Exercise 3

1 Poles and Zeros

(a) Consider the transfer function:

$$H(z) = \frac{0.3 + 0.6z^{-1} + 0.3z^{-2}}{1 + 0.2z^{-2}}$$

Find the zeros and the poles of $H(z)$. Is this system stable? Give an example of another system $G(z)$ which has exactly the same zeros and poles.(6 points)

2 Phase and Group Delay

(a) Consider the filter with the impulse response $h[n] = [1\ 5\ 1]$.Find its group and phase delay using MATLAB. Explain the effect of the system on sine waves of different frequencies in plain words. (Hint: For groupdelay, use the MATLAB function `grpdelay` and for the phase delay, look at its mathematical expression given on page 55 of the lecture notes).(4 points)

3 Linear Phase FIR Filters

(a) Consider the filter $h[n] = [1\ 2\ 2\ 1]$. Is it an FIR or an IIR filter? Is its phase response linear? If so, what type of linear phase FIR $h[n]$ is? Write down its frequency and zero-phase responses. Then plot its frequency and zero-phase responses. For this purpose, you need to write a MATLAB program to calculate its zero-phase response. Do you spot the difference between frequency and zero-phase responses? Plot zeros and poles of the filter with MATLAB.(3 points)

(b) Give an example of type I, II, III and IV linear phase FIR filters. Name them as $h1 \ldots h4$ in MATLAB. Now plot their frequency responses. At which frequencies the frequency response is 0? Plot the zeros and poles of the filters. Is it consistent with the frequency response you got?(3 points)

(c) Show that if $F(z) = (1+z^{-1})/2$, then $F(e^{j\omega}) = e^{-j\omega/2}\cos(\omega/2)$. Is $F(z)$ a linear phase FIR filter? If so, then what type?(2 points)

(d) The transfer function of a type III linear phase FIR filter is

$$H(z) = \sum_{n=0}^{N/2-1} h[n][z^{-n} - z^{-(N-n)}]$$

Prove that the frequency response is zero both at $\omega = 0$ and $\omega = \pi$.(2 points)