Introduction

12 October 2016 14:40

Assume 0 resistance in circuit diagram wires

Node - all the points that are connected together with interconnecting wires in a circuit
A node has 0 resistance so the voltage is the same everywhere along the node

Branches - Components that link a pair of nodes

Passive sign convention - make voltage arrow opposite to current arrow

Quantity	Letter	Unit	Symbol
Charge	Q	Coulomb	C
Conductance	G	Siemens	S
Current	I	Amp	A
Energy	W	Joule	J
Potential	V	Volt	V
Power	P	Watt	W
Resistance	R	Ohm	Ω

Value	Prefix	Symbol
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Value	Prefix	Symbol
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

rcuits (2015-7003) Introduction: 1 – 15 / 16

When charge is free to move, every point in the circuit is electrically neutral

Arrow of current on wire does not matter.
If outcome is -ve then current is flowing in opposite direction to the arrow

Conductance = 1/ resistance

Any node is chosen to be grounded and is referenced to 0V

$V_{AB} = V_A - V_B$

When current flows through a resistor, electrical potential energy > heat energy

P = V.I for any component

$P = V^2/R = I^2.R$ - only for resistors

On Graph
When taking in energy, VI> 0
When supplying energy, VI <0

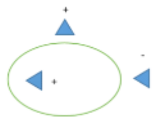
In any circuit, the net power will sum to zero, as elements are supplying and absorbing the same amount overall.

Resistor Circuits

12 October 2016 15:08

KVL - the sum of the voltages around any closed loop is zero

KCL - the sum of the currents entering and exiting any closed loop always sums t

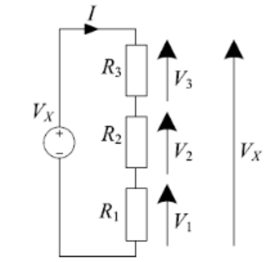


Entering loop	-ve
Exiting loop	+ve

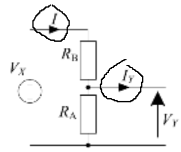
Series:
To spot - each internal node connects to only two branches
Same current flowing through resistors

Parallel:
To spot - components are connected to the same pair of nodes
Same voltage across each resistor

Notation: $3||7 = \dfrac{1}{\frac{1}{3}+\frac{1}{7}} = 2.1$



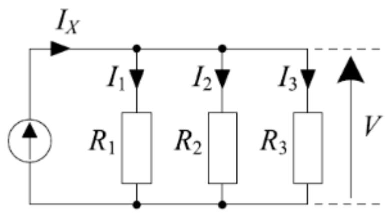
Voltage Divider



If $I_Y = 0$ then voltage divider works as normal
If I_Y is much smaller than I, then voltage divider approximately works.

V_X is divided into $V_1 : V_2 : V_3$ in the proportions $R_1 : R_2 : R_3$.

Current Divider



Special case for TWO resistors - works like a VD

$$I_1 = \frac{R_2}{R_1+R_2} I_X$$

Equivalent Resistors

- Does not affect relationship between V and I
- Series:
  - Nodes disappear
- Parallel:
  - Currents disappear
  - Number of nodes stays the same

I_X is divided into $I_1 : I_2 : I_3$ in the proportions $G_1 : G_2 : G_3$.

$= 1/R_1 : 1/R_2 : 1/R_3$

Parallel Resistors - Formulae

For parallel resistors $G_P = G_1 + G_2 + G_3$

or equivalently $R_P = R_1||R_2||R_3 = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$

These formulae work for any number of resistors.

- For the special case of two parallel resistors

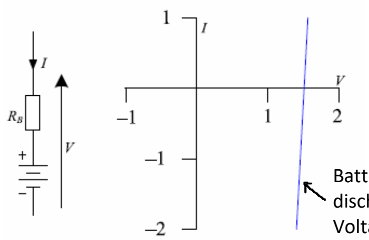
$R_P = \frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2}$ ("product over sum")

- If one resistor is a multiple of the other

Suppose $R_2 = kR_1$, then

$R_P = \frac{R_1 R_2}{R_1+R_2} = \frac{kR_1^2}{(k+1)R_1} = \frac{k}{k+1} R_1 = (1 - \frac{1}{k+1}) R_1$

Internal Resistance



Battery is discharging here - Voltage decreases as current draw increases

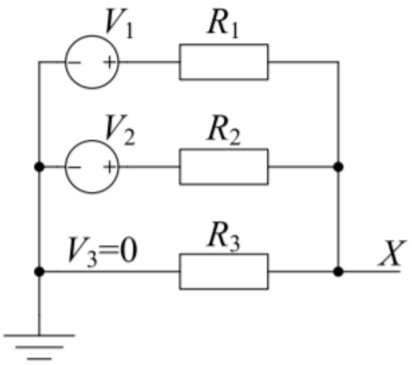
$$V = V_B + IR_B$$

Nodal Analysis

Always label one node as 0V at start

For a circuit with N nodes and S voltage sources, there will always be N - S - 1 KCL simultaneous equations to solve.

Weighted Average Circuit:



$$(X - V_1)G_1 + (X - V_2)G_2 + (X - V_3)G_3 = 0$$

$$X(G_1 + G_2 + G_3) = V_1G_1 + V_2G_2 + V_3G_3$$

$$X = \frac{V_1G_1 + V_2G_2 + V_3G_3}{G_1 + G_2 + G_3} = \frac{\sum V_iG_i}{\sum G_i}$$

Voltage X is the average of V_1, V_2, V_3 weighted by the conductances.

Universal Nodal Analysis Algorithm:

- Pick any node as 0V, label dependant sources as V_S and I_S
- Label nodes next to known voltage sources
- Label any unlabelled nodes with X, Y.. And create super nodes with these terms if any exist
- For any dependent sources, write down an equation in terms of known nodes/ values
- Write down KCL equations for all nodes - include super nodes in normal node KCL's
- Solve simultaneous equations

Summary of Nodal Analysis:

- Simple Circuits (no floating or dependent voltage sources)
- Floating Voltage Sources
  - use supernodes: all the nodes connected by floating voltage sources (independent or dependent)
- Dependent Voltage and Current Sources
  - Label each source with a variable
  - Write extra equations expressing the source values in terms of node voltages
  - Write down the KCL equations as before

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Linearity and Superposition

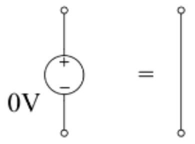
12 November 2016 11:04

Linearity theorem - Any circuit containing resistors and independent voltage and current sources,
Every node voltage and branch current is a linear function of the source values

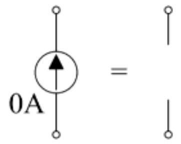
EG -

$$X = \frac{1}{3}U_1 + \frac{2}{3}U_2, \quad Y = \frac{1}{3}U_1 + \frac{11}{3}U_2$$

Zero Value Voltage source - short circuit



Zero Value Current source - open circuit



Superposition - set all but one of the independent sources to 0, and find value of node in terms of one source. Repeat for all sources, then add X_1, X_2, X_3 … to get X

Power DOES NOT obey superposition

If there are dependent sources, treat as independent then eliminate form final expressions

Proportionality - If all independent sources are multiplied by a value K,
then all voltages and currents in the circuit will be multiplied by the value K,
and the power dissipated by any component will be multiplied by K^2

Thevenin and Norton Equivalents

12 November 2016 11:04

Norton - CURRENT SOURCE - PARALLEL

Thevenin - VOLTAGE SOURCE - SERIES

NCP _ TVS

Thevenin Properties:

- $V = R_{TH}I + V_{TH}$
- Open circuit, I = 0, then $V_{OC} = V_{TH}$
- Short circuit, V = 0, then $I_{SC} = -V_{TH} / R_{TH}$
- R_{TH} is represented by 1/ slope of IV graph

Norton Properties:

- $I = V / R_{TH} - I_{VO}$
- Open circuit, I = 0, then $V_{OC} = I_{NO}R_{TH}$
- Short circuit, V = 0, $I_{SC} = -I_{NO}$
- $R_{NO} = R_{TH}$

To find Thevenin Resistance:

- Voltage sources - short circuit
- Current source - open circuit
- Then find resistance between the two terminals

Thevenin via nodal analysis:

1. Label ground as output terminal
2. Write down nodal equations
3. Eliminate all nodes except for input terminal node

Small R_{TH}	Large R_{TH}
Thevenin	Norton

Change between Norton and Thevenin -	$V_{TH} = I_{NO}R_{TH}$	Just like V=IR

Power Transfer from an equivalence circuit, connected to a resistance L.
For a fixed R_{TH}, max power transferred when $R_{TH} = R_L$

Operational Amplifiers

12 November 2016 11:04

Negative feedback adjusts output Y to make V+ ~ V-

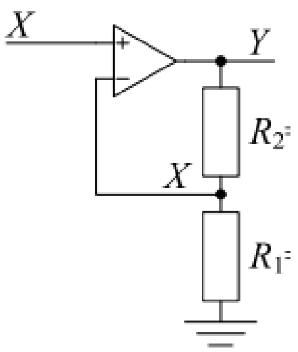
Positive Feedback - VOUT = +- VMAX

To analyse Op-Amp circuit

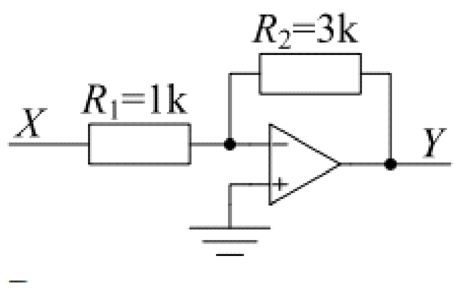
- Assume A ~ ∞
- Assume 0A input current

1. Check for negative feedback, Y connected somehow to V-
2. Assume V+ = V-
3. Apply KCL at each input separately, with input current 0

Non-inverting Amplifier Y/X = 1 + R2 / R1



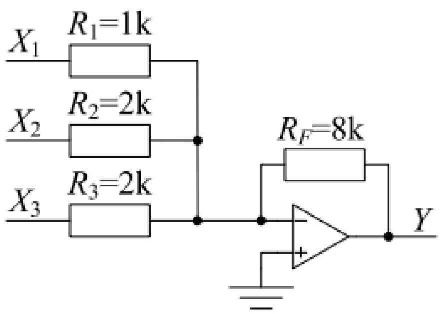
Inverting Amplifier Y/X = -R2 / R1



Inverting Summing Amp

Y is **weighted sum** of input voltages X_i

Where coefficient is $-R_F$ / R_i



Differential Amp

Output = A(Input 1 - Input 2)

So proportional to the difference

Use superposition to work out A

Block Diagrams

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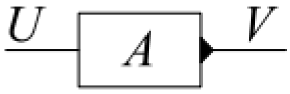
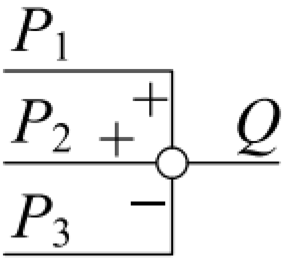
Gain Block: V = U x A



Adder Block: $Q = P_1 + P_2 - P_3$



To solve block diagrams:

1. Label all inputs, outputs and adder outputs
2. Write down equations for the output and all adder outputs
3. Solve by eliminating unwanted variables

Nonlinear Components

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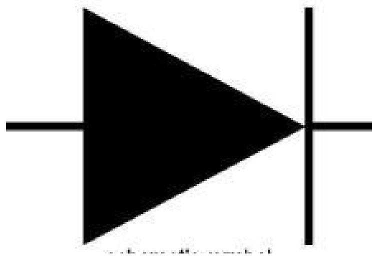
Voltage <----------------



----------------> Current

A diode

To solve a circuit with a diode, guess how it acts, then check if condition is correct

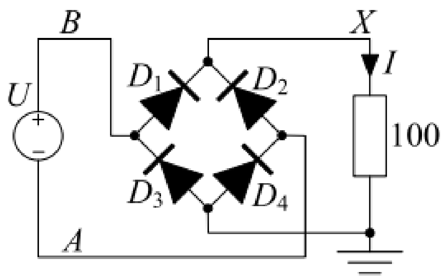
Region	Condition	Equation	Acts as
Forward Bias (On)	I > 0	V = 0	Short Circuit
Reverse Bias (Off)	V < 0	I = 0	Open Circuit
	Check this after	Make the guess	

For real diode, switching occurs around 0.7V

Can high-current circuits, V ~ 1.0V

<u>Bridge Rectifier (Full wave)</u>

4 Diodes,



2 cases:

- D1 and D4 on
- D2 and D3 on

X = |U|

If U is AC, then X has double its frequency

<u>Halfwave Rectifier</u>

- Gives X = max(U-0.7,0)
- U causes Diode to switch on and off

U	Diode	X
<0.7	OFF	0
?0.7	ON	U-0.7

<u>Precision Halfwave Rectifier</u>

- Gives X = max(U, 0)
- Use of op-amps eliminates the 0.7V diode drop

Capacitors and Inductors

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Capacitor Equations:

$$q = Cv$$

differentiate both sides w.r.t. t:

$$i = C\frac{dv}{dt}$$

-> voltage doesn't change instantly

$$W = \frac{1}{2}CV^2$$

Inductor Equations:

$$v = L\frac{di}{dt}$$

-> current doesn't change instantly

$$W = \frac{1}{2}LI^2$$

AVERAGE power absorbed by these components **= 0**

As energy stored is returned over time

	Series	Parallel
Capacitor	// EQN for R	SUM capacitance
Inductor	SAME as R	SAME as R

Can tell from Circuit Symbols

Capacitor	Inductor
Average CURRENT = 0	**Average Voltage = 0**

Sine waves and Phasors

13 November 2016 18:04

Phasor: A complex number representing a sin wave

Phasor $V = X + jY$
Wave $v(t) = X\cos 2\pi ft - Y\sin 2\pi ft$

- **Plus j in phasor > Minus sine in wave**
- **Assume Frequency is known**

The sine wave:

$$A\cos(2\pi ft + \emptyset) = A\cos\emptyset\cos 2\pi ft + A\sin\emptyset\sin 2\pi ft = X\cos 2\pi ft + Y\sin 2\pi ft$$

Complex number $X + jY$ represents the sine wave at time t = 0;

Magnitude of a phasor = $\sqrt{X^2 + Y^2}$, gives the Amplitude of the sine wave

Argument of a phasor = $\tan^{-1}\frac{Y}{X}$, gives Phase Shift **relative to $\cos 2\pi ft$**

Phase Shift	Relative to $\cos 2\pi ft$
<0	**Lagging**
>0	**Leading**

Phasor Arithmetic

Operation	Phasor
ADDITION	$V_1 + V_2$
SCALING	aV
DIFFERENTIATRE	$j\omega V$

Complex Impedance

Component	Z
Resistor	R
Inductor	$j\omega L$
Capacitor	$\frac{1}{j\omega C}$

CIVIL

In a Capacitor I leads V but V leads I in an Inductor

For any network (resistors+capacitors+inductors):

(1) Impedance = Resistance + $j \times$ Reactance

$$Z = R + jX\ (\Omega)$$

(2) Admittance = $\frac{1}{\text{Impedance}}$ = Conductance + $j \times$ Susceptance

$$Y = \frac{1}{Z} = G + jB \text{ Siemens (S)}$$

So $G = \frac{R}{R^2+X^2} = \frac{R}{|Z|^2}$

$B = \frac{-X}{R^2+X^2} = \frac{-X}{|Z|^2}$

Beware: $G \neq \frac{1}{R}$ unless $X = 0$.

Frequency Responses

08 January 2017 19:03

Phasor input into any RCL circuit gives Phasor output, which **varies with input frequency** of input phasor

Transfer function of the circuit is $\frac{output}{input} = gain\ of\ the\ circuit$

Voltage ratio in decibels (dB) $= 20log_{10}\frac{|V_2|}{|V_1|}$

Have to plot two graphs for output,

- **|Gain| vs ω**
- **Phase vs ω**

For **H** = $\boldsymbol{C(j\omega)^R}$

- Magnitude response
  - **C is the y-intercept**
  - **R is the gradient of the straight line**
- Phase response
  - **Horizontal line**
  - **Phase = 90 ∘ x R**
  - **If C < 0, +- 180 ∘**

BEFORE AND AFTER straight line approximations can be found without factorisation:

- **Low Freq. Asymptote**
  - $\frac{Lowest\ power\ of\ jw\ from\ top}{Lowest\ power\ of\ jw\ from\ bottom}$
- **High Freq. Asymptote**
  - $\frac{Highest\ power\ of\ jw\ from\ top}{Highest\ power\ of\ jw\ from\ bottom}$

To plot Magnitude Response for any Transfer function, H(jω)

- **Factorize the polynomials**
- Find **Corner Frequencies**
  - $(j\omega + \alpha)$
- Do approximations as straight line **before, between and after corner frequencies**
  - For frequency less than corner frequency - factor becomes an ω
  - Higher factors become their constant terms

To plot Phase Response for any Transfer function, H(jω)

- **Factorize the polynomials after HF and LF**
- Find **Corner Frequencies**
  - $(j\omega + \alpha)$
    - Take note if from Numerator or Denominator
- If from Denominator
  - At $0.1\omega_c$, Plot decreases by -45 ∘ rad/decade
  - At $10\omega_c$, Plot increases by +45 ∘ rad/decade
- If from Numerator
  - At $0.1\omega_c$, Plot increases by +45 ∘ rad/decade
  - At $10\omega_c$, Plot deacreases by -45 ∘ rad/decade
- For **Intermediate horizontal straight-line** section
  - Value = previous horizontal $\pm \frac{\pi}{4}log_{10}(\frac{current\ corner\ freq}{previous\ corner\ freq})$

Resonance

08 January 2017 19:04

Occurs when quadratic factor in transfer function cannot be factorised into linear factors

For Quadratic factor $F(j\omega) = A(j\omega)^2 + B(j\omega) + C$

- $\omega_c = \sqrt{\frac{C}{A}}$
- Damping factor, $\boldsymbol{\zeta} = \frac{\boldsymbol{B}}{\sqrt{\boldsymbol{4AC}}} = \frac{\boldsymbol{B.Sgn(A)}}{\boldsymbol{2a\omega_c}}$
- Quality Factor, $Q \approx \frac{1}{2\zeta}$
- $Q \triangleq \omega \, x \, W_{stored} \div \bar{P}_R$
    - $W_{stored} = peak\ stored\ energy$
    - $\bar{P}_R = average\ power\ loss\ through\ resistor$

Resonant Frequency, $\boldsymbol{\omega_r}$, occurs when Impedance, Z, is purely real
- Current in L and C cancel out

Half-power bandwidth

- Gain^2 (power)
    - Range of frequencies
    - For which power is greater than half its peak

Magnitude Plot:
- Small ζ > higher peak, smaller bandwidth
- Large ζ > lower peak at lower ω_P , larger bandwidth
    - Peak Frequency, $\omega_P = \omega_C\sqrt{1-2\zeta^2}$
        - True for circuit with only one quadratic factor in denominator

Phase Plot:
- Small ζ > faster phase change around ω_C

Filters

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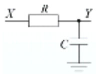
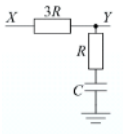
	Low	High
Capacitor	Open Circuit	Short Circuit
Inductor		

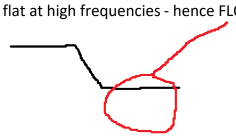
Order of a circuit = highest power of (jw) in the denominator of the Transfer function

Low Pass filter

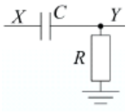


'Gain Floor' - flat at high frequencies - hence FLOOR



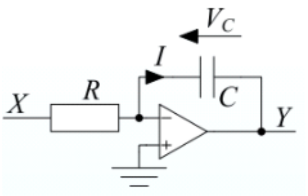


High Pass filter



Integrator

$$\frac{Y}{X} = -\frac{1/j\omega C}{R}$$



Op-Amp Filter advantages:

- **Can have gain > 1**
- **Low output impedance**
- Input impedance does not vary with frequency

Filter Transformations

Impedance Scaling by factor K

R'	C'	L'
KR	$\boldsymbol{K^{-1}}C$	KL

Frequency Shift - To the **LEFT** by a factor of **K**

R'	C'	L'
R	$\boldsymbol{KC}$	$\boldsymbol{KL}$

LR to RC circuit

- $R' = KL$
- $C' = \frac{1}{KR}$

Reflection in Frequency axis around ω_m

- $R' = \frac{K}{\omega_m C}$
- $C' = \frac{1}{\omega_m KR}$
  - Magnitude plot flips
  - Phase plot flips and NEGATES

Power in AC

08 January 2017 19:04

$$< y > = \frac{1}{T}\int_0^T y\,dt$$

Average Power, $P = \frac{1}{T}\int_0^T p(t)dt$

$$\boldsymbol{V_{rms}} \triangleq \sqrt{<\boldsymbol{v^2(t)}>} \equiv DC\ voltage$$

For Voltage phasor $V = |V|e^{j\theta_1}$ and Current phasor $I = |I|e^{j\theta_1}$

Average Power, $P = \frac{1}{2}|V||I|\cos(\theta_1 - \theta_2)$

Power Factor of V and I is $\boldsymbol{\cos(\theta_1 - \theta_2)}$

Complex Power

- S = RMS Voltage * RMS Conjugate Current
- $S = \tilde{V} * \tilde{I}^* = \frac{|\tilde{V}|^2}{Z^*} = |\tilde{I}|^2 * Z$
- $S = P + jQ = |\tilde{V}||\tilde{I}|\cos\emptyset + |\tilde{V}||\tilde{I}|\sin\emptyset$
- Phase of S = Phase of Z
- Apparent Power = |Complex|
- Real(S) = Average Power = P
- Imaginary(S) = **Reactive Power** = Q
- $\cos\emptyset = \frac{P}{|S|}$
- Capacitors generate Reactive Power
- Inductors absorb Reactive Power
- Phase of S = Phase of Z
- Complex Power in any **circuit** = 0 - Tellegen's Theorem

Transients

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y(t) = y_{SS}(t) + y_{Tr}(t)

y_{SS}(t) **same frequency as new input**

Use **nodal/Phasor analysis** to get this term

y_{Tr}(t) if of the form $Ae^{-t/\tau}$

Does not depend on input, only the circuit

$\tau = RC \ or \ \frac{L}{R}$

A depends on initial conditions of input

- RC - find **v_C(0-) ,** RL - find **i_L(0-)**
- Find y(0+) via Nodal Analysis or other
- **A = y(0+) - y_{SS}(0+)**

- Can work out R_{Th}
  - Remove C or L when working out R_{Th}

Sinusoidal input

- For y_{SS}
  - Get transfer Function
  - y_{SS}(t) = X * H(w)

Negative Exponential

Time it takes to get somewhere from a known point

$$T = \tau * \ln\left(\frac{initial\ distance\ to\ Limit}{final\ distance\ to\ Limit}\right)$$

Transient from Transfer Function:

- Calculate H(w)
- **y_{SS}(t) = LF gain * x(t)**
- **y(discontinuity) = HF gain * x(discontinuity)**
- $\boldsymbol{\tau = \frac{1}{Denominator\ CF}}$
- Then work out A using difference

Transmission Lines

08 January 2017 19:05

Transmission Line Equations

- $C_0 \frac{\partial v}{\partial t} = -\frac{\partial i}{\partial x}$
    - Capacitance per unit length, $C_0 = \frac{C}{\partial x}$
- $L_0 \frac{\partial i}{\partial t} = -\frac{\partial v}{\partial x}$
    - Total Inductance per unit length, $L_0 = \frac{L_1 + L_2}{\partial x}$

SOLUTIONS

- $v(t,x) = f\left(t - \frac{x}{u}\right) + g\left(t + \frac{x}{u}\right)$
- $i(t,x) = \frac{f\left(t - \frac{x}{u}\right) + g(t + \frac{x}{u})}{Z_0}$

- u is the propogation delay
- $Z_0 = \sqrt{\frac{L_0}{C_0}}$

Reflections

Backwards wave is a multiple of forward wave

$$g_L = \rho_L f_L$$

Reflection Coefficient

$$\rho_L = \frac{R_L - Z_0}{R_L + Z_0}$$

ρ	Comment
0	No Reflection
+1	Open Circuit - Double Voltage
-1	Closed Circuit - 0V

Power Flow into region

- $P = \frac{f^2}{Z_0} - \frac{g^2}{Z_0}$
    - f carries power into the region
    - g carries power out of the region

Driving a Line:

Voltage on line is from potential divider of Driver Resistance and Z_0

TL and Phasors

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f and g are sinusoidal with same frequency

$F_0 = Ae^{j\emptyset}$ - Phasor at start of line

<u>At point x on the line:</u>

$$k = \frac{\omega}{u}$$

- $F_x = F_0 e^{-jkx}$
- $G_x = G_0 e^{+jkx}$

<u>At Load, L</u>

- Ohms Law applies
- ρ_L still same
- Travelling from x to L
  - $e^{-jk(L-x)}$ *is the phase delay*
    - ^ Multiply Phasor by this

<u>Reflections - Creates a Standing Wave</u>

- Amplitude of Line Voltage ∝ X
- Max Amplitude of (1 + $|\rho_L|$) occurs every $\frac{\lambda}{2}$ away from L