

Exercise 4

1. The transfer function of a second-order filter is given by

$$H(z) = K \frac{2 - z^{-2}}{1 + 0.81z^{-2}}$$

- Determine K such that the amplitude response achieves the value of unity at the zero frequency.
 - Provide the pole-zero plot for the filter and determine the amplitude and phase responses.
2. Consider the filter with the impulse response $h[n] = [1 \ 5 \ 1]$. Find its group and phase delay using MATLAB. Explain the effect of the system on sine waves of different frequencies in plain words. (Hint: For groupdelay, use the MATLAB function `grpdelay` and for the phase delay, look at its mathematical expression given on page 55 of the lecture notes (part I)).
 3. Consider the filter $h[n] = [1 \ 2 \ 2 \ 1]$. Is it an FIR or an IIR filter? Is its phase response linear? If so, what type of linear phase FIR $h[n]$ is? Write down its frequency and zero-phase responses. Then plot its frequency and zero-phase responses. For this purpose, you need to write a MATLAB program to calculate its zero-phase response. Do you spot the difference between frequency and zero-phase responses? Plot zeros and poles of the filter with MATLAB.
 4. Give an example of type I, II, III and IV linear phase FIR filters. Name them as $h1 \dots h4$ in MATLAB. Now plot their frequency responses. At which frequencies the frequency response is 0? Plot the zeros and poles of the filters. Is it consistent with the frequency response you got?
 5. Show that if $F(z) = (1 + z^{-1})/2$, then $F(e^{j\omega}) = e^{-j\omega/2} \cos(\omega/2)$. Is $F(z)$ a linear phase FIR filter? If so, then what type?
 6. The transfer function of a type III linear phase FIR filter is

$$H(z) = \sum_{n=0}^{N/2-1} h[n][z^{-n} - z^{-(N-n)}]$$

Prove that the frequency response is zero both at $\omega = 0$ and $\omega = \pi$.

Food for Thought

Rewrite the following nested **if-else** statement as a **switch** statement that performs exactly in a similar manner. Suppose that *num* is an integer variable that has been initialized, and there are four functions *f1*, *f2*, *f3*, *f4*. You should not use any **if** or **if-else** statements within **switch**, furthermore, only calls to the four functions are allowed as per the following code:

```
if num < -2 || num > 4
    f1(num)
else
    if num <= 2
        if num >= 0
            f2(num)
        else
            f3(num)
        end
    else
        f4(num)
    end
end
```