

Exercise 9

1. By using the bilinear transformation, design a digital Chebyshev filter, which meets the following specifications:

$$\begin{aligned} -1 &\leq 20 \cdot \log_{10} |H(e^{j\omega})| \leq 0, & 0 \leq |\omega| \leq 0.2\pi \\ 20 \cdot \log_{10} |H(e^{j\omega})| &\leq -50, & 0.5\pi \leq |\omega| \leq \pi. \end{aligned}$$

The transfer function of the analog Chebyshev filter is of the form,

$$H_a(s) = \frac{k_0}{B_N(s)} = \frac{k_0}{\prod_{k=1}^N (s - s_k)} \quad (1)$$

The magnitude squared function is,

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \epsilon^2 V_N^2(\Omega)}, \quad (2a)$$

where,

$$V_N(\Omega) = \begin{cases} \cos(N \cos^{-1} \Omega), & |\Omega| \leq 1 \\ \cosh(N \cosh^{-1} \Omega), & |\Omega| > 1 \end{cases} \quad (2b)$$

is the N th order Chebyshev polynomial. If the passband deviation in decibels is A_{\max} then,

$$\epsilon^2 = 10^{A_{\max}/10} - 1. \quad (2c)$$

If the stopband edge is Ω_s and the stopband attenuation is A_{\min} , the degree of the polynomial should satisfy the criterion,

$$N \geq \frac{\cosh^{-1} [(10^{A_{\min}/10} - 1)/\epsilon^2]^{1/2}}{\cosh^{-1}(\Omega_s)}, \quad (3a)$$

where $\cosh^{-1} x$ is given by,

$$\cosh^{-1} x = \ln \left(x + \sqrt{x^2 - 1} \right). \quad (3b)$$

The filter poles are situated at points,

$$s_k = \sigma_k + j\Omega_k, \quad k = 1, 2, \dots, N, \quad (4a)$$

where,

$$\begin{aligned} \sigma_k &= -\frac{\gamma - \gamma^{-1}}{2} \sin \left[\frac{(2k-1)\pi}{2N} \right] \\ \Omega_k &= \frac{\gamma + \gamma^{-1}}{2} \cos \left[\frac{(2k-1)\pi}{2N} \right] \\ \gamma &= \left(\frac{1 + \sqrt{1 + \epsilon^2}}{\epsilon} \right)^{1/N}. \end{aligned} \quad (4b)$$

Verify the analytical results in MATLAB and plot the magnitude response in dB, phase response, passband details, phase delay, group delay, pole-zero plot and impulse response of the filter. (12 Points)

2. Use the transfer function in the above task to obtain a fourth-degree Chebyshev high-pass filter with the passband cutoff frequency of $\omega_p = 0.6\pi$ and the passband ripple of 1dB. Provide the MATLAB plots similar to the ones obtained in the previous task. (8 Points)