Exercise 1

1 Periodicity

A signal $x(n)$ is defined as periodic if there exists an integer N such that $x(n+kN) = x(n)$, where k is an arbitrary integer and N is the period of the periodic sequence.

(a) Consider the sequence $e^{j\omega_0 n}$. What is the period of this sequence?

(b) Consider the signal,

$$x(n) = \cos(0.2\pi n) + \cos(0.5\pi n) + \cos(0.6\pi n).$$

Is this signal periodic? If so, what is the period of this signal?

2 Sampled Signals

Assume a sampling frequency of $F_s = 1000$Hz for this signal, and make it represent a 30Hz sine wave. Plot both the analog and the digitized (sampled) sine wave with MATLAB (hint: `help plot,help stem`). Now make a frequency vector which contains the frequencies from 0 up to 10000Hz. Plot the frequency content of the sampled sine wave for this range of frequencies (hint: `freqz(sig,1,f,Fs)`, where f is a frequency vector and F_s is the sampling frequency). Based on the plot, what can you say about the frequency content of a sampled signal? sampled signals.

3 Convolution

Take the signal

$$x[n] = \delta[n] + 5\delta[n-1] + 8\delta[n-2] + 9\delta[n-3]$$

and the system

$$h[n] = 5\delta[n] + 6\delta[n-1] + 7\delta[n-2]$$

(a) Filter $x[n]$ with $h[n]$ and call it $y[n]$.

(b) Find the frequency response of $x[n]$ and $h[n]$. Then find the product of these frequency responses.

(c) Now compute the frequency response of $y[n]$. Compare the frequency response of $y[n]$ with the product of the frequency responses of $x[n]$ and $h[n]$.

4 Frequency Representation of Discrete Signals

Take the signals,

$$x_1[n] = [1 \quad 2 \quad 3]$$
$$x_2[n] = [1 \quad 2+3j \quad 3-4j]$$

where $j = \sqrt{-1}$. Choose a couple of frequencies ω_1 and ω_2 and find $X_k(e^{j\omega_k})$ and $X_k(e^{j(2\pi-\omega_k)})$. Did you notice any difference? Based on this, explain what range of the frequency response of a real signal is enough to give us information about its whole frequency response? (Note that you also have to deal with the frequencies like -0.7π and 5.3π).

Food for Thought

The following claims to be a mathematical proof that $4 = 5$. Which of the following steps is illegal? Explain why? Proof of $4 = 5$

Step 1: $16 - 36 = 25 - 45$
Step 2: $4^2 - 9 \cdot 4 = 5^2 - 9 \cdot 5$
Step 3: $4^2 - 9 \cdot 4 + 81/4 = 5^2 - 9 \cdot 5 + 81/4$
Step 4: $(4 - 9/2)^2 = (5 - 9/2)^2$
Step 5: $4 - 9/2 = 5 - 9/2$
Step 6: $4 = 5$