0.0 Info about Course

10 January 2017 23:23

Course Page -
http://www.commsp.ee.ic.ac.uk/~kkleung/Intro_Signals_Comm_2017/

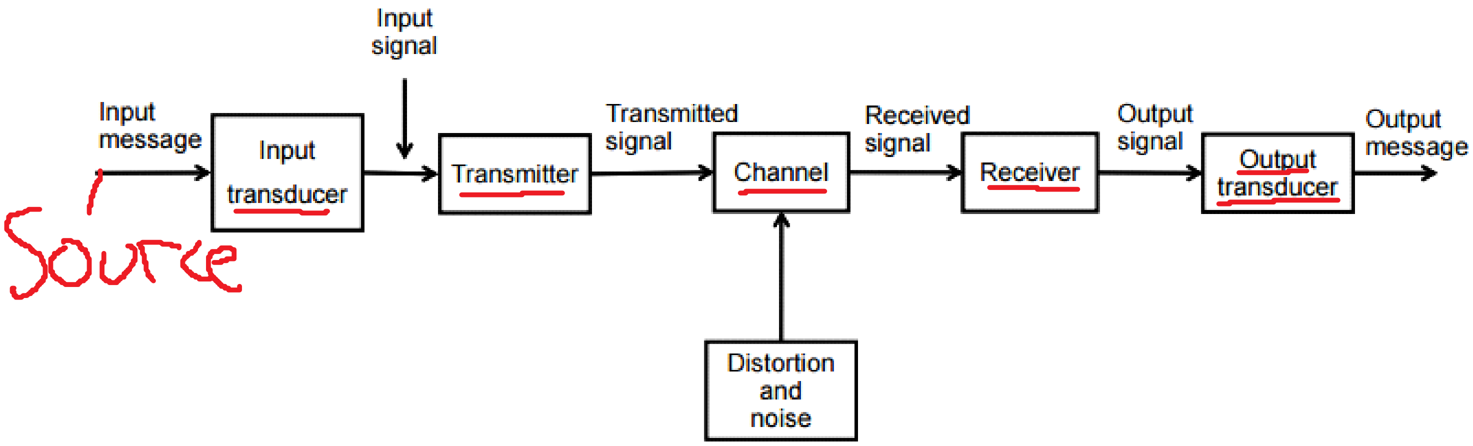
Recommended Book -
B.P Lathi and Z. Ding, Modern Digital and Analog Communication Systems, Oxford University Press

Syllabus

- Fundamentals of Signals and Systems
  - Energy and power
  - Trigonometric and Exponential Fourier Series
  - Fourier transform
  - Linear system and convolution integral
- Modulation
  - Amplitude modulation: DSB, Full AM, SSB
  - Angle modulation: PM, FM
- Advanced Topics: Digital communications, CDMA

1.1 Communication systems process

13 January 2017 21:07



Source - Input Message

Transducer - converts source into an electrical waveform, called the **baseband signal**

Transmitter - alters baseband for a more efficient transmission

Channel - the medium the baseband signal is sent through - e.g. optical fibre, coax etc.

Receiver - undoes modifications on the signal made by channel and transmitter

Output Transducer - converts last form of baseband signal into original form of signal, e.g. soundwave

1.2 Digital vs Analog

13 January 2017 21:18

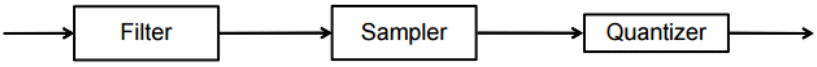
<u>Basics</u>
Digital made from a finite number of symbols
Analog has an infinite amount of values over a range

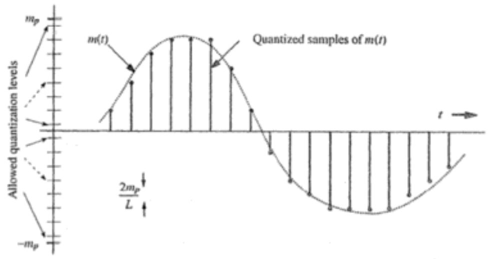
Digital is more robust to noise

<u>Sampling theorem</u>

To reconstruct a signal from samples, the sample rate must be 2x the highest frequency in the original signal spectrum

<u>Analog to Digital conversion (ADC)</u> - as better to transmit Digital signal, so need to convert Analog signal





<u>Sampling</u>
At a set interval time, value of displacement of signal is taken - this is sampling the signal

<u>Quantizing</u>
For each sample value, it is rounded to the closest allowed quantized level

2.0 Classifications of Signals

13 January 2017 21:31

In this course, we deal with signals that are functions of time

Types of classification

Continuous-time	Discrete-time

Analog	Digital

Periodic	Aperiodic

Energy	Power

Deterministic	Probabilistic

NB Analog is not same as continuous-time, and digital is not same as discrete-time

2.1 Continuous / Discrete

13 January 2017 21:44

Definitions

- Continuous - signal that is specified for every value of time
- Discrete - signal that is specified only at discrete values of time

Can obtain a discrete signal from a continuous signal via **sampling.**
Sometimes it's possible to obtain the original continuous from the discrete signal created via sampling. (**lossless**)

2.2 Analog / Digital

13 January 2017 21:45

Definitions

- Analog - signal whose **amplitude** can take on any value in a continuous range
- Digital - signal whose **amplitude** can take on any value from a discrete range

To obtain a Digital signal from Analog - use a **quantizer**

For an Analog signal's value at time t;

- the amplitude is partitioned into **L intervals**
- The **mid-point of the interval** in which the original value falls in, is the (quantized) value taken

Quantization is a **lossy** operation - cannot get back the original amplitude of the Analog signal

2.3 Periodic / Aperiodic

13 January 2017 22:13

<u>Definitions</u>

- A signal g(t) is said to periodic if for some constant T_0

$$g(t) = g(t + T_0) \text{ for all } t$$

- A signal is aperiodic if it is not periodic

If you have the value of g(t), a periodic signal, over a segment equal to its time period T_0
Then you can generate g(t) via **periodic extension**

2.4 Energy / Power

13 January 2017 22:19

Energy

Signal energy, E_g of g(t) is defined as (for g(t) real):

$$E_g = \int_{-\infty}^{\infty} g^2(t)dt.$$

If g(t) is complex, then

$$E_g = \int_{-\infty}^{\infty} g^*(t)g(t)dt = \int_{-\infty}^{\infty} |g(t)|^2 dt.$$

For g(t) to be an energy signal, $E_g < \infty$

So a necessary condition, is that the **signal amplitude tends to 0 as time tends to infinity**

Power

Signal power, P_g of g(t) is defined as (for both real and complex g(t)):

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

For g(t) to be a power signal, $0 < P_g < \infty$

If a signal was to have **infinite energy**, then it is **more suitable to measure it's power.**

A signal cannot be both an energy and power signal at the same time.

DC of a signal - **time average**

$$g_{average} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)dt.$$

Power of periodic signal g(t):

$$P_g = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |g(t)|^2 dt$$

Power of a sinusoid

Let

$$g(t) = A\cos(2\pi f_0 t + \theta)$$

Then

$$P_g = \frac{A^2}{2}$$

2.5 Deterministic / Probabilistic

16 January 2017 19:08

<u>Definitions</u>

- Deterministic - physical description of signal is known completely
- Probabilistic (random signal) - signal is only known in terms of probabilistic descriptions

3.0 Useful Signals

16 January 2017 19:32

Unit Impulse Function

Dirac Delta function - area = 1 - '0 width'

Use to pick out only one value of g(t):

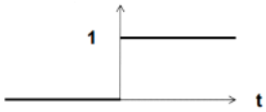
$$g(t)\delta(t-T) = g(T)\delta(t-T)$$

$$\int_{-\infty}^{\infty} g(t)\delta(t-T)dt = g(T)$$

So if you multiply g(t) by $\delta(t-T_0)$ and integrate

The result = $g(T_0)$

Unit Step Function



$$\frac{du(t)}{dt} = \delta(t).$$

Unit step differentiates to the unit impulse

Jump at t=0 gives infinite gradient, hence the impulse

Unit impulse integrates to unit step

Just at t=0, cumulative area = 1

Sinusoids

Important identities

$$e^{\pm jx} = \cos x \pm j\sin x, \cos x = \frac{1}{2}\left[e^{jx} + e^{-jx}\right], \sin x = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right],$$

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

$$a\cos x + b\sin x = C\cos(x+\theta)$$

with $C = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\frac{-b}{a}$

3.1 Inner Product of vectors and signals

19 January 2017 12:58

Notation

<**x**, **y**> = **x** · **y**

Inner Product

<**x** , **y**> = |**x**| |**y**| $\cos\theta$

Therefore

<**x** , **x**> = |**x**|^2

Or in other words, Size of **x** = $\sqrt{dot\ product\ of\ \boldsymbol{x}\ with\ itself}$

When <**x**, **y**> = 0, **x** and **y** are **orthogonal** (perpendicular)

For (energy) signals, [x(t) and y(t)], the inner product is defined as:

$$\langle x(t), y(t) \rangle = \int_{-\infty}^{\infty} x(t)y(t)dt$$

If complex signals, substitute the complex conjugate in the above equation for only one of the signals

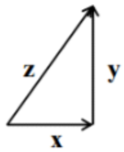
If

$$\langle x(t), y(t) \rangle = 0$$

Then x(t) and y(t) are orthogonal

3.2 Energy / Power of orthogonal signals

19 January 2017 13:23



if x and **y** are orthogonal, and
if z = **x** + **y**
Then you get
$|\mathbf{z}|^2 = |\mathbf{x}|^2 + |\mathbf{y}|^2$ (Pythagorean Theorem)

<u>Energy of orthogonal signals</u>

If **x(t) and y(t) are orthogonal**,
And **z(t) = x(t) + y(t)**,
Then

$$E_z = E_x + E_y$$

<u>Power of orthogonal signals</u>

Same concepts from energy signals apply to power signals

If **g(t) and y(t) are orthogonal**,
And **g(t) = x(t) + y(t)**,
Then

$$P_g = P_x + P_y$$

3.3 Signal correlation

19 January 2017 13:36

Correlation measure for **vectors x** and **y**

$$c_n = \cos\theta = \frac{\langle \mathbf{y}, \mathbf{x} \rangle}{|\mathbf{x}||\mathbf{y}|}$$

In the case of energy signals

$$c_n = \frac{1}{\sqrt{E_y E_x}} \int_{-\infty}^{\infty} y(t)x(t)dt$$

$$-1 \leq c_n \leq 1$$

C_n	Correlation
>0	Positive
0	No correlation
<0	Negative

C_n	Analogy	Cause	Description
1	Best Friends	g(t) = K*x(t) K is positive	Signals are aligned, maximum similarity
-1	Worst Enemies	g(t) = K*x(t) K is negative	Signals are aligned, but in opposite directions
0	Complete Strangers	g(t) and x(t) are orthogonal	Signals are unrelated (!= independent)

3.4 Signals represented as a set of other signals

19 January 2017 19:40

A vector can be represented as the sum of orthogonal vectors, and a **signal** can be **represented** as a **sum of orthogonal signals**.

Orthogonal vector space

Three dimensional vector **g** is described by (a set of) three mutually orthogonal vectors, $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$

$\mathbf{g} = a_1\mathbf{x}_1 + a_2\mathbf{x}_2 + a_3\mathbf{x}_3$

Where $\mathbf{x}_i$ has size that is defined (e.g. 1 unit)

$$a_i = \frac{<\mathbf{g},\mathbf{x}_i>}{|\mathbf{x}_i|^2}$$

Basically, a_i gives scale factor in **g** of $\mathbf{x}_i$

When this occurs, we say this set of vectors is complete-

-Meaning any **g** can be expressed in basis vectors, $\mathbf{x}_i$

$a_i = K * \mathbf{x}_i$

K is in **g**

Inner product of **g** and $\mathbf{x}_i$ gives all 0 except for direction in $\mathbf{x}_i$

And so dividing by $|\mathbf{x}_i|^2$ gives scale factor that a_i represents

Orthogonal signal space

Set of signals **x_i(t) from i=0 to i=N** is **complete** if it can **represent any signal** belonging to a certain space

For example,

$$g(t) \sim a_1x_1(t) + a_2x_2(t) + \ldots + a_Nx_N(t)$$

a_i represents how much $x_i(t)$ is in g(t)

Set of signals $x_i(t)$ is complete when the approximation error is zero for any g(t)

So when g(t) = g(t) represented by set $x_i(t)$

Set $x_i(t)$ is complete

Generally, the set is complete when $N \rightarrow \infty$

4.1 Trigonometric Fourier series

19 January 2017 20:29

Set of sinusoids is complete as all components are mutually orthogonal, so any signal can be represented as an infinite series of sine an cosines waves.

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$a_0 = \frac{1}{T_0}\int_{t_1}^{t_1+T_0} g(t)dt$$

$$a_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} g(t)\cos n\omega_0 t dt$$

$$b_n = \frac{2}{T_0}\int_{t_1}^{t_1+T_0} g(t)\sin n\omega_0 t dt$$

Using the identity

$$a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = C_n \cos(n\omega_0 t + \theta_n)$$

where

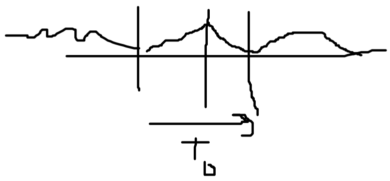
$$C_n = \sqrt{a_n^2 + b_n^2} \qquad \theta_n = \tan^{-1}(-b_n/a_n).$$

The trigonometric Fourier series can be expressed in compact form as

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n) \qquad t_1 \le t \le t_1 + T_0.$$

For consistency, we have denoted a_0 by C_0.

The Fourier series of g(t) over T_0 is periodic with period T_0



Properties:

- Trigonometric Fourier series is periodic, with period $T_0 = 2\pi/\omega_0$
- If g(t) is periodic with period T_0, then the Fourier series for g(t) over T_0 will work for g(t) over all g(t)

4.2 Amplitude and Phase spectra

19 January 2017 20:29

For a compact Fourier series, we can plot **C_n versus ω**, giving the **amplitude spectrum**

Also, we can plot **θ_n versus ω**, giving the **phase spectrum**

These two plots together are the **frequency spectra** of the signal

If a signal has rapid changes in time - very steep gradients, then expect F[signal] to contain high frequencies

4.3 Exponential Fourier Series

19 January 2017 20:29

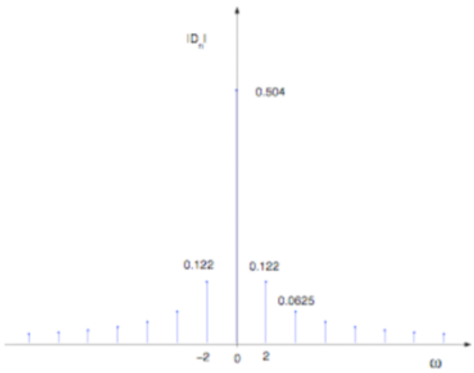
Exponential Fourier series:

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \qquad D_n = \frac{1}{T_0}\int_{T_0} g(t) e^{-jn\omega_0 t} dt$$

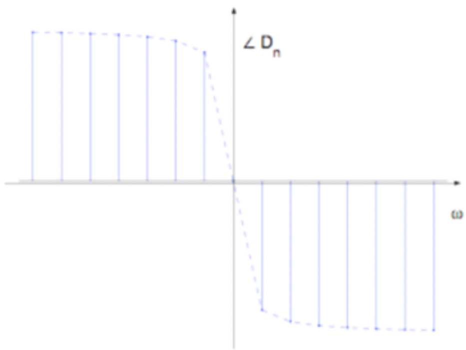
This is related to the compact Fourier series via

$$D_n = \frac{1}{2} C_n e^{j\theta_n} \qquad D_{-n} = \frac{1}{2} C_n e^{-j\theta_n}$$

<u>Frequency response for exponential series:</u>



Symmetrical around y-axis - even function



Odd function, mirrored in y-axis then x-axis

4.4 Parseval's Theorem

19 January 2017 20:29

Trigonometric Fourier series representation $g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$.

The power is given by

$$P_g = C_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2.$$

Exponential Fourier series representation $g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$.

Power for the exponential representation

$$P_g = \sum_{n=-\infty}^{\infty} |D_n|^2$$

D_n can be complex, so need to take | |

5.1 Fourier Transforms

25 January 2017 12:24

$$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}\,dt.$$

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega t}\,d\omega.$$

5.2 Useful functions and examples

25 January 2017 12:25

$$rect(t/\tau) \Leftrightarrow \tau \sin c(\omega\tau/2)$$

Tao represents amplitude scaling from 1

Pair >>

$$\frac{W}{\pi}\operatorname{sinc}(Wt) \qquad \operatorname{rect}\left(\frac{\omega}{2W}\right)$$

Transform pair for TRIANGLE function - sinc^2

$$\Delta\left(\frac{t}{\tau}\right) \qquad \frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$$

$$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right) \qquad \Delta\left(\frac{\omega}{2W}\right)$$

Transform of unit impulse

$$\delta(t) \Leftrightarrow 1$$

Inverse Transform

$$e^{j\omega_0 t} \Leftrightarrow 2\pi\delta(\omega-\omega_0)$$

$$e^{-j\omega_0 t} \Leftrightarrow 2\pi\delta(\omega+\omega_0)$$

Special case when $w_0 = 0$

Transform of cosine -> two unit impulse at +- w_0 in frequency domain

$$\cos(\omega_0 t) \Leftrightarrow \pi\left[\delta\left(\omega+\omega_0\right)+\delta\left(\omega-\omega_0\right)\right]$$

For sin(wt) > times by **j** - **change** middle sign

5.3 Standard Transform to learn (derivation)

10 April 2017 03:43

Transform of infinite unit impulses == infinite unit impulses

For a>0

1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$

Transform of unit step function u(t):

Term to cover DC after t=0 Term to cover the jump

$$\pi\delta(w) + \frac{1}{j\omega}$$

6.0 Basic Properties

10 April 2017 04:31

Add two Signals	>>	Add their separate transforms
Scale a Signal by A	>>	Scale Tran from by A

6.1 Symmetry Property

10 April 2017 03:58

Time	Frequency
$x(t)$	$Y(w)$
$Y(t)$	$2\pi x(-w)$

IF x transforms to Y

THEN

Y transforms to $2\pi x(-w)$

6.2 Scaling Property

10 April 2017 04:03

IF x transforms to Y

THEN

Time	Frequency
x(t)	Y(w)
x(at)	$\frac{1}{\lvert a\rvert} Y(\frac{w}{a})$

6.3 Time-Shifting Property

10 April 2017 04:06

Time	Frequency
$x(t)$	$Y(w)$
$x(t-A)$	$Y(w)e^{-jwA}$

Time Shift >>> Phase Shift

6.4 Frequency Shifting

10 April 2017 04:22

By Multiplication

- Exponential multiplication introduces frequency shift

$$g(t)e^{j\omega_0 t} \Leftrightarrow G(\omega-\omega_0) \qquad g(t)e^{-j\omega_0 t} \Leftrightarrow G(\omega+\omega_0)$$

- Cosine multiplication leads to

AS: $g(t)\cos\omega_0 t = \frac{1}{2}\left[g(t)e^{j\omega_0 t} + g(t)e^{-j\omega_0 t}\right]$,

$$g(t)\cos\omega_0 t \Leftrightarrow \frac{1}{2}\left[G(\omega-\omega_0)+G(\omega+\omega_0)\right]$$ Follows

6.5 Transform of Periodic Signal

10 April 2017 04:27

Transforms to an infinite train of unit impulses

Each impulse at harmonic multiple of Signal Frequency

>>

$$g(t) \Leftrightarrow 2\pi \sum_{n=-\infty}^{\infty} D_n \delta\left(\omega - n\omega_0\right)$$

6.6 Convolution

10 April 2017 04:33

convolution of two functions $g(t)$ and $w(t)$,

$$g(t) * w(t) = \int_{-\infty}^{\infty} g(\tau)w(t-\tau)d\tau$$

To use
- Find t intervals
  - Sliding
- Set t-0
  - Get ends
  - Find limits of Tao
- Use edge of x(tao)
- Edge of t box

- Convolution in time domain

= Multiplication in Frequency

$$g_1(t) * g_2(t) \Leftrightarrow G_1(\omega)G_2(\omega)$$

- Convolution in the frequency domain

= Convolution in Frequency

$$g_1(t)g_2(t) \Leftrightarrow \frac{1}{2\pi}G_1(\omega) * G_2(\omega)$$

6.7 Calculus in Time

10 April 2017 04:33

- The following relationship exists for integration

$$\int_{-\infty}^{t} g(\tau)d\tau \Leftrightarrow \frac{G(\omega)}{j\omega} + \pi G(0)\delta(\omega)$$

Term 1: $\frac{Transform}{j\omega}$

Term 2: π $* Tranform(0) * Unit\ Impulse$

- The following relationship exists differentiation

$$\frac{dg(t)}{dt} \Leftrightarrow j\omega G(\omega) \qquad \frac{d^n g(t)}{dt^n} \Leftrightarrow (j\omega)^n G(\omega)$$

Multiply Transform by (jw)^order of derivative

7.1 Linear Systems

31 March 2017 02:41

A system outputs y(t) for an input g(t)

LINEAR: input sum of signals

- Then output will be sum of single input outputs

$G_1(t) > y_1(t)$ and $G_2(t) > y_2(t)$

Then $\mathbf{g_1(t) + g_2(t) = y_1(t) + y_2(t)}$

TIME INVARIENT:

- If input is shifted in time
- So is output

So system's response is independent of time of input

-show

Let u = tao - T

7.2 Convolution

16 April 2017 15:30

Input into an LTI system - output **h(t) - unit impulse response**

Relate output signal to input via convolution with h(t)

$$y(t) = h(t) * g(t) = \int_{-\infty}^{\infty} \boldsymbol{h(\tau)g(t-\tau)d\tau}$$

Integral of product of two functions

- Where one is:
  - Revered
  - And shifted

At any time T: input is a Dirac delta function

So unit impulse response, scaled by amplitude of input,
Is added onto the output

Current output = SUM of all previous unit impulse responses
Hence integration
Hence $t - \tau$

7.3 System's Transfer Function

16 April 2017 16:01

If in time domain

$$y(t) = h(t) * g(t)$$

And

Time	Frequency
g(t)	G(w)
h(t)	H(w)

Then

$$Y(w) = H(w)G(w)$$

H(w) - transfer function

IDEAL h(t)

- **must = 0**
  - **t < 0**

It's a response to input of a unit impulse

Time	Frequency
Convolution	Product
Product	$\frac{1}{2\pi}$ x Convolution

Product of Two signals

- If product in time domain
- **Bandwidth = Sum of respective bandwidths**

8 ESD

16 April 2017 16:30

Energy/Power of modulated signal = Energy/Power of baseband signal / 2

ESD = $|G(w)|^2$

USE THIS ONE

ESD <-> Time auto correlation ($\int_{-\infty}^{\infty} g(t)g(t+\tau)dt$)

Energy = $\frac{1}{2\pi}\int_{-\infty}^{\infty} ESD d\omega$

For an LTI system

$ESD_{OUTPUT} = |H(w)|^2 * ESD_{INPUT}$

PSD(**S**) <-> Time auto correlation (**R**) ($\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g(t)g(t+\tau)dt$)

Power = $\frac{1}{2\pi}\int_{-\infty}^{\infty} PSD d\omega$

For an LTI system

$PSD_{OUTPUT} = |H(w)|^2 * PSD_{INPUT}$

9.1 Analog Modulation

31 March 2017 02:42

Baseband signal m(t) - modulating signal

Sinusoid - carrier / modulator: $A\cos(\omega_c t + \theta_c)$

Type	A	ω_c	θ_c
AM	m(t)	const	const
FM			

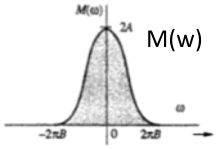
9.2 AM - DSB-SC

18 April 2017 01:25

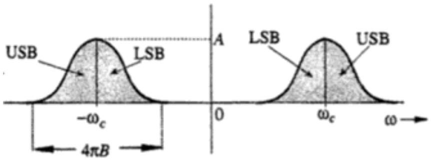
(LSB and USB) DSB-SC signal = $m(t)\cos\omega_c t$

Module - Multiply by carrier

$\Rightarrow$



M(w)



M(W)
half amplitude
centred at +-ω_c

$\boldsymbol{m(t)\cos\omega_c t}$ **<->** $\frac{1}{2}[\boldsymbol{M(\omega+\omega_c)+M(\omega-\omega_c)}]$

<u>Demodulate DSB-SC</u>

- **Multiply DSB by $\cos\omega_c t$**
- $m(t)\,cos^2\,\omega_c t = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t$
- So use Low pass filter ~ Baseband - filter to just get $\frac{1}{2}m(t)$
  - Carrier signal not recovered

9.3 Modulator Types

18 April 2017 01:37

Modulator	Filter	Output
Nonlinear	BPF(ω_c)	4b * DSB-SC
Switching	BPF(ω_c)	$\frac{2}{\pi}$ * DSB-SC
Ring	BPF(ω_c)	$\frac{4}{\pi}$ * DSB-SC

Nonlinear Modulator

- Non-linear systems $y(t) = ax(t) + bx^2(t)$
- input $\cos(\omega_c t) + m(t)\ and\ \cos(\omega_c t) - m(t)$ into systems
- Sum outputs of both systems = $2am(t) + 4bm(t)\cos(\omega_c t)$
- and then pass into Band Pass filer (around ω_c)
    - Output = 4b * $m(t)\cos(\omega_c t)$
        - = 4b * DSB-SC

Ring Modulator

- Diodes
- generate different square pulse train
- BPF at first harmonic - width B Hz bandwidth
    - Signal = $\frac{4}{\pi}$ * DSB-SC

Switching Modulator

- Square Pulse train
    - Fourier SERIES - DC + infinite cosine's at odd harmonics
- m(t) x Square pulse train
- In Frequency -> Convolution
- BPF at first harmonic - width B Hz bandwidth
    - Signal = $\frac{2}{\pi}$ * DSB-SC

10.1 Full AM (DSB)

18 April 2017 20:42

Motivation - need Carrier signal at receiver to be in sync with carrier at transmission
- include carrier in signal

DSB signal = $[A + m(t)]\cos \omega_c t$

$$[\boldsymbol{A} + \boldsymbol{m}(\boldsymbol{t})] \cos \omega_c t \quad <-> \quad \frac{1}{2}[M(\omega + \omega_c) + M(\omega - \omega_c)] + \boldsymbol{\pi A}[\delta(\omega + \omega_c) + \delta(\omega - \omega_c)]$$

Modulation:
- Add DC to m(t)
- Multiply Sum by Carrier

Choose DC level, A, so that
- $A \geq m(t)$ for all t
- #

Power efficiency

$$\eta = \frac{m(t)_{Power}}{A^2 + m(t)_{Power}} 100\%$$

at best, full AM is only 33% efficient

Envelop Detection

Envelope - line matching peaks of DSB signal
- Envelope = A + m(t)

let m_p be the MAXIMIN of $m(t)$

then, $A \geq -m_p$

Modulation index

$$\boldsymbol{\mu} = \frac{\boldsymbol{m_p}}{\boldsymbol{A}}$$

if $0 \leq \mu \leq 1$
then envelope detection possible

10.2 Full AM Modulation

18 April 2017 21:09

<u>Modulation:</u>

- $sum = m(t) + C \cos \omega_c t$

- Pass through a *single diode*
  - C >> m(t)
  - then diode controlled by carrier

- square pulse train generated
- this effectively multiplies itself with the sum

- pass through a BPF
  - $\frac{C}{2} \cos \omega_c t + \frac{2}{\pi} m(t) \cos \omega_c t$

<u>Demodulation</u>:

- pass through a single diode
  - rectifier - gets rid of negative voltage
  - Also multiplies DSB by square pulse train

- pass though LPF
  - block off out of baseband

- pass through capacitor
  - blocks DC

- Output = $\frac{1}{\pi} m(t)$

Alternate:

- After diode;
  - pass through Cap // Resistor

- Get output ~ A + m(t)

Condition

$$\frac{1}{\omega_c} \ll RC < \frac{1}{2\pi B}$$

11.1 SSB

19 April 2017 01:29

SSB	M_+	M_-
LSB	Shift left	Shift right
USB	Shift right	Shift Left

$$\boldsymbol{SSB = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t}$$

USB MINUS LSB PLUS

Frequency	Time	Time(alt)
$M_+(w)$	$m_+(t)$	$\frac{1}{2}(m(t) + jm_h(t))$
$M_-(w)$	$m_-(t)$	$\frac{1}{2}(m - jm_h(t))$

SSB
- Less bandwidth
  - Cheaper
  - Less chance of picking up noise

=> $m_+(t) + m_-(t) = m(t)$

<u>Hilbert Transform</u>

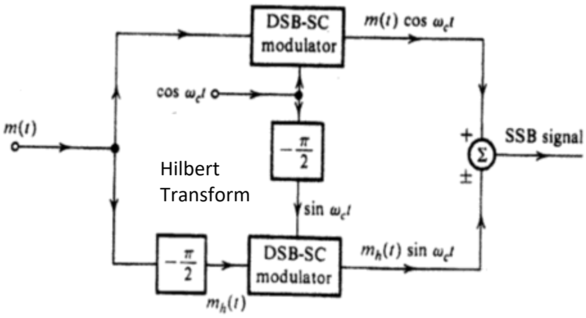
- **pass m(t)** through **filter h(t)**
- Transfer function, $\boldsymbol{H(\omega) = -jsgn(\omega)}$
- **output =** $\boldsymbol{m_h(t)}$

11.2 SSB Modulation

19 April 2017 02:14

Phase-Shift Method

- Phase Shift both m(t) and carrier
  - via Hilbert Transform
- Pass Original and Shifted through DSB-SC modulation
- Sum Outputs
  - Depending on $\mp$
    - Either get USB or LSB



Filter Method

- Just use BPF on DSB-SC
- PROBLEM
  - hard to design filter for baseband around ω_c

Demodulation - same as DSB-SC

- multiply with carrier
- **Produces**
  - **m(t) at baseband**
  - another SSB with carrier $2\omega_c$
  - at **HALF amplitude**

12.1 Angle Modulation

19 April 2017 02:25

Attribute	PM	FM
Linear with m(t)	$\theta(t)$	$\omega_i(t)$
Resulting Calculus	$\omega_i(t)$ Proportional to derivative of m(t)	$\theta(t)$ proportional to integral of m(t)

PHASE: $A\cos[\omega_c t + k_p m(t)]$

FREQUENCY: $A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha]$

Power

- Power is always $\frac{A^2}{2}$

13.1 Angle Modulation Bandwidth

19 April 2017 02:27

Condition: Coefficient of **sin MUST BE << 1**

Narrow-band FM: $A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t\right]$

a(t) is the integral of m(t)

Narrow-band PM: $A\left[\cos\omega_c t - k_p m(t)\sin\omega_c t\right]$

Narrow-Band and AM:

- Can be generated via DSB-SC modulators
- **Requires 2B Hz bandwidth**

If condition not satisfied

Wide-Band FM

- Frequency Deviation
  - $\boldsymbol{\Delta f = \frac{k_f m_p}{2\pi}}$
  - Deviation Ratio
    - $\beta = \frac{\Delta f}{B}$
- **Carson's Rule**
  - $B_{FM} = 2(\Delta f + B)$

Wide-Band PM

Only difference

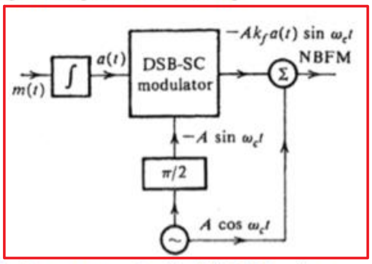
- $\dot{m}(t)$ used instead of $m(t)$

- $\Delta f = \frac{k_f \dot{m}_p}{2\pi}$
- $B_{PM} = 2(\Delta f + B)$

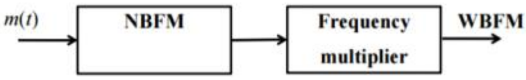
14.1 FM Modulation

19 April 2017 02:27

- Narrowband signal is generated using

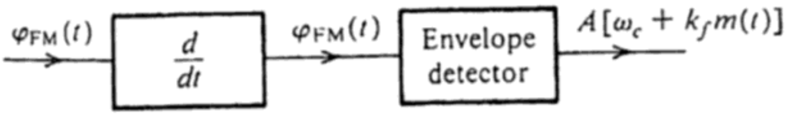


- NBFM signal is then converted to WBFM using



Multiplier
Converter
- subtracts w_c from carrier
- trig

<u>Demodulation</u>

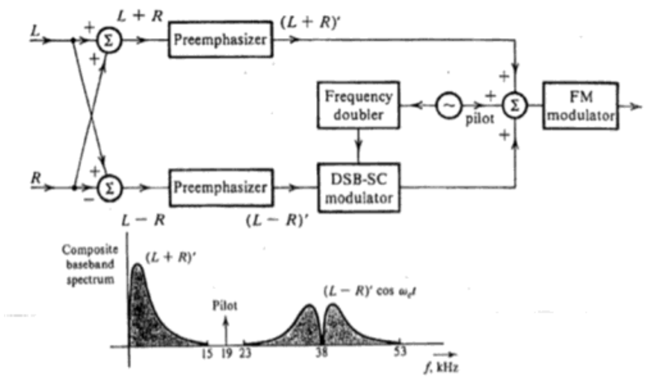


Detect **FM like you detect AM**

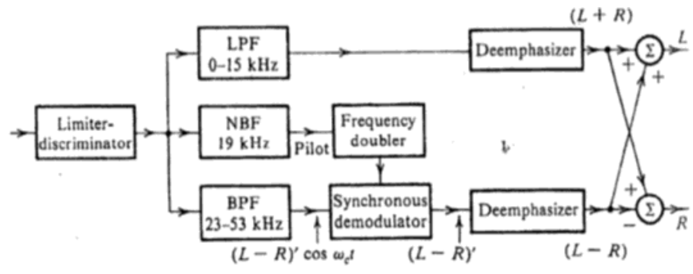
15.1 Channels

19 April 2017 02:28

Transmission



Receiving



16.1 Sampling Frequency

31 March 2017 02:42

$f_s > 2B$

16.2 Quantizing Interval

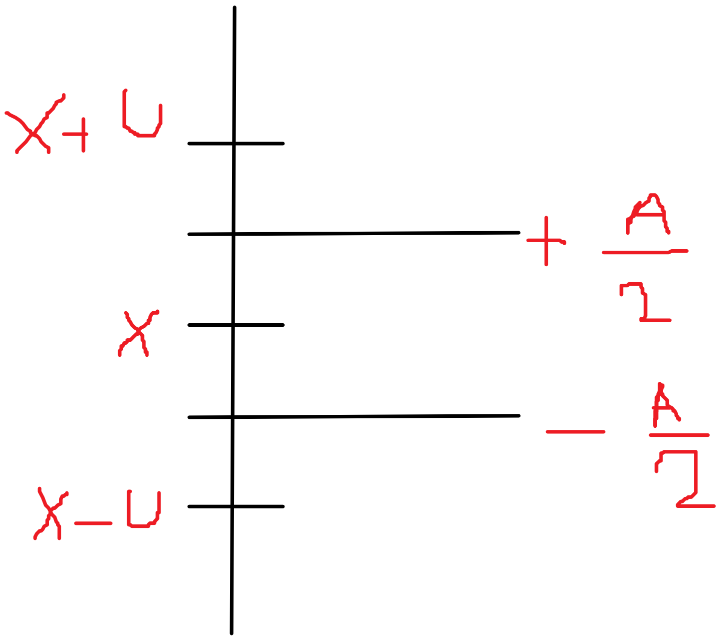
22 April 2017 22:00

Unit = U

Step size = A = $\frac{2 * Amplitude}{Number\ of\ Intervals}$

What is the interval around X?

- $X - \frac{A}{2} < X < X + \frac{A}{2}$



16.3 Transmission Coding

22 April 2017 22:29

Return to zero - between sending bit - signal settle to zero
Non-return - no settling back to zero

17.1 Digital Modulation

22 April 2017 22:39

ASK - Amplitude Shift Keying

PSK - Phase Shift Keying

FSK - Frequency Shift Keying

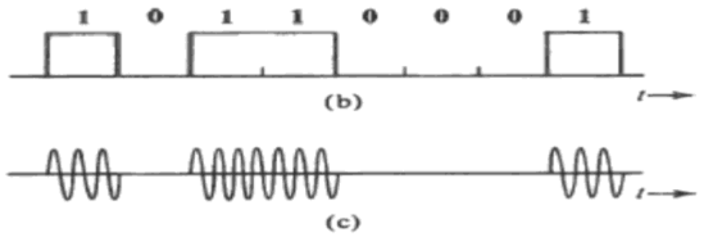
17.2 ASK

22 April 2017 22:42

Modulation

ON-OFF non-return to zero

Bit	Amplitude of Carrier
1	**A**
0	**0**



Demodulation

- Coherent Detection
  - BPF - get rid of noise
  - Multiply by carrier
  - LPF - get original **A(t)**
  - Requires regeneration of carrier

- Non-coherent
  - Envelope Detection

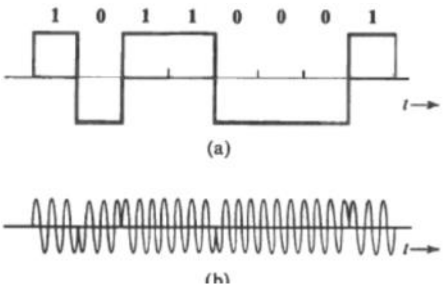
17.3 PSK

22 April 2017 22:55

<u>Modulation</u>

POLAR non-return to zero

Upon a change in bit
Phase is shifted by -90



m(t): Polar non-return-to-zero

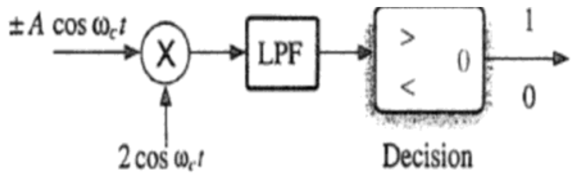
$$\varphi_{PSK} = m(t)\cos(\omega_c t)$$

<u>Detection</u>

- Cannot envelope Detect PSK

Coherent:

- Like AM



17.4 FSK

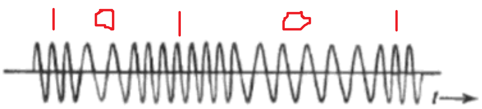
22 April 2017 22:51

Modulation

POLAR non-return to zero

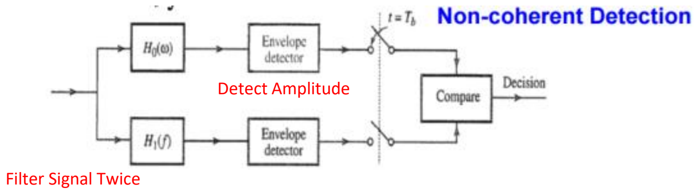
Same modulation technique as FM - $FSK = \cos[w_c t + k_f \int m(t)dt]$

Bit	Frequency of Carrier - **compared to Carrier Frequency**
1	Higher
0	Lower



Non-Coherent Detection

Bit	Frequency	Filter 1	Filter 2
1	High	0	Pass Through
0	Low	Pass Through	0

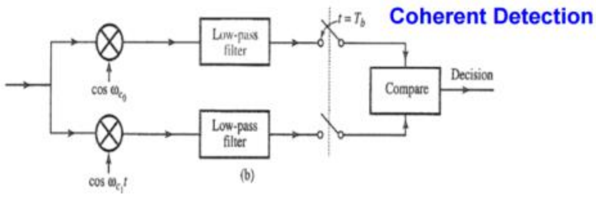


t = T_B When time to detect bit - samples

Coherent Detection

Multiply Signal by 'Carrier' at both HIGHER and LOWER frequency

Low Pass Filter - only get m(t) at baseband when frequency of Input and 'Carrier' MATCHES - due to Trig Identity



18.1 Channels

22 April 2017 23:45

CHANNEL Bandwidth - range of SIGNAL Bandwidth allowed

- Without significant:
  - Loss of energy
  - Distortion
  - Bps / 2 = B

Baud Rate - 2 * Channel Bandwidth

Channel **CAPACITY** =

MAXIMUM Data Rate - in bps

Over a communication channel

$$C = 2B\log_2 M$$

Where M = number of levels
- e.g. 2 for Binary

B = Channel Bandwidth