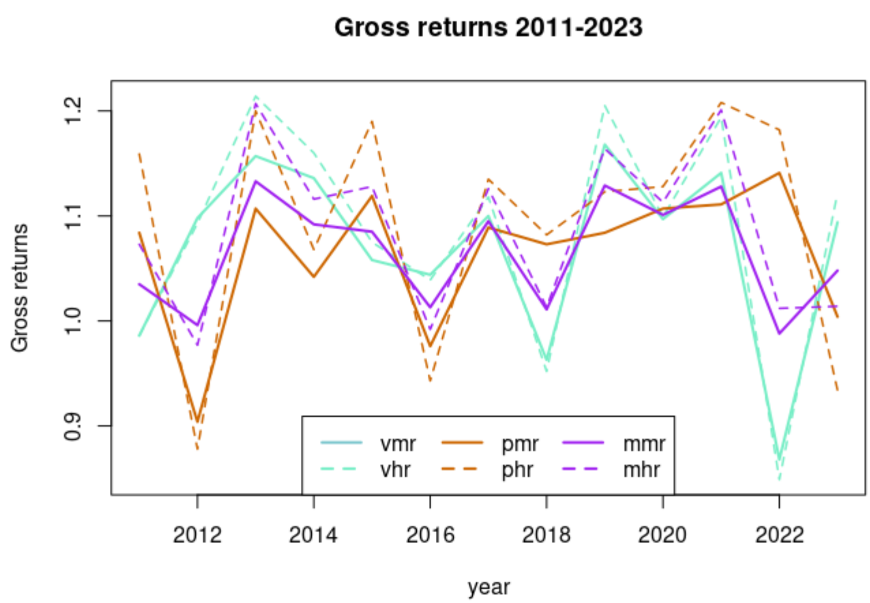
Pension returns analysis

23:32 01 May 2024

Fit log returns to F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated shape parameter (degrees of freedom).
`xi` is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.
`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.



Summary of gross returns

	vmr	vhr	vmrl	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
Min. :	0.868	0.849	0.801	0.904	0.878	0.988	0.977	0.979	0.967
1st Qu.:	1.044	1.039	1.013	1.042	1.068	1.013	1.013	1.021	1.012
Median :	1.097	1.099	1.085	1.084	1.128	1.085	1.113	1.102	1.094
Mean :	1.070	1.085	1.061	1.065	1.095	1.066	1.087	1.081	1.074
3rd Qu.:	1.136	1.160	1.128	1.107	1.182	1.101	1.128	1.121	1.106
Max. :	1.168	1.214	1.193	1.141	1.208	1.133	1.207	1.178	1.163

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.182	phr	1.214	vhr
0.979	vm_ph_r	1.044	vmr	1.113	mhr	1.087	mhr	1.160	vhr	1.208	phr
0.977	mhr	1.042	pmr	1.102	vm_ph_r	1.085	vhr	1.136	vmr	1.207	mhr
0.967	vh_pm_r	1.039	vhr	1.099	vhr	1.081	vm_ph_r	1.128	vmrl	1.193	vmrl
0.904	pmr	1.021	vm_ph_r	1.097	vmr	1.074	vh_pm_r	1.128	mhr	1.178	vm_ph_r
0.878	phr	1.013	vmrl	1.094	vh_pm_r	1.070	vmr	1.121	vm_ph_r	1.168	vmr
0.868	vmr	1.013	mmr	1.085	vmrl	1.066	mmr	1.107	pmr	1.163	vh_pm_r
0.849	vhr	1.013	mhr	1.085	mmr	1.065	pmr	1.106	vh_pm_r	1.141	pmr
0.801	vmrl	1.012	vh_pm_r	1.084	pmr	1.061	vmrl	1.101	mmr	1.133	mmr

Correlations and covariance

Correlations

	vmr	vhr	pmr	phr
vmr	1.000	0.993	-0.197	-0.095
vhr	0.993	1.000	-0.119	-0.016
pmr	-0.197	-0.119	1.000	0.957
phr	-0.095	-0.016	0.957	1.000

Covariances

	vmr	vhr	pmr	phr
vmr	0.007	0.009	-0.001	-0.001
vhr	0.009	0.011	-0.001	0.000
pmr	-0.001	-0.001	0.004	0.007
phr	-0.001	0.000	0.007	0.011

Compare pension plans

Risk of max loss

Risk of max loss of `x` percent for a single period (year).
`x` values are row names.

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
0	21.167	21.333	11.833	14.000	12.333	12.667	16.667	16.000
5	12.167	13.167	5.667	8.333	5.833	3.833	8.667	8.167
10	7.000	8.000	3.000	5.000	2.833	0.500	4.333	4.167
25	1.333	1.500	0.500	1.000	0.333	0.000	0.333	0.333
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.333	vhr	13.167	vhr	8.000	vhr	1.500	vhr	0	vmr	0	vmr	0	vmr
21.167	vmr	12.167	vmr	7.000	vmr	1.333	vmr	0	vhr	0	vhr	0	vhr
16.667	vm_ph_r	8.667	vm_ph_r	5.000	phr	1.000	phr	0	pmr	0	pmr	0	pmr
16.000	vh_pm_r	8.333	phr	4.333	vm_ph_r	0.500	pmr	0	phr	0	phr	0	phr
14.000	phr	8.167	vh_pm_r	4.167	vh_pm_r	0.333	mmr	0	mmr	0	mmr	0	mmr
12.667	mhr	5.833	mmr	3.000	pmr	0.333	vm_ph_r	0	mhr	0	mhr	0	mhr
12.333	mmr	5.667	pmr	2.833	mmr	0.333	vh_pm_r	0	vm_ph_r	0	vm_ph_r	0	vm_ph_r
11.833	pmr	3.833	mhr	0.500	mhr	0.000	mhr	0	vh_pm_r	0	vh_pm_r	0	vh_pm_r

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
0	78.833	78.667	88.167	86.000	87.667	87.333	83.333	84.000
5	63.833	66.667	71.667	76.000	71.667	70.167	69.333	69.000
10	40.833	50.167	32.500	59.667	35.500	46.000	47.167	43.833
25	0.000	0.000	0.000	0.000	0.000	0.833	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
88.167	pmr	76.000	phr	59.667	phr	0.833	mhr	0	vmr	0	vmr
87.667	mmr	71.667	pmr	50.167	vhr	0.000	vmr	0	vhr	0	vhr
87.333	mhr	71.667	mmr	47.167	vm_ph_r	0.000	vhr	0	pmr	0	pmr
86.000	phr	70.167	mhr	46.000	mhr	0.000	pmr	0	phr	0	phr
84.000	vh_pm_r	69.333	vm_ph_r	43.833	vh_pm_r	0.000	phr	0	mmr	0	mmr
83.333	vm_ph_r	69.000	vh_pm_r	40.833	vmr	0.000	mmr	0	mhr	0	mhr
78.833	vmr	66.667	vhr	35.500	mmr	0.000	vm_ph_r	0	vm_ph_r	0	vm_ph_r
78.667	vhr	63.833	vmr	32.500	pmr	0.000	vh_pm_r	0	vh_pm_r	0	vh_pm_r

MC risk percentiles

Risk of loss from first to last period.

`_m` is medium.
`_h` is high.

`a` is simulation from estimated distribution of returns of mix.
`b` is mix of simulations from estimated distribution of returns from individual funds.

`l` for "long", going back to 2007.

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
0	4.91	2.65	1.92	0.98	1.18	0	0.63	0.69
5	4.32	2.37	1.68	0.88	1.04	0	0.56	0.55
10	3.67	2.04	1.47	0.80	0.90	0	0.44	0.47
25	2.33	1.16	1.09	0.56	0.55	0	0.22	0.27
50	0.82	0.38	0.63	0.33	0.24	0	0.04	0.13
90	0.05	0.02	0.14	0.07	0.03	0	0.00	0.02
99	0.00	0.00	0.05	0.01	0.00	0	0.00	0.00

`1e6` simulation paths of `mhr_b`:

	0	5	10	25	50	90	99
prob_pct	0.118	0.095	0.076	0.036	0.008	0	0

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.91	vmr	4.32	vmr	3.67	vmr	2.33	vmr	0.82	vmr	0.14	pmr	0.05	pmr
2.65	vhr	2.37	vhr	2.04	vhr	1.16	vhr	0.63	pmr	0.07	phr	0.01	phr
1.92	pmr	1.68	pmr	1.47	pmr	1.09	pmr	0.38	vhr	0.05	vmr	0.00	vmr
1.18	mmr	1.04	mmr	0.90	mmr	0.56	phr	0.33	phr	0.03	mmr	0.00	vhr
0.98	phr	0.88	phr	0.80	phr	0.55	mmr	0.24	mmr	0.02	vhr	0.00	mmr
0.69	vh_pm_r	0.56	vm_ph_r	0.47	vh_pm_r	0.27	vh_pm_r	0.13	vh_pm_r	0.02	vh_pm_r	0.00	mhr
0.63	vm_ph_r	0.55	vh_pm_r	0.44	vm_ph_r	0.22	vm_ph_r	0.04	vm_ph_r	0.00	mhr	0.00	vm_ph_r
0.00	mhr	0.00	mhr	0.00	mhr	0.00	mhr	0.00	mhr	0.00	vm_ph_r	0.00	vh_pm_r

MC gains percentiles

Chance of gains from first to last period.
`_a` is simulation from estimated distribution of returns of mix.
`_b` is mix of simulations from estimated distribution of returns from individual funds.

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
0	95.09	97.35	98.08	99.02	98.82	100.00	99.37	99.31
5	94.49	96.90	97.89	98.85	98.63	100.00	99.29	99.16
10	93.80	96.46	97.75	98.63	98.45	100.00	99.16	98.99
25	91.24	95.08	97.10	98.27	97.63	100.00	98.60	98.41
50	85.83	92.13	95.35	97.17	95.88	99.99	97.39	96.62

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
100	71.80	83.79	88.74	94.41	89.61	99.59	91.79	90.50
200	39.30	61.73	59.83	85.16	59.41	92.88	69.79	64.10
300	16.30	39.66	23.06	71.21	22.97	71.44	41.76	32.81
400	5.33	23.14	4.29	54.62	3.78	43.88	19.93	12.51
500	1.44	12.68	0.48	38.57	0.25	22.42	7.79	3.76
1000	0.00	0.26	0.02	2.34	0.00	0.23	0.01	0.00

1e6 simulation paths of `mhr_b`:

	0	5	10	25	50	100	200	300	400	500	1000
prob	99.882	99.854	99.824	99.686	99.301	97.513	86.912	65.992	41.486	21.693	0.086

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mhr	100.00	mhr	100.00	mhr	100.00	mhr	99.99	mhr	99.59	mhr
99.37	vm_ph_r	99.29	vm_ph_r	99.16	vm_ph_r	98.60	vm_ph_r	97.39	vm_ph_r	94.41	phr
99.31	vh_pm_r	99.16	vh_pm_r	98.99	vh_pm_r	98.41	vh_pm_r	97.17	phr	91.79	vm_ph_r
99.02	phr	98.85	phr	98.63	phr	98.27	phr	96.62	vh_pm_r	90.50	vh_pm_r
98.82	mmr	98.63	mmr	98.45	mmr	97.63	mmr	95.88	mmr	89.61	mmr
98.08	pmr	97.89	pmr	97.75	pmr	97.10	pmr	95.35	pmr	88.74	pmr
97.35	vhr	96.90	vhr	96.46	vhr	95.08	vhr	92.13	vhr	83.79	vhr
95.09	vmr	94.49	vmr	93.80	vmr	91.24	vmr	85.83	vmr	71.80	vmr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
92.88	mhr	71.44	mhr	54.62	phr	38.57	phr	2.34	phr
85.16	phr	71.21	phr	43.88	mhr	22.42	mhr	0.26	vhr
69.79	vm_ph_r	41.76	vm_ph_r	23.14	vhr	12.68	vhr	0.23	mhr
64.10	vh_pm_r	39.66	vhr	19.93	vm_ph_r	7.79	vm_ph_r	0.02	pmr
61.73	vhr	32.81	vh_pm_r	12.51	vh_pm_r	3.76	vh_pm_r	0.01	vm_ph_r
59.83	pmr	23.06	pmr	5.33	vmr	1.44	vmr	0.00	vmr
59.41	mmr	22.97	mmr	4.29	pmr	0.48	pmr	0.00	mmr
39.30	vmr	16.30	vmr	3.78	mmr	0.25	mmr	0.00	vh_pm_r

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated degrees of freedom, or shape parameter.
`xi` is the estimated skewness parameter.

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
m	0.048	0.063	0.058	0.084	0.059	0.082	0.067	0.062
s	0.120	0.126	0.123	0.121	0.088	0.071	0.091	0.090
nu	3.304	4.390	2.265	3.185	2.773	89.863	4.660	3.892
xi	0.034	0.019	0.477	0.018	0.029	0.770	0.048	0.019
R^2	0.993	0.995	0.991	0.964	0.890	0.961	0.927	0.933

Fit statistics ranking

m	ranking	s	ranking	R^2	ranking
0.084	phr	0.071	mhr	0.995	vhr
0.082	mhr	0.088	mmr	0.993	vmr
0.067	vm_ph_r	0.090	vh_pm_r	0.991	pmr
0.063	vhr	0.091	vm_ph_r	0.964	phr
0.062	vh_pm_r	0.120	vmr	0.961	mhr
0.059	mmr	0.121	phr	0.933	vh_pm_r
0.058	pmr	0.123	pmr	0.927	vm_ph_r
0.048	vmr	0.126	vhr	0.890	mmr

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).
Statistics are given for the final state of all paths.
Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.
0 is defined as any value below a threshold.
`dai_pct` (for down-and-in) is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
mc_m	295.32	409.52	345.08	601.31	345.65	541.75	411.22	378.07
mc_s	135.03	210.06	116.43	272.65	108.44	173.87	156.51	137.95
mc_min	4.82	2.88	0.00	1.04	1.45	160.90	34.17	4.87
mc_max	947.71	1593.81	2517.59	2247.55	748.29	1459.68	1230.60	1069.74
dao_pct	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
dai_pct	4.44	2.39	1.73	0.89	1.03	0.00	0.53	0.62

Ranking

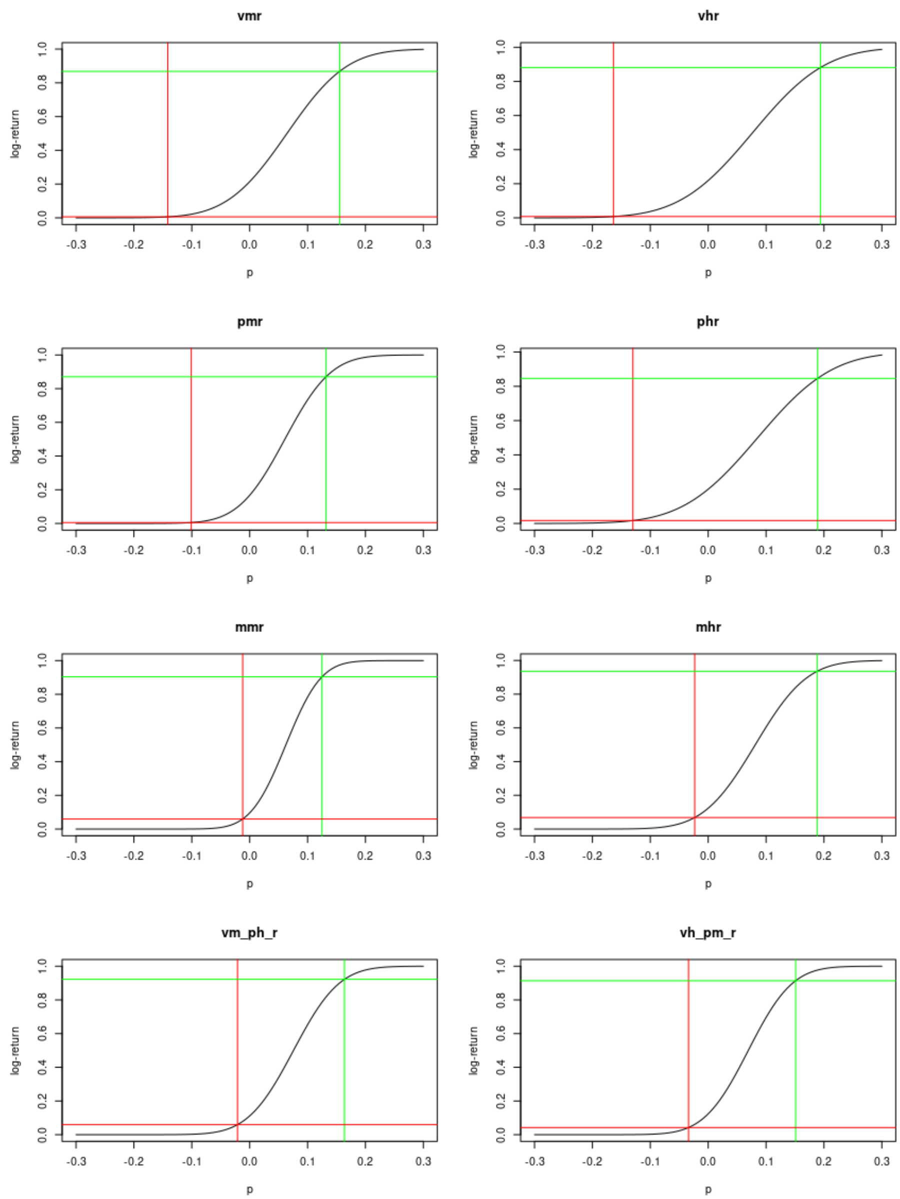
mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
601.31	phr	108.44	mmr	160.90	mhr	2517.59	pmr	0.00	vmr	0.00	mhr
541.75	mhr	116.43	pmr	34.17	vm_ph_r	2247.55	phr	0.00	vhr	0.53	vm_ph_r
411.22	vm_ph_r	135.03	vmr	4.87	vh_pm_r	1593.81	vhr	0.00	phr	0.62	vh_pm_r
409.52	vhr	137.95	vh_pm_r	4.82	vmr	1459.68	mhr	0.00	mmr	0.89	phr
378.07	vh_pm_r	156.51	vm_ph_r	2.88	vhr	1230.60	vm_ph_r	0.00	mhr	1.03	mmr
345.65	mmr	173.87	mhr	1.45	mmr	1069.74	vh_pm_r	0.00	vm_ph_r	1.73	pmr

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
345.08	pmr	210.06	vhr	1.04	phr	947.71	vmr	0.00	vh_pm_r	2.39	vhr
295.32	vmr	272.65	phr	0.00	pmr	748.29	mmr	0.03	pmr	4.44	vmr

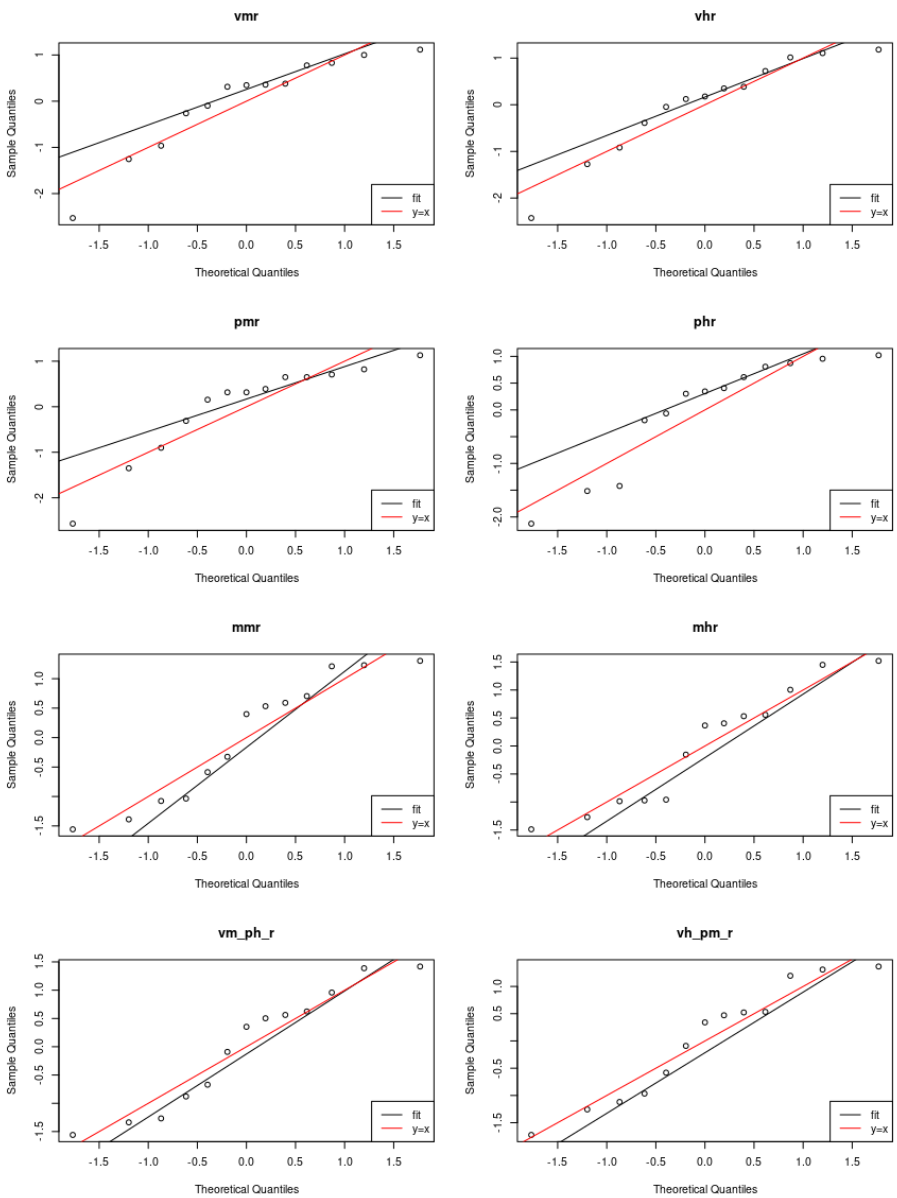
Compare Gaussian and skewed t-distribution fits

Gaussian fits

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
m	0.064	0.077	0.061	0.085	0.062	0.081	0.076	0.069
s	0.081	0.099	0.063	0.101	0.048	0.070	0.062	0.060


vmr
vhr
pmr
phr
mmr
mhr
vm_ph_r
vh_pm_r
log-return
p
0.0 0.2 0.4 0.6 0.8 1.0
-0.3 -0.2 -0.1 0.0 0.1 0.2 0.3

Gaussian QQ plots



Gaussian vs skewed t

Probability in percent that the smallest and largest (respectively) observed return for each fund was generated by a normal distribution:

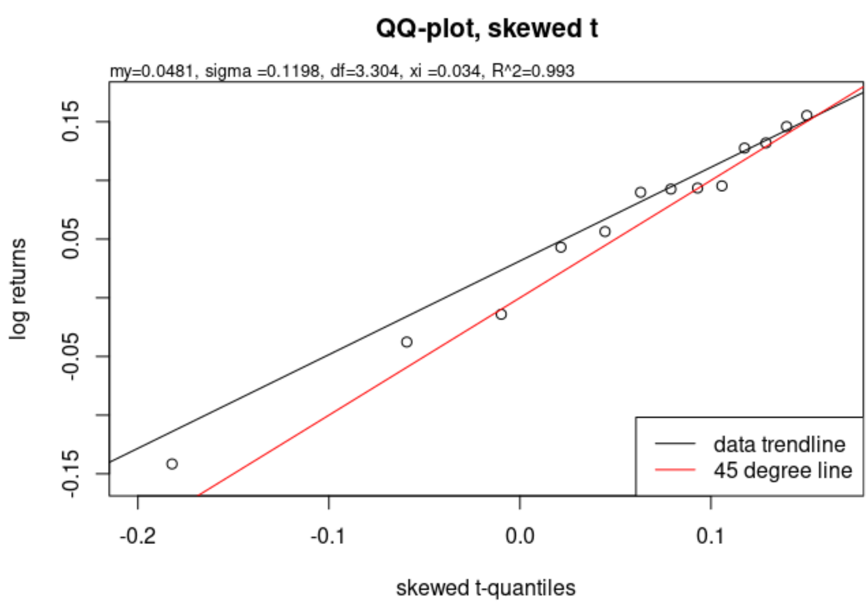
	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
P_norm(X_min)	0.571	0.758	0.511	1.676	5.971	6.842	5.945	4.228
P_norm(X_max)	13.230	11.876	12.922	15.359	9.628	6.429	7.796	8.592
P_t(X_min)	5.377	5.080	3.489	4.315	10.570	8.015	13.008	10.520
P_t(X_max)	0.118	0.156	2.825	0.188	0.488	5.141	0.229	0.175

Average number of years between min or max events (respectively):

	vmr	vhr	pmr	phr	mmr	mhr	vm_ph_r	vh_pm_r
norm: avg yrs btw min	175.248	131.911	195.568	59.669	16.748	14.616	16.820	23.650
norm: avg yrs btw max	7.559	8.420	7.739	6.511	10.386	15.556	12.827	11.639
t: avg yrs btw min	18.596	19.687	28.663	23.173	9.461	12.476	7.688	9.506
t: avg yrs btw max	848.548	640.410	35.400	531.552	205.104	19.450	437.280	572.483

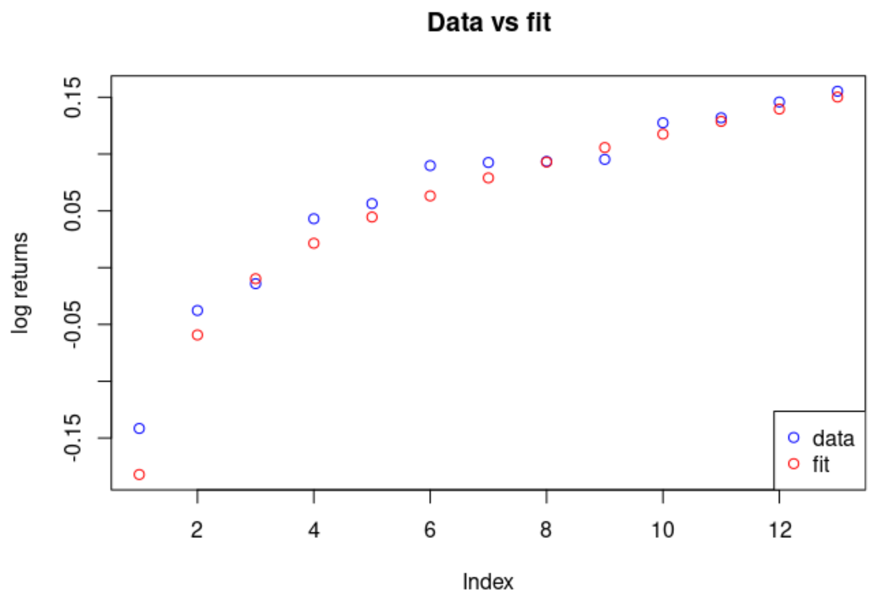
Velliv medium risk (vmr), 2011 - 2023

QQ Plot



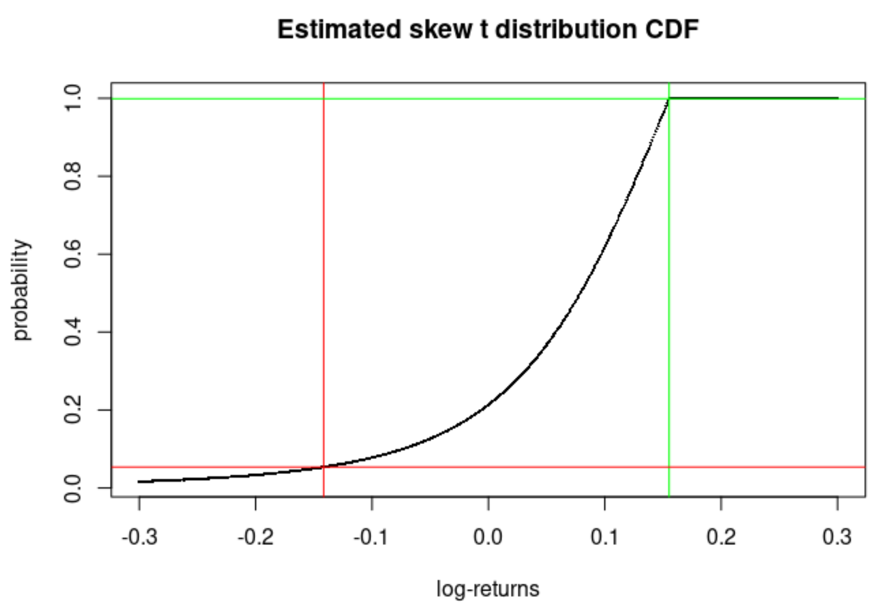
Data vs fit

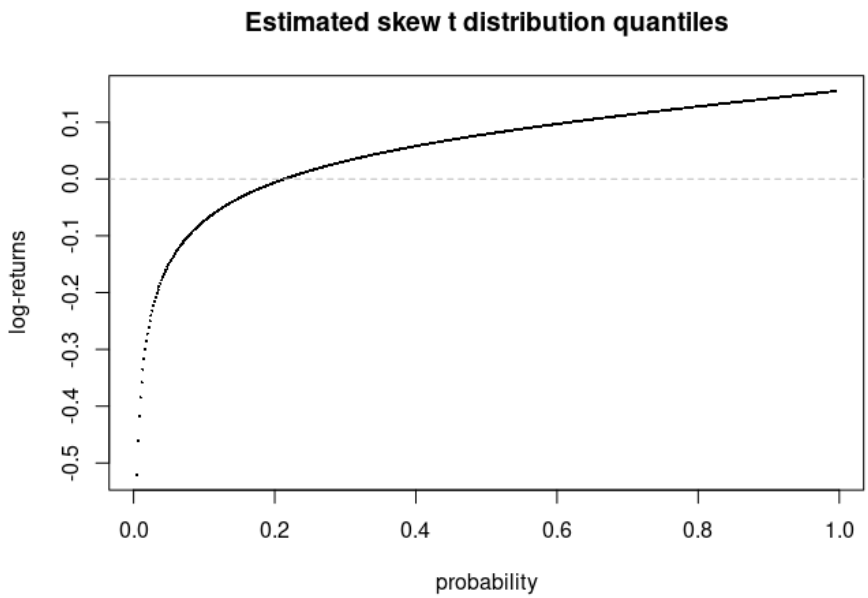
Let’s plot the fit and the observed returns together.

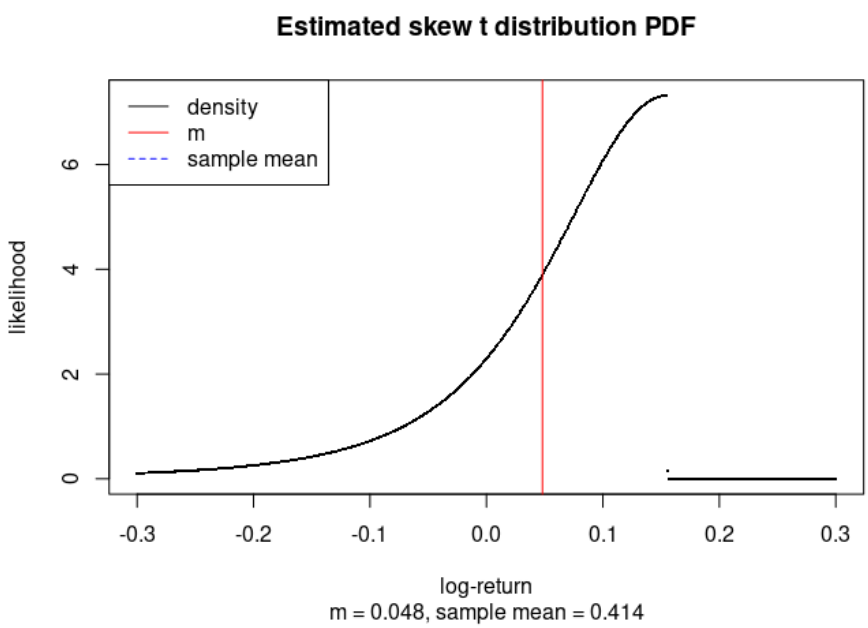


Estimated distribution

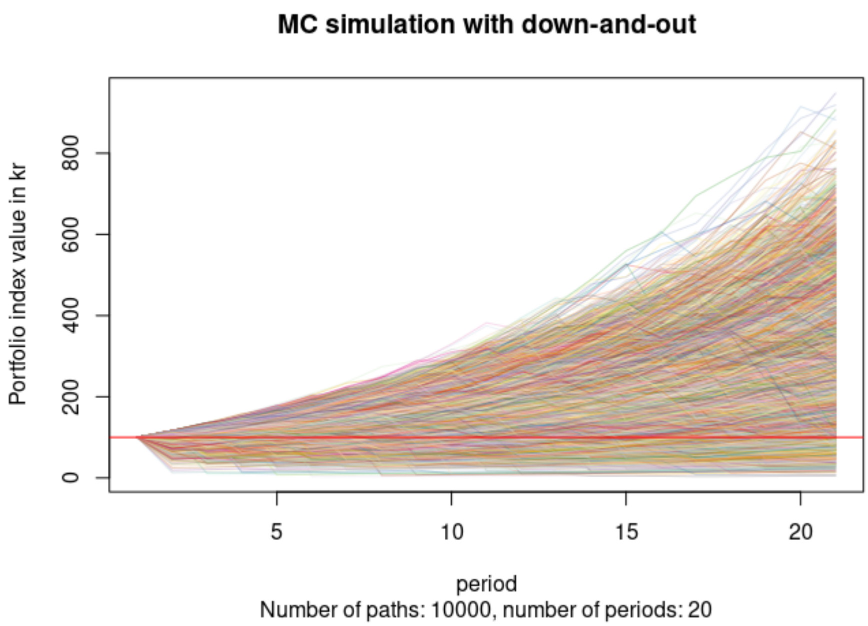
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

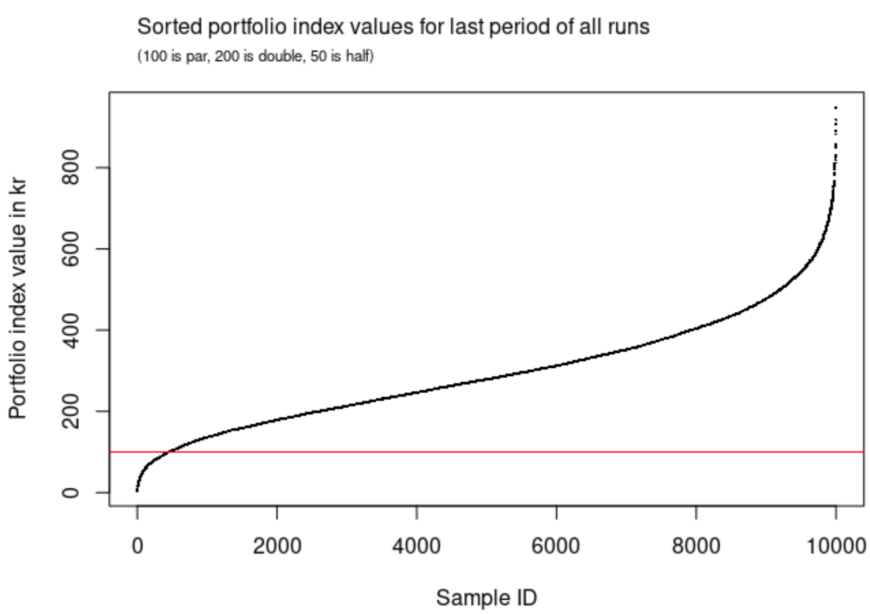



Estimated skew t distribution quantiles
log-returns
probability
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
0.0
0.2
0.4
0.6
0.8
1.0


Estimated skew t distribution PDF
density
m
sample mean
likelihood
log-return
m = 0.048, sample mean = 0.414
0
2
4
6
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3

Monte Carlo

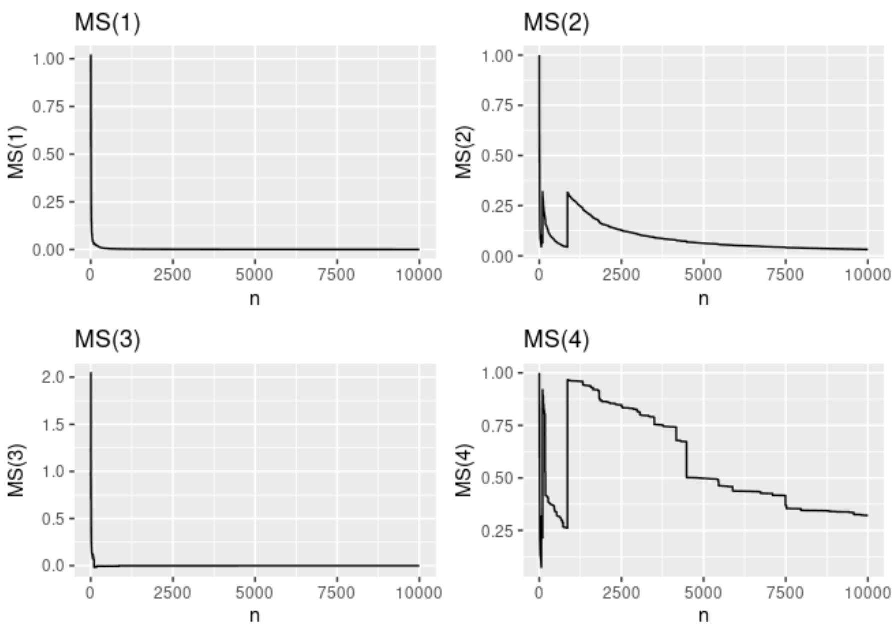

MC simulation with down-and-out
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period
Number of paths: 10000, number of periods: 20


Sorted portfolio index values for last period of all runs
(100 is par, 200 is double, 50 is half)
Portfolio index value in kr
0
200
400
600
800
0
2000
4000
6000
8000
10000
Sample ID

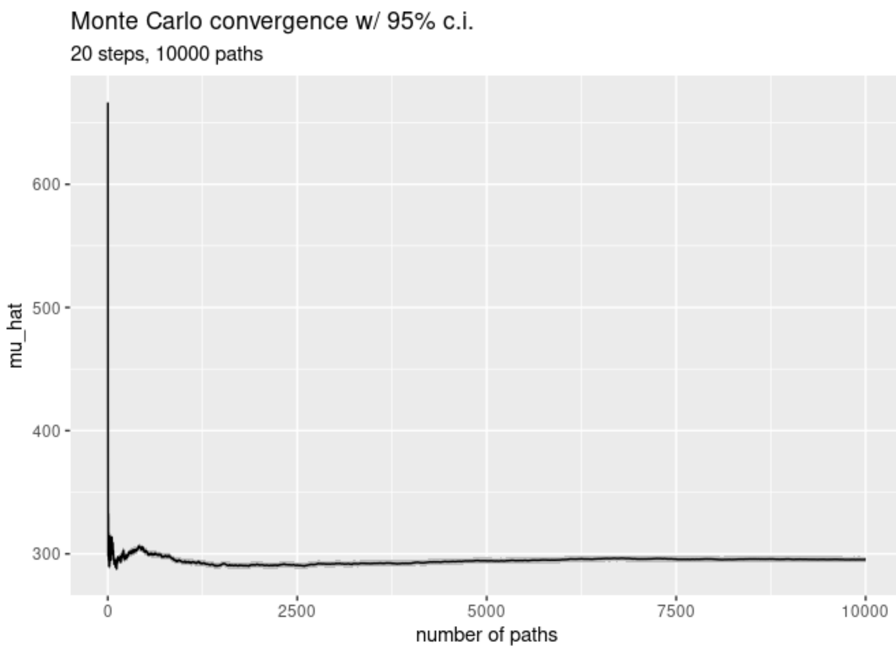
Convergence

Max vs sum

Max vs sum plots for the first four moments:



MC

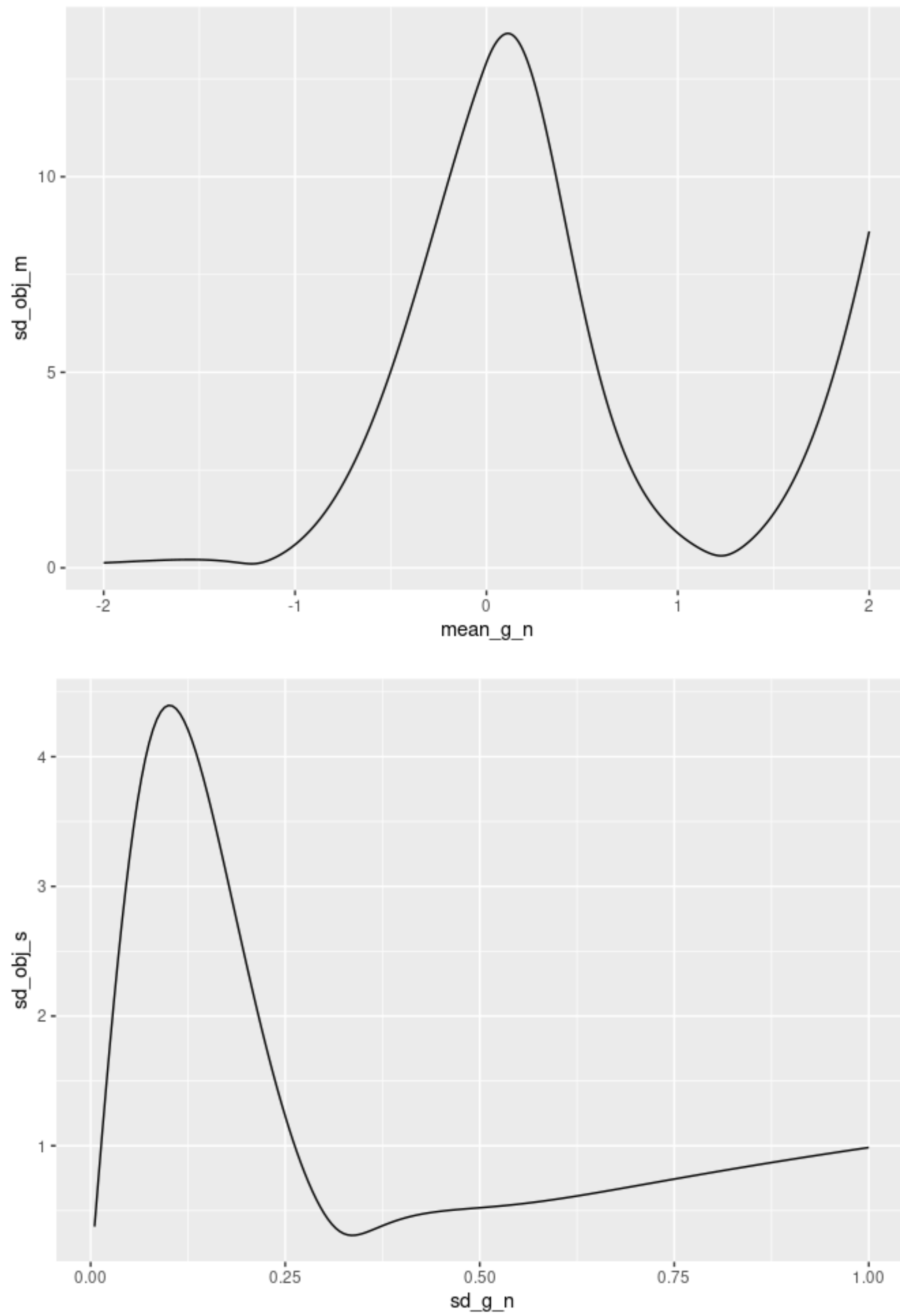


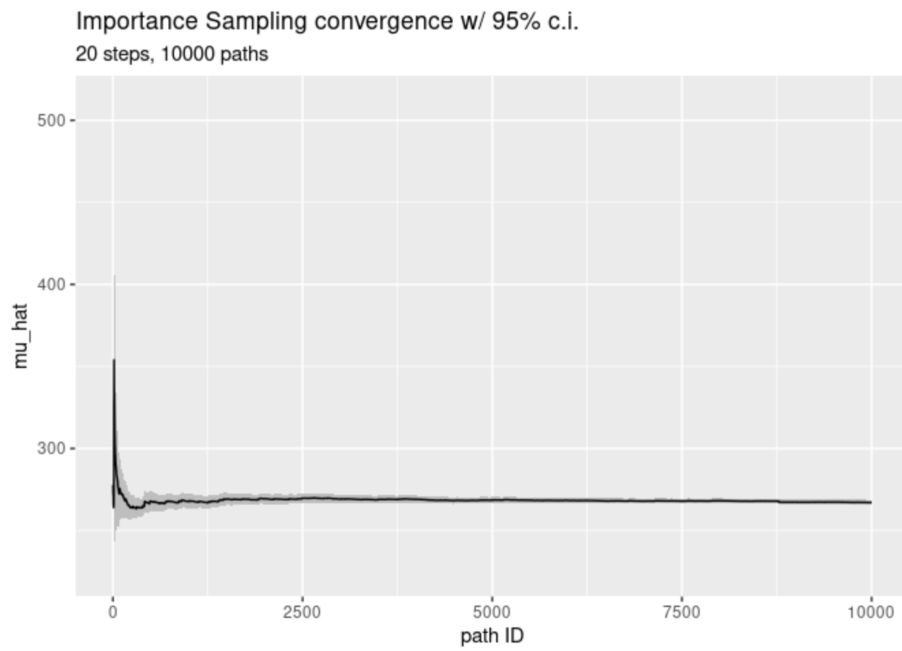
IS

Parameters

```
## [1] 1.2262221 0.3361598
```

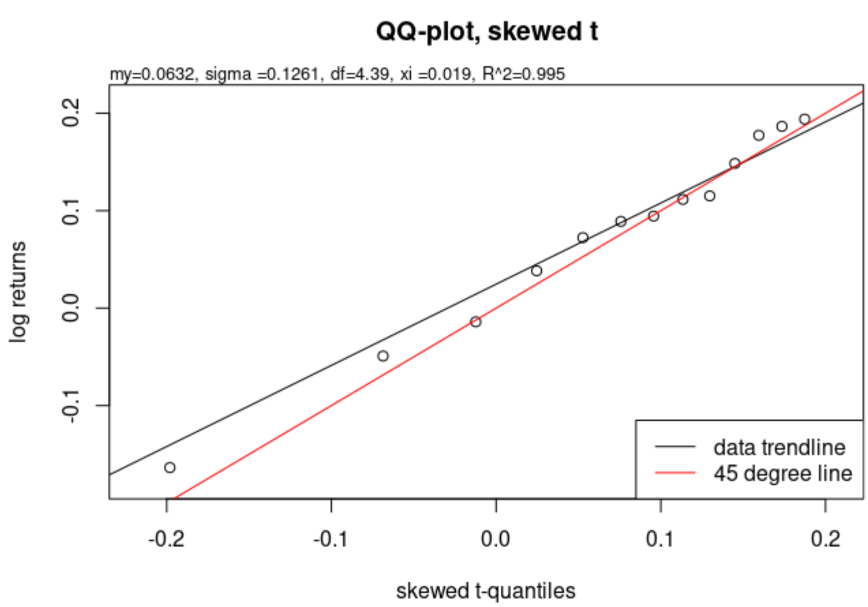
Objective function plots


sd_obj_m
10
5
0
-2
-1
0
1
2
mean_g_n
sd_obj_s
4
3
2
1
0.00
0.25
0.50
0.75
1.00
sd_g_n



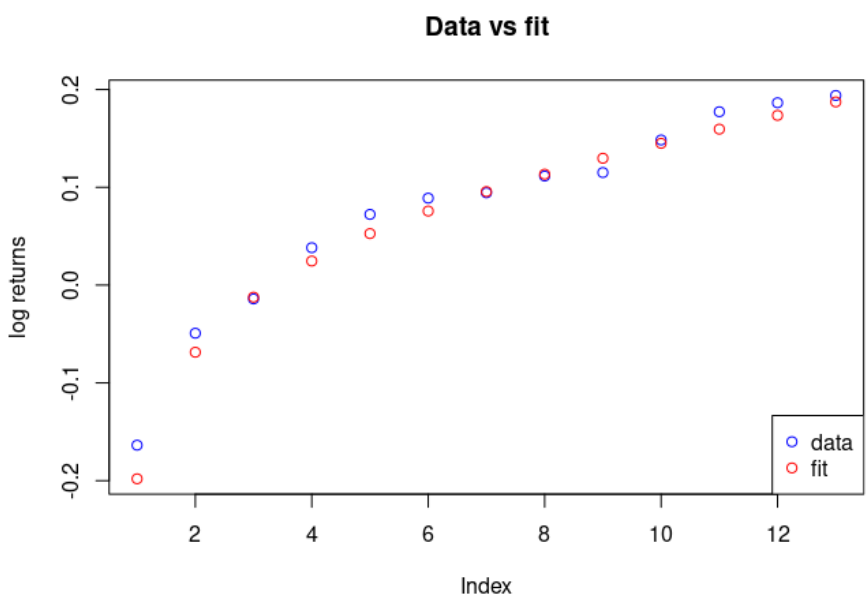
Velliv medium risk (vmrl), 2007 - 2023

QQ Plot



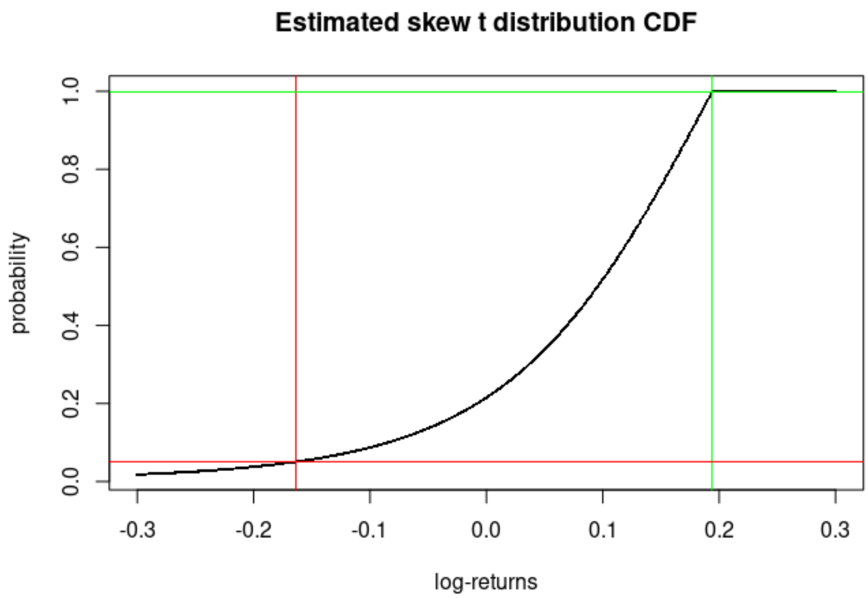
Data vs fit

Let's plot the fit and the observed returns together.

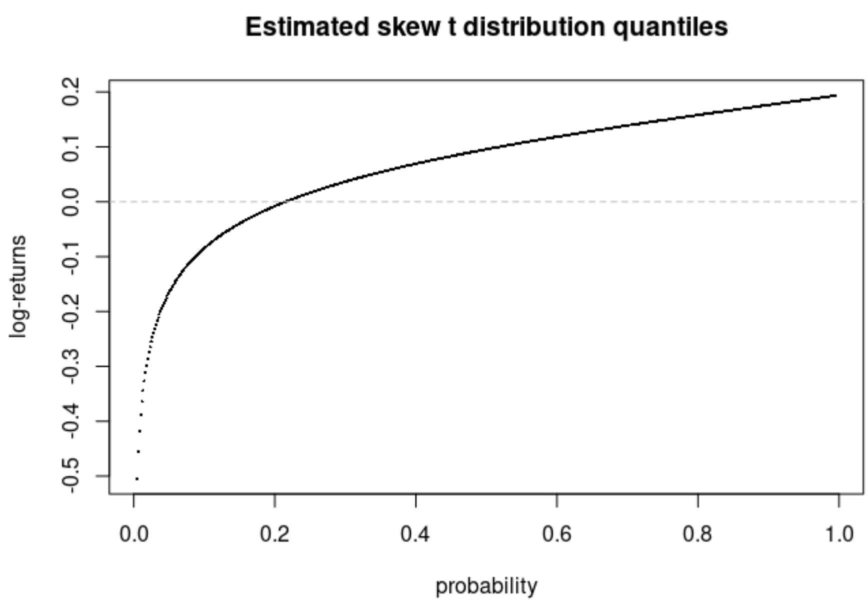


Estimated distribution

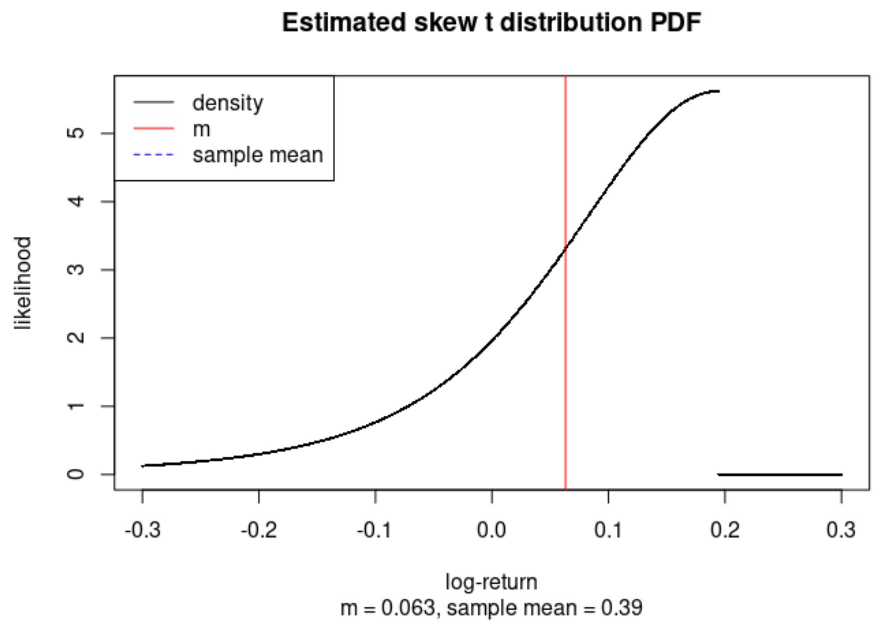
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



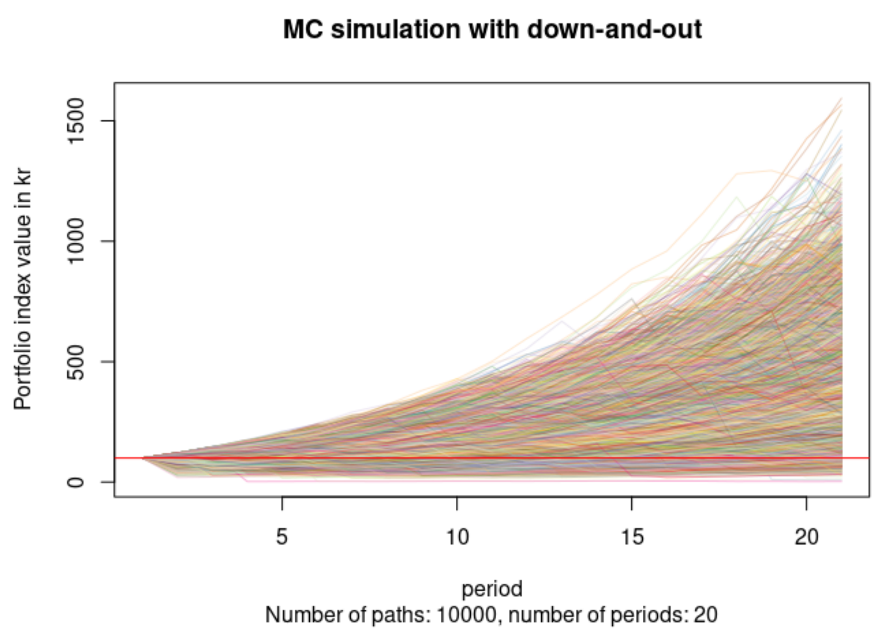
Estimated skew t distribution CDF
probability
log-returns
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
0.0
0.2
0.4
0.6
0.8
1.0

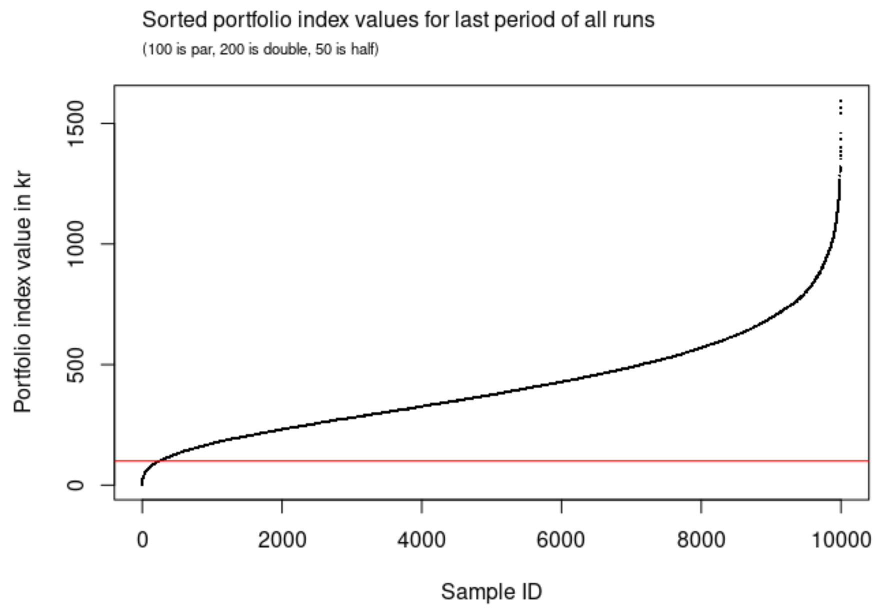


Estimated skew t distribution quantiles
log-returns
probability
0.0
0.2
0.4
0.6
0.8
1.0
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
0.1
0.2


Estimated skew t distribution PDF
density
m
sample mean
likelihood
0
1
2
3
4
5
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
log-return
m = 0.063, sample mean = 0.39

Monte Carlo

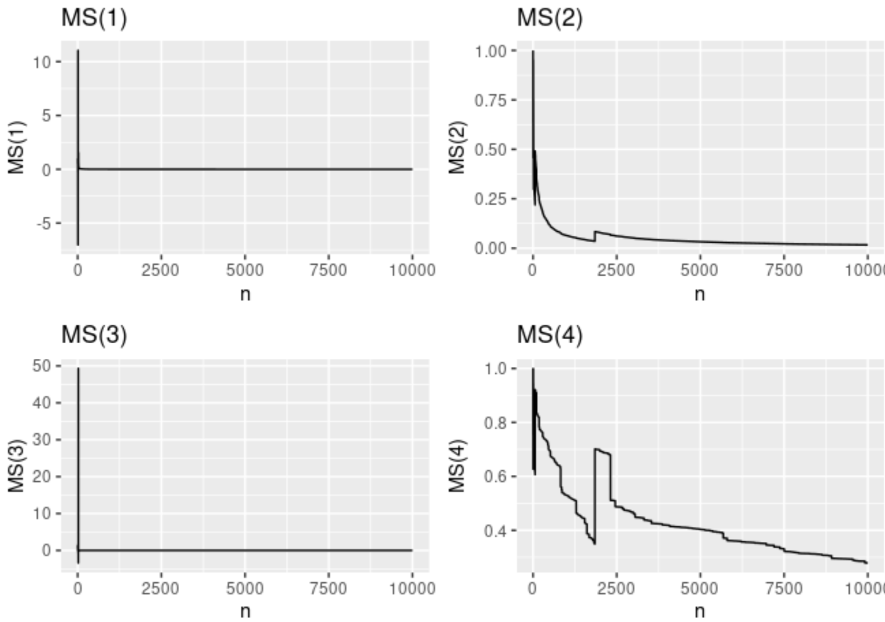

MC simulation with down-and-out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Convergence

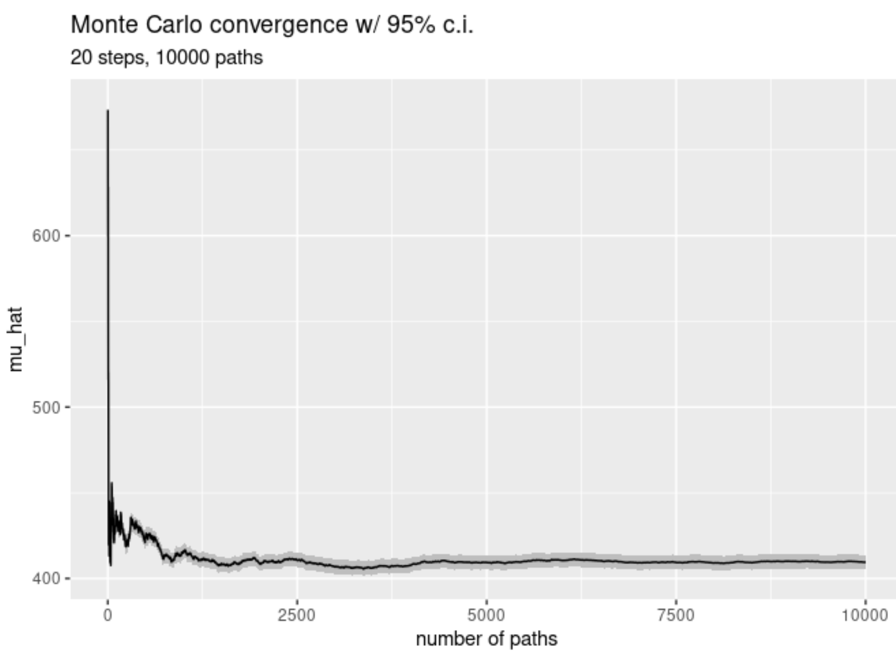
Max vs sum

Max vs sum plots for the first four moments:



MS(1)
MS(2)
MS(3)
MS(4)
n

MC



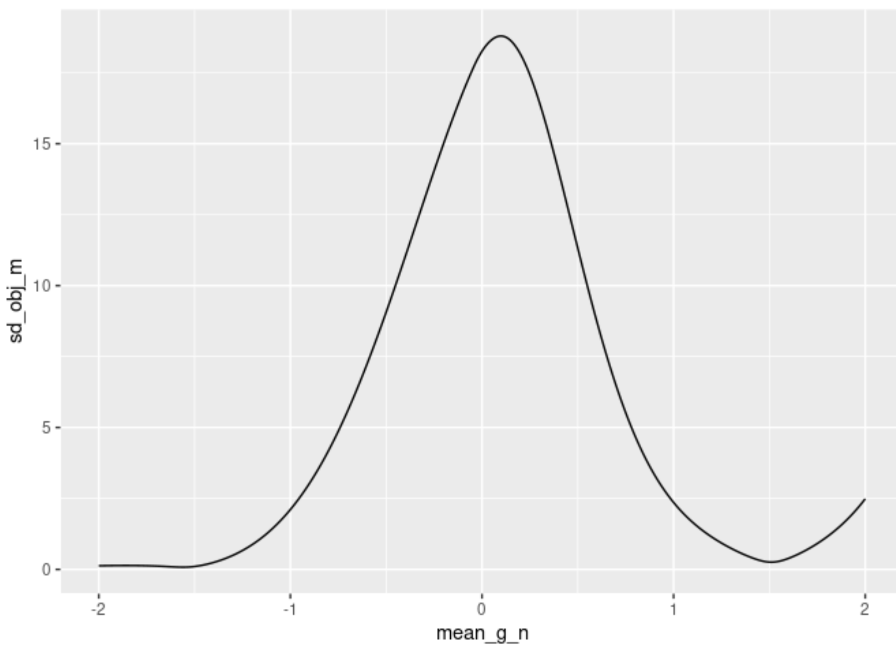
Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
number of paths

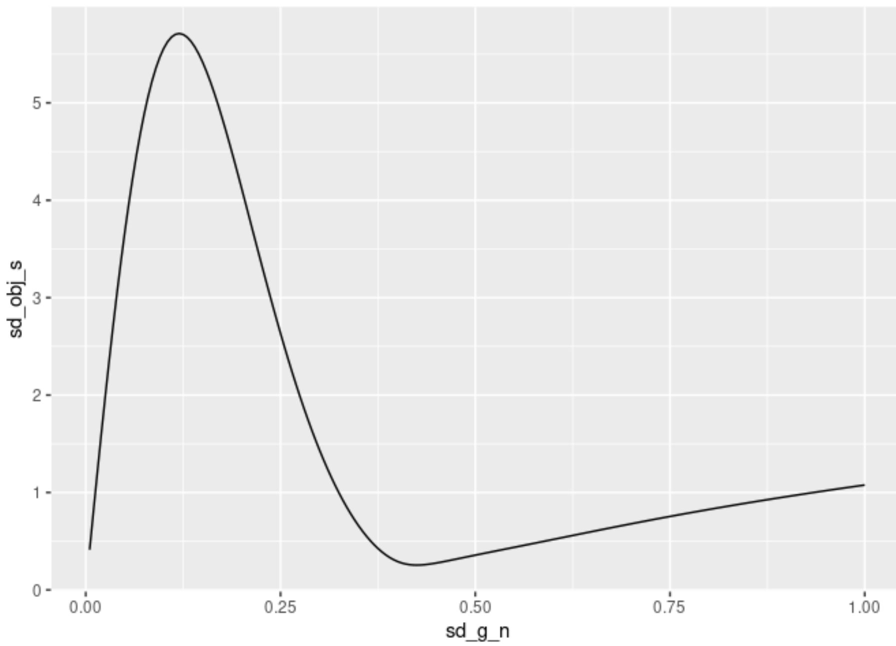
IS

Parameters

```
## [1] 1.5098519 0.4248085
```

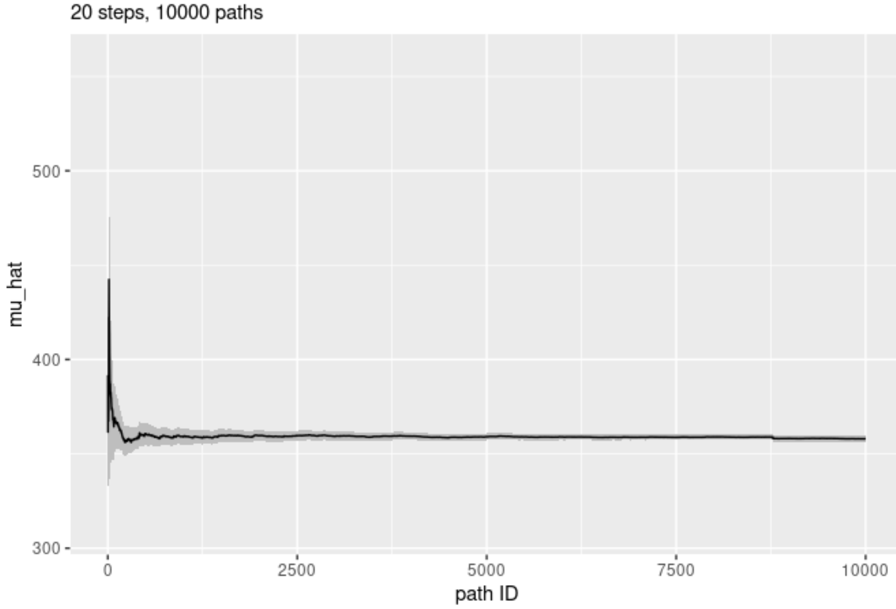
Objective function plots





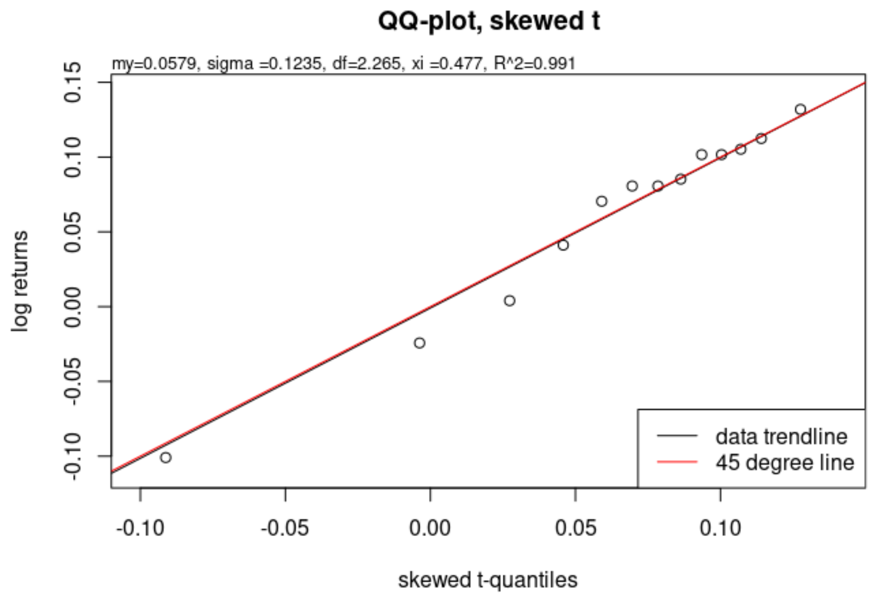
Importance Sampling convergence w/ 95% c.i.

20 steps, 10000 paths



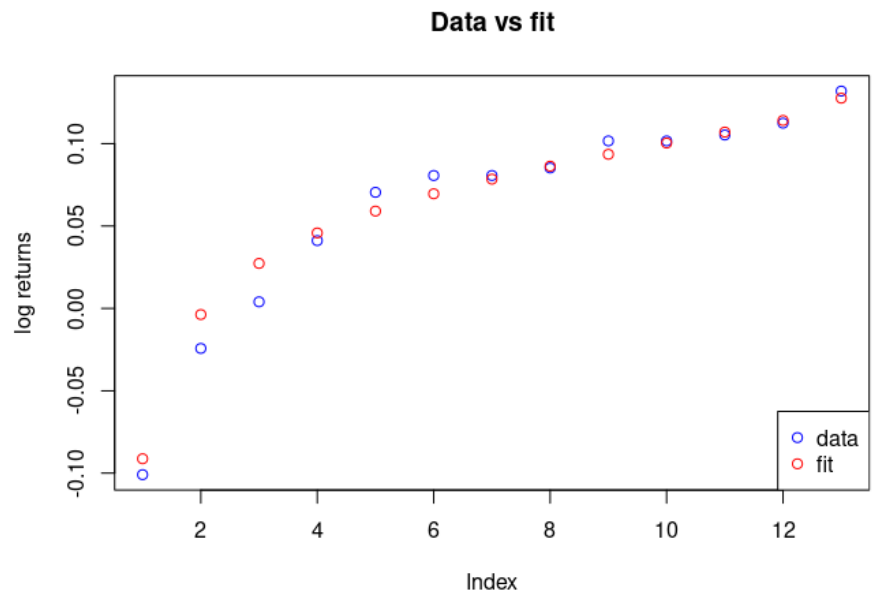
Velliv high risk (vhr), 2011 - 2023

QQ Plot



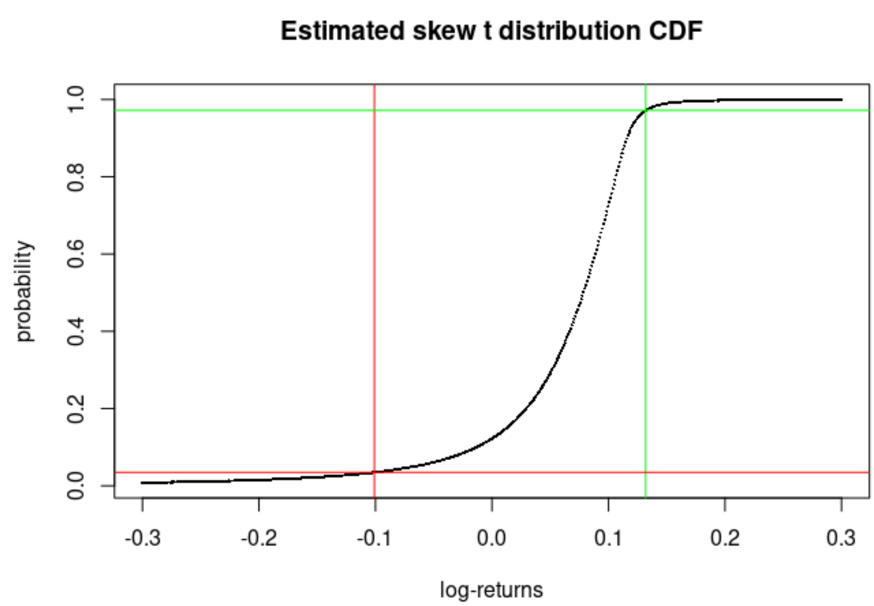
Data vs fit

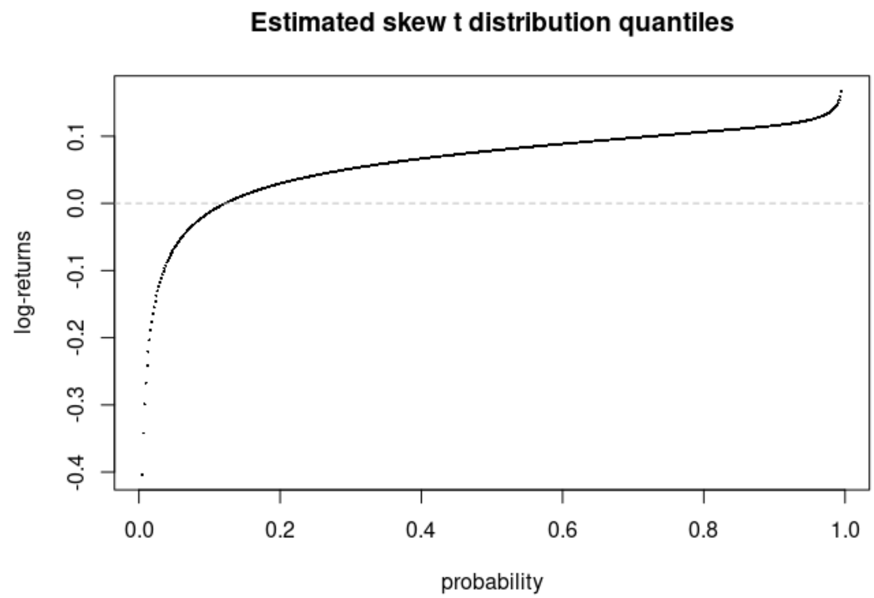
Let’s plot the fit and the observed returns together.

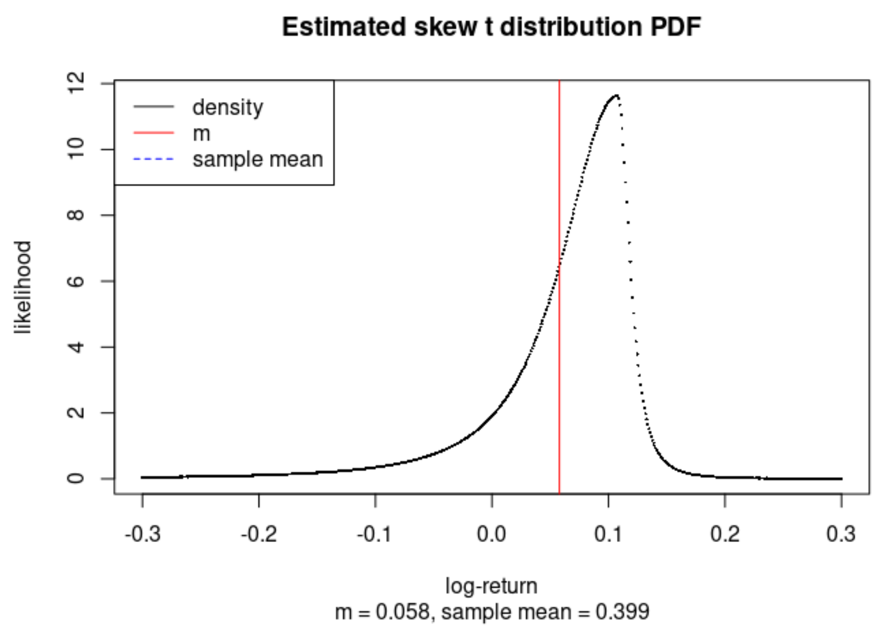


Estimated distribution

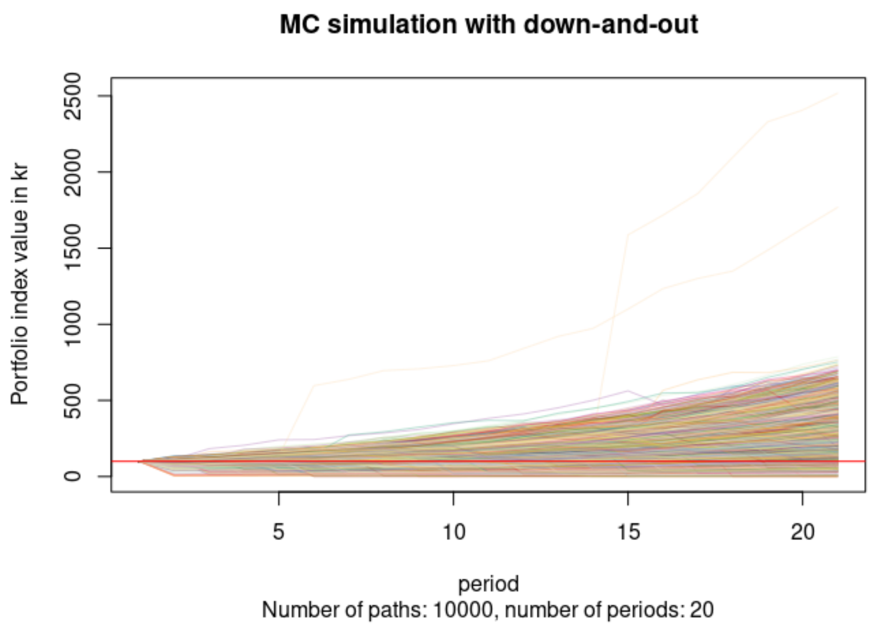
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



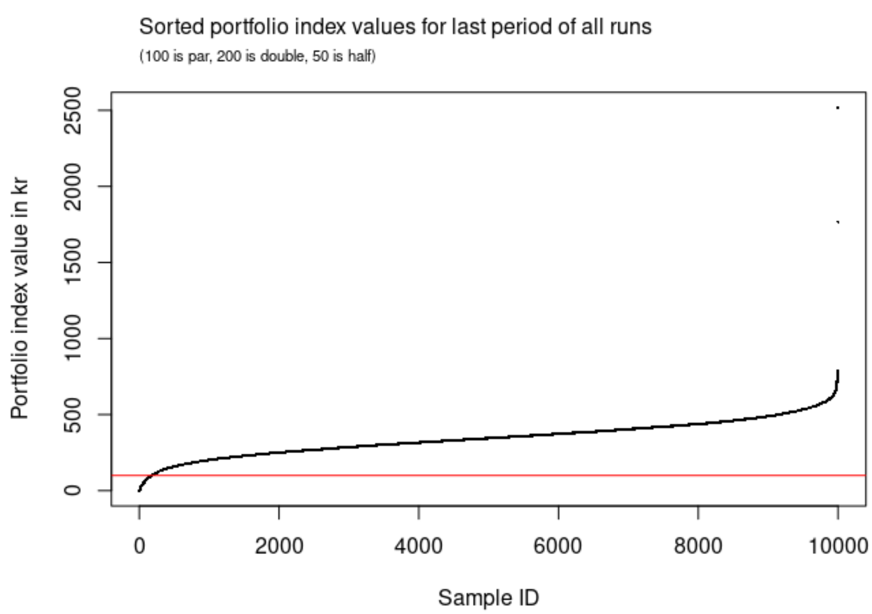

Estimated skew t distribution quantiles
log-returns
probability
0.1
0.0
-0.1
-0.2
-0.3
-0.4
0.0
0.2
0.4
0.6
0.8
1.0


Estimated skew t distribution PDF
density
m
sample mean
likelihood
log-return
m = 0.058, sample mean = 0.399
12
10
8
6
4
2
0
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3

Monte Carlo



MC simulation with down-and-out
Portfolio index value in kr
0
500
1000
1500
2000
2500
5
10
15
20
period
Number of paths: 10000, number of periods: 20

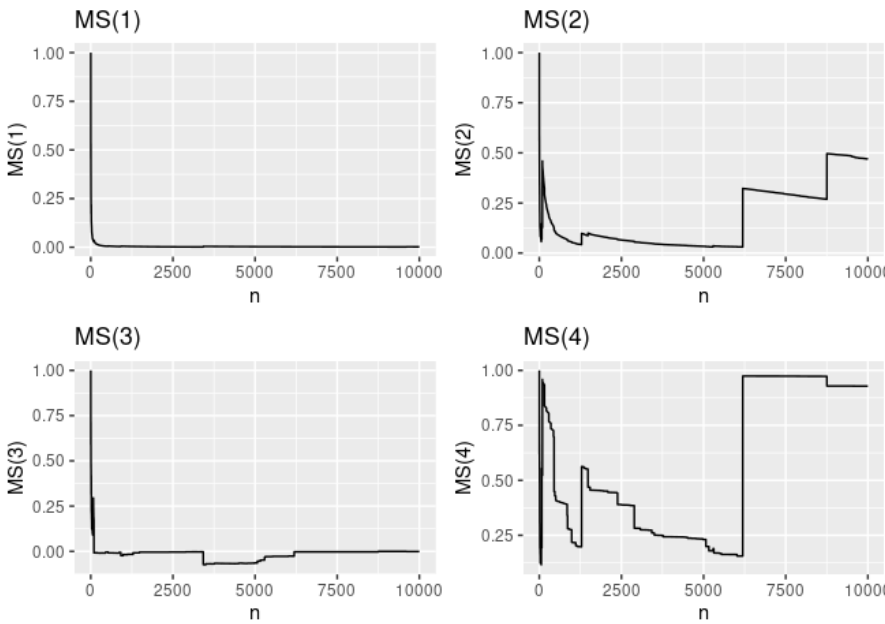


Sorted portfolio index values for last period of all runs
(100 is par, 200 is double, 50 is half)
Portfolio index value in kr
0
500
1000
1500
2000
2500
0
2000
4000
6000
8000
10000
Sample ID

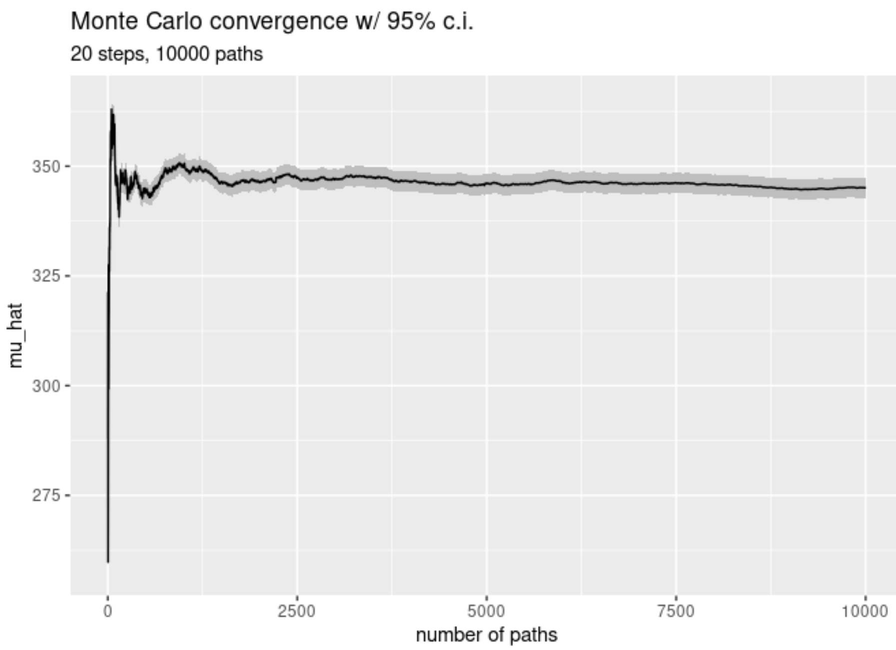
Convergence

Max vs sum

Max vs sum plots for the first four moments:



MC

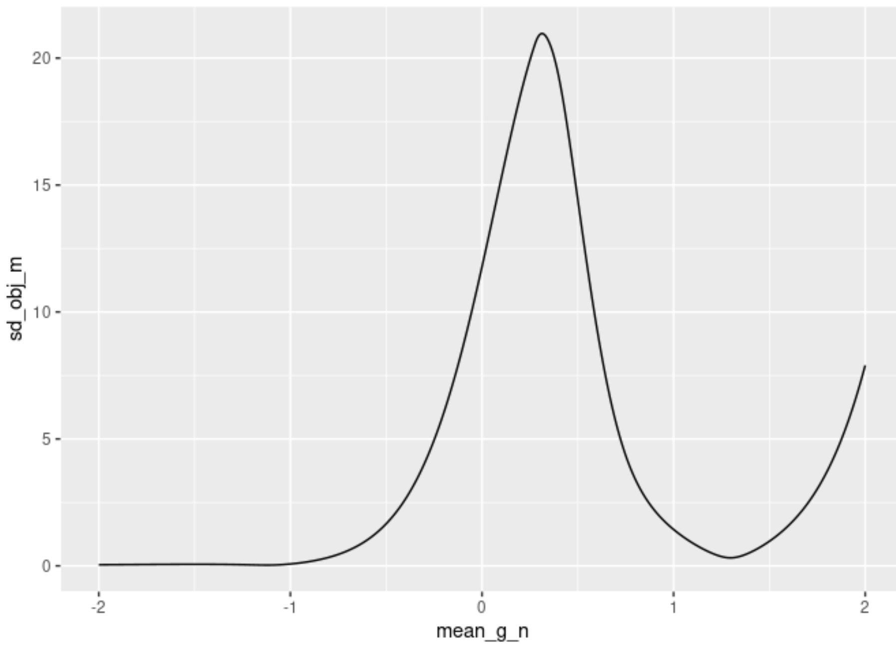


IS

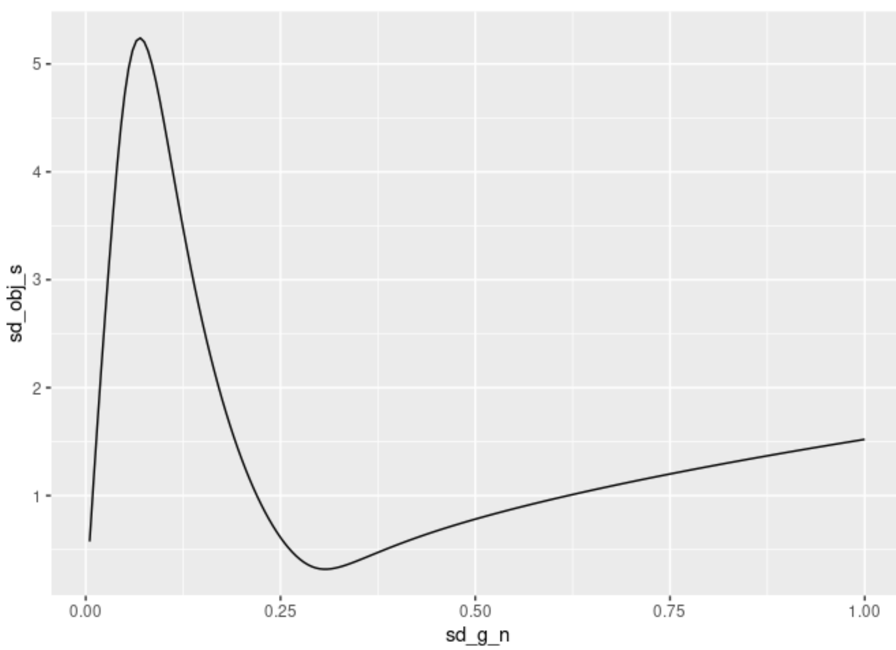
Parameters

```
## [1] 1.2965857 0.3073935
```

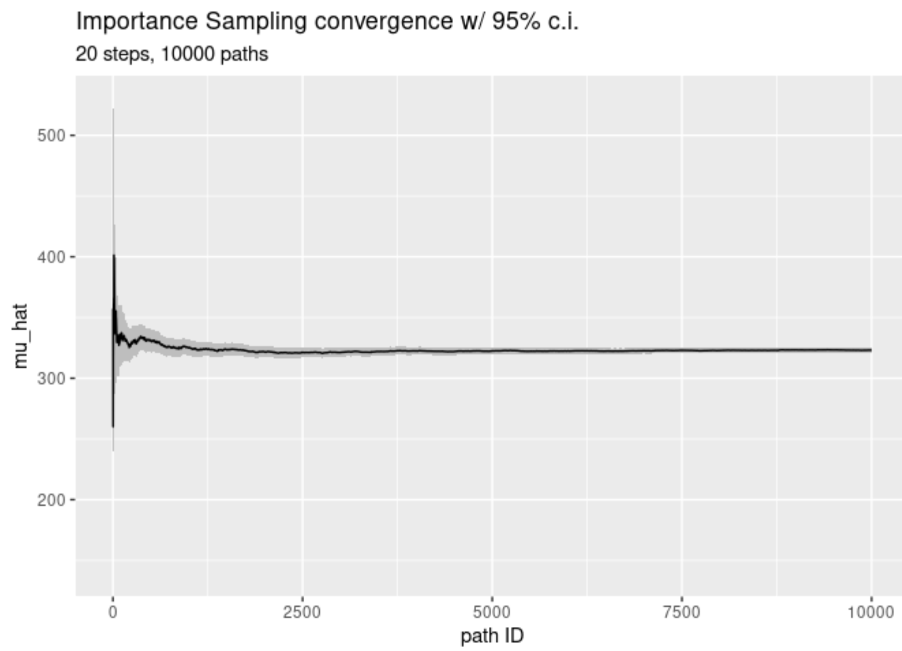
Objective function plots



sd_obj_m
20
15
10
5
0
-2
-1
0
1
2
mean_g_n

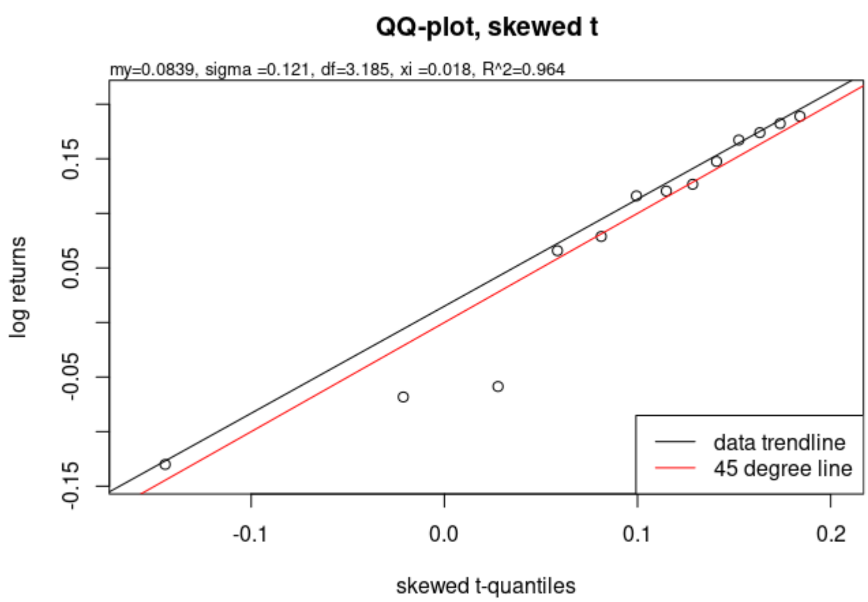


sd_obj_s
5
4
3
2
1
0.00
0.25
0.50
0.75
1.00
sd_g_n



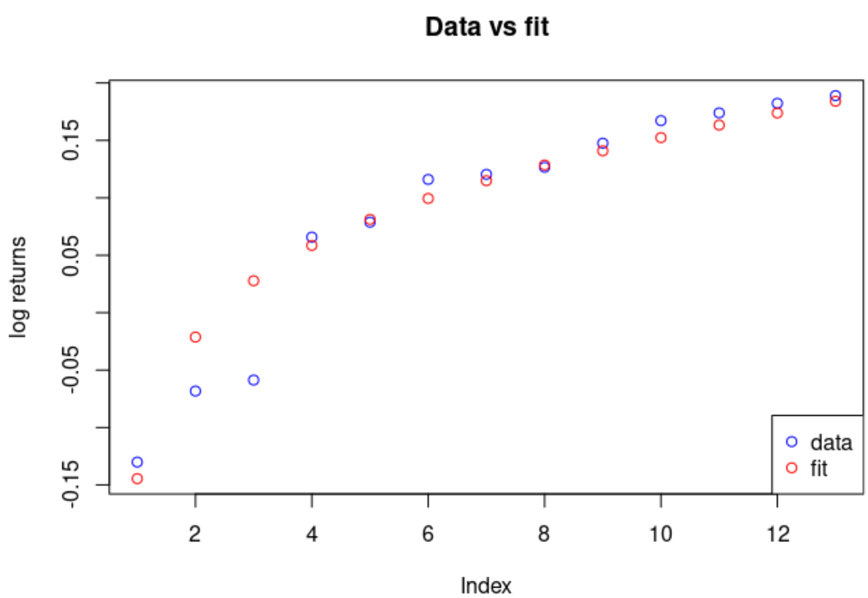
PFA medium risk (pmr), 2011 - 2023

QQ Plot



Data vs fit

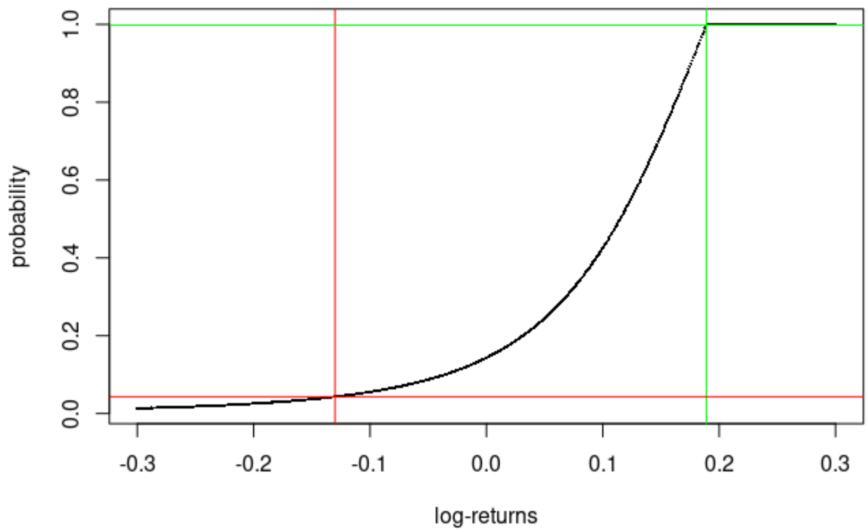
Let's plot the fit and the observed returns together.



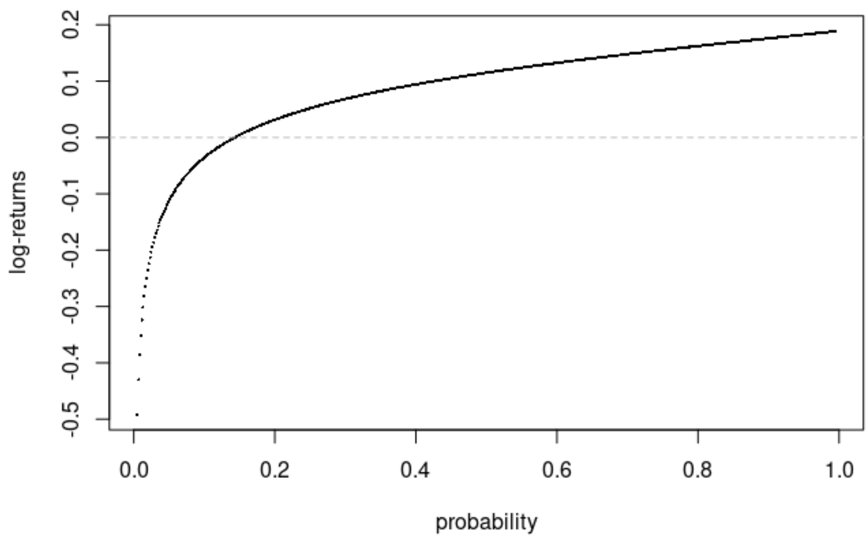
Estimated distribution

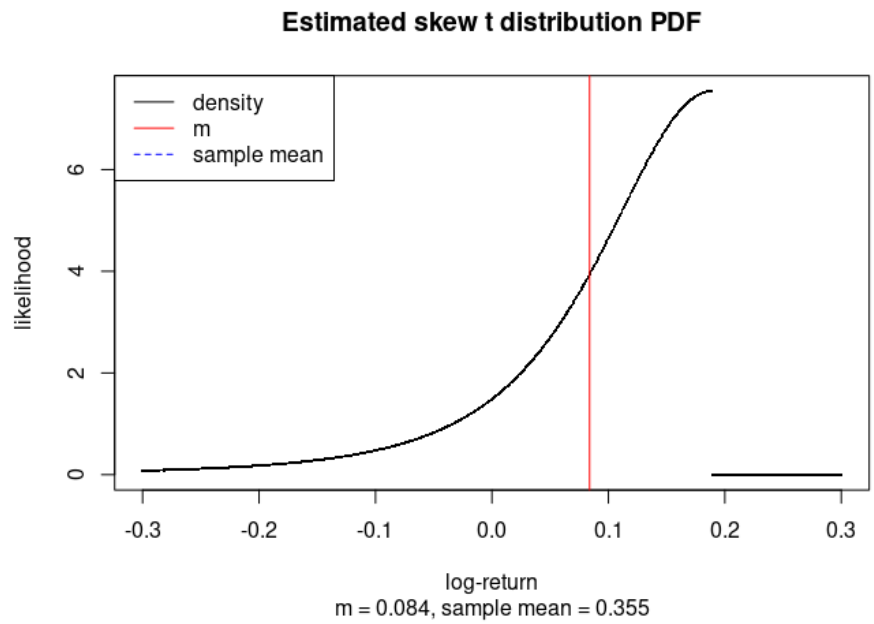
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

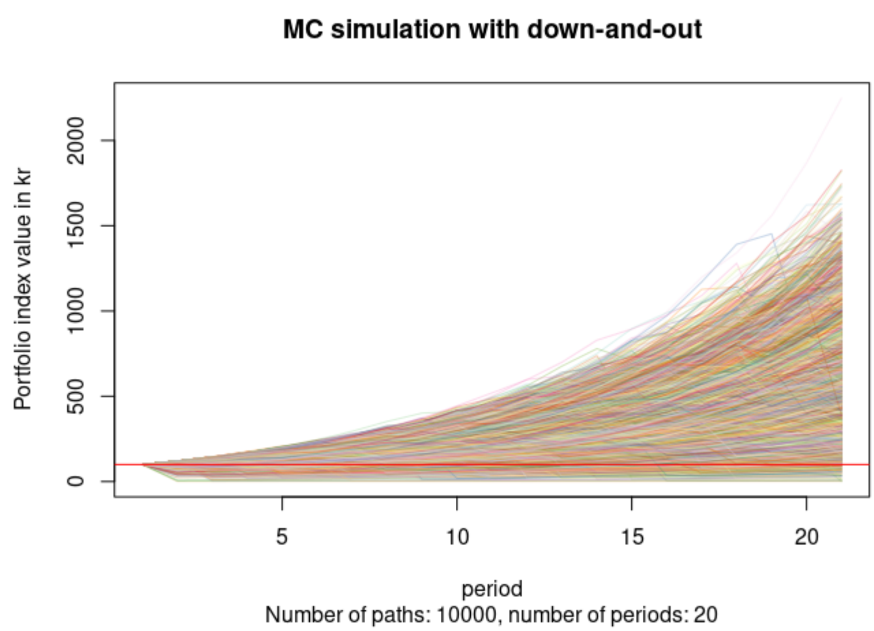


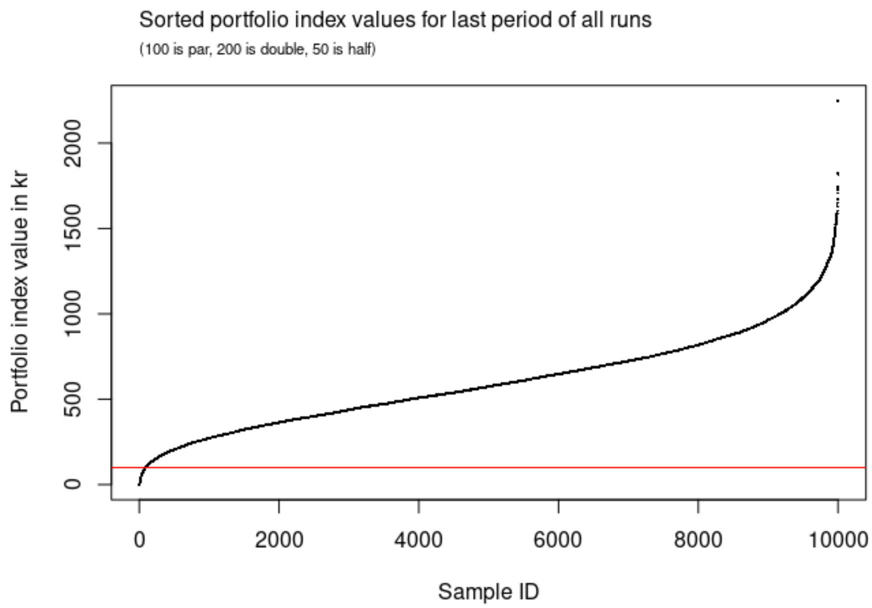
Estimated skew t distribution quantiles




Estimated skew t distribution PDF
density
m
sample mean
likelihood
log-return
m = 0.084, sample mean = 0.355

Monte Carlo

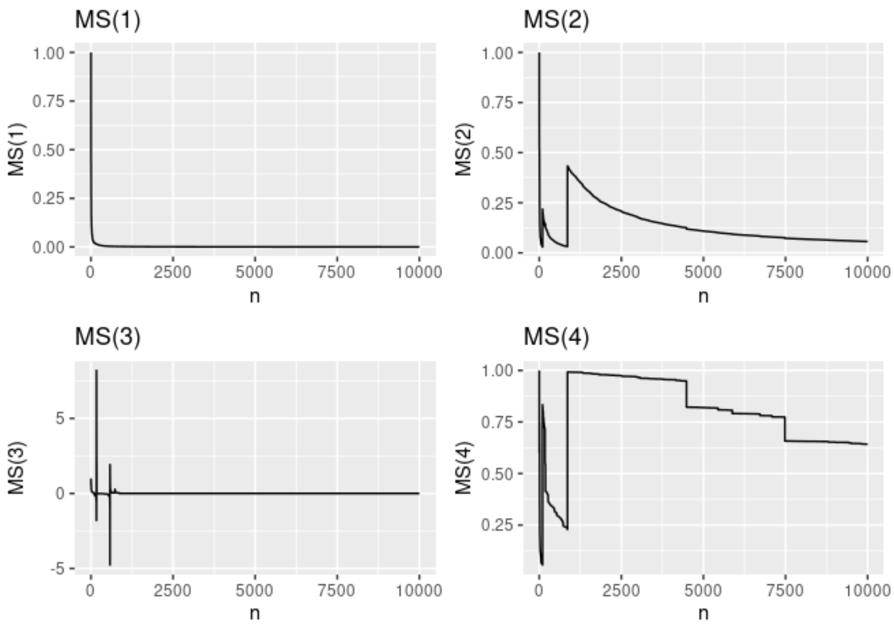

MC simulation with down-and-out
Portfolio index value in kr
period
Number of paths: 10000, number of periods: 20



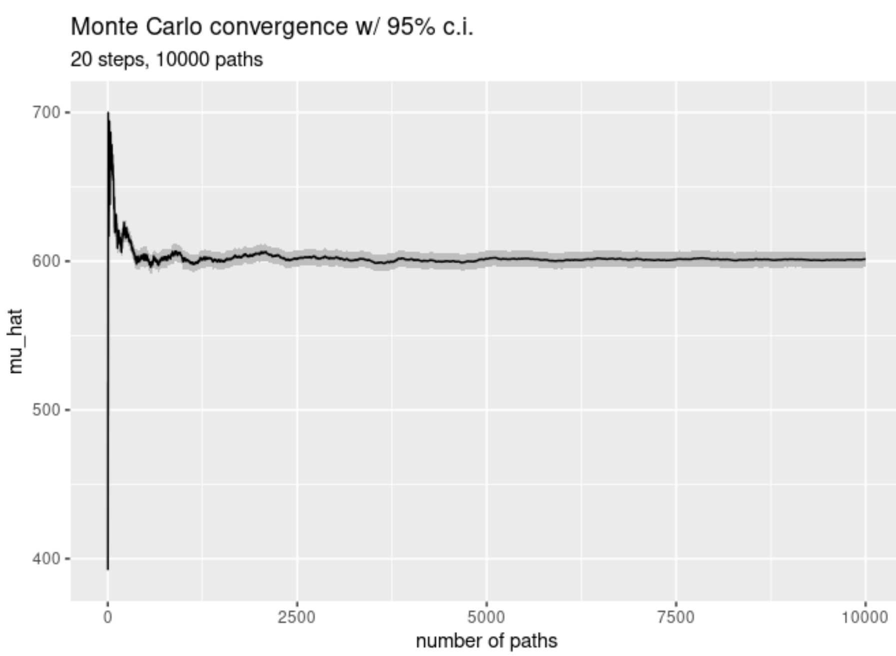
Convergence

Max vs sum

Max vs sum plots for the first four moments:


MS(1)
MS(2)
MS(3)
MS(4)
1.00
0.75
0.50
0.25
0.00
5
0
-5
0
2500
5000
7500
10000
n

MC

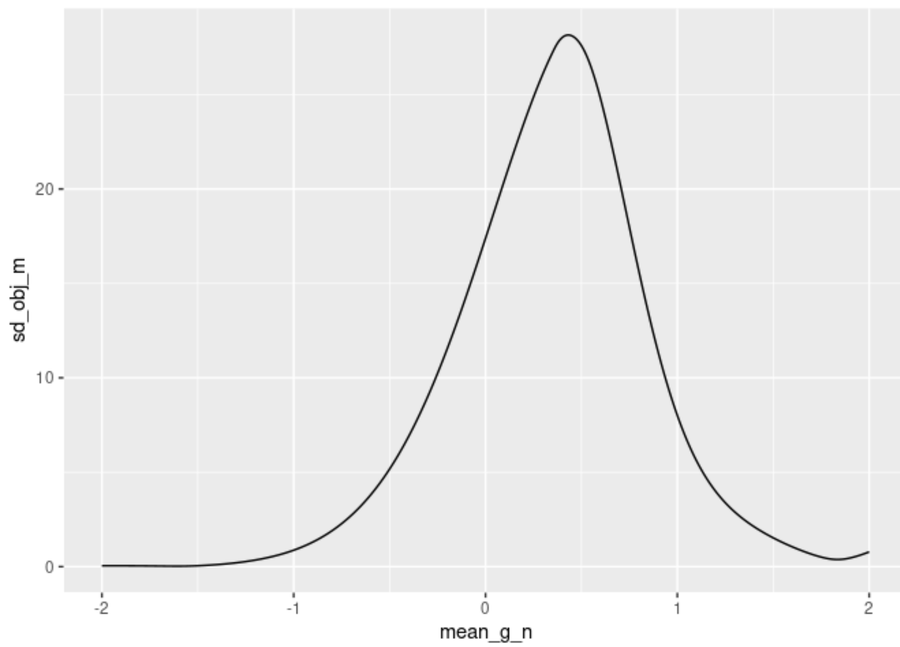

Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
700
600
500
400
0
2500
5000
7500
10000
number of paths

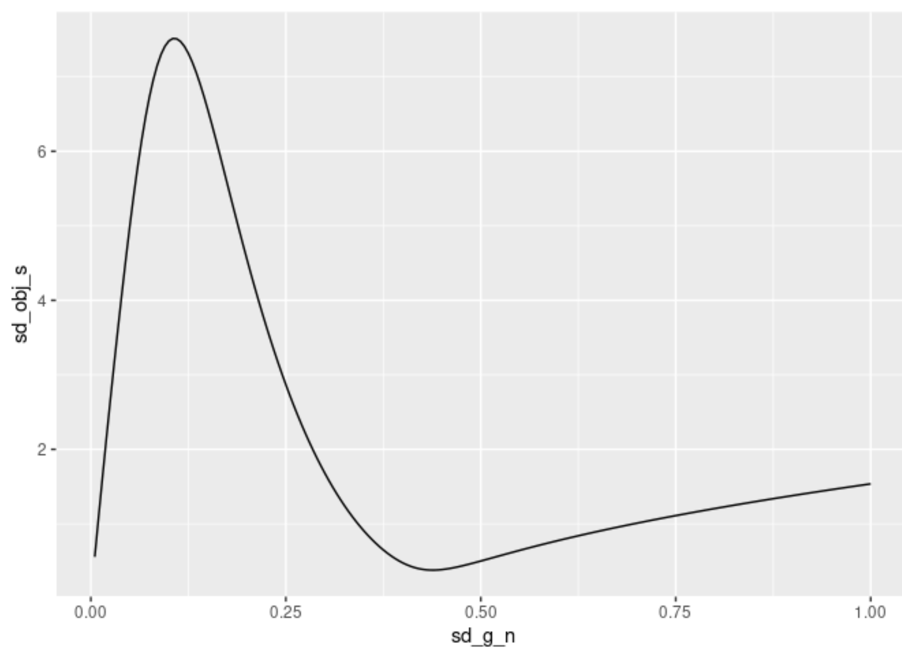
IS

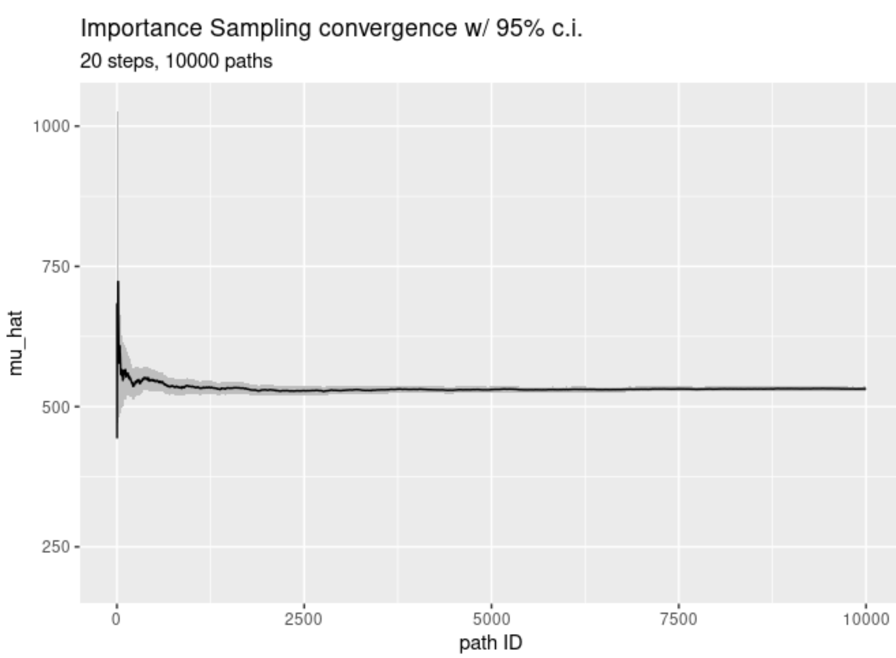
Parameters

```
## [1] 1.8351623 0.4382935
```

Objective function plots

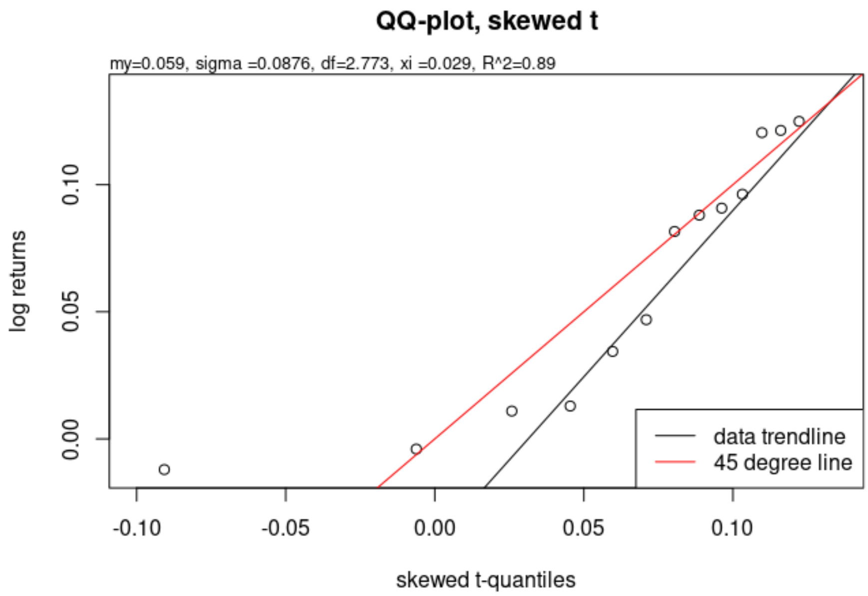



sd_obj_s
6
4
2
0.00
0.25
0.50
0.75
1.00
sd_g_n


Importance Sampling convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
1000
750
500
250
0
2500
5000
7500
10000
path ID

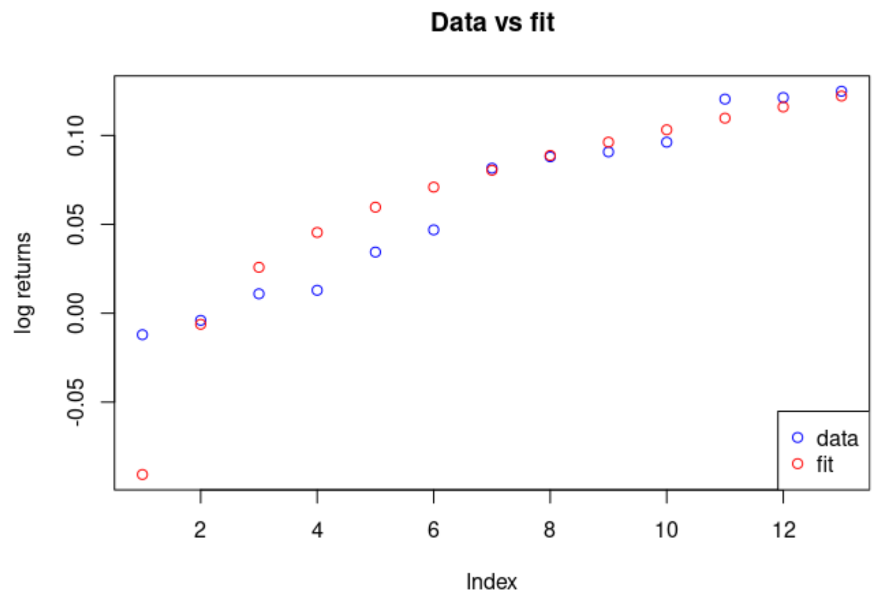
PFA high risk (phr), 2011 - 2023

QQ Plot



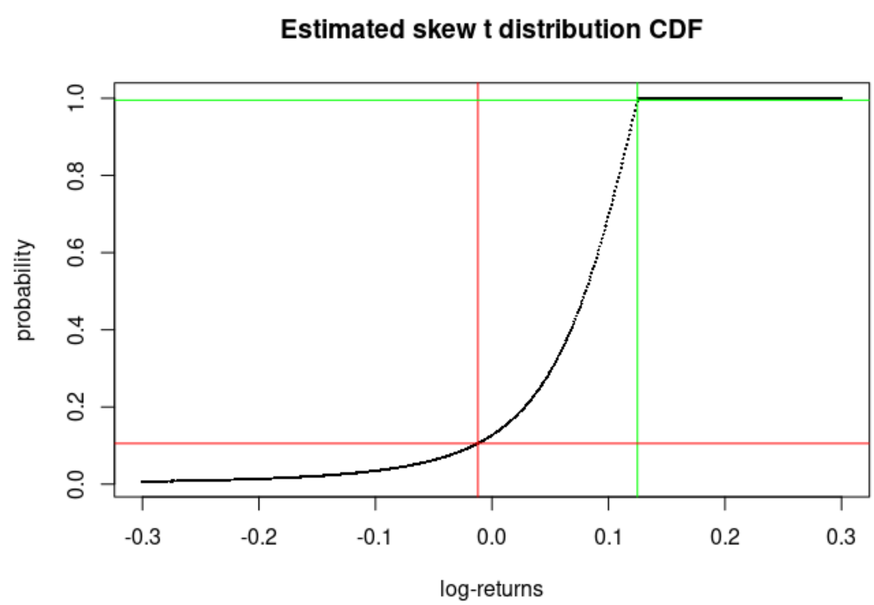
Data vs fit

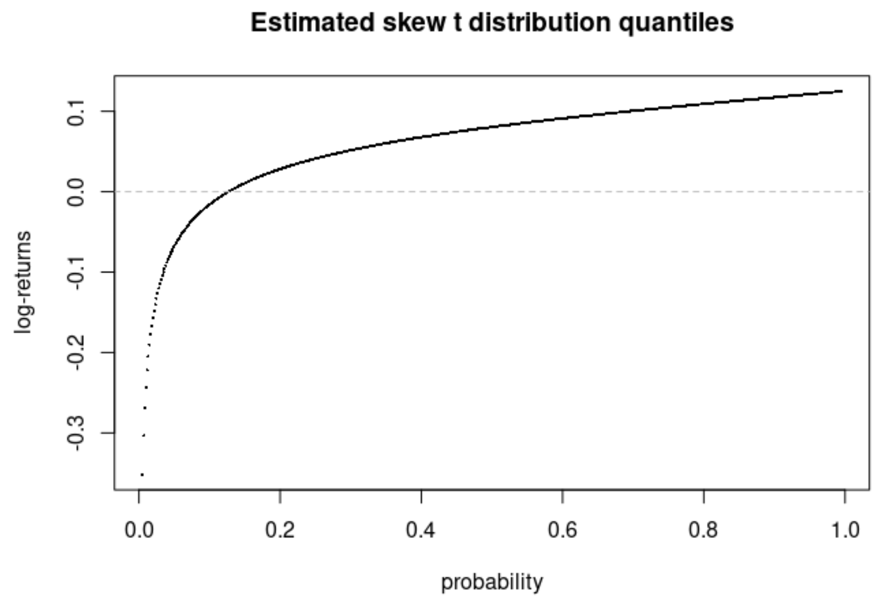
Let’s plot the fit and the observed returns together.



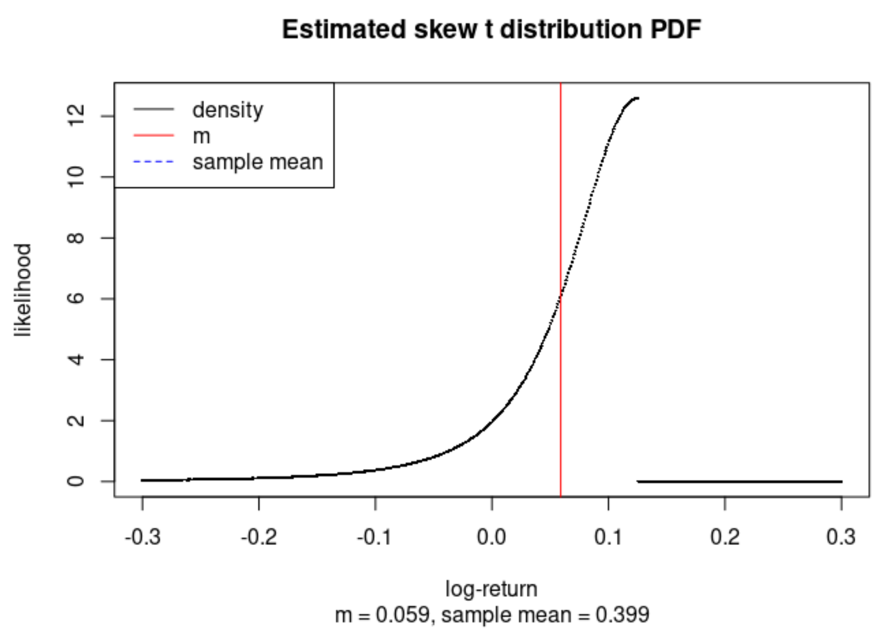
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



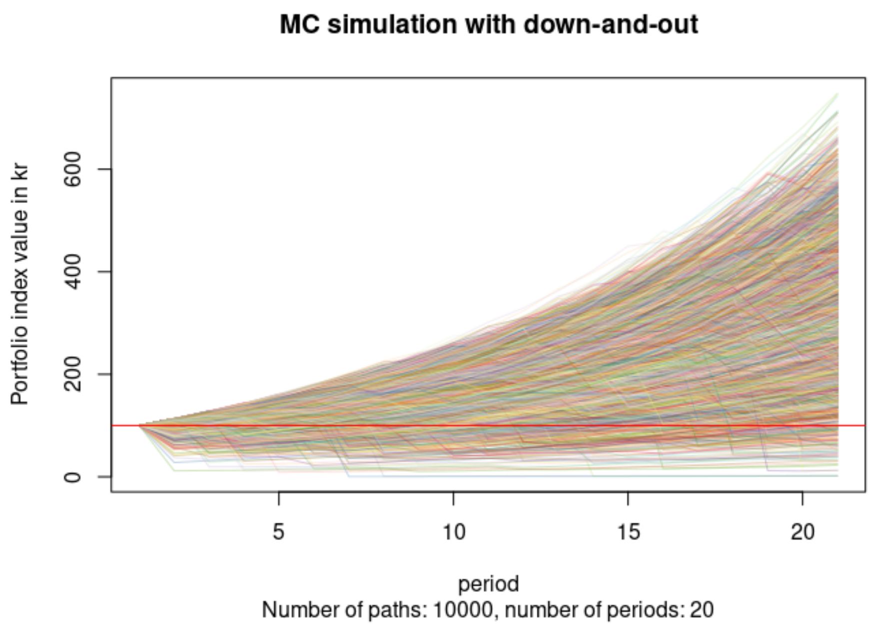


Estimated skew t distribution quantiles
log-returns
0.1
0.0
-0.1
-0.2
-0.3
0.0
0.2
0.4
0.6
0.8
1.0
probability

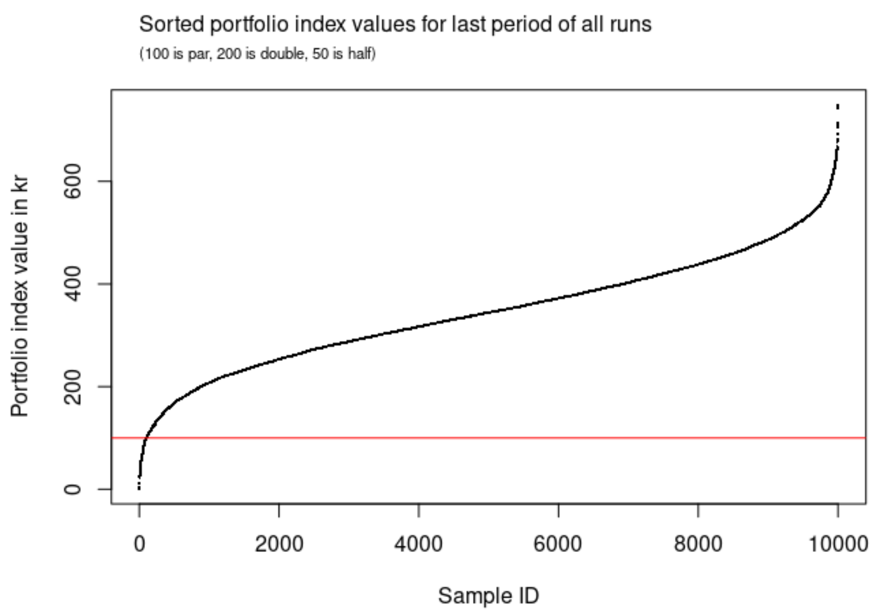


Estimated skew t distribution PDF
density
m
sample mean
likelihood
12
10
8
6
4
2
0
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
log-return
m = 0.059, sample mean = 0.399

Monte Carlo



MC simulation with down-and-out
Portfolio index value in kr
0
200
400
600
5
10
15
20
period
Number of paths: 10000, number of periods: 20

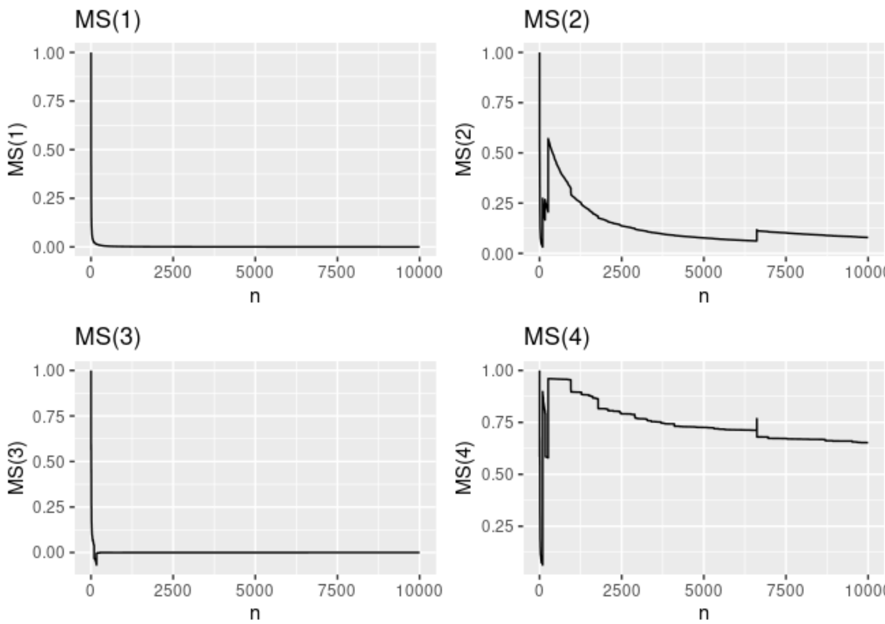


Sorted portfolio index values for last period of all runs
(100 is par, 200 is double, 50 is half)
Portfolio index value in kr
0
200
400
600
0
2000
4000
6000
8000
10000
Sample ID

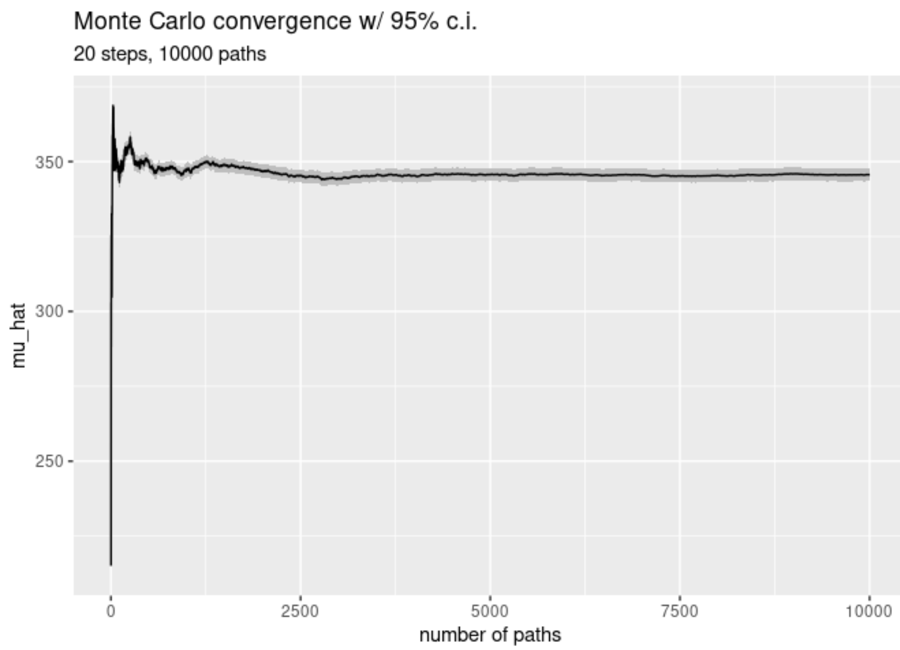
Convergence

Max vs sum

Max vs sum plots for the first four moments:



MC

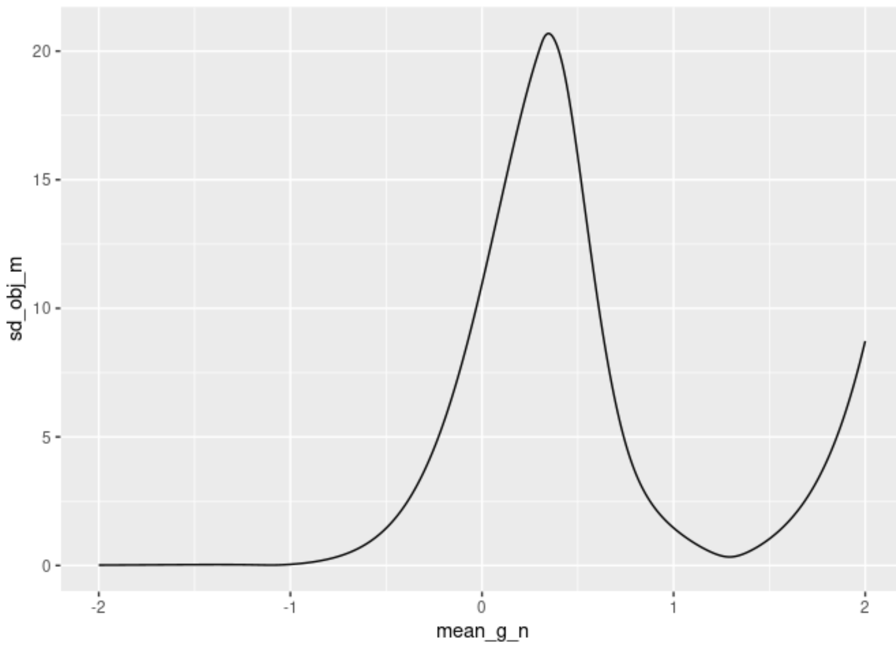


IS

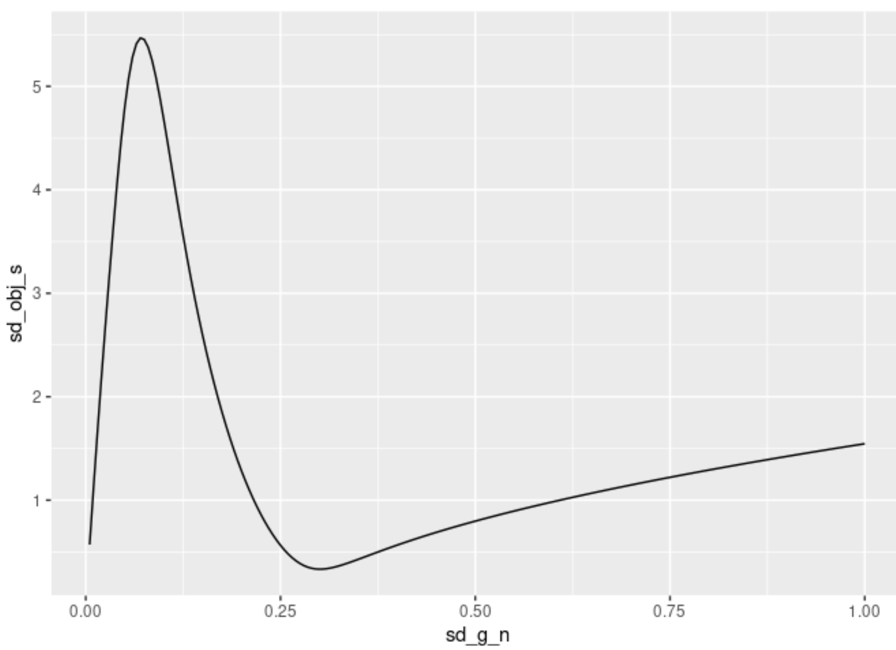
Parameters

```
## [1] 1.2921756 0.3005586
```

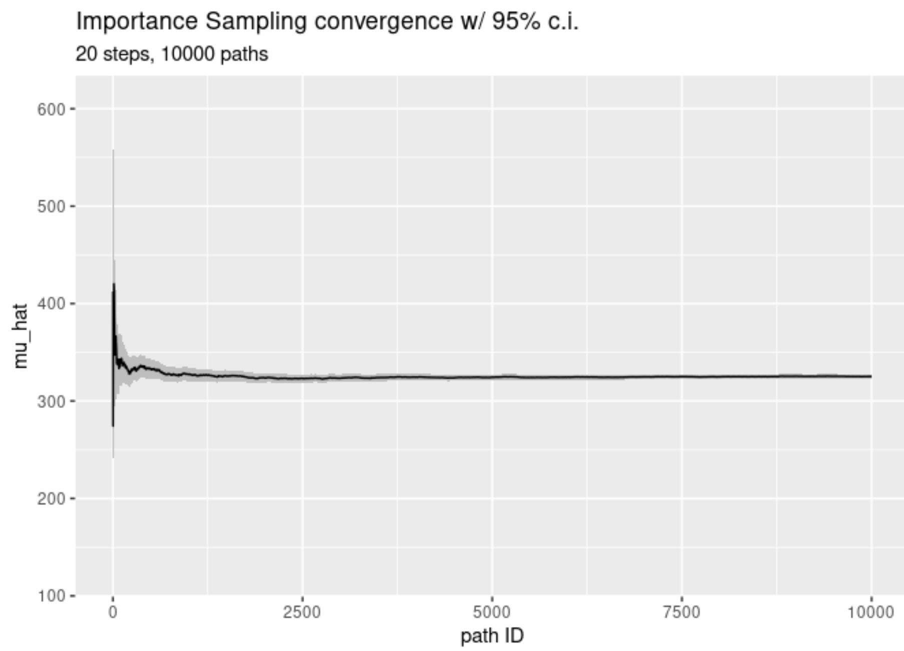
Objective function plots



20
15
10
5
0
sd_obj_m
-2
-1
0
1
2
mean_g_n

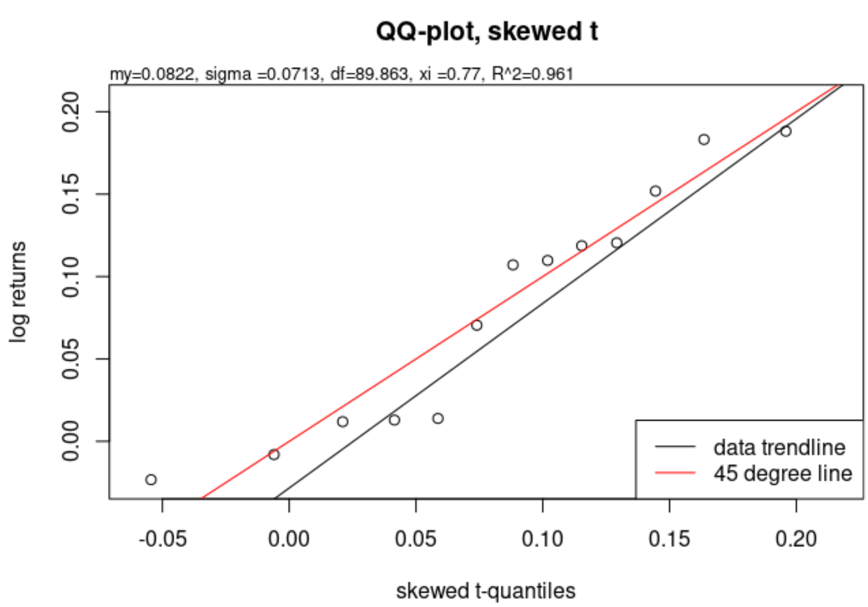


5
4
3
2
1
sd_obj_s
0.00
0.25
0.50
0.75
1.00
sd_g_n



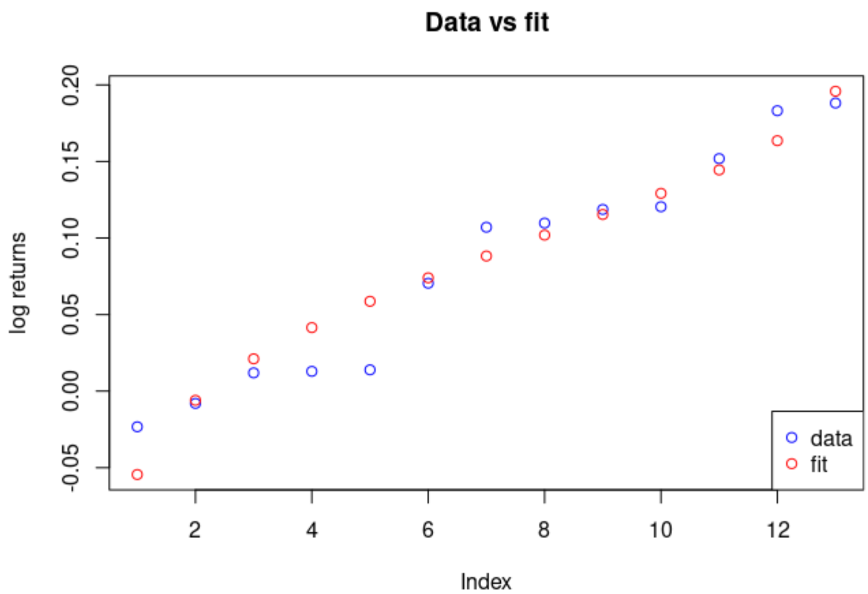
Mix medium risk (mmr), 2011 - 2023

QQ Plot



Data vs fit

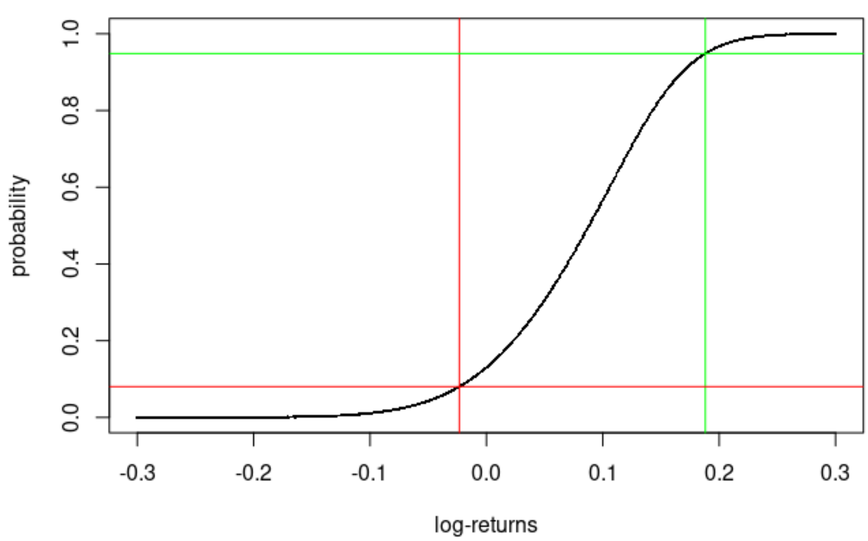
Let's plot the fit and the observed returns together.



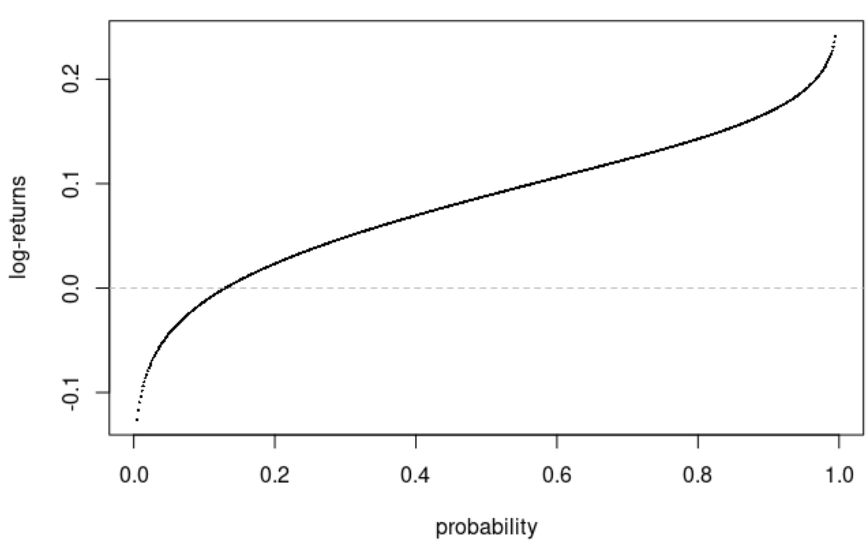
Estimated distribution

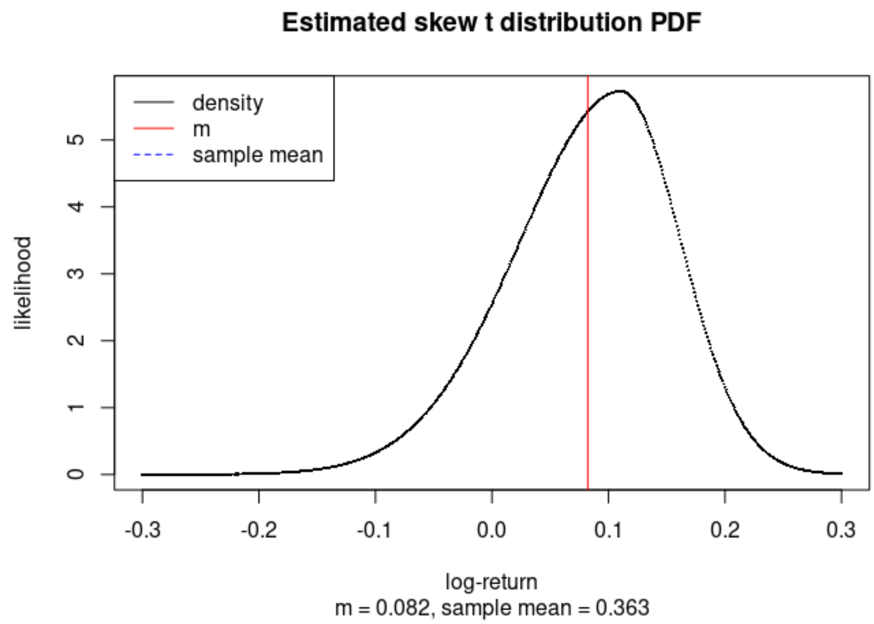
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

Estimated skew t distribution CDF

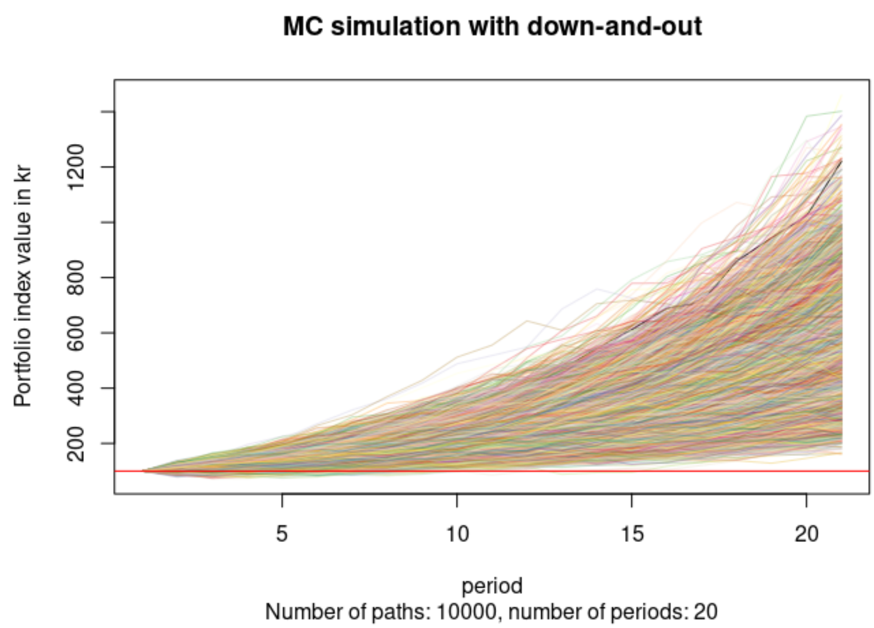


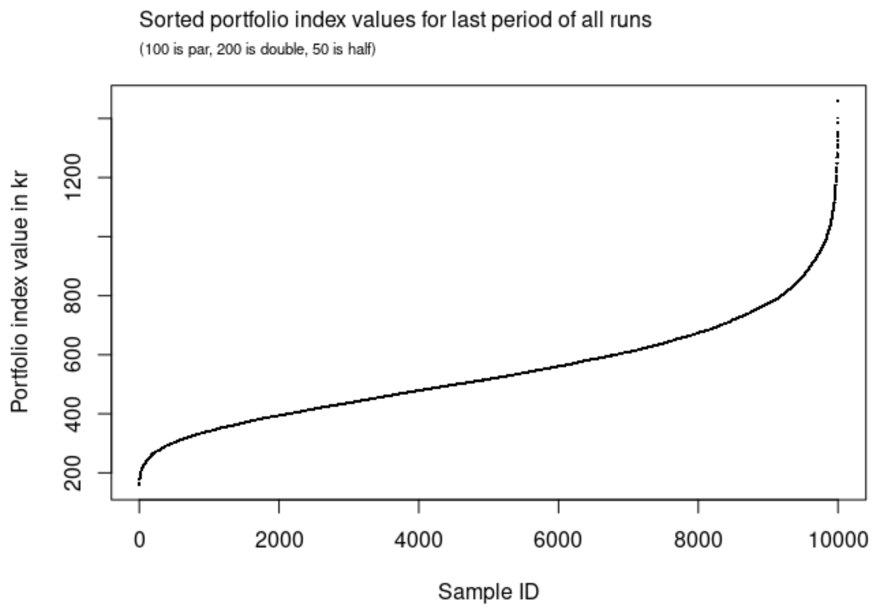
Estimated skew t distribution quantiles




Estimated skew t distribution PDF
density
m
sample mean
likelihood
log-return
m = 0.082, sample mean = 0.363

Monte Carlo

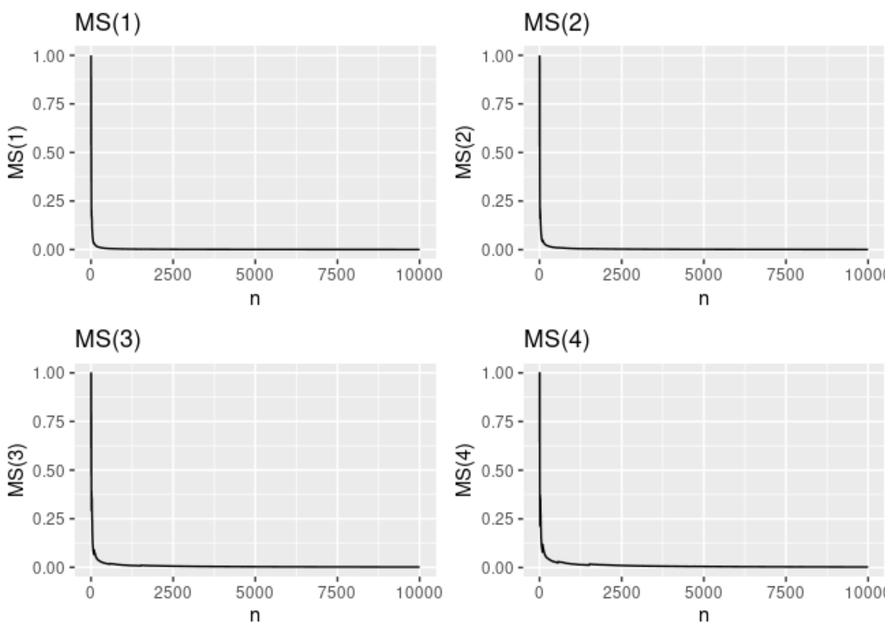

MC simulation with down-and-out
Portfolio index value in kr
period
Number of paths: 10000, number of periods: 20



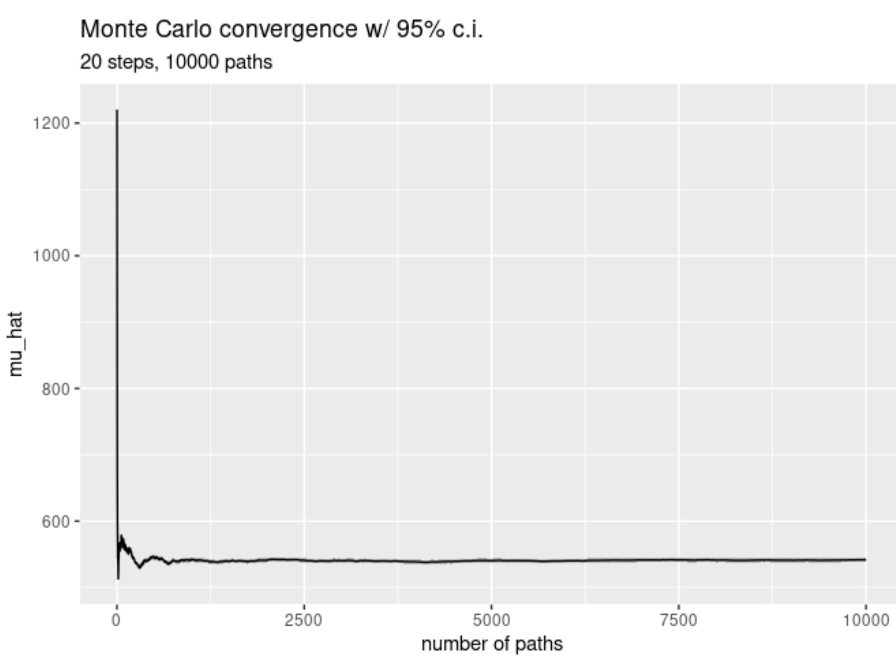
Convergence

Max vs sum

Max vs sum plots for the first four moments:


MS(1)
MS(2)
MS(3)
MS(4)
1.00
0.75
0.50
0.25
0.00
0
2500
5000
7500
10000
n

MC

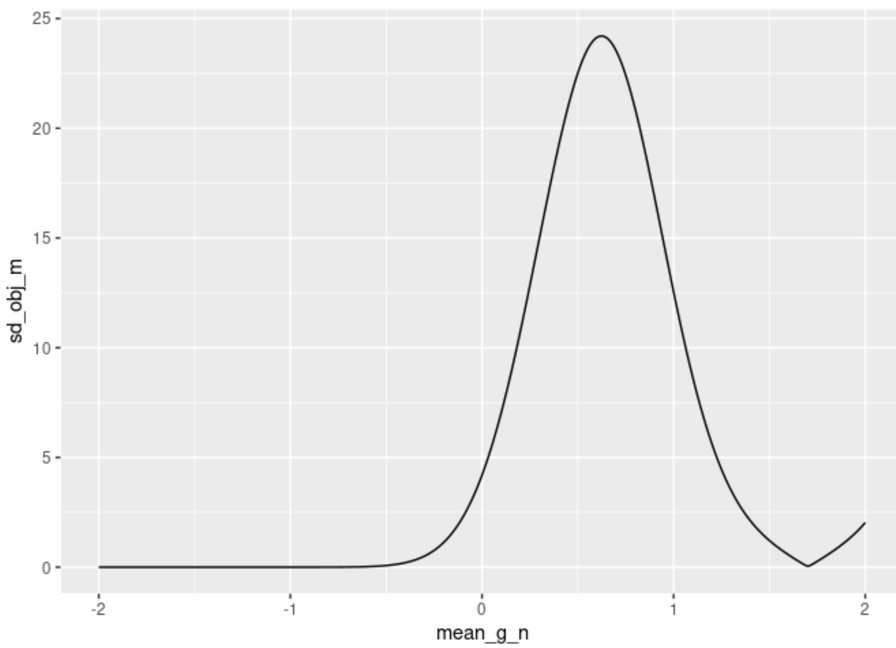

Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
1200
1000
800
600
0
2500
5000
7500
10000
number of paths

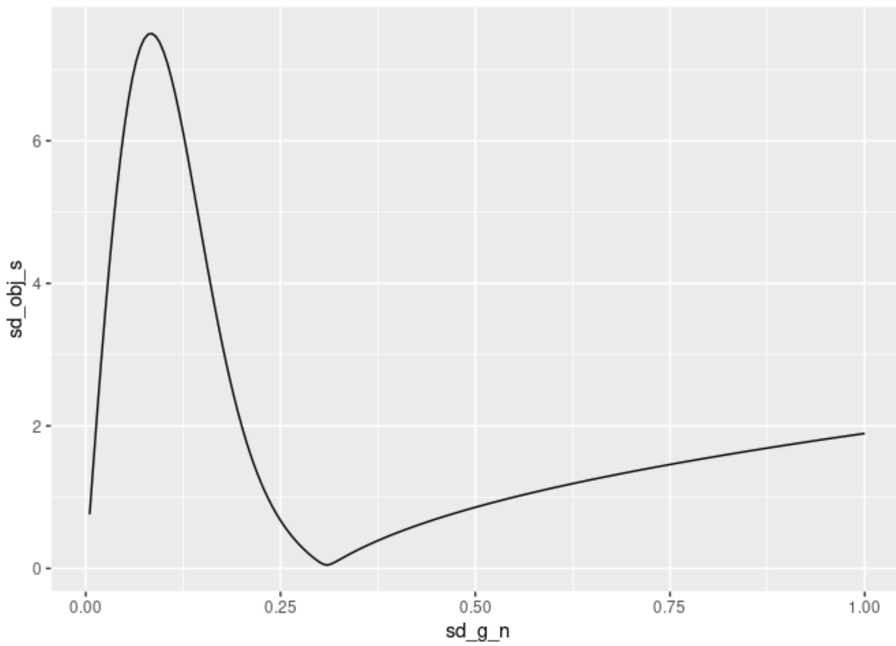
IS

Parameters

```
## [1] 1.7011721 0.3095095
```

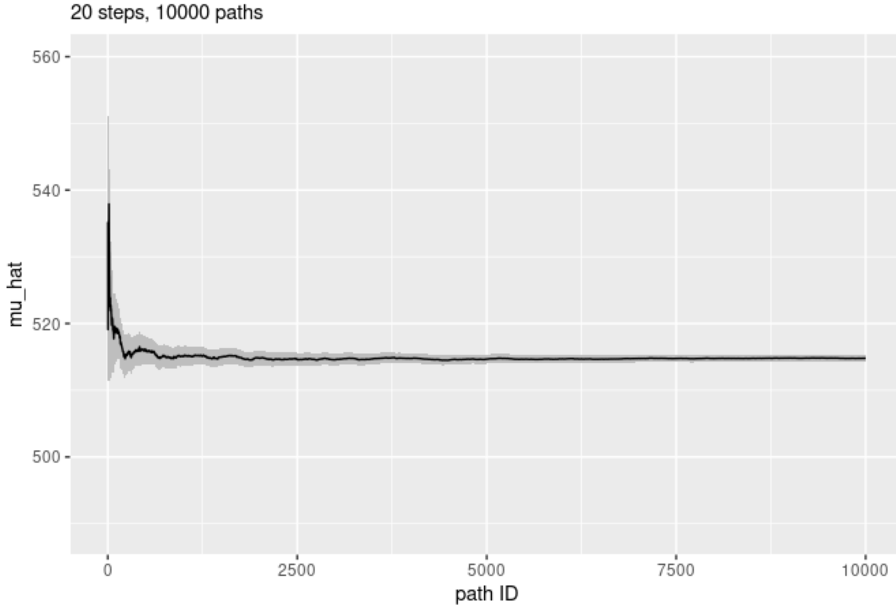
Objective function plots





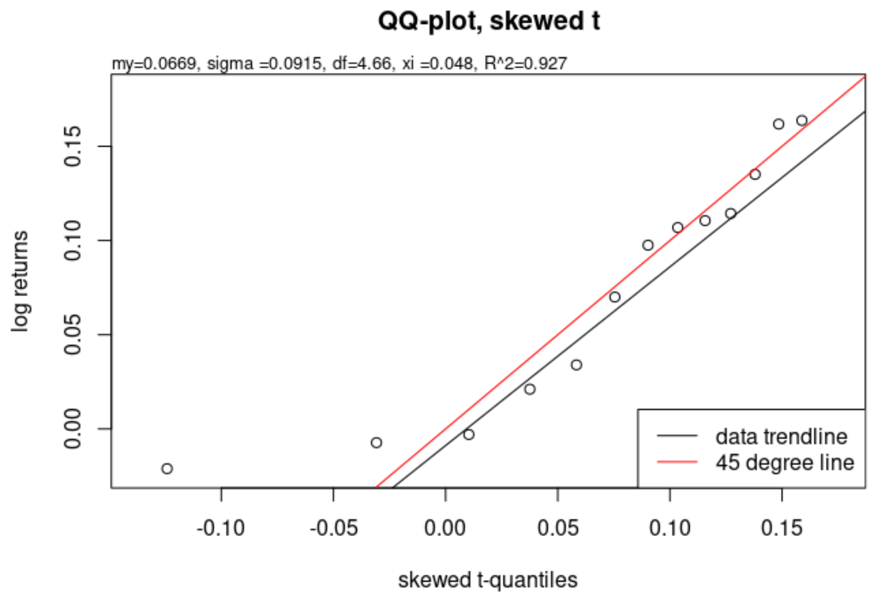
Importance Sampling convergence w/ 95% c.i.

20 steps, 10000 paths



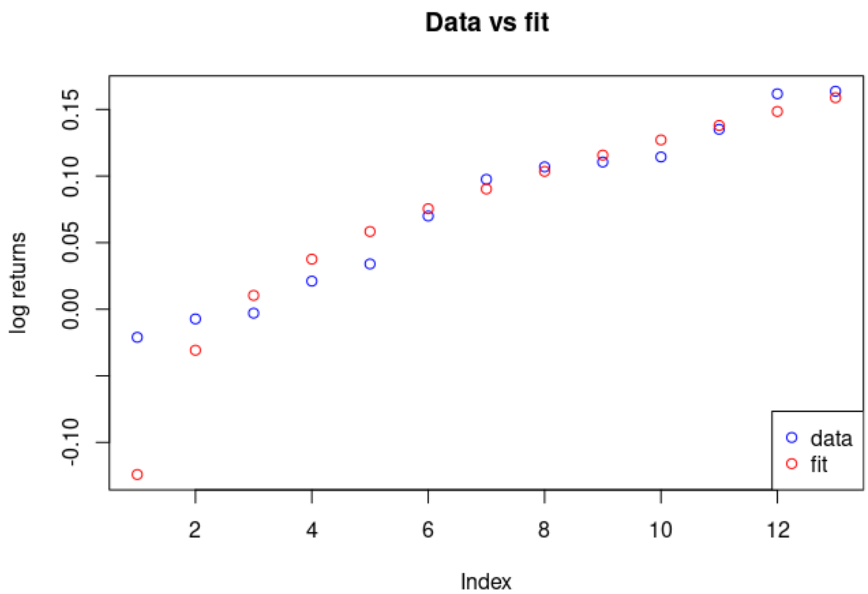
Mix high risk (mhr), 2011 - 2023

QQ Plot



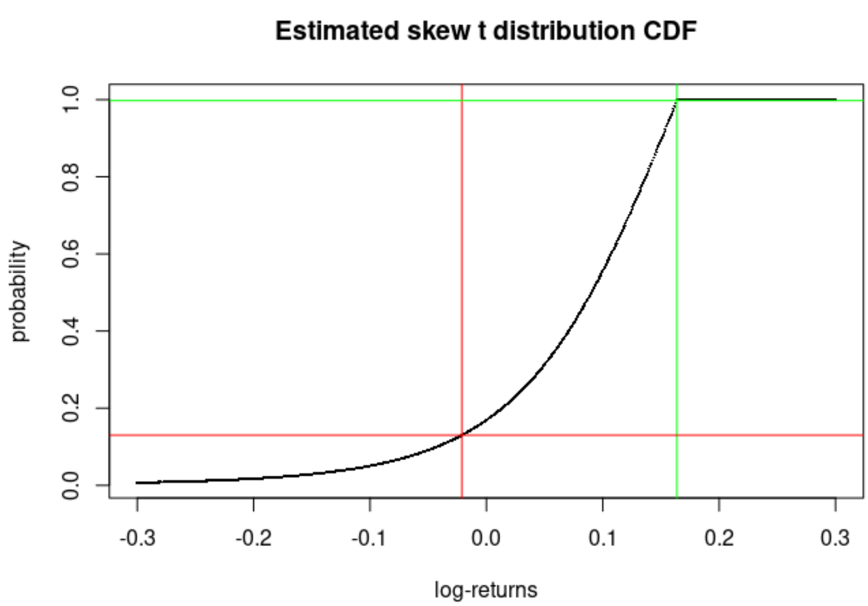
Data vs fit

Let's plot the fit and the observed returns together.

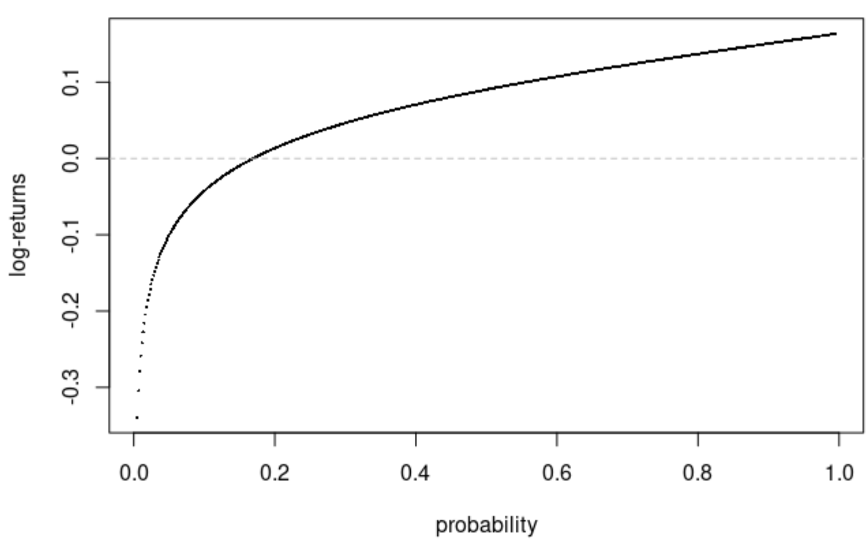


Estimated distribution

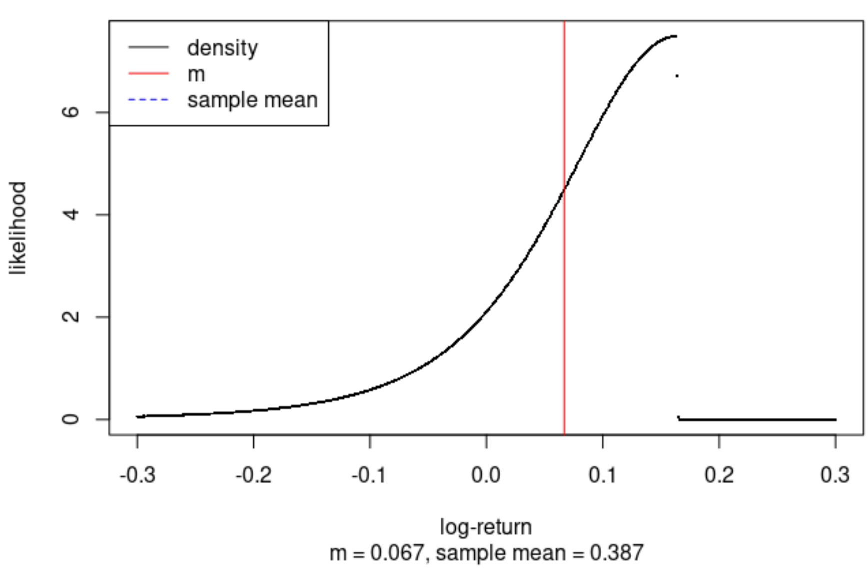
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



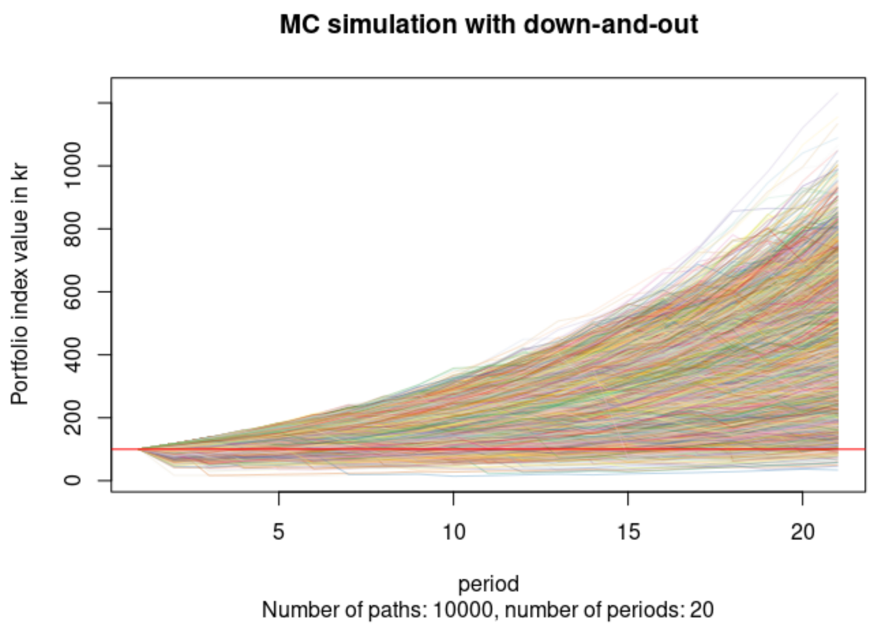
Estimated skew t distribution quantiles

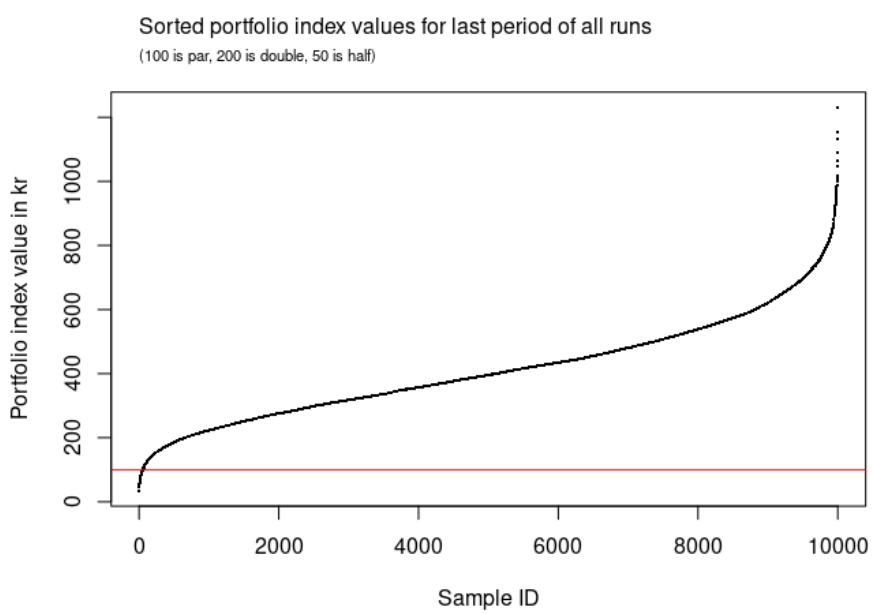


Estimated skew t distribution PDF



Monte Carlo

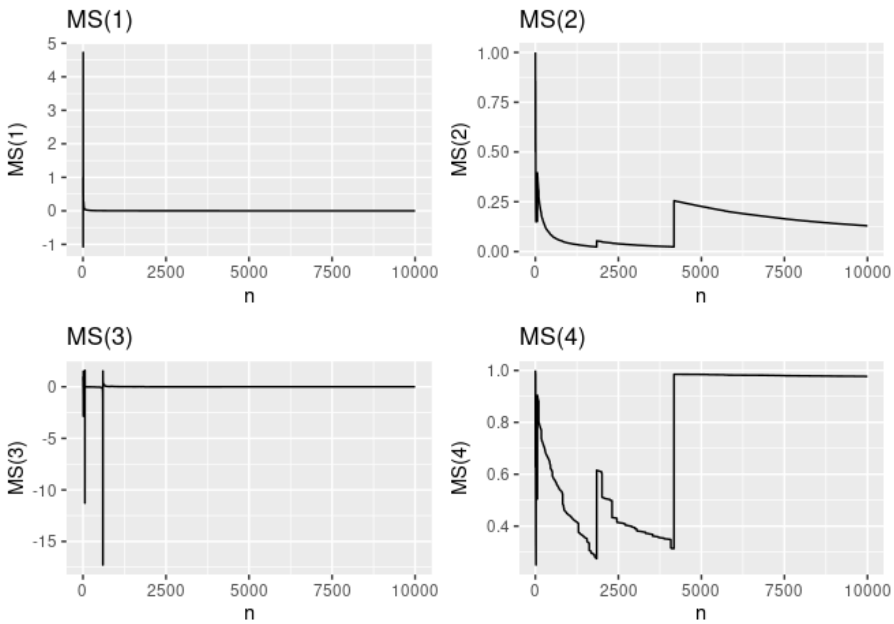

MC simulation with down-and-out
Portfolio index value in kr
0
200
400
600
800
1000
5
10
15
20
period
Number of paths: 10000, number of periods: 20


Sorted portfolio index values for last period of all runs
(100 is par, 200 is double, 50 is half)
Portfolio index value in kr
0
200
400
600
800
1000
0
2000
4000
6000
8000
10000
Sample ID

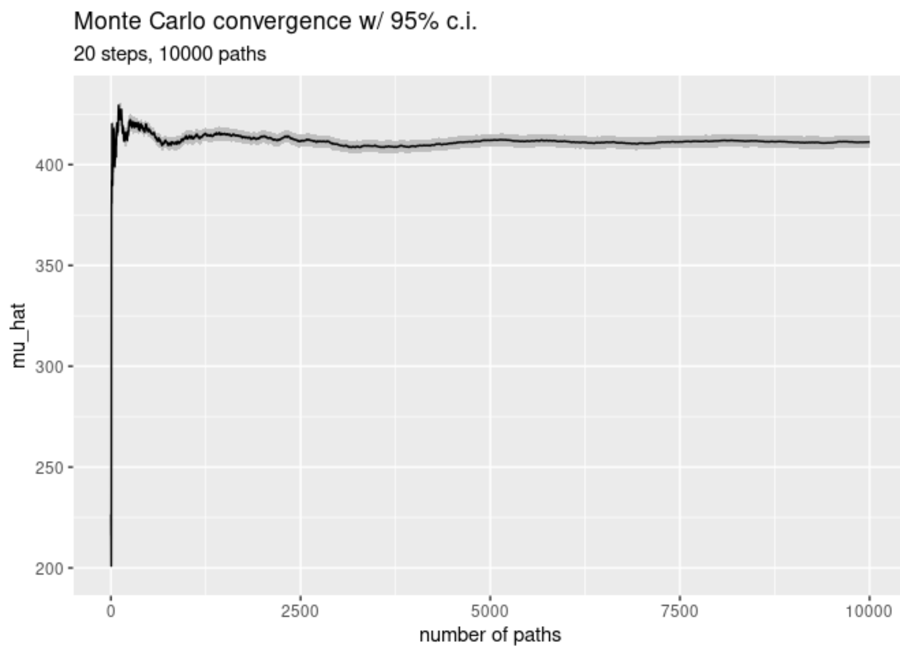
Convergence

Max vs sum

Max vs sum plots for the first four moments:



MC

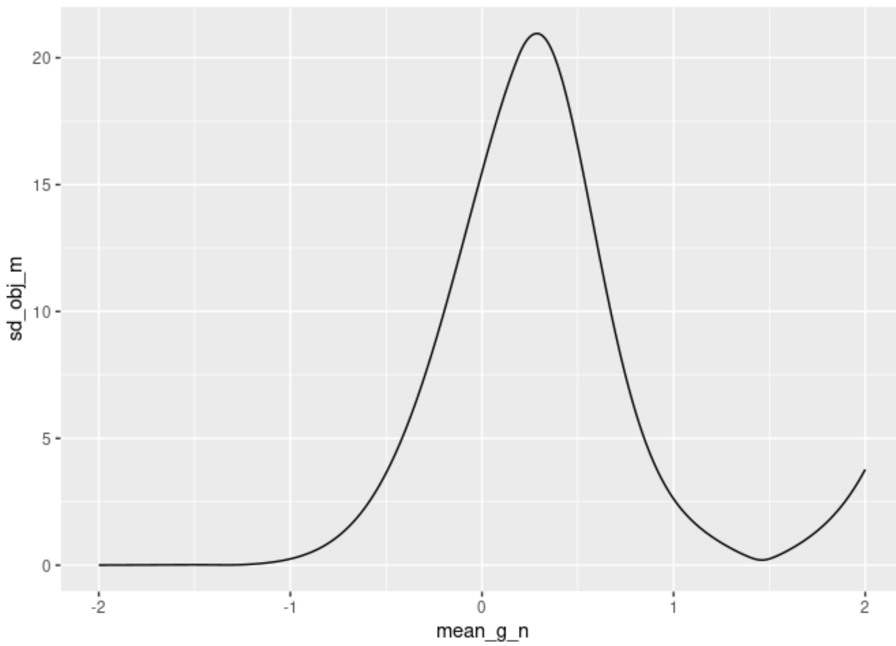


IS

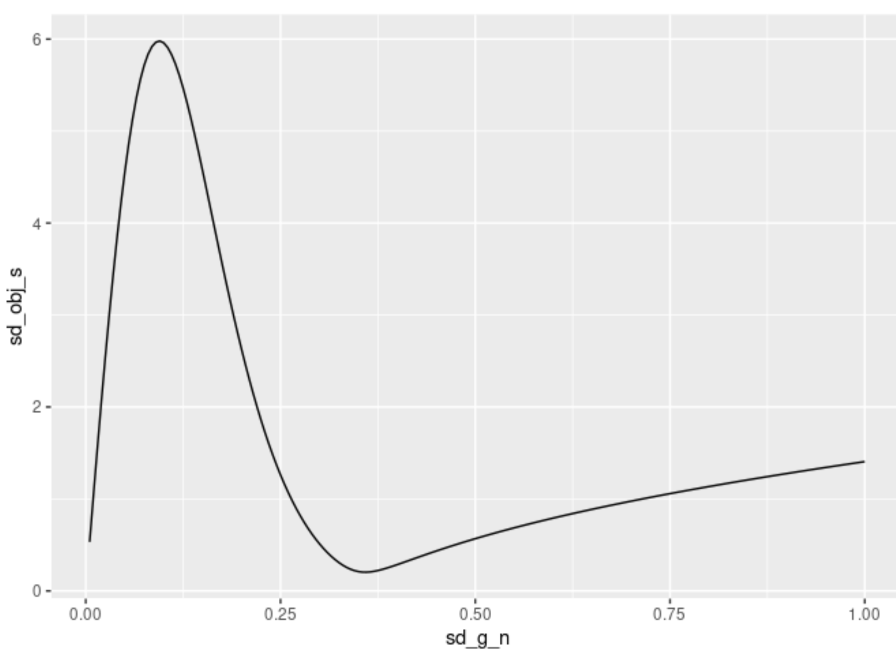
Parameters

```
## [1] 1.4606802 0.3586853
```

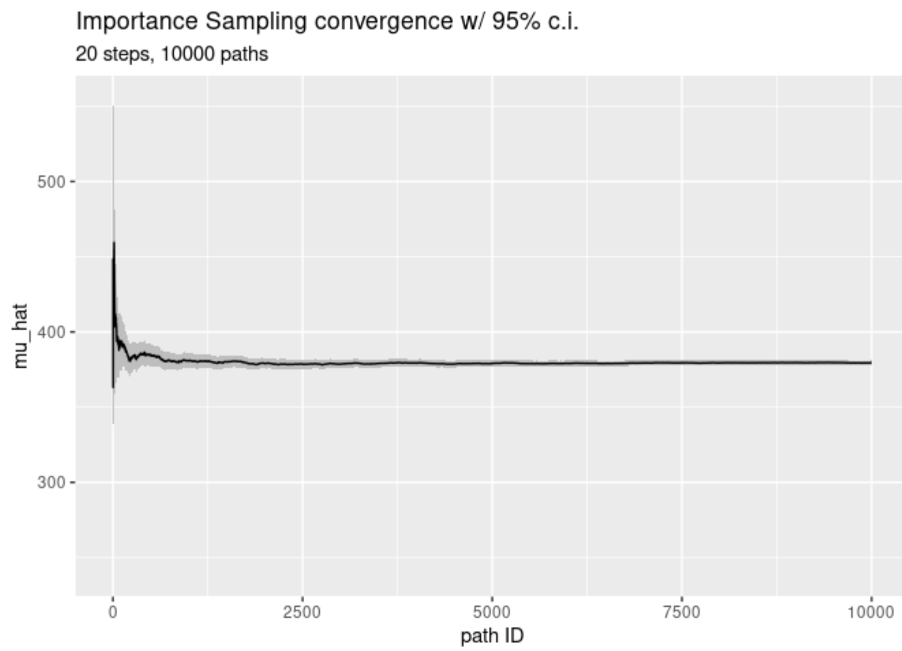
Objective function plots



sd_obj_m
mean_g_n

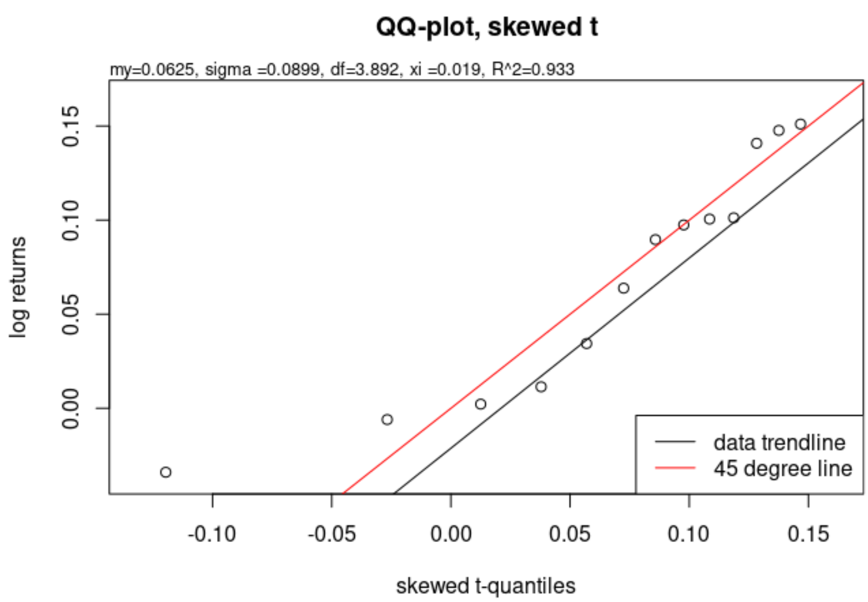


sd_obj_s
sd_g_n



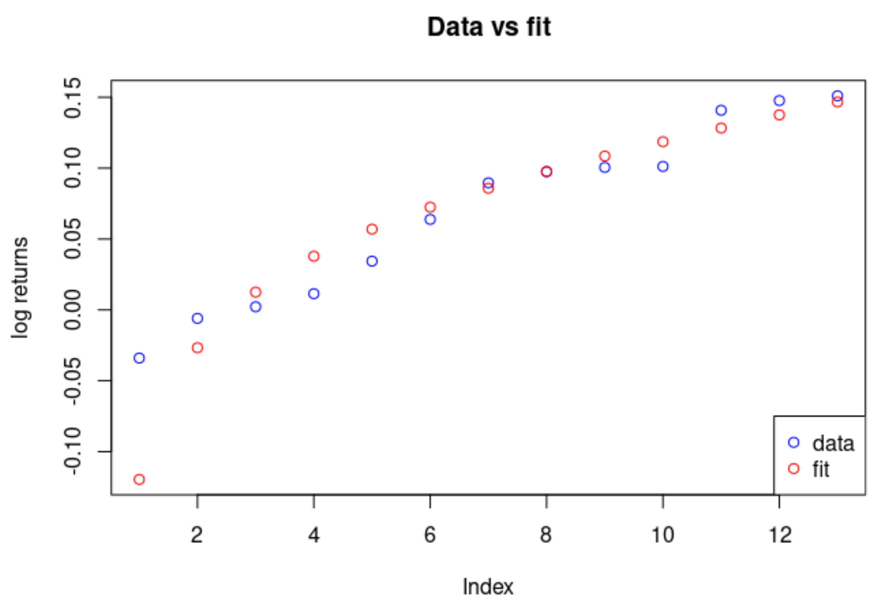
Mix vmr+phr (vm_ph), 2011 - 2023

QQ Plot



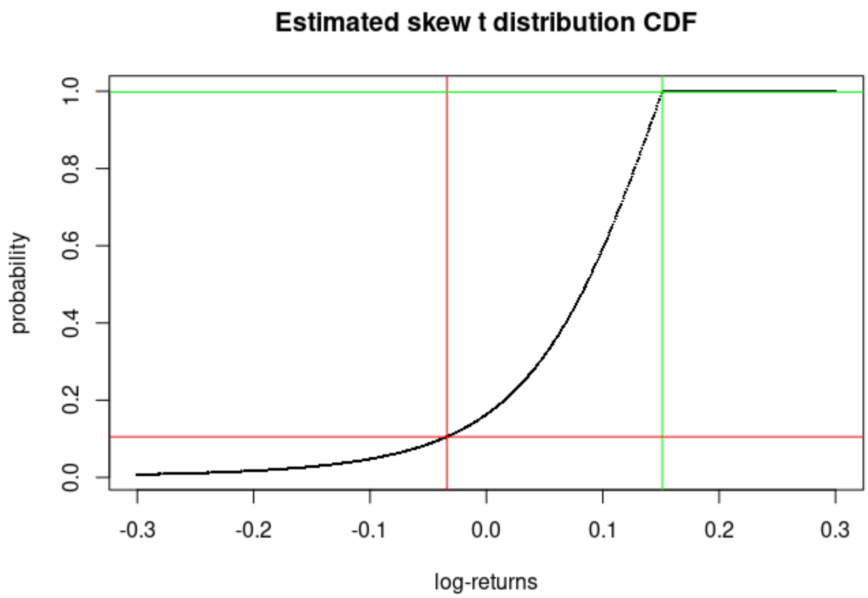
Data vs fit

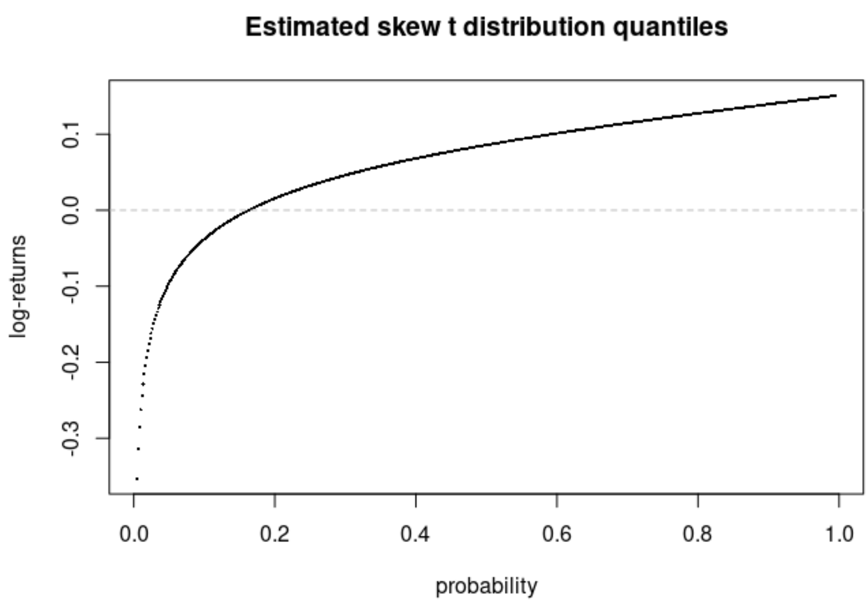
Let's plot the fit and the observed returns together.

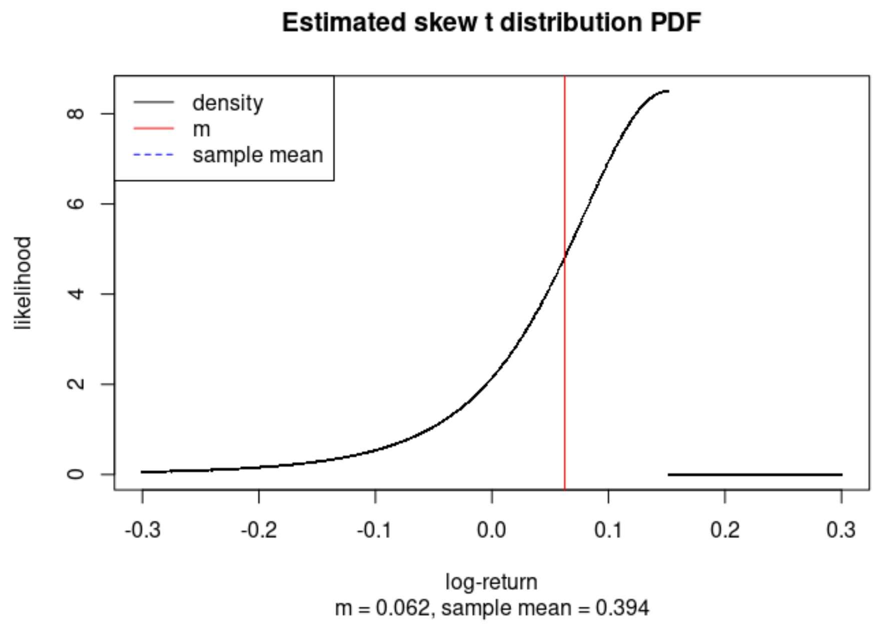


Estimated distribution

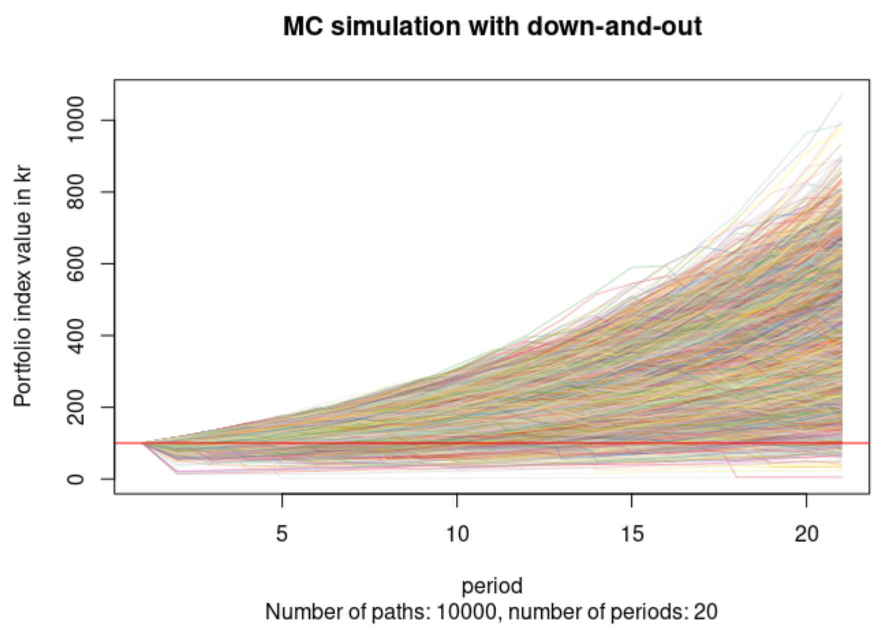
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

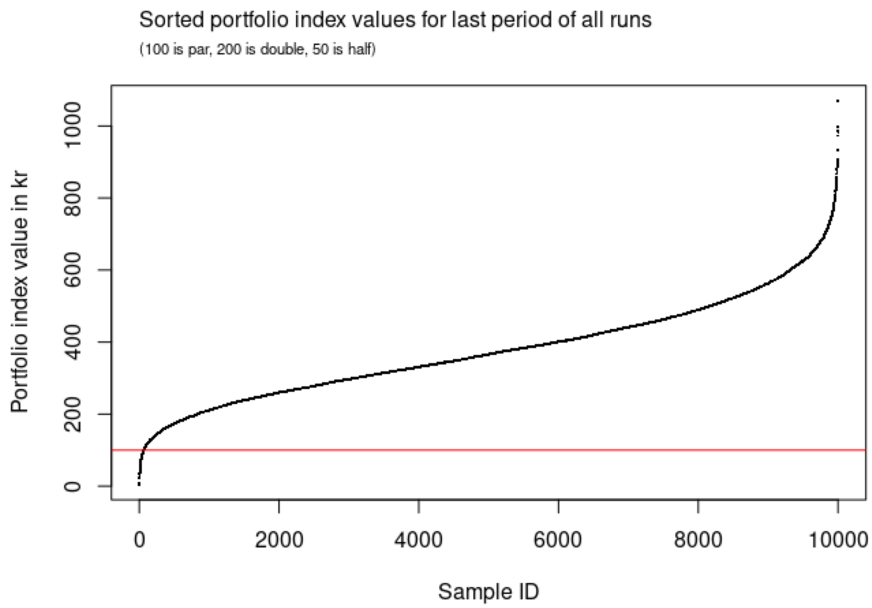

Estimated skew t distribution CDF
probability
log-returns
0.0
0.2
0.4
0.6
0.8
1.0
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3


Estimated skew t distribution quantiles
log-returns
probability
-0.3
-0.2
-0.1
0.0
0.1
0.0
0.2
0.4
0.6
0.8
1.0


Estimated skew t distribution PDF
density
m
sample mean
likelihood
8
6
4
2
0
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
log-return
m = 0.062, sample mean = 0.394

Monte Carlo

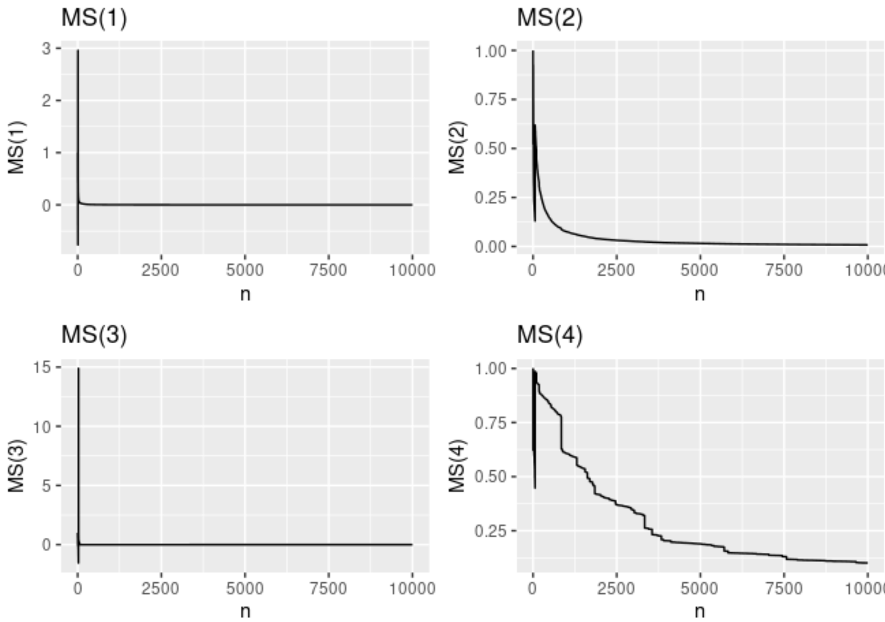

MC simulation with down-and-out
Portfolio index value in kr
1000
800
600
400
200
0
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Convergence

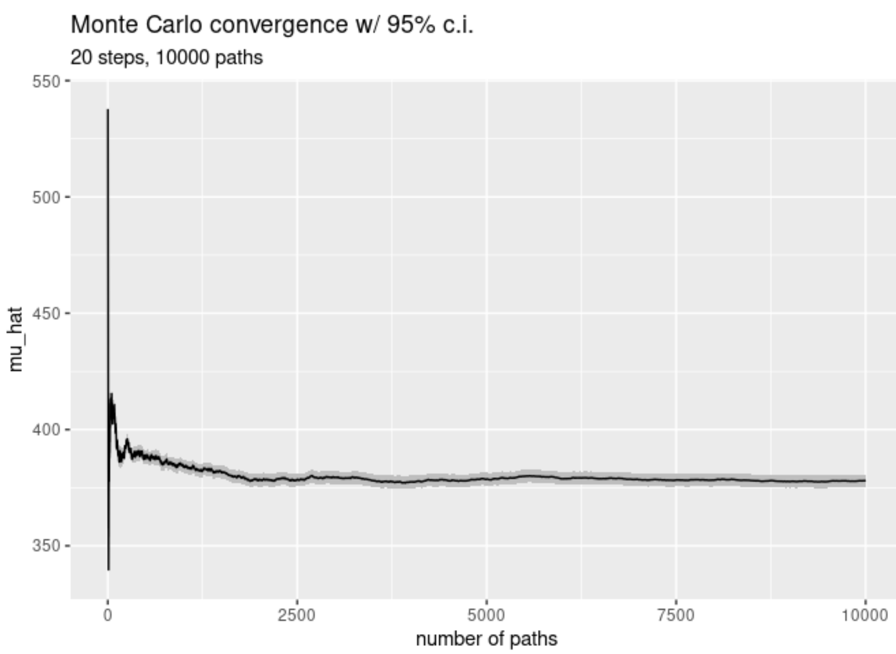
Max vs sum

Max vs sum plots for the first four moments:



MS(1)
MS(2)
MS(3)
MS(4)
n

MC



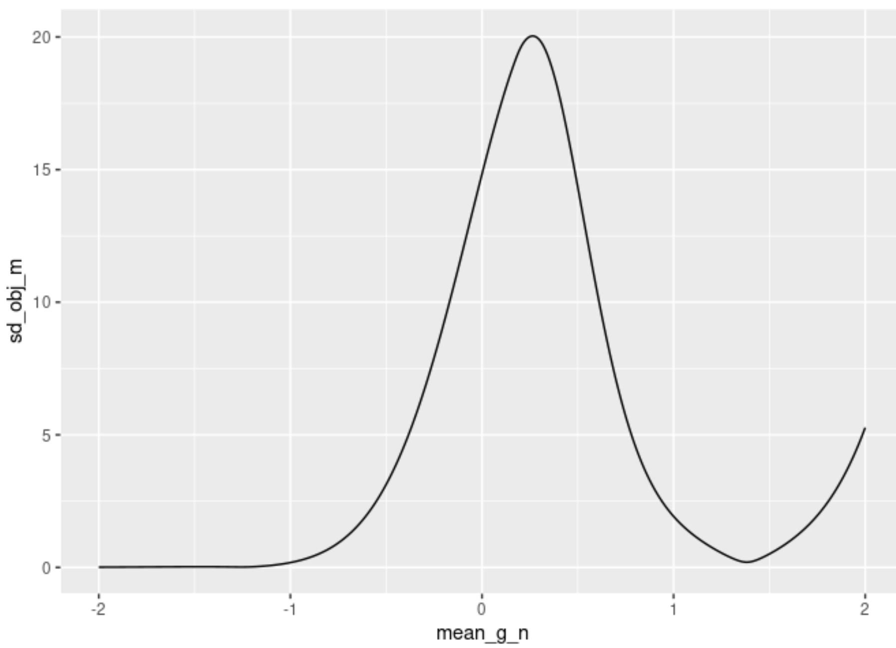
Monte Carlo convergence w/ 95% c.i.
20 steps, 10000 paths
mu_hat
number of paths

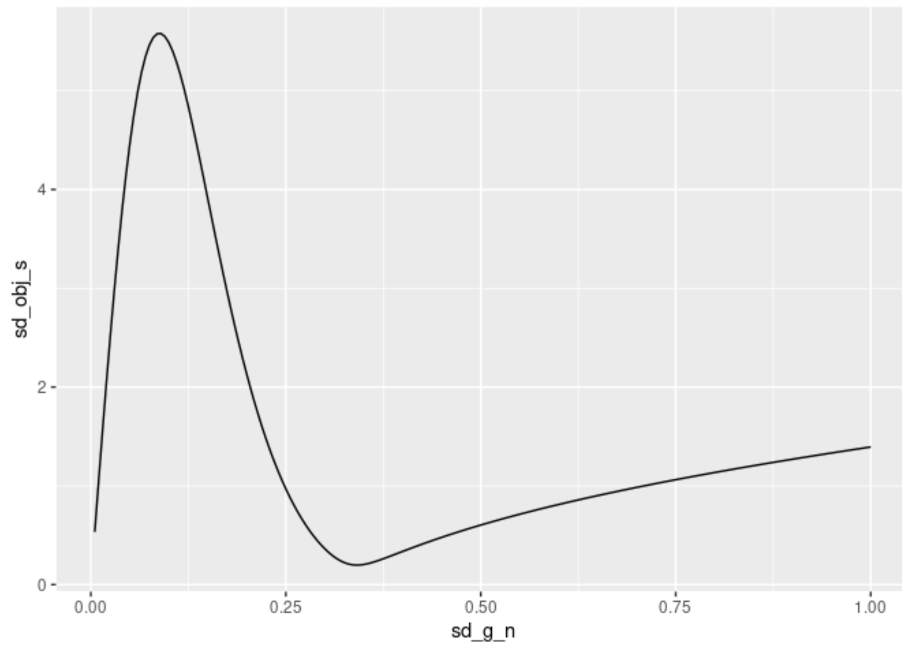
IS

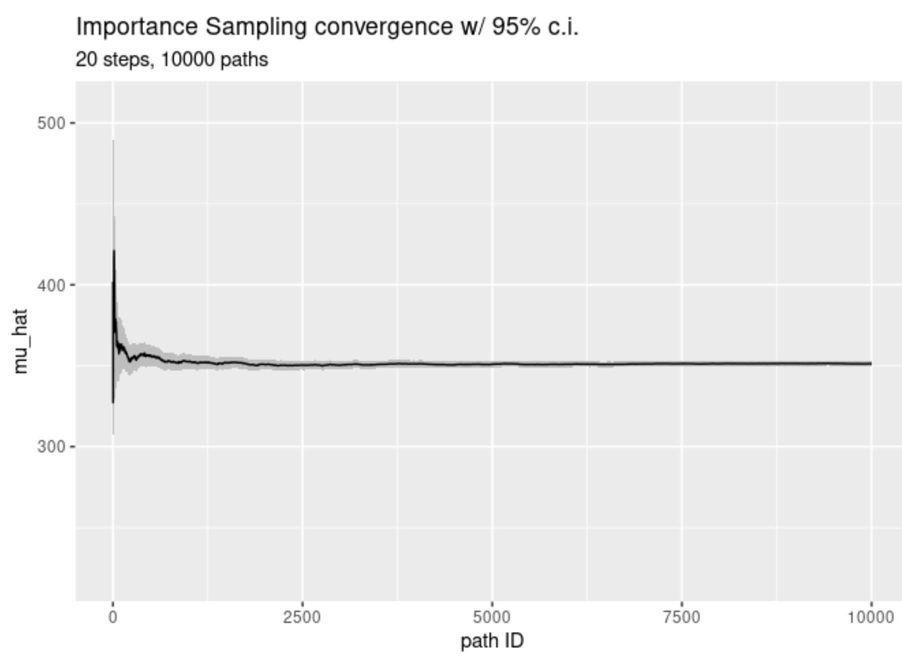
Parameters

```
## [1] 1.3815768 0.3410388
```

Objective function plots







Comments

(Ignoring `mhr_a`...)

`mhr` has some nice properties:
- It has a relatively high `nu` value of 90, which means it is tending more towards exponential tails than polynomial tails. All other funds have `nu` values close to 3, except `phr` which is even worse at close to 2. (Note that for a Gaussian, `nu` is infinite.)
- It has the lowest losing percentage of all simulations, which is better than 1/6 that of `phr`.
- It has a DAO percentage of 0, which is the same as `mmr`, and less than `phr`.
- Only `phr` has a higher `mc_m`.
- It has a smaller `mc_s` than the individual components, `vhr` and `phr`.
- It has the highest `xi` of all fits, suggesting less left skewness. Density plots for `vmr`, `phr` and `mmr` have an extremely sharp drop, as if an upward limiter has been applied, which corresponds to extremely low `xi` values. The density plot for `mhr` is by far the most symmetrical of all the fits. As seen in the section "Compare Gaussian and skewed t-distribution fits", the other skewed t-distribution fits don't capture the max observed returns at all.
- Only `mmr` has as higher `mc_min`. However, that of `mmr` is 18 times higher with 62, so `mmr` is a clear winner here.
- Naturally, it has a `mc_max` smaller than the individual components, `vhr` and `phr`, but ca. 1.5 times higher then `mmr`.
- All the first 4 moments converge nicely. For all other fits, the 4th moment doesn't seem to converge.

Taleb, Statistical Consequences Of Fat Tails, p. 97:
"the variance of a finite variance random variable with tail exponent < 4 will be infinite".

And p. 363:
"The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, un- der a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance."

- Importance Sampling seems to converge to a lower level than Monte Carlo does. Is that because IS catches more observations in the lower tail? Supporting this thesis is that MC for `mhr` with `1e4` paths gives a mean of 520, while `1e6` paths gives a mean of 478 (see under "Many simulations").

- Note: QQ lines by design pass through 1st and 3rd quantiles. They are not trendlines in the sense of linear regression.

Appendix

Many simulations of mc_mhr_b

1e6 paths:

```
# Down-and-out simulation:
# Probability of down-and-out: 0 percent
#
# Mean portfolio index value after 20 years: 478.339 kr.
# SD of portfolio index value after 20 years: 163.093 kr.
# Min total portfolio index value after 20 years: 2.233 kr.
# Max total portfolio index value after 20 years: 1561.965 kr.
#
# Share of paths finishing below 100: 0.1181 percent
```

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2}\right) \bigg/ \left(\frac{x_{t-1} + y_{t-1}}{2}\right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns $=$ Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2\frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1}y_t = 2(x_{t-1}y_{t-1}x_t + x_{t-1}y_{t-1}y_t)$$

$$(x_{t-1}x_1y_{t-1} + y_{t-1}x_ty_{t-1}) + (x_{t-1}x_{t-1}y_t + x_{t-1}y_{t-1}y_t) = 2(x_{t-1}y_{t-1}x_t + x_{t-1}y_{t-1}y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: `R = 1+r`

```
## Let x_0 be 100.
```

```
## Let y_0 be 200.
```

```
## So the initial value of the pf is 300 .
```

```
## Let R_x be 0.5.
```

```
## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.
```

```
## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): 0.02798047
## s(data_x): 0.4736139
## m(data_y): 10.00934
## s(data_y): 3.435938
##
## m(data_x + data_y): 5.018661
## s(data_x + data_y): 1.861301
```

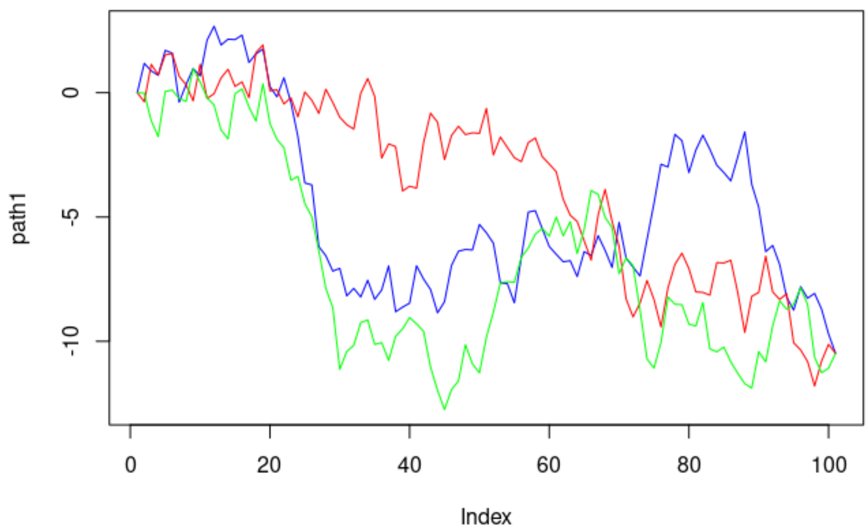
m and s of final state of all paths.
`_a` is mix of simulated returns.
`_b` is simulated mixed returns.

m_a	m_b	s_a	s_b
100.395	100.523	7.828	8.207
100.280	100.324	7.876	8.423
100.678	100.278	7.797	8.348
100.436	100.097	7.717	8.076
100.251	100.164	7.579	8.266
100.202	100.174	7.870	8.530
100.682	100.875	7.833	8.249
100.401	100.435	7.438	8.372
99.944	100.607	7.766	8.203
100.409	100.310	7.787	8.231

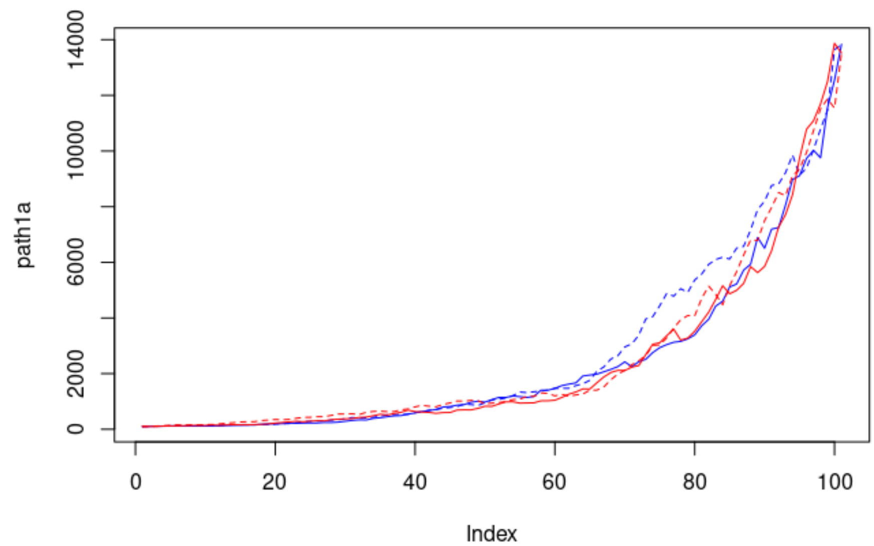
```
##       m_a              m_b              s_a              s_b
##  Min.   : 99.94   Min.   :100.1   Min.   :7.438   Min.   :8.076
##  1st Qu.:100.26   1st Qu.:100.2   1st Qu.:7.730   1st Qu.:8.213
##  Median :100.40   Median :100.3   Median :7.792   Median :8.258
##  Mean   :100.37   Mean   :100.4   Mean   :7.749   Mean   :8.291
##  3rd Qu.:100.43   3rd Qu.:100.5   3rd Qu.:7.831   3rd Qu.:8.366
##  Max.   :100.68   Max.   :100.9   Max.   :7.876   Max.   :8.530
```

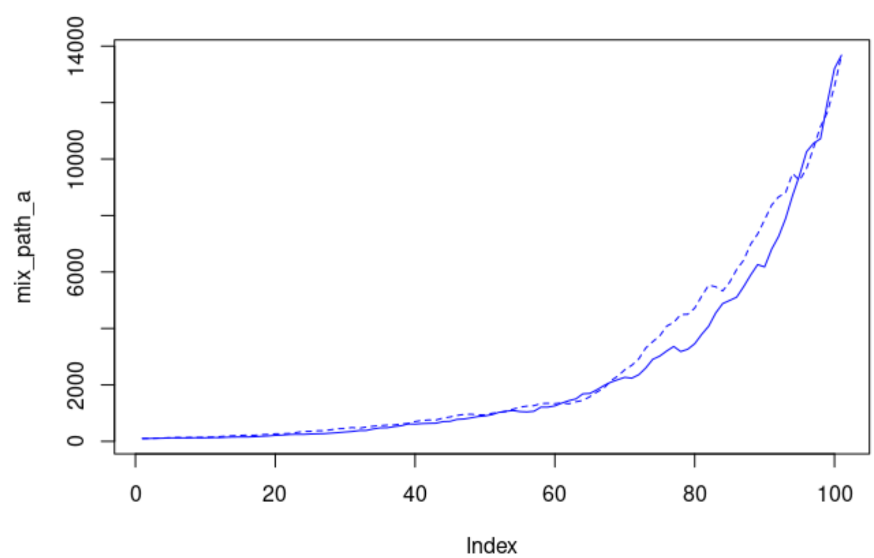
`_a` and `_b` are very close to equal.
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?





The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

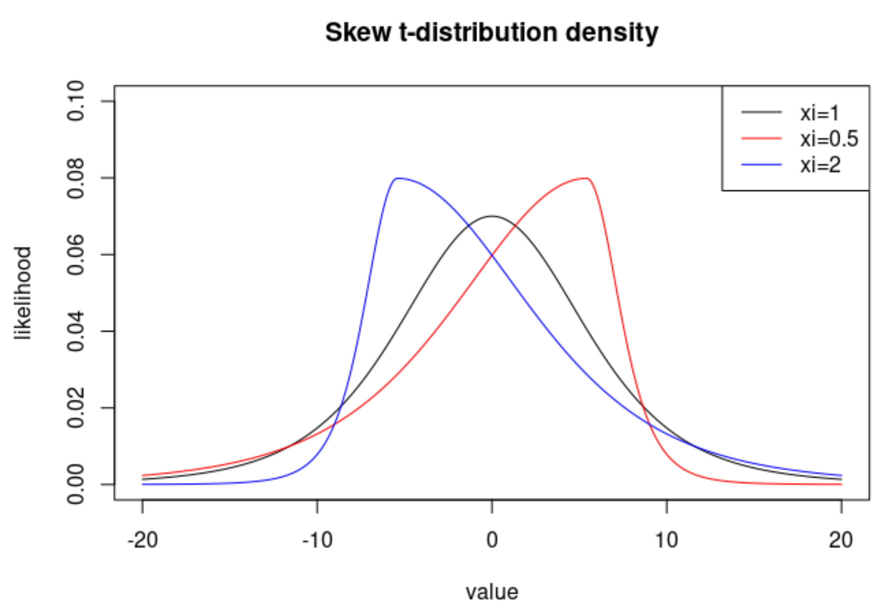
```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?
Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##        m                  s
##  Min.   :0.05943   Min.   :0.03653
##  1st Qu.:0.06521   1st Qu.:0.06151
##  Median :0.06913   Median :0.06847
##  Mean   :0.07044   Mean   :0.06724
##  3rd Qu.:0.07616   3rd Qu.:0.07457
##  Max.   :0.08687   Max.   :0.09149
```

The meaning of `xi`

The fit for `mhr` has the highest `xi` value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, ..., X^p)}{\sum_{i=1}^{n} X_i^p} \rightarrow 0$$

for $n \rightarrow \infty$.

If not, X doesn't have a p'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192