find_patterns

March 8, 2020

```
[1]: import matplotlib.pyplot as plt
     import os
     import numpy as np
```

0.1 Helpers

```
[2]: def clean(matrix, tol=1e-10):
         arr = matrix.copy()

         # Flush small elements to zero
         arr[np.abs(arr) < tol] = 0

         # Coalesce similar elements
         if arr.size > 1:
             amfl = np.abs(arr.flat)
             amix = np.argsort(amfl)

             i, ix = 0, amix[0]
             for j, jx in enumerate(amix[1:], start=1):
                 if amfl[jx] - amfl[ix] >= tol:
                     if j - i > 1:
                         amfl[amix[i:j]] = np.median(amfl[amix[i:j]])
                     i, ix = j, jx

             if i != j:
                 amfl[amix[i:]] = np.median(amfl[amix[i:]])

             # Fix up the signs and assign
             arr.flat = np.copysign(amfl, arr.flat)

         return arr
```

```
[3]: MAT_DIR = "./../../example_mats_med"

     matrices = {
         "quad" : {},
```

```
    "hex"   : {},
    "tet"   : {},
    "tri"   : {}
}
```

[4]:
```python
for d in os.walk(MAT_DIR):
    # mat file names
    for m in d[2]:
        mat_data = np.loadtxt(os.path.join(d[0], m))
        mat_data = clean(mat_data)
        # order of prolem and sub type
        mat_name = d[0].split("/")[-2] + "-" + d[0].split("/")[-1]
        # add size
        mat_name += "-" + m.split(".")[0]

        # store in correct location according to type
        for k in matrices.keys():
            if k in d[0]:
                matrices[k][mat_name] = mat_data
```

0.2 Quad

0.3 see ratios

[5]:
```python
from collections import defaultdict
ratios = defaultdict(int)
for name, data in matrices["quad"].items():
    dims = data.shape
    ratios[dims[0]/dims[1]] += 1
```

[6]: `ratios`

[6]:
```
defaultdict(int,
            {0.5: 12,
             1.0: 5,
             2.0: 14,
             1.5: 4,
             1.333333333333333: 2,
             0.75: 2,
             2.5: 2,
             0.8: 2,
             1.25: 2,
             3.5: 2,
             0.5714285714285714: 2,
             1.75: 2,
             3.0: 2,
```

```
                    0.6666666666666666: 2})
```

0.4 heatmaps

```
[7]: import seaborn as sns
```
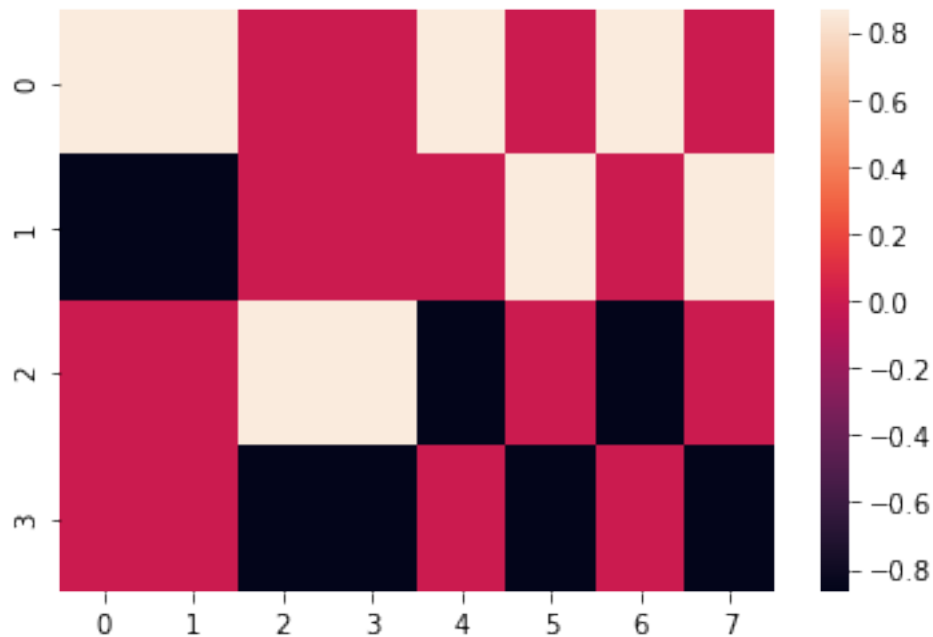
```
[9]: for name, data in matrices["quad"].items():
         dims = data.shape

         if dims[0]/dims[1] == 0.5:
             print(name)
             print("ratio: " + str(dims[0]/dims[1]))
             print(np.unique(data, return_counts=True))
             plt.figure()
             ax = sns.heatmap(data)
             plt.show()
```
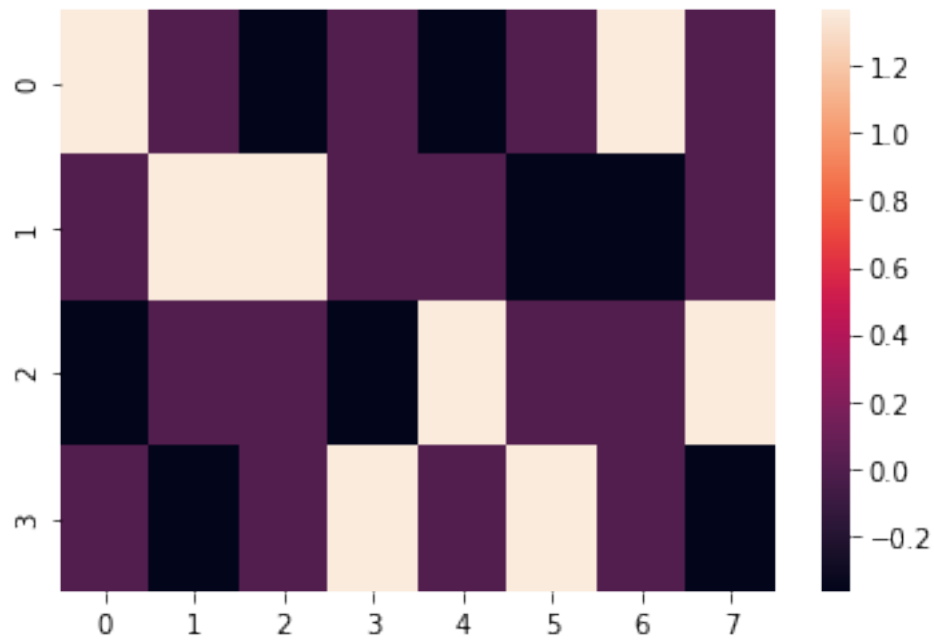
```
p1-gauss-legendre-m132
ratio: 0.5
(array([-0.8660254,  0.        ,  0.8660254]), array([ 8, 16,  8]))
```
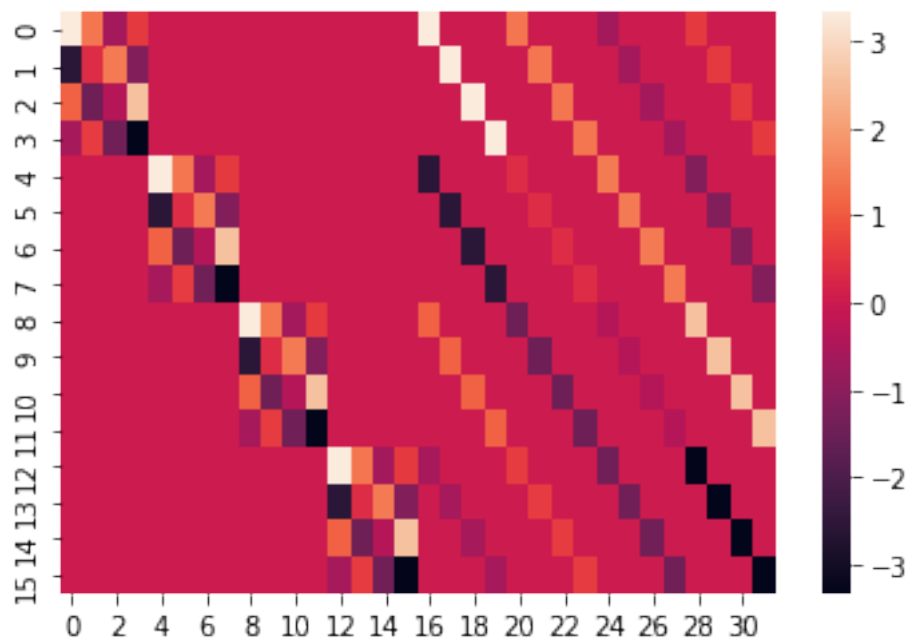


```
p1-gauss-legendre-m3
ratio: 0.5
(array([-0.3660254,  0.        ,  1.3660254]), array([ 8, 16,  8]))
```
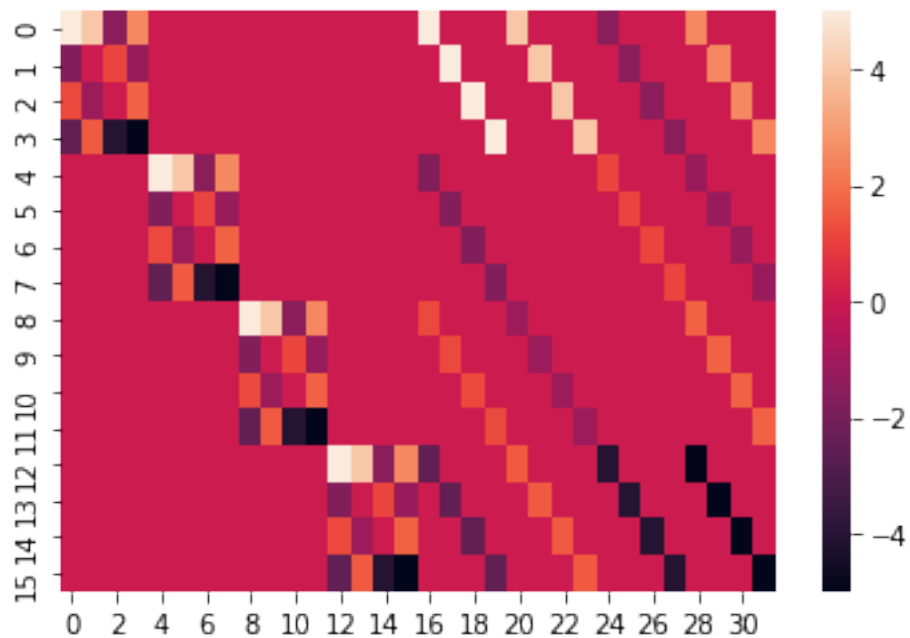
p3-gauss-legendre-m132
ratio: 0.5
(array([-3.33200024, -2.59241098, -1.47067023, -1.42024047, -1.12482651,
       -0.61623104, -0.58062817, -0.38441439, 0.   , 0.38441439,
        0.58062817, 0.61623104, 1.12482651, 1.42024047, 1.47067023,
        2.59241098, 3.33200024]), array([8,   8,   8,   8,   8,   8,   8,
8, 384,   8,   8,   8,   8,
         8,   8,   8,   8]))

p3-gauss-legendre-lobatto-m132
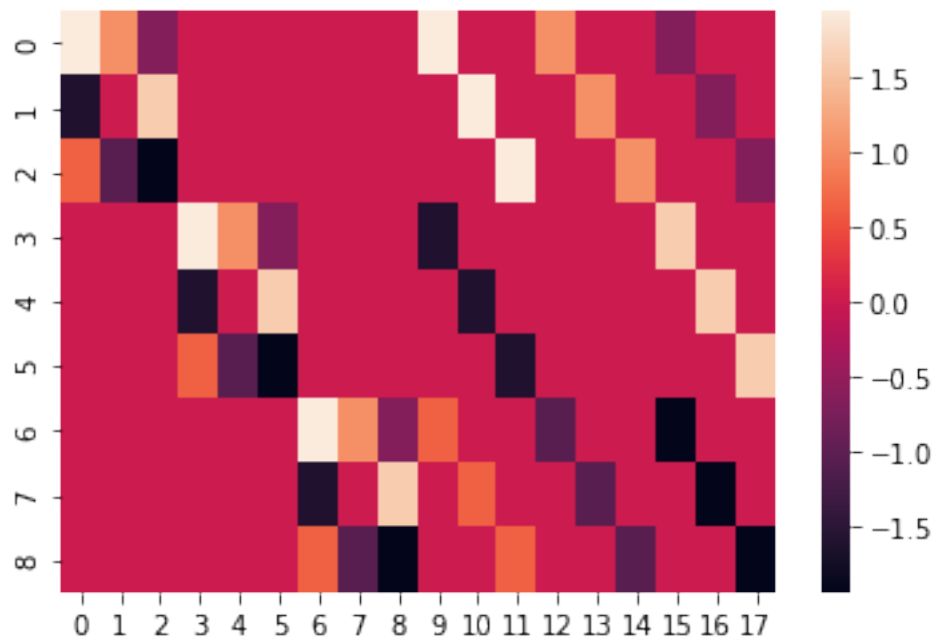ratio: 0.5
(array([-5.   , -4.04508497, -2.5   , -1.70344419, -1.54508497,
        -1.20344419, -1.11803399, 0.   , 1.11803399, 1.20344419,
         1.54508497, 1.70344419, 2.5   , 4.04508497, 5.   ]),
array([8,   8,   8,   8,   8,   8,   8, 400,   8,   8,   8,   8,   8,
         8,   8]))

5

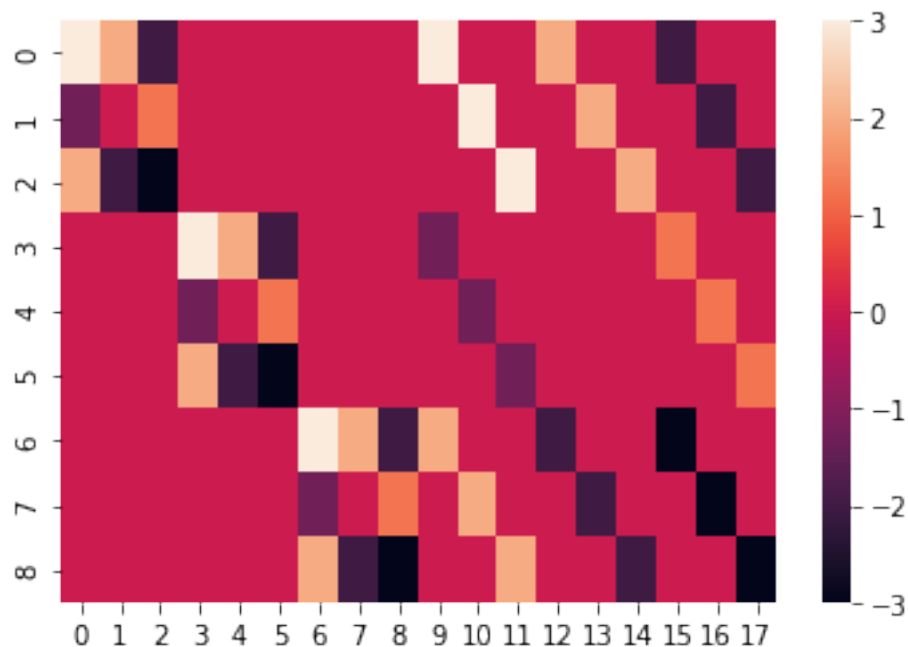p2-gauss-legendre-m132
ratio: 0.5
(array([-1.93649167, -1.61374306, -1.03279556, -0.64549722, 0.   ,
        0.64549722, 1.03279556, 1.61374306, 1.93649167]), array([6,   6,
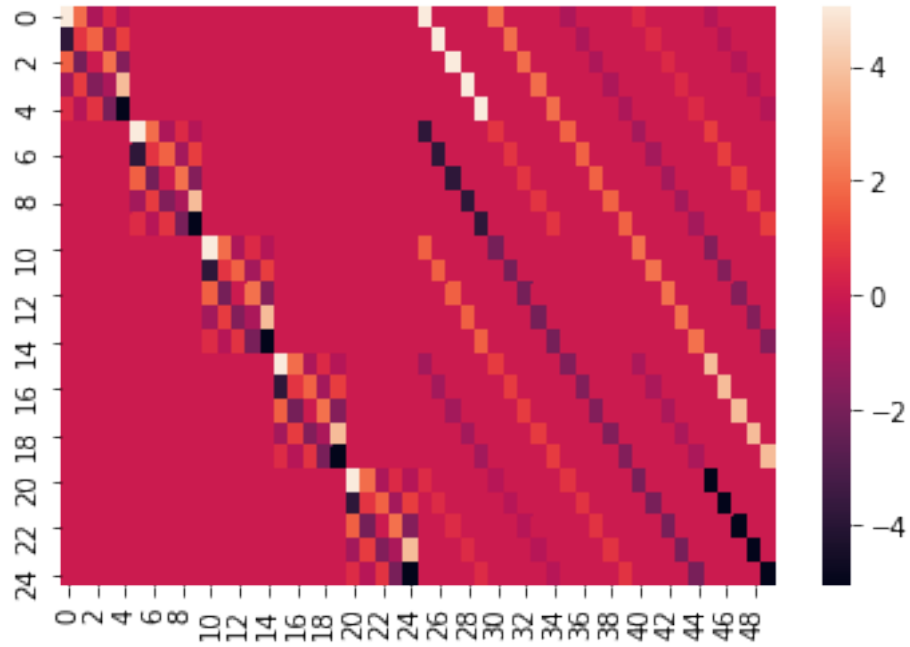6,   6, 114,   6,   6,   6,   6]))

p2-gauss-legendre-lobatto-m132
ratio: 0.5
(array([-3. , -2. , -1.25, 0. , 1.25, 2. , 3.]), array([6, 12,   6,
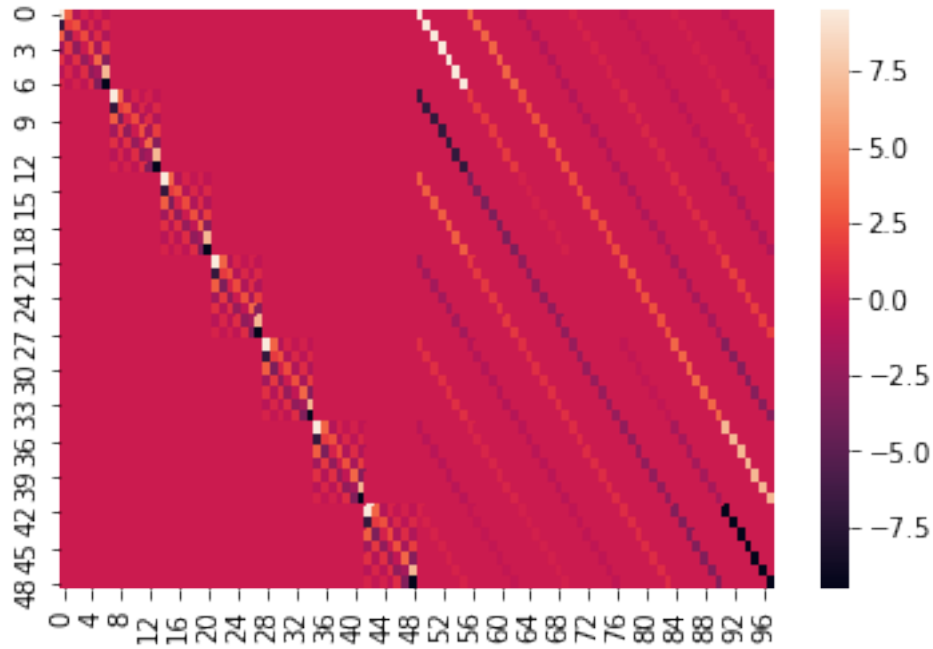114,   6, 12,   6]))

p4-gauss-legendre-m132
ratio: 0.5
(array([-5.0670406 , -3.81255789, -2.02152894, -1.93986454, -1.70607503,
       -1.68402696, -0.97042088, -0.92855803, -0.75835322, -0.72313963,
       -0.55176685, -0.49375908, 0.   , 0.49375908, 0.55176685,
        0.72313963, 0.75835322, 0.92855803, 0.97042088, 1.68402696,
        1.70607503, 1.93986454, 2.02152894, 3.81255789, 5.0670406]),
array([10,   10,   10,   10,   10,   10,   10,   10,   10,   10,   10,
        10, 1010,   10,   10,   10,   10,   10,   10,   10,   10,   10,
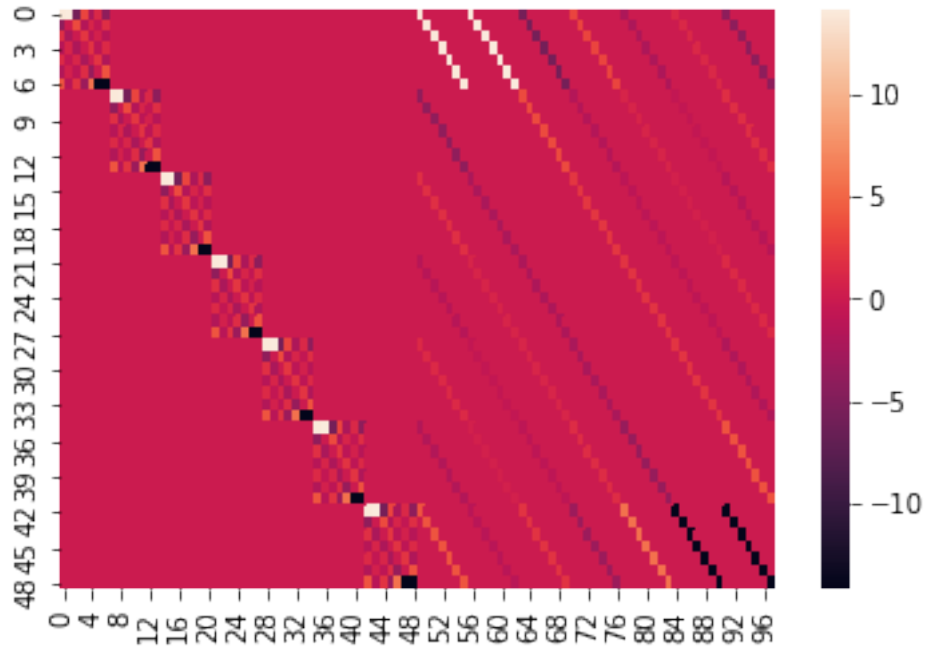   10,   10,   10]))

```
p4-gauss-legendre-lobatto-m132
ratio: 0.5
(array([-7.5       , -6.75650249, -3.        , -2.66666667, -2.31241882,
       -1.74574312, -1.41016418, -1.33658458, -1.33043832, -1.3125    ,
       -0.76376262,  0.        ,  0.76376262,  1.3125    ,  1.33043832,
        1.33658458,  1.41016418,  1.74574312,  2.31241882,  2.66666667,
        3.        ,  6.75650249,  7.5       ]), array([ 10,   10,   10,   10,
 10,   10,   10,   10,   10,   10,   10,
       1030,   10,   10,   10,   10,   10,   10,   10,   10,   10,   10,
         10]))
```

```
p6-gauss-legendre-m132
ratio: 0.5
(array([-9.56818234, -6.98244017, -3.47320931, -3.3238044 , -3.11057141,
       -2.57685029, -2.5550682 , -2.3560803 , -1.86201883, -1.64736568,
       -1.6443125 , -1.24717059, -1.23199698, -1.10600426, -1.08928106,
       -1.01615121, -0.85730425, -0.74753216, -0.67428047, -0.59619008,
       -0.52681049, -0.48587352, -0.43674268, -0.40809682,  0.        ,
        0.40809682,  0.43674268,  0.48587352,  0.52681049,  0.59619008,
        0.67428047,  0.74753216,  0.85730425,  1.01615121,  1.08928106,
        1.10600426,  1.23199698,  1.24717059,  1.6443125 ,  1.64736568,
        1.86201883,  2.3560803 ,  2.5550682 ,  2.57685029,  3.11057141,
        3.3238044 ,  3.47320931,  6.98244017,  9.56818234]), array([ 14,    14,
   14,    14,    14,    14,    14,    14,    14,    14,    14,
       14,    14,    14,    14,    14,    14,    14,    14,    14,    14,
       14,    14, 4130,    14,    14,    14,    14,    14,    14,    14,
       14,    14,    14,    14,    14,    14,    14,    14,    14,    14,
       14,    14,    14,    14,    14]))
```
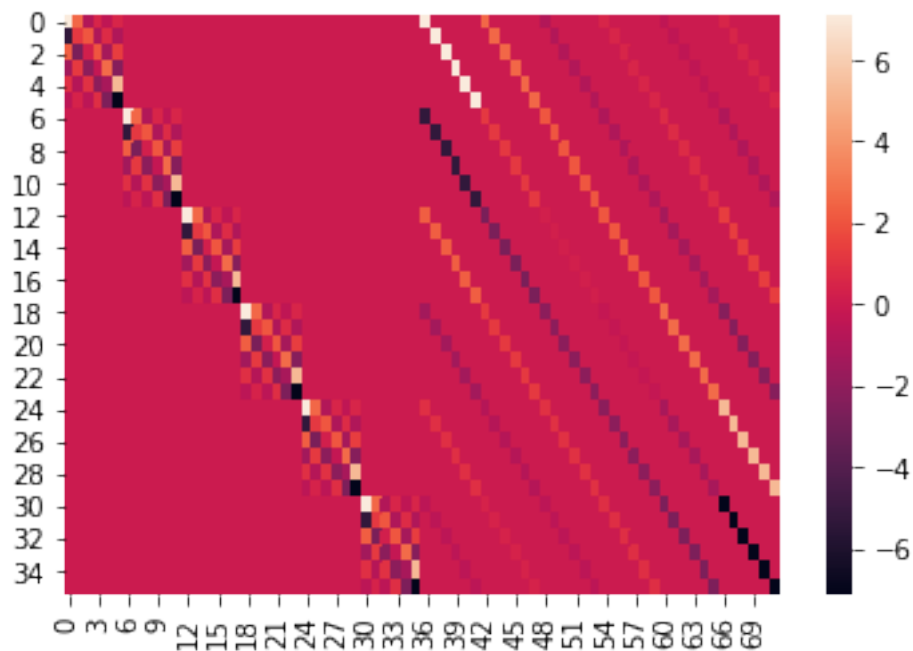
p6-gauss-legendre-lobatto-m132
ratio: 0.5
(array([-14.2015766 , -14.   , -5.66898523, -4.   ,
       -3.89455263, -3.45582821, -3.2   , -2.26669809,
       -2.21580428, -2.04996481, -2.00696924, -1.78762708,
       -1.67823848, -1.59860669, -1.40625   , -1.38846981,
       -1.31737344, -1.0664419 , -0.9613398 , -0.90754447,
       -0.61639084, -0.60224718, 0.   , 0.60224718,
        0.61639084, 0.90754447, 0.9613398 , 1.0664419 ,
        1.31737344, 1.38846981, 1.40625   , 1.59860669,
        1.67823848, 1.78762708, 2.00696924, 2.04996481,
        2.21580428, 2.26669809, 3.2   , 3.45582821,
        3.89455263, 4.   , 5.66898523, 14.   ,
       14.2015766]), array([14,   14,   14,   14,   14,   14,   14,   14,
  14,   14,   14,
        14,   14,   14,   14,   14,   14,   14,   14,   14,   14,   14,
      4186,   14,   14,   14,   14,   14,   14,   14,   14,   14,   14,
        14,   14,   14,   14,   14,   14,   14,   14,   14,   14,
       14]))
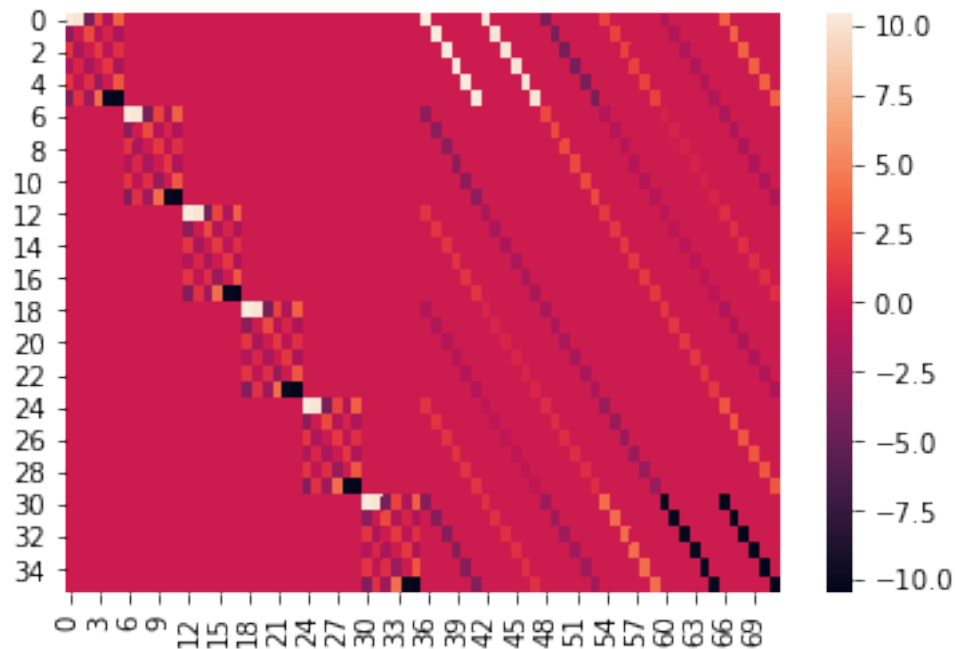
p5-gauss-legendre-m132
ratio: 0.5
(array([-7.14532713, -5.27576253, -2.68967445, -2.57598208, -2.3443613 ,
       -2.09538893, -2.08190754, -1.38899458, -1.2631629 , -1.17485228,
       -0.97773482, -0.89798768, -0.88602069, -0.75619011, -0.53621056,
       -0.52495233, -0.43845798, -0.25302628, 0.   , 0.25302628,
        0.43845798, 0.52495233, 0.53621056, 0.75619011, 0.88602069,
        0.89798768, 0.97773482, 1.17485228, 1.2631629 , 1.38899458,
        2.08190754, 2.09538893, 2.3443613 , 2.57598208, 2.68967445,
        5.27576253, 7.14532713]), array([12,   12,   12,   12,   12,   12,
 12,   12,   12,   12,   12,
        12,   12,   12,   12,   12,   12,   12, 2160,   12,   12,   12,
        12,   12,   12,   12,   12,   12,   12,   12,   12,   12,   12,
        12,   12,   12,   12]))

p5-gauss-legendre-lobatto-m132
ratio: 0.5
(array([-10.5   , -10.14141594, -4.03618727, -3.5   ,
        -3.04545575, -2.52342678, -2.24468465, -1.75296197,
        -1.72125695, -1.52483422, -1.49687198, -1.34991331,
        -1.30958379, -1.15282816, -0.78635667, -0.65354751,
         0.   ,   0.65354751,   0.78635667,   1.15282816,
         1.30958379,   1.34991331,   1.49687198,   1.52483422,
         1.72125695,   1.75296197,   2.24468465,   2.52342678,
         3.04545575,   3.5   ,   4.03618727, 10.14141594,
        10.5   ]), array([12,   12,   12,   12,   12,   12,   12,   12,
 12,   12,   12,
        12,   12,   12,   12,   12, 2208,   12,   12,   12,   12,   12,
        12,   12,   12,   12,   12,   12,   12,   12,   12,   12,   12]))
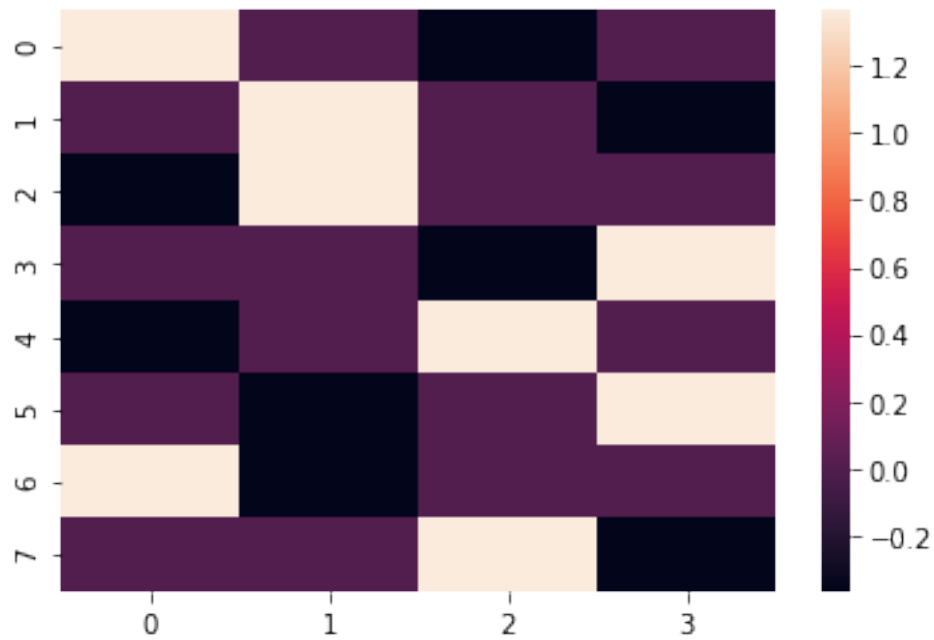
```
[10]: for name, data in matrices["quad"].items():
          dims = data.shape

          if dims[0]/dims[1] == 2.0:
              print(name)
              print("ratio: " + str(dims[0]/dims[1]))
              print(np.unique(data, return_counts=True))
              plt.figure()
              ax = sns.heatmap(data)
              plt.show()
```
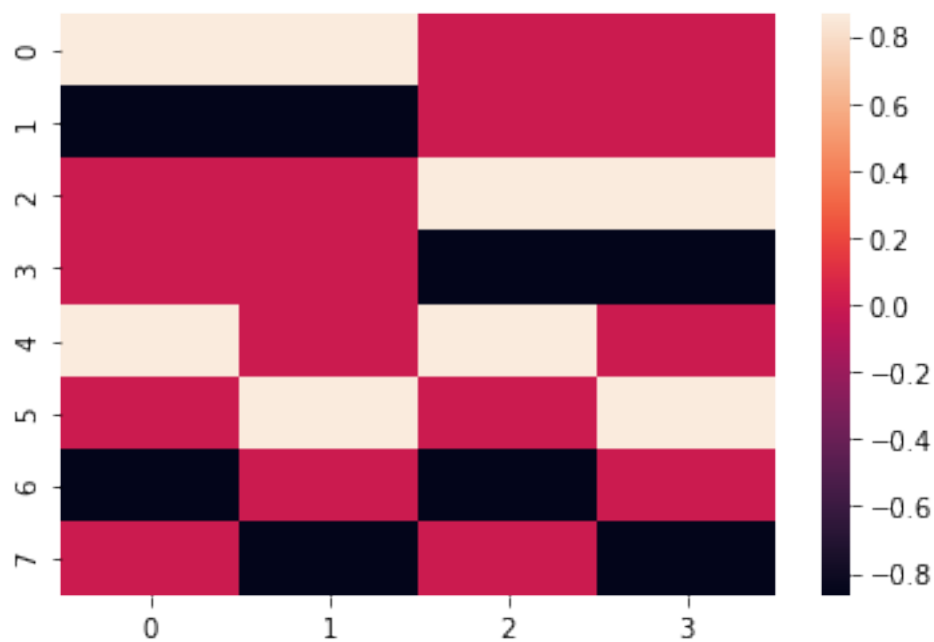
```
p1-gauss-legendre-m0
ratio: 2.0
(array([-0.3660254,  0.       ,  1.3660254]), array([ 8, 16,  8]))
```
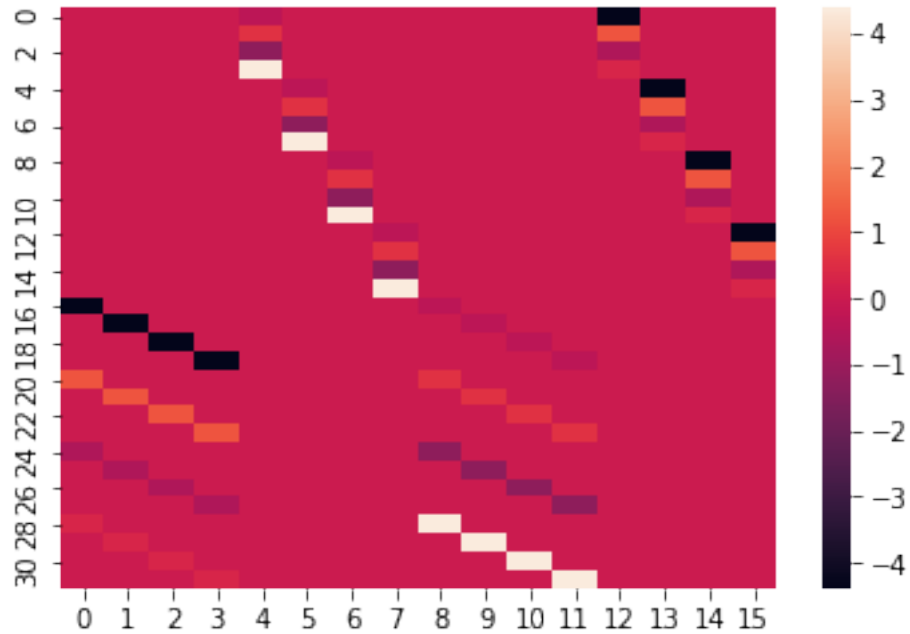
p1-gauss-legendre-m460
ratio: 2.0
(array([-0.8660254, 0.   , 0.8660254]), array([8, 16, 8]))

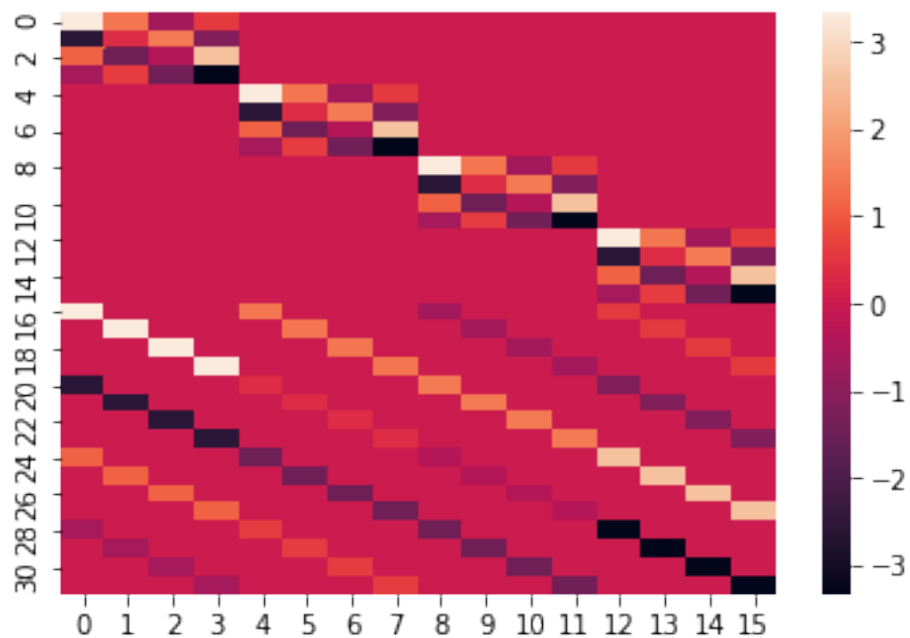p3-gauss-legendre-m6

ratio: 2.0
(array([-4.38915297, -1.24762477, -0.6145281 , -0.32748486, 0.   ,
        0.32748486, 0.6145281 , 1.24762477, 4.38915297]), array([8,   8,
8,   8, 448,   8,   8,   8,   8]))

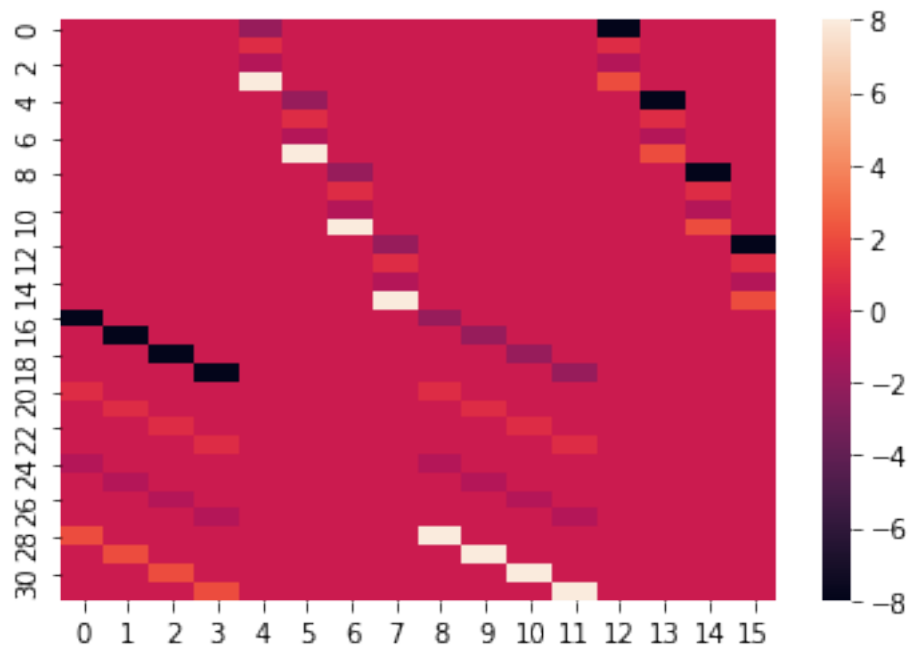

p3-gauss-legendre-m460
ratio: 2.0
(array([-3.33200024, -2.59241098, -1.47067023, -1.42024047, -1.12482651,
       -0.61623104, -0.58062817, -0.38441439, 0.   , 0.38441439,
        0.58062817, 0.61623104, 1.12482651, 1.42024047, 1.47067023,
        2.59241098, 3.33200024]), array([8,   8,   8,   8,   8,   8,   8,
8, 384,   8,   8,   8,   8,
         8,   8,   8,   8]))

p3-gauss-legendre-lobatto-m6
ratio: 2.0
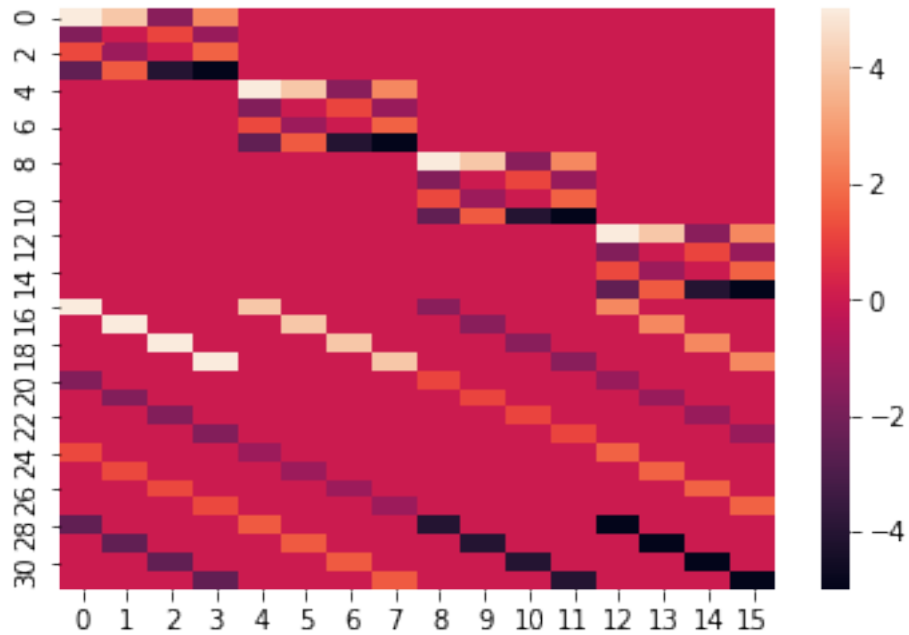(array([-8.   , -2.   , -0.89442719, 0.   , 0.89442719,
        2.   , 8.   ]), array([8,   8, 16, 448, 16,   8,   8]))

```
p3-gauss-legendre-lobatto-m460
ratio: 2.0
(array([-5.        , -4.04508497, -2.5        , -1.70344419, -1.54508497,
        -1.20344419, -1.11803399,  0.        ,  1.11803399,  1.20344419,
         1.54508497,  1.70344419,  2.5        ,  4.04508497,  5.        ]),
array([  8,   8,   8,   8,   8,   8,   8, 400,   8,   8,   8,   8,   8,
          8,   8]))
```
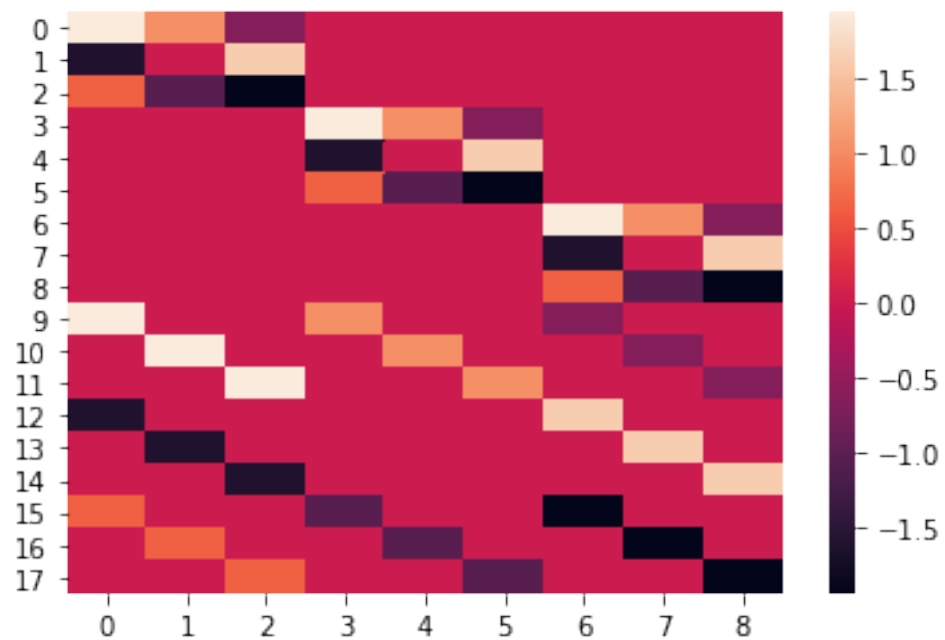


```
p2-gauss-legendre-m460
ratio: 2.0
(array([-1.93649167, -1.61374306, -1.03279556, -0.64549722,  0.        ,
         0.64549722,  1.03279556,  1.61374306,  1.93649167]), array([  6,   6,
  6,   6, 114,   6,   6,   6,   6]))
```
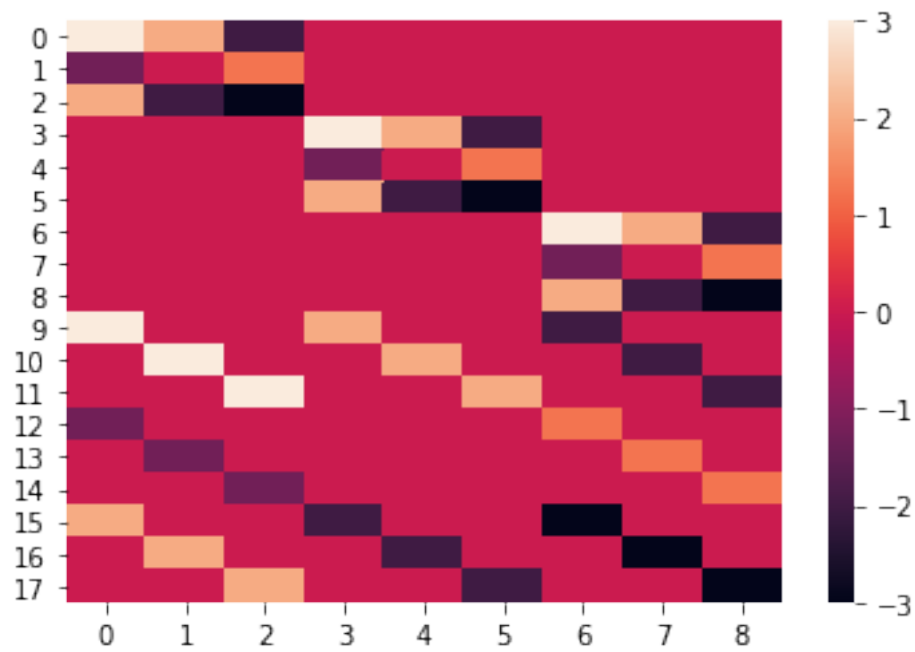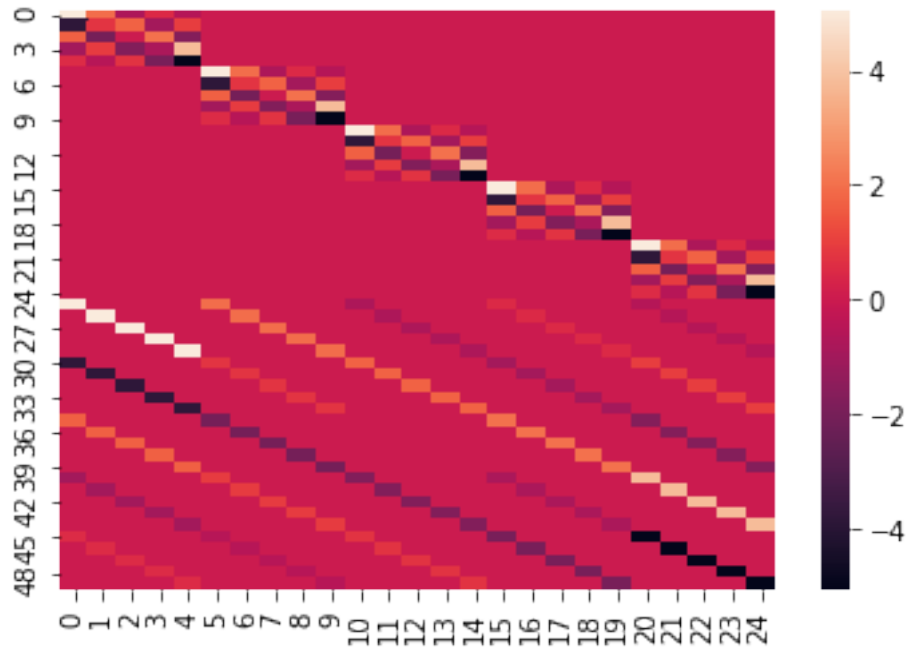
p2-gauss-legendre-lobatto-m460
ratio: 2.0
(array([-3.   , -2.   , -1.25, 0. , 1.25, 2. , 3.]), array([6, 12,   6,
114,   6, 12,   6]))

p4-gauss-legendre-m460
ratio: 2.0
(array([-5.0670406 , -3.81255789, -2.02152894, -1.93986454, -1.70607503,
       -1.68402696, -0.97042088, -0.92855803, -0.75835322, -0.72313963,
       -0.55176685, -0.49375908, 0.   , 0.49375908, 0.55176685,
        0.72313963, 0.75835322, 0.92855803, 0.97042088, 1.68402696,
        1.70607503, 1.93986454, 2.02152894, 3.81255789, 5.0670406]),
array([10,   10,   10,   10,   10,   10,   10,   10,   10,   10,   10,
         10, 1010,   10,   10,   10,   10,   10,   10,   10,   10,   10,
         10,   10,   10]))

p4-gauss-legendre-lobatto-m460
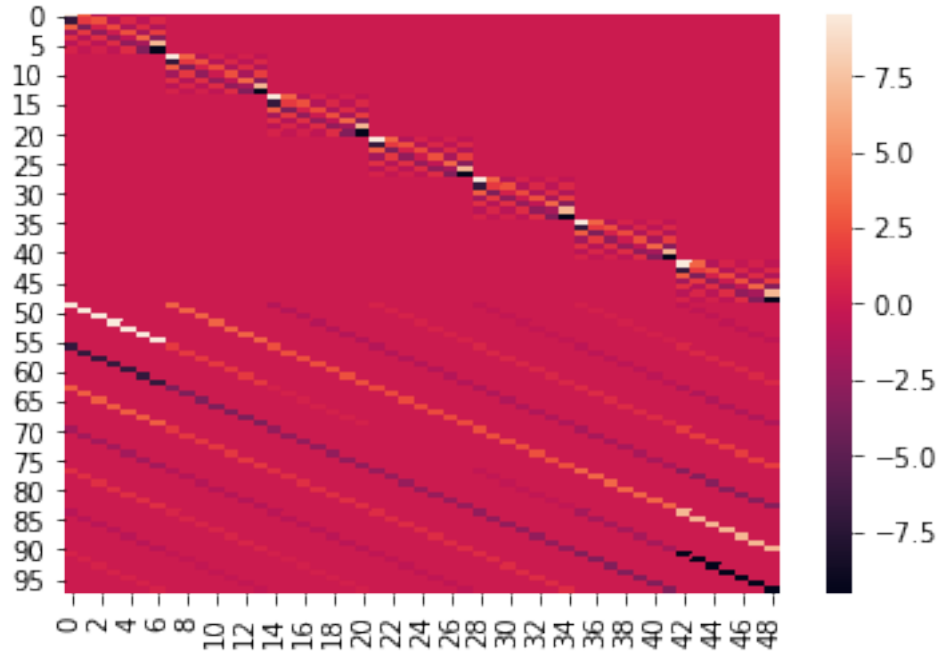ratio: 2.0
(array([-7.5   , -6.75650249, -3.   , -2.66666667, -2.31241882,
       -1.74574312, -1.41016418, -1.33658458, -1.33043832, -1.3125   ,
       -0.76376262, 0.   , 0.76376262, 1.3125   , 1.33043832,
        1.33658458, 1.41016418, 1.74574312, 2.31241882, 2.66666667,
        3.   , 6.75650249, 7.5   ]), array([10,   10,   10,   10,
    10,   10,   10,   10,   10,   10,   10,
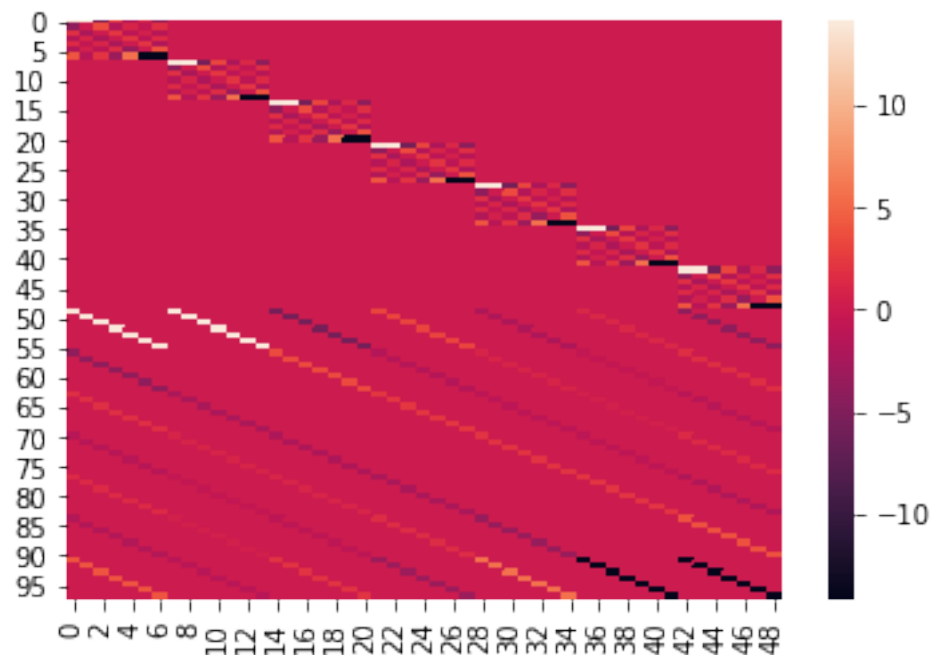   1030,   10,   10,   10,   10,   10,   10,   10,   10,   10,   10,
       10]))
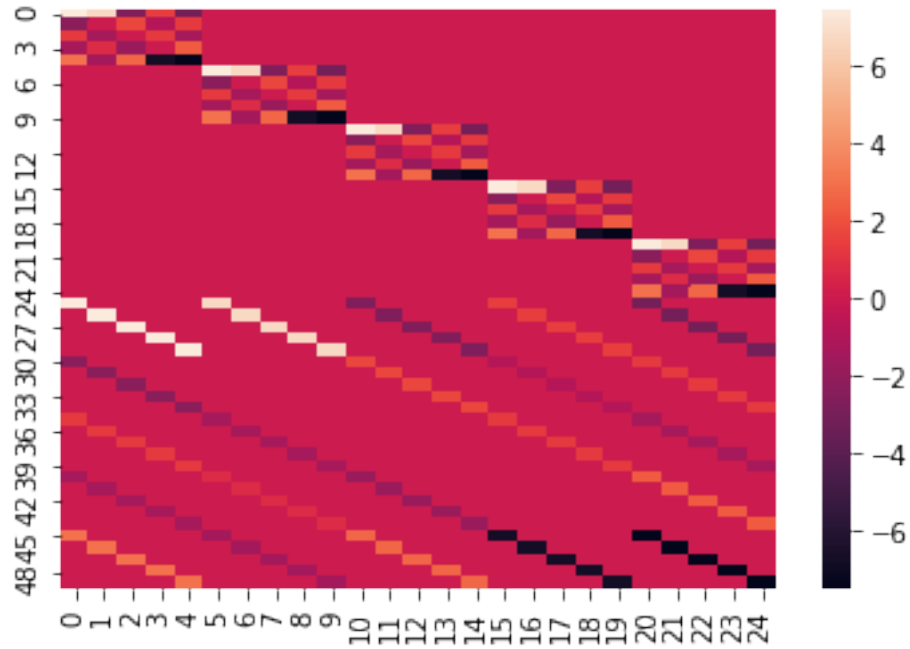
p6-gauss-legendre-m460
ratio: 2.0
(array([-9.56818234, -6.98244017, -3.47320931, -3.3238044 , -3.11057141,
       -2.57685029, -2.5550682 , -2.3560803 , -1.86201883, -1.64736568,
       -1.6443125 , -1.24717059, -1.23199698, -1.10600426, -1.08928106,
       -1.01615121, -0.85730425, -0.74753216, -0.67428047, -0.59619008,
       -0.52681049, -0.48587352, -0.43674268, -0.40809682, 0.   ,
        0.40809682, 0.43674268, 0.48587352, 0.52681049, 0.59619008,
        0.67428047, 0.74753216, 0.85730425, 1.01615121, 1.08928106,
        1.10600426, 1.23199698, 1.24717059, 1.6443125 , 1.64736568,
        1.86201883, 2.3560803 , 2.5550682 , 2.57685029, 3.11057141,
        3.3238044 , 3.47320931, 6.98244017, 9.56818234]), array([14,   14,
   14,   14,   14,   14,   14,   14,   14,   14,   14,
        14,   14,   14,   14,   14,   14,   14,   14,   14,   14,
        14,   14, 4130,   14,   14,   14,   14,   14,   14,   14,
        14,   14,   14,   14,   14,   14,   14,   14,   14,   14,
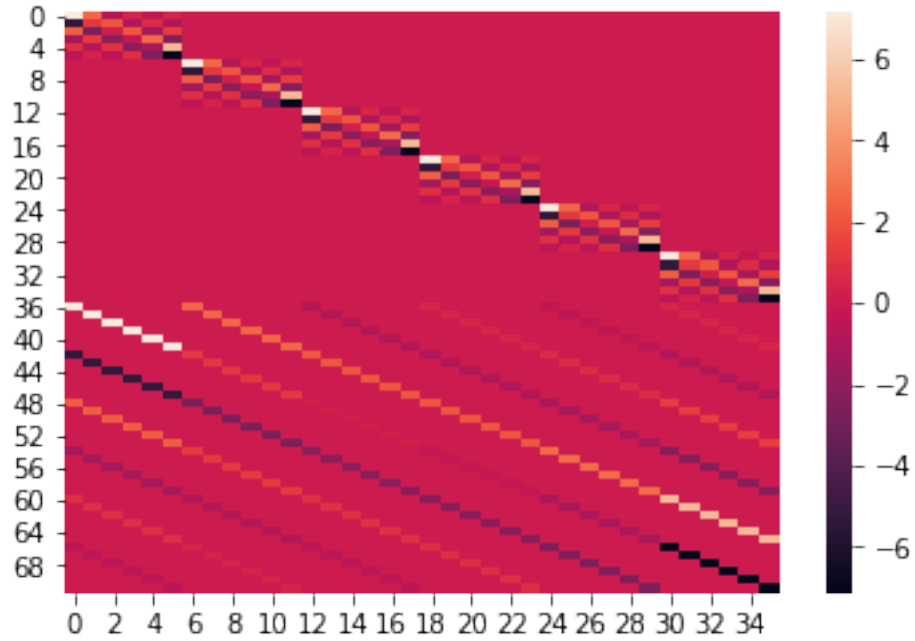   14,   14,   14,   14,   14]))

p6-gauss-legendre-lobatto-m460
ratio: 2.0
(array([-14.2015766 , -14.   , -5.66898523, -4.   ,
       -3.89455263, -3.45582821, -3.2   , -2.26669809,
       -2.21580428, -2.04996481, -2.00696924, -1.78762708,
       -1.67823848, -1.59860669, -1.40625   , -1.38846981,
       -1.31737344, -1.0664419 , -0.9613398 , -0.90754447,
       -0.61639084, -0.60224718, 0.   , 0.60224718,
        0.61639084, 0.90754447, 0.9613398 , 1.0664419 ,
        1.31737344, 1.38846981, 1.40625   , 1.59860669,
        1.67823848, 1.78762708, 2.00696924, 2.04996481,
        2.21580428, 2.26669809, 3.2   , 3.45582821,
        3.89455263, 4.   , 5.66898523, 14.   ,
       14.2015766]), array([14,   14,   14,   14,   14,   14,   14,   14,
  14,   14,   14,
        14,   14,   14,   14,   14,   14,   14,   14,   14,   14,   14,
     4186,   14,   14,   14,   14,   14,   14,   14,   14,   14,   14,
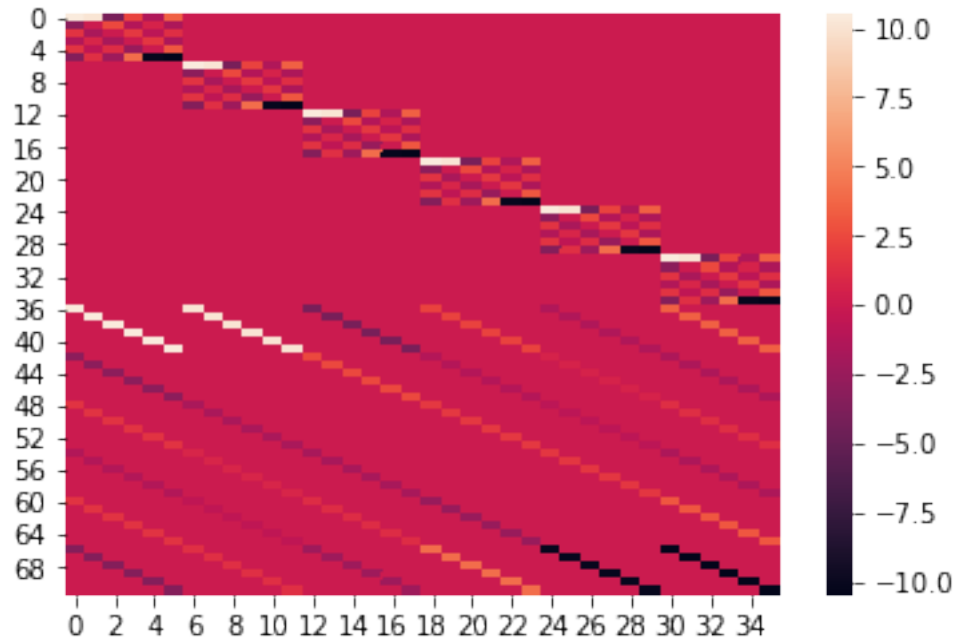   14,   14,   14,   14,   14,   14,   14,   14,   14,   14,   14,
        14]))

p5-gauss-legendre-m460
ratio: 2.0
(array([-7.14532713, -5.27576253, -2.68967445, -2.57598208, -2.3443613 ,
       -2.09538893, -2.08190754, -1.38899458, -1.2631629 , -1.17485228,
       -0.97773482, -0.89798768, -0.88602069, -0.75619011, -0.53621056,
       -0.52495233, -0.43845798, -0.25302628, 0.   , 0.25302628,
        0.43845798, 0.52495233, 0.53621056, 0.75619011, 0.88602069,
        0.89798768, 0.97773482, 1.17485228, 1.2631629 , 1.38899458,
        2.08190754, 2.09538893, 2.3443613 , 2.57598208, 2.68967445,
        5.27576253, 7.14532713]), array([12,   12,   12,   12,   12,   12,
 12,   12,   12,   12,   12,
        12,   12,   12,   12,   12,   12,   12, 2160,   12,   12,   12,
        12,   12,   12,   12,   12,   12,   12,   12,   12,   12,   12,
        12,   12,   12,   12]))

22

p5-gauss-legendre-lobatto-m460
ratio: 2.0
(array([-10.5   , -10.14141594, -4.03618727, -3.5   ,
        -3.04545575, -2.52342678, -2.24468465, -1.75296197,
        -1.72125695, -1.52483422, -1.49687198, -1.34991331,
        -1.30958379, -1.15282816, -0.78635667, -0.65354751,
         0.   ,   0.65354751,   0.78635667,   1.15282816,
         1.30958379,   1.34991331,   1.49687198,   1.52483422,
         1.72125695,   1.75296197,   2.24468465,   2.52342678,
         3.04545575,   3.5   ,   4.03618727, 10.14141594,
        10.5   ]), array([12,   12,   12,   12,   12,   12,   12,   12,
        12,   12,   12,
        12,   12,   12,   12,   12, 2208,   12,   12,   12,   12,   12,
        12,   12,   12,   12,   12,   12,   12,   12,   12,   12,   12]))
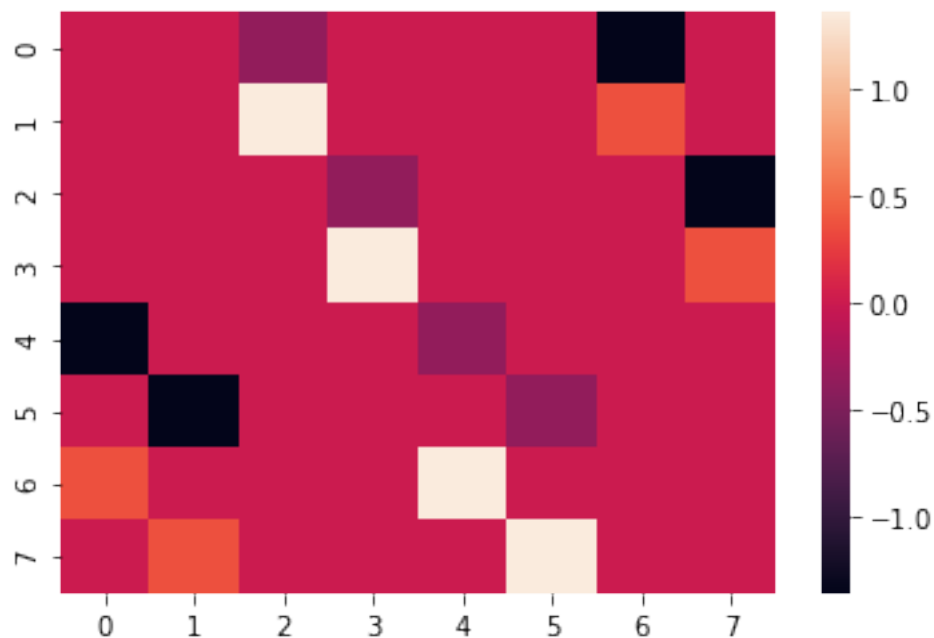
```
[11]: for name, data in matrices["quad"].items():
          dims = data.shape

          if dims[0]/dims[1] != 2.0 and dims[0]/dims[1] != 0.5:
              print(name)
              print("ratio: " + str(dims[0]/dims[1]))
              print(np.unique(data, return_counts=True))
              plt.figure()
              ax = sns.heatmap(data)
              plt.show()
```

```
p1-gauss-legendre-m6
ratio: 1.0
(array([-1.3660254, -0.3660254,  0.        ,  0.3660254,  1.3660254]), array([ 4,
4, 48,  4,  4]))
```
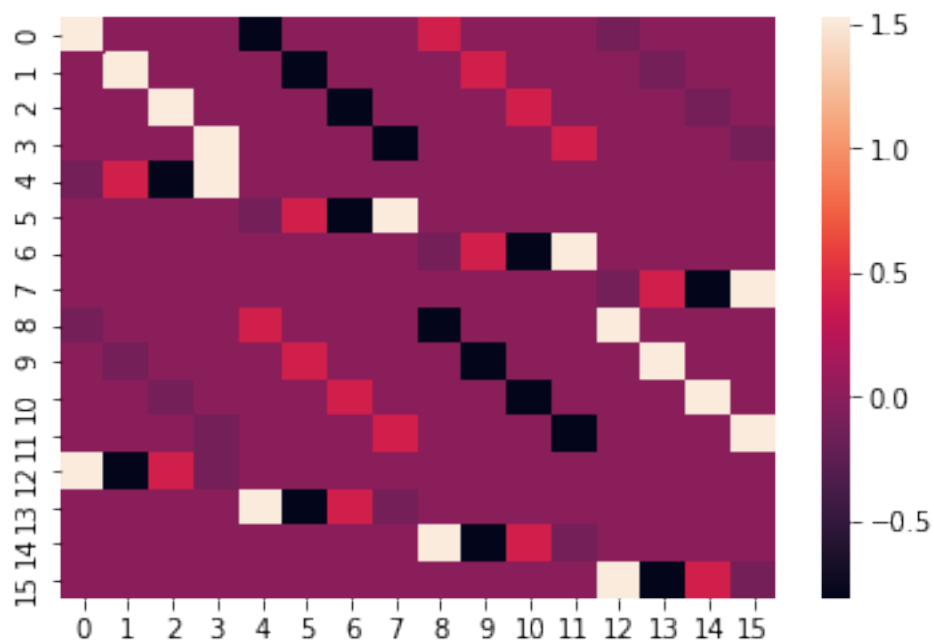
p3-gauss-legendre-m0
ratio: 1.0
(array([-0.81363245, -0.1139172 , 0.   , 0.40076152, 1.52678813]),
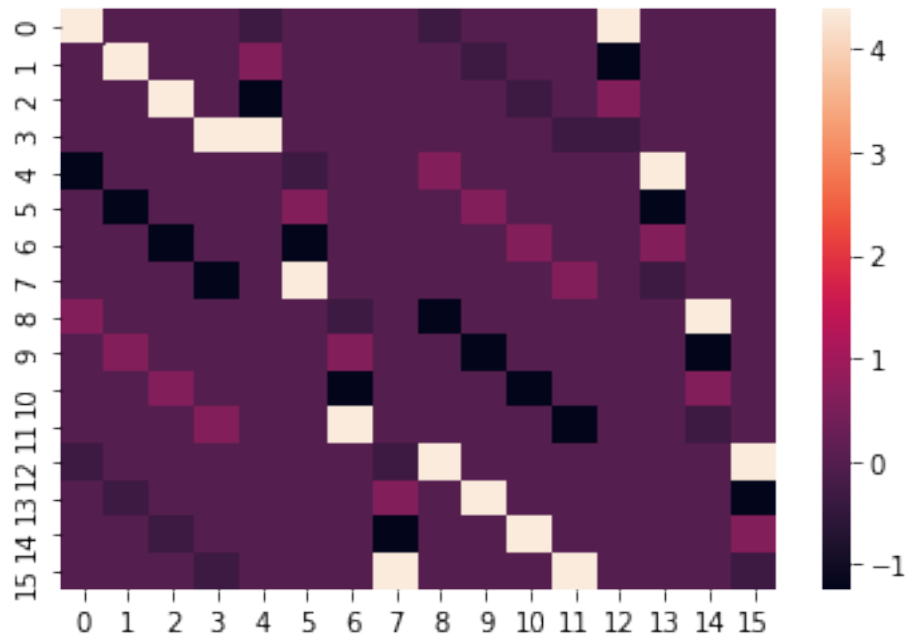array([16, 16, 192, 16, 16]))
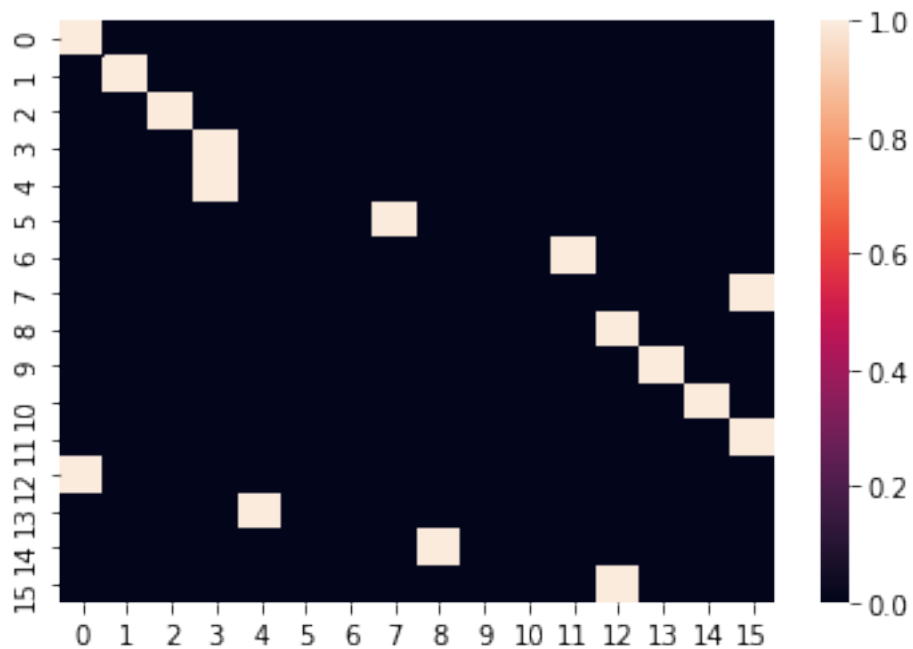
```
p3-gauss-legendre-m3
ratio: 1.0
(array([-1.24762477, -0.32748486,  0.        ,  0.6145281 ,  4.38915297]),
array([ 16,  16, 192,  16,  16]))
```
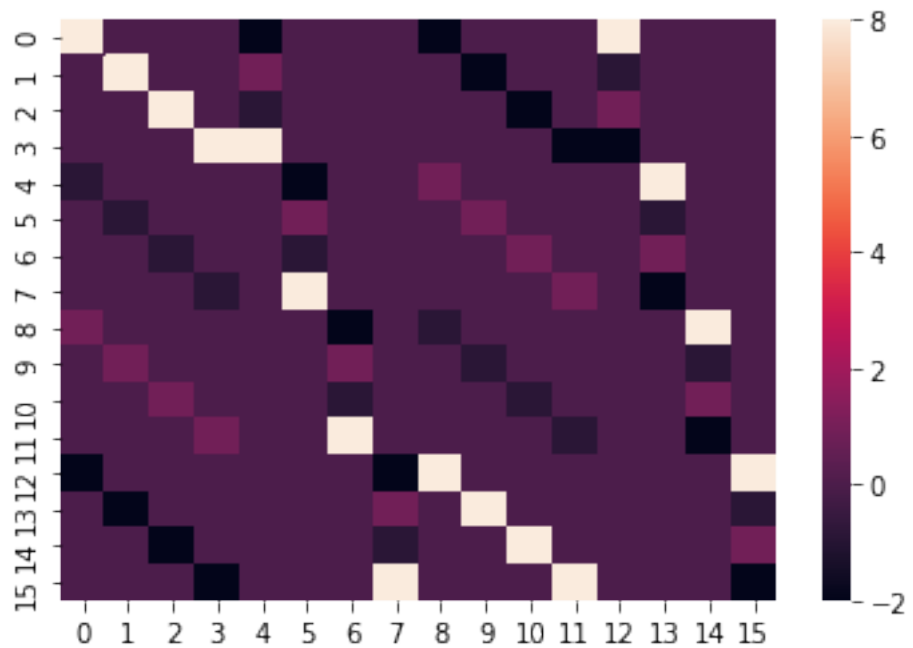


```
p3-gauss-legendre-lobatto-m0
ratio: 1.0
(array([0., 1.]), array([240,  16]))
```

p3-gauss-legendre-lobatto-m3
ratio: 1.0
(array([-2.   , -0.89442719, 0.   , 0.89442719, 8.   ]),
array([16, 16, 192, 16, 16]))

p2-gauss-legendre-m6

ratio: 1.5

(array([-2.661895, -0.75   , -0.338105, 0.   , 0.338105, 0.75   ,
        2.661895]), array([6,   6,   6, 180,   6,   6,   6]))
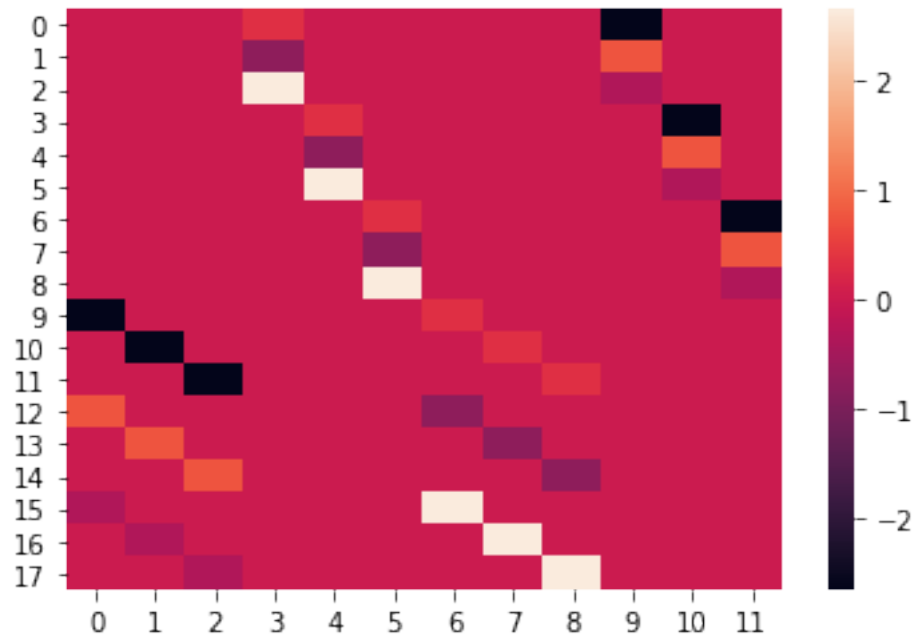
p2-gauss-legendre-m0

ratio: 1.3333333333333333

(array([-0.66666667, 0.   , 0.18783611, 1.47883056]), array([12, 72, 12,
12]))
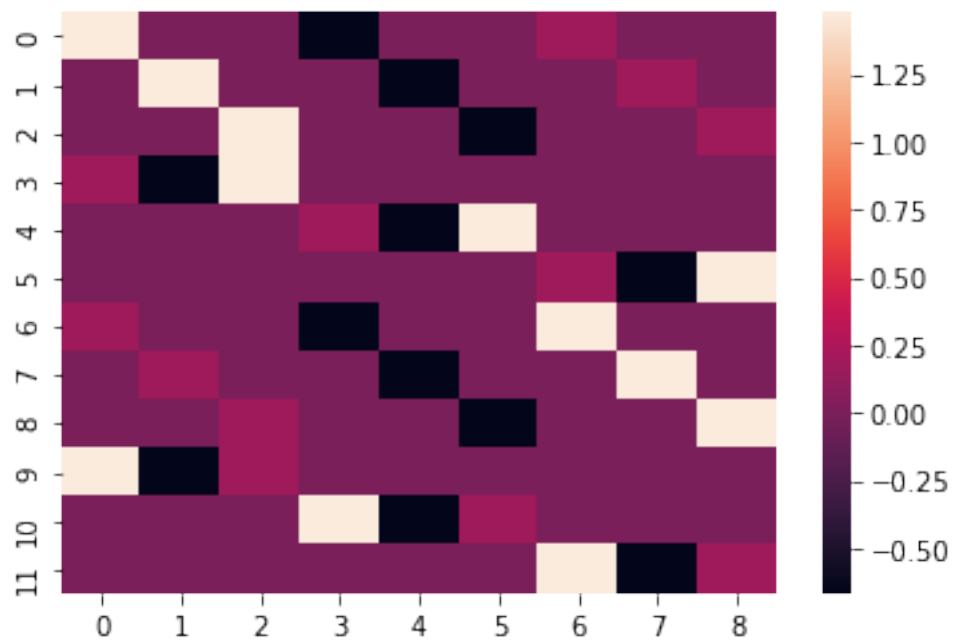
p2-gauss-legendre-m3
ratio: 0.75
(array([-0.75   , 0.   , 0.338105, 2.661895]), array([12, 72, 12, 12]))
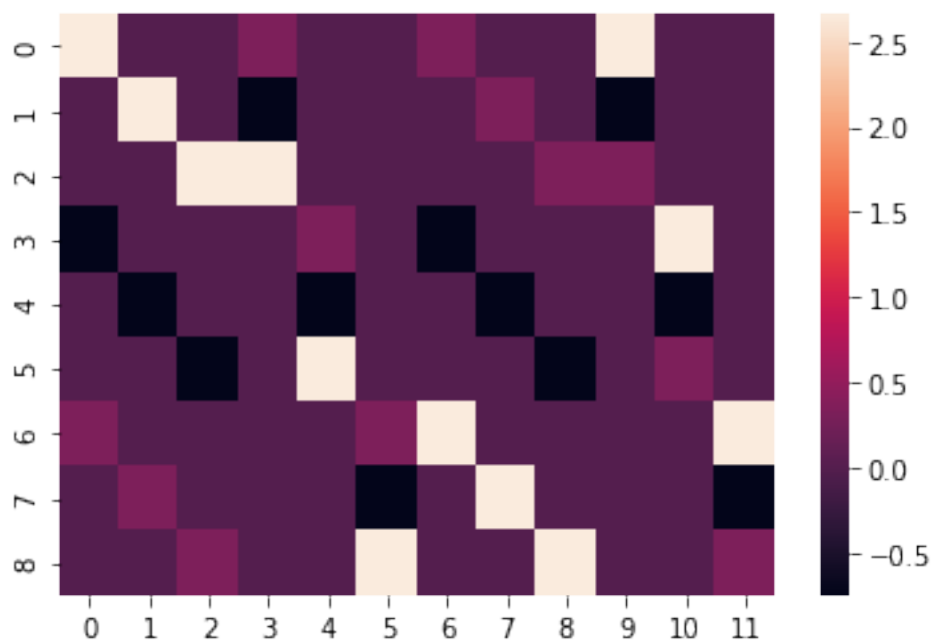
p2-gauss-legendre-lobatto-m6

ratio: 1.5
(array([-4.5 , -1.5 , -0.75, 0. , 0.75, 1.5 , 4.5]), array([6,   6,   6,
180,   6,   6,   6]))
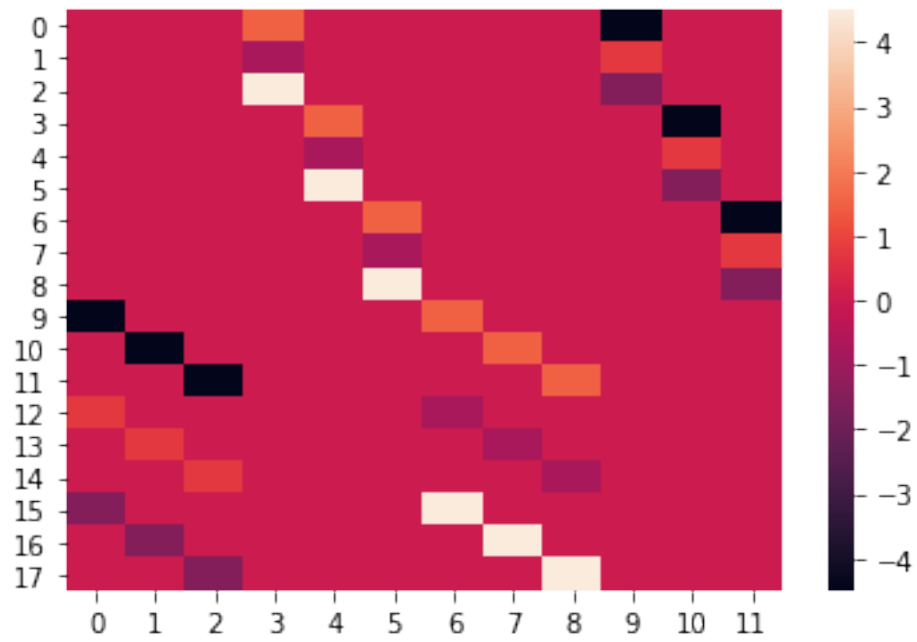
p2-gauss-legendre-lobatto-m0
ratio: 1.3333333333333333
(array([0., 1.]), array([96, 12]))
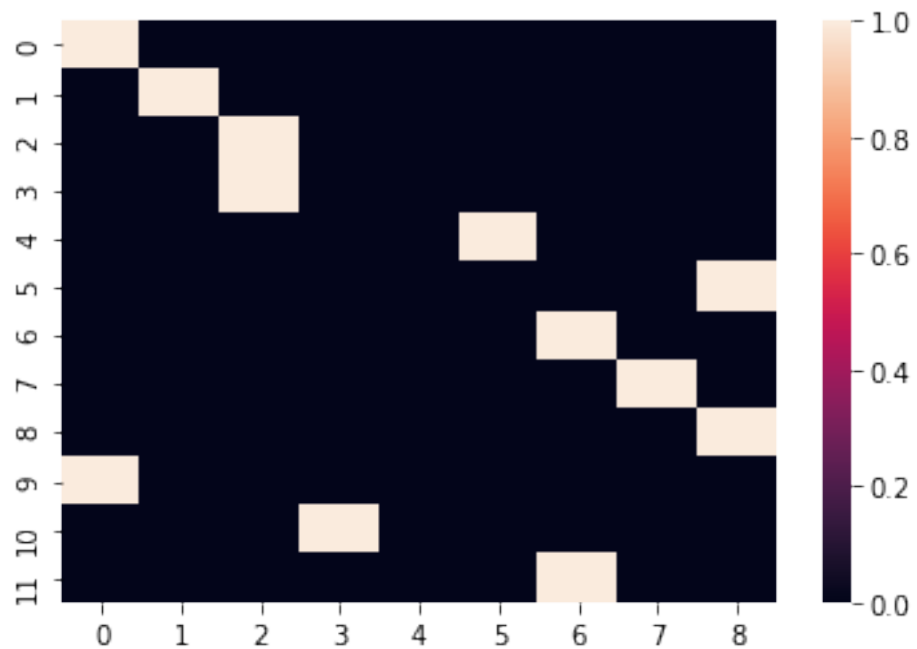
```
p2-gauss-legendre-lobatto-m3
ratio: 0.75
(array([-0.75,  0.  ,  1.5 ,  4.5 ]), array([12, 72, 12, 12]))
```
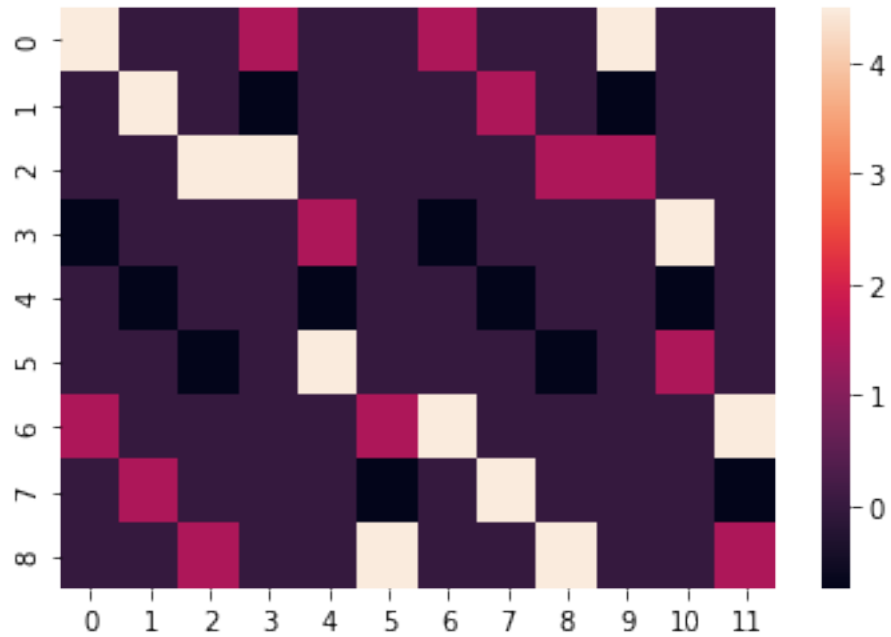


```
p4-gauss-legendre-m6
ratio: 2.5
(array([-6.54804561, -1.86607792, -0.9375    , -0.55981112, -0.32228787,
        0.        ,  0.32228787,  0.55981112,  0.9375    ,  1.86607792,
        6.54804561]), array([ 10,  10,  10,  10,  10, 900,  10,  10,  10,  10,
10]))
```
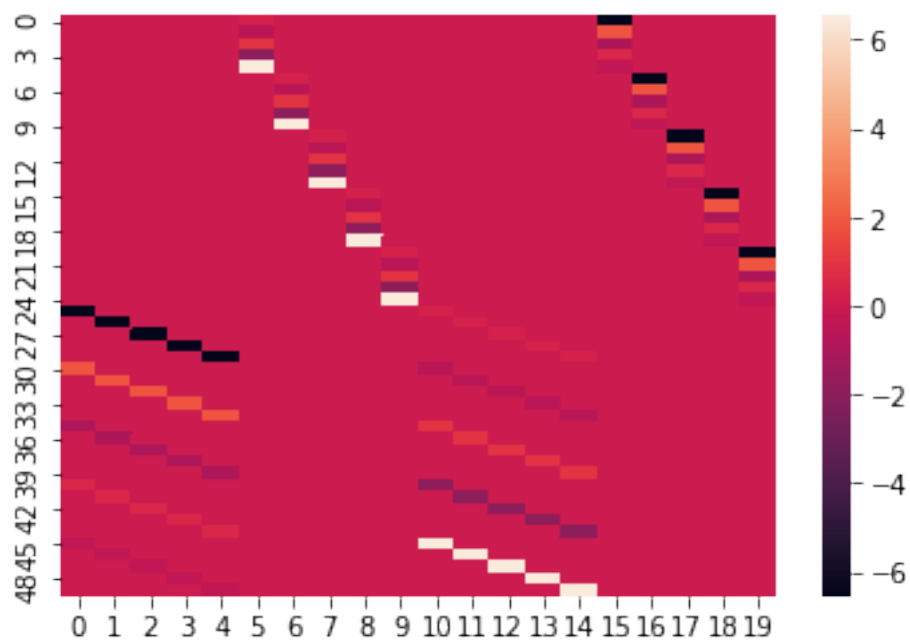
p4-gauss-legendre-m0
ratio: 0.8
(array([-0.89315839, -0.26794165, 0.   , 0.07635866, 0.53333333,
        1.55140805]), array([20, 20, 400, 20, 20, 20]))
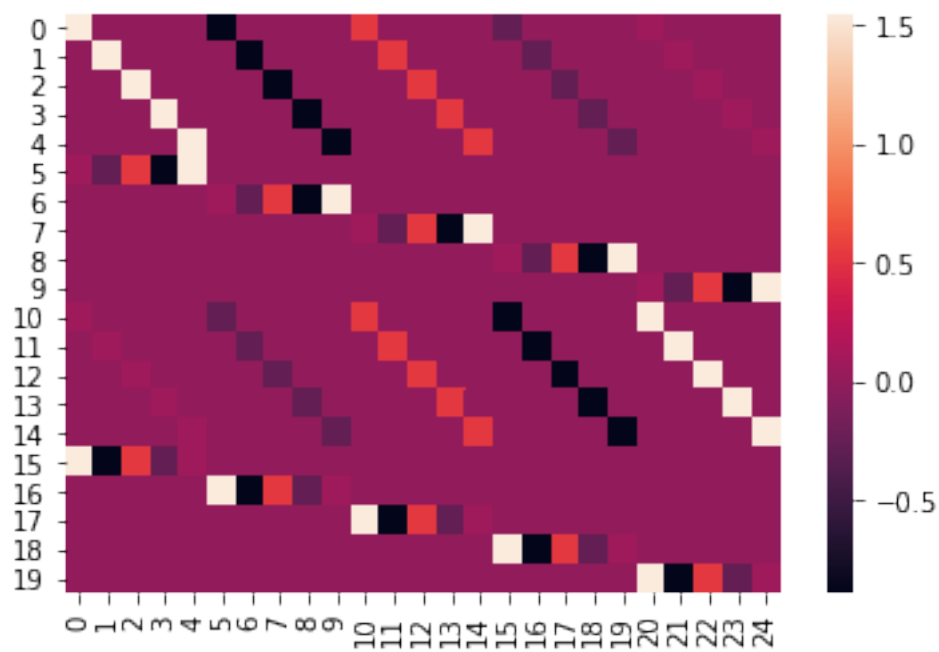
p4-gauss-legendre-m3
ratio: 1.25
(array([-1.86607792, -0.55981112, 0.   , 0.32228787, 0.9375   ,
        6.54804561]), array([20, 20, 400, 20, 20, 20]))
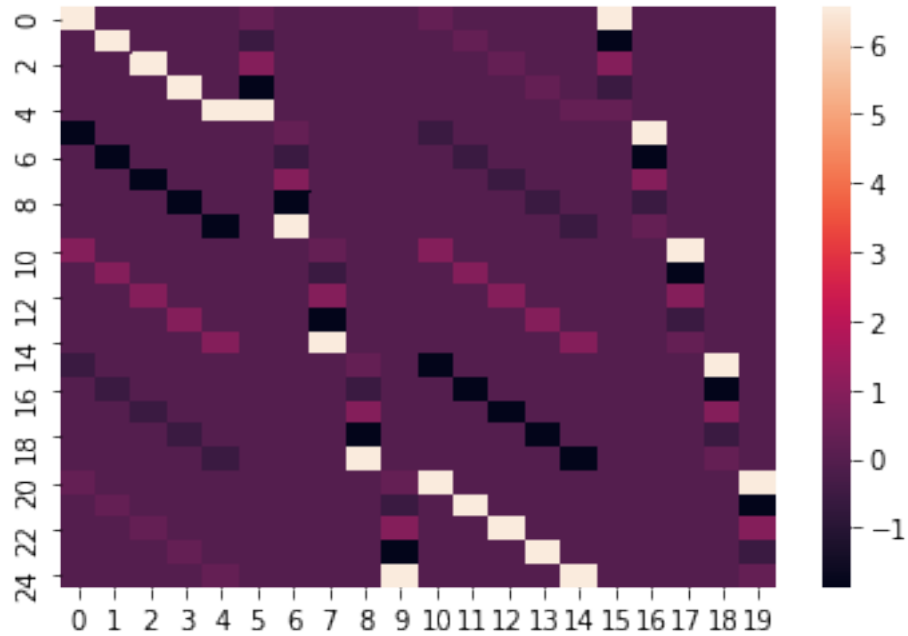
p4-gauss-legendre-lobatto-m6
ratio: 2.5
(array([-12.5   , -2.5   , -1.07142857, -0.9375   ,
         0.   , 0.9375   , 1.07142857, 2.5   ,
        12.5   ]), array([10, 10, 20, 10, 900, 10, 20, 10, 10]))
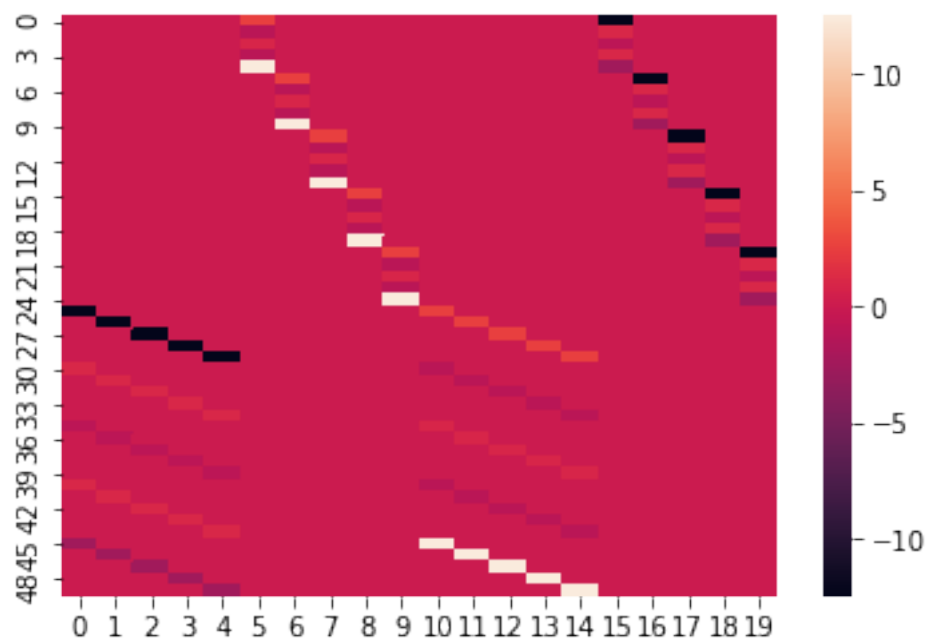
```
p4-gauss-legendre-lobatto-m0
ratio: 0.8
(array([0., 1.]), array([480,  20]))
```
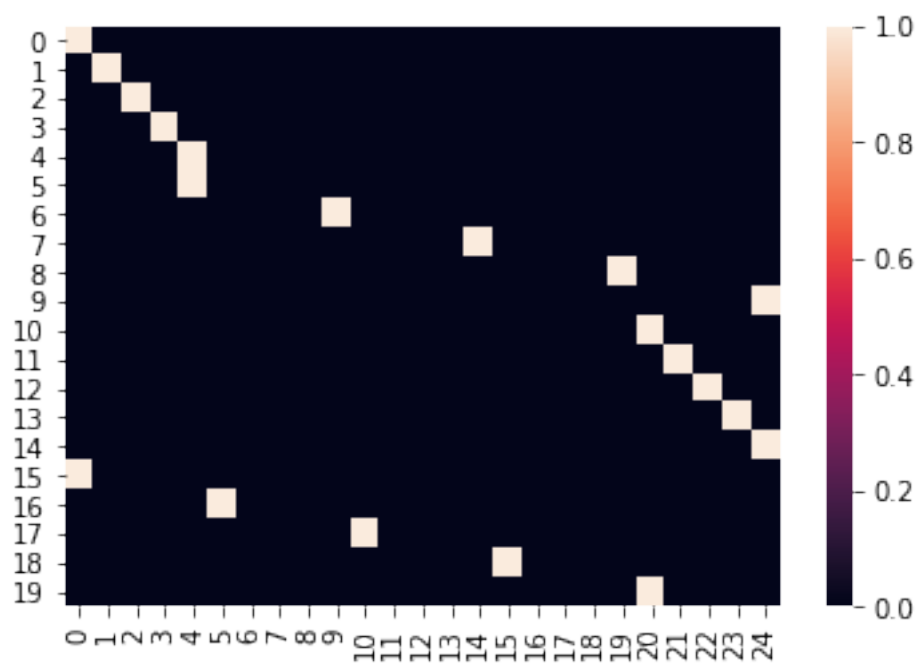
```
p4-gauss-legendre-lobatto-m3
ratio: 1.25
(array([-1.07142857,  0.        ,  0.9375    ,  2.5       , 12.5       ]),
array([ 40, 400,  20,  20,  20]))
```
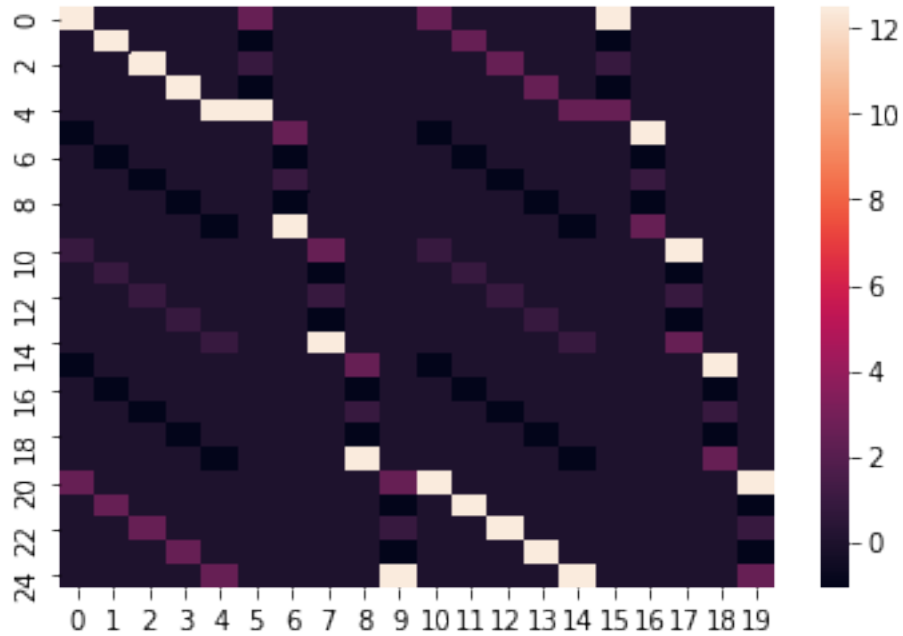


```
p6-gauss-legendre-m6
ratio: 3.5
(array([-12.16096777,  -3.47053266,  -1.76022778,  -1.09375   ,
         -0.74392821,  -0.51507804,  -0.31752836,   0.        ,
          0.31752836,   0.51507804,   0.74392821,   1.09375   ,
          1.76022778,   3.47053266,  12.16096777]), array([ 14,   14,   14,
 14,   14,   14,   14, 2548,   14,   14,   14,
         14,   14,   14,   14]))
```
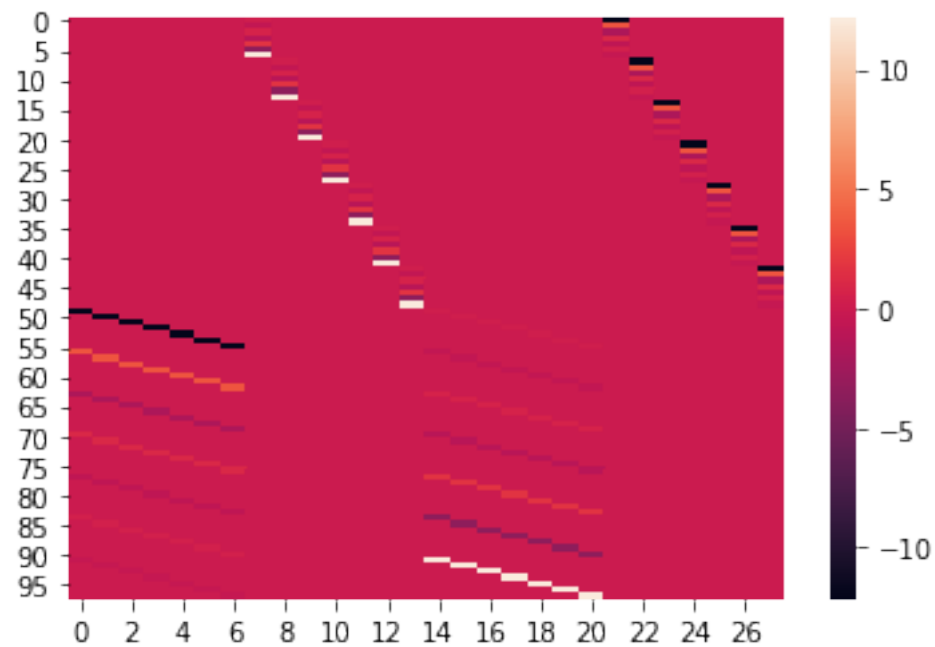
p6-gauss-legendre-m0
ratio: 0.5714285714285714
(array([-0.9707267 , -0.45714286, -0.1440701 , 0.   , 0.04111515,
        0.28405415, 0.67210786, 1.5746625]), array([28,   28,   28, 1176,
   28,   28,   28,   28]))
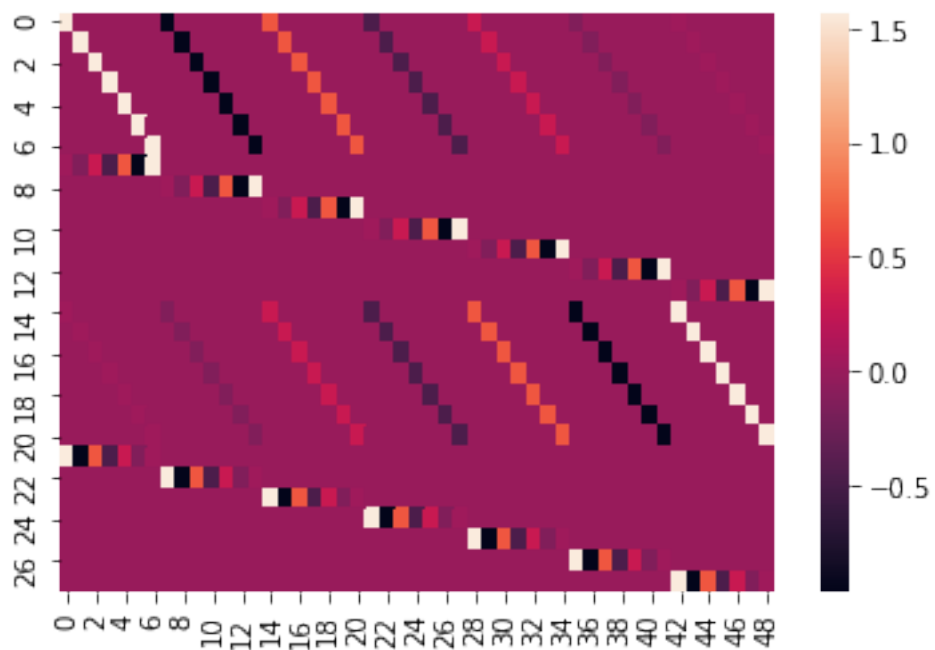
p6-gauss-legendre-m3
ratio: 1.75
(array([-3.47053266, -1.09375   , -0.51507804, 0.   , 0.31752836,
        0.74392821, 1.76022778, 12.16096777]), array([28,   28,   28, 1176,
   28,   28,   28,   28]))
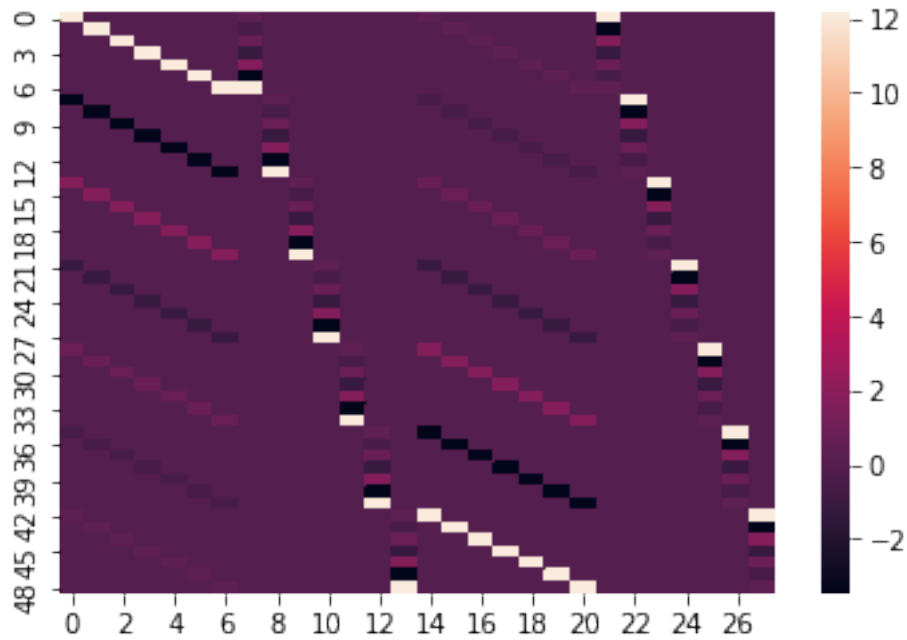
p6-gauss-legendre-lobatto-m6
ratio: 3.5
(array([-24.5   , -3.5   , -1.45162661, -1.16237041,
       -1.09375   ,   0.   ,   1.09375   ,   1.16237041,
        1.45162661,   3.5   , 24.5   ]), array([14,   14,   28,
   28,   14, 2548,   14,   28,   28,   14,   14]))
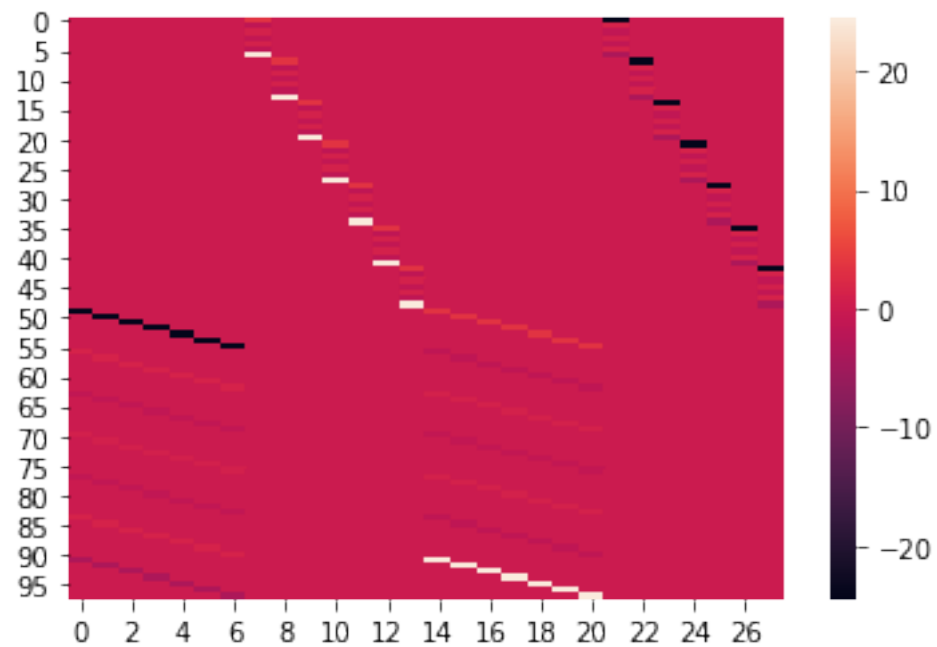
p6-gauss-legendre-lobatto-m0
ratio: 0.5714285714285714
(array([0., 1.]), array([1344,   28]))
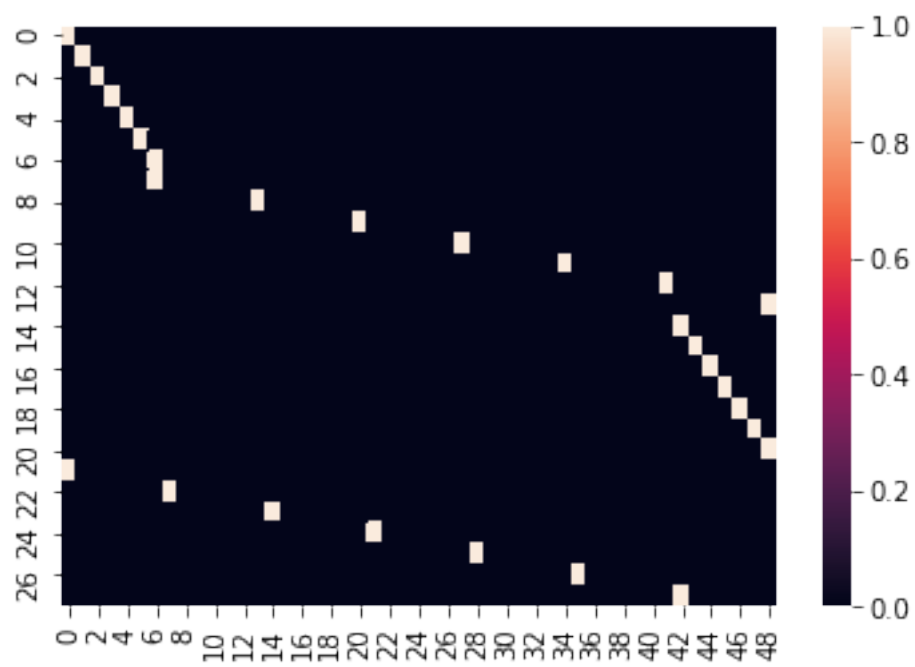
```
p6-gauss-legendre-lobatto-m3
ratio: 1.75
(array([-1.45162661, -1.09375   ,  0.         ,  1.16237041,  3.5        ,
        24.5        ]), array([  56,   28, 1176,   56,   28,   28]))
```
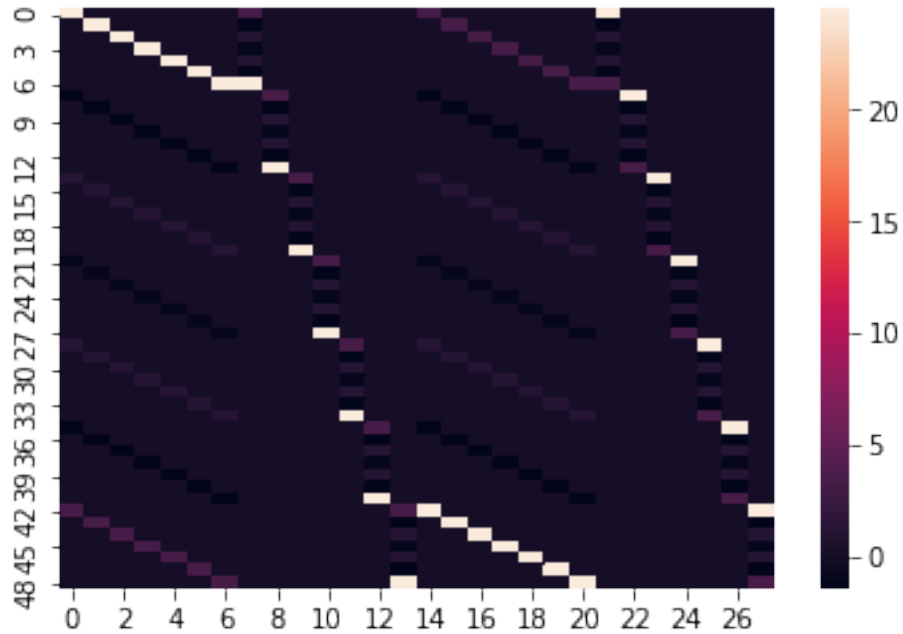


```
p5-gauss-legendre-m6
ratio: 3.0
(array([-9.13864199, -2.60688198, -1.31846908, -0.81046465, -0.53165311,
        -0.31935145,  0.         ,  0.31935145,  0.53165311,  0.81046465,
         1.31846908,  2.60688198,  9.13864199]), array([  12,   12,   12,   12,
   12,   12, 1584,   12,   12,   12,   12,
         12,   12]))
```
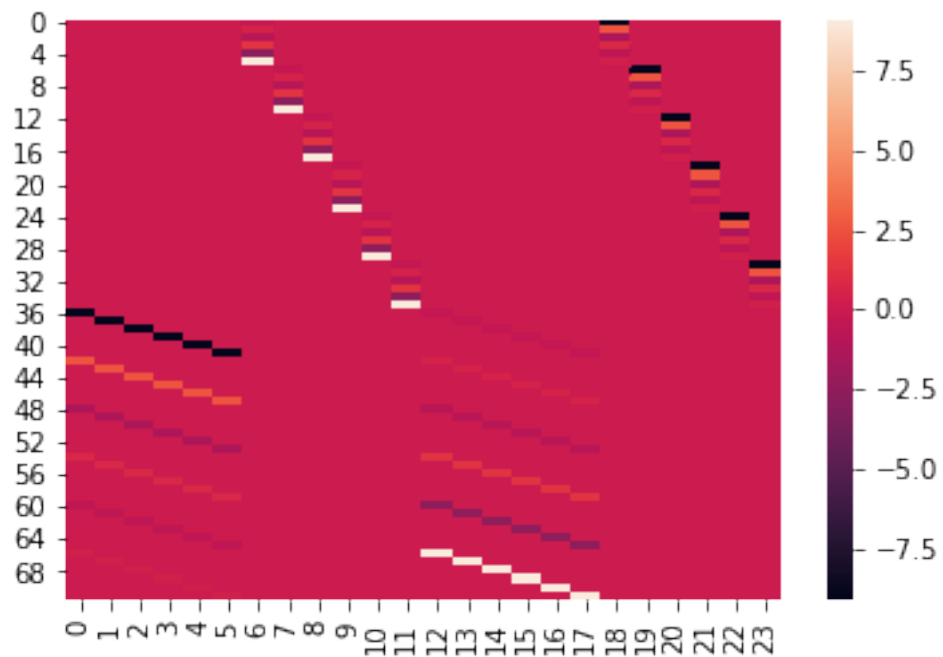
p5-gauss-legendre-m0
ratio: 0.6666666666666666
(array([-0.94046284, -0.3792277 , -0.05471272, 0.   , 0.19180001,
        0.61693006, 1.5656732]), array([24,   24,   24, 720,   24,   24,   24]))
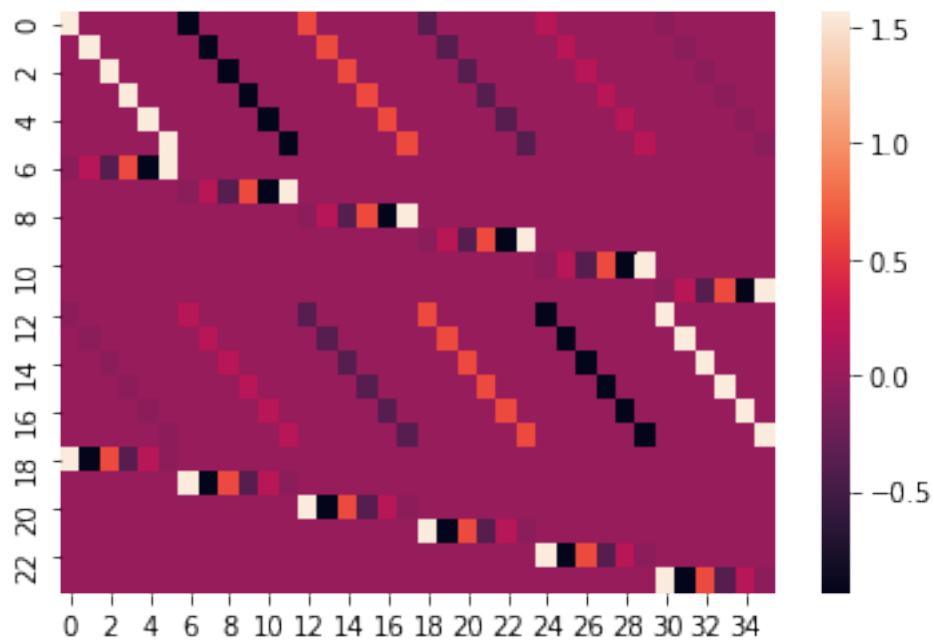
```
p5-gauss-legendre-m3
ratio: 1.5
(array([-2.60688198, -0.81046465, -0.31935145,  0.        ,  0.53165311,
        1.31846908,  9.13864199]), array([ 24,  24,  24, 720,  24,  24,  24]))
```
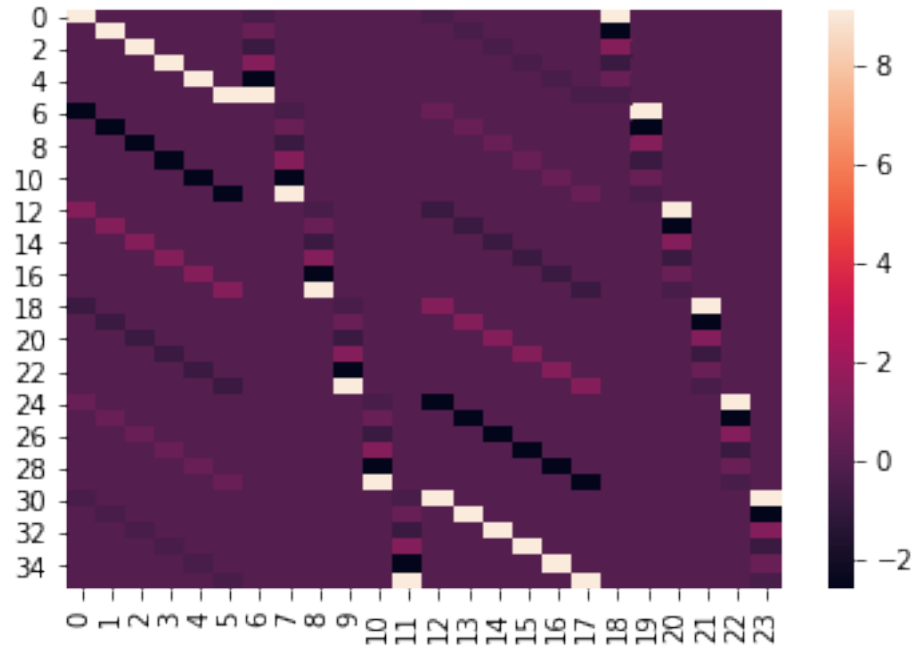


```
p5-gauss-legendre-lobatto-m6
ratio: 3.0
(array([-18.        ,  -3.        ,  -1.2590908 ,  -1.03988318,
         0.        ,   1.03988318,   1.2590908 ,   3.        ,
        18.        ]), array([  12,   12,   24,   24, 1584,   24,   24,   12,
12]))
```
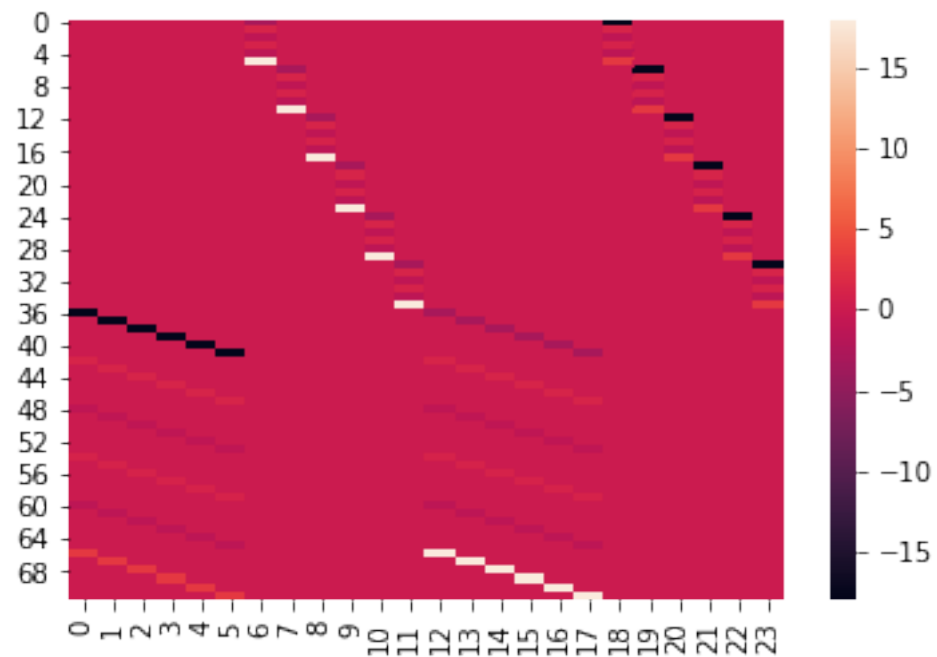
p5-gauss-legendre-lobatto-m0
ratio: 0.6666666666666666
(array([0., 1.]), array([840, 24]))
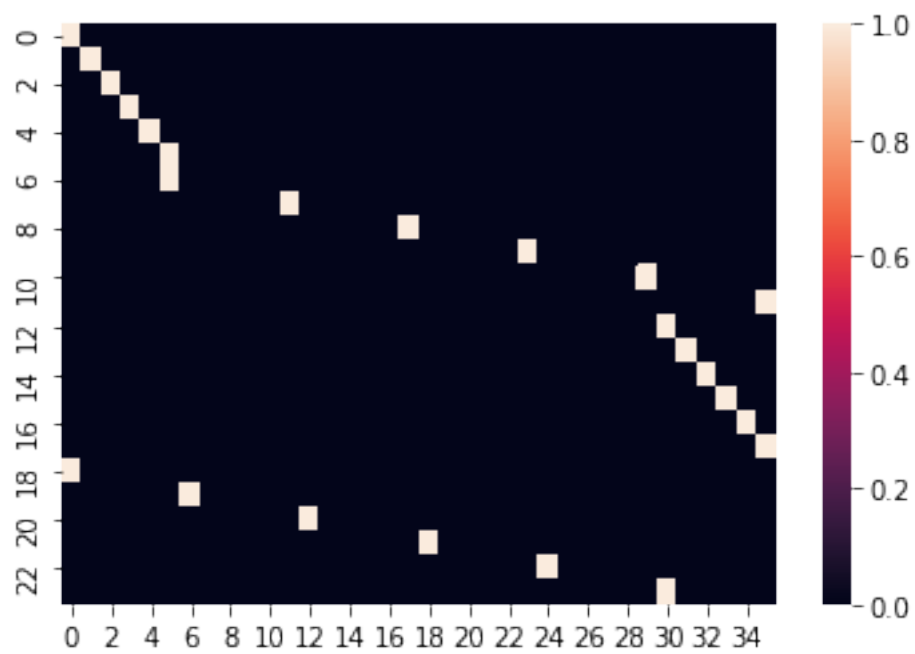
```
p5-gauss-legendre-lobatto-m3
ratio: 1.5
(array([-3.        , -1.2590908 , -1.03988318,  0.        ,  1.03988318,
         1.2590908 , 18.        ]), array([ 24,  24,  24, 720,  24,  24,  24]))
```
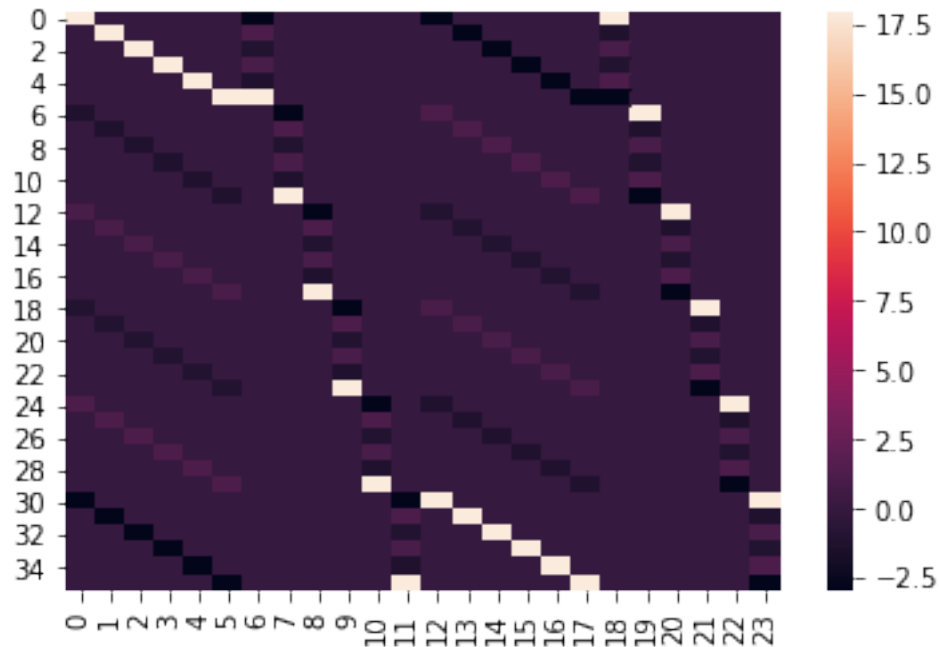

0.4.1 thoughts

- m132 and m460 have distinct dim ratio and pattern
- vary diag

0.5 Hex

```
[12]: from collections import defaultdict
      ratios_hex = defaultdict(int)
      for name, data in matrices["hex"].items():
          dims = data.shape
          ratios_hex[dims[0]/dims[1]] += 1
```

```
[13]: ratios_hex
```

```
[13]: defaultdict(int,
                  {0.3333333333333333: 12,
                   1.0: 5,
```

```
                3.0: 14,
                2.0: 4,
                1.5: 4,
                0.6666666666666666: 2,
                0.5: 2,
                2.5: 2,
                1.2: 2,
                0.8333333333333334: 2,
                3.5: 2,
                0.8571428571428571: 2,
                1.1666666666666667: 2})
```

```python
[14]: for name, data in matrices["hex"].items():
          dims = data.shape

          if dims[0]/dims[1] == 1/3.0:
              print(name)
              print("ratio: " + str(dims[0]/dims[1]))
              print(np.unique(data, return_counts=True))
              plt.figure()
              ax = sns.heatmap(data)
              plt.show()
```

```
p1-gauss-legendre-m132
ratio: 0.3333333333333333
(array([-0.8660254,  0.       ,  0.8660254]), array([ 24, 144,  24]))
```
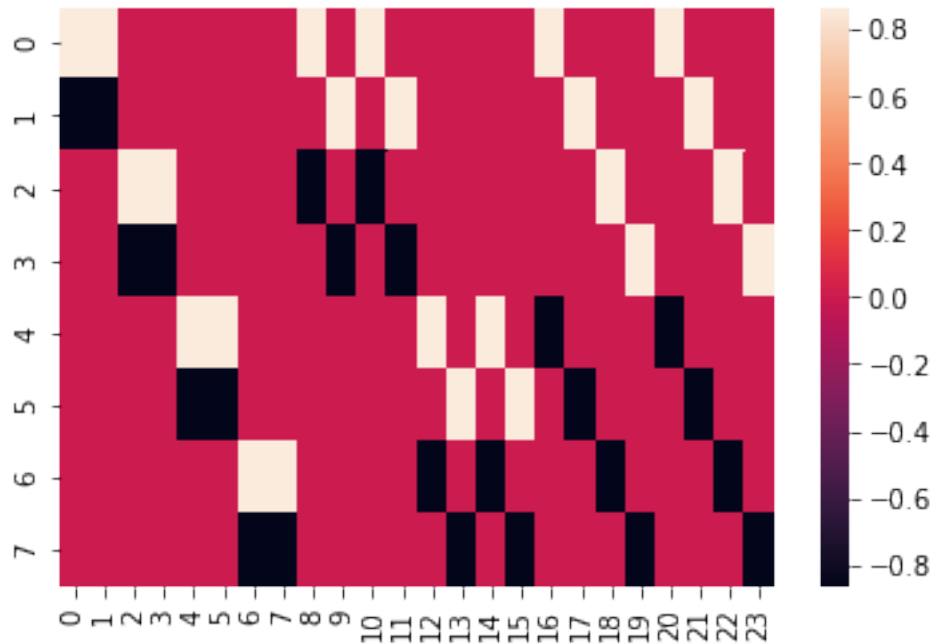
p1-gauss-legendre-m3
ratio: 0.3333333333333333
(array([-0.3660254, 0.   , 1.3660254]), array([24, 144, 24]))
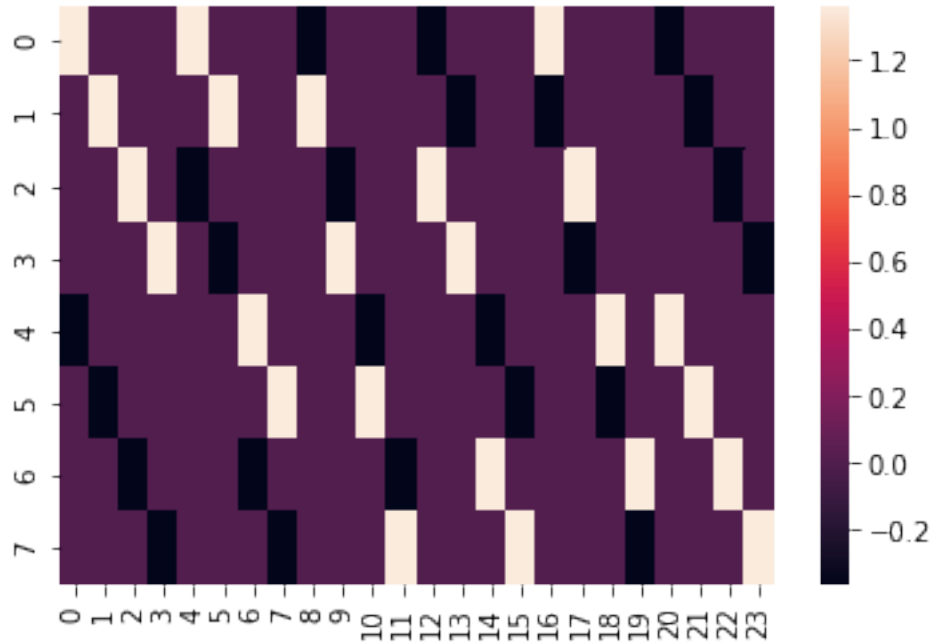
p3-gauss-legendre-m132
ratio: 0.3333333333333333
(array([-3.33200024, -2.59241098, -1.47067023, -1.42024047, -1.12482651,
       -0.61623104, -0.58062817, -0.38441439, 0.   , 0.38441439,
        0.58062817, 0.61623104, 1.12482651, 1.42024047, 1.47067023,
        2.59241098, 3.33200024]), array([48,   48,   48,   48,   48,
   48,   48,   48, 11520,
         48,   48,   48,   48,   48,   48,   48,   48]))
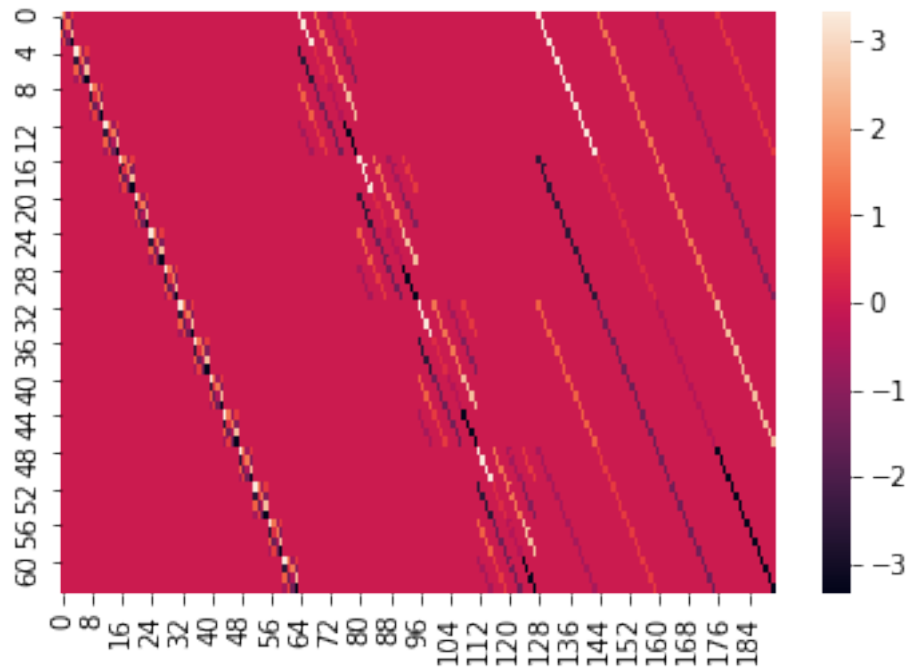
p3-gauss-legendre-lobatto-m132
ratio: 0.3333333333333333
(array([-5.   , -4.04508497, -2.5   , -1.70344419, -1.54508497,
        -1.20344419, -1.11803399, 0.   , 1.11803399, 1.20344419,
         1.54508497, 1.70344419, 2.5   , 4.04508497, 5.   ]),
array([48,   48,   48,   48,   48,   48,   48, 11616,   48,
          48,   48,   48,   48,   48,   48]))
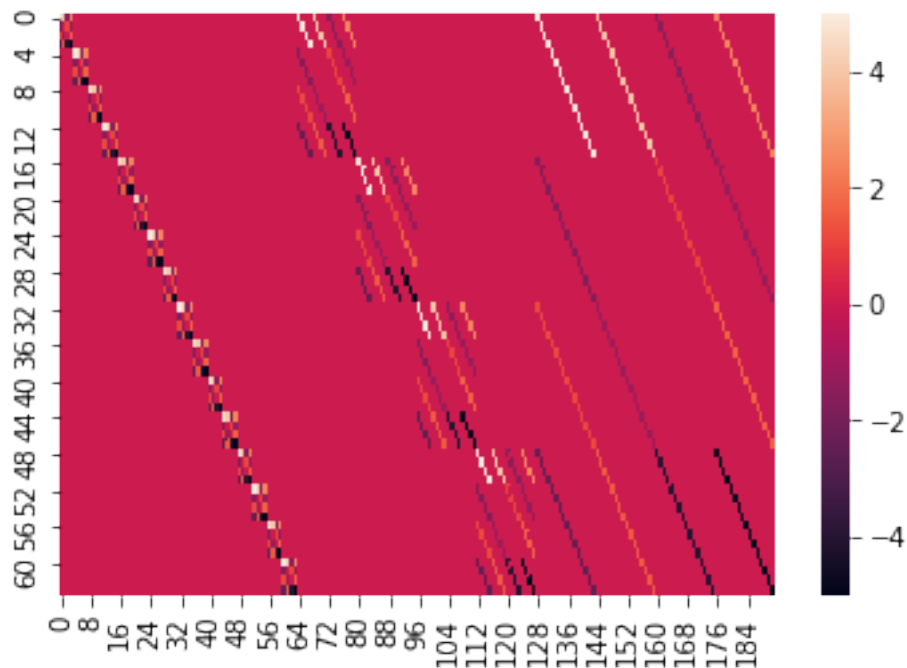
p2-gauss-legendre-m132
ratio: 0.3333333333333333
(array([-1.93649167, -1.61374306, -1.03279556, -0.64549722, 0.   ,
        0.64549722, 1.03279556, 1.61374306, 1.93649167]), array([27,   27,
27,   27, 1971,   27,   27,   27,   27]))
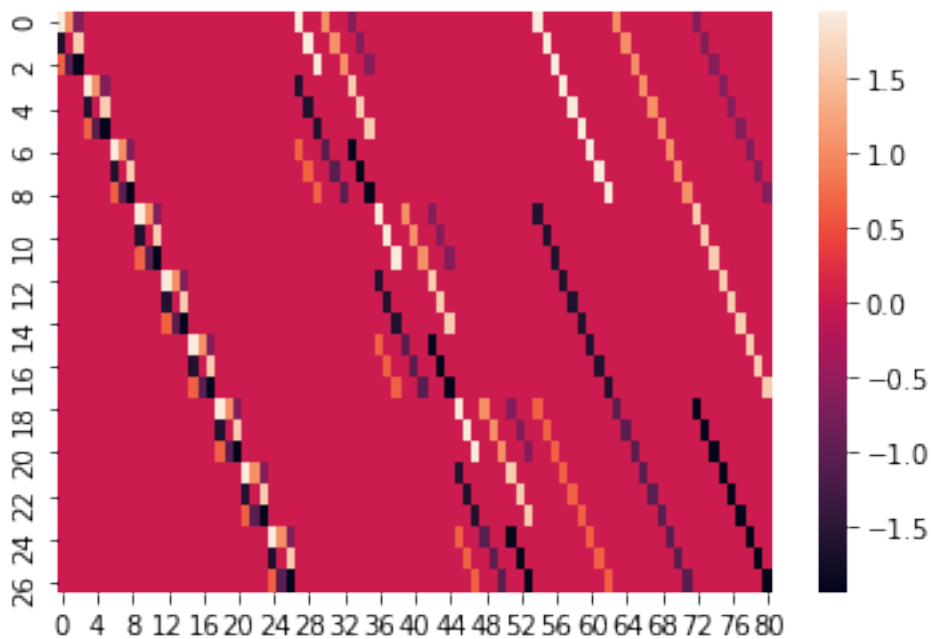
```
p2-gauss-legendre-lobatto-m132
ratio: 0.3333333333333333
(array([-3.  , -2.  , -1.25,  0.  ,  1.25,  2.  ,  3. ]), array([ 27,   54,
27, 1971,   27,   54,   27]))
```
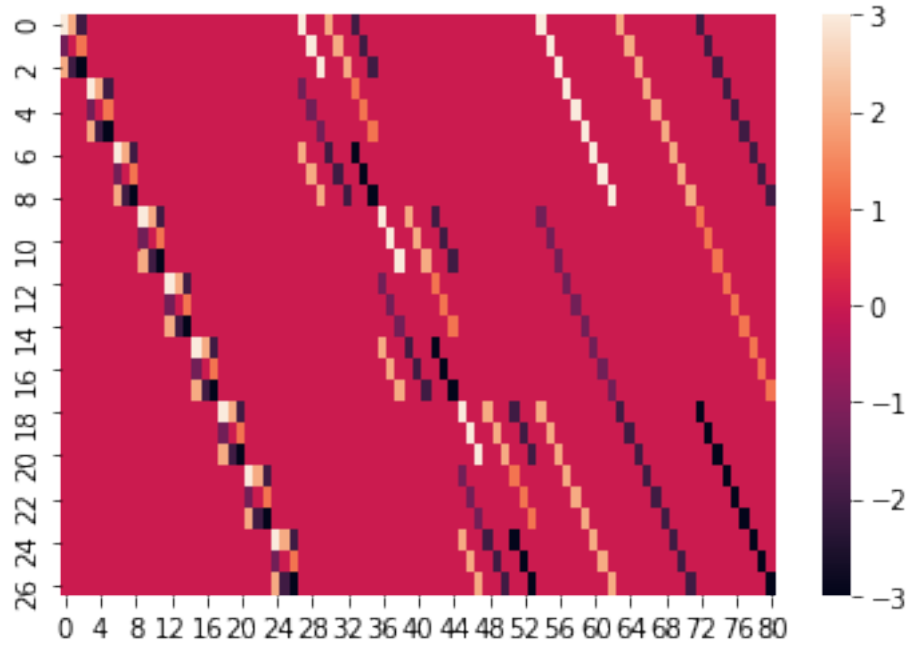


```
p4-gauss-legendre-m132
ratio: 0.3333333333333333
(array([-5.0670406 , -3.81255789, -2.02152894, -1.93986454, -1.70607503,
       -1.68402696, -0.97042088, -0.92855803, -0.75835322, -0.72313963,
       -0.55176685, -0.49375908,  0.        ,  0.49375908,  0.55176685,
        0.72313963,  0.75835322,  0.92855803,  0.97042088,  1.68402696,
        1.70607503,  1.93986454,  2.02152894,  3.81255789,  5.0670406 ]),
array([   75,     75,     75,     75,     75,     75,     75,     75,     75,
          75,     75,     75, 45075,     75,     75,     75,     75,     75,
          75,     75,     75,     75,     75,     75,     75]))
```
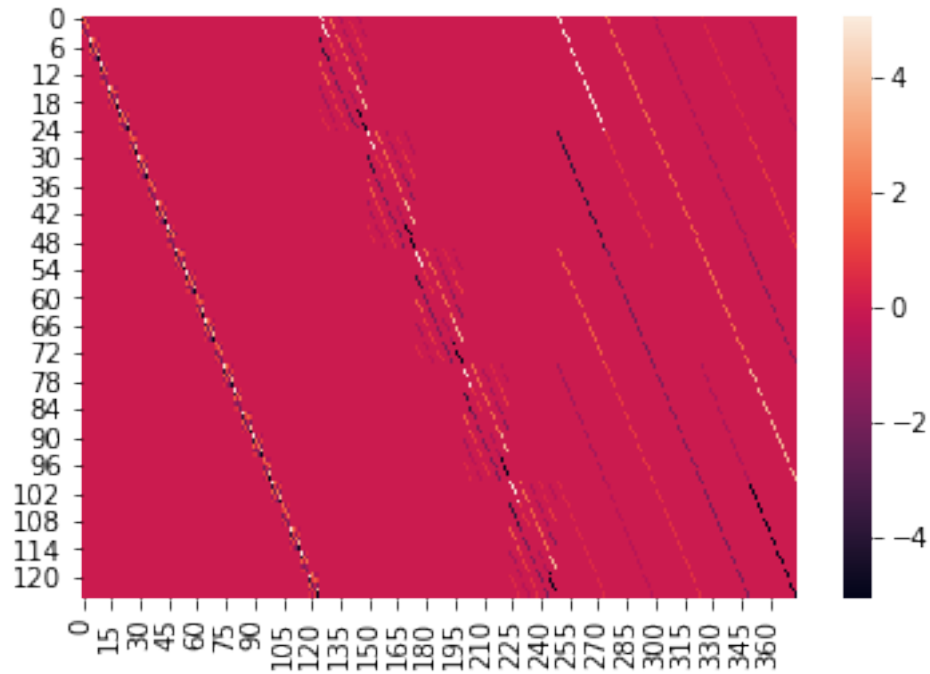
p4-gauss-legendre-lobatto-m132
ratio: 0.3333333333333333
(array([-7.5   , -6.75650249, -3.   , -2.66666667, -2.31241882,
       -1.74574312, -1.41016418, -1.33658458, -1.33043832, -1.3125   ,
       -0.76376262, 0.   , 0.76376262, 1.3125   , 1.33043832,
        1.33658458, 1.41016418, 1.74574312, 2.31241882, 2.66666667,
        3.   , 6.75650249, 7.5   ]), array([75,   75,   75,
    75,   75,   75,   75,   75,   75,
        75,   75, 45225,   75,   75,   75,   75,   75,   75,
        75,   75,   75,   75,   75]))

49
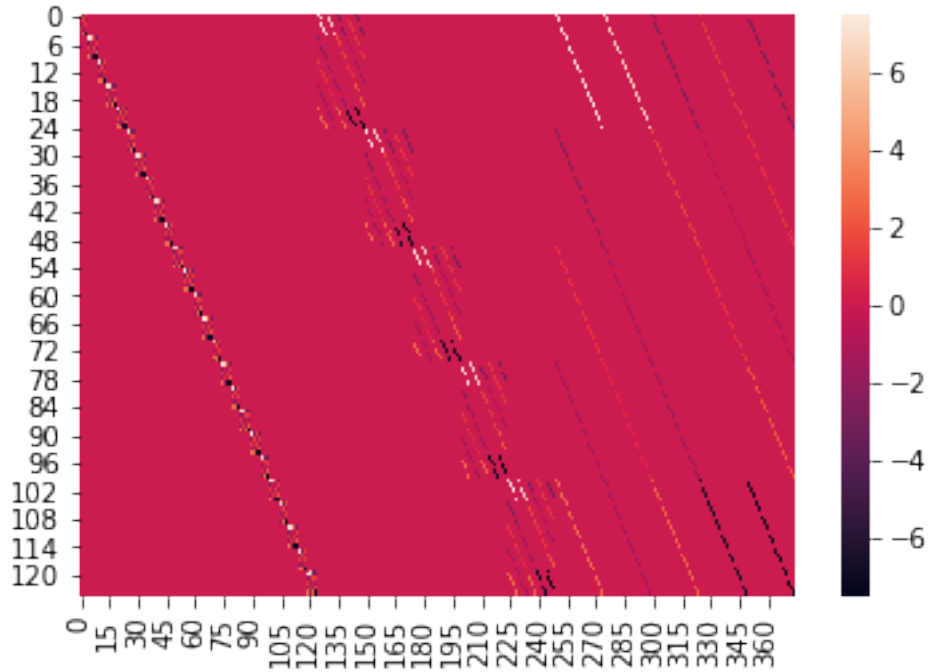
p6-gauss-legendre-m132
ratio: 0.3333333333333333
(array([-9.56818234, -6.98244017, -3.47320931, -3.3238044 , -3.11057141,
        -2.57685029, -2.5550682 , -2.3560803 , -1.86201883, -1.64736568,
        -1.6443125 , -1.24717059, -1.23199698, -1.10600426, -1.08928106,
        -1.01615121, -0.85730425, -0.74753216, -0.67428047, -0.59619008,
        -0.52681049, -0.48587352, -0.43674268, -0.40809682, 0.   ,
         0.40809682, 0.43674268, 0.48587352, 0.52681049, 0.59619008,
         0.67428047, 0.74753216, 0.85730425, 1.01615121, 1.08928106,
         1.10600426, 1.23199698, 1.24717059, 1.6443125 , 1.64736568,
         1.86201883, 2.3560803 , 2.5550682 , 2.57685029, 3.11057141,
         3.3238044 , 3.47320931, 6.98244017, 9.56818234]), array([147,
    147,   147,   147,   147,   147,   147,   147,
          147,   147,   147,   147,   147,   147,   147,   147,
          147,   147,   147,   147,   147,   147,   147,   147,
       345891,   147,   147,   147,   147,   147,   147,   147,
          147,   147,   147,   147,   147,   147,   147,   147,
          147,   147,   147,   147,   147,   147,   147,   147,
          147]))

p6-gauss-legendre-lobatto-m132
ratio: 0.3333333333333333
(array([-14.2015766 , -14.   , -5.66898523, -4.   ,
        -3.89455263, -3.45582821, -3.2   , -2.26669809,
        -2.21580428, -2.04996481, -2.00696924, -1.78762708,
        -1.67823848, -1.59860669, -1.40625   , -1.38846981,
        -1.31737344, -1.0664419 , -0.9613398 , -0.90754447,
        -0.61639084, -0.60224718, 0.   , 0.60224718,
         0.61639084, 0.90754447, 0.9613398 , 1.0664419 ,
         1.31737344, 1.38846981, 1.40625   , 1.59860669,
         1.67823848, 1.78762708, 2.00696924, 2.04996481,
         2.21580428, 2.26669809, 3.2   , 3.45582821,
         3.89455263, 4.   , 5.66898523, 14.   ,
        14.2015766]), array([147,   147,   147,   147,   147,   147,
       147,   147,
         147,   147,   147,   147,   147,   147,   147,   147,
         147,   147,   147,   147,   147,   147, 346479,   147,
         147,   147,   147,   147,   147,   147,   147,   147,
         147,   147,   147,   147,   147,   147,   147,   147,
         147,   147,   147,   147,   147]))

p5-gauss-legendre-m132
ratio: 0.3333333333333333
(array([-7.14532713, -5.27576253, -2.68967445, -2.57598208, -2.3443613 ,
       -2.09538893, -2.08190754, -1.38899458, -1.2631629 , -1.17485228,
       -0.97773482, -0.89798768, -0.88602069, -0.75619011, -0.53621056,
       -0.52495233, -0.43845798, -0.25302628, 0.   , 0.25302628,
        0.43845798, 0.52495233, 0.53621056, 0.75619011, 0.88602069,
        0.89798768, 0.97773482, 1.17485228, 1.2631629 , 1.38899458,
        2.08190754, 2.09538893, 2.3443613 , 2.57598208, 2.68967445,
        5.27576253, 7.14532713]), array([108,   108,   108,   108,
108,   108,   108,   108,
          108,   108,   108,   108,   108,   108,   108,   108,
          108,   108, 136080,   108,   108,   108,   108,   108,
          108,   108,   108,   108,   108,   108,   108,   108,
          108,   108,   108,   108,   108]))

p5-gauss-legendre-lobatto-m132
ratio: 0.3333333333333333
(array([-10.5   , -10.14141594, -4.03618727, -3.5   ,
        -3.04545575, -2.52342678, -2.24468465, -1.75296197,
        -1.72125695, -1.52483422, -1.49687198, -1.34991331,
        -1.30958379, -1.15282816, -0.78635667, -0.65354751,
         0.   ,   0.65354751,   0.78635667,   1.15282816,
         1.30958379,   1.34991331,   1.49687198,   1.52483422,
         1.72125695,   1.75296197,   2.24468465,   2.52342678,
         3.04545575,   3.5   ,   4.03618727, 10.14141594,
        10.5   ]), array([108,   108,   108,   108,   108,   108,
   108,   108,
         108,   108,   108,   108,   108,   108,   108,
      136512,   108,   108,   108,   108,   108,   108,
         108,   108,   108,   108,   108,   108,   108,
         108]))

```
[15]: for name, data in matrices["hex"].items():
          dims = data.shape

          if dims[0]/dims[1] == 3.0:
              print(name)
              print("ratio: " + str(dims[0]/dims[1]))
              print(np.unique(data, return_counts=True))
              plt.figure()
              ax = sns.heatmap(data)
              plt.show()
```

```
p1-gauss-legendre-m0
ratio: 3.0
(array([-0.3660254,  0.       ,  1.3660254]), array([ 24, 144,  24]))
```
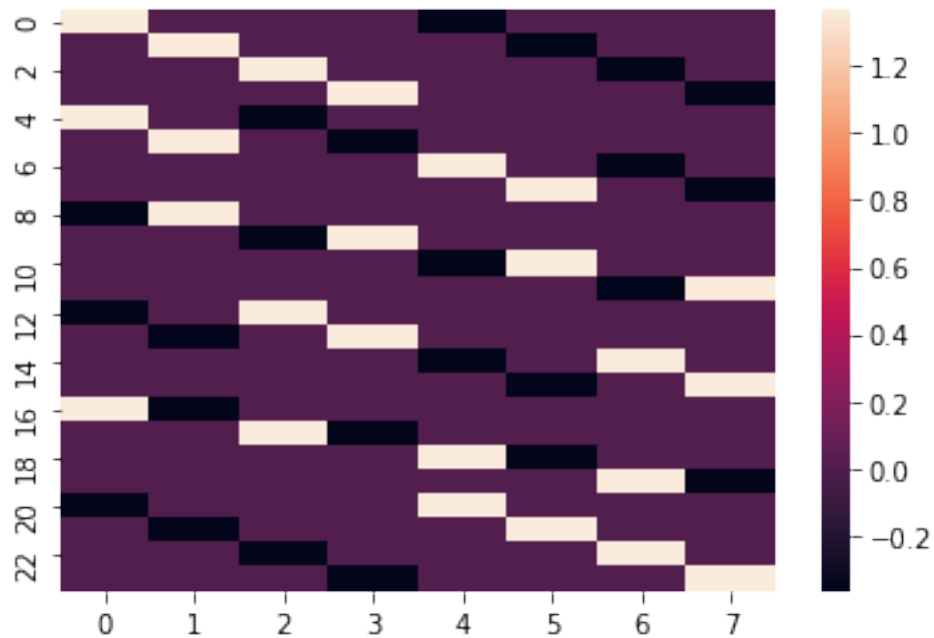
p1-gauss-legendre-m460
ratio: 3.0
(array([-0.8660254, 0.   , 0.8660254]), array([24, 144, 24]))
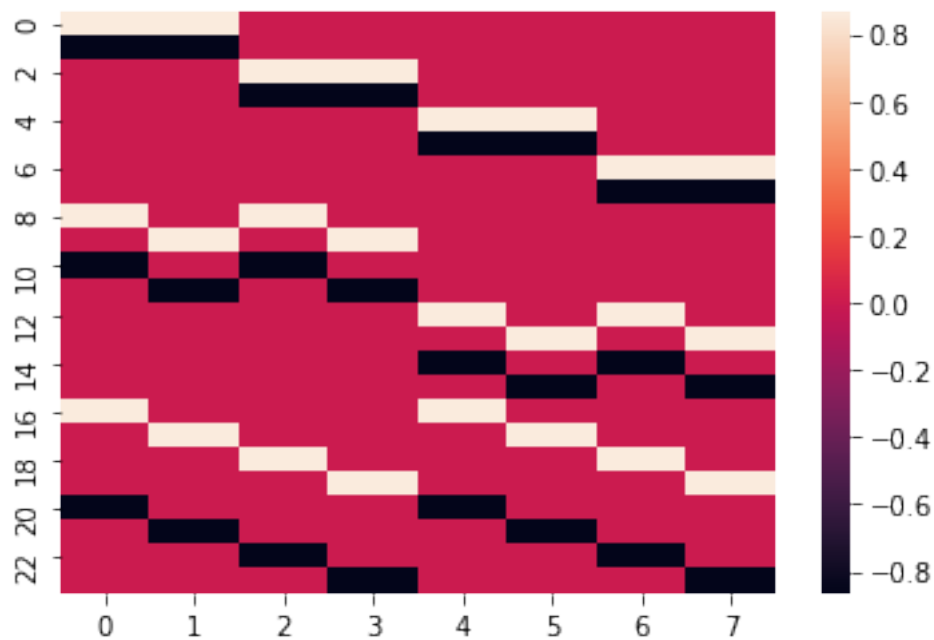
p3-gauss-legendre-m460

```
ratio: 3.0
(array([-3.33200024, -2.59241098, -1.47067023, -1.42024047, -1.12482651,
       -0.61623104, -0.58062817, -0.38441439,  0.        ,  0.38441439,
        0.58062817,  0.61623104,  1.12482651,  1.42024047,  1.47067023,
        2.59241098,  3.33200024]), array([   48,    48,    48,    48,    48,
   48,    48,    48, 11520,
         48,    48,    48,    48,    48,    48,    48,    48]))
```
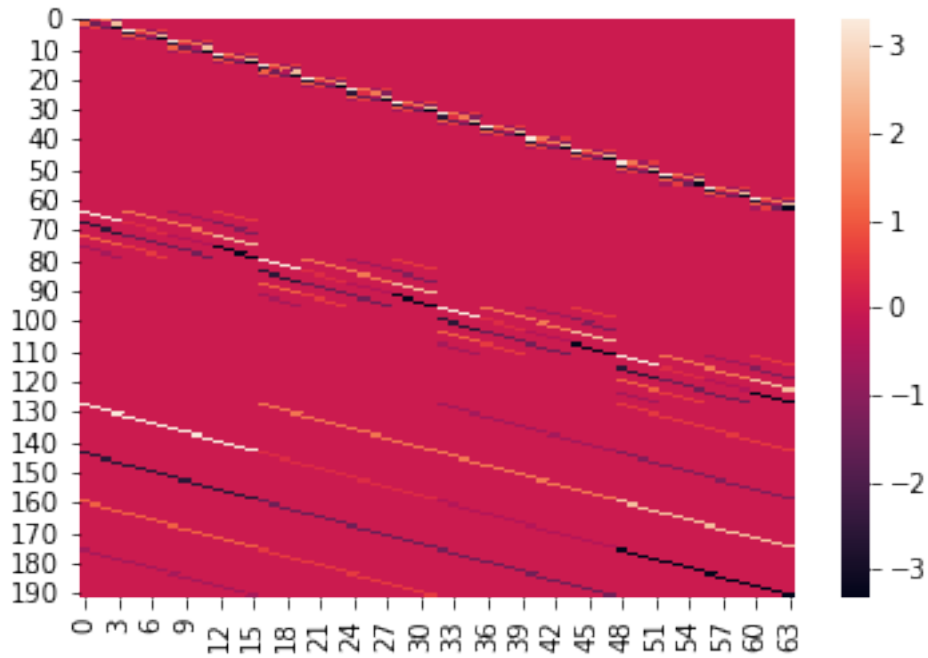


```
p3-gauss-legendre-lobatto-m460
ratio: 3.0
(array([-5.        , -4.04508497, -2.5       , -1.70344419, -1.54508497,
       -1.20344419, -1.11803399,  0.        ,  1.11803399,  1.20344419,
        1.54508497,  1.70344419,  2.5       ,  4.04508497,  5.        ]),
array([   48,    48,    48,    48,    48,    48,    48, 11616,    48,
         48,    48,    48,    48,    48,    48]))
```
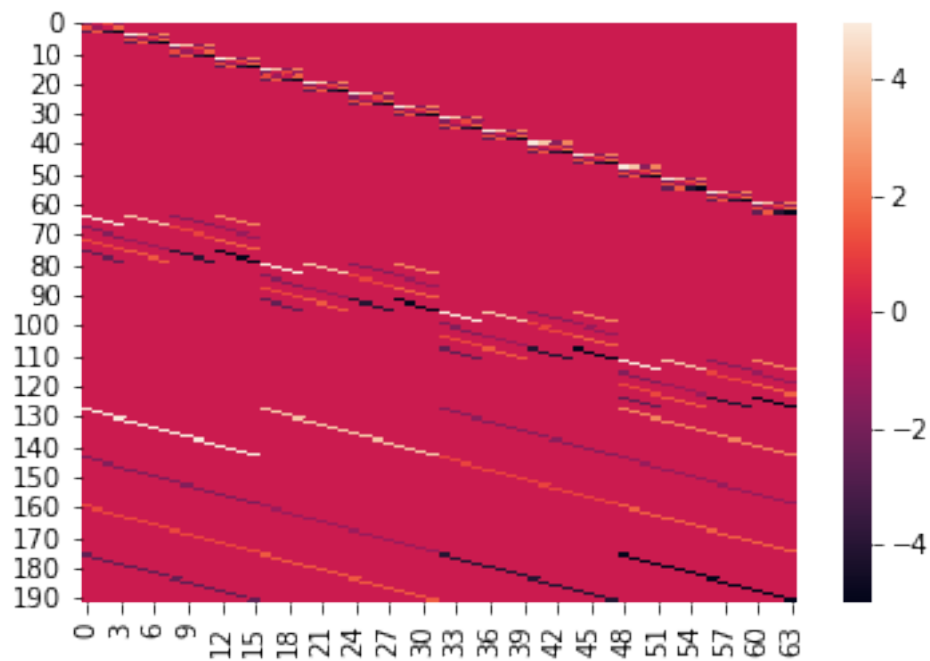
p2-gauss-legendre-m460
ratio: 3.0
(array([-1.93649167, -1.61374306, -1.03279556, -0.64549722, 0.   ,
        0.64549722, 1.03279556, 1.61374306, 1.93649167]), array([27,   27,
   27,   27, 1971,   27,   27,   27,   27]))
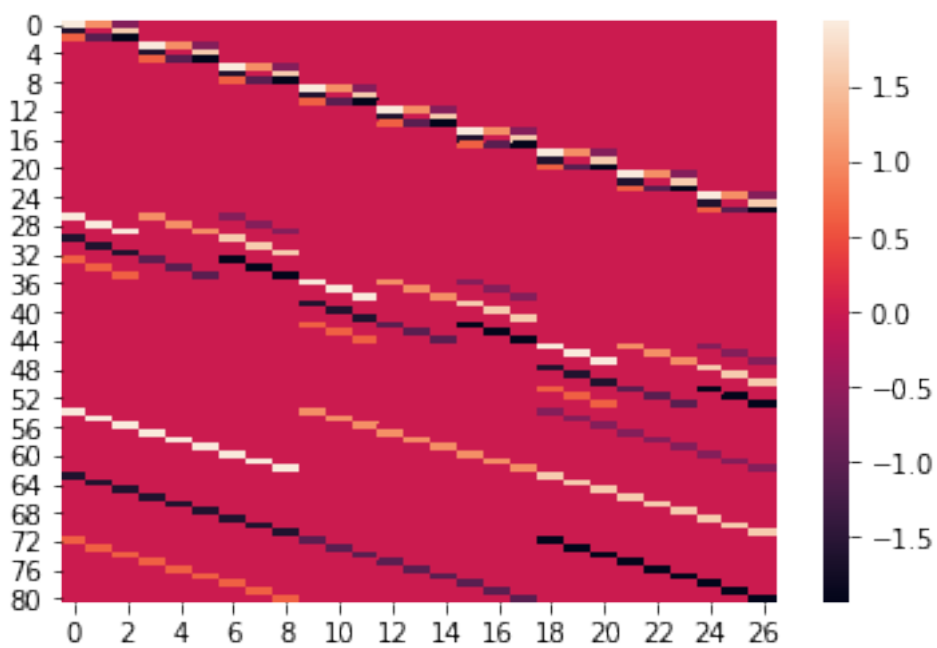
```
p2-gauss-legendre-lobatto-m460
ratio: 3.0
(array([-3.  , -2.  , -1.25,  0.  ,  1.25,  2.  ,  3.  ]), array([  27,   54,
  27, 1971,   27,   54,   27]))
```
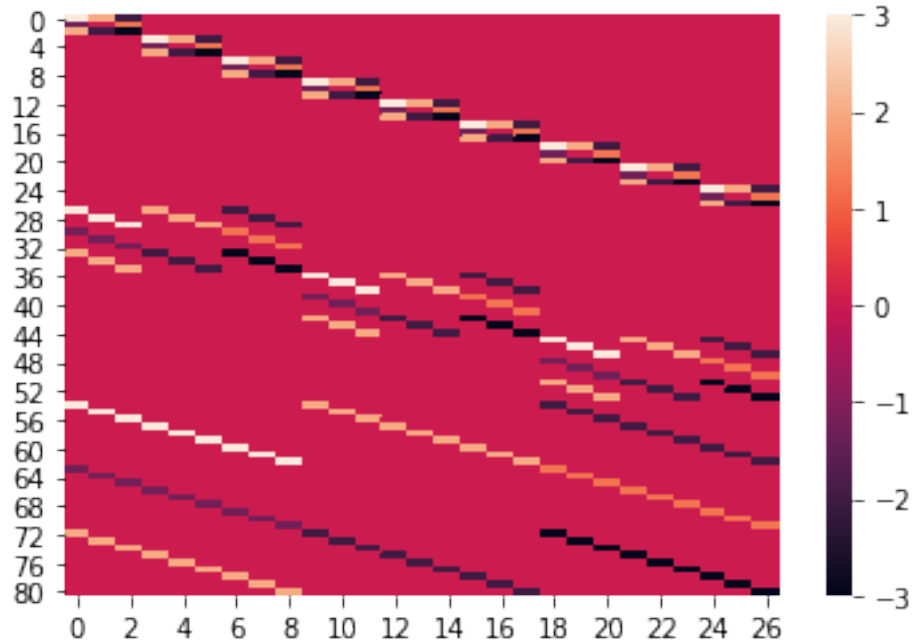


```
p4-gauss-legendre-m460
ratio: 3.0
(array([-5.0670406 , -3.81255789, -2.02152894, -1.93986454, -1.70607503,
       -1.68402696, -0.97042088, -0.92855803, -0.75835322, -0.72313963,
       -0.55176685, -0.49375908,  0.        ,  0.49375908,  0.55176685,
        0.72313963,  0.75835322,  0.92855803,  0.97042088,  1.68402696,
        1.70607503,  1.93986454,  2.02152894,  3.81255789,  5.0670406 ]),
array([   75,    75,    75,    75,    75,    75,    75,    75,    75,
         75,    75,    75, 45075,    75,    75,    75,    75,    75,
         75,    75,    75,    75,    75,    75,    75]))
```

p4-gauss-legendre-lobatto-m460
ratio: 3.0
(array([-7.5   , -6.75650249, -3.   , -2.66666667, -2.31241882,
       -1.74574312, -1.41016418, -1.33658458, -1.33043832, -1.3125   ,
       -0.76376262, 0.   , 0.76376262, 1.3125   , 1.33043832,
        1.33658458, 1.41016418, 1.74574312, 2.31241882, 2.66666667,
        3.   , 6.75650249, 7.5   ]), array([75,   75,   75,
   75,   75,   75,   75,   75,   75,
       75,   75, 45225,   75,   75,   75,   75,   75,   75,
       75,   75,   75,   75,   75]))
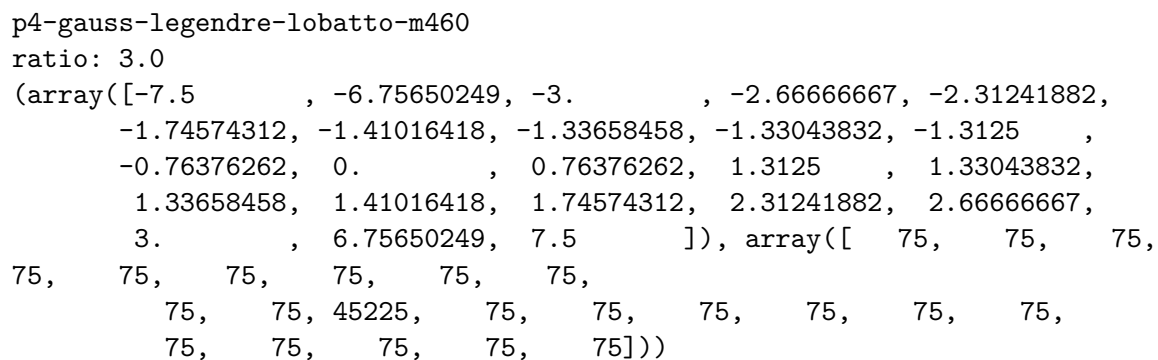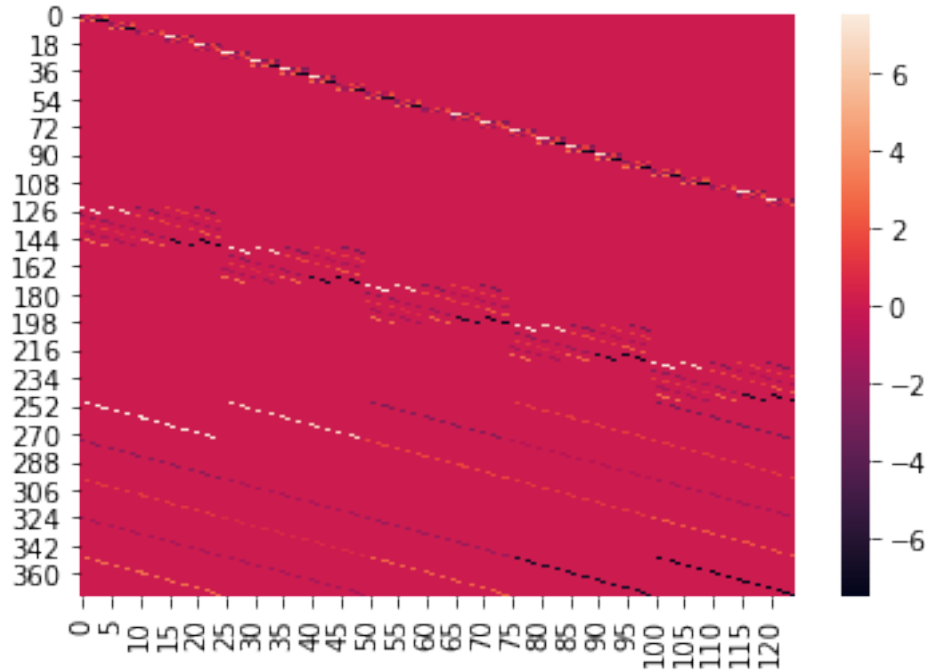
p6-gauss-legendre-m460
ratio: 3.0
(array([-9.56818234, -6.98244017, -3.47320931, -3.3238044 , -3.11057141,
       -2.57685029, -2.5550682 , -2.3560803 , -1.86201883, -1.64736568,
       -1.6443125 , -1.24717059, -1.23199698, -1.10600426, -1.08928106,
       -1.01615121, -0.85730425, -0.74753216, -0.67428047, -0.59619008,
       -0.52681049, -0.48587352, -0.43674268, -0.40809682, 0.   ,
        0.40809682, 0.43674268, 0.48587352, 0.52681049, 0.59619008,
        0.67428047, 0.74753216, 0.85730425, 1.01615121, 1.08928106,
        1.10600426, 1.23199698, 1.24717059, 1.6443125 , 1.64736568,
        1.86201883, 2.3560803 , 2.5550682 , 2.57685029, 3.11057141,
        3.3238044 , 3.47320931, 6.98244017, 9.56818234]), array([147,
    147,   147,   147,   147,   147,   147,   147,
        147,   147,   147,   147,   147,   147,   147,   147,
        147,   147,   147,   147,   147,   147,   147,   147,
    345891,   147,   147,   147,   147,   147,   147,   147,
        147,   147,   147,   147,   147,   147,   147,   147,
        147,   147,   147,   147,   147,   147,   147,   147,
        147]))

```
p6-gauss-legendre-lobatto-m460
ratio: 3.0
(array([-14.2015766 , -14.        , -5.66898523, -4.        ,
        -3.89455263, -3.45582821, -3.2       , -2.26669809,
        -2.21580428, -2.04996481, -2.00696924, -1.78762708,
        -1.67823848, -1.59860669, -1.40625   , -1.38846981,
        -1.31737344, -1.0664419 , -0.9613398 , -0.90754447,
        -0.61639084, -0.60224718,  0.        ,  0.60224718,
         0.61639084,  0.90754447,  0.9613398 ,  1.0664419 ,
         1.31737344,  1.38846981,  1.40625   ,  1.59860669,
         1.67823848,  1.78762708,  2.00696924,  2.04996481,
         2.21580428,  2.26669809,  3.2       ,  3.45582821,
         3.89455263,  4.        ,  5.66898523, 14.        ,
        14.2015766 ]), array([  147,    147,    147,    147,    147,    147,
   147,    147,
         147,    147,    147,    147,    147,    147,    147,    147,
         147,    147,    147,    147,    147,    147, 346479,    147,
         147,    147,    147,    147,    147,    147,    147,    147,
         147,    147,    147,    147,    147,    147,    147,    147,
         147,    147,    147,    147,    147]))
```

p5-gauss-legendre-m6
ratio: 3.0
(array([-9.13864199, -2.60688198, -1.31846908, -0.81046465, -0.53165311,
       -0.31935145, 0.   , 0.31935145, 0.53165311, 0.81046465,
        1.31846908, 2.60688198, 9.13864199]), array([108,   108,   108,
    108,   108,   108, 138672,   108,
        108,   108,   108,   108,   108]))
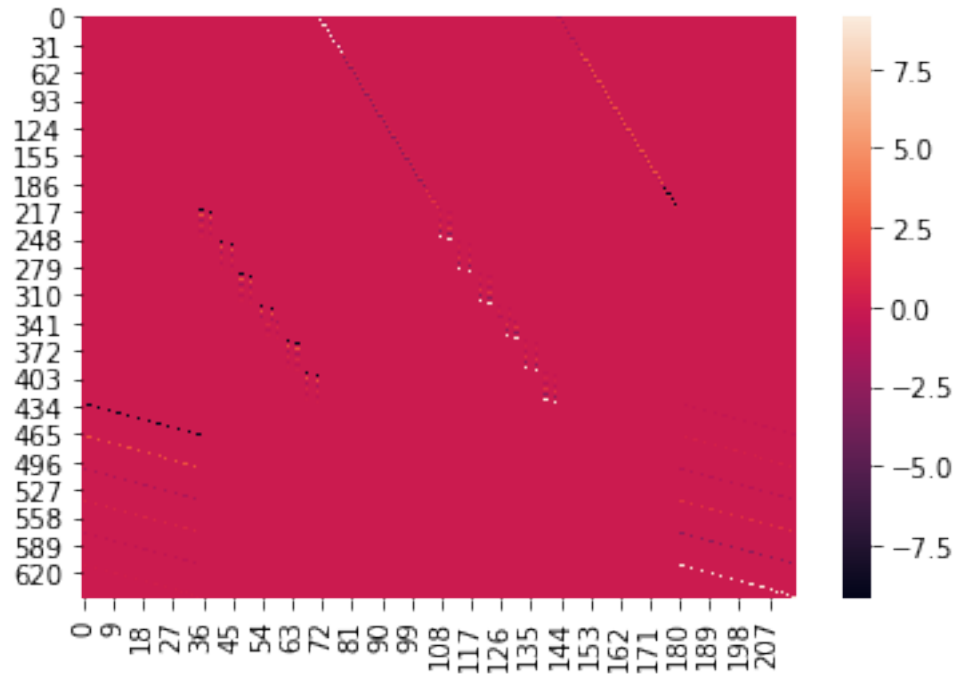
```
p5-gauss-legendre-m460
ratio: 3.0
(array([-7.14532713, -5.27576253, -2.68967445, -2.57598208, -2.3443613 ,
       -2.09538893, -2.08190754, -1.38899458, -1.2631629 , -1.17485228,
       -0.97773482, -0.89798768, -0.88602069, -0.75619011, -0.53621056,
       -0.52495233, -0.43845798, -0.25302628,  0.        ,  0.25302628,
        0.43845798,  0.52495233,  0.53621056,  0.75619011,  0.88602069,
        0.89798768,  0.97773482,  1.17485228,  1.2631629 ,  1.38899458,
        2.08190754,  2.09538893,  2.3443613 ,  2.57598208,  2.68967445,
        5.27576253,  7.14532713]), array([  108,    108,    108,    108,
108,    108,    108,    108,
        108,    108,    108,    108,    108,    108,    108,    108,
        108,    108, 136080,    108,    108,    108,    108,    108,
        108,    108,    108,    108,    108,    108,    108,    108,
        108,    108,    108,    108,    108]))
```

p5-gauss-legendre-lobatto-m6
ratio: 3.0
(array([-18.   , -3.   , -1.2590908 , -1.03988318,
          0.   , 1.03988318, 1.2590908 , 3.   ,
         18.   ]), array([108,   108,   216,   216, 138672,   216,
        216,   108,
          108]))
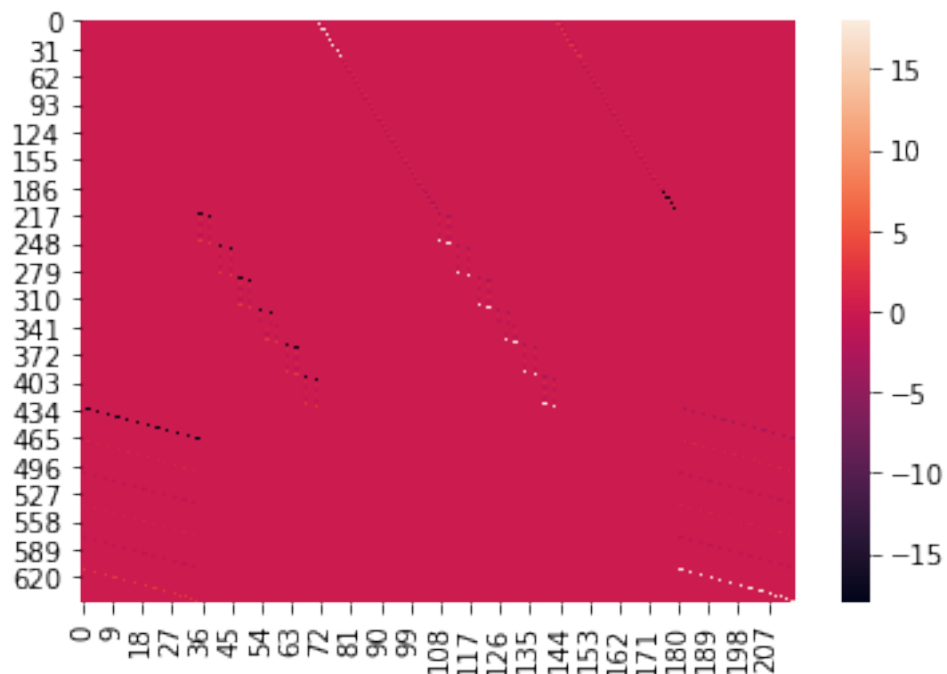
```
p5-gauss-legendre-lobatto-m460
ratio: 3.0
(array([-10.5       , -10.14141594,  -4.03618727,  -3.5       ,
        -3.04545575,  -2.52342678,  -2.24468465,  -1.75296197,
        -1.72125695,  -1.52483422,  -1.49687198,  -1.34991331,
        -1.30958379,  -1.15282816,  -0.78635667,  -0.65354751,
         0.        ,   0.65354751,   0.78635667,   1.15282816,
         1.30958379,   1.34991331,   1.49687198,   1.52483422,
         1.72125695,   1.75296197,   2.24468465,   2.52342678,
         3.04545575,   3.5       ,   4.03618727,  10.14141594,
        10.5       ]), array([  108,    108,    108,    108,    108,     108,
    108,    108,
         108,    108,    108,    108,    108,    108,    108,    108,
      136512,    108,    108,    108,    108,    108,    108,    108,
         108,    108,    108,    108,    108,    108,    108,    108,
         108]))
```

```
[16]: for name, data in matrices["hex"].items():
          dims = data.shape

          if dims[0]/dims[1] != 1/3.0 and dims[0]/dims[1] != 3.0:
              print(name)
              print("ratio: " + str(dims[0]/dims[1]))
              print(np.unique(data, return_counts=True))
              plt.figure()
              ax = sns.heatmap(data)
              plt.show()
```

```
p1-gauss-legendre-m6
ratio: 1.0
(array([-1.3660254, -0.3660254,  0.       ,  0.3660254,  1.3660254]), array([
12,  12, 528,  12,  12]))
```
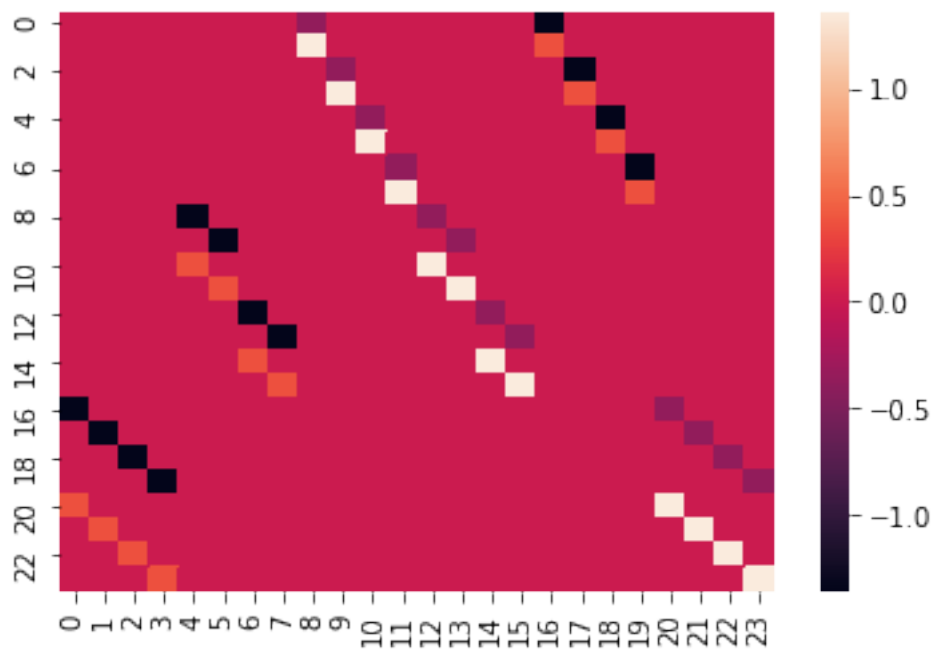
p3-gauss-legendre-m6
ratio: 2.0
(array([-4.38915297, -1.24762477, -0.6145281 , -0.32748486, 0.   ,
        0.32748486, 0.6145281 , 1.24762477, 4.38915297]), array([48,
   48,   48,   48, 18048,   48,   48,   48,   48]))
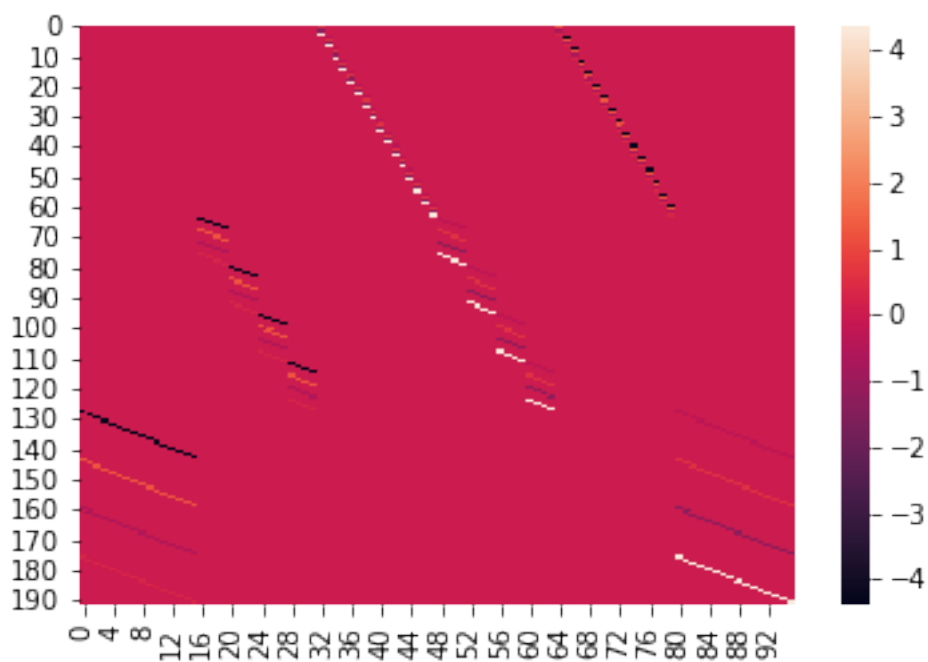
p3-gauss-legendre-m0
ratio: 1.5
(array([-0.81363245, -0.1139172 , 0.   , 0.40076152, 1.52678813]),
array([96,   96, 5760,   96,   96]))
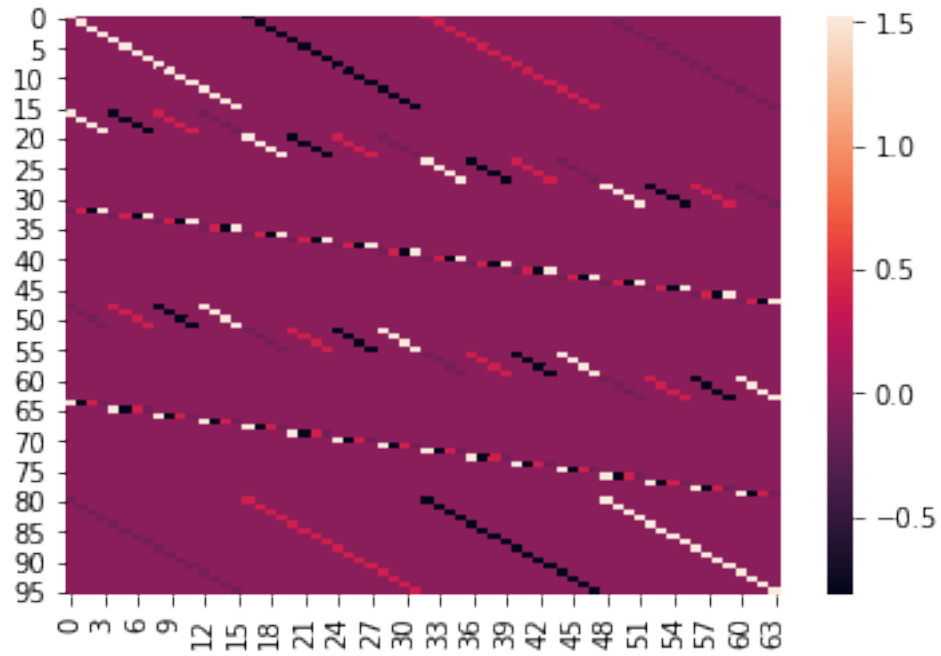
p3-gauss-legendre-m3
ratio: 0.6666666666666666
(array([-1.24762477, -0.32748486, 0.   , 0.6145281 , 4.38915297]),
array([96,   96, 5760,   96,   96]))
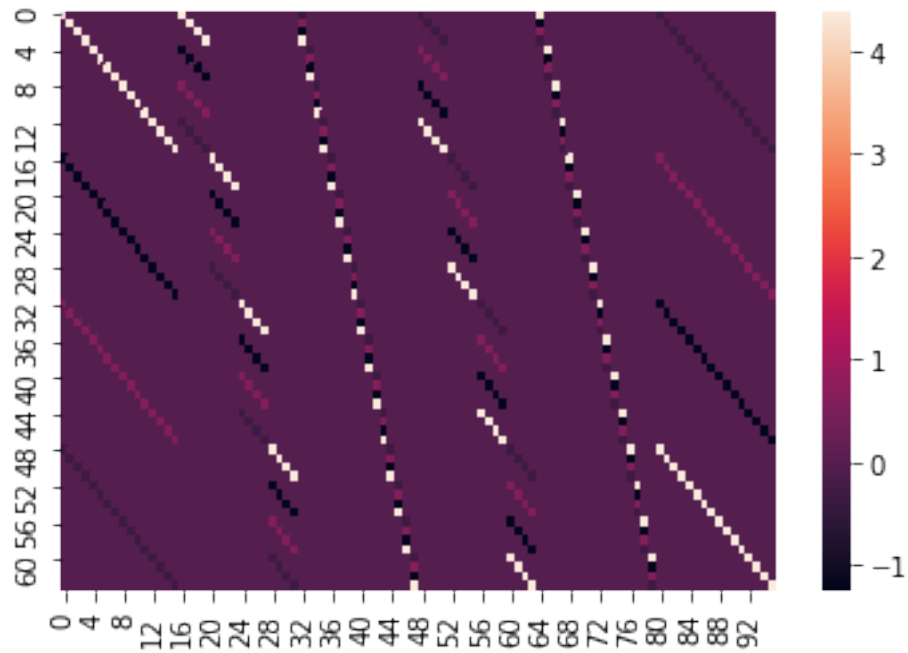
p3-gauss-legendre-lobatto-m6
ratio: 2.0
(array([-8.   , -2.   , -0.89442719, 0.   , 0.89442719,
         2.   , 8.   ]), array([48,   48,   96, 18048,   96,
      48,   48]))
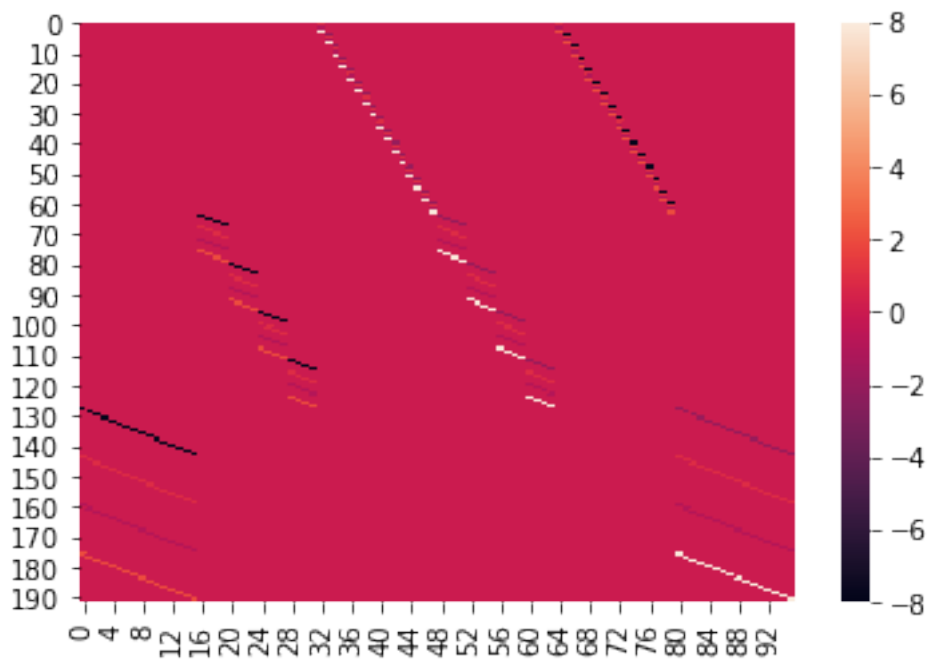
p3-gauss-legendre-lobatto-m0
ratio: 1.5
(array([0., 1.]), array([6048,   96]))
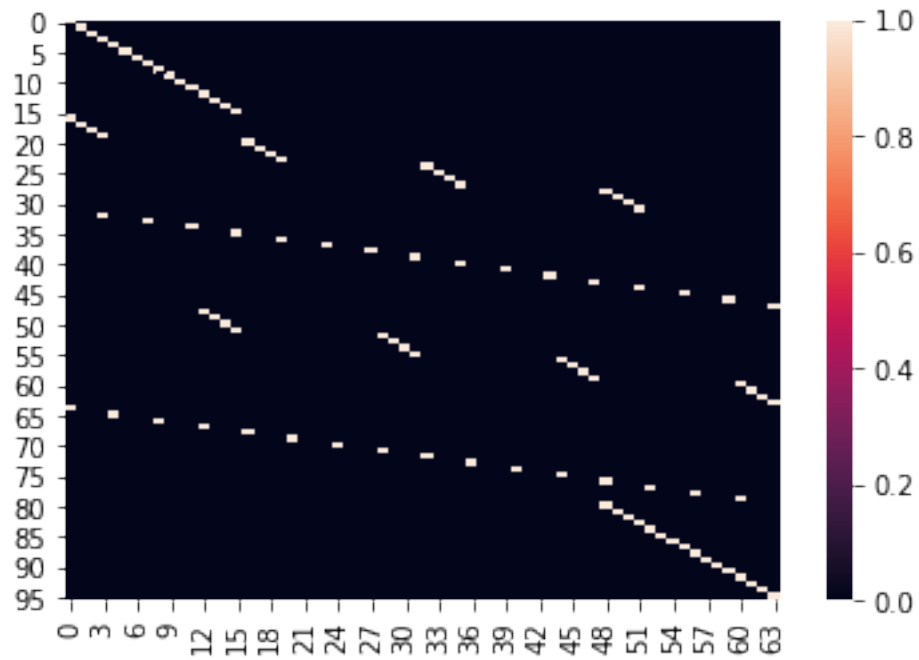
p3-gauss-legendre-lobatto-m3
ratio: 0.6666666666666666
(array([-2.   , -0.89442719, 0.   , 0.89442719, 8.   ]),
array([96,   96, 5760,   96,   96]))
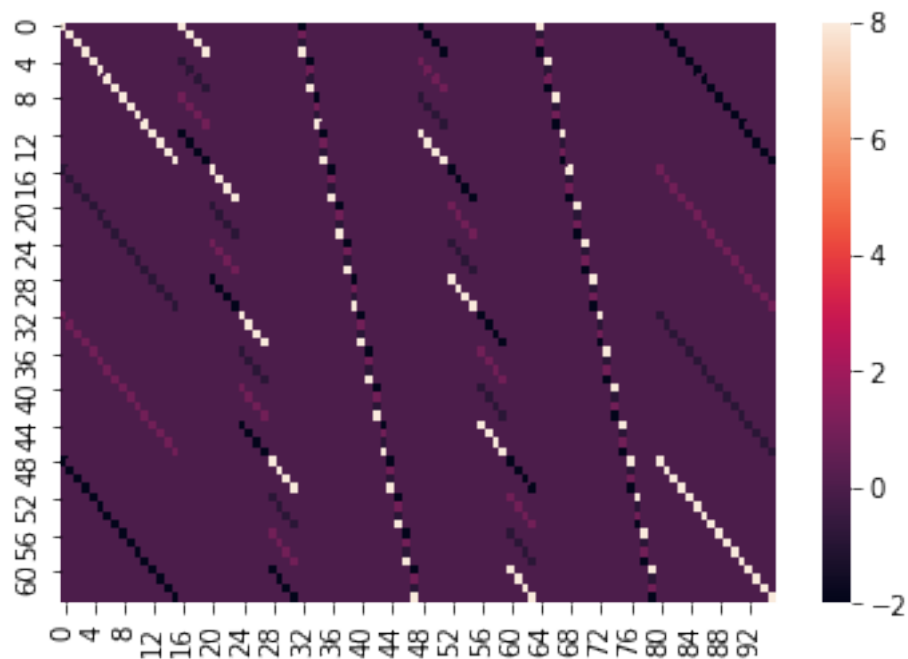
p2-gauss-legendre-m6
ratio: 1.5
(array([-2.661895, -0.75   , -0.338105, 0.   , 0.338105, 0.75   ,
         2.661895]), array([27,   27,   27, 4212,   27,   27,   27]))
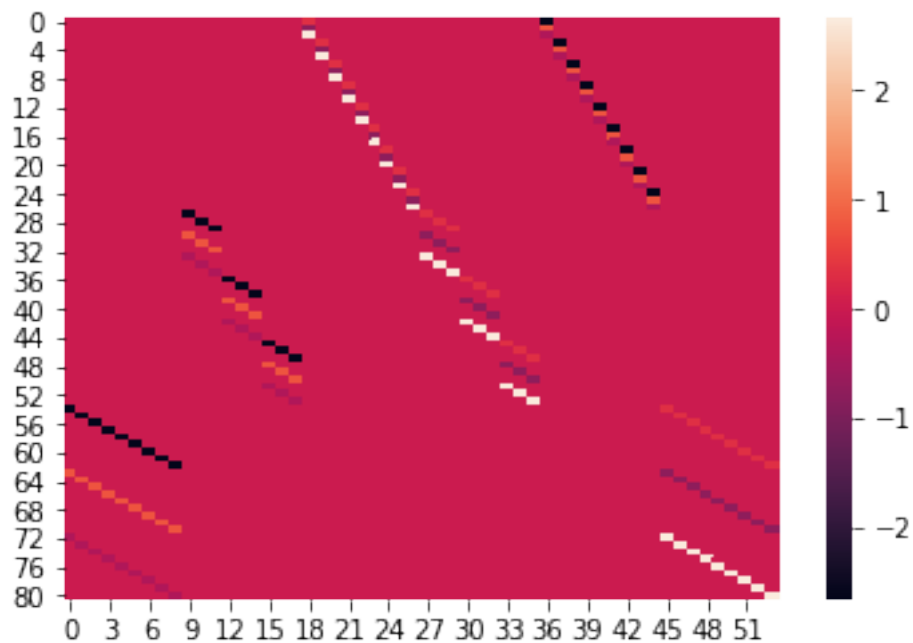
```
p2-gauss-legendre-m0
ratio: 2.0
(array([-0.66666667,  0.        ,  0.18783611,  1.47883056]), array([  54, 1296,
 54,    54]))
```
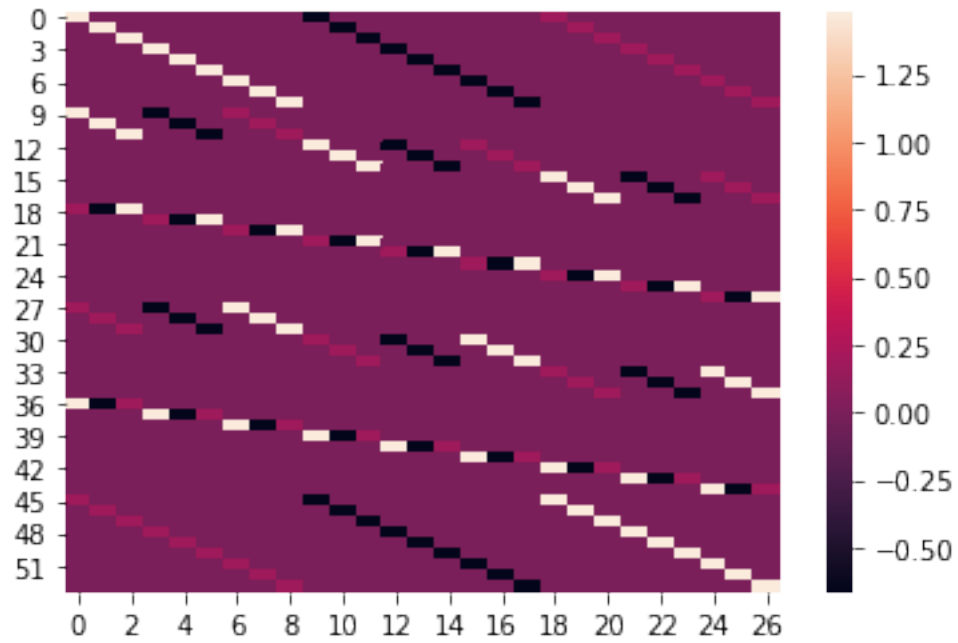


```
p2-gauss-legendre-m3
ratio: 0.5
(array([-0.75     ,  0.       ,  0.338105,  2.661895]), array([  54, 1296,   54,
 54]))
```
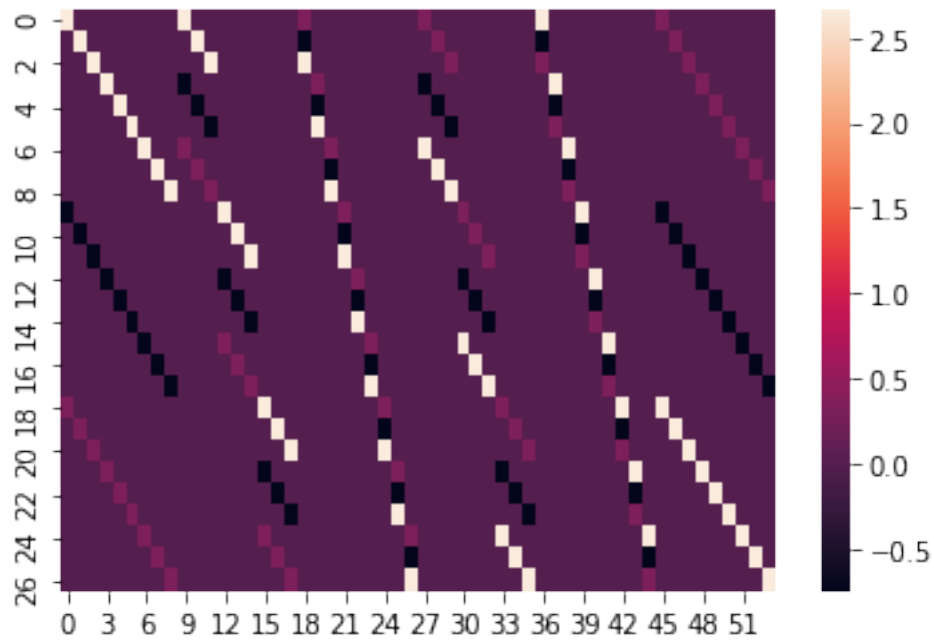
p2-gauss-legendre-lobatto-m6
ratio: 1.5
(array([-4.5 , -1.5 , -0.75, 0. , 0.75, 1.5 , 4.5]), array([27,   27,
27, 4212,   27,   27,   27]))
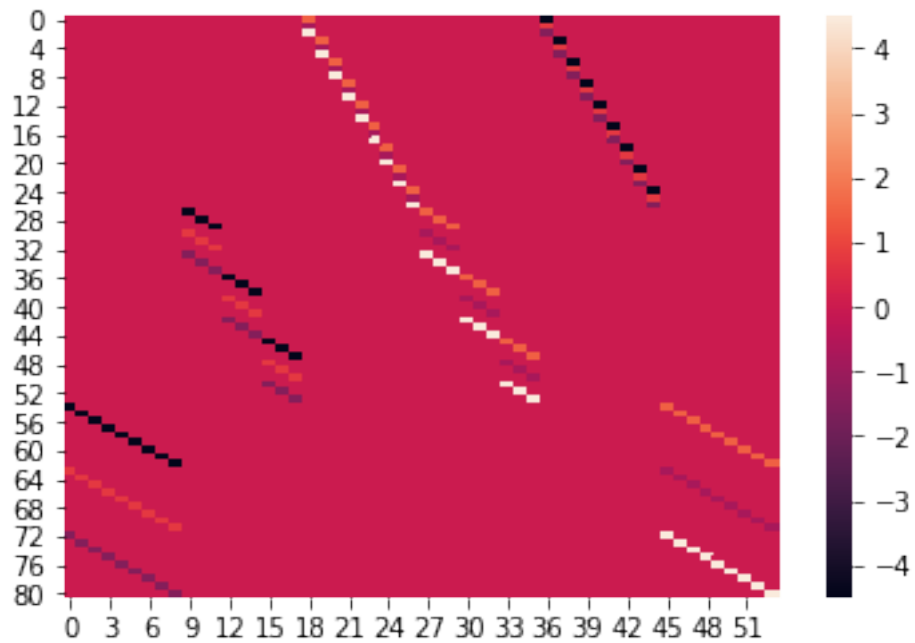
p2-gauss-legendre-lobatto-m0
ratio: 2.0
(array([0., 1.]), array([1404,   54]))
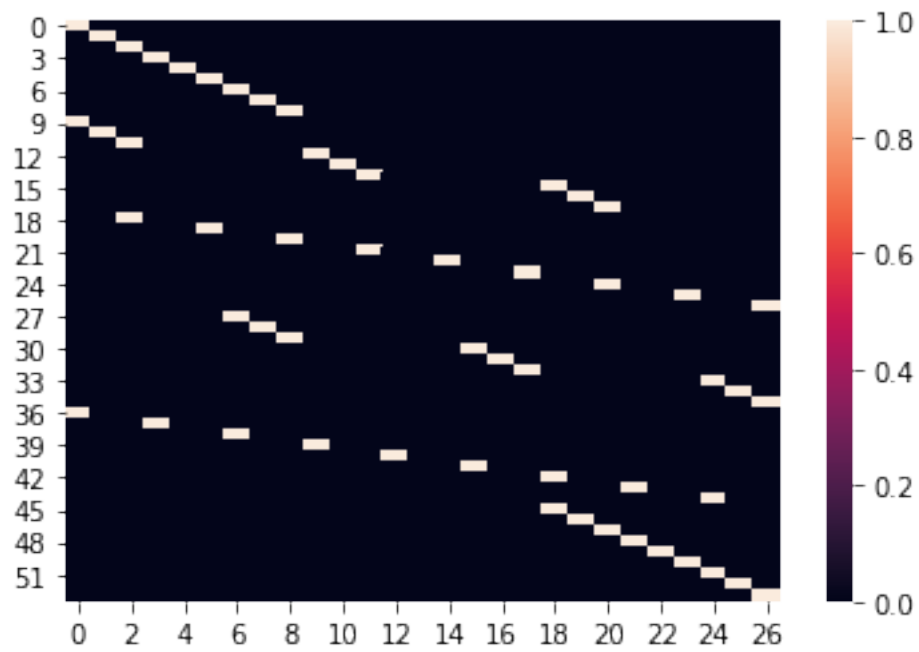
p2-gauss-legendre-lobatto-m3
ratio: 0.5
(array([-0.75, 0. , 1.5 , 4.5]), array([54, 1296,   54,   54]))
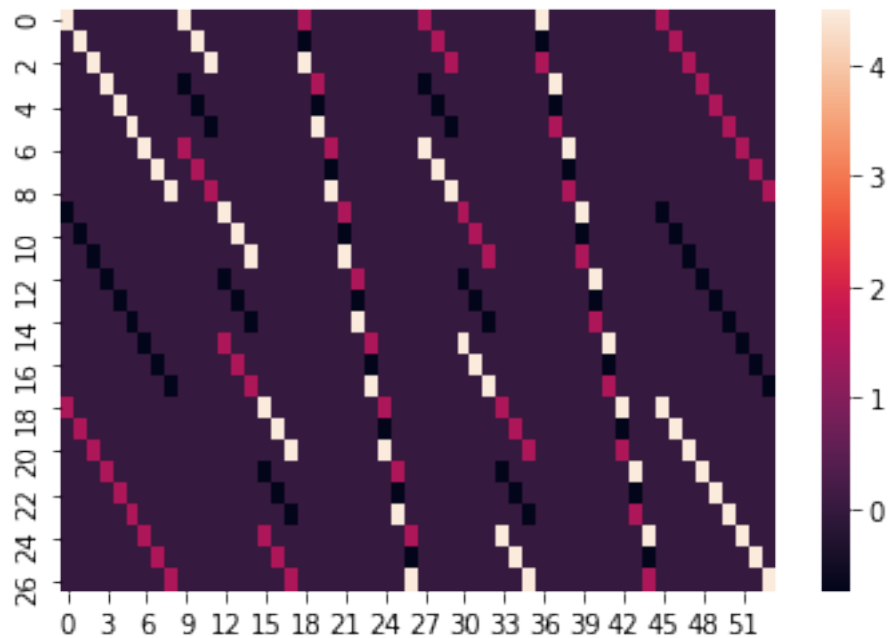
```
p4-gauss-legendre-m6
ratio: 2.5
(array([-6.54804561, -1.86607792, -0.9375    , -0.55981112, -0.32228787,
        0.        ,  0.32228787,  0.55981112,  0.9375    ,  1.86607792,
        6.54804561]), array([   75,     75,     75,     75,     75, 55500,     75,
       75,     75,
         75,     75]))
```
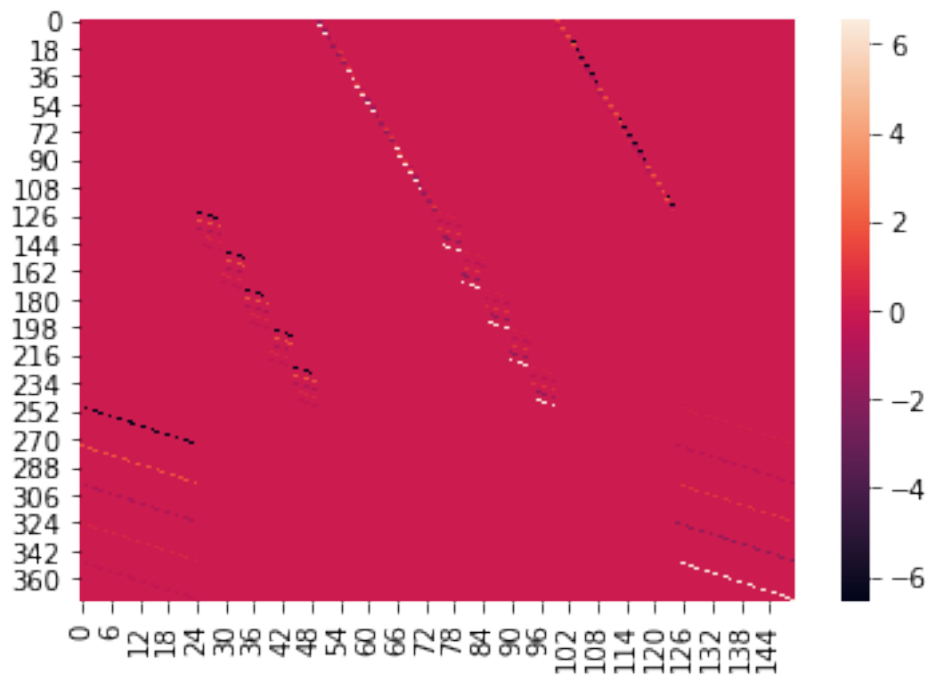


```
p4-gauss-legendre-m0
ratio: 1.2
(array([-0.89315839, -0.26794165,  0.        ,  0.07635866,  0.53333333,
        1.55140805]), array([  150,    150, 18000,    150,    150,    150]))
```
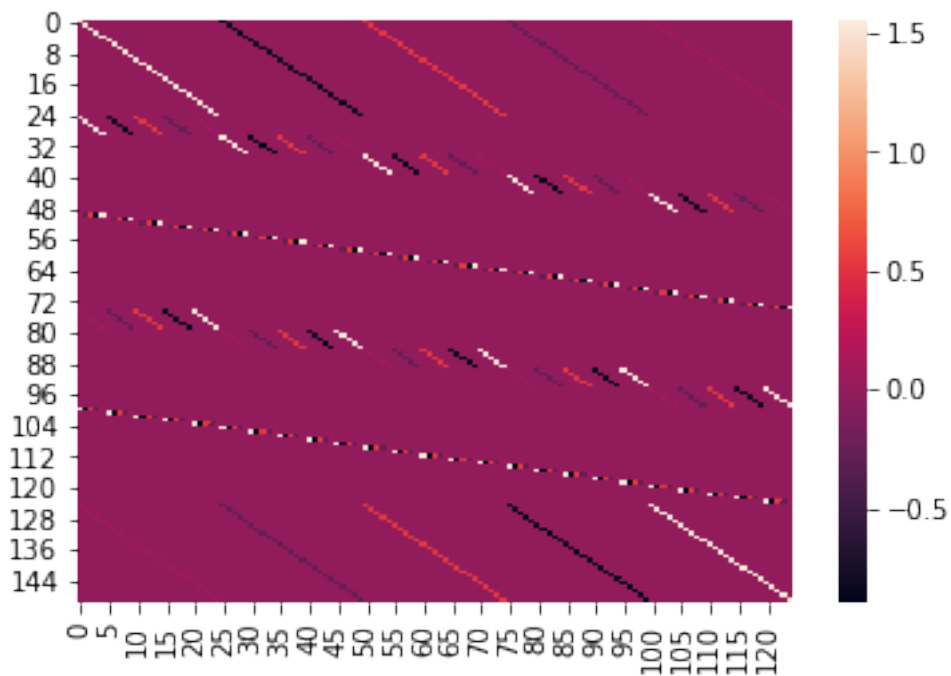
p4-gauss-legendre-m3
ratio: 0.8333333333333334
(array([-1.86607792, -0.55981112, 0.   , 0.32228787, 0.9375   ,
        6.54804561]), array([150,   150, 18000,   150,   150,   150]))
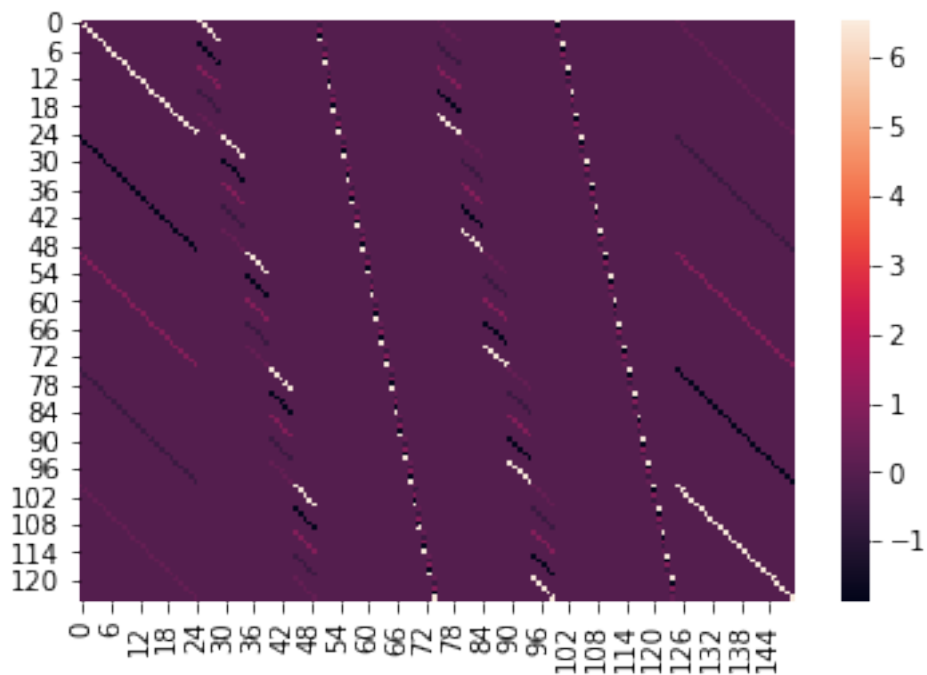
p4-gauss-legendre-lobatto-m6

ratio: 2.5

(array([-12.5   , -2.5   , -1.07142857, -0.9375   ,
          0.   ,   0.9375   ,   1.07142857,   2.5   ,
         12.5   ]), array([75,   75,   150,   75, 55500,   75,   150,
    75,   75]))
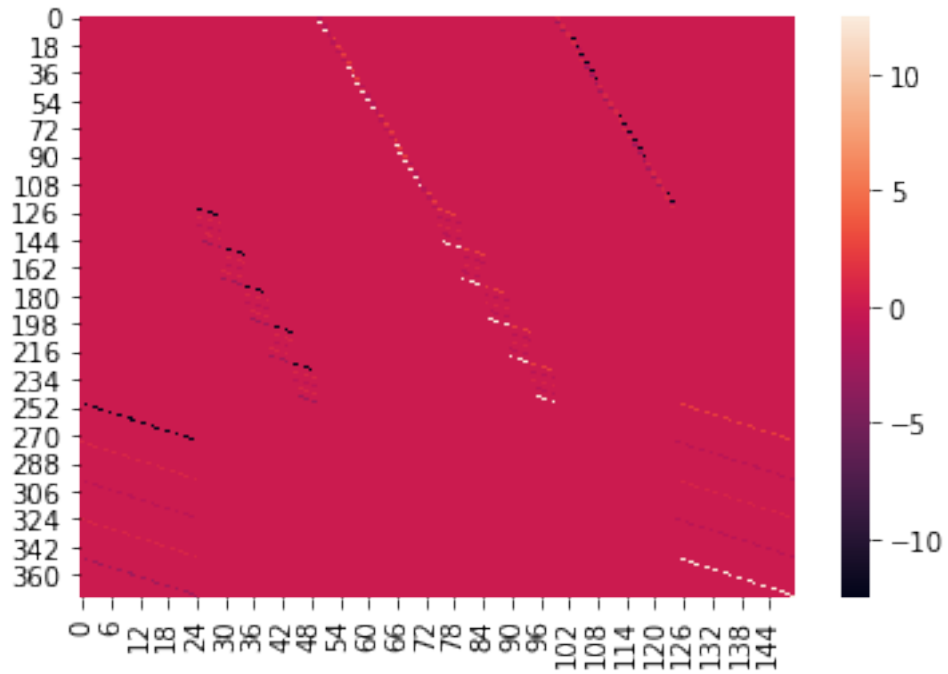
p4-gauss-legendre-lobatto-m0

ratio: 1.2

(array([0., 1.]), array([18600,   150]))
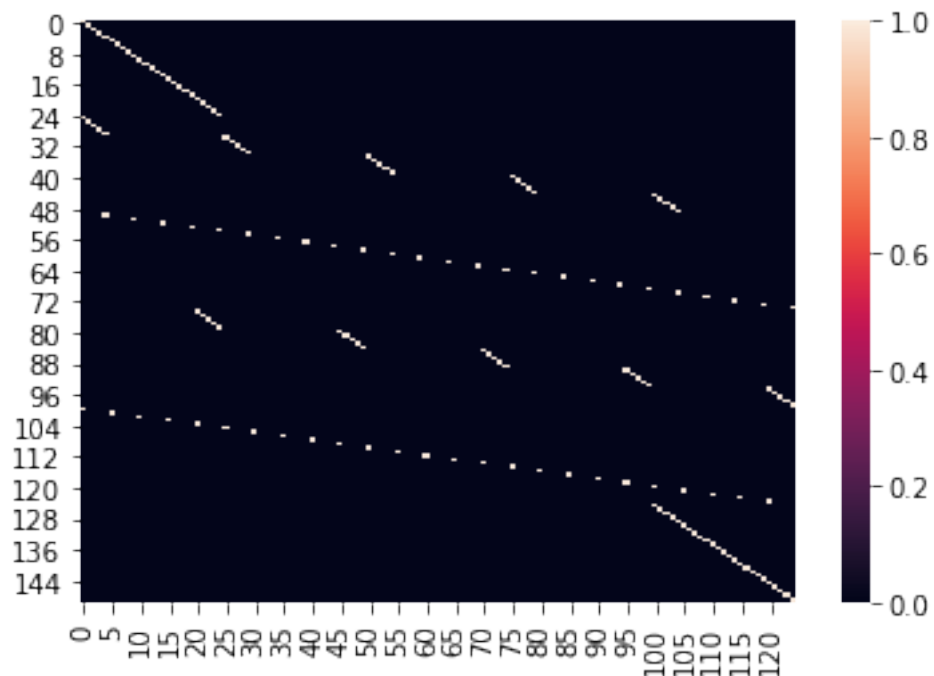
p4-gauss-legendre-lobatto-m3
ratio: 0.8333333333333334
(array([-1.07142857, 0.   , 0.9375   , 2.5   , 12.5   ]),
array([300, 18000,   150,   150,   150]))
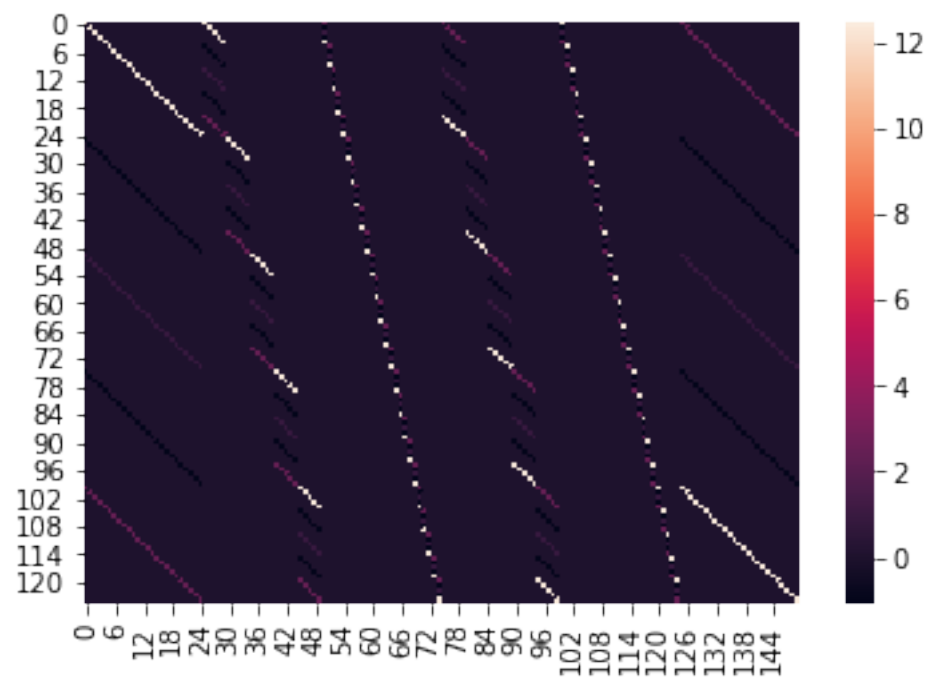
```
p6-gauss-legendre-m6
ratio: 3.5
(array([-12.16096777,  -3.47053266,  -1.76022778,  -1.09375    ,
         -0.74392821,  -0.51507804,  -0.31752836,   0.         ,
          0.31752836,   0.51507804,   0.74392821,   1.09375    ,
          1.76022778,   3.47053266,  12.16096777]), array([  147,     147,
147,      147,     147,      147,     147, 300468,
          147,     147,      147,     147,      147,     147,     147]))
```
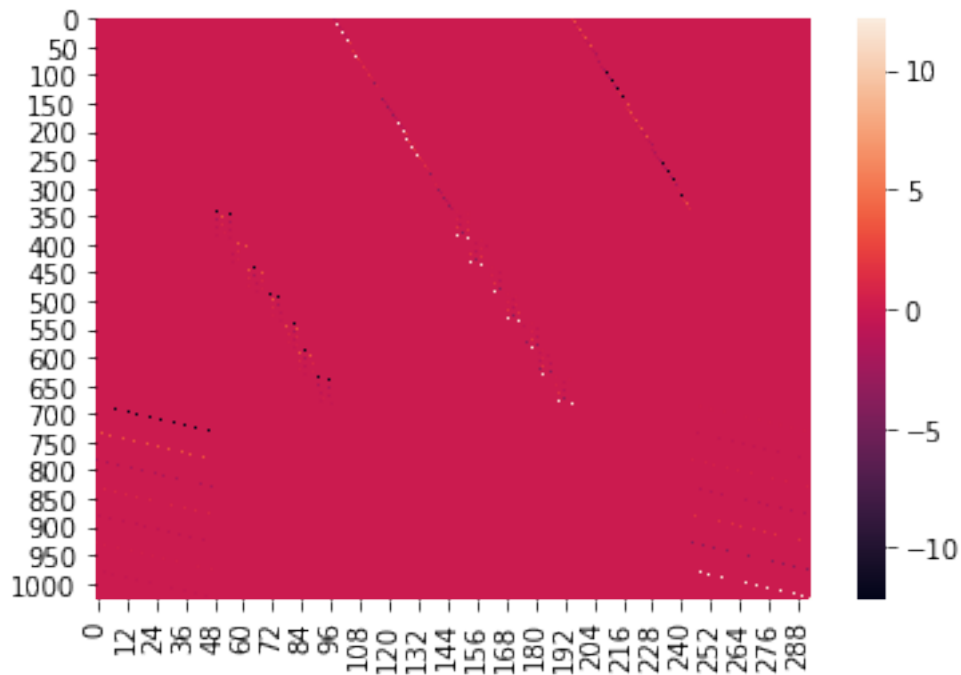


```
p6-gauss-legendre-m0
ratio: 0.8571428571428571
(array([-0.9707267 , -0.45714286, -0.1440701 ,  0.        ,  0.04111515,
        0.28405415,  0.67210786,  1.5746625 ]), array([ 294,    294,    294,
98784,    294,    294,    294,    294]))
```
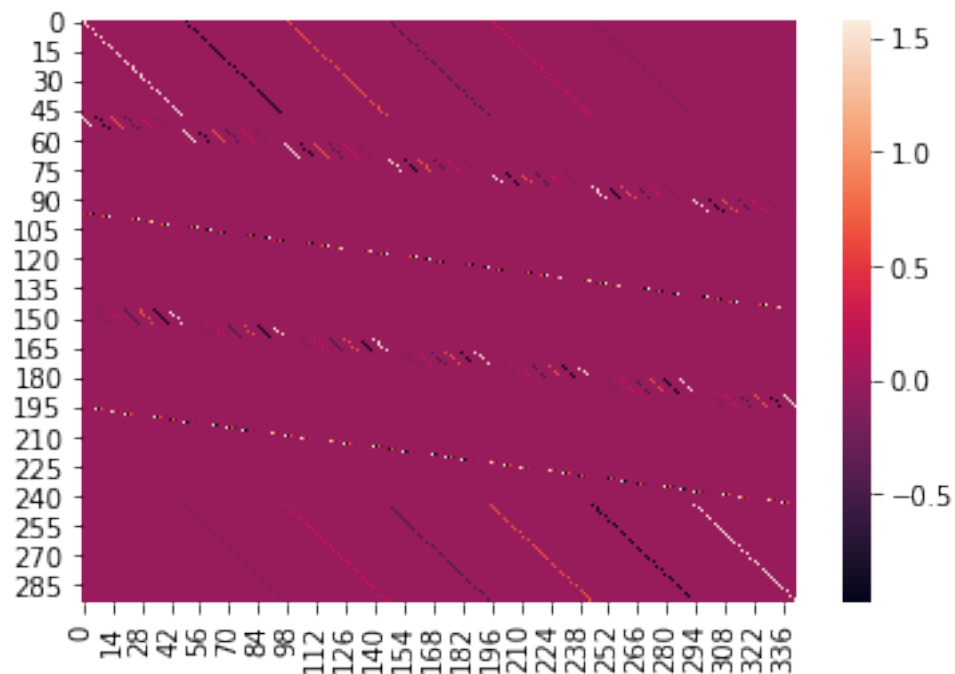
p6-gauss-legendre-m3
ratio: 1.1666666666666667
(array([-3.47053266, -1.09375   , -0.51507804, 0.   , 0.31752836,
        0.74392821, 1.76022778, 12.16096777]), array([294,   294,   294,
98784,   294,   294,   294,   294]))
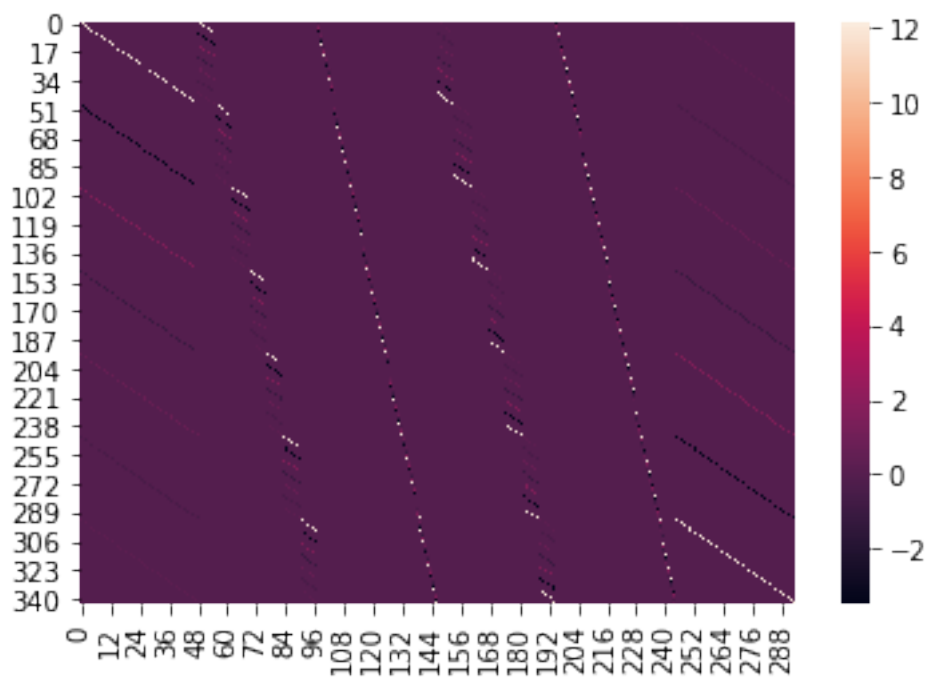
```
p6-gauss-legendre-lobatto-m6
ratio: 3.5
(array([-24.5       , -3.5       , -1.45162661, -1.16237041,
        -1.09375  ,  0.        ,  1.09375   ,  1.16237041,
         1.45162661,  3.5       , 24.5       ]), array([ 147,     147,
  294,     294,     147, 300468,     147,     294,
         294,     147,     147]))
```
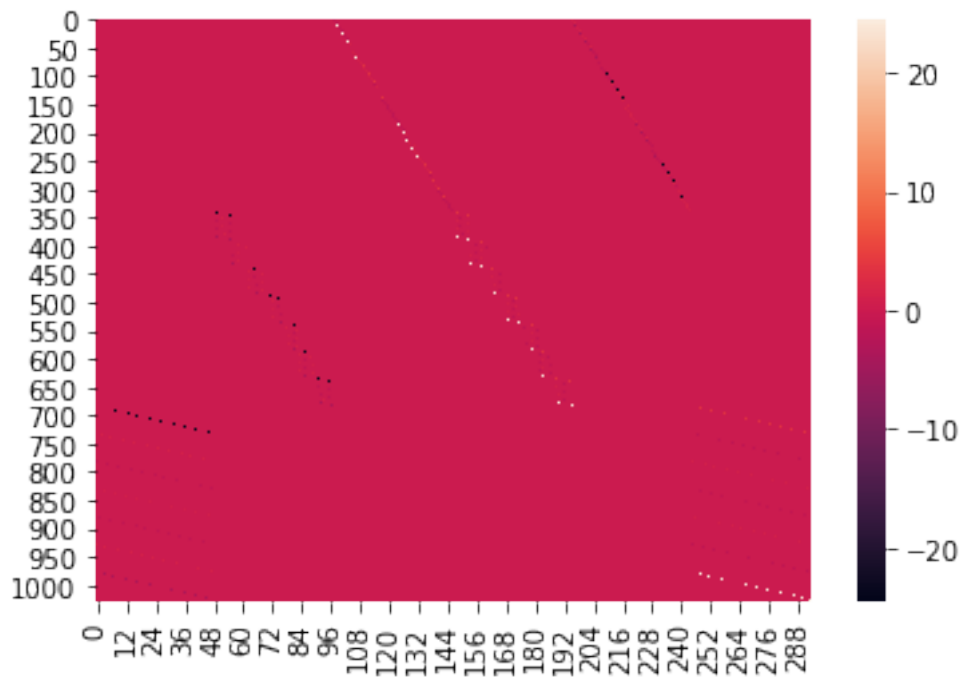


```
p6-gauss-legendre-lobatto-m0
ratio: 0.8571428571428571
(array([0., 1.]), array([100548,    294]))
```
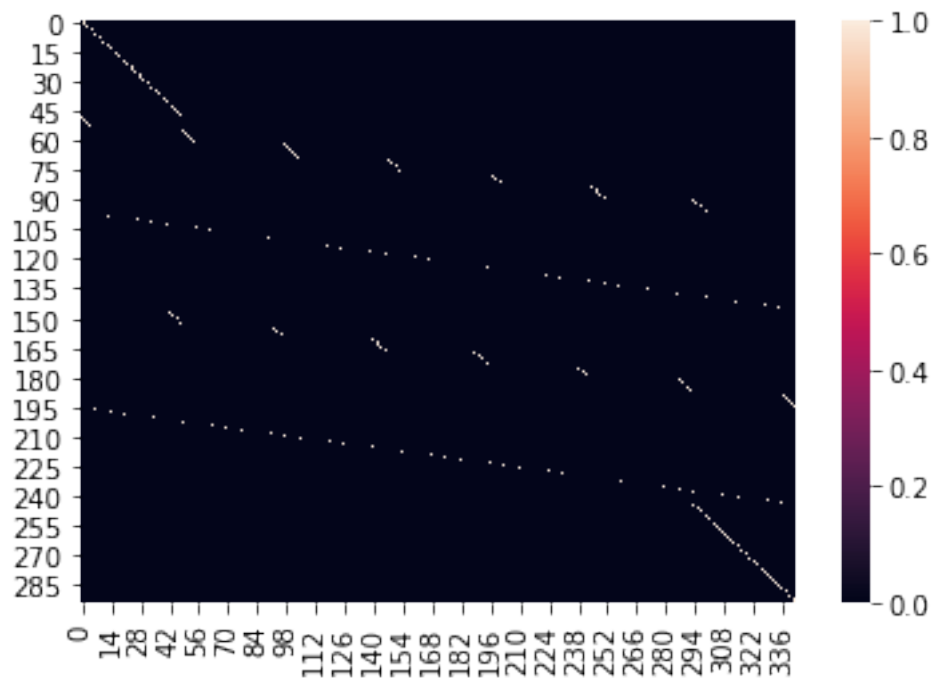
p6-gauss-legendre-lobatto-m3
ratio: 1.1666666666666667
(array([-1.45162661, -1.09375   , 0.   , 1.16237041, 3.5   ,
        24.5   ]), array([588,   294, 98784,   588,   294,   294]))
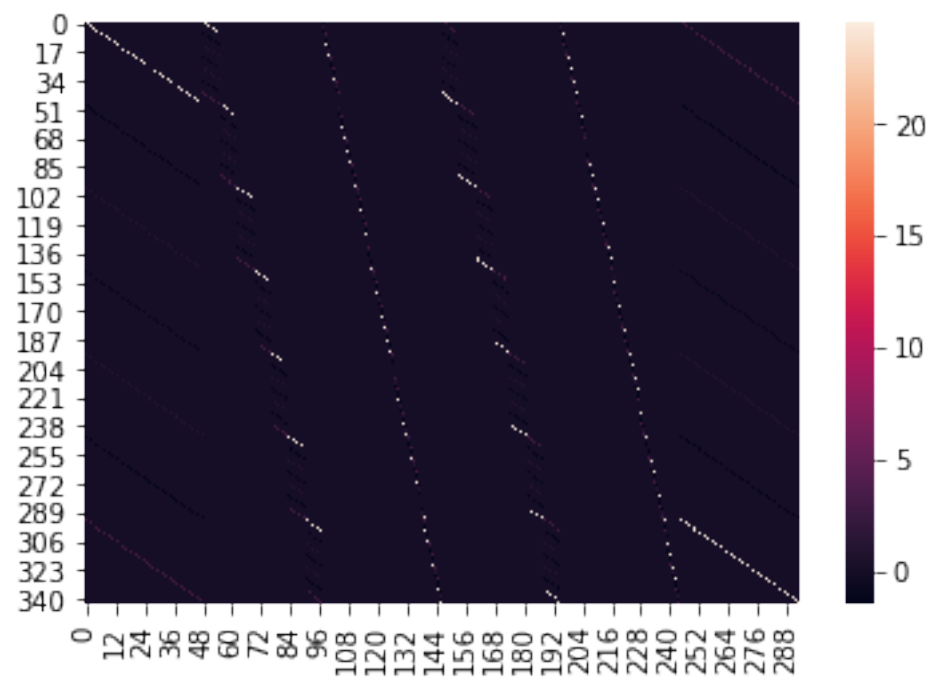
```
p5-gauss-legendre-m0
ratio: 1.0
(array([-0.94046284, -0.3792277 , -0.05471272,  0.        ,  0.19180001,
        0.61693006,  1.5656732 ]), array([  216,    216,    216, 45360,    216,
216,    216]))
```
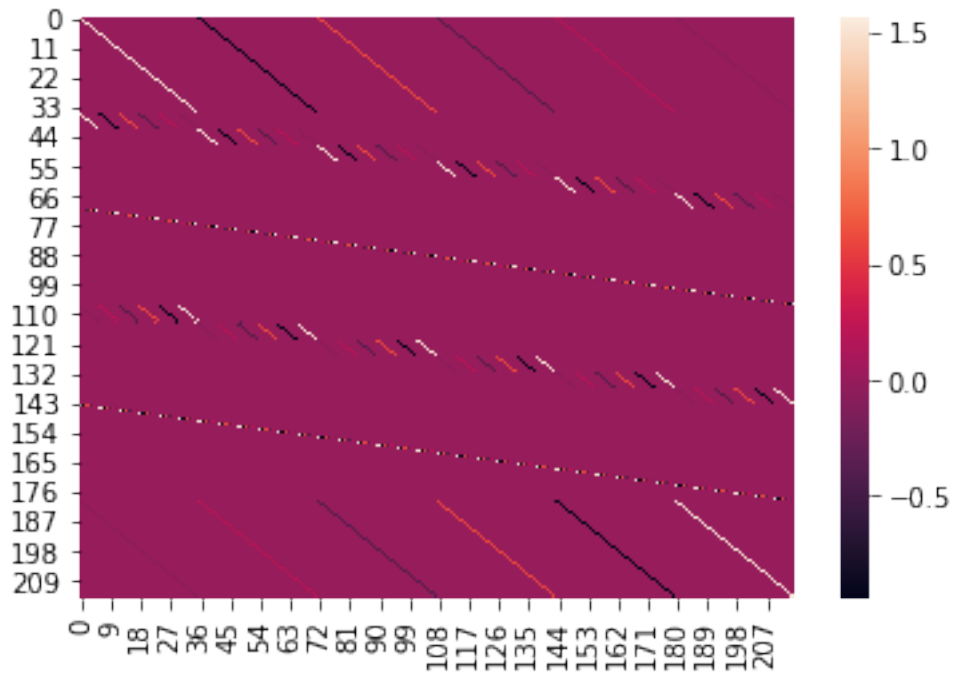


```
p5-gauss-legendre-m3
ratio: 1.0
(array([-2.60688198, -0.81046465, -0.31935145,  0.        ,  0.53165311,
        1.31846908,  9.13864199]), array([  216,    216,    216, 45360,    216,
216,    216]))
```
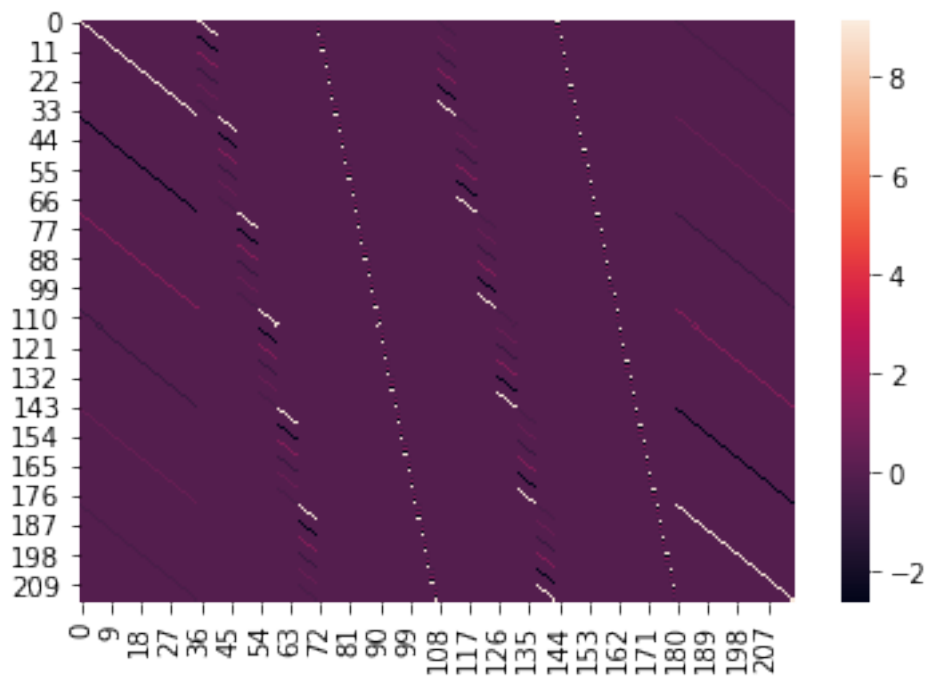
p5-gauss-legendre-lobatto-m0
ratio: 1.0
(array([0., 1.]), array([46440,   216]))
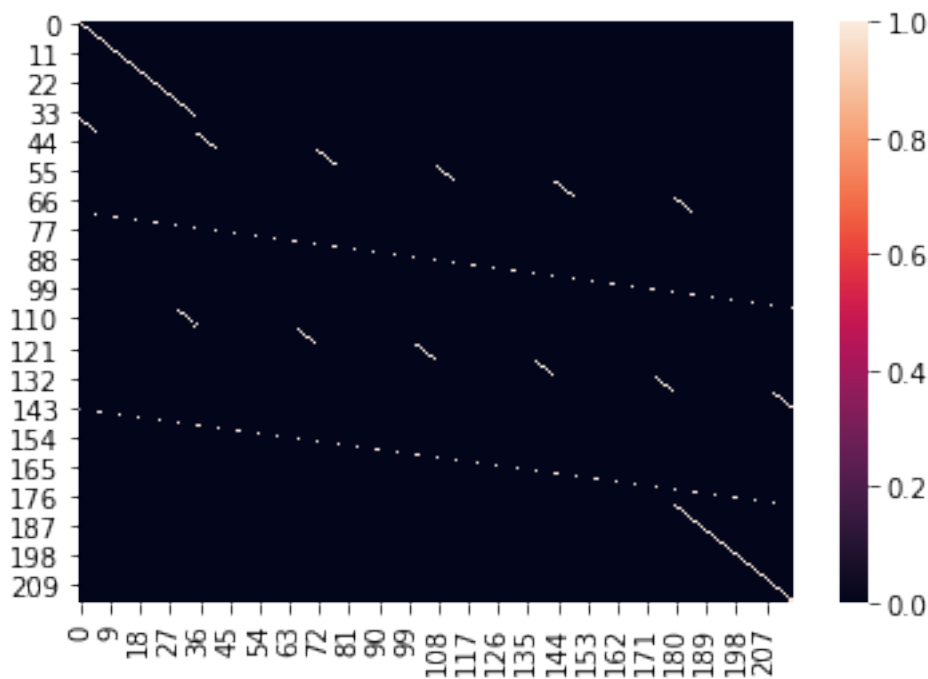
```
p5-gauss-legendre-lobatto-m3
ratio: 1.0
(array([-3.       , -1.2590908 , -1.03988318,  0.        ,  1.03988318,
         1.2590908 , 18.        ]), array([  216,    216,    216, 45360,    216,
  216,    216]))
```
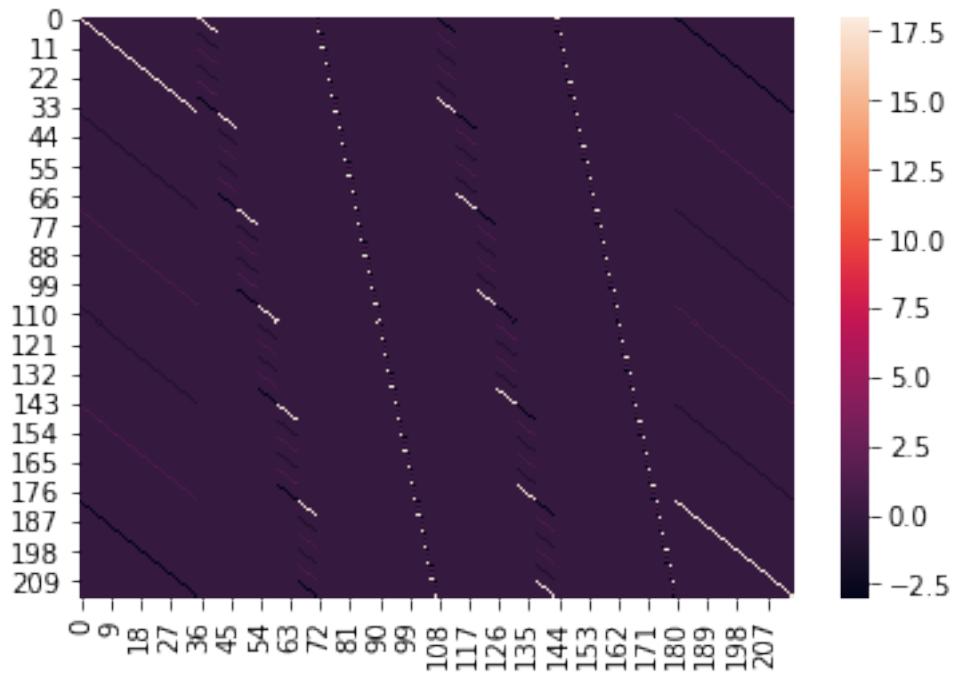

0.5.1 Thoughts

- m132 and m460 - quite sparse
- m0, m3, m6 - intersecting diags

0.6 Tri

```
[17]: from collections import defaultdict
      ratios_tri = defaultdict(int)
      for name, data in matrices["tri"].items():
          dims = data.shape
          ratios_tri[dims[0]/dims[1]] += 1
```

```
[18]: ratios_tri
```

```
[18]: defaultdict(int,
                  {0.5: 4,
                   1.0: 3,
                   2.0: 5,
                   1.6666666666666667: 1,
                   1.2: 1,
                   0.8333333333333334: 1,
                   1.3333333333333333: 1,
                   1.5: 1,
                   0.6666666666666666: 1,
                   0.75: 1,
                   0.8571428571428571: 1})
```

```python
[19]: for name, data in matrices["tri"].items():
          dims = data.shape

          print(name)
          print("ratio: " + str(dims[0]/dims[1]))
          print(np.unique(data, return_counts=True))
          plt.figure()
          ax = sns.heatmap(data)
          plt.show()
```

```
p1-williams-shunn-m132
ratio: 0.5
(array([-1.,  0.,  1.]), array([6, 6, 6]))
```
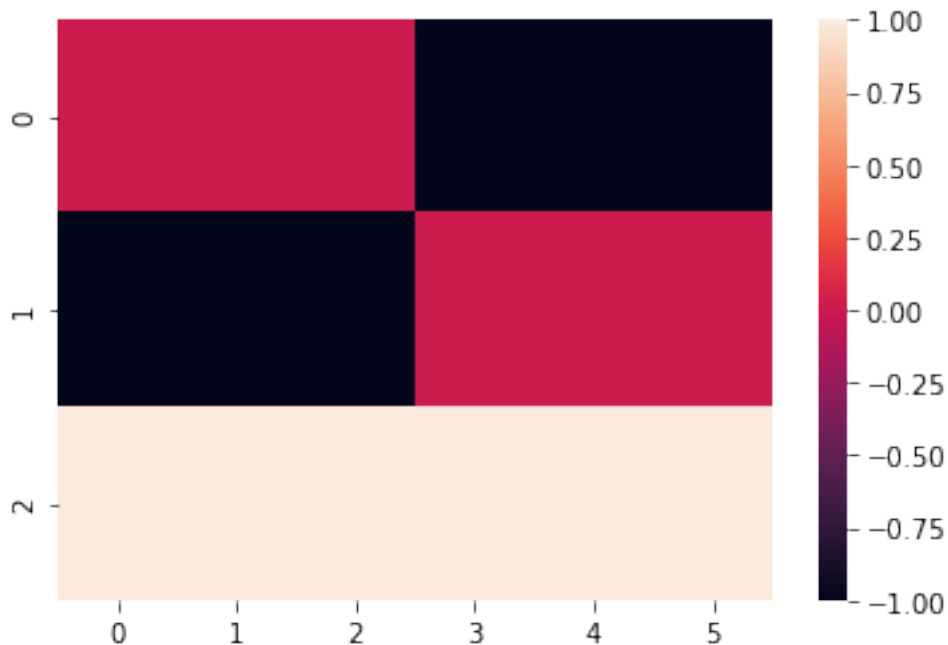
p1-williams-shunn-m6

ratio: 1.0

(array([-1.8660254, -0.5   , -0.1339746, 0.   , 0.1339746,
        0.5   , 1.8660254]), array([4, 4, 4, 12, 4, 4, 4]))
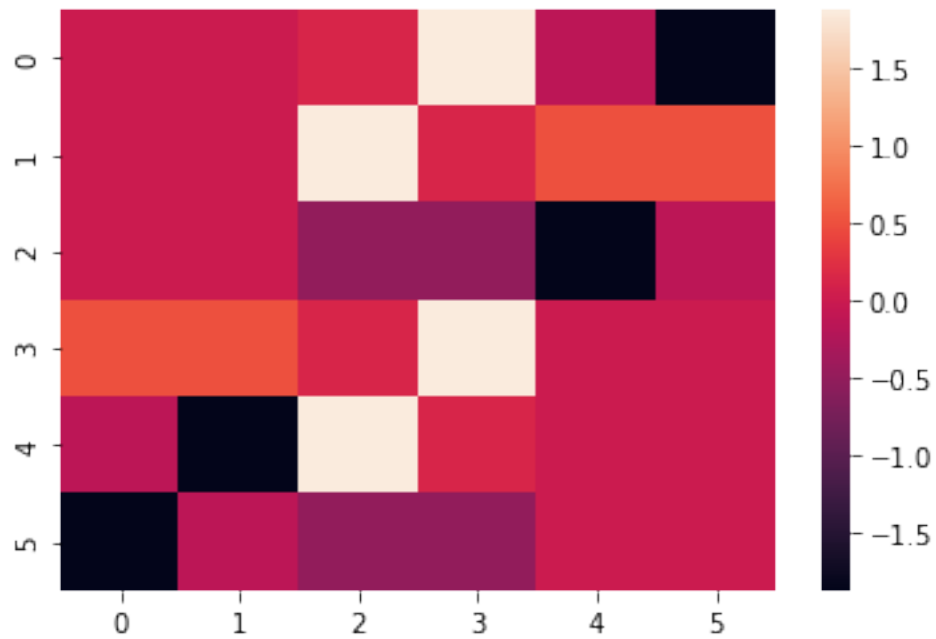
p1-williams-shunn-m0

ratio: 2.0

(array([-0.33333333, 0.0893164 , 1.24401694]), array([6, 6, 6]))
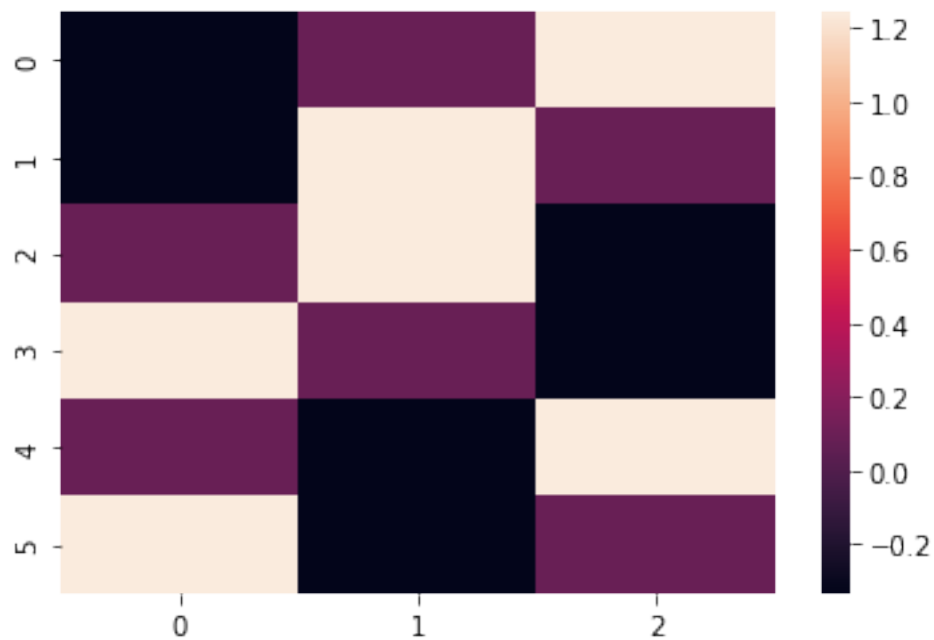
p1-williams-shunn-m3
ratio: 0.5
(array([-0.70710678, -0.5   , 0.1339746 , 0.18946869, 1.8660254 ,
        2.63895843]), array([2, 4, 4, 2, 4, 2]))
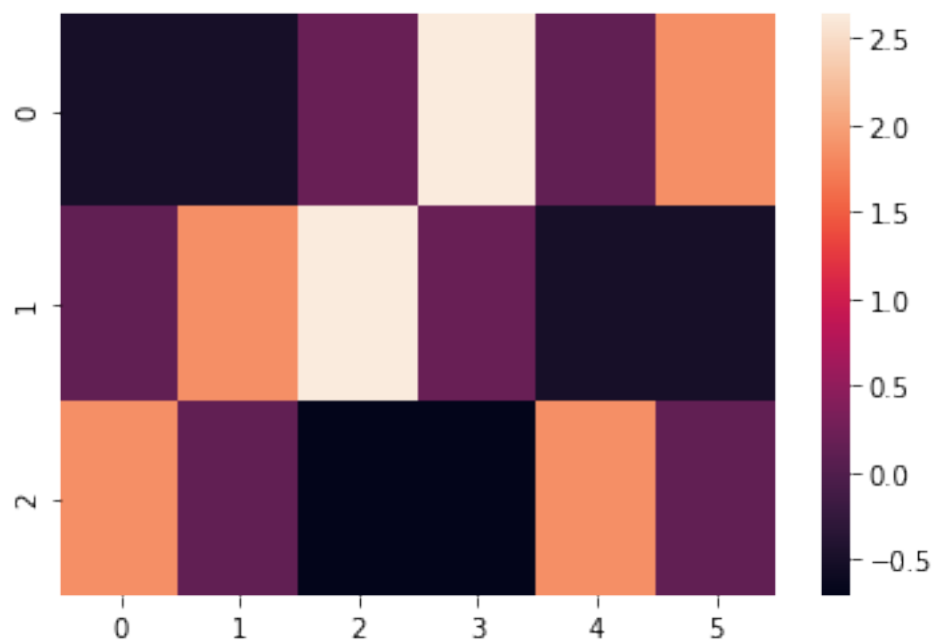
p1-williams-shunn-m460
ratio: 2.0
(array([-1., 0., 1.]), array([6, 6, 6]))
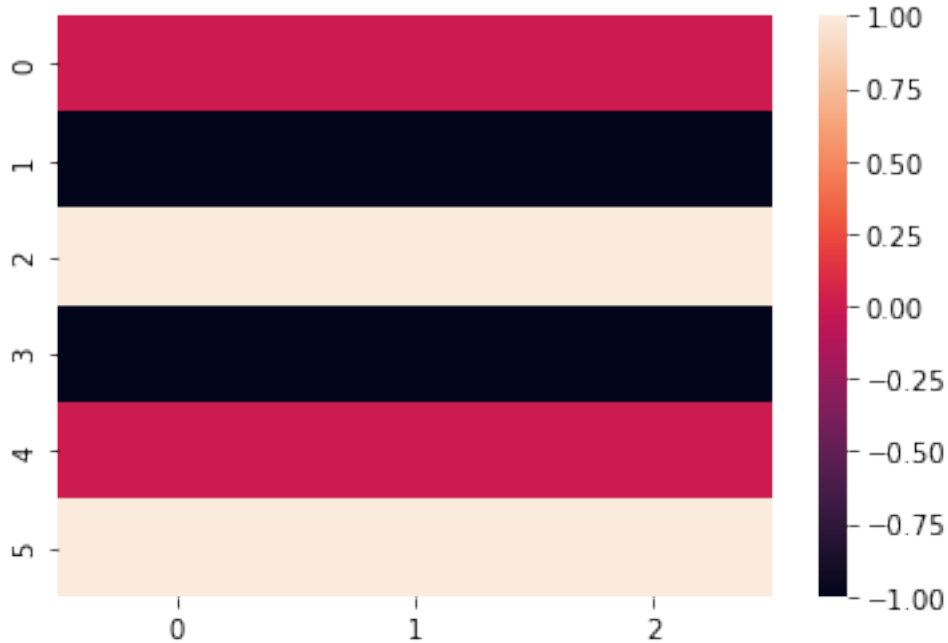
p3-williams-shunn-m132
ratio: 0.5
(array([-3.35205463, -2.68438394, -2.41092917, -2.13546192, -2.05926946,
        -2.05520728, -1.97089607, -1.92836375, -1.88014544, -1.83332026,
        -1.7966788 , -1.5477945 , -1.5418831 , -1.52860008, -1.52010698,
        -1.50282181, -1.46886676, -1.46850504, -1.44208707, -1.28830824,
        -0.97091421, -0.90886871, -0.82263206, -0.81333662, -0.62200876,
        -0.60184466, -0.57645687, -0.54483942, -0.27345477, -0.25948626,
        -0.24962445, -0.17506184, -0.16478474, -0.16456586, -0.14828215,
        -0.13090571, -0.11467199, -0.0955321 , -0.08483972, -0.07145144,
        -0.06009504, -0.04322055, -0.03664146, -0.03161745, -0.03017698,
        -0.02677969, -0.02177612, -0.0201641 , 0.   , 0.0201641 ,
         0.02177612, 0.02677969, 0.03017698, 0.03161745, 0.03664146,
         0.04322055, 0.06009504, 0.07145144, 0.08483972, 0.0955321 ,
         0.11467199, 0.13090571, 0.14828215, 0.16456586, 0.16478474,
         0.17506184, 0.24962445, 0.25948626, 0.27345477, 0.54483942,
         0.57645687, 0.60184466, 0.62200876, 0.81333662, 0.82263206,
         0.90886871, 0.97091421, 1.28830824, 1.44208707, 1.46850504,
         1.46886676, 1.50282181, 1.52010698, 1.52860008, 1.5418831 ,
         1.5477945 , 1.7966788 , 1.83332026, 1.88014544, 1.92836375,
         1.97089607, 2.05520728, 2.05926946, 2.13546192, 2.41092917,
         2.68438394, 3.35205463]), array([2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,

```
2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 8, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2]))
```



```
p3-williams-shunn-m6
ratio: 1.6666666666666667
(array([-5.35339889, -4.0659391 , -1.01654961, -0.98992608, -0.87995339,
       -0.87358235, -0.84130301, -0.50036306, -0.45597692, -0.37077408,
       -0.36566639, -0.35855909, -0.32426292, -0.24211985, -0.13739944,
       -0.08108871, -0.05630924, -0.05511438, -0.04597254, -0.02349354,
        0.        ,  0.02349354,  0.04597254,  0.05511438,  0.05630924,
        0.08108871,  0.13739944,  0.24211985,  0.32426292,  0.35855909,
        0.36566639,  0.37077408,  0.45597692,  0.50036306,  0.84130301,
        0.87358235,  0.87995339,  0.98992608,  1.01654961,  4.0659391 ,
        5.35339889]), array([ 4,  4,  4,  4,  4,  4,  4,  4,  4,  4,  4,  4,  4,
  4,  4,  4,  4,
        4,  4,  4, 80,  4,  4,  4,  4,  4,  4,  4,  4,  4,  4,  4,  4,
        4,  4,  4,  4,  4,  4,  4]))
```

p3-williams-shunn-m0
ratio: 1.2
(array([-0.66889879, -0.60002949, -0.3142121 , -0.12080501, -0.08480139,
        -0.08035418, -0.03774133, -0.01360123, -0.00931764, 0.01069078,
         0.01909215, 0.02376128, 0.0478322 , 0.14437936, 0.15168967,
         0.20558978, 0.30259288, 0.31395692, 1.30764361, 1.40253253]),
array([6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6]))
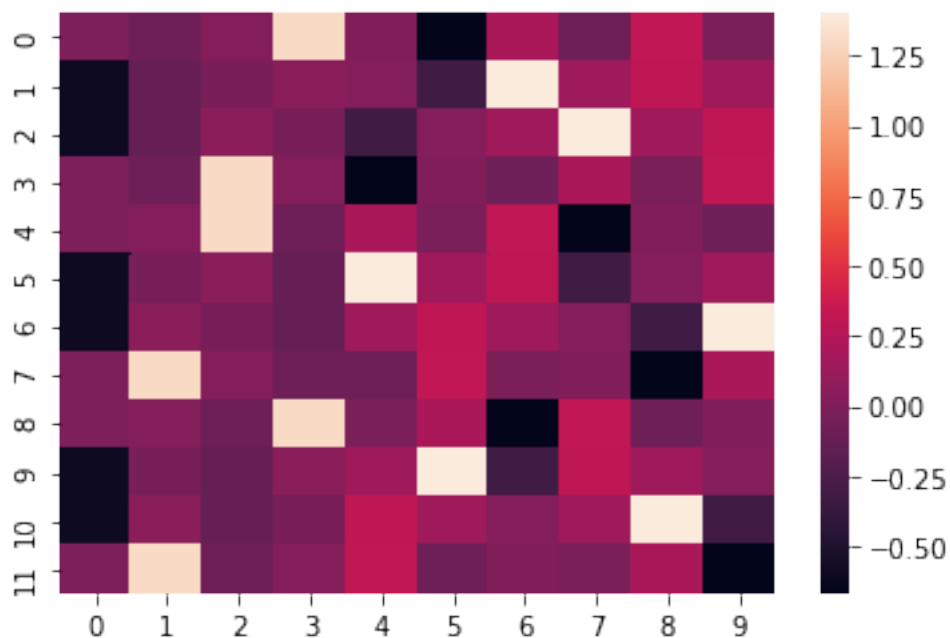
```
p3-williams-shunn-m3
ratio: 0.8333333333333334
(array([-1.43761825, -1.39996689, -1.24444201, -1.18978213, -1.01654961,
        -0.98992608, -0.87995339, -0.84130301, -0.50707913, -0.35855909,
        -0.34240918, -0.24211985, -0.19431216, -0.13739944, -0.06501499,
        -0.04597254, -0.03322488, -0.02349354,  0.05511438,  0.05630924,
         0.07794351,  0.07963329,  0.08108871,  0.11467676,  0.32426292,
         0.36566639,  0.37077408,  0.45597692,  0.45857702,  0.50036306,
         0.51713037,  0.52435374,  0.64484874,  0.70762023,  0.87358235,
         1.23543201,  4.0659391 ,  5.35339889,  5.75010622,  7.57084931]),
array([2, 2, 2, 2, 4, 4, 4, 4, 2, 4, 2, 4, 2, 4, 2, 4, 2, 4, 4, 4, 2, 2,
       4, 2, 4, 4, 4, 4, 2, 4, 2, 2, 2, 2, 4, 2, 4, 4, 2, 2]))
```
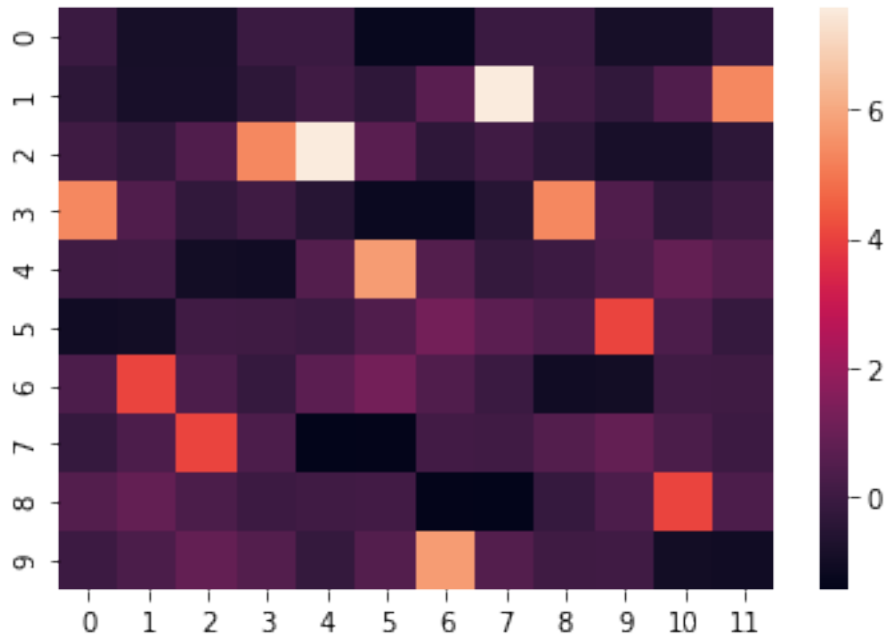


```
p3-williams-shunn-m460
ratio: 2.0
(array([-3.35205463, -2.68438394, -2.41092917, -2.13546192, -2.05926946,
        -2.05520728, -1.97089607, -1.92836375, -1.88014544, -1.83332026,
        -1.7966788 , -1.5477945 , -1.5418831 , -1.52860008, -1.52010698,
        -1.50282181, -1.46886676, -1.46850504, -1.44208707, -1.28830824,
        -0.97091421, -0.90886871, -0.82263206, -0.81333662, -0.62200876,
        -0.60184466, -0.57645687, -0.54483942, -0.27345477, -0.25948626,
        -0.24962445, -0.17506184, -0.16478474, -0.16456586, -0.14828215,
        -0.13090571, -0.11467199, -0.0955321 , -0.08483972, -0.07145144,
        -0.06009504, -0.04322055, -0.03664146, -0.03161745, -0.03017698,
```

```
       -0.02677969, -0.02177612, -0.0201641 ,  0.          ,  0.0201641 ,
        0.02177612,  0.02677969,  0.03017698,  0.03161745,  0.03664146,
        0.04322055,  0.06009504,  0.07145144,  0.08483972,  0.0955321 ,
        0.11467199,  0.13090571,  0.14828215,  0.16456586,  0.16478474,
        0.17506184,  0.24962445,  0.25948626,  0.27345477,  0.54483942,
        0.57645687,  0.60184466,  0.62200876,  0.81333662,  0.82263206,
        0.90886871,  0.97091421,  1.28830824,  1.44208707,  1.46850504,
        1.46886676,  1.50282181,  1.52010698,  1.52860008,  1.5418831 ,
        1.5477945 ,  1.7966788 ,  1.83332026,  1.88014544,  1.92836375,
        1.97089607,  2.05520728,  2.05926946,  2.13546192,  2.41092917,
        2.68438394,  3.35205463]), array([2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
        2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
        2, 2, 2, 2, 8, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
        2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
        2, 2, 2, 2, 2, 2, 2, 2, 2]))
```



```
p2-williams-shunn-m132
ratio: 0.5
(array([-2.04015464, -1.54015464, -1.49089629, -1.42729767, -1.41977847,
        -1.3507575 , -1.33528231, -1.27832187, -0.72154845, -0.66136036,
        -0.06902097, -0.06359862, -0.06018808, -0.05696044,  0.          ,
         0.05696044,  0.06018808,  0.06359862,  0.06902097,  0.66136036,
         0.72154845,  1.27832187,  1.33528231,  1.3507575 ,  1.41977847,
         1.42729767,  1.49089629,  1.54015464,  2.04015464]), array([2, 2, 2, 2,
2, 2, 2, 4, 2, 2, 2, 4, 2, 2, 8, 2, 2, 4, 2, 2, 2, 4,
```

```
2, 2, 2, 2, 2, 2, 2]))
```
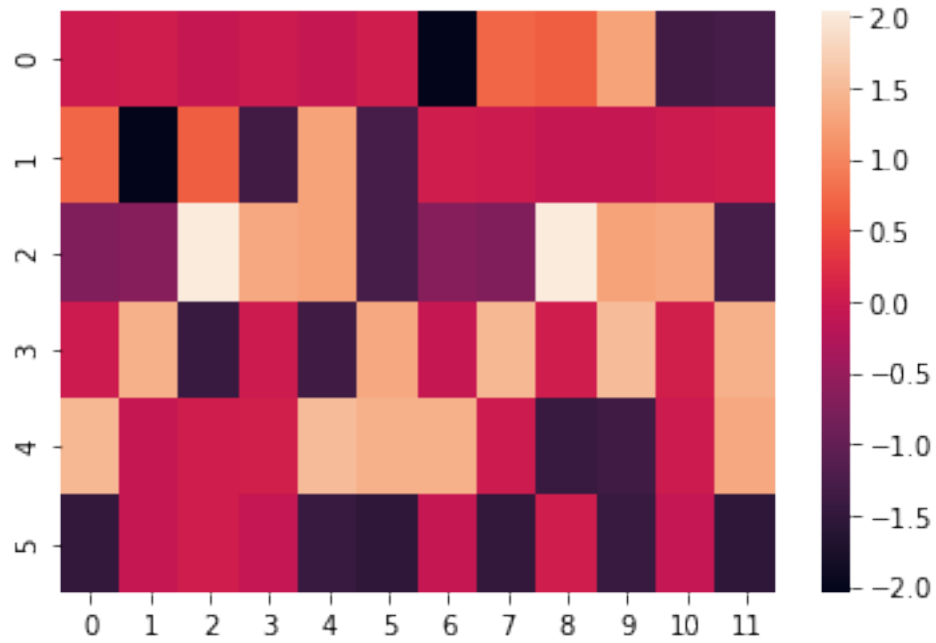


```
p2-williams-shunn-m6
ratio: 1.3333333333333333
(array([-3.29848568, -2.7069103 , -0.73116613, -0.72780648, -0.68678848,
       -0.39198253, -0.30276343, -0.20029351, -0.09694943, -0.00560158,
        0.        ,  0.00560158,  0.09694943,  0.20029351,  0.30276343,
        0.39198253,  0.68678848,  0.72780648,  0.73116613,  2.7069103 ,
        3.29848568]), array([ 4,  2,  4,  2,  4,  4,  4,  4,  4,  4, 36,  4,  4,
 4,  4,  4,  4,
        2,  4,  2,  4]))
```
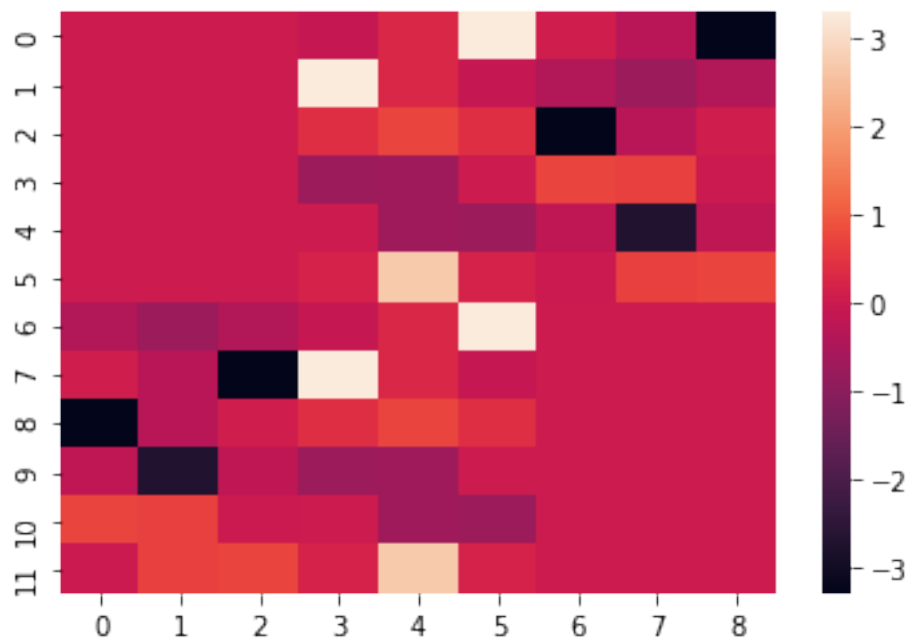
p2-williams-shunn-m0
ratio: 1.5
(array([-0.58798459, -0.34518578, -0.03837513, 0.00450465, 0.07490108,
        0.15515698, 0.16107078, 0.18005308, 1.30562731, 1.36051633]),
array([6, 6, 6, 6, 6, 6, 6, 3, 6, 3]))
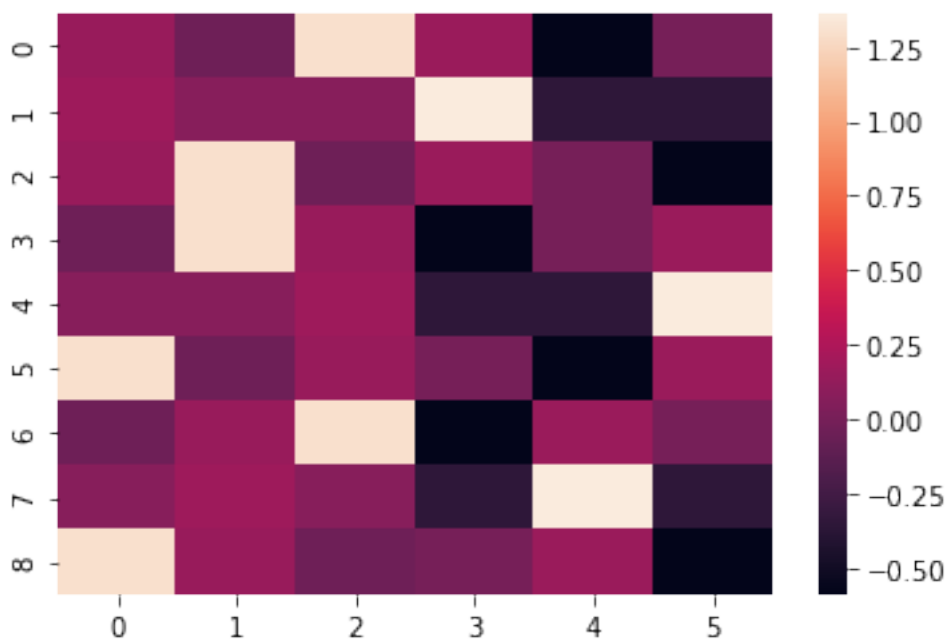
p2-williams-shunn-m3
ratio: 0.6666666666666666
(array([-1.03402506, -0.97126559, -0.73116613, -0.68678848, -0.13710719,
        -0.09694943, 0.00560158, 0.00792184, 0.20029351, 0.2832578 ,
         0.30276343, 0.39198253, 0.42817214, 0.55434701, 0.72780648,
         1.02927379, 2.7069103 , 3.29848568, 3.82814926, 4.66476318]),
array([2, 2, 4, 4, 2, 4, 4, 2, 4, 2, 4, 4, 2, 2, 2, 1, 2, 4, 1, 2]))
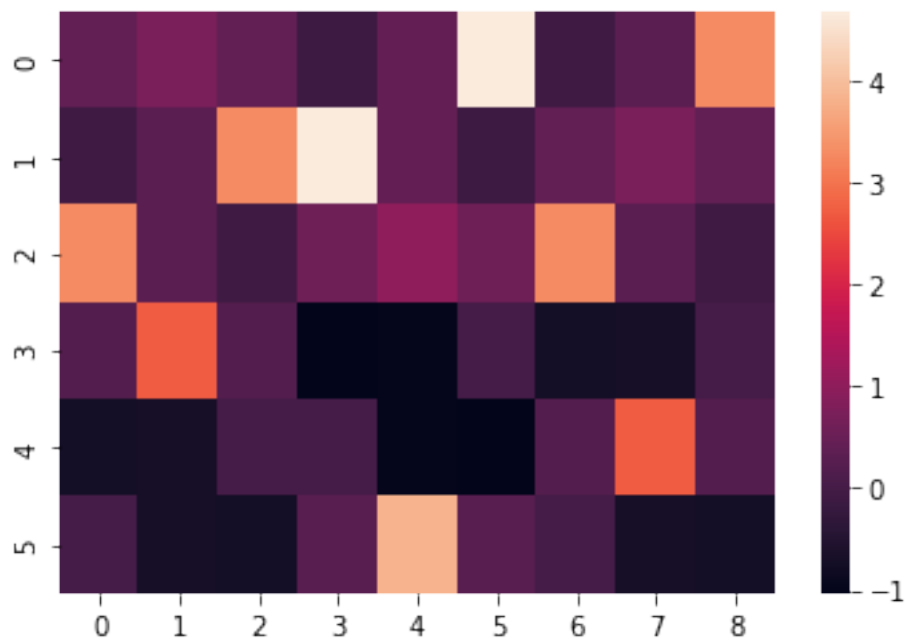
p2-williams-shunn-m460
ratio: 2.0
(array([-2.04015464, -1.54015464, -1.49089629, -1.42729767, -1.41977847,
        -1.3507575 , -1.33528231, -1.27832187, -0.72154845, -0.66136036,
        -0.06902097, -0.06359862, -0.06018808, -0.05696044, 0.   ,
         0.05696044, 0.06018808, 0.06359862, 0.06902097, 0.66136036,
         0.72154845, 1.27832187, 1.33528231, 1.3507575 , 1.41977847,
         1.42729767, 1.49089629, 1.54015464, 2.04015464]), array([2, 2, 2, 2,
2, 2, 2, 4, 2, 2, 2, 4, 2, 2, 8, 2, 2, 4, 2, 2, 2, 4,
        2, 2, 2, 2, 2, 2, 2]))
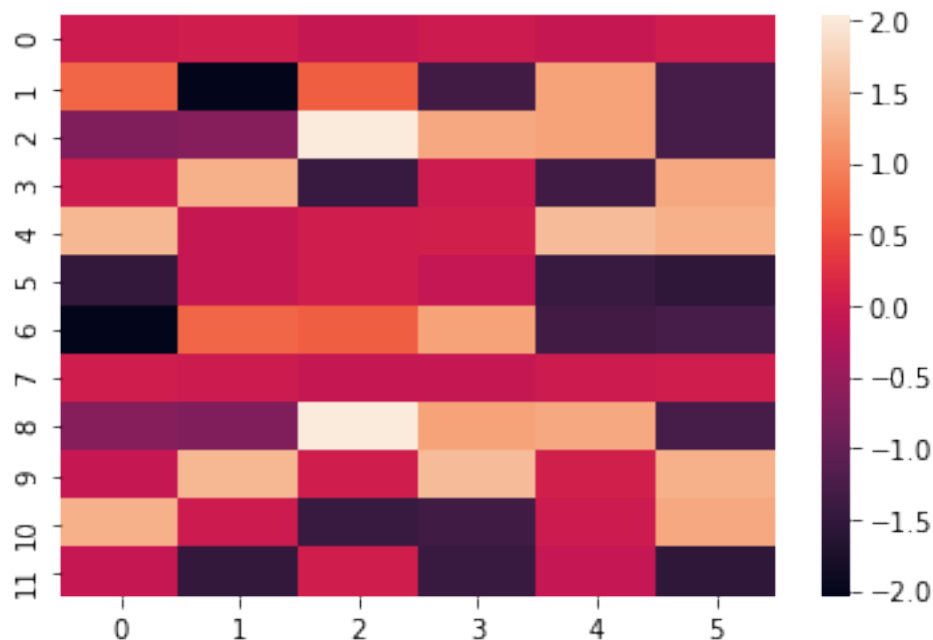
p4-williams-shunn-m6
ratio: 2.0
(array([-8.17798628, -5.9351833 , -5.45445852, -1.42627153, -1.33618961,
        -1.20584154, -1.17290095, -1.12531082, -1.0722364 , -0.97941096,
        -0.97589301, -0.92491869, -0.75403024, -0.69823501, -0.61438083,
        -0.58563893, -0.57871403, -0.43793491, -0.40508372, -0.40443234,
        -0.37668499, -0.37325346, -0.31585491, -0.298143 , -0.23564033,
        -0.21116871, -0.15204942, -0.14147099, -0.10925634, -0.10651087,
        -0.10547656, -0.09218614, -0.07404784, -0.05850348, -0.04441552,
        -0.03493992, -0.03407069, -0.03078013, -0.01797338, 0.   ,
         0.01797338, 0.03078013, 0.03407069, 0.03493992, 0.04441552,
         0.05850348, 0.07404784, 0.09218614, 0.10547656, 0.10651087,
         0.10925634, 0.14147099, 0.15204942, 0.21116871, 0.23564033,
         0.298143 , 0.31585491, 0.37325346, 0.37668499, 0.40443234,
         0.40508372, 0.43793491, 0.57871403, 0.58563893, 0.61438083,
         0.69823501, 0.75403024, 0.92491869, 0.97589301, 0.97941096,
         1.0722364 , 1.12531082, 1.17290095, 1.20584154, 1.33618961,
         1.42627153, 5.45445852, 5.9351833 , 8.17798628]), array([4,   4,
   2,   4,   4,   2,   4,   4,   4,   4,   2,   4,   4,
         4,   4,   4,   4,   4,   4,   4,   4,   4,   4,   4,
         4,   4,   4,   4,   4,   4,   4,   4,   4,   4,   4,
       150,   4,   4,   4,   4,   4,   4,   4,   4,   4,   4,   4,
         4,   4,   4,   4,   4,   4,   4,   4,   4,   4,   4,
         4,   4,   4,   2,   4,   4,   4,   4,   2,   4,   4,   2,   4,
         4]))
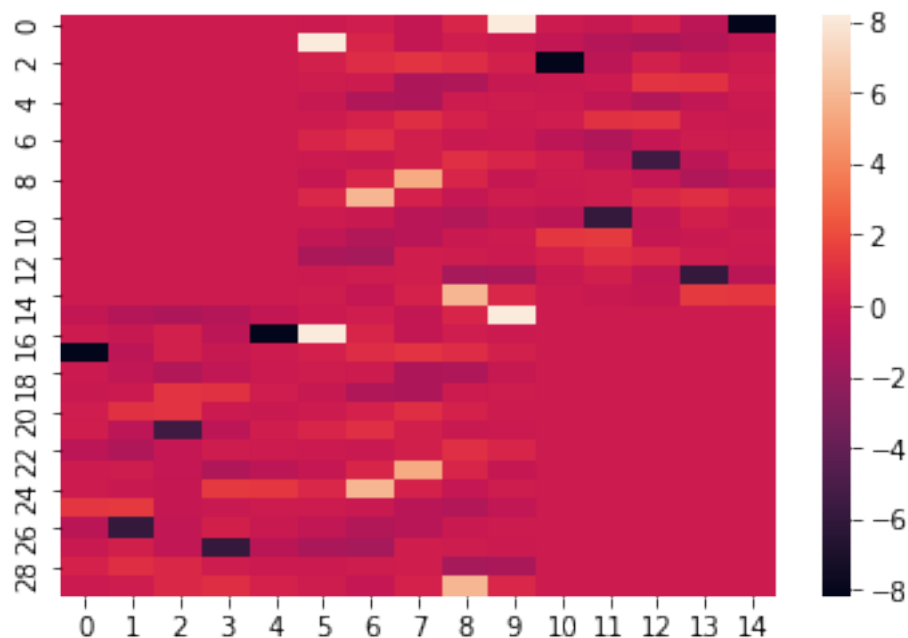
p4-williams-shunn-m0
ratio: 1.0
(array([-0.69744491, -0.67233012, -0.56475005, -0.27169671, -0.23898215,
        -0.19251477, -0.18053925, -0.10748769, -0.08515381, -0.06972436,
        -0.03258816, -0.02742354, -0.02715144, -0.01242883, -0.0113506 ,
        -0.00599464, -0.00303887, 0.00931605, 0.0110788 , 0.01539912,
         0.01661907, 0.02039085, 0.02066118, 0.04349218, 0.04705261,
         0.0508158 , 0.08730963, 0.10336184, 0.10678938, 0.10969481,
         0.21702186, 0.25146279, 0.30569695, 0.3130712 , 0.36215781,
         0.51339976, 1.27736279, 1.40533041, 1.44641914]), array([6, 6, 6, 6,
6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
       6, 6, 6, 6, 6, 6, 3, 6, 6, 6, 6, 6, 3, 6, 6, 3]))
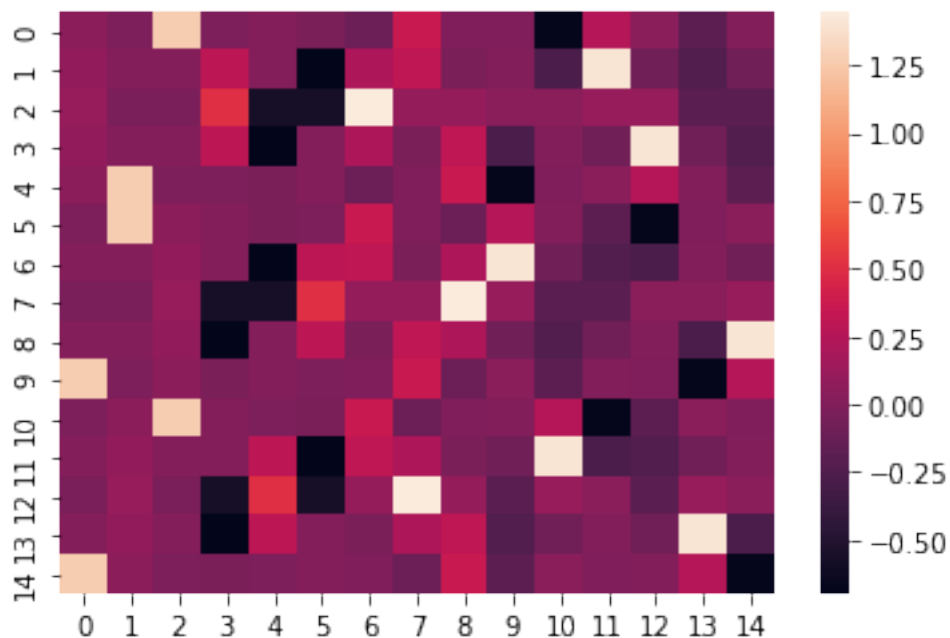
p4-williams-shunn-m3
ratio: 1.0
(array([-2.01705254, -1.88965747, -1.65873243, -1.59142983, -1.42627153,
        -1.38509626, -1.33618961, -1.17290095, -1.12531082, -1.06635978,
        -0.97941096, -0.75403024, -0.61933349, -0.52786011, -0.43793491,
        -0.42163788, -0.37325346, -0.29863765, -0.298143 , -0.21503036,
        -0.21116871, -0.1545118 , -0.15204942, -0.14916638, -0.10925634,
        -0.10547656, -0.08273641, -0.06281303, -0.05850348, -0.04818323,
        -0.04441552, -0.03407069, 0.01797338, 0.0254182 , 0.03078013,
         0.03493992, 0.04352968, 0.04941252, 0.07404784, 0.09218614,
         0.10471945, 0.10651087, 0.13037089, 0.14147099, 0.15062912,
         0.2000702 , 0.23564033, 0.31585491, 0.33324575, 0.37668499,
         0.40443234, 0.40508372, 0.4466863 , 0.53271302, 0.5719537 ,
         0.57287489, 0.57871403, 0.58563893, 0.61438083, 0.69823501,
         0.81842523, 0.82821852, 0.8688657 , 0.92491869, 0.97589301,
         0.98745343, 1.0722364 , 1.20584154, 1.30803255, 1.38012113,
         1.51637126, 1.70531745, 5.45445852, 5.9351833 , 7.71376922,
         8.17798628, 8.39361672, 11.56541912]), array([2, 2, 2, 2, 4, 2, 4, 4,
4, 2, 4, 4, 2, 2, 4, 2, 4, 2, 4, 2, 4, 2,
       4, 2, 4, 4, 2, 2, 4, 2, 4, 4, 4, 2, 4, 4, 2, 2, 4, 4, 2, 4, 2, 4,
       2, 2, 4, 4, 2, 4, 4, 4, 2, 2, 2, 2, 4, 4, 4, 4, 2, 2, 2, 4, 2, 2,
       4, 2, 2, 1, 2, 1, 2, 4, 1, 4, 2, 2]))

p6-williams-shunn-m0
ratio: 0.75
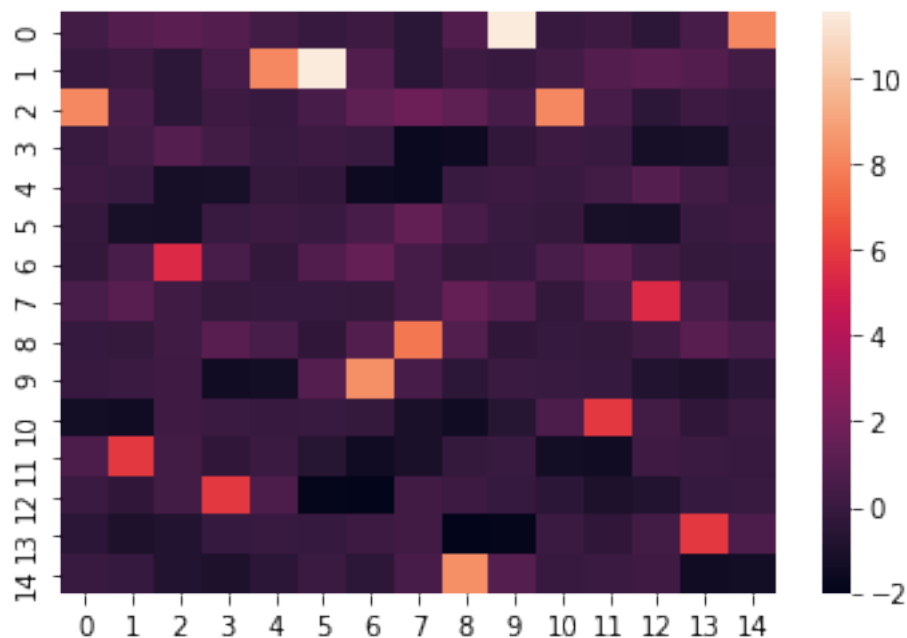(array([-7.41044169e-01, -7.38289664e-01, -7.25767872e-01, -6.38646802e-01,
        -4.77147528e-01, -4.53632738e-01, -4.12587193e-01, -2.54117997e-01,
        -2.48291504e-01, -2.19806321e-01, -2.15611594e-01, -1.95383585e-01,
        -1.88070629e-01, -1.54365873e-01, -1.43013072e-01, -1.21108481e-01,
        -1.18476245e-01, -1.07956755e-01, -1.05794734e-01, -1.02247568e-01,
        -8.75907367e-02, -6.25762860e-02, -5.34367241e-02, -5.01200807e-02,
        -4.03670126e-02, -3.25100222e-02, -3.22270469e-02, -2.90870970e-02,
        -2.65501477e-02, -2.58193683e-02, -2.40714060e-02, -1.56172771e-02,
        -1.48667876e-02, -1.46989743e-02, -1.27664863e-02, -1.24519034e-02,
        -1.17185130e-02, -6.96412768e-03, -5.33272229e-03, -4.64683053e-03,
        -4.30138531e-03, -4.29596094e-03, -3.11016731e-03, -2.40244872e-03,
        -1.26653376e-03, -1.20221015e-03, -8.75968607e-04, -6.74831817e-04,
         2.12902983e-04, 1.68997918e-03, 1.79788484e-03, 3.05187106e-03,
         3.29347463e-03, 3.86884947e-03, 5.47398312e-03, 5.68913026e-03,
         7.62625621e-03, 9.49071388e-03, 1.00556570e-02, 1.15487698e-02,
         1.30064544e-02, 1.48751272e-02, 1.75496630e-02, 2.24128076e-02,
         2.49133645e-02, 2.67791244e-02, 2.71982107e-02, 2.74277069e-02,
         2.87703805e-02, 3.29107435e-02, 5.12621469e-02, 5.19019476e-02,
         5.55615970e-02, 6.35085293e-02, 7.15892459e-02, 8.29488514e-02,
         9.49158802e-02, 1.01677254e-01, 1.13805867e-01, 1.15522613e-01,
         1.16092478e-01, 1.56401357e-01, 1.90132305e-01, 2.20481558e-01,
         2.20690345e-01, 2.35048601e-01, 2.39212121e-01, 2.59847577e-01,
         2.94177912e-01, 3.14292504e-01, 3.43459146e-01, 4.05287493e-01,
         4.13324857e-01, 4.33036739e-01, 5.48741352e-01, 6.51019189e-01,

```
        1.32528104e+00,  1.38126921e+00,  1.45334388e+00,  1.47672725e+00]),
array([6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
       6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
       6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
       6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 3, 6, 6, 6, 6, 6, 6, 6, 6, 6,
       6, 6, 6, 6, 6, 3, 6, 3, 6, 6, 6, 3]))
```


p5-williams-shunn-m0
ratio: 0.8571428571428571
```
(array([-7.65894028e-01, -7.38708994e-01, -6.80742415e-01, -5.13852470e-01,
        -4.05548339e-01, -2.81129273e-01, -2.60198092e-01, -2.33940466e-01,
        -1.88528665e-01, -1.72622727e-01, -1.01293177e-01, -9.76972473e-02,
        -8.69088221e-02, -7.26334079e-02, -7.04942428e-02, -4.15974137e-02,
        -4.03378214e-02, -3.34896744e-02, -3.19680928e-02, -3.12297537e-02,
        -2.66915981e-02, -2.05291746e-02, -1.18605072e-02, -9.94564171e-03,
        -9.30556437e-03, -7.10427213e-03, -4.92625080e-03, -4.50387726e-03,
        -6.87330481e-04,  3.63601285e-05,  2.04972514e-04,  1.06482964e-03,
         1.55018104e-03,  2.76614589e-03,  4.82550105e-03,  5.70951546e-03,
         1.51472297e-02,  1.55191163e-02,  1.64842028e-02,  1.90214540e-02,
         2.18085144e-02,  4.34052853e-02,  4.34558224e-02,  4.34919937e-02,
         5.02467102e-02,  5.87467865e-02,  6.59949605e-02,  6.96494762e-02,
         1.36054272e-01,  1.42309879e-01,  1.66429628e-01,  1.91265517e-01,
         1.97305222e-01,  2.15770668e-01,  2.26725095e-01,  2.55044578e-01,
         3.05027580e-01,  3.83471293e-01,  4.45022708e-01,  5.66461807e-01,
         1.37310461e+00,  1.40761104e+00,  1.45363638e+00]), array([6, 6, 6, 6,
6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
```

```
6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6]))
```


0.6.1 Thoughts

- no dominant dimension ratio

- can get corners of zeros

- repeat many constants for same amount

- seemingly random placement

- although can get clusters of a constant

- prob best to randomly place

0.7 Tet

```
[20]: from collections import defaultdict
      ratios_tet = defaultdict(int)
      for name, data in matrices["tet"].items():
          dims = data.shape
          ratios_tet[dims[0]/dims[1]] += 1
```

```
[21]: ratios_tet
```

```
[21]: defaultdict(int,
                  {0.3333333333333333: 3,
                   1.0: 1,
                   3.0: 3,
                   1.5: 1,
                   2.0: 1,
                   0.5: 1,
                   1.25: 1,
                   2.4: 1,
                   0.4166666666666667: 1,
                   1.7142857142857142: 1})
```

```
[22]: for name, data in matrices["tet"].items():
          dims = data.shape

          print(name)
          print("ratio: " + str(dims[0]/dims[1]))
          print(np.unique(data, return_counts=True))
          plt.figure()
          ax = sns.heatmap(data)
          plt.show()
```

```
p1-shunn-ham-m132
ratio: 0.3333333333333333
(array([-1.11803399,  0.        ,  1.11803399]), array([12, 24, 12]))
```
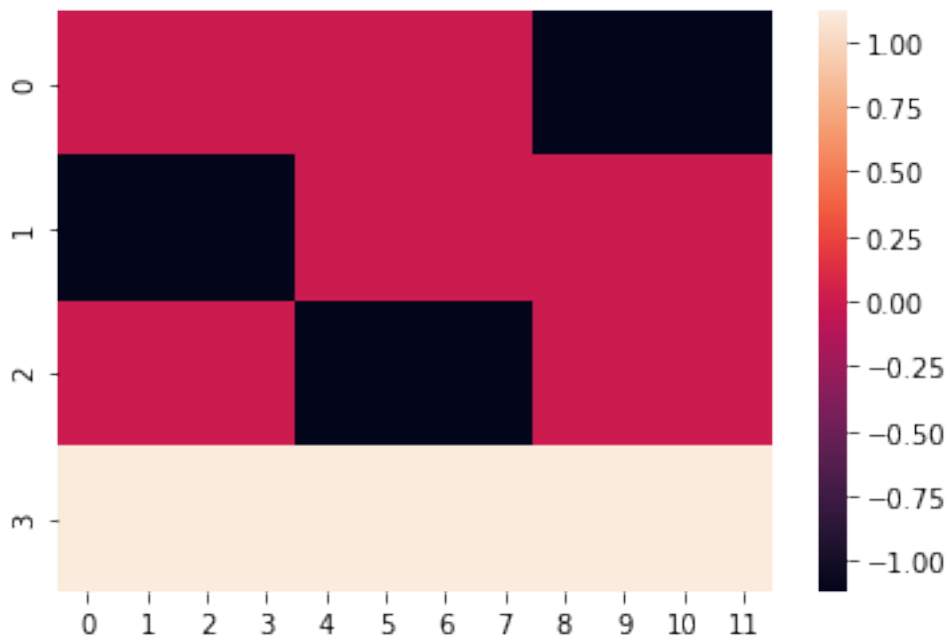
p1-shunn-ham-m6

ratio: 1.0

(array([-2.36338998, -0.61803399, -0.127322 , 0.   , 0.127322 ,
        0.61803399, 2.36338998]), array([9, 9, 18, 72, 18, 9, 9]))
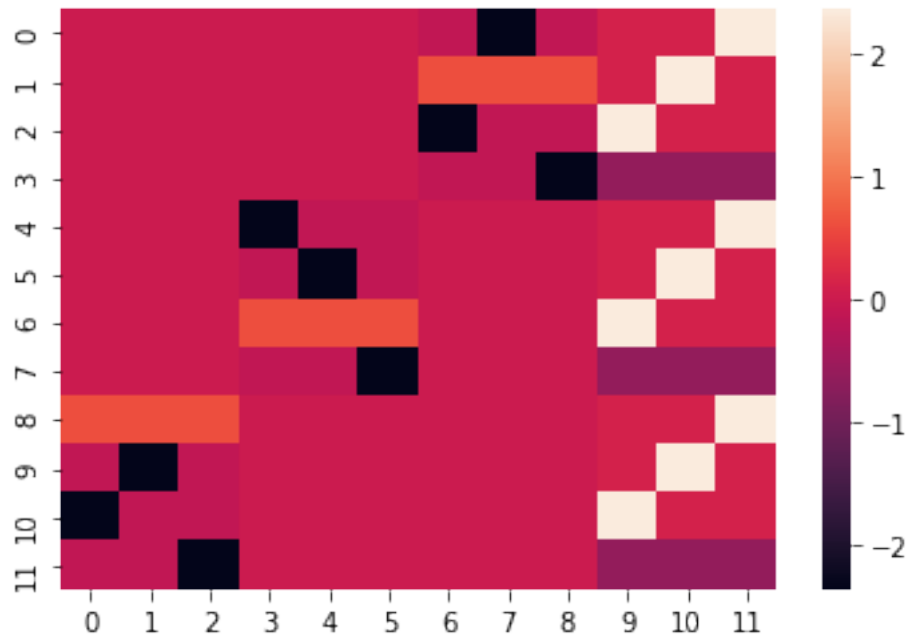
p1-shunn-ham-m0

ratio: 3.0

(array([-0.30901699, 0.063661 , 1.18169499]), array([12, 24, 12]))

p1-shunn-ham-m3
ratio: 0.3333333333333333
(array([-1.07046627, -0.61803399, 0.127322 , 0.22052818, 2.36338998,
        4.09351153]), array([3, 9, 18, 6, 9, 3]))

p1-shunn-ham-m460
ratio: 3.0
(array([-1.11803399, 0.   , 1.11803399]), array([12, 24, 12]))

p3-shunn-ham-m6
ratio: 1.5
(array([-7.67317874e+00, -4.82515868e+00, -4.19680096e+00, -2.76502047e+00,
       -1.49774568e+00, -1.21666464e+00, -9.25672626e-01, -9.02137346e-01,
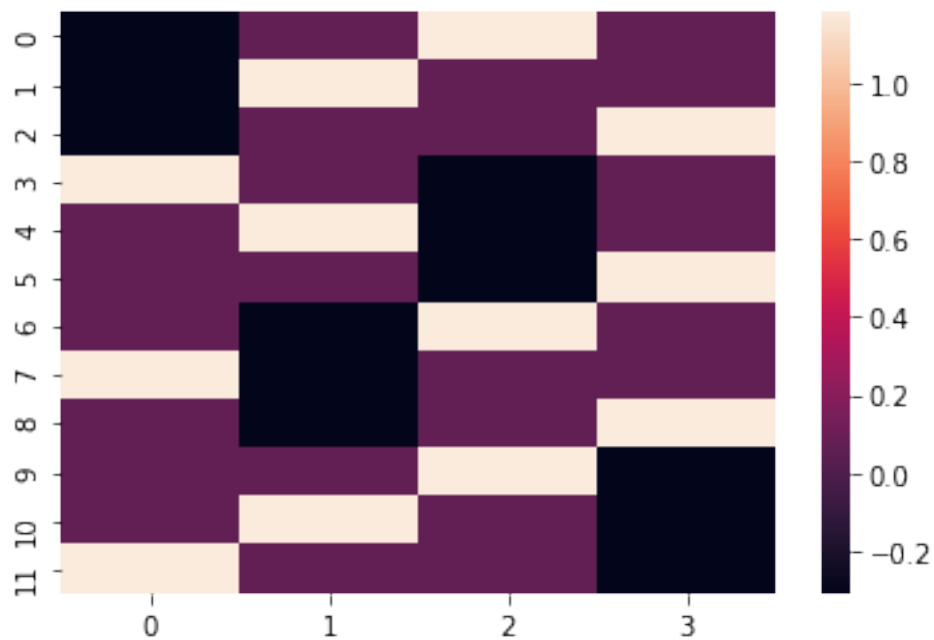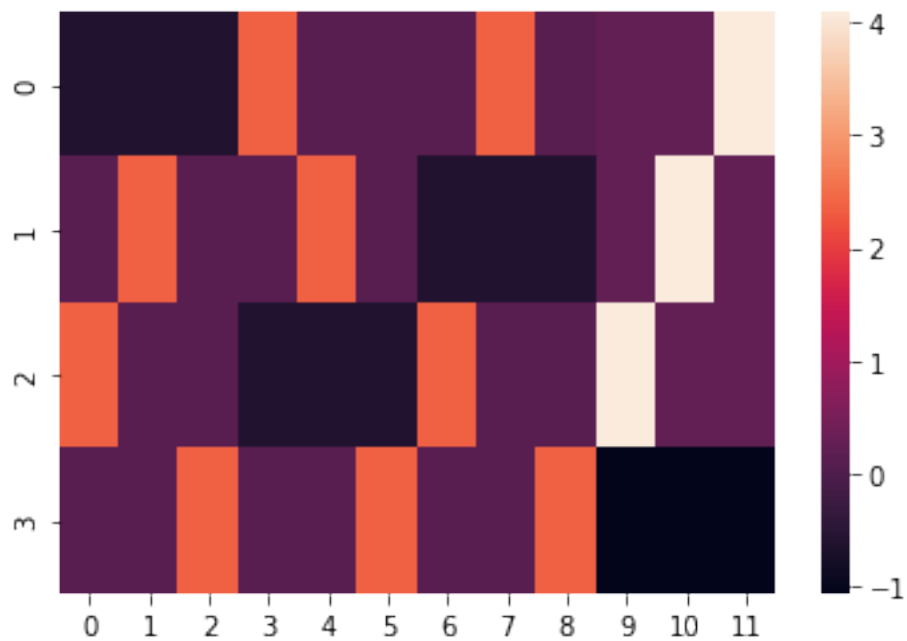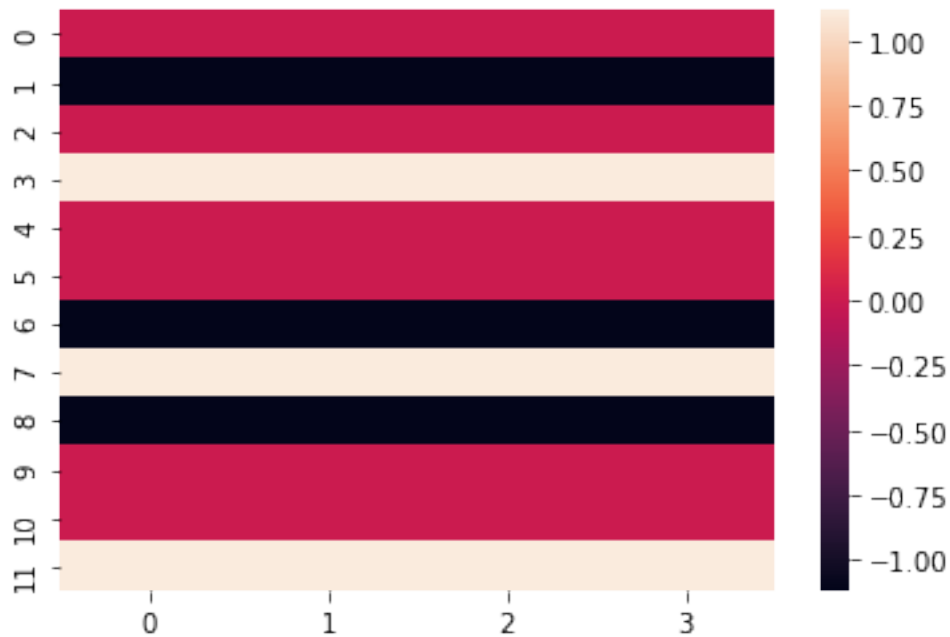   -8.37457798e-01, -7.03186498e-01, -6.90537714e-01, -6.56770646e-01,
       -5.47548032e-01, -5.30818522e-01, -4.75237563e-01, -3.65181021e-01,
       -3.24471383e-01, -3.24149807e-01, -2.61656644e-01, -2.46983272e-01,
       -1.79436063e-01, -1.59906209e-01, -1.43650268e-01, -1.35681212e-01,
       -1.31709608e-01, -1.26877479e-01, -1.15273131e-01, -9.17257660e-02,
       -8.72615087e-02, -7.93524066e-02, -7.75993461e-02, -7.60145555e-02,
       -6.31507997e-02, -6.28622859e-02, -3.79836236e-02, -3.32984468e-02,
       -1.76756946e-02, -9.49759940e-03, -3.05169500e-03, -1.22538693e-03,
        0.00000000e+00, 1.22538693e-03, 3.05169500e-03, 9.49759940e-03,
        1.76756946e-02, 3.32984468e-02, 3.79836236e-02, 6.28622859e-02,
        6.31507997e-02, 7.60145555e-02, 7.75993461e-02, 7.93524066e-02,
        8.72615087e-02, 9.17257660e-02, 1.15273131e-01, 1.26877479e-01,
        1.31709608e-01, 1.35681212e-01, 1.43650268e-01, 1.59906209e-01,
        1.79436063e-01, 2.46983272e-01, 2.61656644e-01, 3.24149807e-01,
        3.24471383e-01, 3.65181021e-01, 4.75237563e-01, 5.30818522e-01,
        5.47548032e-01, 6.56770646e-01, 6.90537714e-01, 7.03186498e-01,
        8.37457798e-01, 9.02137346e-01, 9.25672626e-01, 1.21666464e+00,
        1.49774568e+00, 2.76502047e+00, 4.19680096e+00, 4.82515868e+00,

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       7.67317874e+00]), array([   9,    18,     3,     3,    18,    18,    18,     9,
  18,     9,     9,
          18,     9,     9,    18,     9,    18,    18,    18,     9,    18,    18,
          18,    18,    18,    18,    18,     9,    18,    18,    18,    18,     9,
          18,    18,    18,    18,    18,    18,    18, 1200,    18,    18,    18,
          18,    18,    18,    18,     9,    18,    18,    18,    18,     9,    18,
          18,    18,    18,    18,    18,    18,     9,    18,    18,    18,     9,
          18,     9,     9,    18,     9,     9,    18,     9,    18,    18,    18,
           3,     3,    18,     9]))
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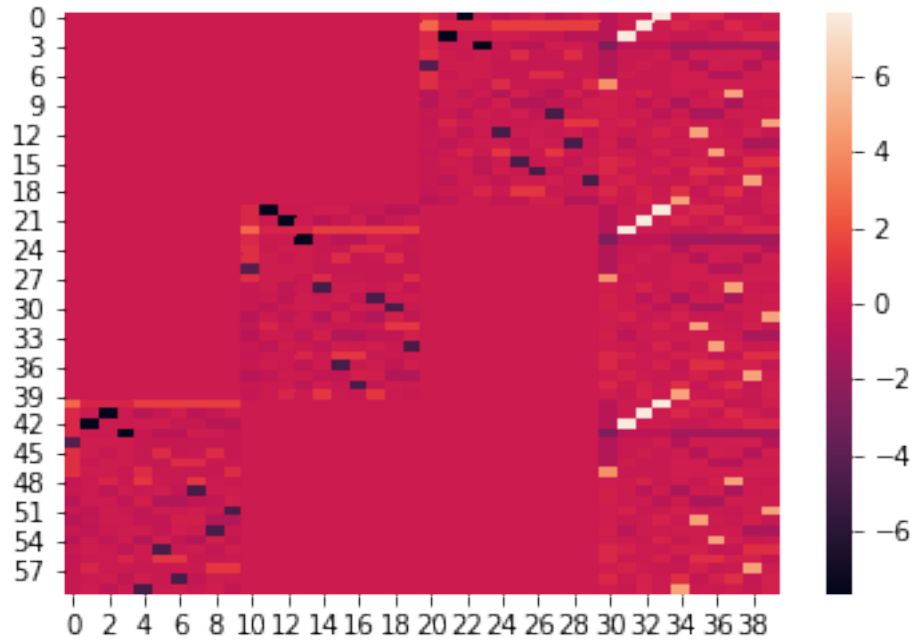


```
p3-shunn-ham-m0
ratio: 2.0
(array([-0.52600721, -0.43125275, -0.31416444, -0.26943946, -0.09618673,
        -0.0935334 , -0.0761862 , -0.04076538, -0.039667  , -0.03767137,
        -0.02680521, -0.02426688, -0.02017264, -0.01957271, -0.01906866,
        -0.01728259, -0.01474506, -0.00304354,  0.0047247 ,  0.00981748,
         0.01019143,  0.01093024,  0.0121453 ,  0.01234691,  0.01448161,
         0.01571394,  0.01697567,  0.02603404,  0.04759603,  0.04968805,
         0.10752904,  0.10760102,  0.12988772,  0.13863615,  0.19247882,
         0.23801822,  0.28560425,  0.91241758,  1.29776873,  1.35246873]),
array([24, 12, 12, 24,  4, 24, 24, 12, 24, 12, 24, 24, 24, 12, 24, 24, 24,
       24, 12, 24, 24, 24, 24, 24, 12, 24, 24, 24, 24, 24, 24, 24, 24, 12,
       12, 24, 24, 12, 24,  4]))
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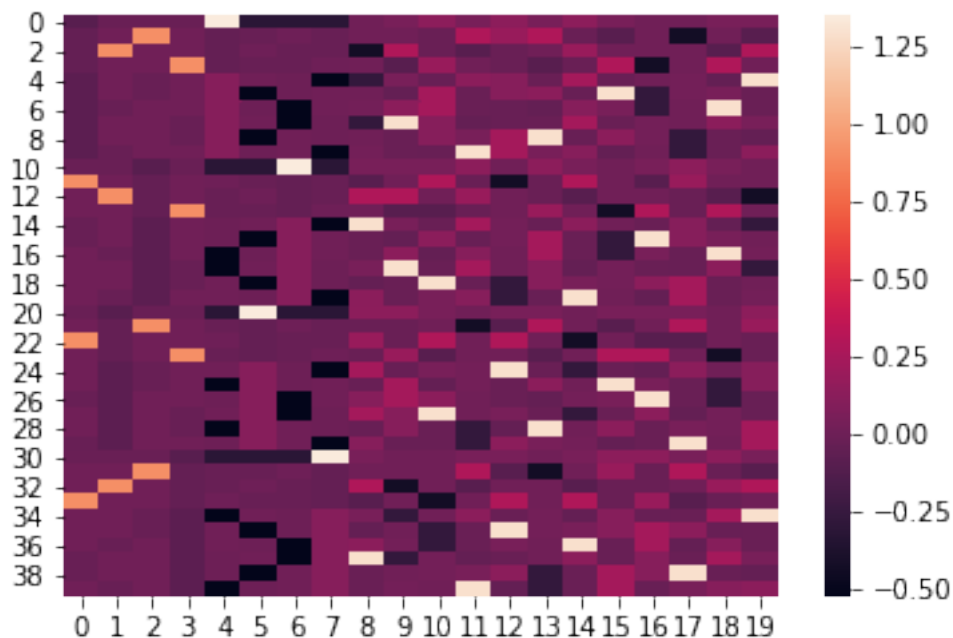
p3-shunn-ham-m3
ratio: 0.5
(array([-4.78915594e+00, -2.76502047e+00, -2.59417161e+00, -2.10732496e+00,
        -1.56254772e+00, -1.49774568e+00, -1.45051946e+00, -1.21666464e+00,
        -1.19604641e+00, -9.48381011e-01, -9.19404649e-01, -9.02137346e-01,
        -8.37457798e-01, -6.90537714e-01, -5.47548032e-01, -5.30818522e-01,
        -4.53202601e-01, -2.76965679e-01, -2.61656644e-01, -2.48809562e-01,
        -1.99658919e-01, -1.59906209e-01, -1.58873687e-01, -1.43650268e-01,
        -1.34406010e-01, -1.31661072e-01, -1.15273131e-01, -9.17257660e-02,
        -7.75993461e-02, -7.60145555e-02, -5.76746017e-02, -3.32984468e-02,
        -5.28569078e-03, -3.05169500e-03, 1.22538693e-03, 2.12243243e-03,
         9.49759940e-03, 1.64503247e-02, 1.76756946e-02, 3.06152011e-02,
         3.79836236e-02, 6.28622859e-02, 6.31507997e-02, 6.57895660e-02,
         7.93524066e-02, 8.72615087e-02, 1.08880673e-01, 1.09380394e-01,
         1.26877479e-01, 1.31709608e-01, 1.35681212e-01, 1.37442400e-01,
         1.51141367e-01, 1.79436063e-01, 2.19758239e-01, 2.28127733e-01,
         2.35006752e-01, 2.46983272e-01, 3.10792378e-01, 3.24149807e-01,
         3.24471383e-01, 3.65181021e-01, 4.27787577e-01, 4.75237563e-01,
         5.61443936e-01, 5.62000920e-01, 6.32512082e-01, 6.56770646e-01,
         7.03186498e-01, 8.23135605e-01, 9.25672626e-01, 1.13756013e+00,
         1.21795474e+00, 1.60331202e+00, 4.19680096e+00, 4.82515868e+00,
         7.26907250e+00, 7.67317874e+00, 8.35741999e+00, 1.32903354e+01]),
 array([1, 3, 6, 6, 3, 18, 6, 18, 3, 3, 3, 9, 18, 9, 9, 9, 6,
         6, 18, 6, 6, 18, 3, 18, 6, 6, 18, 9, 18, 18, 6, 18, 6, 18,
        18, 6, 18, 6, 18, 6, 18, 18, 9, 6, 18, 18, 6, 3, 18, 18, 18,
         6, 6, 18, 6, 6, 6, 9, 6, 18, 18, 9, 3, 18, 6, 6, 3, 18,
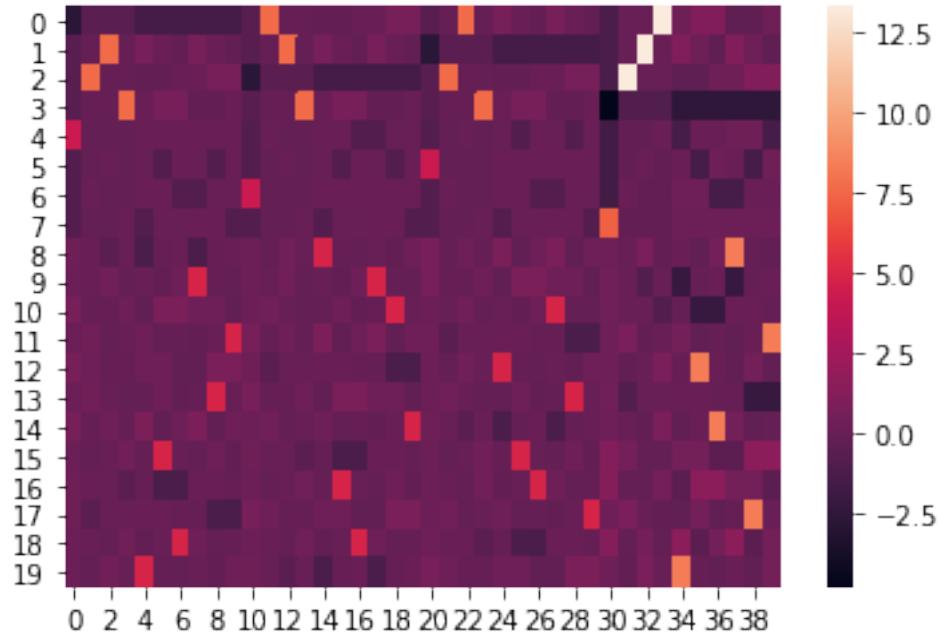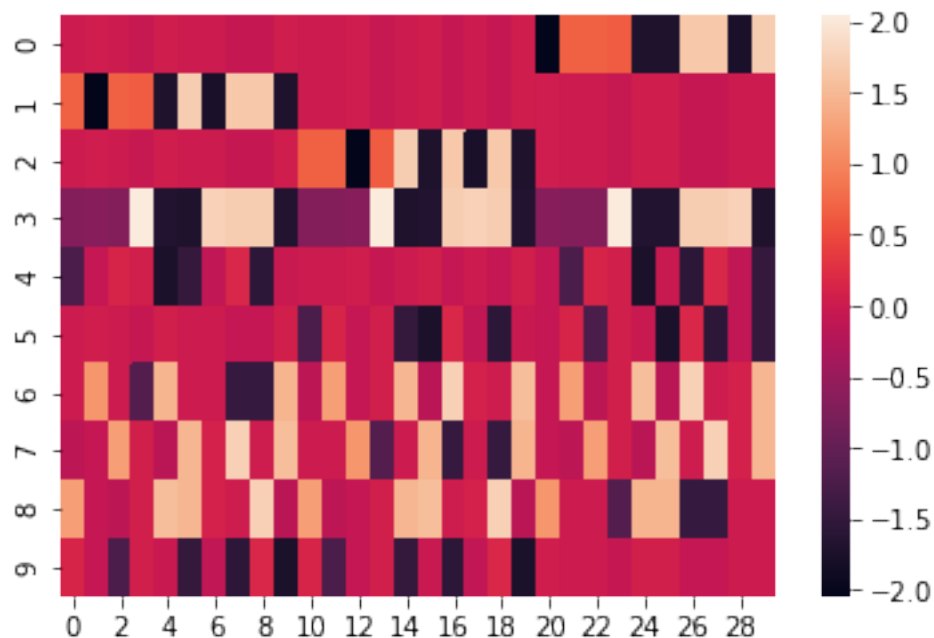
9, 6, 18, 6, 3, 6, 3, 18, 1, 9, 6, 3]))
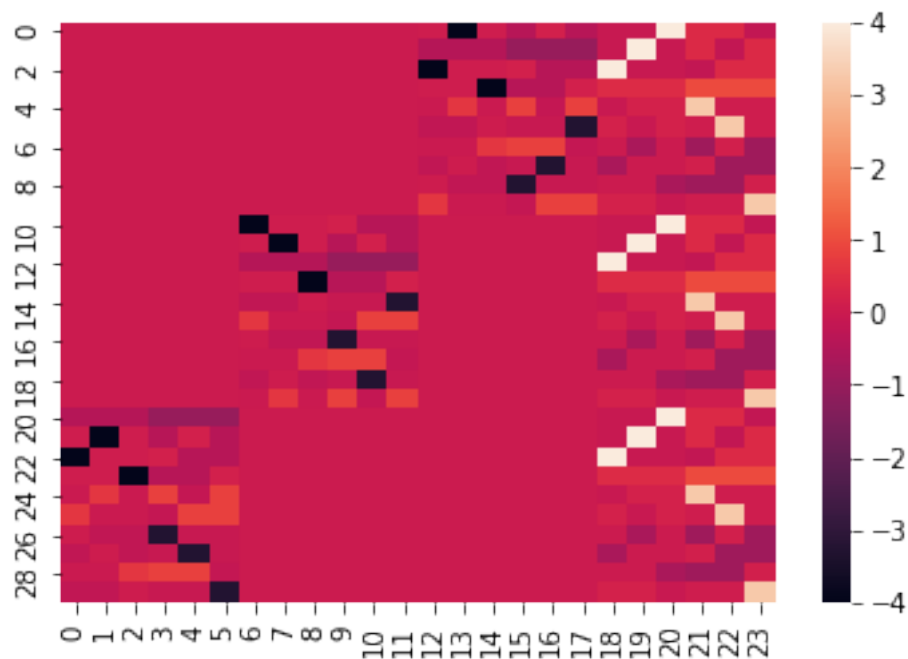

p2-shunn-ham-m132
ratio: 0.3333333333333333
(array([-2.04495479, -1.76141856, -1.7500337 , -1.71337861, -1.71315436,
       -1.66511441, -1.55992148, -1.4821005 , -1.46108807, -1.24271838,
       -1.16327656, -0.68315281, -0.64472623, -0.1690998 , -0.13387893,
       -0.09883341, -0.07944183, -0.07782098, -0.07192009, -0.06195884,
       -0.0482642 , -0.03842658, -0.02101243, 0.   , 0.02101243,
        0.03842658, 0.0482642 , 0.06195884, 0.07192009, 0.07782098,
        0.07944183, 0.09883341, 0.13387893, 0.1690998 , 0.64472623,
        0.68315281, 1.16327656, 1.24271838, 1.46108807, 1.4821005 ,
        1.55992148, 1.66511441, 1.71315436, 1.71337861, 1.7500337 ,
        1.76141856, 2.04495479]), array([3, 3, 6, 3, 6, 6, 6, 6, 6,
6, 3, 6, 3, 6, 6, 6, 6,
        6, 6, 3, 12, 6, 6, 48, 6, 6, 12, 3, 6, 6, 6, 6, 6, 6,
        3, 6, 3, 6, 6, 6, 6, 6, 6, 3, 6, 3, 3]))

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p2-shunn-ham-m6
ratio: 1.25
(array([-4.00272788, -3.27178567, -0.99206734, -0.81465504, -0.6394707 ,
       -0.43646441, -0.37957603, -0.1891731 , -0.16506007, -0.12998823,
       -0.07558059, -0.06659091, -0.05881613, -0.01105329, 0.   ,
        0.01105329, 0.05881613, 0.06659091, 0.07558059, 0.12998823,
        0.16506007, 0.1891731 , 0.37957603, 0.43646441, 0.6394707 ,
        0.81465504, 0.99206734, 3.27178567, 4.00272788]), array([9,   9,
9, 18,   9,   9, 18, 18,   9,   9, 18, 18,   9,
       18, 360, 18,   9, 18, 18,   9,   9, 18, 18,   9,   9, 18,
        9,   9,   9]))

p2-shunn-ham-m0
ratio: 2.4
(array([-0.51465891, -0.31063802, -0.02988924, -0.02322603, -0.01351974,
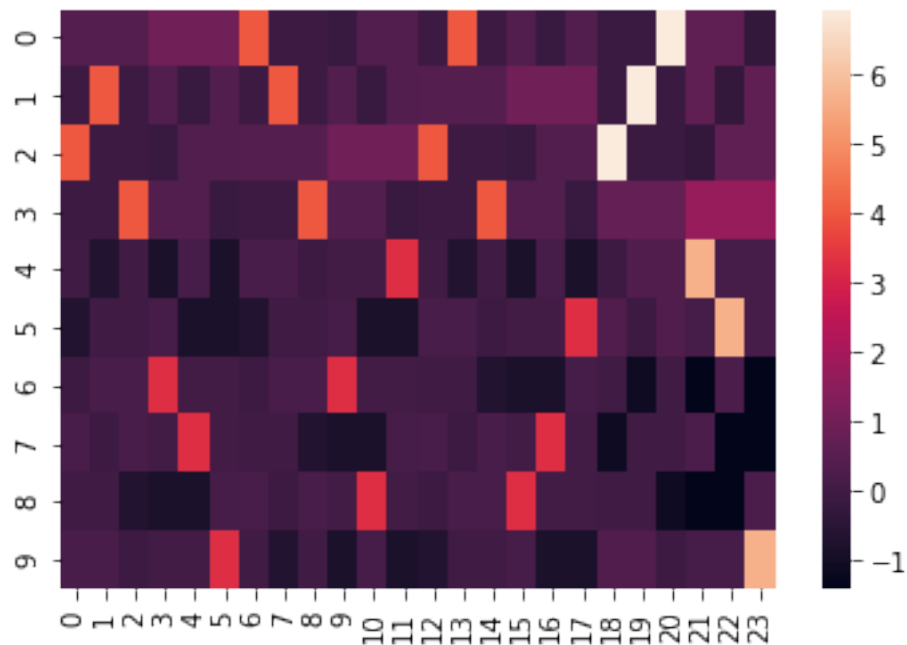   0.00316332, 0.0101803 , 0.05032626, 0.05472887, 0.12075897,
        0.14180657, 0.14741365, 1.16322444, 1.28381482]), array([12, 24, 24,
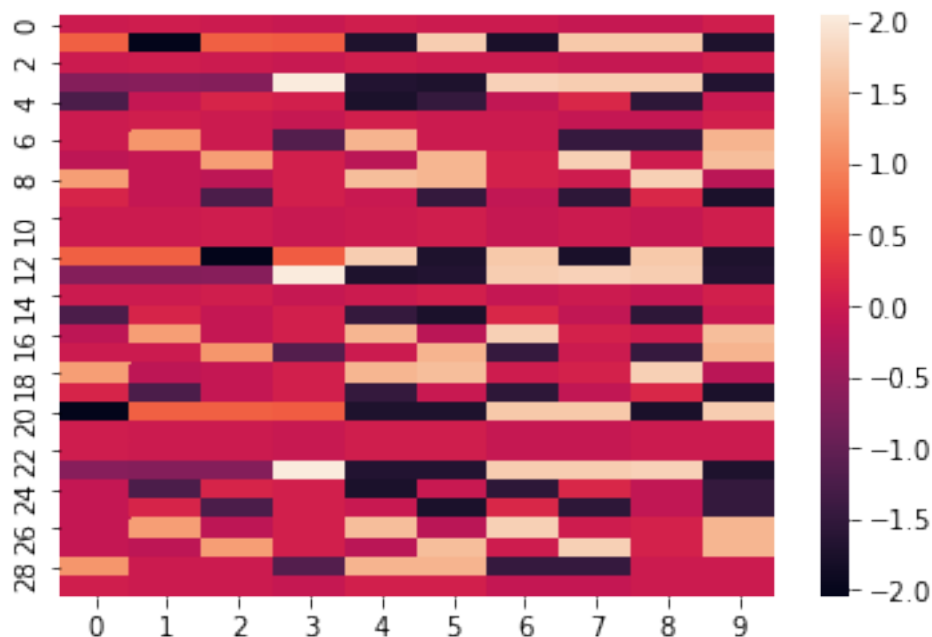12, 12, 12, 24, 24, 24, 12, 24, 12, 12, 12]))
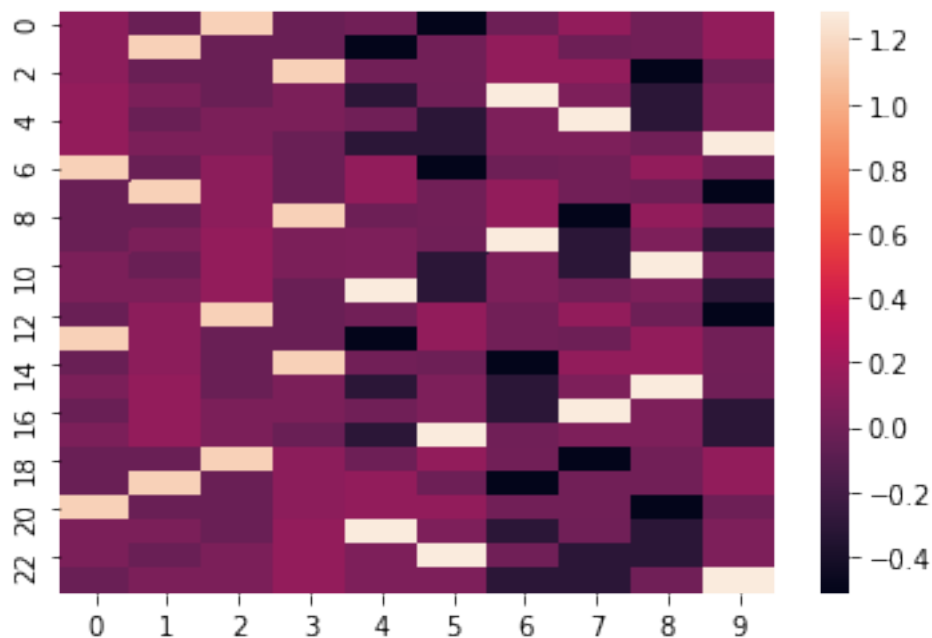
```
p2-shunn-ham-m3
ratio: 0.4166666666666667
(array([-1.41102391, -1.10759575, -0.81465504, -0.6394707 , -0.28589244,
        -0.16506007, -0.11533884, -0.10187253, -0.06659091, -0.05881613,
         0.01105329,  0.01914486,  0.07558059,  0.12998823,  0.13090943,
         0.1891731 ,  0.22514622,  0.32765742,  0.37957603,  0.43646441,
         0.65744496,  0.75597853,  0.99206734,  1.71831104,  3.27178567,
         4.00272788,  5.66689902,  6.93292806]), array([ 6,  3, 18,  9,  3,  9,
        6,  3, 18,  9, 18,  6, 18,  9,  6, 18,  3,
        6, 18,  9,  6,  3,  9,  3,  9,  9,  3,  3]))
```



```
p2-shunn-ham-m460
ratio: 3.0
(array([-2.04495479, -1.76141856, -1.7500337 , -1.71337861, -1.71315436,
        -1.66511441, -1.55992148, -1.4821005 , -1.46108807, -1.24271838,
        -1.16327656, -0.68315281, -0.64472623, -0.1690998 , -0.13387893,
        -0.09883341, -0.07944183, -0.07782098, -0.07192009, -0.06195884,
        -0.0482642 , -0.03842658, -0.02101243,  0.        ,  0.02101243,
         0.03842658,  0.0482642 ,  0.06195884,  0.07192009,  0.07782098,
         0.07944183,  0.09883341,  0.13387893,  0.1690998 ,  0.64472623,
         0.68315281,  1.16327656,  1.24271838,  1.46108807,  1.4821005 ,
         1.55992148,  1.66511441,  1.71315436,  1.71337861,  1.7500337 ,
         1.76141856,  2.04495479]), array([ 3,  3,  6,  3,  6,  6,  6,  6,  6,
        6,  3,  6,  3,  6,  6,  6,  6,
```

```
6,  6,  3, 12,  6,  6, 48,  6,  6, 12,  3,  6,  6,  6,  6,  6,  6,
3,  6,  3,  6,  6,  6,  6,  6,  6,  3,  6,  3,  3]))
```



```
p4-shunn-ham-m0
ratio: 1.7142857142857142
(array([-5.74613811e-01, -5.39665641e-01, -5.13991877e-01, -5.03391360e-01,
        -2.91709818e-01, -2.33444109e-01, -1.94001487e-01, -1.78130052e-01,
        -1.54416777e-01, -1.14172569e-01, -8.53187241e-02, -7.16810200e-02,
        -6.99274278e-02, -6.38926333e-02, -4.77953190e-02, -4.65333649e-02,
        -4.50496821e-02, -4.09847594e-02, -2.59106780e-02, -2.46085920e-02,
        -2.36983984e-02, -2.34238007e-02, -1.91048007e-02, -1.85252382e-02,
        -1.64897402e-02, -1.58056742e-02, -1.54453899e-02, -1.42159222e-02,
        -1.32598758e-02, -1.31347801e-02, -1.26644723e-02, -1.26083153e-02,
        -1.21768716e-02, -1.17624156e-02, -1.10291555e-02, -1.09631800e-02,
        -1.00506445e-02, -8.97098501e-03, -7.40196254e-03, -5.38730036e-03,
        -5.26203469e-03, -4.69546640e-03, -2.55523286e-03, -1.28621116e-03,
        -9.81029397e-04, -5.83003419e-04, -5.12047775e-04,  3.48511664e-04,
         8.69537319e-04,  1.03790198e-03,  3.10006563e-03,  3.37058189e-03,
         4.13557676e-03,  4.64648771e-03,  5.14790153e-03,  5.22563316e-03,
         5.66916552e-03,  6.17301102e-03,  7.53708698e-03,  8.82222631e-03,
         9.39495704e-03,  9.46762582e-03,  1.00711323e-02,  1.21752642e-02,
         1.43212346e-02,  1.51165333e-02,  1.60611807e-02,  1.74437492e-02,
         1.76164238e-02,  2.15732536e-02,  2.40333240e-02,  2.41363244e-02,
         2.46189934e-02,  2.59962765e-02,  2.97624168e-02,  3.02696999e-02,
         3.07480871e-02,  4.47889442e-02,  4.84054312e-02,  4.86941571e-02,
         6.35099515e-02,  7.95892612e-02,  8.22488856e-02,  8.54106788e-02,
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        8.96335245e-02,  1.04144440e-01,  1.04884805e-01,  1.09756978e-01,
        1.15828426e-01,  1.42823167e-01,  1.77358644e-01,  1.95173445e-01,
        2.28801621e-01,  2.41239546e-01,  2.41298686e-01,  2.52836481e-01,
        4.30430880e-01,  1.08899825e+00,  1.27028863e+00,  1.34207969e+00,
        1.37100480e+00]), array([24, 12, 12, 24, 24, 24, 12, 24, 24, 12, 24, 24,
24, 12, 24, 24, 24,
        24, 24, 24, 24, 24, 24, 24, 12, 24, 24, 24, 12, 24, 12, 24, 24, 24,
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        24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24,
        12, 24, 24, 24, 12, 24, 24, 24, 12, 24, 24, 24, 24, 24, 24, 12, 24,
        24, 12, 12, 12, 24, 24, 24, 24, 24, 12, 24, 12, 12, 24, 12, 12]))
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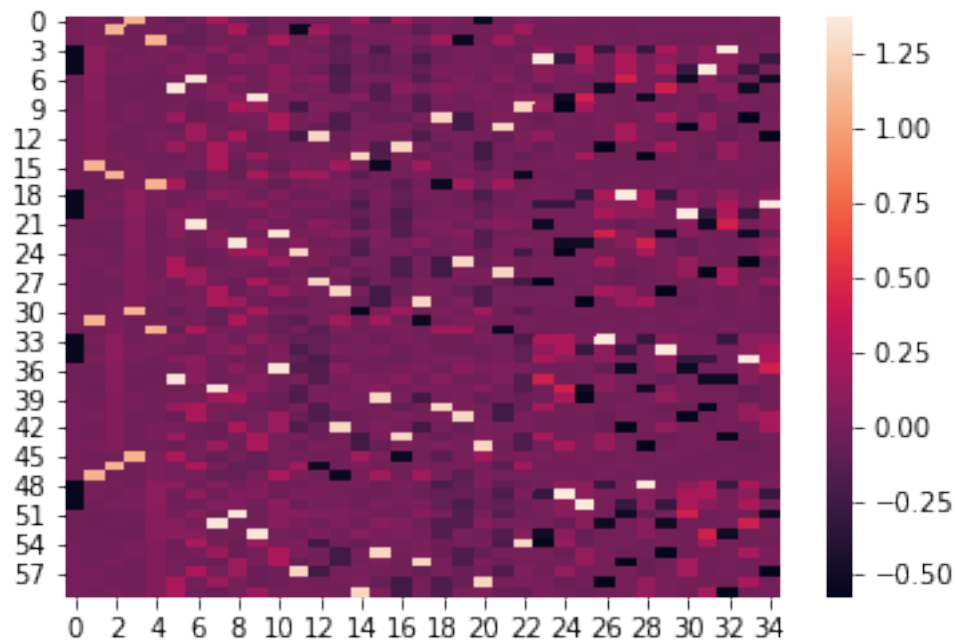

0.7.1 Thoughts

- m* has its own pattern style

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[ ]:
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