Exercise 8

IIR Filter Design

1. Where in the z-plane are the following s-plane poles mapped as a result of bilinear transformation with $c = 0.3$:

   - $p_1 = -2 + 3j$
   - $p_2 = 2 + 3j$
   - $p_3 = 0$
   - $p_4 = 8j$
   - $p_5 = -18j$
   - $p_6 = -8000j$
   - $p_7 = -0.3$
   - $p_8 = \infty j$

   Express $p_1, p_2 \ldots p_8$ as complex exponential.

2. Use pen and paper (and probably a calculator) to design a digital IIR Butterworth filter using bilinear transformation, which meets the following specifications:

$$\begin{aligned} \omega_p &= 0.2\pi \\ \omega_s &= 0.4\pi \\ A_p &= 3\text{dB} \\ A_s &= 20\text{dB} \end{aligned}$$

3. Using MATLAB, design an IIR Butterworth, Chebyshev I, Chebyshev II and elliptic filter which meet the following specifications:

$$\begin{aligned} \omega_p &= 0.2\pi \\ \omega_s &= 0.24\pi \\ A_p &= 3\text{dB} \\ A_s &= 20\text{dB} \end{aligned}$$

   Confirm that the solutions given by MATLAB do meet the specifications.

4. Consider the transfer function:

$$H(z) = \frac{0.3375 + 0.3375z^{-1}}{1 - 0.3249z^{-1}}$$

   What is the location of its poles and zeros? What kind of filter is $H(z)$? Using the equations on page 62 of the lecture notes, design a highpass filter with passband edge at 0.7π, assuming that the passband edge of $H(z)$ is located at 0.3π. How about a highpass filter with passband edge at 0.3π? (Note: Equation 39a, Lecture notes, Part IV, Page 62, which is given below).

$$Z^{-1} = G(z^{-1}) = -\frac{z^{-1} + \alpha}{1 + \alpha z^{-1}}$$