Exercise 3

1 Implementing Digital Filters and Z-Transform (Cont.)

Consider the difference equation:

$$x[n] = y[n] - 5y[n-1] + 6y[n-2]$$

(a) Present a direct form implementation of the above system. Next implement the system both as a cascade and as a parallel connection of two sub-systems. (Hint: Lecture Notes, Part I, Page 38 - 39).

(b) Implement the system

$$x[n] = y[n] - 3y[n-1] + 4y[n-2] - 2y[n-3]$$

as a cascade of two sub-systems. (Hint: $z = 1$ is the root of the polynomial $1 - 3z + 4z^2 - 2z^3$, Lecture Notes Part I, page 33 - 34)

2 Zeros and Poles

(a) Consider the transfer function:

$$H(z) = \frac{0.3 + 0.6z^{-1} + 0.3z^{-2}}{1 + 0.2z^{-2}}$$

Find the zeros and the poles of $H(z)$. Is this system stable? Give an example of another system $G(z)$ which has exactly the same zeros and poles.

(b) Determine the unit sample response of the following system:

$$y(n) = 2.5y(n-1) - y(n-2) + x(n) - 5x(n-1) + 6x(n-2)$$

What can you say about the poles and zeros of this system.(hint: Z-transform,Partial fractions)

3 All-Pass Systems

Find the coefficients a, b, c and d to make $H(z)$ an All-pass system:

$$H(z) = \frac{8 - 3z^{-1} + 5z^{-2} + z^{-3}}{a + bz^{-1} + cz^{-2} + dz^{-3}}$$

Plot the linear-scale amplitude response of $H(z)$. Also plot and calculate the zeros and poles of $H(z)$. How the poles and zeros are related to each other? Is this system stable?(Hint:Lecture Notes Part II, Page 83)