Exercise 9

1. By using the bilinear transformation, design a digital Chebyshev filter, which meets the following specifications:

$$-1 \leq 20 \cdot \log_{10} |H(e^{j\omega})| \leq 0, \qquad 0 \leq |\omega| \leq 0.2\pi$$
$$20 \cdot \log_{10} |H(e^{j\omega})| \leq -50, \qquad 0.5\pi \leq |\omega| \leq \pi.$$

The transfer function of the analog Chebyshev filter is of the form,

$$H_a(s) = \frac{k_0}{B_N(s)} = \frac{k_0}{\prod_{k=1}^{N}(s - s_k)} \tag{1}$$

The magnitude squared function is,

$$|H_a(j\Omega)|^2 = \frac{1}{1 + \epsilon^2 V_N^2(\Omega)}, \tag{2a}$$

where,

$$V_N(\Omega) = \begin{cases} \cos(N\cos^{-1}\Omega), & |\Omega| \leq 1 \\ \cosh(N\cosh^{-1}\Omega), & |\Omega| > 1 \end{cases} \tag{2b}$$

is the Nth order Chebyshev polynomial. If the passband deviation in decibels is $A_{\max}$ then,

$$\epsilon^2 = 10^{A_{\max}/10} - 1. \tag{2c}$$

If the stopband edge is Ω_s and the stopband attenuation is $A_{\min}$, the degree of the polynomial should satisfy the criterion,

$$N \geq \frac{\cosh^{-1}\left[(10^{A_{\min}/10} - 1)/\epsilon^2\right]^{1/2}}{\cosh^{-1}(\Omega_s)}, \tag{3a}$$

where $\cosh^{-1} x$ is given by,

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right). \tag{3b}$$

The filter poles are situated at points,

$$s_k = \sigma_k + j\Omega_k, \qquad k = 1, 2, \ldots, N, \tag{4a}$$

where,

$$\begin{aligned} \sigma_k &= -\frac{\gamma - \gamma^{-1}}{2}\sin\left[\frac{(2k-1)\pi}{2N}\right] \\ \Omega_k &= \frac{\gamma + \gamma^{-1}}{2}\cos\left[\frac{(2k-1)\pi}{2N}\right] \\ \gamma &= \left(\frac{1 + \sqrt{1 + \epsilon^2}}{\epsilon}\right)^{1/N}. \end{aligned} \tag{4b}$$

Verify the analytical results in MATLAB and plot the magnitude response in dB, phase response, passband details, phase delay, group delay, pole-zero plot and impulse response of the filter.

2. Use the transfer function in the above task to obtain a fourth-degree Chebyshev high-pass filter with the passband cutoff frequency of $\omega_p = 0.6\pi$ and the passband ripple of 1dB. Provide the MATLAB plots similar to the ones obtained in the previous task.