Sharif University of Technology

Numerical Analysis of Roll Deflection during Plate Rolling using Finite Element Methods & Artificial Neural Networks

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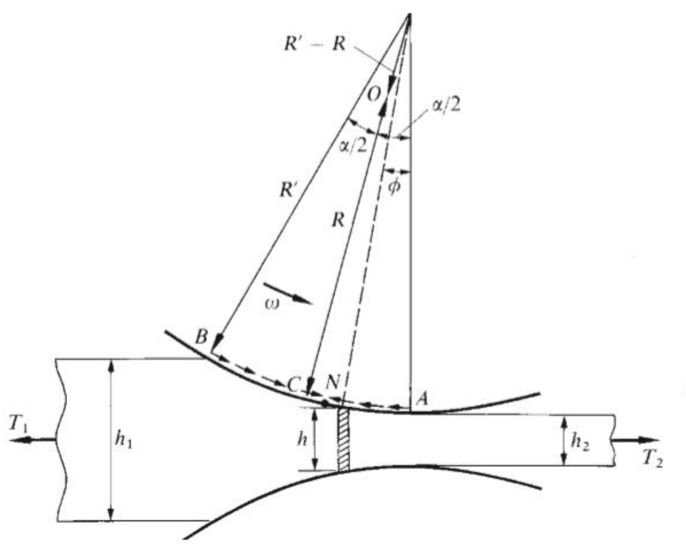
1-1 Introduction

In **Mathematical Simulation** Part of my Master thesis, the **work-roll deflection** during multi-pass rolling operations was predicted employing a **combined model** using **finite element method**, **artificial neural network** (ANN) and **finite difference scheme**. The ANN modeling was utilized for calculating the flow stress of the steel being rolled while the mean pressure was calculated based on cold rolling theories coupled with the finite difference approach. Finally, the work-roll deflection was determined by means of the predicted mean roll pressure and finite element formulation. It was found that that an **increase** in the **number of passes** as well as **reducing** rolling **temperature** led to an **increase** in work-roll the **deflection**.

1-2 Cold Rolling of Strips

We shall investigate the process in which a metal sheet or strip is reduced in thickness by passing it between a pair of cylindrical rolls shaving their axes parallel to one another. In cold rolling, the radius R of the rolls is usually more than 50 times the initial strip thickness. If the width of the strip is at least five times the length of the arc of contact, the non-plastic material prevents the lateral spread, and the deformation takes place effectively under plane strain condition. Due to the pressure of the rolled stock, the rolls are themselves flattened so as to increase the arc of contact by as much as 20 to 25 percent or even more. It will be assumed, for simplicity, that the part of the rolls in contact with the strip is deformed into a cylindrical surface of a larger radius $R^{/}$.



1-1 Geometry of strip rolling, showing the forces acting on a slice considered on the exit side.

Since the volume of the material passing through each vertical plane per unit time is the same, the speed of the strip steadily increases as it moves through the roll gap. On the entry side, the peripheral speed of the rolls is higher than that of the strip, and consequently the frictional forces draw the strip into the roll gap. On the exit side, the strip moves faster than the rolls, and the frictional forces therefore oppose the delivery of the strip. It follows that there is **a neutral point** N somewhere on the arc of contact where the strip moves at the same speed as that of the rolls.

1-3 Equation of Equilibrium

It is assumed at the outset that each element of the strip is uniformly compressed between the rolls while passing through the roll gap. The vertical compression of the strip is accompanied by a horizontal force which is increasingly compressive as the neutral point is approached from either side. the equation of equilibrium may be written as:

$$\frac{d}{d\phi}(hp) = 2qR'(sin\phi \mp \mu cos\phi) \qquad 1\text{-}1$$

Where the upper sign applies to the exit side and the lower sign to the entry side of the neutral point. We now assume that the material is everywhere plastic between the planes of entry and exit, and the principal compressive stresses at each point on a vertical section are approximately equal to p and q. Then the yield criterion may be written in the form:

$$q - p = 2k \qquad 1\text{-}2$$

Where the shear yield stress k generally varies along the arc of contact. The value of 2k at a generic point on the arc of contact is approximately equal to the ordinate of the compressive stress–strain curve, obtained under plane strain condition, corresponding to an abscissa equal to ln(h1/h). Alternatively, the variation of the yield stress can be estimated by rolling a length of the strip in a succession of passes and carrying out a tensile test at the end of each pass. This gives the tensile yield stress √3k as a function of the thickness ratio h/h1. since the angle of contact is small, we arrive at the governing equation:

$$\frac{d}{d\phi}(q - 2k) \mp 2\mu R' q = 4kR'\phi \qquad 1\text{-}3$$

Where:

$$h = h_2 + 2R^{'}(1 - \cos\phi) \simeq h_2 + R^{'}\phi^2 \qquad 1\text{-}4$$

1-4 Finite Difference Formulation

In order to solve 1-3 equation by Finite Difference method, 3-5 equation is obtained by **Forward Difference** approach for the exit side (when $p = 0$ then $2k = q_0$). By using the initial and final strip thickness in 1-4 equation ϕ_1 for first step is obtained.

$$\frac{h_i(q_{i+1} - 2k_{i+1} - q_i + 2k_i)}{\phi_{i+1} - \phi_i} + 2\mu R q_0 = 4kR\phi_i \qquad 1\text{-}5$$

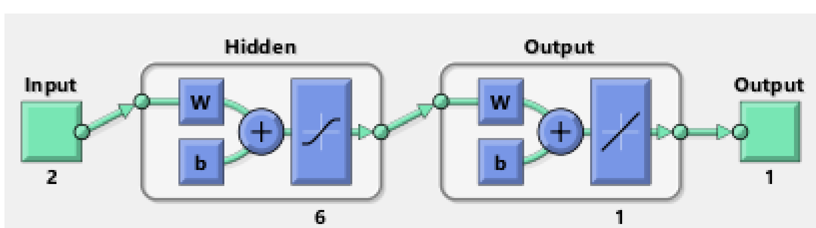
Like the exist side the **Forward Difference** approach is used for entry side. The equation of entry side may be written as below:

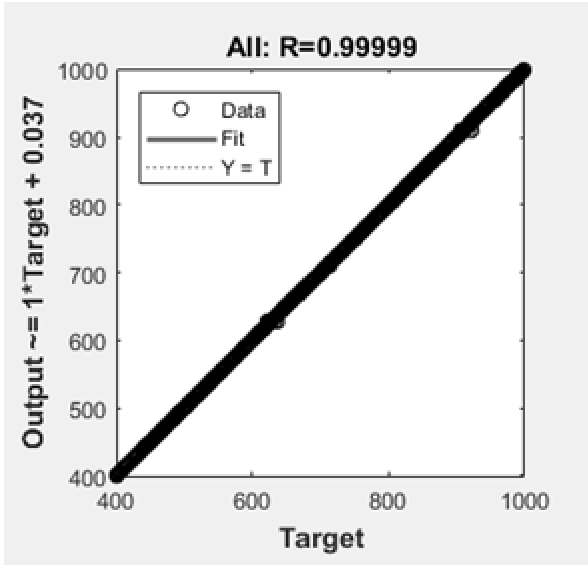
$$\frac{h_i(q_{i+1} - 2k_{i+1})}{\phi_{i+1}} - 2\mu R q_0 = 0 \qquad 1\text{-}6$$

After that, in every thickness the amount of pressure update until the final thickness. When the amount of calculated pressure in tow mentioned side are equal, it can be assumed as Neutral point pressure at corresponded degree.

1-5 Artificial Neural Network

In any stage of thickness reduction, the flow stress is different. Therefore, artificial **neural network** (ANN) is utilized for calculating the flow stress. By conducting mechanical tensile test in different strain rate and temperature, the needed data have been obtained.





1-2 The utilized artificial neural network and its regression.

Back propagation method and **linear function** have been used for creating artificial neutral network while:

Train	Validation	Test
70%	15%	15%

Finally, the below equation can be used for calculating the mean pressure that is going to be used for the **work-roll deflection** during multi-pass rolling operations.

$$\bar{P} = \frac{\int_{\alpha}^{0} q \, d\phi}{\Delta\phi} \qquad 1\text{-}7$$

Finally, the work-roll deflection was determined by means of the predicted mean roll pressure and finite element formulation (Galerkin) as below:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) = p(x) \qquad 1\text{-}8$$

Where:

$$I = \frac{\pi d^4}{64} \qquad 1\text{-}9$$

1-6 Finite Element Formulation

Cantilevered Beam is used as Boundary condition.

$$K^e a^e = f^e \tag{1-10}$$

$$f^e = f^e_{VM} + f^e_p \tag{1-11}$$

$$f^e_{VM} = \begin{bmatrix} -V(x_i) \\ -M(x_i) \\ V(x_j) \\ M(x_j) \end{bmatrix} \tag{1-12}$$

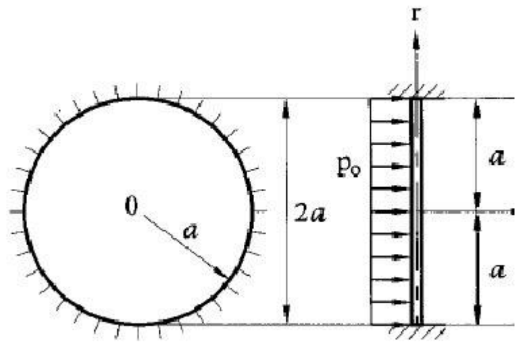
$$f^e_p = \bar{p}L \int_0^1 N^T \, d\xi = \bar{p}L \begin{bmatrix} \int_0^1 (1 - 3\xi^2 + 2\xi^2) d\xi \\ \int_1^0 L(\xi - 2\xi^2 + \xi^3) \, d\xi \\ \int_1^0 (3\xi^2 - 2\xi^3) d\xi \\ \int_1^0 L(-\xi^2 + \xi^3) d\xi \end{bmatrix} = \begin{bmatrix} \bar{p}L/2 \\ \bar{p}L^2/12 \\ \bar{p}L/2 \\ -\bar{p}L^2/12 \end{bmatrix} \tag{1-13}$$

$$K^e = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{1-14}$$

$$a^e = \begin{bmatrix} w_i \\ \theta_i \\ w_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} 1 & x_i & x_i^2 & x_i^3 \\ 0 & 1 & 2x_i & 3x_i^2 \\ 1 & x_j & x_j^2 & x_j^3 \\ 0 & 1 & 2x_j & 3x_j^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \tag{1-15}$$

1-7 Validation

The below model is utilized for validation of mentioned model:



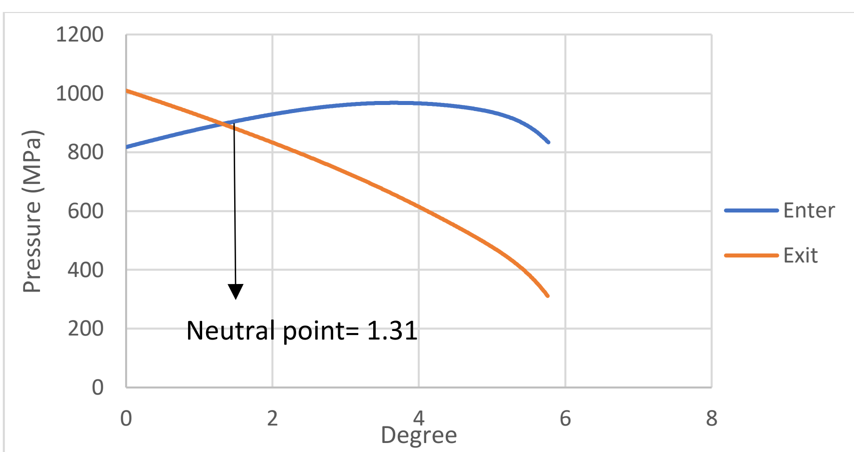
1-3 Circular shape plate under pressure.

$$Deflection\ (Max) = \frac{pa^2}{16\pi D} \qquad at\ \ r = 0 \tag{1-16}$$

$$D = \frac{Et^3}{12(1-\nu)} \tag{1-17}$$

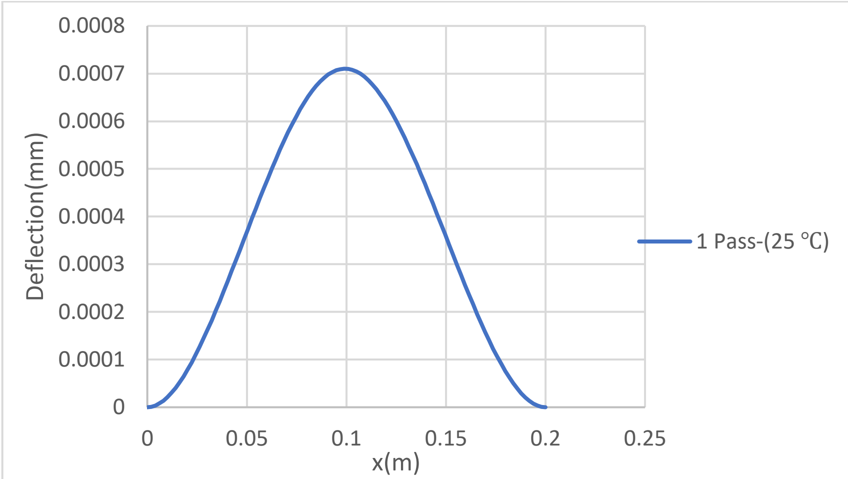
1-8 Result

As it mentioned earlier the model has been used for multi-pass rolling. Final results of mentioned model on the 1 pass Forward rolling at 25 ℃ are represented here. Further results can be found on my full script of master thesis.



1-4 Neutral point position of 1 pass rolled strip at 25 ℃.

The mean pressure is 121 MPa and the deflection on work-roll can be seen in the following feature.



1-4 Work-roll Deflection after 1 pass rolled strip at 25 ℃.

MATLAB codes of mentioned **combined model can be seen at GitHub.**