Basic Signals and Signal Properties

14 October 2017 14:51

Refer to Signals and Communications Year 1!

A Pulse Signal:

- Generated using two step functions
  - One with negative amplitude and time delayed

Odd part:

- $\frac{1}{2}[x(t) - x(-t)]$

Even part:

- $\frac{1}{2}[x(t) + x(-t)]$

Exponential Function - e^{st}

- $s = \sigma + j\omega$
- $e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t}e^{j\omega t} = e^{\sigma t}(\cos \omega t + \sin\omega t)$
- Can use to model:
  - A constant
  - Regular exponential
  - Any sinusoid
  - Exponentially varying sinusoid

Deterministic Signals - value of signal at time t, can be obtained from mathematical expressions

Stochastic Signals - random values, only know probability of a value being the output

Can calculate

- Mean value - simplest calculation
- Variance in Value
  - $\sigma_x{}^2 = E\{x^2(t)\} - [E\{x(t)\}]^2$

Continuous - Laplace Transforms - FT is a subset

Discrete - Z-Transforms - DFT is a subset

Discrete-Time Exponential - $e^{\lambda n}$

- λ is complex
- map λ to γ -> $\gamma = e^{\lambda}$

- Re{λ} < 0 - exponential decay
- Re{λ} > 0 - exponential growth
- Re{λ} = 0 - constant amplitude - oscillates based on Im{λ}

Systems Basics

21 October 2017 00:18

Show that a system is linear:
- show that output scales the same as input is scaled
- show that outputs can be summed if inputs are summed

Small-signal analysis - approximate into small linear systems

<u>Superposition:</u>
- zero-input component
  - t = 0
  - initial conditions
- zero-state component
  - t > 0
  - input of x(t)

total response = zero-input response + zero-state response
(Decomposition property)

Dynamic System
- output depends on entire past inputs
- stores history - memory

Finite-memory System
- memory lasts on past T units of time

Instantaneous System
- no memory
- just depends on current input

Casual, $x(t) = 0, t < 0$
Non-casual, above rule doesn't apply
Anti-casual, $x(t) = 0, t \geq 0$

Casual System
- depends on
  - x(t)
  - $t <= t_0$
- practical systems must be casual

Non-Casual
- can still be realised
  - when independent variable is not time
- post-process - already have data values

Time - Continuous vs Discrete

Amplitude - Analogue vs Digital

Invertible System
- **one-to-one** mapping of input to output exist

Stable System
- bounded input gives bounded output - BIBO

Linear Differential System
- D operator
  - $D^N y \ldots = D^M x \ldots$
- M <= N
  - else
  - system is a (M-N)th differentiator
    - unstable - unbounded output

Linear
test with $ax_1 + bx_2$
Casual
must have x(t)
no future values
Time-varying
do test with $x(t-t_o)$

Essential bandwidth:

- $|H(\omega_0)|^2 = \left[\frac{1}{\sqrt{2}}|H(0)|\right]^2$

Total response

21 October 2017 02:13

zero-input

- set up DE from circuit / system
- auxiliary equation for DE
- find y(t) in terms of auxiliary terms
  - must be real

zero-state

- compute x(t) from DE
  - sub in y(t)
  - gives input required to sustain system state

Input = characteristic mode (auxiliary / natural frequencies)

- causes resonance type behaviour
- system has no obstacle to this input

<u>For an LTI system,</u>

$$input\ of\ e^{j\omega_0 t}\ and\ TF\ H(s)$$
$$output\ is\ H(j\omega_0)e^{j\omega_0 t}$$

Convolution

31 October 2017 20:33

Unit Impulse Response:

- For a system, $Q(D)y(t) = P(D)x(t)$
- the $\delta(t)$ response,
- $h(t) = [P(D)y_n(t)]u(t)$
  - $Q(D)y_n(t) = 0$
  - **Initial Conditions:**
    - $y_n^{(N-1)}(0) = \mathbf{1}$
    - **rest >>> = 0**

LTI System Output

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} \boldsymbol{x(\tau) * h(t-\tau)d\tau}$$

Convolution Properties:

- $x_1(t) * x_2(t) = x_2(t) * x_1(t)$
- $x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$
- $x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$
- $x_1(t-T_1) * x_2(t-T_2) = c(t-T_1-T_2)$
- $x_1(t) * \delta(t) = x_1(t)$
- two signals have duration T_1 and T_2 ,
  - then duration of convolution of these two signals = $T_1 + T_2$

Properties of convolution

No	$x_1(t)$	$x_2(t)$	$x_1(t) * x_2(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t),\ \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$

Convolution in Systems

04 November 2017 20:46

Natural vs Forced Responses

- characteristic nodes can also appear in zero-state response
- after grouping
  - **natural response**
    - sum of characteristics nodes
  - **forced response**
    - remaining nodes

Two LTI systems connected:

In Parallel:
- $y(t) = h_1(t) * x_1(t) + h_2(t) * x_2(t)$

In Series:
- $y(t) = h_1(t) * h_2(t) * x(t)$
  - order is not important

Integrator:
- Can be before or after LTI in series
  - same output

Laplace Transformations

07 November 2017 21:47

Frequency Domain

Components of the form e^{st}

$$s = \alpha + j\omega$$

Summary of most important Laplace transform pairs

No	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$

The Linear Laplace Transform

(casual signals)

$$X(s) = \mathcal{L}\{x(t)\} = \int_0^\infty x(t)e^{-st}dt$$

Region Of Convergence

Values of s for which $\mathcal{L}\{x(t)\}$ are **valid**

Finding Inverse Laplace Transforms

1. **Manipulate X(s)**
2. find **matches** to known Laplace Transforms

Properties:

1. Time Shifting
$$\mathcal{L}\{x(t-t_0)\} = e^{-st_0}X(s)$$
2. Frequency Shifting
$$\mathcal{L}\{e^{s_0 t}x(t)\} = X(s-s_0)$$
3. Time-Differentiation
$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0-)$$
4. Frequency-Differentiation
$$\mathcal{L}\{tx(t)\} = -\frac{dX(s)}{ds}$$
5. Time-Integration
$$\mathcal{L}\left\{\int_{0-}^{\infty} x(\tau)d\tau\right\} = \frac{X(s)}{s}$$
6. Frequency-Integration
$$\mathcal{L}\left\{\frac{x(t)}{t}\right\} = \int_s^\infty X(z)dz$$
7. Scaling
$$\mathcal{L}\{x(at)\} = \frac{1}{a}X\left(\frac{s}{a}\right)$$

Time-Convolution

$$\mathcal{L}\{x_1(t) * x_2(t)\} = X_1(s)X_2(s)$$

Frequency Convolution

$$\mathcal{L}\{x_1(t)x_2(t)\} = \frac{1}{2\pi j}X_1(s) * X_2(s)$$

Impulse Response h(t)

$$\mathcal{L}\{h(t)\} = H(s)$$

then

$$Y(s) = H(s)X(s)$$

Initial Value Theorem

$$y(0+) = \lim_{s\to\infty} sY(s)$$

Final Value Theorem

$$y(\infty) = \lim_{s\to 0} sY(s)$$

Repeated Time-Differentiation

$$\mathcal{L}\left\{\frac{d^n x(t)}{dt^n}\right\} = s^n X(s) - s^{n-1}x(0^-) - s^{n-2}\dot{x}(0^-) - \cdots - x^{(n-1)}(0^-)$$

Laplace for DE

19 November 2017 19:24

LTI System DE

1. Convert terms into Freq using Laplace Transform (Time-Differentiation property)
2. get into from Y(s) = ..
    a. terms transferred from RHS to LHS are zero-input
    b. LHS come from zero-state
3. convert back to y(t) using matching method for inverse Laplace

Initial Conditions for Capacitor

$$V(s) = \frac{1}{Cs} I(s) + \frac{v(0^-)}{s}.$$

Initial Conditions for Inductor

$$V(s) = L(sI(s) - i(0^-)) = LsI(s) - Li(0^-)$$

Fourier Transform

19 November 2017 19:25

$$X(\omega) = \mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

Duality

If $x(t) \Leftrightarrow X(\omega)$ then $X(t) \Leftrightarrow 2\pi x(-\omega)$

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\,du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

System Stability

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$$X(j\omega) = X(s)|_{s=j\omega}$$

Fourier -- Laplace

$$X(\omega) = X(j\omega)?$$

if $x(t)$ is absolutely integrable

Area Under is FINITE

$$\int_{-\infty}^{\infty} |x(t)|dt < \infty$$

- BIBO stability exists when

$$\int_{\tau=-\infty}^{\infty} |h(\tau)|d\tau < \infty$$

- Recall that very often $h(t)$ is a linear combination of causal exponential functions of the form $x(t) = e^{at}u(t)$.
- For stability we require that $\text{Re}\{a\} < 0$.
- These function contribute with the term $\frac{1}{s-a}$ to the transfer function in the Laplace domain. The constant a which zeroes the denominator of $\frac{1}{s-a}$ or, in other words, makes the term $\frac{1}{s-a}$ infinite is called a **pole** of the transfer function.
- Therefore, in order to achieve stability, the poles of the transfer function of a causal system must lie on the left half of the s −plane.

Frequency Response Plots

23 November 2017 23:18

To find frequency response:

- System has transfer function H(s)
- sub: s = jw , to get $H(jw) = \frac{j\omega+\alpha}{j\omega+\beta}$
- Amplitude Response = |H(jw)|
- Phase Response = $\angle H(jw) = \tan^{-1}\left(\frac{\omega}{\alpha}\right) - \tan^{-1}\left(\frac{\omega}{\beta}\right)$

$$H(s) = \frac{K(s+a_1)(s+a_2)}{s(s+b_1)(s^2+b_2 s+b_3)}$$

Poles - roots of the denominator polynomial

Zeros - roots of the numerator polynomial

Using $s = jw$
And rearrange

Express in decibel - log components are SUMMED

Amplitude:

- find new horizontal
- get asymptotes separately
- sum asymptotes
- apply corrections
  - a CF 2x away - 1dB
  - a CF 5x away - 0.17 dB
  - a CF at, 3dB

Pole at the origin

Amplitude:

- $-20\log w$
  - -20dB / decade
  - crosses axis at $\omega = 1$

Phase:

- $-j\omega \rightarrow -90$

Zero at the origin

Amplitude:

- $20\log w$
  - 20dB / decade
  - crosses axis at $\omega = 1$

Phase:

- $j\omega \rightarrow +90$

1st order pole at -a

Amplitude:

- $w \ll a$, 0
- $w \gg a$, $-20log\omega + 20loga$
  - -20dB / decade
  - crosses axis at $\omega = a$
    - is also the corner frequency
    - max error is -3dB at CF

Phase:

- $-\tan^{-1}\left(\frac{\omega}{a}\right)$
- $\omega \le a/10$, 0
- $\omega \ge 10a$, -90
- slope of $-45°$ *per decade connects*
  - crosses w-axis at $\omega = a/10$

1st order zero at -a

Amplitude:

- $w \ll a$, 0
- $w \gg a$, $20log\omega + 20loga$
  - 20dB / decade
  - crosses axis at $\omega = a$
    - is also the corner frequency
    - max error is +3dB at CF

Phase:

- $\tan^{-1}\left(\frac{\omega}{a}\right)$
- $\omega \le a/10$, 0
- $\omega \ge 10a$, $+90$
- slope of $+45°$ *per decade connects*
  - crosses w-axis at $\omega = a/10$

2nd order pole

Amplitude: (zero is reflection in x-axis)

- *express as* $s^2 + 2\zeta\omega_n s + \omega_n^2$
- $w \ll \omega_n$, 0
- $w \gg \omega_n$, $-40log\omega - 40loga$
  - -40dB / decade
  - crosses axis at $\omega = \omega_n$
    - is also the corner frequency
  - max error depends on ζ (< 1)

Phase:

-
  - actual depends on ζ
- mirror image for a zero

Zeros-Poles-Filters

23 November 2017 23:18

$$|H(s)|_{s=p} = b_0 \frac{product\ of\ the\ distances\ of\ zeros\ to\ p}{product\ of\ the\ distances\ of\ poles\ to\ p}$$

$$\angle H(s)_{s=p} = sum\ of\ zeros'angles\ to\ p\ - sum\ of\ poles'angles\ to\ p$$
$$add\ \pi to\ phase\ if\ b_0\ is\ negative$$

$POLES\ - \max gain\ at\ \omega_0$

$ZEROS\ - \min gain\ at\ \omega_0$

moving -a closer to Im axis increases the enhancement/suppression

Single pole: $|H(j\omega)| = \frac{K}{d}$

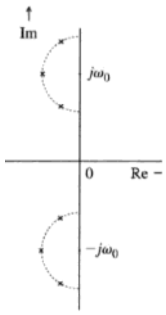
Complex conjugate poles: $|H(j\omega)| = \frac{K}{dd'}$

Phase effect:
- starts at 0
- increases, and tends to $-\pi$ as $\omega \to \infty$

Complex conjugate zeros:
- gain suppression
- $|H(j\omega)| = Krr'$

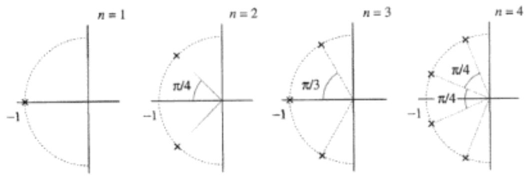
Phase effect:
- starts at 0
- increases, and tends to π as $\omega \to \infty$

Bandpass Filters:
- like wall of poles, but now around $\omega = \omega_0$, $not\ \omega = 0$
  - (+-w0)



Low Pass:
- simplest case - gain 1 at $\omega = 0$, 0 elsewhere
  - one pole on the real axis
- Wall of poles (Butterworth)
  - want gain 1 at $0 \le \omega \le \omega_c$
  - 0 gain for $\omega > \omega_c$
  - ideal - has a semicircle of infinite poles

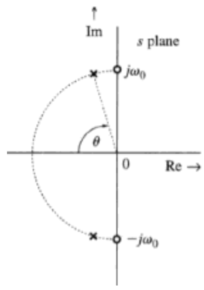
Butterworth:
- filters with poles evenly distributed around left half of unit circle



Sallen-Key:
- $H(s) = \dfrac{1}{1 + C_2(R_1 + R_2)s + C_1C_2R_1R_2s^2}$
- assuming $\omega_c = 1, chose\ C_1C_2R_1R_2 = 1$
- n even, $C_2(R_1 + R_2) = -2\cos\left(\frac{2k+n-1}{2n}\pi\right)$

To get a Butterworth filter of order n, cascade n/2 Sallen-Keys
- set n = n, and k=1,2… as adding more Sallen-Keys
then add an RC at end if n is odd

Notch:
- 0 gain around ω_0
- zeros at $\pm\omega_0$
  - forces two poles, for gain to be 1 elsewhere
  - complex conjugate poles, along the semicircle of $\pm\omega_0$



Signal Transmission

29 November 2017 18:45

Distortionless system

$|H(\omega)| = G_0 \ - constant$

$\angle H(\omega) = -\omega t_d \ - linear, passes\ through\ origin, slope\ of\ t_d$

Group delay: $\boldsymbol{t_g(\omega) = -\frac{d}{d\omega}\angle H(\omega)} \ - if\ constant\ - phase\ is\ linear$

LP: linear and pass through origin

BP:

- only liner through band on interest
- $H(\omega) = G_0 e^{j[\phi_0 - \omega t_g]}$
- output envelope remains undistorted
- output **carrier gets extra $\boldsymbol{\phi_0}$**
  - considered distortionless as message contained in envelope

- for input
  - $x(t)e^{j\omega_c t}$
- output
  - $y(t) = G_0 x(t - t_g) e^{j[\omega_c(t - t_g) + \phi_0]}$
  - $G_0\ gain\ at\ \omega_c$
  - $t_g\ slope\ of\ tangent\ at\ \omega_c$
  - $\phi_0\ y\ intercept\ by\ tangent\ at\ \omega_c$

In a real signal.

derived from Parseval's

$$\boldsymbol{\Delta E_x = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} |X(\omega)|^2 d\omega}$$

Windowing:

- multiply a signal by rectangular window
- energy leaks out from mainlobe to sidelobe
- mainlobe is $\frac{4\pi}{T}\ around\ \omega = 0\ for\ a\ window\ - amplitude\ T$

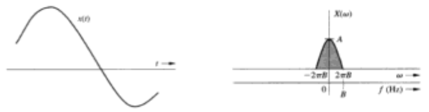
  To **reduce truncation - increase width**

  To **reduce leakage - avoid big discontinuity**

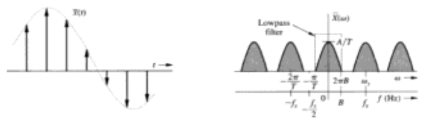
Sampling - Quantization

09 December 2017 17:25

BHz, with Fourier transform $X(\omega)$ (depicted real for convenience).



ıe sampled signal has the following spectrum.



Reconstruct - **lowpass** filter with $B \leq \omega_c \leq f_s - B$

- filter should have **gain of** $\boldsymbol{T_s}$ *as sampled has ampltidue* $\boldsymbol{A/T_s}$
- **convolution with sinc** function in time domain

<u>Ideal Reconstruction:</u>

- using a LPF with $\boldsymbol{h(t)} = \boldsymbol{sinc(2\pi Bt)}$
- **Interpolation** Formula:
  - $recovered\ \boldsymbol{x(t)} = \sum_{\boldsymbol{n}} \boldsymbol{x(nT_s)sinc(2\pi Bt - n\pi)}$
  - **summing** the weighted, and shifted sinc caused by each sample
    - gives the original signal

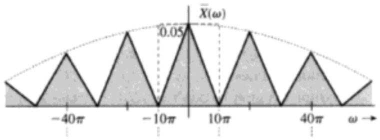
<u>Aliasing:</u>

- sampling lower than Nyquist frequency
- can't distinguish between different signals

- before sampling, **bandlimit input to** $\boldsymbol{\frac{f_s}{2} Hz}$ with a low pass filter
  - this is an **anti-aliasing filter**

<u>Practical Sampling:</u>

- **train of pulses** instead of impulse
- baseband copy has **amplitude A/4 = pulse width / period**
  - rest have decreasing amplitude



DFT

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Spectral width = 2B

<u>Spectral sampling theorem: - sample in frequency domain</u>

- x(t) is time-limited to τ seconds
- $then\ sampling\ rate\ = \boldsymbol{R\ Hz > \tau\ s}$
- periodic extension of $x(t) \to x_{T_0}(t)$, $\boldsymbol{T_0 > \tau}$
- $\boldsymbol{X_{T_0}(\omega)}$ $is\ the\ sampled\ version\ of\ X(\omega)$
    - samples separated by $\boldsymbol{f_0 = \frac{1}{T_o}}$
    - amplitude scaled by $\boldsymbol{\frac{1}{T_0}}$

<u>Reconstruct:</u>

- $spectral\ sample\ rate\ R = T_0 > \tau\ \ samples/Hz$
- Spectral interpolation formula
    - $X(\omega) = \sum_{n=-\infty} X(n\omega_0) sinc\left(\frac{\omega T_0}{2} - n\pi\right) e^{-j(\omega - n\omega_0)T_0}$, $T_0 > \tau$

<u>DFT:</u>

- repeat a time-limited signal and take it's samples
    - the spectrum is also sampled and periodically repeated
- relate samples of X(w) to samples of x(t)
- spectral sample rate $R = T_0$
    - spectral samples spaced out by $f_0 = \frac{1}{T_0}$
- sample rate $f_s = \frac{1}{T_s}$
    - signal samples spaced out by T_s
- for discrete signal
    - in one period T_0
    - # samples, $N_0 = \frac{T_0}{T_s}$
- for discrete spectrum
    - in one period f_s
    - # samples, $N_0' = \frac{f_s}{f_0}$
- $N_0' = N_0$

if $x(nT)$ and $X(r\omega_0)$ $are\ the\ n^{th}\ and\ r^{th}\ samples$

DFT: $X_r = \sum_{n=i}^{i+N_0-1} \left[\boldsymbol{\frac{T_0}{N_0} x(nT)}\right] e^{-jnr\Omega_0}$

IDFT: $x_n = \frac{1}{N_0} \sum_{r=i}^{i+N_0-1} [\boldsymbol{X(r\omega_0)}] e^{jnr\Omega_0}$, $\omega_0 = \frac{2\pi}{T_0}$

$$\boldsymbol{\Omega_0} = \omega_0 T_s = \boldsymbol{\frac{2\pi}{N_0}}$$

z - Transform

09 December 2017 17:26

$z = e^{sT}$

- time advance by T seconds

$\Rightarrow \boldsymbol{z^{-1} = e^{-sT}} \rightarrow \textit{sampling period delay}$

All discrete-time systems can be expressed via z

for discrete-time $x_n = x[n]$

Unilateral z-transform: (casual)

$$X[z] = \sum_{n=0}^{n=\infty} x[n]z^{-n}$$

Bilateral z-transform:

$$X[z] = \sum_{n=-\infty}^{n=\infty} x[n]z^{-n}$$

Union of ROC's covers z-plane

Inverse found same as Laplace inverse method

Useful series to remember: $1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$, $|x| < 1$

System	Signal	Continuous-time	Discrete-time
Both	Both	Laplace	z
Stable	Convergent	Fourier	DFT

s-plane to z-plane

s	z
Im-axis	unit circle
LHS plane	inner unit-circle
RHS-plane	outer unit-circle

Stable only if ROC of H(z) is within unit-circle

if **CASUAL**, stable if poles of H(z) lie within unit-circle

$\boldsymbol{Shift\ property}$:

$$\boldsymbol{Z\{x[n-m]\} = z^{-m}X(Z)}$$

A few key transforms:

$x[n]$	$X[z]$	ROC
$\gamma^n u[n]$	$\frac{z}{z-\gamma}$	$\lvert z\rvert > \lvert\gamma\rvert$
$-\gamma^n u[-n-1]$ (casual)	$\frac{z}{z-\gamma}$	$\lvert z\rvert < \lvert\gamma\rvert$
$\delta[n]$	1	
u[n]	$\frac{z}{z-1}$	$\lvert z\rvert > 1$
$\cos \beta n\, u[n]$	$\frac{z(z-\cos\beta)}{z^2 - 2z\cos\beta + 1}$	$\lvert z\rvert > 1$

Tables

11 June 2018 18:16

z –transform Table

No.	$x[n]$	$X[z]$
1	$\delta[n-k]$	z^{-k}
2	$u[n]$	$\frac{z}{z-1}$
3	$nu[n]$	$\frac{z}{(z-1)^2}$
4	$n^2u[n]$	$\frac{z(z+1)}{(z-1)^3}$
5	$n^3u[n]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^n u[n]$	$\frac{z}{z-\gamma}$
7	$\gamma^{n-1}u[n-1]$	$\frac{1}{z-\gamma}$
8	$n\gamma^n u[n]$	$\frac{\gamma z}{(z-\gamma)^2}$

z –transform Table

No.	$x[n]$	$X[z]$
10	$\frac{n(n-1)(n-2)\cdots(n-m+1)}{\gamma^m m!}\gamma^n u[n]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\|\gamma\|^n \cos \beta n\, u[n]$	$\frac{z(z-\|\gamma\|\cos\beta)}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
11b	$\|\gamma\|^n \sin \beta n\, u[n]$	$\frac{z\|\gamma\|\sin\beta}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
12a	$r\|\gamma\|^n\cos(\beta n+\theta)u[n]$	$\frac{rz[z\cos\theta-\|\gamma\|\cos(\beta-\theta)]}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
12b	$r\|\gamma\|^n\cos(\beta n+\theta)u[n] \quad \gamma=\|\gamma\|e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\|\gamma\|^n\cos(\beta n+\theta)u[n]$	$\frac{z(Az+B)}{z^2+2az+\|\gamma\|^2}$

$$r=\sqrt{\frac{A^2|\gamma|^2+B^2-2AaB}{|\gamma|^2-a^2}} \qquad \beta=\cos^{-1}\frac{-a}{|\gamma|} \qquad \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{|\gamma|^2-a^2}}$$

Fourier:

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t)+x_2(t)$	$X_1(\omega)+X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t-t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega-\omega_0)$
Time convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\,du$	$\frac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$

Summary of most important Laplace transform pairs

No	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$

Properties of convolution

No	$x_1(t)$	$x_2(t)$	$x_1(t)*x_2(t)$
1	$x(t)$	$\delta(t-T)$	$x(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t),\ \lambda_1\neq\lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$

Exams

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Discrete Fourier Series:

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jkw_0 n}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jkw_0 n}$$

$$w_0 = \frac{2\pi}{N}$$

train of impulse FS:

$$\frac{1}{T} \sum e^{-jk\omega_0 t}$$

$$u[n-a] * u[n-b] = (n-a-b+\mathbf{1})u[n-a-b]$$

$$\delta[n-a] * \delta[n-b] = \delta[n-a-b]$$

1st order: amplitude w>>a +-20dB/decade
phase +- 90, at 0.1x 10x

Origin:
constant phase +-90
amplitude : +-20db at w=1

2nd order pole:
$s^2 + 2\zeta\omega_n s + \omega_n^2$
$w \gg \omega_n, -40\ dB/decade$
phase
$\omega \ll \omega_n, 0°$
$\omega \gg \omega_n, -180°$