Exercise 4

1. Determine the unit sample response of the following system:

$$y(n) = 2.5y(n-1) - y(n-2) + x(n) - 5x(n-1) + 6x(n-2)$$

What can you say about the poles and zeros of this system.(hint: Z-transform,Partial fractions)(4 points)

2. Find the coefficients a, b c and d to make $H(z)$ an All-pass system:

$$H(z) = \frac{8 - 3z^{-1} + 5z^{-2} + z^{-3}}{a + bz^{-1} + cz^{-2} + dz^{-3}}$$

Plot the linear-scale amplitude response of $H(z)$. Also plot and calculate the zeros and poles of $H(z)$. How the poles and zeros are related to each other? Is this system stable?(hint:Lecture Notes Part II, Page 83)(3 points)

3. The transfer function of a second-order filter is given by

$$H(z) = K\frac{1 + z^{-2}}{1 + 0.81z^{-2}}$$

- Determine K such that the amplitude response achieves the value of unity at the zero frequency.(2 points)
- Provide the pole-zero plot for the filter and determine the amplitude and phase responses.(2 points)

4. Consider the signal:

$$x[n] = \sin\left(\frac{\pi}{6}n\right) - \sin\left(\frac{\pi}{12}n\right) + 2\sin\left(\frac{\pi}{8}n\right)$$

(a) Generate and plot $x[n]$ using MATLAB.(1 point)

(b) If the sampling frequency is 2000, what continuous sines do $x[n]$ represent?(1 point)

5. Consider the difference equation:

$$x[n] = y[n] - 5y[n-1] + 6y[n-2]$$

(a) Present a direct form implementation of the above system. Next implement the system both as a cascade and as a parallel connection of two sub-systems. (Hint: Lecture Notes, Part I, Page 38 - 39).(2.5 points)

(b) Implement the system

$$x[n] = y[n] - 3y[n-1] + 4y[n-2] - 2y[n-3]$$

as a cascade of two sub-systems. (Hint: $z = 1$ is the root of the polynomial $1 - 3z + 4z^2 - 2z^3$, Lecture Notes Part I, page 33 - 34)(2.5 points)

6. Assume $g[n] = [1\ 2\ 3\ 2\ 1]$ is the common term for $h_1[n] \ldots h_4[n]$ which are type I,II, III and IV linear phase FIR filters. Use MATLAB to implement $h_1 \ldots h_4$ and plot the frequency response, zero diagram and impulse response for each case.(Hint: Lecture Notes Part III, Page 42 - 47)(2 points)