Exercise 11

Quantization

1. Design an FIR filter that meets the following specifications:

$$\begin{aligned}\omega_p &= 0.4\pi\\ \omega_s &= 0.6\pi\\ \delta_p &= 0.01\\ \delta_s &= 0.001\end{aligned}$$

Quantize the filter with different number of bits $(8, 10, 12, 14, \ldots$ etc.) and different filter orders. How many bits are required to achieve the prescribed stopband attenuation? Plot the frequency responses of the original and the quantized filters. Plot also the quantization error in each case.

Scaling

1. Consider the system with transfer function:

$$H(z) = \frac{1}{1 - 0.8z^{-1}}$$

The purpose of this task is to clarify the concept of scaling. One of the objectives of scaling is to prevent overflow at the output. We want to find the value K, such that filtering of a signal $x[n], |x[n]| \leq 1, \forall\ n$ with $KH(z)$ does not cause overflow.

(a) Determine the impulse response of this filter.

(b) Generate a random signal consisting of 1000 samples, staying between -1 and 1. Filter the signal with $H(z)$, and check if there is any sample in the output which exceeds -1 or 1.

(c) Next scale $H(z)$ such that overflow never occurs. This is the so called *worst case scaling*. Generate a random signal of length 1000, and confirm that the output signal stays within the required limits.

(d) Repeat the experiment a few times. Increase the value of K gradually, and see if overflow happens.

(e) It seems that the value found for K is too pessimistic, and K can be larger. Show that this is not the case. In other words, assume that the amplitude of the input signal is restricted to be between -1 and 1 and find an example signal which causes overflow at the output of $H(z)$, if K is larger than what you found at part c).

(f) Now scale $H(z)$ according to L_2 norm, and repeat part b) for this scaled version of the filter. How often does overflow occur?

(g) Determine the output noise variance of the scaled filter, if $1+6$ bits are available.