

Monthly pension returns analysis: Velliv June 2012 - April 2024

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Fit log returns to F-S skew standardized Student-t distribution.

\bar{m} is the location parameter.

s is the scale parameter.

ν is the estimated shape parameter (degrees of freedom).

ξ is the estimated skewness parameter.

Returns data

The long version of Velliv medium risk data runs from January 2007 to April 2024 (incl).

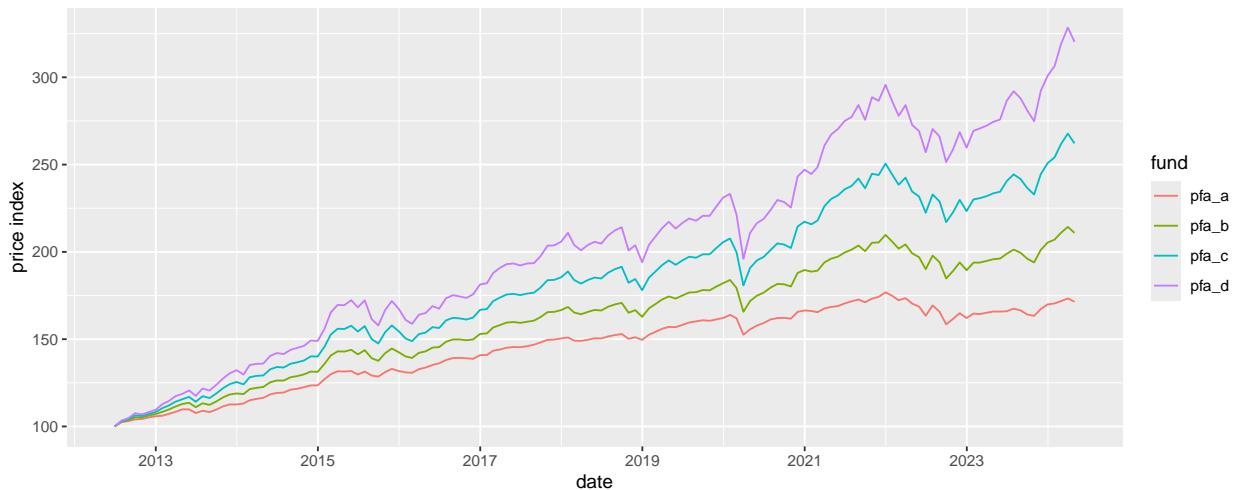
For January 2007 to May 2012 no low risk and high risk funds existed. For this period the medium risk data is copied into the other funds.

The short version runs from June 2012 to April 2024.

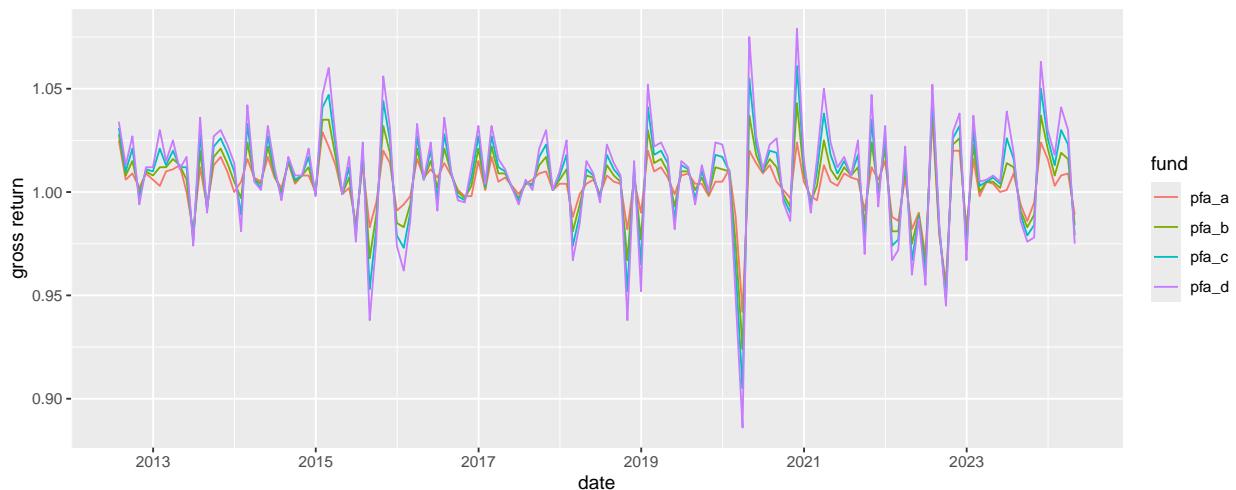
Velliv returns are including bonus and “DinKapital.

PFA returns are including ”KundeKapital”.

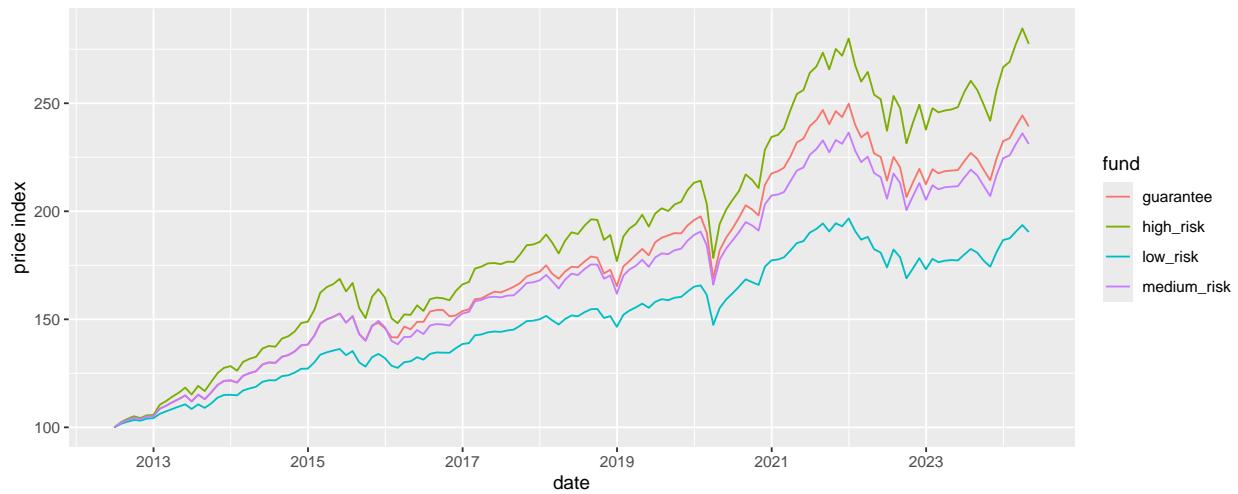
PFA monthly prices



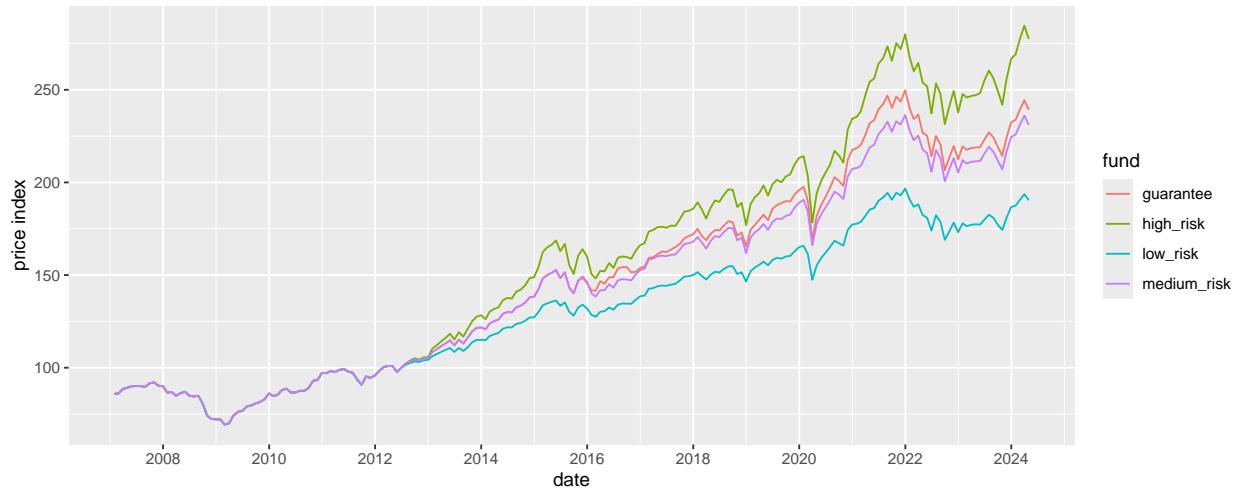
PFA monthly gross returns



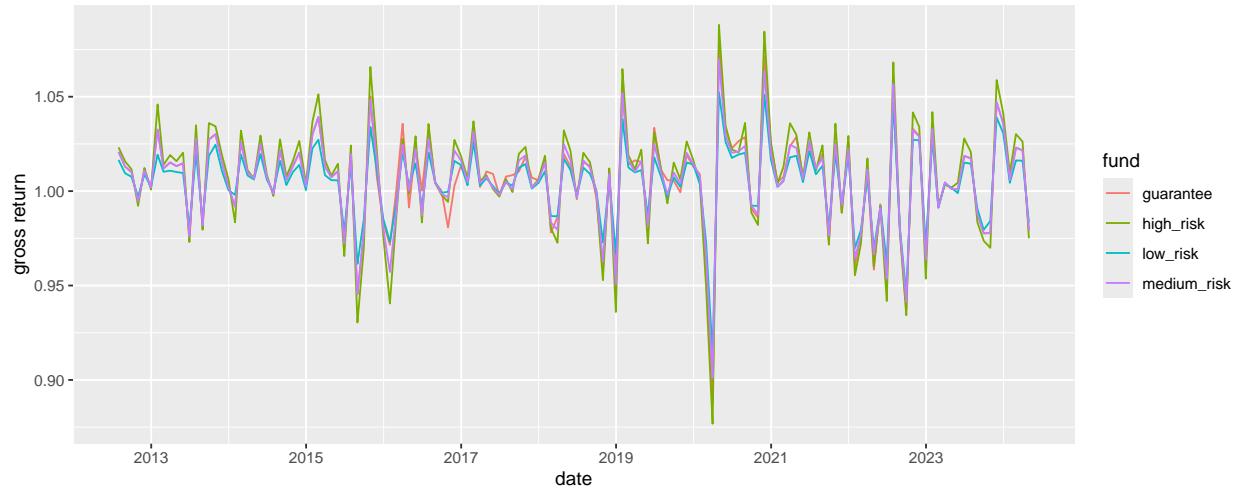
Velliv monthly prices



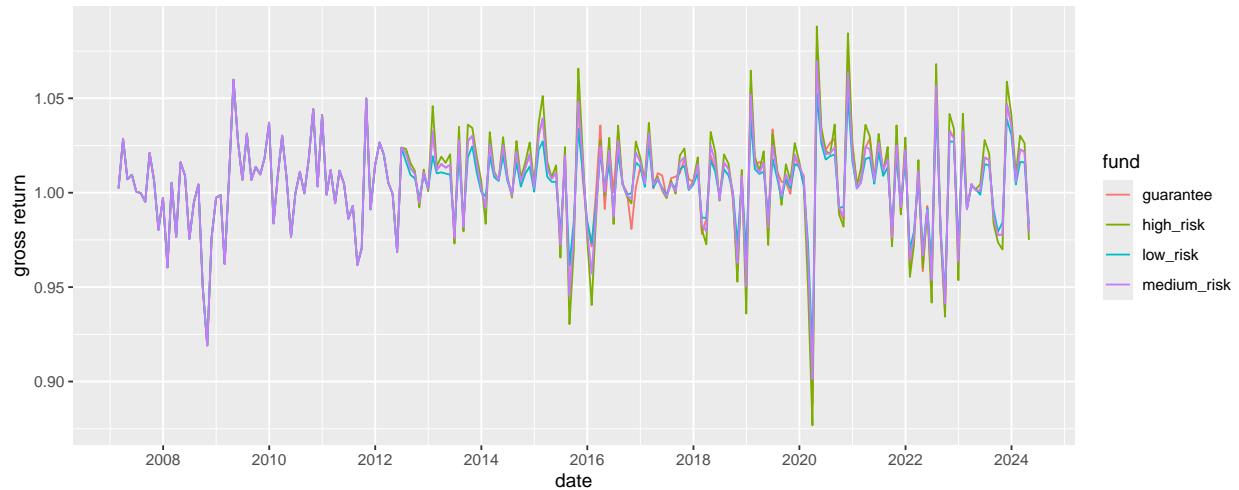
Velliv monthly prices – long



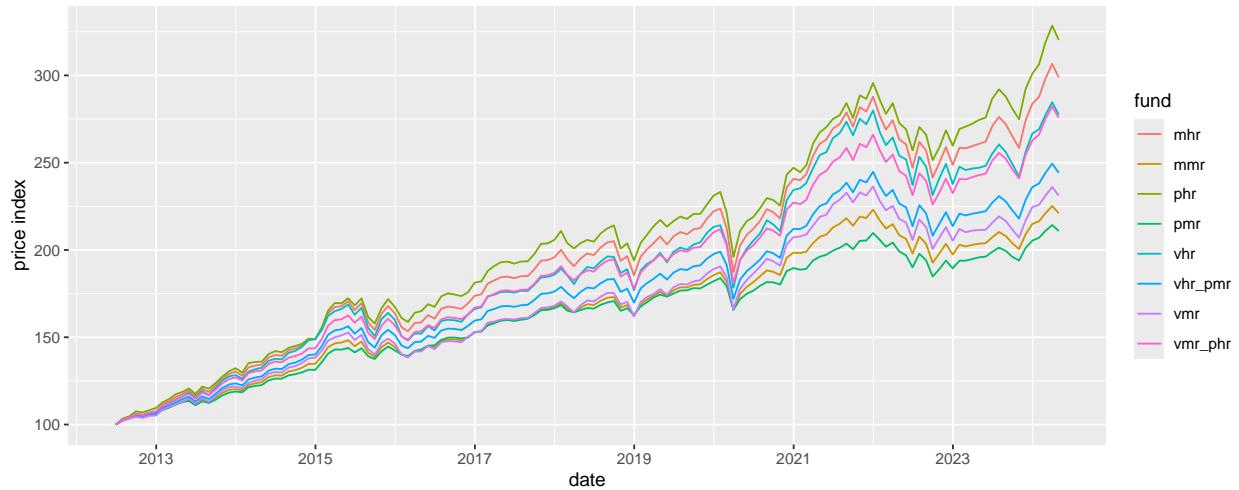
Velliv monthly gross returns



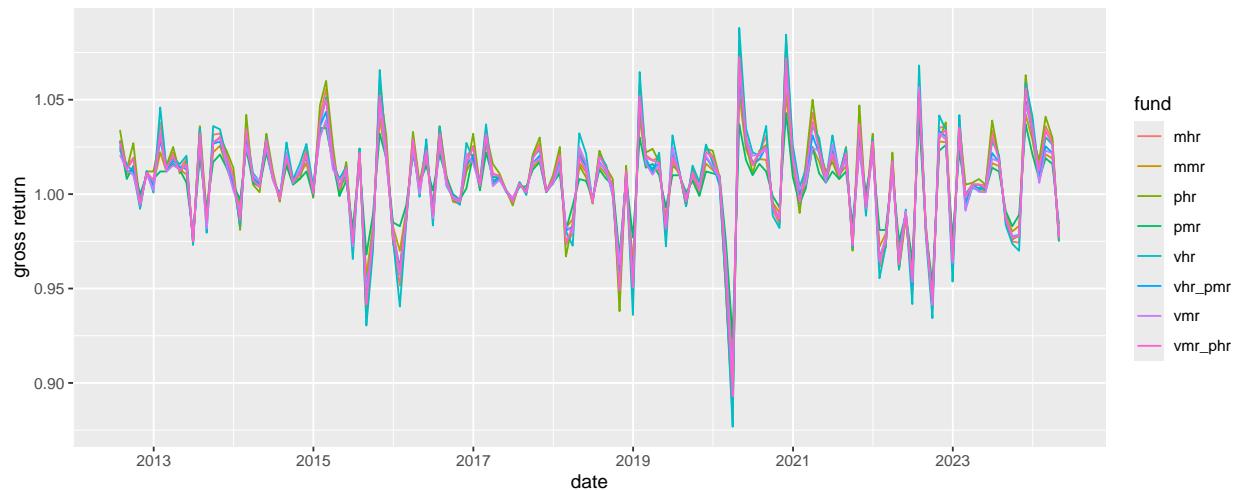
Velliv monthly gross returns – long

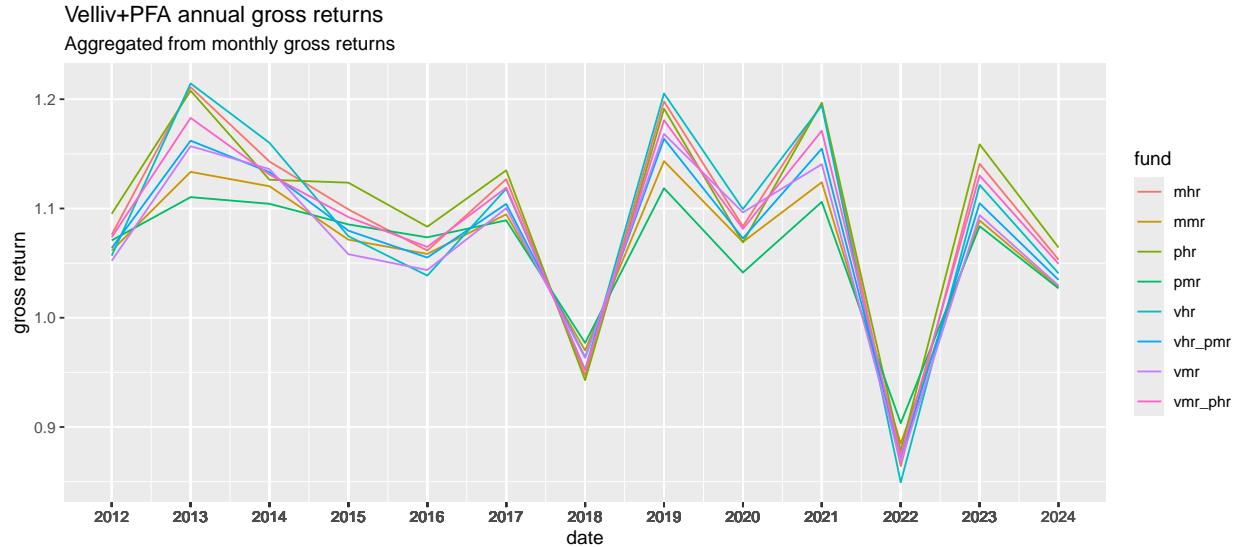


Velliv and PFA monthly prices



Velliv and PFA monthly gross returns





Summary of log-returns

The summary statistics are transformed back to the scale of gross returns by taking $\exp()$ of each summary statistic. (Note: Taking arithmetic mean of gross returns directly is no good. Must be geometric mean.)

	vmr	vhr	vmrl	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Min. :	0.901	0.877	0.901	0.924	0.886	0.912	0.882	0.893	0.899
1st Qu.:	0.996	0.994	0.995	0.999	0.994	0.997	0.995	0.996	0.996
Median :	1.010	1.012	1.007	1.008	1.012	1.009	1.013	1.011	1.011
Mean :	1.006	1.007	1.005	1.005	1.008	1.006	1.008	1.007	1.006
3rd Qu.:	1.021	1.027	1.020	1.015	1.025	1.018	1.025	1.022	1.021
Max. :	1.070	1.088	1.070	1.043	1.079	1.054	1.082	1.073	1.065

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.924	pmr	0.999	pmr	1.013	mhr	1.008	phr	1.027	vhr	1.088	vhr
0.912	mmr	0.997	mmr	1.012	phr	1.008	mhr	1.025	phr	1.082	mhr
0.901	vmr	0.996	vmr	1.012	vhr	1.007	vhr	1.025	mhr	1.079	phr
0.901	vmrl	0.996	vhr_pmr	1.011	vmr_phr	1.007	vmr_phr	1.022	vmr_phr	1.073	vmr_phr
0.899	vhr_pmr	0.996	vmr_phr	1.011	vhr_pmr	1.006	vhr_pmr	1.021	vmr	1.070	vmr
0.893	vmr_phr	0.995	mhr	1.010	vmr	1.006	vmr	1.021	vhr_pmr	1.070	vmrl
0.886	phr	0.995	vmrl	1.009	mmr	1.006	mmr	1.020	vmrl	1.065	vhr_pmr
0.882	mhr	0.994	phr	1.008	pmr	1.005	pmr	1.018	mmr	1.054	mmr
0.877	vhr	0.994	vhr	1.007	vmrl	1.005	vmrl	1.015	pmr	1.043	pmr

Correlations and covariance

Correlations

	vmr	vhr	pmr	phr
vmr	1.000	0.997	0.961	0.964
vhr	0.997	1.000	0.951	0.967
pmr	0.961	0.951	1.000	0.977
phr	0.964	0.967	0.977	1.000

Covariances

	vmr	vhr	pmr	phr
vmr	0.001	0.001	0	0.001
vhr	0.001	0.001	0	0.001
pmr	0.000	0.000	0	0.000
phr	0.001	0.001	0	0.001

Compare pension plans

Risk of loss

Risk of loss at least as big as row name in percent for a single period (year).

Skewed *t*-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	32.333	33.000	30.167	31.667	31.167	32.167	31.833	31.667
5	2.500	4.167	0.667	3.167	1.500	3.500	2.667	2.500
10	0.167	0.500	0.000	0.167	0.000	0.333	0.167	0.167
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Standardized *t*-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	28.833	29.500	22.667	29.333	25.667	29.333	29	27.333
5	1.833	3.167	0.833	2.500	1.167	2.667	2	1.833
10	0.000	0.333	0.000	0.000	0.000	0.167	0	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0	0.000

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	40.333	40.667	37.833	38.500	39.167	39.500	39.167	39.667
5	0.500	2.333	0.000	1.333	0.000	1.667	0.833	0.500
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Worst ranking for loss percentiles

Skewed *t*-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
33.000	vhr	4.167	vhr	0.500	vhr	0	vmr	0	vmr	0	vmr	0	vmr
32.333	vmr	3.500	mhr	0.333	mhr	0	vhr	0	vhr	0	vhr	0	vhr
32.167	mhr	3.167	phr	0.167	vmr	0	pmr	0	pmr	0	pmr	0	pmr
31.833	vmr_phr	2.667	vmr_phr	0.167	phr	0	phr	0	phr	0	phr	0	phr
31.667	phr	2.500	vmr	0.167	vmr_phr	0	mmr	0	mmr	0	mmr	0	mmr
31.667	vhr_pmr	2.500	vhr_pmr	0.167	vhr_pmr	0	mhr	0	mhr	0	mhr	0	mhr
31.167	mmr	1.500	mmr	0.000	pmr	0	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
30.167	pmr	0.667	pmr	0.000	mmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Standardized *t*-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
29.500	vhr	3.167	vhr	0.333	vhr	0	vmr	0	vmr	0	vmr	0	vmr
29.333	phr	2.667	mhr	0.167	mhr	0	vhr	0	vhr	0	vhr	0	vhr
29.333	mhr	2.500	phr	0.000	vmr	0	pmr	0	pmr	0	pmr	0	pmr
29.000	vmr_phr	2.000	vmr_phr	0.000	pmr	0	phr	0	phr	0	phr	0	phr
28.833	vmr	1.833	vmr	0.000	phr	0	mmr	0	mmr	0	mmr	0	mmr
27.333	vhr_pmr	1.833	vhr_pmr	0.000	mmr	0	mhr	0	mhr	0	mhr	0	mhr
25.667	mmr	1.167	mmr	0.000	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
22.667	pmr	0.833	pmr	0.000	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
40.667	vhr	2.333	vhr	0	vmr								
40.333	vmr	1.667	mhr	0	vhr								
39.667	vhr_pmr	1.333	phr	0	pmr								
39.500	mhr	0.833	vmr_phr	0	phr								
39.167	mmr	0.500	vmr	0	mmr								
39.167	vmr_phr	0.500	vhr_pmr	0	mhr								
38.500	phr	0.000	pmr	0	vmr_phr								
37.833	pmr	0.000	mmr	0	vhr_pmr								

Chance of min gains

Chance of gains of at least x percent for a single period (year).
 x values are row names.

Skewed *t*-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	67.667	67.000	69.833	68.333	68.833	67.833	68.167	68.333
5	1.167	3.833	0.167	3.667	0.500	3.333	2.167	1.333
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Standardized *t*-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	71.167	70.500	77.333	70.667	74.333	70.667	71.000	72.667
5	7.833	13.500	3.667	11.500	5.500	12.500	9.667	8.167
10	0.667	1.833	0.167	1.167	0.333	1.333	0.833	0.833
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	59.667	59.333	62.167	61.500	60.833	60.5	60.833	60.333
5	3.333	8.333	0.167	7.167	1.333	7.5	5.167	3.500
10	0.000	0.000	0.000	0.000	0.000	0.0	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.0	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.0	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.0	0.000	0.000

Best ranking for gains percentiles

Skewed *t*-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
69.833	pmr	3.833	vhr	0	vmr	0	vmr	0	vmr	0	vmr
68.833	mmr	3.667	phr	0	vhr	0	vhr	0	vhr	0	vhr
68.333	phr	3.333	mhr	0	pmr	0	pmr	0	pmr	0	pmr
68.333	vhr_pmr	2.167	vmr_phr	0	phr	0	phr	0	phr	0	phr
68.167	vmr_phr	1.333	vhr_pmr	0	mmr	0	mmr	0	mmr	0	mmr
67.833	mhr	1.167	vmr	0	mhr	0	mhr	0	mhr	0	mhr
67.667	vmr	0.500	mmr	0	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
67.000	vhr	0.167	pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Standardized *t*-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
77.333	pmr	13.500	vhr	1.833	vhr	0	vmr	0	vmr	0	vmr
74.333	mmr	12.500	mhr	1.333	mhr	0	vhr	0	vhr	0	vhr
72.667	vhr_pmr	11.500	phr	1.167	phr	0	pmr	0	pmr	0	pmr
71.167	vmr	9.667	vmr_phr	0.833	vmr_phr	0	phr	0	phr	0	phr
71.000	vmr_phr	8.167	vhr_pmr	0.833	vhr_pmr	0	mmr	0	mmr	0	mmr
70.667	phr	7.833	vmr	0.667	vmr	0	mhr	0	mhr	0	mhr
70.667	mhr	5.500	mmr	0.333	mmr	0	vmr_phr	0	vmr_phr	0	vmr_phr
70.500	vhr	3.667	pmr	0.167	pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
62.167	pmr	8.333	vhr	0	vmr	0	vmr	0	vmr	0	vmr
61.500	phr	7.500	mhr	0	vhr	0	vhr	0	vhr	0	vhr
60.833	mmr	7.167	phr	0	pmr	0	pmr	0	pmr	0	pmr
60.833	vmr_phr	5.167	vmr_phr	0	phr	0	phr	0	phr	0	phr
60.500	mhr	3.500	vhr_pmr	0	mmr	0	mmr	0	mmr	0	mmr
60.333	vhr_pmr	3.333	vmr	0	mhr	0	mhr	0	mhr	0	mhr
59.667	vmr	1.333	mmr	0	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
59.333	vhr	0.167	pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

MC risk percentiles

Risk of loss at least as big as row name in percent from first to last period.

Skewed *t*-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	17.71	18.08	10.00	11.89	7.65	7.23	7.11	8.27
5	9.25	11.04	3.51	6.44	2.25	2.58	2.60	2.78
10	4.54	6.57	1.19	3.42	0.62	0.90	0.75	0.85
25	0.47	0.88	0.09	0.35	0.02	0.02	0.04	0.03
50	0.02	0.03	0.01	0.00	0.00	0.00	0.00	0.00

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Standardized t -distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	4.21	4.55	2.33	3.44	0.64	0.58	0.60	0.62
5	2.12	2.64	1.13	1.67	0.30	0.18	0.18	0.23
10	1.08	1.32	0.57	0.73	0.11	0.11	0.08	0.06
25	0.10	0.21	0.15	0.06	0.02	0.01	0.00	0.01
50	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	14.05	15.07	9.03	9.83	4.83	4.39	4.46	5.75
5	6.01	8.08	2.22	4.34	0.64	1.03	1.01	1.15
10	1.97	3.53	0.29	1.66	0.02	0.23	0.14	0.12
25	0.00	0.07	0.00	0.02	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Worst ranking for MC loss percentiles

Skewed t -distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
18.08	vhr	11.04	vhr	6.57	vhr	0.88	vhr	0.03	vhr	0	vmr	0	vmr
17.71	vmr	9.25	vmr	4.54	vmr	0.47	vmr	0.02	vmr	0	vhr	0	vhr
11.89	phr	6.44	phr	3.42	phr	0.35	phr	0.01	pmr	0	pmr	0	pmr
10.00	pmr	3.51	pmr	1.19	pmr	0.09	pmr	0.00	phr	0	phr	0	phr
8.27	vhr_pmr	2.78	vhr_pmr	0.90	mhr	0.04	vmr_phr	0.00	mmr	0	mmr	0	mmr
7.65	mmr	2.60	vmr_phr	0.85	vhr_pmr	0.03	vhr_pmr	0.00	mhr	0	mhr	0	mhr
7.23	mhr	2.58	mhr	0.75	vmr_phr	0.02	mmr	0.00	vmr_phr	0	vmr_phr	0	vmr_phr
7.11	vmr_phr	2.25	mmr	0.62	mmr	0.02	mhr	0.00	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Standardized t -distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.55	vhr	2.64	vhr	1.32	vhr	0.21	vhr	0.04	pmr	0.01	pmr	0	vmr
4.21	vmr	2.12	vmr	1.08	vmr	0.15	pmr	0.00	vmr	0.00	vmr	0	vhr
3.44	phr	1.67	phr	0.73	phr	0.10	vmr	0.00	vhr	0.00	vhr	0	pmr
2.33	pmr	1.13	pmr	0.57	pmr	0.06	phr	0.00	phr	0.00	phr	0	phr
0.64	mmr	0.30	mmr	0.11	mmr	0.02	mmr	0.00	mmr	0.00	mmr	0	mmr
0.62	vhr_pmr	0.23	vhr_pmr	0.11	mhr	0.01	mhr	0.00	mhr	0.00	mhr	0	mhr
0.60	vmr_phr	0.18	mhr	0.08	vmr_phr	0.01	vhr_pmr	0.00	vmr_phr	0.00	vmr_phr	0	vmr_phr
0.58	mhr	0.18	vmr_phr	0.06	vhr_pmr	0.00	vmr_phr	0.00	vhr_pmr	0.00	vhr_pmr	0	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
15.07	vhr	8.08	vhr	3.53	vhr	0.07	vhr	0	vmr	0	vmr	0	vmr
14.05	vmr	6.01	vmr	1.97	vmr	0.02	phr	0	vhr	0	vhr	0	vhr
9.83	phr	4.34	phr	1.66	phr	0.00	vmr	0	pmr	0	pmr	0	pmr
9.03	pmr	2.22	pmr	0.29	pmr	0.00	pmr	0	phr	0	phr	0	phr
5.75	vhr_pmr	1.15	vhr_pmr	0.23	mhr	0.00	mmr	0	mmr	0	mmr	0	mmr
4.83	mmr	1.03	mhr	0.14	vmr_phr	0.00	mhr	0	mhr	0	mhr	0	mhr
4.46	vmr_phr	1.01	vmr_phr	0.12	vhr_pmr	0.00	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
4.39	mhr	0.64	mmr	0.02	mmr	0.00	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

MC gains percentiles

Skewed t -distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	82.29	81.92	90.00	88.11	92.35	92.77	92.89	91.73
5	70.65	72.69	75.97	79.76	78.86	84.34	82.99	80.04
10	55.47	61.27	55.27	68.75	55.72	70.44	66.97	61.66
25	13.20	24.67	4.82	29.65	3.28	20.10	13.41	9.13
50	0.14	1.52	0.02	1.64	0.00	0.21	0.02	0.05
100	0.00	0.04	0.01	0.01	0.00	0.00	0.00	0.00
200	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00
300	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
400	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
500	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
1000	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00

Standardized t -distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	95.79	95.45	97.67	96.56	99.36	99.42	99.40	99.38
5	92.09	92.35	94.81	93.66	98.00	98.15	98.21	98.12
10	86.07	87.80	89.06	88.99	94.76	95.63	94.87	95.25
25	54.92	65.59	49.09	64.12	54.65	72.05	64.24	65.03
50	10.26	24.55	4.50	18.98	2.96	14.88	7.98	8.18
100	0.28	1.31	0.17	0.50	0.05	0.12	0.04	0.11
200	0.03	0.04	0.02	0.00	0.01	0.00	0.00	0.01
300	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00
400	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
500	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
1000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	85.95	84.93	90.97	90.17	95.17	95.61	95.54	94.25
5	72.53	74.85	75.33	81.58	82.78	87.77	85.46	82.92
10	56.07	62.19	52.17	69.19	58.26	74.22	69.32	62.77
25	13.98	25.16	4.79	29.60	4.08	21.63	13.74	9.72
50	0.40	2.19	0.00	2.08	0.00	0.21	0.05	0.01
100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
200	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
300	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
400	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
500	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Best ranking for MC gains percentiles

Skewed t -distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
92.89	vmr_phr	84.34	mhr	70.44	mhr	29.65	phr	1.64	phr	0.04	vhr
92.77	mhr	82.99	vmr_phr	68.75	phr	24.67	vhr	1.52	vhr	0.01	pmr
92.35	mmr	80.04	vhr_pmr	66.97	vmr_phr	20.10	mhr	0.21	mhr	0.01	phr
91.73	vhr_pmr	79.76	phr	61.66	vhr_pmr	13.41	vmr_phr	0.14	vmr	0.00	vmr
90.00	pmr	78.86	mmr	61.27	vhr	13.20	vmr	0.05	vhr_pmr	0.00	mmr
88.11	phr	75.97	pmr	55.72	mmr	9.13	vhr_pmr	0.02	pmr	0.00	mhr
82.29	vmr	72.69	vhr	55.47	vmr	4.82	pmr	0.02	vmr_phr	0.00	vmr_phr
81.92	vhr	70.65	vmr	55.27	pmr	3.28	mmr	0.00	mmr	0.00	vhr_pmr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
0.01	vhr								
0.01	phr	0.00	vmr	0.00	vmr	0.00	vmr	0.00	vmr
0.00	vmr	0.00	pmr	0.00	pmr	0.00	pmr	0.00	pmr
0.00	pmr	0.00	phr	0.00	phr	0.00	phr	0.00	phr
0.00	mmr								
0.00	mhr								
0.00	vmr_phr								
0.00	vhr_pmr								

Standardized t -distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
99.42	mhr	98.21	vmr_phr	95.63	mhr	72.05	mhr	24.55	vhr	1.31	vhr
99.40	vmr_phr	98.15	mhr	95.25	vhr_pmr	65.59	vhr	18.98	phr	0.50	phr
99.38	vhr_pmr	98.12	vhr_pmr	94.87	vmr_phr	65.03	vhr_pmr	14.88	mhr	0.28	vmr
99.36	mmr	98.00	mmr	94.76	mmr	64.24	vmr_phr	10.26	vmr	0.17	pmr
97.67	pmr	94.81	pmr	89.06	pmr	64.12	phr	8.18	vhr_pmr	0.12	mhr
96.56	phr	93.66	phr	88.99	phr	54.92	vmr	7.98	vmr_phr	0.11	vhr_pmr
95.79	vmr	92.35	vhr	87.80	vhr	54.65	mmr	4.50	pmr	0.05	mmr
95.45	vhr	92.09	vmr	86.07	vmr	49.09	pmr	2.96	mmr	0.04	vmr_phr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
0.04	vhr	0.02	vmr	0.01	pmr	0.01	pmr	0	vmr
0.03	vmr	0.02	vhr	0.00	vmr	0.00	vmr	0	vhr
0.02	pmr	0.02	pmr	0.00	vhr	0.00	vhr	0	pmr
0.01	mmr	0.00	phr	0.00	phr	0.00	phr	0	phr
0.01	vhr_pmr	0.00	mmr	0.00	mmr	0.00	mmr	0	mmr
0.00	phr	0.00	mhr	0.00	mhr	0.00	mhr	0	mhr
0.00	mhr	0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0	vmr_phr
0.00	vmr_phr	0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr	0	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
95.61	mhr	87.77	mhr	74.22	mhr	29.60	phr	2.19	vhr	0	vmr
95.54	vmr_phr	85.46	vmr_phr	69.32	vmr_phr	25.16	vhr	2.08	phr	0	vhr
95.17	mmr	82.92	vhr_pmr	69.19	phr	21.63	mhr	0.40	vmr	0	pmr
94.25	vhr_pmr	82.78	mmr	62.77	vhr_pmr	13.98	vmr	0.21	mhr	0	phr
90.97	pmr	81.58	phr	62.19	vhr	13.74	vmr_phr	0.05	vmr_phr	0	mmr
90.17	phr	75.33	pmr	58.26	mmr	9.72	vhr_pmr	0.01	vhr_pmr	0	mhr
85.95	vmr	74.85	vhr	56.07	vmr	4.79	pmr	0.00	pmr	0	vmr_phr

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
84.93	vhr	72.53	vmr	52.17	pmr	4.08	mmr	0.00	mmr	0	vhr_pmr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
0	vmr	0	vmr	0	vmr	0	vmr	0	vmr
0	vhr	0	vhr	0	vhr	0	vhr	0	vhr
0	pmr	0	pmr	0	pmr	0	pmr	0	pmr
0	phr	0	phr	0	phr	0	phr	0	phr
0	mmr	0	mmr	0	mmr	0	mmr	0	mmr
0	mhr	0	mhr	0	mhr	0	mhr	0	mhr
0	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
0	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.

\bar{m} is the location parameter.

s is the scale parameter.

ν is the estimated degrees of freedom, or shape parameter.

ξ is the estimated skewness parameter.

Skewed t -distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.005	0.007	0.005	0.008	0.005	0.007	0.007	0.006
s	0.027	0.034	0.019	0.030	0.023	0.031	0.028	0.027
ν	3.384	3.488	3.474	3.959	3.344	3.702	3.726	3.369
ξ	0.699	0.708	0.770	0.737	0.716	0.714	0.715	0.709
R^2	0.993	0.992	0.994	0.996	0.993	0.993	0.994	0.993

Standardized t -distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.013	0.015	0.012	0.014	0.012	0.015	0.014	0.013
s	0.032	0.040	0.027	0.035	0.029	0.037	0.033	0.033
ν	3.446	3.510	2.629	4.002	3.035	3.780	3.760	3.260
R^2	0.978	0.978	0.962	0.981	0.971	0.977	0.978	0.974

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.006	0.007	0.005	0.008	0.006	0.008	0.007	0.006
s	0.024	0.031	0.017	0.028	0.021	0.029	0.026	0.024
R^2	0.968	0.969	0.962	0.973	0.965	0.969	0.969	0.966

AIC and BIC AIC

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
sstd	-671.412	-603.343	-768.368	-622.474	-718.914	-617.207	-647.901	-672.555
std	-628.069	-561.055	-728.782	-585.156	-674.537	-574.620	-605.443	-628.239
normal	-646.514	-579.369	-743.179	-603.088	-692.459	-593.830	-624.670	-646.870

BIC

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
sstd	-659.588	-591.520	-756.545	-610.651	-707.091	-605.384	-636.077	-660.732
std	-616.246	-549.232	-716.959	-573.333	-662.714	-562.796	-593.620	-616.416
normal	-634.691	-567.546	-731.356	-591.265	-680.636	-582.006	-612.847	-635.046

Kappa Let $\{X_{g,i}\}$ be Gaussian distributed with mean μ and scale σ .

Let $\{X_{\nu,i}\}$ be t -distributed, scaled such that $\mathbb{M}^{\nu}(1) = \mathbb{M}^g(1) = \sqrt{\frac{2}{\pi}}\sigma$.

Given n_g , we want to determine and n_{ν}^* such that

$$\text{Var} \left[\sum_i^{n_g} X_{g,i} \right] = \text{Var} \left[\sum_i^{n_{\nu}^*} X_{\nu,i} \right]$$

For iid. r.v $\{X_i\}$:

$$S_n = X_1 + X_2 + \cdots + X_n$$

$$\mathbb{M}(n) = \mathbb{E}(|S_n - \mathbb{E}(S_n)|)$$

Taleb's convergence metric (κ):

The “rate” of convergence for n summands vs n_0 , i.e. the improved convergence achieved by $n - n_0$ additional terms, is given by $\kappa(n_0, n)$:

$$\kappa(n_0, n) = 2 - \frac{\log(n) - \log(n_0)}{\log \left(\frac{\mathbb{M}(n)}{\mathbb{M}(n_0)} \right)}$$

κ

vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0.16	0.16	0.16	0.13	0.15	0.14	0.13	0.17

n_{min}

What is the minimum value of n_{ν} , the number of observations from a given skewed t -distribution, we need to achieve the same degree of convergence as with $n_g = 30$ observations from a Gaussian distribution with the same mean and standard deviation?

vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
58	55	55	48	58	51	50	57

Fit statistics ranking Skewed t -distribution (sstd):

m	ranking	s	ranking	R^2	ranking
0.008	phr	0.019	pmr	0.996	phr
0.007	mhr	0.023	mmr	0.994	vmr_phr
0.007	vmr_phr	0.027	vhr_pmr	0.994	pmr
0.007	vhr	0.027	vmr	0.993	mmr
0.006	vhr_pmr	0.028	vmr_phr	0.993	mhr
0.005	vmr	0.030	phr	0.993	vmr
0.005	pmr	0.031	mhr	0.993	vhr_pmr
0.005	mmr	0.034	vhr	0.992	vhr

Standardized t -distribution (std):

m	ranking	s	ranking	R^2	ranking
0.015	vhr	0.027	pmr	0.981	phr
0.015	mhr	0.029	mmr	0.978	vmr
0.014	phr	0.032	vmr	0.978	vhr
0.014	vmr_phr	0.033	vhr_pmr	0.978	vmr_phr
0.013	vhr_pmr	0.033	vmr_phr	0.977	mhr
0.013	vmr	0.035	phr	0.974	vhr_pmr
0.012	mmr	0.037	mhr	0.971	mmr
0.012	pmr	0.040	vhr	0.962	pmr

Normal distribution:

m	ranking	s	ranking	R^2	ranking
0.008	phr	0.017	pmr	0.973	phr
0.008	mhr	0.021	mmr	0.969	vmr_phr
0.007	vhr	0.024	vhr_pmr	0.969	vhr
0.007	vmr_phr	0.024	vmr	0.969	mhr
0.006	vhr_pmr	0.026	vmr_phr	0.968	vmr
0.006	vmr	0.028	phr	0.966	vhr_pmr
0.006	mmr	0.029	mhr	0.965	mmr
0.005	pmr	0.031	vhr	0.962	pmr

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).

Statistics are given for the final state of all paths.

Probability of down-and-out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.

0 is defined as any value below a threshold.

dai_pct (for down-and-in) is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	390.56	546.54	368.99	744.90	379.50	649.19	569.75	458.13
mc_s	162.67	310.60	106.60	358.90	98.41	230.77	195.66	154.95
mc_min	9.95	47.50	41.88	67.17	120.88	157.64	134.70	146.99
mc_max	1962.47	9418.03	964.46	3790.71	969.20	2350.77	2262.60	1819.14
dai_pct	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dai_pct	0.32	0.32	0.03	0.03	0.00	0.00	0.00	0.00

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	2281.79	4769.87	1795.80	3664.55	2345.79	4253.11	2979.65	3285.27
mc_s	1214.65	3240.61	1239.38	2099.78	31176.36	3565.88	1263.15	1921.35
mc_min	221.36	374.70	41.43	193.68	557.15	783.35	689.69	499.82
mc_max	16109.79	45965.08	94908.01	28483.53	3118787.05	295755.16	30086.97	59511.22
dai_pct	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dai_pct	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	441.92	627.78	365.42	789.64	403.75	711.18	614.07	498.41
mc_s	169.65	319.17	98.48	362.10	98.81	244.56	197.97	164.59
mc_min	90.51	80.75	123.03	120.90	165.90	188.76	136.86	160.53
mc_max	1791.46	3370.41	1109.65	3404.35	1102.37	2538.96	1961.78	1606.02
dao_pct	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
dai_pct	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00

Ranking Skewed *t*-distribution (sstd):

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
744.90	phr	98.41	mmr	157.64	mhr	9418.03	vhr	0	vmr	0.00	mmr
649.19	mhr	106.60	pmr	146.99	vhr_pmr	3790.71	phr	0	vhr	0.00	mhr
569.75	vmr_phr	154.95	vhr_pmr	134.70	vmr_phr	2350.77	mhr	0	pmr	0.00	vmr_phr
546.54	vhr	162.67	vmr	120.88	mmr	2262.60	vmr_phr	0	phr	0.00	vhr_pmr
458.13	vhr_pmr	195.66	vmr_phr	67.17	phr	1962.47	vmr	0	mmr	0.03	pmr
390.56	vmr	230.77	mhr	47.50	vhr	1819.14	vhr_pmr	0	mhr	0.03	phr
379.50	mmr	310.60	vhr	41.88	pmr	969.20	mmr	0	vmr_phr	0.32	vmr
368.99	pmr	358.90	phr	9.95	vmr	964.46	pmr	0	vhr_pmr	0.32	vhr

Standardized *t*-distribution (std):

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
4769.87	vhr	1214.65	vmr	783.35	mhr	3118787.05	mmr	0	vmr	0.00	vmr
4253.11	mhr	1239.38	pmr	689.69	vmr_phr	295755.16	mhr	0	vhr	0.00	vhr
3664.55	phr	1263.15	vmr_phr	557.15	mmr	94908.01	pmr	0	pmr	0.00	phr
3285.27	vhr_pmr	1921.35	vhr_pmr	499.82	vhr_pmr	59511.22	vhr_pmr	0	phr	0.00	mmr
2979.65	vmr_phr	2099.78	phr	374.70	vhr	45965.08	vhr	0	mmr	0.00	mhr
2345.79	mmr	3240.61	vhr	221.36	vmr	30086.97	vmr_phr	0	mhr	0.00	vmr_phr
2281.79	vmr	3565.88	mhr	193.68	phr	28483.53	phr	0	vmr_phr	0.00	vhr_pmr
1795.80	pmr	31176.36	mmr	41.43	pmr	16109.79	vmr	0	vhr_pmr	0.01	pmr

Normal distribution:

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
789.64	phr	98.48	pmr	188.76	mhr	3404.35	phr	0	vmr	0.00	pmr
711.18	mhr	98.81	mmr	165.90	mmr	3370.41	vhr	0	vhr	0.00	phr
627.78	vhr	164.59	vhr_pmr	160.53	vhr_pmr	2538.96	mhr	0	pmr	0.00	mmr
614.07	vmr_phr	169.65	vmr	136.86	vmr_phr	1961.78	vmr_phr	0	phr	0.00	mhr
498.41	vhr_pmr	197.97	vmr_phr	123.03	pmr	1791.46	vmr	0	mmr	0.00	vmr_phr
441.92	vmr	244.56	mhr	120.90	phr	1606.02	vhr_pmr	0	mhr	0.00	vhr_pmr
403.75	mmr	319.17	vhr	90.51	vmr	1109.65	pmr	0	vmr_phr	0.01	vmr
365.42	pmr	362.10	phr	80.75	vhr	1102.37	mmr	0	vhr_pmr	0.01	vhr

Compare Gaussian and skewed t-distribution fits

Gaussian fits

Gaussian QQ plots

Gaussian vs skewed t

Probability in percent that the smallest and largest (respectively) observed return for each fund was generated by a normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
P_norm(X_min)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
P_norm(X_max)	0.546	0.580	1.599	0.796	1.161	0.746	0.793	0.903
P_t(X_min)	0.556	0.523	0.342	0.387	0.476	0.443	0.433	0.499

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
P_t(X_max)	0.448	0.469	1.135	0.614	0.739	0.518	0.543	0.613

Average number of years between min or max events (respectively):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
norm: avg yrs btw min	1.157446e+77	3.872665e+82	3.449915e+57	4.076881e+55	2.913573e+67	1.048138e+67	2.446247e+63	4.894371e+71
norm: avg yrs btw max	1.830960e+02	1.724260e+02	6.252800e+01	1.256870e+02	8.614100e+01	1.341240e+02	1.261390e+02	1.107200e+02
t: avg yrs btw min	1.798360e+02	1.912300e+02	2.924120e+02	2.584920e+02	2.099400e+02	2.257430e+02	2.309140e+02	2.002190e+02
t: avg yrs btw max	2.233340e+02	2.130540e+02	8.811500e+01	1.628680e+02	1.352750e+02	1.928930e+02	1.843230e+02	1.630560e+02

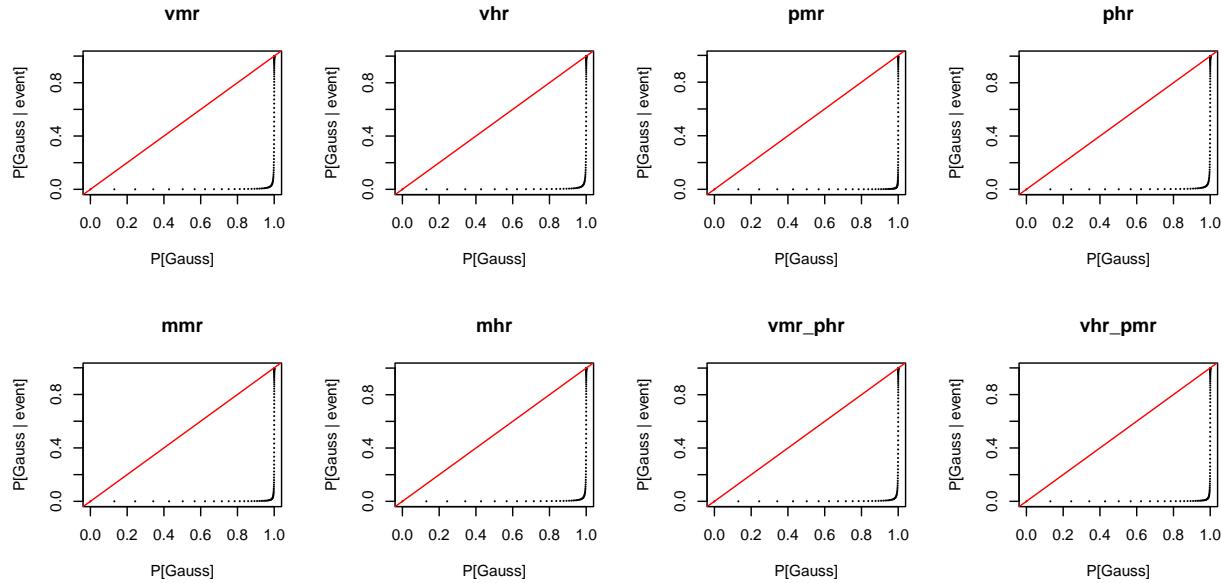
Lilliefors test p-values for Lilliefors test.

Testing H_0 , that log-returns are Gaussian.

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
p value	0	0	0	0	0	0.001	0.001	0

Wittgenstein's Ruler For different given probabilities that returns are Gaussian, what is the probability that the distribution is Gaussian rather than skewed t-distributed, given the smallest/largest observed log-returns?

Conditional probabilities for smallest observed log-returns:



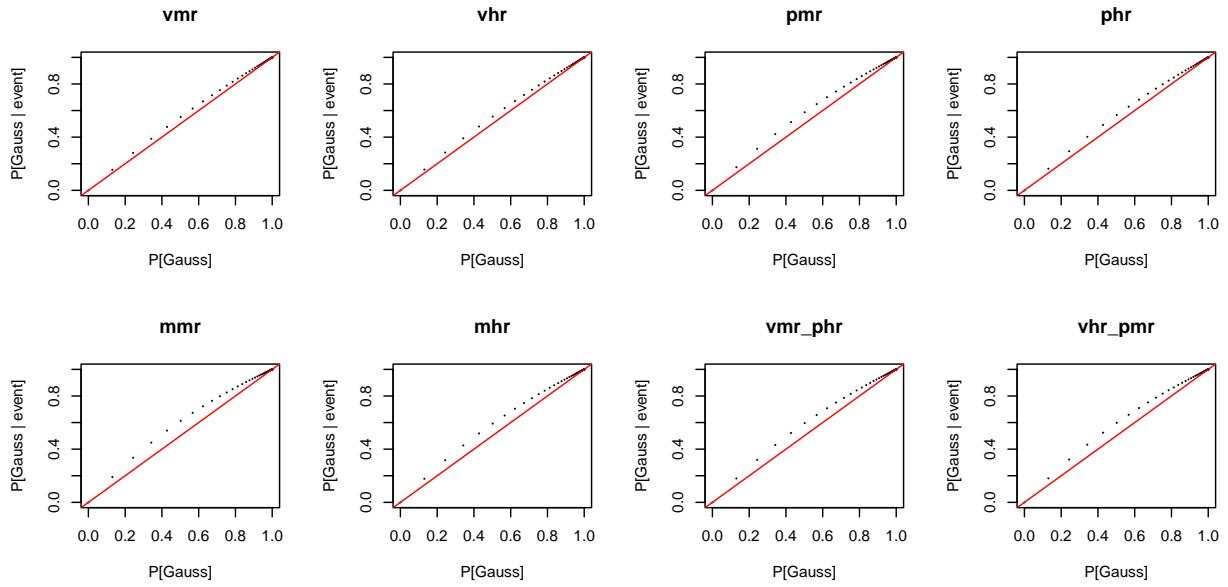
Use $1 - \text{p-value}$ from Lilliefors test as prior probability that the distribution is Gaussian.

$x_{\text{obs}} = \min(x)$ and $P[\text{Event} | \text{Gaussian}] = P_{\text{Gauss}}[X \leq x_{\text{min}}]$:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Lillie p-val	0.000	0.00	0.000	0.00	0.000	0.001	0.001	0.000
Prior prob	1.000	1.00	1.000	1.00	1.000	0.999	0.999	1.000
$P[\text{Gauss} \text{Event}]$	0.838	0.82	0.768	0.69	0.815	0.486	0.418	0.965

Use $1 - \text{p-value}$ from Lilliefors test as prior probability that the distribution is Gaussian.

$x_{\text{obs}} = \max(x)$ and $P[\text{Event} | \text{Gaussian}] = P_{\text{Gauss}}[X \geq x_{\text{max}}]$:

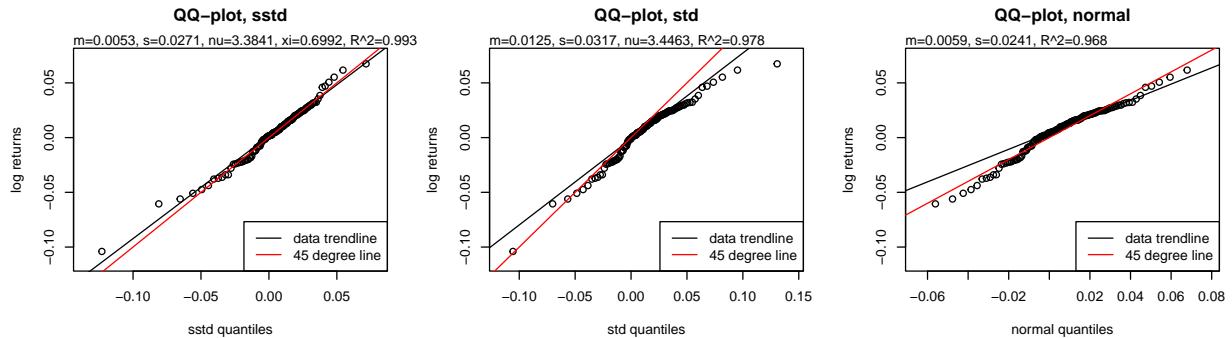


	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Lillie p-val	0	0	0	0	0	0.001	0.001	0
Prior prob	1	1	1	1	1	0.999	0.999	1
P[Gauss Event]	1	1	1	1	1	1.000	1.000	1

Velliv medium risk (vmr), June 2012 - April 2024

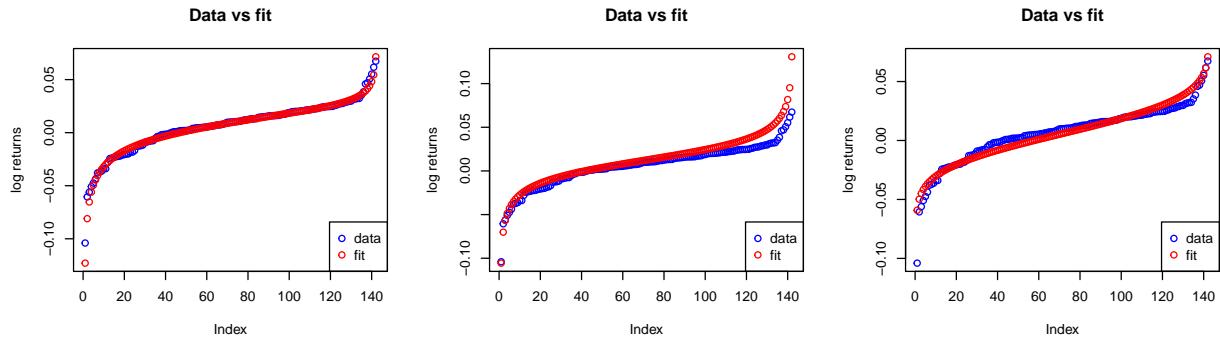
QQ Plot

Skewed t -distribution (sstd):



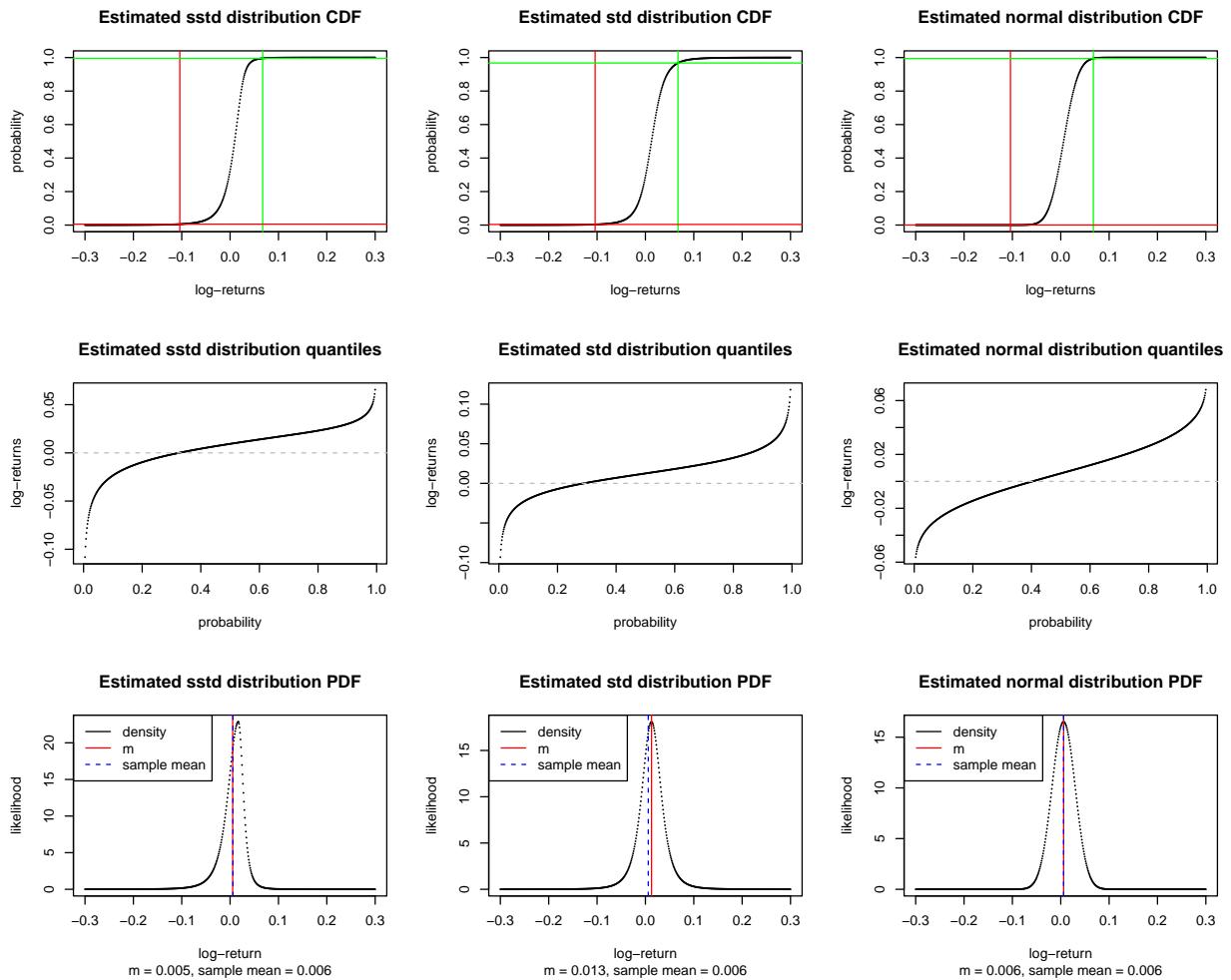
Data vs fit

Let's plot the fit and the observed returns together.



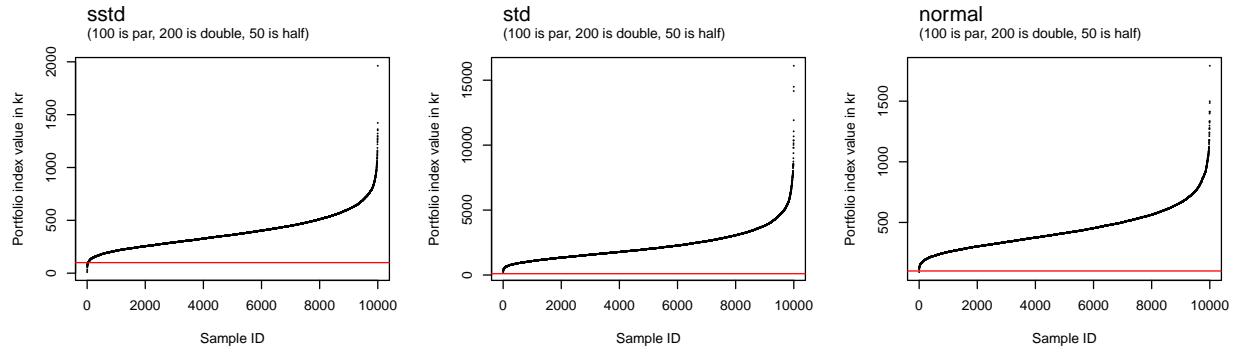
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

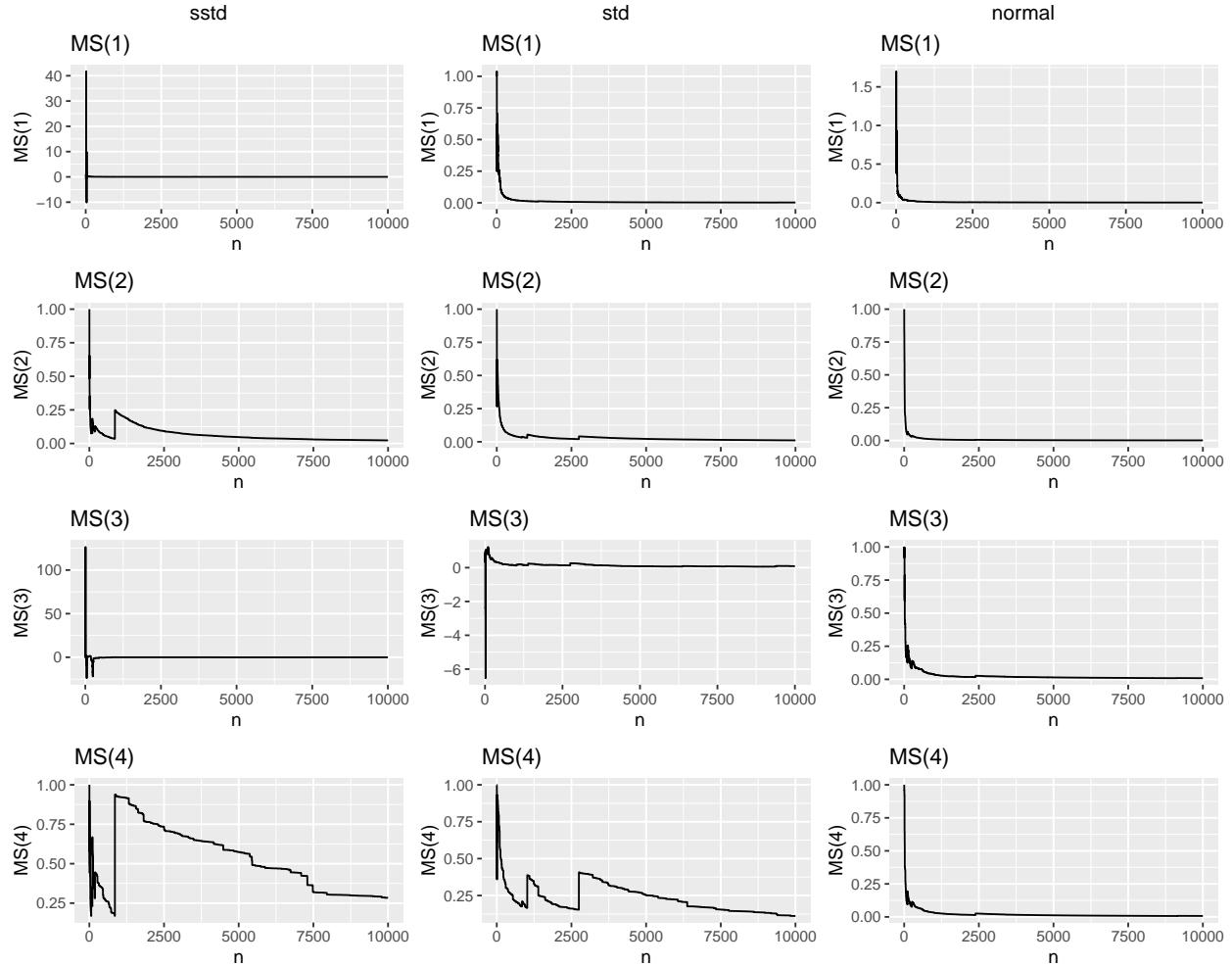
Sorted portfolio index values for last period of all runs

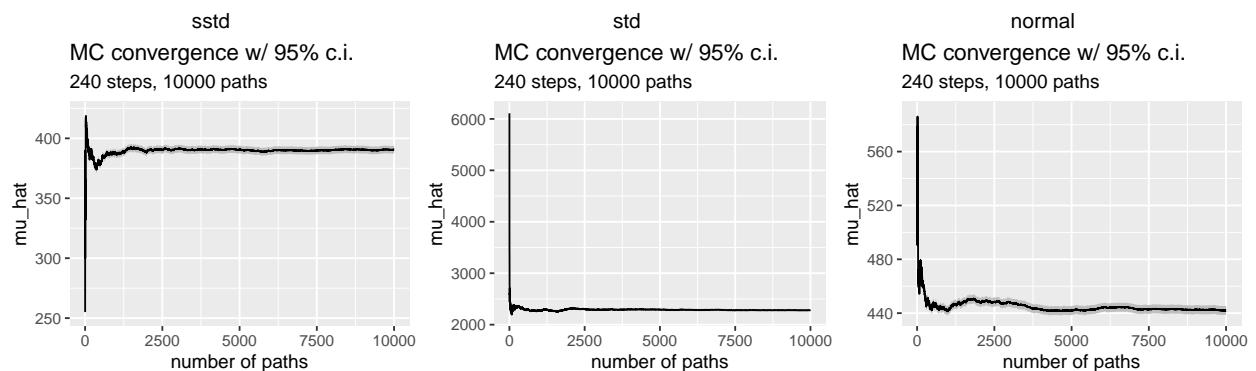


Convergence

Max vs sum

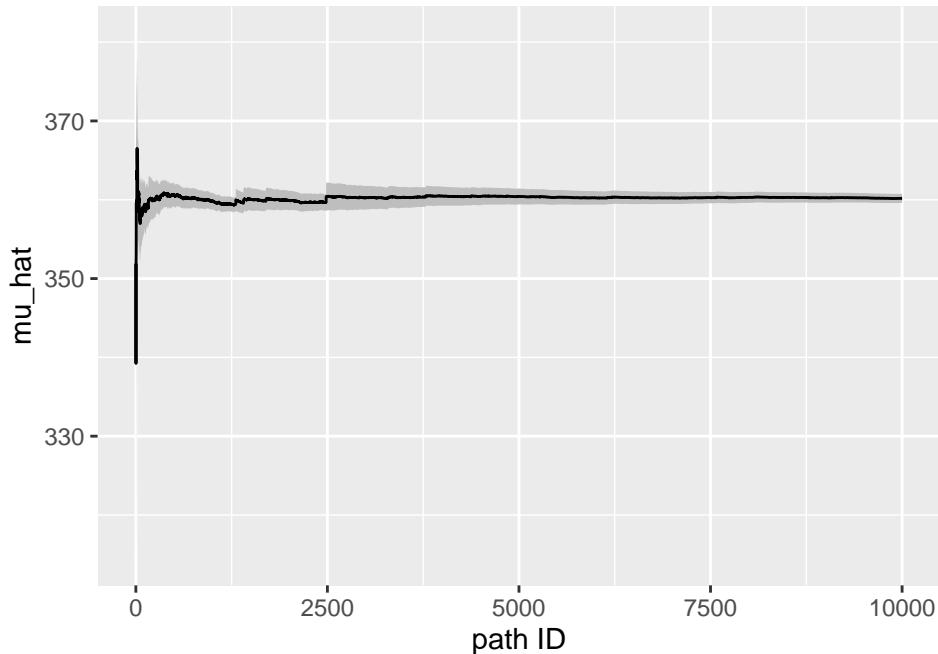
Max vs sum plots for the first four moments:



MC**IS**

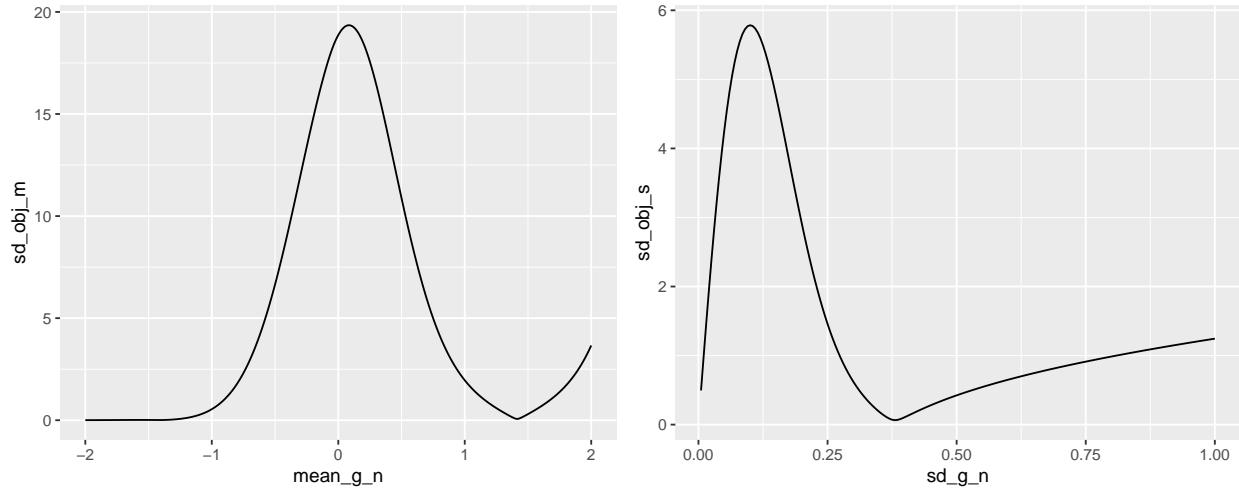
Skewed t -distribution with a normal proposal distribution.

Importance Sampling convergence w/ 95% c.i.
240 steps, 10000 paths

**Parameters**

```
## [1] 1.4145605 0.3807834
```

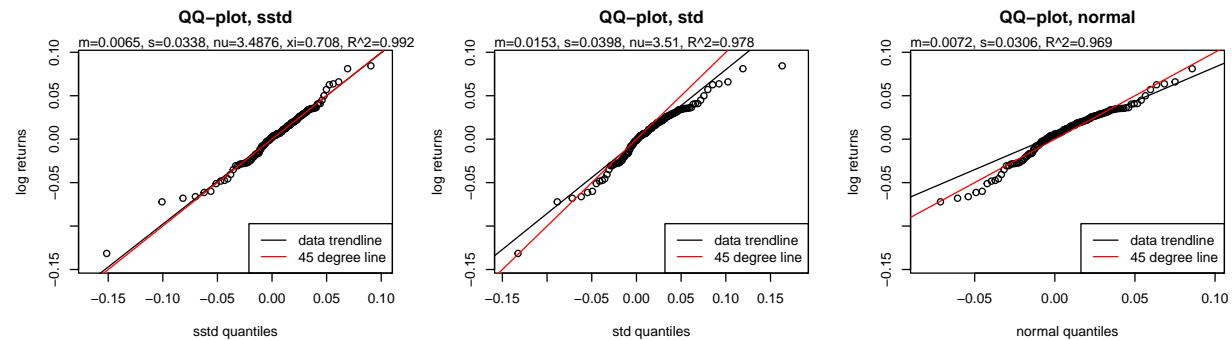
Objective function plots



Velliv high risk (vhr), June 2012 - April 2024

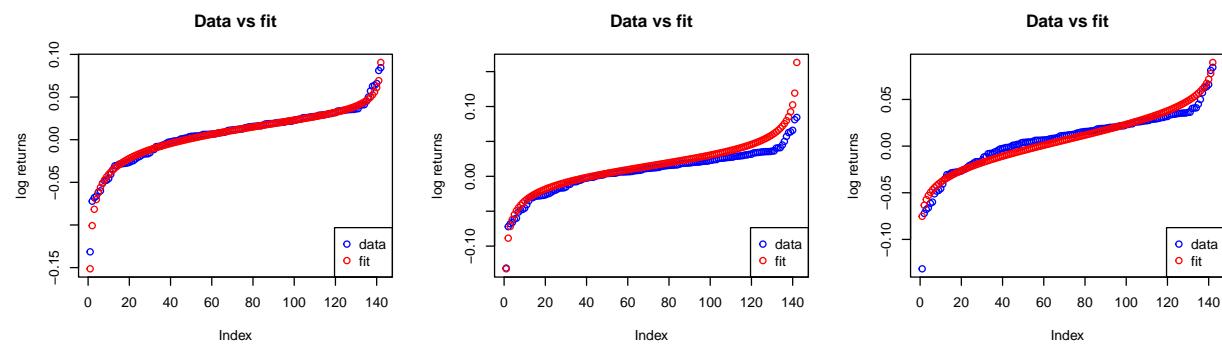
QQ Plot

Skewed t -distribution (sstd):



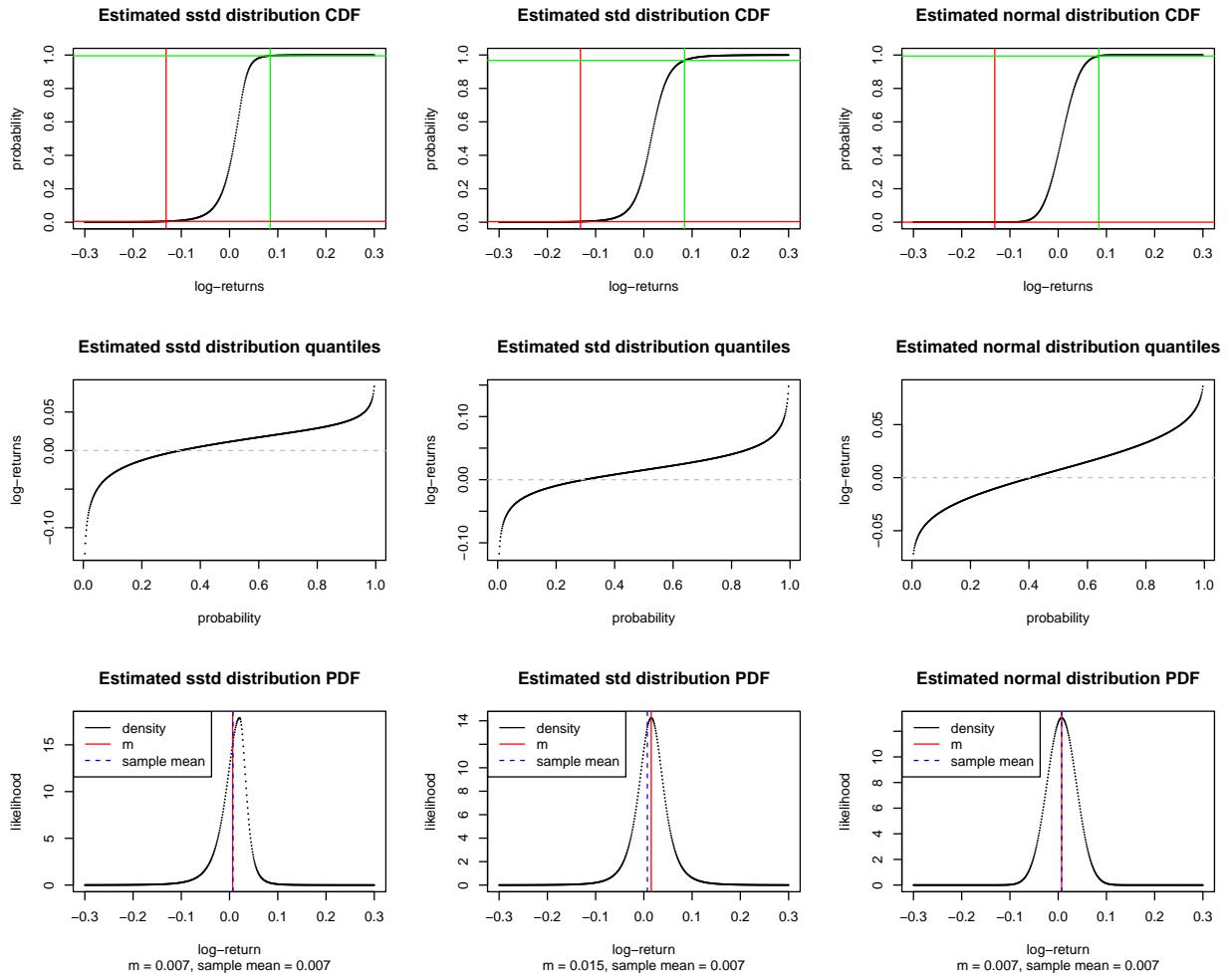
Data vs fit

Let's plot the fit and the observed returns together.



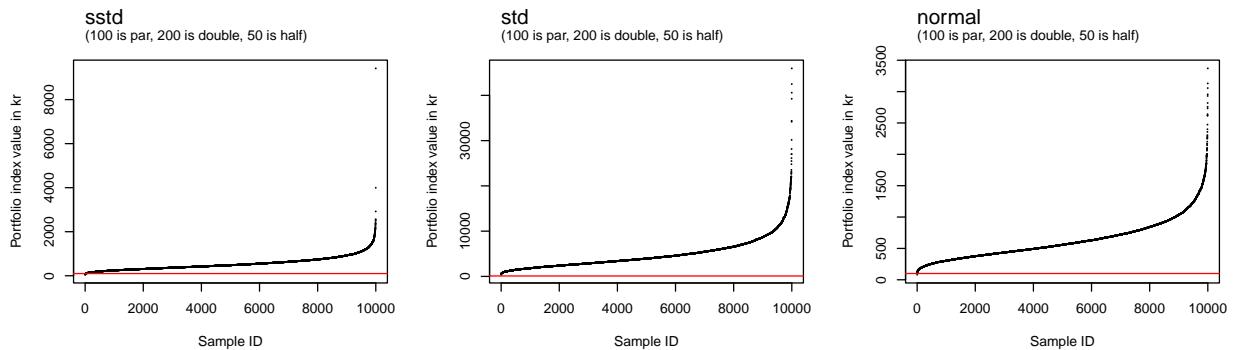
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

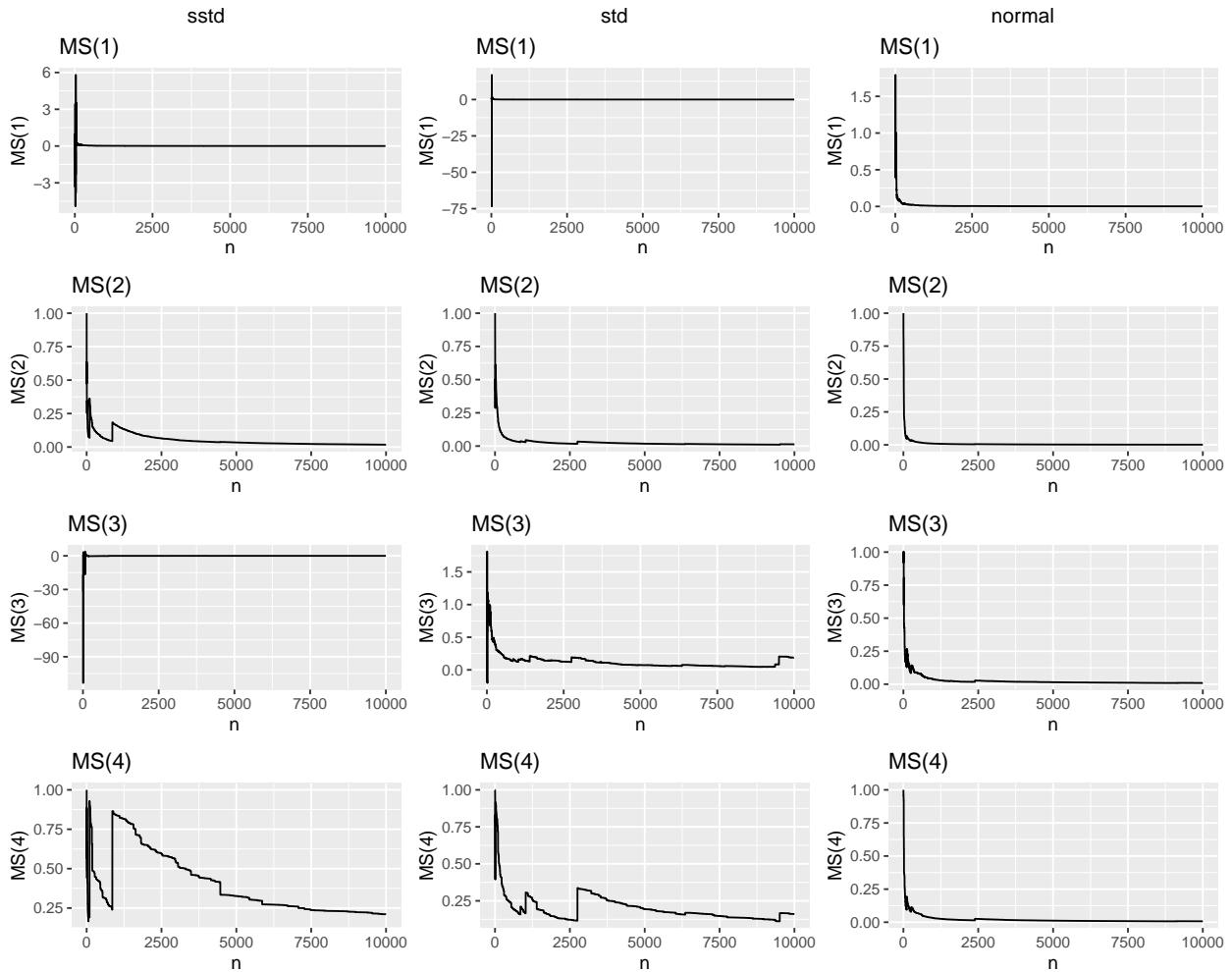
Sorted portfolio index values for last period of all runs



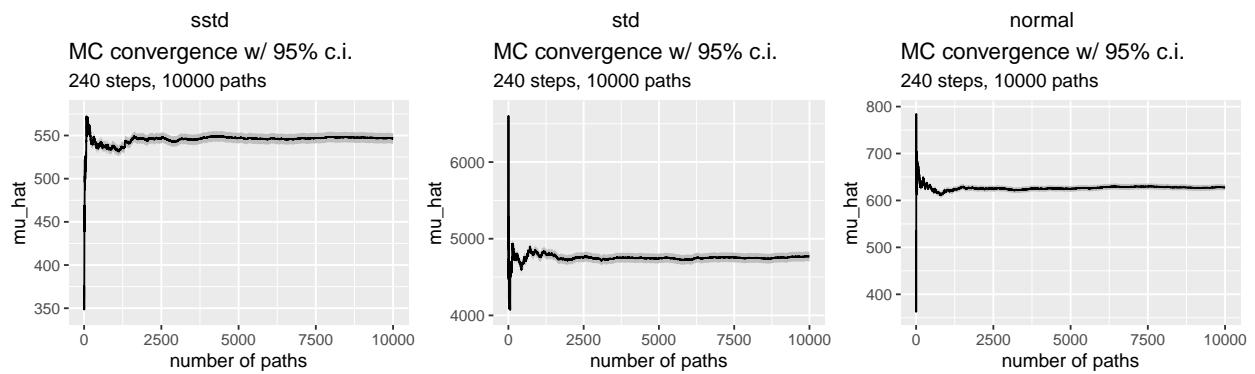
Convergence

Max vs sum

Max vs sum plots for the first four moments:



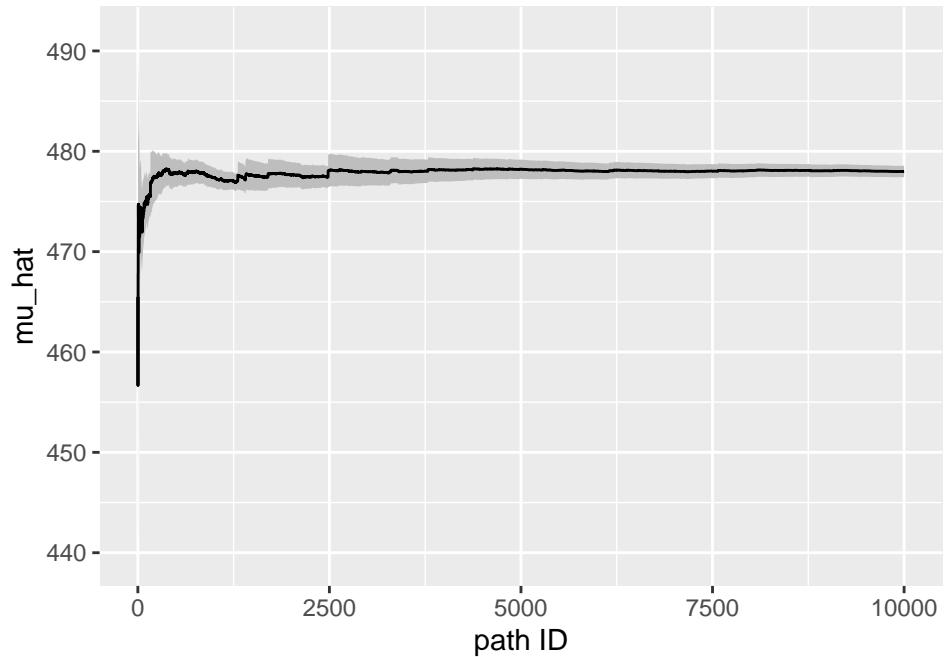
MC



IS

Skewed t -distribution with a normal proposal distribution.

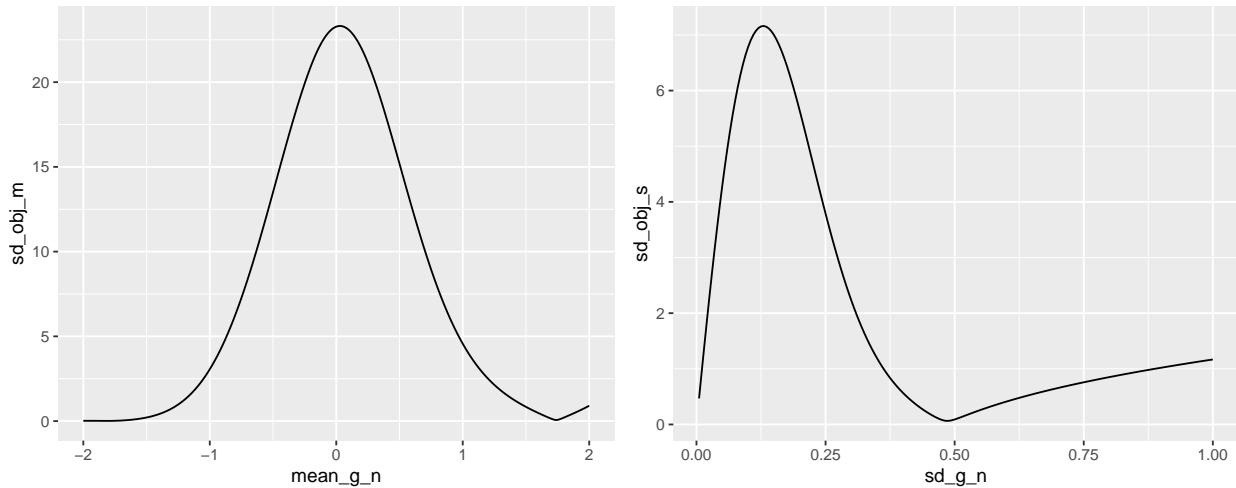
Importance Sampling convergence w/ 95% c.i.
 240 steps, 10000 paths



Parameters

```
## [1] 1.7391222 0.4858909
```

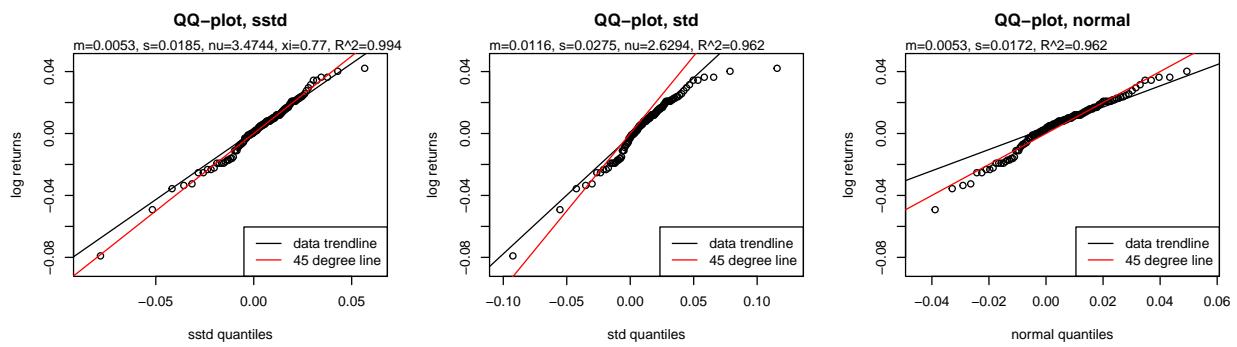
Objective function plots



PFA medium risk (pmr), June 2012 - April 2024

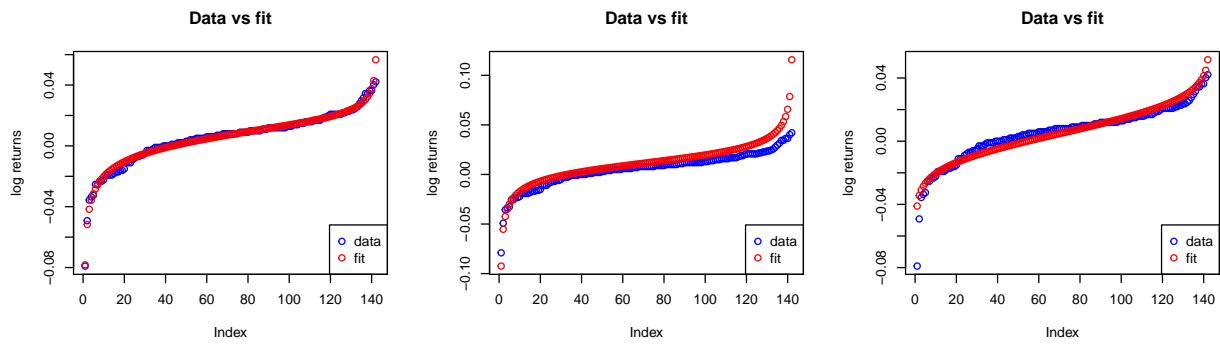
QQ Plot

Skewed t -distribution (sstd):



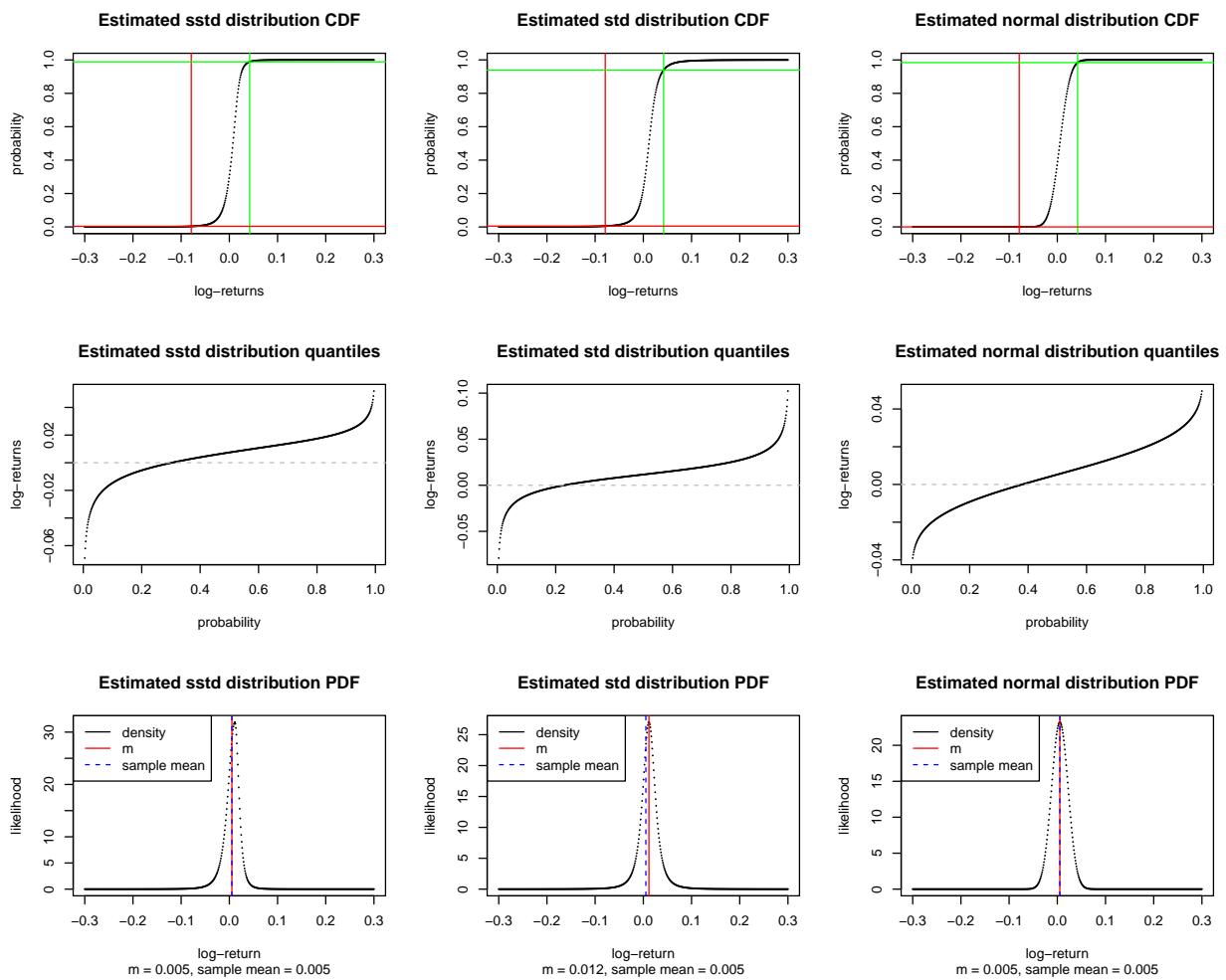
Data vs fit

Let's plot the fit and the observed returns together.



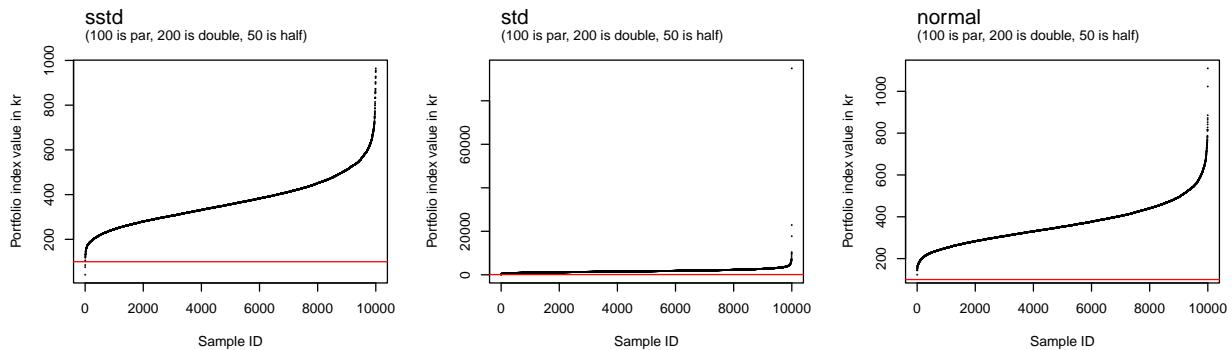
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

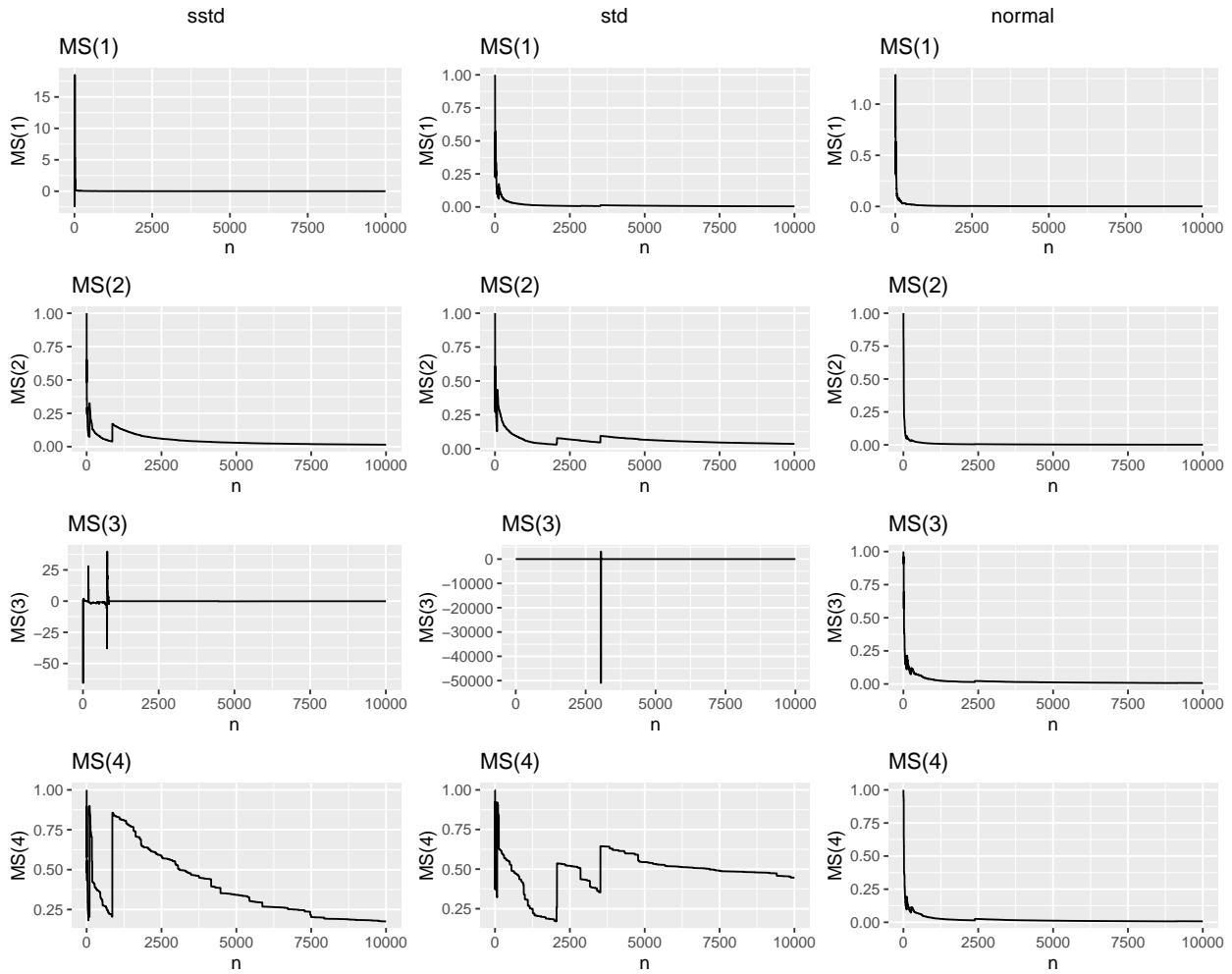
Sorted portfolio index values for last period of all runs



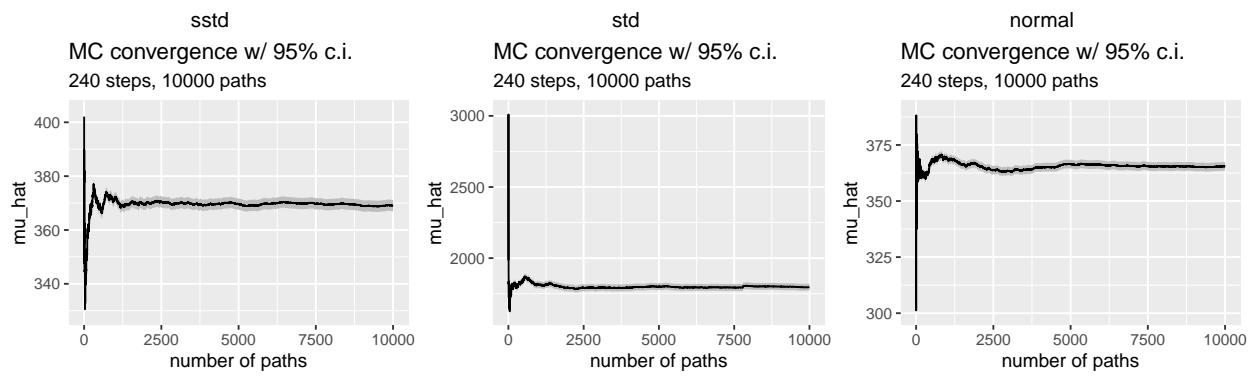
Convergence

Max vs sum

Max vs sum plots for the first four moments:



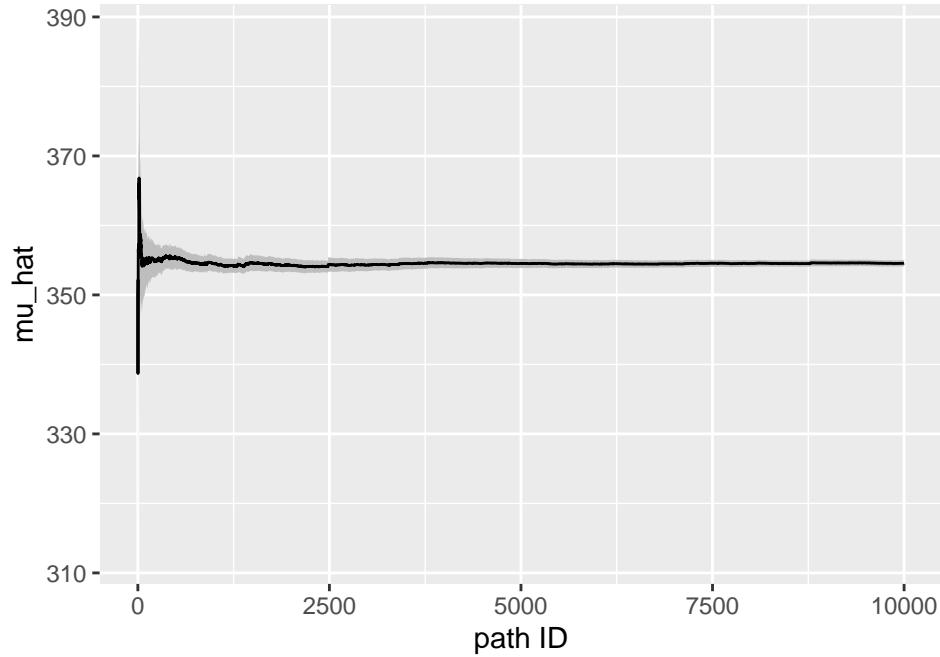
MC



IS

Skewed t -distribution with a normal proposal distribution.

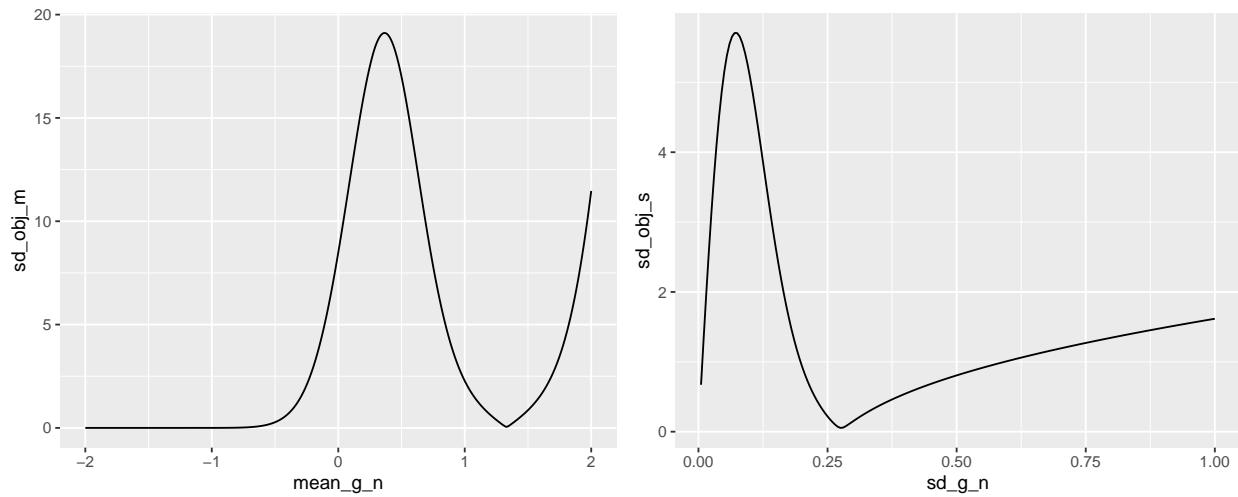
**Importance Sampling convergence w/ 95% c.i.
240 steps, 10000 paths**



Parameters

```
## [1] 1.3304634 0.2764028
```

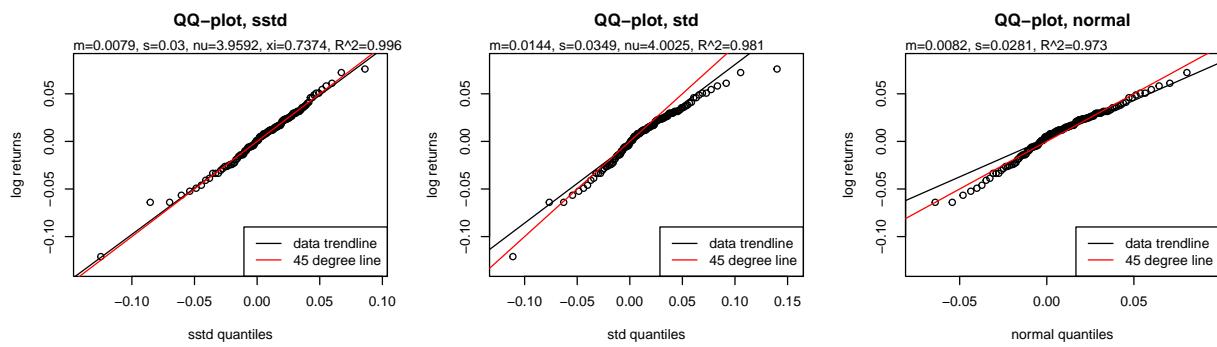
Objective function plots



PFA high risk (phr), June 2012 - April 2024

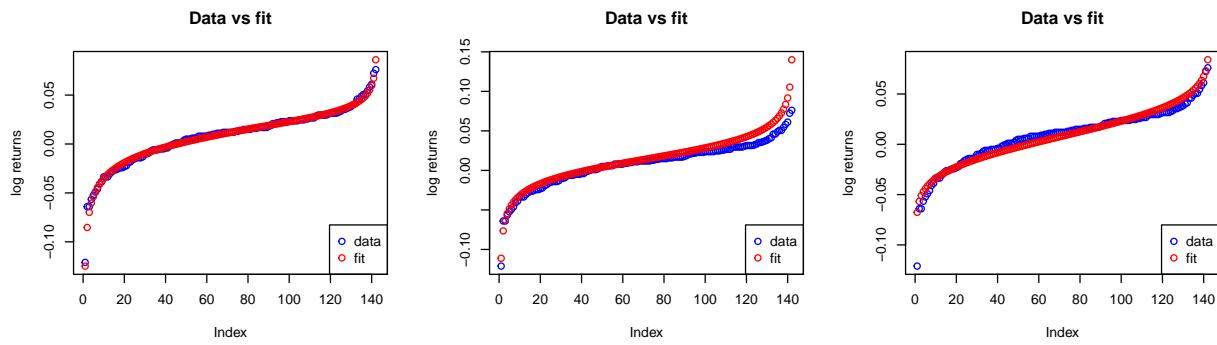
QQ Plot

Skewed t -distribution (sstd):



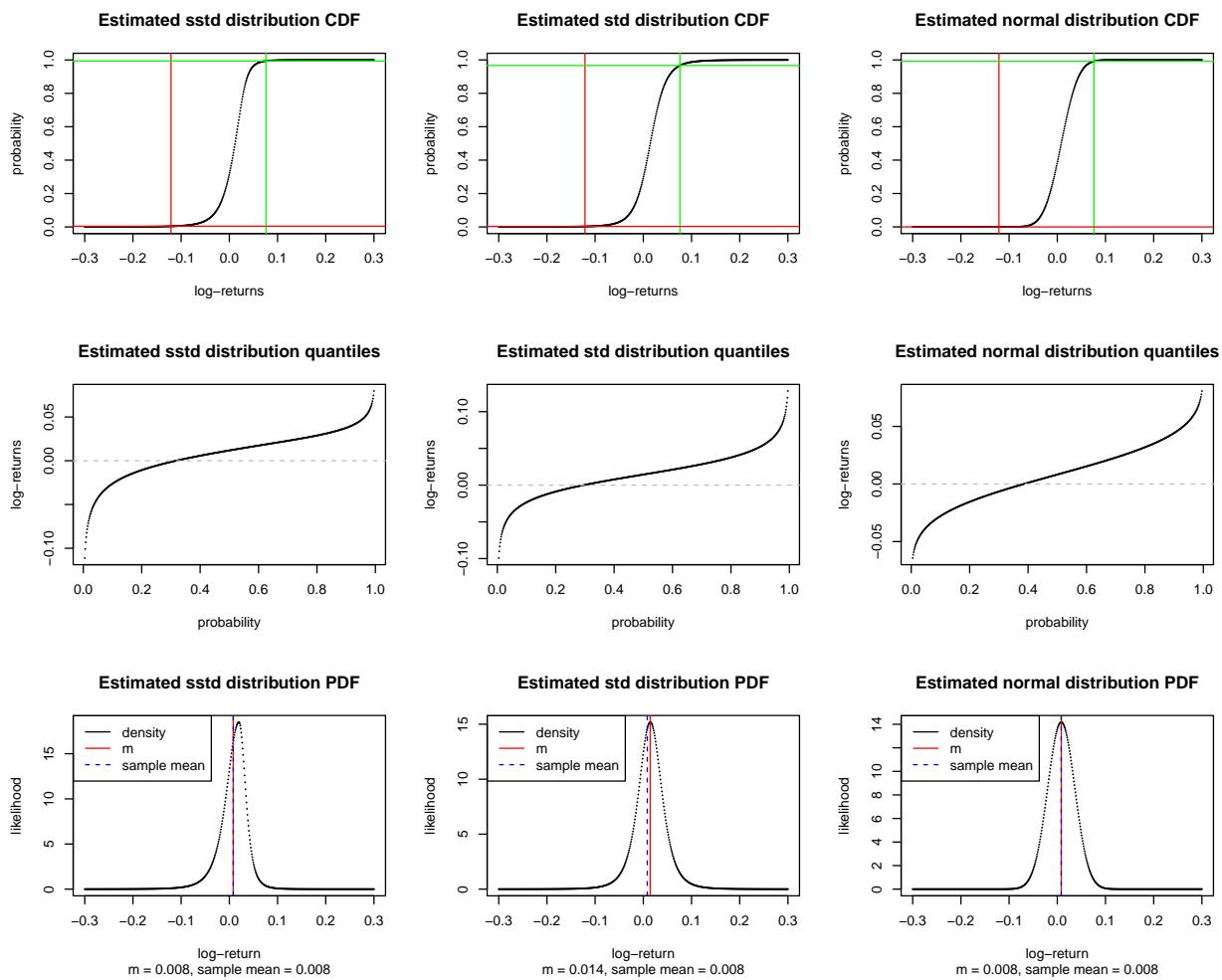
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

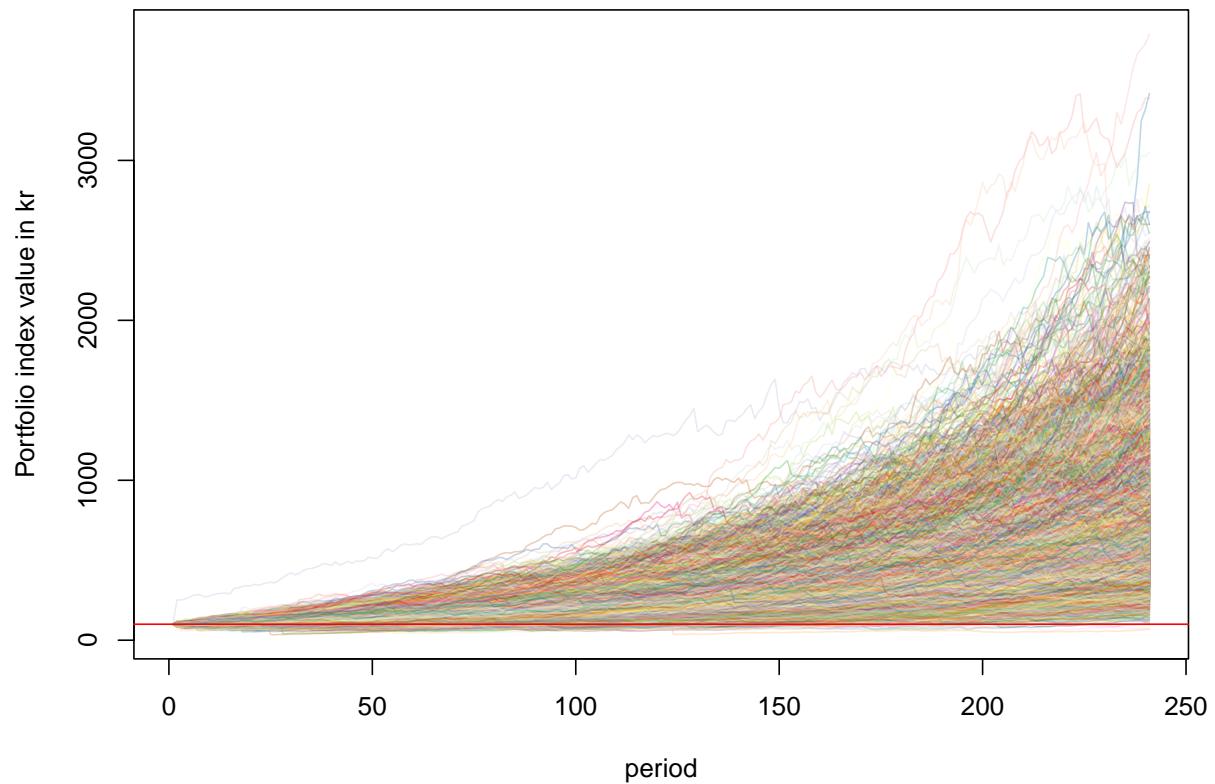


Monte Carlo

phr has the sstd fit with the highest sstd fit with the value of nu. Compare with other distributions:

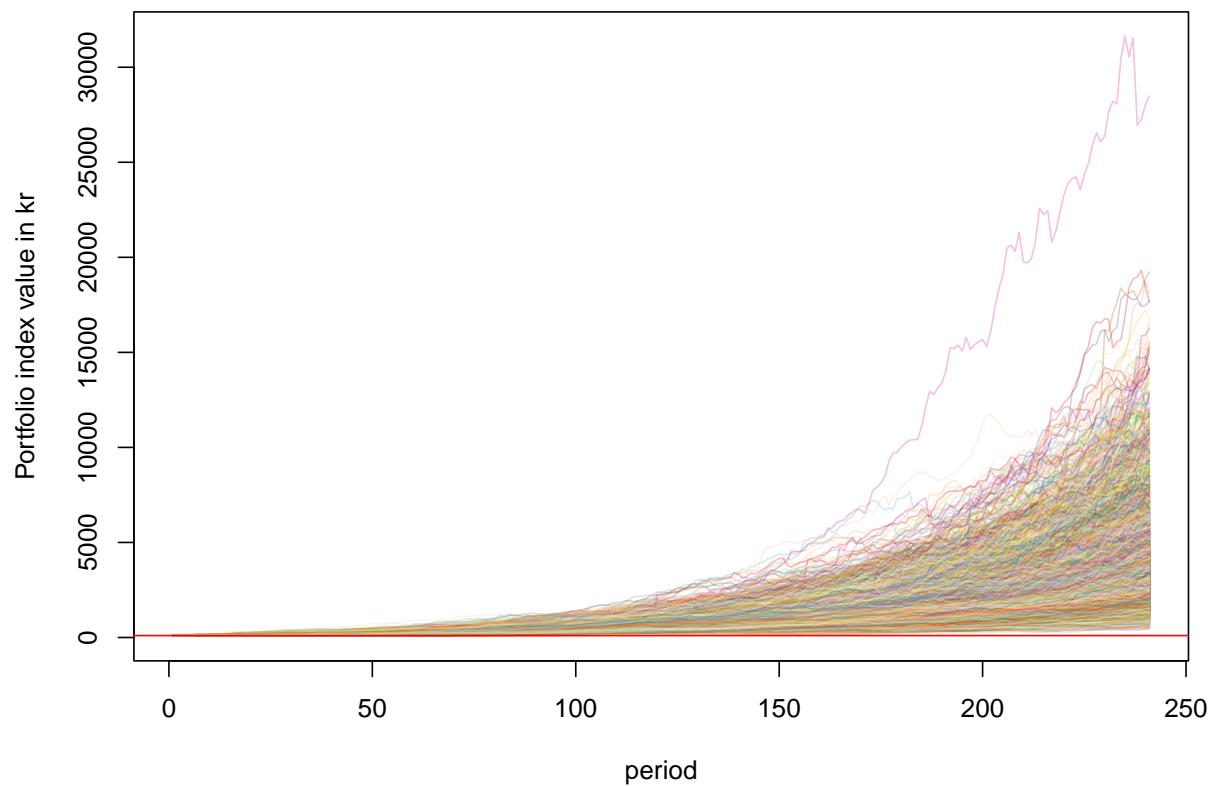
MC simulation with down-and-out

sstd distribution, number of paths: 10000, number of periods: 240



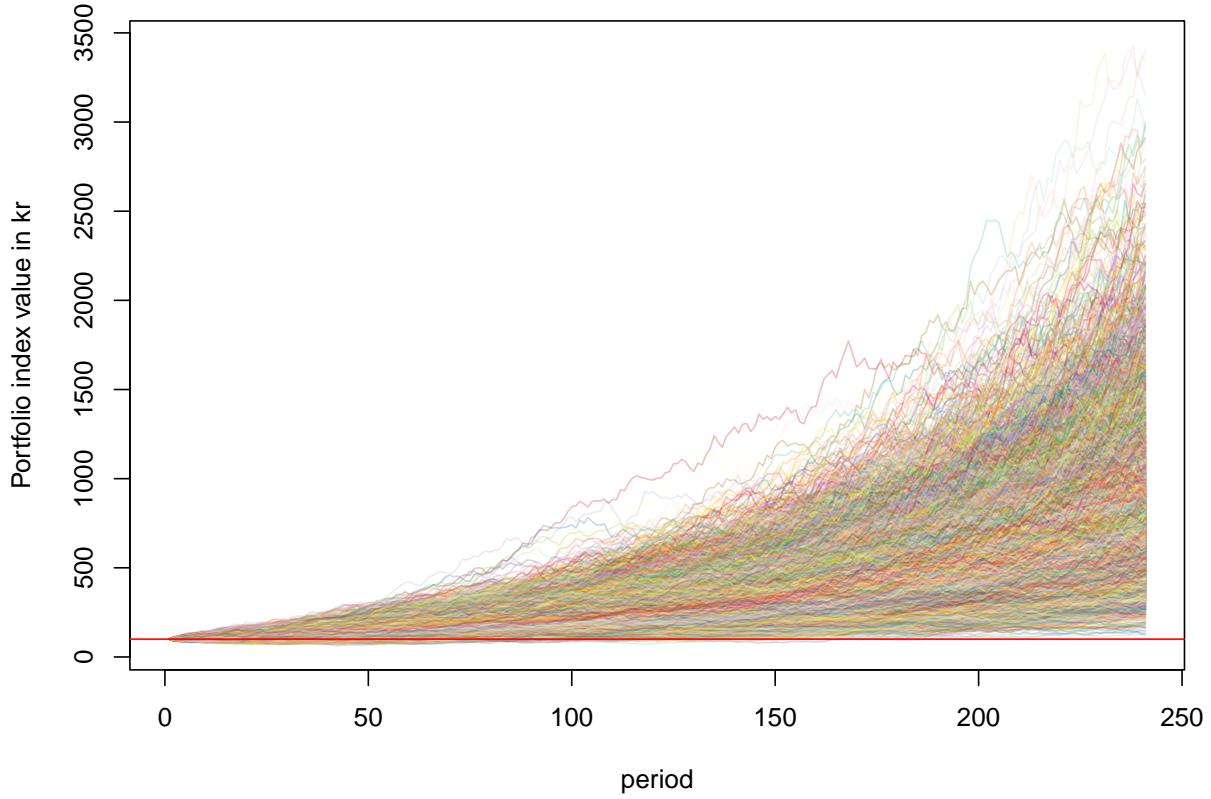
MC simulation with down-and-out

std distribution, number of paths: 10000, number of periods: 240

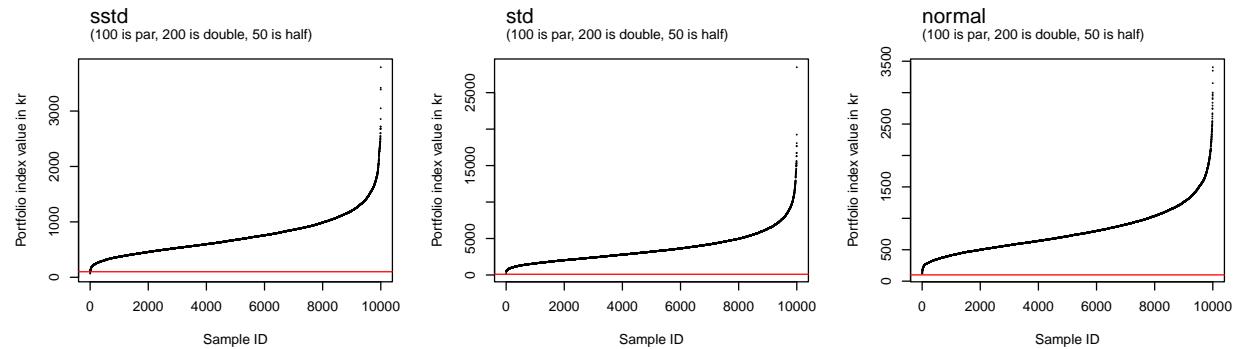


MC simulation with down-and-out

normal distribution, number of paths: 10000, number of periods: 240



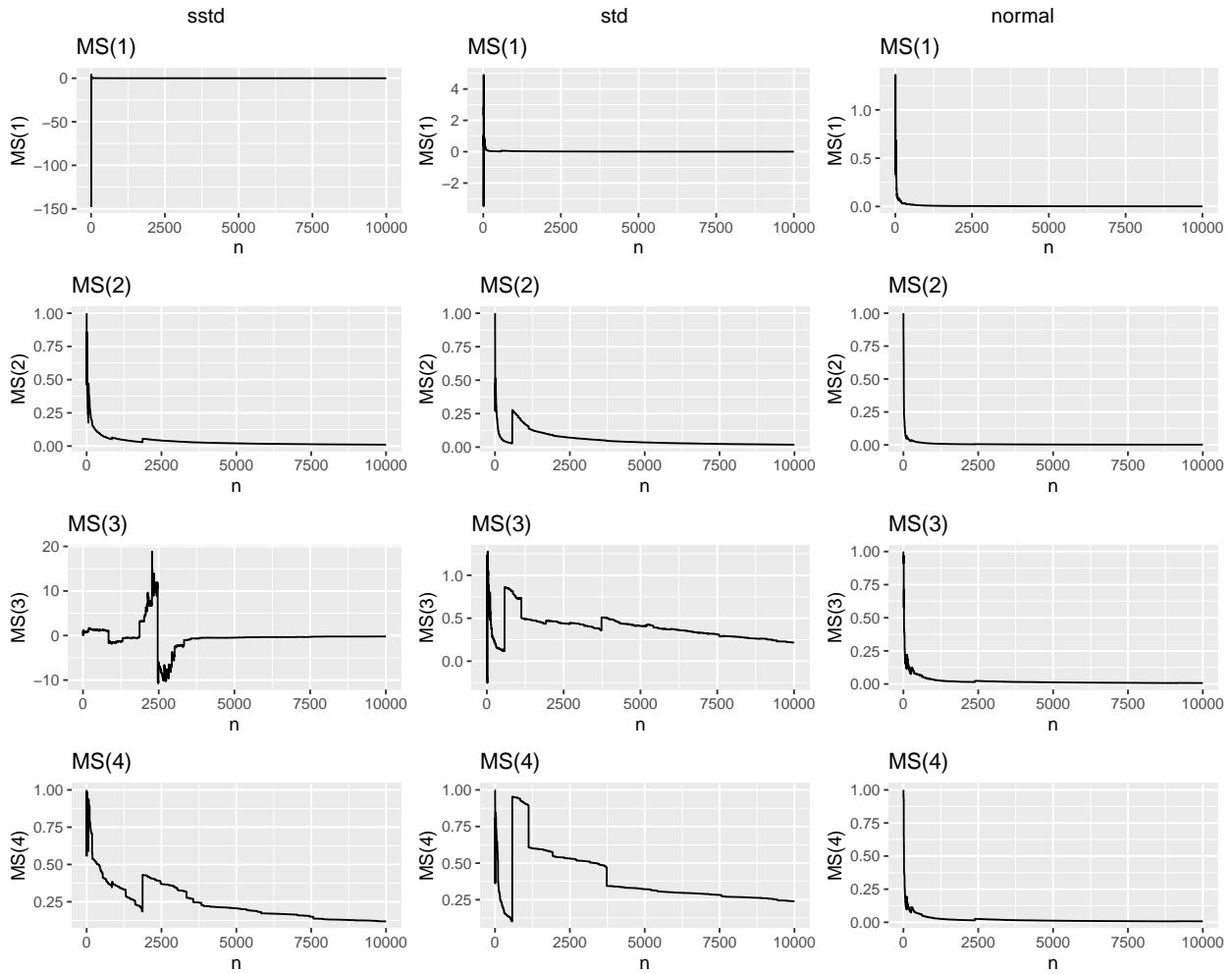
Sorted portfolio index values for last period of all runs



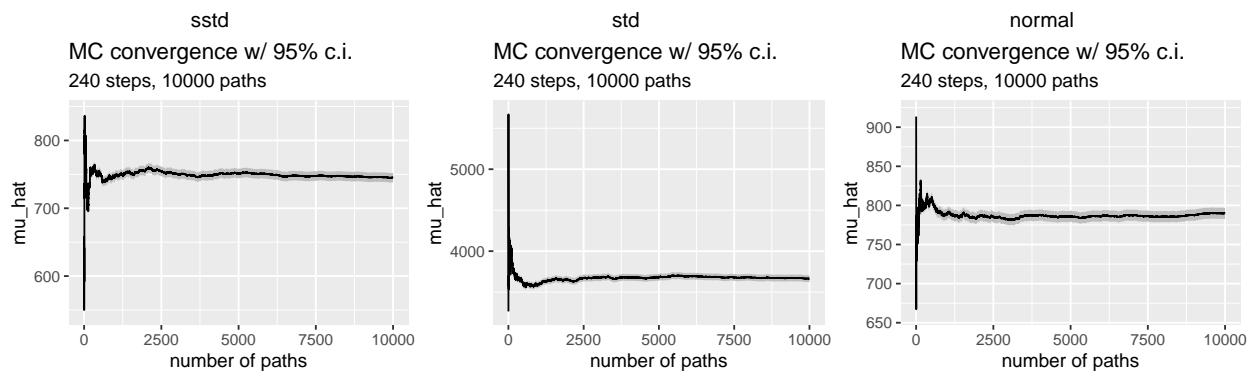
Convergence

Max vs sum

Max vs sum plots for the first four moments:



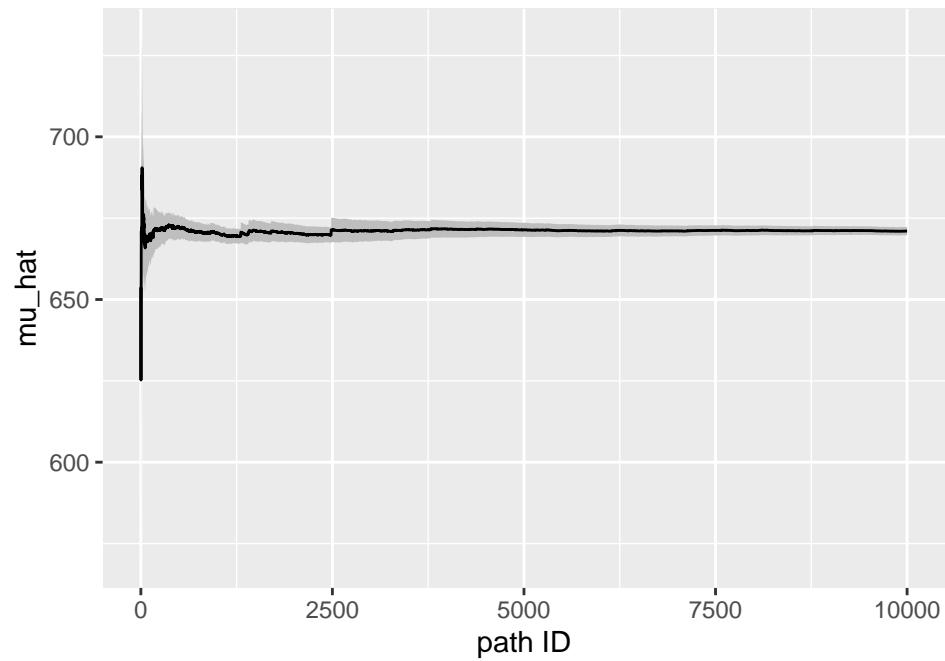
MC



IS

Skewed t -distribution with a normal proposal distribution.

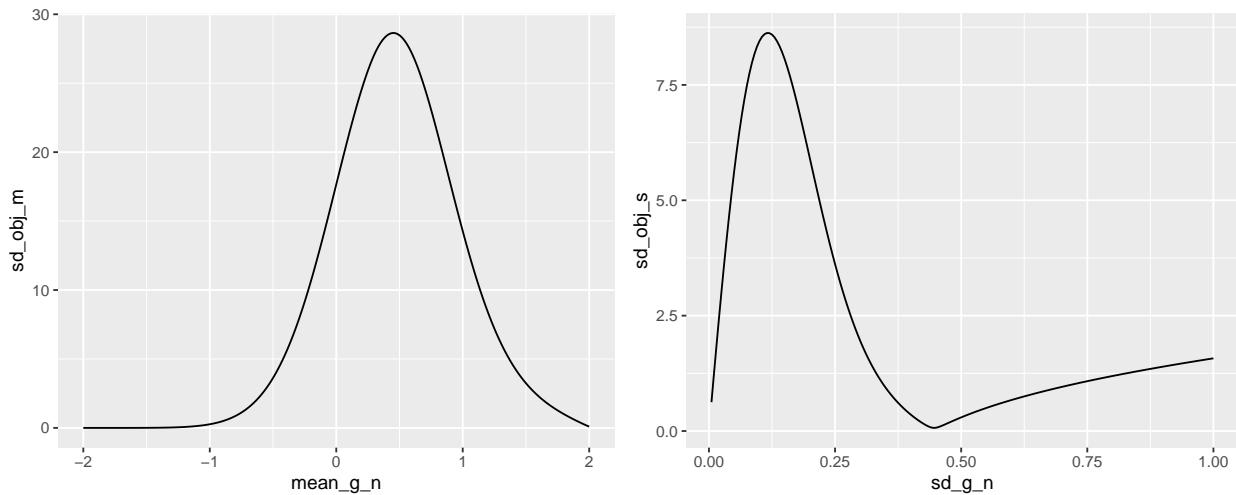
Importance Sampling convergence w/ 95% c.i.
240 steps, 10000 paths



Parameters

```
## [1] 2.0162301 0.4463226
```

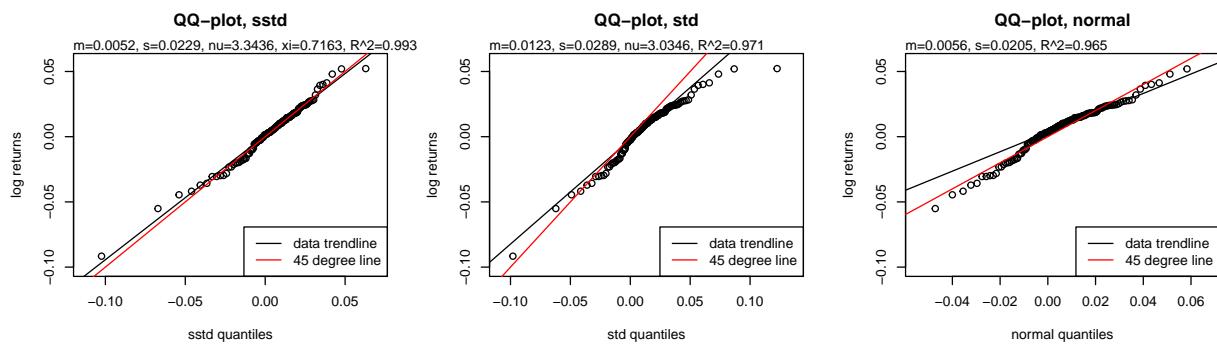
Objective function plots



Mix medium risk (mmr), June 2012 - April 2024

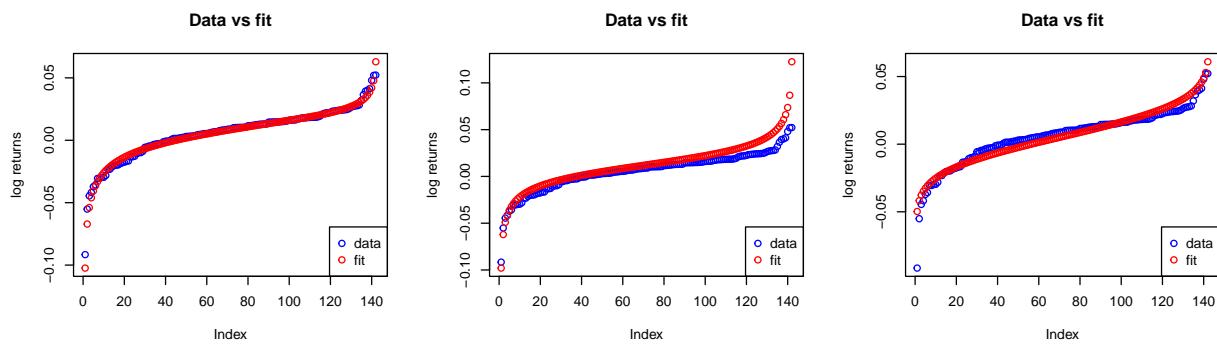
QQ Plot

Skewed t -distribution (sstd):



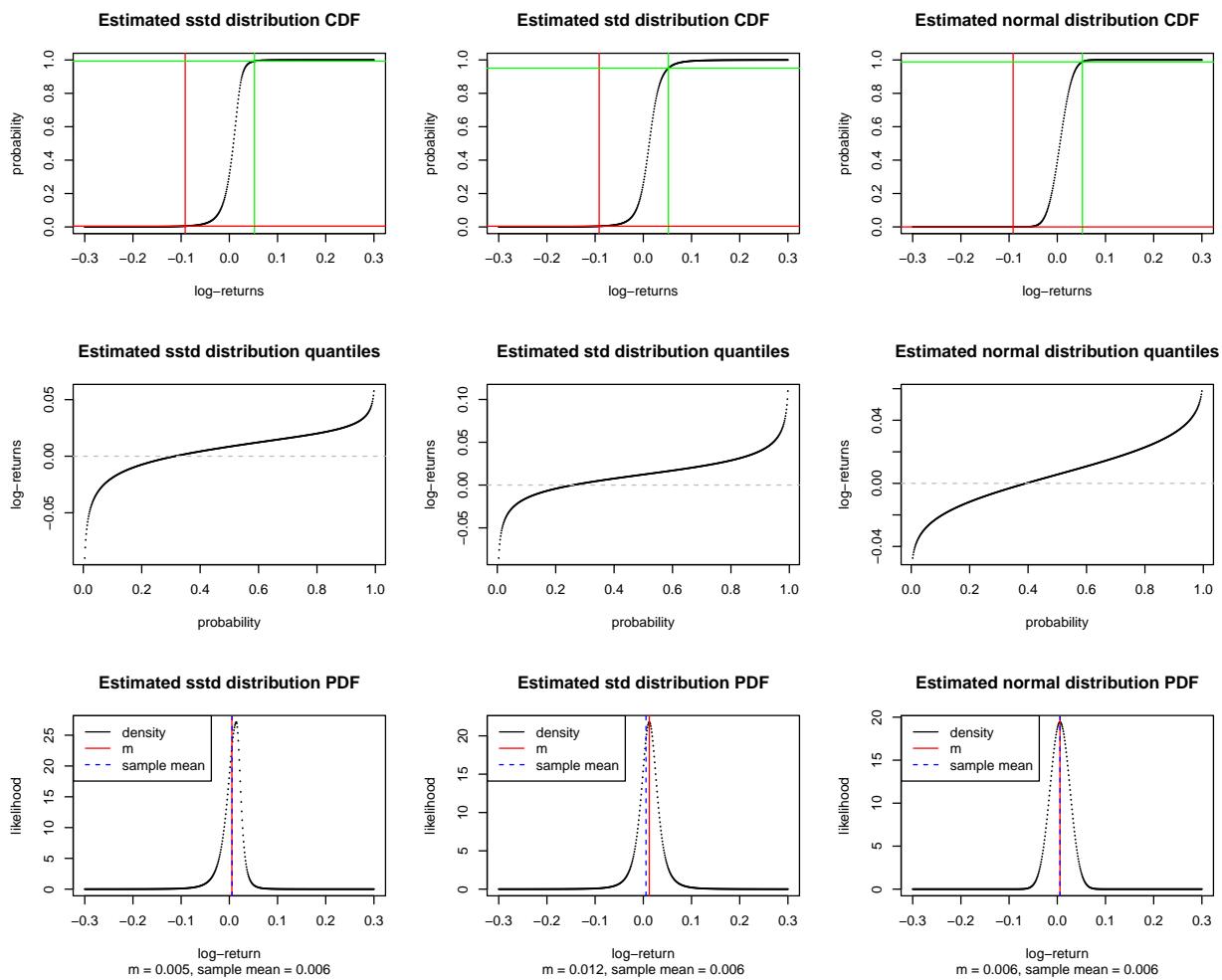
Data vs fit

Let's plot the fit and the observed returns together.



Estimated distribution

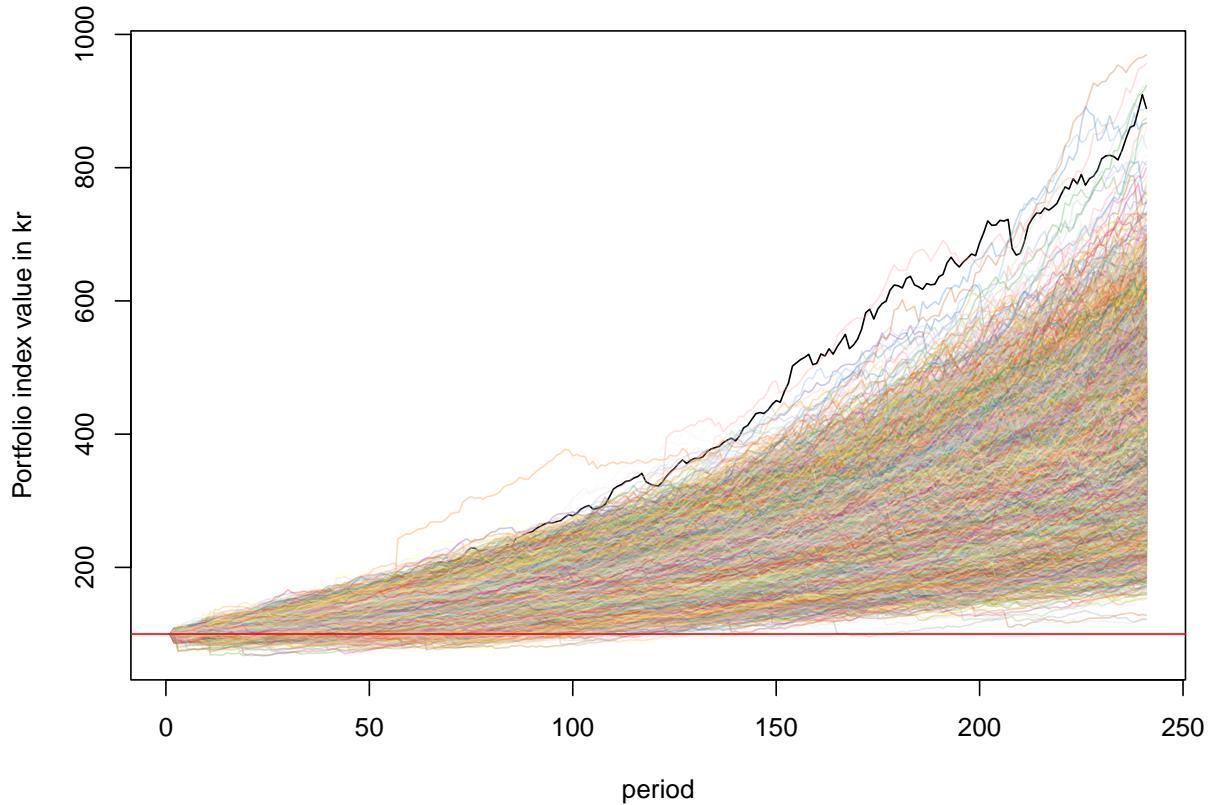
Now let's look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

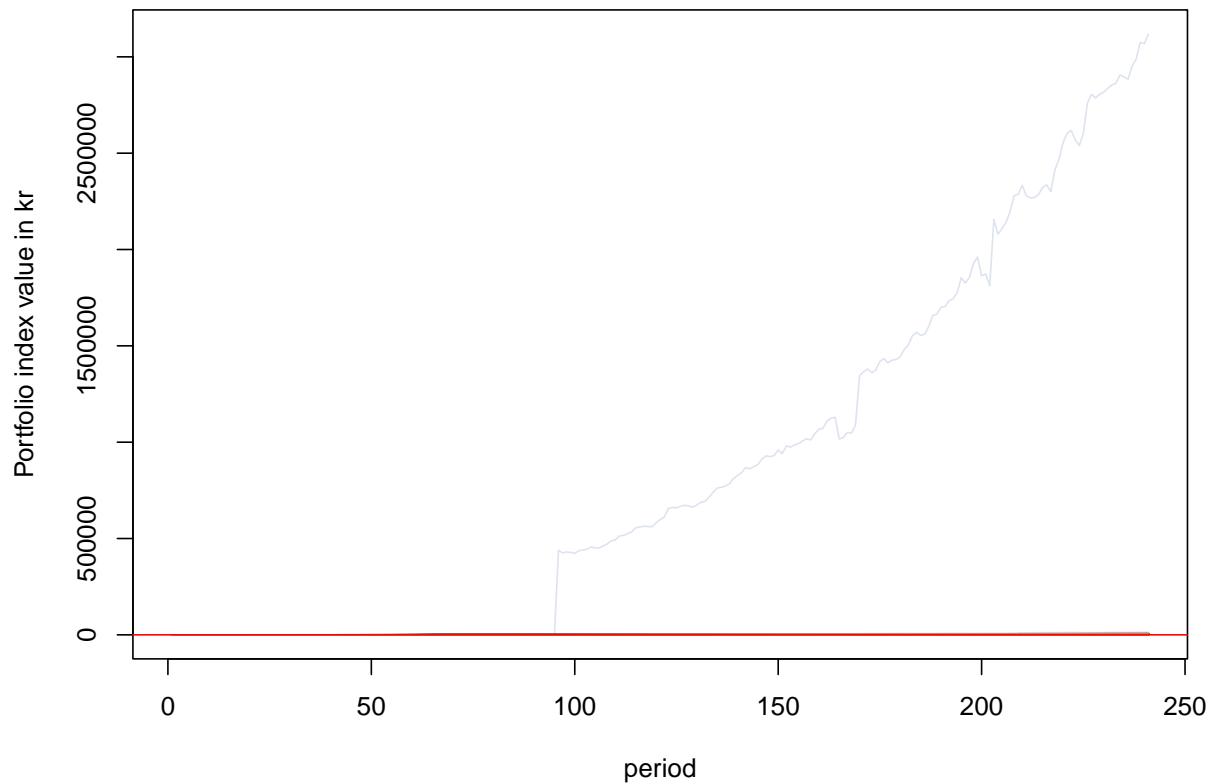
mmr has the sstd fit with the lowest value of nu. Compare with other distributions:

MC simulation with down-and-out
sstd distribution, number of paths: 10000, number of periods: 240



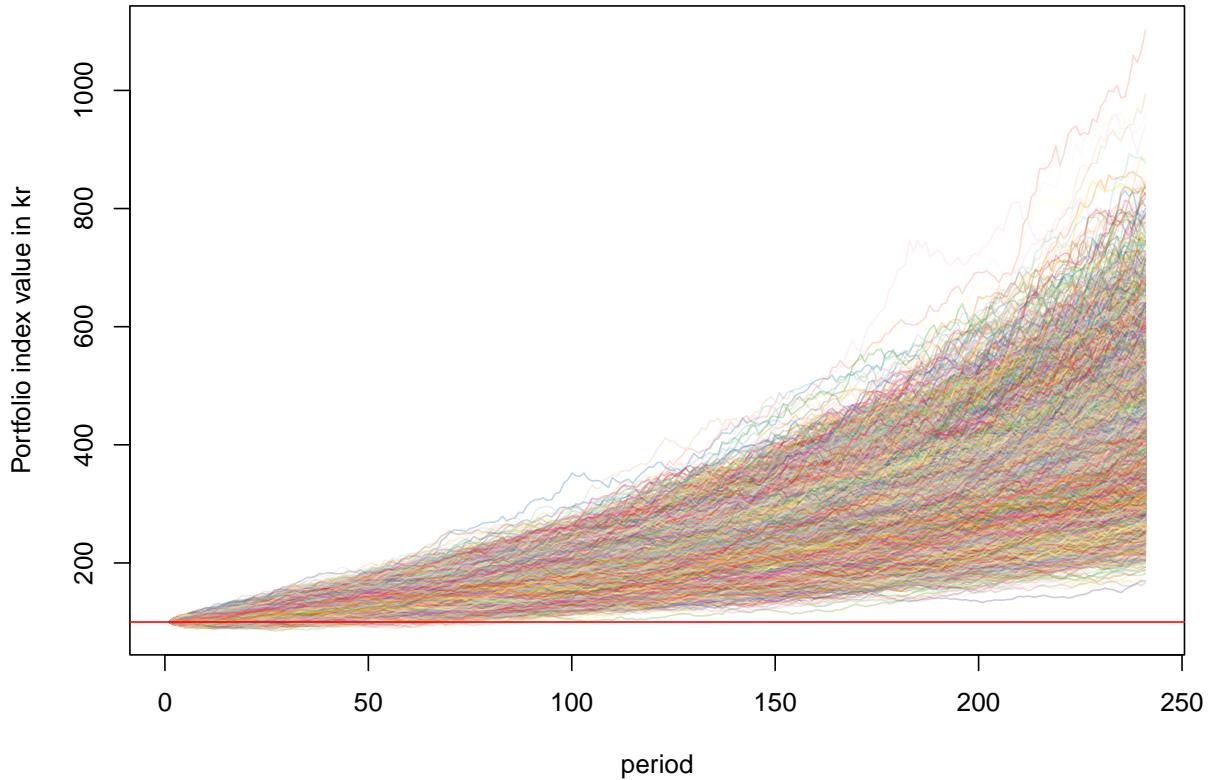
MC simulation with down-and-out

std distribution, number of paths: 10000, number of periods: 240

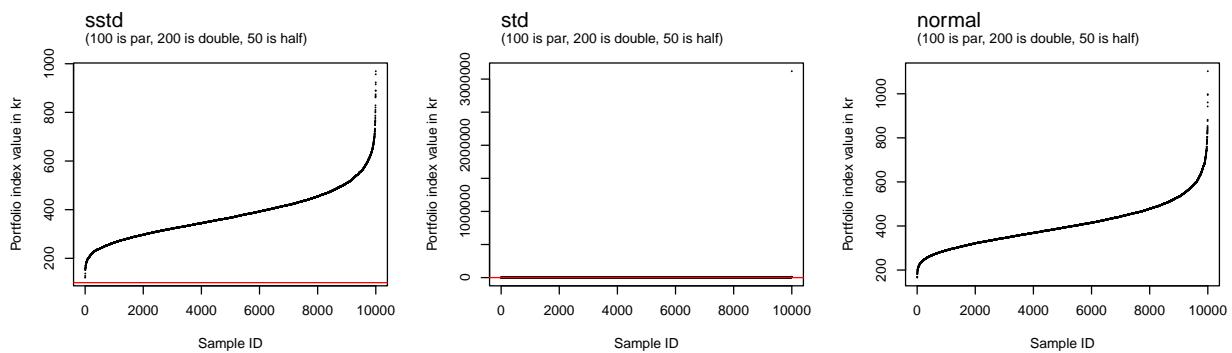


MC simulation with down-and-out

normal distribution, number of paths: 10000, number of periods: 240



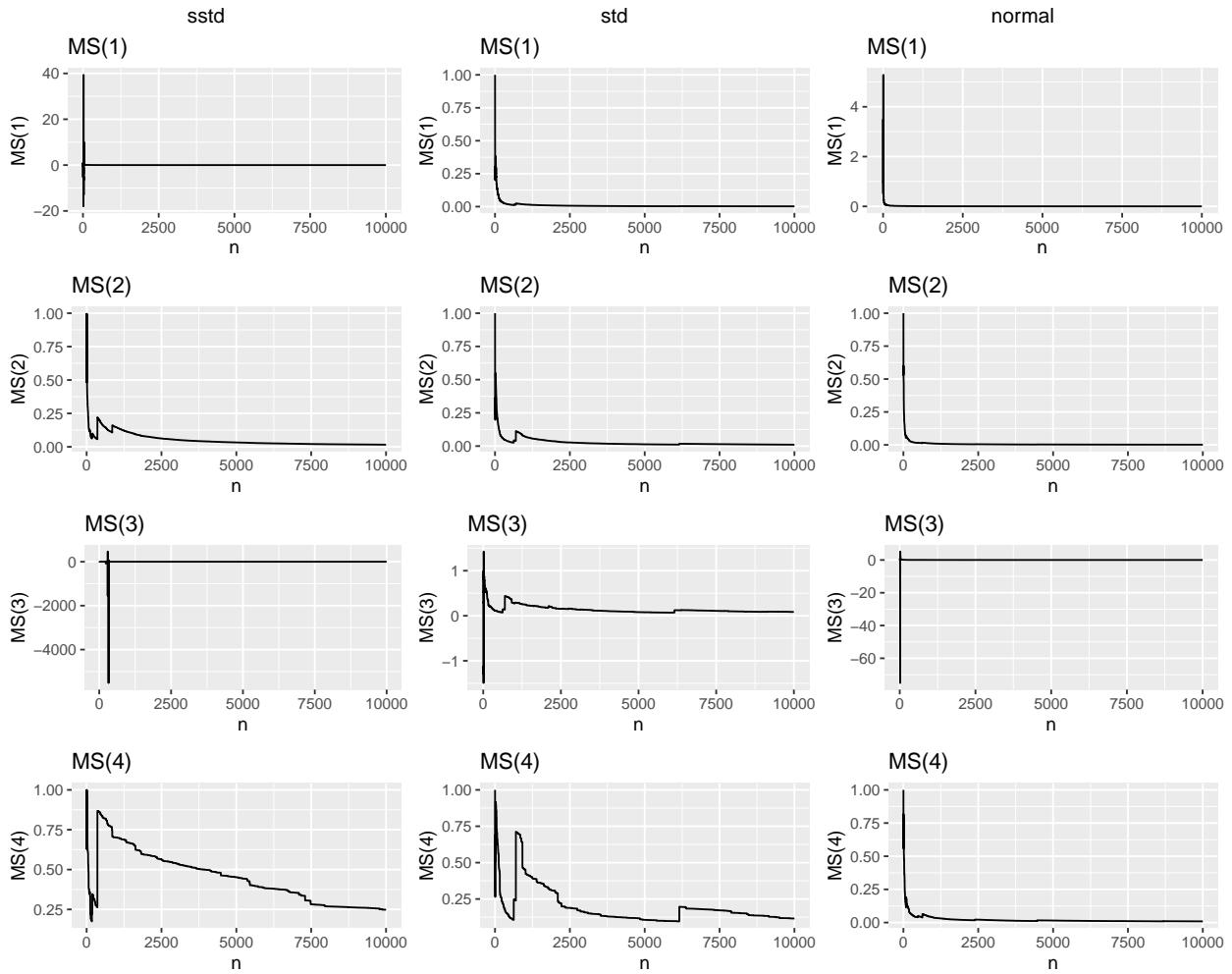
Sorted portfolio index values for last period of all runs



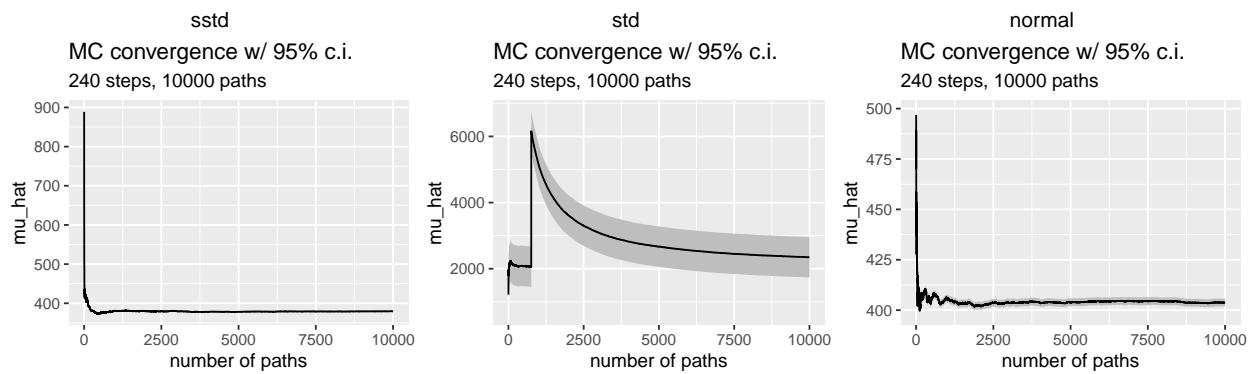
Convergence

Max vs sum

Max vs sum plots for the first four moments:



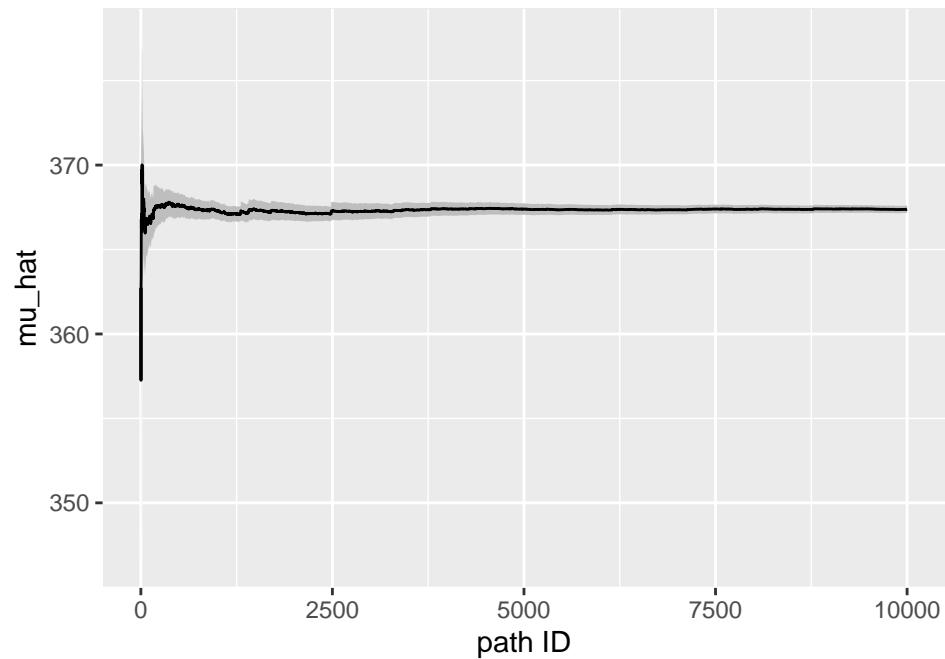
MC



IS

Skewed t -distribution with a normal proposal distribution.

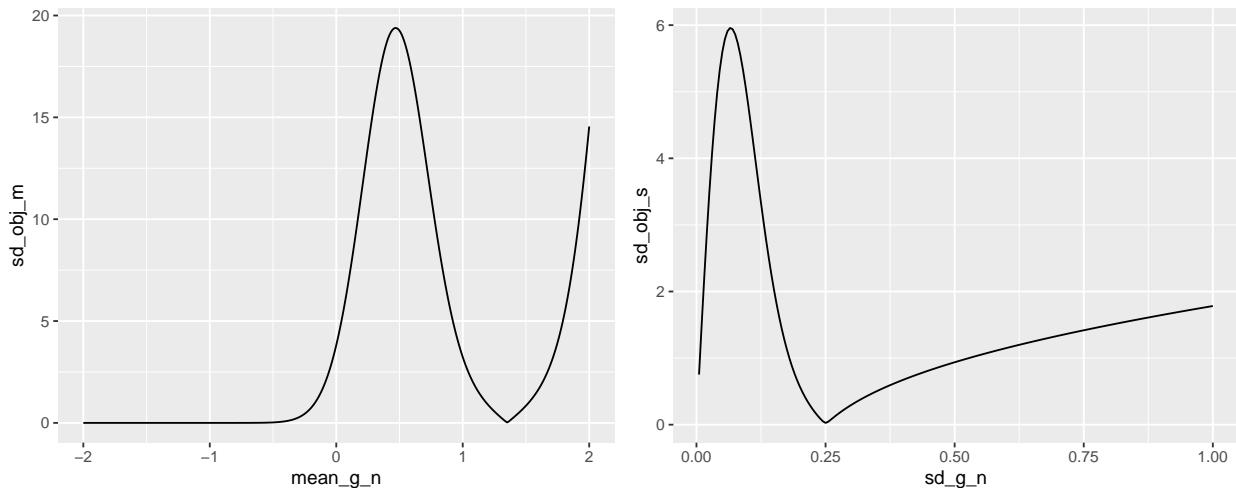
Importance Sampling convergence w/ 95% c.i.
 240 steps, 10000 paths



Parameters

```
## [1] 1.3516393 0.2503782
```

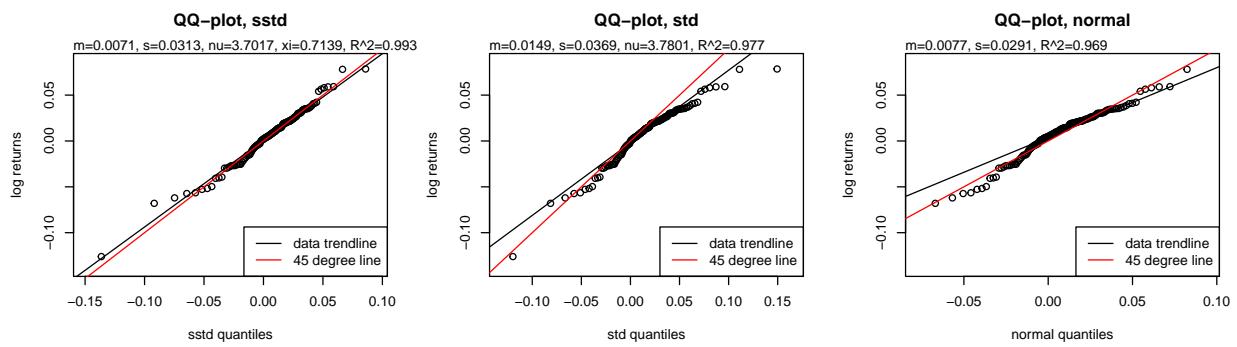
Objective function plots



Mix high risk (mhr), June 2012 - April 2024

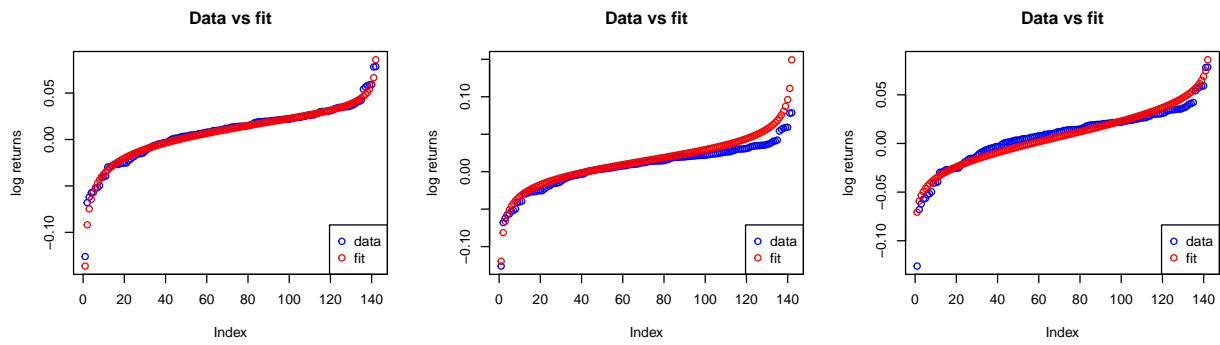
QQ Plot

Skewed t -distribution (sstd):



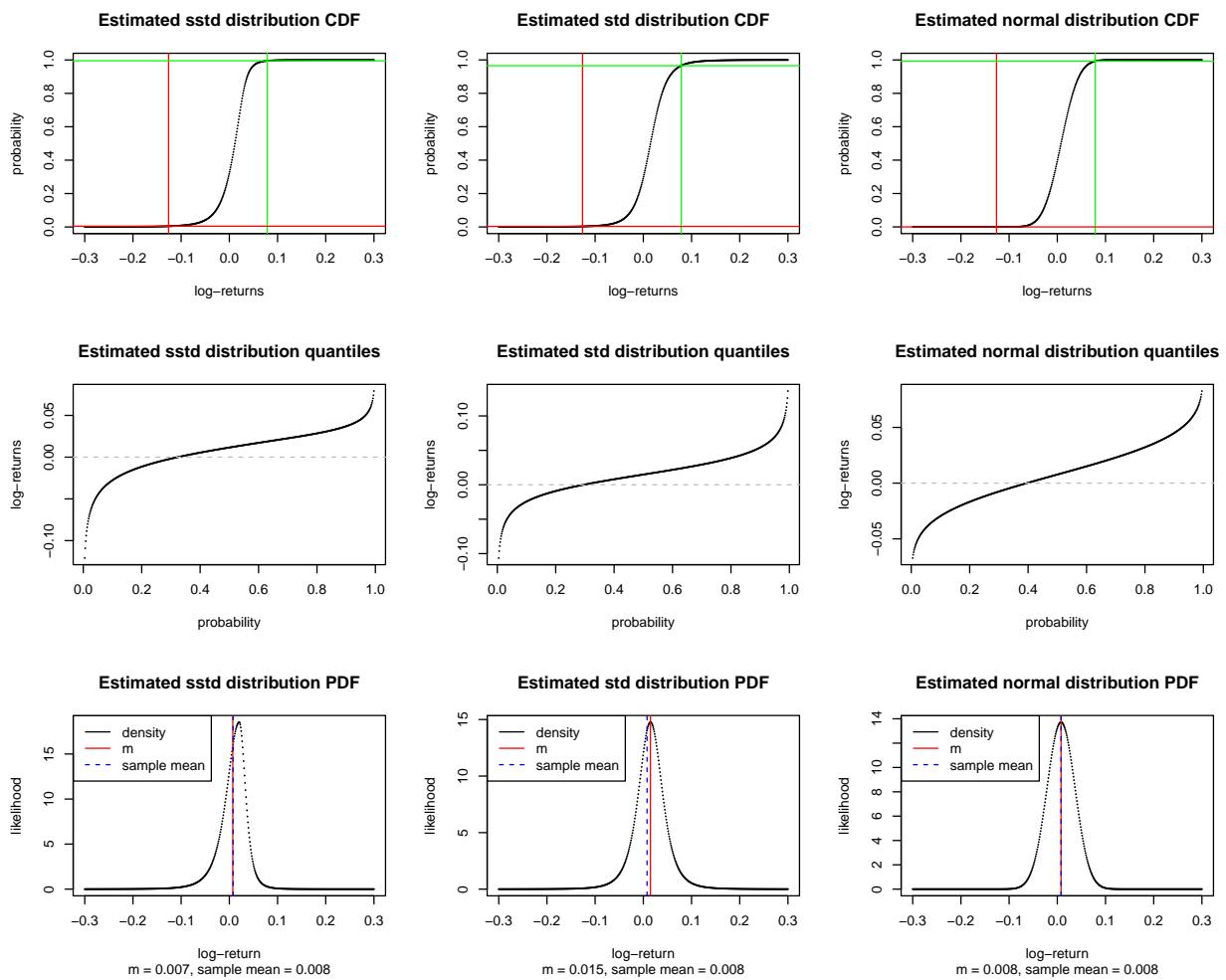
Data vs fit

Let's plot the fit and the observed returns together.



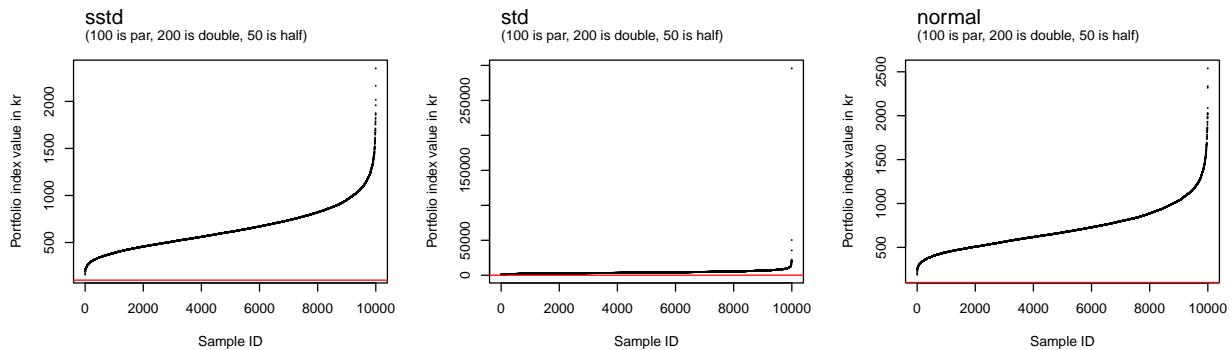
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

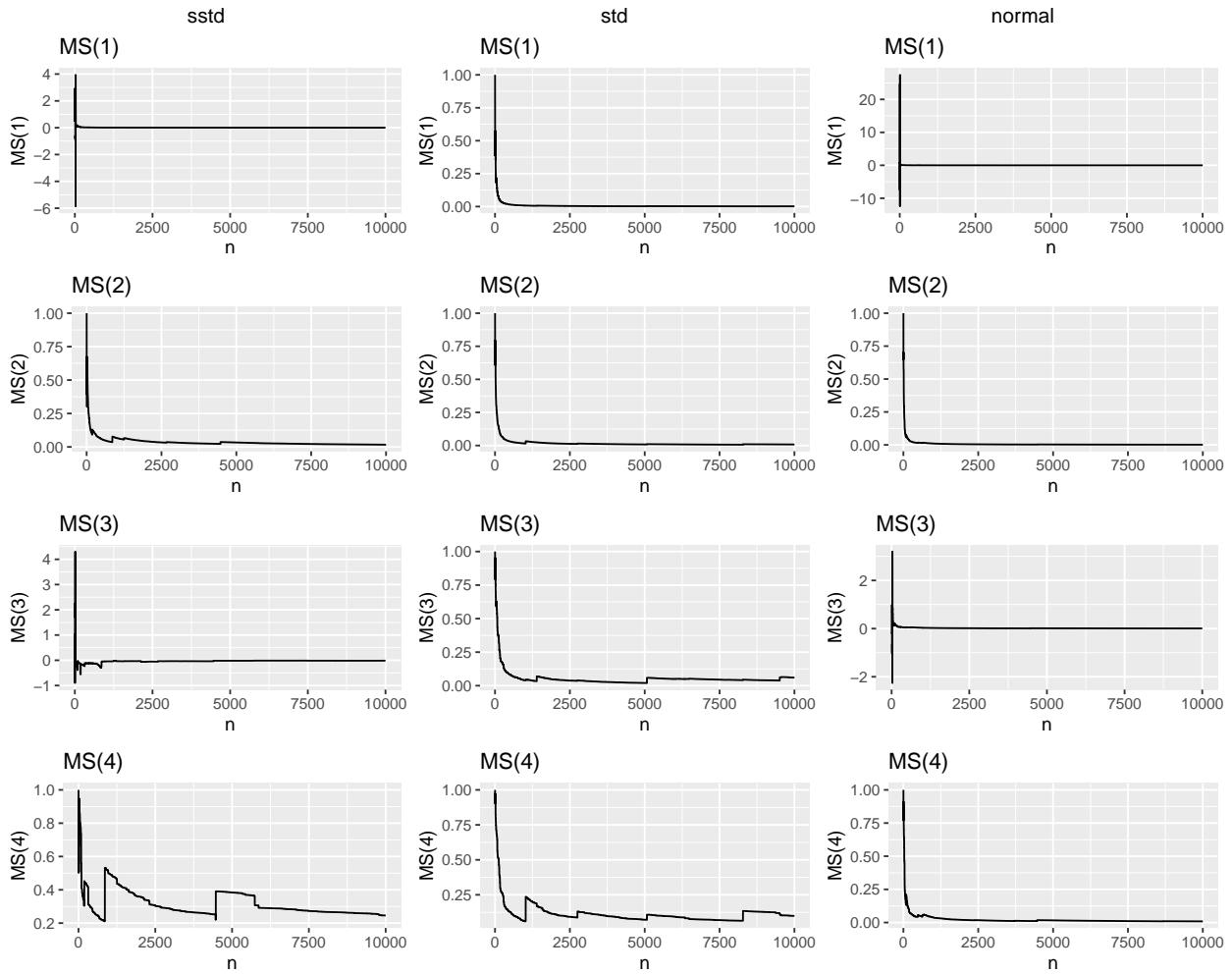
Sorted portfolio index values for last period of all runs



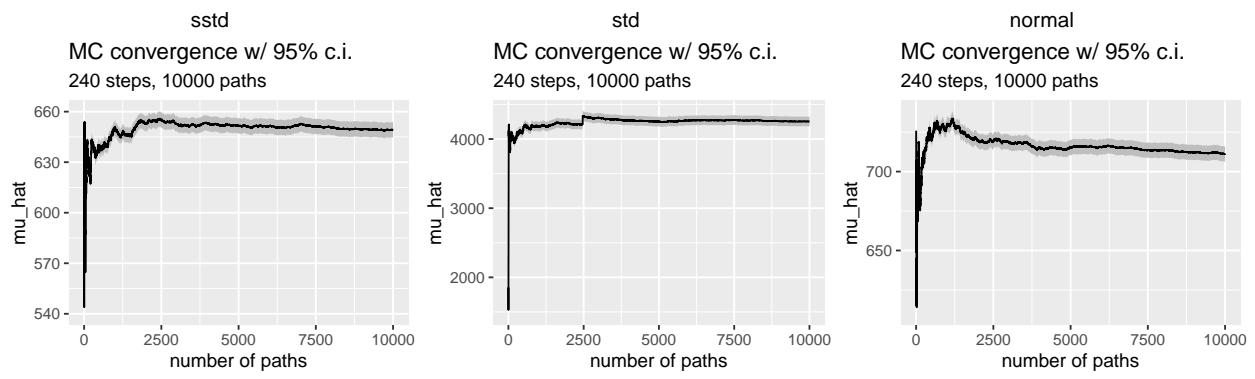
Convergence

Max vs sum

Max vs sum plots for the first four moments:



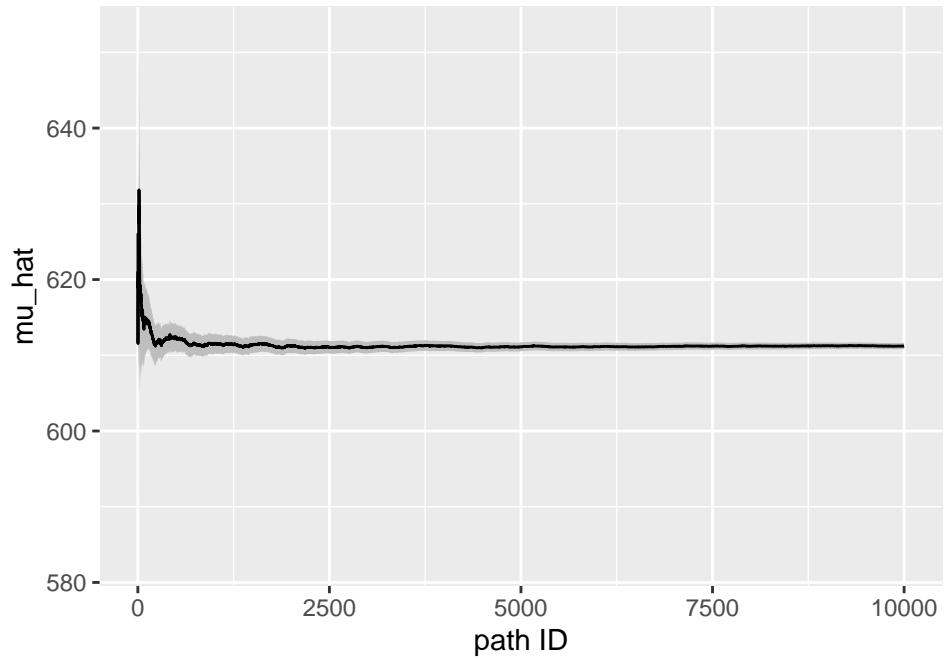
MC



IS

Skewed t -distribution with a normal proposal distribution.

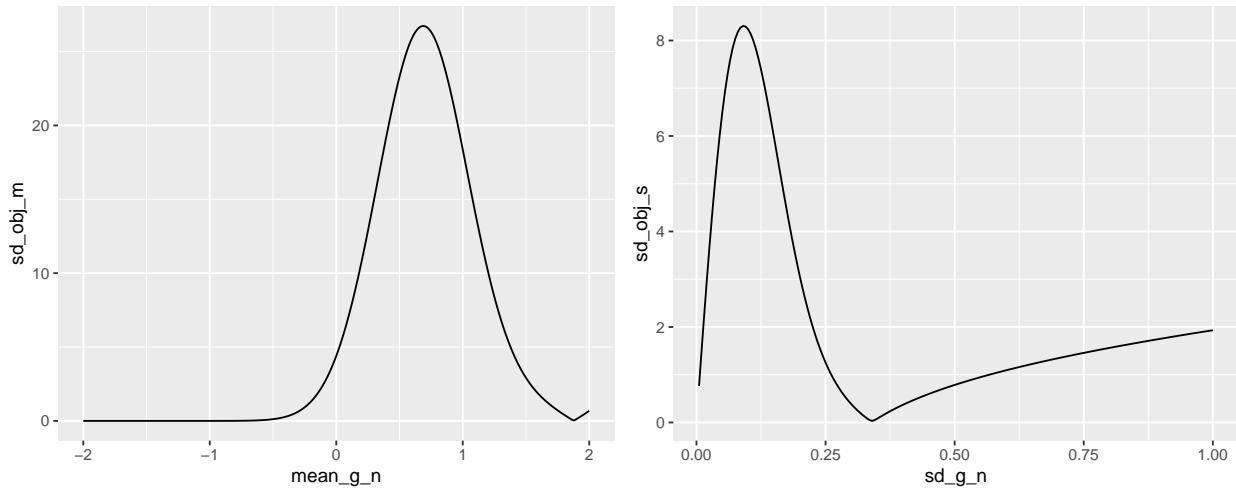
Importance Sampling convergence w/ 95% c.i.
 240 steps, 10000 paths



Parameters

```
## [1] 1.8775189 0.3400818
```

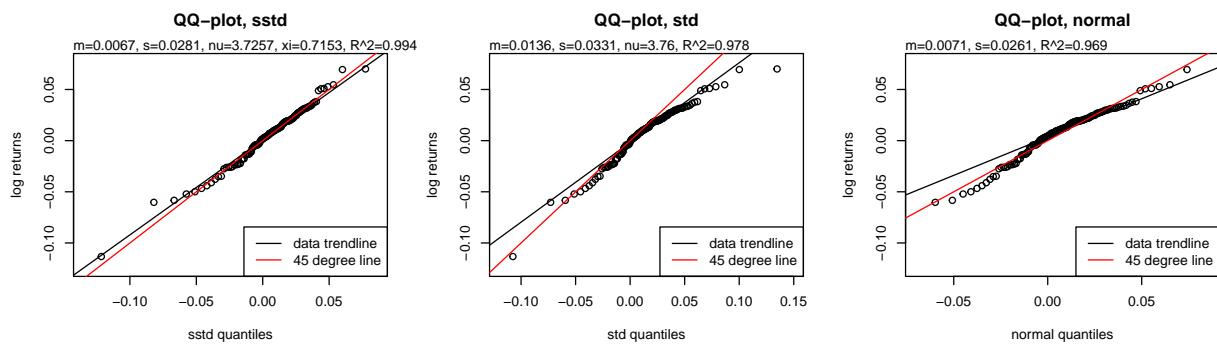
Objective function plots



Mix vmr+phr (vm_ph), June 2012 - April 2024

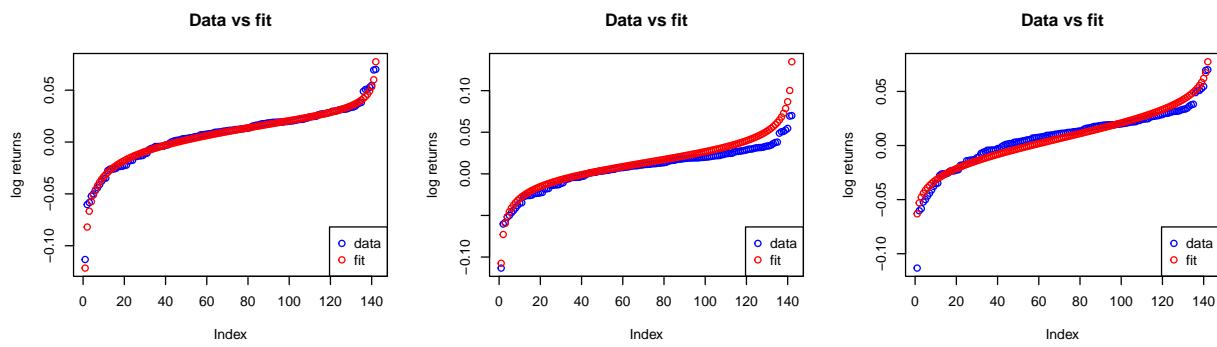
QQ Plot

Skewed t -distribution (sstd):



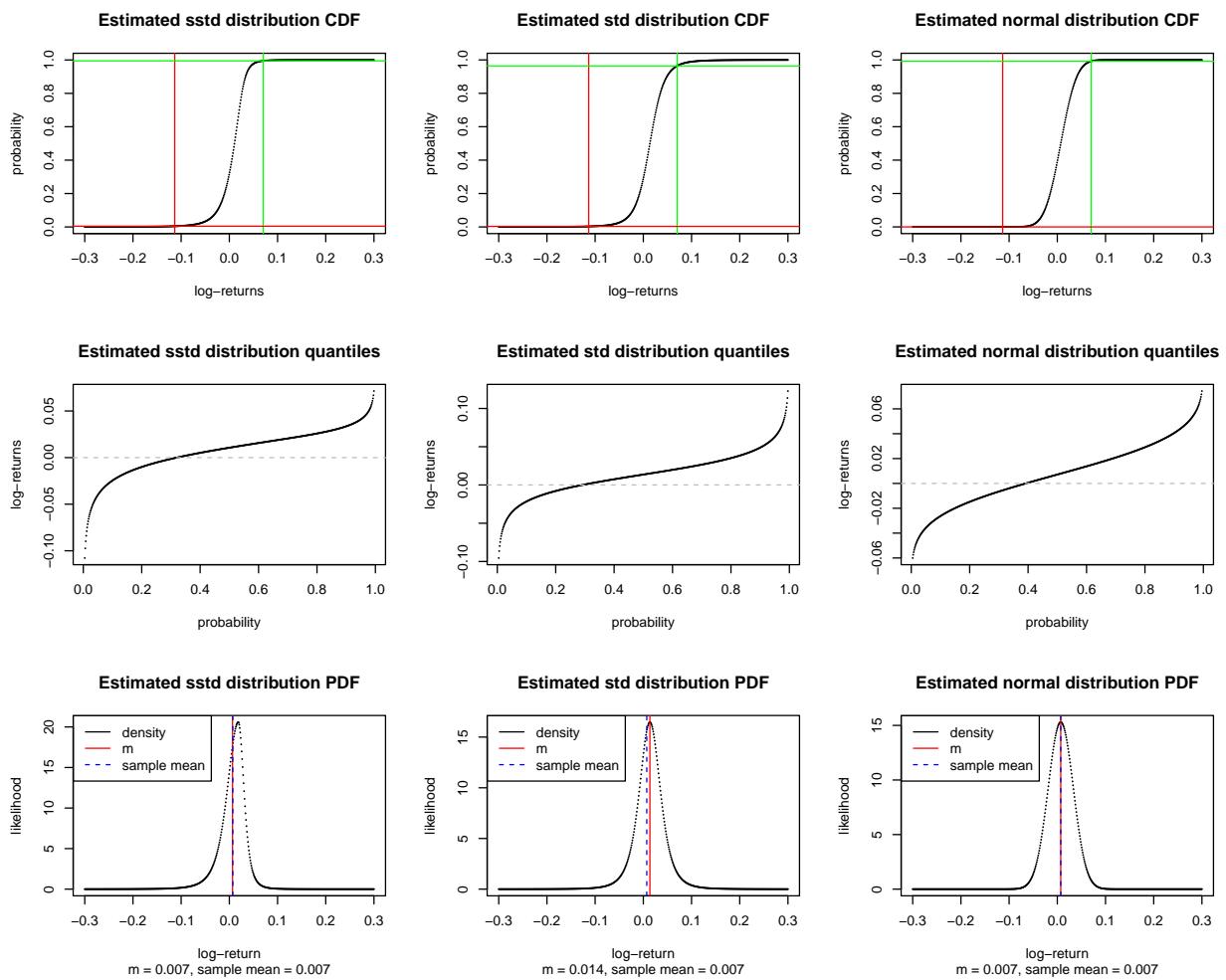
Data vs fit

Let's plot the fit and the observed returns together.



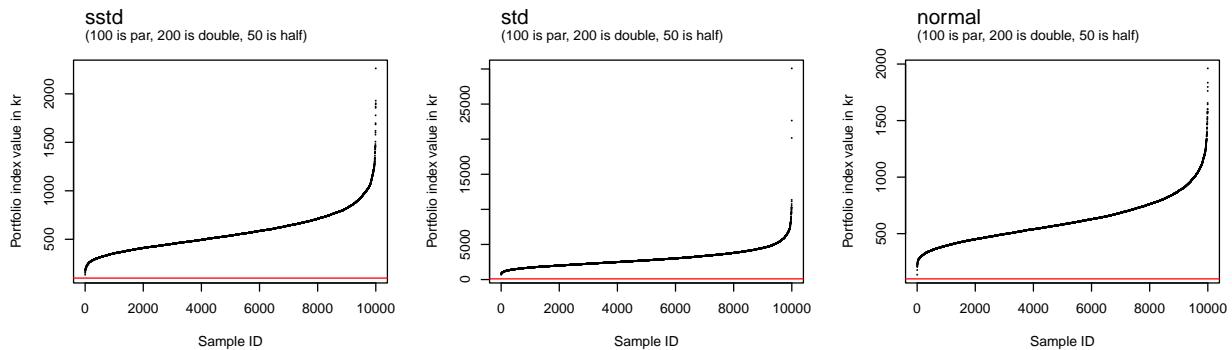
Estimated distribution

Now let's look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

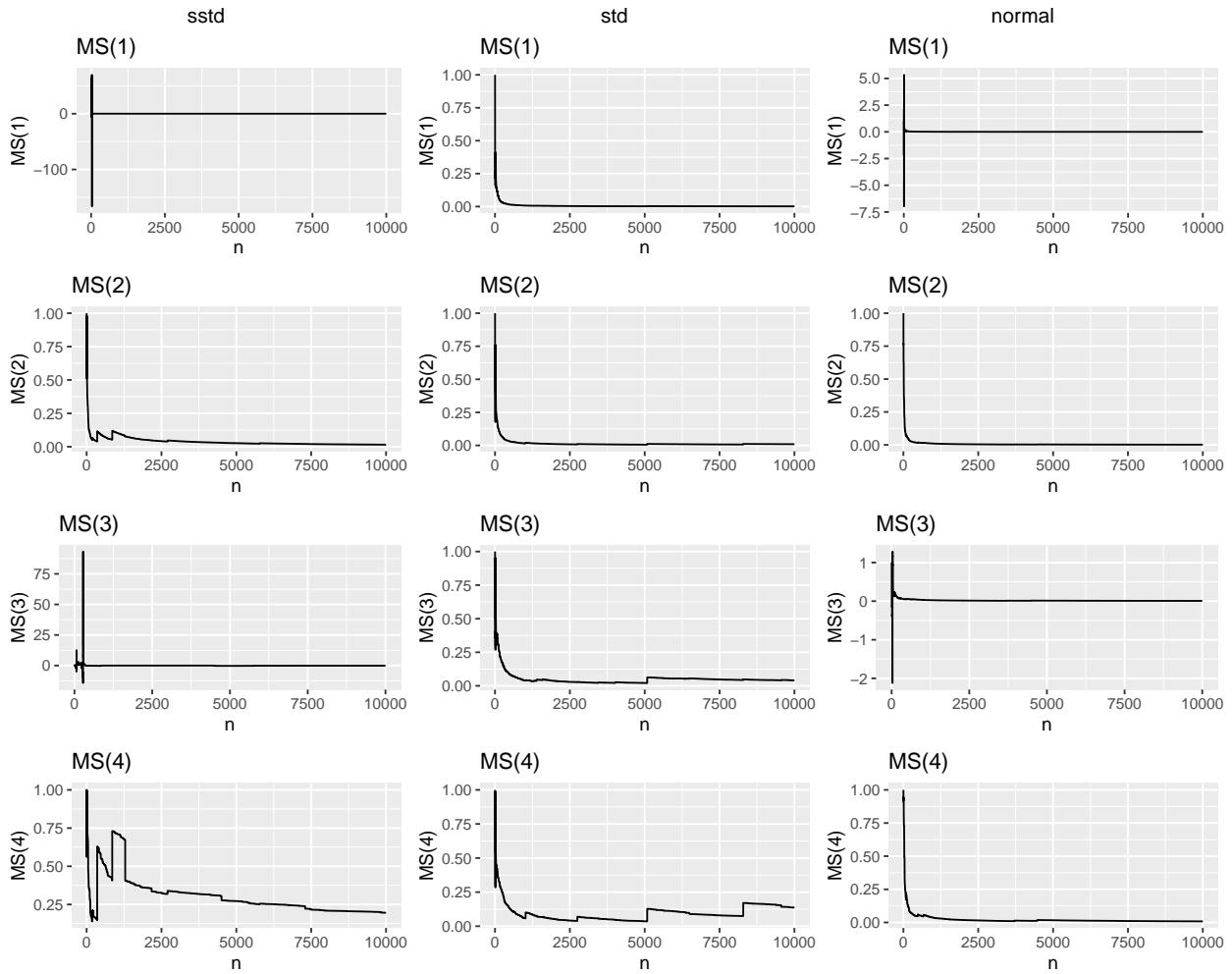
Sorted portfolio index values for last period of all runs



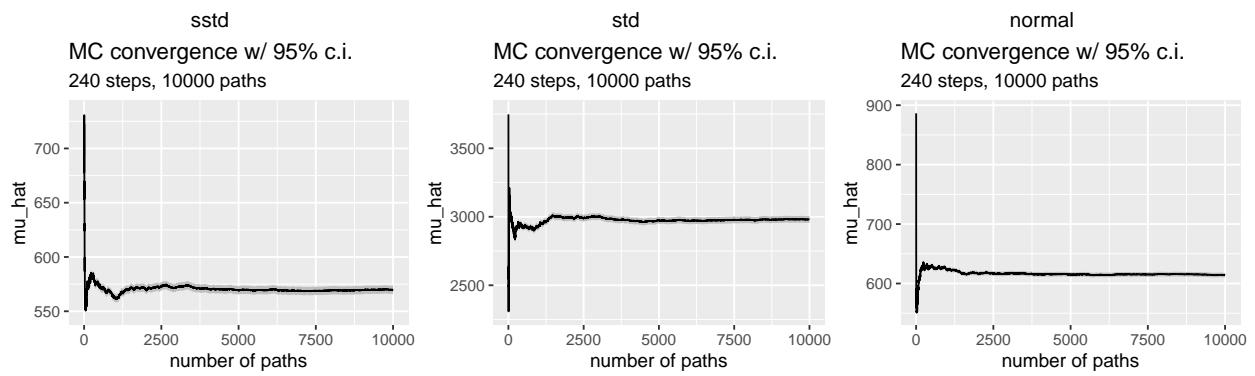
Convergence

Max vs sum

Max vs sum plots for the first four moments:



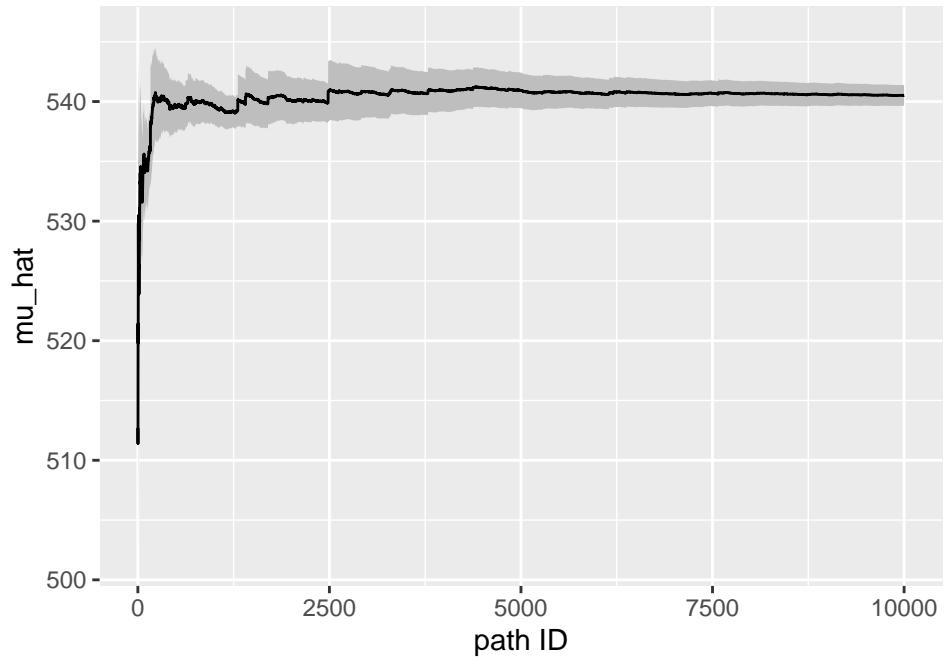
MC



IS

Skewed t -distribution with a normal proposal distribution.

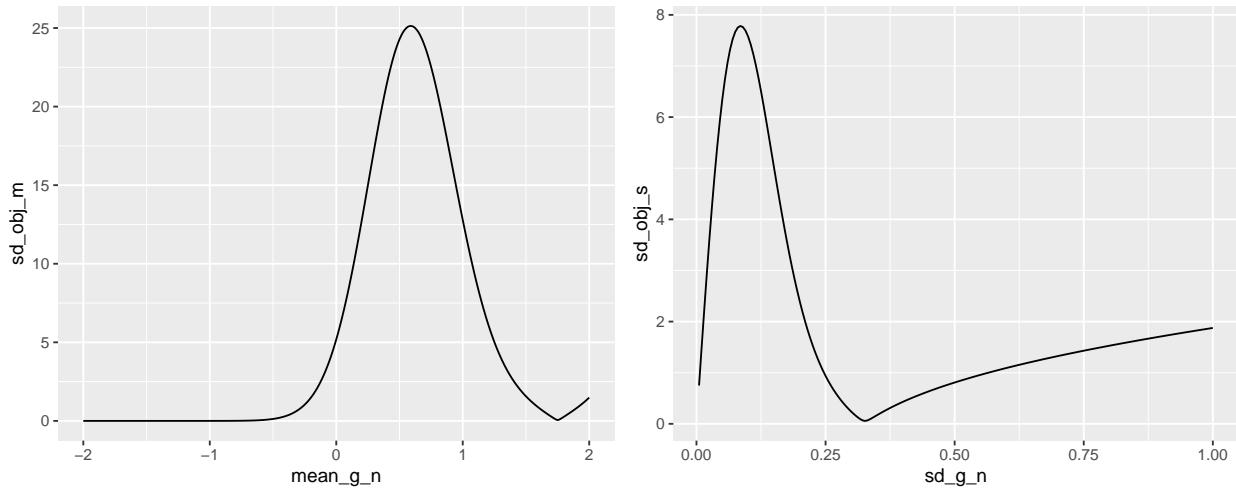
Importance Sampling convergence w/ 95% c.i.
 240 steps, 10000 paths



Parameters

```
## [1] 1.7507161 0.3263777
```

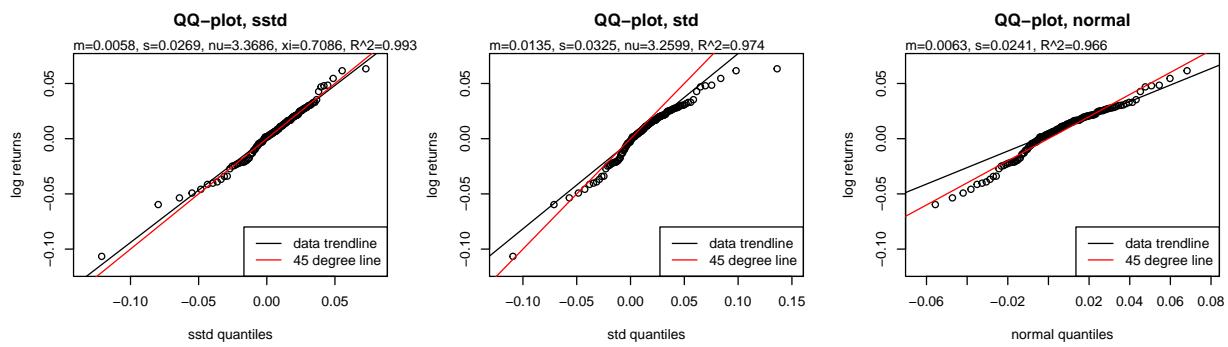
Objective function plots



Mix vhr+pmr (mh_pm), June 2012 - April 2024

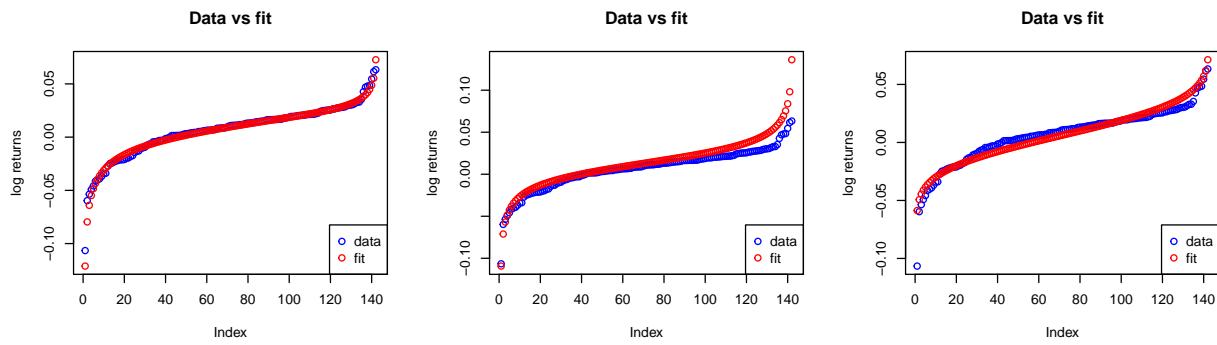
QQ Plot

Skewed t -distribution (sstd):



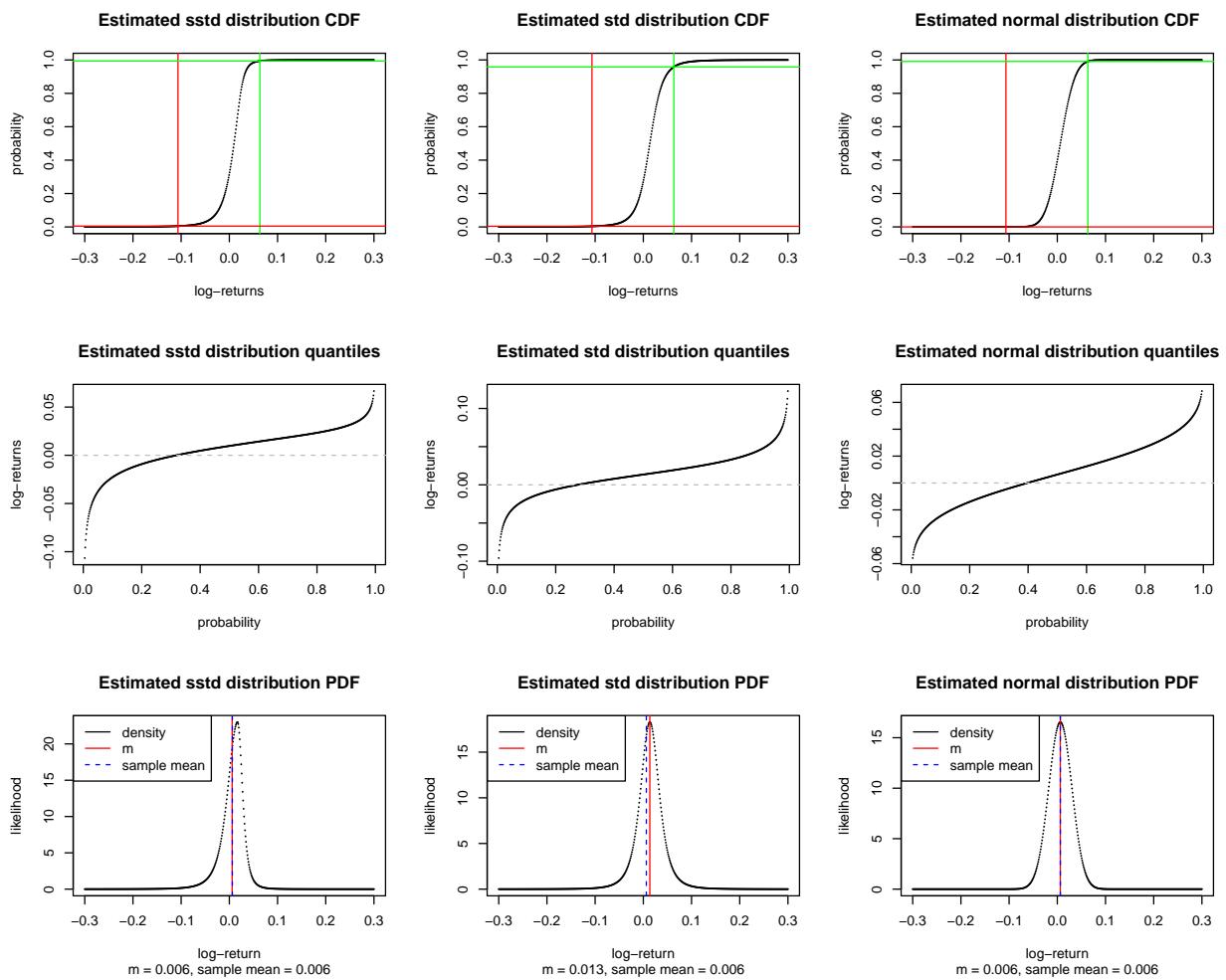
Data vs fit

Let's plot the fit and the observed returns together.



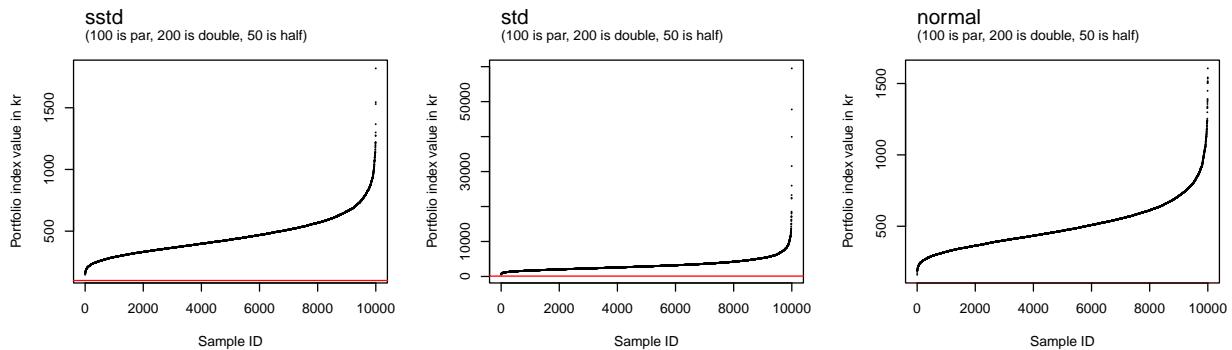
Estimated distribution

Now let's look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

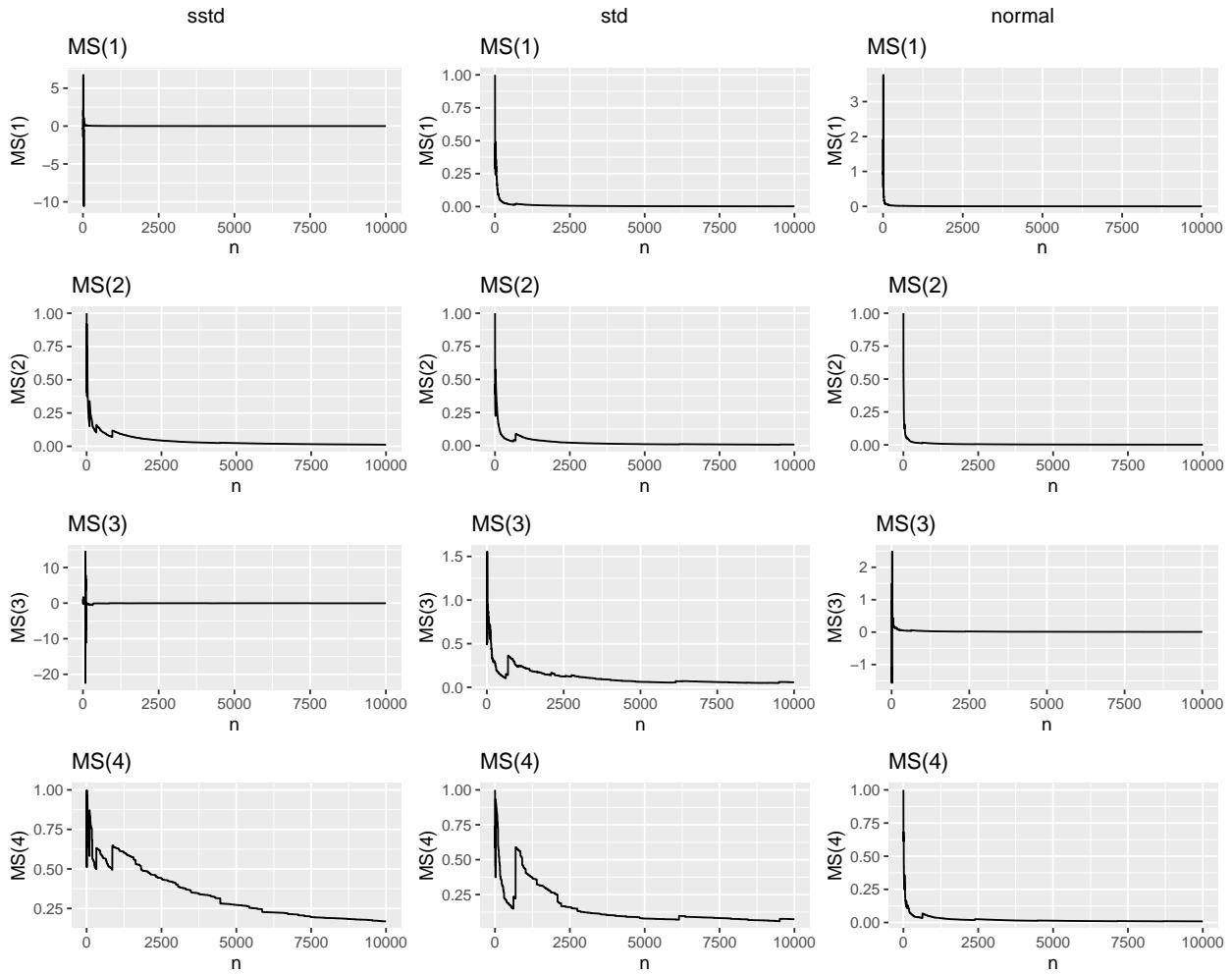
Sorted portfolio index values for last period of all runs



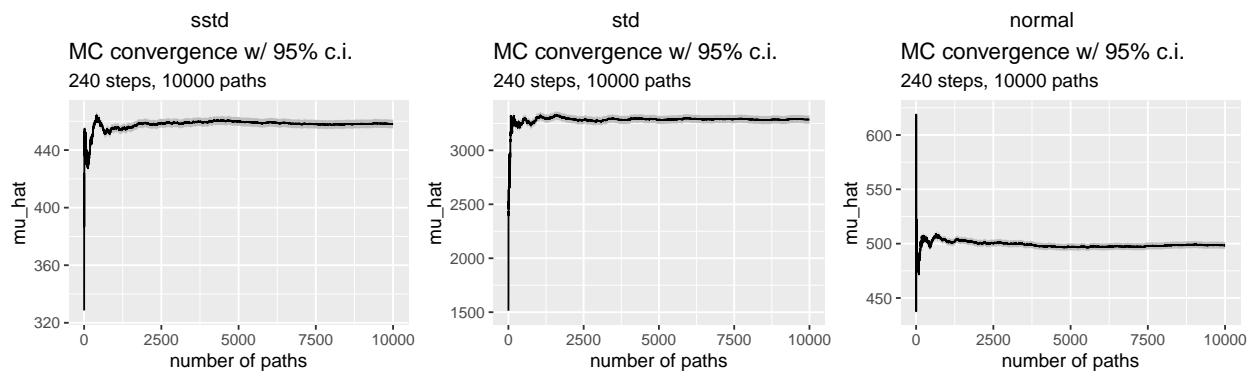
Convergence

Max vs sum

Max vs sum plots for the first four moments:



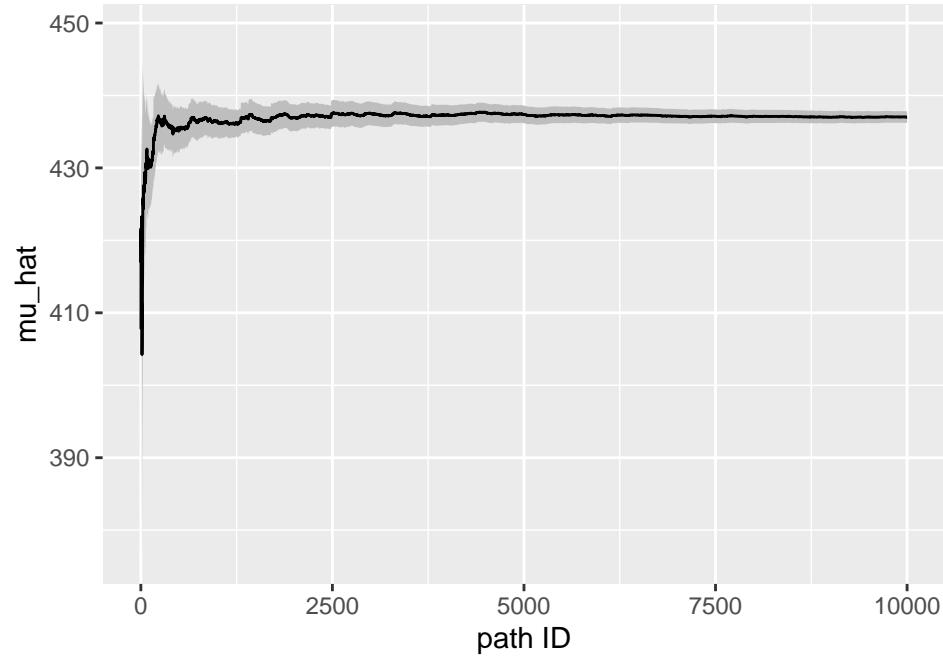
MC



IS

Skewed t -distribution with a normal proposal distribution.

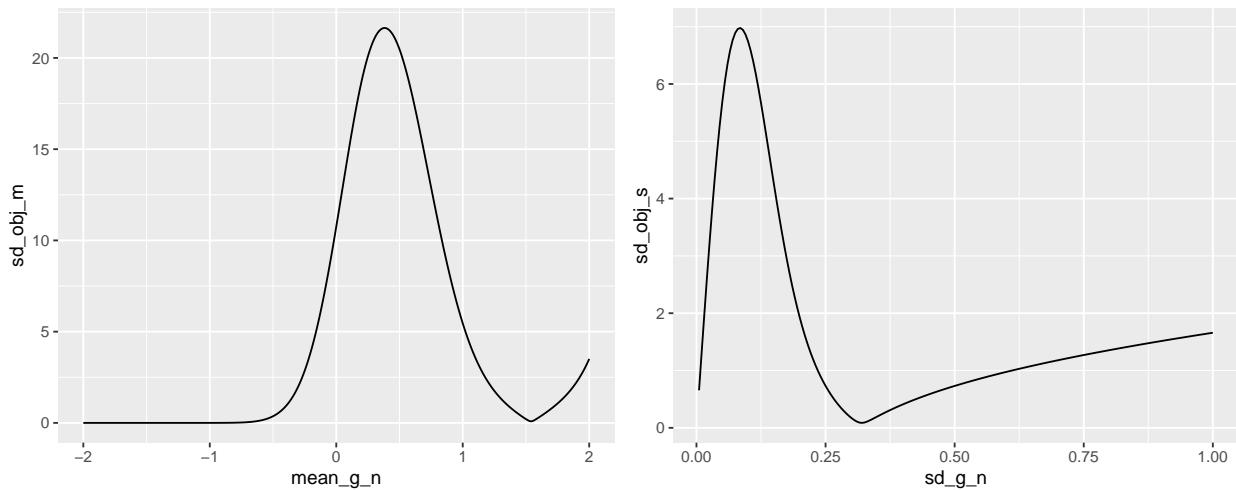
Importance Sampling convergence w/ 95% c.i.
240 steps, 10000 paths



Parameters

```
## [1] 1.540135 0.320090
```

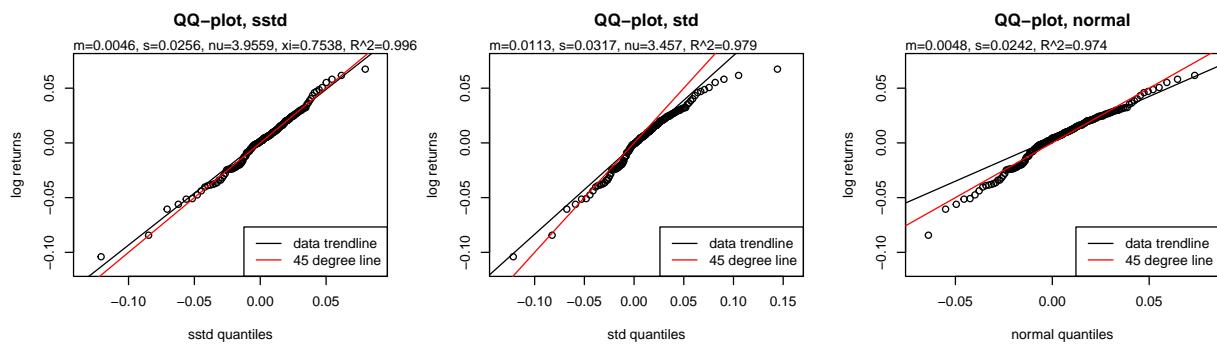
Objective function plots



Velliv medium risk (vmr), June 2012 - April 2024

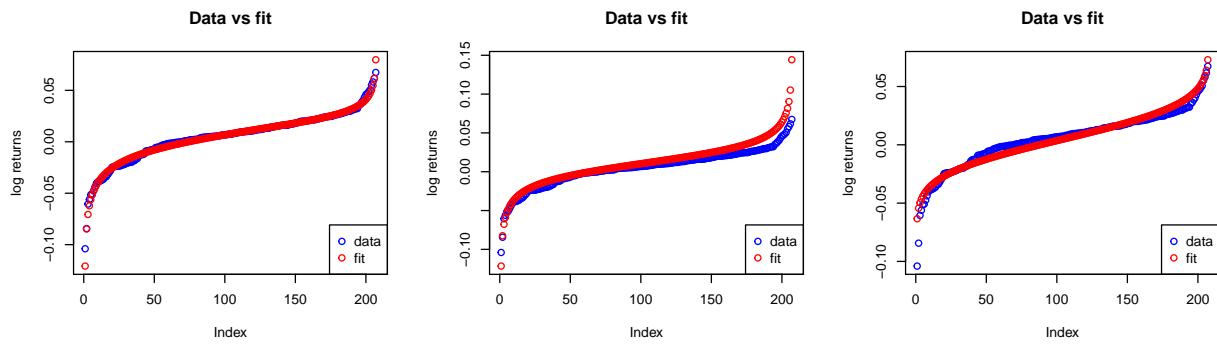
QQ Plot

Skewed t -distribution (sstd):



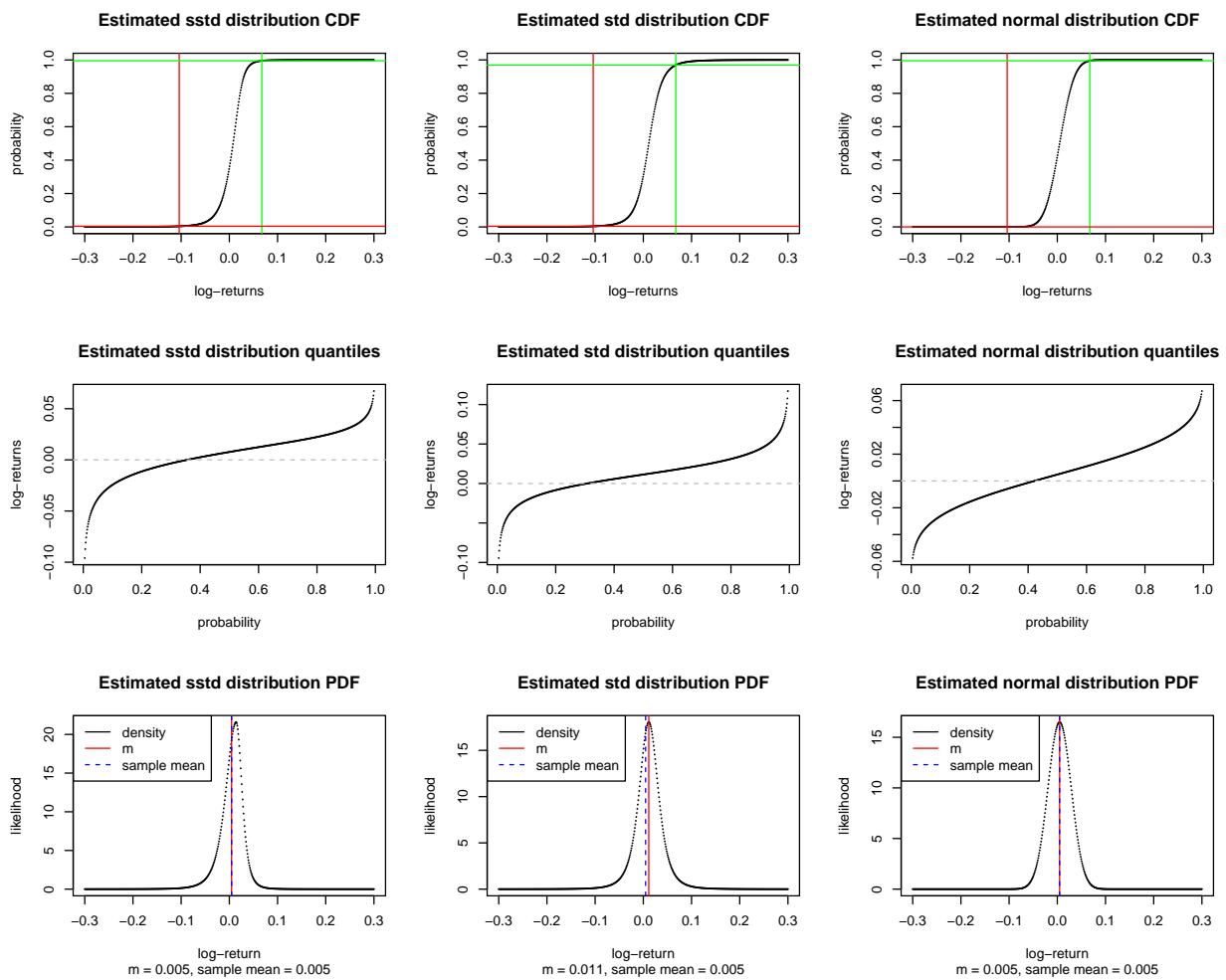
Data vs fit

Let's plot the fit and the observed returns together.



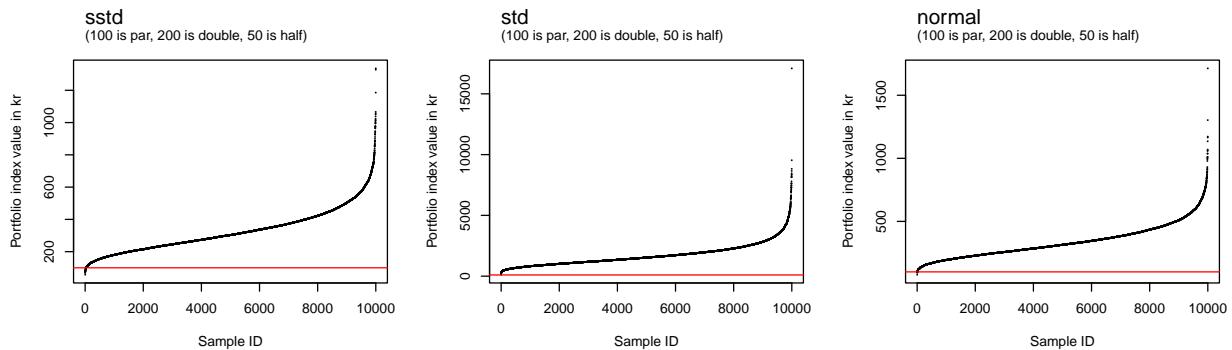
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

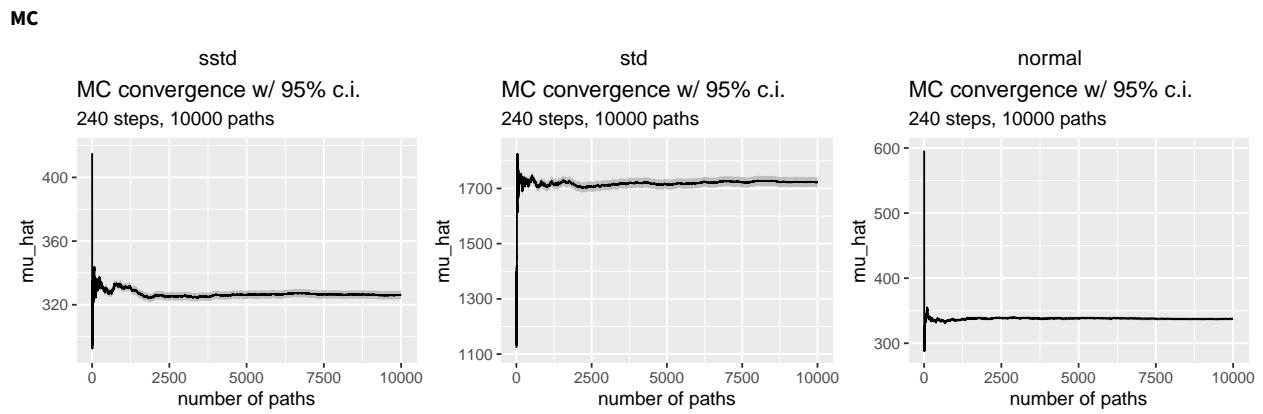
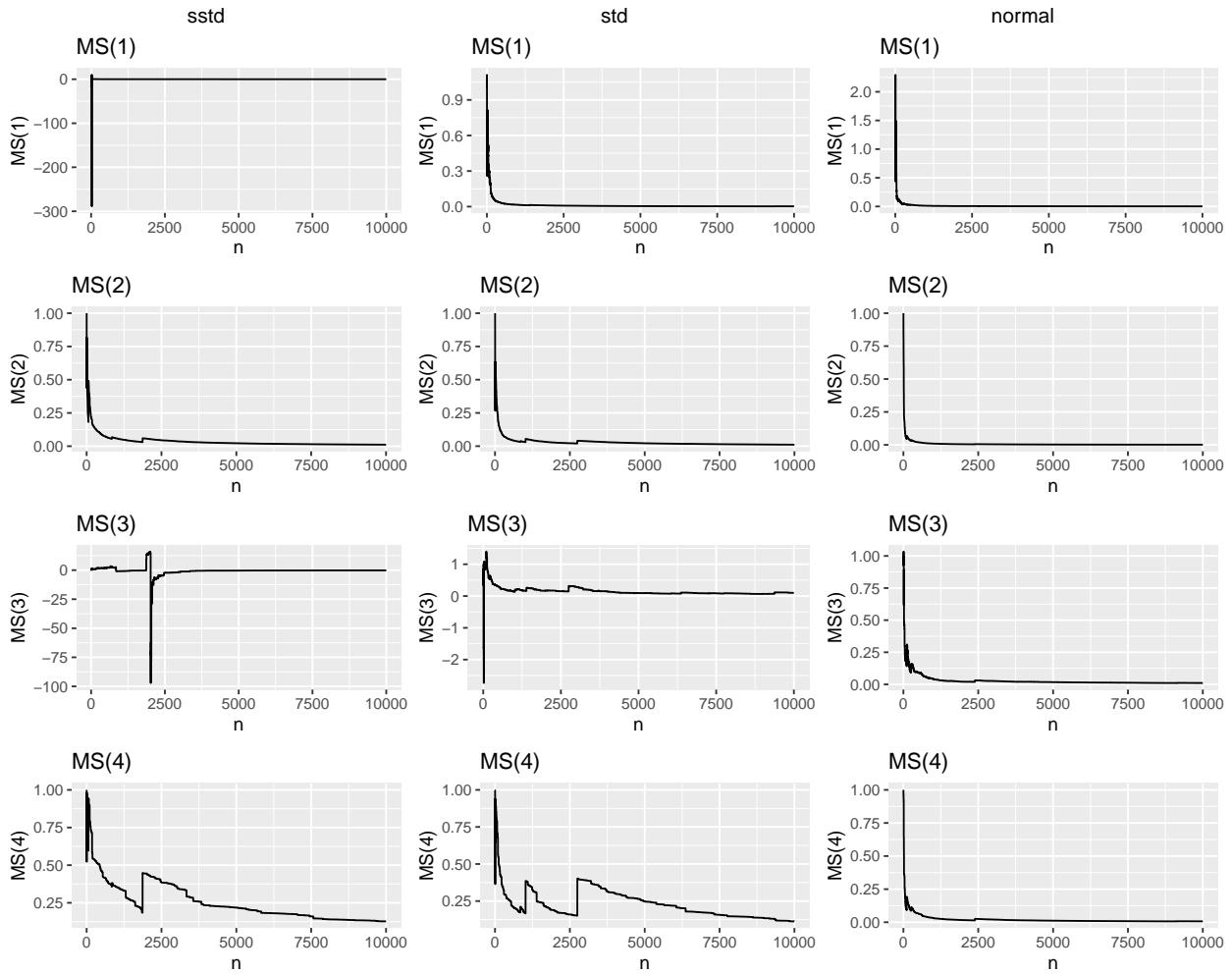
Sorted portfolio index values for last period of all runs



Convergence

Max vs sum

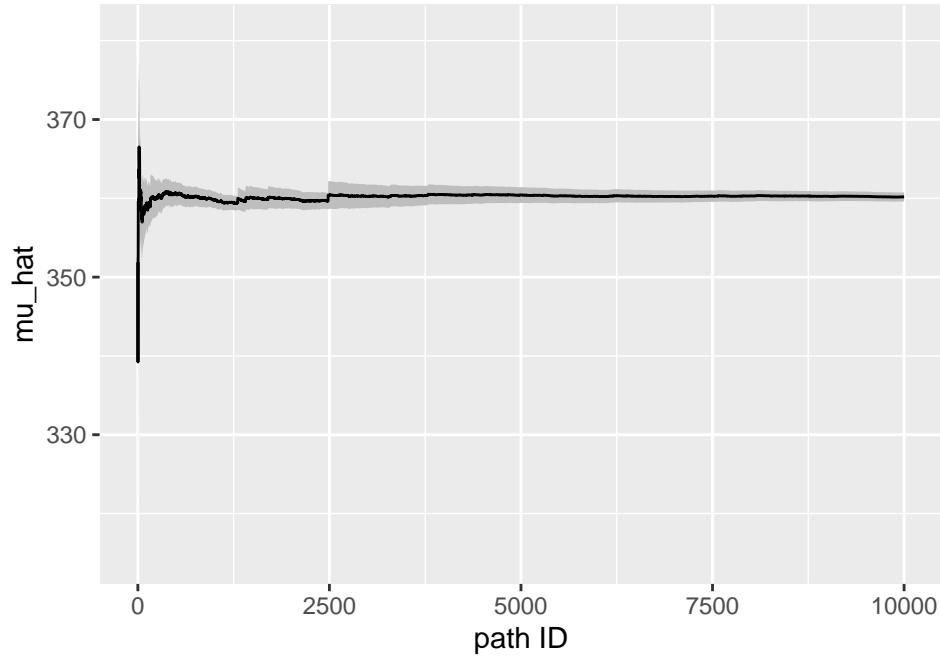
Max vs sum plots for the first four moments:



IS

Skewed t -distribution with a normal proposal distribution.

Importance Sampling convergence w/ 95% c.i.
240 steps, 10000 paths



Parameters

```
## [1] 1.4145605 0.3807834
```

Objective function plots

