Exercise 5

Design of FIR filters with windowing Method

1. Prove that,

$$h_{id}^{0}[n] = \begin{cases} 1 - \dfrac{\omega_c}{\pi}, & n = 0 \\ \dfrac{-\sin(\omega_c n)}{\pi n}, & |n| > 0 \end{cases}$$

(refer to lecture notes, Part III, Page 86–89).(4 points)

2. Consider $h_{id}^{0}[n]$ in problem 1 with only 1001 sample points. This is basically a truncated version of the impulse response of an ideal highpass filter. Assume $\omega_c = \pi/4$ and plot the linear amplitude response of $h_{id}^{0}[n]$ using `freqz`. Is $h_{id}^{0}[n]$ defined above causal? How to implement an anti-causal system in MATLAB?(4 points)

3. Now window your 1001 sample points of the ideal highpass impulse response ($\omega_c = \pi/4$) with Bartlett window function give by,

$$w[n] = 1 - \frac{|n|}{M+1}.$$

Plot the linear amplitude response of the windowed ideal highpass response. Compare it with the amplitude response you got in problem 2. Do you notice any improvement? Did you use any window whatsoever in problem 2?(4 points)

4. A signal is sampled at 100 samples/second. What are the minimum orders needed for the linear-phase FIR filters using rectangular and Hann windows, which preserve the frequencies from 0 to 20Hz and remove the frequencies from 30 to 50Hz? Using the approach of page 85 - 89 (Lecture notes: Part III) and tables on the page 101, plot the impulse response of the filters. What type of FIR filter have you got? What do you expect for the group delay of these filters.(4 points)

5. It is required to design a type II linear-phase FIR filter using Kaiser window which meets the following specifications:

$$\begin{aligned} \omega_p &= 0.4\pi \\ \omega_s &= 0.6\pi \\ \delta_p &= 0.01 \\ \delta_s &= 0.001 \end{aligned}$$

What is the required length and the parameter α for the window? What would be your answer if there is no constaint on the type of the filter to be designed? Confirm your results with the MATLAB function `kaiserord`.(4 points)