Complex systems pervade the world we live in, and a common method of describing complex systems for study is using complex networks. Complex networks use a graph representation which describes entities participating in the system as nodes, and interactions or relationships between entities as edges. A common question to ask about complex networks is which nodes have the most influence over the behavior of the system, and one way to attempt to answer this question is using a technique called transfer entropy. Given two processes, Process A and Process B, transfer entropy is a statistical measure of the extent to which knowledge of both Process A’s past and Process B’s past helps to predict Process B’s future, compared to predictions about Process B’s future made only using knowledge of Process B’s past. It is possible to describe influence dynamics in a complex system by describing as a type of complex network called a transfer entropy network, where nodes represent entities participating in the system and the edges describe transfer entropy between those entities, with a directed edge from Node A to Node B existing if the transfer entropy from Process A to Process B exceeds a given threshold.

Transfer entropy networks were criticized on theoretical grounds by James et al. in 2016 [1]. This criticism rested on the grounds that both graph notation and transfer entropy are both only capable of explicitly describing how one entity relates one other entity at a time, resulting in linear relationships between entities that can be described as weighted sums of incoming edges or outgoing edges, while complex systems may have more complex relationships that involve larger numbers of entities and non-linear relationships [1]. James et al. demonstrate a pair of hypothetical scenarios in which transfer entropy networks either ignore the influence of such relationships, or falsely attribute the influence of such relationships to only one of the nodes participating in them [1]. Despite the theoretical arguments made, however, the practical impact of problems associated with higher order relationships has not been quantified by James et al. or in the literature that has followed.

When using transfer entropy networks to rank entities in a complex system based on their localizable influence over the behavior of other entities, how sensitive is the produced ranking to higher order relationships among the entities in the system?

To test this, we have produced a multi-agent system that simulates social media activity. At each time step, an agent may post a message without external prompting, will review some of the messages sent by other agents, and may respond to them. Which agents can see which other agents’ messages is determined by a network describing the ground truth of the layout of the social network. The simulation includes higher order relationships between nodes that may drive additional message sending. The message sending behavior of each agent is recorded as a time series, which are used to construct a transfer entropy network. Each node in the network represents one agent, and edges between them describe the transfer entropy of their message sending behaviors. Rankings of nodes based on influence can be produced for both the transfer entropy network and the ground truth network, and both the rankings and the graphs themselves can be compared to see how accurately the ground truth rankings and network can be retrieved by transfer entropy. The experiments to be conducted using this system will consist of measuring the accuracy of retrieved rankings and graphs as the number and size of higher order relationships are varied, and as the size and complexity of the simulated social network are varied. This experimentation will commence in the coming weeks. The results will help to establish thresholds for the conditions under which transfer entropy networks remain an effective tool in practice despite their theoretical limitations.

References:

[1] R. G. James, N. Barnett, and J. P. Crutchfield, “Information Flows? A Critique of Transfer Entropies,” *Phys. Rev. Lett.*, vol. 116, no. 23, p. 238701, Jun. 2016, doi: [10.1103/PhysRevLett.116.238701](https://doi.org/10.1103/PhysRevLett.116.238701).