به نام خدا

دانشگاه تهران

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رباتیک و مکاترونیک

مینی پروژه شماره ۲

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شماره دانشجویی	۸۱۰۱۹۹۵۹۸
تاریخ ارسال گزارش	۱ خرداد ۱۴۰۳

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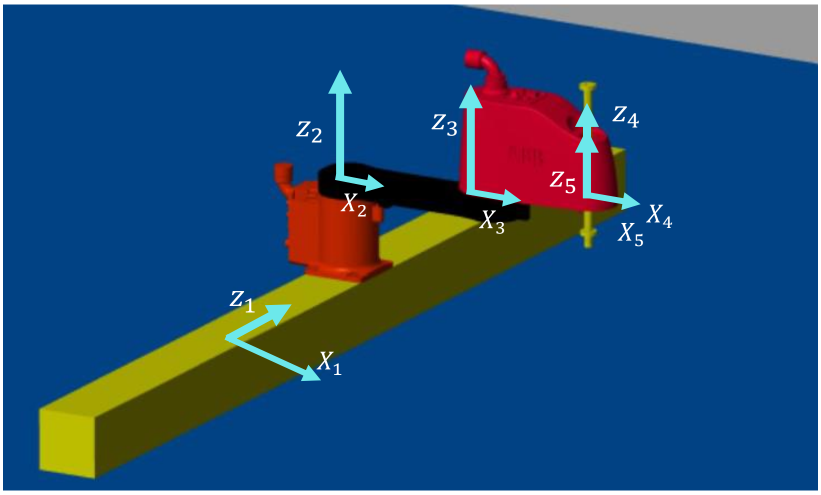

Figure1: DH axes

i	a_i	b_i	α_i	θ_i	$q_i^{initial}$
1	0	b_1	$\frac{\pi}{2}$	0	0.08
2	400mm	257.7mm	0	θ_2	0
3	250mm	0	0	θ_3	0
4	0	b_4	0	0	0

Table 1: DH Table

Part 2 – Forward Kinematic Problem

$$\overrightarrow{a_1} = \begin{bmatrix} 0 \\ 0 \\ b_1 \end{bmatrix}, \overrightarrow{a_2} = \begin{bmatrix} 400\cos\theta_2 \\ 400\sin\theta_2 \\ 257.7 \end{bmatrix}, \overrightarrow{a_3} = \begin{bmatrix} 250\cos\theta_3 \\ 250\sin\theta_3 \\ 0 \end{bmatrix}, \overrightarrow{a_4} = \begin{bmatrix} 0 \\ 0 \\ b_4 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} \cos\theta_2 & -sin\theta_2 & 0 \\ sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} \cos\theta_3 & -sin\theta_3 & 0 \\ sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_{EE} = Q_1 Q_2 Q_3 Q_4$$

$$\vec{P} = a_1 + Q_1 a_2 + Q_1 Q_2 a_3 + Q_1 Q_2 Q_3 a_4$$

Using MATLAB we obtained:

$$Q_{EE} = \begin{pmatrix} \cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3) & -\cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2) & 0 \\ 0 & 0 & -1 \\ \cos(\theta_2)\sin(\theta_3) + \cos(\theta_3)\sin(\theta_2) & \cos(\theta_2)\cos(\theta_3) - \sin(\theta_2)\sin(\theta_3) & 0 \end{pmatrix}$$

$$\boxed{Q_{EE} = \begin{pmatrix} \cos(\theta_2 + \theta_3) & -\sin(\theta_2 + \theta_3) & 0 \\ 0 & 0 & -1 \\ \sin(\theta_2 + \theta_3) & \cos(\theta_2 + \theta_3) & 0 \end{pmatrix}}$$

$$P = \begin{pmatrix} 400\cos(\theta_2) + 250\cos(\theta_2)\cos(\theta_3) - 250\sin(\theta_2)\sin(\theta_3) \\ -b_4 - \frac{2577}{10} \\ b_1 + 400\sin(\theta_2) + 250\cos(\theta_2)\sin(\theta_3) + 250\cos(\theta_3)\sin(\theta_2) \end{pmatrix}$$

$$\boxed{P = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 400\cos(\theta_2) + 250\cos(\theta_2 + \theta_3) \\ -b_4 - 257.7 \\ b_1 + 400\sin(\theta_2) + 250\cos(\theta_2 - \theta_3) \end{pmatrix}}$$

Part 3 – Inverse Kinematic Problem

IKP:

$$b_4 = -257.7 - y$$

$$there\ is\ one\ answer\ for\ b_4.$$

$$\theta_2 = \cos^{-1}\frac{x - 250\ \mathrm{Q}_{\mathrm{EE}_{11}}}{400}$$

$$there\ are\ two\ answers\ for\ \theta_2.$$

$$\gamma = \theta_1 + \theta_2 = \tan^{-1}\frac{\mathrm{Q}_{\mathrm{EE}_{13}}}{\mathrm{Q}_{\mathrm{EE}_{11}}}$$

$$there\ is\ one\ answer\ for\ \gamma.$$

$$\theta_1 = \gamma - \theta_2 = \tan^{-1}\frac{\mathrm{Q}_{\mathrm{EE}_{13}}}{\mathrm{Q}_{\mathrm{EE}_{11}}} - \cos^{-1}\frac{x - 250\ \mathrm{Q}_{\mathrm{EE}_{11}}}{400}$$

$$so, there\ are\ two\ answers\ for\ \theta_1.$$

$$b_1 = z - 400\sin(\theta_2) - 250\cos(\theta_2 - \theta_3)$$

$$two\ answers\ for\ b_1, too.$$

IKP has two answers.

Part 4 – Jacobian Matrix for the Manipulator

In general terms, Jacobian matrix is shown below:

$$J = \begin{bmatrix} [\overrightarrow{e_1}]_1 & [\overrightarrow{e_2}]_1 & [\overrightarrow{e_3}]_1 & [\overrightarrow{e_4}]_1 \\ [\overrightarrow{e_1}]_1 \times \overrightarrow{r_1} & [\overrightarrow{e_2}]_1 \times \overrightarrow{r_2} & [\overrightarrow{e_3}]_1 \times \overrightarrow{r_3} & [\overrightarrow{e_4}]_1 \times \overrightarrow{r_4} \end{bmatrix}$$

Scara robot has 3 transitional degrees of freedom and one rotational degree of freedom. With respect to two prismatic joints, the first joint and the fourth one, we can right the Jacobian matrix as shown below:

$$J = \begin{bmatrix} 0 & [\overrightarrow{e_2}]_1 & [\overrightarrow{e_3}]_1 & 0 \\ [\overrightarrow{e_1}]_1 & [\overrightarrow{e_2}]_1 \times [\overrightarrow{r_2}]_1 & [\overrightarrow{e_3}]_1 \times [\overrightarrow{r_3}]_1 & [\overrightarrow{e_4}]_1 \end{bmatrix}$$

$$\overrightarrow{[e_1]}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \overrightarrow{[e_2]}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, [\overrightarrow{e_3}]_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \overrightarrow{[e_4]}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We calculated e in the first coordinate:

$$[\overrightarrow{e_2}]_1 = Q_1 \overrightarrow{e_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$[\overrightarrow{e_3}]_1 = Q_1 Q_2 \overrightarrow{e_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & sin\theta_2 \\ sin\theta_2 & 0 & -\cos\theta_2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$[\overrightarrow{e_4}]_1 = Q_1 Q_2 Q_3 \overrightarrow{e_4} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

Now r is calculated in the first coordinate, using MATLAB:

$$[\overrightarrow{r_4}]_1 = \overrightarrow{[a_4]_1} = Q_1 Q_2 Q_3 \overrightarrow{a_4} = \begin{pmatrix} 0 \\ -b_4 \\ 0 \end{pmatrix}$$

$$[\overrightarrow{r_3}]_1 = \overrightarrow{[a_3]_1} + \overrightarrow{[a_4]_1} = Q_1 Q_2 \overrightarrow{a_3} + Q_1 Q_2 Q_3 \overrightarrow{a_4} = \begin{pmatrix} 250\cos(\theta_2)\cos(\theta_3) - 250\sin(\theta_2)\sin(\theta_3) \\ -b_4 \\ 250\cos(\theta_2)\sin(\theta_3) + 250\cos(\theta_3)\sin(\theta_2) \end{pmatrix}$$

$$[\overrightarrow{r_2}]_1 = \overrightarrow{[a_2]_1} + \overrightarrow{[a_3]_1} + \overrightarrow{[a_4]_1} = Q_1 \overrightarrow{a_2} + Q_1 Q_2 \overrightarrow{a_3} + Q_1 Q_2 Q_3 \overrightarrow{a_4}$$

$$[\overrightarrow{r_2}]_1 = \begin{pmatrix} 400\ \cos(\theta_2) + 250\ \cos(\theta_2)\ \cos(\theta_3) - 250\ \sin(\theta_2)\ \sin(\theta_3) \\ -b_4 - 257.7 \\ 400\ \sin(\theta_2) + 250\ \cos(\theta_2)\ \sin(\theta_3) + 250\ \cos(\theta_3)\ \sin(\theta_2) \end{pmatrix}$$

$$[\overrightarrow{r_1}]_1 = \overrightarrow{[a_1]_1} + \overrightarrow{[a_2]_1} + \overrightarrow{[a_3]_1} + \overrightarrow{[a_4]_1} = \overrightarrow{a_1} + Q_1\overrightarrow{a_2} + Q_1Q_2\overrightarrow{a_3} + Q_1Q_2Q_3\overrightarrow{a_4}$$

$$[\overrightarrow{r_1}]_1 = \begin{pmatrix} 400\cos(\theta_2) + 250\cos(\theta_2)\cos(\theta_3) - 250\sin(\theta_2)\sin(\theta_3) \\ -b_4 - 257.7 \\ b_1 + 400\sin(\theta_2) + 250\cos(\theta_2)\sin(\theta_3) + 250\cos(\theta_3)\sin(\theta_2) \end{pmatrix}$$

Jacobian Matrix obtained by these equations is shown below:

$$J = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -400\sin(\theta_2) - \sigma_3 - \sigma_2 & -\sigma_3 - \sigma_2 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 400\cos(\theta_2) + \sigma_4 - \sigma_1 & \sigma_4 - \sigma_1 & 0 \end{pmatrix}$$

Where:

$$\sigma_1 = 250\sin(\theta_2)\sin(\theta_3)$$

$$\sigma_2 = 250\cos(\theta_3)\sin(\theta_2)$$

$$\sigma_3 = 250\cos(\theta_2)\sin(\theta_3)$$

$$\sigma_4 = 250\cos(\theta_2)\cos(\theta_3)$$

Part 5 - Dynamic Model

Dynamic model can be obtained by using the Euler-Lagrange method. The kinetic energy T and the potential energy of the system V are the sums of the kinetic energies and the potential energies of all the manipulator's Links.

$$T = \sum_{i=1}^{4} T_i \,, V = \sum_{i=1}^{4} V_i$$

$$M = \frac{\partial^2 T}{\partial \dot{\vec{\theta}}^2}$$

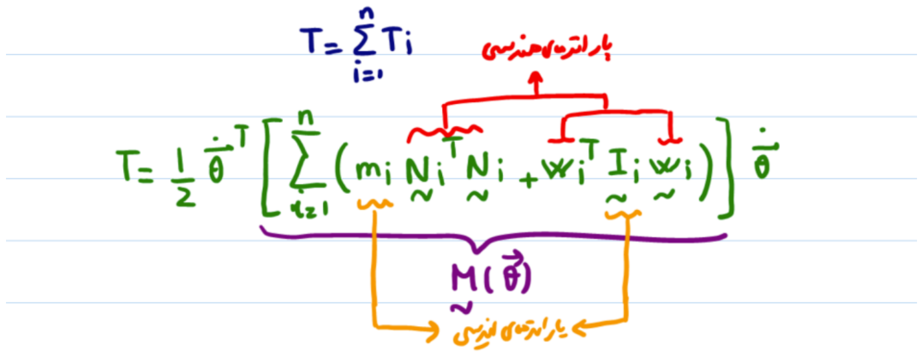

Figure 2: T calculation

$$M\ddot{\theta} + \dot{M}\dot{\theta} - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = \vec{\tau}$$

$$\frac{\partial T}{\partial \theta} = \begin{bmatrix} \frac{\partial T}{\partial \theta_2} \\ \frac{\partial T}{\partial \theta_3} \end{bmatrix}$$

$$\frac{\partial V}{\partial \theta} = \begin{bmatrix} \frac{\partial V}{\partial \theta_2} \\ \frac{\partial V}{\partial \theta_3} \end{bmatrix}$$

To use equations in the paper, we need to have I in DH frame. We have these in the center of mass frame. To calculate these, we must follow the equations below.

Q is the rotation matrix from link to DH frame.

T is the rotation and position matrix from center of mass to DH frame.

$$[CoM]_{DH} = T[CoM]_{link}$$

$$[I]_{DH} = Q[I]_{link}\, Q^{-1}$$

$$T = \begin{pmatrix} Q & b \\ 0 & 1 \end{pmatrix}$$

To find Q, and then T matrixes, I checked the direction of coordinates in link block in the Simulink file. The T matrix, also known as the homogeneous transformation matrix, is a 4x4 matrix that represents the transformation from one coordinate frame to another. It is composed of a 3x3 rotation matrix (Q) and a 3x1 translation vector (b). The last row of the T matrix is always [0 0 0 1].

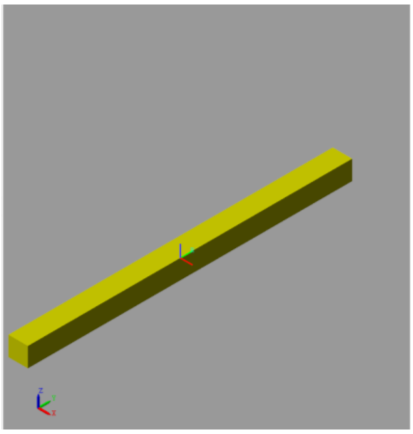

Figure 3: link fixed coordinate- link 0

As we can see there is a $-\frac{\pi}{2}$ difference between this z axis and what we have set as z in DH frame. The rotation is around x axis.

$$Q = Q_x(-\frac{\pi}{2})$$

$$[I]_{DH} = Q[I]_{link}\, Q^{-1}$$

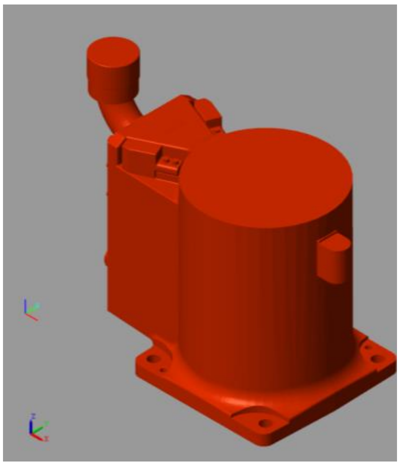

Figure 4: link fixed coordinate- link 1

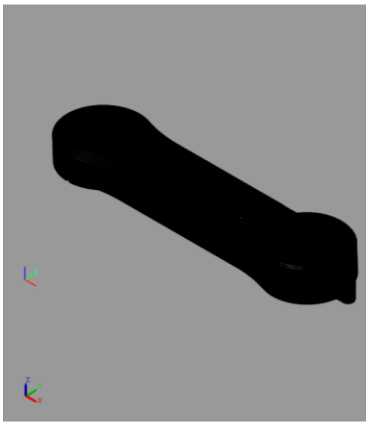

Figure 5: link fixed coordinate- link 2

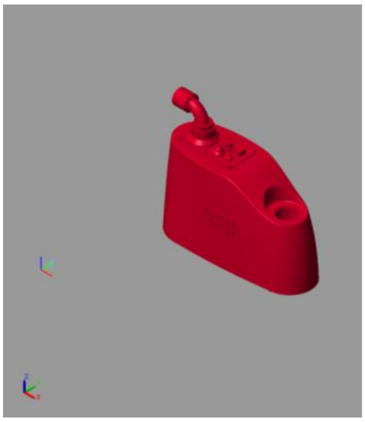

Figure 6: link fixed coordinate- link 3

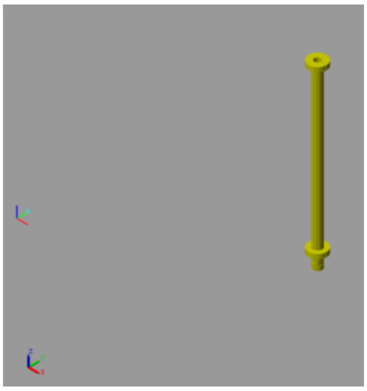

Figure 7: link fixed coordinate- link 4

So, for using the equations in paper it was needed to apply changes, to calculate the kinematic energy in DH frame.

With the help of transition matrixes, applying like picture below, I was able to obtain the dynamic model.

$$\boldsymbol{Link_0} \xrightarrow{+\begin{bmatrix}0\\0\\73.5mm\end{bmatrix}\ then\ Rot_x(-90^\circ)} \boldsymbol{Joint_1} \xrightarrow{Rot_x(90^\circ)\ then\ +\begin{bmatrix}-200mm\\0\\0\end{bmatrix}} \boldsymbol{Link_1}$$

$$\xrightarrow{+\begin{bmatrix}200mm\\80mm\\78mm\end{bmatrix}} \boldsymbol{Joint_2} \xrightarrow{+\begin{bmatrix}-73mm\\-83mm\\-79mm\end{bmatrix}} \boldsymbol{Link_2} \xrightarrow{+\begin{bmatrix}470mm\\90mm\\200mm\end{bmatrix}} \boldsymbol{Joint_3} \xrightarrow{+\begin{bmatrix}-400mm\\-90mm\\-200mm\end{bmatrix}} \boldsymbol{Link_3}$$

$$\xrightarrow{+\begin{bmatrix}650mm\\70mm\\280mm\end{bmatrix}} \boldsymbol{Joint_4} \xrightarrow{+\begin{bmatrix}-650mm\\0\\-345mm\end{bmatrix}} \boldsymbol{Link_4}$$

Figure 8: Needed transitions to obtain dynamic model.

I’ve also generated MATLAB functions for different components of the SCARA robot's dynamic model. Specifically, functions for the inertia matrix M, the potential energy P, the kinetic energy T, and the Euler-Lagrange equations τ_{av}. These functions can be utilized to perform various analyses and simulations of the SCARA robot's dynamics.

Part 6 - Verify the Dynamic Model using simspace.

Plotting the simspace output:

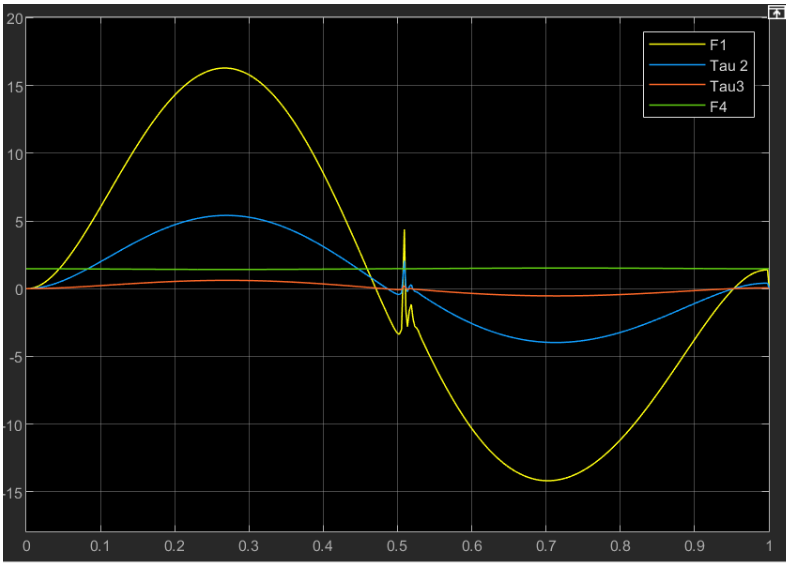

Figure 9: Simspace simulation

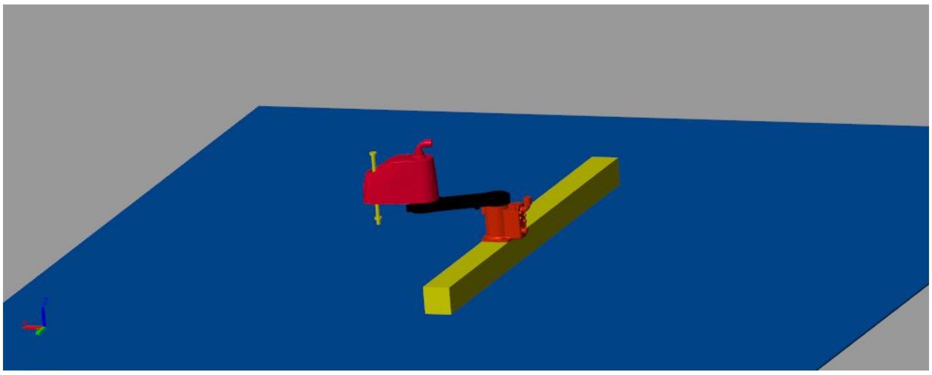

Figure 10: Simspace visualization

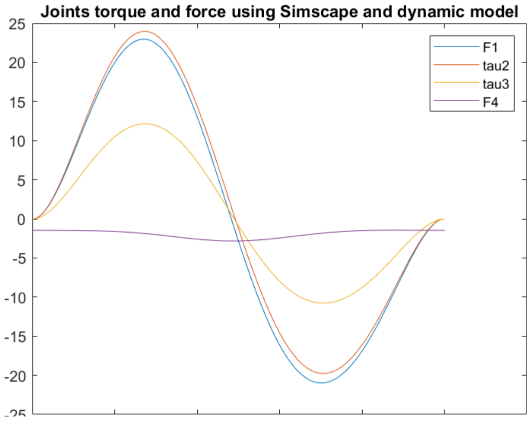

Figure 11: Simulation using dynamic model.

In conclusion, comparing the dynamic model and Simscape simulation for the SCARA robot revealed mostly consistent torque and force outputs, despite starting from different initial points. We can see in simspace and dynamic model, F4 has low oscillation. Tau2 and F1 are sinusoids and Tau 3 in both graphs has lower amplitude.

While there were slight variations, likely due to simplifications in the model and the unique complexities of Simscape, overall, the dynamic model did a good job of capturing the robot's behavior.