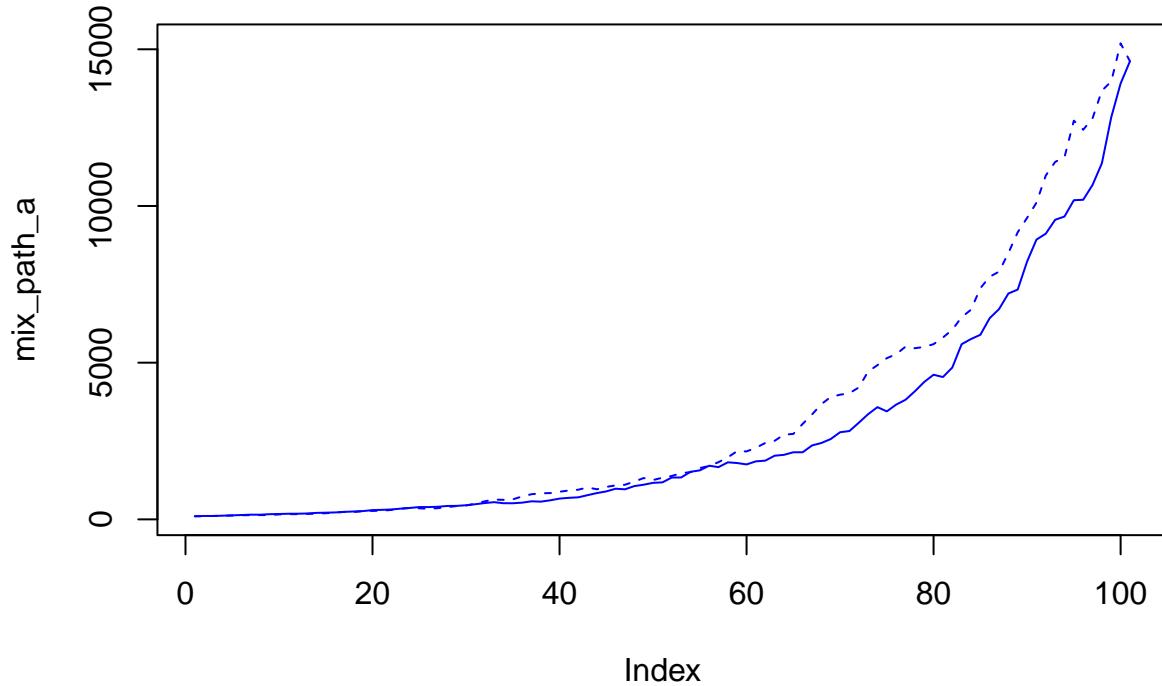
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?





The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?

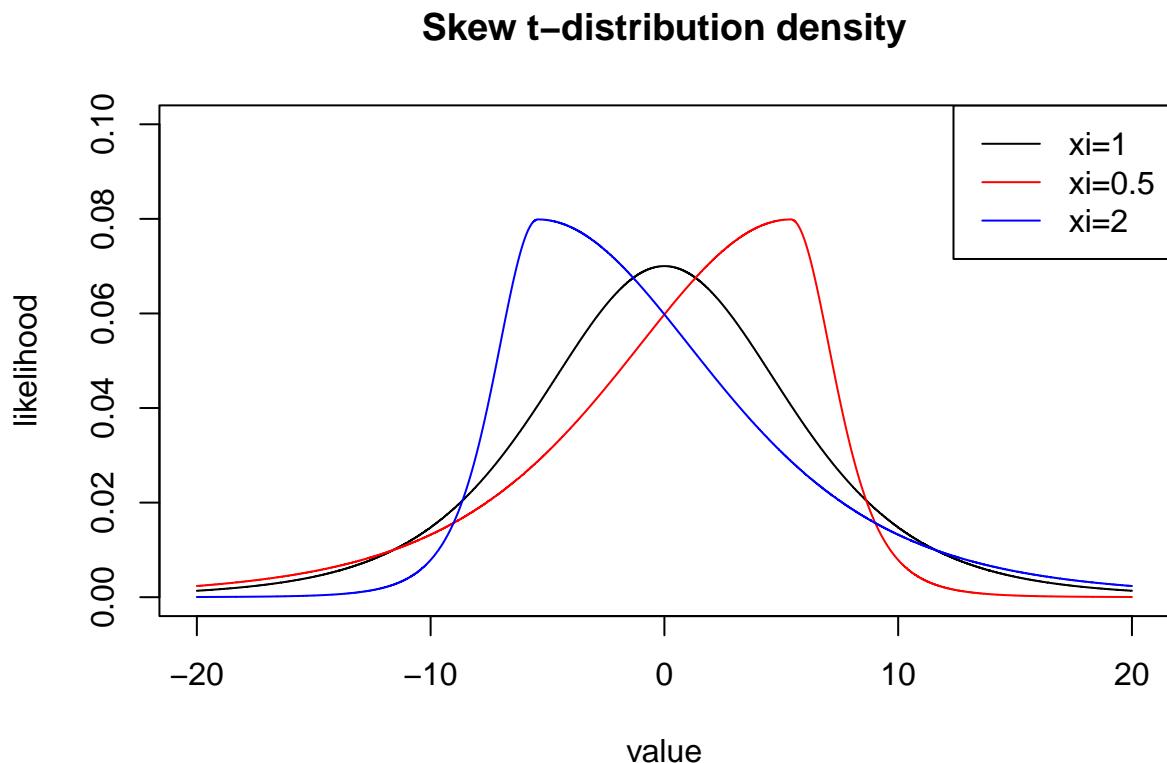
Let's try to include the Velliv data from 2007-2010.

We do this by sampling 13 observations from `vmrl`.

```
##          m            s
##  Min. :0.05965  Min.  :0.04459
##  1st Qu.:0.06545  1st Qu.:0.05941
##  Median :0.06898  Median :0.06651
##  Mean   :0.07028  Mean   :0.06727
##  3rd Qu.:0.07459  3rd Qu.:0.07112
##  Max.   :0.08492  Max.   :0.08828
```

The meaning of ξ_i

The fit for `mhr` has the highest ξ_i value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, \dots, X_n^p)}{\sum_{i=1}^n X_i^p} \rightarrow 0$$

for $n \rightarrow \infty$.

If not, X doesn't have a p 'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192