بسمه تعالی
دانشکده مهندسی کامپیوتر، دانشگاه صنعتی شریف

- مشخصات : عطیه جمشیدپور
- شماره دانشجویی:۹۴۱۰۳۸۳۵

تمرین پنجم محاسبات عددی

موعد تحویل :۰۳/۱۲

تمرین اول

الف)

$a_0=1, b_{-1}=0.5, b_0=0.5$

با حل معادلات داریم :

$y(x)=1, y'(x)=f(x,y)=0 \rightarrow$

$y(x_n+h)=y(x_n)a_0+h(b_{-1}y'(x_n+h)+b_0y'(x_n)) \rightarrow$

$1=1*a_0+h(b_{-1}*0+b_0*0) \rightarrow a_0=1$

$y(x)=x, y'(x)=f(x,y)=1 \rightarrow$

$y(x_n+h)=y(x_n)a_0+h(b_{-1}y'(x_n+h)+b_0y'(x_n)) \rightarrow$

$(x_n+h)=x_n*1+h(b_{-1}*1+b_0*1) \rightarrow b_{-1}+b_0=1$

$y(x)=x^{2,} y'(x)=f(x,y)=2x \rightarrow$

$y(x_n+h)=y(x_n)a_0+h(b_{-1}y'(x_n+h)+b_0y'(x_n)) \rightarrow$

$(x_n+h)^2=(x_n)^2*1+h(b_{-1}*2(x_n+h)+b_0*2x_n) \rightarrow b_{-1}=0.5, b_0=0.5$

ب)

$y_0=3, y'=f(x,y)=e^x+y \rightarrow$

$f'(x,y)=\frac{\partial(e^x+y)}{\partial x}=e^x+y'=2e^x+y, f''(x,y)=\frac{\partial(2e^x+y)}{\partial x}=2e^x+y'=3e^x+y$

$y_{n+1}=y_n+hf(x_n,y_n)+\frac{h^2}{2!}f'(x_n,y_n)+\frac{h^3}{3!}f''(x_n,y_n)$

$y_1=y_0+hf(x_0,y_0)+\frac{h^2}{2!}f'(x_{0,}y_0)+\frac{h^3}{3!}f'(x_{0,}y_0) \rightarrow$

$y_1=3+(0.1)(e^0+3)+\frac{(0.1)^2}{2!}(2e^0+3)+\frac{(0.1)^3}{3!}(3e^0+3) \rightarrow y_1=y(0.1)=3.426$

تمرین دوم

الف

$$y_0=1, y'=f(x,y)=y; y_{n+1}=y_n+hf(x_n,y_n)=y_n+hy_n \rightarrow y_{n+1}=(1+h)y_n$$

$$\rightarrow y_4=(1+h)^4 y_0=(1+0.01)^4(1)\rightarrow y_4=y(0.04)=1.0406$$

ب)

$$y_0=1, y'=f(x,y)=\sin(x)+\sin(y); y_{n+1}=y_n+hf(x_n,y_n)=y_n+h\sin(y_n)+h\sin(x_n)\rightarrow$$

$$y_1=y_0+h\sin(y_0)+h\sin(x_0)\rightarrow y_1=(0.25)\sin(1)+(0.25)\sin(0)=1.2104$$

$$y_2=y_1+h\sin(y_1)+h\sin(x_1)\rightarrow y_2=(0.25)\sin(1.2104)+(0.25)\sin(0.25)=1.5062$$

$$y_3=y_2+h\sin(y_2)+h\sin(x_2)\rightarrow y_3=(0.25)\sin(1.5062)+(0.25)\sin(0.5)=1.8755$$

$$y_4=y_3+h\sin(y_3)+h\sin(x_3)\rightarrow y_4=(0.25)\sin(1.8755)+(0.25)\sin(0.75)=2.2844$$

$$y(0)=1, y(0.25)=1.2104, y(0.5)=1.5062, y(0.75)=1.8755, y(1)=2.2844$$

تمرین سوم

$$y_0=1, y'_0=0, y''+y'-6y=0\rightarrow y'=p=f_1(x,y,p), p'=6y-p=f_2(x,y,p); y_0=1, p_0=0$$

$$y_{n+1}=y_n+\frac{1}{6}[k_1+2k_2+2k_3+k_4]\rightarrow 1+\frac{1}{6}[0+2(0.03)+2(0.0285)+0.0580]=1.0292$$

$$p_{n+1}=p_n+\frac{1}{6}[l_1+2l_2+2l_3+l_4]\rightarrow p_1=0+\frac{1}{6}[0.6+2(0.57)+2(0.5805)+0.5590]=0.5767$$

$$y(0.1)\approx 1.0292, y'(0.1)\approx 0.5767$$

جزئیات محاسبات در ادامه آمده است.

$$k_1=hf_1(x_n,y_n,p_n)=hp_n=(0.1)(0)=0$$

$$l_1=hf_2(x_n,y_n,p_n)=h(6y_n-p_n)=(0.1)(6(1)-0)=0.6$$

$$k_2=hf_1(x_n+\frac{h}{2}, y_n+\frac{k_1}{2}, p_n+\frac{l_1}{2})=h(p_n+\frac{l_1}{2})=(0.1)(0+\frac{0.6}{2})=0.03$$

$$l_2=hf_2(x_n+\frac{h}{2}, y_n+\frac{k_1}{2}, p_n+\frac{l_1}{2})=h(6y_n-p_n+3k_1-\frac{l_1}{2})=(0.1)(6(1)-0+3(0)-\frac{0.6}{2})=0.57$$

$$k_3=hf_1(x_n+\frac{h}{2}, y_n+\frac{k_2}{2}, p_n+\frac{l_2}{2})=h(p_n+\frac{l_2}{2})=(0.1)(0+\frac{0.57}{2})=0.0285$$

$$l_3=hf_2(x_n+\frac{h}{2},y_n+\frac{k_2}{2},p_n+\frac{l_2}{2})=h(6y_n-p_n+3k_2-\frac{l_2}{2})=(0.1)(6(1)-0+3(0.03)-\frac{0.57}{2})=0.5805$$

$$k_4=hf_1(x_n+h,y_n+k_3,p_n+l_3)=h(p_n+l_3)(0.1)(0+0.5805)=0.0580$$

$$l_4=hf_2(x_n+h,y_n+k_3,p_n+l_3)=h(6y_n-p_n+3k_3-\frac{l_3}{2})=(0.1)(6(1)-0+3(0.0285)-0.5955)=0.5590$$

تمرین چهارم

$$y'=f(t,y)=1+(t-y)^2,y(2)=1$$

$$y_{i+1}=y_i+\frac{1}{6}(k_1+4k_2+k_3)$$

$$k_1=hf(t_i,y_i)$$

$$k_2=hf(t_i+\frac{h}{2},y_i+\frac{k_1}{2})$$

$$k_3=hf(t_i+h,y_i+2k_2-k_1)$$

$$y_1=y_0+\frac{1}{6}(k_1+4k_2+k_3)\rightarrow 1+\frac{1}{6}(0.4+4(0.362)+0.3535)=1.3669$$

$$k_1=hf(t_0,y_0)=0.2(1+(2-1)^2)=0.4$$

$$k_2=hf(t_0+\frac{h}{2},y_0+\frac{k_1}{2})=(0.2)(1+(2.1-1.2)^2)=0.362$$

$$k_3=hf(t_0+h,y_0+2k_2-k_1)=(0.2)(1+(2.2-1.324)^2)=0.3535$$

$$y_2=y_1+\frac{1}{6}(k_1+4k_2+k_3)=1.3669+\frac{1}{6}(0.3388+4(0.3166)+0.3091)=1.686$$

$$k_1=hf(t_1,y_1)=0.2(1+(2.2-1.3669)^2)=0.3388$$

$$k_2=hf(t_i+\frac{h}{2},y_i+\frac{k_1}{2})=0.2(1+(2.3-1.5363)^2)=0.3166$$

$$k_3=hf(t_i+h,y_i+2k_2-k_1)=0.2(1+(2.4-1.6613)^2)=0.3091$$

$$y_3=y_2+\frac{1}{6}(k_1+4k_2+k_3)=1.9753+\frac{1}{6}(0.302+4(0.2879)+0.282)=1.9753$$

$$k_1=hf(t_i,y_i)=0.2(1+(2.4-1.686)^2)=0.302$$

$$k_2=hf(t_i+\frac{h}{2},y_i+\frac{k_1}{2})0.2(1+(2.5-1.837)^2)=0.2879$$

$$k_3=hf(t_i+h,y_i+2k_2-k_1)0.2(1+(2.6-1.9598)^2)=0.282$$

$$y(2.6)\approx 1.9753$$

تمرین پنجم

$$y'(t_i)=f(t_i,y(t_i))\rightarrow y(t_{i+1})=y(t_i)+ahy'(t_i)+bhy'(t_{i-1})+chy'(t_{i-2})$$

با بسط هر دو طرف با سری تیلور داریم :

$$y(t_i)+hy'(t_i)+\frac{1}{2}h^2y''(t_i)+\frac{1}{6}h^3y'''(t_i)+O(h^4)=y(t_i)+ahy'(t_i)+¿$$

$$bh(y'(t_i)-hy''(t_i)+\frac{1}{2}y'''(t_i)+O(h^3))+ch(y'(t_i)-2hy''(t_i)+\frac{4}{2}y'''(t_i)+O(h^3))\rightarrow$$

$$\rightarrow y(t_i)+(a+b+c)hy'(t_i)+(-b-2c)h^2y''(t_i)+(\frac{1}{2}b+2c)h^3y'''(t_i)+O(h^4)$$

با در نظر گرفتن اتحاد ضرایب داریم :

$$1=a+b+c,\frac{1}{2}=-b-2c,\frac{1}{6}=\frac{1}{2}b+2c\rightarrow a=\frac{23}{12},b=\frac{-6}{12},c=\frac{5}{12}$$

بنابراین فرم کلی به ترتیب زیر خواهد بود :

$$y(t_{i+1})=y(t_i)+\frac{h}{12}[23f(t_i,y(t_i))-16f(t_{i-1},y(t_{i-1}))5f(t_{i-2},y(t_{i-2}))]+O(h^4)$$

بتابراین مرتبه خطا h^4 است.

تمرین ششم

کد مربوط به این بخش در ضمیه این سند در attachments/codes/6.py آمده است.

[1] : http://www2.cs.uh.edu/~gabriel/courses/cosc3361_f05/exercises3.pdf