Pension returns analysis

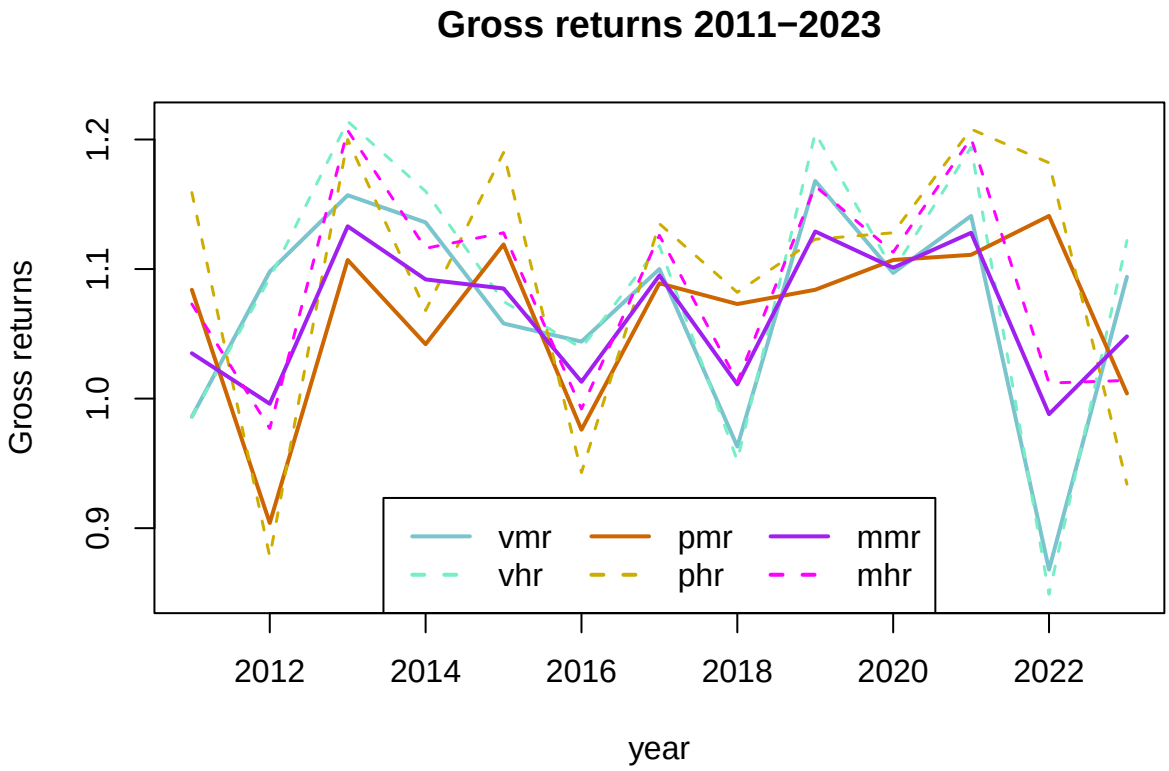
13:04 18 April 2024

Contents

Fit log returns to F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated shape parameter (degrees of freedom).
`xi` is the estimated skewness parameter.

Log returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.
`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.



Summary of gross returns

```
##       vmr              pmr              mmr              vhr        
##  Min.   :0.868   Min.   :0.904   Min.   :0.988   Min.   :0.849  
##  1st Qu.:1.044   1st Qu.:1.042   1st Qu.:1.013   1st Qu.:1.039  
##  Median :1.097   Median :1.084   Median :1.085   Median :1.099  
##  Mean   :1.070   Mean   :1.065   Mean   :1.066   Mean   :1.085  
##  3rd Qu.:1.136   3rd Qu.:1.107   3rd Qu.:1.101   3rd Qu.:1.160  
##  Max.   :1.168   Max.   :1.141   Max.   :1.133   Max.   :1.214  
##       phr              mhr        
##  Min.   :0.878   Min.   :0.977  
##  1st Qu.:1.068   1st Qu.:1.013  
##  Median :1.128   Median :1.113  
##  Mean   :1.095   Mean   :1.087  
##  3rd Qu.:1.182   3rd Qu.:1.128  
##  Max.   :1.208   Max.   :1.207
```

```
##       vmrl        
##  Min.   :0.801  
##  1st Qu.:1.013  
##  Median :1.085  
##  Mean   :1.061  
##  3rd Qu.:1.128  
##  Max.   :1.193
```

```
##            vmr   pmr   mmr   vhr   phr   mhr
## Min.   : 0.868 0.904 0.988 0.849 0.878 0.977
## 1st Qu.: 1.044 1.042 1.013 1.039 1.068 1.013
## Median : 1.097 1.084 1.085 1.099 1.128 1.113
## Mean   : 1.070 1.065 1.066 1.085 1.095 1.087
## 3rd Qu.: 1.136 1.107 1.101 1.160 1.182 1.128
## Max.   : 1.168 1.141 1.133 1.214 1.208 1.207
```

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.095	phr	1.182	phr	1.214	vhr
0.977	mhr	1.044	vmr	1.113	mhr	1.087	mhr	1.160	vhr	1.208	phr
0.904	pmr	1.042	pmr	1.099	vhr	1.085	vhr	1.136	vmr	1.207	mhr
0.878	phr	1.039	vhr	1.097	vmr	1.070	vmr	1.128	mhr	1.168	vmr
0.868	vmr	1.013	mmr	1.085	mmr	1.066	mmr	1.107	pmr	1.141	pmr
0.849	vhr	1.013	mhr	1.084	pmr	1.065	pmr	1.101	mmr	1.133	mmr

Covariance

```
## cov(vmr, pmr) =  -0.001094875
```

```
## cov(vhr, phr) =  -0.0001730651
```

Gaussian fits

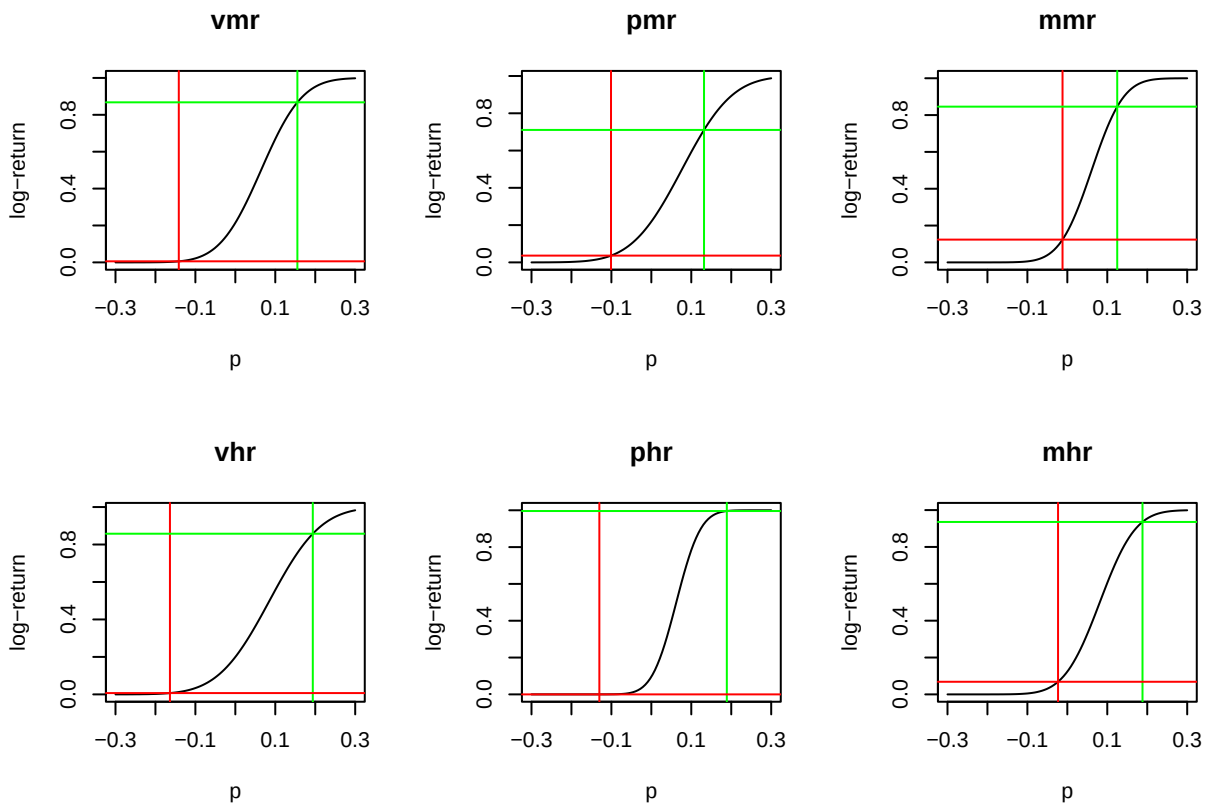
	vmr	vhr	pmr	phr	mmr	mhr
m	0.064	0.077	0.061	0.085	0.062	0.081
s	0.081	0.099	0.063	0.101	0.048	0.070

Probability in percent that the smallest and largest (respectively) observed return for each fund was generated by a normal distribution:

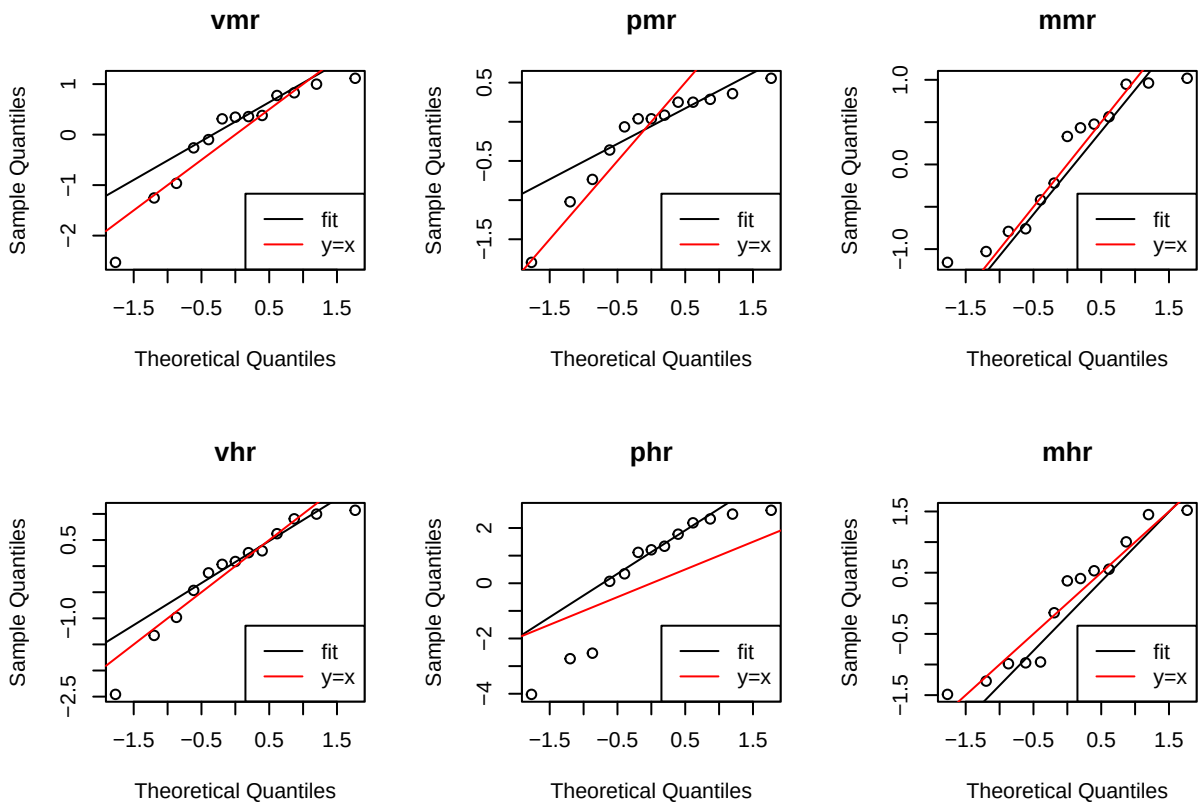
	vmr	vhr	pmr	phr	mmr	mhr
P(X_min)	0.754	0.982	0.68	2.055	6.729	7.645
P(X_max)	14.190	12.822	13.88	16.327	10.528	7.212

Average number of years between min or max events (respectively):

	vmr	vhr	pmr	phr	mmr	mhr
avg yrs btw min	132.699	101.874	146.958	48.659	14.860	13.080
avg yrs btw max	7.047	7.799	7.205	6.125	9.499	13.866



Gaussian QQ plots



Velliv medium risk, 2011 - 2023

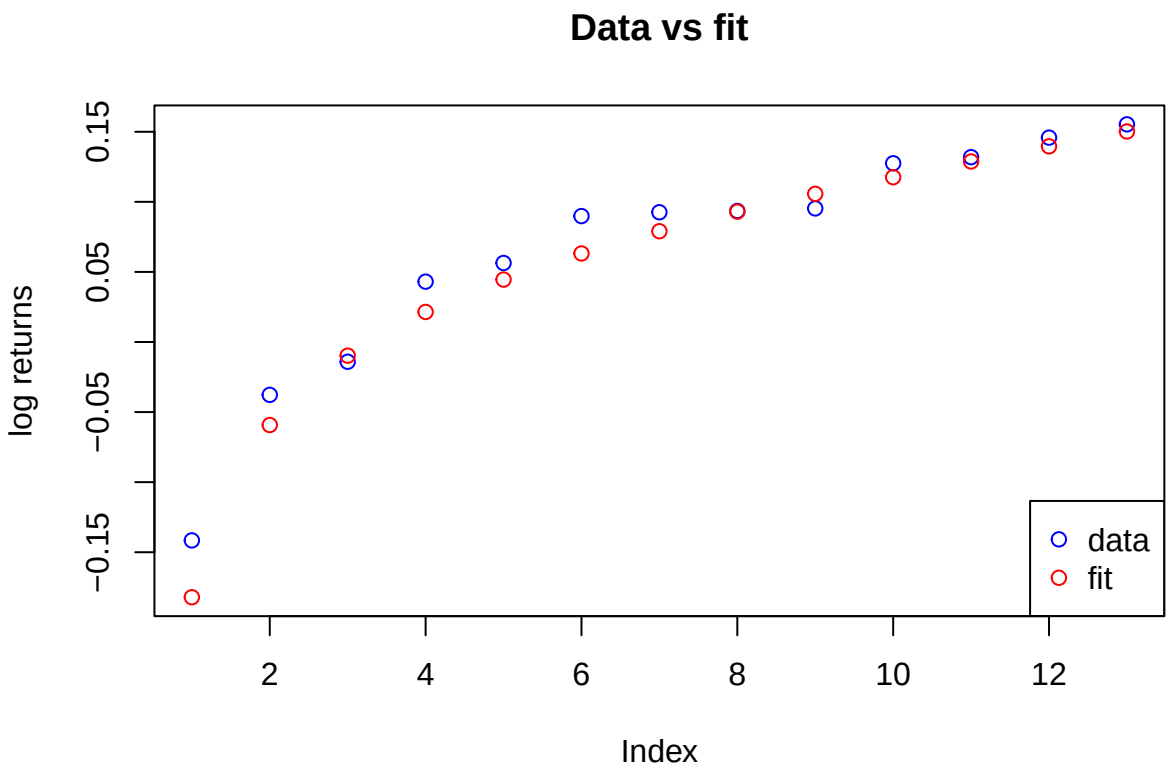
```
##
## AIC: -27.8497
## BIC: -25.58991
## m: 0.0480931
## s: 0.1198426
## nu (df): 3.303595
## xi: 0.03361192
## R^2: 0.993
##
## An R^2 of 0.993 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 7.4 percent
## What is the risk of losing max 25 %? =< 1.8 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 41 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

QQ Plot

The qq plot looks great. Log returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

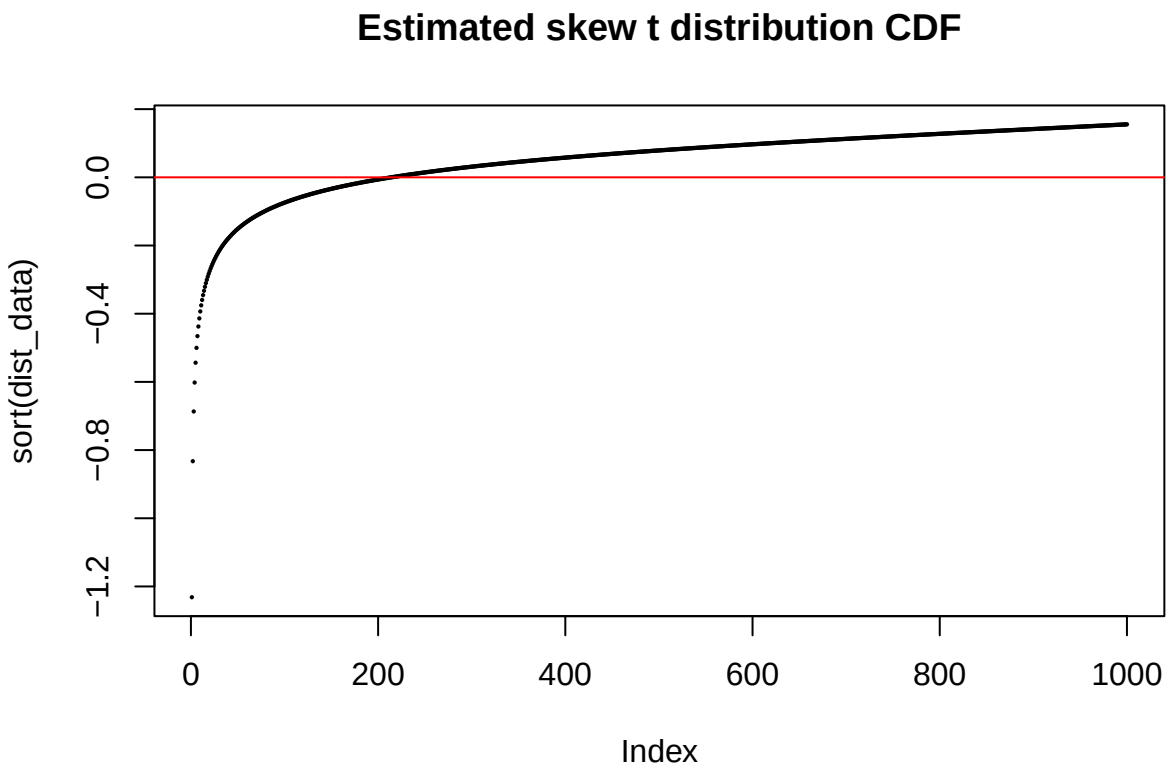
Data vs fit

Let's plot the fit and the observed returns together.

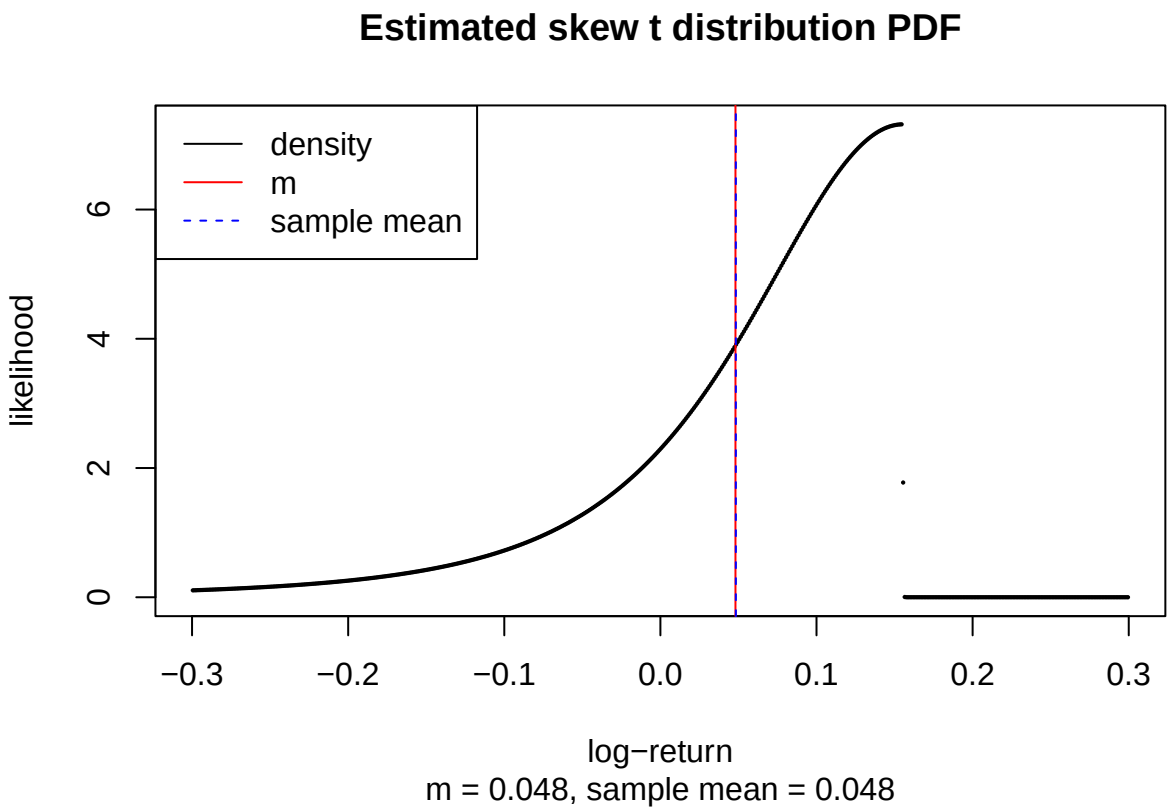


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

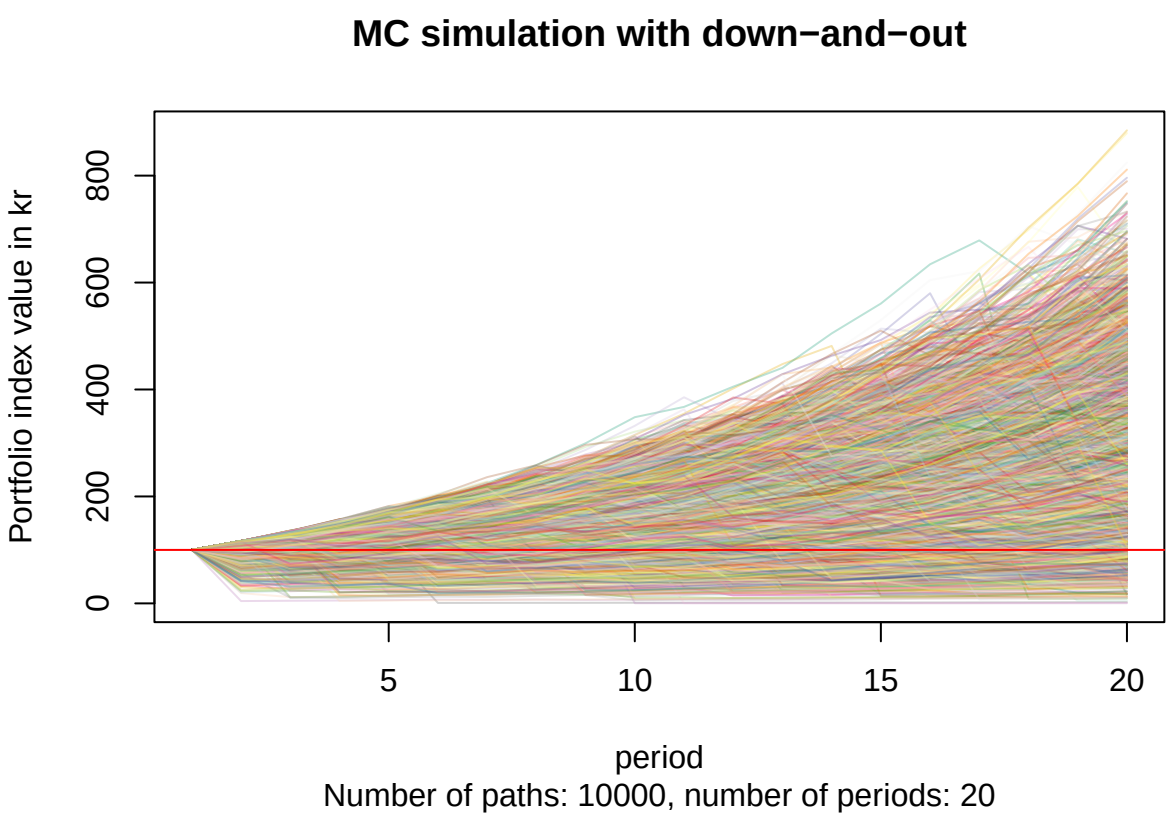


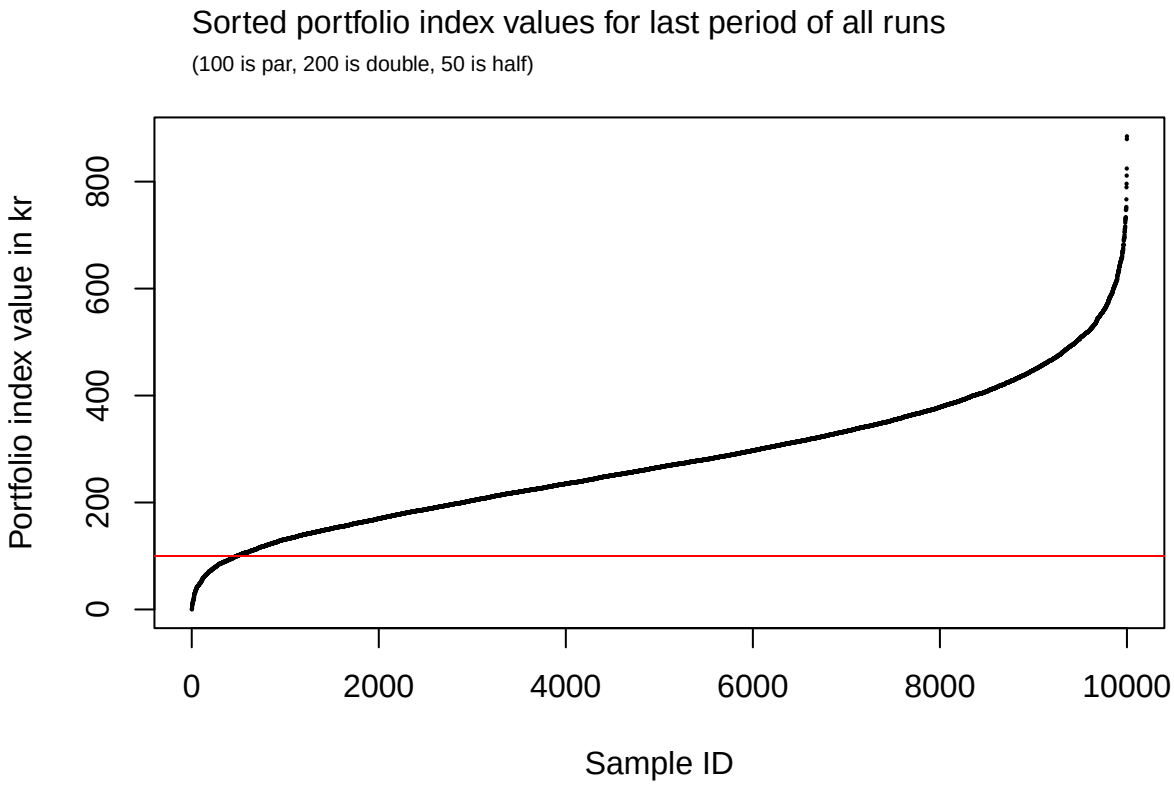
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.



Monte Carlo

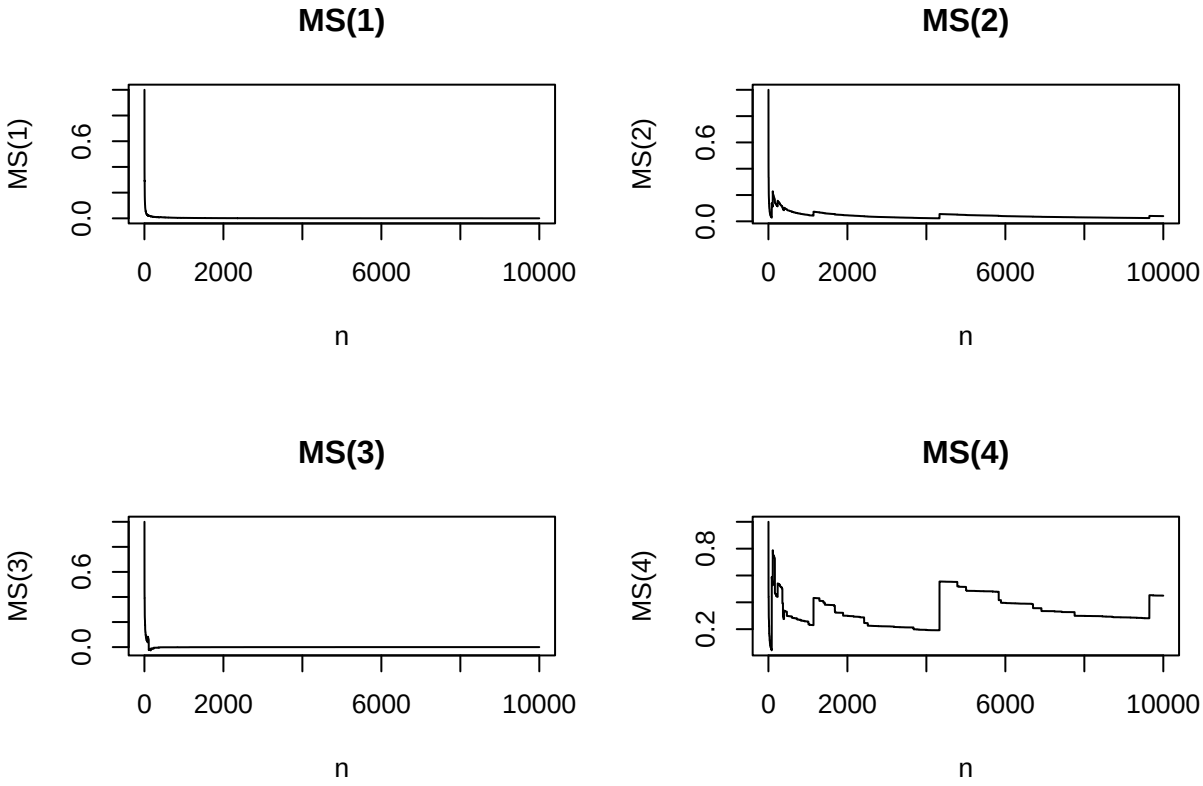
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 278.909 kr.
## SD of portfolio index value after 20 years: 124.477 kr.
## Min total portfolio index value after 20 years: 0.426 kr.
## Max total portfolio index value after 20 years: 884.639 kr.
##
## Share of paths finishing below 100: 4.82 percent
```

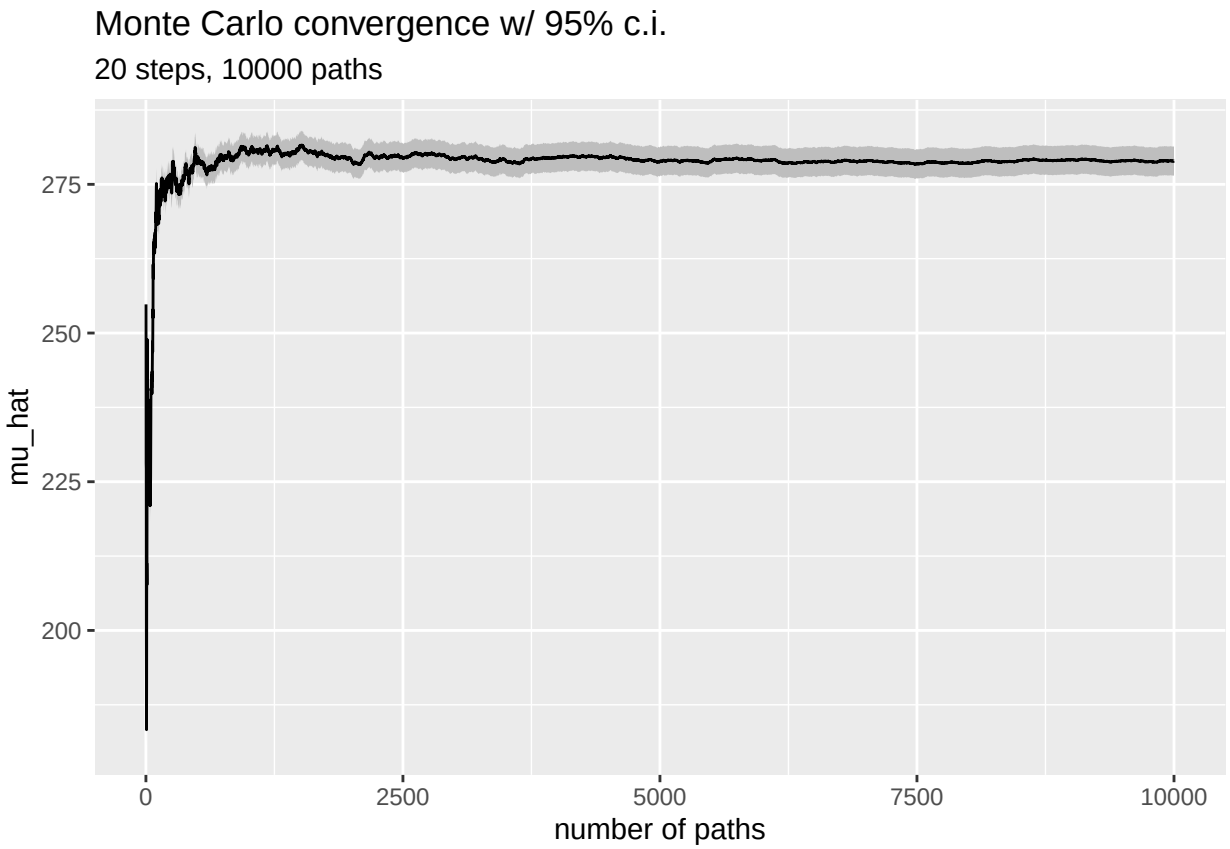

MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Convergence

Max vs sum plots for the first four moments:


MS(1)
MS(1)
0.0
0.6
0
2000
6000
10000
n
MS(2)
MS(2)
0.0
0.6
0
2000
6000
10000
n
MS(3)
MS(3)
0.0
0.6
0
2000
6000
10000
n
MS(4)
MS(4)
0.2
0.8
0
2000
6000
10000
n

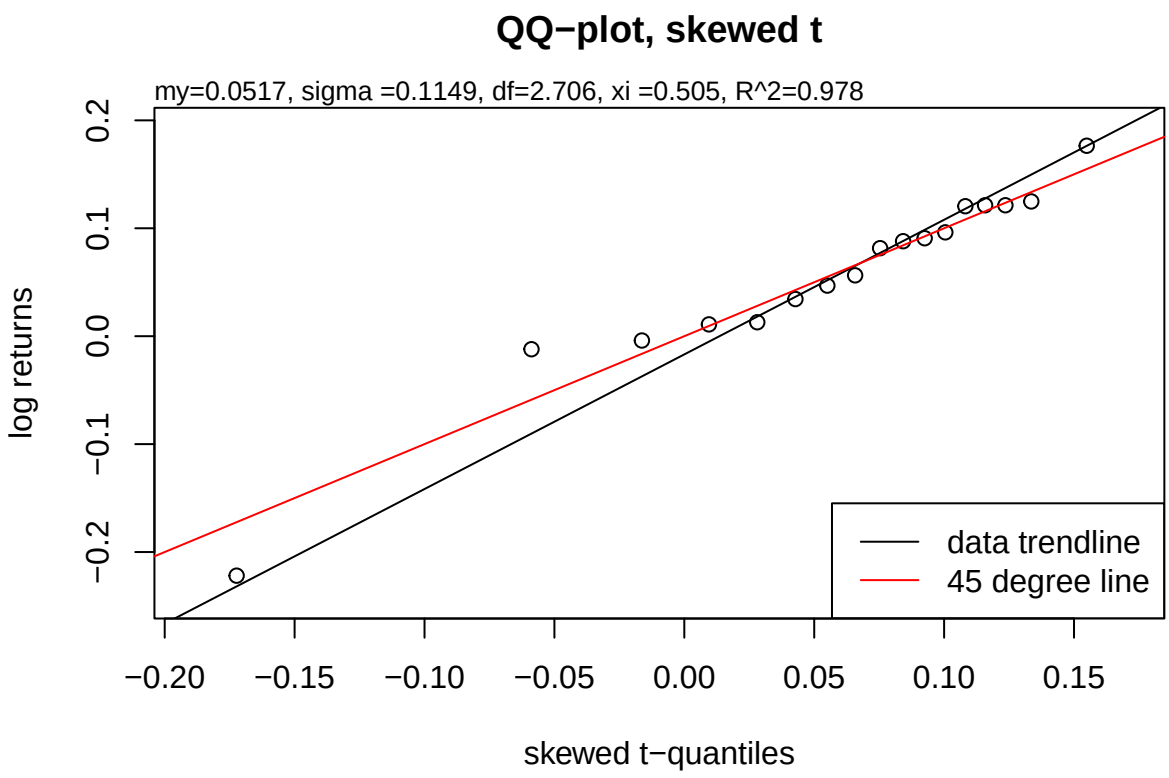


Velliv medium risk, 2007 - 2023

Fit to skew t distribution

```
##
## AIC: -34.35752
## BIC: -31.02467
## m: 0.05171176
## s: 0.1149408
## nu (df): 2.706099
## xi: 0.5049945
## R^2: 0.978
##
## An R^2 of 0.978 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 5.4 percent
## What is the risk of losing max 25 %? =< 1.3 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 36.2 percent
## What is the chance of gaining min 25 %? >= 0.3 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

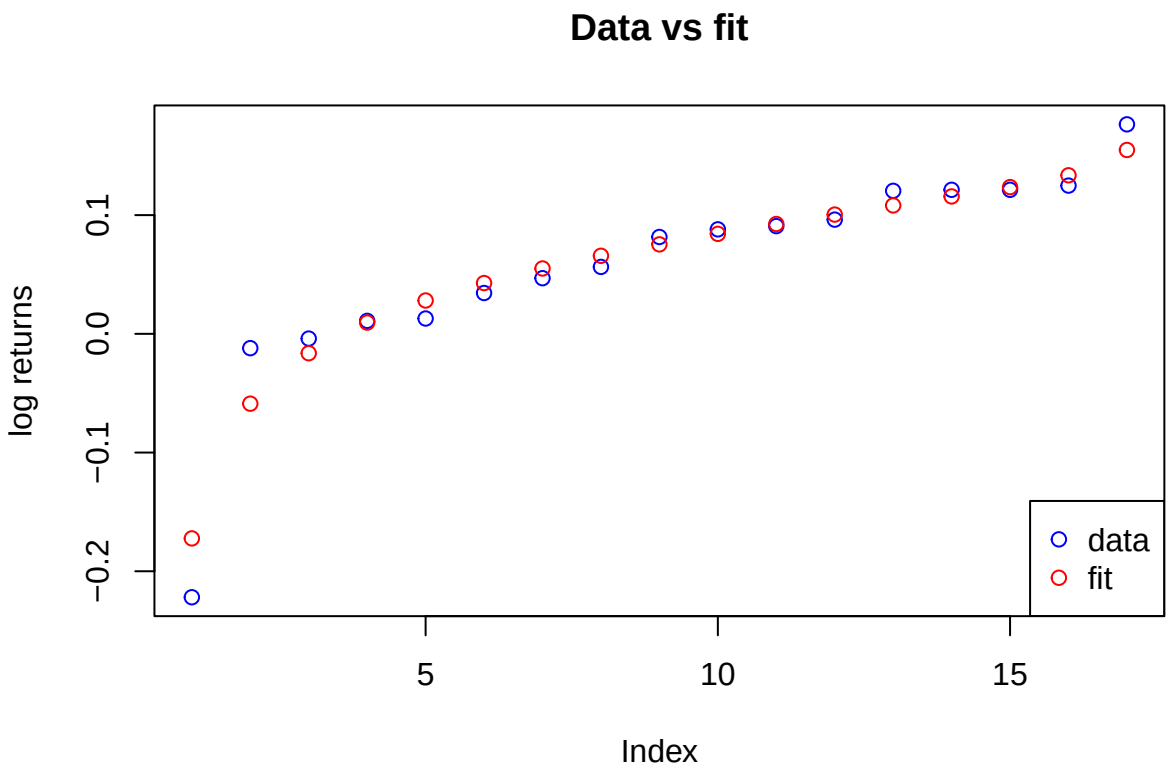
QQ Plot



The qq plot looks good. Log returns for Velliv high risk seems to be consistent with a skewed t-distribution.

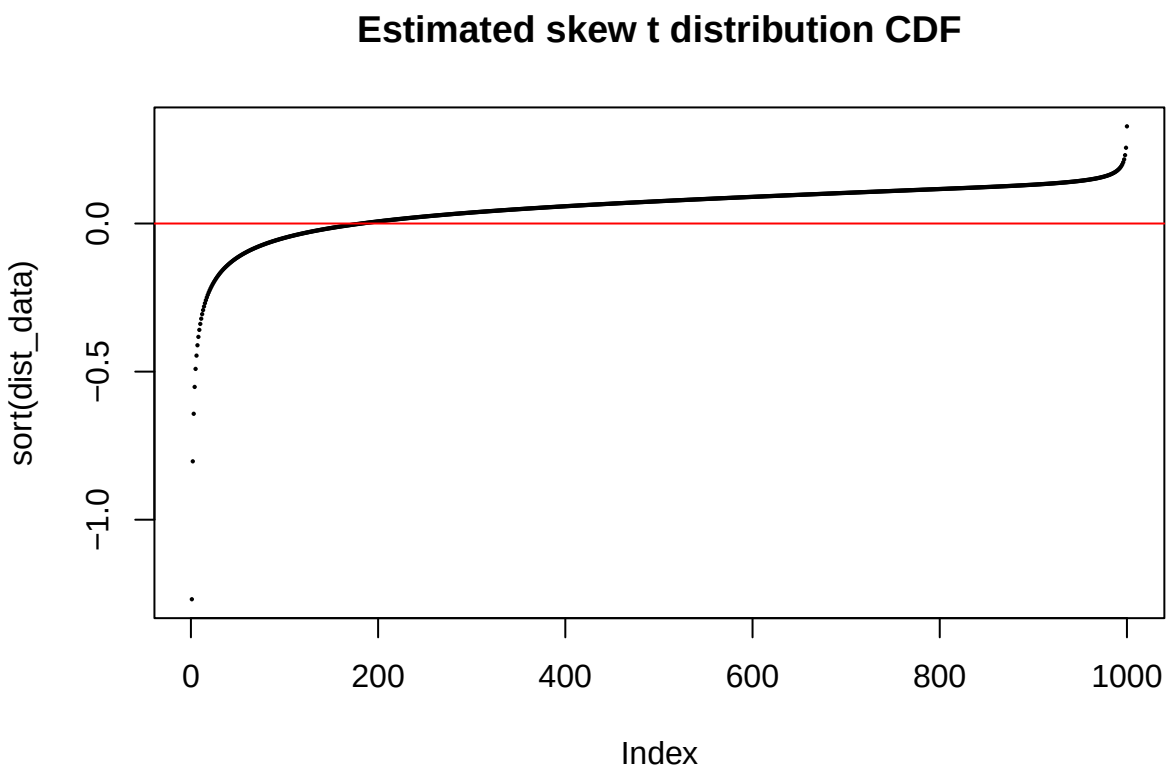
Data vs fit

Let's plot the fit and the observed returns together.

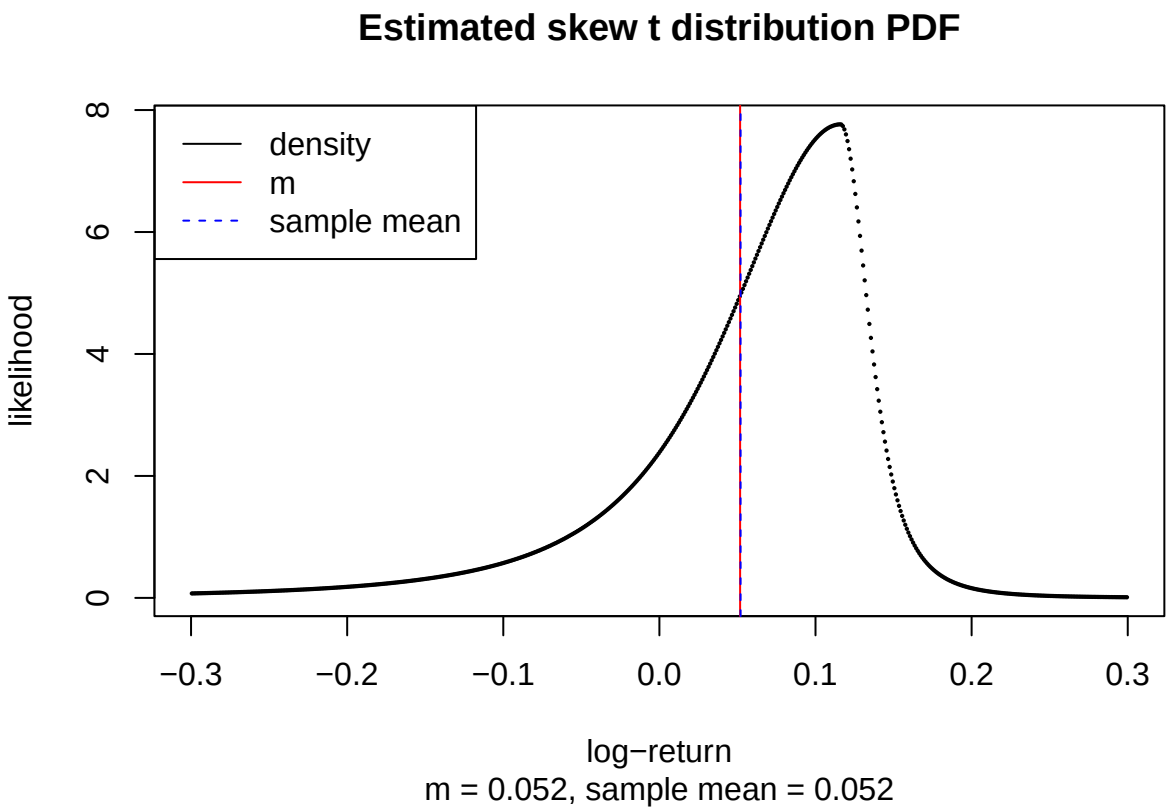


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

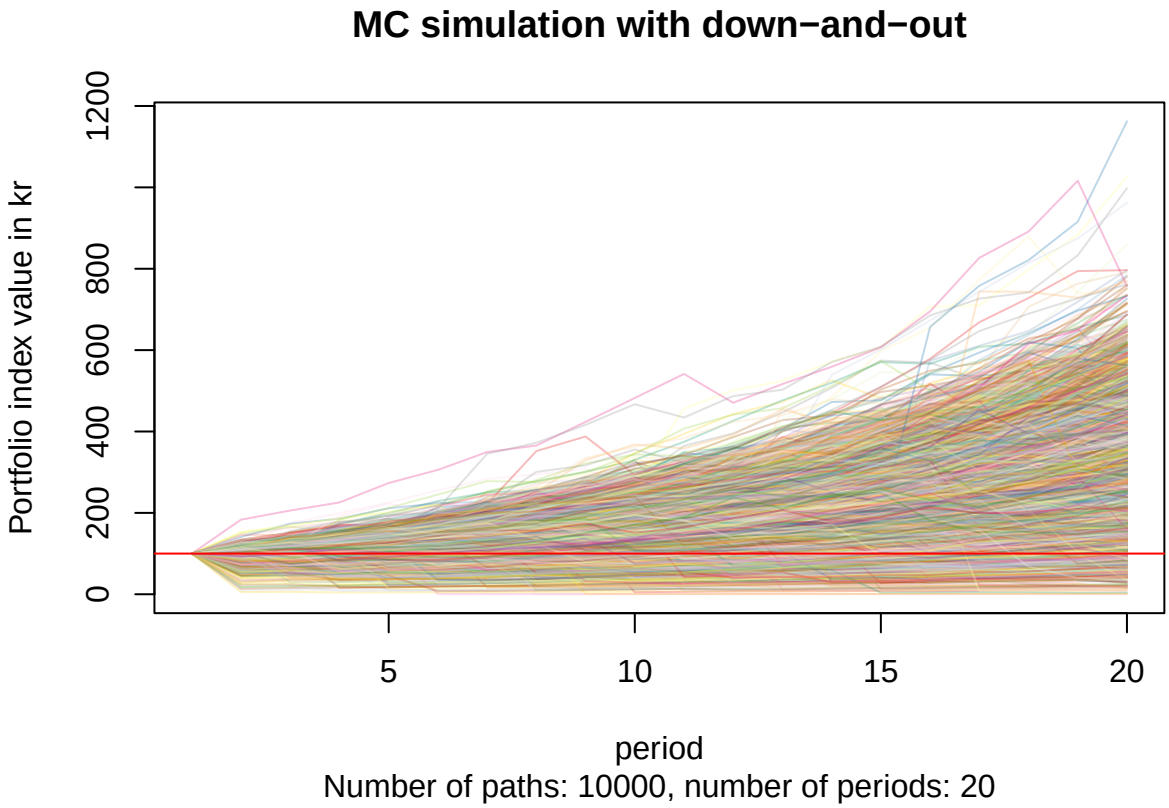


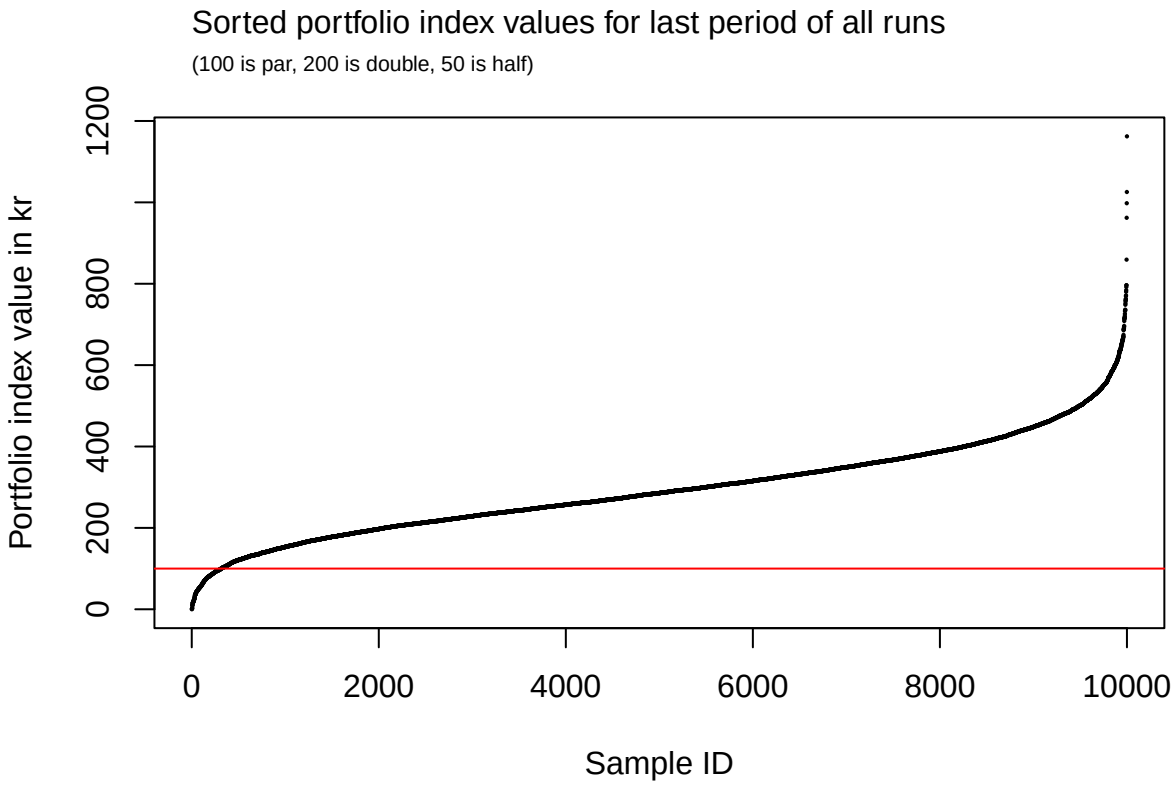
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened. But because the disastrous loss in 2008 was followed by a large profit the following year, we see some increased upside for the top percentiles. Beware: A 1.2 return following a 0.8 return doesn't take us back where we were before the loss. Path dependency! So if returns more or less average out, but high returns have a tendency to follow high losses, that's bad!



Monte Carlo

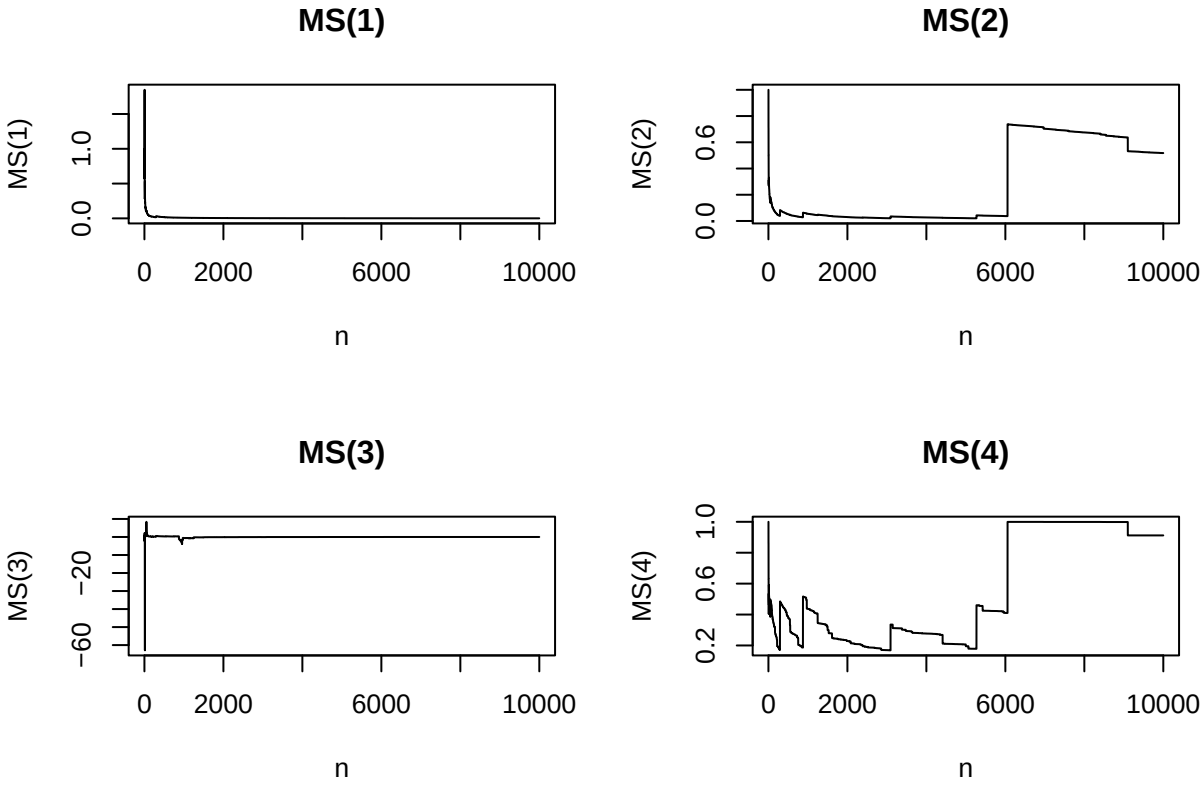
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 295.263 kr.
## SD of portfolio index value after 20 years: 117.56 kr.
## Min total portfolio index value after 20 years: 0.066 kr.
## Max total portfolio index value after 20 years: 1162.234 kr.
##
## Share of paths finishing below 100: 3.16 percent
```

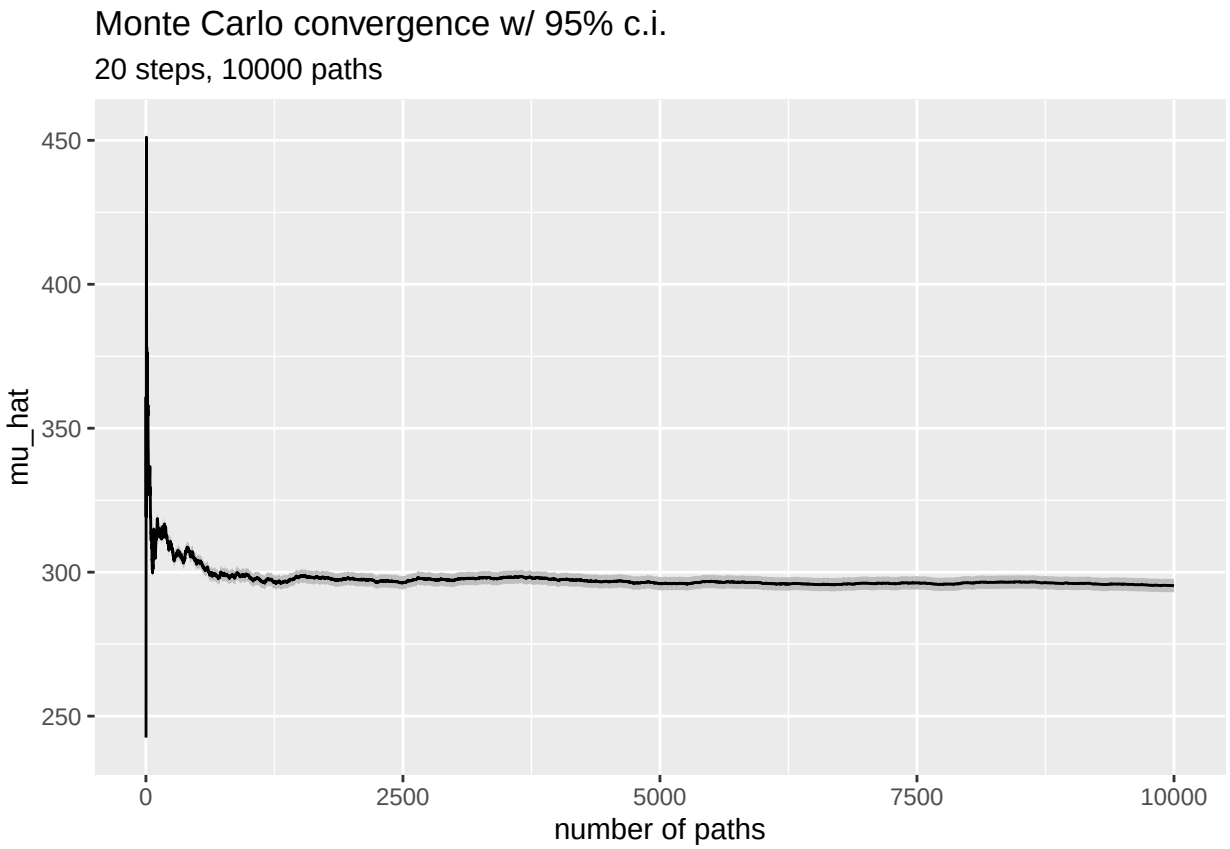

MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
1200
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Convergence

Max vs sum plots for the first four moments:


MS(1)
MS(1)
0.0
1.0
0
2000
6000
10000
n
MS(2)
MS(2)
0.0
0.6
0
2000
6000
10000
n
MS(3)
MS(3)
−60
−20
0
2000
6000
10000
n
MS(4)
MS(4)
0.2
0.6
1.0
0
2000
6000
10000
n

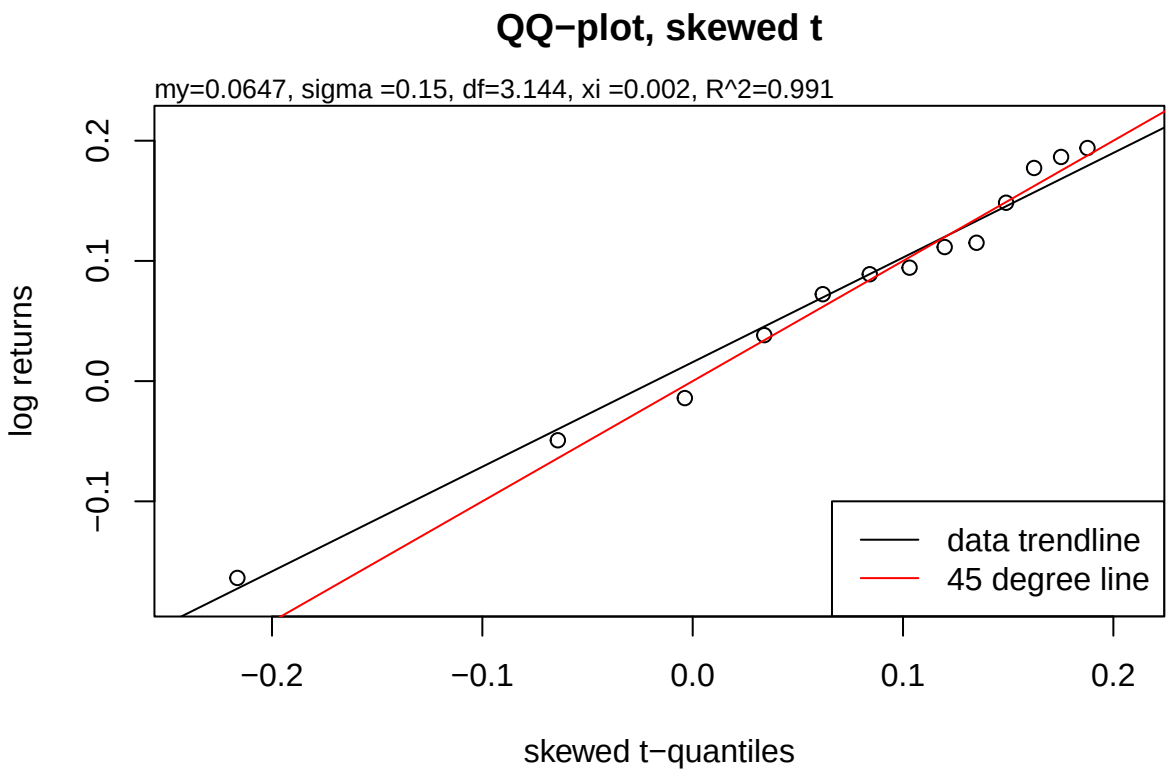


Velliv high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -21.42488
## BIC: -19.16508
## m: 0.06471454
## s: 0.1499924
## nu (df): 3.144355
## xi: 0.002367034
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 8.3 percent
## What is the risk of losing max 25 %? =< 2.5 percent
## What is the risk of losing max 50 %? =< 0.4 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 53.3 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

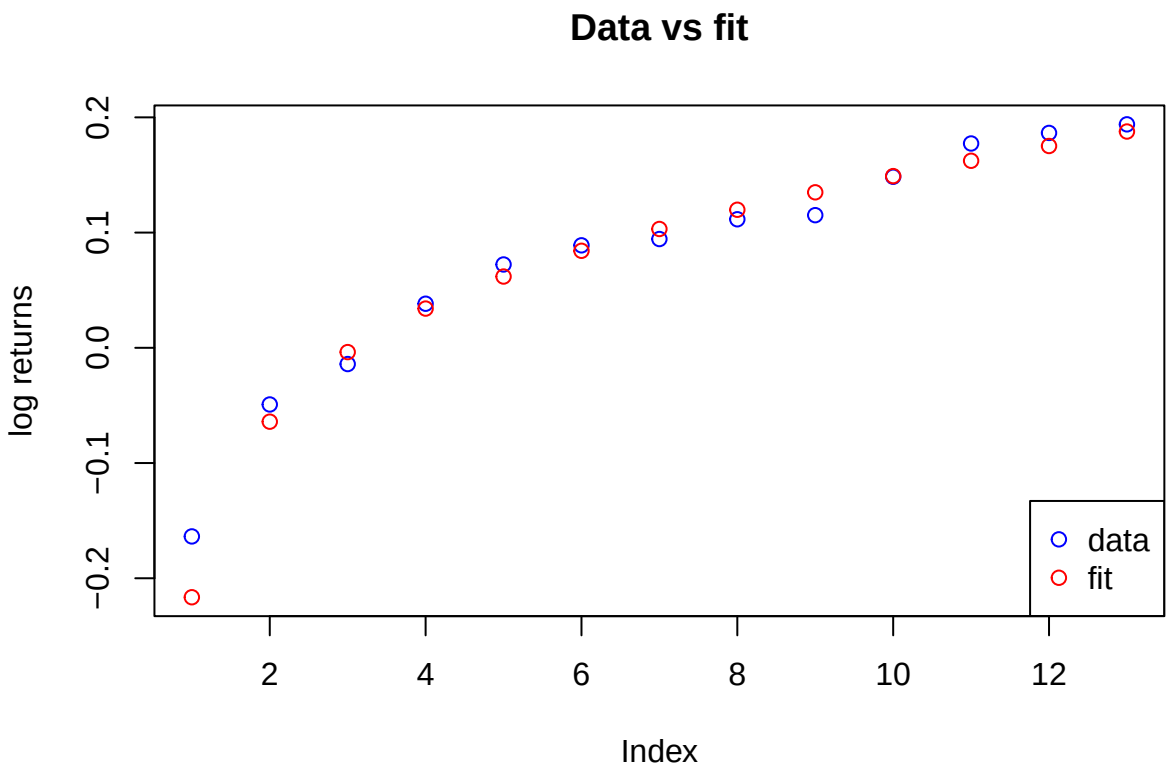
QQ Plot



The qq plot looks great. Returns for Velliv medium risk seems to be consistent with a skewed t-distribution.

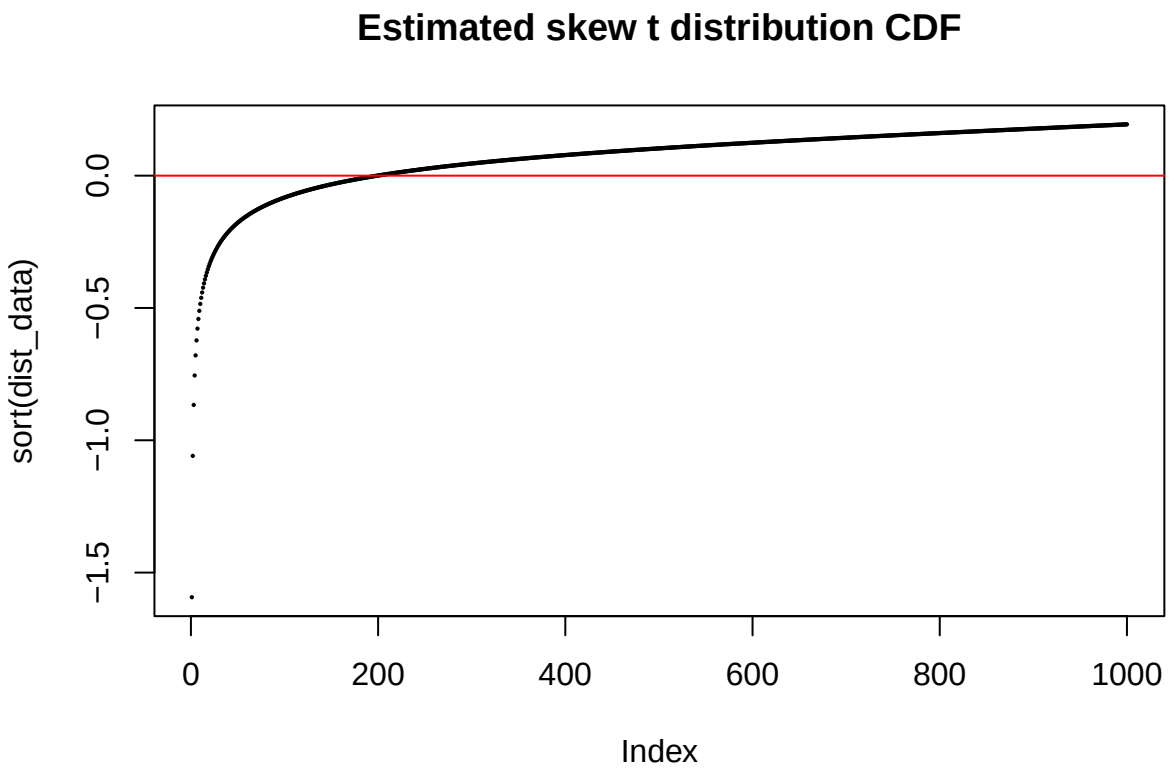
Data vs fit

Let's plot the fit and the observed returns together.

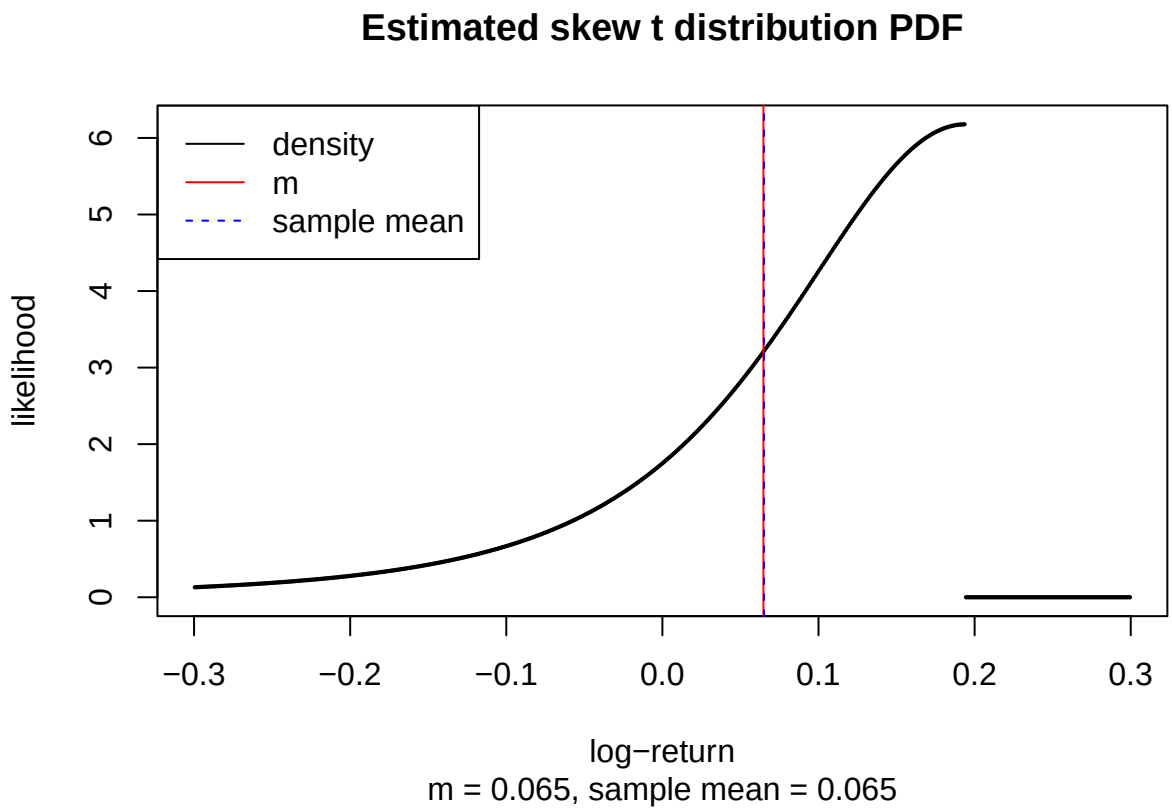


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

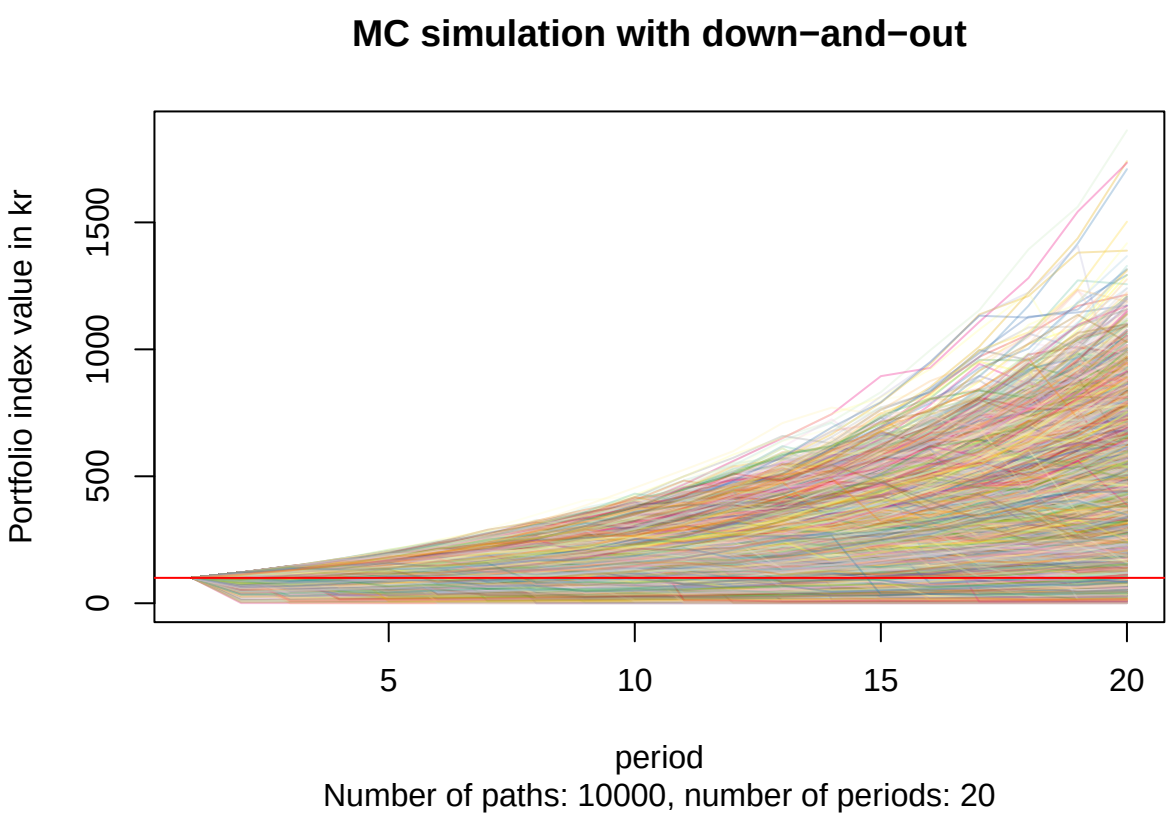


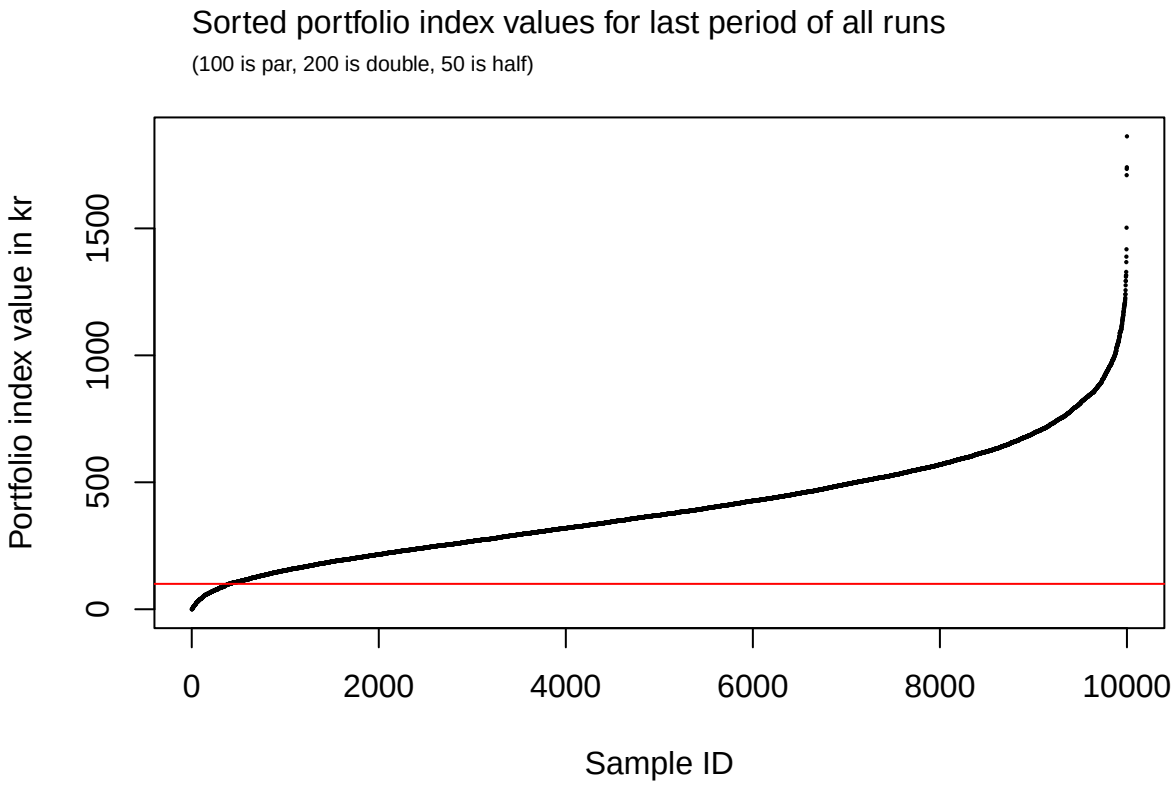
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.



Monte Carlo

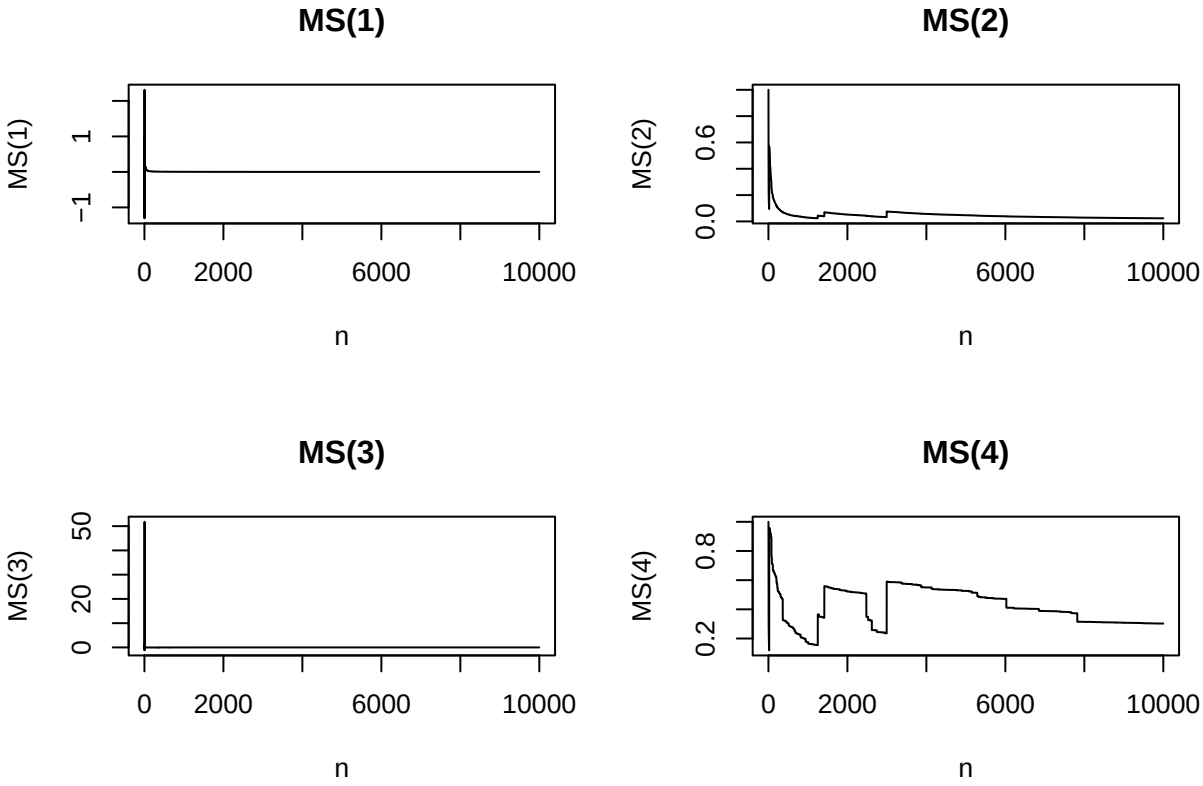
```
## Down-and-out simulation:
## Probability of down-and-out: 0.01 percent
##
## Mean portfolio index value after 20 years: 402.534 kr.
## SD of portfolio index value after 20 years: 217.34 kr.
## Min total portfolio index value after 20 years: 0 kr.
## Max total portfolio index value after 20 years: 1862.587 kr.
##
## Share of paths finishing below 100: 4.02 percent
```

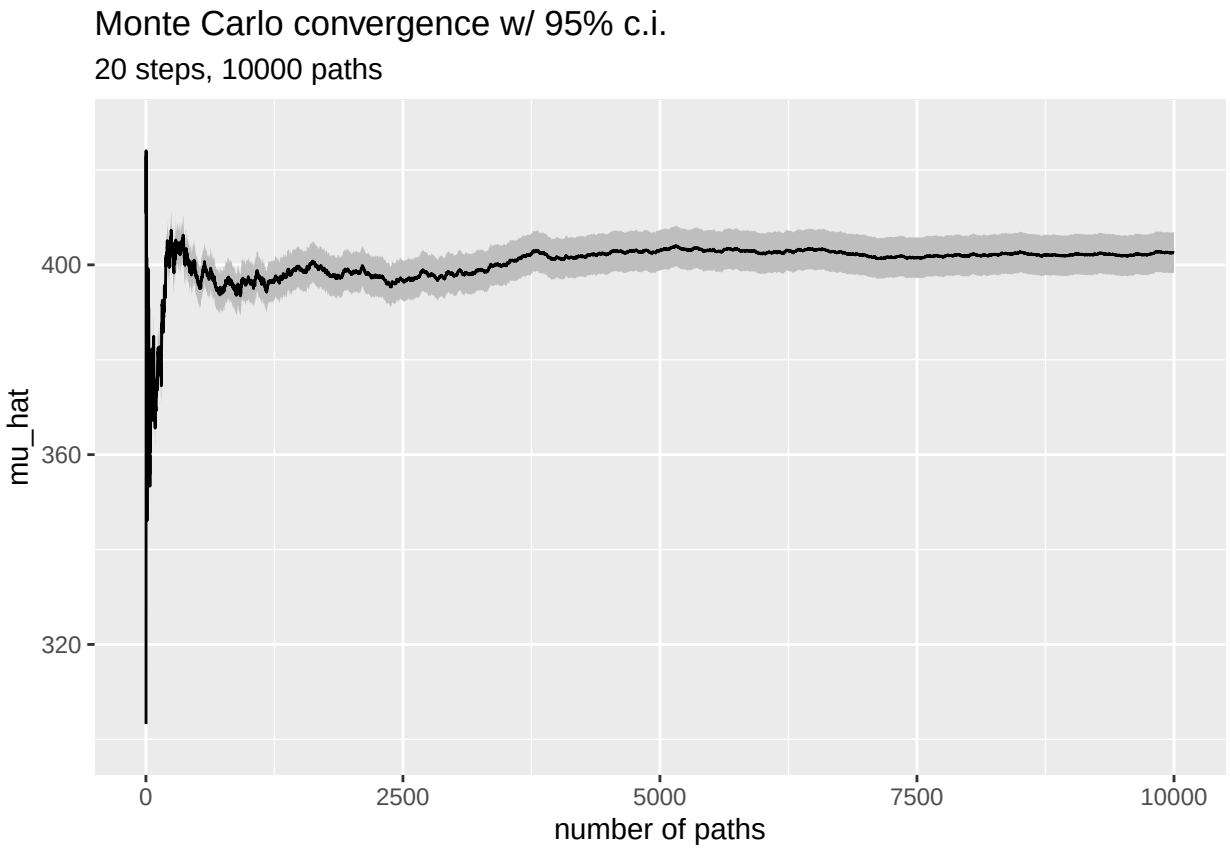

MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Convergence

Max vs sum plots for the first four moments:


MS(1)
MS(1)
1
−1
0
2000
6000
10000
n
MS(2)
MS(2)
0.6
0.0
0
2000
6000
10000
n
MS(3)
MS(3)
50
20
0
0
2000
6000
10000
n
MS(4)
MS(4)
0.8
0.2
0
2000
6000
10000
n

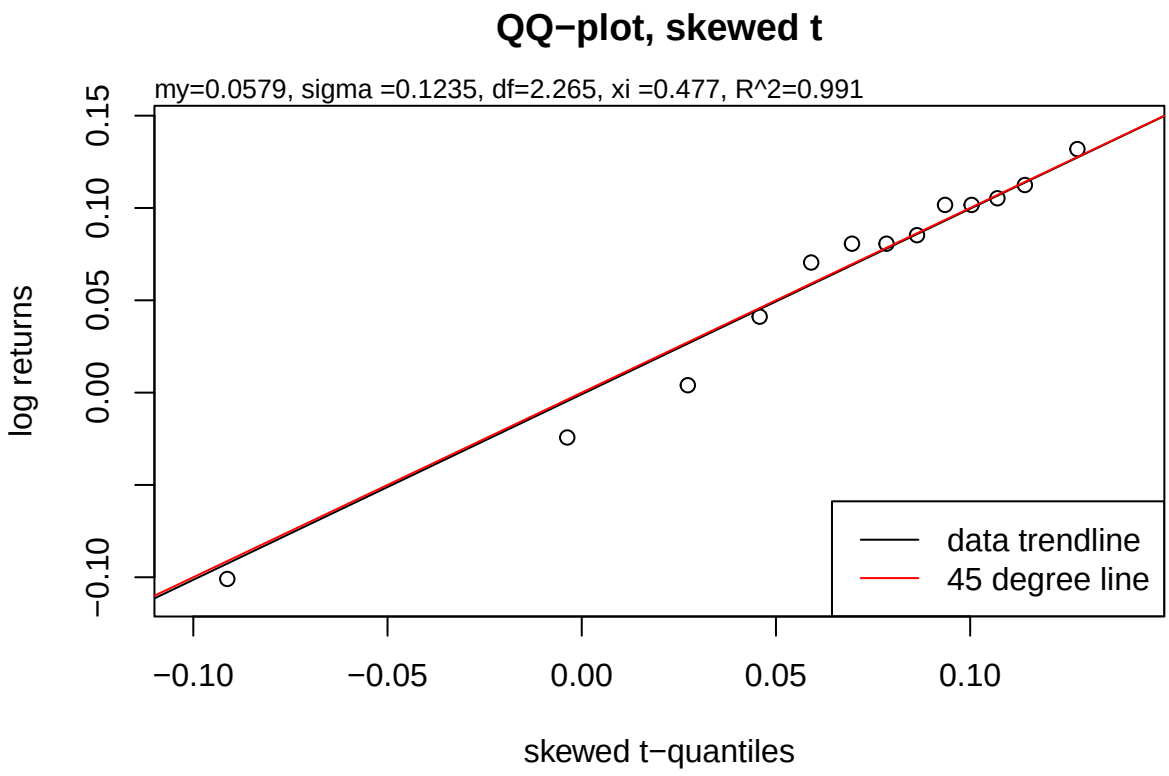


PFA medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -33.22998
## BIC: -30.97018
## m: 0.05789224
## s: 0.1234592
## nu (df): 2.265273
## xi: 0.477324
## R^2: 0.991
##
## An R^2 of 0.991 suggests that the fit is extremely good.
##
## What is the risk of losing max 10 %? =< 3.3 percent
## What is the risk of losing max 25 %? =< 0.9 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 32.7 percent
## What is the chance of gaining min 25 %? >= 0.1 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

QQ Plot

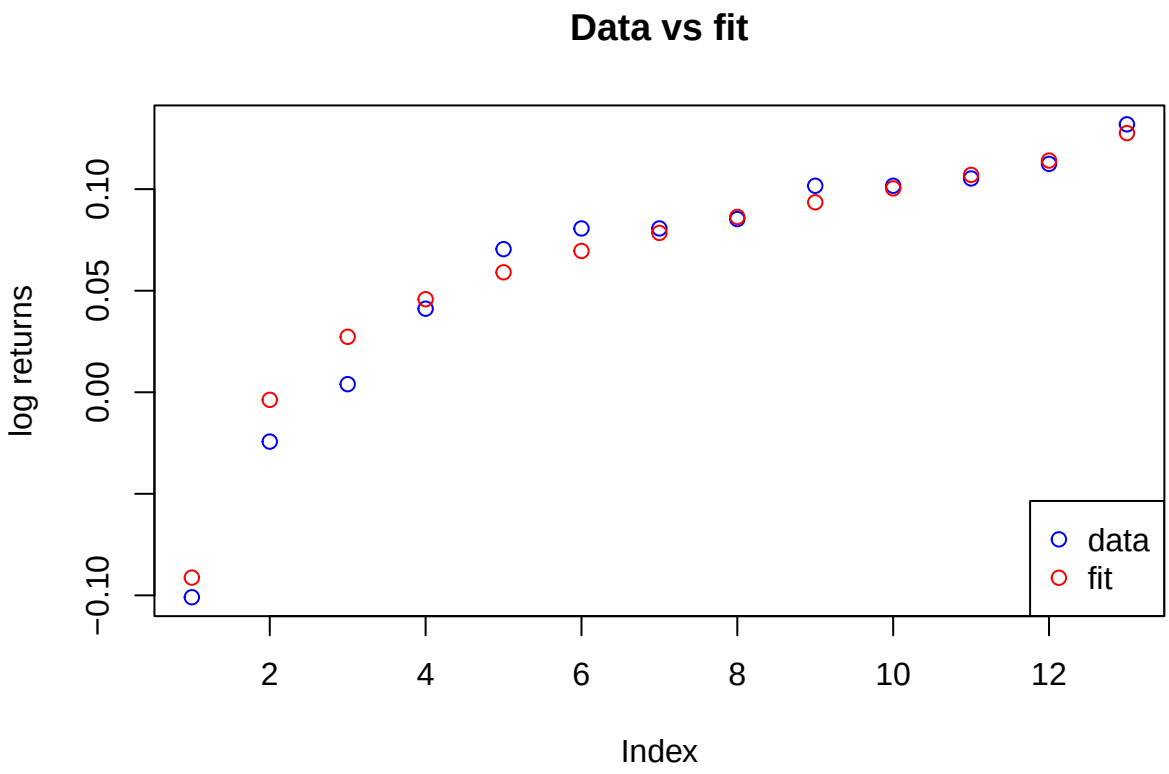


The qq plot looks great. Log returns for PFA medium risk seems to be consistent with a skewed t-distribution.

```
##  [1] -0.091256521 -0.003731241  0.027312079  0.045808232  0.059068633
##  [6]  0.069575113  0.078454727  0.086316936  0.093536451  0.100370932
## [11]  0.107018607  0.114081432  0.127604387
```

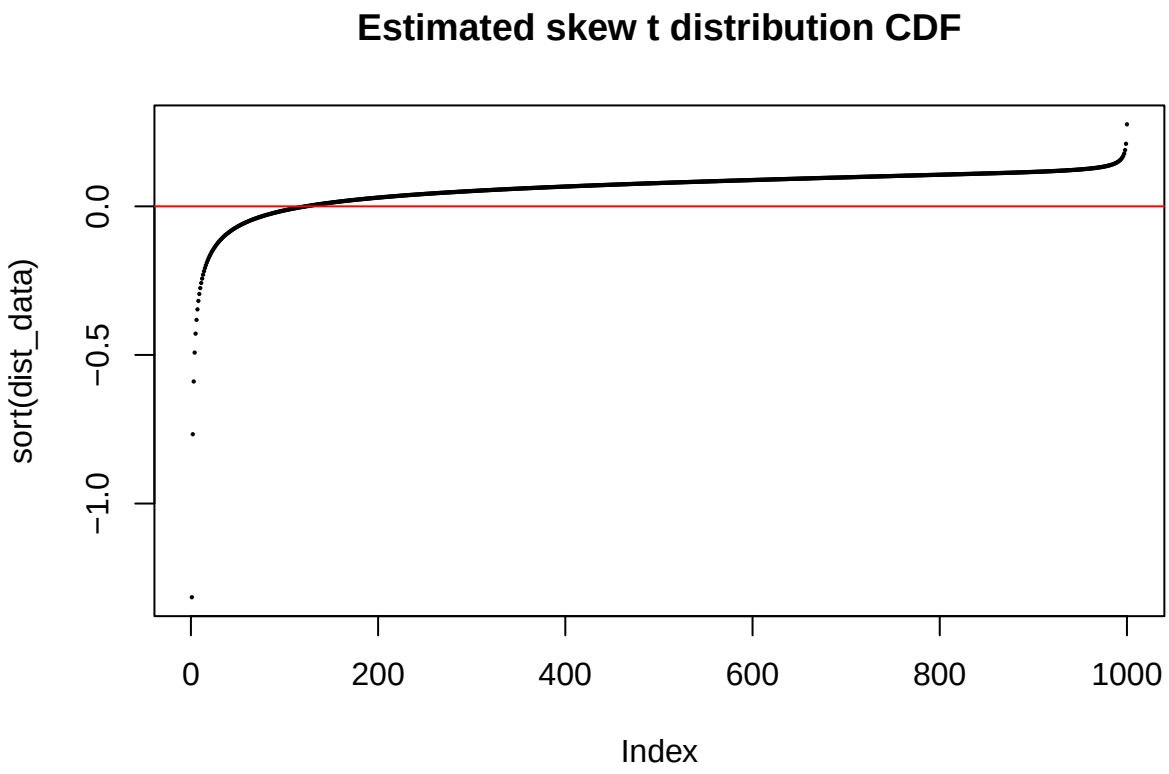
Data vs fit

Let's plot the fit and the observed returns together.

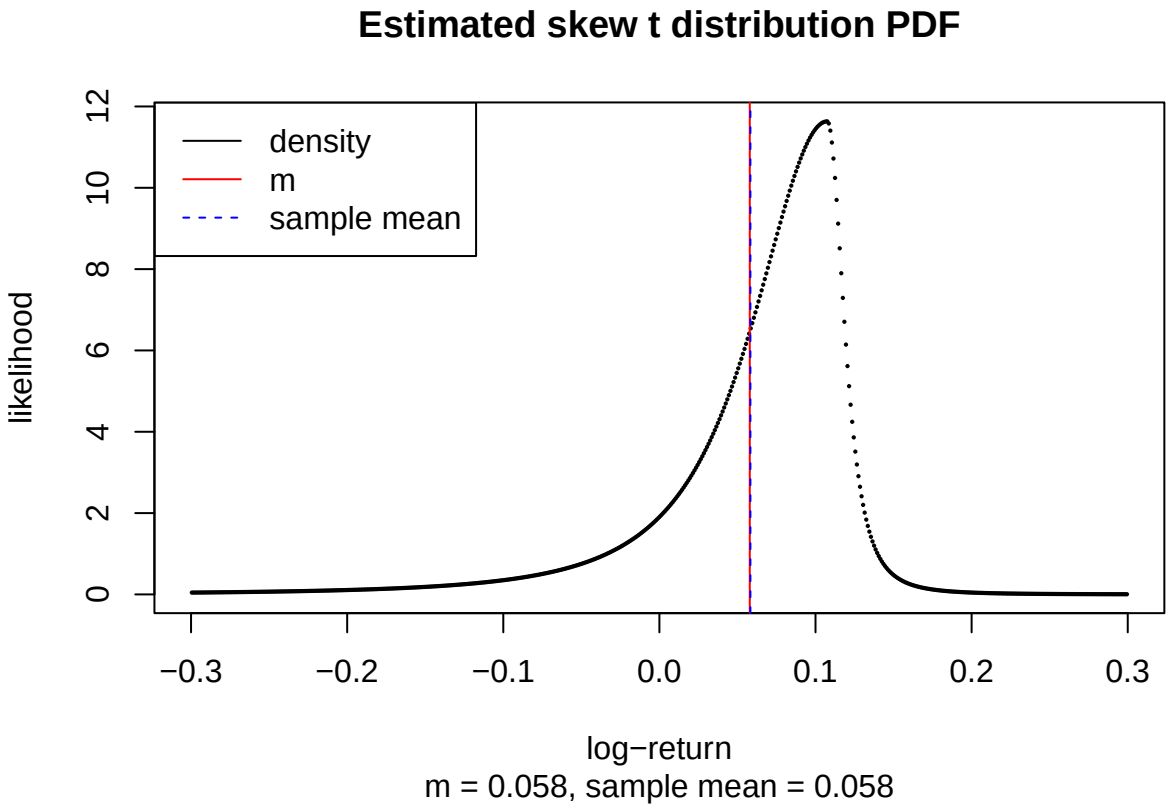


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

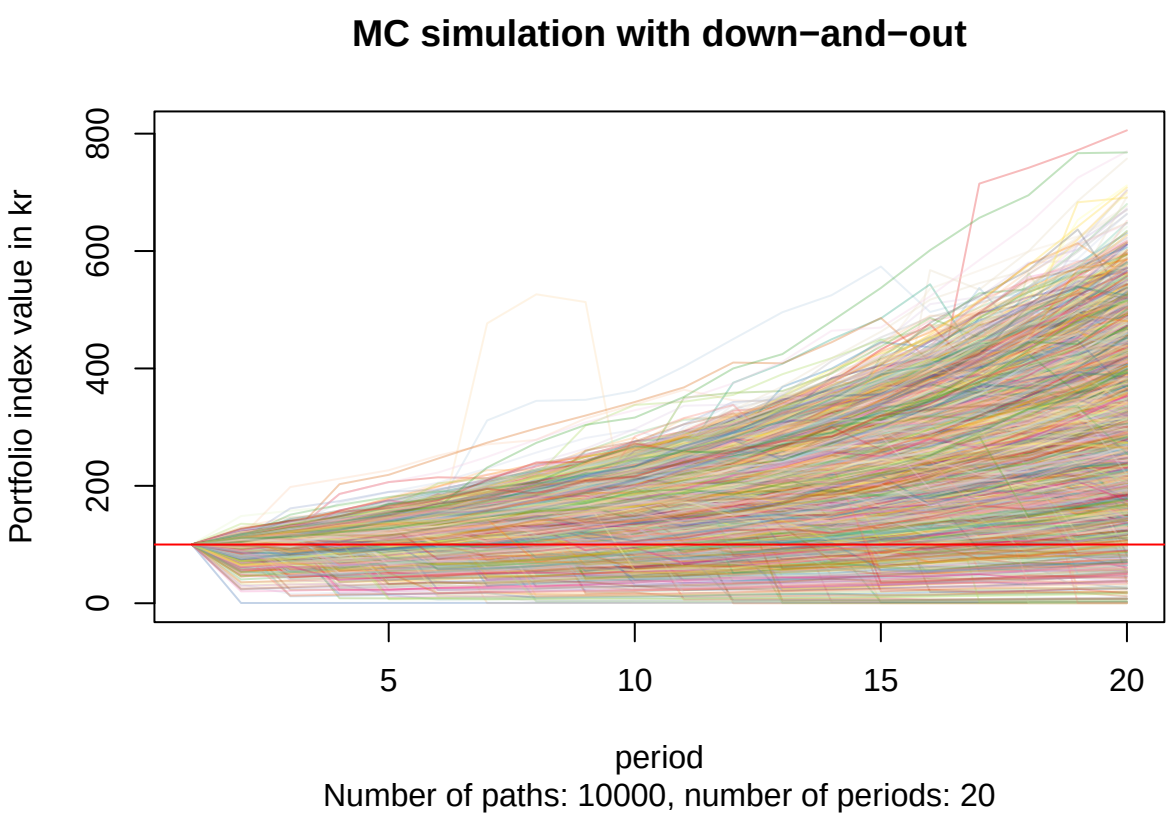


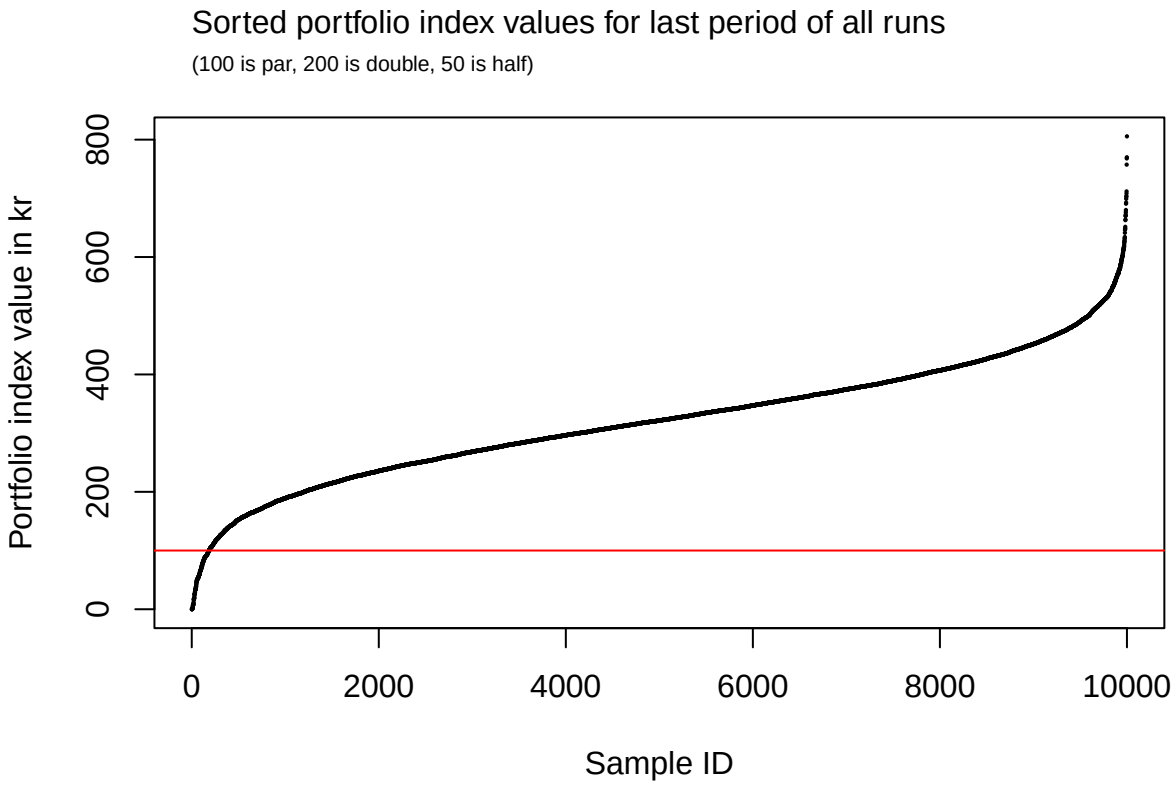
We see that for a few observations out of a 1000, the losses are disastrous. While there is some uptick at the top percentiles, the curve basically flattens out.



Monte Carlo

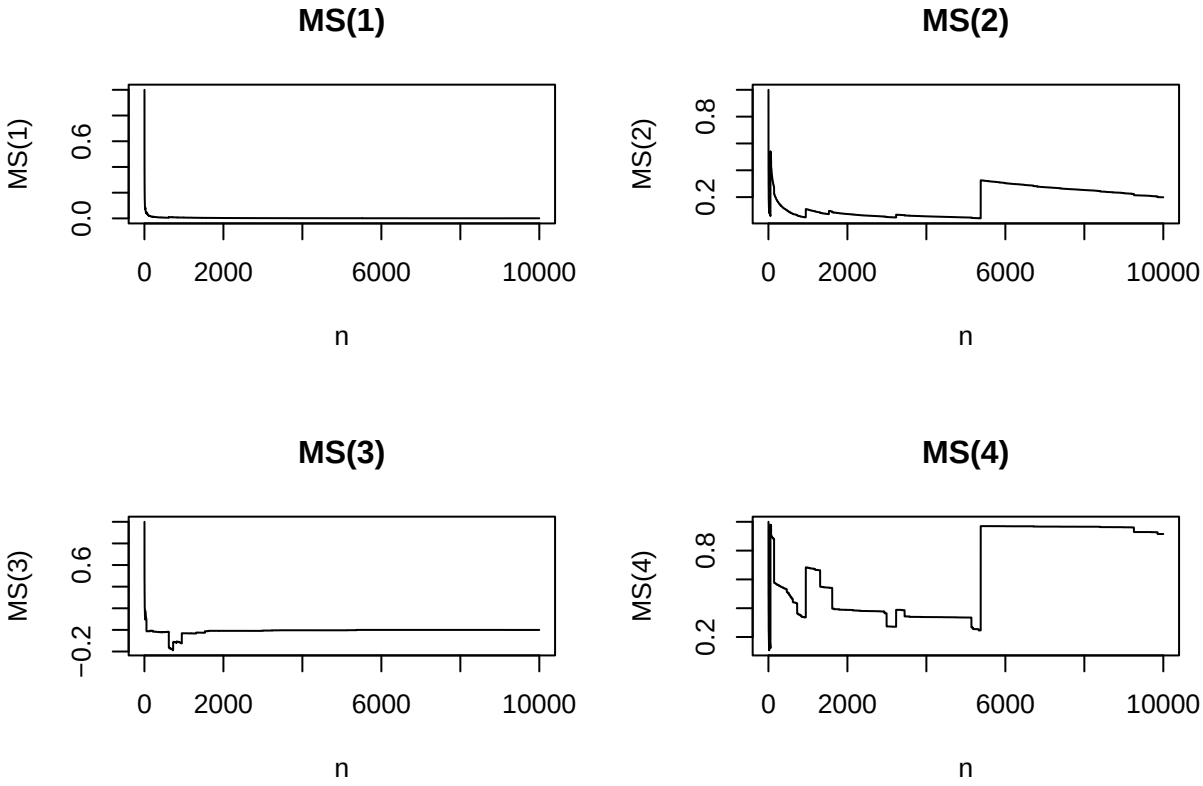
```
## Down-and-out simulation:
## Probability of down-and-out: 0.01 percent
##
## Mean portfolio index value after 20 years: 321.284 kr.
## SD of portfolio index value after 20 years: 103.871 kr.
## Min total portfolio index value after 20 years: 0 kr.
## Max total portfolio index value after 20 years: 805.612 kr.
##
## Share of paths finishing below 100: 1.85 percent
```

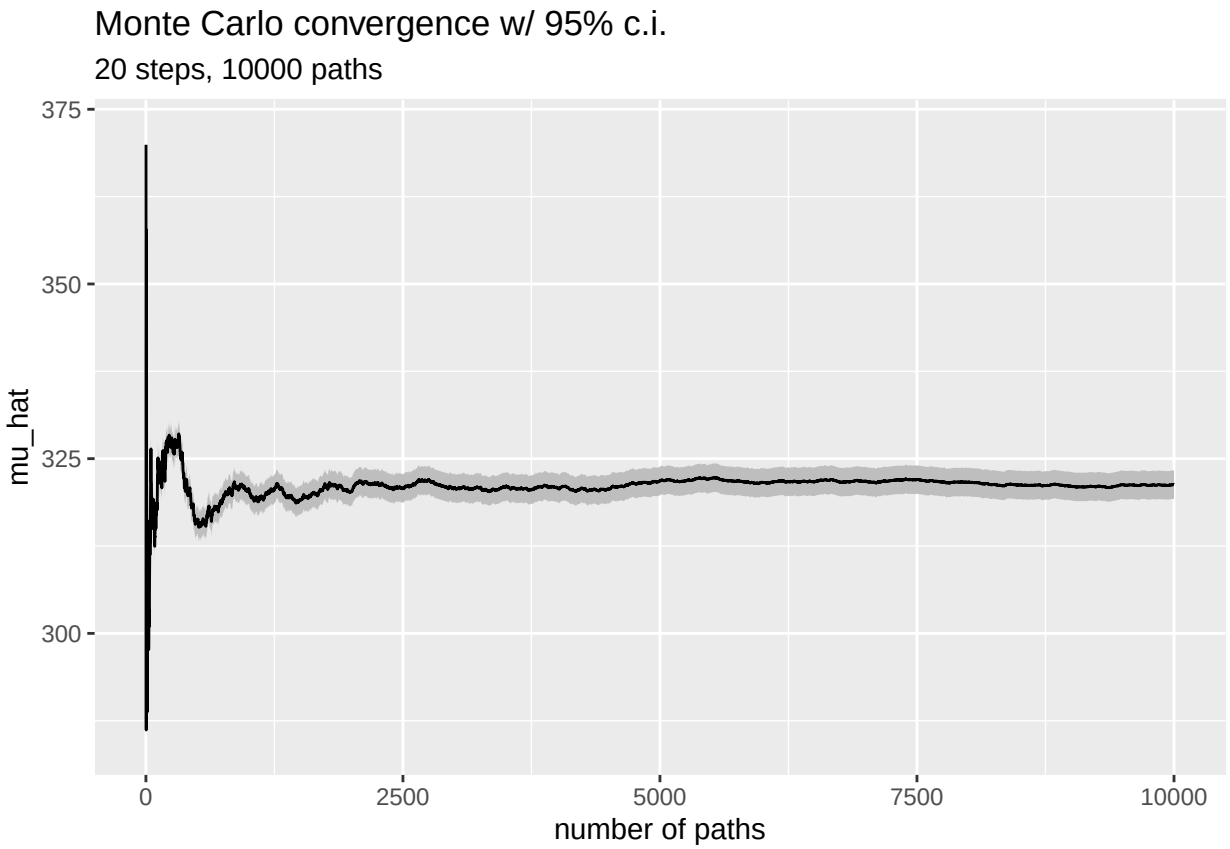

MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Convergence

Max vs sum plots for the first four moments:


MS(1)
MS(1)
0.0
0.6
0
2000
6000
10000
n
MS(2)
MS(2)
0.2
0.8
0
2000
6000
10000
n
MS(3)
MS(3)
−0.2
0.6
0
2000
6000
10000
n
MS(4)
MS(4)
0.2
0.8
0
2000
6000
10000
n

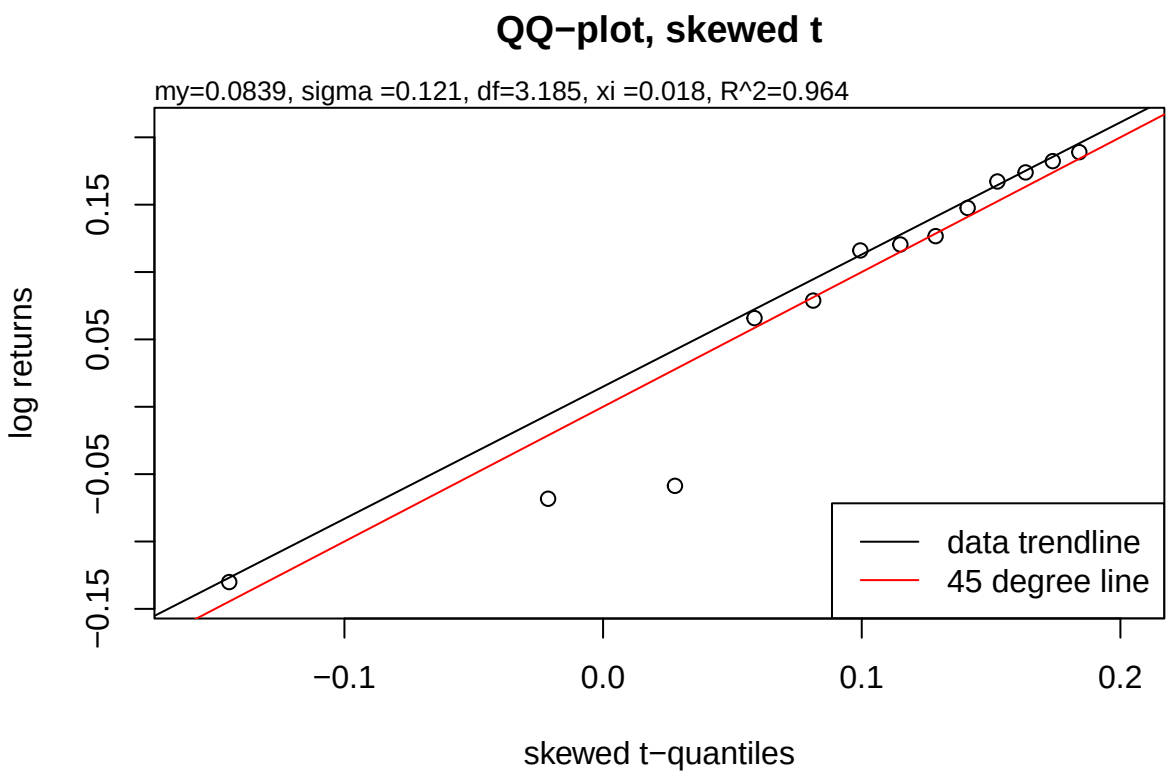


PFA high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -23.72565
## BIC: -21.46585
## m: 0.08386034
## s: 0.1210107
## nu (df): 3.184569
## xi: 0.01790306
## R^2: 0.964
##
## An R^2 of 0.964 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 5.3 percent
## What is the risk of losing max 25 %? =< 1.4 percent
## What is the risk of losing max 50 %? =< 0.2 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 59.6 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

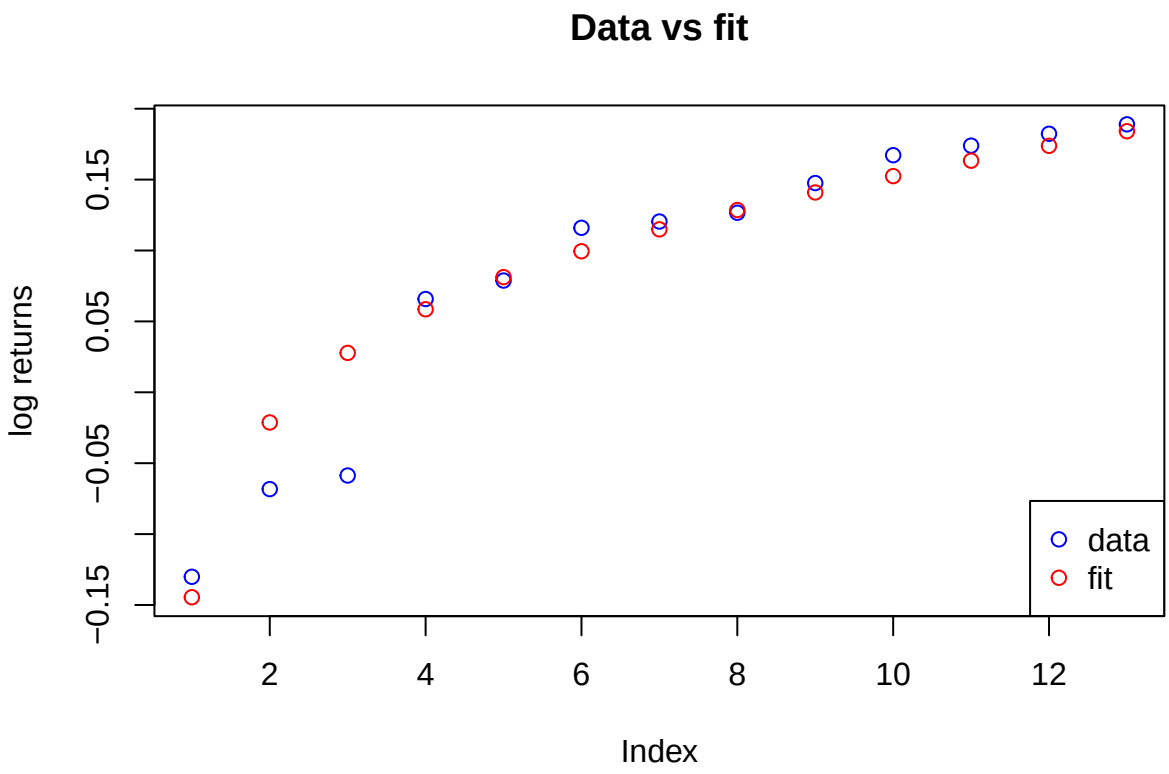
QQ Plot



The qq plot looks ok. Returns for PFA high risk seems to be consistent with a skewed t-distribution.

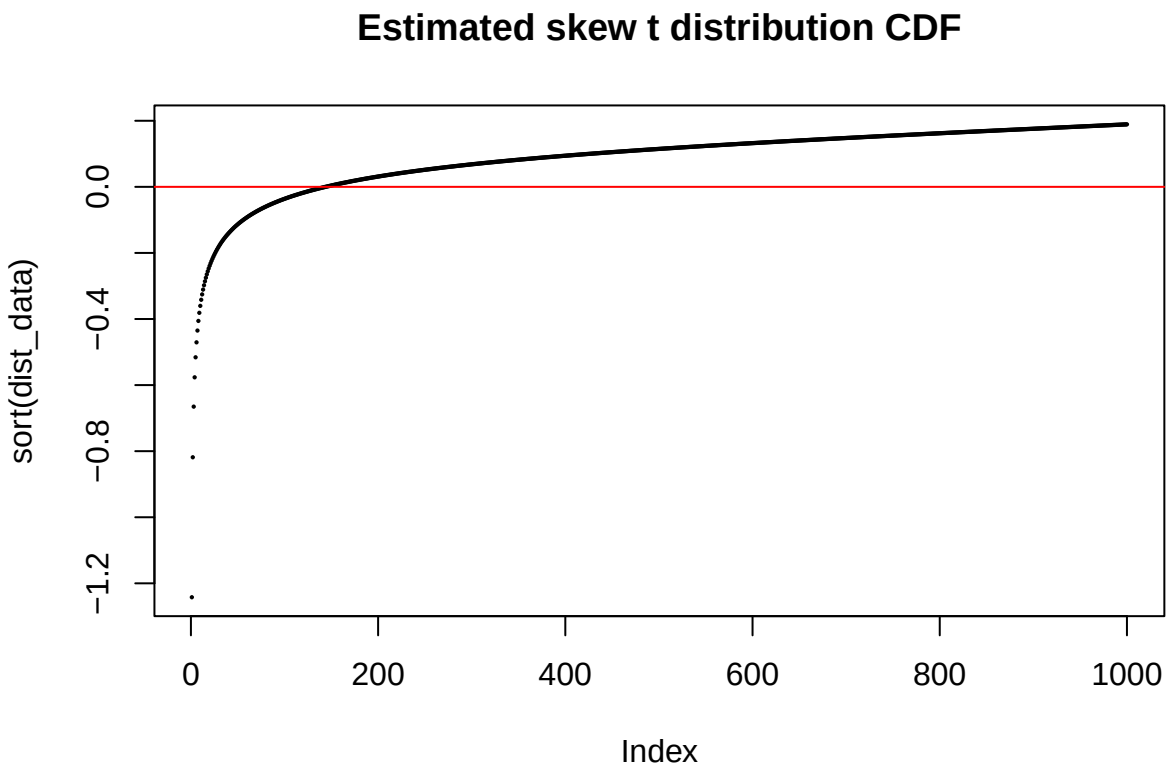
Data vs fit

Let's plot the fit and the observed returns together.

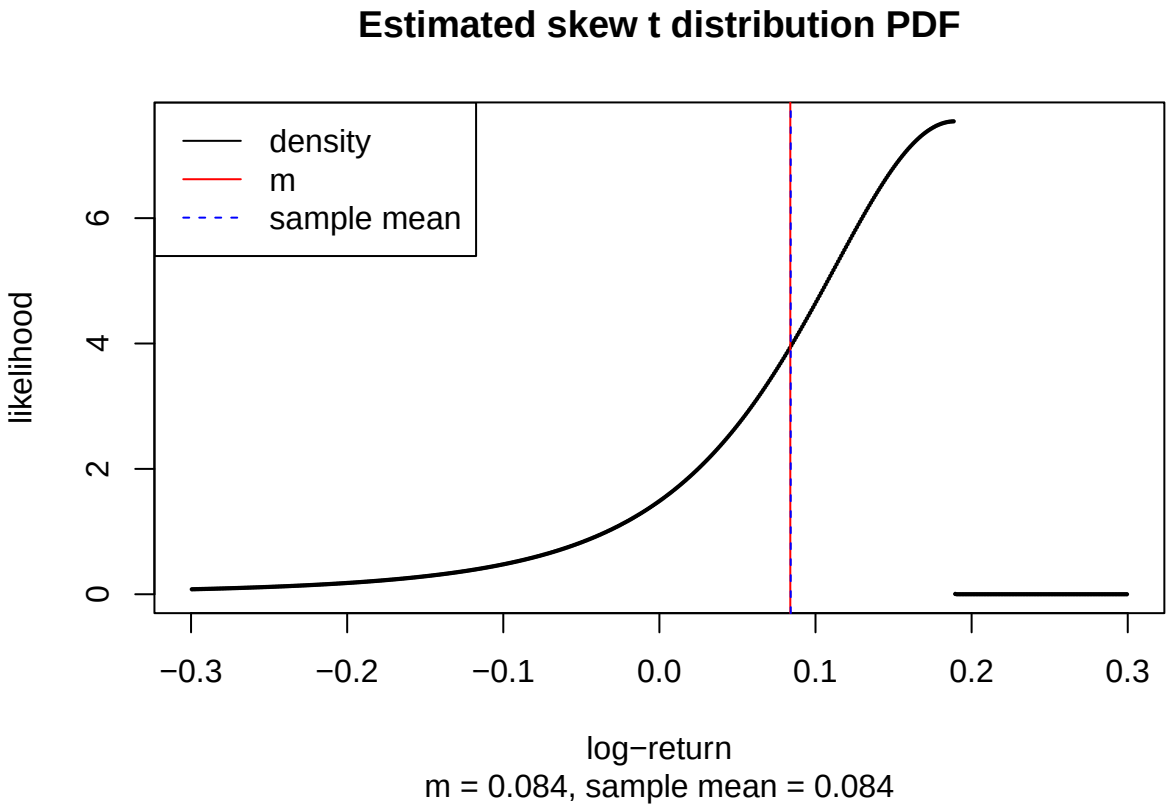


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

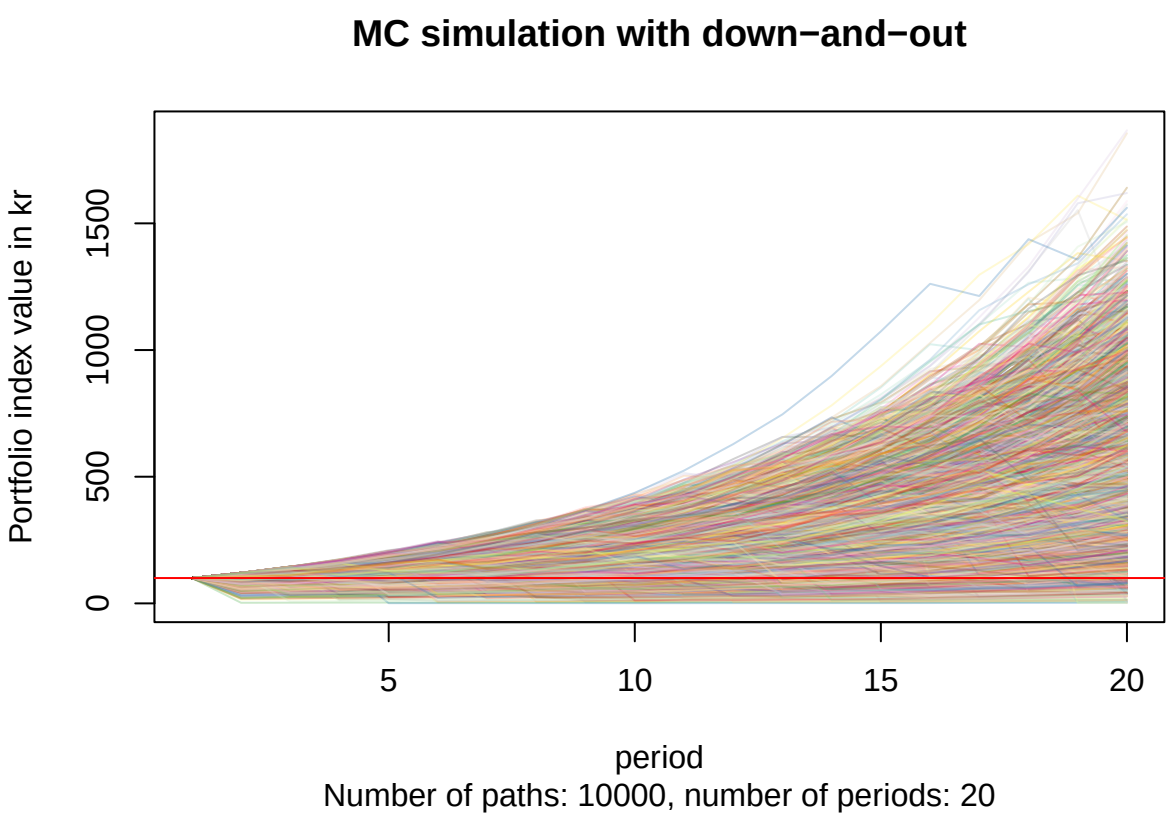


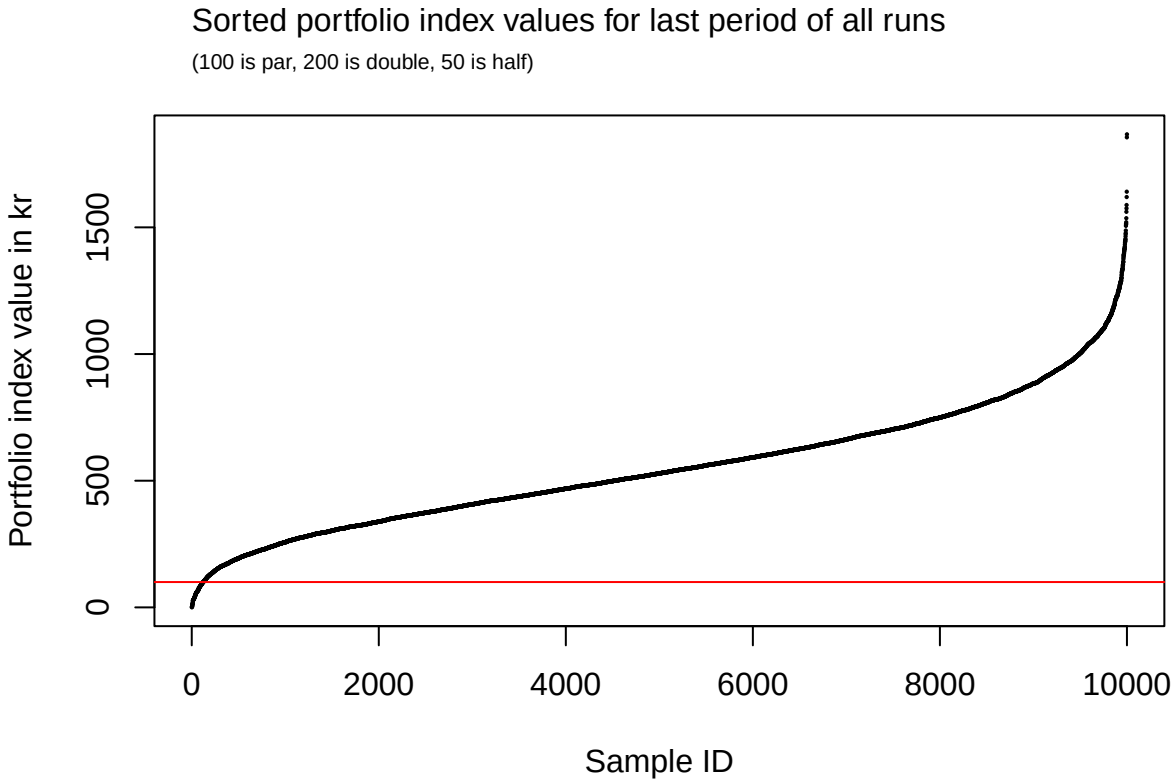
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.



Monte Carlo

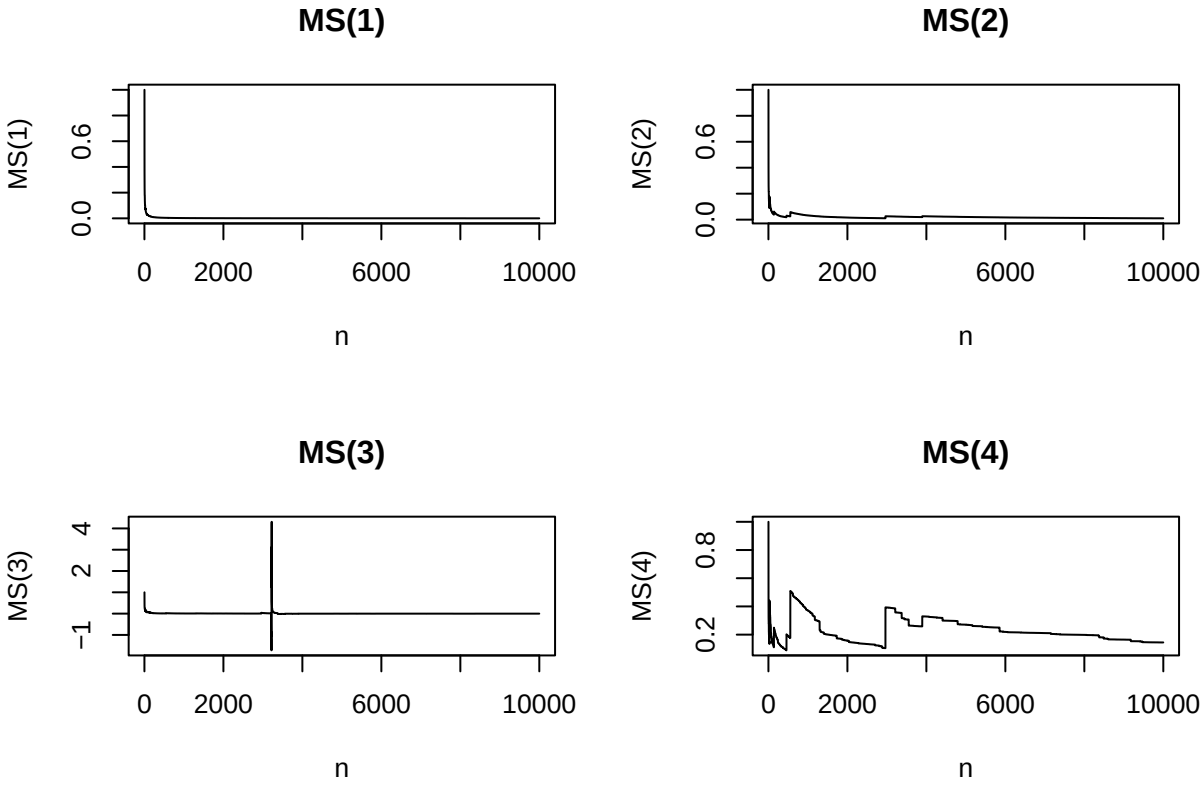
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 553.404 kr.
## SD of portfolio index value after 20 years: 247.536 kr.
## Min total portfolio index value after 20 years: 0.575 kr.
## Max total portfolio index value after 20 years: 1867.129 kr.
##
## Share of paths finishing below 100: 1.22 percent
```

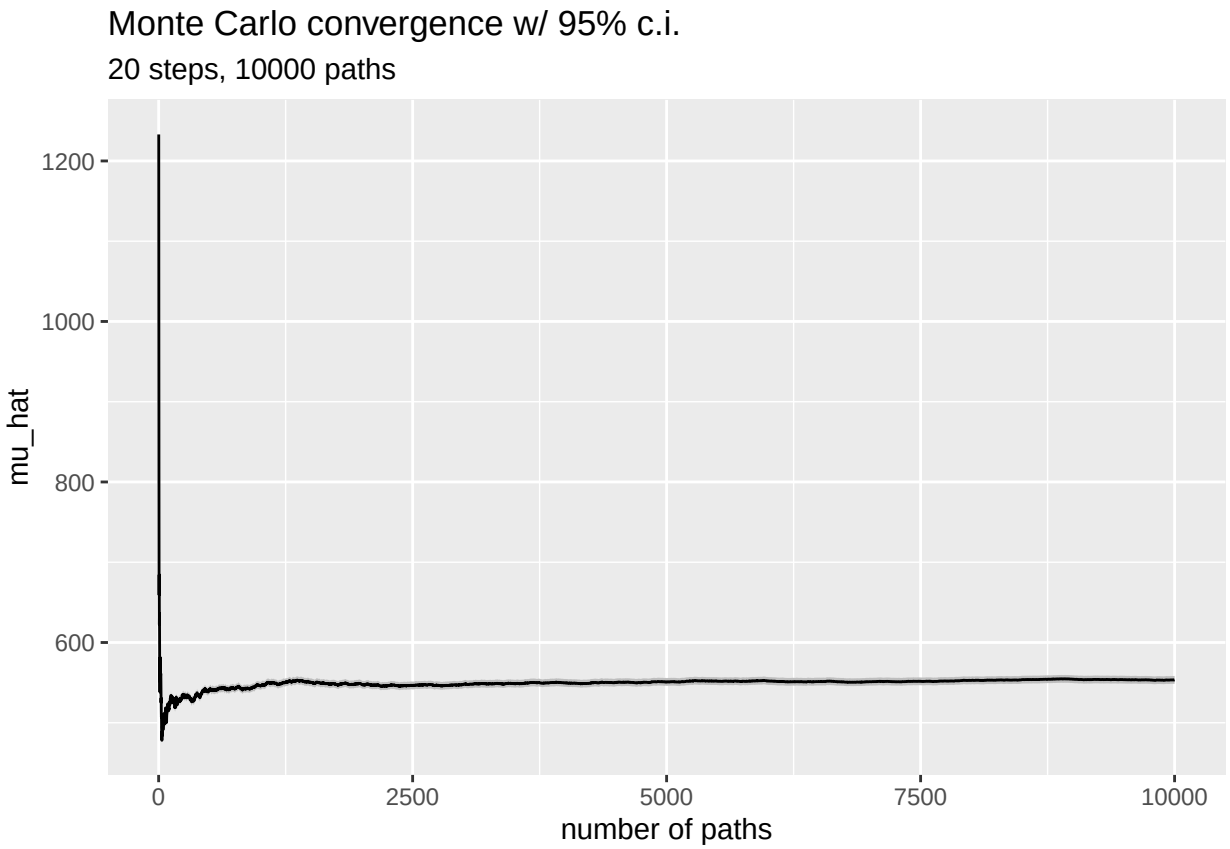

MC simulation with down−and−out
Portfolio index value in kr
0
500
1000
1500
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Convergence

Max vs sum plots for the first four moments:


MS(1)
MS(1)
0.0
0.6
0
2000
6000
10000
n
MS(2)
MS(2)
0.0
0.6
0
2000
6000
10000
n
MS(3)
MS(3)
−1
2
4
0
2000
6000
10000
n
MS(4)
MS(4)
0.2
0.8
0
2000
6000
10000
n

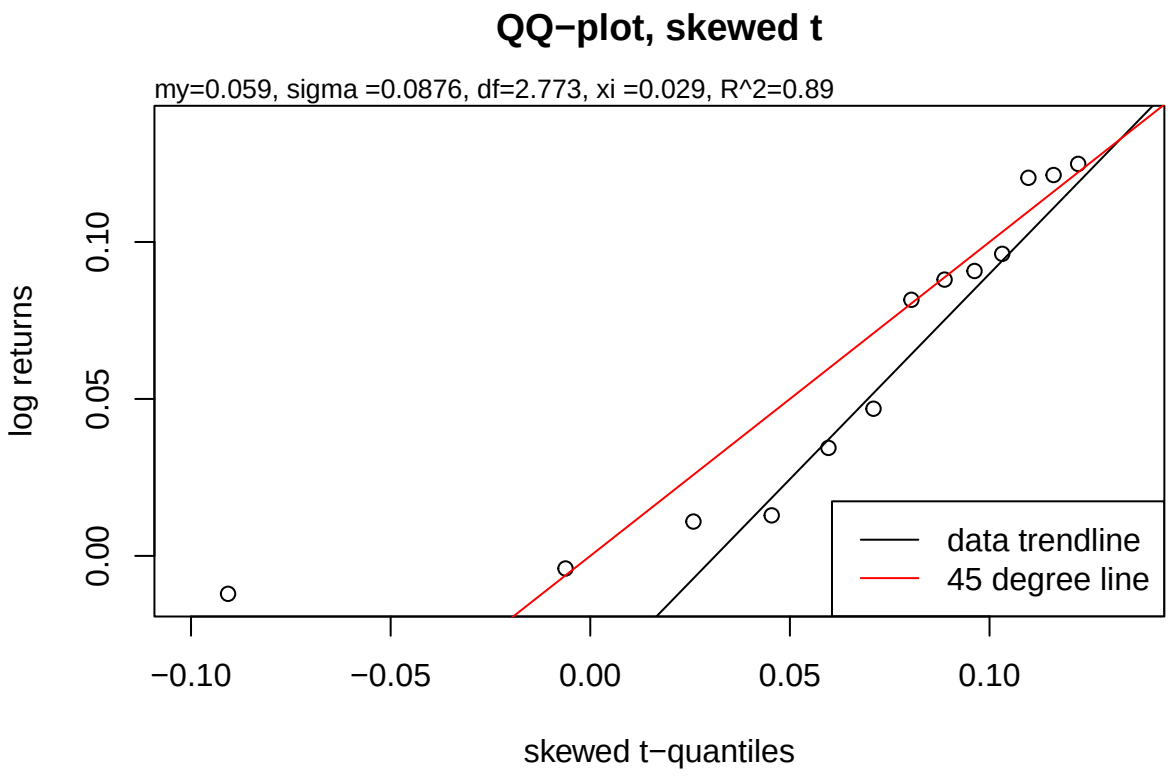


Mix medium risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -36.9603
## BIC: -34.7005
## m: 0.05902873
## s: 0.08757749
## nu (df): 2.772621
## xi: 0.02904471
## R^2: 0.89
##
## An R^2 of 0.89 suggests that the fit is not completely random.
##
## What is the risk of losing max 10 %? =< 3.3 percent
## What is the risk of losing max 25 %? =< 0.7 percent
## What is the risk of losing max 50 %? =< 0.1 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 35.6 percent
## What is the chance of gaining min 25 %? >= 0 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

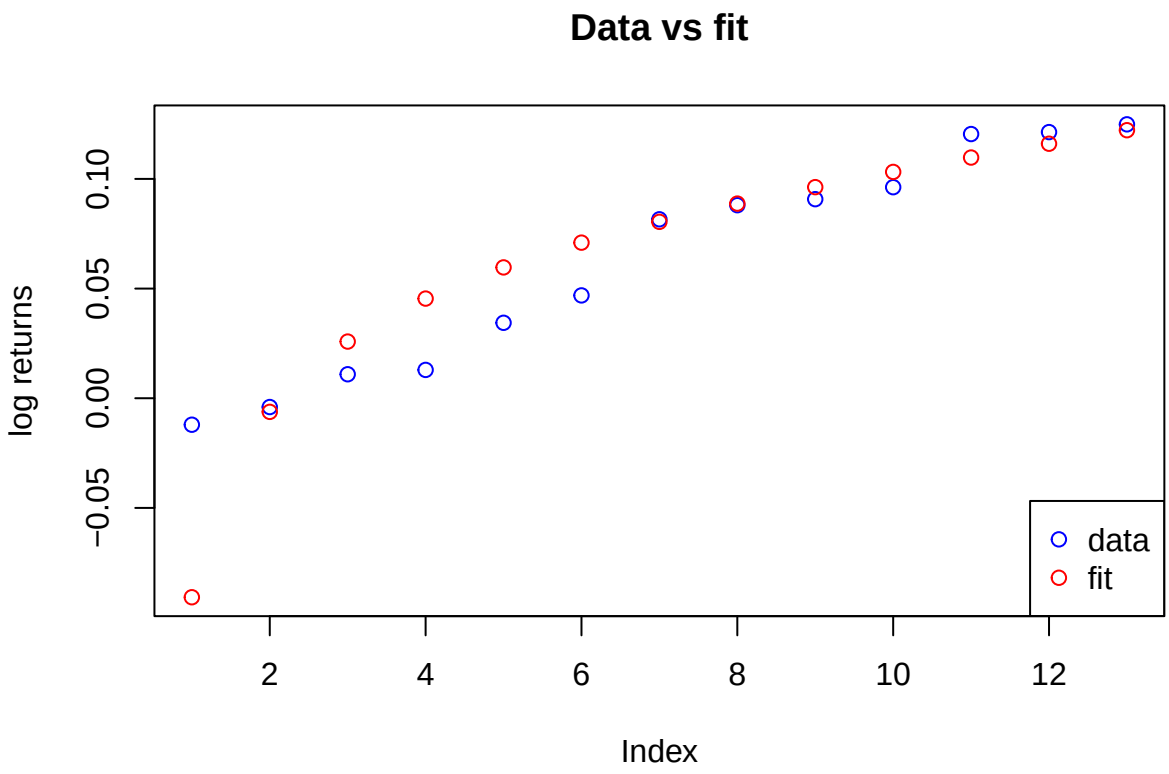
QQ Plot



The fit suggests big losses for the lowest percentiles, which are not present in the data.
So the fit is actually a very cautious estimate.

Data vs fit

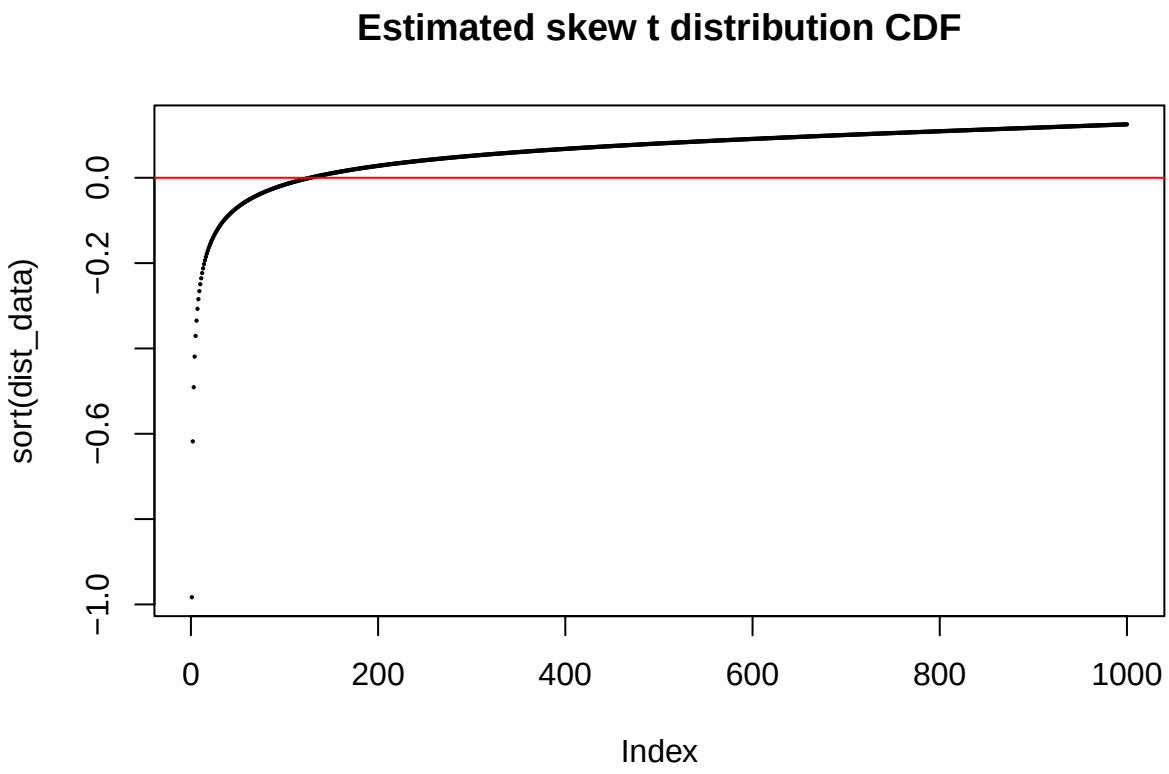
Let's plot the fit and the observed returns together.



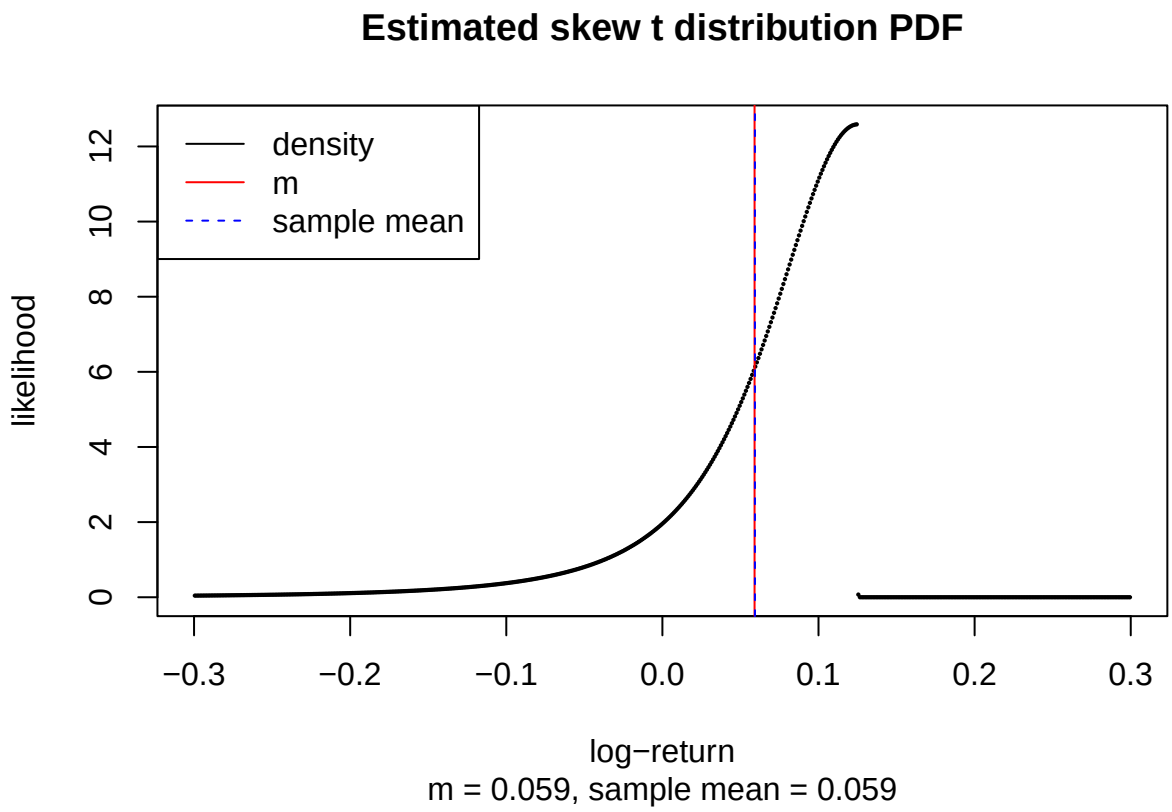
Interestingly, the fit predicts a much bigger “biggest loss” than the actual data. This is the main reason that R^2 is 0.90 and not higher.

Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



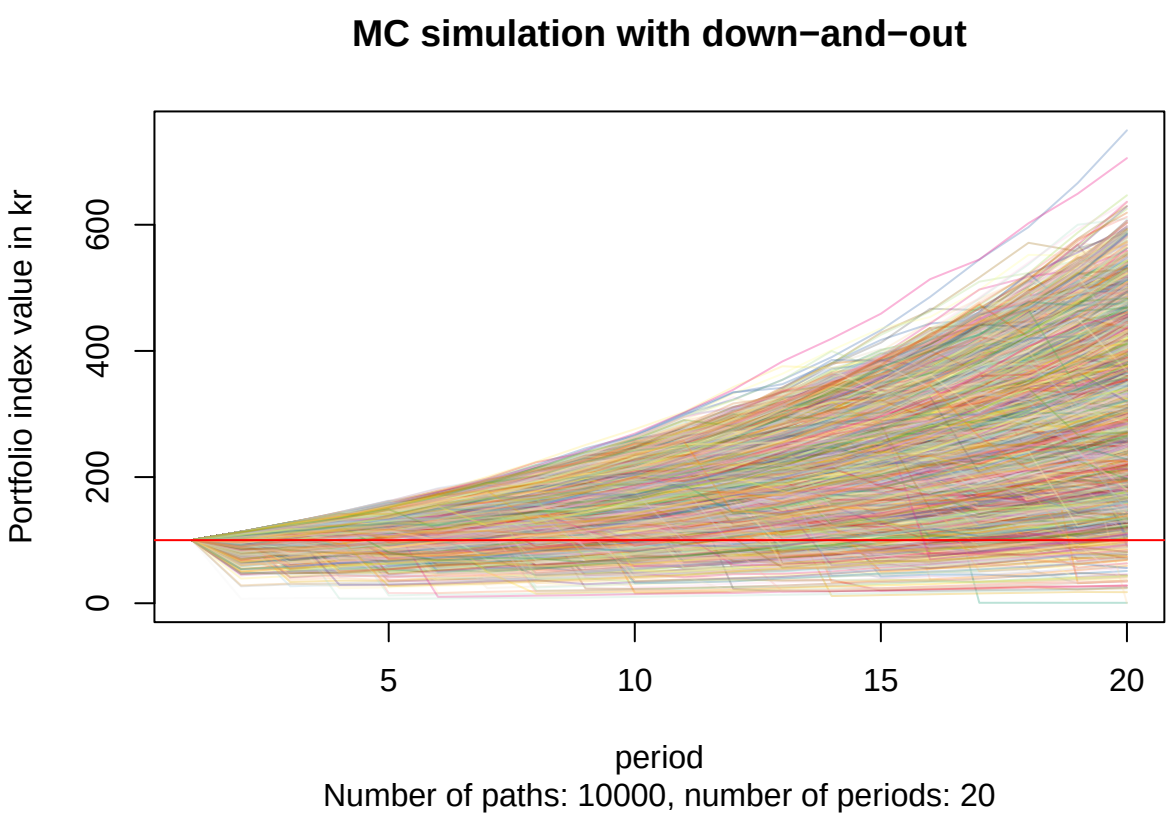
We see that for a few observations out of a 1000, the losses are disastrous, while the upside is very dampened.

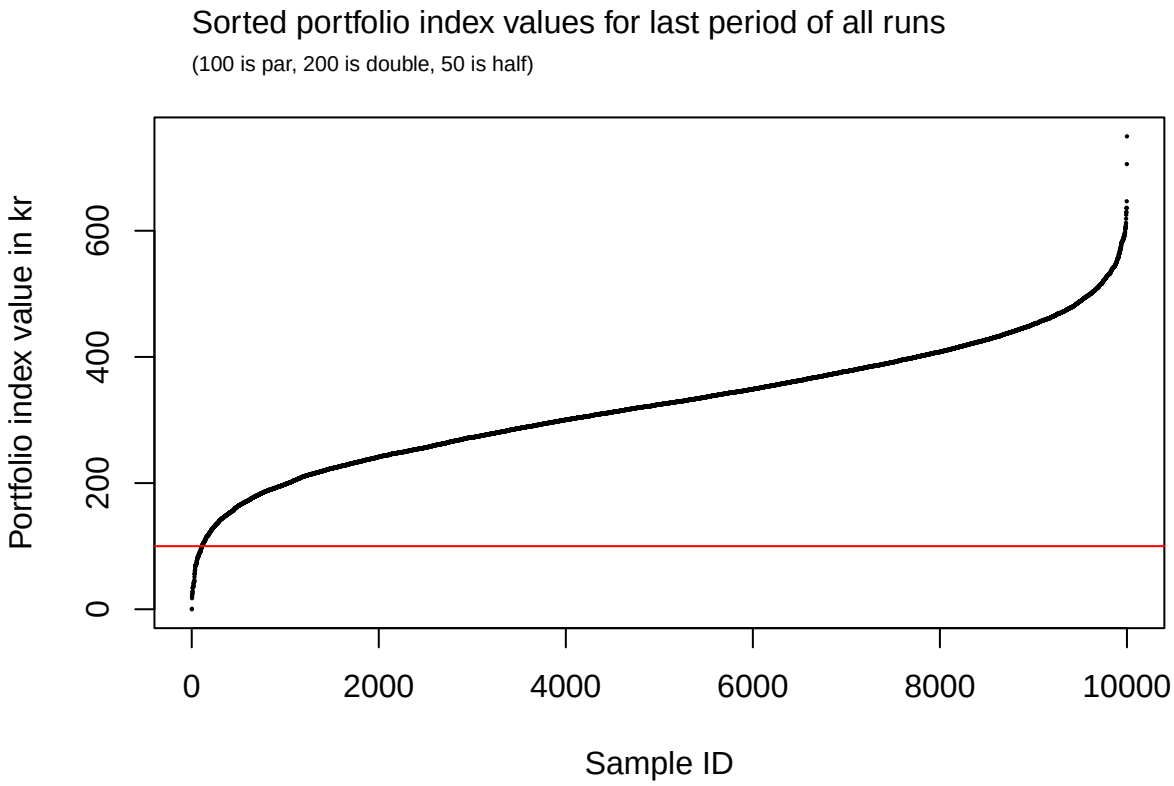


Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

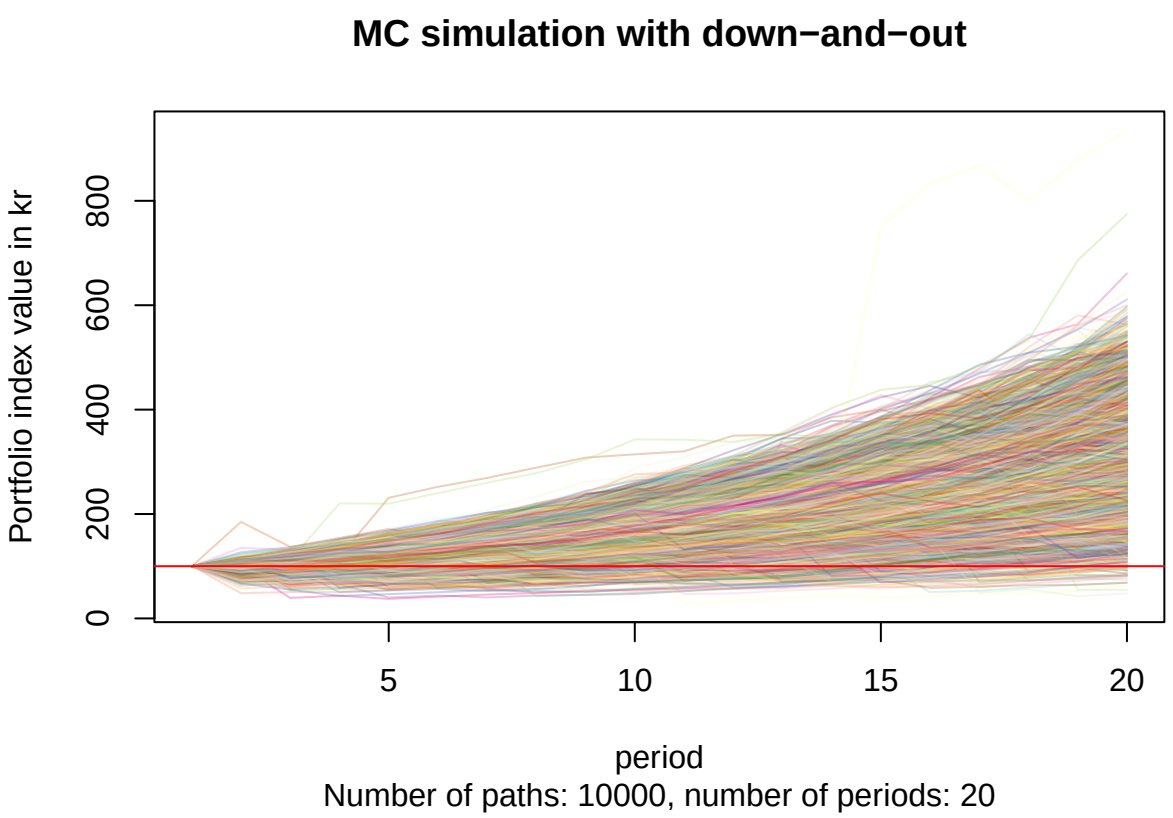
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 324.935 kr.
## SD of portfolio index value after 20 years: 98.607 kr.
## Min total portfolio index value after 20 years: 0 kr.
## Max total portfolio index value after 20 years: 749.655 kr.
##
## Share of paths finishing below 100: 1.09 percent
```

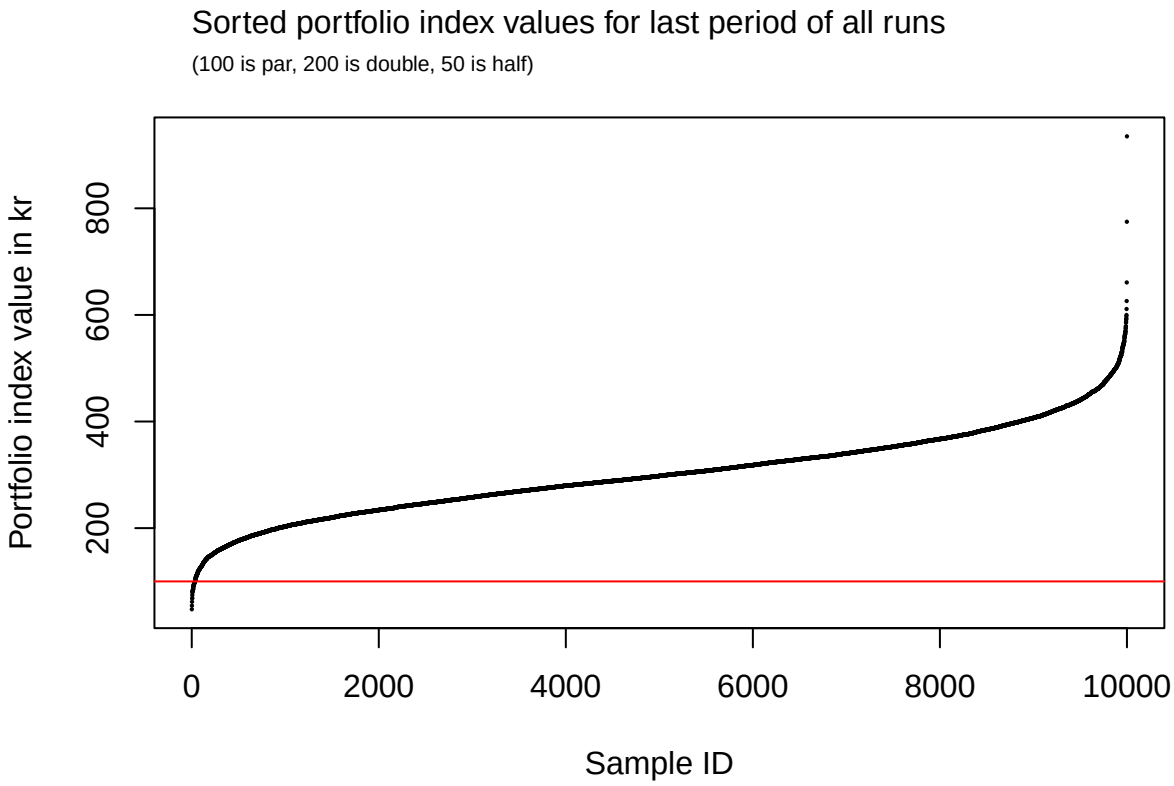

MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Version b: Mix of simulations from estimated distribution of returns from individual funds.

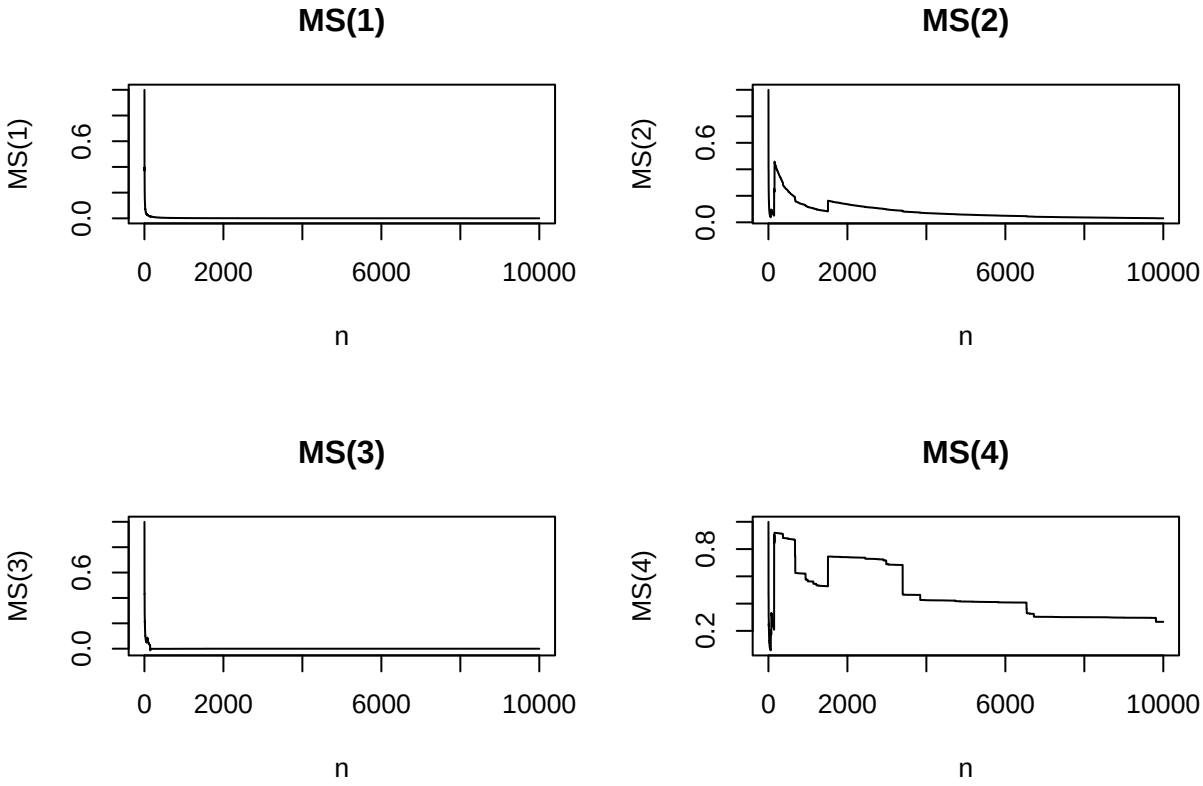
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 302.035 kr.
## SD of portfolio index value after 20 years: 80.643 kr.
## Min total portfolio index value after 20 years: 47.795 kr.
## Max total portfolio index value after 20 years: 935.034 kr.
##
## Share of paths finishing below 100: 0.34 percent
```

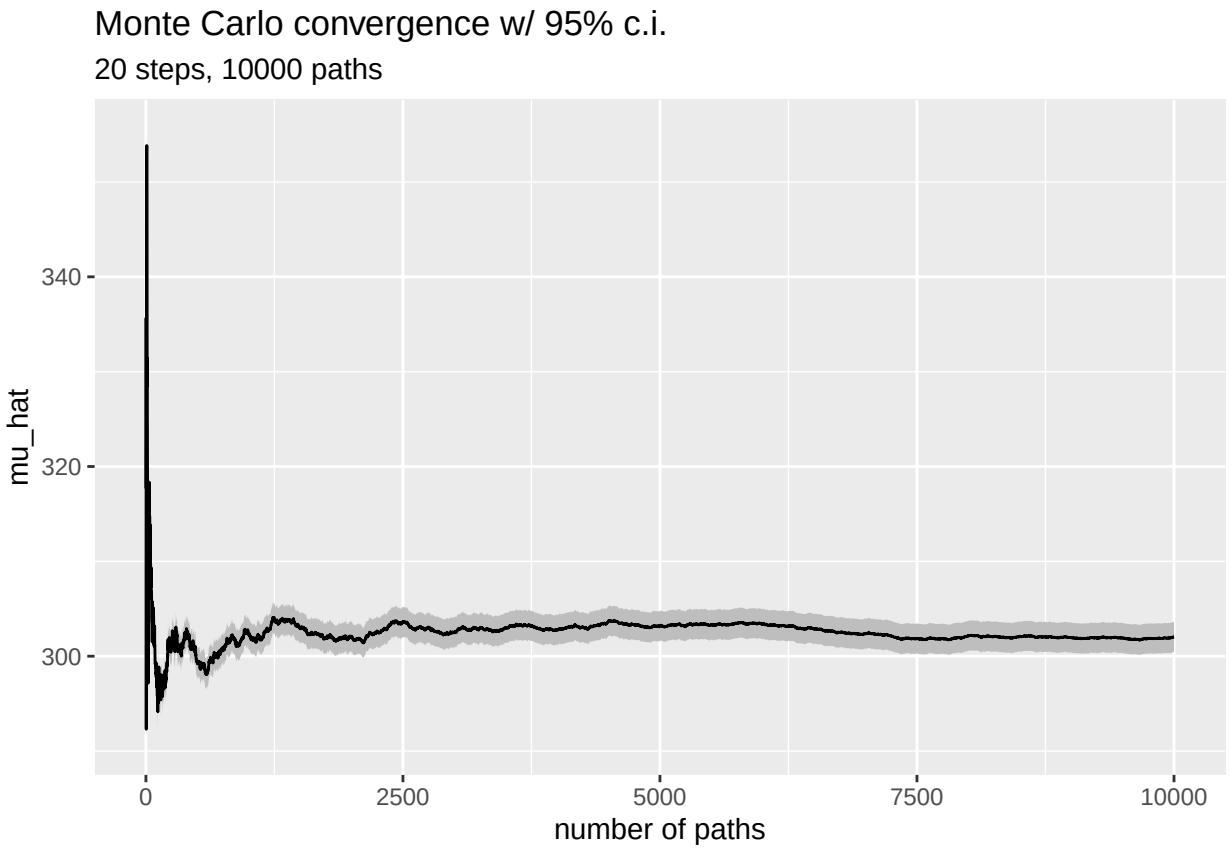

MC simulation with down−and−out
Portfolio index value in kr
0
200
400
600
800
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Convergence

Max vs sum plots for the first four moments:


MS(1)
MS(1)
0.0
0.6
0
2000
6000
10000
n
MS(2)
MS(2)
0.0
0.6
0
2000
6000
10000
n
MS(3)
MS(3)
0.0
0.6
0
2000
6000
10000
n
MS(4)
MS(4)
0.2
0.8
0
2000
6000
10000
n

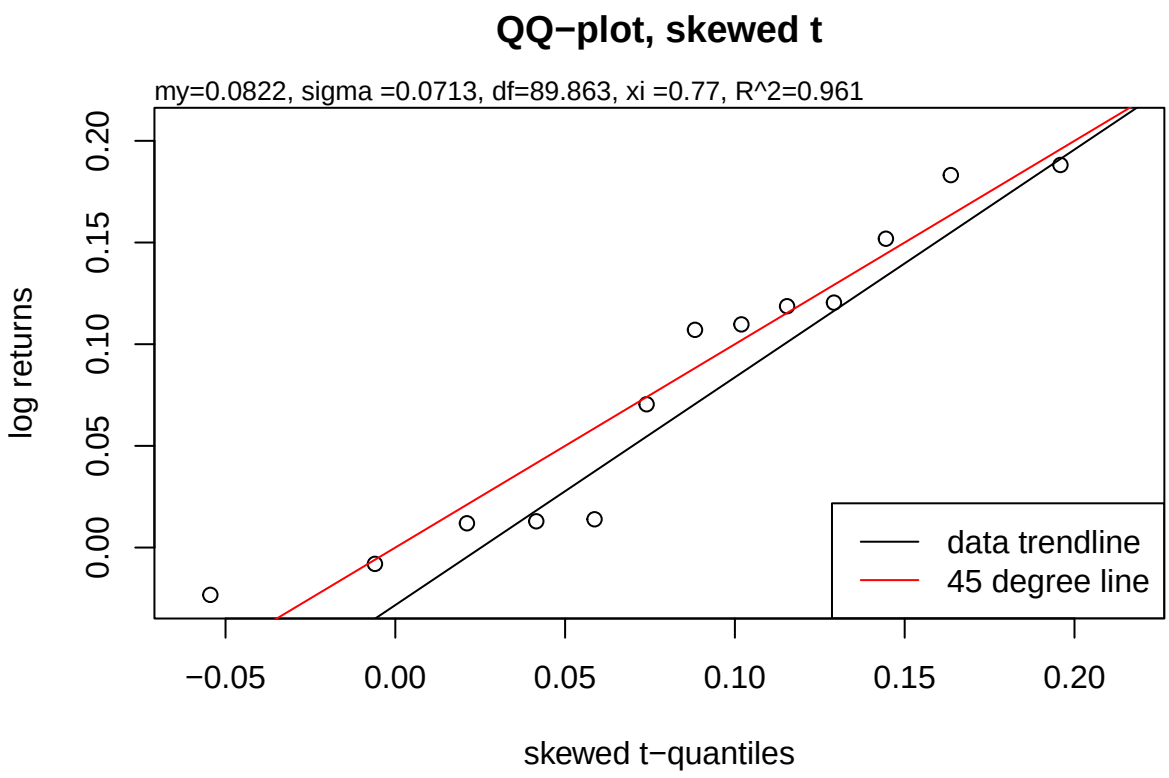


Mix high risk, 2011 - 2023

Fit to skew t distribution

```
##
## AIC: -24.26084
## BIC: -22.00104
## m: 0.0822419
## s: 0.07129843
## nu (df): 89.86289
## xi: 0.7697502
## R^2: 0.961
##
## An R^2 of 0.961 suggests that the fit is very good.
##
## What is the risk of losing max 10 %? =< 0.9 percent
## What is the risk of losing max 25 %? =< 0 percent
## What is the risk of losing max 50 %? =< 0 percent
## What is the risk of losing max 90 %? =< 0 percent
## What is the risk of losing max 99 %? =< 0 percent
##
## What is the chance of gaining min 10 %? >= 46.1 percent
## What is the chance of gaining min 25 %? >= 1.2 percent
## What is the chance of gaining min 50 %? >= 0 percent
## What is the chance of gaining min 90 %? >= 0 percent
## What is the chance of gaining min 99 %? >= 0 percent
```

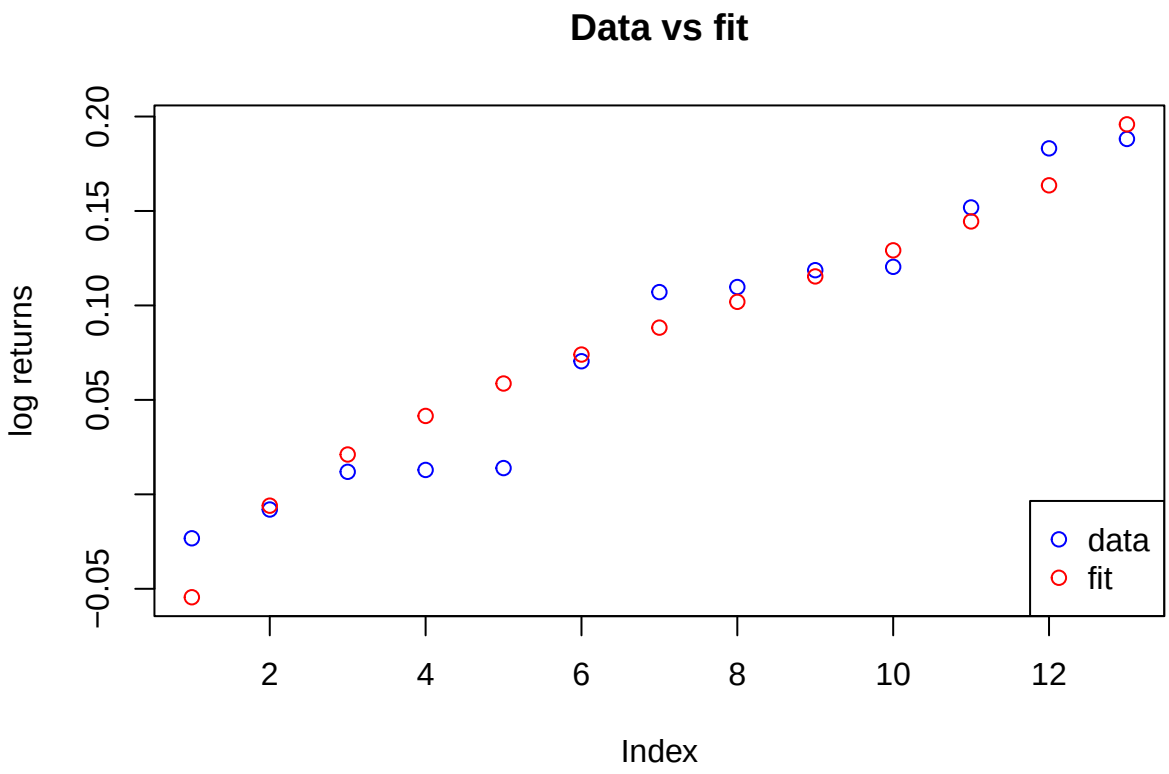
QQ Plot



The qq plot looks good Returns for mixed medium risk portfolios seems to be consistent with a skewed t-distribution.

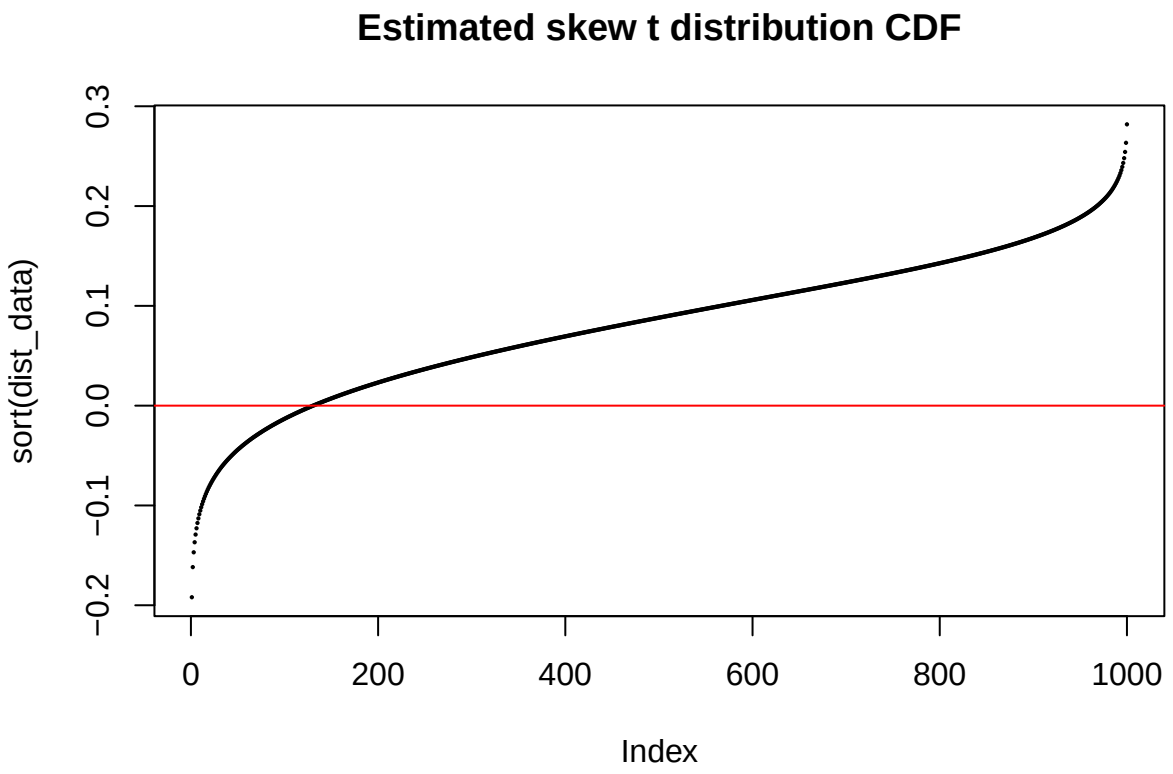
Data vs fit

Let's plot the fit and the observed returns together.

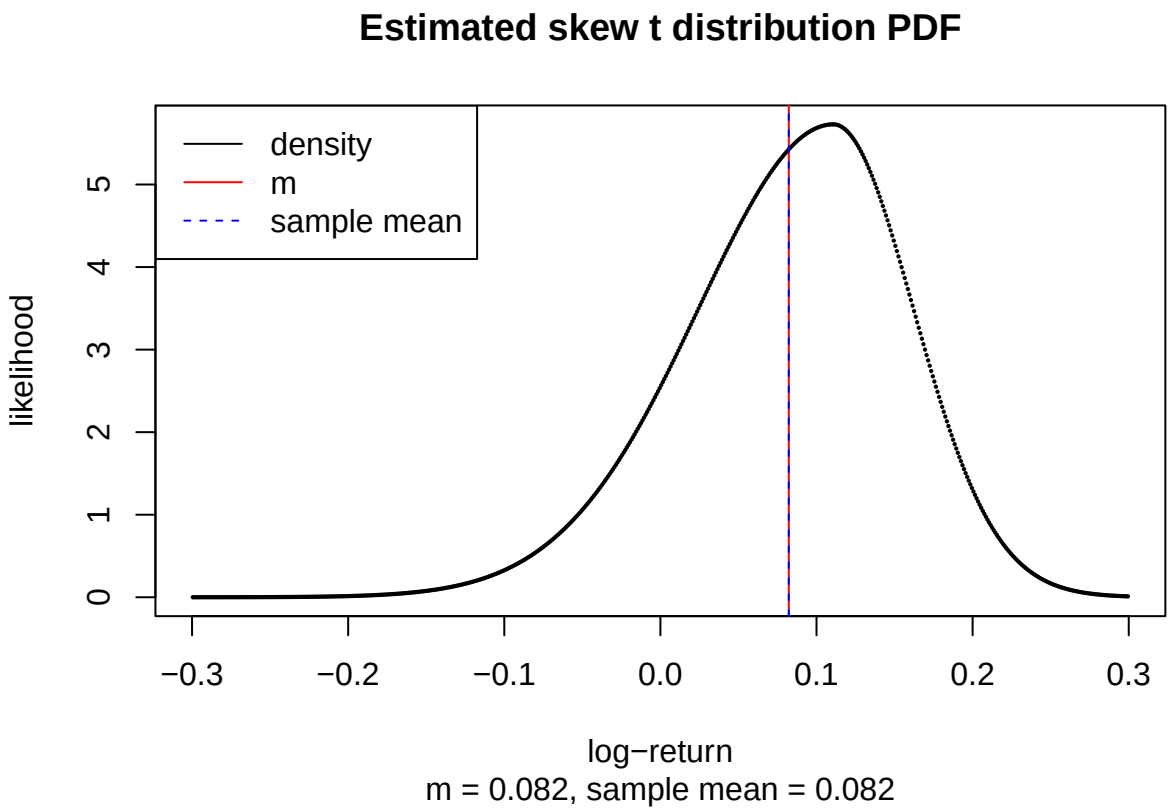


Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



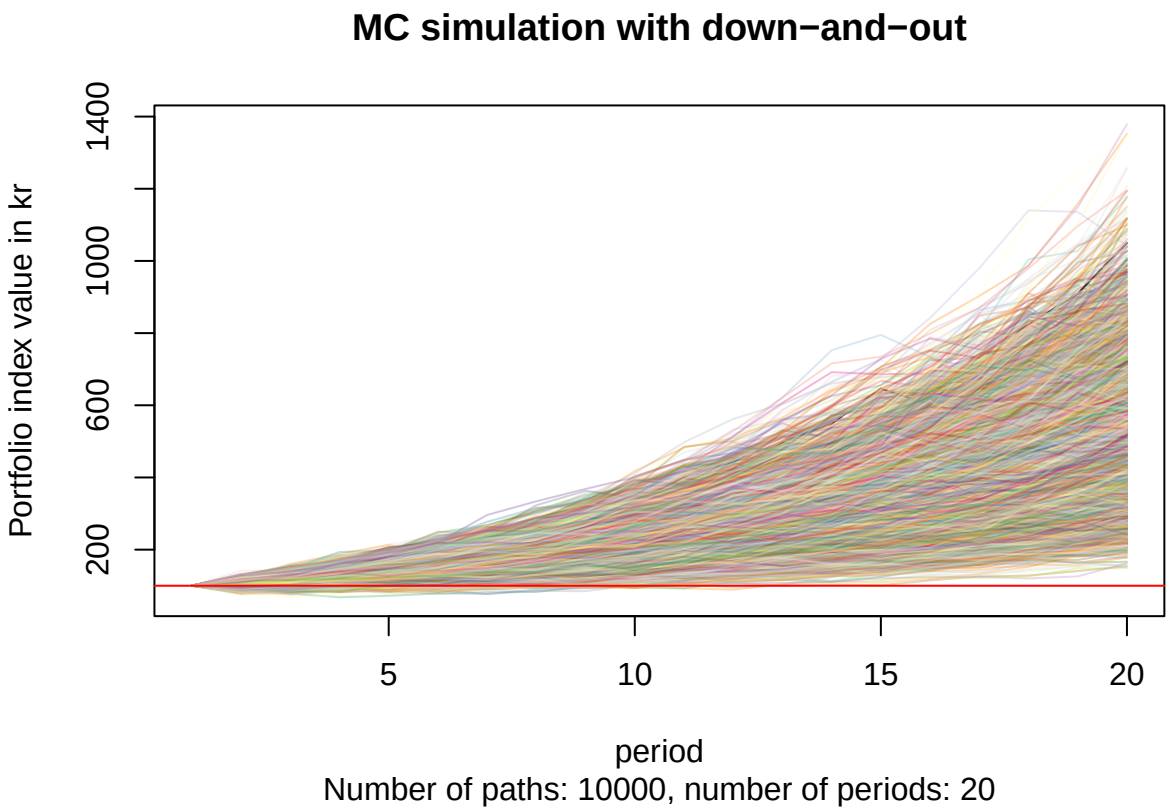
We see that the high risk mix provides a much better upside and smaller downside.

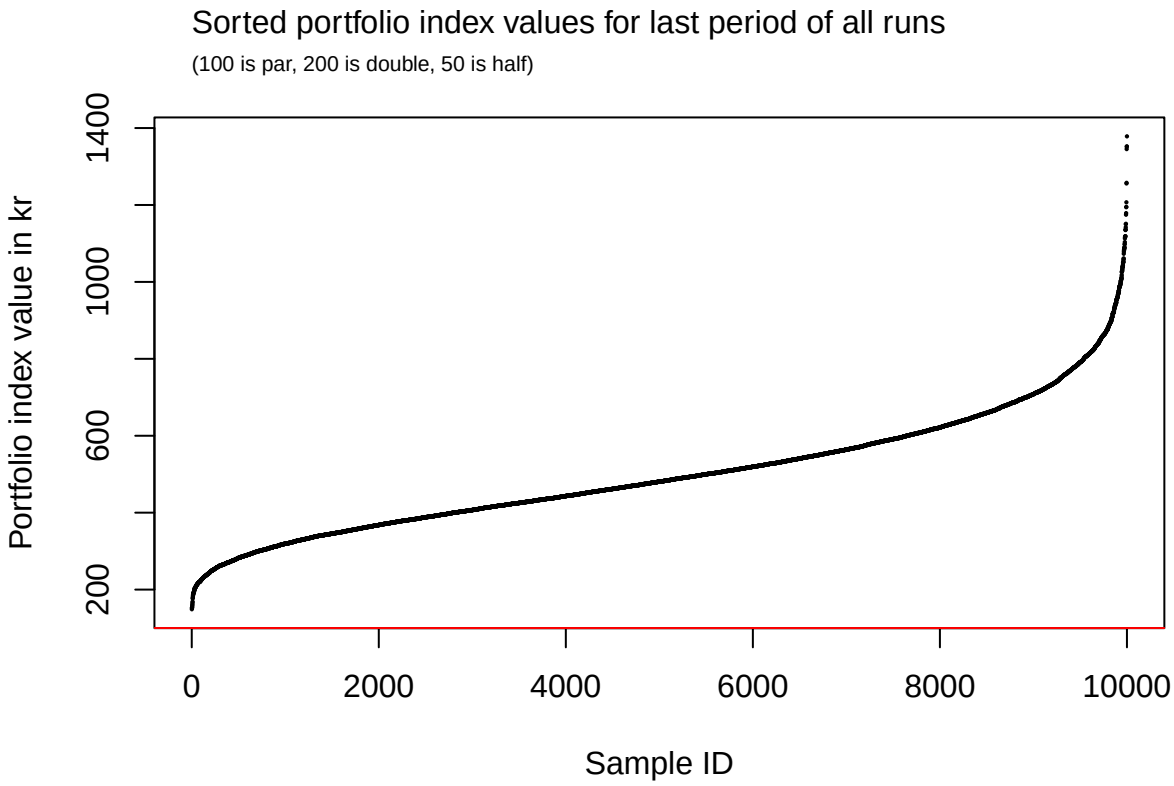


Monte Carlo

Version a: Simulation from estimated distribution of returns of mix.

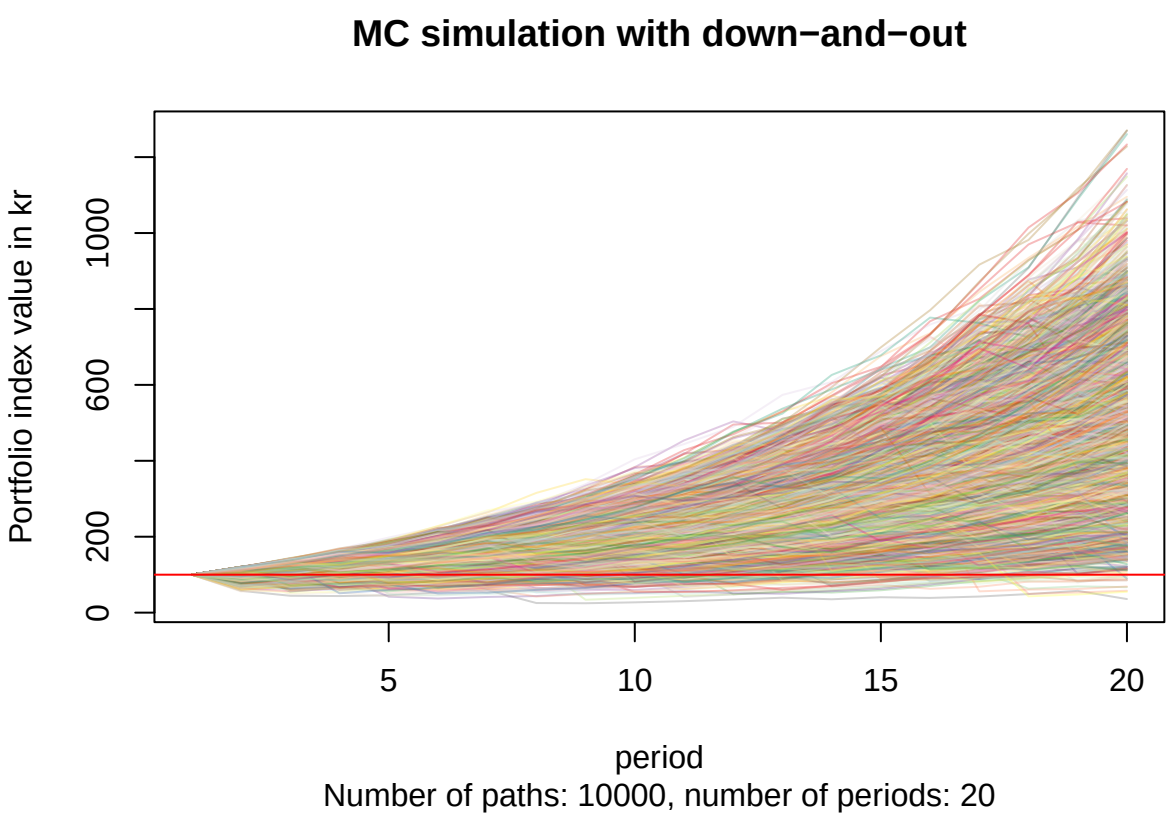
```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 500.807 kr.
## SD of portfolio index value after 20 years: 156.996 kr.
## Min total portfolio index value after 20 years: 148.911 kr.
## Max total portfolio index value after 20 years: 1378.438 kr.
##
## Share of paths finishing below 100: 0 percent
```

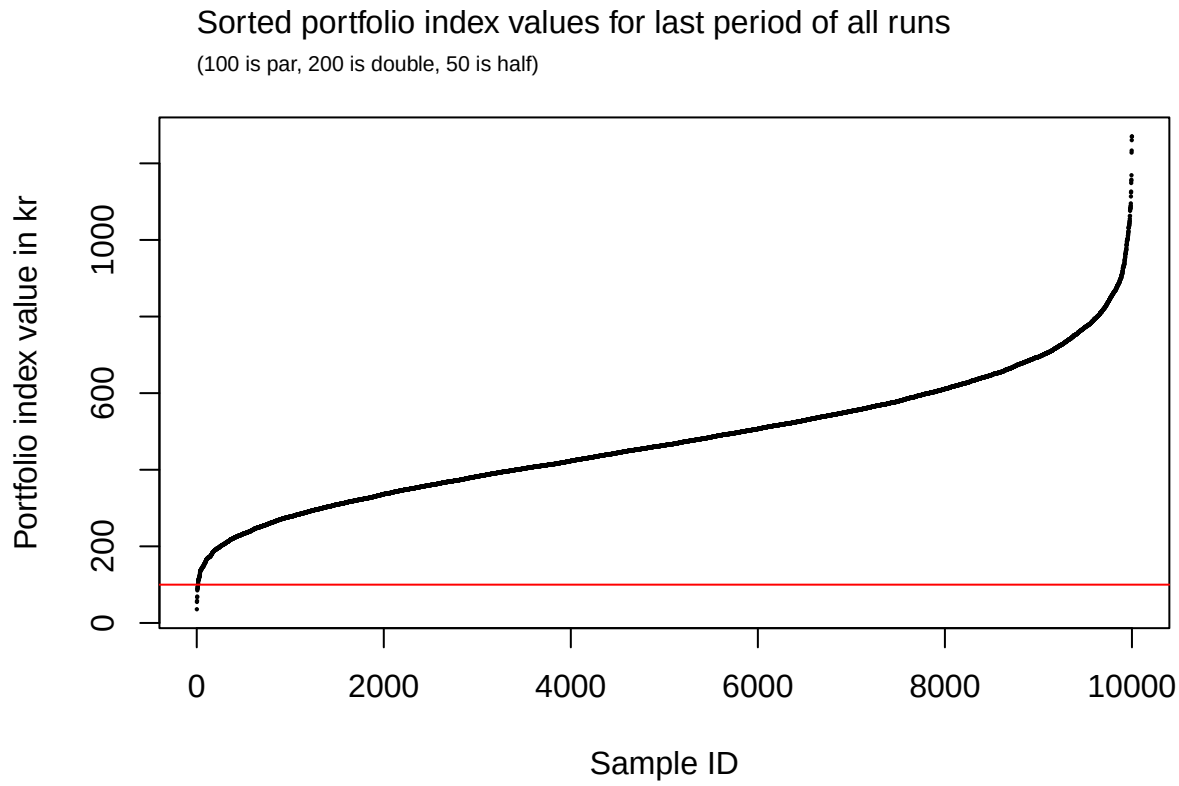

MC simulation with down−and−out
Portfolio index value in kr
200
600
1000
1400
5
10
15
20
period
Number of paths: 10000, number of periods: 20



Version b: Mix of simulations from estimated distribution of returns from individual funds.

```
## Down-and-out simulation:
## Probability of down-and-out: 0 percent
##
## Mean portfolio index value after 20 years: 478.043 kr.
## SD of portfolio index value after 20 years: 164.919 kr.
## Min total portfolio index value after 20 years: 35.802 kr.
## Max total portfolio index value after 20 years: 1270.507 kr.
##
## Share of paths finishing below 100: 0.13 percent
```


MC simulation with down−and−out
Portfolio index value in kr
0
200
600
1000
5
10
15
20
period
Number of paths: 10000, number of periods: 20

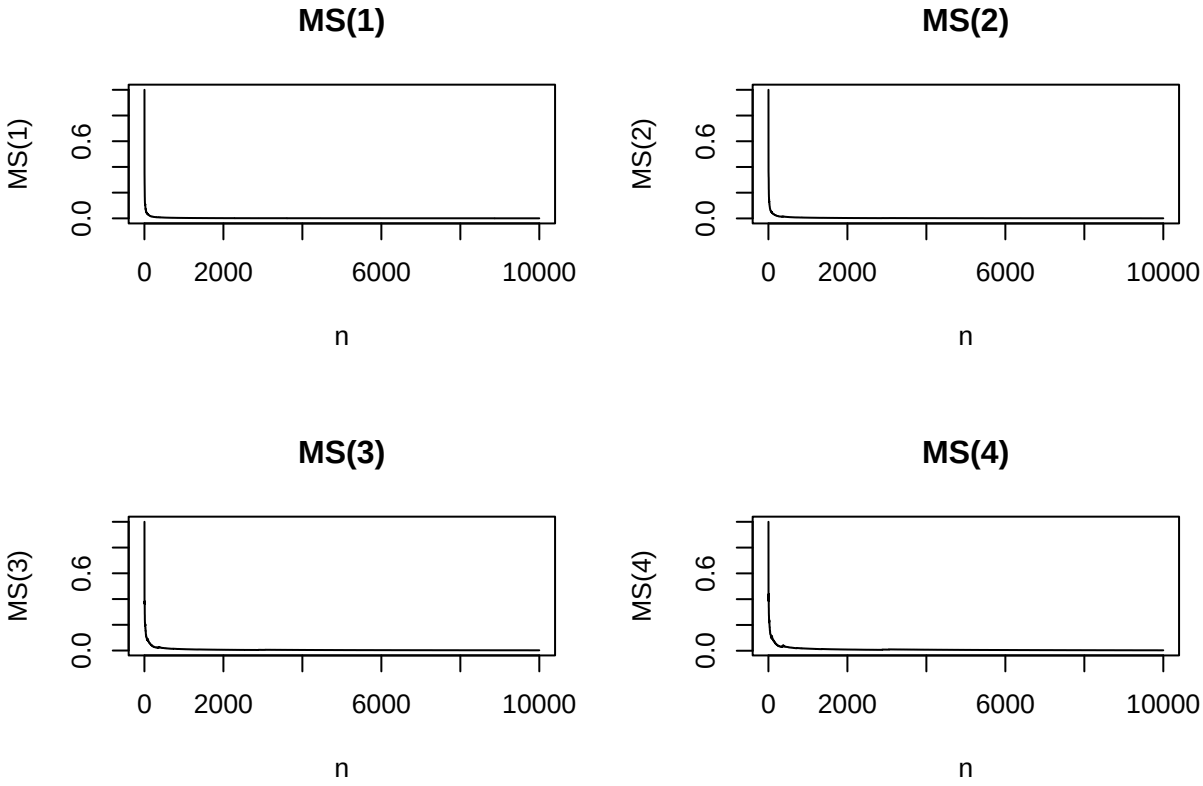


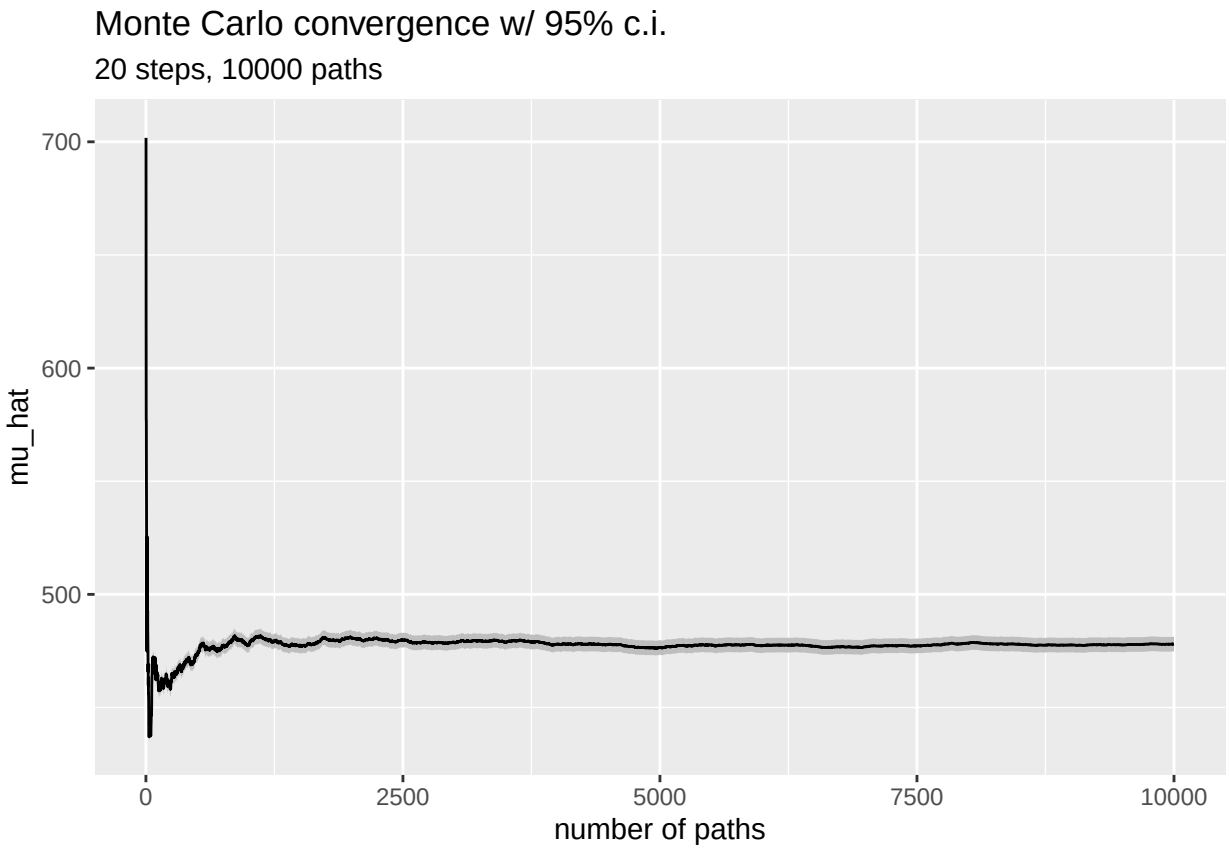
Many simulations 1e6 paths:

```
# Down-and-out simulation:
# Probability of down-and-out: 0 percent
#
# Mean portfolio index value after 20 years: 478.339 kr.
# SD of portfolio index value after 20 years: 163.093 kr.
# Min total portfolio index value after 20 years: 2.233 kr.
# Max total portfolio index value after 20 years: 1561.965 kr.
#
# Share of paths finishing below 100: 0.1181 percent
```

Convergence

Max vs sum plots for the first four moments:


MS(1)
MS(1)
0.0
0.6
0
2000
6000
10000
n
MS(2)
MS(2)
0.0
0.6
0
2000
6000
10000
n
MS(3)
MS(3)
0.0
0.6
0
2000
6000
10000
n
MS(4)
MS(4)
0.0
0.6
0
2000
6000
10000
n



Compare pension plans

Risk of max loss

Risk of max loss of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	21.3	18.2	19.9	12.2	14.3	12.7	13.0
5	12.5	9.6	12.8	6.0	8.6	6.2	4.2
10	7.4	5.4	8.3	3.3	5.3	3.3	0.9
25	1.8	1.3	2.5	0.9	1.4	0.7	0.0
50	0.2	0.2	0.4	0.2	0.2	0.1	0.0
90	0.0	0.0	0.0	0.0	0.0	0.0	0.0
99	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Worst ranking for loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.3	Velliv_m	12.8	Velliv_h	8.3	Velliv_h	2.5	Velliv_h	0.4	Velliv_h	0	Velliv_m	0	Velliv_m
19.9	Velliv_h	12.5	Velliv_m	7.4	Velliv_m	1.8	Velliv_m	0.2	Velliv_m	0	Velliv_m_l	0	Velliv_m_l
18.2	Velliv_m_l	9.6	Velliv_m_l	5.4	Velliv_m_l	1.4	PFA_h	0.2	Velliv_m_l	0	Velliv_h	0	Velliv_h
14.3	PFA_h	8.6	PFA_h	5.3	PFA_h	1.3	Velliv_m_l	0.2	PFA_m	0	PFA_m	0	PFA_m
13.0	mix_h	6.2	mix_m	3.3	PFA_m	0.9	PFA_m	0.2	PFA_h	0	PFA_h	0	PFA_h
12.7	mix_m	6.0	PFA_m	3.3	mix_m	0.7	mix_m	0.1	mix_m	0	mix_m	0	mix_m
12.2	PFA_m	4.2	mix_h	0.9	mix_h	0.0	mix_h	0.0	mix_h	0	mix_h	0	mix_h

Chance of min gains

Chance of min gains of x percent for a single period (year).
x values are row names.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m	mix_h
0	78.7	81.8	80.1	87.8	85.7	87.3	87.0
5	63.8	64.9	69.2	71.5	75.8	71.4	69.9
10	41.0	36.2	53.3	32.7	59.6	35.6	46.1
25	0.0	0.3	0.0	0.1	0.0	0.0	1.2
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Best ranking for gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
87.8	PFA_m	75.8	PFA_h	59.6	PFA_h	1.2	mix_h	0	Velliv_m	0	Velliv_m
87.3	mix_m	71.5	PFA_m	53.3	Velliv_h	0.3	Velliv_m_l	0	Velliv_m_l	0	Velliv_m_l
87.0	mix_h	71.4	mix_m	46.1	mix_h	0.1	PFA_m	0	Velliv_h	0	Velliv_h
85.7	PFA_h	69.9	mix_h	41.0	Velliv_m	0.0	Velliv_m	0	PFA_m	0	PFA_m
81.8	Velliv_m_l	69.2	Velliv_h	36.2	Velliv_m_l	0.0	Velliv_h	0	PFA_h	0	PFA_h
80.1	Velliv_h	64.9	Velliv_m_l	35.6	mix_m	0.0	PFA_h	0	mix_m	0	mix_m
78.7	Velliv_m	63.8	Velliv_m	32.7	PFA_m	0.0	mix_m	0	mix_h	0	mix_h

MC risk percentiles

Risk of loss from first to last period.

`_a` is simulation from estimated distribution of returns of mix.
`_b` is mix of simulations from estimated distribution of returns from individual funds.

`_m` is medium.
`_h` is high.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	4.82	3.16	4.02	1.85	1.22	1.09	0	0.34	0.13
5	4.22	2.77	3.68	1.70	1.10	0.97	0	0.24	0.11
10	3.64	2.40	3.48	1.51	1.04	0.87	0	0.17	0.08
25	2.18	1.55	2.50	1.12	0.75	0.53	0	0.06	0.05
50	0.95	0.72	1.25	0.58	0.40	0.28	0	0.01	0.01
90	0.07	0.08	0.19	0.19	0.06	0.02	0	0.00	0.00
99	0.02	0.01	0.04	0.07	0.01	0.02	0	0.00	0.00

1e6 simulation paths of `mhr_b`:

	0	5	10	25	50	90	99
prob_pct	0.118	0.095	0.076	0.036	0.008	0	0

Worst ranking for MC loss percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.82	Velliv_m	4.22	Velliv_m	3.64	Velliv_m	2.50	Velliv_h	1.25	Velliv_h	0.19	Velliv_h	0.07	PFA_m
4.02	Velliv_h	3.68	Velliv_h	3.48	Velliv_h	2.18	Velliv_m	0.95	Velliv_m	0.19	PFA_m	0.04	Velliv_h
3.16	Velliv_m_l	2.77	Velliv_m_l	2.40	Velliv_m_l	1.55	Velliv_m_l	0.72	Velliv_m_l	0.08	Velliv_m_l	0.02	Velliv_m
1.85	PFA_m	1.70	PFA_m	1.51	PFA_m	1.12	PFA_m	0.58	PFA_m	0.07	Velliv_m	0.02	mix_m_a
1.22	PFA_h	1.10	PFA_h	1.04	PFA_h	0.75	PFA_h	0.40	PFA_h	0.06	PFA_h	0.01	Velliv_m_l
1.09	mix_m_a	0.97	mix_m_a	0.87	mix_m_a	0.53	mix_m_a	0.28	mix_m_a	0.02	mix_m_a	0.01	PFA_h
0.34	mix_m_b	0.24	mix_m_b	0.17	mix_m_b	0.06	mix_m_b	0.01	mix_m_b	0.00	mix_h_a	0.00	mix_h_a

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.13	mix_h_b	0.11	mix_h_b	0.08	mix_h_b	0.05	mix_h_b	0.01	mix_h_b	0.00	mix_m_b	0.00	mix_m_b
0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_a	0.00	mix_h_b	0.00	mix_h_b

MC gains percentiles

Chance of gains from first to last period.
_a is simulation from estimated distribution of returns of mix.
_b is mix of simulations from estimated distribution of returns from individual funds.

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_h_a	mix_m_b	mix_h_b
0	95.18	96.84	95.98	98.15	98.78	98.91	100.00	99.66	99.87
5	94.46	96.45	95.43	97.99	98.67	98.76	100.00	99.61	99.85
10	93.61	96.04	94.92	97.73	98.55	98.59	100.00	99.51	99.83
25	91.09	94.52	93.35	97.00	98.18	97.92	100.00	99.11	99.68
50	85.38	90.51	90.42	95.26	97.34	96.21	99.99	97.85	99.26
100	70.94	79.32	82.74	87.99	94.64	89.69	99.71	90.78	97.49
200	39.22	45.02	63.75	58.58	85.27	60.11	92.83	49.00	86.55
300	16.41	17.40	44.63	21.95	71.02	22.47	71.94	11.45	65.80
400	5.46	5.06	28.95	4.10	54.80	3.89	44.95	1.20	41.56
500	1.43	1.27	16.88	0.52	38.73	0.20	23.55	0.05	21.71
1000	0.00	0.01	0.64	0.00	2.51	0.00	0.24	0.00	0.12

1e6 simulation paths of `mhr_b`:

	0	5	10	25	50	100	200	300	400	500	1000
prob	99.882	99.854	99.824	99.686	99.301	97.513	86.912	65.992	41.486	21.693	0.086

Best ranking for MC gains percentiles

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	100.00	mix_h_a	99.99	mix_h_a	99.71	mix_h_a
99.87	mix_h_b	99.85	mix_h_b	99.83	mix_h_b	99.68	mix_h_b	99.26	mix_h_b	97.49	mix_h_b
99.66	mix_m_b	99.61	mix_m_b	99.51	mix_m_b	99.11	mix_m_b	97.85	mix_m_b	94.64	PFA_h
98.91	mix_m_a	98.76	mix_m_a	98.59	mix_m_a	98.18	PFA_h	97.34	PFA_h	90.78	mix_m_b
98.78	PFA_h	98.67	PFA_h	98.55	PFA_h	97.92	mix_m_a	96.21	mix_m_a	89.69	mix_m_a
98.15	PFA_m	97.99	PFA_m	97.73	PFA_m	97.00	PFA_m	95.26	PFA_m	87.99	PFA_m
96.84	Velliv_m_l	96.45	Velliv_m_l	96.04	Velliv_m_l	94.52	Velliv_m_l	90.51	Velliv_m_l	82.74	Velliv_h
95.98	Velliv_h	95.43	Velliv_h	94.92	Velliv_h	93.35	Velliv_h	90.42	Velliv_h	79.32	Velliv_m_l
95.18	Velliv_m	94.46	Velliv_m	93.61	Velliv_m	91.09	Velliv_m	85.38	Velliv_m	70.94	Velliv_m

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
92.83	mix_h_a	71.94	mix_h_a	54.80	PFA_h	38.73	PFA_h	2.51	PFA_h
86.55	mix_h_b	71.02	PFA_h	44.95	mix_h_a	23.55	mix_h_a	0.64	Velliv_h
85.27	PFA_h	65.80	mix_h_b	41.56	mix_h_b	21.71	mix_h_b	0.24	mix_h_a
63.75	Velliv_h	44.63	Velliv_h	28.95	Velliv_h	16.88	Velliv_h	0.12	mix_h_b
60.11	mix_m_a	22.47	mix_m_a	5.46	Velliv_m	1.43	Velliv_m	0.01	Velliv_m_l
58.58	PFA_m	21.95	PFA_m	5.06	Velliv_m_l	1.27	Velliv_m_l	0.00	Velliv_m
49.00	mix_m_b	17.40	Velliv_m_l	4.10	PFA_m	0.52	PFA_m	0.00	PFA_m
45.02	Velliv_m_l	16.41	Velliv_m	3.89	mix_m_a	0.20	mix_m_a	0.00	mix_m_a
39.22	Velliv_m	11.45	mix_m_b	1.20	mix_m_b	0.05	mix_m_b	0.00	mix_m_b

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.
`m` is the location parameter.

`s` is the scale parameter.
`nu` is the estimated degrees of freedom, or shape parameter.
`xi` is the estimated skewness parameter.

	Velliv_medium	Velliv_medium_long	Velliv_high	PFA_medium	PFA_high	mix_medium	mix_high
m	0.048	0.052	0.065	0.058	0.084	0.059	0.082
s	0.120	0.115	0.150	0.123	0.121	0.088	0.071
nu	3.304	2.706	3.144	2.265	3.185	2.773	89.863
xi	0.034	0.505	0.002	0.477	0.018	0.029	0.770
R-squared	0.993	0.978	0.991	0.991	0.964	0.890	0.961

Fit statistics ranking

m	ranking	s	ranking	R-squared	ranking
0.084	PFA_high	0.071	mix_high	0.993	Velliv_medium
0.082	mix_high	0.088	mix_medium	0.991	Velliv_high
0.065	Velliv_high	0.115	Velliv_medium_long	0.991	PFA_medium
0.059	mix_medium	0.120	Velliv_medium	0.978	Velliv_medium_long
0.058	PFA_medium	0.121	PFA_high	0.964	PFA_high
0.052	Velliv_medium_long	0.123	PFA_medium	0.961	mix_high
0.048	Velliv_medium	0.150	Velliv_high	0.890	mix_medium

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).
Statistics are given for the final state of all paths.
Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.
0 is defined as any value below a threshold.
`losing_prob_pct` is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

	Velliv_m	Velliv_m_l	Velliv_h	PFA_m	PFA_h	mix_m_a	mix_m_b	mix_h_a	mix_h_b
mc_m	278.909	295.263	402.534	321.284	553.404	324.935	302.035	500.807	478.043
mc_s	124.477	117.560	217.340	103.871	247.536	98.607	80.643	156.996	164.919
mc_min	0.426	0.066	0.000	0.000	0.575	0.000	47.795	148.911	35.802
mc_max	884.639	1162.234	1862.587	805.612	1867.129	749.655	935.034	1378.438	1270.507
dao_pct	0.000	0.000	0.010	0.010	0.000	0.000	0.000	0.000	0.000
losing_pct	4.820	3.160	4.020	1.850	1.220	1.090	0.340	0.000	0.130

Ranking

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	losing_pct	ranking
553.404	PFA_h	80.643	mix_m_b	148.911	mix_h_a	1867.129	PFA_h	0.00	Velliv_m	0.00	mix_h_a
500.807	mix_h_a	98.607	mix_m_a	47.795	mix_m_b	1862.587	Velliv_h	0.00	Velliv_m_l	0.13	mix_h_b
478.043	mix_h_b	103.871	PFA_m	35.802	mix_h_b	1378.438	mix_h_a	0.00	PFA_h	0.34	mix_m_b
402.534	Velliv_h	117.560	Velliv_m_l	0.575	PFA_h	1270.507	mix_h_b	0.00	mix_m_a	1.09	mix_m_a
324.935	mix_m_a	124.477	Velliv_m	0.426	Velliv_m	1162.234	Velliv_m_l	0.00	mix_m_b	1.22	PFA_h
321.284	PFA_m	156.996	mix_h_a	0.066	Velliv_m_l	935.034	mix_m_b	0.00	mix_h_a	1.85	PFA_m
302.035	mix_m_b	164.919	mix_h_b	0.000	mix_m_a	884.639	Velliv_m	0.00	mix_h_b	3.16	Velliv_m_l
295.263	Velliv_m_l	217.340	Velliv_h	0.000	Velliv_h	805.612	PFA_m	0.01	Velliv_h	4.02	Velliv_h
278.909	Velliv_m	247.536	PFA_h	0.000	PFA_m	749.655	mix_m_a	0.01	PFA_m	4.82	Velliv_m

Comments

(Ignoring `mhr_a`...)

`mhr` has some nice properties:
- It has a relatively high `nu` value of 90, which means it is tending more towards exponential tails than polynomial tails. All other funds have `nu` values close to 3, except `phr` which is even worse at close to 2. (Note that for a Gaussian, `nu` is infinite.)
- It has the lowest losing percentage of all simulations, which is better than 1/6 that of `phr`.
- It has a DAO percentage of 0, which is the same as `mmr`, and less than `phr`.
- Only `phr` has a higher `mc_m`.
- It has a smaller `mc_s` than the individual components, `vhr` and `phr`.
- It has the highest `xi` of all fits, suggesting less left skewness. Density plots for `vmr`, `phr` and `mmr` have an extremely sharp drop, as if an upward limiter has been applied, which corresponds to extremely low `xi` values. The density plot for `mhr` is by far the most symmetrical of all the fits.
- Only `mmr` has as higher `mc_min`. However, that of `mmr` is 18 times higher with 62, so `mmr` is a clear winner here.
- Naturally, it has a `mc_max` smaller than the individual components, `vhr` and `phr`, but ca. 1.5 times higher then `mmr`.
- All the first 4 moments converge nicely. For all other fits, the 4th moment doesn't seem to converge.

Taleb, Statistical Consequences Of Fat Tails, p. 97:
"the variance of a finite variance random variable with tail exponent < 4 will be infinite".

And p. 363:
"The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, un- der a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance."

Appendix

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2}\right) \Big/ \left(\frac{x_{t-1} + y_{t-1}}{2}\right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns $=$ Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2\frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1})x_t y_{t-1} + (x_{t-1} + y_{t-1})x_{t-1} y_t = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

$$(x_{t-1} x_1 y_{t-1} + y_{t-1} x_t y_{t-1}) + (x_{t-1} x_{t-1} y_t + x_{t-1} y_{t-1} y_t) = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: `R = 1+r`

```
## Let x_0 be 100.

## Let y_0 be 200.

## So the initial value of the pf is 300 .

## Let R_x be 0.5.

## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.
```

```
## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): -0.09094363
## s(data_x): 0.3843497
## m(data_y): 11.14102
## s(data_y): 2.903419
##
## m(data_x + data_y): 5.525036
## s(data_x + data_y): 1.483679
```

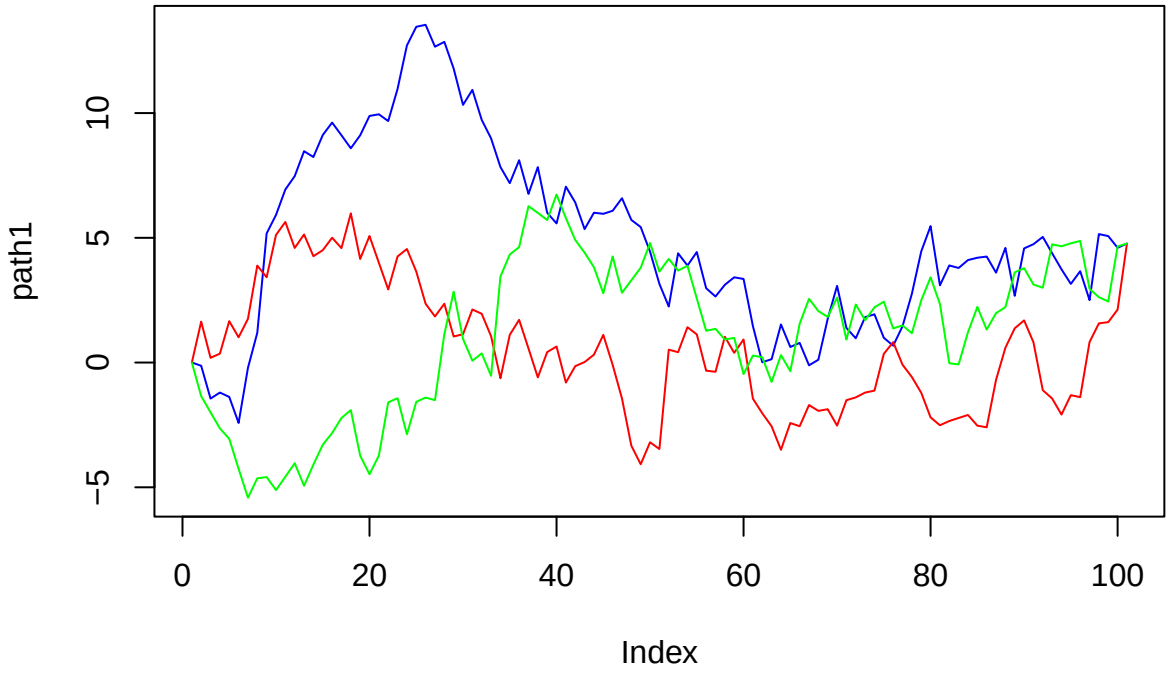
m and s of final state of all paths.
_a is mix of simulated returns.
_b is simulated mixed returns.

m_a	m_b	s_a	s_b
110.575	110.118	6.586	6.512
110.672	110.236	6.552	6.591
110.669	110.763	6.604	6.524
110.664	110.768	6.389	6.608
110.467	110.446	6.402	6.390
110.560	110.526	6.574	6.588
110.501	110.204	6.397	6.429
110.447	110.373	6.570	6.585
110.588	110.519	6.708	6.681
110.570	110.661	6.593	6.755

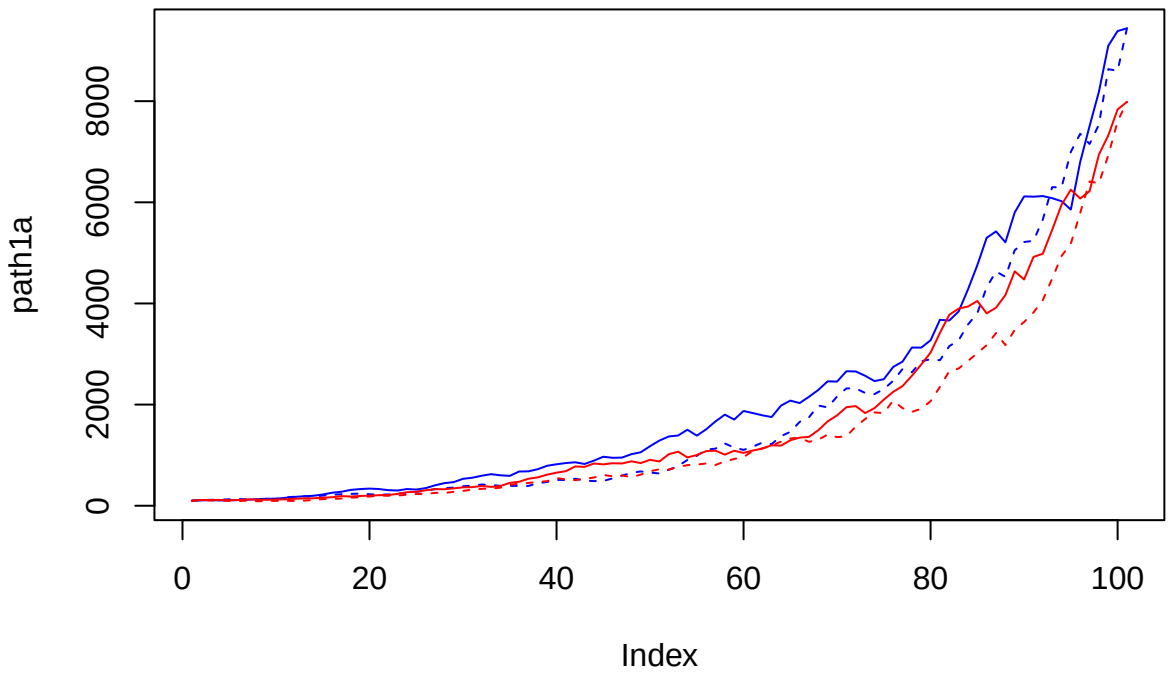
```
##       m_a              m_b              s_a              s_b        
##  Min.   :110.4   Min.   :110.1   Min.   :6.389   Min.   :6.390  
##  1st Qu.:110.5   1st Qu.:110.3   1st Qu.:6.440   1st Qu.:6.515  
##  Median :110.6   Median :110.5   Median :6.572   Median :6.586  
##  Mean   :110.6   Mean   :110.5   Mean   :6.538   Mean   :6.566  
##  3rd Qu.:110.6   3rd Qu.:110.6   3rd Qu.:6.592   3rd Qu.:6.604  
##  Max.   :110.7   Max.   :110.8   Max.   :6.708   Max.   :6.755
```

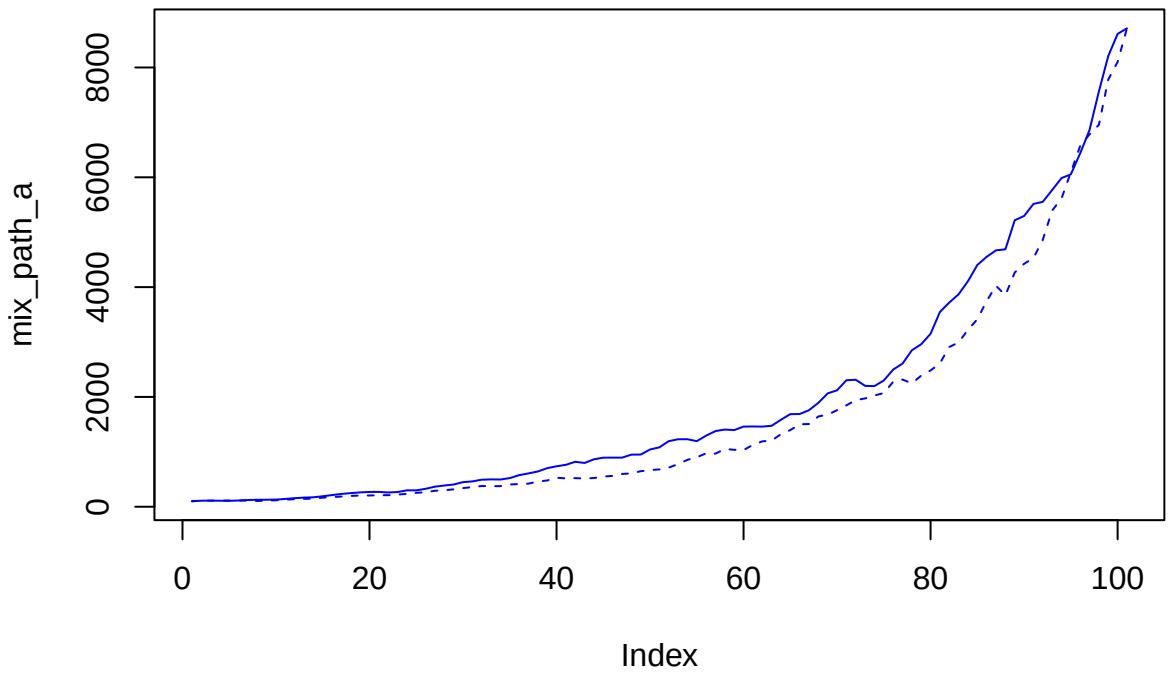
_a and _b are very close to equal.
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?


path1a
0
2000
4000
6000
8000
0
20
40
60
80
100
Index



The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

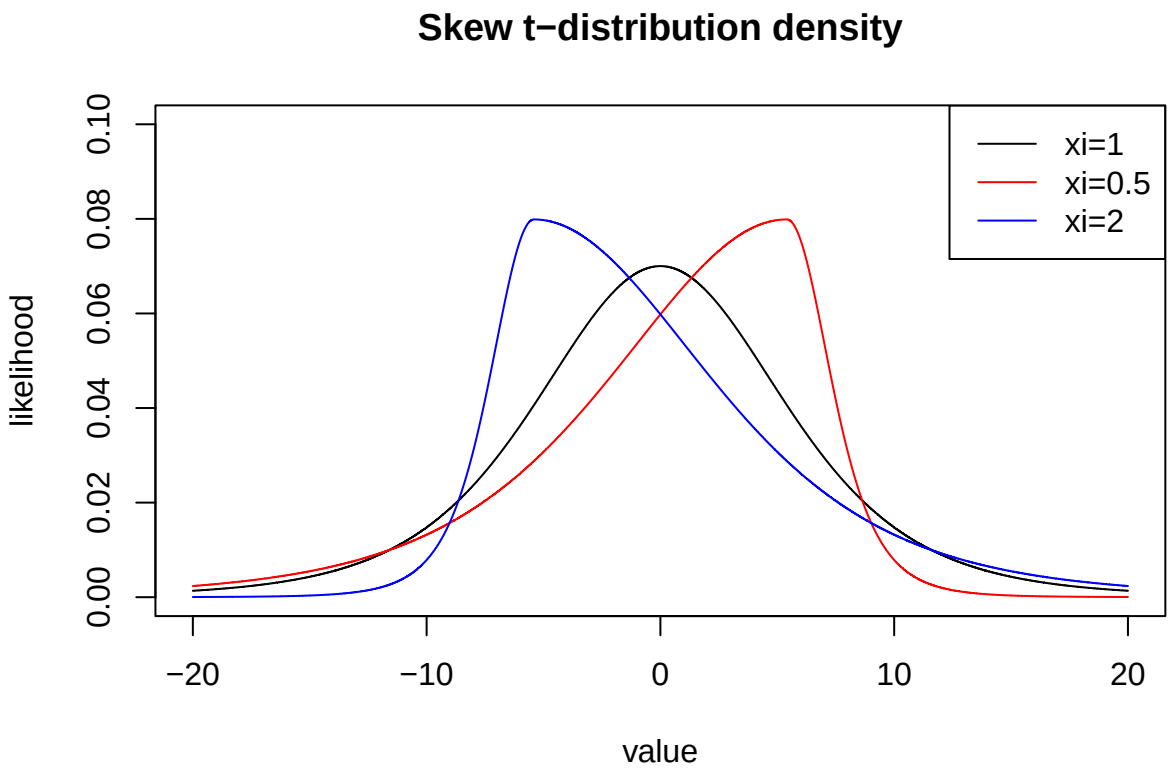
```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?
Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##        m                s          
##  Min.   :0.05930   Min.   :0.03995  
##  1st Qu.:0.06651   1st Qu.:0.06139  
##  Median :0.06930   Median :0.06685  
##  Mean   :0.07073   Mean   :0.06809  
##  3rd Qu.:0.07307   3rd Qu.:0.07603  
##  Max.   :0.08382   Max.   :0.09479
```

The meaning of `xi`

The fit for `mhr` has the highest `xi` value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, ..., X^p)}{\sum_{i=1}^{n} X_i^p} \to 0$$

for $n \to \infty$.

If not, X doesn't have a p'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192