Exercise 1

1 Periodicity

A signal $x(n)$ is defined as periodic if there exists an integer N such that $x(n+kN) = x(n)$, where k is an arbitrary integer and N is the period of the periodic sequence.

(a) Consider the sequence $e^{j\omega_0 n}$. What is the period of this sequence? (2 points)

(b) Consider the signal,

$$x(n) = \cos(0.2\pi n) + \cos(0.5\pi n) + \cos(0.6\pi n).$$

Is this signal periodic? If so, what is the period of this signal? (2 points)

2 Complex Plane

(a) Sketch the following numbers in the complex plane:

- 5
- $2e^{j0.1\pi}$
- $3e^{j\pi}$.

How can you represent these numbers in the form of $a + jb$?(1 point)

(b) Express the following numbers through their magnitude and phase:(1 point)

- $2 - 2j$
- j
- $-3 + 2j$
- $-4 - j$.

3 Sampled Signals

(a) In practical applications, sampled signals are typically represented as arrays of real numbers. In addition to the sample values themselves, what do you need to know before the properties of a sampled signal are uniquely defined?(1 point)

(b) Assume a sampling frequency of $F_s = 1000$Hz for this signal, and make it represent a 30Hz sine wave. Plot both the analog and the digitized (sampled) sine wave with MATLAB (hint: `help plot,help stem`). Now make a frequency vector which contains the frequencies from 0 up to 10000Hz. Plot the frequency content of the sampled sine wave for this range of frequencies (hint: `freqz(sig,1,f,Fs)`, where f is a frequency vector and F_s is the sampling frequency). Based on the plot, what can you say about the frequency content of a sampled signal? Note that in MATLAB, we always deal with sampled signals.(4 points)

4 Convolution

Take the signal

$$x[n] = \delta[n] + 5\delta[n-1] + 8\delta[n-2] + 9\delta[n-3]$$

and the system

$$h[n] = 5\delta[n] + 6\delta[n-1] + 7\delta[n-2]$$

(a) Filter $x[n]$ with $h[n]$ and call it $y[n]$.(1 point)

(b) Find the frequency response of $x[n]$ and $h[n]$. Then find the product of these frequency responses.(1 point)

(c) Now compute the frequency response of $y[n]$. Compare the frequency response of $y[n]$ with the product of the frequency responses of $x[n]$ and $h[n]$.(2 points)

5 Frequency Representation of Discrete Signals

Take the signals,

$$x_1[n] = [1\ 2\ 3]$$
$$x_2[n] = [1\ 2+3j\ 3-4j]$$

where $j = \sqrt{-1}$. Choose a couple of frequencies ω_1 and ω_2 and find $X_k(e^{j\omega_k})$ and $X_k(e^{j(2\pi-\omega_k)})$. Did you notice any difference? Based on this, explain what range of the frequency response of a real signal is enough to give us information about its whole frequency response? (Note that you also have to deal with the frequencies like -0.7π and 5.3π).(5 points)