Roots

26 April 2017 14:03

For a cubic:

- Differentiate to find turning points
    - Can then explore properties

Trig Identities to Know

26 April 2017 14:25

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \sin\beta\cos\alpha \\
\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \sin\beta\cos\alpha \\
\cos(\alpha+\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\
\cos(\alpha-\beta) &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \\
\tan(\alpha+\beta) &= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \\
\tan(\alpha-\beta) &= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}.
\end{aligned}$$

Complex

16 October 2016 18:10

- Multiply by i - rotation by 90 degrees anti-clockwise
- $e^{ix} = \cos x + \text{i}\sin x$
- $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

Locus

- | | = constant - CIRCLE
- | | = | | - Perpendicular Bisector
- Arg(z) = constant - HALF LINE

Hyperbolic Trig

- $\cos(ix) = cosh(x) = \frac{1}{2}(e^x + e^{-x})$
- $\sin(ix) = isinh(x) = i\frac{1}{2}(e^x - e^{-x})$
- $sinh^{-1}(x) = \ln\left(x + \sqrt{1 + x^2}\right)$
- $cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$

Functions

26 April 2017 16:01

EVEN: $f(-x) = f(x)$

ODD: $f(-x) = -f(x)$

$$let\ Y(x) = \frac{1}{2}[f(x) \pm f(-x)]$$

For +: Y(-x) = Y(x) -> EVEN part of F

For -: Y(-x) = -Y(x) -> ODD part of F

HEAVISIDE Function

- Unit Step
-

X	H(x)
<0	0
>=0	1

MULTIPLY FUNCIONS

F	G	Type
Odd	Odd	EVEN
Even	Even	EVEN
O/E	E/O	**ODD**

EXTENSIONS
- Periodic
- Even
- Odd

INVERSE
- Only take inverse of Normal RESTRICTED
- e.g. cos^-1(cos(-pi/2)) - NOT VALID
  - As -pi/2 out of restriction

Limits

27 April 2017 14:45

- Can Factor out a Scalar

- Limits are Linear

- **$LIM(F.G) = Lim_L * Lim_G$**

- Limit of F(G) = Lim of F
        - as x-> lim of G

$$
\begin{aligned}
a^2 - b^2 &= (a-b)(a+b) \\
a^3 - b^3 &= (a-b)(a^b + ab + b^2) \\
a^4 - b^4 &= (a-b)(a^3 + a^2b + ab^2 + b^3) \\
\text{etc}
\end{aligned}
$$

Approach:

- If some smooth Fn: input value
    - Get value
        - Sensible - limit found
        - Otherwise doesn't exist
    - **Except**
        - If one of the 4 special cases
        - Explore
            - Factorisation
            - Divide by highest power of x
            - Use Binomial / Maclaurin Series
            - Sandwich Theorem
                - Use of LIM of sinc
            - Multiply by Conjugate

Sandwich Theorem:

Interval Containing A

LIM of H

as x->a

- IF
    - LIM of F = L
    - LIM of G = L
- AND
    - F < H < G
- THEN
- LIM of H = L

Differentiation

27 April 2017 14:45

Parametric

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Stationary Points:

- $f^{(k)}(x_0) = 0$ for k = 0,1,2..n-1 but $f^{(n)}(x_0) \neq 0$
  - n is odd - inflection
  - n is even
    - < 0 = max
    - > 0 = min

L'Hopital's Rule:

- Case "$\frac{0}{0}$"
  - $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$
- Case "$\frac{\infty}{\infty}$"
  - $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)} = \lim\limits_{x \to \infty} \frac{f'(x)}{g'(x)}$
  - $x \to \infty$

Integration

27 April 2017 14:45

Riemann Sum:

- $$\int_a^b f(x)dx = \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\sum_{i=0}^{n-1} f(x_i)$$

Trig Substitutions:

- 1-x^2 = x=sinu
- **x^2 +- 1** = x=**sinhu**

- acosx + bsinx + c **= tan(x/2)**

Volumes of Revolution:

- x-axis
  - $\pi \int_a^b f^2(x)dx$
- y-axis
  - $\pi \int_a^b g^2(y)dy$

Arc Length:

- $\int_{t=a}^{b} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$

- $\int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

Surface Area of Revolution:

- $2\pi \int_a^b f(x)\sqrt{1+[f'(x)]^2}dx$

Integral Recurrence Relation:

STEPS

- let I_n = $\int_a^b [f(x)]^n dx$

- **By Parts**

  $\int_a^b [\boldsymbol{f(x)}]^{\boldsymbol{n-1}} [f(x)]dx$

Series

08 January 2017 19:07

Integral Test:

$a_n = f(x)$

- $\int_{1}^{\infty} f(x)dx$
- If integral exists
  - series converges
- Otherwise,
  - series diverges

Comparison Test:

$a_n < b_n$ for all n
then if b_n series converges
a_n series converges

Alternating Series Test:

a_n -> 0
and terms get smaller
then
$(-1)^n a_n$ converges

Absolutely Converges:

if $|a_n|$ converges
then
a_n converges

Ratio Test:

- $k = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

k	Conclusion
<1	Converges absolutely
>1	Diverges
0	Can't tell

Radius of Convergence

$|x| < 1 / L$

Fourier Series

08 January 2017 19:07

USE 1/2 a_0 in series

<u>Conditions:</u>

- bounded
- finite number of extrema and discontinuity
- periodic

To Differentiate:

- continuous
- smooth

Fourier Transforms

08 January 2017 19:06

Transform a Derivative -> **multiply Transform on f(t) by (iw)^n**

Dirac - Isolate a value at that point in time

Vectors

01 May 2017 20:15

$\boldsymbol{x} \cdot \boldsymbol{y} = |x||y| \cos\theta$

Unit Vector

$\widehat{\boldsymbol{x}} = \frac{1}{|\boldsymbol{x}|}\boldsymbol{x}$

Vector Product Property - Area
$|\boldsymbol{a}\ x\ \boldsymbol{b}| = absin\theta$

a x a = 0

a x b - perpendicular to BOTH a and b

Point to a plane:

- Line
  - passing through P
  - perpendicular
- find intersection of L and P

Gives:

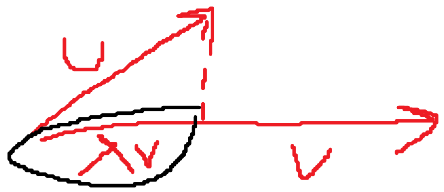
- point
- distance

Projection - $\lambda\boldsymbol{v}$

$\lambda = \frac{\boldsymbol{u} \cdot \boldsymbol{v}}{\boldsymbol{v} \cdot \boldsymbol{v}}$

v - DIRECTION

u - vector being projected onto v



SHORTEST Distance between lines

- Cross Product lines
- new line is a multiple of this
- Find $P_1 > P_2$ = The above multiple
- Solve

Equation of a plane R

- $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{n} \cdot \boldsymbol{x_0}$
- r - Plane
- n - any normal (can find via **vector product** of **vectors on the plane**)
- x_0 - any point on the plane

Matrices

01 May 2017 20:55

$A = A^T$	Symmetric
$A = -A^T$	antisymmetric
$A^T = A^{-1}$	**Orthogonal**

Comparison	Det
A^T	Same
Swap 2 R/C	Change Sign
Same 2 R/C	= 0
Multiple R/C by C	Multiply by C
Row Ops	No Change
det(AB)	det(A)det(B)
R/C Dep.	=0 (n x n)
R/C Indep.	!=0 (n x n)

Eigen:

- Values - $\det(A-\lambda I) = 0$
- Vectors - sub in values
  - $(A-\lambda I)\mathbf{x} = \mathbf{0}$

Finding A^{-1}

- Gauss-Jordan Elim.
- (A : I)
  - Into $(I : A^{-1})$

Diagonalization: $A = PDP^{-1}$

$D = P^{-1}AP$

- A symmetric - orthogonal diagonalization
- P - contains NORMALised eigenvectors
- D - Diagonal Matrix with eigen values

Reduced Echelon:

- ALL Pivots = 1
- Identity Matrix

Rank:

- m x n
- # of linearly independent R/C's
- Use Gaussian Elimination to find Rank
  - Row Ops
  - Echelon Form - Pivots

Cases:

- Right most column contains a pivot
  - No solutions
- Otherwise
  - Rank = N - Unique Solution
  - Rank < N - Infinite Solutions

1st ODE

01 May 2017 21:41

Homogenous:

- Terms of the form $\frac{y}{x}$
- Solve with the substitution $\frac{y}{x} = v$

Linear: - y' + P(x)y = Q(x)

- Use Integrating Factor
  - $\boldsymbol{\mu(x)} = e^{\int P(x)dx}$
- $y\mu(x) = \int Q(x)\mu(x)$

Bernoulli's: - y' + P(x)y = Q(x)y^n

- Multiply Eqn by y^{-n}
- Use Sub. - $v = y^{1-n}$
- Get Linear FO ODE

$$f(x,y) = \frac{\boldsymbol{ax + by + c}}{\boldsymbol{ex + fy + g}}$$

Case 1: c = g = 0

- Convert to Homogenous

Case 2: af = be

- Manipulate with $\frac{b}{f}\left[\frac{f}{b}.f(x,y)\right]$
- Get $F[\alpha x + \beta y]$
- < ----- Solve

$$f(x,y) = F(ax + by), \qquad v = ax + by$$

$$\frac{\boldsymbol{dv}}{\boldsymbol{dx}} = \boldsymbol{a + bF(v)}$$

Case 3: af != be

- $x = X + \alpha$
- $y = Y + \beta$
  - Turn $\frac{dY}{dX}$ into Case 1

2nd ODE

01 May 2017 21:41

Homogenous: = 0

- Auxiliary Equation

k_1 and k_2 real and unequal	$k_1 = k_2$ equal roots	$K_1 = p+qi$ $K_2 = p-qi$ (non real)
$y = Ae^{k_1x} + Be^{k_2x}$	$y = e^{k_1x}(A + Bx)$	$y = e^{px}(Acosqx + Bsinqx)$

Choosing a Particular Integral

$f(x) = ce^{\lambda x}$	y_c - contains $ae^{\lambda x}$ but **not** $axe^{\lambda x}$	$y = axe^{\lambda x}$
	y_c - contains $axe^{\lambda x}$	$y = ax^2e^{\lambda x}$
	y_c - does not contain $ae^{\lambda x}$ or $axe^{\lambda x}$	$y = ae^{\lambda x}$
$f(x) = ccos\lambda x$ or $f(x) = csin\lambda x$	y_c - $Acos\lambda x + Bsin\lambda x$	$y = axsin\lambda x$ if $f(x) = ccos\lambda x$ $y = axcos\lambda x$ if $f(x) = csin\lambda x$
	y_c – does **not** contain $Acos\lambda x + Bsin\lambda x$	$y = acos\lambda x + bsin\lambda x$
$f(x)$ polynomial f degree n		$y = ax^n + bx^{n-1} + \cdots$

More on ODE

02 May 2017 03:31

Linear ODE with Varying Coefficients

e^t - Euler's Equation: $x^2y'' + Axy' + By = f(x)$

- Sub. $x = e^t$

Legendre Equation: $(\alpha x + \beta)^2 y'' + (\alpha x + \beta)y' + a_0 y = f(x)$

- $\alpha x + \beta = e^t$

Series Solution:

- Based on Maclaurin Series
- Use Leibnitz' theorem to find $y^n(0)$ constants
  - Via a recurrence relation
- find derivatives of y at zero through recurrence
  - use these values for Maclaurin series

Coupled 1st Order:

- $\dot{\underline{x}} = A\underline{x}$
  - $\underline{x} = \underline{a}_1 c_1 e^{\lambda_1 t} + \underline{a}_2 c_2 e^{\lambda_2 t}$
  - $\underline{a}_i$: eigen vectors of A
  - c_i : find through initial conditions
  - λ_i : eigen values of A

Multivariable Calculus

01 May 2017 21:41

Contour Curves - closer lines = steeper

Total Differential:

- $\boldsymbol{df} = \frac{\partial f}{\partial x}\boldsymbol{dx} + \frac{\partial f}{\partial y}\boldsymbol{dy}$

Radially Symmetric

- df/dthi = 0

Relative Error:

- Get total differential
  - Divide this by function
- Triangle inequality with Magnitudes

Chain Rule:

- Go through variables
  - **Divide total derivative by dx_i**
- When dealing with non-trivial
  - f(x,y), x(u,v), y(u,v)
  - All derivatives become **PARTIAL**

PDE's

02 May 2017 04:36

<u>Wave Equation:</u>

- $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

- Change on Variables
  - x +- ct
- Chain Rule

- **d'Alembert's Solution**
  - $y(x,t) = f(x+ct) + g(x-ct)$

Equation	How to Solve
Wave	Change Variables + Chain Rule
Laplace	Chain
Heat	Direct Evaluation

<u>Laplace's Equation:</u>

- $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

- $f(x,y) = \mathbf{\tan^{-1}}(\boldsymbol{\frac{y}{x}})$
  - Solution

More MV

02 May 2017 05:24

Implicitly Defined F

- $F(x, y, z) = 0$
  - $\frac{\partial A}{\partial B} = -\frac{F_B}{F_A}$

Change of Coordinates:

chain rule

Stationary Points:

- find F_x and F_y

  set to =0

  Solve - coordinates
- hessian Det
  - D=0 - more info needed
  - Stationary Point
    - D > 0
    - Min
      - $f_{xx} > 0$ and $f_{yy} > 0$
    - Max
      - $f_{xx} < 0$ and $f_{yy} < 0$
  - Saddle
    - D < 0

Exact ODE:

- $P + \frac{dy}{dx} Q = 0$

IF

- $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

THEN

- $f = \int P \, dx + \boldsymbol{g(y)}$
- $f = \int Q \, dy + \boldsymbol{h(x)}$

Equate

df = 0

f = Constant

Inexact ODE:

- $\frac{\partial P}{\partial y} \; ! = \frac{\partial Q}{\partial x}$
- $\frac{\partial}{\partial y}(\lambda P) = \frac{\partial}{\partial x}(\lambda Q)$
- λ either λ(x) or λ(y)
  - Solve above equation to obtain λ
- Multiply original inexact ODE with λ

Lagrange Multipliers:

- f(x,y) find stationary point
  - subject to g(x,y) =0

- Create
  - $\Phi(x,y,\lambda) = f(x,y) - \lambda g(x,y)$

- $\Delta\Phi = \begin{pmatrix} \Phi_x \\ \Phi_y \\ \Phi_\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$