Annual pension returns analysis

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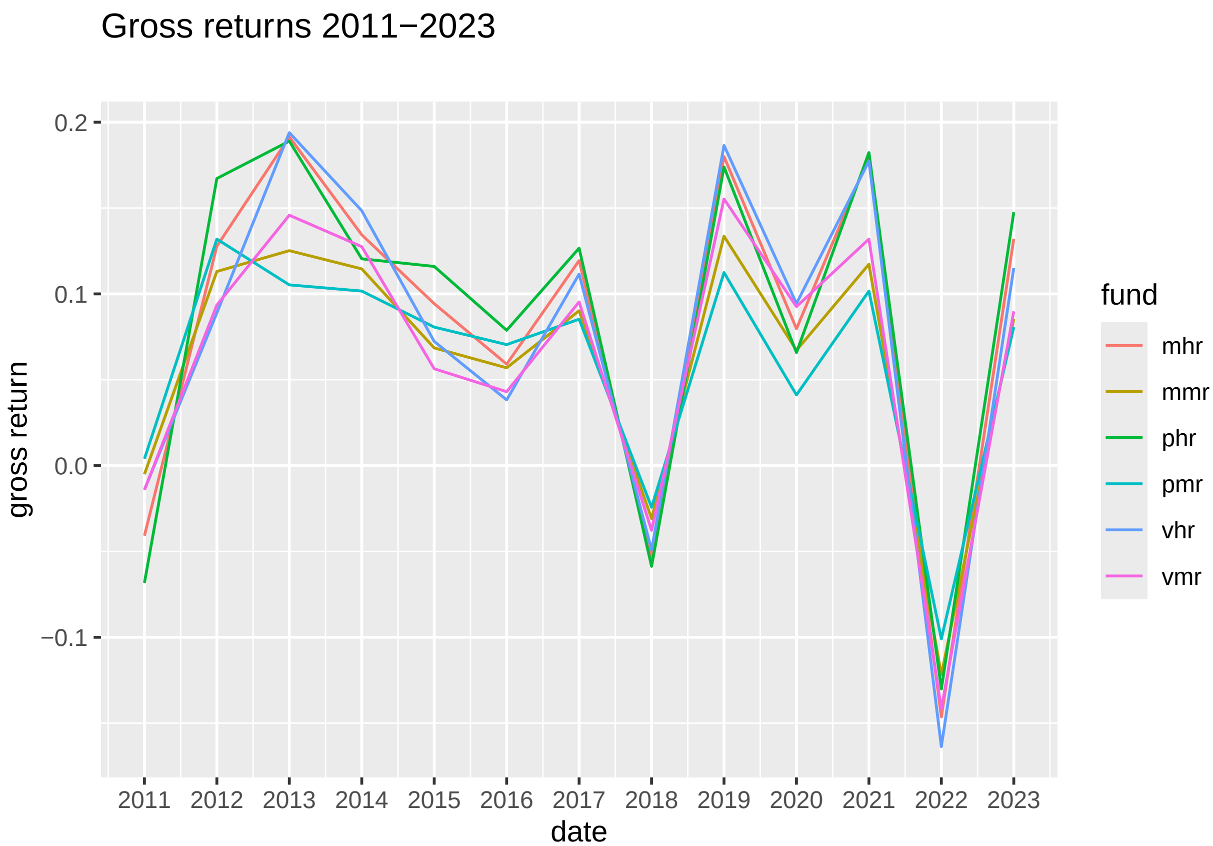
Contents

Fit log returns to F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated shape parameter (degrees of freedom).
`xi` is the estimated skewness parameter.

Returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.
`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.

PFA medium risk is risk profile B.
PFA high risk is risk profile D.



Summary of log-returns

The summary statistics are transformed back to the scale of gross returns by taking $exp()$ of each summary statistic. (Note: Taking arithmetic mean of gross returns directly is no good. Must be geometric mean.)

	vmr	vhr	vmrl	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Min. :	0.868	0.849	0.801	0.904	0.878	0.885	0.864	0.874	0.873
1st Qu.:	1.044	1.039	1.013	1.042	1.068	1.059	1.061	1.064	1.055
Median :	1.097	1.099	1.085	1.084	1.128	1.089	1.127	1.119	1.104
Mean :	1.067	1.080	1.057	1.063	1.089	1.065	1.085	1.079	1.072
3rd Qu.:	1.136	1.160	1.128	1.107	1.182	1.121	1.144	1.139	1.134
Max. :	1.168	1.214	1.193	1.141	1.208	1.143	1.211	1.183	1.163

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.904	pmr	1.068	phr	1.128	phr	1.089	phr	1.182	phr	1.214	vhr
0.885	mmr	1.064	vmr_phr	1.127	mhr	1.085	mhr	1.160	vhr	1.211	mhr
0.878	phr	1.061	mhr	1.119	vmr_phr	1.080	vhr	1.144	mhr	1.208	phr
0.874	vmr_phr	1.059	mmr	1.104	vhr_pmr	1.079	vmr_phr	1.139	vmr_phr	1.193	vmrl
0.873	vhr_pmr	1.055	vhr_pmr	1.099	vhr	1.072	vhr_pmr	1.136	vmr	1.183	vmr_phr
0.868	vmr	1.044	vmr	1.097	vmr	1.067	vmr	1.134	vhr_pmr	1.168	vmr
0.864	mhr	1.042	pmr	1.089	mmr	1.065	mmr	1.128	vmrl	1.163	vhr_pmr
0.849	vhr	1.039	vhr	1.085	vmrl	1.063	pmr	1.121	mmr	1.143	mmr
0.801	vmrl	1.013	vmrl	1.084	pmr	1.057	vmrl	1.107	pmr	1.141	pmr

Correlations and covariance

Correlations

	vmr	vhr	pmr	phr
vmr	1.000	0.993	0.938	0.941
vhr	0.993	1.000	0.917	0.939
pmr	0.938	0.917	1.000	0.957
phr	0.941	0.939	0.957	1.000

Covariances

	vmr	vhr	pmr	phr
vmr	0.007	0.009	0.005	0.008
vhr	0.009	0.011	0.006	0.010
pmr	0.005	0.006	0.004	0.007
phr	0.008	0.010	0.007	0.011

Compare pension plans

Risk of loss

Risk of loss at least as big as row name in percent for a single period (year).

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	17.167	19.667	11.833	16.000	16.667	17.000	15.000	14.500
5	9.167	12.500	5.667	9.333	8.500	10.500	8.667	7.333
10	5.000	8.000	3.000	5.333	4.500	6.500	5.000	3.833
25	0.667	2.167	0.500	0.833	0.667	1.667	1.000	0.333
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	17.333	20.333	8.833	26.667	7.167	26.500	26.667	8.833
5	7.667	10.333	4.333	14.500	3.667	14.167	13.333	5.000
10	3.000	4.667	2.333	6.333	2.000	6.000	5.167	2.833
25	0.000	0.000	0.333	0.000	0.333	0.000	0.000	0.667
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	21.167	21.667	16.500	19.667	18.667	20.167	19.833	19.167
5	7.333	9.500	3.333	8.500	5.167	8.667	7.667	6.333
10	1.500	2.833	0.000	2.667	0.500	2.500	1.833	1.167
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Worst ranking for loss percentiles

Skewed t-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
19.667	vhr	12.500	vhr	8.000	vhr	2.167	vhr	0	vmr	0	vmr	0	vmr
17.167	vmr	10.500	mhr	6.500	mhr	1.667	mhr	0	vhr	0	vhr	0	vhr
17.000	mhr	9.333	phr	5.333	phr	1.000	vmr_phr	0	pmr	0	pmr	0	pmr
16.667	mmr	9.167	vmr	5.000	vmr	0.833	phr	0	phr	0	phr	0	phr
16.000	phr	8.667	vmr_phr	5.000	vmr_phr	0.667	vmr	0	mmr	0	mmr	0	mmr
15.000	vmr_phr	8.500	mmr	4.500	mmr	0.667	mmr	0	mhr	0	mhr	0	mhr
14.500	vhr_pmr	7.333	vhr_pmr	3.833	vhr_pmr	0.500	pmr	0	vmr_phr	0	vmr_phr	0	vmr_phr
11.833	pmr	5.667	pmr	3.000	pmr	0.333	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Standardized t-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
26.667	phr	14.500	phr	6.333	phr	0.667	vhr_pmr	0	vmr	0	vmr	0	vmr
26.667	vmr_phr	14.167	mhr	6.000	mhr	0.333	pmr	0	vhr	0	vhr	0	vhr
26.500	mhr	13.333	vmr_phr	5.167	vmr_phr	0.333	mmr	0	pmr	0	pmr	0	pmr
20.333	vhr	10.333	vhr	4.667	vhr	0.000	vmr	0	phr	0	phr	0	phr
17.333	vmr	7.667	vmr	3.000	vmr	0.000	vhr	0	mmr	0	mmr	0	mmr
8.833	pmr	5.000	vhr_pmr	2.833	vhr_pmr	0.000	phr	0	mhr	0	mhr	0	mhr
8.833	vhr_pmr	4.333	pmr	2.333	pmr	0.000	mhr	0	vmr_phr	0	vmr_phr	0	vmr_phr
7.167	mmr	3.667	mmr	2.000	mmr	0.000	vmr_phr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.667	vhr	9.500	vhr	2.833	vhr	0	vmr	0	vmr	0	vmr	0	vmr
21.167	vmr	8.667	mhr	2.667	phr	0	vhr	0	vhr	0	vhr	0	vhr
20.167	mhr	8.500	phr	2.500	mhr	0	pmr	0	pmr	0	pmr	0	pmr
19.833	vmr_phr	7.667	vmr_phr	1.833	vmr_phr	0	phr	0	phr	0	phr	0	phr
19.667	phr	7.333	vmr	1.500	vmr	0	mmr	0	mmr	0	mmr	0	mmr
19.167	vhr_pmr	6.333	vhr_pmr	1.167	vhr_pmr	0	mhr	0	mhr	0	mhr	0	mhr
18.667	mmr	5.167	mmr	0.500	mmr	0	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
16.500	pmr	3.333	pmr	0.000	pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Chance of min gains

Chance of gains of at least x percent for a single period (year).
x values are row names.

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	82.833	80.333	88.167	84.000	83.333	83.000	85.000	85.500
5	68.333	69.333	71.667	73.000	66.500	72.500	73.167	71.167
10	44.667	53.333	32.500	55.833	35.667	56.167	53.000	46.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	82.667	79.667	91.167	73.333	92.833	73.500	73.333	91.167
5	65.833	65.000	80.000	58.167	84.000	58.000	56.667	82.667
10	44.500	48.000	54.833	42.500	62.333	42.167	39.500	65.500
25	7.000	11.667	6.667	10.000	7.500	9.500	7.000	12.167
50	0.167	0.500	0.833	0.000	1.000	0.000	0.000	1.833
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.167

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	78.833	78.333	83.500	80.333	81.333	79.833	80.167	80.833
5	57.667	61.333	57.667	64.167	57.833	63.000	61.833	60.167
10	35.167	42.500	29.000	46.167	32.167	44.333	41.333	37.167
25	2.167	6.667	0.000	8.333	0.833	7.167	4.833	2.333
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Best ranking for gains percentiles

Skewed t-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
88.167	pmr	73.167	vmr_phr	56.167	mhr	0	vmr	0	vmr	0	vmr
85.500	vhr_pmr	73.000	phr	55.833	phr	0	vhr	0	vhr	0	vhr
85.000	vmr_phr	72.500	mhr	53.333	vhr	0	pmr	0	pmr	0	pmr
84.000	phr	71.667	pmr	53.000	vmr_phr	0	phr	0	phr	0	phr
83.333	mmr	71.167	vhr_pmr	46.000	vhr_pmr	0	mmr	0	mmr	0	mmr
83.000	mhr	69.333	vhr	44.667	vmr	0	mhr	0	mhr	0	mhr
82.833	vmr	68.333	vmr	35.667	mmr	0	vmr_phr	0	vmr_phr	0	vmr_phr
80.333	vhr	66.500	mmr	32.500	pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Standardized t-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
92.833	mmr	84.000	mmr	65.500	vhr_pmr	12.167	vhr_pmr	1.833	vhr_pmr	0.167	vhr_pmr
91.167	pmr	82.667	vhr_pmr	62.333	mmr	11.667	vhr	1.000	mmr	0.000	vmr
91.167	vhr_pmr	80.000	pmr	54.833	pmr	10.000	phr	0.833	pmr	0.000	vhr
82.667	vmr	65.833	vmr	48.000	vhr	9.500	mhr	0.500	vhr	0.000	pmr
79.667	vhr	65.000	vhr	44.500	vmr	7.500	mmr	0.167	vmr	0.000	phr
73.500	mhr	58.167	phr	42.500	phr	7.000	vmr	0.000	phr	0.000	mmr
73.333	phr	58.000	mhr	42.167	mhr	7.000	vmr_phr	0.000	mhr	0.000	mhr
73.333	vmr_phr	56.667	vmr_phr	39.500	vmr_phr	6.667	pmr	0.000	vmr_phr	0.000	vmr_phr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
83.500	pmr	64.167	phr	46.167	phr	8.333	phr	0	vmr	0	vmr
81.333	mmr	63.000	mhr	44.333	mhr	7.167	mhr	0	vhr	0	vhr
80.833	vhr_pmr	61.833	vmr_phr	42.500	vhr	6.667	vhr	0	pmr	0	pmr
80.333	phr	61.333	vhr	41.333	vmr_phr	4.833	vmr_phr	0	phr	0	phr
80.167	vmr_phr	60.167	vhr_pmr	37.167	vhr_pmr	2.333	vhr_pmr	0	mmr	0	mmr
79.833	mhr	57.833	mmr	35.167	vmr	2.167	vmr	0	mhr	0	mhr
78.833	vmr	57.667	vmr	32.167	mmr	0.833	mmr	0	vmr_phr	0	vmr_phr
78.333	vhr	57.667	pmr	29.000	pmr	0.000	pmr	0	vhr_pmr	0	vhr_pmr

MC risk percentiles

Risk of loss at least as big as row name in percent from first to last period.

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	1.46	4.12	1.91	0.82	0.10	0.15	0.05	0.28
5	1.22	3.69	1.71	0.71	0.08	0.12	0.02	0.24
10	1.01	3.27	1.56	0.63	0.08	0.08	0.02	0.22
25	0.67	2.32	1.13	0.35	0.02	0.03	0.00	0.11
50	0.18	1.10	0.62	0.10	0.00	0.01	0.00	0.02
90	0.02	0.11	0.16	0.01	0.00	0.00	0.00	0.00
99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	0.08	0.07	0.71	0.43	0	0	0	0.02
5	0.07	0.05	0.65	0.34	0	0	0	0.01
10	0.06	0.03	0.60	0.22	0	0	0	0.01
25	0.03	0.00	0.50	0.07	0	0	0	0.00
50	0.00	0.00	0.32	0.00	0	0	0	0.00
90	0.00	0.00	0.11	0.00	0	0	0	0.00
99	0.00	0.00	0.00	0.00	0	0	0	0.00

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	0.02	0.03	0.01	0	0	0	0	0
5	0.01	0.02	0.00	0	0	0	0	0
10	0.00	0.02	0.00	0	0	0	0	0
25	0.00	0.01	0.00	0	0	0	0	0
50	0.00	0.00	0.00	0	0	0	0	0
90	0.00	0.00	0.00	0	0	0	0	0
99	0.00	0.00	0.00	0	0	0	0	0

Worst ranking for MC loss percentiles

Skewed t-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.12	vhr	3.69	vhr	3.27	vhr	2.32	vhr	1.10	vhr	0.16	pmr	0	vmr
1.91	pmr	1.71	pmr	1.56	pmr	1.13	pmr	0.62	pmr	0.11	vhr	0	vhr
1.46	vmr	1.22	vmr	1.01	vmr	0.67	vmr	0.18	vmr	0.02	vmr	0	pmr
0.82	phr	0.71	phr	0.63	phr	0.35	phr	0.10	phr	0.01	phr	0	phr
0.28	vhr_pmr	0.24	vhr_pmr	0.22	vhr_pmr	0.11	vhr_pmr	0.02	vhr_pmr	0.00	mmr	0	mmr
0.15	mhr	0.12	mhr	0.08	mmr	0.03	mhr	0.01	mhr	0.00	mhr	0	mhr

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.10	mmr	0.08	mmr	0.08	mhr	0.02	mmr	0.00	mmr	0.00	vmr_phr	0	vmr_phr
0.05	vmr_phr	0.02	vmr_phr	0.02	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0.00	vhr_pmr	0	vhr_pmr

Standardized t-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.71	pmr	0.65	pmr	0.60	pmr	0.50	pmr	0.32	pmr	0.11	pmr	0	vmr
0.43	phr	0.34	phr	0.22	phr	0.07	phr	0.00	vmr	0.00	vmr	0	vhr
0.08	vmr	0.07	vmr	0.06	vmr	0.03	vmr	0.00	vhr	0.00	vhr	0	pmr
0.07	vhr	0.05	vhr	0.03	vhr	0.00	vhr	0.00	phr	0.00	phr	0	phr
0.02	vhr_pmr	0.01	vhr_pmr	0.01	vhr_pmr	0.00	mmr	0.00	mmr	0.00	mmr	0	mmr
0.00	mmr	0.00	mmr	0.00	mmr	0.00	mhr	0.00	mhr	0.00	mhr	0	mhr
0.00	mhr	0.00	mhr	0.00	mhr	0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0	vmr_phr
0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr	0	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.03	vhr	0.02	vhr	0.02	vhr	0.01	vhr	0	vmr	0	vmr	0	vmr
0.02	vmr	0.01	vmr	0.00	vmr	0.00	vmr	0	vhr	0	vhr	0	vhr
0.01	pmr	0.00	pmr	0.00	pmr	0.00	pmr	0	pmr	0	pmr	0	pmr
0.00	phr	0.00	phr	0.00	phr	0.00	phr	0	phr	0	phr	0	phr
0.00	mmr	0.00	mmr	0.00	mmr	0.00	mmr	0	mmr	0	mmr	0	mmr
0.00	mhr	0.00	mhr	0.00	mhr	0.00	mhr	0	mhr	0	mhr	0	mhr
0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

MC gains percentiles

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	98.54	95.88	98.09	99.18	99.90	99.85	99.95	99.72
5	98.24	95.55	97.81	99.06	99.87	99.79	99.94	99.64
10	97.91	95.06	97.60	98.89	99.82	99.78	99.92	99.57
25	96.96	93.67	96.73	98.42	99.60	99.68	99.82	99.26
50	94.42	90.66	94.89	97.19	98.89	99.11	99.53	98.35
100	86.53	84.01	88.48	93.76	94.62	96.52	97.40	93.64
200	59.55	64.45	58.53	80.88	63.21	83.14	81.81	67.25
300	30.57	45.25	22.12	62.83	20.52	57.69	51.89	34.04
400	12.38	29.22	4.27	43.97	2.68	32.80	24.25	13.33
500	4.05	17.64	0.42	28.05	0.14	15.30	8.30	3.96
1000	0.00	0.69	0.00	0.90	0.00	0.08	0.01	0.00

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	99.92	99.93	99.29	99.57	100.00	100.00	100.00	99.98
5	99.91	99.89	99.26	99.48	100.00	100.00	100.00	99.97
10	99.90	99.86	99.16	99.36	100.00	100.00	100.00	99.97
25	99.79	99.77	99.07	98.68	99.97	99.99	99.98	99.95
50	99.44	99.43	98.71	97.08	99.93	99.86	99.89	99.91
100	97.24	97.56	97.82	91.37	99.72	99.15	99.27	99.76
200	85.25	87.19	94.22	71.23	97.47	92.05	90.13	97.86
300	66.73	73.08	87.24	50.68	89.35	73.72	67.15	91.22
400	47.04	57.48	75.62	34.36	73.25	52.35	43.09	77.78

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
500	32.45	44.15	62.30	22.57	52.86	33.00	24.39	60.20
1000	4.20	9.59	16.84	2.48	6.20	2.30	0.98	9.95

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	99.98	99.97	99.99	100.00	100.00	100.00	100.00	100.00
5	99.97	99.94	99.99	100.00	100.00	100.00	100.00	100.00
10	99.94	99.91	99.98	99.98	100.00	100.00	100.00	100.00
25	99.82	99.81	99.95	99.95	100.00	100.00	100.00	100.00
50	98.97	99.26	99.66	99.76	99.97	99.99	100.00	100.00
100	93.16	96.31	95.60	98.23	98.98	99.78	99.68	99.44
200	63.54	78.98	58.13	88.14	69.29	93.87	89.96	81.07
300	32.36	56.15	20.52	70.34	22.63	73.77	62.19	42.63
400	13.97	36.76	4.82	50.71	4.22	47.44	33.40	16.63
500	5.63	22.34	0.89	34.42	0.87	26.11	15.81	5.72
1000	0.07	1.64	0.00	4.11	0.00	0.51	0.23	0.07

Best ranking for MC gains percentiles

Skewed t-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
99.95	vmr_phr	99.94	vmr_phr	99.92	vmr_phr	99.82	vmr_phr	99.53	vmr_phr	97.40	vmr_phr
99.90	mmr	99.87	mmr	99.82	mmr	99.68	mhr	99.11	mhr	96.52	mhr
99.85	mhr	99.79	mhr	99.78	mhr	99.60	mmr	98.89	mmr	94.62	mmr
99.72	vhr_pmr	99.64	vhr_pmr	99.57	vhr_pmr	99.26	vhr_pmr	98.35	vhr_pmr	93.76	phr
99.18	phr	99.06	phr	98.89	phr	98.42	phr	97.19	phr	93.64	vhr_pmr
98.54	vmr	98.24	vmr	97.91	vmr	96.96	vmr	94.89	pmr	88.48	pmr
98.09	pmr	97.81	pmr	97.60	pmr	96.73	pmr	94.42	vmr	86.53	vmr
95.88	vhr	95.55	vhr	95.06	vhr	93.67	vhr	90.66	vhr	84.01	vhr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
83.14	mhr	62.83	phr	43.97	phr	28.05	phr	0.90	phr
81.81	vmr_phr	57.69	mhr	32.80	mhr	17.64	vhr	0.69	vhr
80.88	phr	51.89	vmr_phr	29.22	vhr	15.30	mhr	0.08	mhr
67.25	vhr_pmr	45.25	vhr	24.25	vmr_phr	8.30	vmr_phr	0.01	vmr_phr
64.45	vhr	34.04	vhr_pmr	13.33	vhr_pmr	4.05	vmr	0.00	vmr
63.21	mmr	30.57	vmr	12.38	vmr	3.96	vhr_pmr	0.00	pmr
59.55	vmr	22.12	pmr	4.27	pmr	0.42	pmr	0.00	mmr
58.53	pmr	20.52	mmr	2.68	mmr	0.14	mmr	0.00	vhr_pmr

Standardized t-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mmr	100.00	mmr	100.00	mmr	99.99	mhr	99.93	mmr	99.76	vhr_pmr
100.00	mhr	100.00	mhr	100.00	mhr	99.98	vmr_phr	99.91	vhr_pmr	99.72	mmr
100.00	vmr_phr	100.00	vmr_phr	100.00	vmr_phr	99.97	mmr	99.89	vmr_phr	99.27	vmr_phr
99.98	vhr_pmr	99.97	vhr_pmr	99.97	vhr_pmr	99.95	vhr_pmr	99.86	mhr	99.15	mhr
99.93	vhr	99.91	vmr	99.90	vmr	99.79	vmr	99.44	vmr	97.82	pmr
99.92	vmr	99.89	vhr	99.86	vhr	99.77	vhr	99.43	vhr	97.56	vhr
99.57	phr	99.48	phr	99.36	phr	99.07	pmr	98.71	pmr	97.24	vmr
99.29	pmr	99.26	pmr	99.16	pmr	98.68	phr	97.08	phr	91.37	phr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
97.86	vhr_pmr	91.22	vhr_pmr	77.78	vhr_pmr	62.30	pmr	16.84	pmr
97.47	mmr	89.35	mmr	75.62	pmr	60.20	vhr_pmr	9.95	vhr_pmr
94.22	pmr	87.24	pmr	73.25	mmr	52.86	mmr	9.59	vhr
92.05	mhr	73.72	mhr	57.48	vhr	44.15	vhr	6.20	mmr
90.13	vmr_phr	73.08	vhr	52.35	mhr	33.00	mhr	4.20	vmr
87.19	vhr	67.15	vmr_phr	47.04	vmr	32.45	vmr	2.48	phr
85.25	vmr	66.73	vmr	43.09	vmr_phr	24.39	vmr_phr	2.30	mhr
71.23	phr	50.68	phr	34.36	phr	22.57	phr	0.98	vmr_phr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	phr	100.00	phr	100.00	mmr	100.00	mmr	100.00	vmr_phr	99.78	mhr
100.00	mmr	100.00	mmr	100.00	mhr	100.00	mhr	100.00	vhr_pmr	99.68	vmr_phr
100.00	mhr	100.00	mhr	100.00	vmr_phr	100.00	vmr_phr	99.99	mhr	99.44	vhr_pmr
100.00	vmr_phr	100.00	vmr_phr	100.00	vhr_pmr	100.00	vhr_pmr	99.97	mmr	98.98	mmr
100.00	vhr_pmr	100.00	vhr_pmr	99.98	pmr	99.95	pmr	99.76	phr	98.23	phr
99.99	pmr	99.99	pmr	99.98	phr	99.95	phr	99.66	pmr	96.31	vhr
99.98	vmr	99.97	vmr	99.94	vmr	99.82	vmr	99.26	vhr	95.60	pmr
99.97	vhr	99.94	vhr	99.91	vhr	99.81	vhr	98.97	vmr	93.16	vmr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
93.87	mhr	73.77	mhr	50.71	phr	34.42	phr	4.11	phr
89.96	vmr_phr	70.34	phr	47.44	mhr	26.11	mhr	1.64	vhr
88.14	phr	62.19	vmr_phr	36.76	vhr	22.34	vhr	0.51	mhr
81.07	vhr_pmr	56.15	vhr	33.40	vmr_phr	15.81	vmr_phr	0.23	vmr_phr
78.98	vhr	42.63	vhr_pmr	16.63	vhr_pmr	5.72	vhr_pmr	0.07	vmr
69.29	mmr	32.36	vmr	13.97	vmr	5.63	vmr	0.07	vhr_pmr
63.54	vmr	22.63	mmr	4.82	pmr	0.89	pmr	0.00	pmr
58.13	pmr	20.52	pmr	4.22	mmr	0.87	mmr	0.00	mmr

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated degrees of freedom, or shape parameter.
`xi` is the estimated skewness parameter.

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.060	0.065	0.058	0.078	0.052	0.074	0.070	0.066
s	0.101	0.150	0.121	0.113	0.098	0.139	0.119	0.090
nu	3.574	3.144	2.275	3.856	3.032	3.095	2.965	3.569
xi	0.000	0.002	0.477	0.015	0.023	0.006	0.002	0.003
R^2	0.993	0.991	0.991	0.968	0.992	0.979	0.977	0.995

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.084	0.090	0.102	0.073	0.113	0.072	0.067	0.124
s	0.106	0.122	0.345	0.119	0.895	0.117	0.108	0.793
nu	4.844	7.368	2.045	5682540.710	2.006	8971817.739	9916906.918	2.012

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
R^2	0.935	0.955	0.918	0.923	0.908	0.937	0.924	0.919

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.064	0.077	0.061	0.085	0.063	0.081	0.076	0.069
s	0.081	0.099	0.063	0.101	0.071	0.099	0.090	0.081
R^2	0.933	0.954	0.916	0.923	0.911	0.937	0.924	0.927

AIC and BIC AIC

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
sstd	-28.098	-21.425	-33.230	-23.741	-33.930	-22.790	-26.979	-30.291
std	-16.385	-11.623	-22.924	-11.324	-20.406	-11.817	-13.869	-16.796
normal	-20.316	-15.218	-27.005	-14.616	-23.809	-15.345	-17.593	-20.613

BIC

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
sstd	-25.838	-19.165	-30.970	-21.482	-31.670	-20.530	-24.720	-28.031
std	-14.125	-9.363	-20.664	-9.064	-18.146	-9.558	-11.609	-14.536
normal	-18.056	-12.958	-24.746	-12.357	-21.550	-13.085	-15.333	-18.353

Kappa Let $\{X_{g,i}\}$ be Gaussian distributed with mean μ and scale σ.

Let $\{X_{\nu,i}\}$ be t-distributed, scaled such that $\mathbb{M}^{\nu}(1) = \mathbb{M}^{g}(1) = \sqrt{\frac{2}{\pi}}\sigma$.

Given n_g, we want to determine and n_ν^* such that

$$\text{Var}\left[\sum_i^{n_g} X_{g,i}\right] = \text{Var}\left[\sum_i^{n_\nu^*} X_{\nu,i}\right]$$

For iid. r.v $\{X_i\}$:

$$S_n = X_1 + X_2 + \cdots + X_n$$
$$\mathbb{M}(n) = \mathbb{E}(|S_n - \mathbb{E}(S_n)|)$$

Taleb's convergence metric (κ):

The "rate" of convergence for n summands vs n_0, i.e. the improved convergence achieved by $n - n_0$ additional terms, is given by $\kappa(n_0, n)$:

$$\kappa(n_0, n) = 2 - \frac{\log(n) - \log(n_0)}{\log\left(\frac{\mathbb{M}(n)}{\mathbb{M}(n_0)}\right)}$$

κ

vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0.17	0.21	0.33	0.15	0.22	0.22	0.23	0.17

n_{min}

What is the minimum value of n_ν, the number of observations from a given skewed t-distribution, we need to achieve the same degree of convergence as with $n_g = 30$ observations from a Gaussian distribution with the same mean and standard deviation?

vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
62	78	164	57	78	78	81	62

Fit statistics ranking Skewed t-distribution (sstd):

m	ranking	s	ranking	R^2	ranking
0.078	phr	0.090	vhr_pmr	0.995	vhr_pmr
0.074	mhr	0.098	mmr	0.993	vmr
0.070	vmr_phr	0.101	vmr	0.992	mmr
0.066	vhr_pmr	0.113	phr	0.991	vhr
0.065	vhr	0.119	vmr_phr	0.991	pmr
0.060	vmr	0.121	pmr	0.979	mhr
0.058	pmr	0.139	mhr	0.977	vmr_phr
0.052	mmr	0.150	vhr	0.968	phr

Standardized t-distribution (std):

m	ranking	s	ranking	R^2	ranking
0.124	vhr_pmr	0.106	vmr	0.955	vhr
0.113	mmr	0.108	vmr_phr	0.937	mhr
0.102	pmr	0.117	mhr	0.935	vmr
0.090	vhr	0.119	phr	0.924	vmr_phr
0.084	vmr	0.122	vhr	0.923	phr
0.073	phr	0.345	pmr	0.919	vhr_pmr
0.072	mhr	0.793	vhr_pmr	0.918	pmr
0.067	vmr_phr	0.895	mmr	0.908	mmr

Normal distribution:

m	ranking	s	ranking	R^2	ranking
0.085	phr	0.063	pmr	0.954	vhr
0.081	mhr	0.071	mmr	0.937	mhr
0.077	vhr	0.081	vhr_pmr	0.933	vmr
0.076	vmr_phr	0.081	vmr	0.927	vhr_pmr
0.069	vhr_pmr	0.090	vmr_phr	0.924	vmr_phr
0.064	vmr	0.099	mhr	0.923	phr
0.063	mmr	0.099	vhr	0.916	pmr
0.061	pmr	0.101	phr	0.911	mmr

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).
Statistics are given for the final state of all paths.
Probability of down-and-out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.
0 is defined as any value below a threshold.
`dai_pct` (for down-and-in) is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	365.22	438.43	342.27	534.24	352.73	480.78	449.93	389.07
mc_s	144.99	242.02	113.87	240.41	92.05	171.16	141.79	134.00
mc_min	3.00	3.00	0.04	9.81	76.71	57.75	78.93	9.02

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_max	1061.35	1733.76	933.98	1791.85	932.38	1698.31	1317.63	1447.45
dao_pct	0.02	0.06	0.12	0.00	0.00	0.00	0.00	0.00
dai_pct	1.36	3.89	1.83	0.66	0.09	0.09	0.03	0.30

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	594.24	701.71	1.293393e+20	498.88	1439.39	601.20	546.42	4.300181e+05
mc_s	312.33	413.32	1.293393e+22	283.05	54541.36	248.89	211.17	4.192550e+07
mc_min	65.73	78.35	3.000000e-02	57.83	122.67	122.23	127.60	9.973000e+01
mc_max	5603.32	5079.93	1.293393e+24	3108.62	5410256.83	2395.18	2186.83	4.191411e+09
dao_pct	0.00	0.00	9.000000e-02	0.00	0.00	0.00	0.00	0.000000e+00
dai_pct	0.08	0.09	6.900000e-01	0.40	0.00	0.00	0.00	1.000000e-02

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	387.42	513.18	351.29	610.65	369.91	561.39	501.78	428.70
mc_s	147.82	243.69	100.63	294.23	89.78	187.95	164.02	130.13
mc_min	92.96	82.74	102.84	99.94	131.02	163.84	160.98	158.70
mc_max	1434.40	3114.30	892.69	3527.40	944.67	1814.66	1700.44	2093.70
dao_pct	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dai_pct	0.01	0.02	0.00	0.01	0.00	0.00	0.00	0.00

Ranking Skewed t-distribution (sstd):

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
534.24	phr	92.05	mmr	78.93	vmr_phr	1791.85	phr	0.00	phr	0.03	vmr_phr
480.78	mhr	113.87	pmr	76.71	mmr	1733.76	vhr	0.00	mmr	0.09	mmr
449.93	vmr_phr	134.00	vhr_pmr	57.75	mhr	1698.31	mhr	0.00	mhr	0.09	mhr
438.43	vhr	141.79	vmr_phr	9.81	phr	1447.45	vhr_pmr	0.00	vmr_phr	0.30	vhr_pmr
389.07	vhr_pmr	144.99	vmr	9.02	vhr_pmr	1317.63	vmr_phr	0.00	vhr_pmr	0.66	phr
365.22	vmr	171.16	mhr	3.00	vmr	1061.35	vmr	0.02	vmr	1.36	vmr
352.73	mmr	240.41	phr	3.00	vhr	933.98	pmr	0.06	vhr	1.83	pmr
342.27	pmr	242.02	vhr	0.04	pmr	932.38	mmr	0.12	pmr	3.89	vhr

Standardized t-distribution (std):

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
1.293393e+20	pmr	2.111700e+02	vmr_phr	127.60	vmr_phr	1.293393e+24	pmr	0.00	vmr	0.00	mmr
4.300181e+05	vhr_pmr	2.488900e+02	mhr	122.67	mmr	4.191411e+09	vhr_pmr	0.00	vhr	0.00	mhr
1.439390e+03	mmr	2.830500e+02	phr	122.23	mhr	5.410257e+06	mmr	0.00	phr	0.00	vmr_phr
7.017100e+02	vhr	3.123300e+02	vmr	99.73	vhr_pmr	5.603320e+03	vmr	0.00	mmr	0.01	vhr_pmr
6.012000e+02	mhr	4.133200e+02	vhr	78.35	vhr	5.079930e+03	vhr	0.00	mhr	0.08	vmr
5.942400e+02	vmr	5.454136e+04	mmr	65.73	vmr	3.108620e+03	phr	0.00	vmr_phr	0.09	vhr
5.464200e+02	vmr_phr	4.192550e+07	vhr_pmr	57.83	phr	2.395180e+03	mhr	0.00	vhr_pmr	0.40	phr
4.988800e+02	phr	1.293393e+22	pmr	0.03	pmr	2.186830e+03	vmr_phr	0.09	pmr	0.69	pmr

Normal distribution:

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
610.65	phr	89.78	mmr	163.84	mhr	3527.40	phr	0	vmr	0.00	pmr
561.39	mhr	100.63	pmr	160.98	vmr_phr	3114.30	vhr	0	vhr	0.00	mmr
513.18	vhr	130.13	vhr_pmr	158.70	vhr_pmr	2093.70	vhr_pmr	0	pmr	0.00	mhr

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
501.78	vmr_phr	147.82	vmr	131.02	mmr	1814.66	mhr	0	phr	0.00	vmr_phr
428.70	vhr_pmr	164.02	vmr_phr	102.84	pmr	1700.44	vmr_phr	0	mmr	0.00	vhr_pmr
387.42	vmr	187.95	mhr	99.94	phr	1434.40	vmr	0	mhr	0.01	vmr
369.91	mmr	243.69	vhr	92.96	vmr	944.67	mmr	0	vmr_phr	0.01	phr
351.29	pmr	294.23	phr	82.74	vhr	892.69	pmr	0	vhr_pmr	0.02	vhr

Compare Gaussian and skewed t-distribution fits

Gaussian fits

Gaussian QQ plots

Gaussian vs skewed t

Probability in percent that the smallest and largest (respectively) observed return for each fund was generated by a normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
P_norm(X_min)	0.070	0.088	0.389	0.582	0.161	0.255	0.265	0.155
P_norm(X_max)	13.230	11.876	12.922	15.359	15.936	13.198	15.397	15.556
P_t(X_min)	3.743	5.457	3.475	4.567	4.100	5.035	4.182	3.022
P_t(X_max)	0.000	0.001	2.818	0.023	0.052	0.004	0.000	0.001

Average number of years between min or max events (respectively):

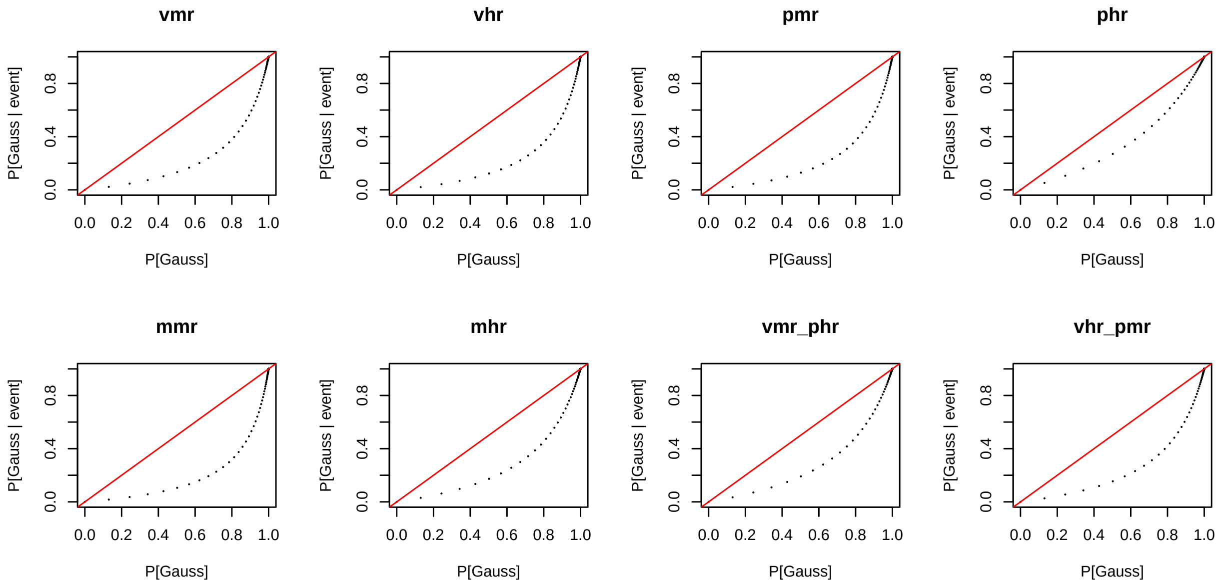
	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
norm: avg yrs btw min	1.438131e+03	1139.205	256.817	171.880	620.586	392.517	376.706	644.455
norm: avg yrs btw max	7.559000e+00	8.420	7.739	6.511	6.275	7.577	6.495	6.428
t: avg yrs btw min	2.671500e+01	18.324	28.775	21.898	24.387	19.862	23.914	33.089
t: avg yrs btw max	3.834699e+08	178349.076	35.487	4439.617	1930.115	23903.982	236209.123	124926.545

Lilliefors test p-values for Lilliefors test.
Testing H_0, that log-returns are Gaussian.

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
p value	0.052	0.343	0.024	0.06	0.041	0.251	0.113	0.183

Wittgenstein's Ruler For different given probabilities that returns are Gaussian, what is the probability that the distribution is Gaussian rather than skewed t-distributed, given the smallest/largest observed log-returns?

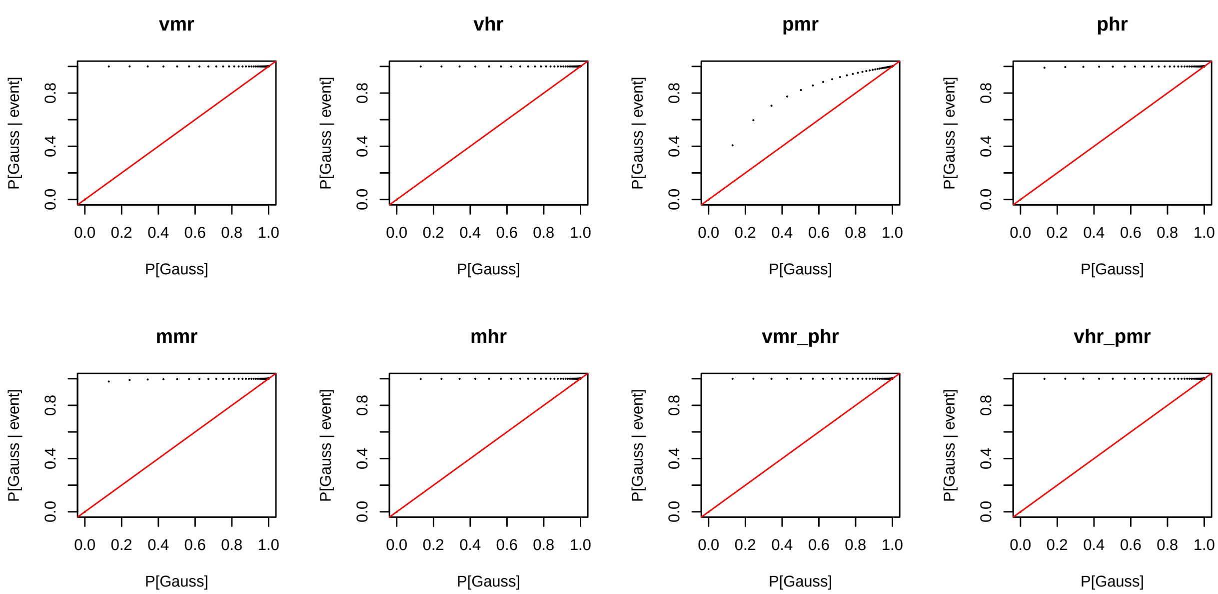
Conditional probabilities for smallest observed log-returns:



Use $1 -$ p-value from Lilliefors test as prior probability that the distribution is Gaussian.
$x_{\text{obs}} = \min(x)$ and $P[\text{Event} \mid \text{Gaussian}] = P_{\text{Gauss}}[X \leq x_{\text{min}}]$:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Lillie p-val	0.052	0.343	0.024	0.060	0.041	0.251	0.113	0.183
Prior prob	0.948	0.657	0.976	0.940	0.959	0.749	0.887	0.817
P[Gauss | Event]	0.737	0.210	0.855	0.852	0.730	0.383	0.649	0.448

Use $1 -$ p-value from Lilliefors test as prior probability that the distribution is Gaussian.
$x_{\text{obs}} = \max(x)$ and $P[\text{Event} \mid \text{Gaussian}] = P_{\text{Gauss}}[X \geq x_{\text{max}}]$:



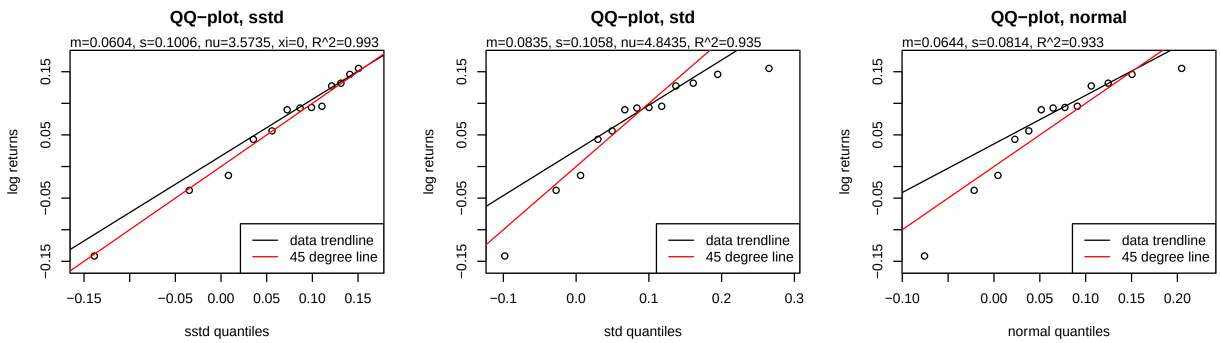
	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Lillie p-val	0.052	0.343	0.024	0.06	0.041	0.251	0.113	0.183

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Prior prob	0.948	0.657	0.976	0.94	0.959	0.749	0.887	0.817
P[Gauss | Event]	1.000	1.000	0.995	1.00	1.000	1.000	1.000	1.000

Velliv medium risk (vmr), 2011 - 2023

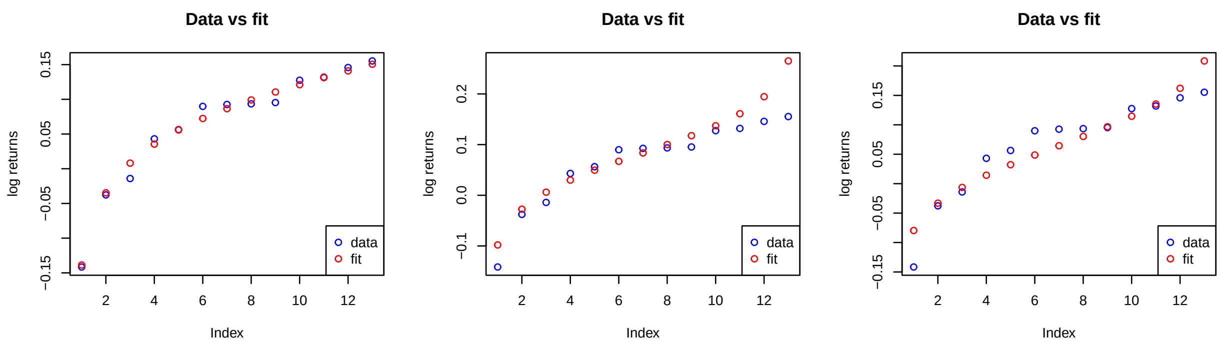
QQ Plot

Skewed t-distribution (sstd):



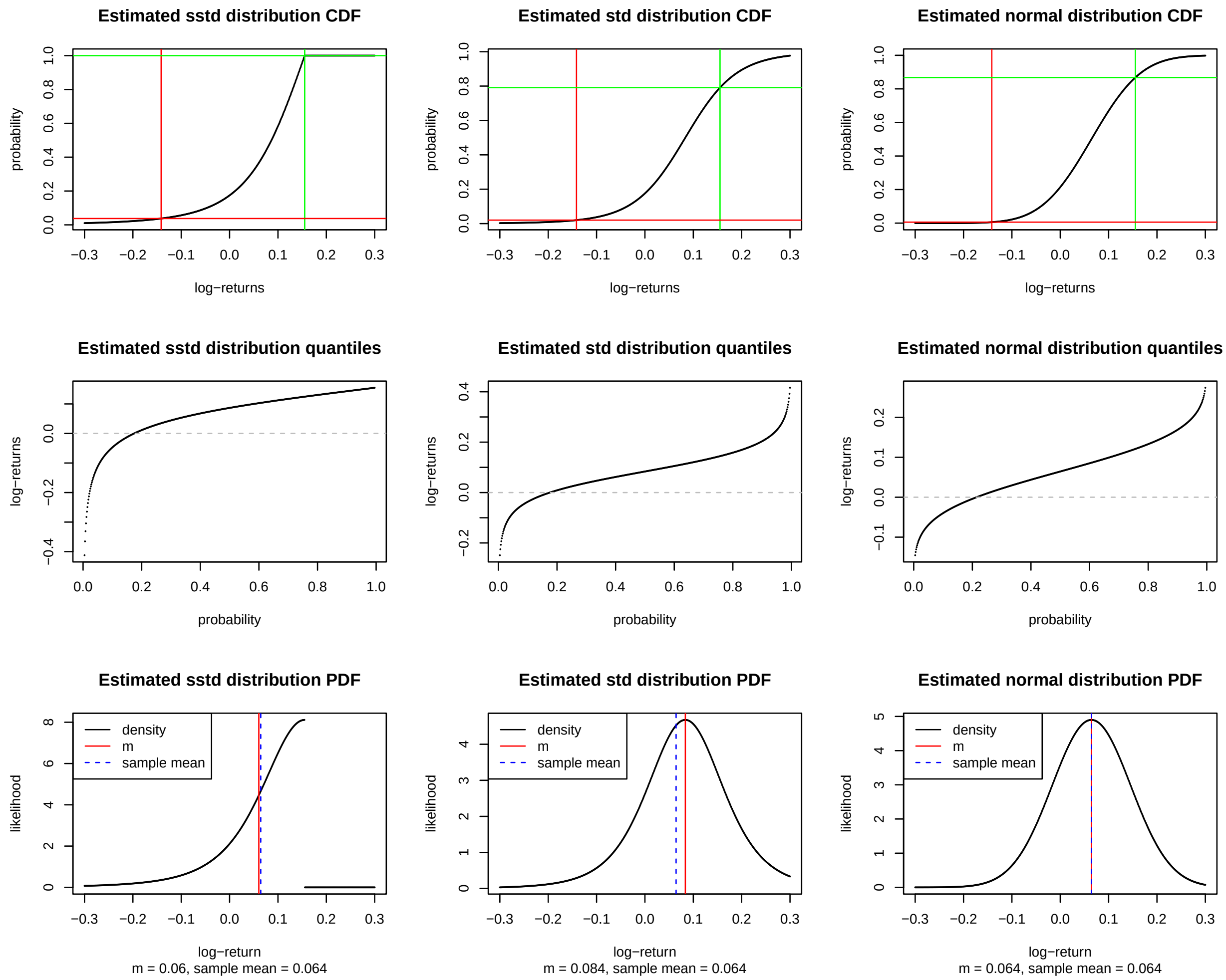
Data vs fit

Let's plot the fit and the observed returns together.



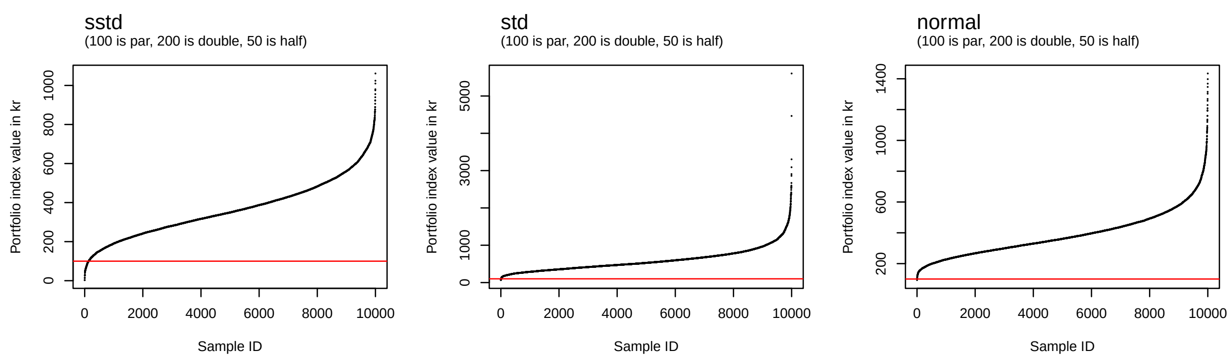
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

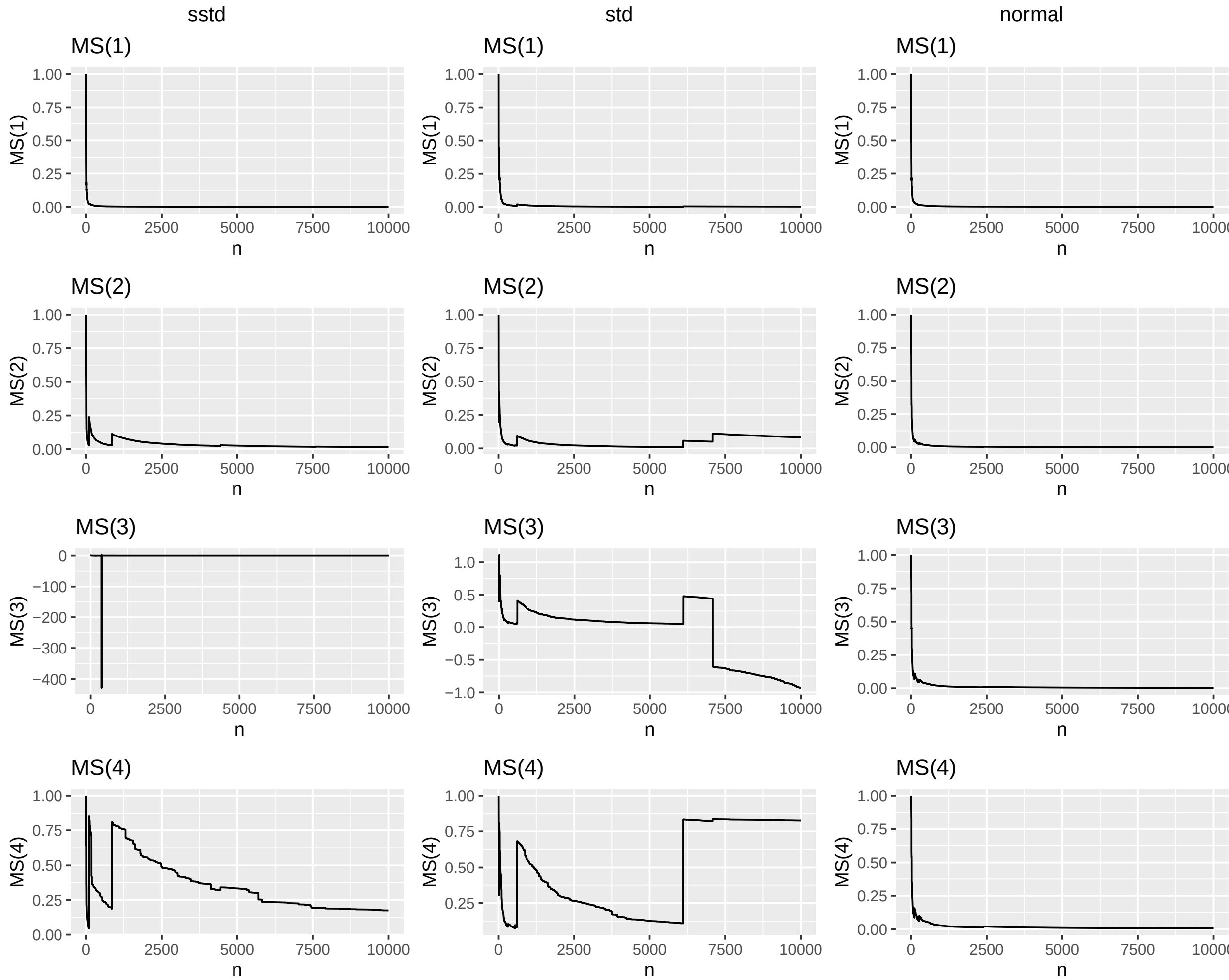
Sorted portfolio index values for last period of all runs



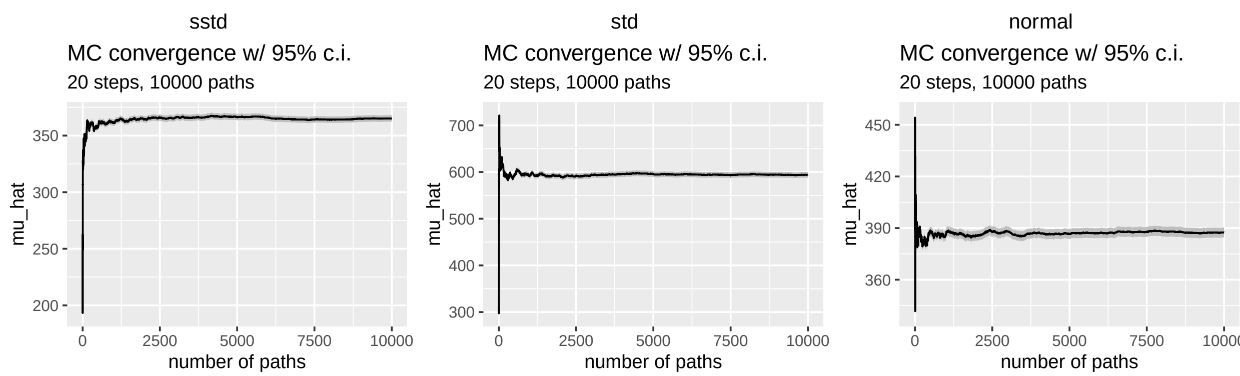
Convergence

Max vs sum

Max vs sum plots for the first four moments:

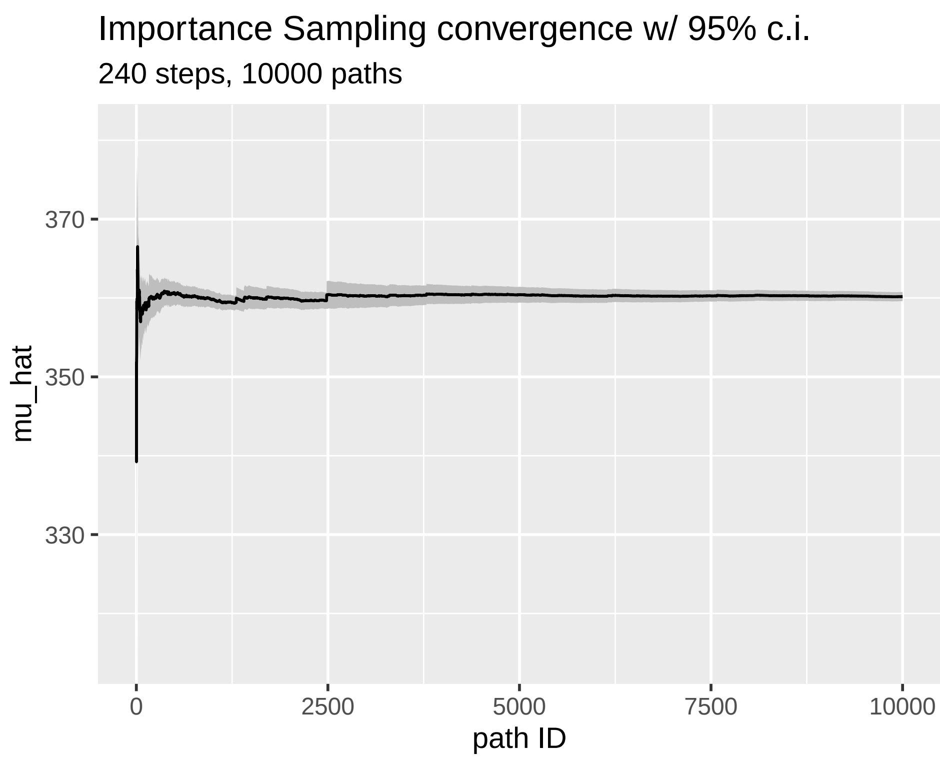


MC



IS

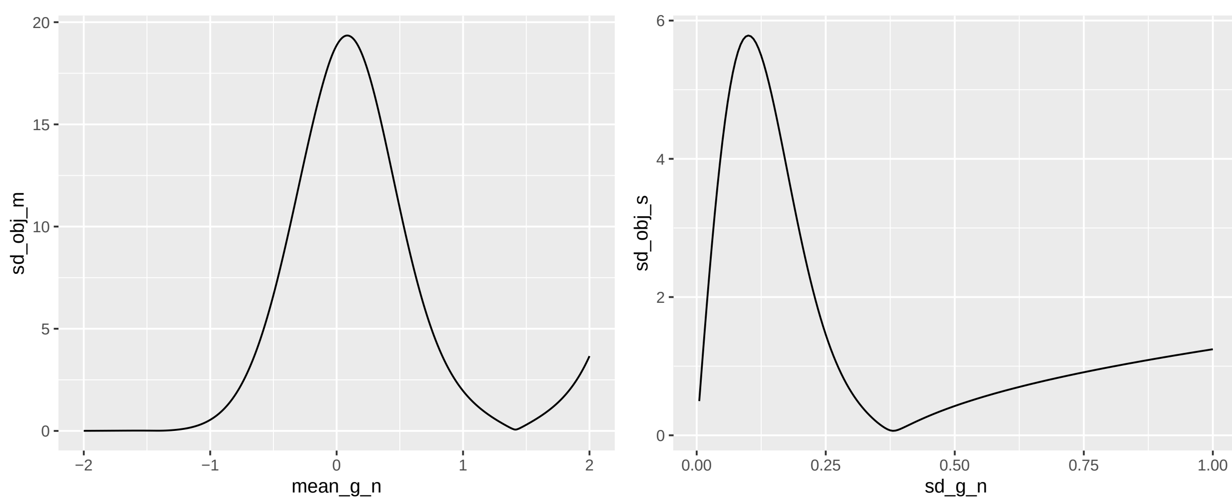
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.4145605 0.3807834
```

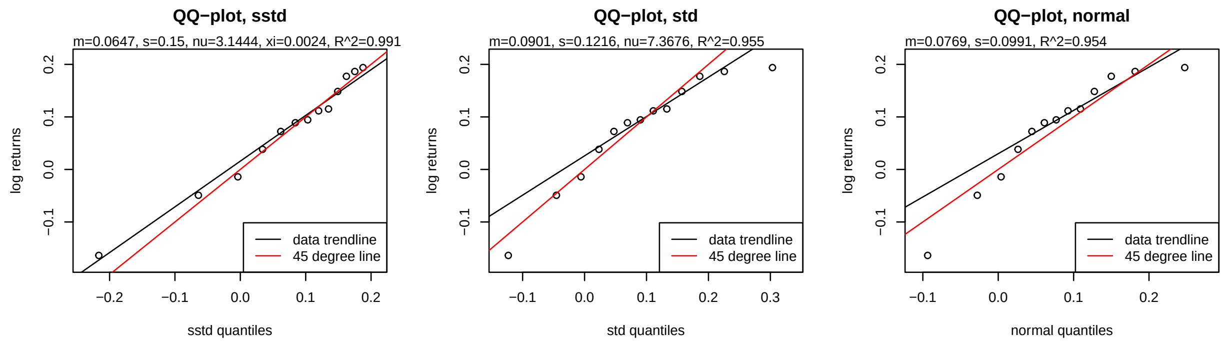
Objective function plots



Velliv high risk (vhr), 2011 - 2023

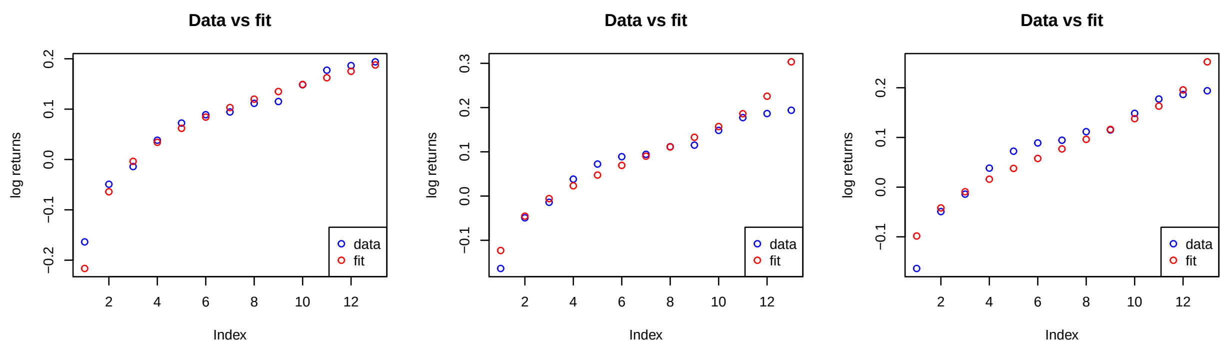
QQ Plot

Skewed t-distribution (sstd):



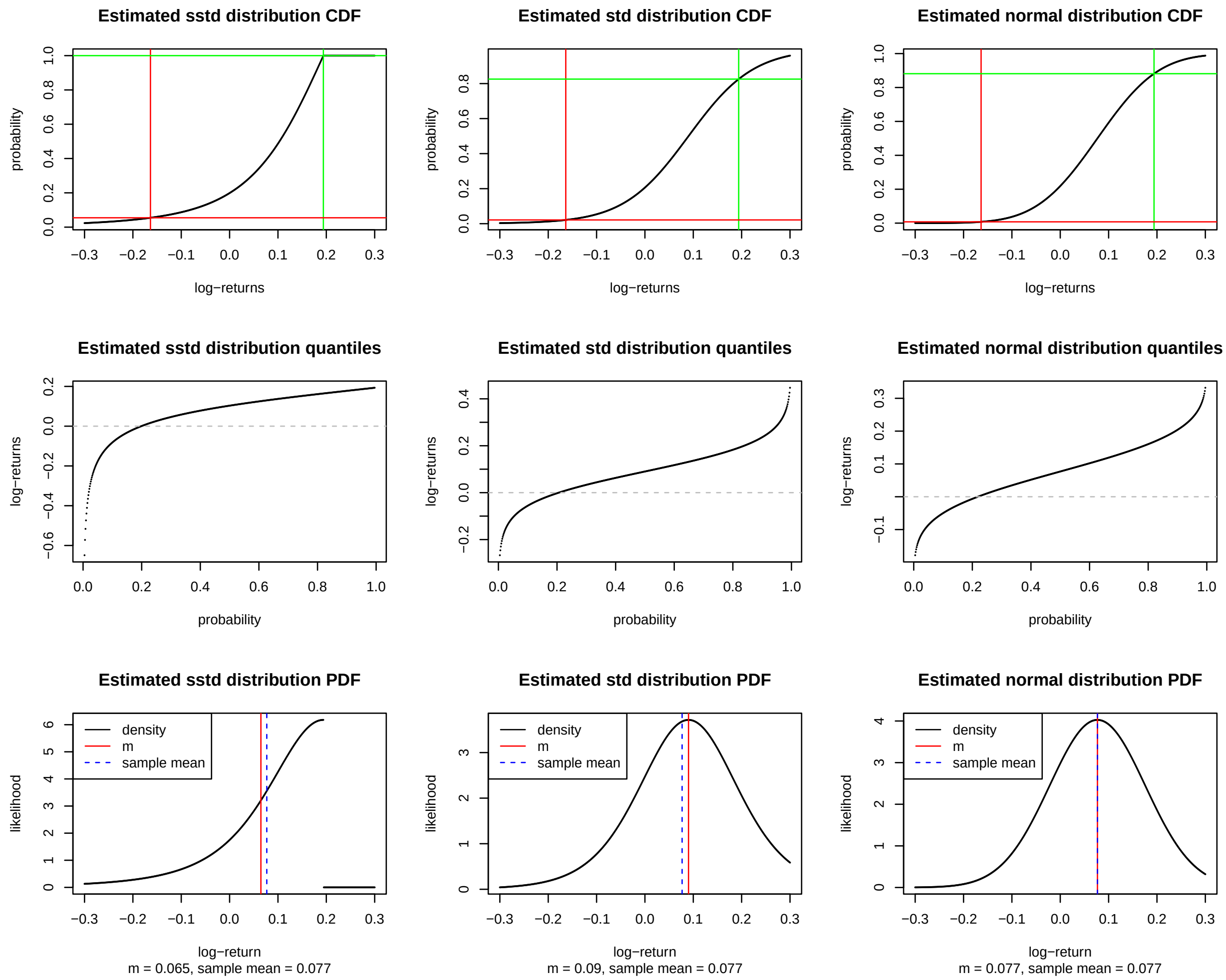
Data vs fit

Let's plot the fit and the observed returns together.



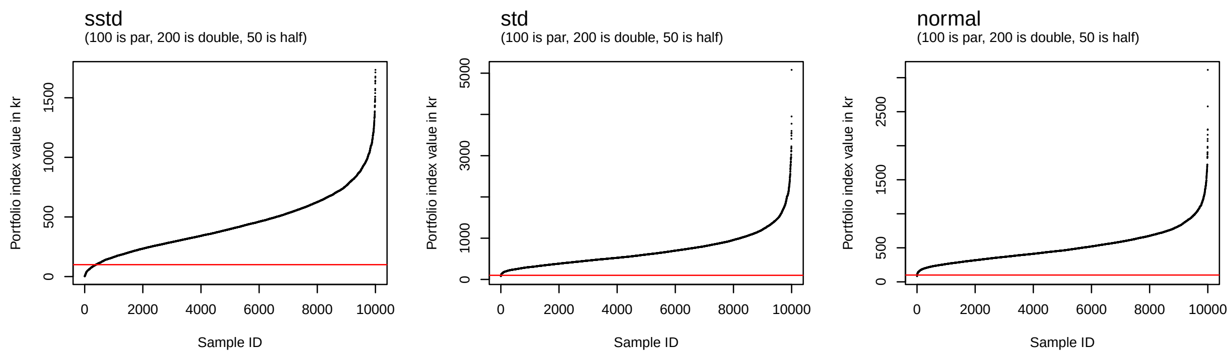
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

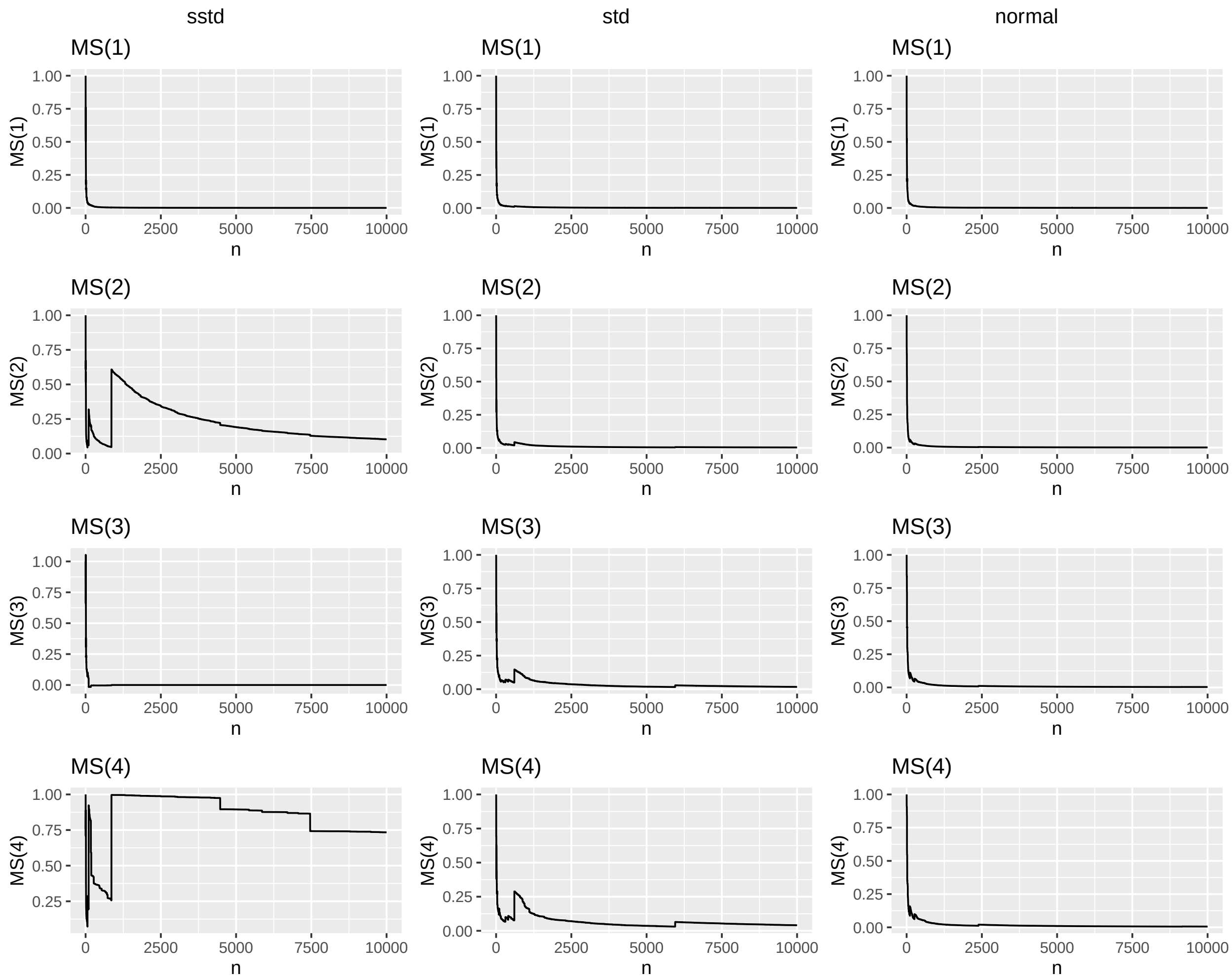
Sorted portfolio index values for last period of all runs



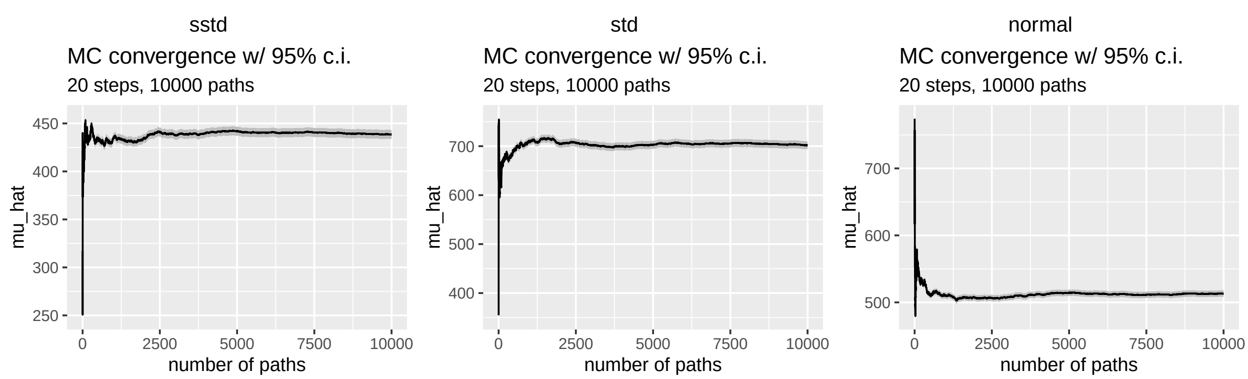
Convergence

Max vs sum

Max vs sum plots for the first four moments:

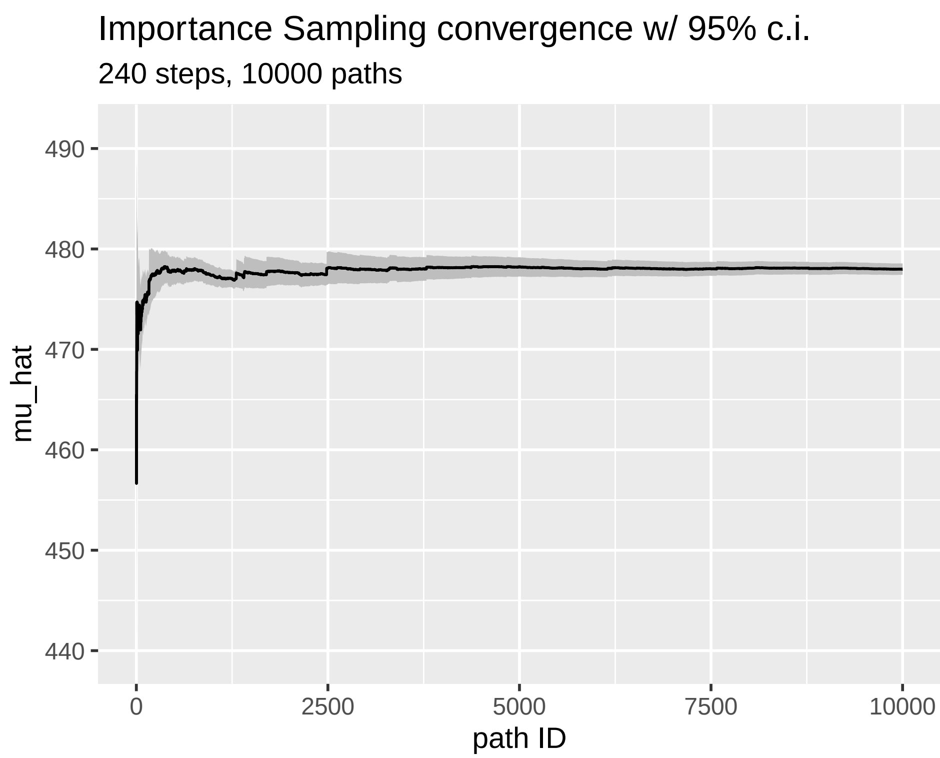


MC



IS

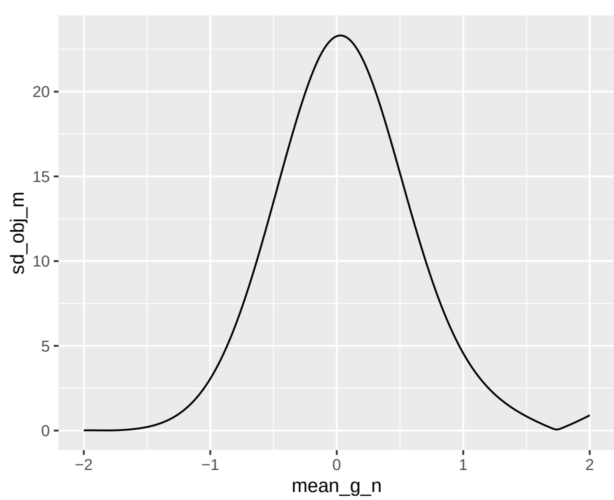
Skewed t-distribution with a normal proposal distribution.

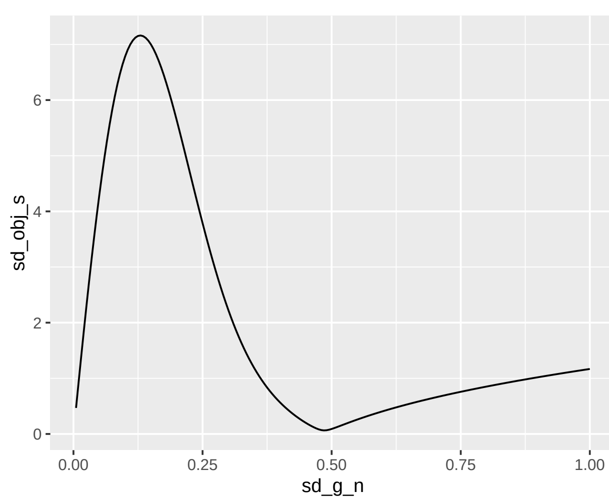


Parameters

```
## [1] 1.7391222 0.4858909
```

Objective function plots

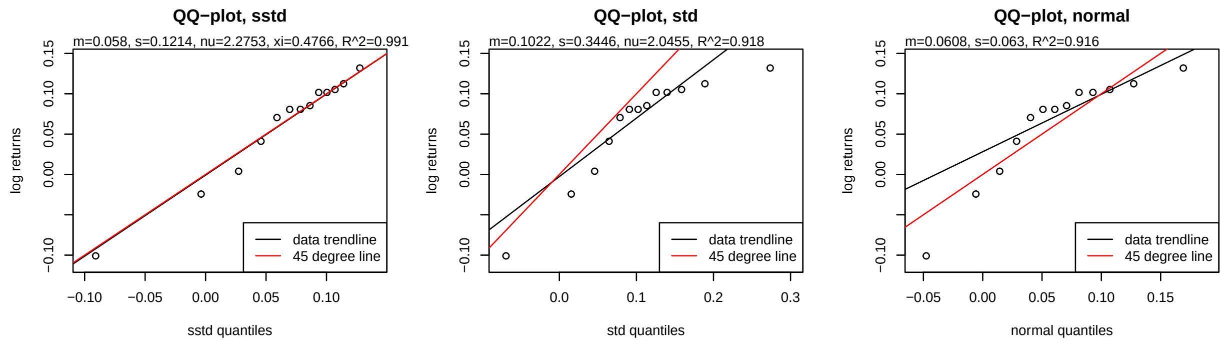




PFA medium risk (pmr), 2011 - 2023

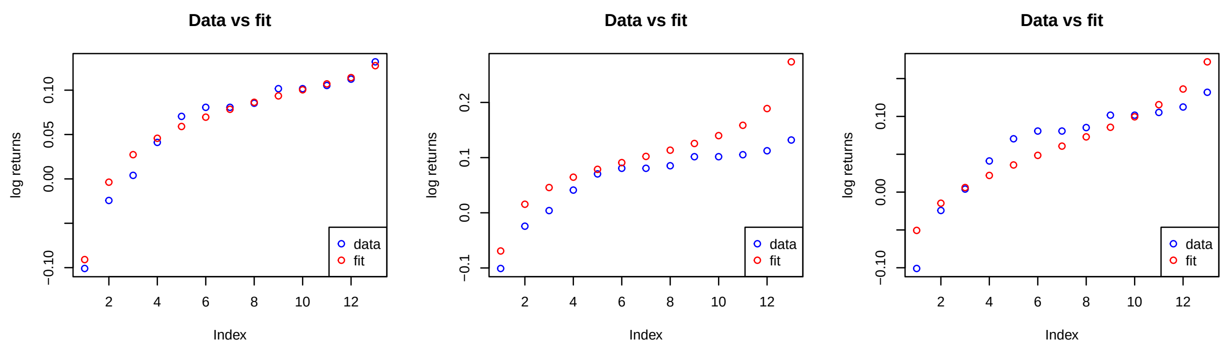
QQ Plot

Skewed t-distribution (sstd):



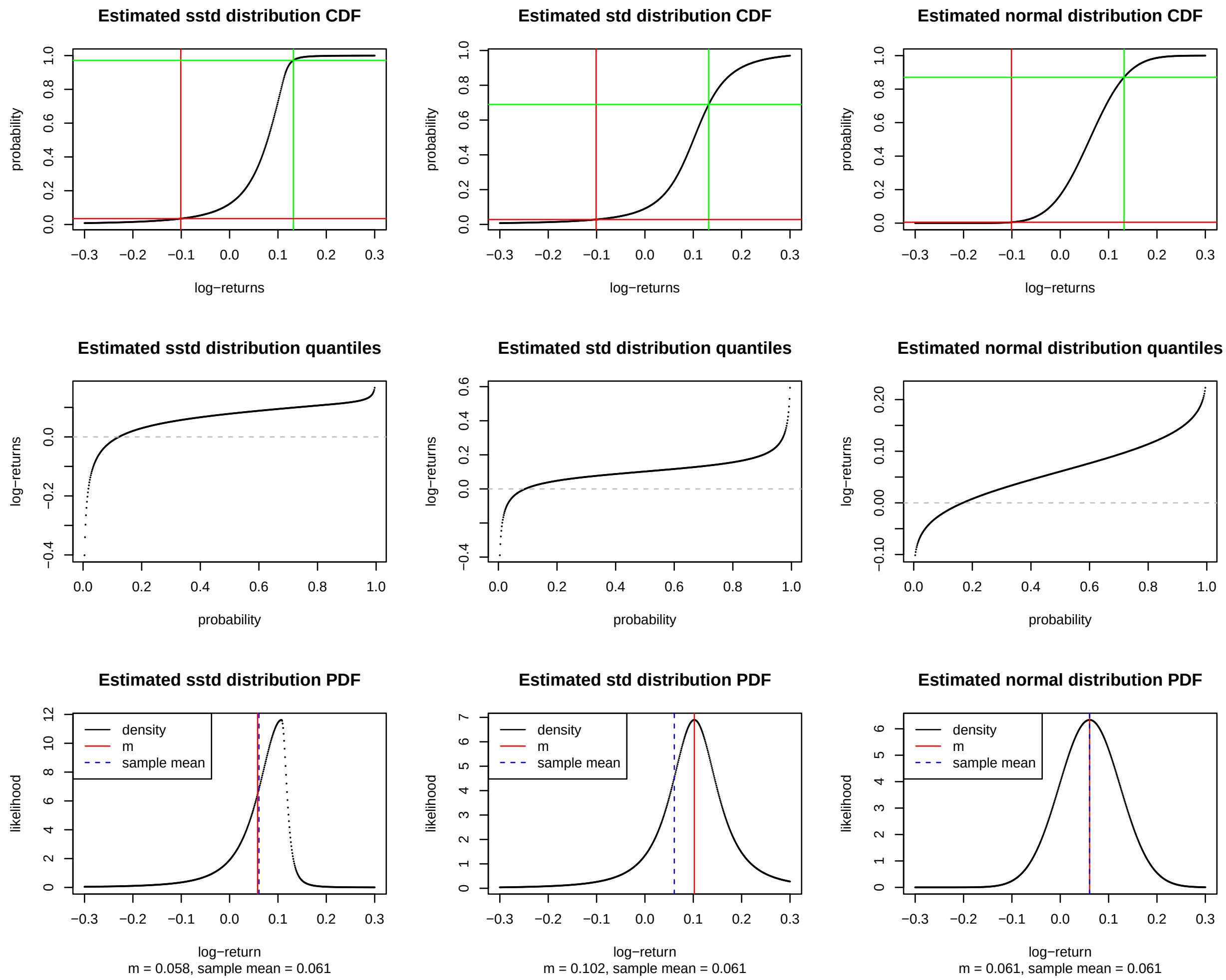
Data vs fit

Let's plot the fit and the observed returns together.



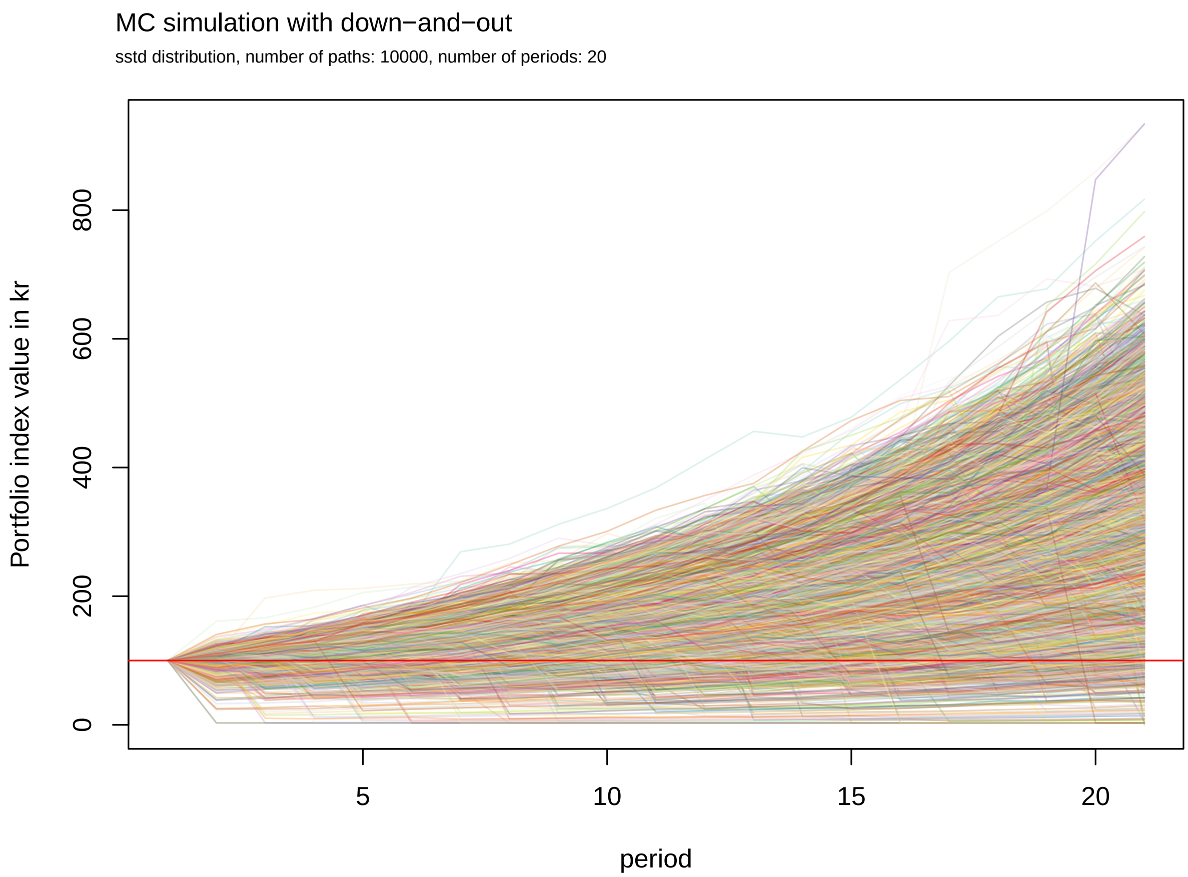
Estimated distribution

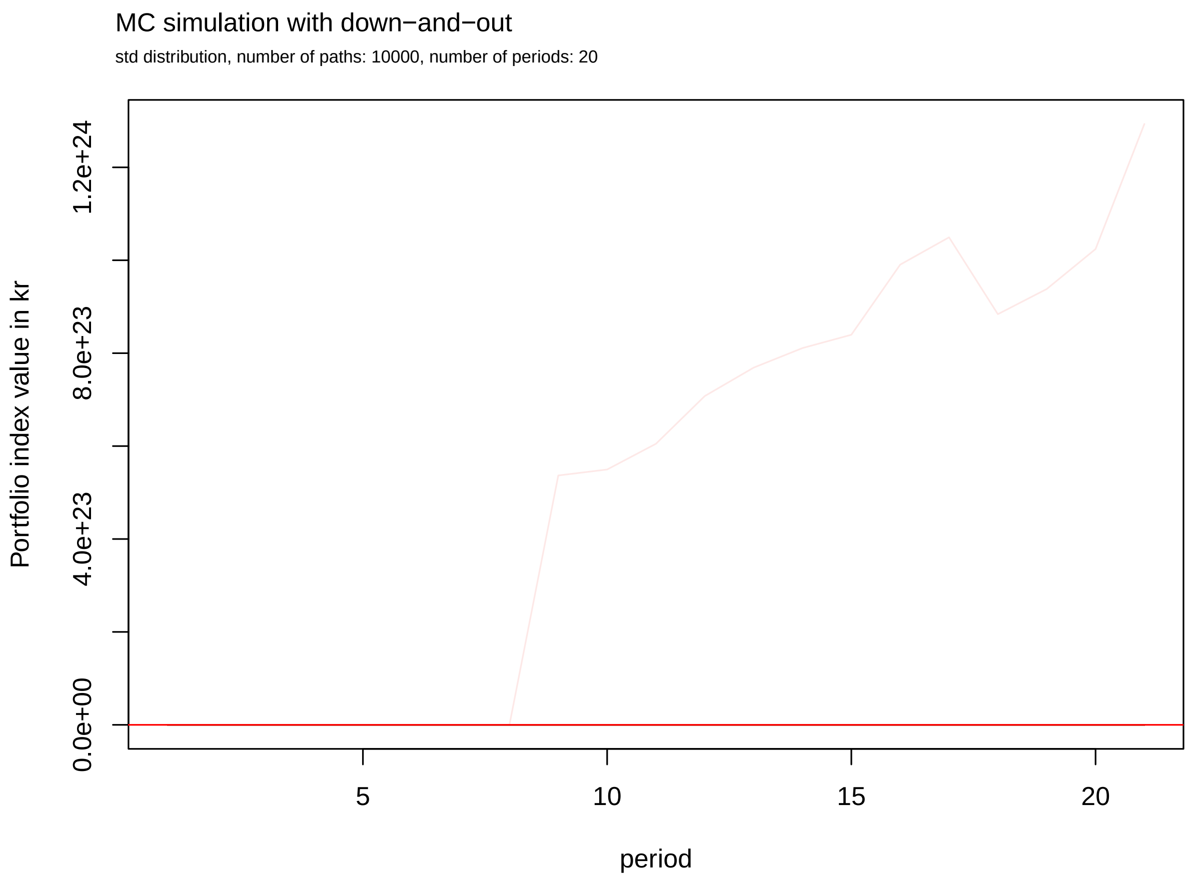
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



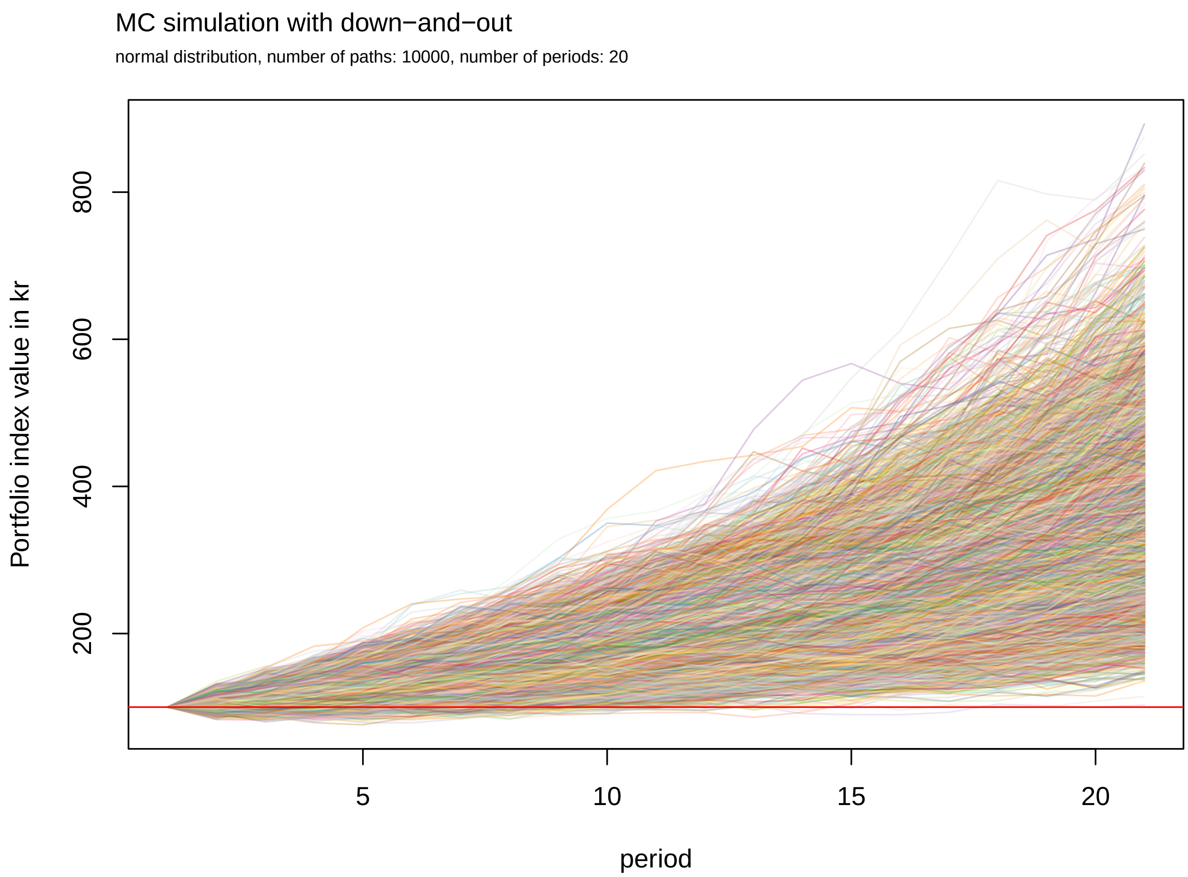
Monte Carlo

pmr has the sstd fit with the lowest value of nu. Compare with other distributions:

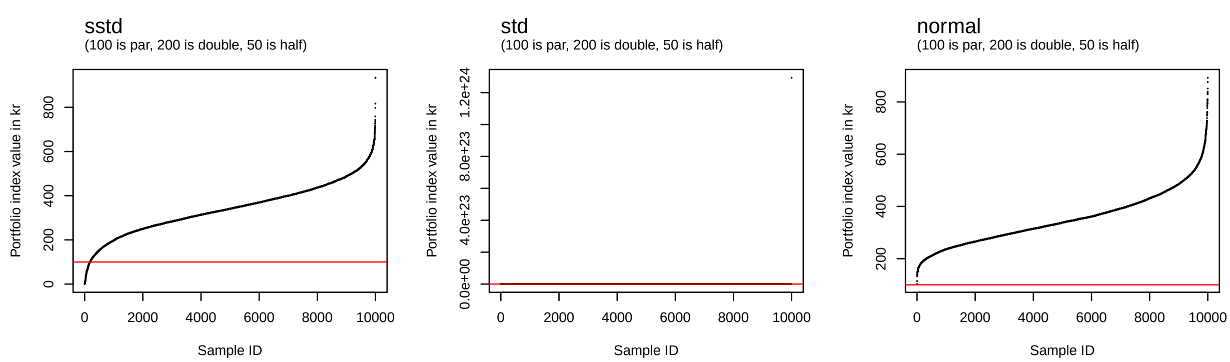

MC simulation with down−and−out
sstd distribution, number of paths: 10000, number of periods: 20
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period



MC simulation with down−and−out
std distribution, number of paths: 10000, number of periods: 20
Portfolio index value in kr
0.0e+00
4.0e+23
8.0e+23
1.2e+24
5
10
15
20
period



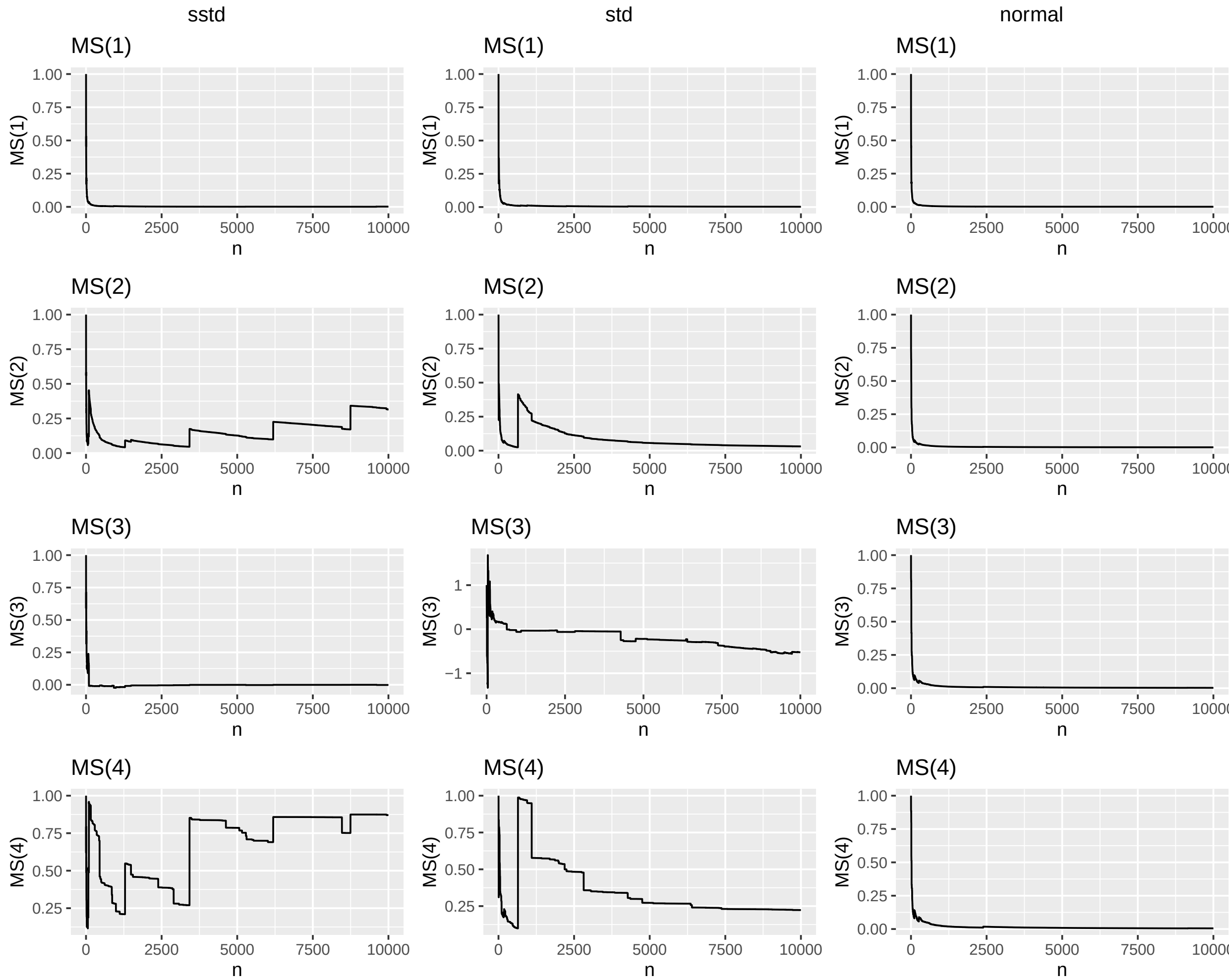
Sorted portfolio index values for last period of all runs



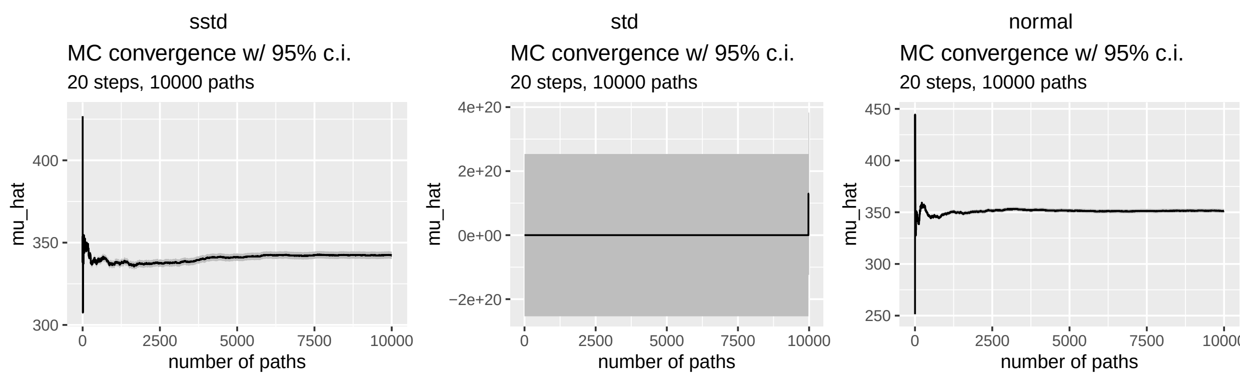
Convergence

Max vs sum

Max vs sum plots for the first four moments:

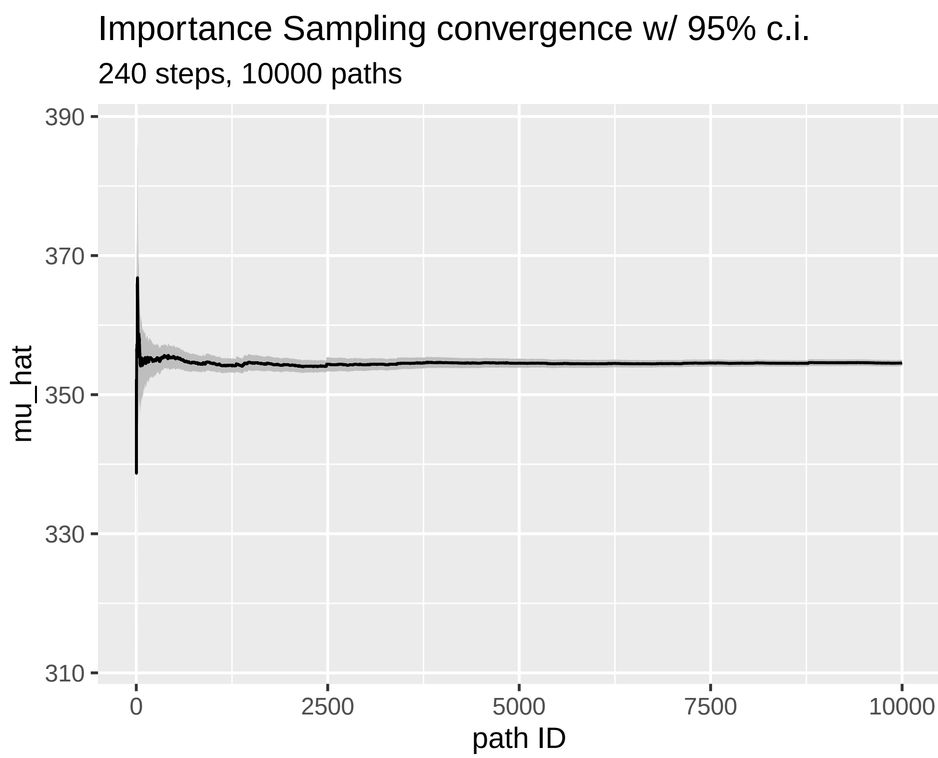


MC



IS

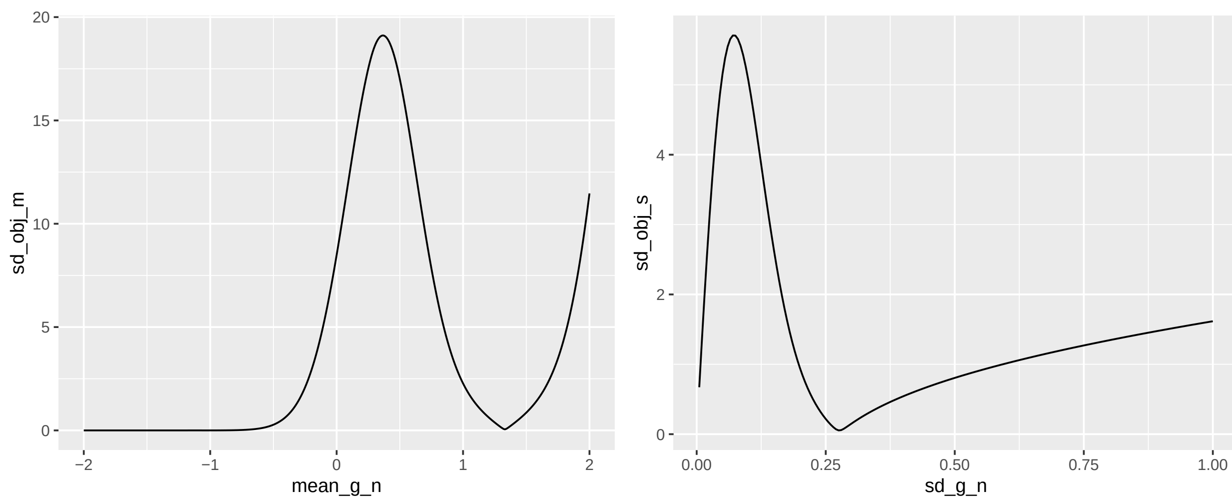
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.3304634 0.2764028
```

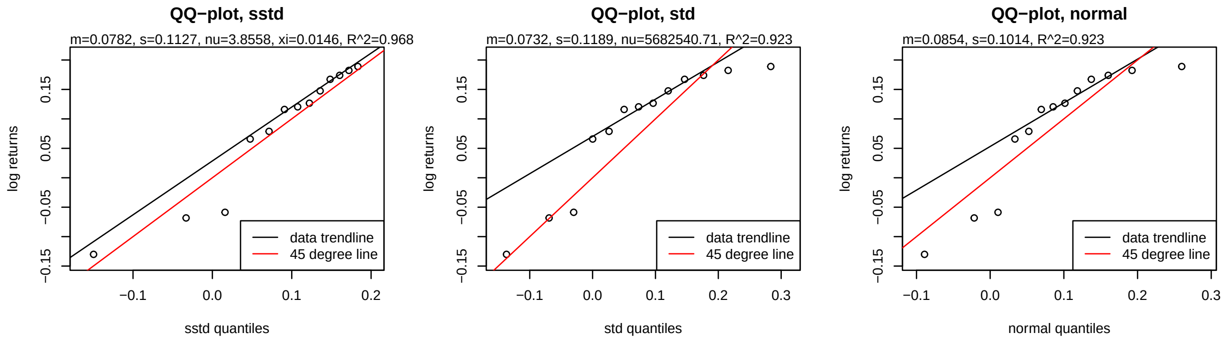
Objective function plots



PFA high risk (phr), 2011 - 2023

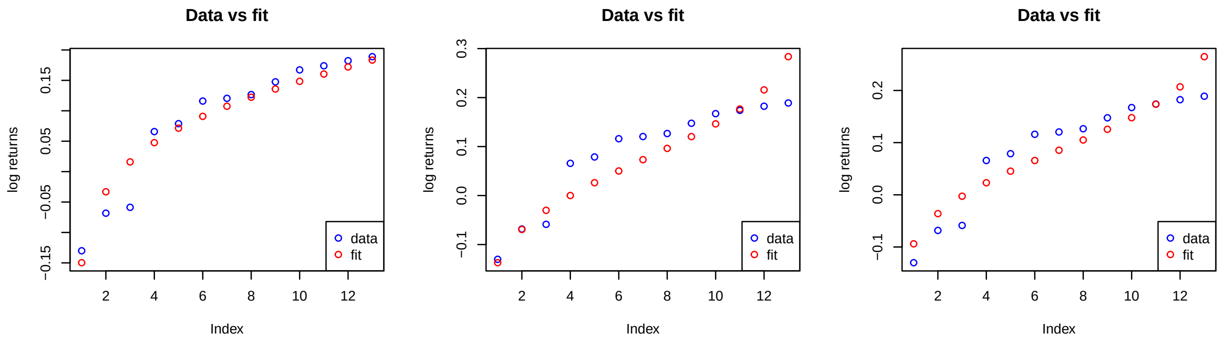
QQ Plot

Skewed t-distribution (sstd):



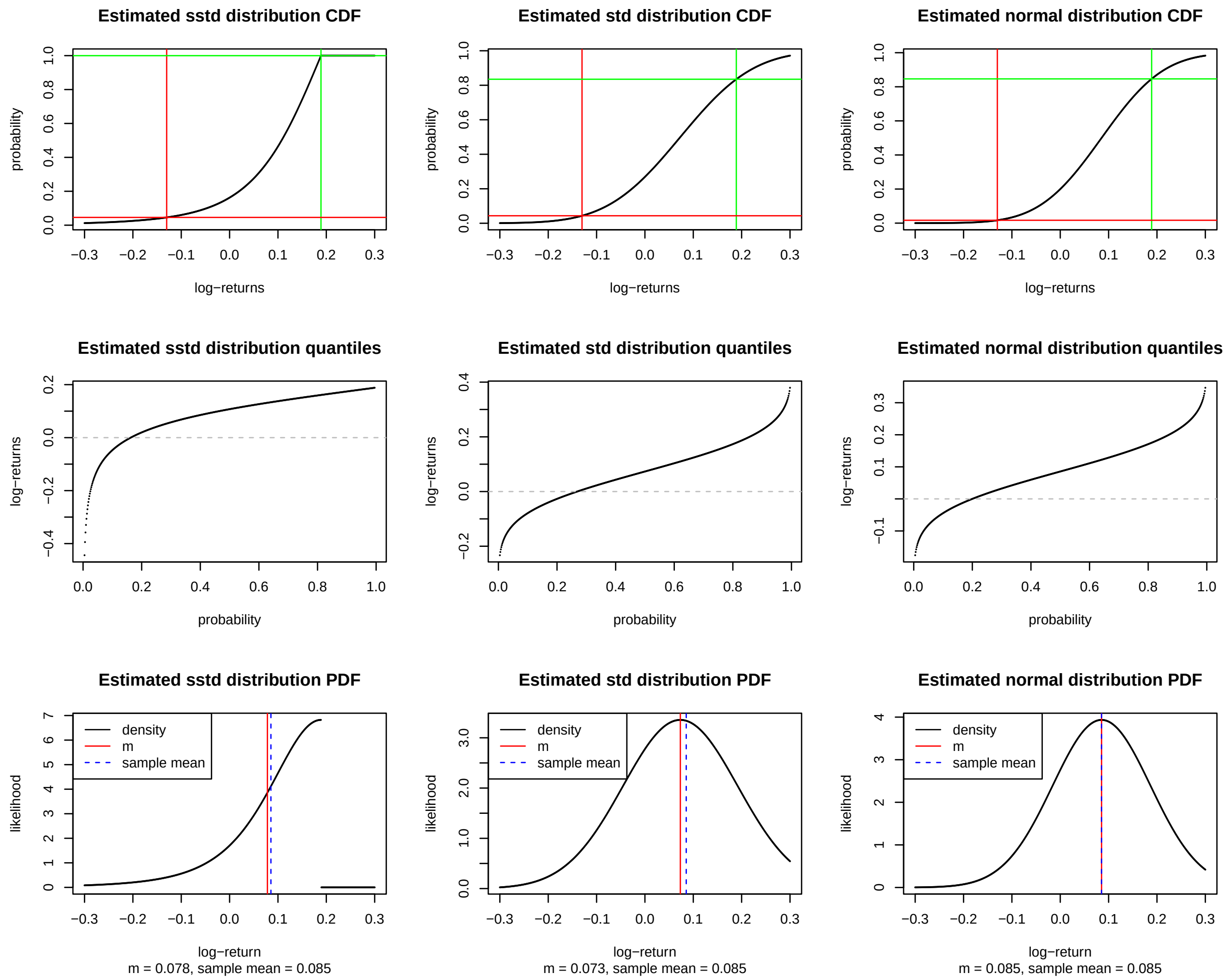
Data vs fit

Let's plot the fit and the observed returns together.



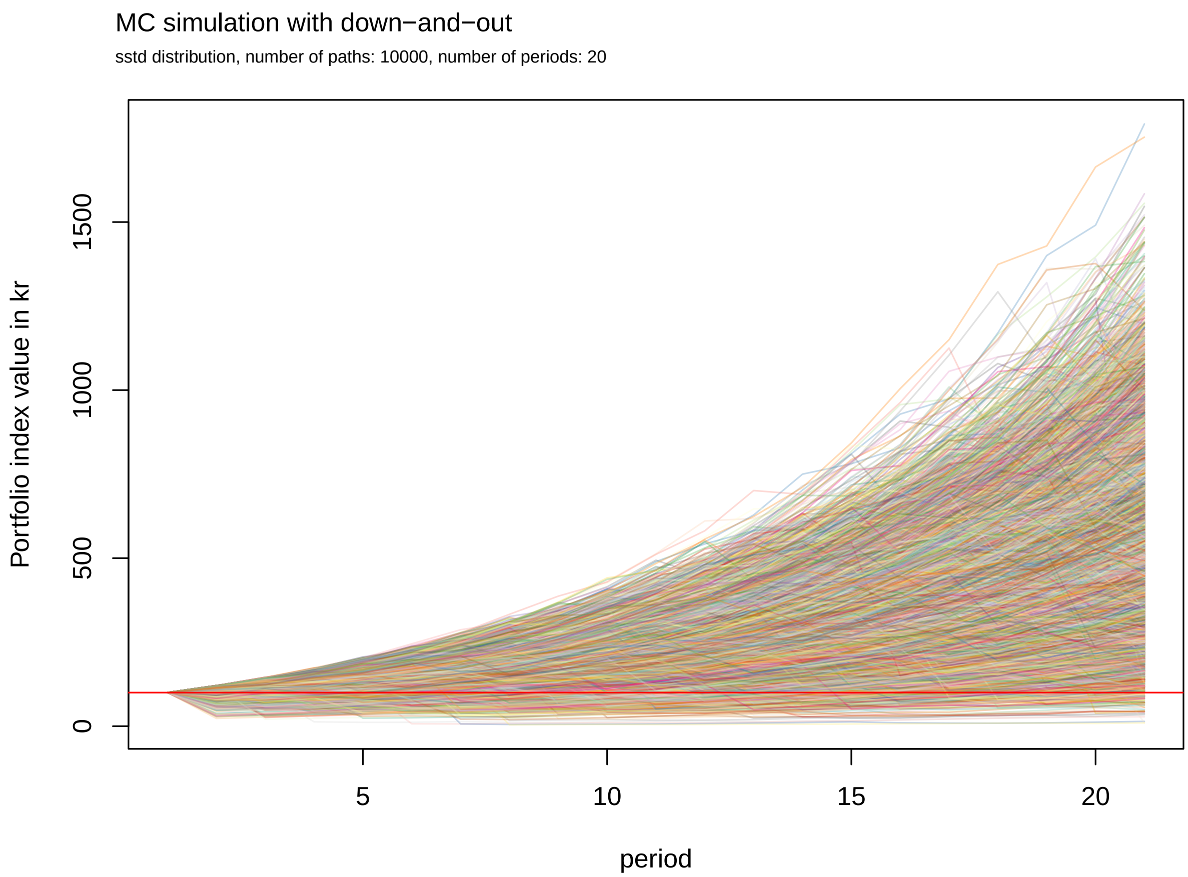
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

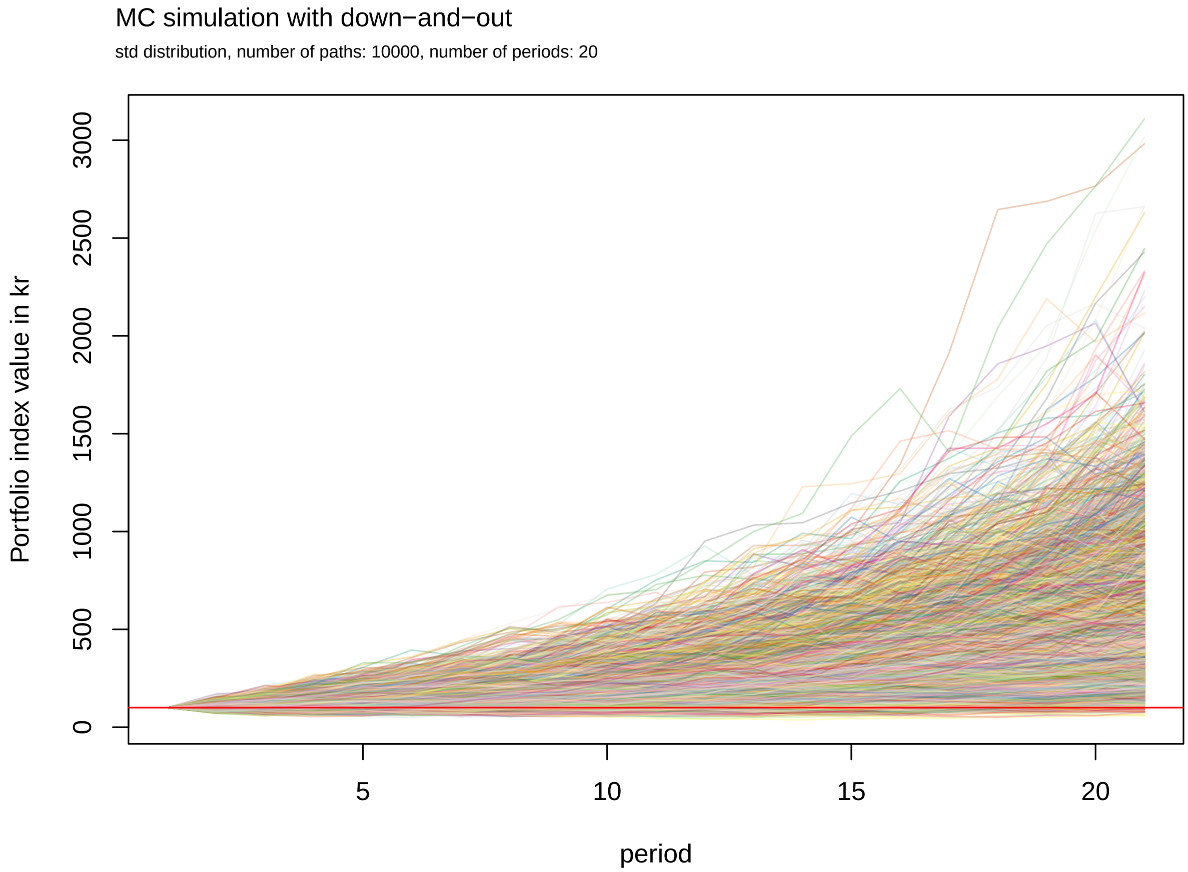


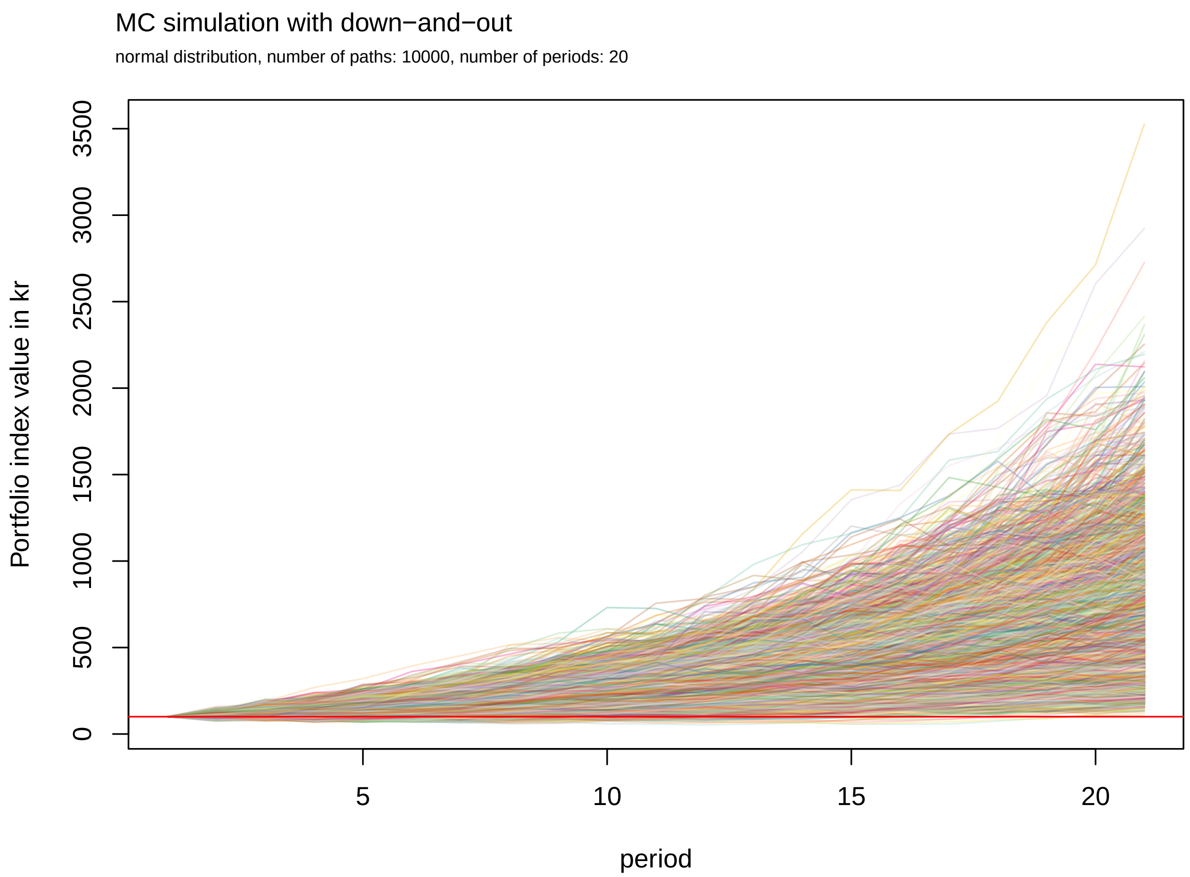
Monte Carlo

phr has the sstd fit with the highest sstd fit with thevalue of nu. Compare with other distributions:

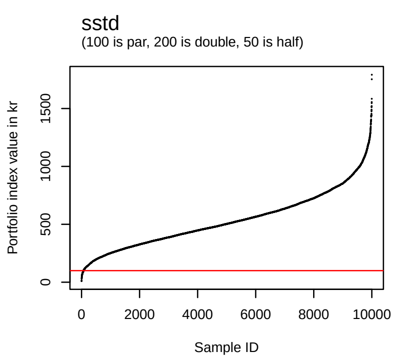


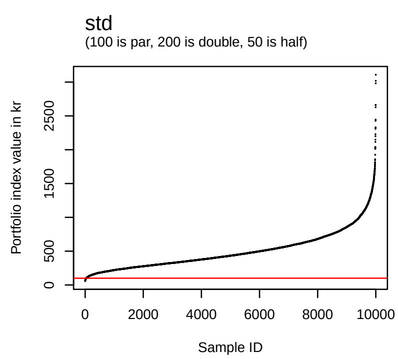
MC simulation with down−and−out
sstd distribution, number of paths: 10000, number of periods: 20
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period

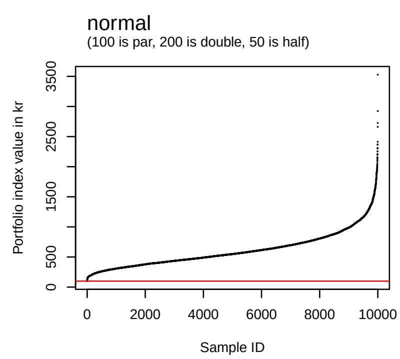

MC simulation with down−and−out
std distribution, number of paths: 10000, number of periods: 20
Portfolio index value in kr
0
500
1000
1500
2000
2500
3000
5
10
15
20
period



Sorted portfolio index values for last period of all runs



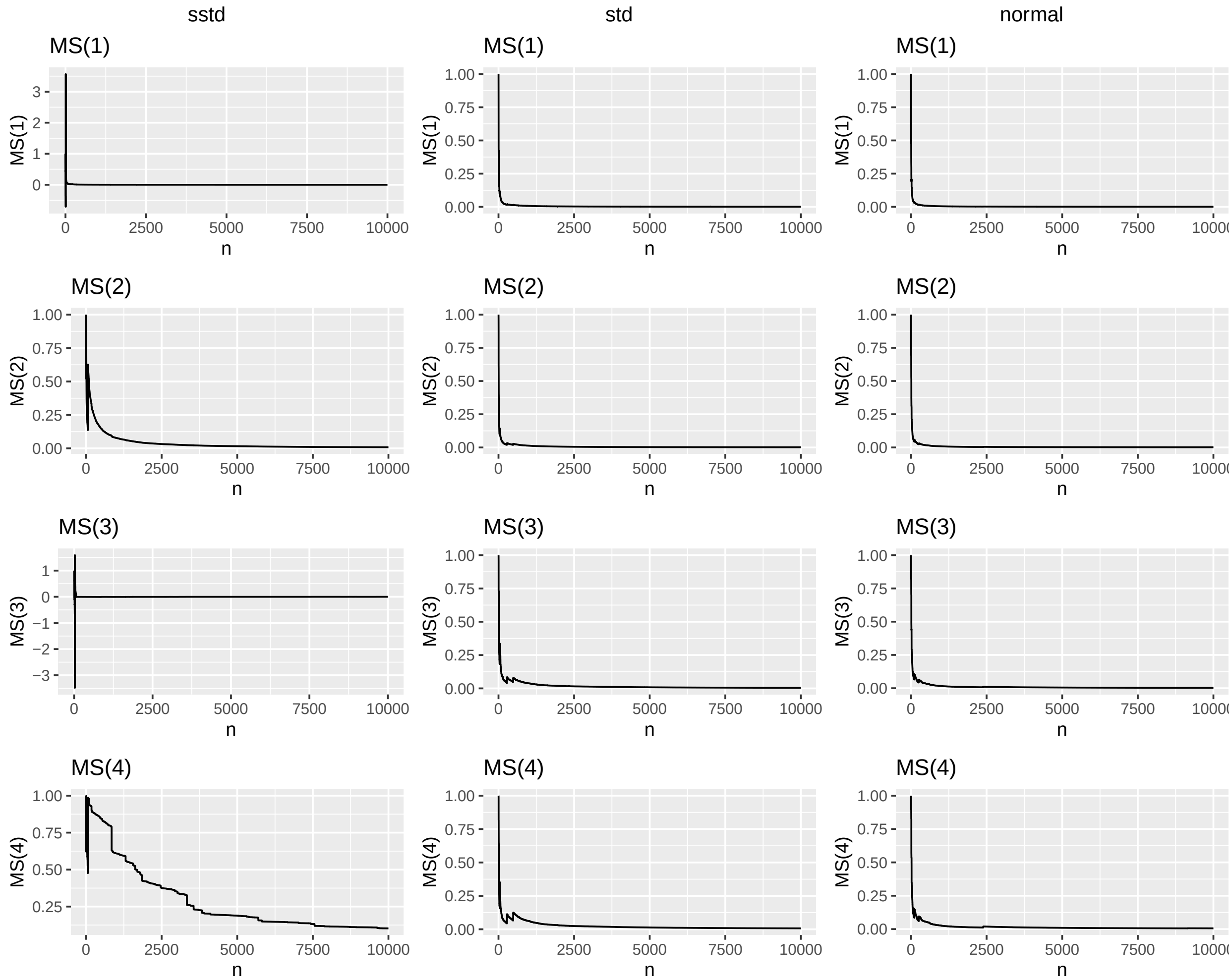




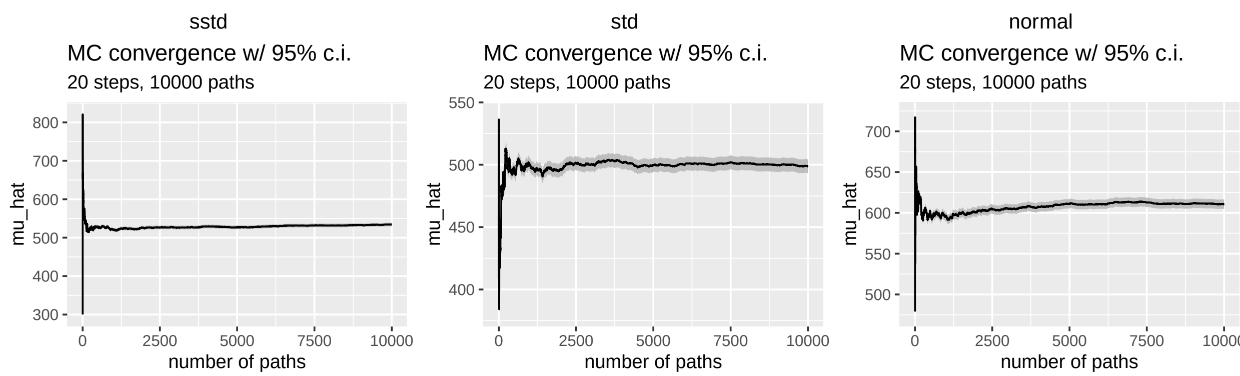
Convergence

Max vs sum

Max vs sum plots for the first four moments:

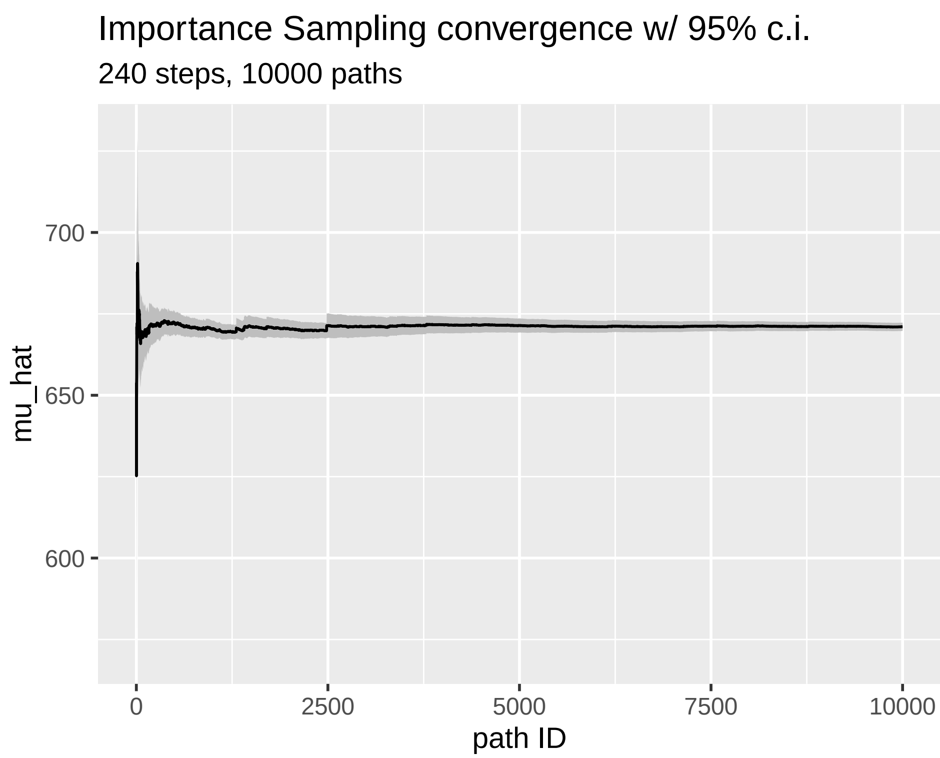


MC



IS

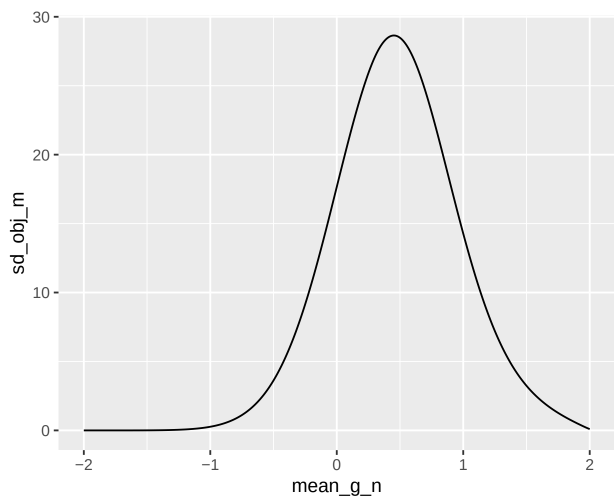
Skewed t-distribution with a normal proposal distribution.

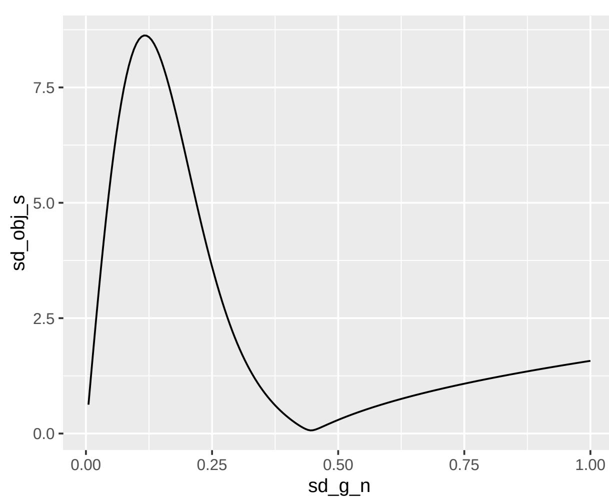


Parameters

```
## [1] 2.0162301 0.4463226
```

Objective function plots

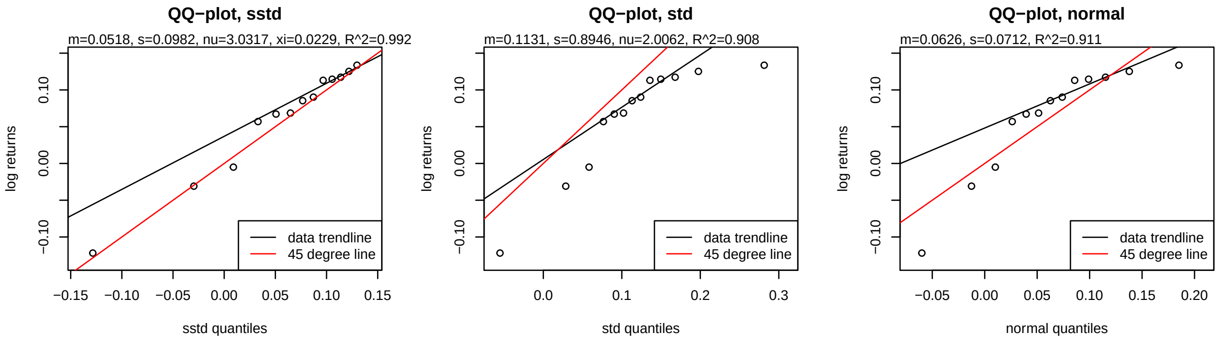




Mix medium risk (mmr), 2011 - 2023

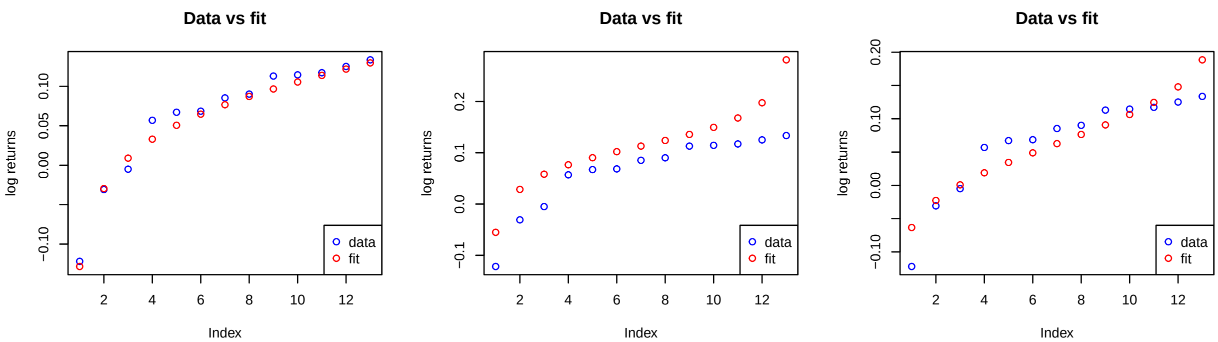
QQ Plot

Skewed t-distribution (sstd):



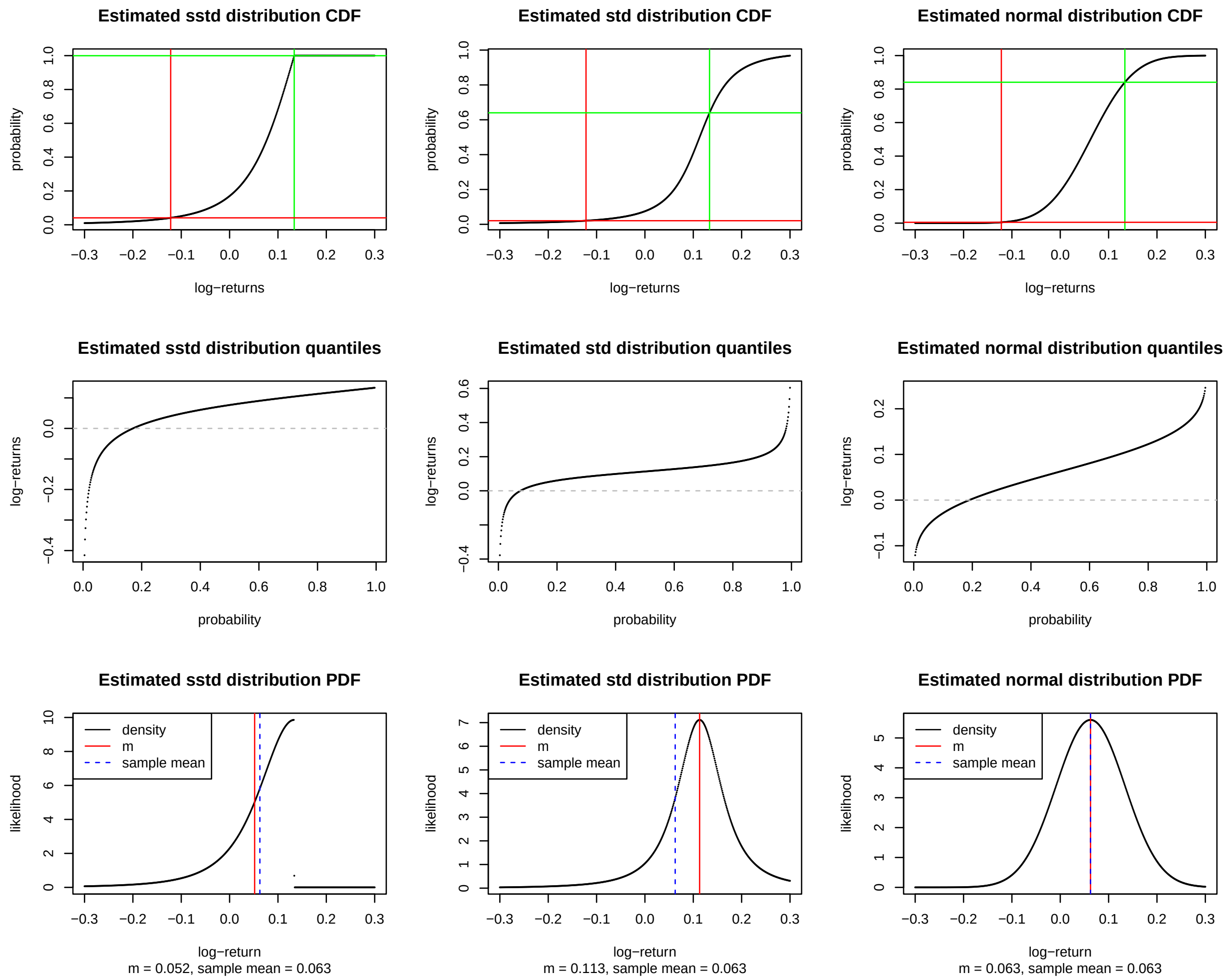
Data vs fit

Let’s plot the fit and the observed returns together.



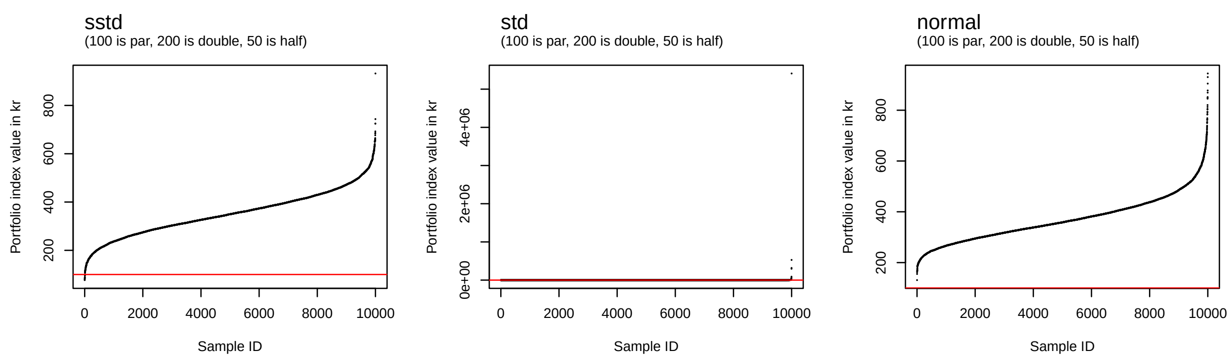
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

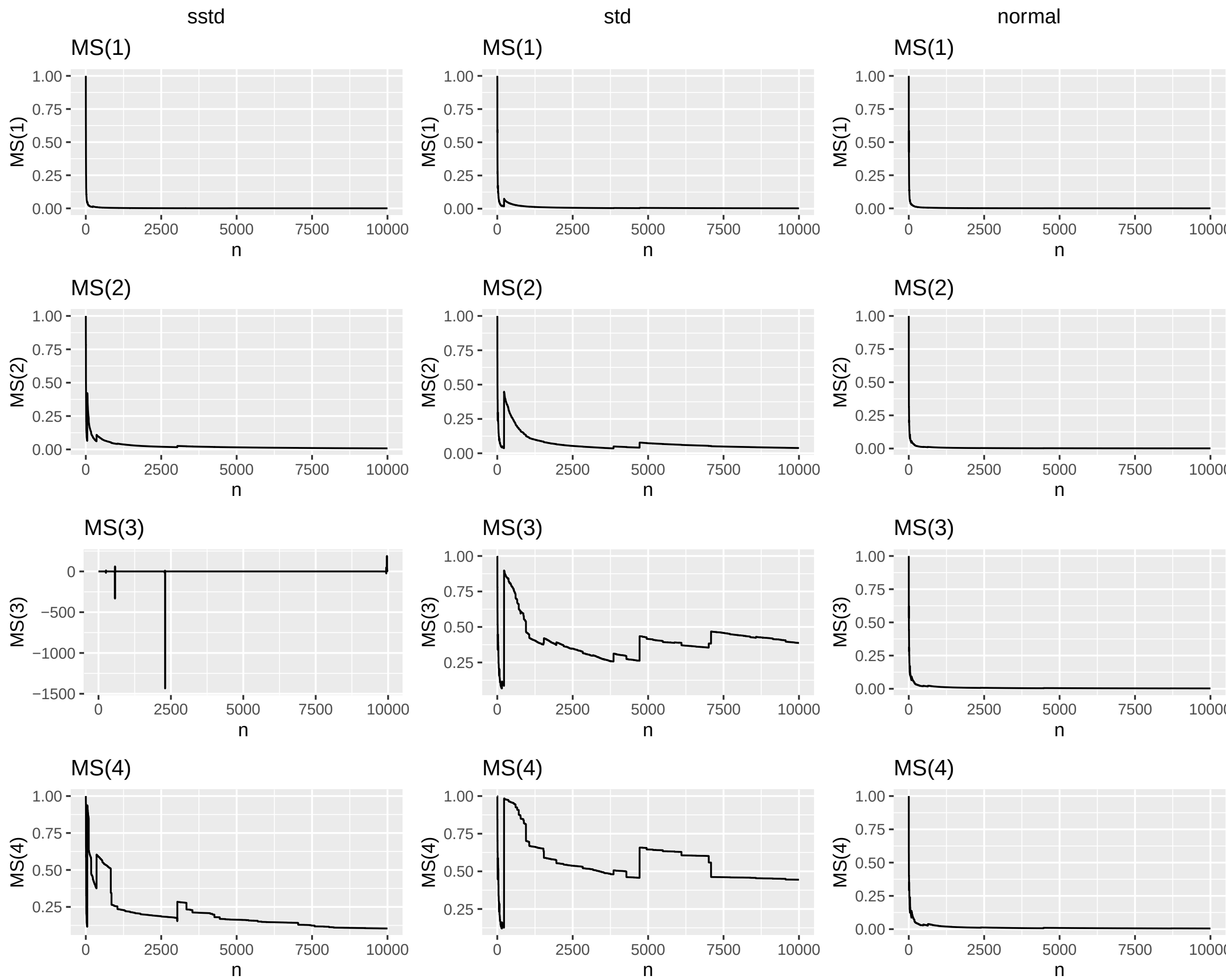
Sorted portfolio index values for last period of all runs



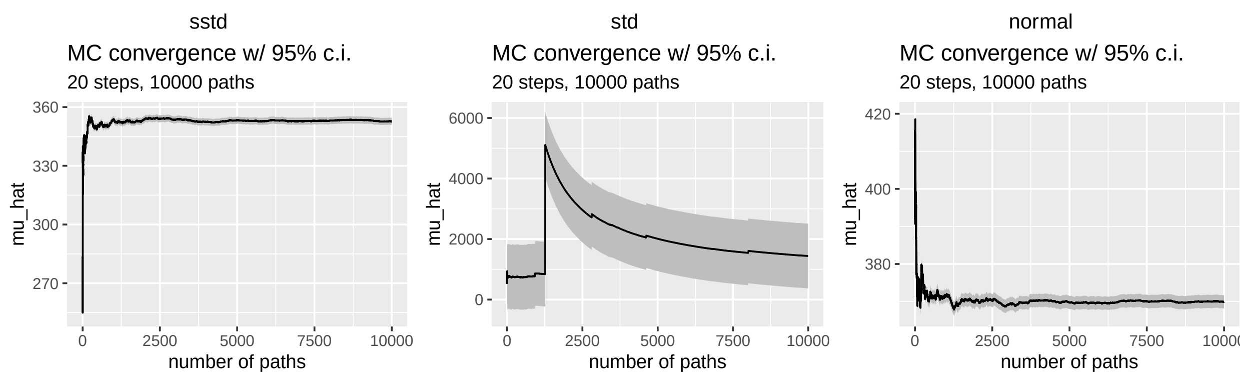
Convergence

Max vs sum

Max vs sum plots for the first four moments:

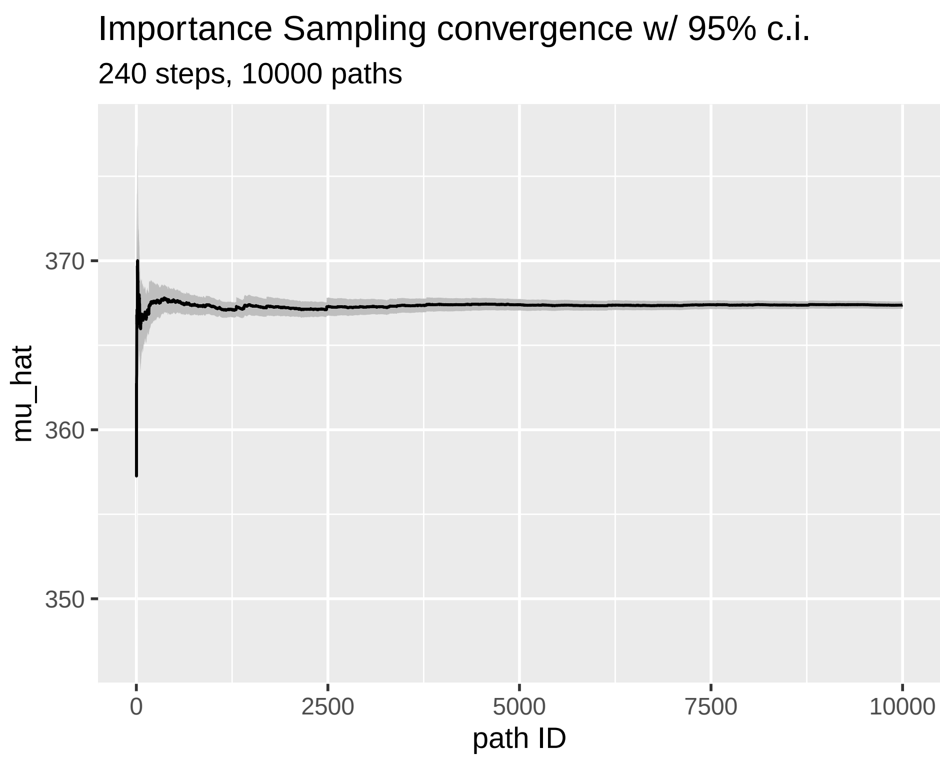


MC



IS

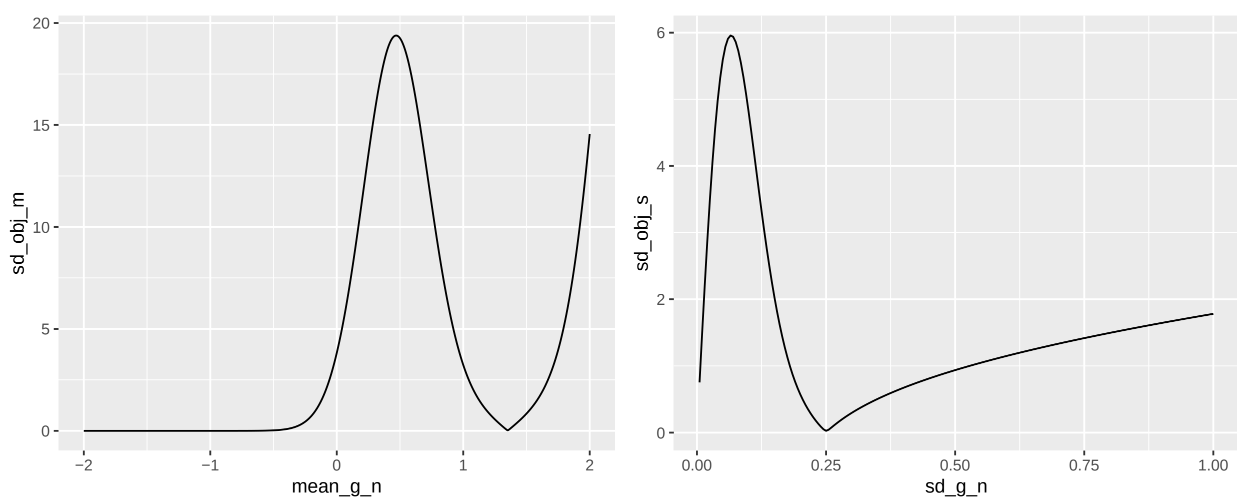
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.3516393 0.2503782
```

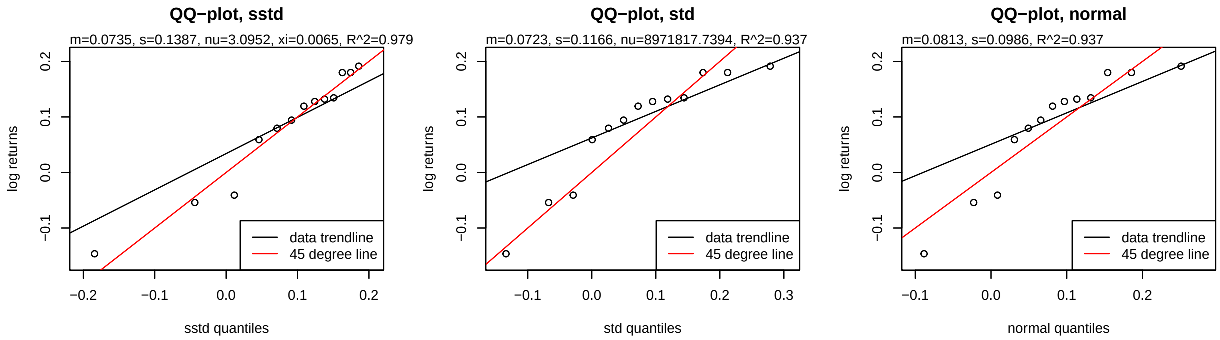
Objective function plots



Mix high risk (mhr), 2011 - 2023

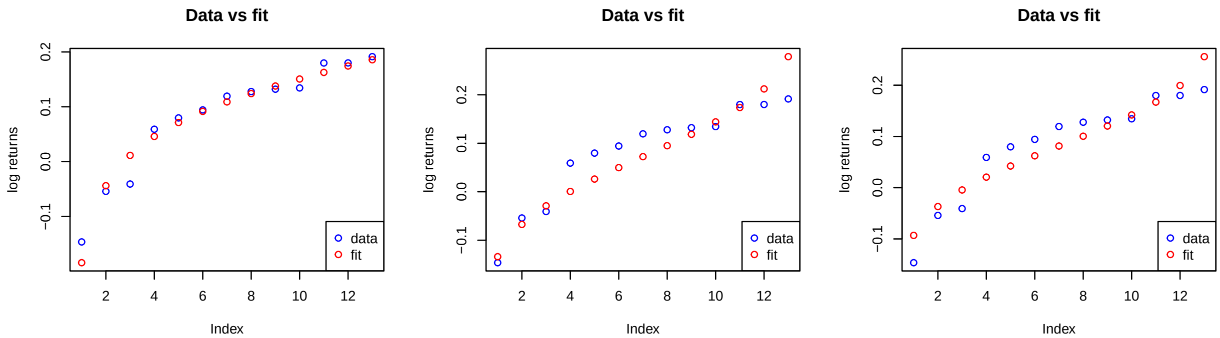
QQ Plot

Skewed t-distribution (sstd):



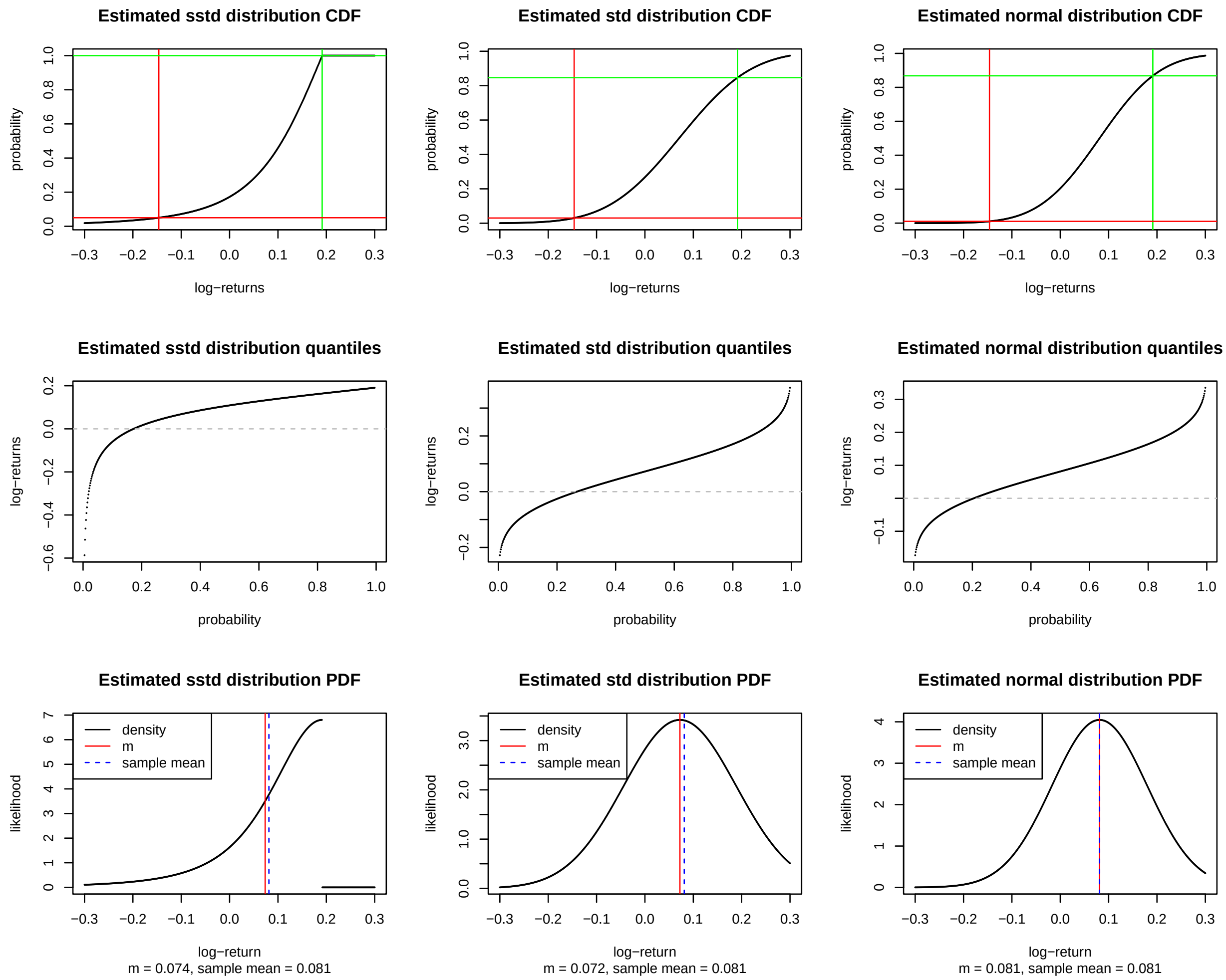
Data vs fit

Let's plot the fit and the observed returns together.



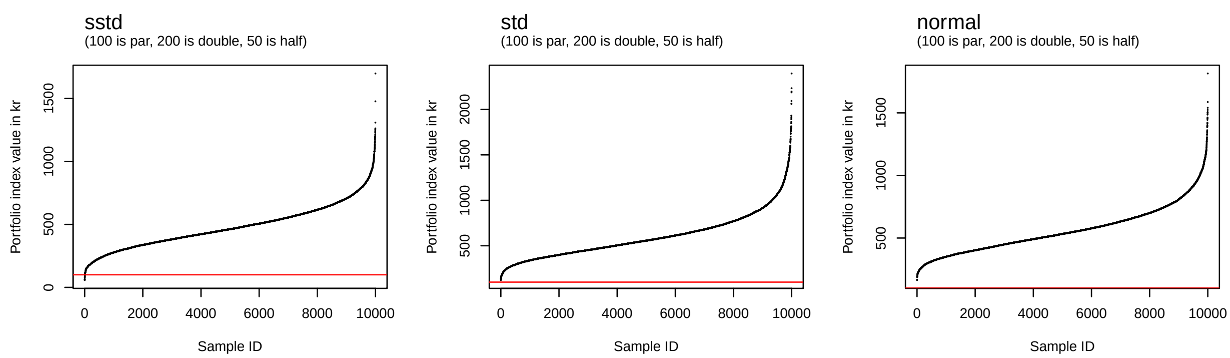
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

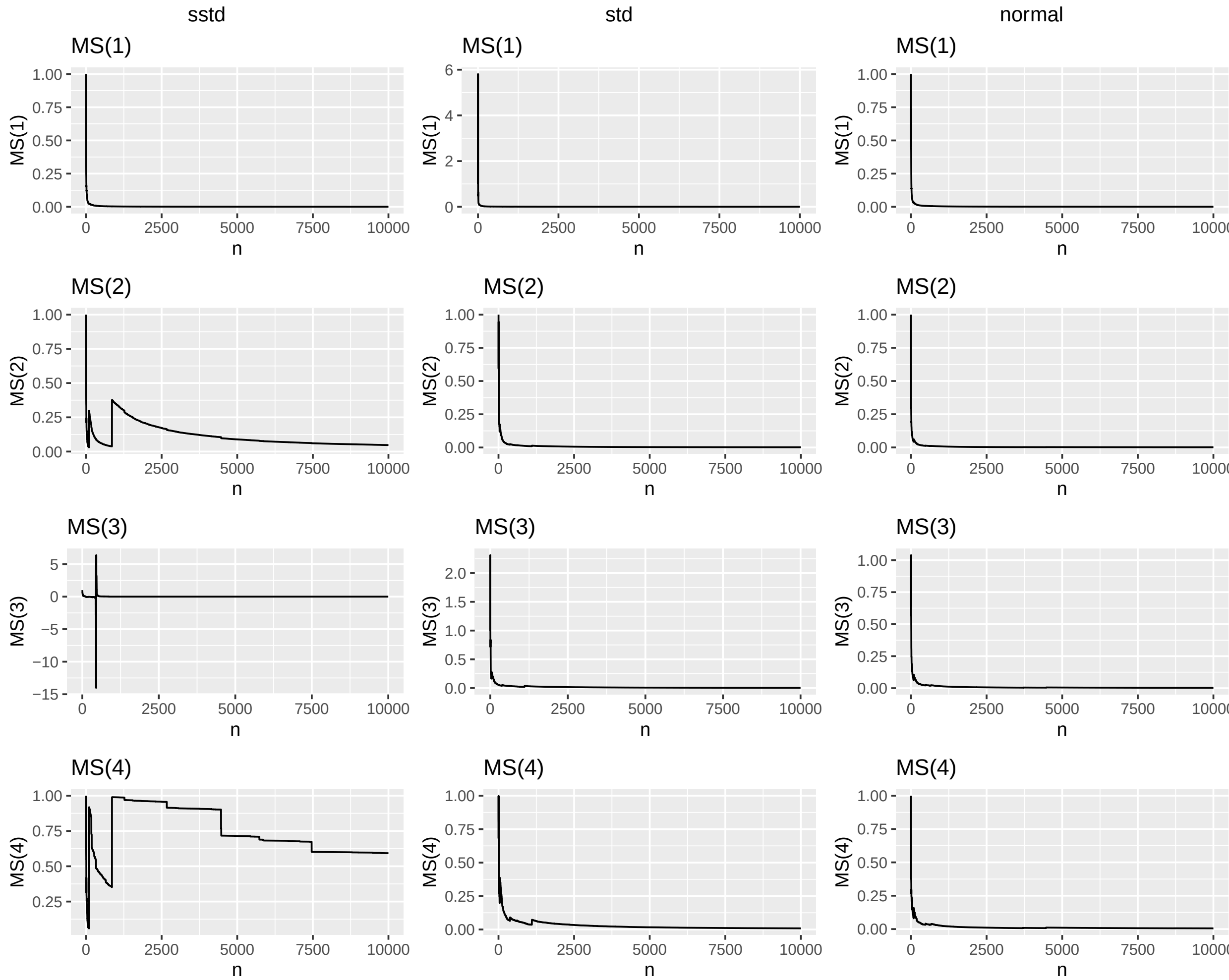
Sorted portfolio index values for last period of all runs



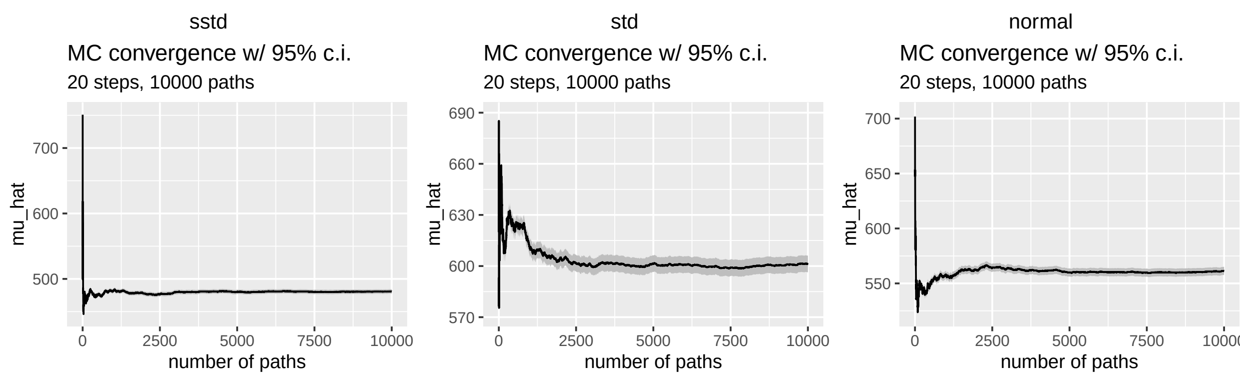
Convergence

Max vs sum

Max vs sum plots for the first four moments:

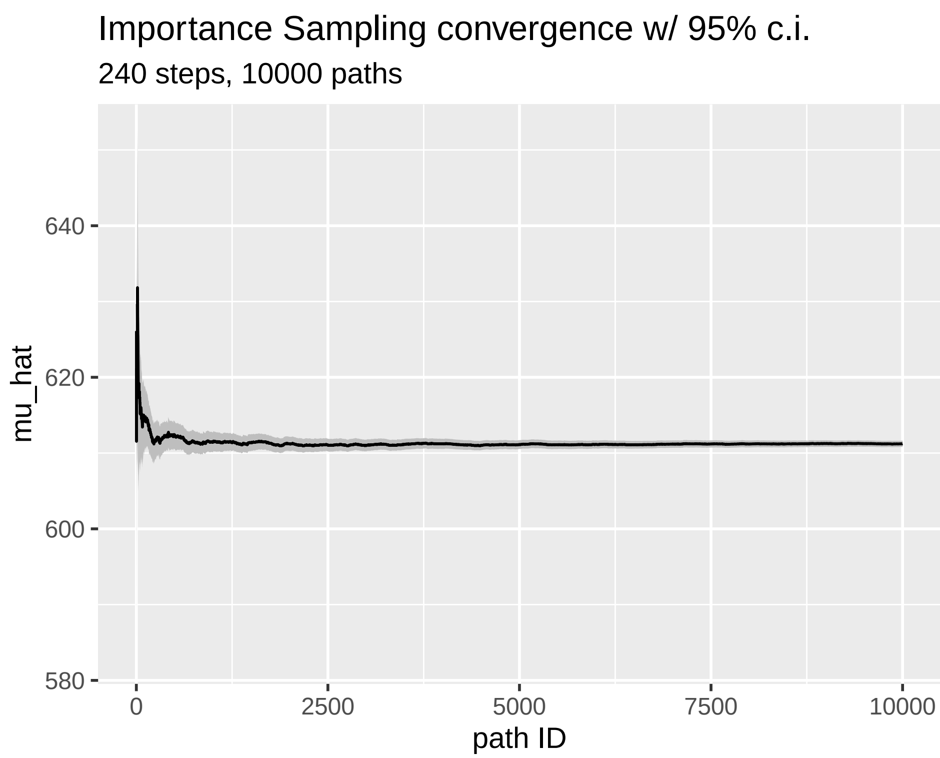


MC



IS

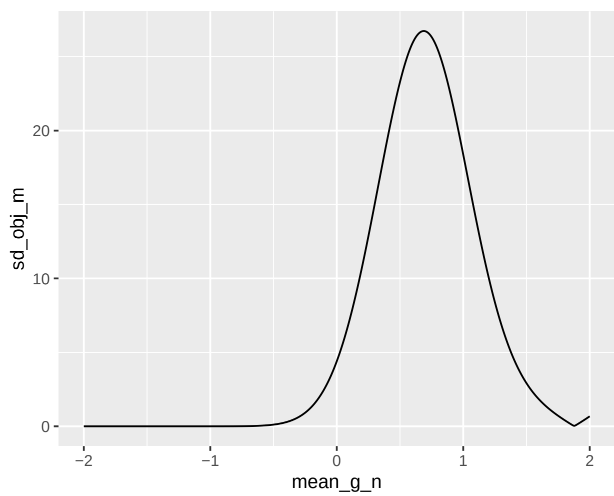
Skewed t-distribution with a normal proposal distribution.

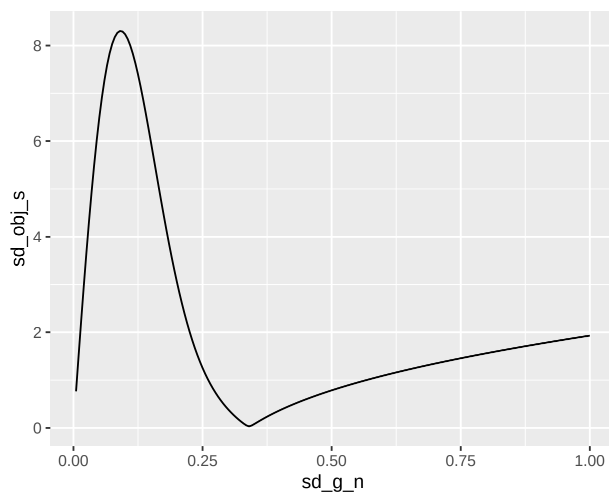


Parameters

```
## [1] 1.8775189 0.3400818
```

Objective function plots

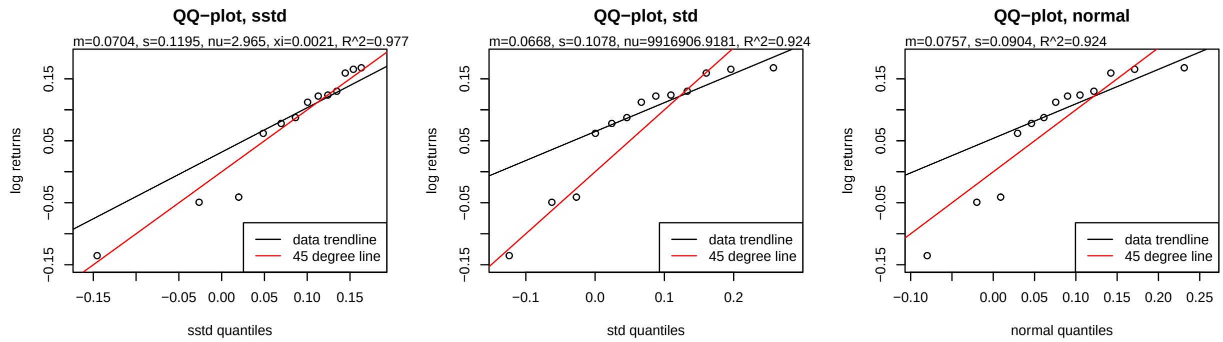




Mix vmr+phr (vm_ph), 2011 - 2023

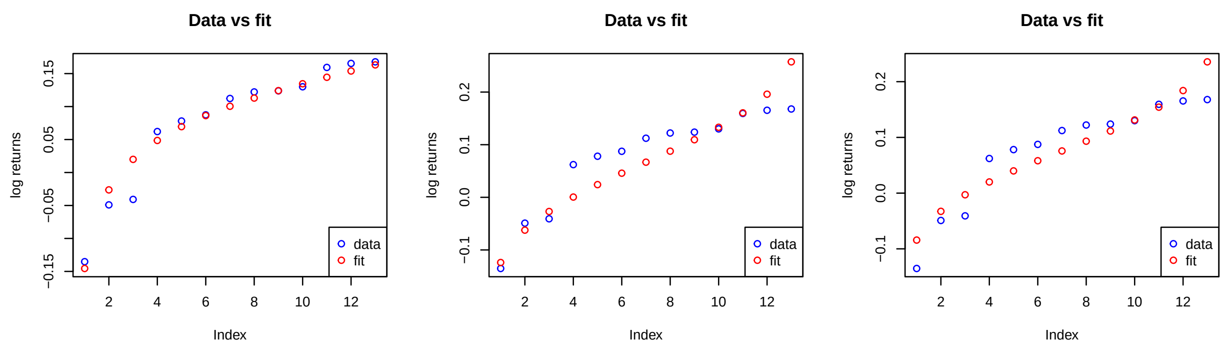
QQ Plot

Skewed t-distribution (sstd):



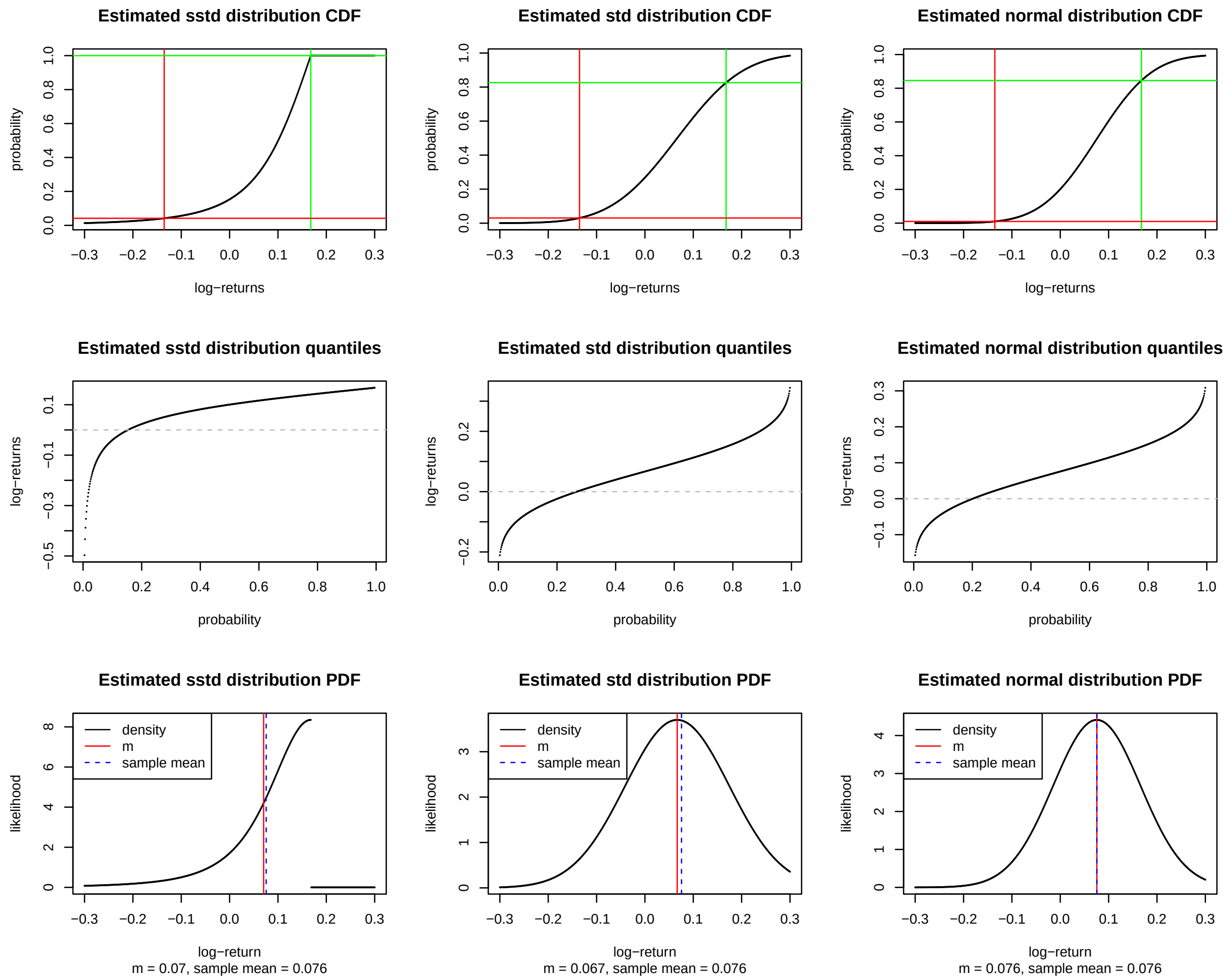
Data vs fit

Let's plot the fit and the observed returns together.



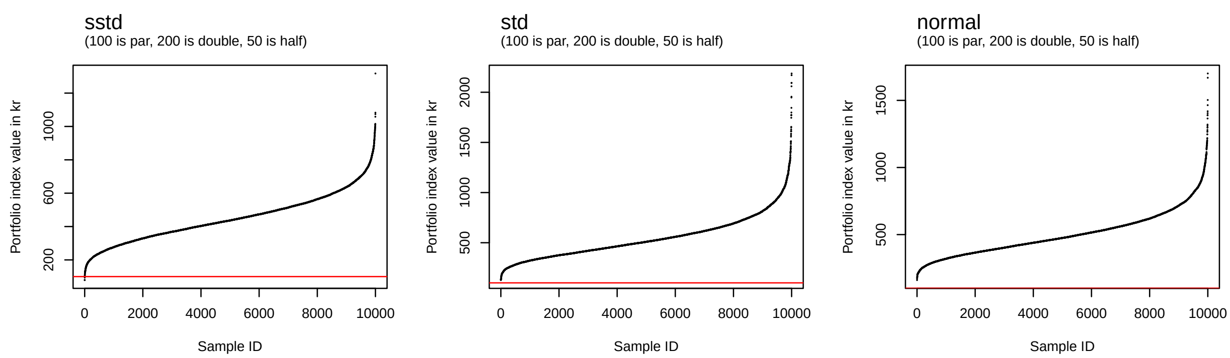
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

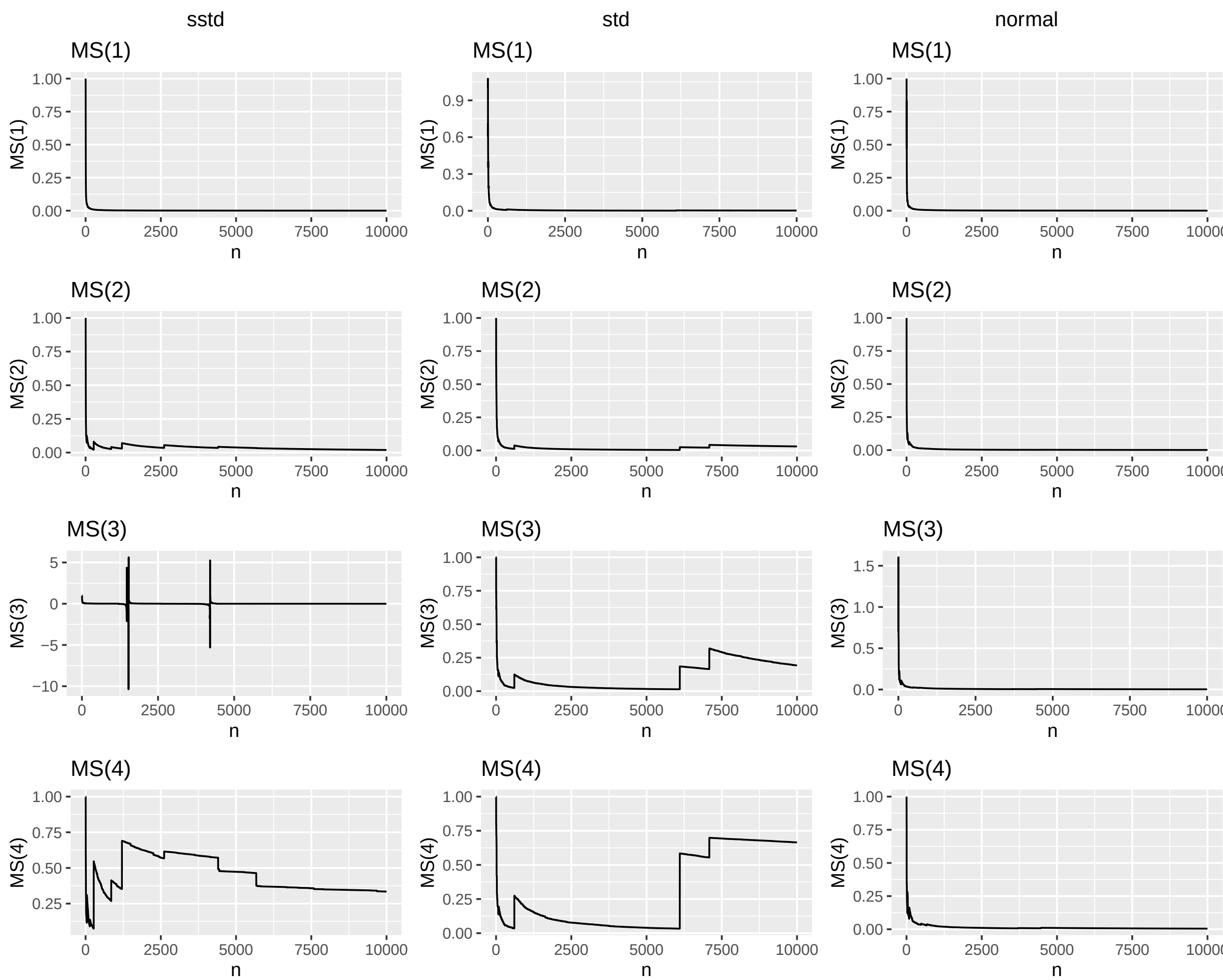
Sorted portfolio index values for last period of all runs



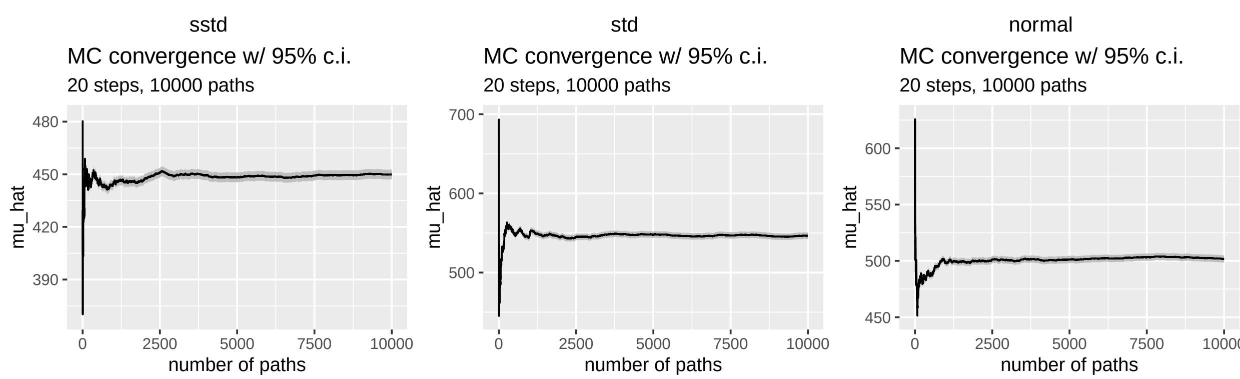
Convergence

Max vs sum

Max vs sum plots for the first four moments:

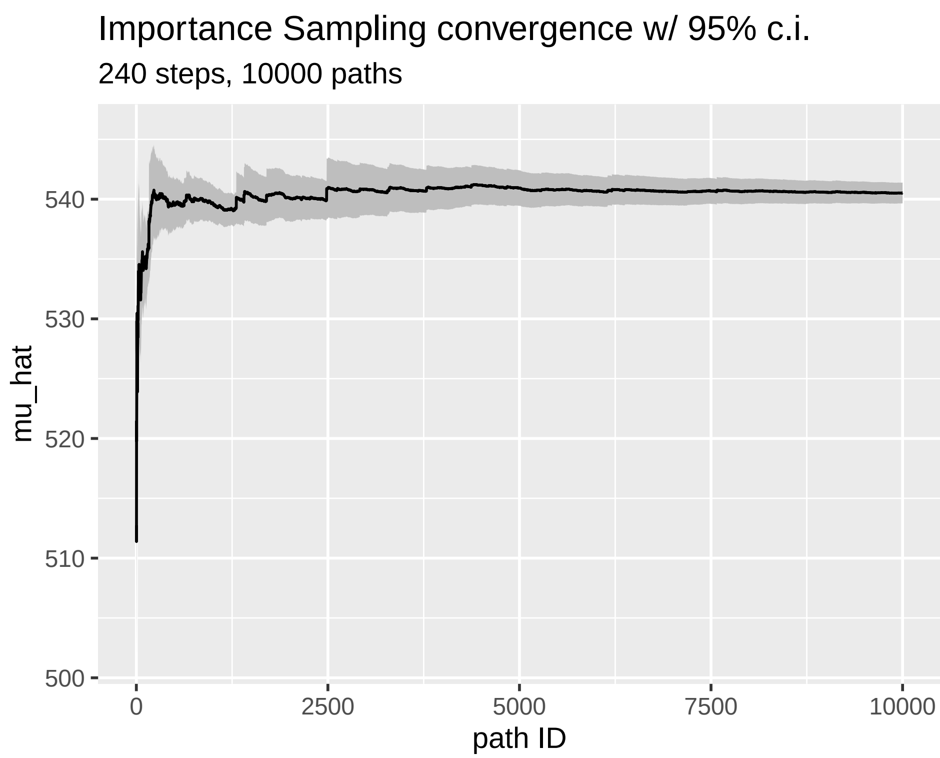


MC



IS

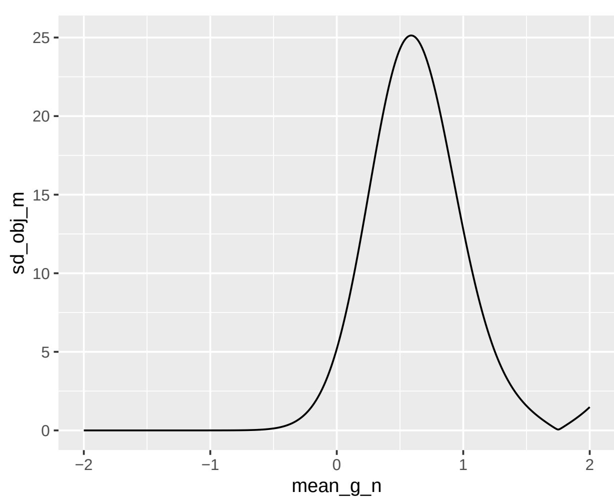
Skewed t-distribution with a normal proposal distribution.

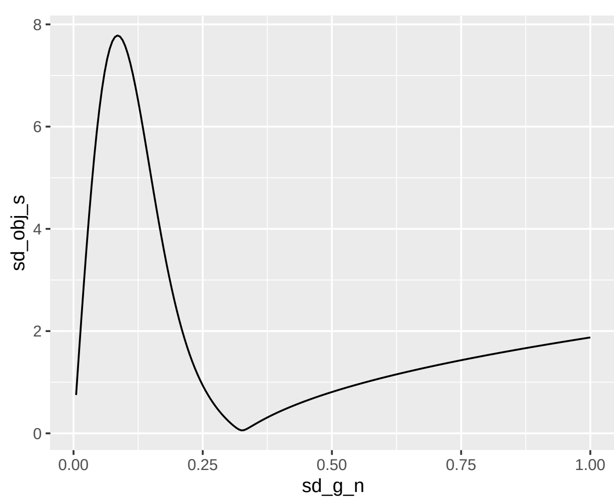


Parameters

```
## [1] 1.7507161 0.3263777
```

Objective function plots

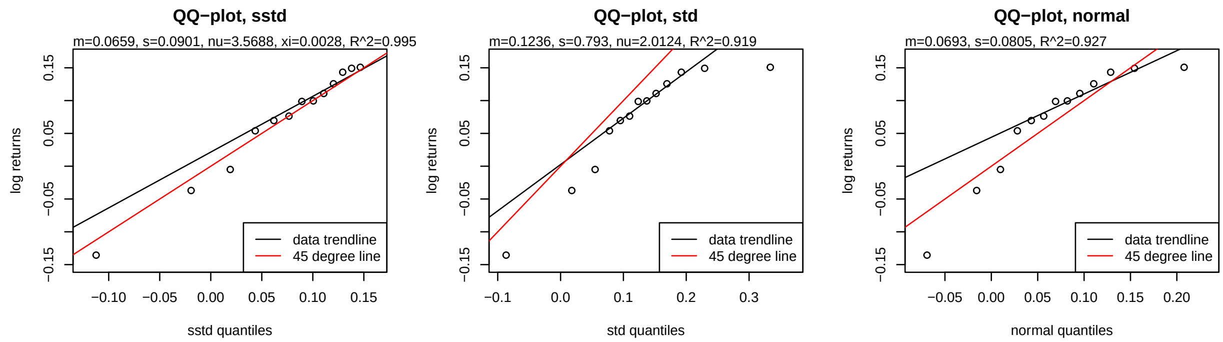




Mix vhr+pmr (mh_pm), 2011 - 2023

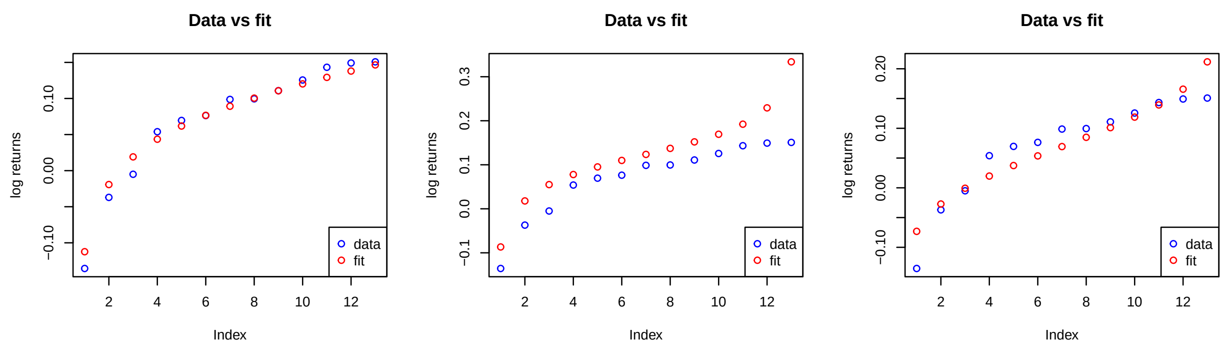
QQ Plot

Skewed t-distribution (sstd):



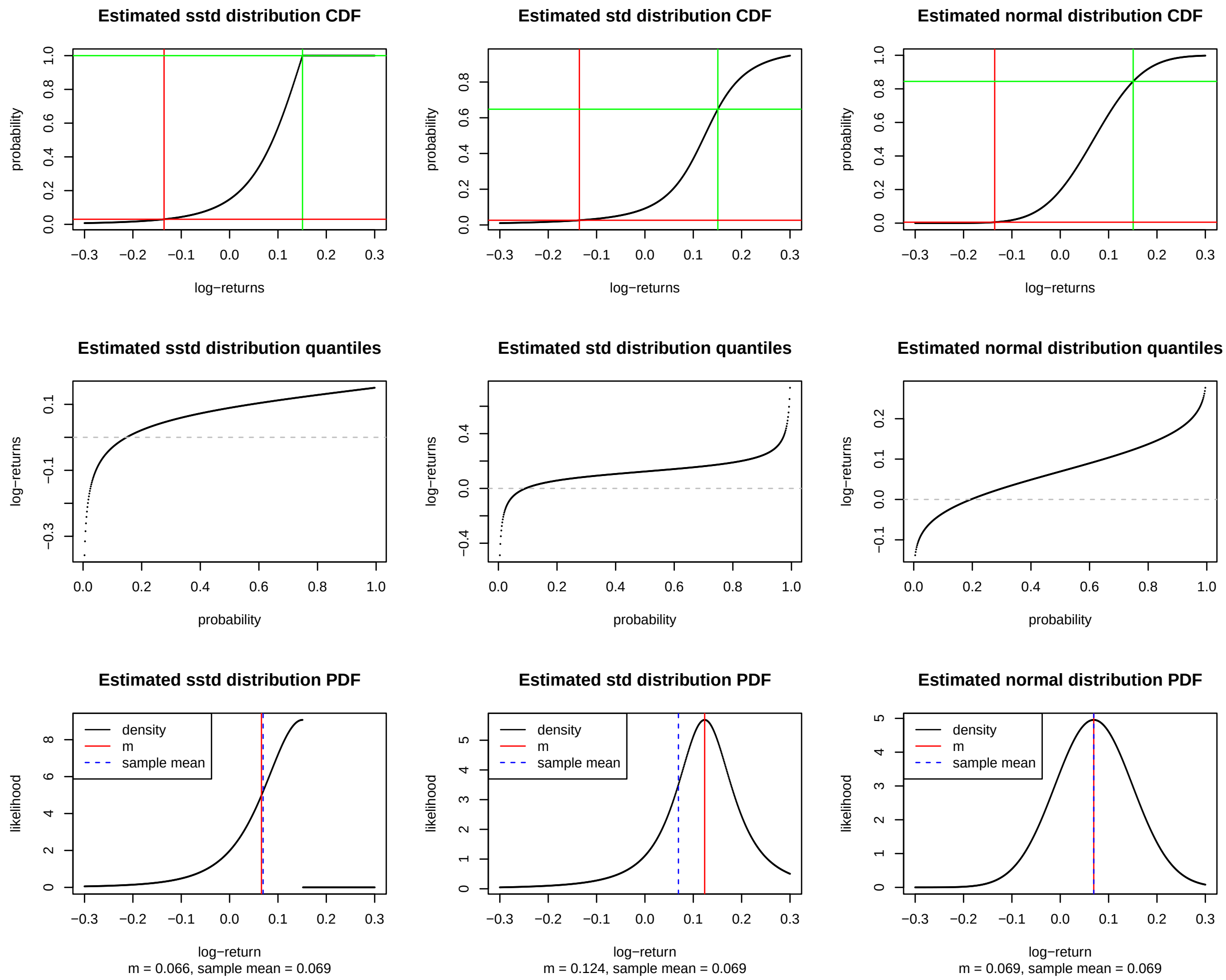
Data vs fit

Let’s plot the fit and the observed returns together.



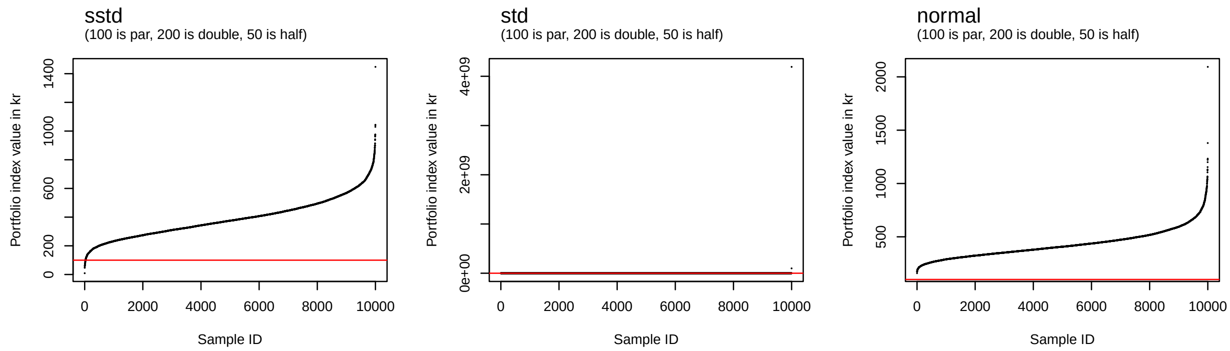
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

Sorted portfolio index values for last period of all runs



Convergence

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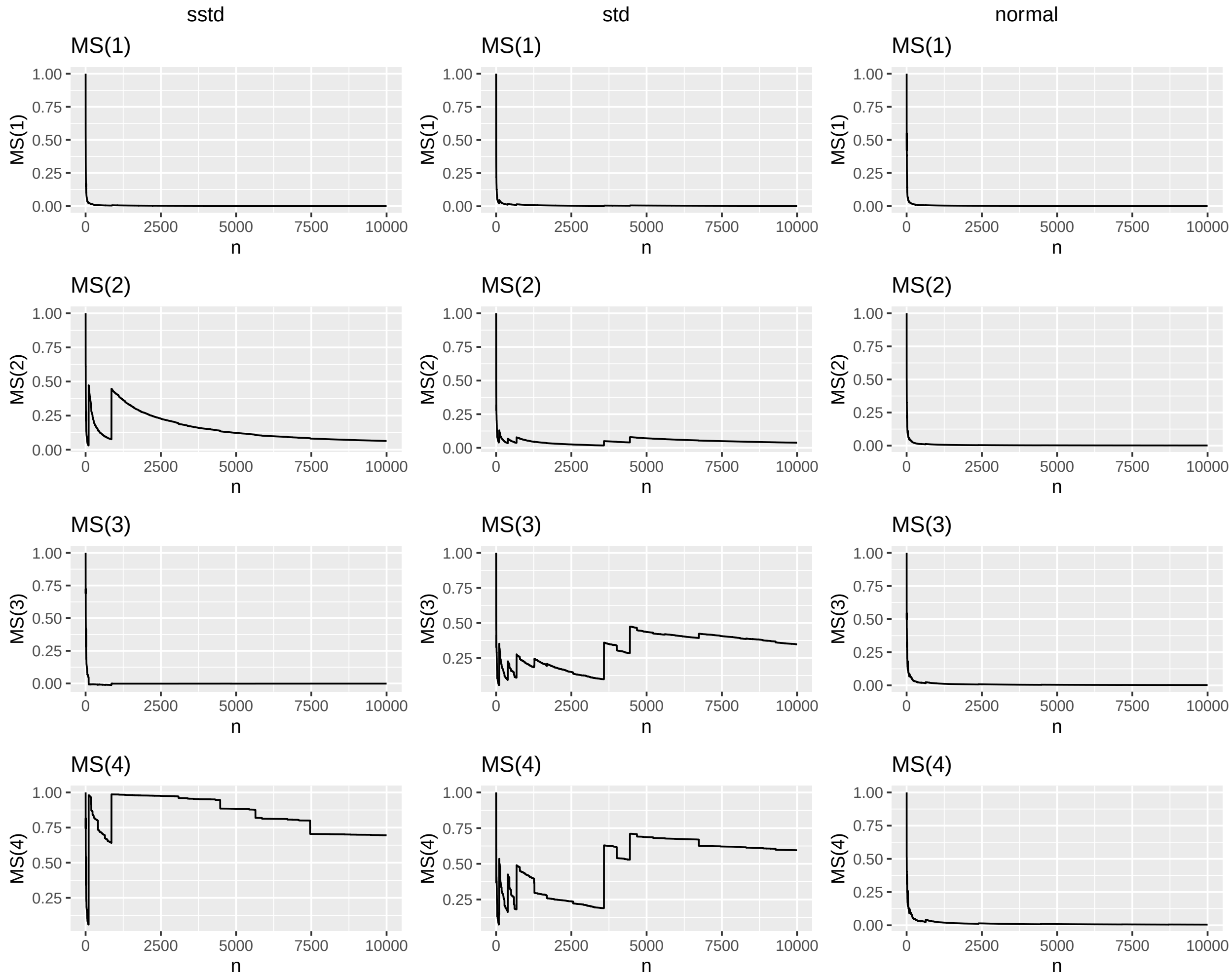
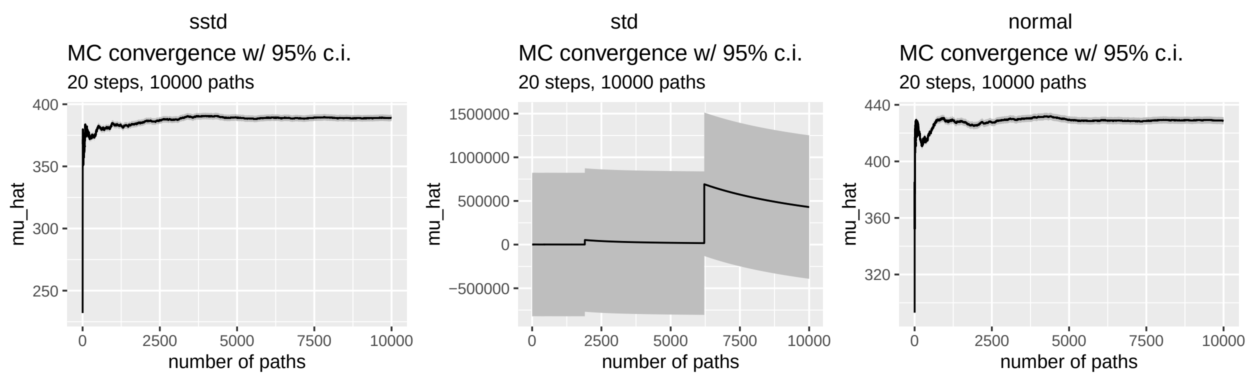
Max vs sum plots for the first four moments:

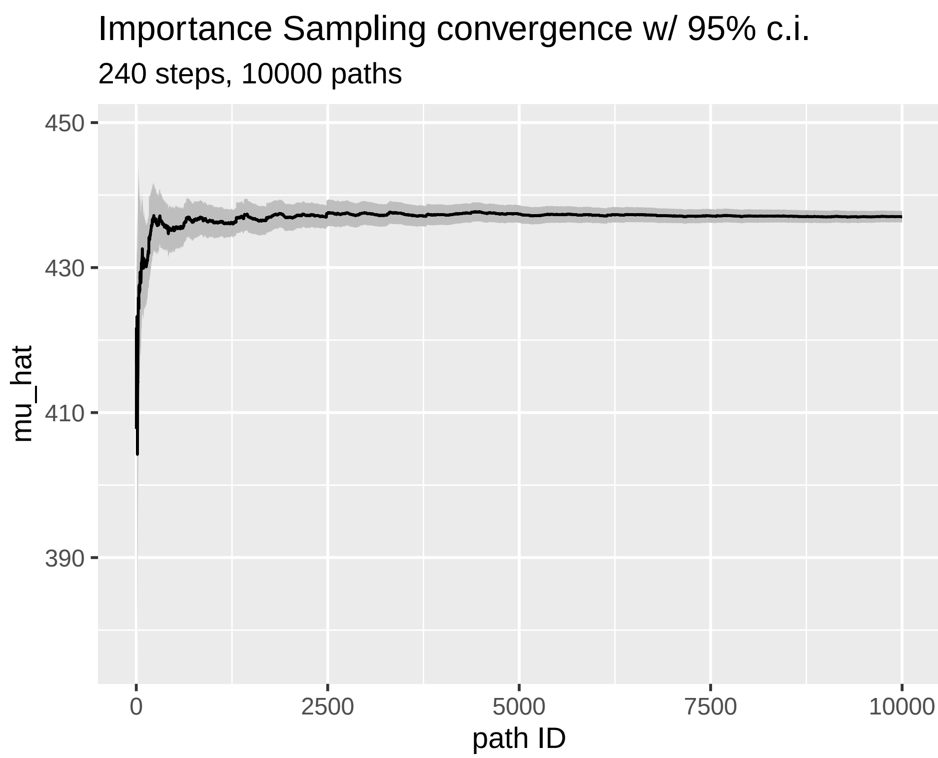


MC



IS

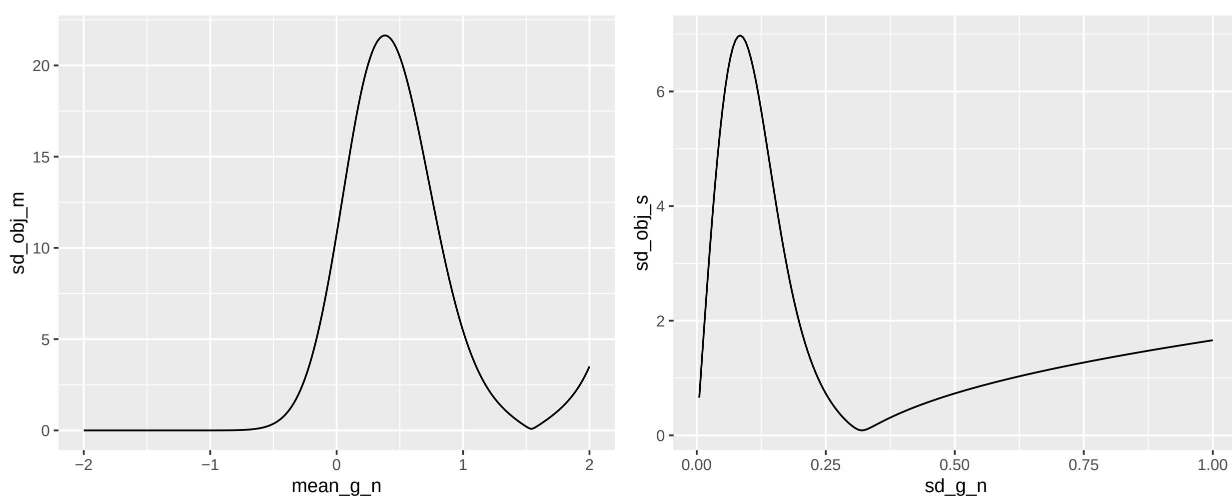
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.540135 0.320090
```

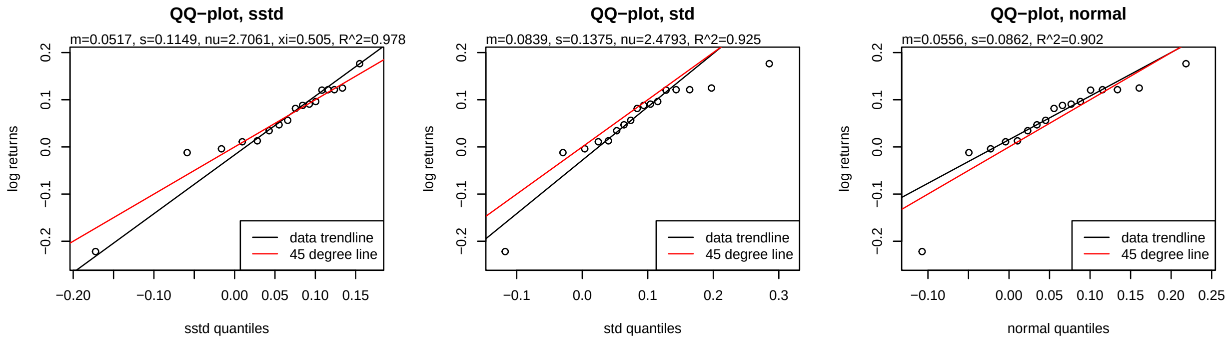
Objective function plots



Velliv medium risk (vmr), 2011 - 2023

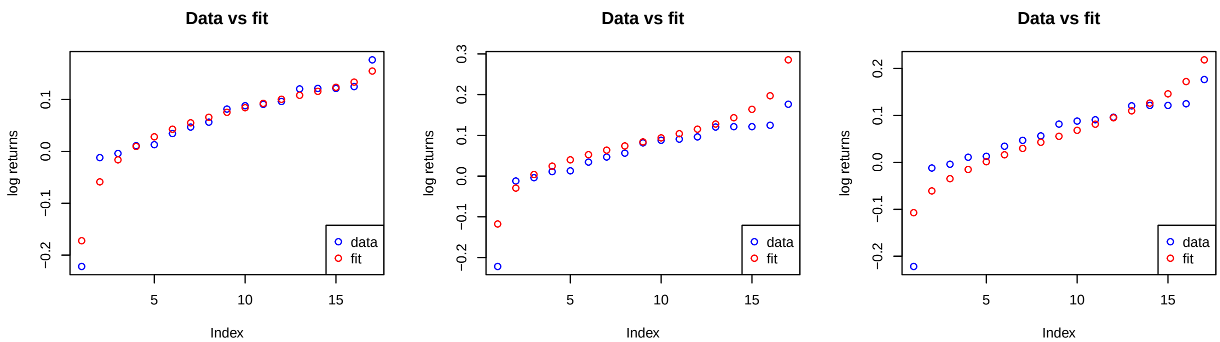
QQ Plot

Skewed t-distribution (sstd):



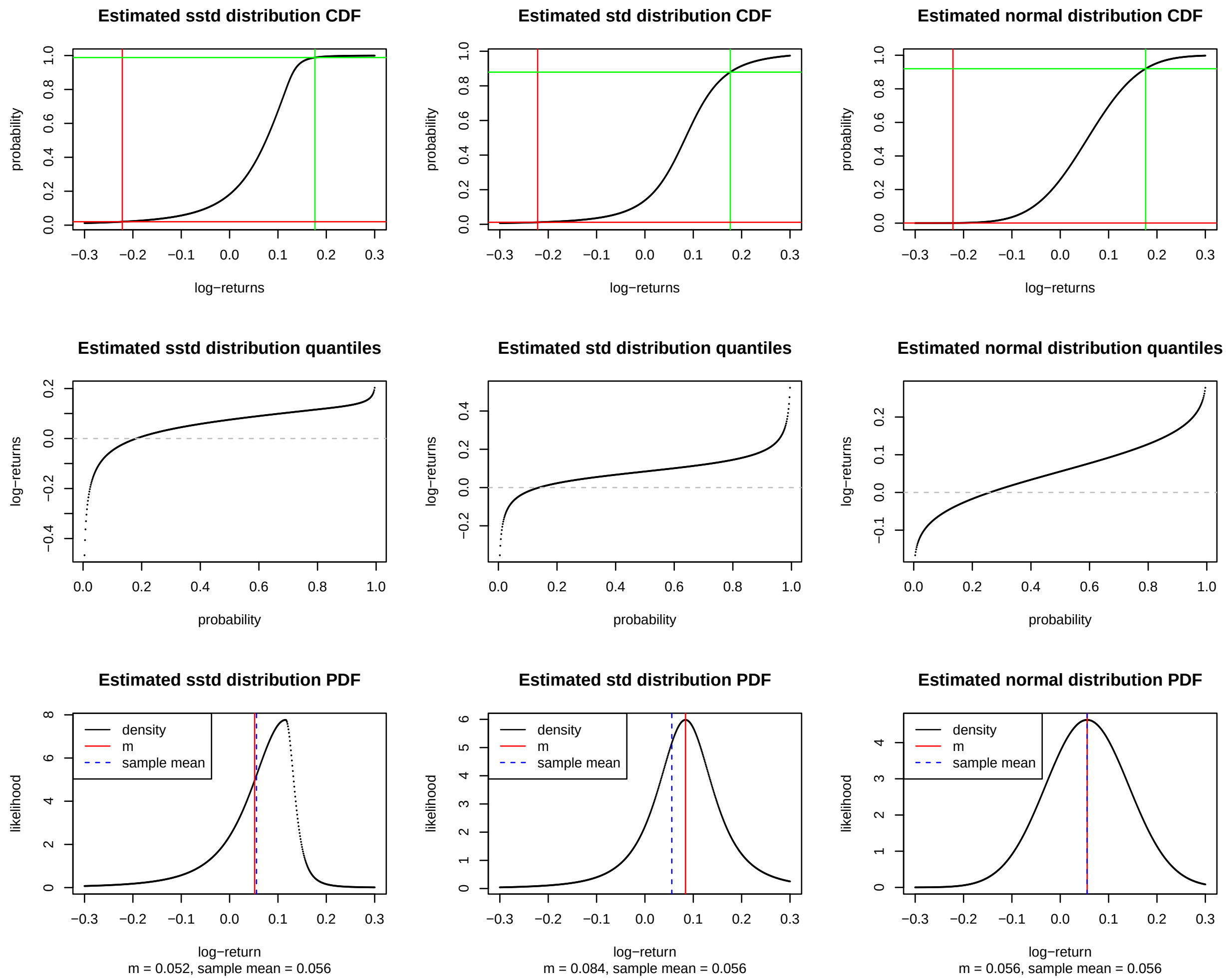
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

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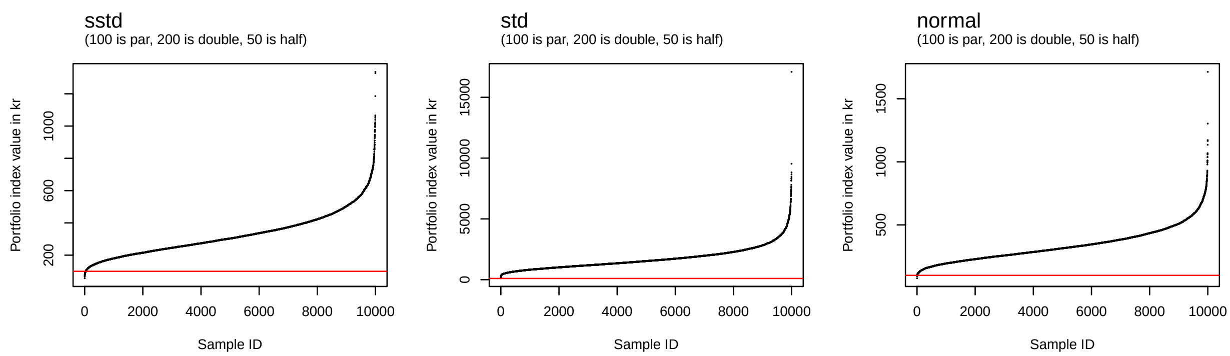

Convergence

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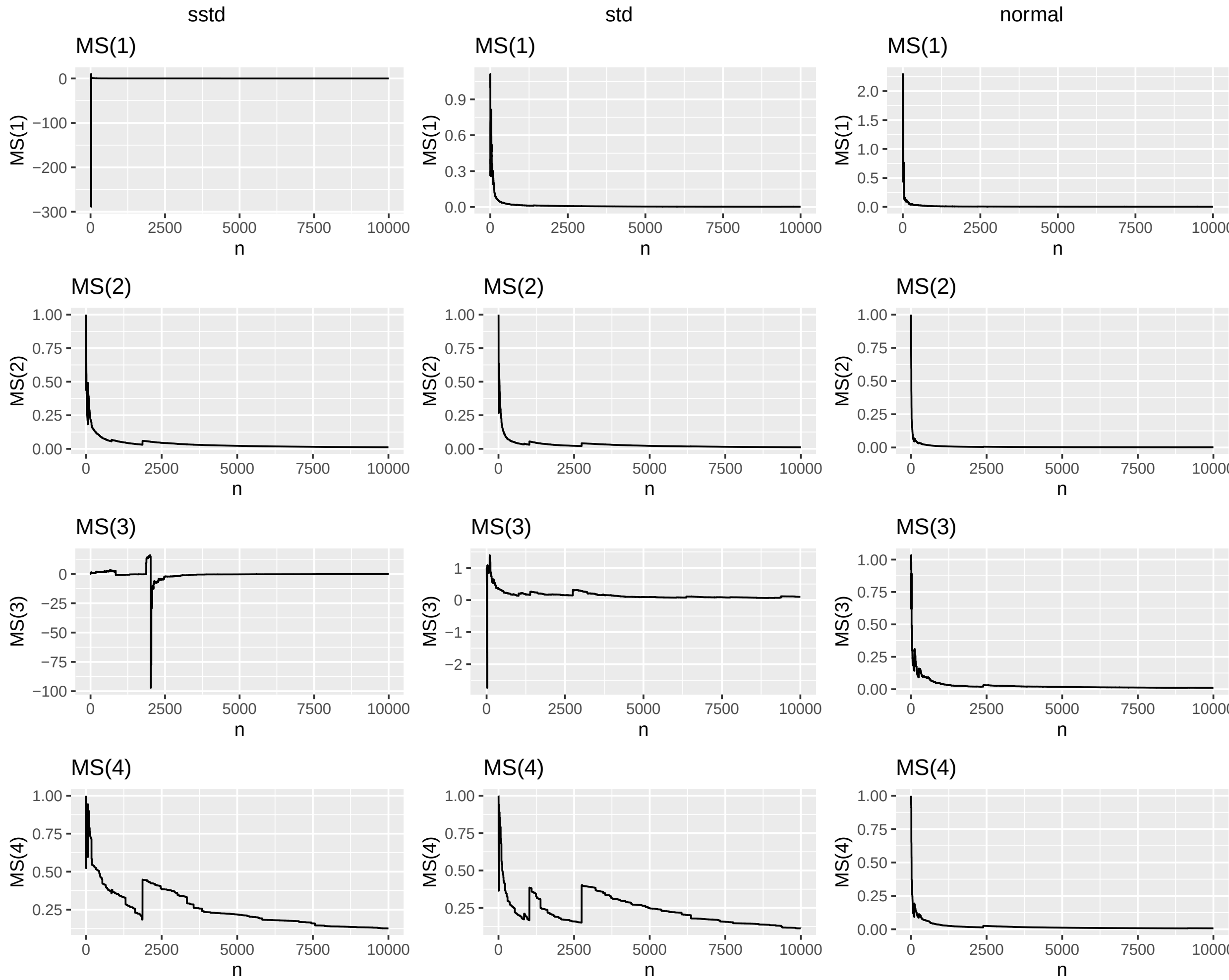
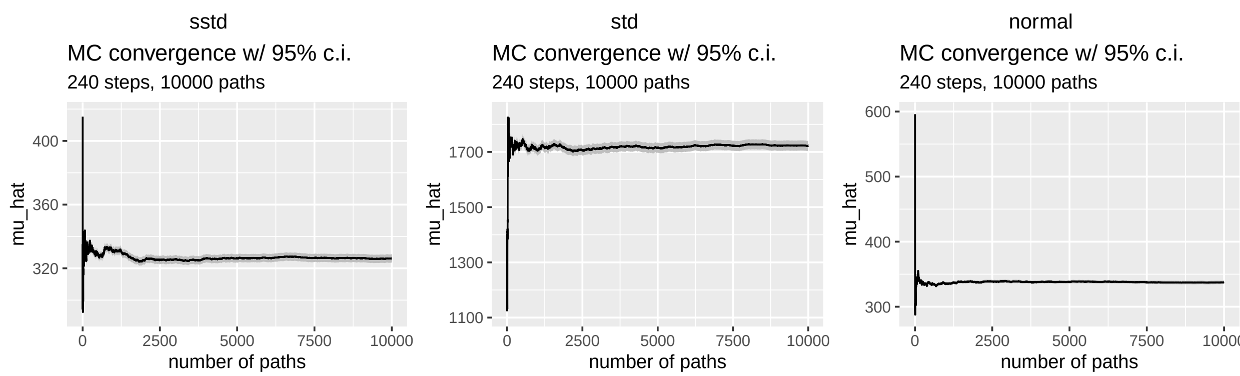
Max vs sum plots for the first four moments:

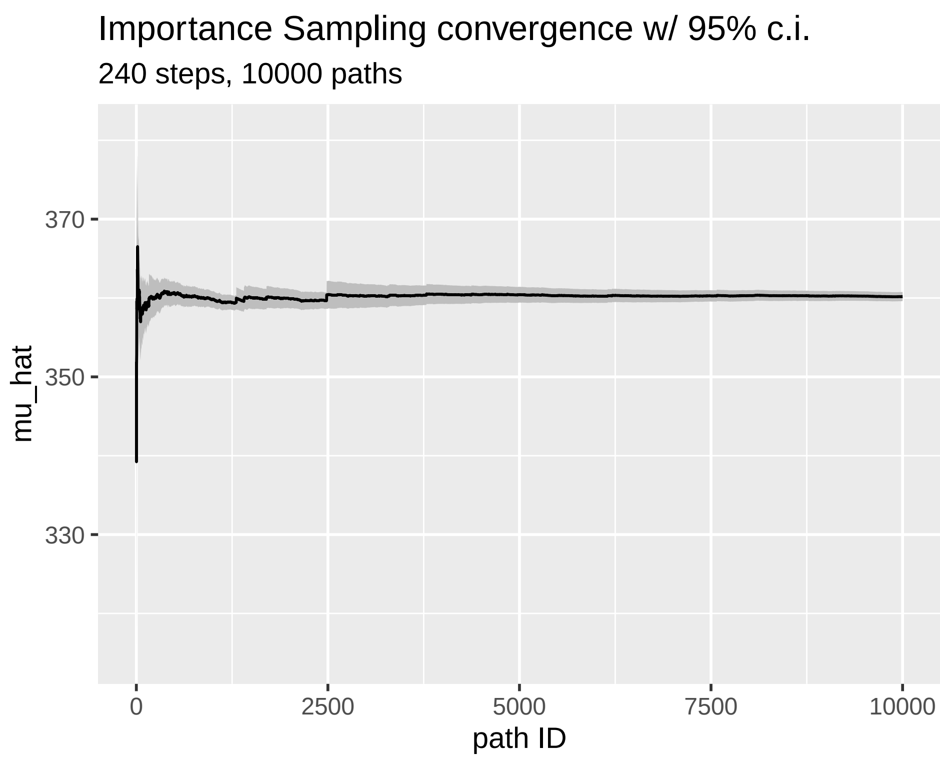


MC



IS

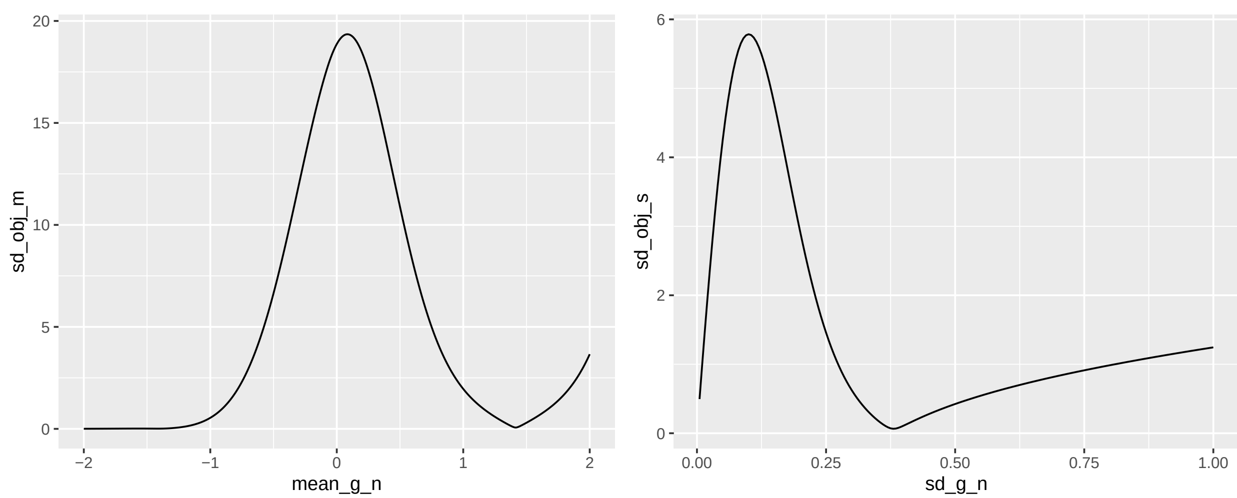
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.4145605 0.3807834
```

Objective function plots



Appendix

Infinite variance

Taleb, Statistical Consequences Of Fat Tails, p. 97:
"the variance of a finite variance random variable with tail exponent < 4 will be infinite".

And p. 363:
"The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, under a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance."

QQ lines

Note: QQ lines by design pass through 1st and 3rd quantiles. They are not trendlines in the sense of linear regression.

Arithmetic vs geometric mean

Let m be the number of steps in each path and n be the number of paths. a is the initial capital. Use arithmetic mean for mean of all paths at time t:

$$\frac{a(e^{z_1} + e^{z_2} + \cdots + e^{z_n})}{n}$$

where

$$z_i := x_{i,1} + x_{i,2} + \cdots + x_{i,m}$$

Use geometric mean for mean of all steps in a single path i:

$$ae^{\frac{x_{i,1}+x_{i,2}+\cdots+x_{i,m}}{m}} = a\sqrt[m]{e^{x_{i,1}+x_{i,2}+\cdots+x_{i,m}}}$$

So for **Monte Carlo** of returns after m periods, we

- fit a skewed t-distribution to log-returns and use that distribution to simulate $\{x_{i,j}\}_j^m$,
- for each path i, calculate $100 \cdot e^{z_i}$,
- calculate the mean of $\{z_i\}_i^n$:
  - $$\bar{z} = 100\frac{e^{z_1} + e^{z_2} + \cdots + e^{z_n}}{n}$$

For **Importance Sampling**, we

- model log-returns on a skewed t-distribution,
- for each path i, calculate $100 \cdot e^{z_i}$,
- fit a skewed t-distribution to $\{z_i\}_i^n$ and use it as our f density function from which we simulate $\{h_i\}_i^n$,
  - In our case h and z are identical, because we have an idea for a distribution to simulate z, but in general for IS h could be a function of z.
- calculate $w* = \frac{f}{g^*}$, where $g*$ is our proposal distribution, which minimizes the variance of $h \cdot w$.
- calculate the arithmetic mean of $\{h_i w_i^*\}_i^n$:
  - $$100\frac{e^{h_1 w_1^*} + e^{h_2 w_2^*} + \cdots + e^{h_n w_n^*}}{n}$$

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2}\right) \Big/ \left(\frac{x_{t-1} + y_{t-1}}{2}\right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns $=$ Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2\frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1} y_t = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

$$(x_{t-1} x_t y_{t-1} + y_{t-1} x_t y_{t-1}) + (x_{t-1} x_{t-1} y_t + x_{t-1} y_{t-1} y_t) = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: `R = 1+r`

```
## Let x_0 be 100.
```

```
## Let y_0 be 200.
```

```
## So the initial value of the pf is 300 .
```

```
## Let R_x be 0.5.
```

```
## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.
```

```
## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): -0.06320383
## s(data_x): 0.4880426
## m(data_y): 10.04932
## s(data_y): 2.530568
##
## m(data_x + data_y): 4.993058
## s(data_x + data_y): 1.338536
```

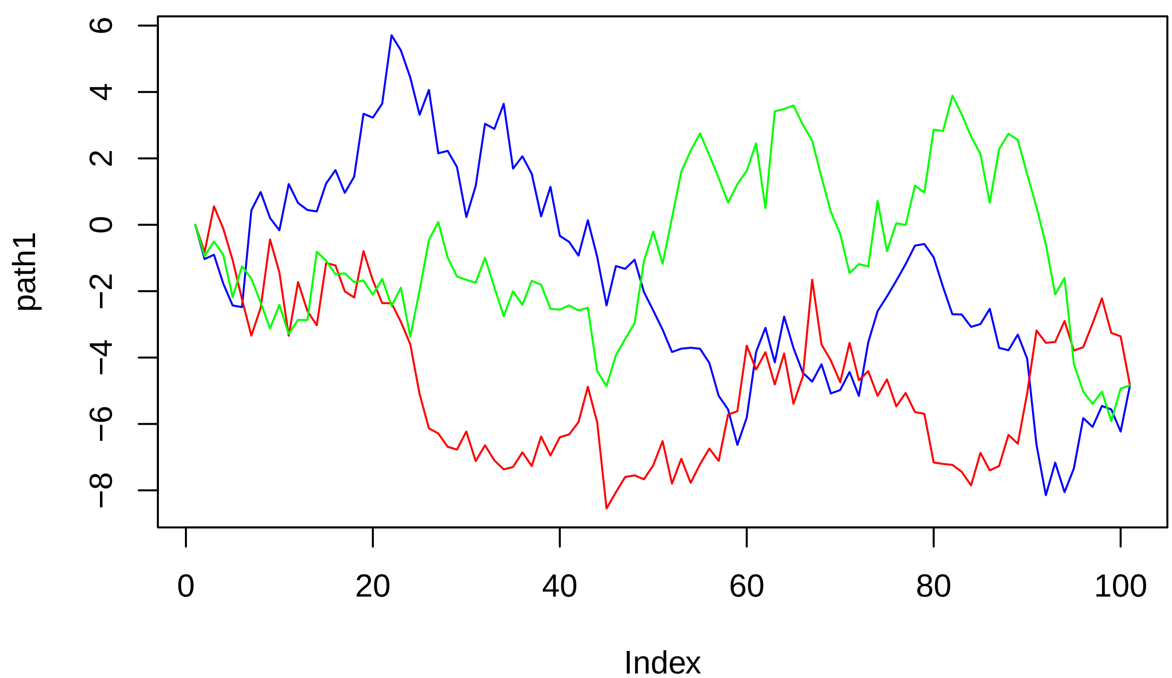
m and s of final state of all paths.
_a is mix of simulated returns.
_b is simulated mixed returns.

m_a	m_b	s_a	s_b
99.624	99.767	5.913	5.954
99.888	99.846	5.682	5.990
99.881	99.488	5.852	6.229
99.567	100.238	5.825	6.203
99.651	99.955	5.641	5.713
99.853	99.855	5.605	6.067
99.687	99.774	5.912	6.105
100.064	100.217	5.720	6.041
99.637	99.896	5.870	6.038
99.658	99.660	5.669	5.857

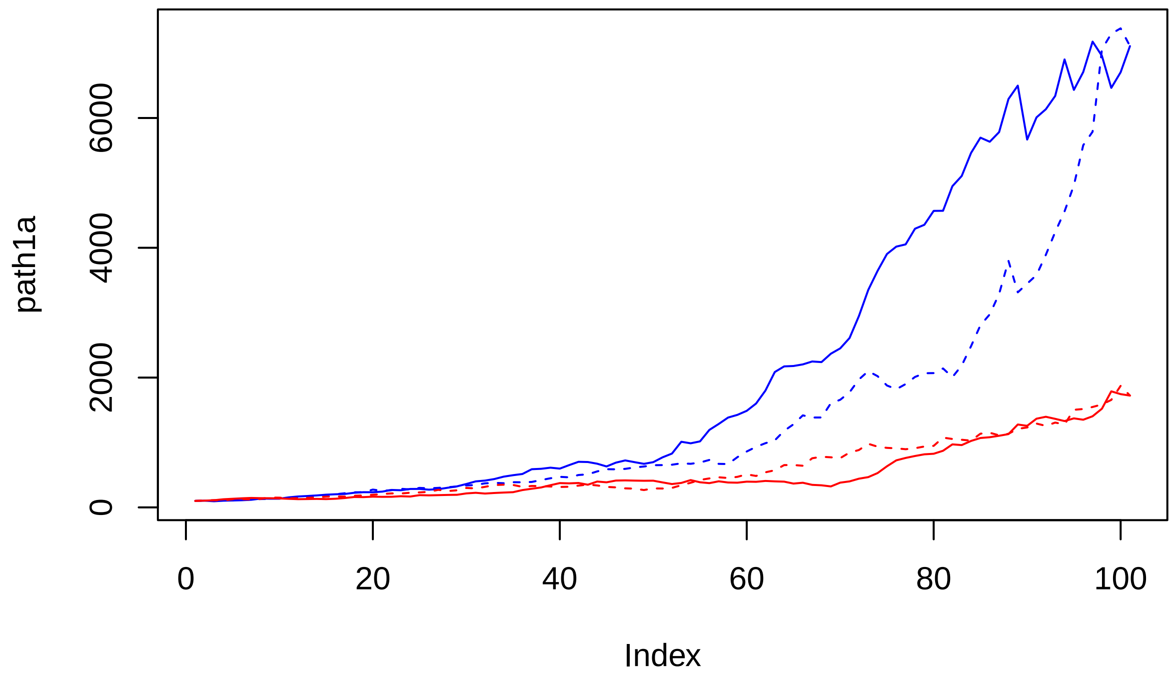
```
##       m_a              m_b              s_a              s_b
##  Min.   : 99.57   Min.   : 99.49   Min.   :5.605   Min.   :5.713
##  1st Qu.: 99.64   1st Qu.: 99.77   1st Qu.:5.672   1st Qu.:5.963
##  Median : 99.67   Median : 99.85   Median :5.772   Median :6.039
##  Mean   : 99.75   Mean   : 99.87   Mean   :5.769   Mean   :6.020
##  3rd Qu.: 99.87   3rd Qu.: 99.94   3rd Qu.:5.866   3rd Qu.:6.096
##  Max.   :100.06   Max.   :100.24   Max.   :5.913   Max.   :6.229
```

_a and _b are very close to equal.
We attribute the differences to differences in estimating the distributions in version a and b.

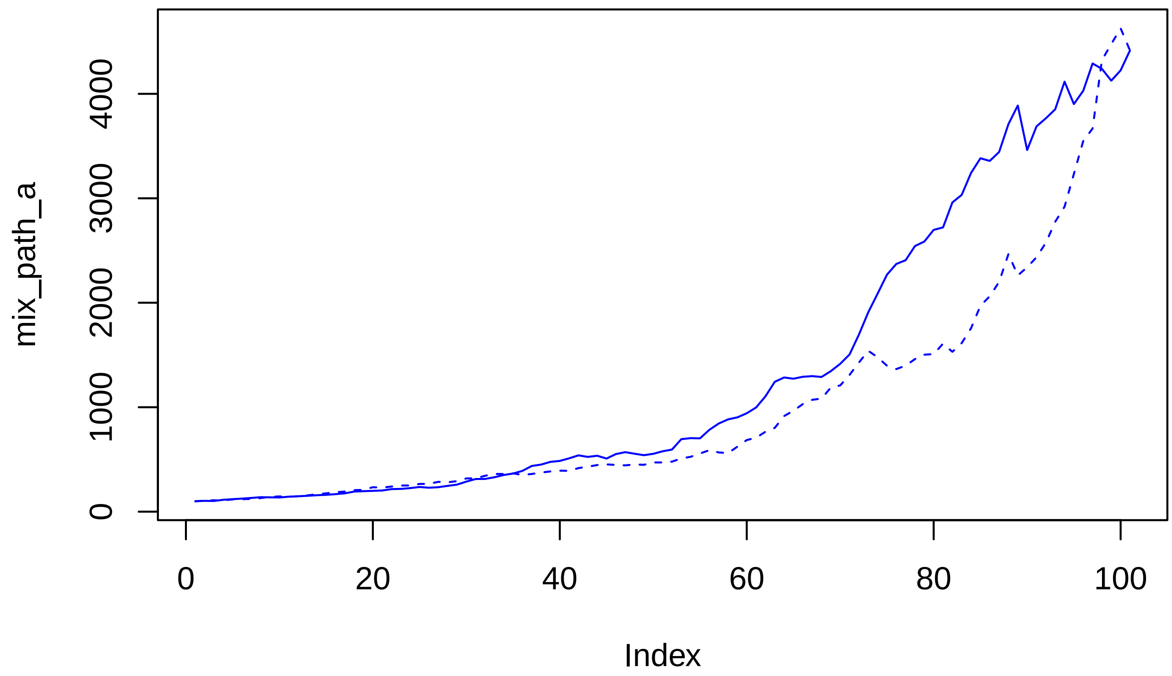
The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?



path1a
0
2000
4000
6000
0
20
40
60
80
100
Index



The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.01055146
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

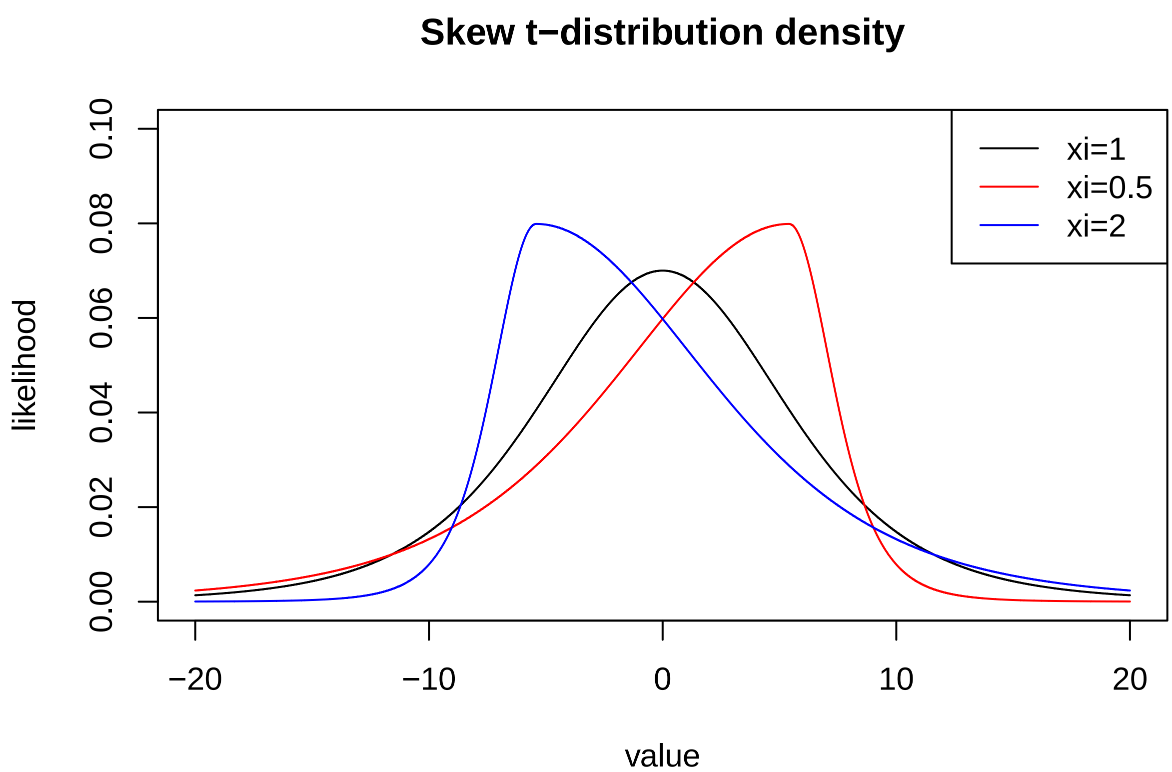
```
## [1] 0.01055146
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?
Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##        m                 s
##  Min.   :0.06193   Min.   :0.05402
##  1st Qu.:0.06674   1st Qu.:0.06154
##  Median :0.06960   Median :0.06767
##  Mean   :0.07098   Mean   :0.06873
##  3rd Qu.:0.07350   3rd Qu.:0.07559
##  Max.   :0.08358   Max.   :0.08828
```

The meaning of `xi`

The fit for `mhr` has the highest `xi` value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, ..., X^p)}{\sum_{i=1}^n X_i^p} \to 0$$

for $n \to \infty$.

If not, X doesn't have a p'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192