Pension returns analysis

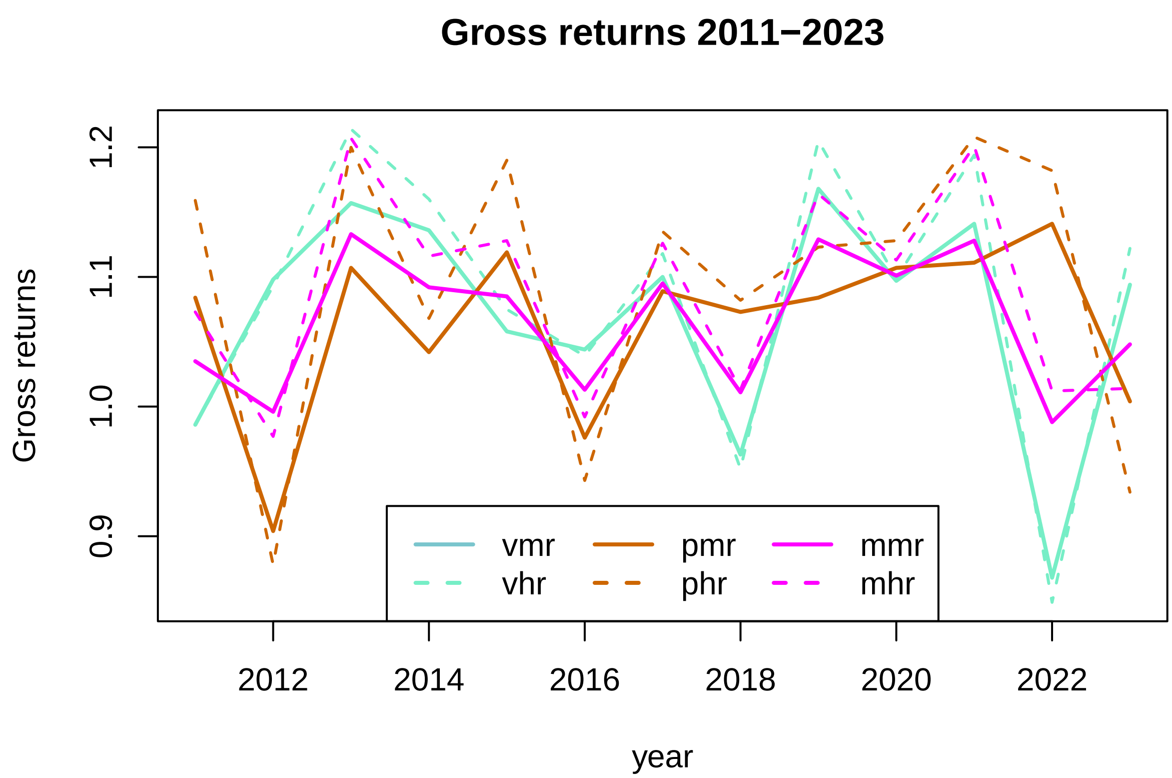
14:01 22 May 2024

Contents

Fit log returns to F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated shape parameter (degrees of freedom).
`xi` is the estimated skewness parameter.

Returns data 2011-2023.

For 2011, medium risk data is used in the high risk data set, as no high risk fund data is available prior to 2012.
`vmrl` is a long version of Velliv medium risk data, from 2007 to 2023. For 2007 to 2011 (both included) no high risk data is available.



Summary of log-returns

The summary statistics are transformed back to the scale of gross returns by taking $exp()$ of each summary statistic. (Note: Taking arithmetic mean of gross returns directly is no good. Must be geometric mean.)

	vmr	vhr	vmrl	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Min. :	0.868	0.849	0.801	0.904	0.878	0.988	0.977	0.979	0.967
1st Qu.:	1.044	1.039	1.013	1.042	1.068	1.013	1.013	1.021	1.011
Median :	1.097	1.099	1.085	1.084	1.128	1.085	1.113	1.102	1.094
Mean :	1.067	1.080	1.057	1.063	1.089	1.064	1.085	1.079	1.072
3rd Qu.:	1.136	1.160	1.128	1.107	1.182	1.101	1.128	1.121	1.107
Max. :	1.168	1.214	1.193	1.141	1.208	1.133	1.207	1.178	1.163

Ranking

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.988	mmr	1.068	phr	1.128	phr	1.089	phr	1.182	phr	1.214	vhr
0.979	vmr_phr	1.044	vmr	1.113	mhr	1.085	mhr	1.160	vhr	1.208	phr
0.977	mhr	1.042	pmr	1.102	vmr_phr	1.080	vhr	1.136	vmr	1.207	mhr
0.967	vhr_pmr	1.039	vhr	1.099	vhr	1.079	vmr_phr	1.128	vmrl	1.193	vmrl
0.904	pmr	1.021	vmr_phr	1.097	vmr	1.072	vhr_pmr	1.128	mhr	1.178	vmr_phr
0.878	phr	1.013	vmrl	1.094	vhr_pmr	1.067	vmr	1.121	vmr_phr	1.168	vmr
0.868	vmr	1.013	mmr	1.085	vmrl	1.064	mmr	1.107	pmr	1.163	vhr_pmr
0.849	vhr	1.013	mhr	1.085	mmr	1.063	pmr	1.107	vhr_pmr	1.141	pmr

Min. :	ranking	1st Qu.:	ranking	Median :	ranking	Mean :	ranking	3rd Qu.:	ranking	Max. :	ranking
0.801	vmrl	1.011	vhr_pmr	1.084	pmr	1.057	vmrl	1.101	mmr	1.133	mmr

Correlations and covariance

Correlations

	vmr	vhr	pmr	phr
vmr	1.000	0.993	-0.197	-0.095
vhr	0.993	1.000	-0.119	-0.016
pmr	-0.197	-0.119	1.000	0.957
phr	-0.095	-0.016	0.957	1.000

Covariances

	vmr	vhr	pmr	phr
vmr	0.007	0.009	-0.001	-0.001
vhr	0.009	0.011	-0.001	0.000
pmr	-0.001	-0.001	0.004	0.007
phr	-0.001	0.000	0.007	0.011

Compare pension plans

Risk of loss

Risk of loss at least as big as x percent for a single period (year).
x values are row names.

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	21.167	21.333	11.833	14.000	12.333	12.667	16.667	16.000
5	12.167	13.167	5.667	8.333	5.833	3.833	8.667	8.167
10	7.000	8.000	3.000	5.000	2.833	0.500	4.333	4.167
25	1.333	1.500	0.500	1.000	0.333	0.000	0.333	0.333
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	17.333	20.333	8.833	26.667	11.667	14.333	13.500	15.000
5	7.667	10.333	4.333	14.500	1.000	3.500	2.667	2.833
10	3.000	4.667	2.333	6.333	0.000	0.167	0.000	0.000
25	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	21.167	21.667	16.500	19.667	9.333	12.0	10.833	12.000
5	7.333	9.500	3.333	8.500	0.500	2.5	1.667	1.833

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
10	1.500	2.833	0.000	2.667	0.000	0.0	0.000	0.000
25	0.000	0.000	0.000	0.000	0.000	0.0	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.0	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.0	0.000	0.000
99	0.000	0.000	0.000	0.000	0.000	0.0	0.000	0.000

Worst ranking for loss percentiles

Skewed t-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.333	vhr	13.167	vhr	8.000	vhr	1.500	vhr	0	vmr	0	vmr	0	vmr
21.167	vmr	12.167	vmr	7.000	vmr	1.333	vmr	0	vhr	0	vhr	0	vhr
16.667	vmr_phr	8.667	vmr_phr	5.000	phr	1.000	phr	0	pmr	0	pmr	0	pmr
16.000	vhr_pmr	8.333	phr	4.333	vmr_phr	0.500	pmr	0	phr	0	phr	0	phr
14.000	phr	8.167	vhr_pmr	4.167	vhr_pmr	0.333	mmr	0	mmr	0	mmr	0	mmr
12.667	mhr	5.833	mmr	3.000	pmr	0.333	vmr_phr	0	mhr	0	mhr	0	mhr
12.333	mmr	5.667	pmr	2.833	mmr	0.333	vhr_pmr	0	vmr_phr	0	vmr_phr	0	vmr_phr
11.833	pmr	3.833	mhr	0.500	mhr	0.000	mhr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Standardized t-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
26.667	phr	14.500	phr	6.333	phr	0.333	pmr	0	vmr	0	vmr	0	vmr
20.333	vhr	10.333	vhr	4.667	vhr	0.000	vmr	0	vhr	0	vhr	0	vhr
17.333	vmr	7.667	vmr	3.000	vmr	0.000	vhr	0	pmr	0	pmr	0	pmr
15.000	vhr_pmr	4.333	pmr	2.333	pmr	0.000	phr	0	phr	0	phr	0	phr
14.333	mhr	3.500	mhr	0.167	mhr	0.000	mmr	0	mmr	0	mmr	0	mmr
13.500	vmr_phr	2.833	vhr_pmr	0.000	mmr	0.000	mhr	0	mhr	0	mhr	0	mhr
11.667	mmr	2.667	vmr_phr	0.000	vmr_phr	0.000	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
8.833	pmr	1.000	mmr	0.000	vhr_pmr	0.000	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
21.667	vhr	9.500	vhr	2.833	vhr	0	vmr	0	vmr	0	vmr	0	vmr
21.167	vmr	8.500	phr	2.667	phr	0	vhr	0	vhr	0	vhr	0	vhr
19.667	phr	7.333	vmr	1.500	vmr	0	pmr	0	pmr	0	pmr	0	pmr
16.500	pmr	3.333	pmr	0.000	pmr	0	phr	0	phr	0	phr	0	phr
12.000	mhr	2.500	mhr	0.000	mmr	0	mmr	0	mmr	0	mmr	0	mmr
12.000	vhr_pmr	1.833	vhr_pmr	0.000	mhr	0	mhr	0	mhr	0	mhr	0	mhr
10.833	vmr_phr	1.667	vmr_phr	0.000	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
9.333	mmr	0.500	mmr	0.000	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Chance of min gains

Chance of gains of at least x percent for a single period (year).
x values are row names.

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	78.833	78.667	88.167	86.000	87.667	87.333	83.333	84.000
5	63.833	66.667	71.667	76.000	71.667	70.167	69.333	69.000
10	40.833	50.167	32.500	59.667	35.500	46.000	47.167	43.833
25	0.000	0.000	0.000	0.000	0.000	0.833	0.000	0.000
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	82.667	79.667	91.167	73.333	88.333	85.667	86.500	85.000
5	65.833	65.000	80.000	58.167	57.833	64.500	63.333	60.000
10	44.500	48.000	54.833	42.500	22.833	38.833	35.000	31.167
25	7.000	11.667	6.667	10.000	0.000	1.500	0.500	0.167
50	0.167	0.500	0.833	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	78.833	78.333	83.500	80.333	90.667	88.000	89.167	88.000
5	57.667	61.333	57.667	64.167	61.333	68.000	66.833	63.500
10	35.167	42.500	29.000	46.167	24.500	42.000	37.500	33.000
25	2.167	6.667	0.000	8.333	0.000	1.833	0.500	0.167
50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Best ranking for gains percentiles

Skewed t-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
88.167	pmr	76.000	phr	59.667	phr	0.833	mhr	0	vmr	0	vmr
87.667	mmr	71.667	pmr	50.167	vhr	0.000	vmr	0	vhr	0	vhr
87.333	mhr	71.667	mmr	47.167	vmr_phr	0.000	vhr	0	pmr	0	pmr
86.000	phr	70.167	mhr	46.000	mhr	0.000	pmr	0	phr	0	phr
84.000	vhr_pmr	69.333	vmr_phr	43.833	vhr_pmr	0.000	phr	0	mmr	0	mmr
83.333	vmr_phr	69.000	vhr_pmr	40.833	vmr	0.000	mmr	0	mhr	0	mhr
78.833	vmr	66.667	vhr	35.500	mmr	0.000	vmr_phr	0	vmr_phr	0	vmr_phr
78.667	vhr	63.833	vmr	32.500	pmr	0.000	vhr_pmr	0	vhr_pmr	0	vhr_pmr

Standardized t-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
91.167	pmr	80.000	pmr	54.833	pmr	11.667	vhr	0.833	pmr	0	vmr
88.333	mmr	65.833	vmr	48.000	vhr	10.000	phr	0.500	vhr	0	vhr
86.500	vmr_phr	65.000	vhr	44.500	vmr	7.000	vmr	0.167	vmr	0	pmr
85.667	mhr	64.500	mhr	42.500	phr	6.667	pmr	0.000	phr	0	phr
85.000	vhr_pmr	63.333	vmr_phr	38.833	mhr	1.500	mhr	0.000	mmr	0	mmr
82.667	vmr	60.000	vhr_pmr	35.000	vmr_phr	0.500	vmr_phr	0.000	mhr	0	mhr
79.667	vhr	58.167	phr	31.167	vhr_pmr	0.167	vhr_pmr	0.000	vmr_phr	0	vmr_phr
73.333	phr	57.833	mmr	22.833	mmr	0.000	mmr	0.000	vhr_pmr	0	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
90.667	mmr	68.000	mhr	46.167	phr	8.333	phr	0	vmr	0	vmr
89.167	vmr_phr	66.833	vmr_phr	42.500	vhr	6.667	vhr	0	vhr	0	vhr
88.000	mhr	64.167	phr	42.000	mhr	2.167	vmr	0	pmr	0	pmr

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
88.000	vhr_pmr	63.500	vhr_pmr	37.500	vmr_phr	1.833	mhr	0	phr	0	phr
83.500	pmr	61.333	vhr	35.167	vmr	0.500	vmr_phr	0	mmr	0	mmr
80.333	phr	61.333	mmr	33.000	vhr_pmr	0.167	vhr_pmr	0	mhr	0	mhr
78.833	vmr	57.667	vmr	29.000	pmr	0.000	pmr	0	vmr_phr	0	vmr_phr
78.333	vhr	57.667	pmr	24.500	mmr	0.000	mmr	0	vhr_pmr	0	vhr_pmr

MC risk percentiles

Risk of loss at least as big as row name in percent from first to last period.

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	4.94	2.74	2.00	1.08	0.29	0.05	0.16	0.11
5	4.28	2.34	1.86	0.97	0.23	0.03	0.15	0.08
10	3.75	2.03	1.66	0.81	0.16	0.01	0.13	0.04
25	2.24	1.28	1.29	0.47	0.10	0.01	0.09	0.01
50	0.89	0.41	0.75	0.23	0.01	0.00	0.00	0.00
90	0.05	0.01	0.23	0.02	0.00	0.00	0.00	0.00
99	0.00	0.00	0.07	0.00	0.00	0.00	0.00	0.00

1e6 sstd simulation paths of `mhr`:

	0	5	10	25	50	90	99
prob_pct	0.118	0.095	0.076	0.036	0.008	0	0

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	0.06	0.09	0.82	0.32	0	0	0	0.02
5	0.05	0.08	0.78	0.23	0	0	0	0.02
10	0.05	0.03	0.77	0.21	0	0	0	0.01
25	0.01	0.00	0.63	0.07	0	0	0	0.00
50	0.00	0.00	0.46	0.00	0	0	0	0.00
90	0.00	0.00	0.14	0.00	0	0	0	0.00
99	0.00	0.00	0.04	0.00	0	0	0	0.00

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	0.04	0.03	0	0.03	0	0	0	0
5	0.03	0.01	0	0.02	0	0	0	0
10	0.01	0.01	0	0.01	0	0	0	0
25	0.00	0.01	0	0.00	0	0	0	0
50	0.00	0.00	0	0.00	0	0	0	0
90	0.00	0.00	0	0.00	0	0	0	0
99	0.00	0.00	0	0.00	0	0	0	0

Worst ranking for MC loss percentiles

Skewed t-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
4.94	vmr	4.28	vmr	3.75	vmr	2.24	vmr	0.89	vmr	0.23	pmr	0.07	pmr
2.74	vhr	2.34	vhr	2.03	vhr	1.29	pmr	0.75	pmr	0.05	vmr	0.00	vmr

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
2.00	pmr	1.86	pmr	1.66	pmr	1.28	vhr	0.41	vhr	0.02	phr	0.00	vhr
1.08	phr	0.97	phr	0.81	phr	0.47	phr	0.23	phr	0.01	vhr	0.00	phr
0.29	mmr	0.23	mmr	0.16	mmr	0.10	mmr	0.01	mmr	0.00	mmr	0.00	mmr
0.16	vmr_phr	0.15	vmr_phr	0.13	vmr_phr	0.09	vmr_phr	0.00	mhr	0.00	mhr	0.00	mhr
0.11	vhr_pmr	0.08	vhr_pmr	0.04	vhr_pmr	0.01	mhr	0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr
0.05	mhr	0.03	mhr	0.01	mhr	0.01	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr

Standardized t-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.82	pmr	0.78	pmr	0.77	pmr	0.63	pmr	0.46	pmr	0.14	pmr	0.04	pmr
0.32	phr	0.23	phr	0.21	phr	0.07	phr	0.00	vmr	0.00	vmr	0.00	vmr
0.09	vhr	0.08	vhr	0.05	vmr	0.01	vmr	0.00	vhr	0.00	vhr	0.00	vhr
0.06	vmr	0.05	vmr	0.03	vhr	0.00	vhr	0.00	phr	0.00	phr	0.00	phr
0.02	vhr_pmr	0.02	vhr_pmr	0.01	vhr_pmr	0.00	mmr	0.00	mmr	0.00	mmr	0.00	mmr
0.00	mmr	0.00	mmr	0.00	mmr	0.00	mhr	0.00	mhr	0.00	mhr	0.00	mhr
0.00	mhr	0.00	mhr	0.00	mhr	0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr
0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	90	ranking	99	ranking
0.04	vmr	0.03	vmr	0.01	vmr	0.01	vhr	0	vmr	0	vmr	0	vmr
0.03	vhr	0.02	phr	0.01	vhr	0.00	vmr	0	vhr	0	vhr	0	vhr
0.03	phr	0.01	vhr	0.01	phr	0.00	pmr	0	pmr	0	pmr	0	pmr
0.00	pmr	0.00	pmr	0.00	pmr	0.00	phr	0	phr	0	phr	0	phr
0.00	mmr	0.00	mmr	0.00	mmr	0.00	mmr	0	mmr	0	mmr	0	mmr
0.00	mhr	0.00	mhr	0.00	mhr	0.00	mhr	0	mhr	0	mhr	0	mhr
0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0.00	vmr_phr	0	vmr_phr	0	vmr_phr	0	vmr_phr
0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr	0.00	vhr_pmr	0	vhr_pmr	0	vhr_pmr	0	vhr_pmr

MC gains percentiles

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	95.06	97.26	98.00	98.92	99.71	99.95	99.84	99.89
5	94.32	96.90	97.74	98.79	99.62	99.94	99.77	99.87
10	93.61	96.43	97.57	98.68	99.53	99.93	99.71	99.84
25	91.02	94.88	96.82	98.36	99.14	99.80	99.51	99.61
50	85.77	91.56	94.85	97.57	97.74	99.37	98.83	98.87
100	72.15	83.27	88.04	94.65	90.32	97.44	96.09	94.53
200	40.32	61.23	59.24	84.78	49.18	86.21	78.99	65.32
300	16.58	39.46	23.29	70.32	11.45	63.67	50.32	29.21
400	5.42	22.99	4.80	54.11	1.09	38.72	24.53	9.21
500	1.49	12.41	0.58	38.19	0.08	19.04	9.33	2.32
1000	0.00	0.26	0.02	2.36	0.00	0.05	0.00	0.01

1e6 sstd simulation paths of `mhr`:

	0	5	10	25	50	100	200	300	400	500	1000
prob	99.882	99.854	99.824	99.686	99.301	97.513	86.912	65.992	41.486	21.693	0.086

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	99.94	99.91	99.18	99.68	100.00	100.00	100.00	99.98
5	99.93	99.86	99.14	99.56	100.00	100.00	100.00	99.98
10	99.90	99.84	99.11	99.45	100.00	100.00	100.00	99.98
25	99.78	99.69	98.96	98.90	99.99	99.99	99.98	99.97
50	99.42	99.14	98.66	97.10	99.94	99.95	99.95	99.93
100	97.46	97.22	97.78	91.56	99.79	99.45	99.37	99.74
200	85.85	88.22	94.25	71.42	97.68	91.44	90.15	98.17
300	67.15	73.58	87.69	50.88	89.51	73.73	67.48	91.84
400	47.20	58.07	76.67	33.68	73.57	51.15	42.46	78.45
500	31.51	44.35	63.42	22.19	54.01	32.42	24.06	60.16
1000	3.95	9.81	17.22	2.62	6.55	2.23	0.76	9.20

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
0	99.96	99.97	100.00	99.97	100.00	100.00	100.00	100.00
5	99.92	99.97	100.00	99.97	100.00	100.00	100.00	100.00
10	99.89	99.97	100.00	99.97	100.00	100.00	100.00	100.00
25	99.74	99.83	99.94	99.90	100.00	99.99	100.00	100.00
50	98.87	99.42	99.68	99.68	100.00	99.98	99.99	100.00
100	93.02	96.54	95.09	98.10	98.96	99.90	99.58	99.39
200	64.16	80.34	57.26	88.20	68.67	94.01	89.86	81.54
300	32.76	57.39	19.82	70.42	22.22	74.48	62.15	43.44
400	13.85	37.25	4.78	51.39	4.38	47.03	33.23	16.91
500	5.67	22.92	0.98	35.22	0.69	25.61	15.85	5.96
1000	0.03	1.58	0.00	3.92	0.01	0.52	0.26	0.06

Best ranking for MC gains percentiles

Skewed t-distribution (sstd):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
99.95	mhr	99.94	mhr	99.93	mhr	99.80	mhr	99.37	mhr	97.44	mhr
99.89	vhr_pmr	99.87	vhr_pmr	99.84	vhr_pmr	99.61	vhr_pmr	98.87	vhr_pmr	96.09	vmr_phr
99.84	vmr_phr	99.77	vmr_phr	99.71	vmr_phr	99.51	vmr_phr	98.83	vmr_phr	94.65	phr
99.71	mmr	99.62	mmr	99.53	mmr	99.14	mmr	97.74	mmr	94.53	vhr_pmr
98.92	phr	98.79	phr	98.68	phr	98.36	phr	97.57	phr	90.32	mmr
98.00	pmr	97.74	pmr	97.57	pmr	96.82	pmr	94.85	pmr	88.04	pmr
97.26	vhr	96.90	vhr	96.43	vhr	94.88	vhr	91.56	vhr	83.27	vhr
95.06	vmr	94.32	vmr	93.61	vmr	91.02	vmr	85.77	vmr	72.15	vmr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
86.21	mhr	70.32	phr	54.11	phr	38.19	phr	2.36	phr
84.78	phr	63.67	mhr	38.72	mhr	19.04	mhr	0.26	vhr
78.99	vmr_phr	50.32	vmr_phr	24.53	vmr_phr	12.41	vhr	0.05	mhr
65.32	vhr_pmr	39.46	vhr	22.99	vhr	9.33	vmr_phr	0.02	pmr
61.23	vhr	29.21	vhr_pmr	9.21	vhr_pmr	2.32	vhr_pmr	0.01	vhr_pmr
59.24	pmr	23.29	pmr	5.42	vmr	1.49	vmr	0.00	vmr
49.18	mmr	16.58	vmr	4.80	pmr	0.58	pmr	0.00	mmr
40.32	vmr	11.45	mmr	1.09	mmr	0.08	mmr	0.00	vmr_phr

Standardized t-distribution (std):

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	mmr	100.00	mmr	100.00	mmr	99.99	mmr	99.95	mhr	99.79	mmr
100.00	mhr	100.00	mhr	100.00	mhr	99.99	mhr	99.95	vmr_phr	99.74	vhr_pmr

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	vmr_phr	100.00	vmr_phr	100.00	vmr_phr	99.98	vmr_phr	99.94	mmr	99.45	mhr
99.98	vhr_pmr	99.98	vhr_pmr	99.98	vhr_pmr	99.97	vhr_pmr	99.93	vhr_pmr	99.37	vmr_phr
99.94	vmr	99.93	vmr	99.90	vmr	99.78	vmr	99.42	vmr	97.78	pmr
99.91	vhr	99.86	vhr	99.84	vhr	99.69	vhr	99.14	vhr	97.46	vmr
99.68	phr	99.56	phr	99.45	phr	98.96	pmr	98.66	pmr	97.22	vhr
99.18	pmr	99.14	pmr	99.11	pmr	98.90	phr	97.10	phr	91.56	phr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
98.17	vhr_pmr	91.84	vhr_pmr	78.45	vhr_pmr	63.42	pmr	17.22	pmr
97.68	mmr	89.51	mmr	76.67	pmr	60.16	vhr_pmr	9.81	vhr
94.25	pmr	87.69	pmr	73.57	mmr	54.01	mmr	9.20	vhr_pmr
91.44	mhr	73.73	mhr	58.07	vhr	44.35	vhr	6.55	mmr
90.15	vmr_phr	73.58	vhr	51.15	mhr	32.42	mhr	3.95	vmr
88.22	vhr	67.48	vmr_phr	47.20	vmr	31.51	vmr	2.62	phr
85.85	vmr	67.15	vmr	42.46	vmr_phr	24.06	vmr_phr	2.23	mhr
71.42	phr	50.88	phr	33.68	phr	22.19	phr	0.76	vmr_phr

Normal distribution:

0	ranking	5	ranking	10	ranking	25	ranking	50	ranking	100	ranking
100.00	pmr	100.00	pmr	100.00	pmr	100.00	mmr	100.00	mmr	99.90	mhr
100.00	mmr	100.00	mmr	100.00	mmr	100.00	vmr_phr	100.00	vhr_pmr	99.58	vmr_phr
100.00	mhr	100.00	mhr	100.00	mhr	100.00	vhr_pmr	99.99	vmr_phr	99.39	vhr_pmr
100.00	vmr_phr	100.00	vmr_phr	100.00	vmr_phr	99.99	mhr	99.98	mhr	98.96	mmr
100.00	vhr_pmr	100.00	vhr_pmr	100.00	vhr_pmr	99.94	pmr	99.68	pmr	98.10	phr
99.97	vhr	99.97	vhr	99.97	vhr	99.90	phr	99.68	phr	96.54	vhr
99.97	phr	99.97	phr	99.97	phr	99.83	vhr	99.42	vhr	95.09	pmr
99.96	vmr	99.92	vmr	99.89	vmr	99.74	vmr	98.87	vmr	93.02	vmr

200	ranking	300	ranking	400	ranking	500	ranking	1000	ranking
94.01	mhr	74.48	mhr	51.39	phr	35.22	phr	3.92	phr
89.86	vmr_phr	70.42	phr	47.03	mhr	25.61	mhr	1.58	vhr
88.20	phr	62.15	vmr_phr	37.25	vhr	22.92	vhr	0.52	mhr
81.54	vhr_pmr	57.39	vhr	33.23	vmr_phr	15.85	vmr_phr	0.26	vmr_phr
80.34	vhr	43.44	vhr_pmr	16.91	vhr_pmr	5.96	vhr_pmr	0.06	vhr_pmr
68.67	mmr	32.76	vmr	13.85	vmr	5.67	vmr	0.03	vmr
64.16	vmr	22.22	mmr	4.78	pmr	0.98	pmr	0.01	mmr
57.26	pmr	19.82	pmr	4.38	mmr	0.69	mmr	0.00	pmr

Summary statistics

Fit summary

Summary for fit of log returns to an F-S skew standardized Student-t distribution.
`m` is the location parameter.
`s` is the scale parameter.
`nu` is the estimated degrees of freedom, or shape parameter.
`xi` is the estimated skewness parameter.

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.048	0.063	0.058	0.084	0.059	0.082	0.067	0.062
s	0.120	0.126	0.123	0.121	0.088	0.071	0.091	0.090
nu	3.304	4.390	2.265	3.185	2.773	89.863	4.660	3.892
xi	0.034	0.019	0.477	0.018	0.029	0.770	0.048	0.019

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Rˆ2	0.993	0.995	0.991	0.964	0.890	0.961	0.927	0.933

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.084	0.090	0.102	0.073	0.058	0.075	0.071	0.065
s	0.106	0.122	0.345	0.119	0.050	0.071	0.065	0.063
nu	4.844	7.368	2.045	5682540.710	5283545.362	15657038.400	2680674.834	7710686.839
Rˆ2	0.935	0.955	0.918	0.923	0.960	0.965	0.969	0.972

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
m	0.064	0.077	0.061	0.085	0.062	0.081	0.076	0.069
s	0.081	0.099	0.063	0.101	0.048	0.070	0.062	0.060
Rˆ2	0.933	0.954	0.916	0.923	0.960	0.965	0.969	0.972

AIC and BIC AIC

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
sstd	-27.850	-21.575	-33.230	-23.726	-36.960	-24.261	-29.651	-31.100
std	-16.385	-11.623	-22.924	-11.324	-33.923	-24.564	-27.112	-27.818
normal	-20.316	-15.218	-27.005	-14.616	-34.127	-24.140	-27.388	-28.318

BIC

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
sstd	-25.590	-19.315	-30.970	-21.466	-34.701	-22.001	-27.391	-28.841
std	-14.125	-9.363	-20.664	-9.064	-31.663	-22.304	-24.852	-25.558
normal	-18.056	-12.958	-24.746	-12.357	-31.867	-21.880	-25.129	-26.058

Fit statistics ranking Skewed t-distribution (sstd):

m	ranking	s	ranking	Rˆ2	ranking
0.084	phr	0.071	mhr	0.995	vhr
0.082	mhr	0.088	mmr	0.993	vmr
0.067	vmr_phr	0.090	vhr_pmr	0.991	pmr
0.063	vhr	0.091	vmr_phr	0.964	phr
0.062	vhr_pmr	0.120	vmr	0.961	mhr
0.059	mmr	0.121	phr	0.933	vhr_pmr
0.058	pmr	0.123	pmr	0.927	vmr_phr
0.048	vmr	0.126	vhr	0.890	mmr

Standardized t-distribution (std):

m	ranking	s	ranking	Rˆ2	ranking
0.102	pmr	0.050	mmr	0.972	vhr_pmr
0.090	vhr	0.063	vhr_pmr	0.969	vmr_phr
0.084	vmr	0.065	vmr_phr	0.965	mhr
0.075	mhr	0.071	mhr	0.960	mmr
0.073	phr	0.106	vmr	0.955	vhr

m	ranking	s	ranking	R^2	ranking
0.071	vmr_phr	0.119	phr	0.935	vmr
0.065	vhr_pmr	0.122	vhr	0.923	phr
0.058	mmr	0.345	pmr	0.918	pmr

Normal distribution:

m	ranking	s	ranking	R^2	ranking
0.085	phr	0.048	mmr	0.972	vhr_pmr
0.081	mhr	0.060	vhr_pmr	0.969	vmr_phr
0.077	vhr	0.062	vmr_phr	0.965	mhr
0.076	vmr_phr	0.063	pmr	0.960	mmr
0.069	vhr_pmr	0.070	mhr	0.954	vhr
0.064	vmr	0.081	vmr	0.933	vmr
0.062	mmr	0.099	vhr	0.923	phr
0.061	pmr	0.101	phr	0.916	pmr

Monte Carlo simulations summary

Monte Carlo simulations of portfolio index values (currency values).
Statistics are given for the final state of all paths.
Probability of down-and_out is calculated as the share of paths that reach 0 at some point. All subsequent values for a path are set to 0, if the path reaches at any point.
0 is defined as any value below a threshold.
`dai_pct` (for down-and-in) is the probability of losing money. This is calculated as the share of paths finishing below index 100.

```
## Number of paths: 10000
```

Skewed t-distribution (sstd):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	296.42	406.29	344.96	600.86	319.52	505.48	446.06	375.72
mc_s	134.29	210.50	119.95	274.68	88.20	172.23	151.43	120.31
mc_min	3.03	2.37	0.01	5.08	35.43	51.71	56.16	52.44
mc_max	915.54	1474.60	2824.80	1922.91	796.58	1326.58	1087.53	1319.52
dao_pct	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
dai_pct	4.67	2.47	1.94	0.92	0.29	0.04	0.14	0.09

Standardized t-distribution (std):

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	592.50	709.13	6.012997e+05	500.16	40290.88	597.63	544.36	3.552074e+26
mc_s	306.65	419.62	5.951009e+07	288.20	3902356.86	244.94	203.81	3.552074e+28
mc_min	74.74	90.15	1.000000e-02	63.30	117.24	125.28	131.47	8.999000e+01
mc_max	6365.75	5689.44	5.950808e+09	4376.28	390227286.46	2398.10	2311.76	3.552074e+30
dao_pct	0.00	0.00	2.000000e-02	0.00	0.00	0.00	0.00	0.000000e+00
dai_pct	0.04	0.02	8.100000e-01	0.27	0.00	0.00	0.00	1.000000e-02

Normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
mc_m	387.99	517.74	349.54	610.41	368.48	559.96	501.75	431.08
mc_s	145.35	244.18	101.36	288.89	89.04	186.52	165.28	129.46
mc_min	91.52	71.65	106.12	72.91	139.12	148.34	142.53	151.90
mc_max	1442.08	3034.63	972.84	3328.70	1167.49	2199.87	1503.21	1373.78
dao_pct	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
dai_pct	0.04	0.01	0.00	0.01	0.00	0.00	0.00	0.00

Ranking Skewed t-distribution (sstd):

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
600.86	phr	88.20	mmr	56.16	vmr_phr	2824.80	pmr	0.00	vmr	0.04	mhr
505.48	mhr	119.95	pmr	52.44	vhr_pmr	1922.91	phr	0.00	vhr	0.09	vhr_pmr
446.06	vmr_phr	120.31	vhr_pmr	51.71	mhr	1474.60	vhr	0.00	phr	0.14	vmr_phr
406.29	vhr	134.29	vmr	35.43	mmr	1326.58	mhr	0.00	mmr	0.29	mmr
375.72	vhr_pmr	151.43	vmr_phr	5.08	phr	1319.52	vhr_pmr	0.00	mhr	0.92	phr
344.96	pmr	172.23	mhr	3.03	vmr	1087.53	vmr_phr	0.00	vmr_phr	1.94	pmr
319.52	mmr	210.50	vhr	2.37	vhr	915.54	vmr	0.00	vhr_pmr	2.47	vhr
296.42	vmr	274.68	phr	0.01	pmr	796.58	mmr	0.01	pmr	4.67	vmr

Standardized t-distribution (std):

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
3.552074e+26	vhr_pmr	2.038100e+02	vmr_phr	131.47	vmr_phr	3.552074e+30	vhr_pmr	0.00	vmr	0.00	mmr
6.012997e+05	pmr	2.449400e+02	mhr	125.28	mhr	5.950808e+09	pmr	0.00	vhr	0.00	mhr
4.029088e+04	mmr	2.882000e+02	phr	117.24	mmr	3.902273e+08	mmr	0.00	phr	0.00	vmr_phr
7.091300e+02	vhr	3.066500e+02	vmr	90.15	vhr	6.365750e+03	vmr	0.00	mmr	0.01	vhr_pmr
5.976300e+02	mhr	4.196200e+02	vhr	89.99	vhr_pmr	5.689440e+03	vhr	0.00	mhr	0.02	vhr
5.925000e+02	vmr	3.902357e+06	mmr	74.74	vmr	4.376280e+03	phr	0.00	vmr_phr	0.04	vmr
5.443600e+02	vmr_phr	5.951009e+07	pmr	63.30	phr	2.398100e+03	mhr	0.00	vhr_pmr	0.27	phr
5.001600e+02	phr	3.552074e+28	vhr_pmr	0.01	pmr	2.311760e+03	vmr_phr	0.02	pmr	0.81	pmr

Normal distribution:

mc_m	ranking	mc_s	ranking	mc_min	ranking	mc_max	ranking	dao_pct	ranking	dai_pct	ranking
610.41	phr	89.04	mmr	151.90	vhr_pmr	3328.70	phr	0	vmr	0.00	pmr
559.96	mhr	101.36	pmr	148.34	mhr	3034.63	vhr	0	vhr	0.00	mmr
517.74	vhr	129.46	vhr_pmr	142.53	vmr_phr	2199.87	mhr	0	pmr	0.00	mhr
501.75	vmr_phr	145.35	vmr	139.12	mmr	1503.21	vmr_phr	0	phr	0.00	vmr_phr
431.08	vhr_pmr	165.28	vmr_phr	106.12	pmr	1442.08	vmr	0	mmr	0.00	vhr_pmr
387.99	vmr	186.52	mhr	91.52	vmr	1373.78	vhr_pmr	0	mhr	0.01	vhr
368.48	mmr	244.18	vhr	72.91	phr	1167.49	mmr	0	vmr_phr	0.01	phr
349.54	pmr	288.89	phr	71.65	vhr	972.84	pmr	0	vhr_pmr	0.04	vmr

Compare Gaussian and skewed t-distribution fits

Gaussian fits

Gaussian QQ plots

Gaussian vs skewed t

Probability in percent that the smallest and largest (respectively) observed return for each fund was generated by a normal distribution:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
P_norm(X_min)	0.070	0.088	0.389	0.582	11.639	9.919	10.048	6.801
P_norm(X_max)	13.230	11.876	12.922	15.359	9.628	6.429	7.796	8.592
P_t(X_min)	5.377	5.080	3.489	4.315	10.570	8.015	13.008	10.520
P_t(X_max)	0.118	0.156	2.825	0.188	0.488	5.141	0.229	0.175

Average number of years between min or max events (respectively):

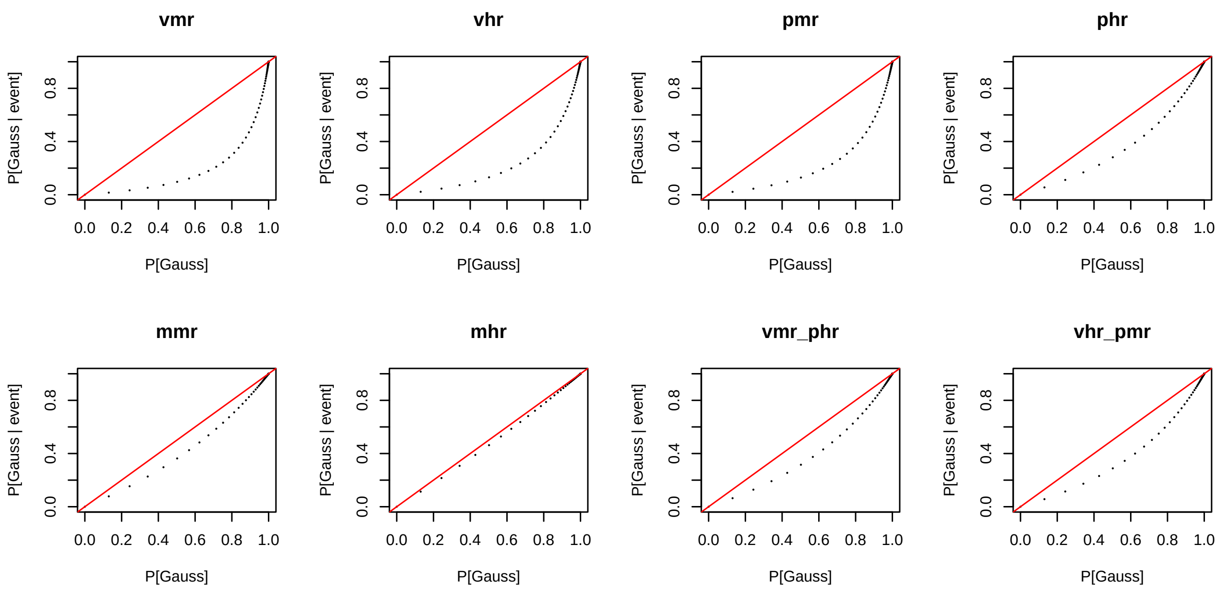
	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
norm: avg yrs btw min	1438.131	1139.205	256.817	171.880	8.592	10.082	9.952	14.705
norm: avg yrs btw max	7.559	8.420	7.739	6.511	10.386	15.556	12.827	11.639
t: avg yrs btw min	18.596	19.687	28.663	23.173	9.461	12.476	7.688	9.506
t: avg yrs btw max	848.548	640.410	35.400	531.552	205.104	19.450	437.280	572.483

Lilliefors test p-values for Lilliefors test.
Testing H_0, that log-returns are Gaussian.

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
p value	0.052	0.343	0.024	0.06	0.24	0.137	0.375	0.415

Wittgenstein's Ruler For different given probabilities that returns are Gaussian, what is the probability that the distribution is Gaussian rather than skewed t-distributed, given the smallest/largest observed log-returns?

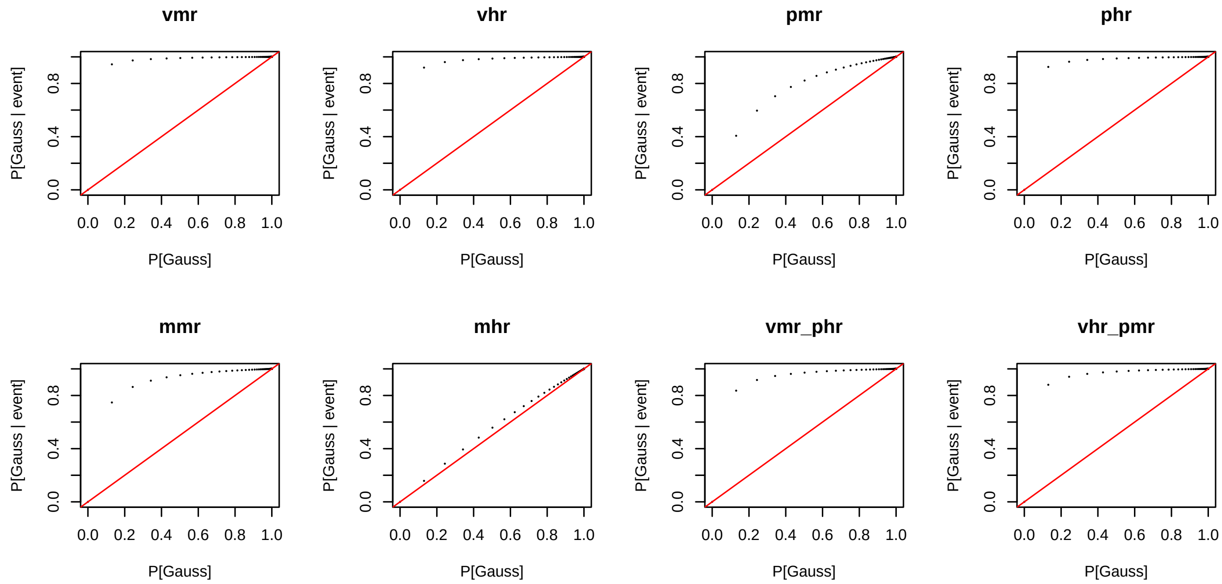
Conditional probabilities for smallest observed log-returns:



Use $1 -$ p-value from Lilliefors test as prior probability that the distribution is Gaussian.
$x_{\text{obs}} = \min(x)$ and $P[\text{Event} \mid \text{Gaussian}] = P_{\text{Gauss}}[X \leq x_{\text{min}}]$:

	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Lillie p-val	0.052	0.343	0.024	0.060	0.240	0.137	0.375	0.415
Prior prob	0.948	0.657	0.976	0.940	0.760	0.863	0.625	0.585
P[Gauss | Event]	0.661	0.223	0.854	0.859	0.642	0.844	0.433	0.362

Use $1 -$ p-value from Lilliefors test as prior probability that the distribution is Gaussian.
$x_{\text{obs}} = \max(x)$ and $P[\text{Event} \mid \text{Gaussian}] = P_{\text{Gauss}}[X \geq x_{\text{max}}]$:

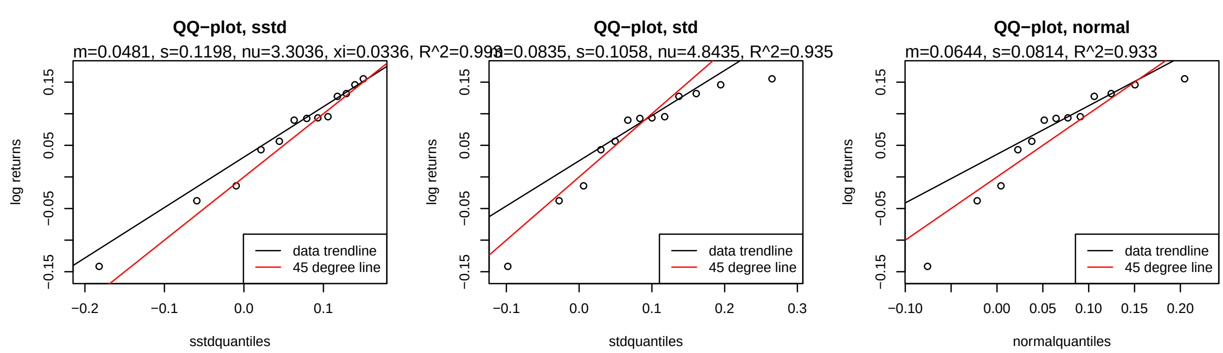


	vmr	vhr	pmr	phr	mmr	mhr	vmr_phr	vhr_pmr
Lillie p-val	0.052	0.343	0.024	0.060	0.240	0.137	0.375	0.415
Prior prob	0.948	0.657	0.976	0.940	0.760	0.863	0.625	0.585
P[Gauss | Event]	1.000	0.993	0.995	0.999	0.984	0.888	0.983	0.986

Velliv medium risk (vmr), 2011 - 2023

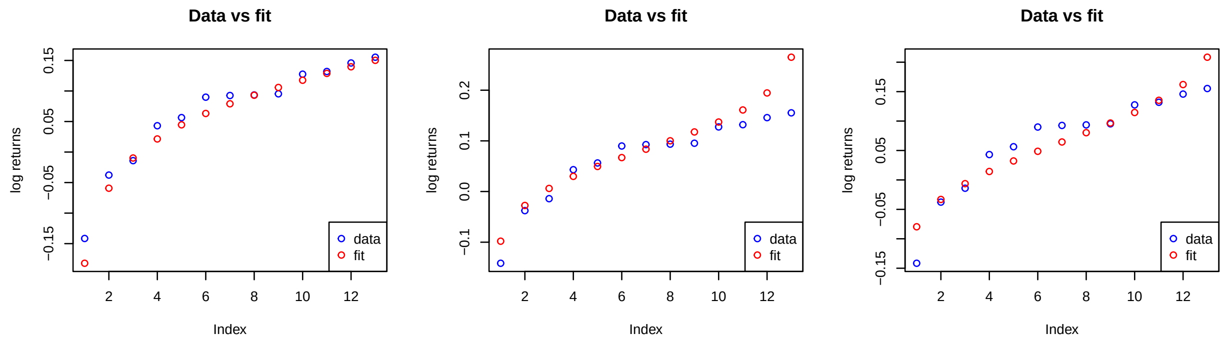
QQ Plot

Skewed t-distribution (sstd):



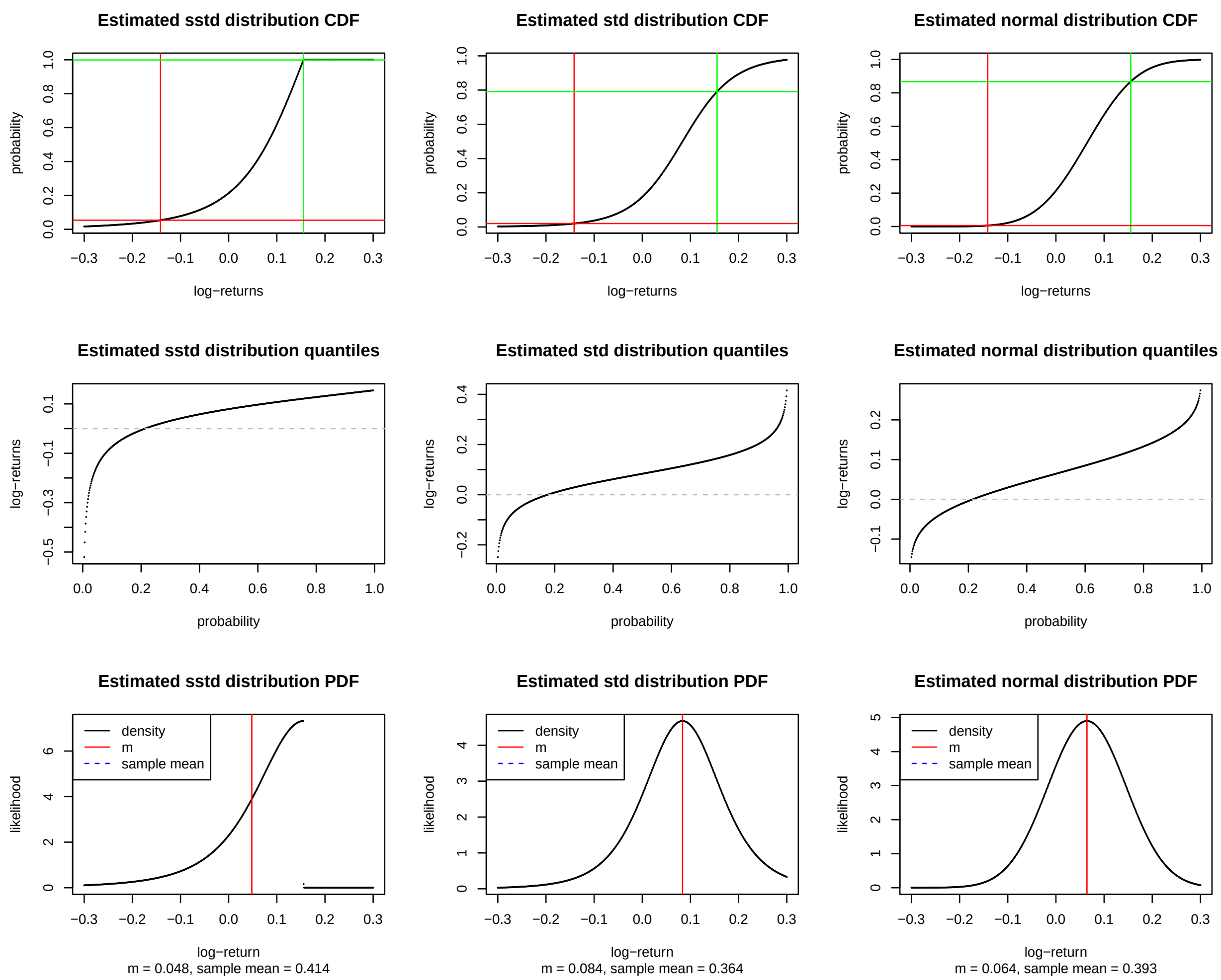
Data vs fit

Let's plot the fit and the observed returns together.



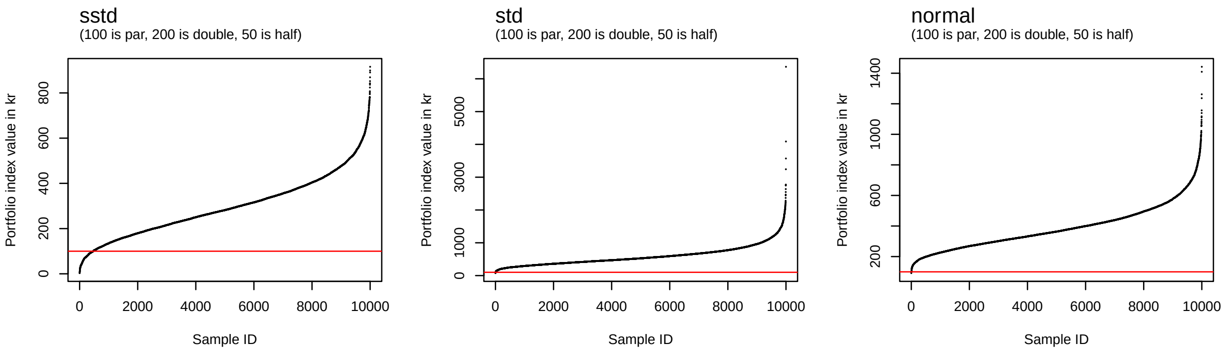
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

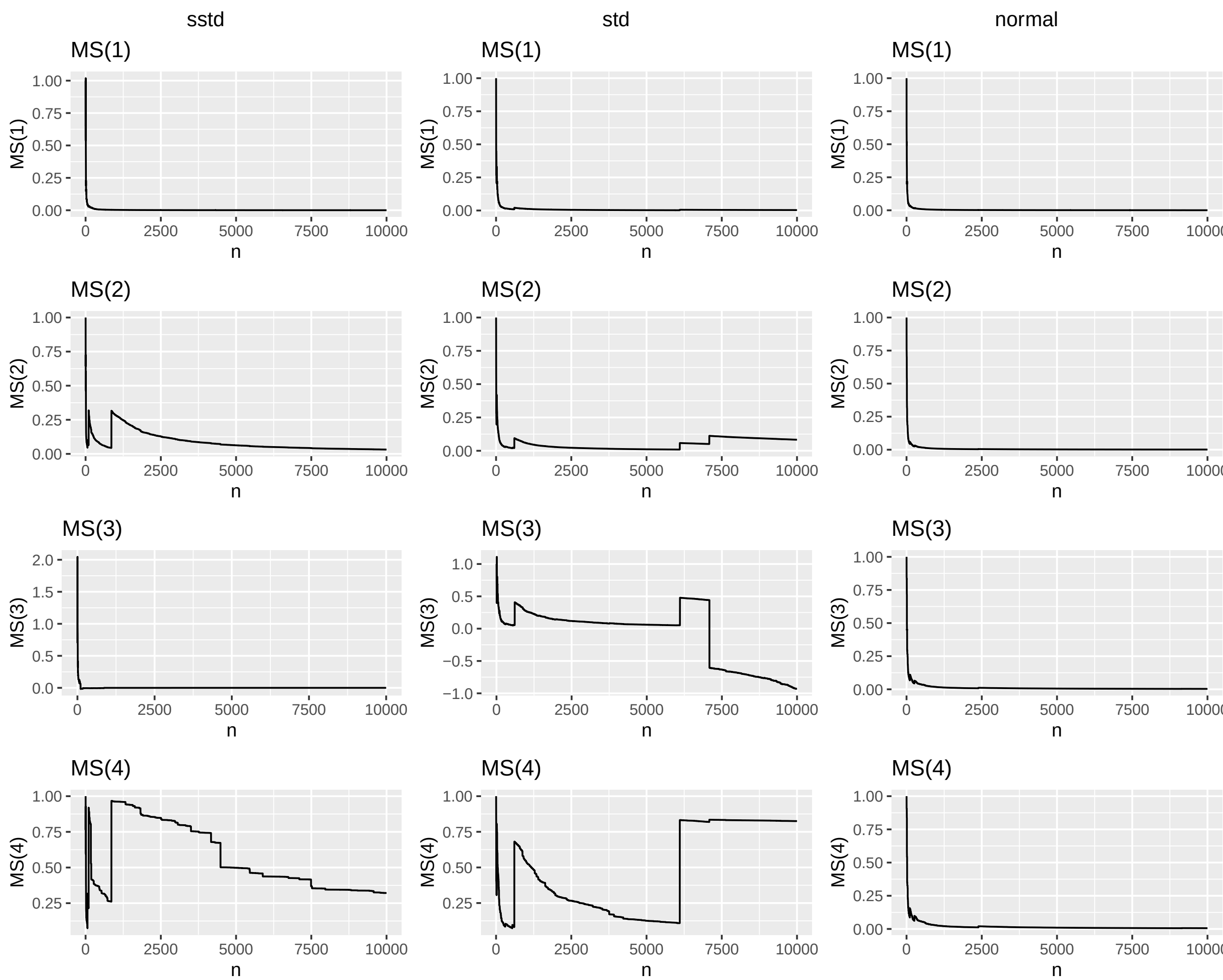
Sorted portfolio index values for last period of all runs



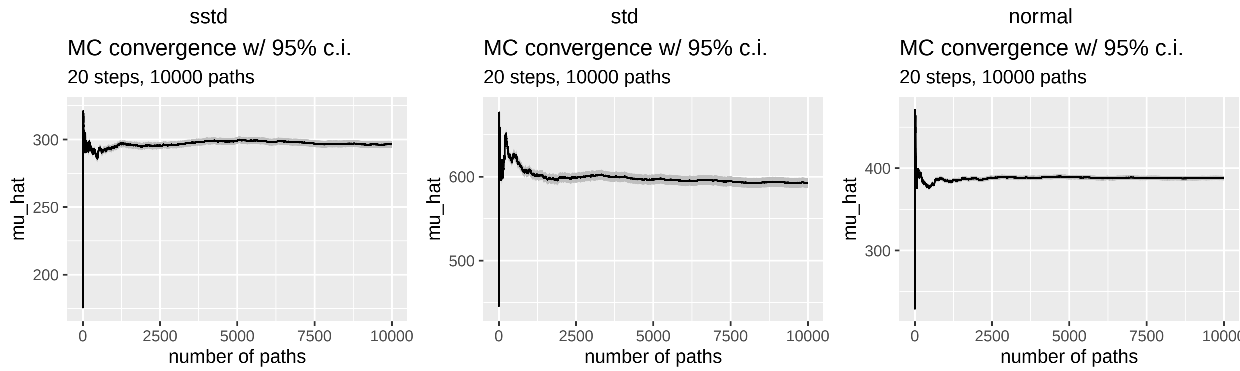
Convergence

Max vs sum

Max vs sum plots for the first four moments:

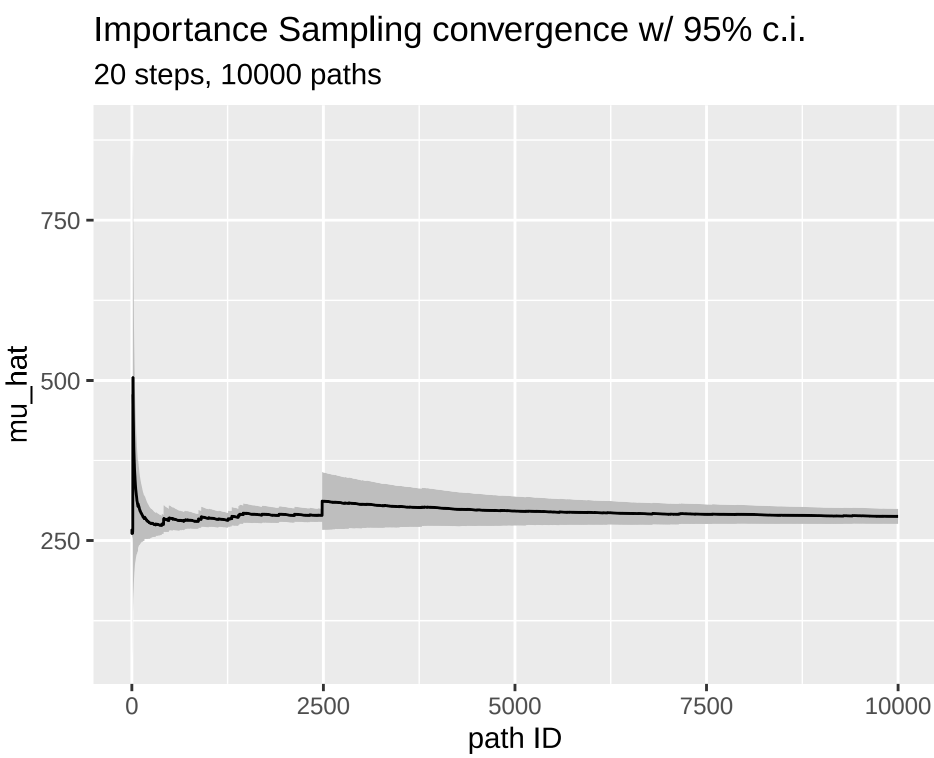


MC



IS

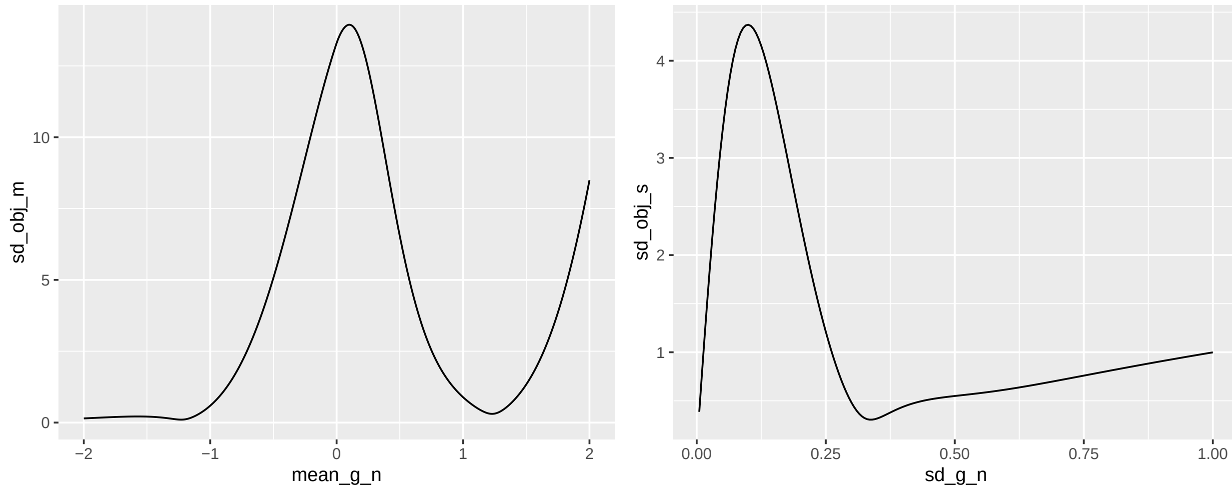
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.2294983 0.3373312
```

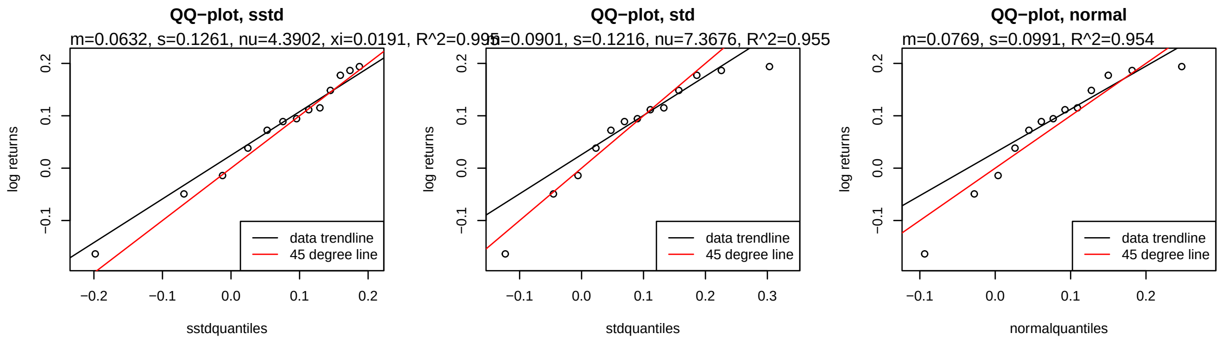
Objective function plots



Velliv high risk (vhr), 2011 - 2023

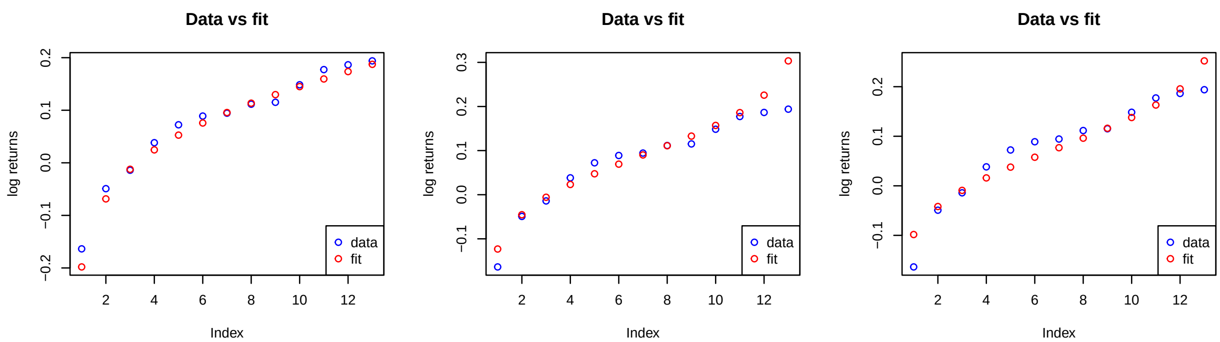
QQ Plot

Skewed t-distribution (sstd):



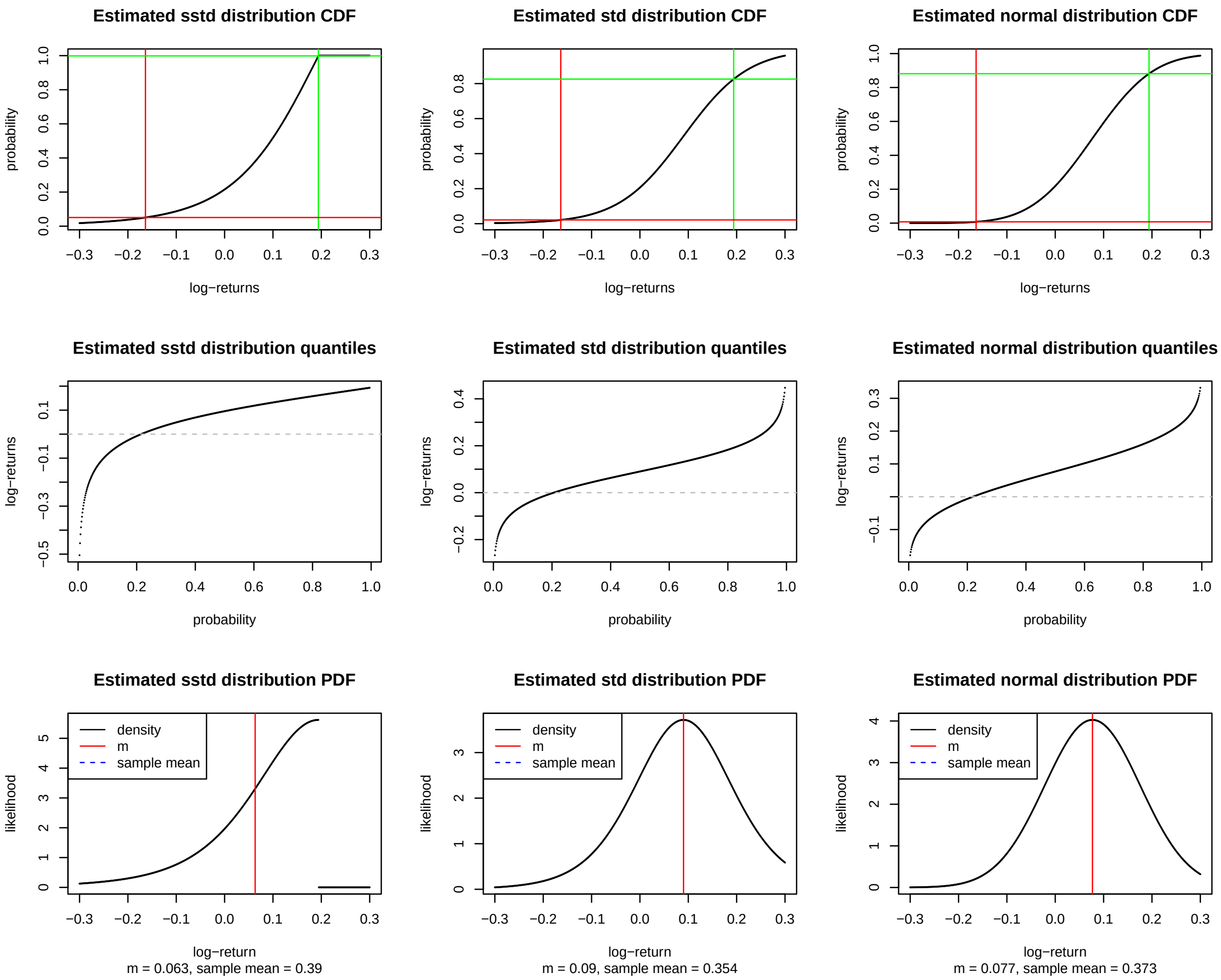
Data vs fit

Let's plot the fit and the observed returns together.



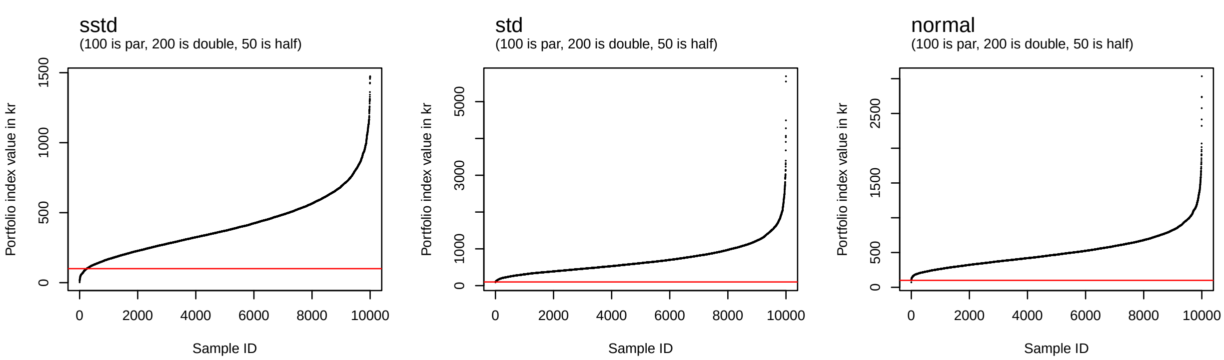
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

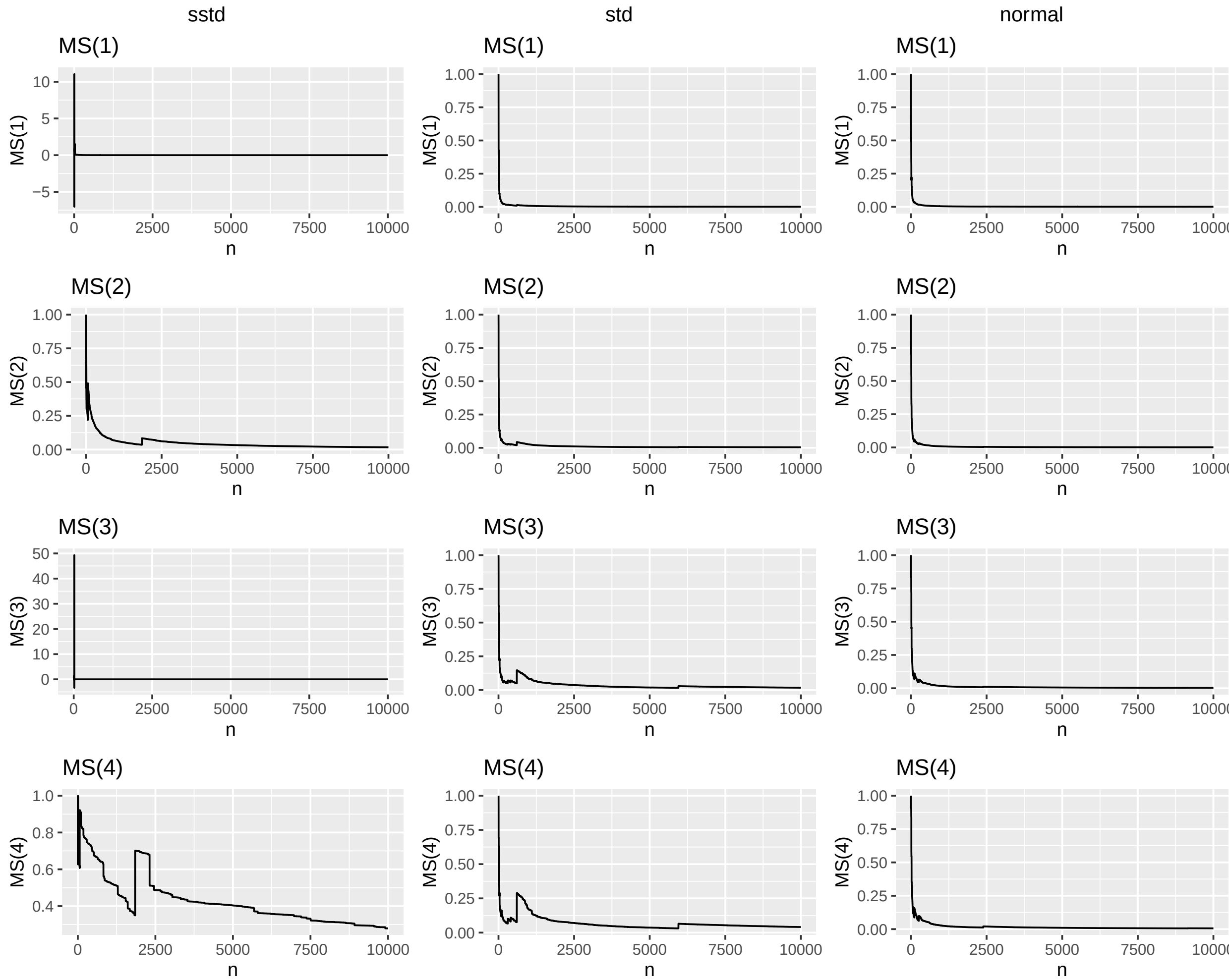
Sorted portfolio index values for last period of all runs



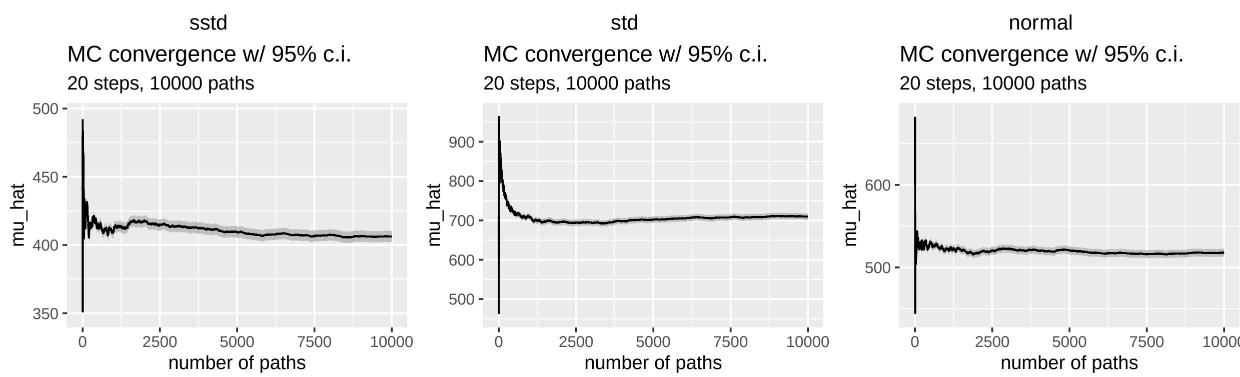
Convergence

Max vs sum

Max vs sum plots for the first four moments:

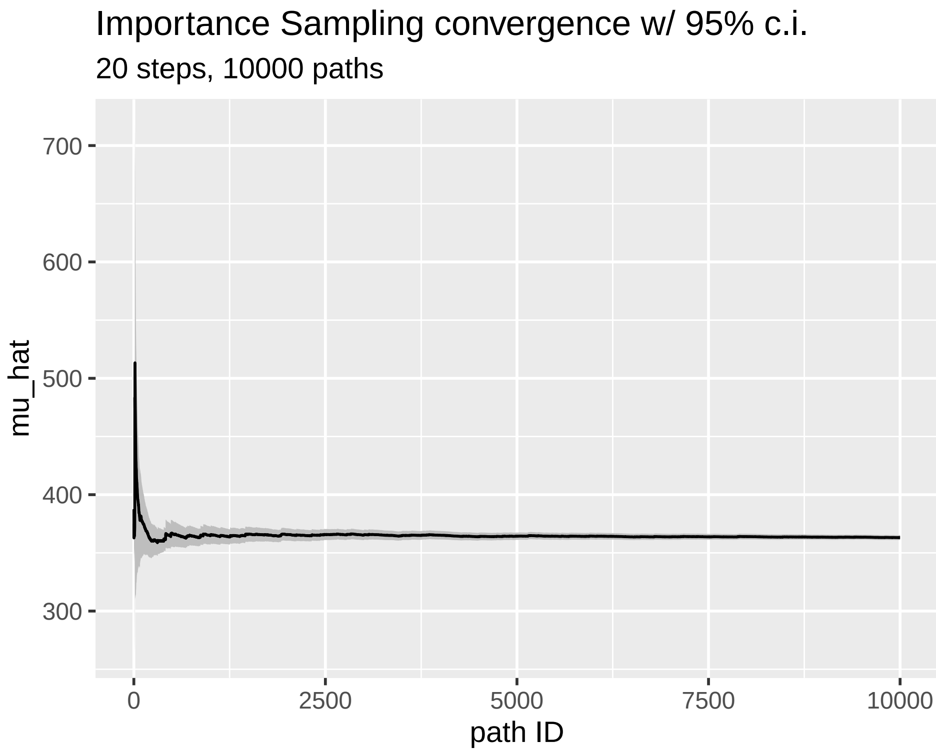


MC



IS

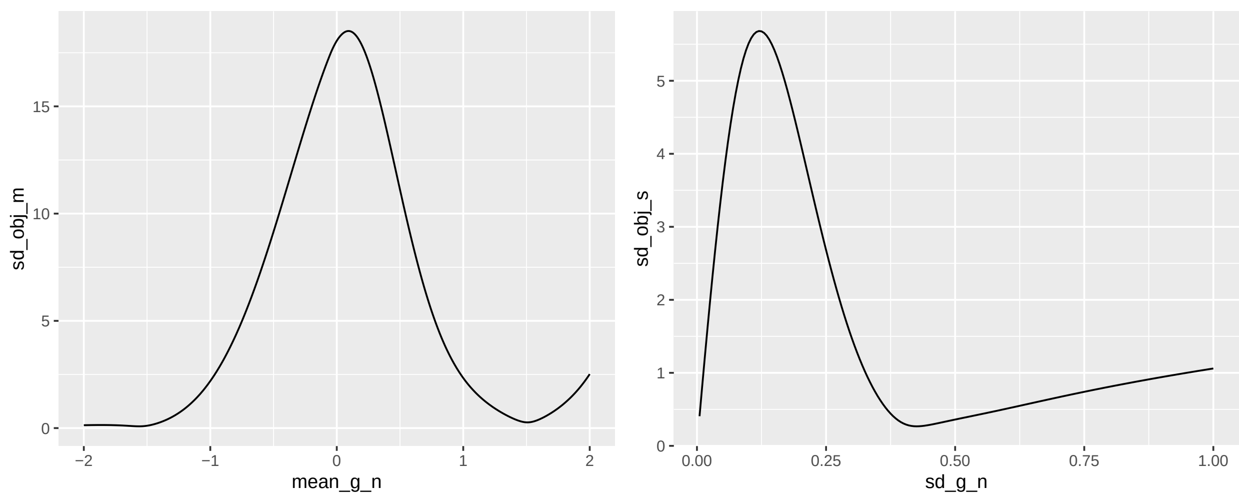
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.5074609 0.4255322
```

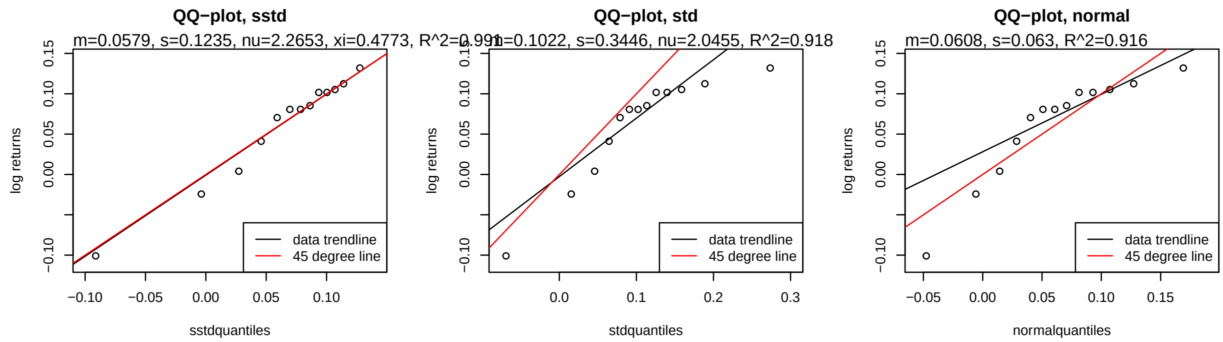
Objective function plots



PFA medium risk (pmr), 2011 - 2023

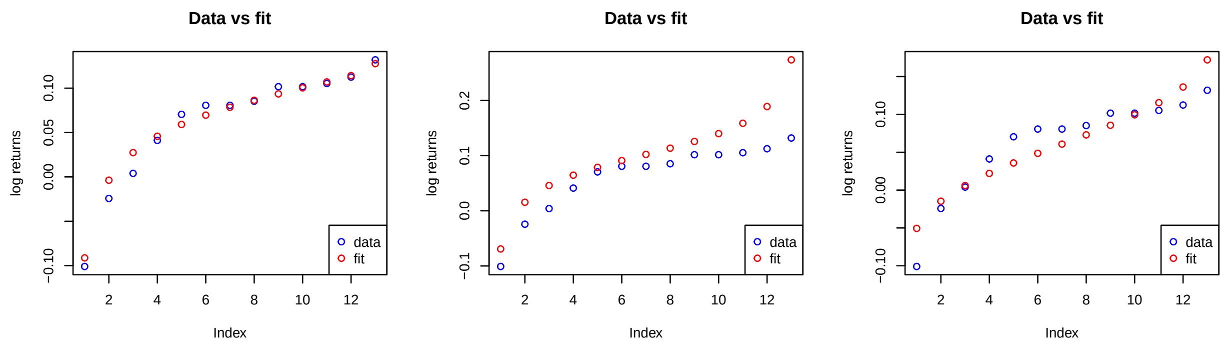
QQ Plot

Skewed t-distribution (sstd):



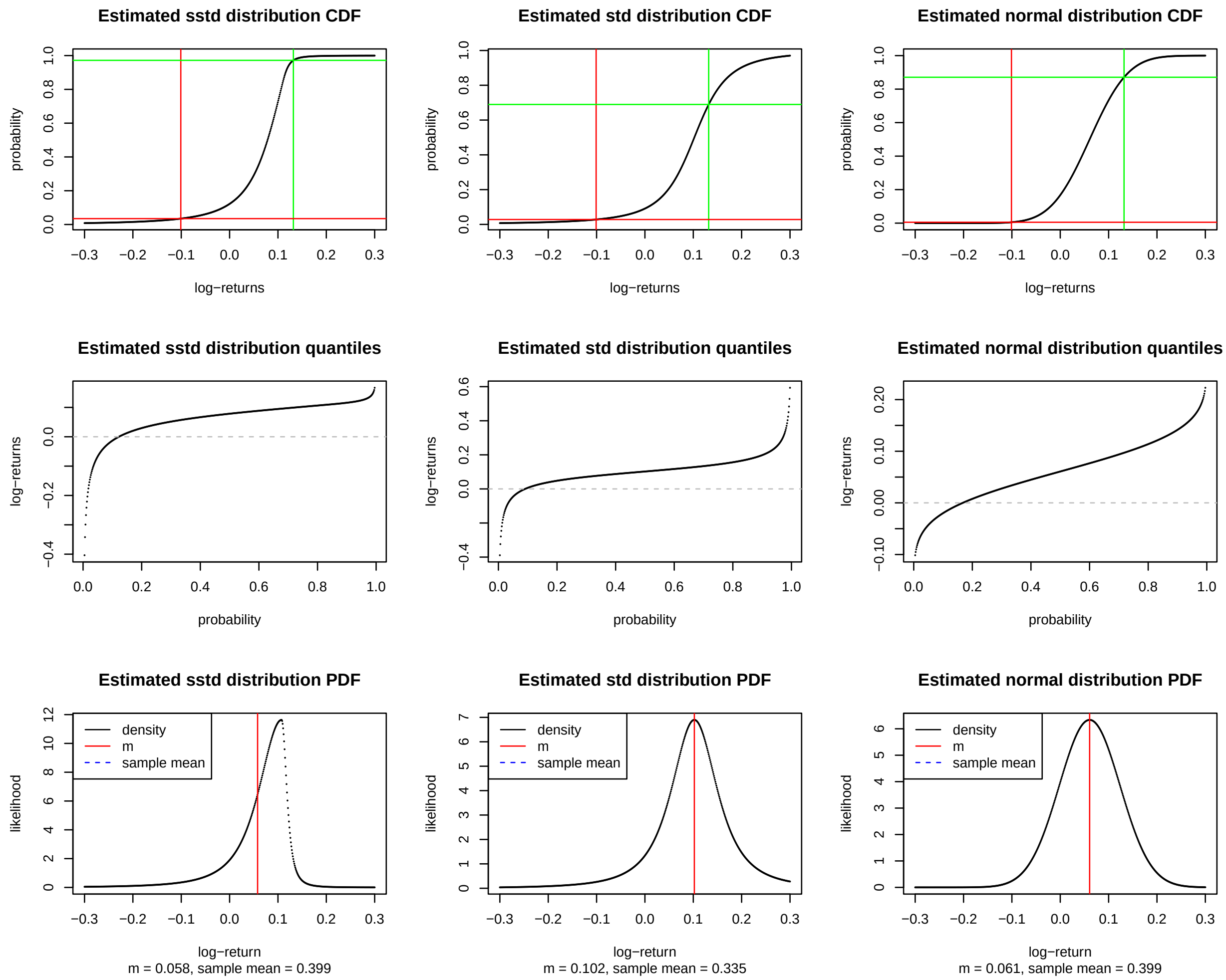
Data vs fit

Let's plot the fit and the observed returns together.



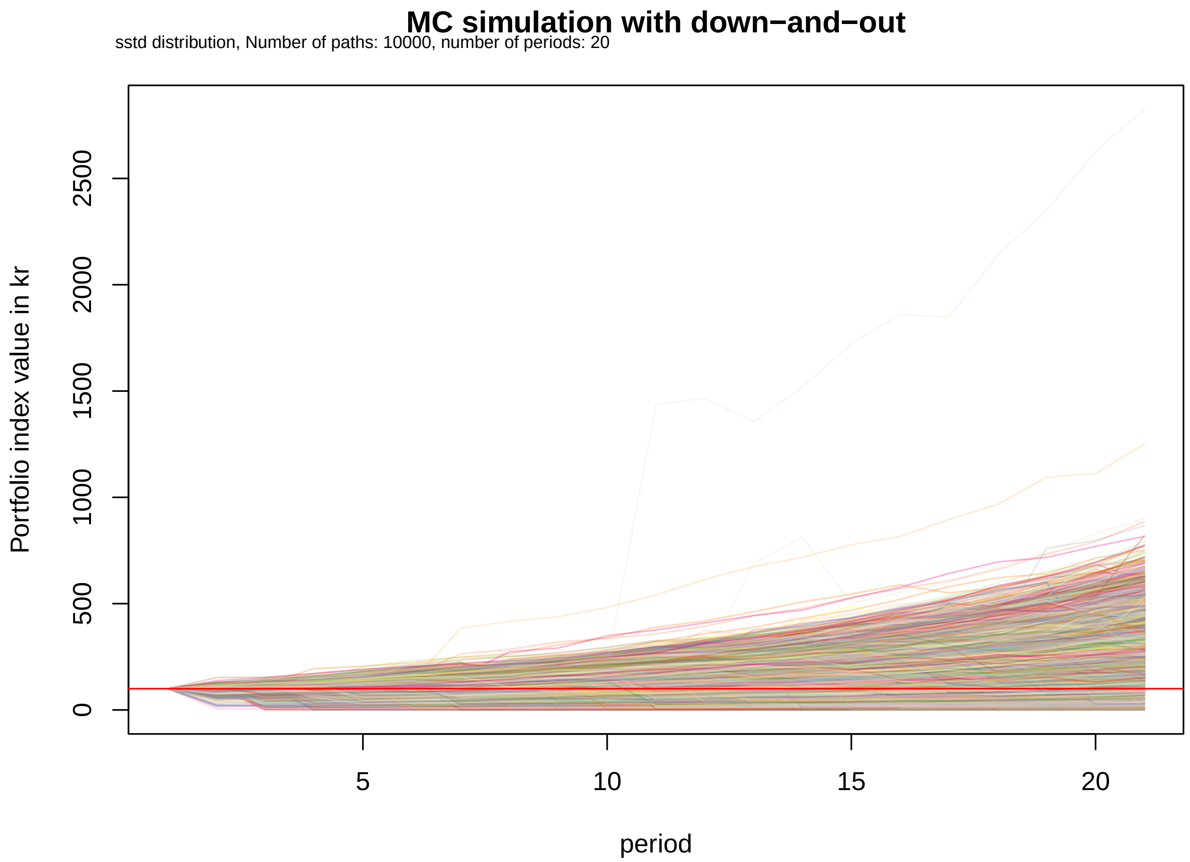
Estimated distribution

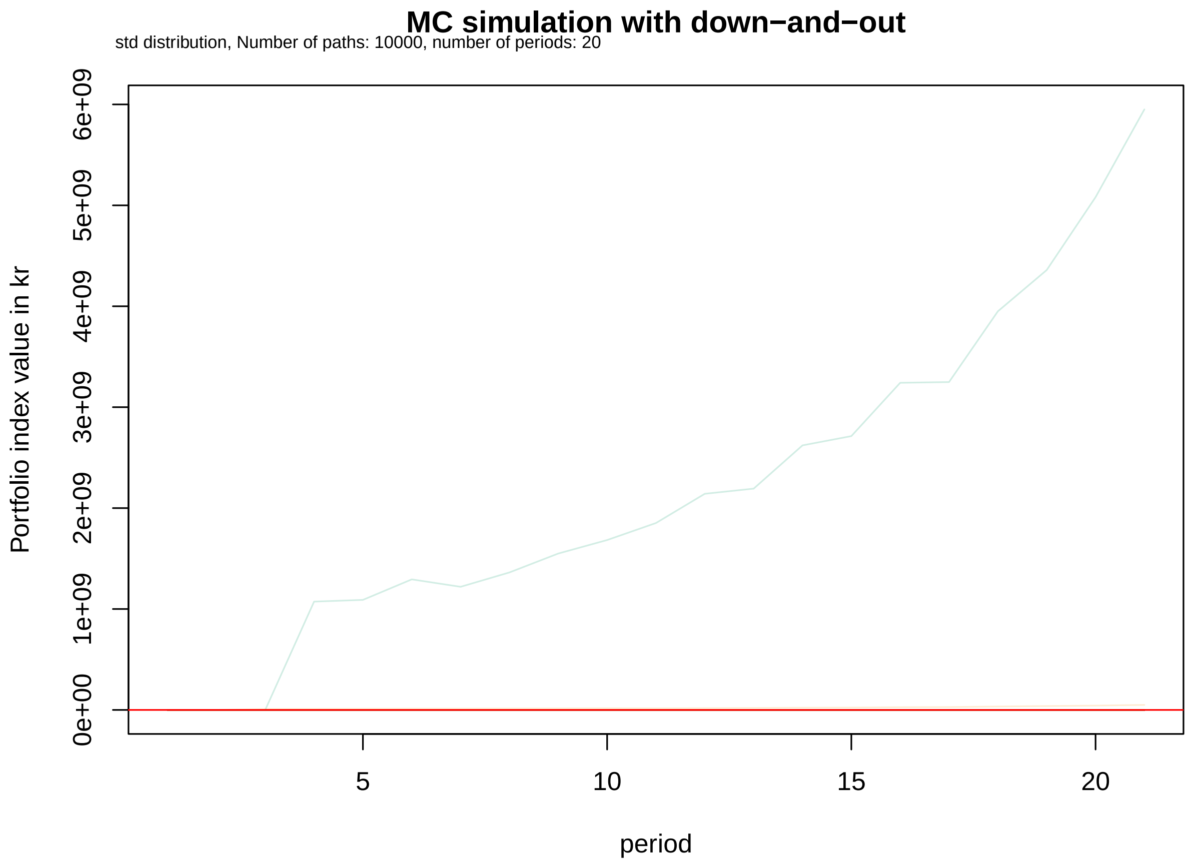
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



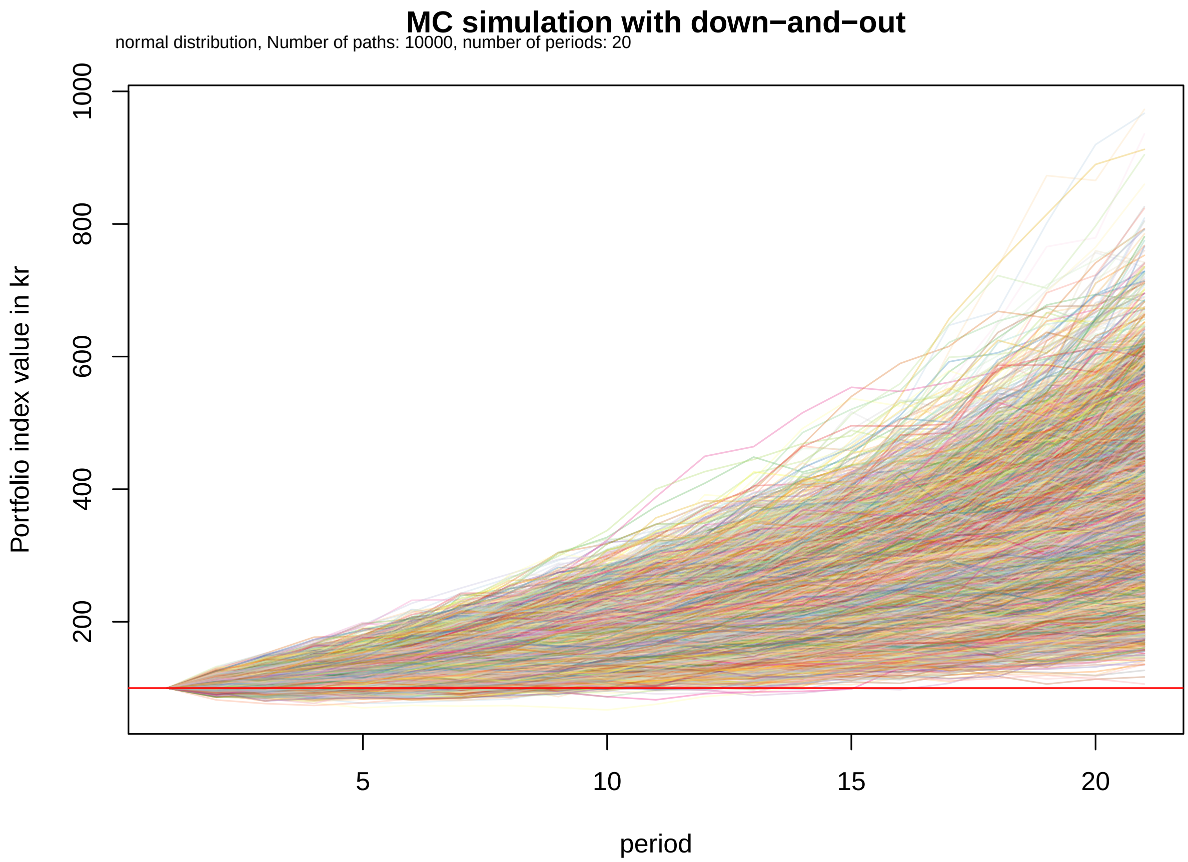
Monte Carlo

pmr has the sstd fit with the lowest value of nu. Inspect simulations:

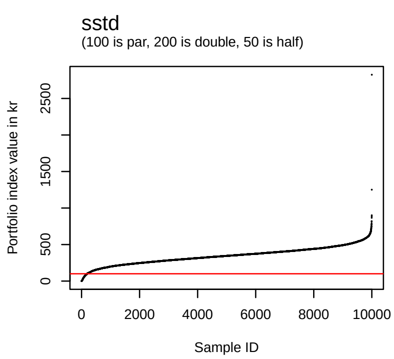

MC simulation with down−and−out
sstd distribution, Number of paths: 10000, number of periods: 20
Portfolio index value in kr
0
500
1000
1500
2000
2500
5
10
15
20
period

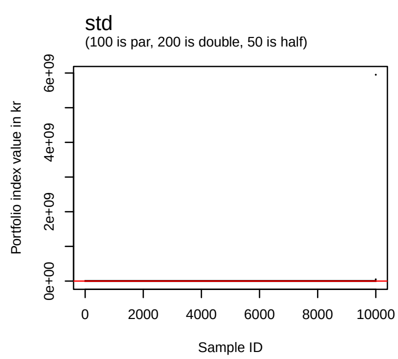


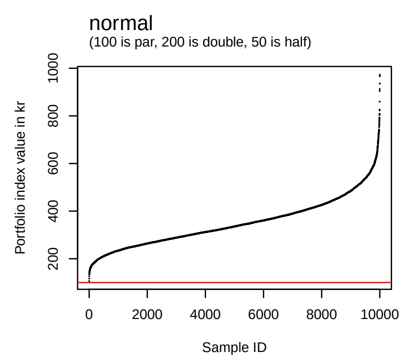
MC simulation with down−and−out
std distribution, Number of paths: 10000, number of periods: 20
Portfolio index value in kr
0e+00
1e+09
2e+09
3e+09
4e+09
5e+09
6e+09
5
10
15
20
period



Sorted portfolio index values for last period of all runs



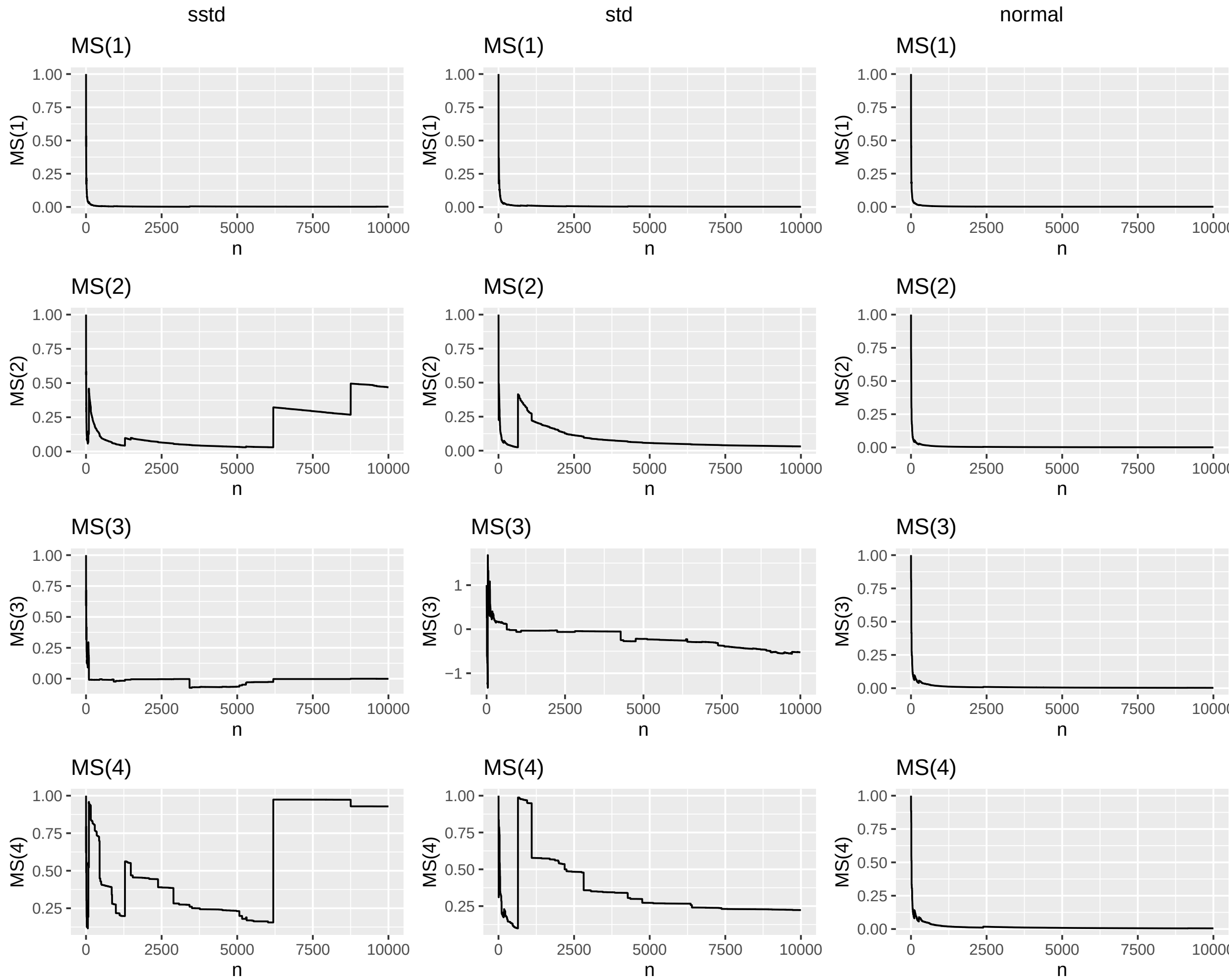




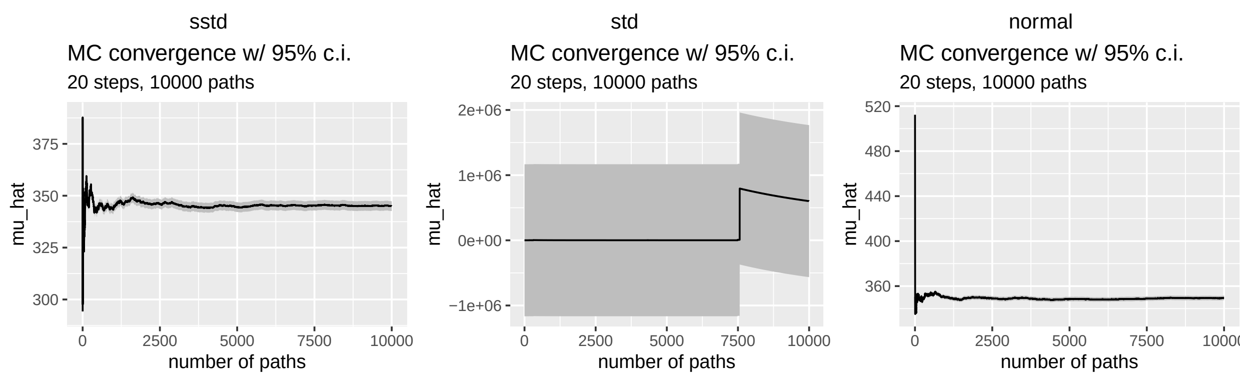
Convergence

Max vs sum

Max vs sum plots for the first four moments:

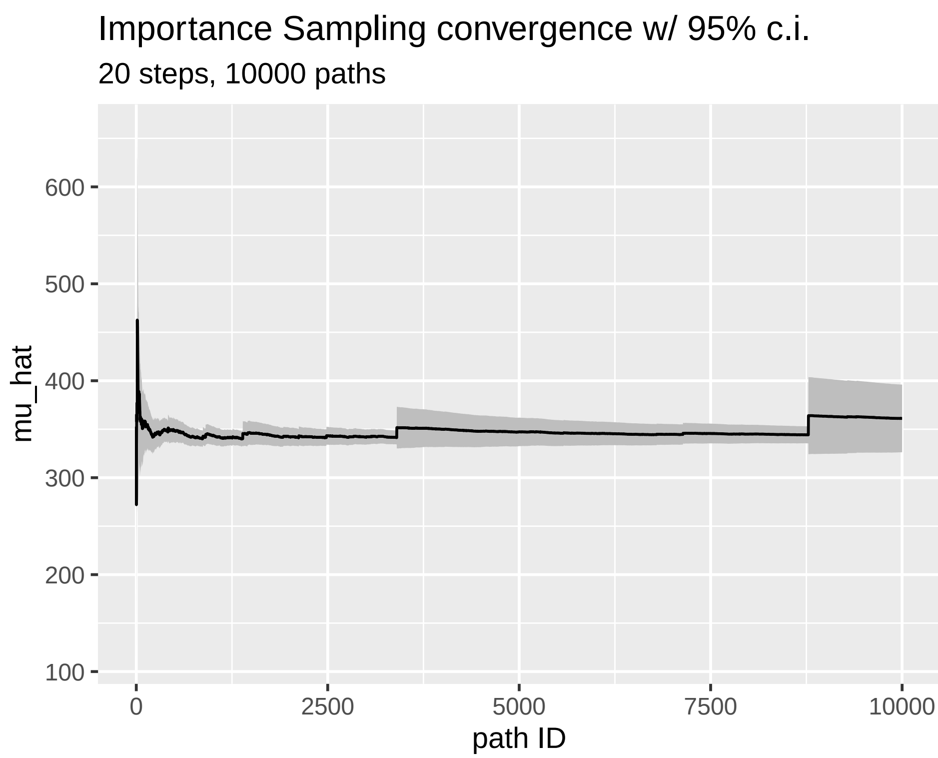


MC



IS

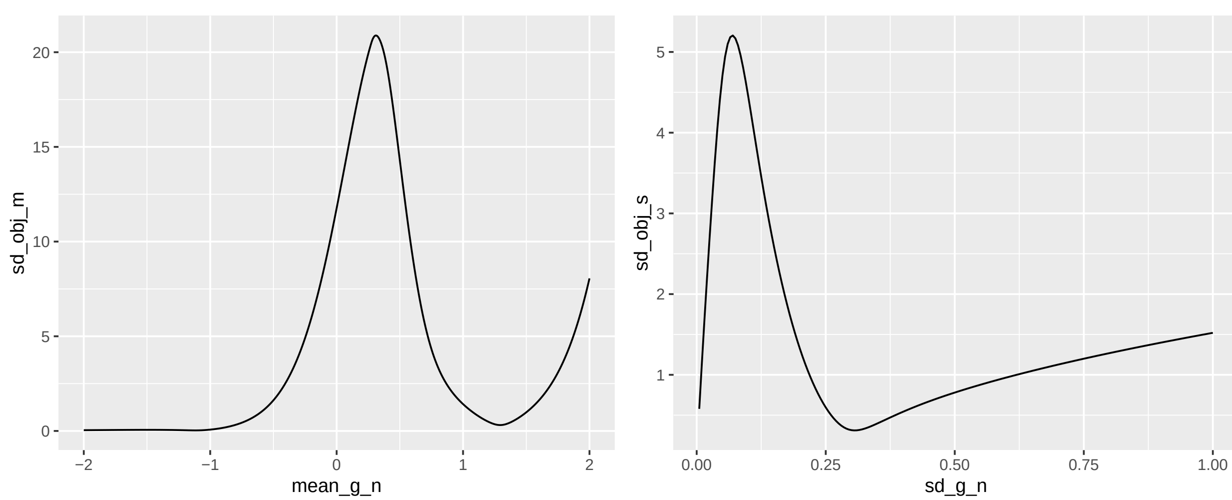
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.2936284 0.3062685
```

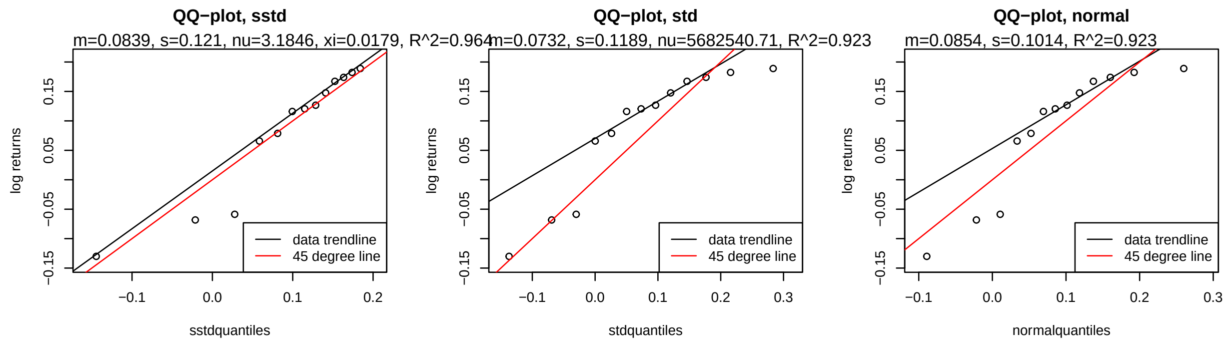
Objective function plots



PFA high risk (phr), 2011 - 2023

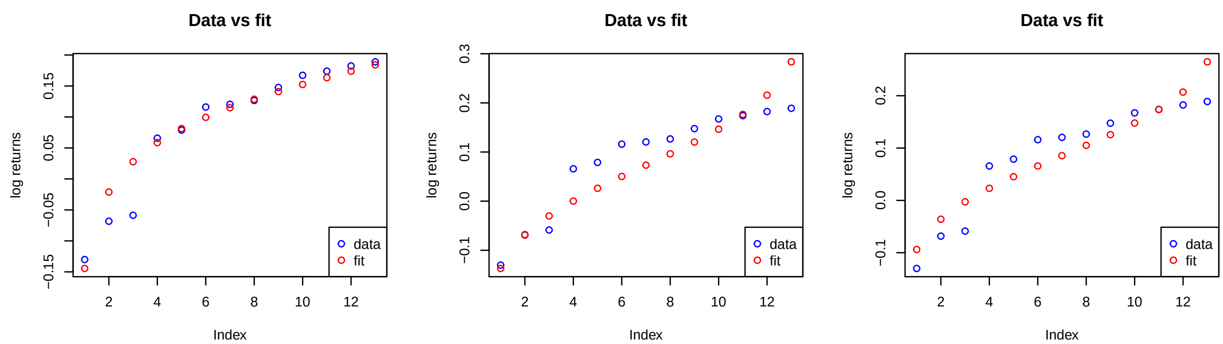
QQ Plot

Skewed t-distribution (sstd):



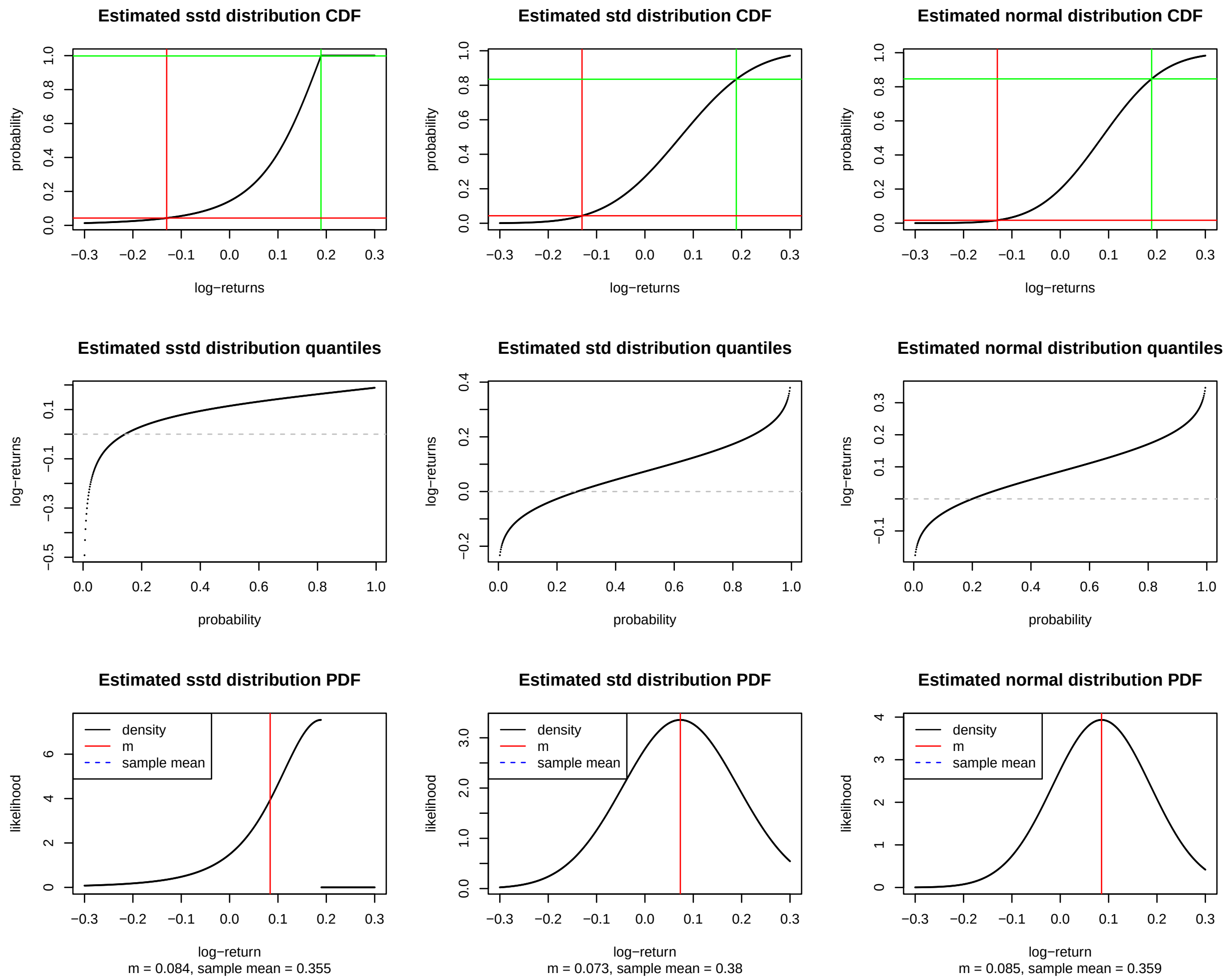
Data vs fit

Let's plot the fit and the observed returns together.



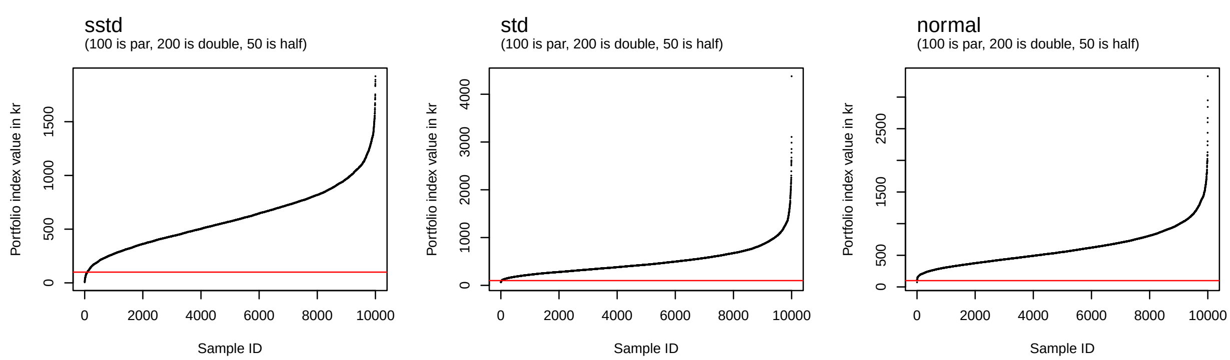
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

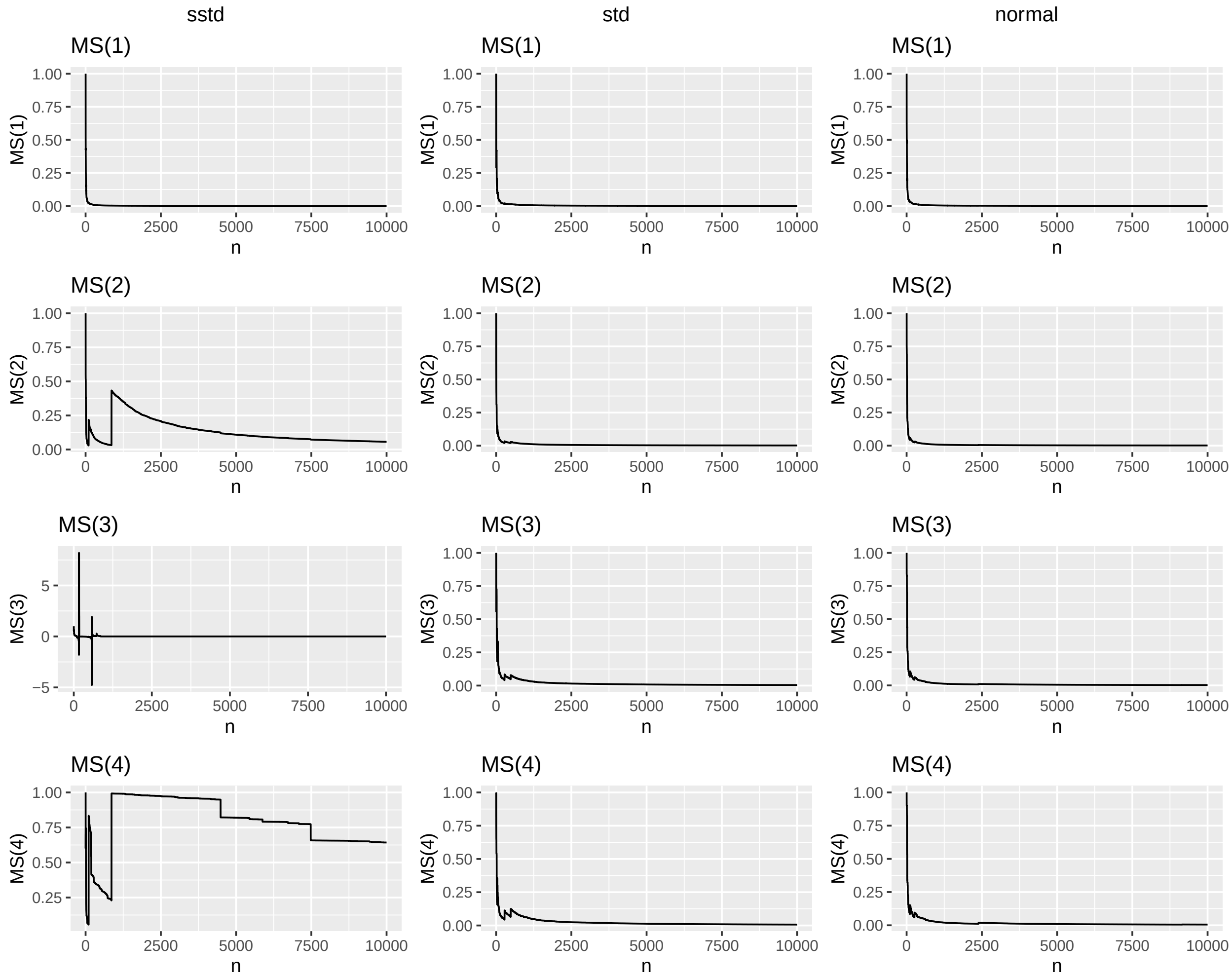
Sorted portfolio index values for last period of all runs



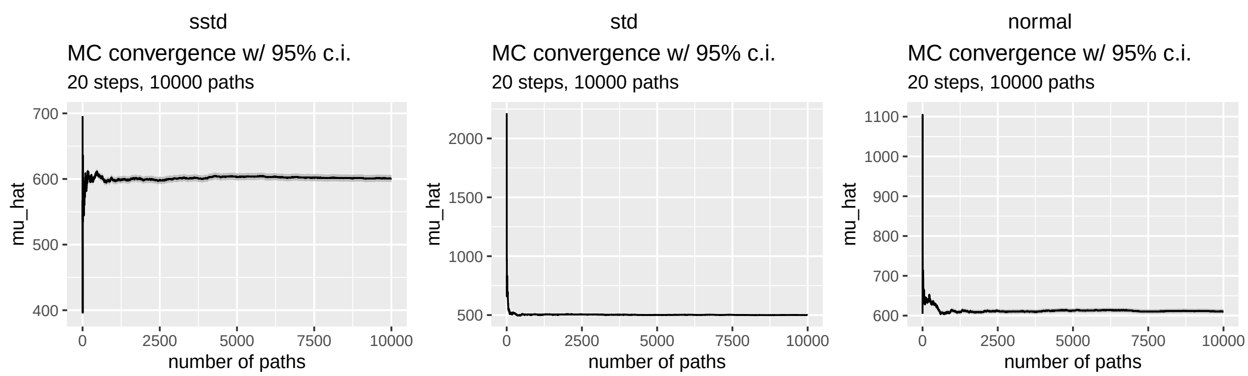
Convergence

Max vs sum

Max vs sum plots for the first four moments:

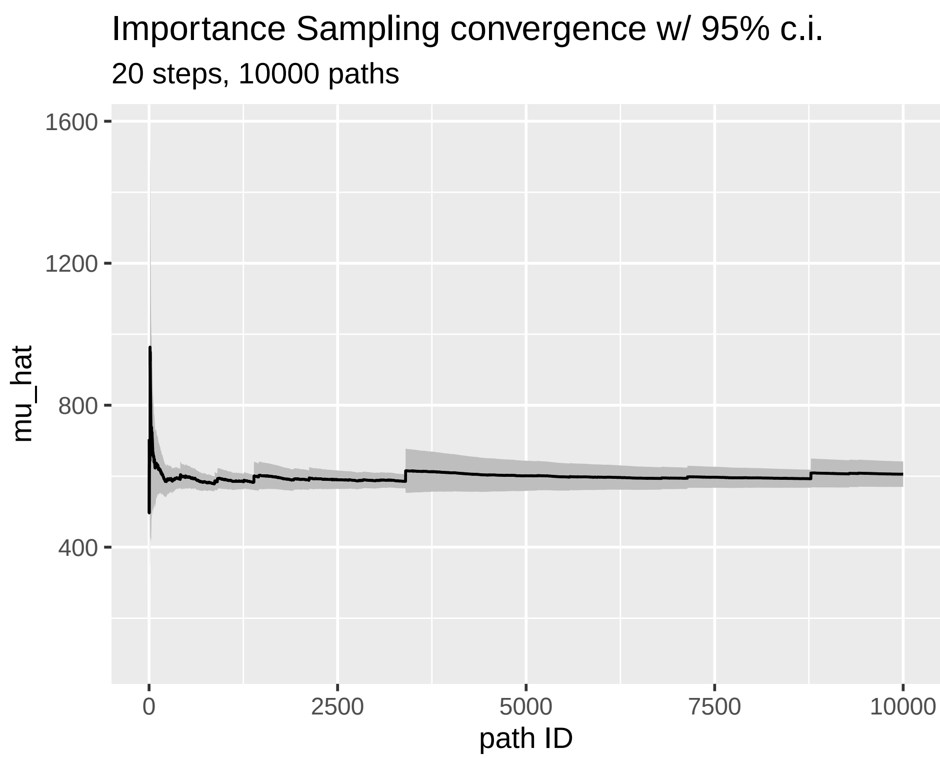


MC



IS

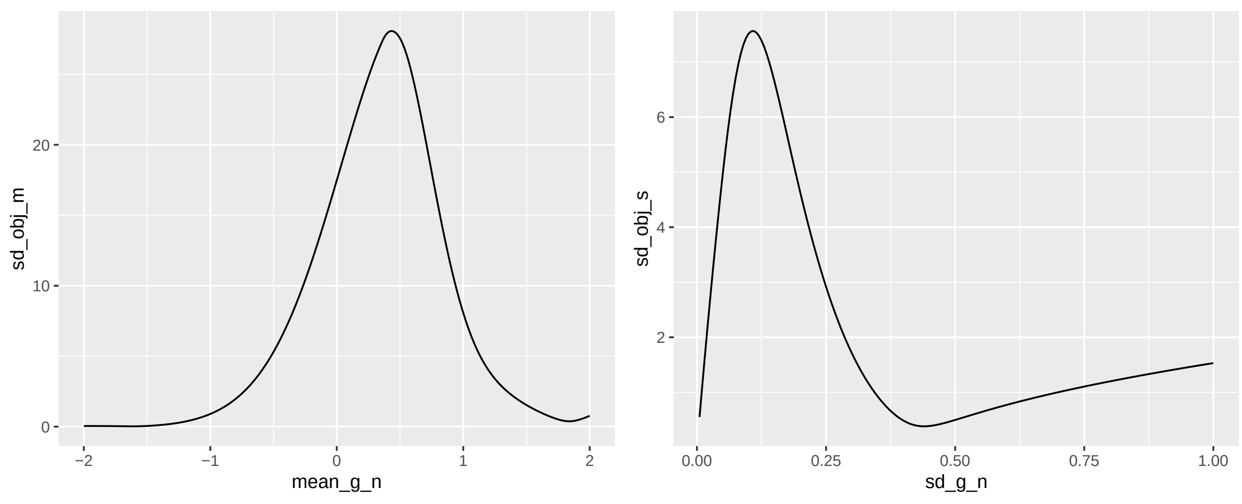
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.8379614 0.4397688
```

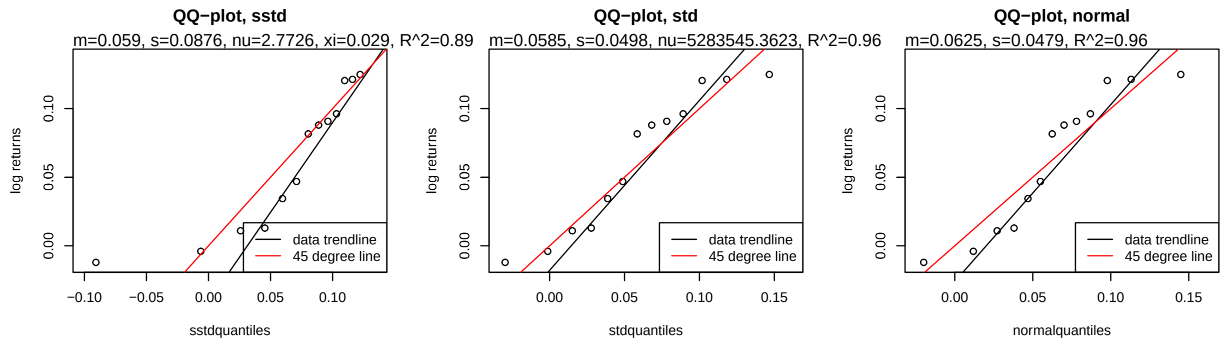
Objective function plots



Mix medium risk (mmr), 2011 - 2023

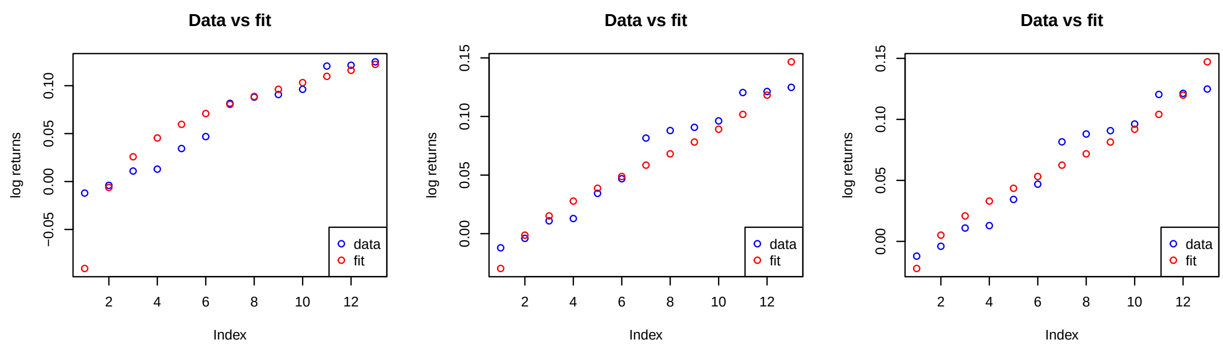
QQ Plot

Skewed t-distribution (sstd):



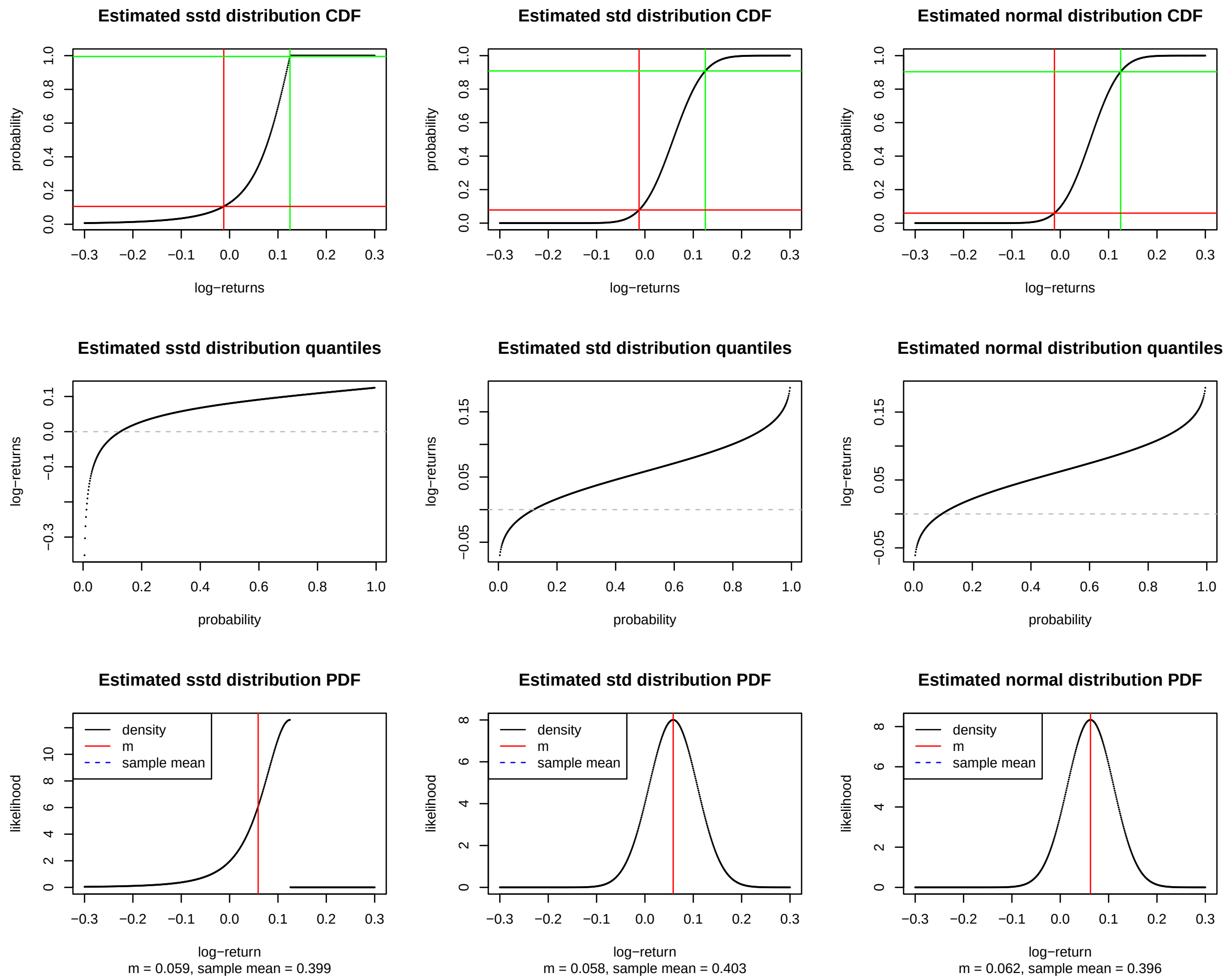
Data vs fit

Let's plot the fit and the observed returns together.



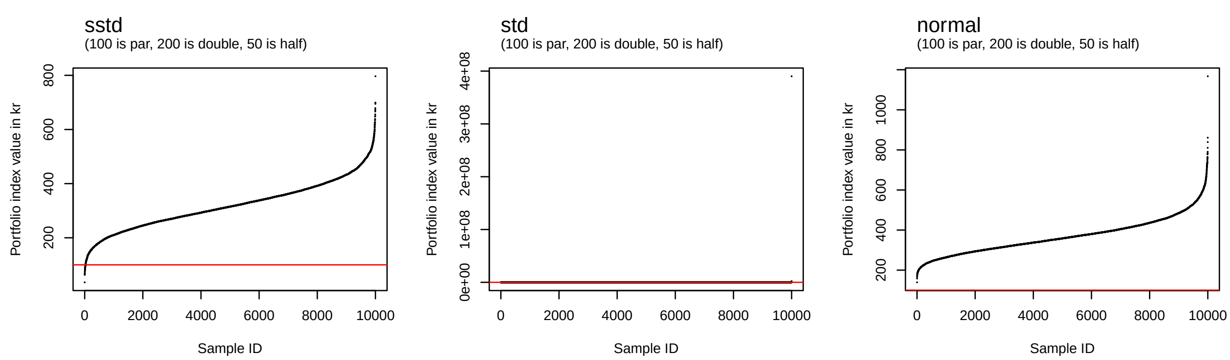
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

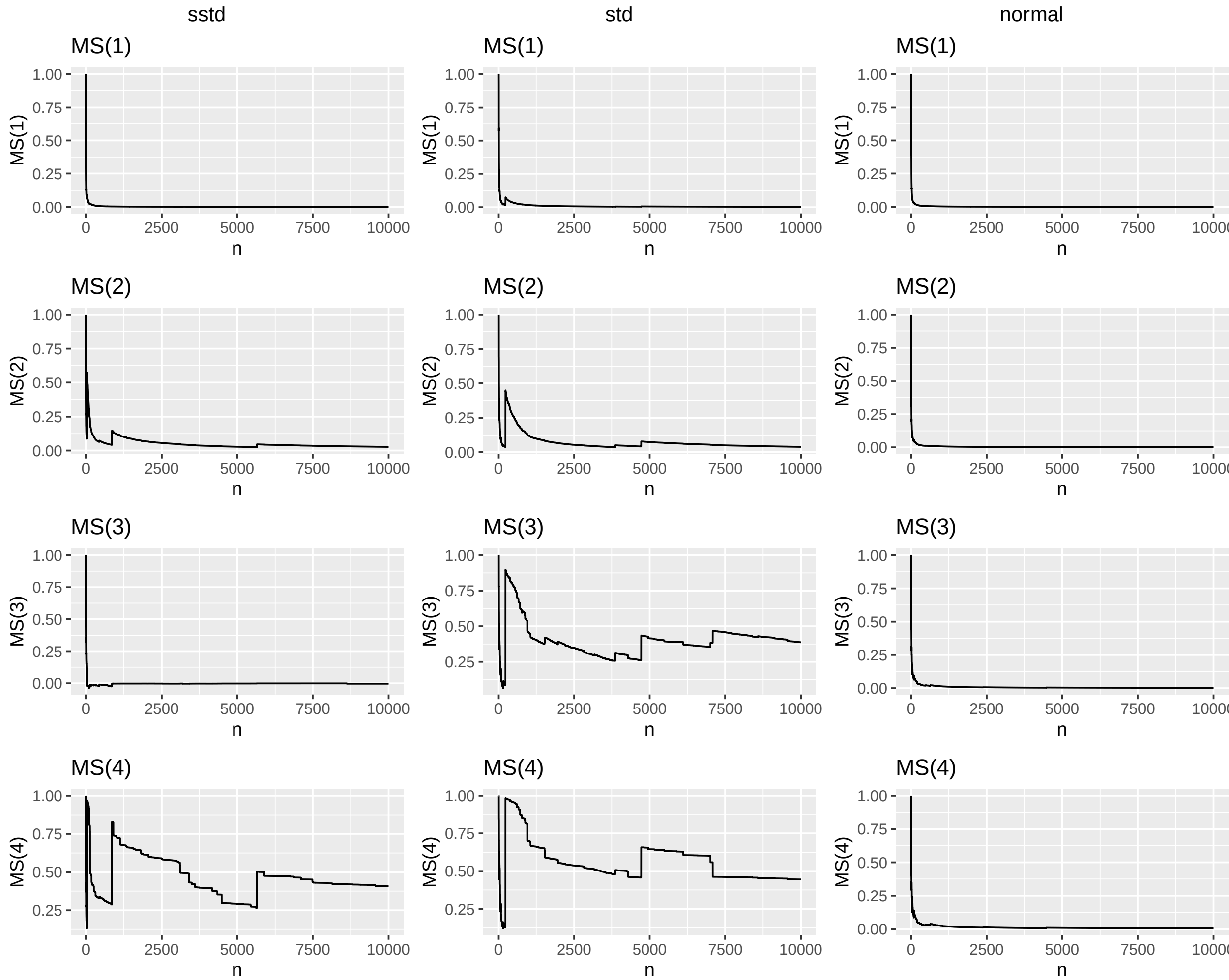
Sorted portfolio index values for last period of all runs



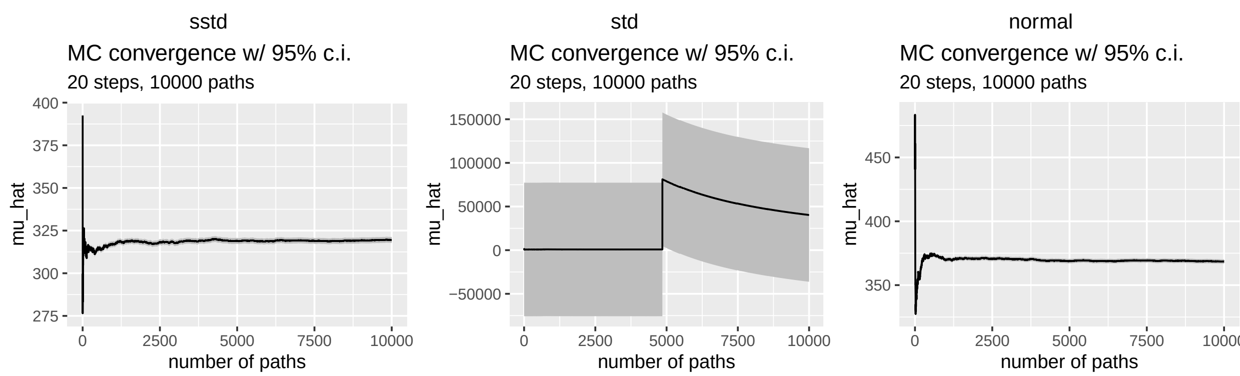
Convergence

Max vs sum

Max vs sum plots for the first four moments:

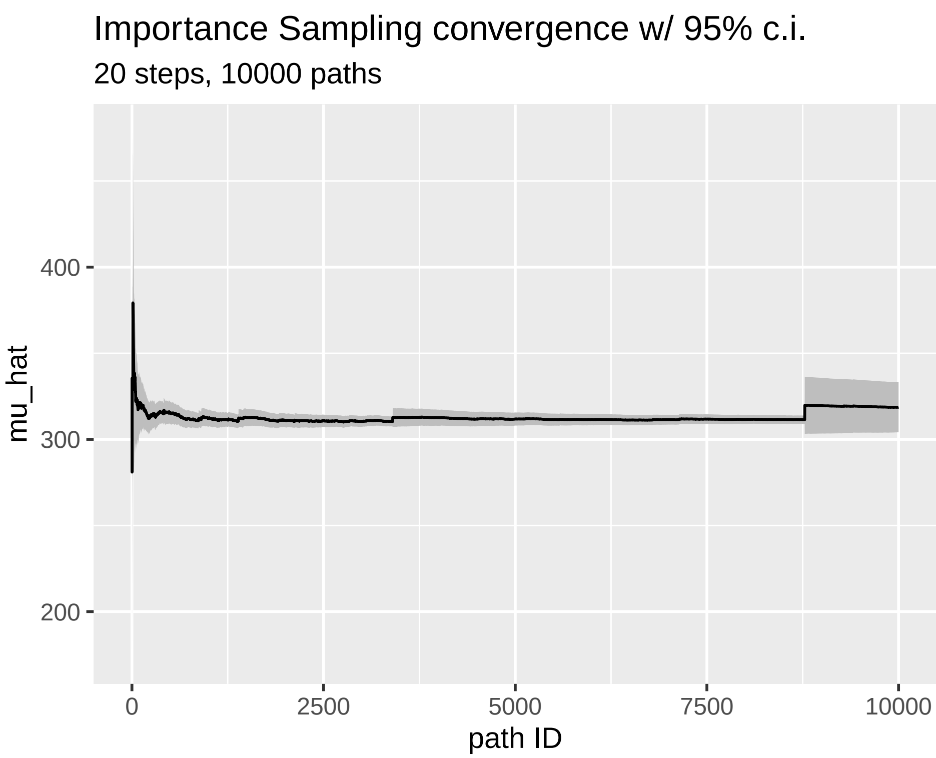


MC



IS

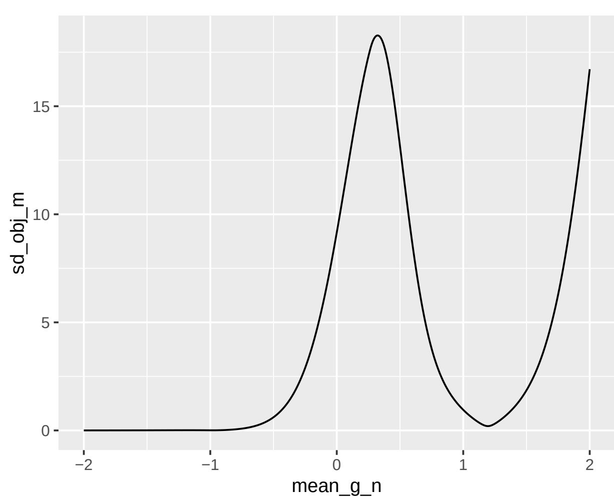
Skewed t-distribution with a normal proposal distribution.

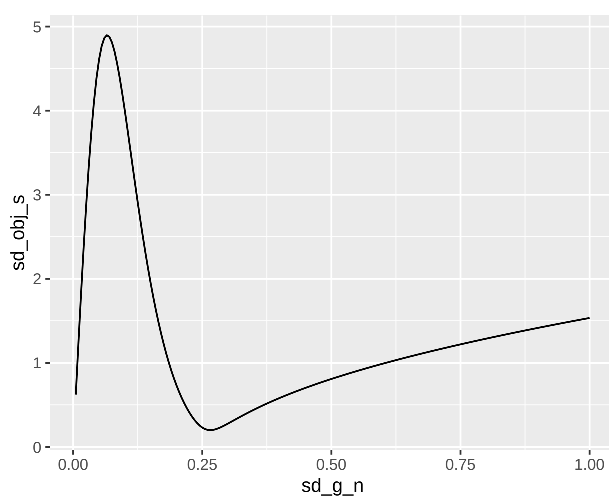


Parameters

```
## [1] 1.1948623 0.2654885
```

Objective function plots

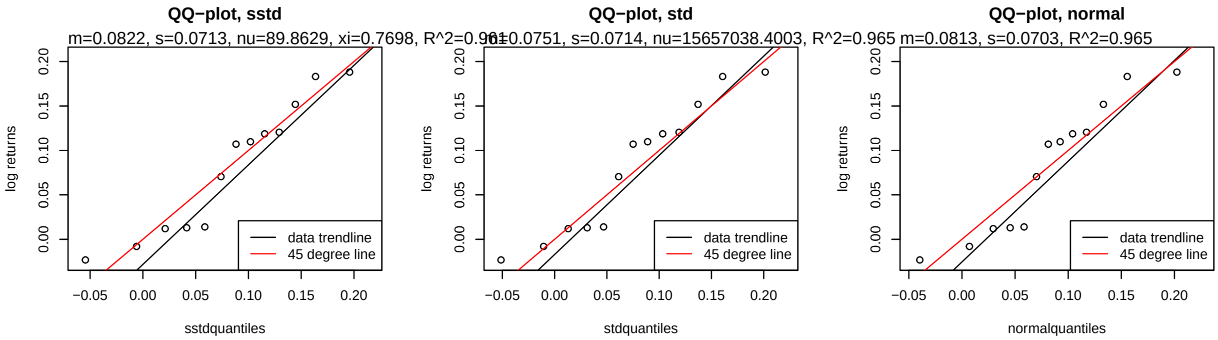




Mix high risk (mhr), 2011 - 2023

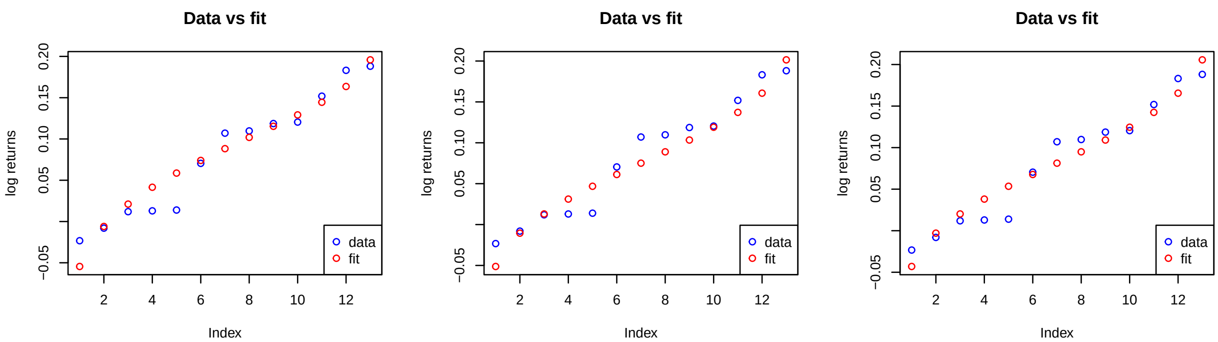
QQ Plot

Skewed t-distribution (sstd):



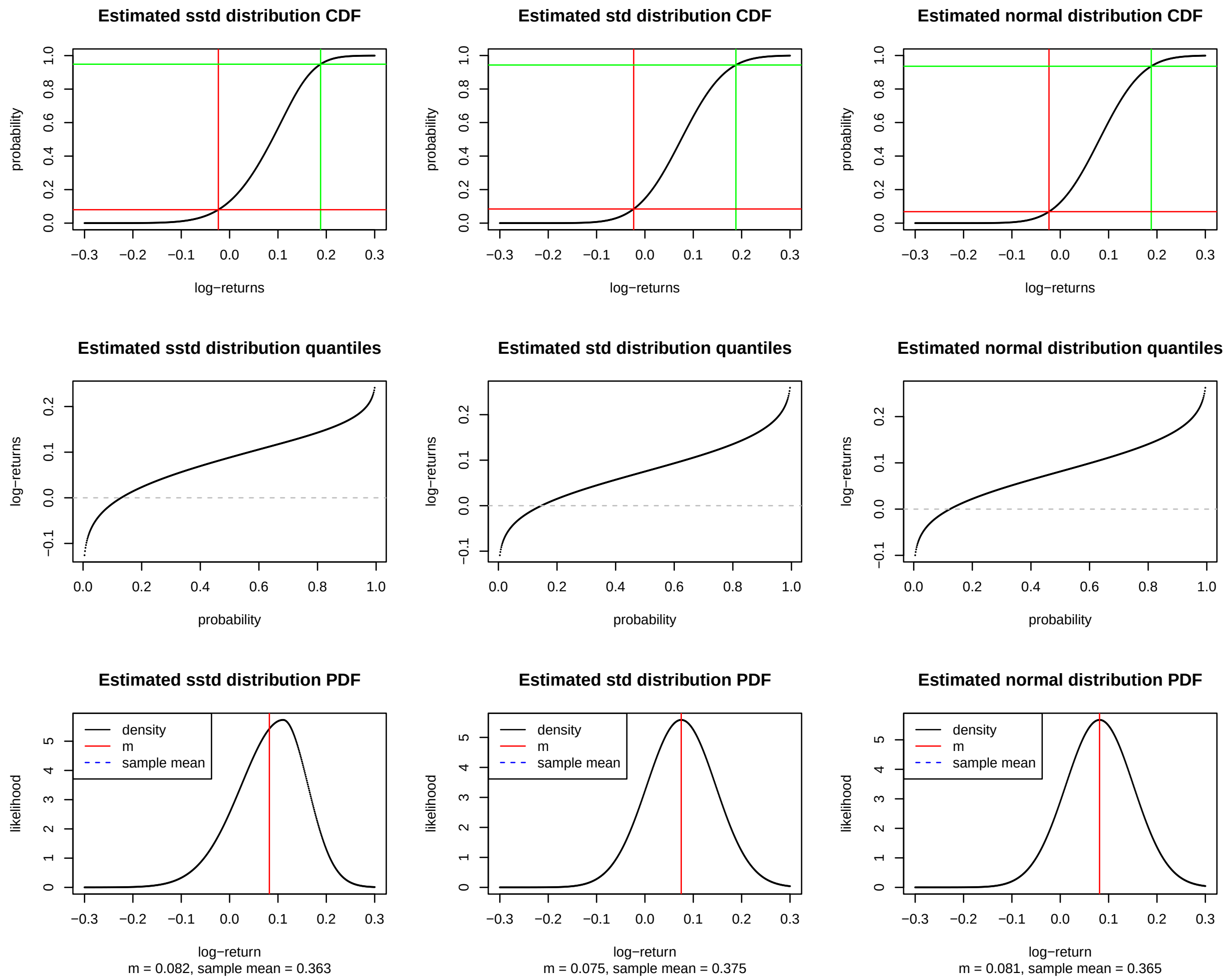
Data vs fit

Let's plot the fit and the observed returns together.



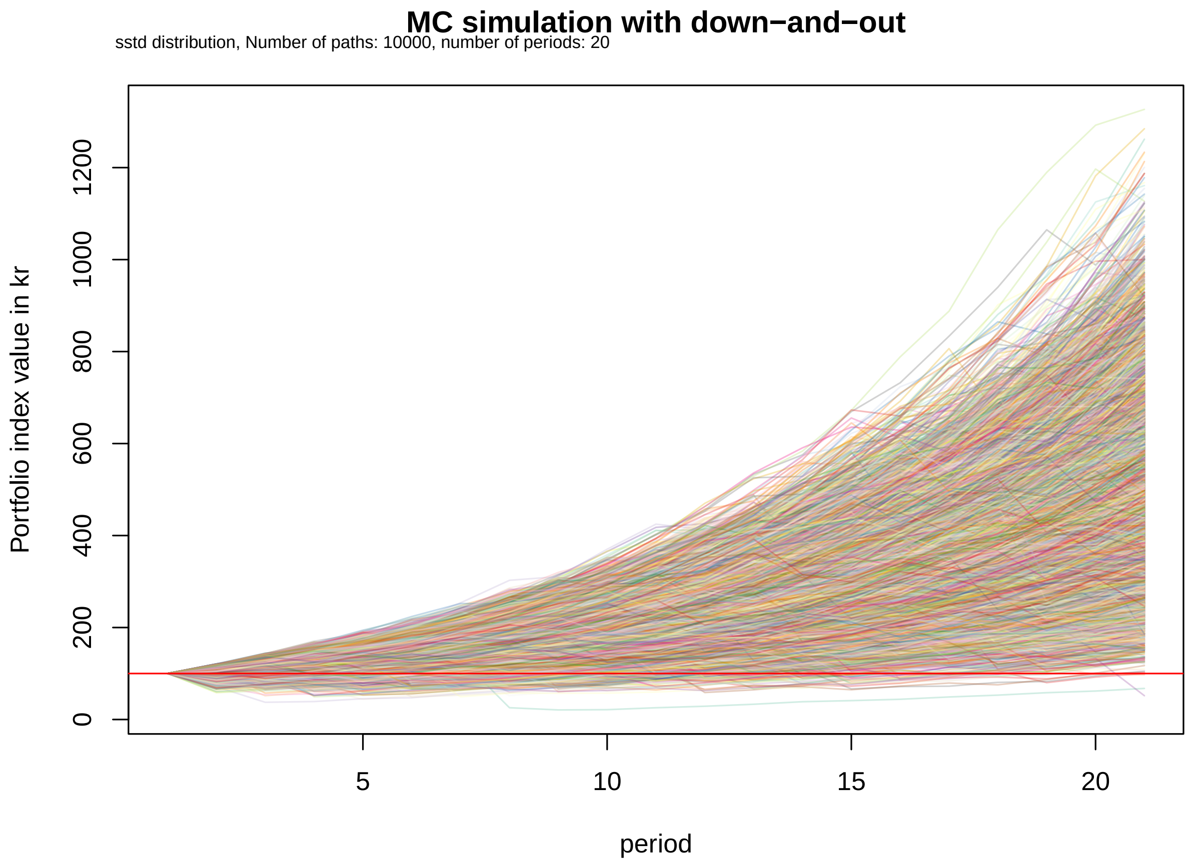
Estimated distribution

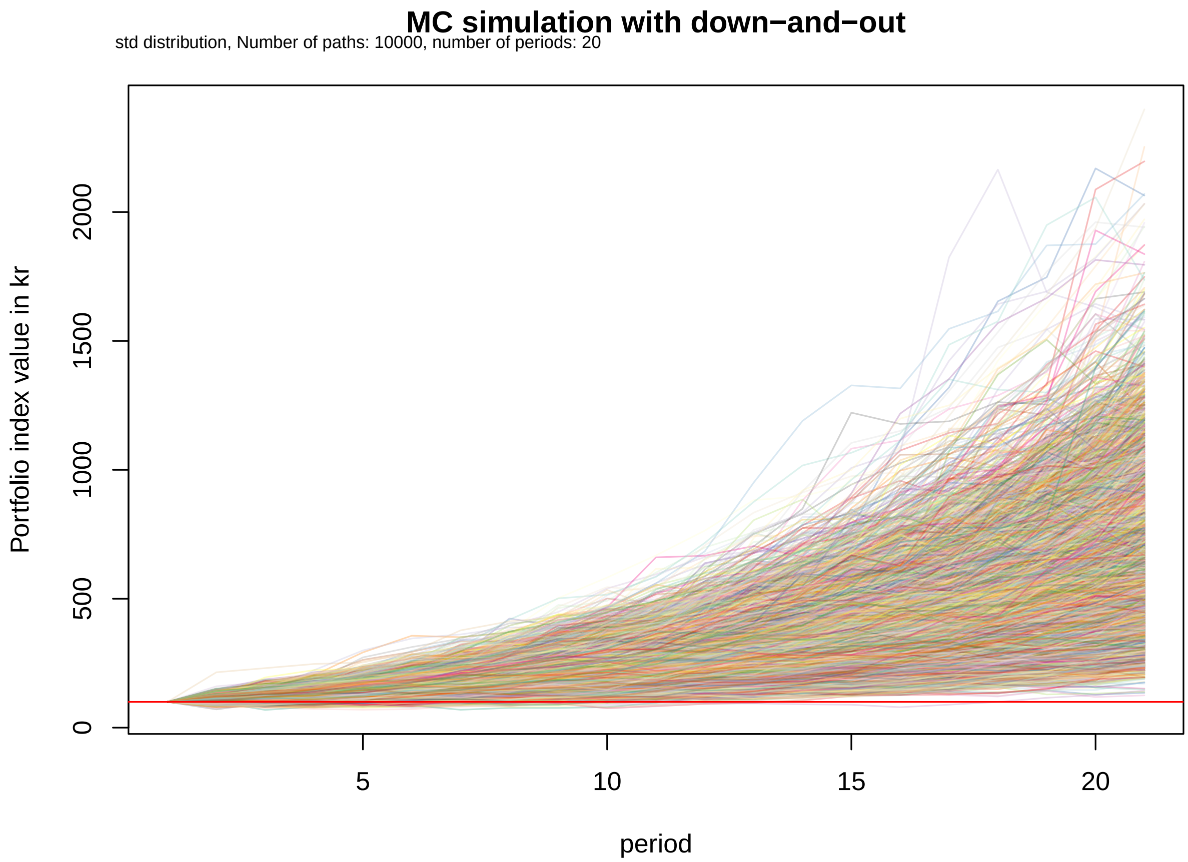
Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:

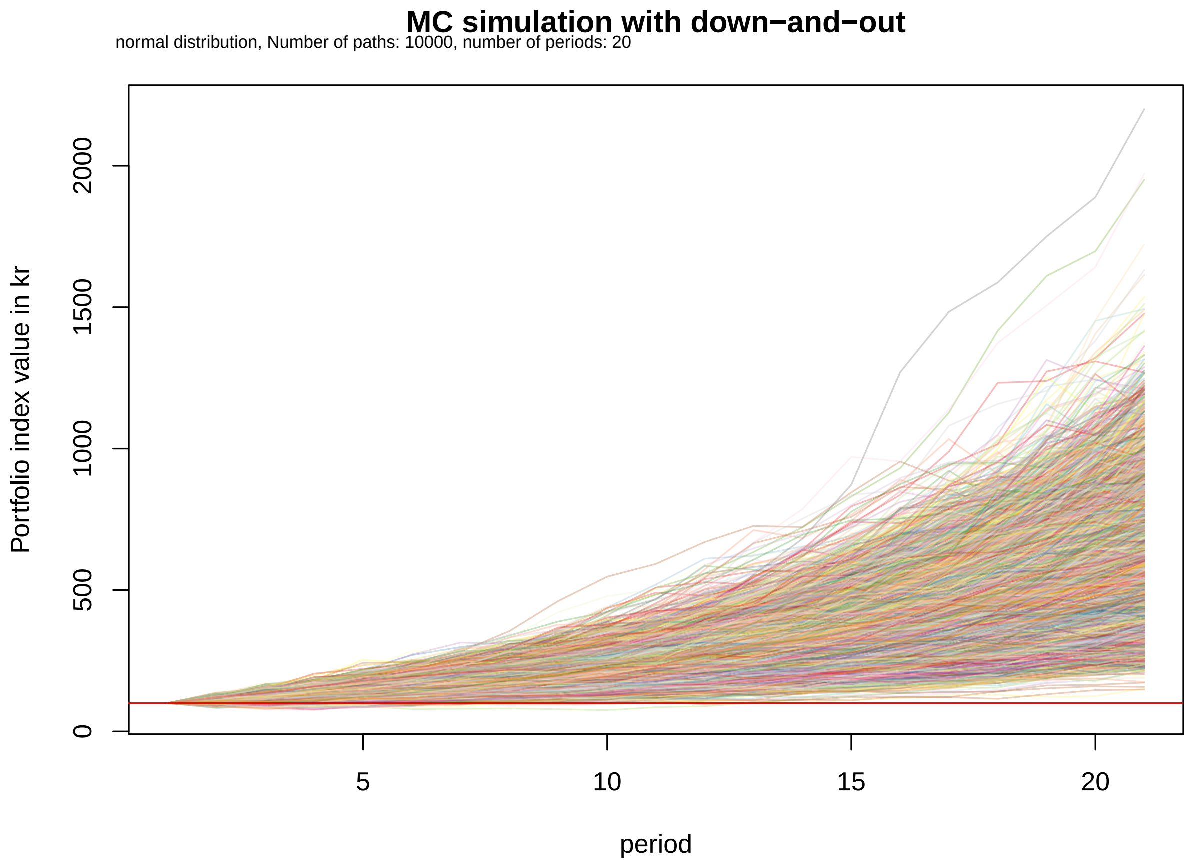


Monte Carlo

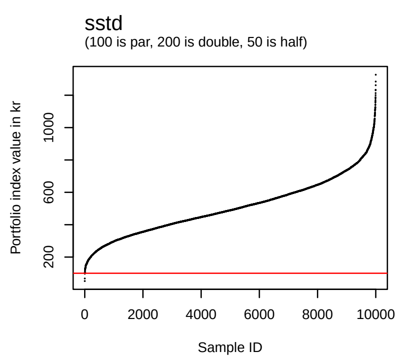
mhr has the sstd fit with thehighest sstd fit with thevalue of nu. Inspect simulations:

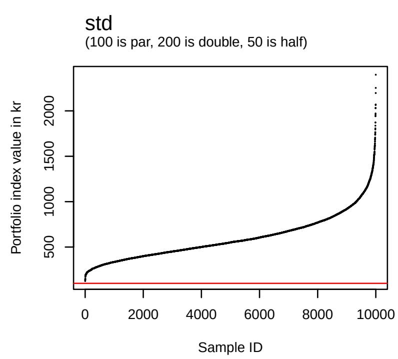

MC simulation with down−and−out
sstd distribution, Number of paths: 10000, number of periods: 20
Portfolio index value in kr
0
200
400
600
800
1000
1200
5
10
15
20
period

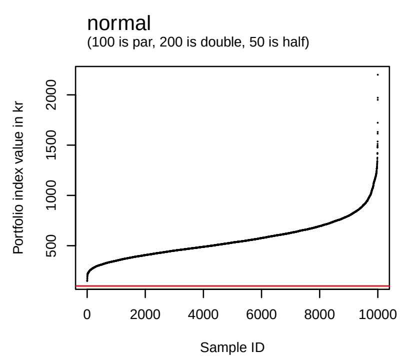

MC simulation with down−and−out
std distribution, Number of paths: 10000, number of periods: 20
Portfolio index value in kr
0
500
1000
1500
2000
5
10
15
20
period



Sorted portfolio index values for last period of all runs



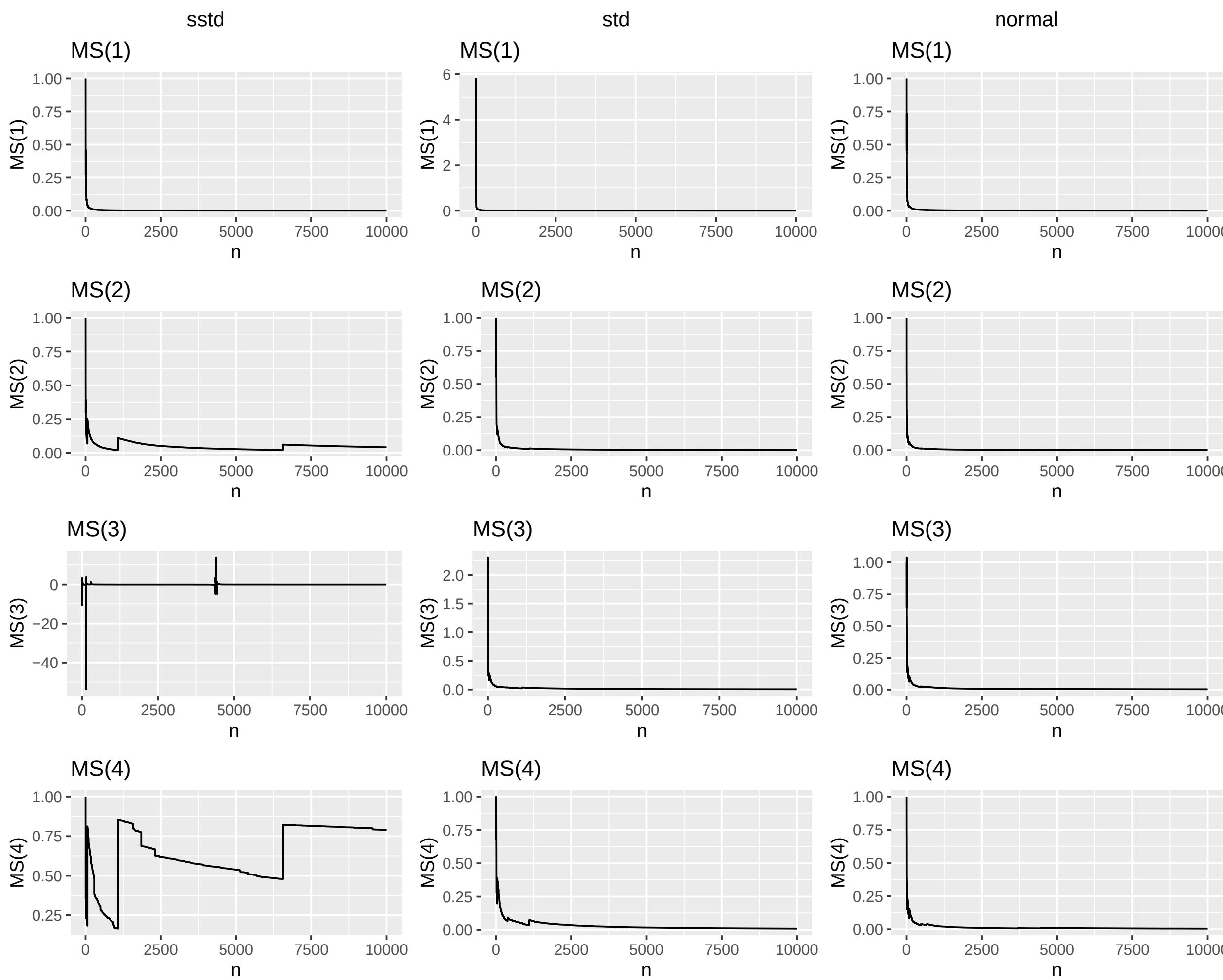




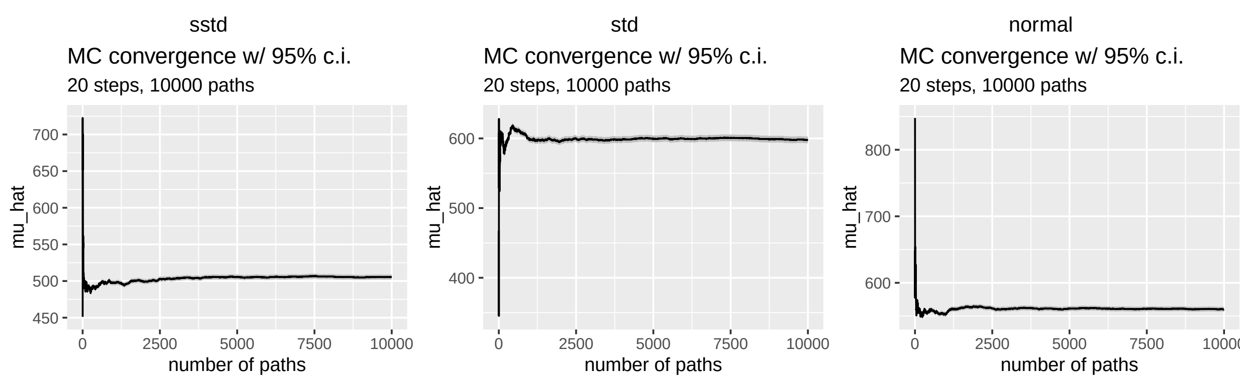
Convergence

Max vs sum

Max vs sum plots for the first four moments:

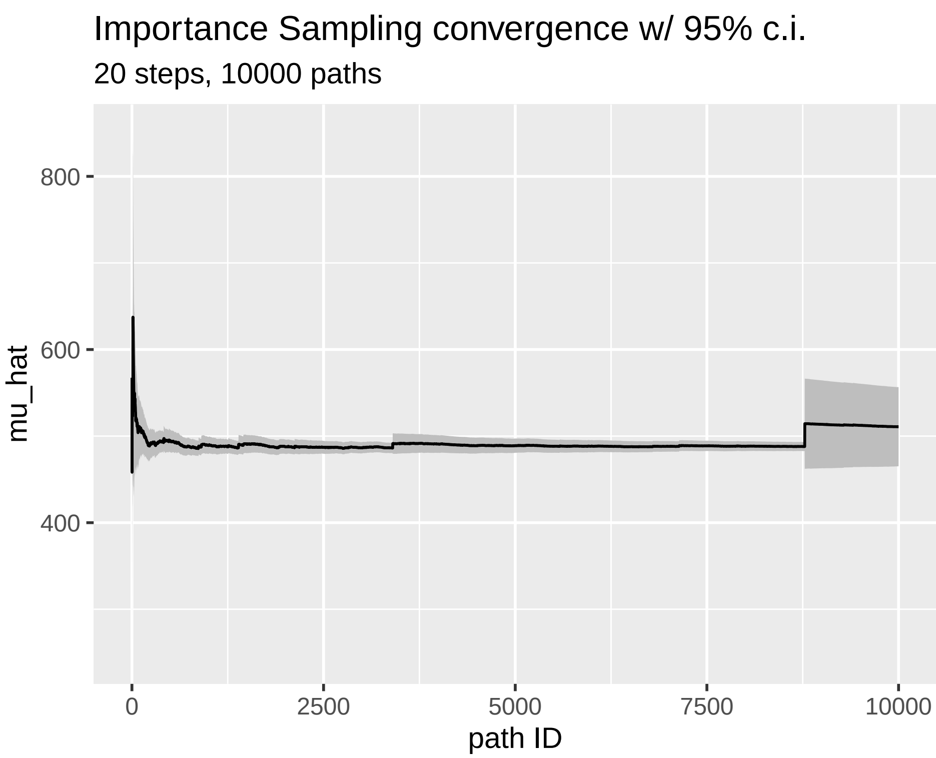


MC



IS

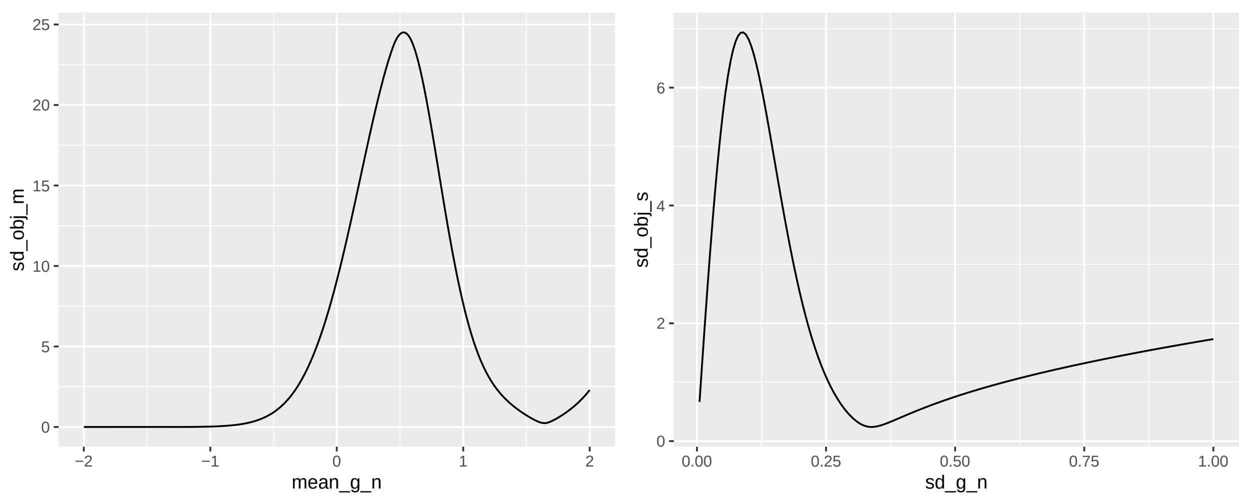
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.6413478 0.3380133
```

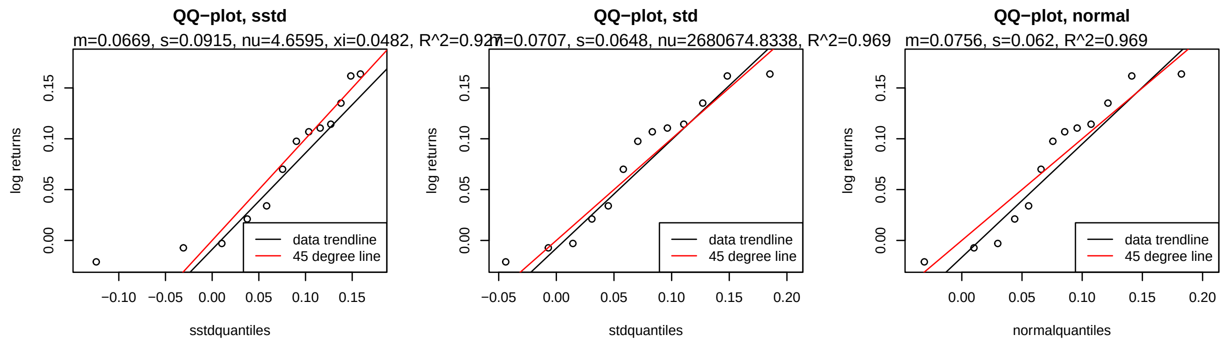
Objective function plots



Mix vmr+phr (vm_ph), 2011 - 2023

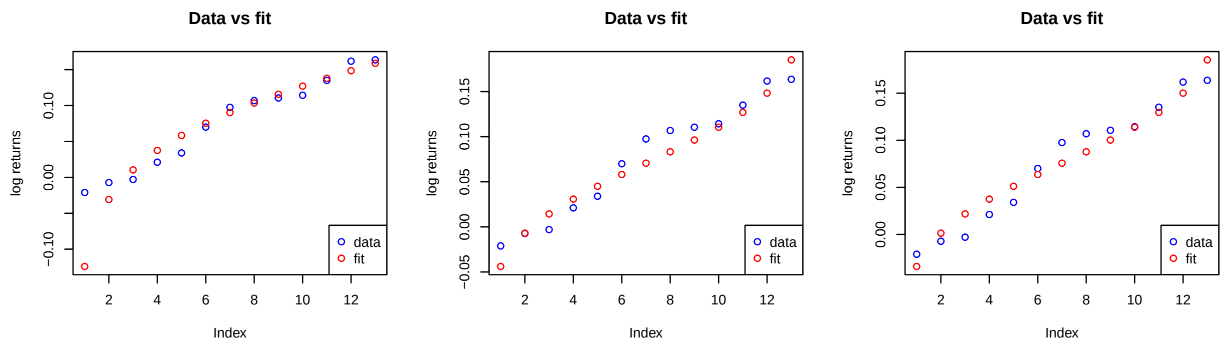
QQ Plot

Skewed t-distribution (sstd):



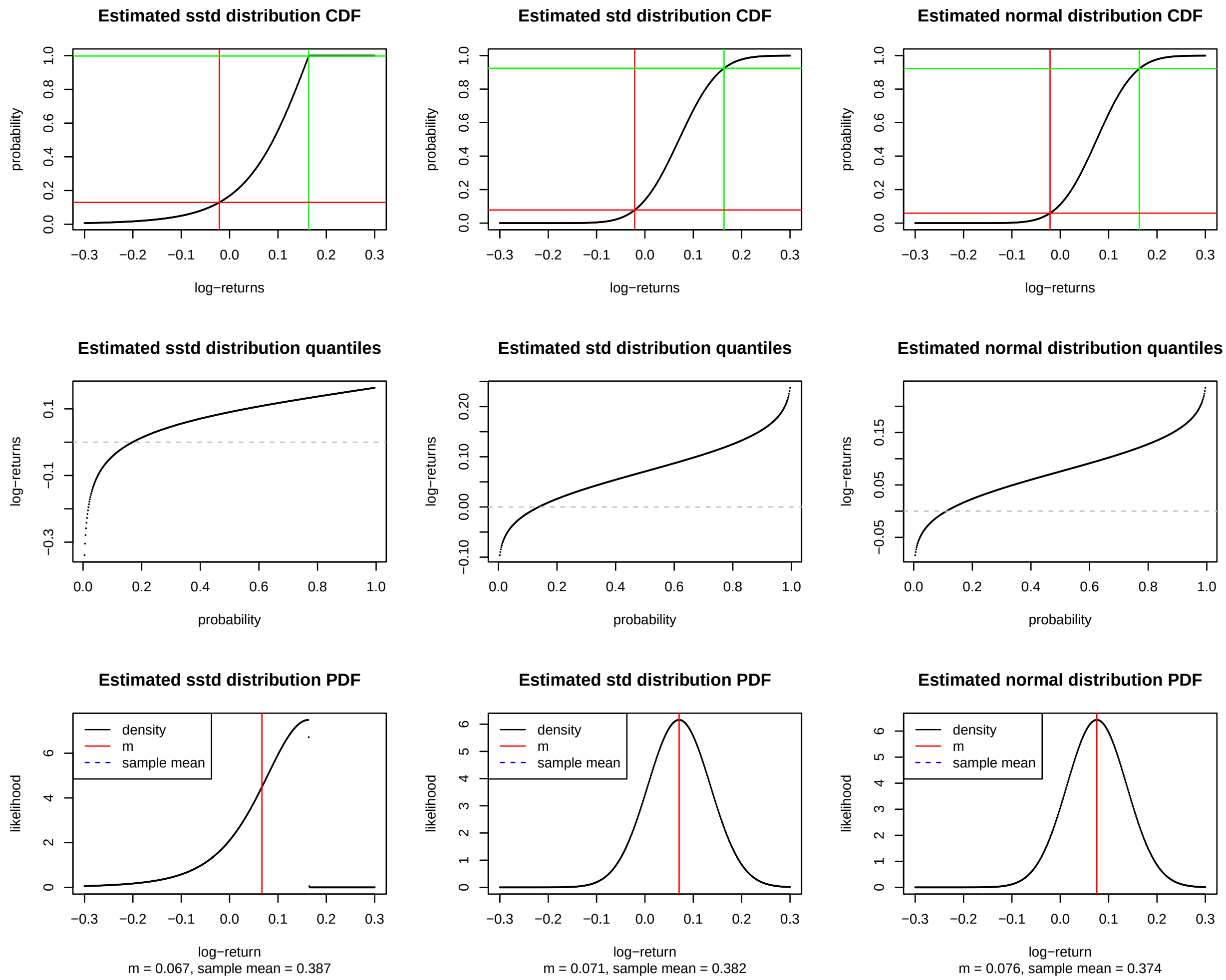
Data vs fit

Let's plot the fit and the observed returns together.



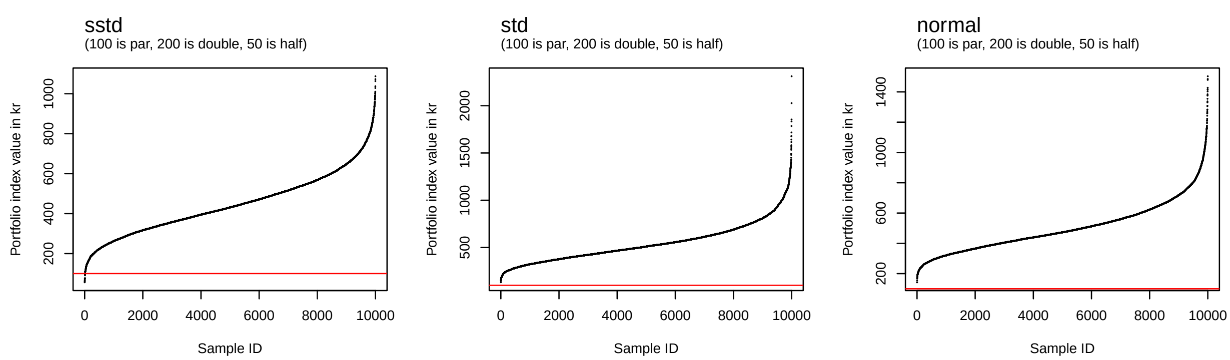
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

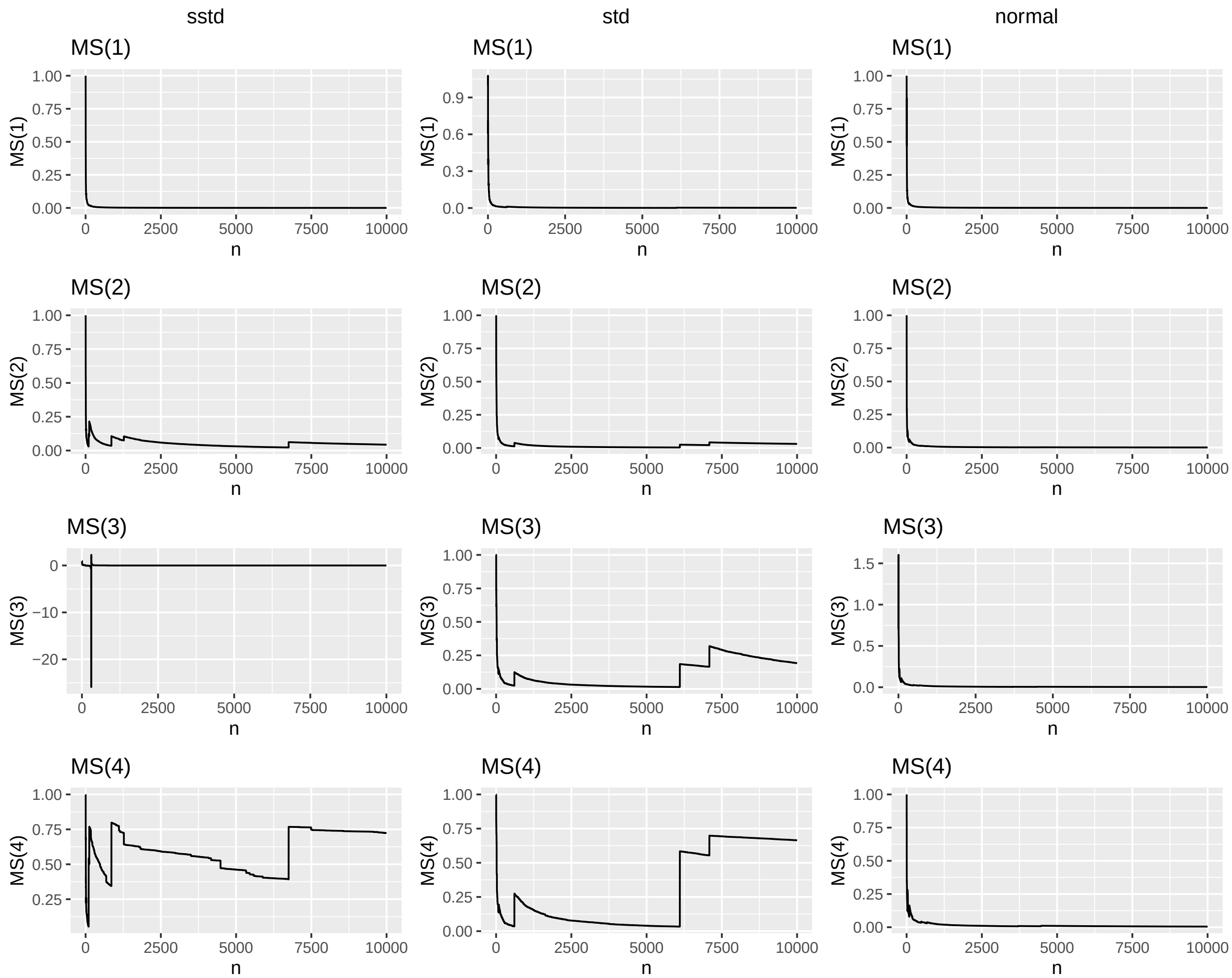
Sorted portfolio index values for last period of all runs



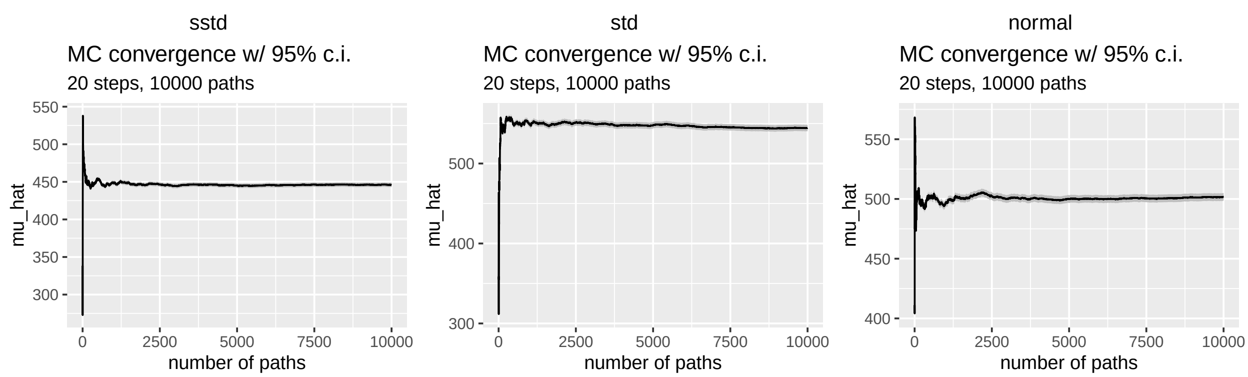
Convergence

Max vs sum

Max vs sum plots for the first four moments:

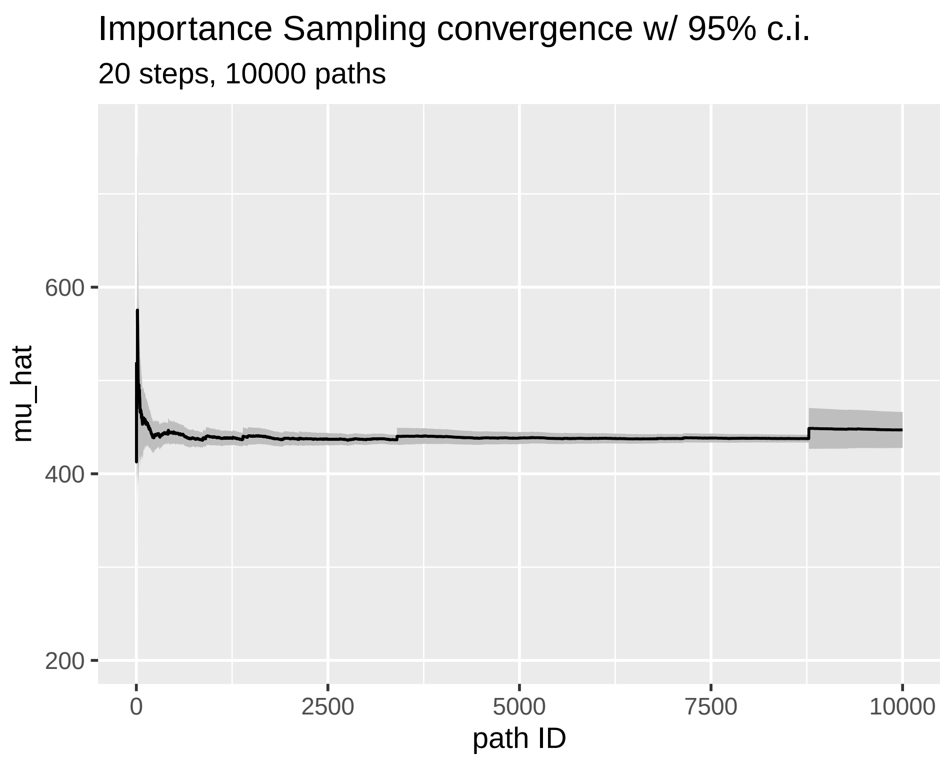


MC



IS

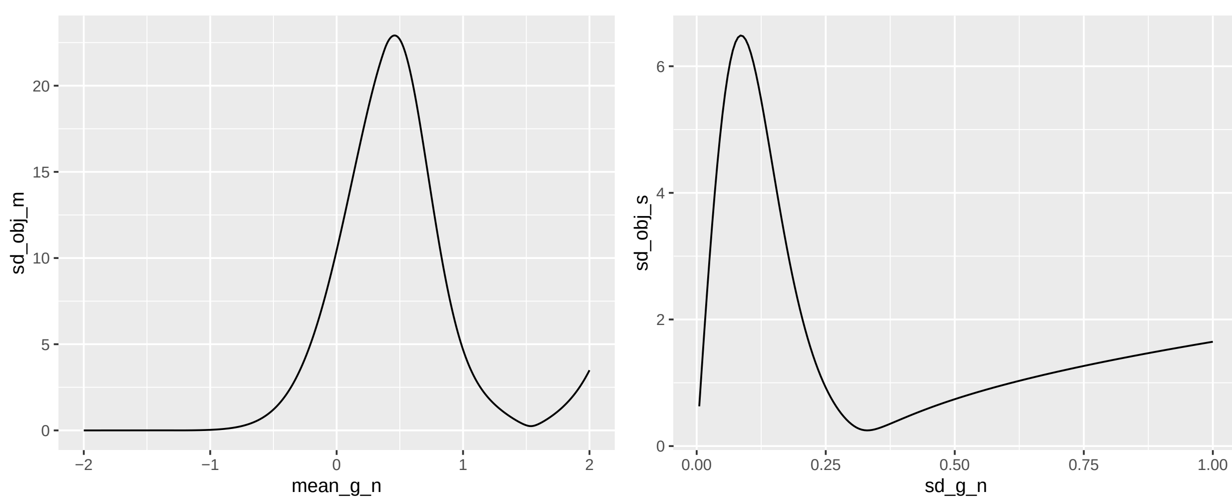
Skewed t-distribution with a normal proposal distribution.



Parameters

```
## [1] 1.5363616 0.3304634
```

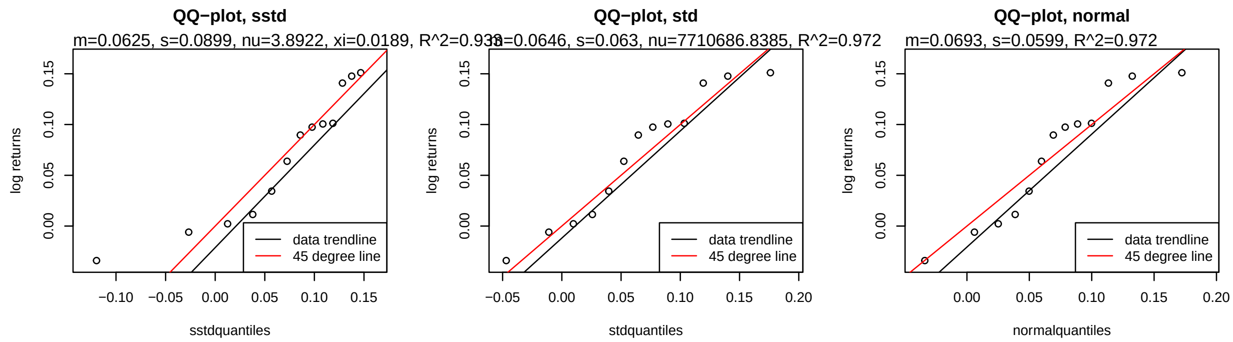
Objective function plots



Mix vhr+pmr (mh_pm), 2011 - 2023

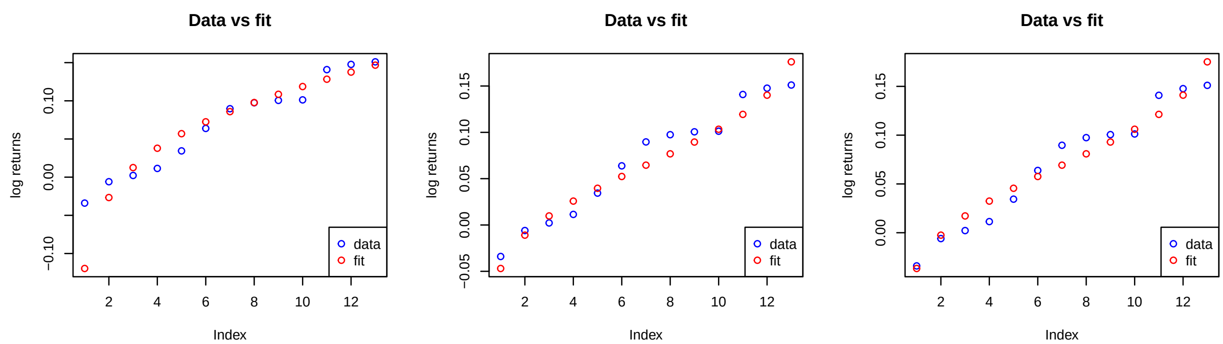
QQ Plot

Skewed t-distribution (sstd):



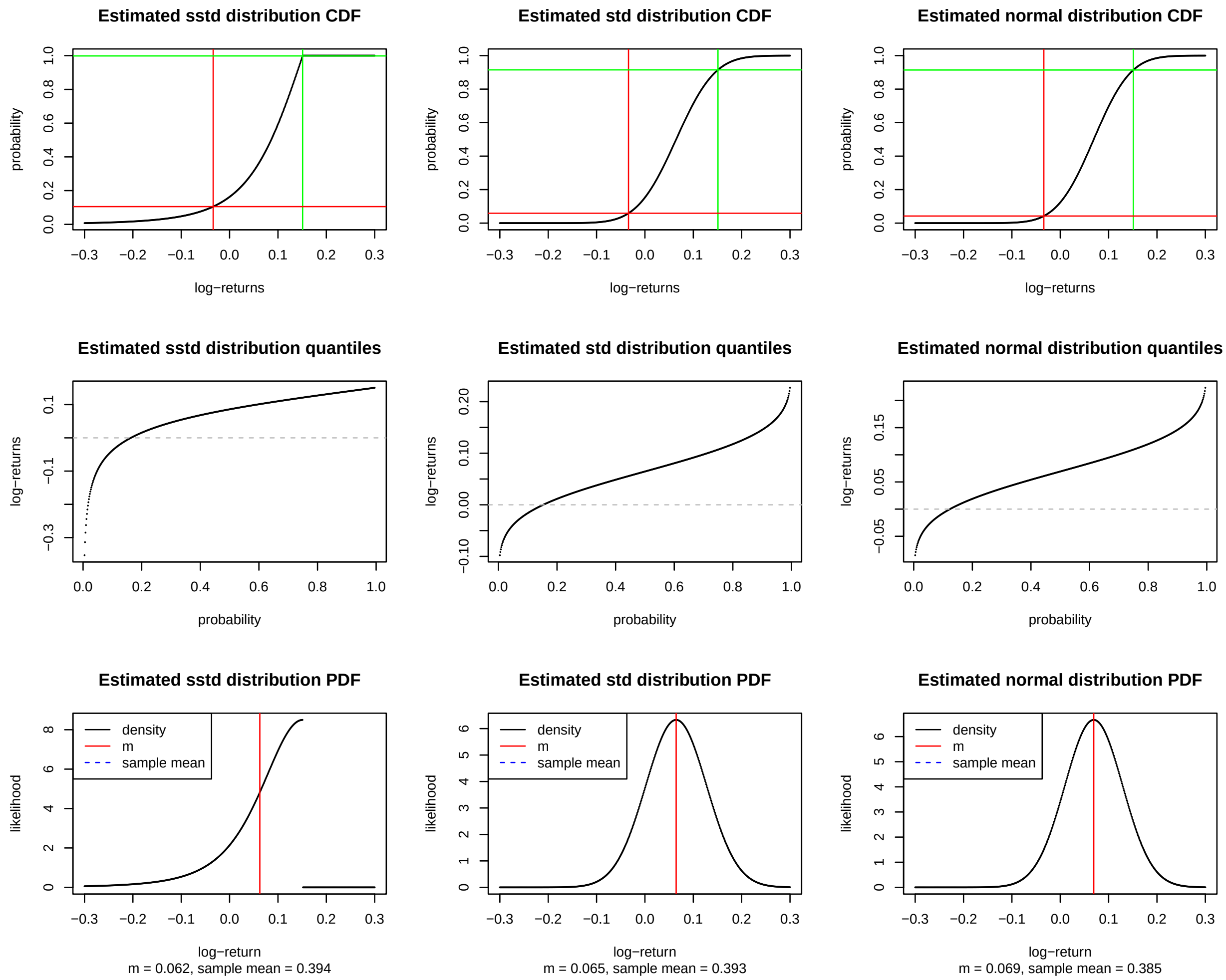
Data vs fit

Let's plot the fit and the observed returns together.



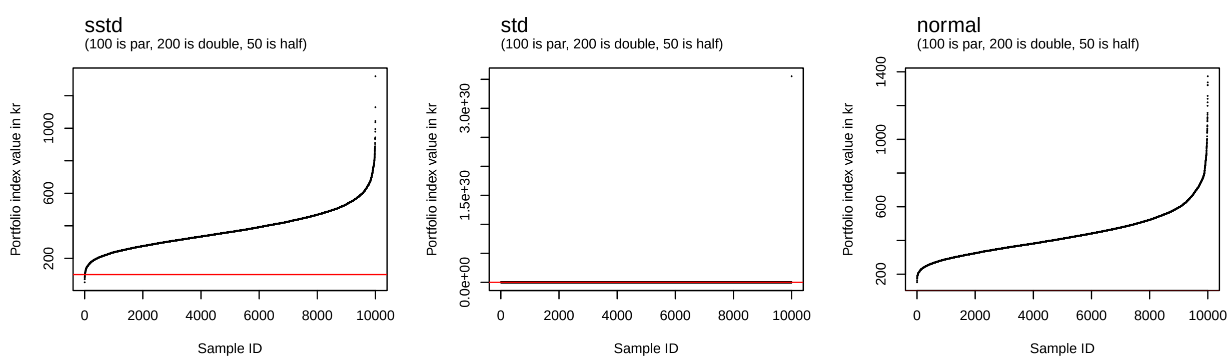
Estimated distribution

Now lets look at the CDF of the estimated distribution for each 0.1% increment between 0.5% and 99.5% for the estimated distribution:



Monte Carlo

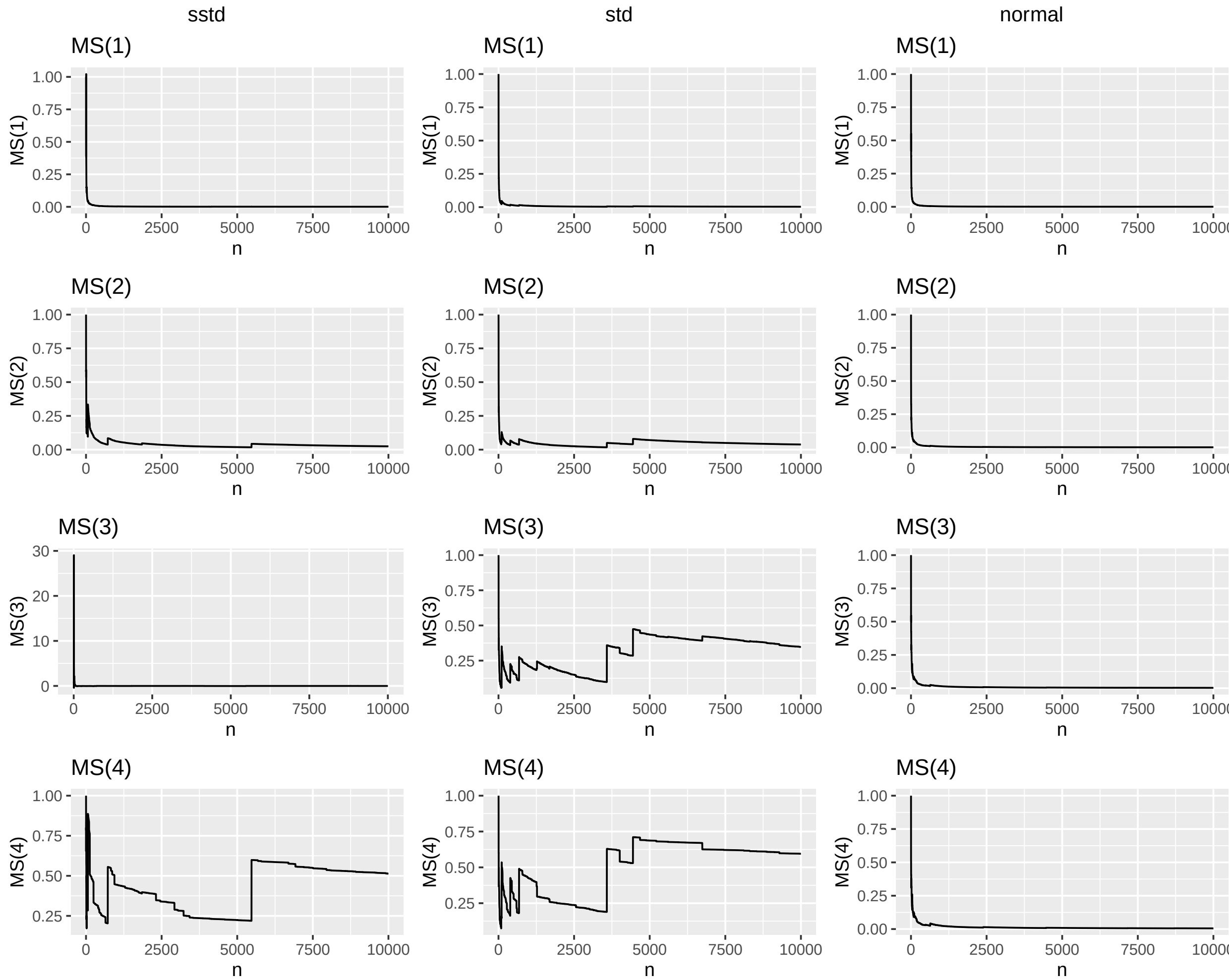
Sorted portfolio index values for last period of all runs



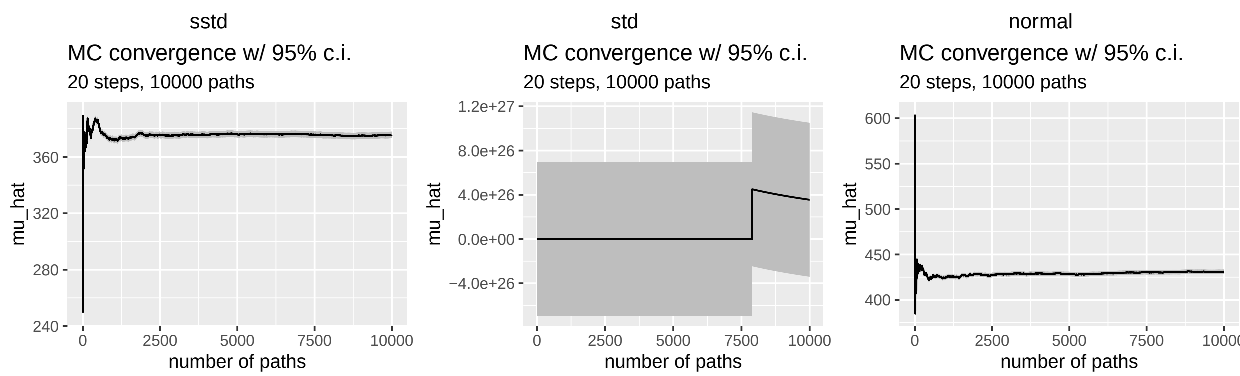
Convergence

Max vs sum

Max vs sum plots for the first four moments:

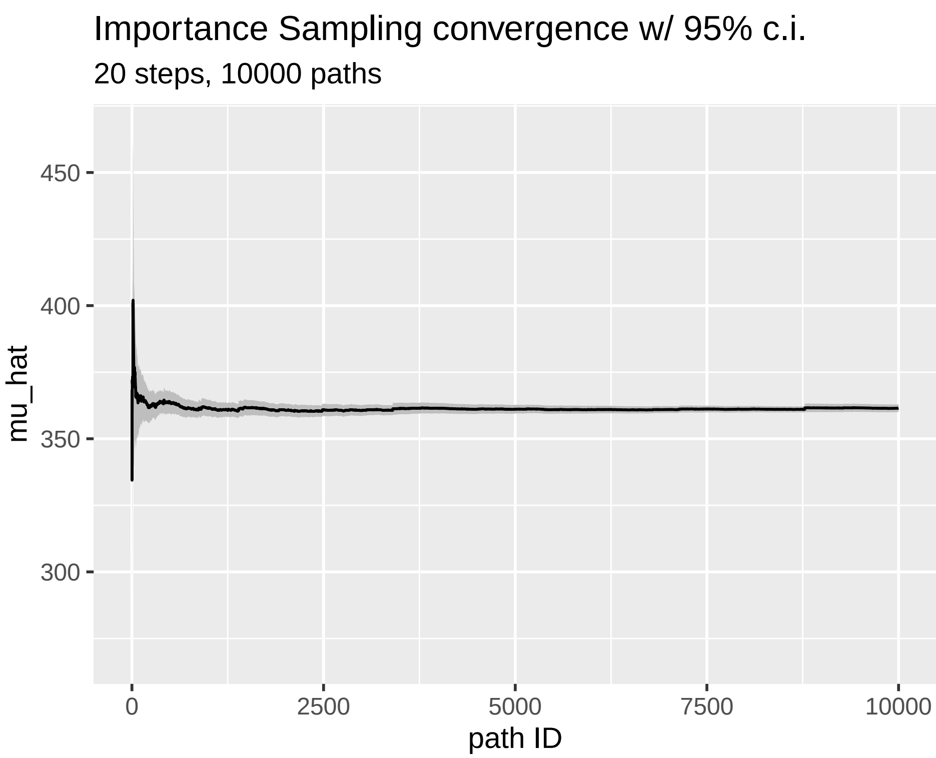


MC



IS

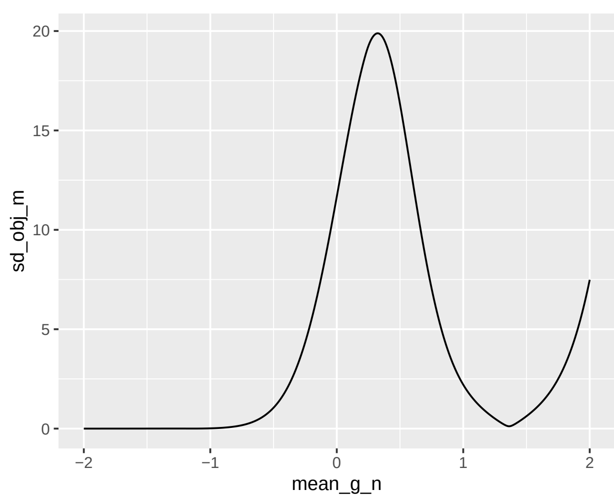
Skewed t-distribution with a normal proposal distribution.

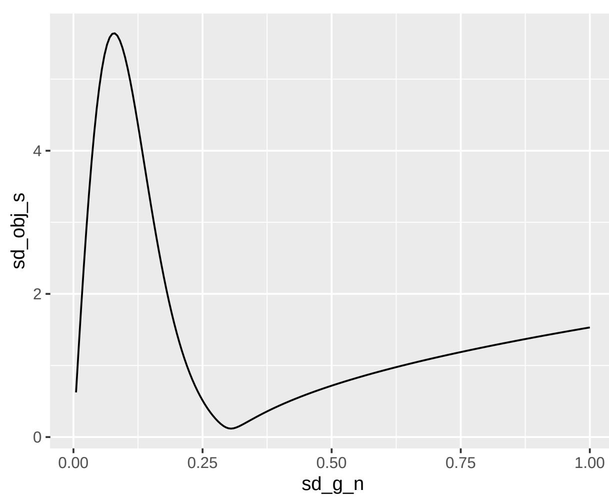


Parameters

```
## [1] 1.3625460 0.3050122
```

Objective function plots





Comments

`mhr` has some nice properties:

- It has a relatively high `nu` value of 90, which means it is tending more towards exponential tails than polynomial tails. All other funds have `nu` values close to 3, except `phr` which is even worse at close to 2. (Note that for a Gaussian, `nu` is infinite.)

- It has the lowest losing percentage of all simulations, which is better than 1/6 that of `phr`.

- It has a DAO percentage of 0, which is the same as `mmr`, and less than `phr`.
- Only `phr` has a higher `mc_m`.
- It has a smaller `mc_s` than the individual components, `vhr` and `phr`.
- It has the highest `xi` of all fits, suggesting less left skewness. Density plots for `vmr`, `phr` and `mmr` have an extremely sharp drop, as if an upward limiter has been applied, which corresponds to extremely low `xi` values. The density plot for `mhr` is by far the most symmetrical of all the fits. As seen in the section "Compare Gaussian and skewed t-distribution fits", the other skewed t-distribution fits don't capture the max observed returns at all.
- Only `mmr` has as higher `mc_min`. However, that of `mmr` is 18 times higher with 62, so `mmr` is a clear winner here.
- Naturally, it has a `mc_max` smaller than the individual components, `vhr` and `phr`, but ca. 1.5 times higher then `mmr`.
- All the first 4 moments converge nicely. For all other fits, the 4th moment doesn't seem to converge.

Taleb, Statistical Consequences Of Fat Tails, p. 97:
"the variance of a finite variance random variable with tail exponent < 4 will be infinite".

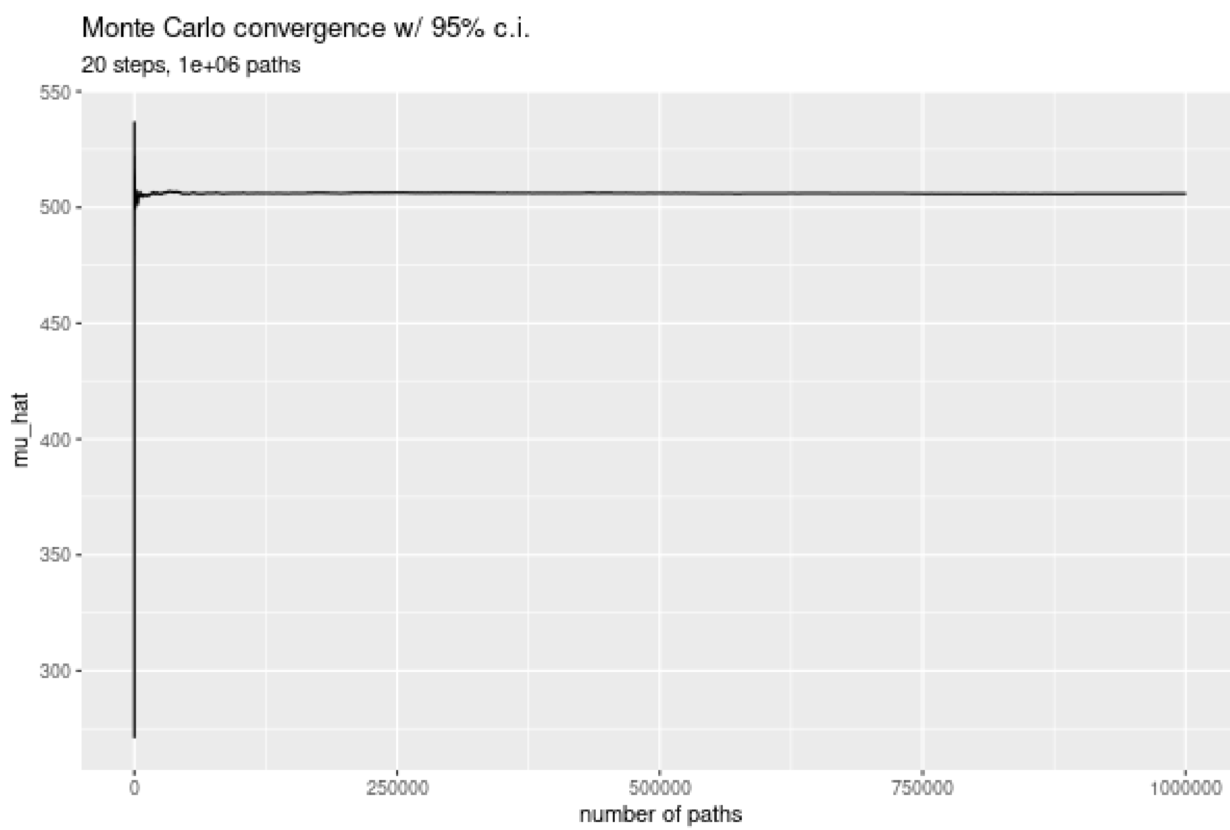
And p. 363:
"The hedging errors for an option portfolio (under a daily revision regime) over 3000 days, under a constant volatility Student T with tail exponent $\alpha = 3$. Technically the errors should not converge in finite time as their distribution has infinite variance."

- Note: QQ lines by design pass through 1st and 3rd quantiles. They are not trendlines in the sense of linear regression.

Appendix

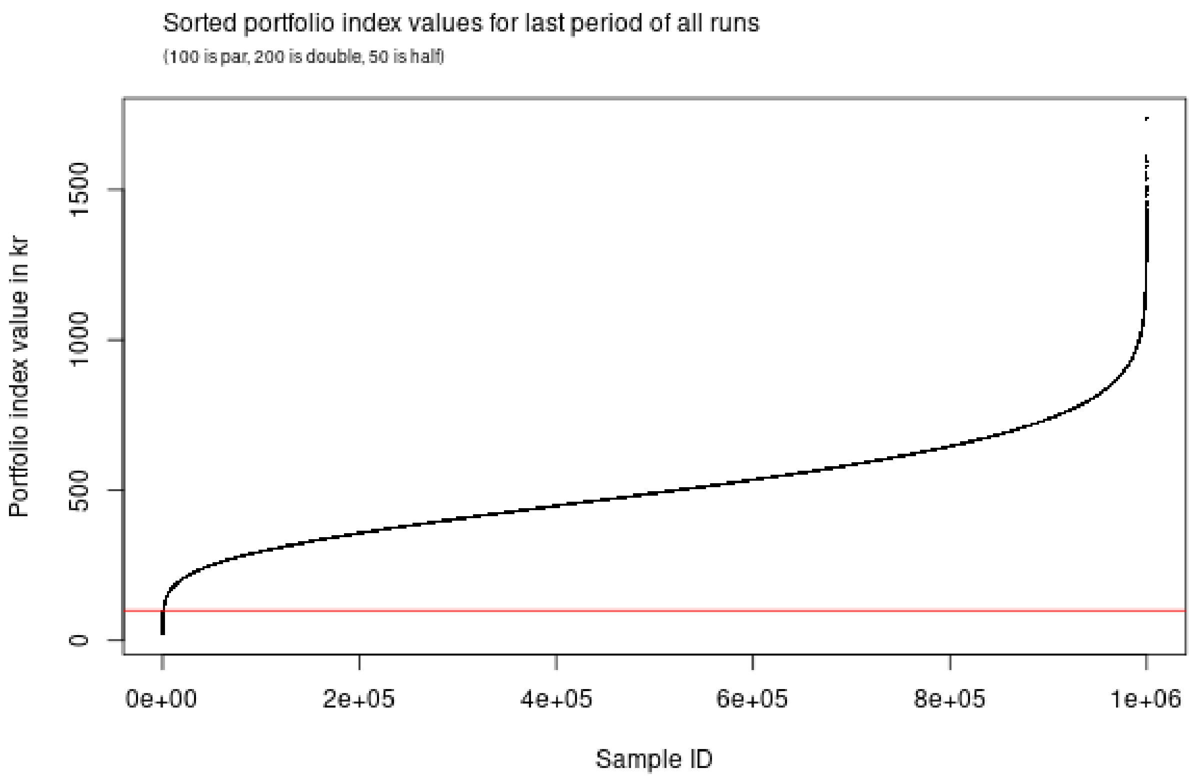
Many simulations of mc_mhr: `num_paths = 1e6`

1e6 paths:



Compare 10^6 and 10^4 paths for mhr:

	mc_m	mc_s	mc_min	mc_max	dao_pct	dai_pct
mc_mhr_1e6	505.90695	173.22176	21.09569	1734.83520	0.00000	0.07330
mc_mhr_1e4	505.47920	172.23152	51.70735	1326.58266	0.00000	0.04000
is_mhr_1e4	510.836	2331.167	205.398	232384.846	ibid.	ibid.



Arithmetic vs geometric mean

Let m be the number of steps in each path and n be the number of paths. a is the initial capital. Use arithmetic mean for mean of all paths at time t:

$$\frac{a(e^{z_1} + e^{z_2} + \cdots + e^{z_n})}{n}$$

where

$$z_i := x_{i,1} + x_{i,2} + \cdots + x_{i,m}$$

Use geometric mean for mean of all steps in a single path i:

$$ae^{\frac{x_{i,1}+x_{i,2}+\cdots+x_{i,m}}{m}} = a\sqrt[m]{e^{x_{i,1}+x_{i,2}+\cdots+x_{i,m}}}$$

So for **Monte Carlo** of returns after m periods, we

- fit a skewed t-distribution to log-returns and use that distribution to simulate $\{x_{i,j}\}_j^m$,
- for each path i, calculate $100 \cdot e^{z_i}$,
- calculate the mean of $\{z_i\}_i^n$:
  -
$$\bar{z} = 100\frac{e^{z_1} + e^{z_2} + \cdots + e^{z_n}}{n}$$

For **Importance Sampling**, we

- model log-returns on a skewed t-distribution,

- for each path i, calculate $100 \cdot e^{z_i}$,
- fit a skewed t-distribution to $\{z_i\}_i^n$ and use it as our f density function from which we simulate $\{h_i\}_i^n$,
  - In our case h and z are identical, because we have an idea for a distribution to simulate z, but in general for IS h could be a function of z.
- calculate $w* = \frac{f}{g^*}$, where $g*$ is our proposal distribution, which minimizes the variance of $h \cdot w$.
- calculate the arithmetic mean of $\{h_i w_i^*\}_i^n$:
  - $$100 \frac{e^{h_1 w_1^*} + e^{h_2 w_2^*} + \cdots + e^{h_n w_n^*}}{n}$$

Average of returns vs returns of average

Math

$$\text{Avg. of returns} := \frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2}$$

$$\text{Returns of avg.} := \left(\frac{x_t + y_t}{2}\right) \bigg/ \left(\frac{x_{t-1} + y_{t-1}}{2}\right) \equiv \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

For which x_1 and y_1 are Avg. of returns $=$ Returns of avg.?

$$\frac{\left(\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}}\right)}{2} = \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$\frac{x_t}{x_{t-1}} + \frac{y_t}{y_{t-1}} = 2 \frac{x_t + y_t}{x_{t-1} + y_{t-1}}$$

$$(x_{t-1} + y_{t-1}) x_t y_{t-1} + (x_{t-1} + y_{t-1}) x_{t-1} y_t = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

$$(x_{t-1} x_1 y_{t-1} + y_{t-1} x_t y_{t-1}) + (x_{t-1} x_{t-1} y_t + x_{t-1} y_{t-1} y_t) = 2(x_{t-1} y_{t-1} x_t + x_{t-1} y_{t-1} y_t)$$

This is not generally true, but true if for instance $x_{t-1} = y_{t-1}$.

Example

Definition: `R = 1+r`

```
## Let x_0 be 100.
```

```
## Let y_0 be 200.
```

```
## So the initial value of the pf is 300 .
```

```
## Let R_x be 0.5.
```

```
## Let R_y be 1.5.
```

Then,

```
## x_1 is R_x * x_0 = 50.
```

```
## y_1 is R_y * y_0 = 300.
```

Average of returns:

```
## 0.5 * (R_x + R_y) = 1
```

So here the value of the pf at t=1 should be unchanged from t=0:

```
## (x_0 + y_0) * 0.5 * (R_x + R_y) = 300
```

But this is clearly not the case:

```
## 0.5 * (x_1 + y_1) = 0.5 * (R_x * x_0 + R_y * y_0) = 175
```

Therefore we should take returns of average, not average of returns!

Let's take the average of log returns instead:

```
## 0.5 * (log(R_x) + log(R_y)) = -0.143841
```

We now get:

```
## (x_0 + y_0) * exp(0.5 * (log(Rx) + log(Ry))) = 259.8076
```

So taking the average of log returns doesn't work either.

Simulation of mix vs mix of simulations

Test if a simulation of a mix (average) of two returns series has the same distribution as a mix of two simulated returns series.

```
## m(data_x): 0.02347733
## s(data_x): 0.3099663
## m(data_y): 9.923634
## s(data_y): 2.748977
##
## m(data_x + data_y): 4.973555
## s(data_x + data_y): 1.408525
```

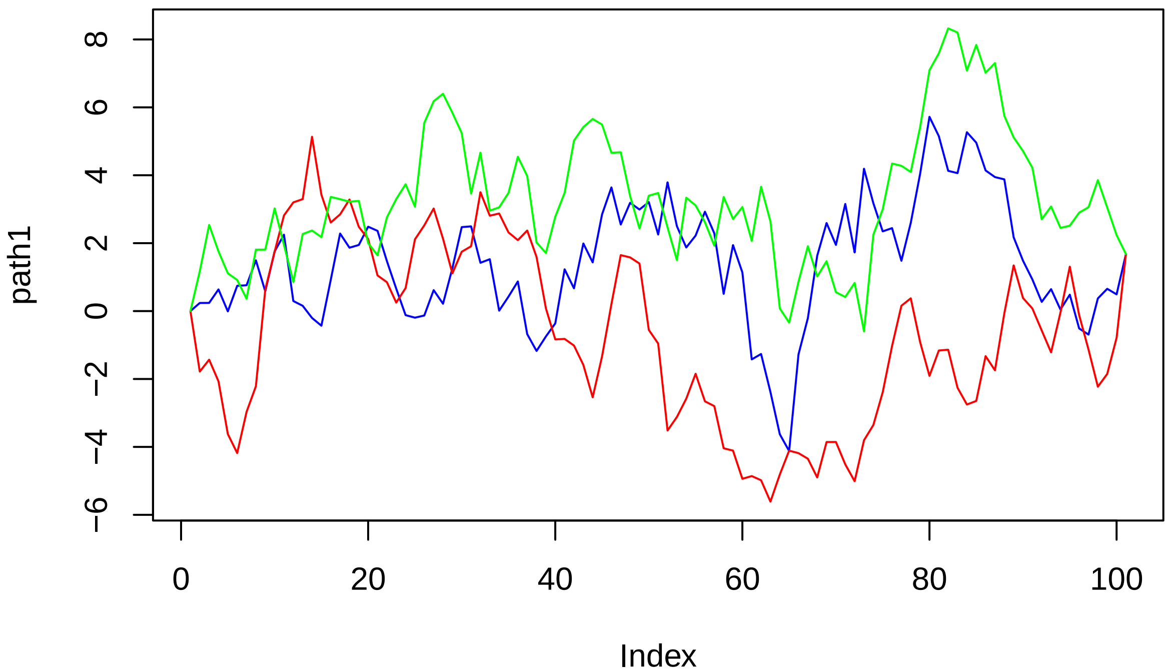
m and s of final state of all paths.
`_a` is mix of simulated returns.
`_b` is simulated mixed returns.

m_a	m_b	s_a	s_b
99.736	99.478	6.213	6.469
99.424	99.458	6.131	6.220
99.266	99.738	6.192	6.088
98.937	99.183	6.251	6.436
99.369	99.297	6.194	6.265
99.470	99.529	6.233	6.106
99.315	99.411	6.357	6.477
99.466	99.543	6.276	6.365
99.234	99.757	6.153	6.341
99.431	99.191	6.513	6.327

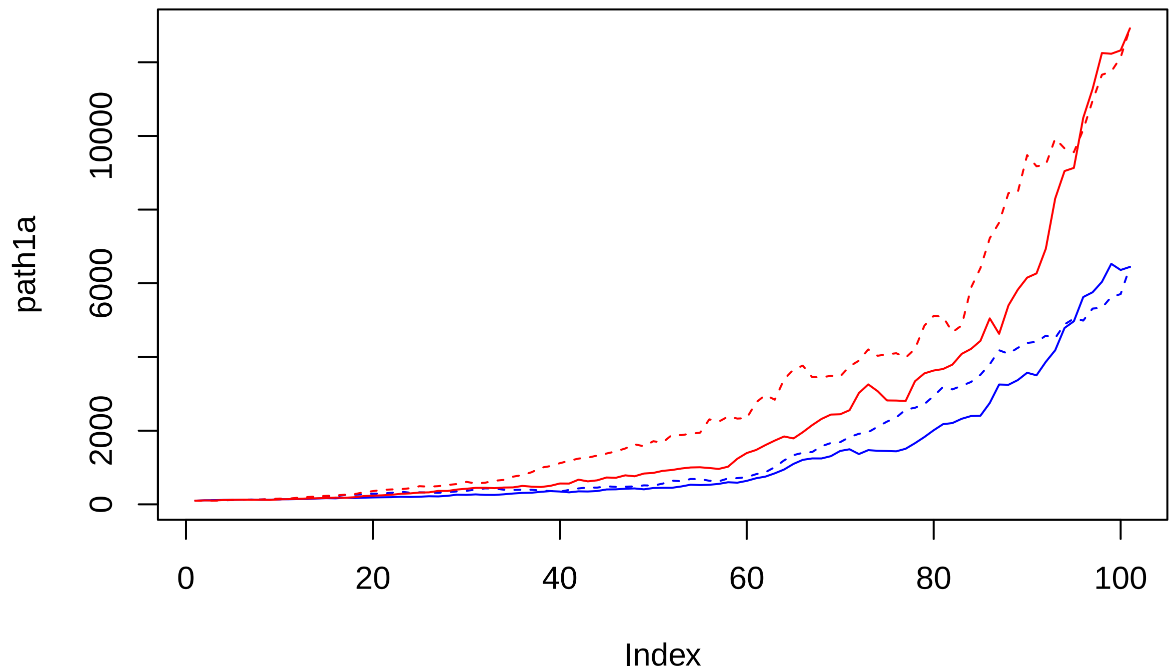
```
##       m_a             m_b             s_a             s_b
##  Min.   :98.94   Min.   :99.18   Min.   :6.131   Min.   :6.088
##  1st Qu.:99.28   1st Qu.:99.33   1st Qu.:6.193   1st Qu.:6.231
##  Median :99.40   Median :99.47   Median :6.223   Median :6.334
##  Mean   :99.36   Mean   :99.46   Mean   :6.251   Mean   :6.309
##  3rd Qu.:99.46   3rd Qu.:99.54   3rd Qu.:6.270   3rd Qu.:6.418
##  Max.   :99.74   Max.   :99.76   Max.   :6.513   Max.   :6.477
```

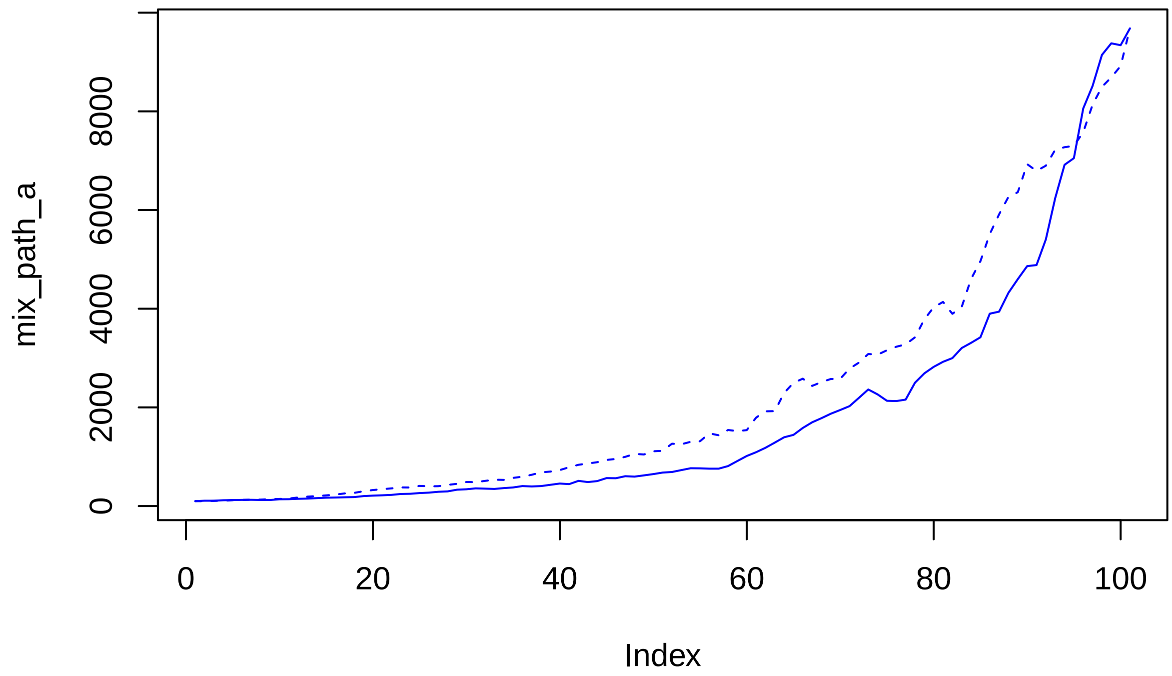
`_a` and `_b` are very close to equal.
We attribute the differences to differences in estimating the distributions in version a and b.

The final state is independent of the order of the preceding steps:



So does the order of the steps in the two processes matter, when mixing simulated returns?


path1a
0
2000
6000
10000
0
20
40
60
80
100
Index



The order of steps in the individual paths do not matter, because the mix of simulated paths is a sum of a sum, so the order of terms doesn't affect the sum. If there is variation it is because the sets preceding steps are not the same. For instance, the steps between step 1 and 60 in the plot above are not the same for the two lines.

Recall,

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(a, b)$$

```
var(0.5 * vhr + 0.5 * phr)
```

```
## [1] 0.005355618
```

```
0.5^2 * var(vhr) + 0.5^2 * var(phr) + 2 * 0.5 * 0.5 * cov(vhr, phr)
```

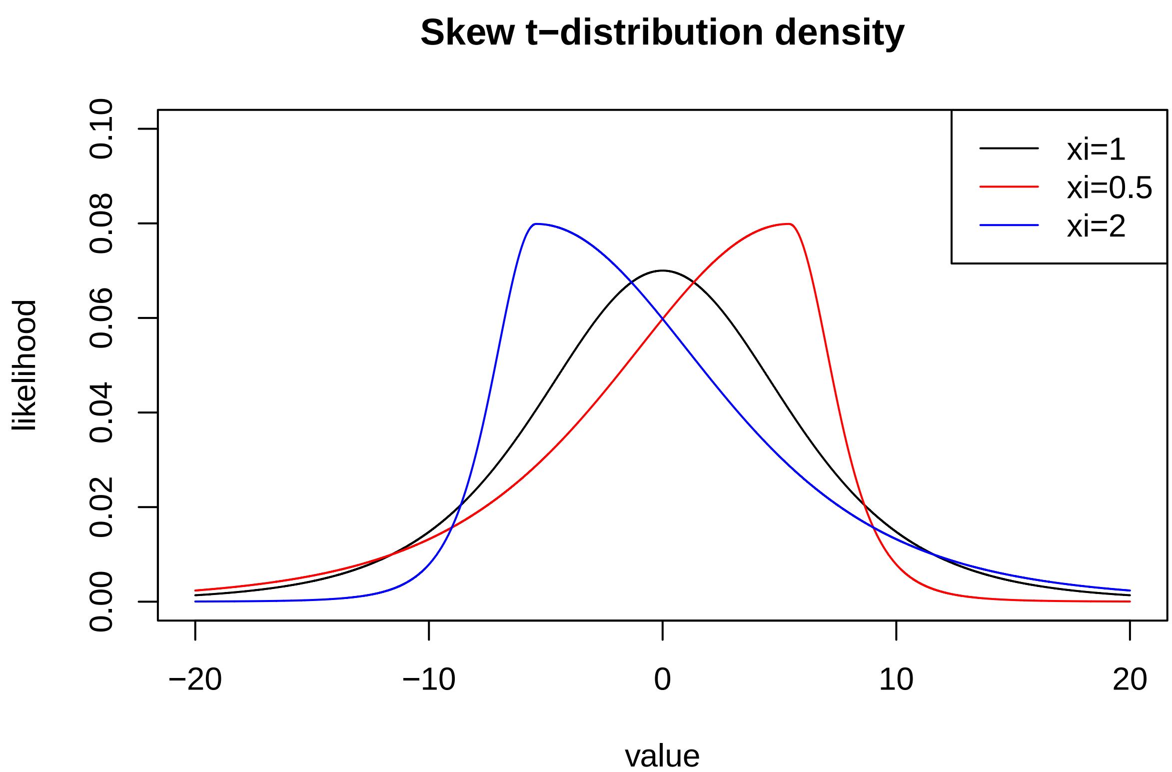
```
## [1] 0.005355618
```

Our distribution estimate is based on 13 observations. Is that enough for a robust estimate? What if we suddenly hit a year like 2008? How would that affect our estimate?
Let's try to include the Velliv data from 2007-2010.
We do this by sampling 13 observations from `vmrl`.

```
##        m                 s
##  Min.   :0.05909   Min.   :0.04494
##  1st Qu.:0.06606   1st Qu.:0.05999
##  Median :0.06953   Median :0.06460
##  Mean   :0.07013   Mean   :0.06629
##  3rd Qu.:0.07428   3rd Qu.:0.07374
##  Max.   :0.08514   Max.   :0.08718
```

The meaning of `xi`

The fit for `mhr` has the highest `xi` value of all. This suggests right-skew:



Max vs sum plot

If the Law Of Large Numbers holds true,

$$\frac{\max(X_1^p, ..., X^p)}{\sum_{i=1}^{n} X_i^p} \to 0$$

for $n \to \infty$.

If not, X doesn't have a p'th moment.

See Taleb: The Statistical Consequences Of Fat Tails, p. 192