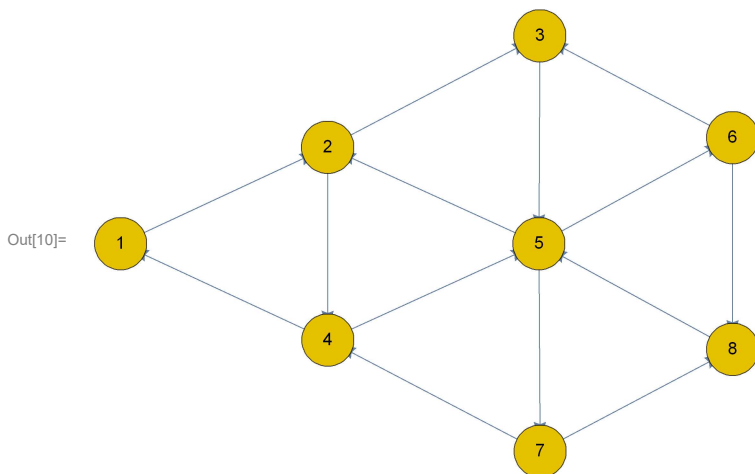


```
In[1]:= Clear[w1N, w2N, w3N, mu1N, mu1Half, mu2N, mu2Half, mu2Full, mu3N, mu3Half, f, ε]
```

```
In[2]:= (*This sheet shows that TH(w1,w2,w3)≠
      TH(w3,w2,w1) particularly if w1>w3 TH(w1,w2,w3)>TH(w3,w2,w1)*)
mu1N := 1/w1N
mu1Half := 1/(w1N - f w1N/2)
mu2N := 1
mu2Half := 1/(1 - f/2)
mu2Full := 1/(1 - f)
mu3N := 1/w3N
mu3Half := 1/(w3N - f w3N/2)
```

```
In[9]:= P := ContinuousMarkovProcess[1,
      {
        {-mu1Half, mu1Half, 0, 0},
        {0, -mu1N - mu2Half, mu1N, mu2Half},
        {0, 0, -mu2Full, 0},
        {mu3N, 0, 0, -mu1Half - mu3N},
        {0, mu3N, 0, 0},
        {0, 0, mu3N, 0},
        {0, 0, 0, mu3Half},
        {0, 0, 0, 0}
      }
```

```
In[10]:= Graph[P]
```



```
In[11]:= Clear[w1N, w2N, w3N, f]
```

```
In[13]:= TH := (∑_{i=4}^6 PDF[P[∞], i]) mu3N + (∑_{i=7}^8 PDF[P[∞], i]) mu3Half
```

```
In[16]:= THSimple[f_, w1N_, w3N_] :=
- ((2 (f - 2 (1 + w3N)) (-2 (-2 + f)^2 w3N^3 + 4 (-2 + f) w1N^4 (1 + w3N) +
(-2 + f) w1N w3N^2 (12 + f^2 + 8 w3N - 2 f (3 + w3N)) -
2 w1N^3 (-4 f (1 + 3 w3N) + f^2 (1 + 3 w3N) + 4 (1 + 4 w3N + w3N^2)) + w1N^2 w3N
(f^3 (1 + w3N) - 2 f^2 (4 + 3 w3N) - 8 (3 + 4 w3N + w3N^2) + 4 f (6 + 5 w3N + w3N^2))) /
(-4 (-2 + f)^3 w3N^3 (-1 + f - w3N - w3N^2) + 4 (-2 + f)^2 w1N^5 (1 + w3N) (f - 2 (1 + w3N)) -
4 (-2 + f) w1N^4 (f^3 w3N - f^2 (1 + 7 w3N + 2 w3N^2) + 4 f (1 + 5 w3N + 3 w3N^2) -
4 (1 + 5 w3N + 5 w3N^2 + w3N^3)) + 2 (-2 + f)^2 w1N w3N^2 (f^3 - f^2 (7 + w3N + 2 w3N^2) +
2 f (9 + 7 w3N + 4 w3N^2 + w3N^3) - 4 (3 + 5 w3N + 5 w3N^2 + 2 w3N^3)) +
(-2 + f) w1N^2 w3N (f^4 (2 + w3N^2) - 2 f^3 (9 + 3 w3N + 4 w3N^2 + w3N^3) +
4 f^2 (16 + 10 w3N + 9 w3N^2 + 4 w3N^3) + 16 (3 + 7 w3N + 8 w3N^2 + 5 w3N^3 + w3N^4) -
8 f (12 + 16 w3N + 12 w3N^2 + 6 w3N^3 + w3N^4)) - 2 w1N^3
(12 f^2 (1 + w3N)^2 (3 + 2 w3N) + 16 (1 + w3N)^2 (1 + 3 w3N + w3N^2) + f^4 (2 + 3 w3N + w3N^2) -
2 f^3 (7 + 14 w3N + 9 w3N^2 + w3N^3) - 8 f (5 + 18 w3N + 21 w3N^2 + 9 w3N^3 + w3N^4)))
```

```
In[18]:= TH1 := THSimple[f, 1 + ε, 1]
TH2 := THSimple[f, 1, 1 + ε]
```

```
In[20]:= FullSimplify[TH1 - TH2]
```

```
Out[20]= - ((2 (-2 + f) f ε (1 + ε) (f - 2 (2 + ε))
(f^6 (1 + ε)^2 (3 + 2 ε) - 64 (2 + ε) (3 + 2 ε)^3 - 2 f^5 (1 + ε) (5 + 4 ε) (5 + ε (5 + ε)) +
4 f^4 (1 + ε) (82 + ε (164 + ε (113 + 2 ε (15 + ε)))) -
32 f (-59 + 2 ε (-43 + ε (4 + ε (37 + 22 ε + 4 ε^2)))) +
16 f^2 (52 + ε (259 + 2 ε (213 + ε (161 + 58 ε + 8 ε^2)))) -
8 f^3 (122 + ε (410 + ε (525 + 2 ε (161 + ε (47 + 5 ε)))))) /
((f^5 (1 + ε) (5 + 3 ε) - 32 (2 + ε) (39 + 4 ε (19 + ε (15 + ε (6 + ε)))) -
4 f^4 (22 + ε (40 + ε (25 + ε (7 + ε)))) +
16 f (227 + ε (525 + ε (513 + 2 ε (137 + 5 ε (8 + ε)))) +
4 f^3 (153 + ε (306 + ε (241 + ε (99 + 2 ε (11 + ε)))) -
8 f^2 (264 + ε (571 + 2 ε (257 + 2 ε (63 + ε (17 + 2 ε))))))
(f^5 (1 + ε) (5 + ε (4 + ε)) - 32 (2 + ε) (39 + 4 ε (19 + ε (15 + ε (6 + ε)))) -
2 f^4 (44 + ε (82 + ε (57 + ε (20 + 3 ε)))) +
4 f^3 (153 + ε (303 + ε (243 + 2 ε (53 + ε (12 + ε)))) -
8 f^2 (264 + ε (561 + 4 ε (125 + ε (62 + ε (17 + 2 ε)))) +
16 f (227 + ε (519 + ε (502 + ε (268 + ε (79 + 10 ε))))))
```

```
In[21]:= Numerator[%20]
```

```
Out[21]= - 2 (-2 + f) f ε (1 + ε) (f - 2 (2 + ε))
(f^6 (1 + ε)^2 (3 + 2 ε) - 64 (2 + ε) (3 + 2 ε)^3 - 2 f^5 (1 + ε) (5 + 4 ε) (5 + ε (5 + ε)) +
4 f^4 (1 + ε) (82 + ε (164 + ε (113 + 2 ε (15 + ε)))) -
32 f (-59 + 2 ε (-43 + ε (4 + ε (37 + 22 ε + 4 ε^2)))) +
16 f^2 (52 + ε (259 + 2 ε (213 + ε (161 + 58 ε + 8 ε^2)))) -
8 f^3 (122 + ε (410 + ε (525 + 2 ε (161 + ε (47 + 5 ε)))))
```

In[22]:= **Factor[%21]**

Out[22]= $-2 (-2 + f) f (-4 + f - 2 \epsilon) \epsilon (1 + \epsilon)$
 $(-3456 + 1888 f + 832 f^2 - 976 f^3 + 328 f^4 - 50 f^5 + 3 f^6 - 8640 \epsilon + 2752 f \epsilon +$
 $4144 f^2 \epsilon - 3280 f^3 \epsilon + 984 f^4 \epsilon - 140 f^5 \epsilon + 8 f^6 \epsilon - 8064 \epsilon^2 - 256 f \epsilon^2 +$
 $6816 f^2 \epsilon^2 - 4200 f^3 \epsilon^2 + 1108 f^4 \epsilon^2 - 140 f^5 \epsilon^2 + 7 f^6 \epsilon^2 - 3328 \epsilon^3 - 2368 f \epsilon^3 +$
 $5152 f^2 \epsilon^3 - 2576 f^3 \epsilon^3 + 572 f^4 \epsilon^3 - 58 f^5 \epsilon^3 + 2 f^6 \epsilon^3 - 512 \epsilon^4 - 1408 f \epsilon^4 +$
 $1856 f^2 \epsilon^4 - 752 f^3 \epsilon^4 + 128 f^4 \epsilon^4 - 8 f^5 \epsilon^4 - 256 f \epsilon^5 + 256 f^2 \epsilon^5 - 80 f^3 \epsilon^5 + 8 f^4 \epsilon^5)$
n1 := 2 (-2 + f) f (-4 + f - 2 \epsilon) \epsilon (1 + \epsilon)
n2 := - (-3456 + 1888 f + 832 f^2 - 976 f^3 + 328 f^4 - 50 f^5 + 3 f^6 - 8640 \epsilon +
 $2752 f \epsilon + 4144 f^2 \epsilon - 3280 f^3 \epsilon + 984 f^4 \epsilon - 140 f^5 \epsilon + 8 f^6 \epsilon - 8064 \epsilon^2 - 256 f \epsilon^2 +$
 $6816 f^2 \epsilon^2 - 4200 f^3 \epsilon^2 + 1108 f^4 \epsilon^2 - 140 f^5 \epsilon^2 + 7 f^6 \epsilon^2 - 3328 \epsilon^3 - 2368 f \epsilon^3 +$
 $5152 f^2 \epsilon^3 - 2576 f^3 \epsilon^3 + 572 f^4 \epsilon^3 - 58 f^5 \epsilon^3 + 2 f^6 \epsilon^3 - 512 \epsilon^4 - 1408 f \epsilon^4 +$
 $1856 f^2 \epsilon^4 - 752 f^3 \epsilon^4 + 128 f^4 \epsilon^4 - 8 f^5 \epsilon^4 - 256 f \epsilon^5 + 256 f^2 \epsilon^5 - 80 f^3 \epsilon^5 + 8 f^4 \epsilon^5)$

(*let NUM=n1*n2 *)

(*n1>0*)

Collect[n2, \epsilon]

$3456 - 1888 f - 832 f^2 + 976 f^3 - 328 f^4 + 50 f^5 - 3 f^6 +$
 $(8640 - 2752 f - 4144 f^2 + 3280 f^3 - 984 f^4 + 140 f^5 - 8 f^6) \epsilon +$
 $(8064 + 256 f - 6816 f^2 + 4200 f^3 - 1108 f^4 + 140 f^5 - 7 f^6) \epsilon^2 +$
 $(3328 + 2368 f - 5152 f^2 + 2576 f^3 - 572 f^4 + 58 f^5 - 2 f^6) \epsilon^3 +$
 $(512 + 1408 f - 1856 f^2 + 752 f^3 - 128 f^4 + 8 f^5) \epsilon^4 + (256 f - 256 f^2 + 80 f^3 - 8 f^4) \epsilon^5$

(*n2>0-->NUM>0*)

In[23]:= **Denominator[%20]**

Out[23]= $(f^5 (1 + \epsilon) (5 + 3 \epsilon) -$
 $32 (2 + \epsilon) (39 + 4 \epsilon (19 + \epsilon (15 + \epsilon (6 + \epsilon)))) - 4 f^4 (22 + \epsilon (40 + \epsilon (25 + \epsilon (7 + \epsilon)))) +$
 $16 f (227 + \epsilon (525 + \epsilon (513 + 2 \epsilon (137 + 5 \epsilon (8 + \epsilon)))) +$
 $4 f^3 (153 + \epsilon (306 + \epsilon (241 + \epsilon (99 + 2 \epsilon (11 + \epsilon)))) -$
 $8 f^2 (264 + \epsilon (571 + 2 \epsilon (257 + 2 \epsilon (63 + \epsilon (17 + 2 \epsilon))))))$
 $(f^5 (1 + \epsilon) (5 + \epsilon (4 + \epsilon)) - 32 (2 + \epsilon) (39 + 4 \epsilon (19 + \epsilon (15 + \epsilon (6 + \epsilon)))) -$
 $2 f^4 (44 + \epsilon (82 + \epsilon (57 + \epsilon (20 + 3 \epsilon)))) +$
 $4 f^3 (153 + \epsilon (303 + \epsilon (243 + 2 \epsilon (53 + \epsilon (12 + \epsilon)))) -$
 $8 f^2 (264 + \epsilon (561 + 4 \epsilon (125 + \epsilon (62 + \epsilon (17 + 2 \epsilon)))) +$
 $16 f (227 + \epsilon (519 + \epsilon (502 + \epsilon (268 + \epsilon (79 + 10 \epsilon))))))$

In[24]:= **Factor[%23]**

Out[24]=
$$\begin{aligned} & (-2496 + 3632 f - 2112 f^2 + 612 f^3 - 88 f^4 + 5 f^5 - 6112 \epsilon + 8304 f \epsilon - 4488 f^2 \epsilon + \\ & \quad 1212 f^3 \epsilon - 164 f^4 \epsilon + 9 f^5 \epsilon - 6272 \epsilon^2 + 8032 f \epsilon^2 - 4000 f^2 \epsilon^2 + 972 f^3 \epsilon^2 - 114 f^4 \epsilon^2 + \\ & \quad 5 f^5 \epsilon^2 - 3456 \epsilon^3 + 4288 f \epsilon^3 - 1984 f^2 \epsilon^3 + 424 f^3 \epsilon^3 - 40 f^4 \epsilon^3 + f^5 \epsilon^3 - 1024 \epsilon^4 + \\ & \quad 1264 f \epsilon^4 - 544 f^2 \epsilon^4 + 96 f^3 \epsilon^4 - 6 f^4 \epsilon^4 - 128 \epsilon^5 + 160 f \epsilon^5 - 64 f^2 \epsilon^5 + 8 f^3 \epsilon^5) \\ & (-2496 + 3632 f - 2112 f^2 + 612 f^3 - 88 f^4 + 5 f^5 - 6112 \epsilon + 8400 f \epsilon - 4568 f^2 \epsilon + \\ & \quad 1224 f^3 \epsilon - 160 f^4 \epsilon + 8 f^5 \epsilon - 6272 \epsilon^2 + 8208 f \epsilon^2 - 4112 f^2 \epsilon^2 + 964 f^3 \epsilon^2 - \\ & \quad 100 f^4 \epsilon^2 + 3 f^5 \epsilon^2 - 3456 \epsilon^3 + 4384 f \epsilon^3 - 2016 f^2 \epsilon^3 + 396 f^3 \epsilon^3 - 28 f^4 \epsilon^3 - 1024 \epsilon^4 + \\ & \quad 1280 f \epsilon^4 - 544 f^2 \epsilon^4 + 88 f^3 \epsilon^4 - 4 f^4 \epsilon^4 - 128 \epsilon^5 + 160 f \epsilon^5 - 64 f^2 \epsilon^5 + 8 f^3 \epsilon^5) \end{aligned}$$

(*Denominator=a*b*)

a :=

$$\begin{aligned} & - (-2496 + 3632 f - 2112 f^2 + 612 f^3 - 88 f^4 + 5 f^5 - 6112 \epsilon + 8304 f \epsilon - 4488 f^2 \epsilon + 1212 f^3 \epsilon - \\ & \quad 164 f^4 \epsilon + 9 f^5 \epsilon - 6272 \epsilon^2 + 8032 f \epsilon^2 - 4000 f^2 \epsilon^2 + 972 f^3 \epsilon^2 - 114 f^4 \epsilon^2 + 5 f^5 \epsilon^2 - \\ & \quad 3456 \epsilon^3 + 4288 f \epsilon^3 - 1984 f^2 \epsilon^3 + 424 f^3 \epsilon^3 - 40 f^4 \epsilon^3 + f^5 \epsilon^3 - 1024 \epsilon^4 + \\ & \quad 1264 f \epsilon^4 - 544 f^2 \epsilon^4 + 96 f^3 \epsilon^4 - 6 f^4 \epsilon^4 - 128 \epsilon^5 + 160 f \epsilon^5 - 64 f^2 \epsilon^5 + 8 f^3 \epsilon^5) \end{aligned}$$

b :=
$$\begin{aligned} & - (-2496 + 3632 f - 2112 f^2 + 612 f^3 - 88 f^4 + 5 f^5 - 6112 \epsilon + 8400 f \epsilon - 4568 f^2 \epsilon + \\ & \quad 1224 f^3 \epsilon - 160 f^4 \epsilon + 8 f^5 \epsilon - 6272 \epsilon^2 + 8208 f \epsilon^2 - 4112 f^2 \epsilon^2 + 964 f^3 \epsilon^2 - \\ & \quad 100 f^4 \epsilon^2 + 3 f^5 \epsilon^2 - 3456 \epsilon^3 + 4384 f \epsilon^3 - 2016 f^2 \epsilon^3 + 396 f^3 \epsilon^3 - 28 f^4 \epsilon^3 - 1024 \epsilon^4 + \\ & \quad 1280 f \epsilon^4 - 544 f^2 \epsilon^4 + 88 f^3 \epsilon^4 - 4 f^4 \epsilon^4 - 128 \epsilon^5 + 160 f \epsilon^5 - 64 f^2 \epsilon^5 + 8 f^3 \epsilon^5) \end{aligned}$$

In[27]:= **Factor[a - b]**

Out[27]=
$$-(-2 + f) f \epsilon (1 + \epsilon) (48 - 16 f - 2 f^2 + f^3 + 40 \epsilon + 4 f \epsilon - 8 f^2 \epsilon + f^3 \epsilon + 8 \epsilon^2 + 4 f \epsilon^2 - 2 f^2 \epsilon^2)$$

In[38]:= **(*a-b=x**

x=x1*x2*)

x1 :=
$$-(-2 + f) f \epsilon (1 + \epsilon)$$

x2 :=
$$(48 - 16 f - 2 f^2 + f^3 + 40 \epsilon + 4 f \epsilon - 8 f^2 \epsilon + f^3 \epsilon + 8 \epsilon^2 + 4 f \epsilon^2 - 2 f^2 \epsilon^2)$$

(*We can see x1 and x2 are positive--> x positive*)

(*Denominator=a*b=(x+b)b=b^2+xb, positive if xb>0,

we know x>0 from above b>0 sufficient*)

b

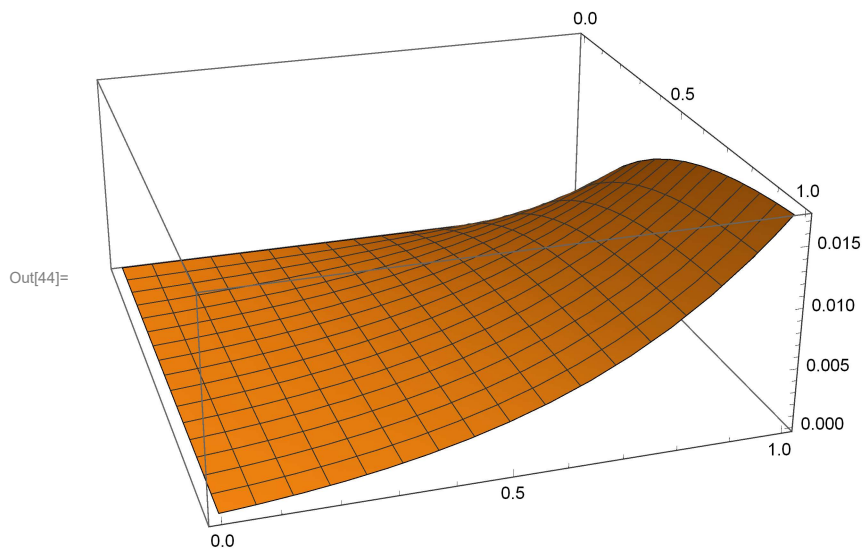
Out[40]=
$$\begin{aligned} & 2496 - 3632 f + 2112 f^2 - 612 f^3 + 88 f^4 - 5 f^5 + 6112 \epsilon - 8400 f \epsilon + 4568 f^2 \epsilon - \\ & \quad 1224 f^3 \epsilon + 160 f^4 \epsilon - 8 f^5 \epsilon + 6272 \epsilon^2 - 8208 f \epsilon^2 + 4112 f^2 \epsilon^2 - 964 f^3 \epsilon^2 + \\ & \quad 100 f^4 \epsilon^2 - 3 f^5 \epsilon^2 + 3456 \epsilon^3 - 4384 f \epsilon^3 + 2016 f^2 \epsilon^3 - 396 f^3 \epsilon^3 + 28 f^4 \epsilon^3 + 1024 \epsilon^4 - \\ & \quad 1280 f \epsilon^4 + 544 f^2 \epsilon^4 - 88 f^3 \epsilon^4 + 4 f^4 \epsilon^4 + 128 \epsilon^5 - 160 f \epsilon^5 + 64 f^2 \epsilon^5 - 8 f^3 \epsilon^5 \end{aligned}$$

In[41]:= **Collect[%40, \epsilon]**

Out[41]=
$$\begin{aligned} & 2496 - 3632 f + 2112 f^2 - 612 f^3 + 88 f^4 - \\ & \quad 5 f^5 + (6112 - 8400 f + 4568 f^2 - 1224 f^3 + 160 f^4 - 8 f^5) \epsilon + \\ & \quad (6272 - 8208 f + 4112 f^2 - 964 f^3 + 100 f^4 - 3 f^5) \epsilon^2 + \\ & \quad (3456 - 4384 f + 2016 f^2 - 396 f^3 + 28 f^4) \epsilon^3 + \\ & \quad (1024 - 1280 f + 544 f^2 - 88 f^3 + 4 f^4) \epsilon^4 + (128 - 160 f + 64 f^2 - 8 f^3) \epsilon^5 \end{aligned}$$

(*b>0*)

In[44]:= **Plot3D**[TH1 - TH2, { ϵ , 0, 1}, {f, 0, 1}]



(*SUMMARY_1*)

Assuming[$\epsilon \geq 0 \ \&\& \ 1 \geq f \geq 0$, **FullSimplify**[TH1 - TH2 > 0]]

Out[45]= $f \in > 0$

(*SUMMARY_2*)

In[46]:= **Assuming**[$\epsilon > 0 \ \&\& \ f == 0$, **FullSimplify**[TH1 - TH2 == 0]]

Out[46]= True