# Optimal Control Of Energy Hub Systems Using Model Predictive Control (MPC)

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Schematic of a typical energy hub. (Source: T. Ding et al., "Review of Optimization Methods for Energy Hub Planning, Operation, Trading, and Control," *IEEE Transactions on Sustainable Energy*, vol. 13, no. 3, pp. 1802–1818, July 2022)

What is an Energy Hub?

- Energy Hubs: Localized units within MES that focus on converting, storing, and optimizing these energy carriers for specific outputs.
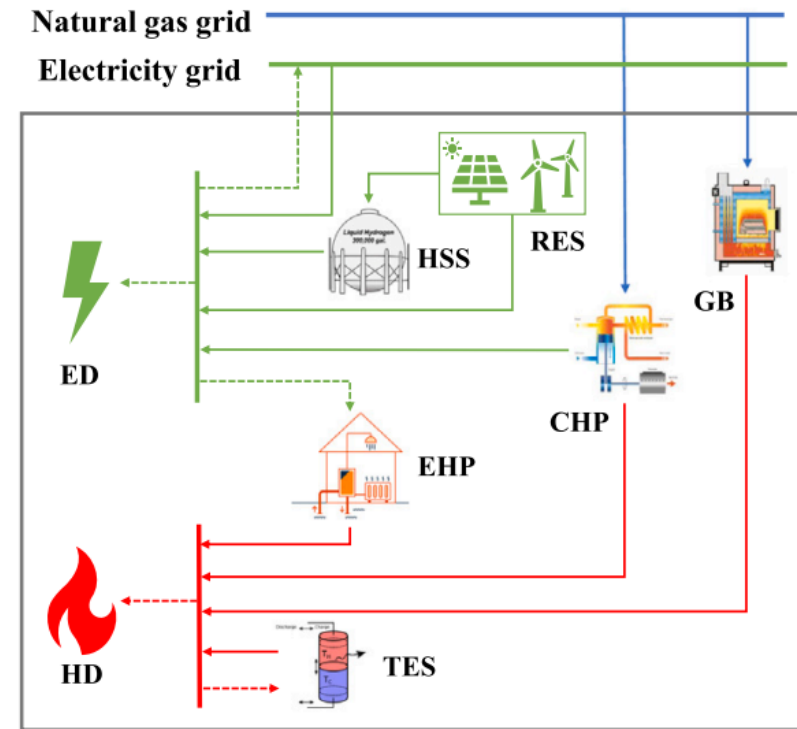
Figure 1. Structure of the proposed smart energy hub (EH) system, as presented in Qiu et al. (2022)
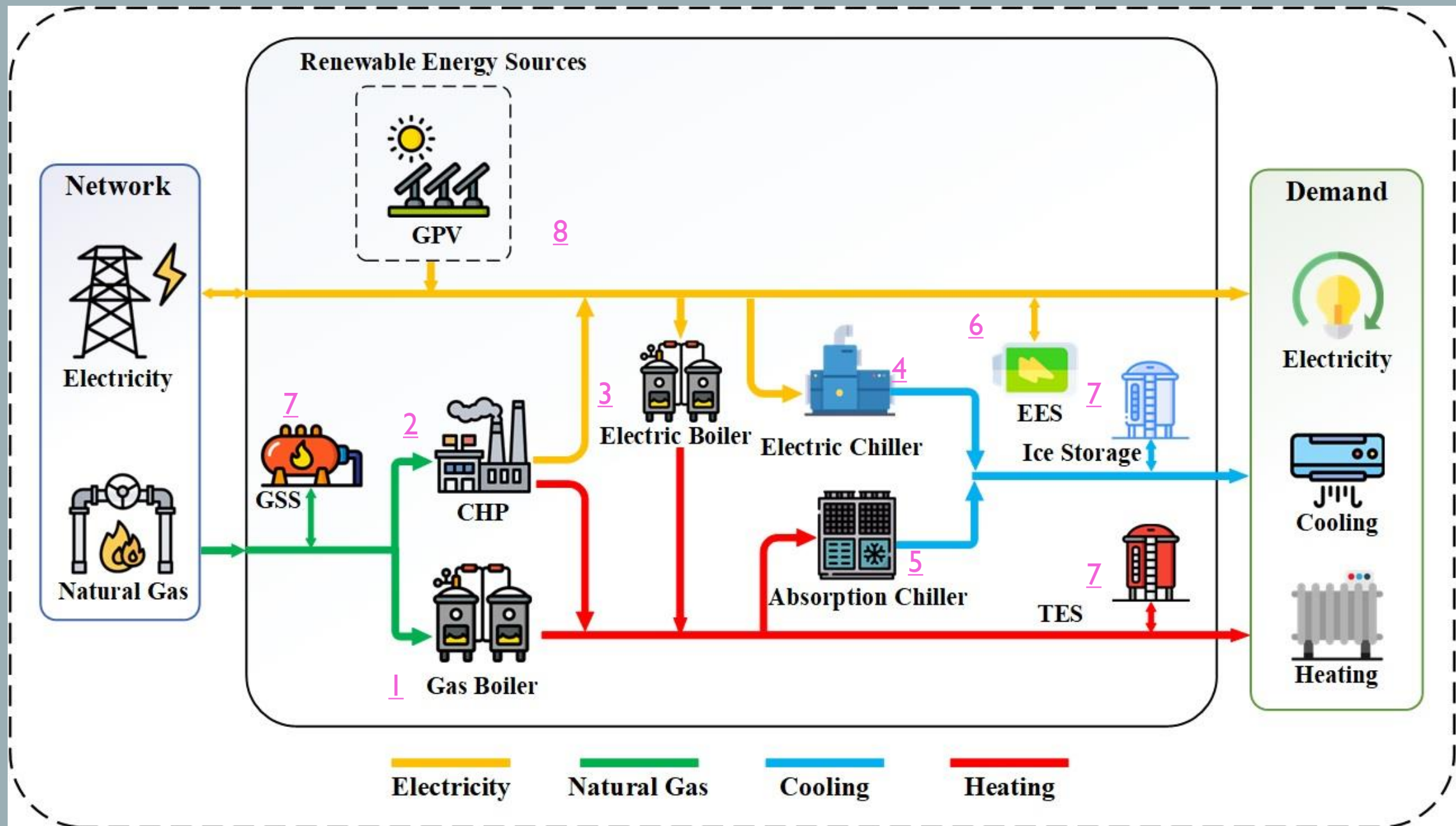
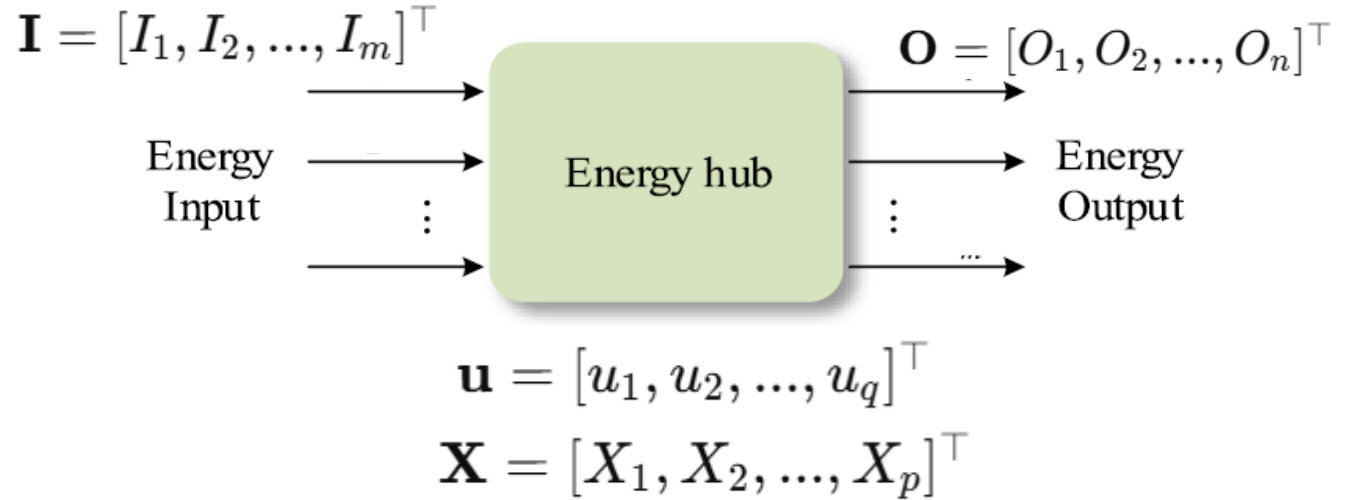**Figure 2.** Schematic of the Energy Hub Studied in This Project

Model

Variables Definition:

I→ input vector (e.g., electricity from the grid, fuel, solar energy, wind)

O→ output vector (e.g., electricity demand, heat, cooling)

X→ internal state variables (e.g., battery state-of-charge, boiler temperature, thermal storage level)

u→ control/decision variables (e.g., unit dispatch, storage charge/discharge rates)

$$\mathbf{I} = [I_1, I_2, ..., I_m]^\top$$

$$\mathbf{O} = [O_1, O_2, ..., O_n]^\top$$

Energy Input → Energy hub → Energy Output

$$\mathbf{u} = [u_1, u_2, ..., u_q]^\top$$

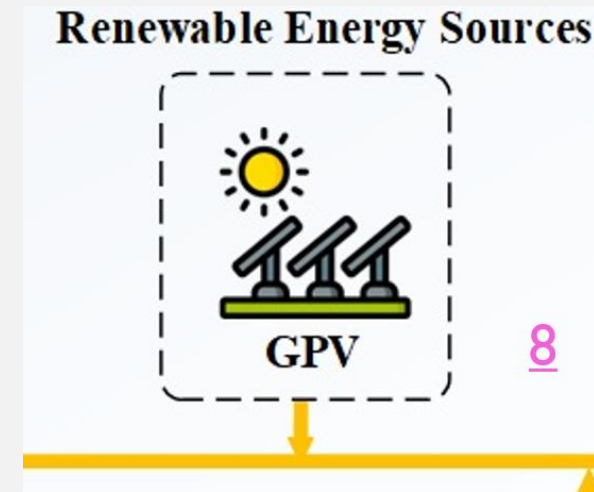$$\mathbf{X} = [X_1, X_2, ..., X_p]^\top$$

General Energy Hub Model:

$$\mathbf{O}(t) = f\big(\mathbf{I}(t), \mathbf{X}(t), \mathbf{u}(t)\big)$$
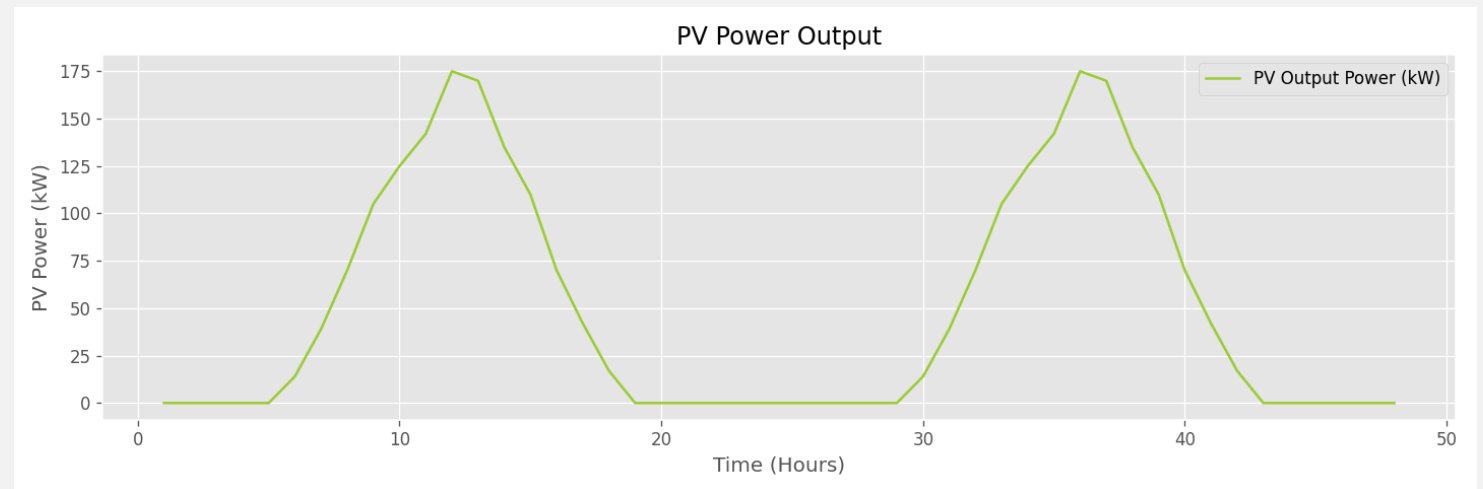
$$\mathbf{X}(t+1) = g\big(\mathbf{X}(t), \mathbf{I}(t), \mathbf{u}(t)\big)$$

Components Modulation

Photovoltaic System:

$$P_{PV}(t) = N_{PV} \times A_{PV} \times \eta_{PV} \times Irr(t)$$

Renewable Energy Sources

GPV

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Figure 3. Photovoltaic System in the EH

PV Power Output

Figure 4. Photovoltaic System Output

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Components Modulation

1. Gas Boiler:

$$H_{GB}(t) = \eta_{GB} F_{GB}(t)$$

$$F_{GB}^{min} \times I_{GB}(t) \leq F_{GB}(t) \leq F_{GB}^{max} \times I_{GB}(t)$$

2. CHP:

$$P_{CHP}(t) = \sum_{j \in S} \omega_j(t) \times P_{CHP}^j, \qquad \omega_j(t) \in [0,1], S = \{A, B, C, D\}$$

$$H_{CHP}(k, i) = \sum_{j \in S} \omega_j(t) \times H_{CHP}^j, \qquad \omega_j(t) \in [0,1], S = \{A, B, C, D\}$$

$$I_{CHP}(k, i) = \sum_{j \in S} \omega_j(t), \qquad \omega_j(t) \in [0,1], I_{CHP}(t) \in \{0,1\}$$

$$F_{CHP}(t) = \frac{P_{CHP}(t)}{\eta_{chp}}$$

Components Modulation

2. CHP:

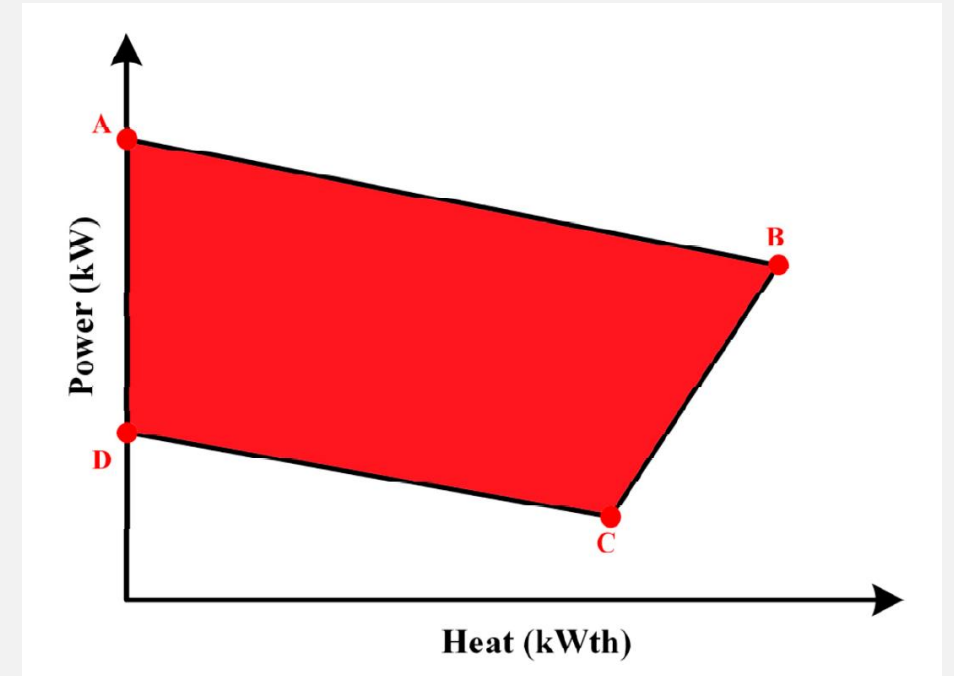$$I_{chp}(t,s) - I_{chp}(t-1,s) \leq I_{chp}(t + UT_{chp}(u), s)$$

$$UT_{chp}(u) = \begin{cases} u & u \leq T_{chp}^{on} \\ 0 & u > T_{chp}^{on} \end{cases}$$

$$I_{chp}(t-1,s) - I_{chp}(t,s) \leq 1 - I_{chp}(t + DT_{chp}(u), s)$$

$$DT_{chp}(u) = \begin{cases} u & u \leq T_{chp}^{off} \\ 0 & u > T_{chp}^{off} \end{cases}$$

$$P_e^{chp}(t,s) - P_e^{chp}(t-1,s) \leq R_{chp}^{Up}$$

$$P_e^{chp}(t-1,s) - P_e^{chp}(t,s) \leq R_{chp}^{Down}$$

Figure 5. Single-zone CHP system with operational axes for heat and power

Components Modulation

3. Heat Pump and Electric Boiler:

$$H_{HP}(t) = \eta_{HP}P_{HP}(t)$$

$$P_{HP}^{min} \times I_{HP}(t) \leq P_{HP}(t) \leq P_{HP}^{max} \times I_{HP}(t)$$

$$P_{EB}(t) - P_{EB}(t-1) \leq R_{EB}^{Up}$$

$$P_{EB}(t-1) - P_{EB}(t) \leq R_{EB}^{Down}$$

4. Electric Chiller:

$$C_{EC}(t,s) = COP_{EC}P_{EC}(t,s)$$

$$C_{EC}^{min} \times I_{EC}(t) \leq C_{EC}(t) \leq C_{EC}^{max} \times I_{EC}(t)$$

5. Absorption Chiller:

$$C_{AC}(t) = COP_{AC}H_{AC}(t)$$

$$C_{AC}^{min} \times I_{AC}(t) \leq C_{AC}(t) \leq C_{AC}^{max} \times I_{AC}(t)$$

6. EES:

$$SOC_{ESS}(t) = (1 - \alpha_{ESS})SOC_{ESS}(t-1) + \eta_{ch}^{ESS}P_{ch}^{ESS}(t) - \frac{P_{dis}^{ESS}(t)}{\eta_{dis}^{ESS}}$$

$$SOC_{ESS}^{min} \leq SOC_{ESS}(t) \leq SOC_{ESS}^{max}$$

$$SOC_{ESS}(t_0) = SOC_{ESS}(t_{24}) = SOC_{ESS}^{initial}$$

Components Modulation

6. EES:

$$P_{ch}^{min} \times I_{ch}^{ESS}(t) \leq P_{ch}^{ESS}(t) \leq P_{ch}^{max} \times I_{ch}^{ESS}(t)$$

$$P_{dis}^{min} \times I_{dis}^{ESS}(t) \leq P_{dis}^{ESS}(t) \leq P_{dis}^{max} \times I_{dis}^{ESS}(t)$$

$$I_{ch}^{ESS}(t) + I_{dis}^{ESS}(t) \leq 1$$

7. Ice Storage, Thermal Storage, Gas Storage

The corresponding formulas for these three components are not repeated for brevity, as they are similar to the EES formulas.

LIMITATIONS AND ENERGY EQUATION

$$P_{buy,Grid}^{min} \leq P_{buy}^{Grid}(t) \leq P_{buy,Grid}^{max}$$
$$P_{sell,Grid}^{min} \leq P_{sell}^{Grid}(t) \leq P_{sell,Grid}^{max}$$

$$F_{buy,Grid}^{min} \leq F_{buy}^{Grid}(t,s) \leq F_{buy,Grid}^{max}$$

$$P_{Load}(t) = P_{PV}(t) + P_{CHP}(t) - P_{EC}(t) - P_{EB}(t) + \left(P_{buy}^{Grid}(t) - P_{sell}^{Grid}(t)\right) + \left(P_{dis}^{ESS}(t) - P_{ch}^{ESS}(t)\right)$$

$$C_{Load}(t) = C_{AC}(t) + C_{EC}(t) + \left(C_{dis}^{CSS}(t) - C_{ch}^{CSS}(t)\right)$$

$$H_{Load}(t) = H_{GB}(t) + H_{CHP}(t) + H_{EB}(t) - H_{AC}(t) + \left(H_{dis}^{HSS}(t) - H_{ch}^{HSS}(t)\right)$$
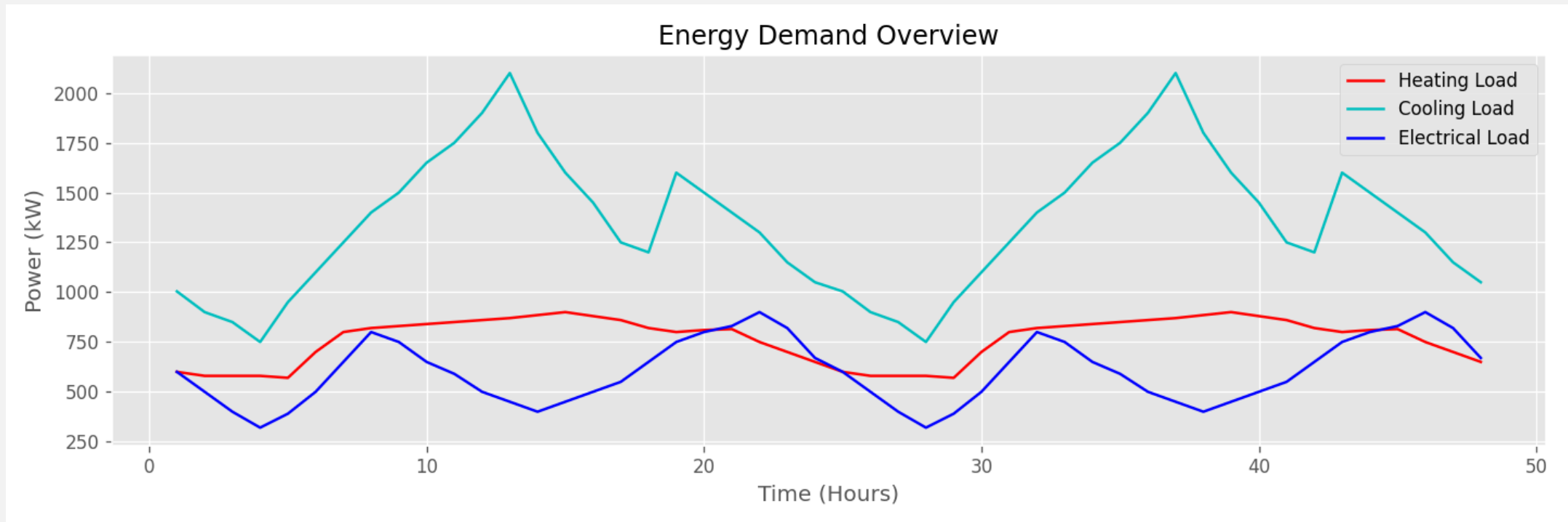
$$F_{buy}^{Grid}(t) = F_{GB}(t) + F_{CHP}(t) + \left(F_{dis}^{GSS}(t) - G_{ch}^{GSS}(t)\right)$$

OBJECTIVE FUNCTION

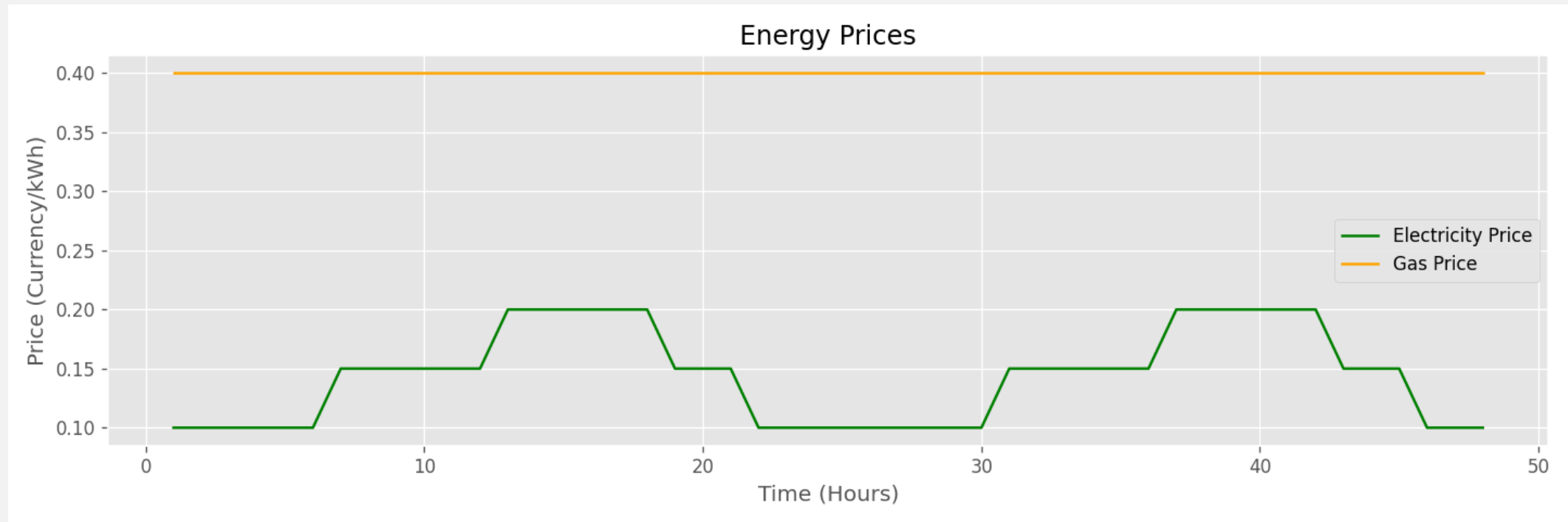$$Cost(t) = c_e^{buy}(t).P_{buy}^{Grid}(t) - c_e^{sell}(t).P_{sell}^{Grid}(t) + c_g^{buy}(t).F_{buy}^{Grid}(t)$$

$$J = \sum_{t=0}^{N-1}\left(Cost(t)\right)$$
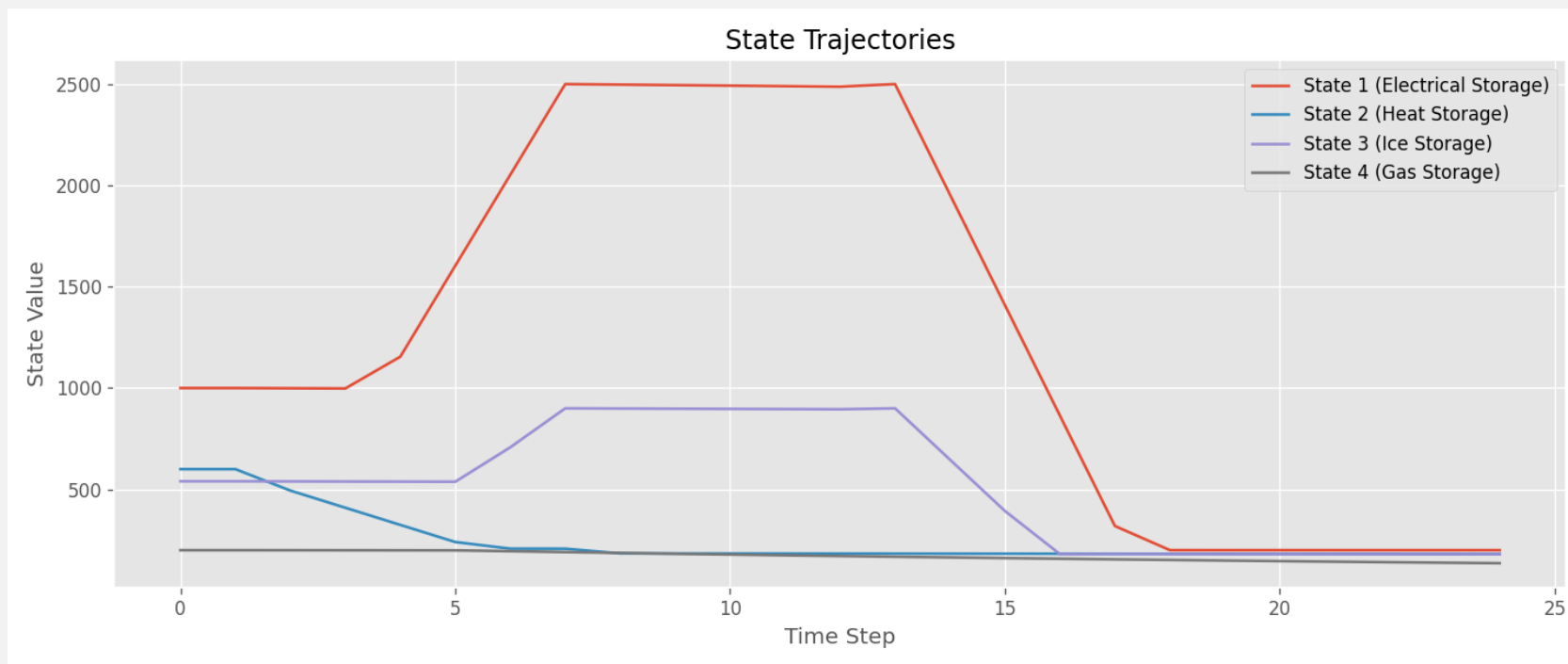
Required Energy Outputs

Figure 6. Energy Demand
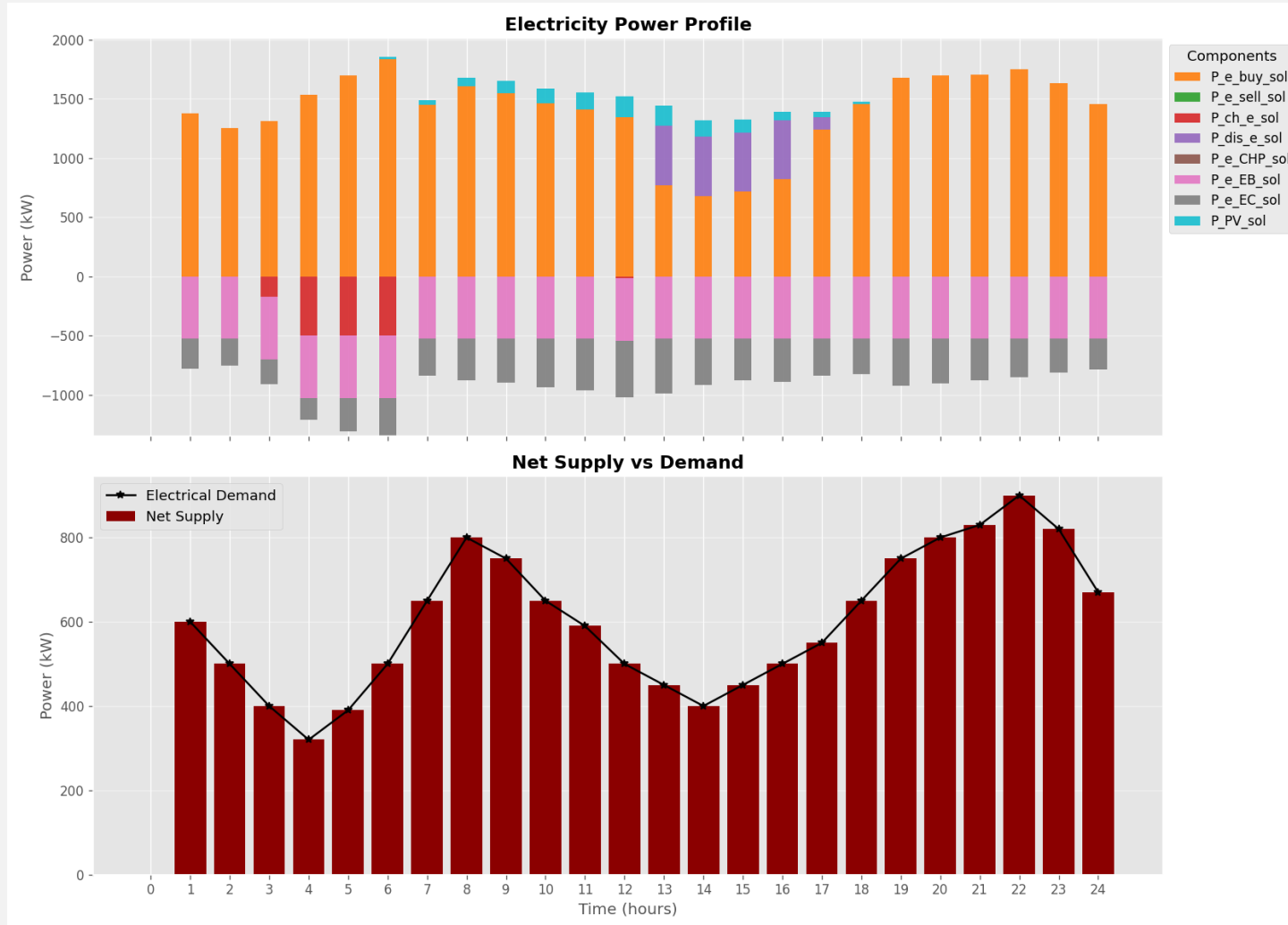
Specifications

Figure 7. Hourly variations of electricity and gas prices procured from the upstream network

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Optimization Results

Figure 8. Dynamic Evolution of System State Variables

Optimization Results

Figure 9. Electricity Power Profile
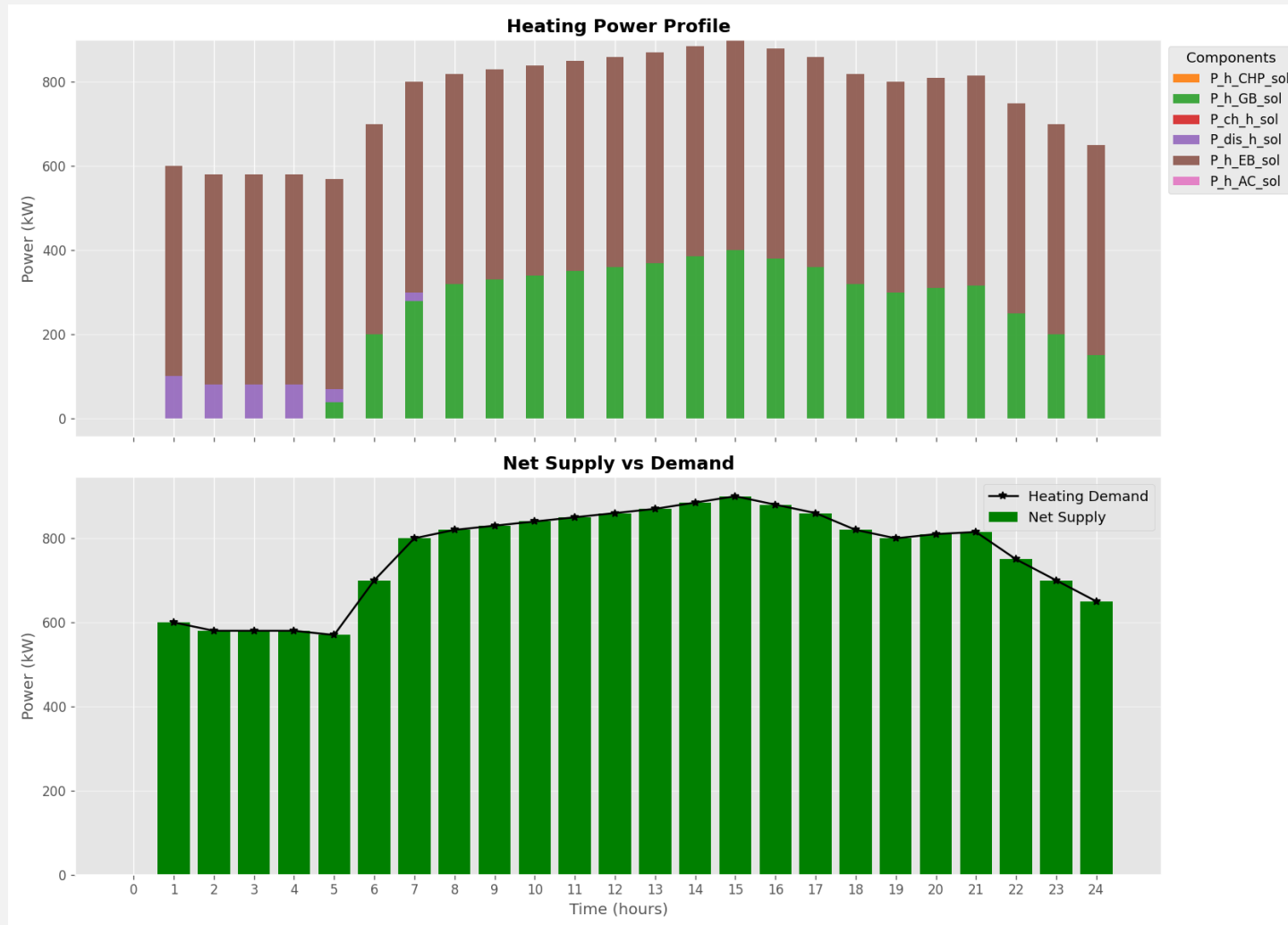
Optimization Results
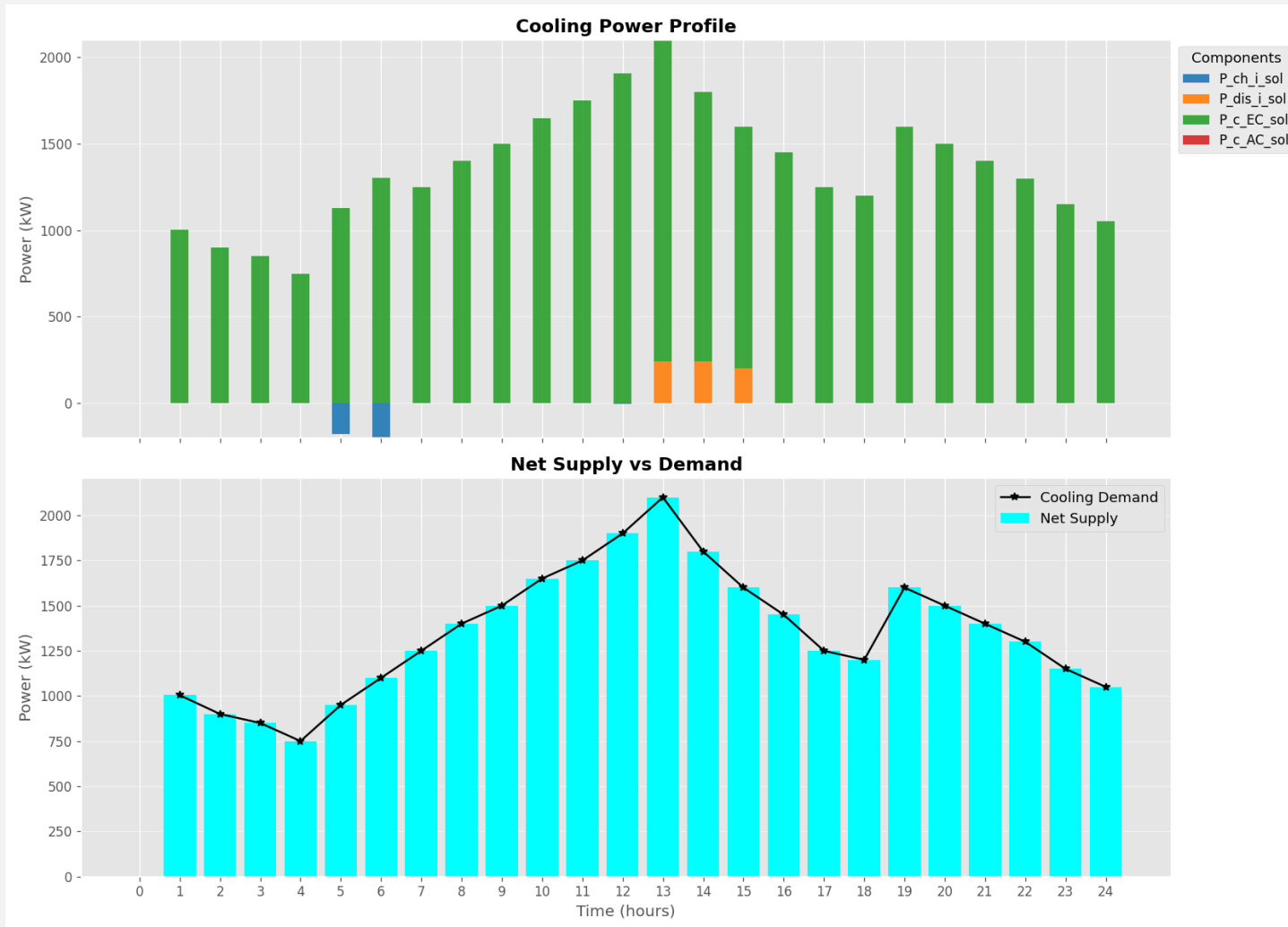

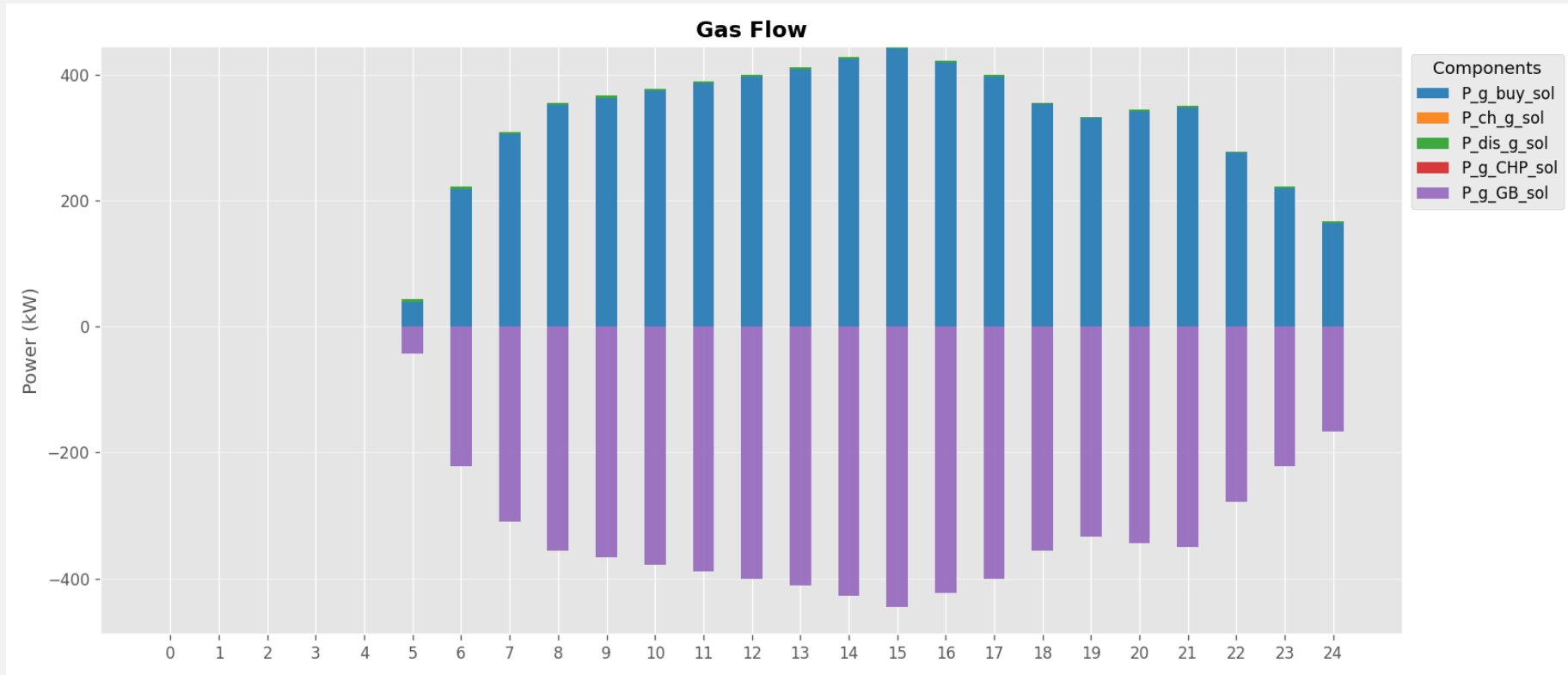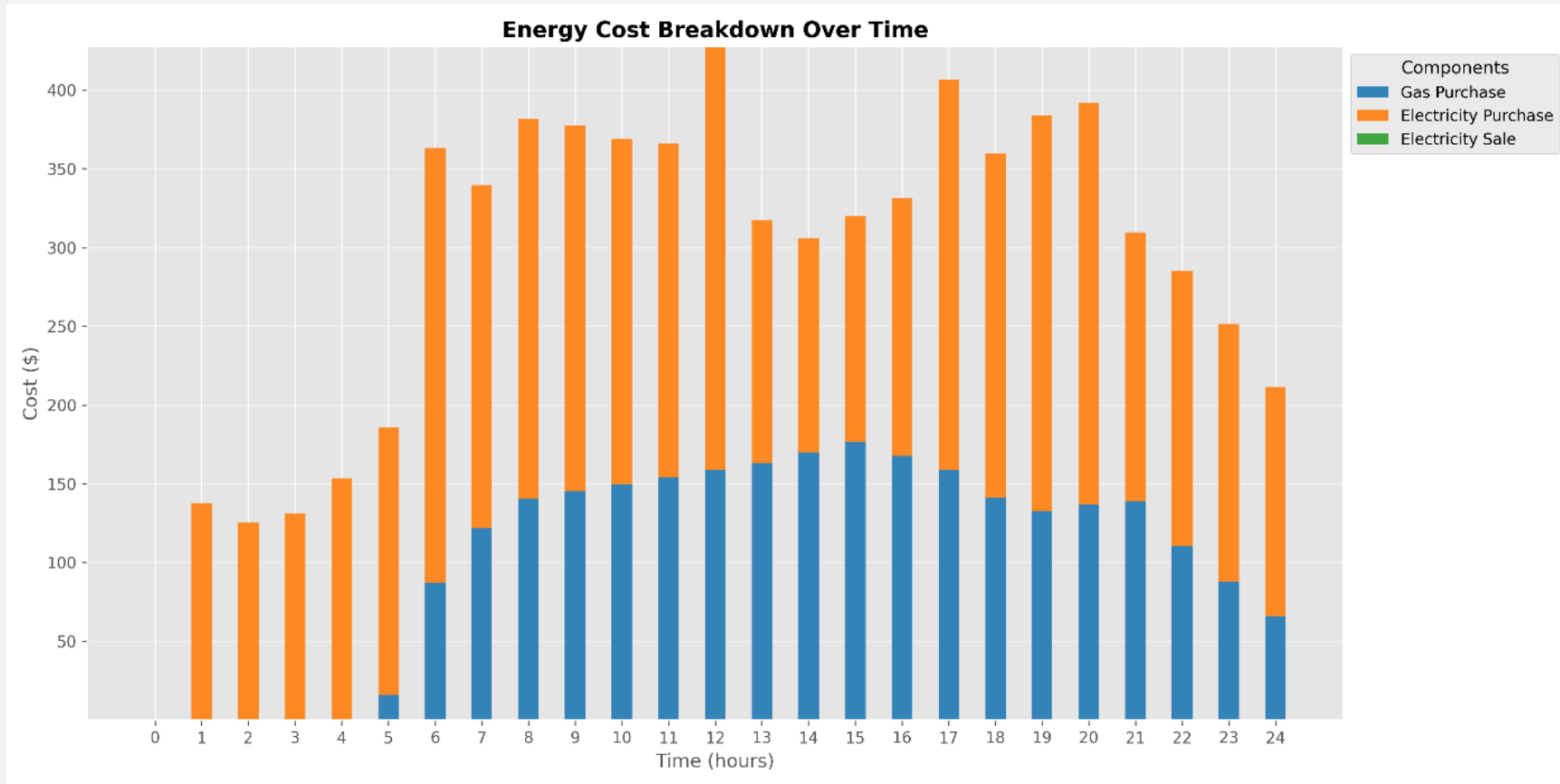**Figure 10.** Heat Power Profile

Figure 11. Cooling Power Profile
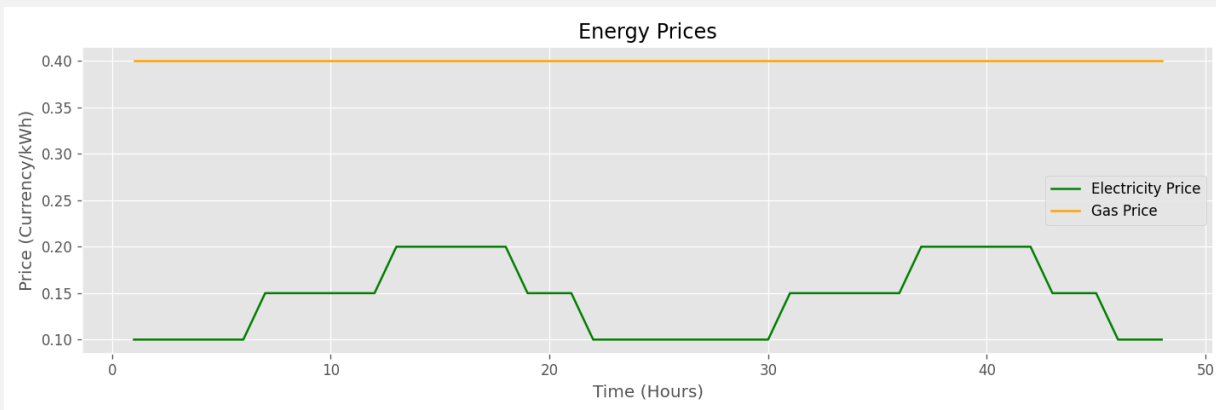
Optimization Results

Figure 12. Gas Power Flow
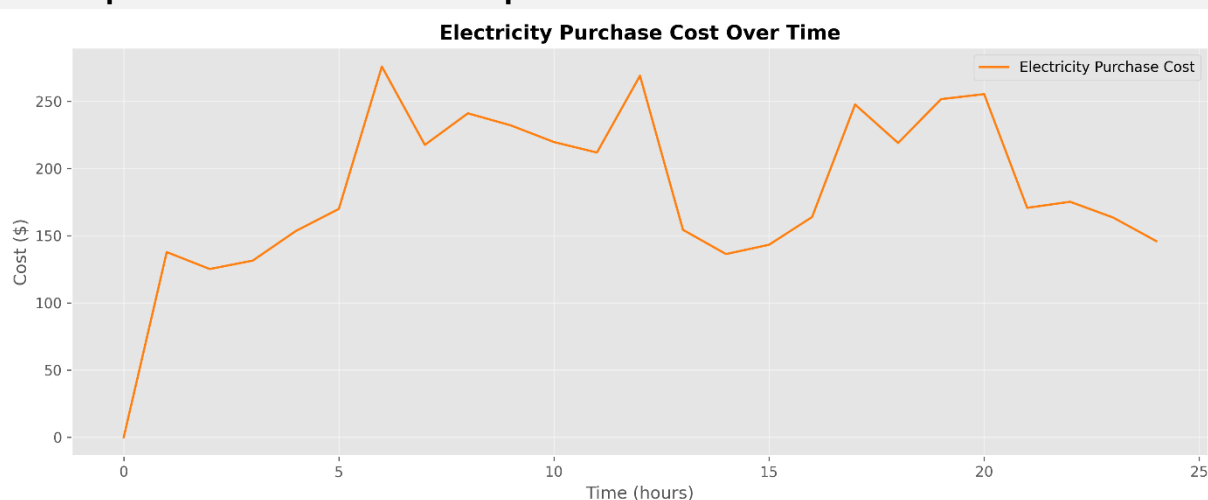
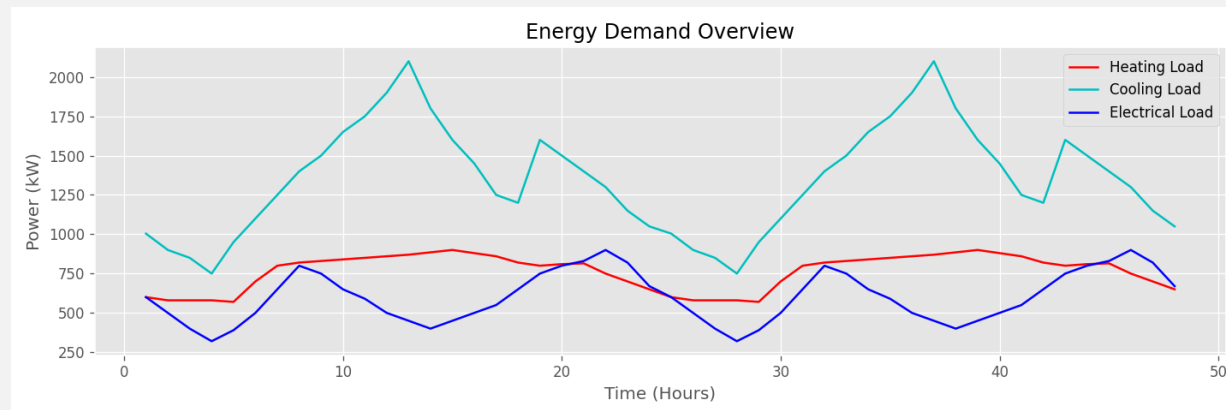# Optimization Results

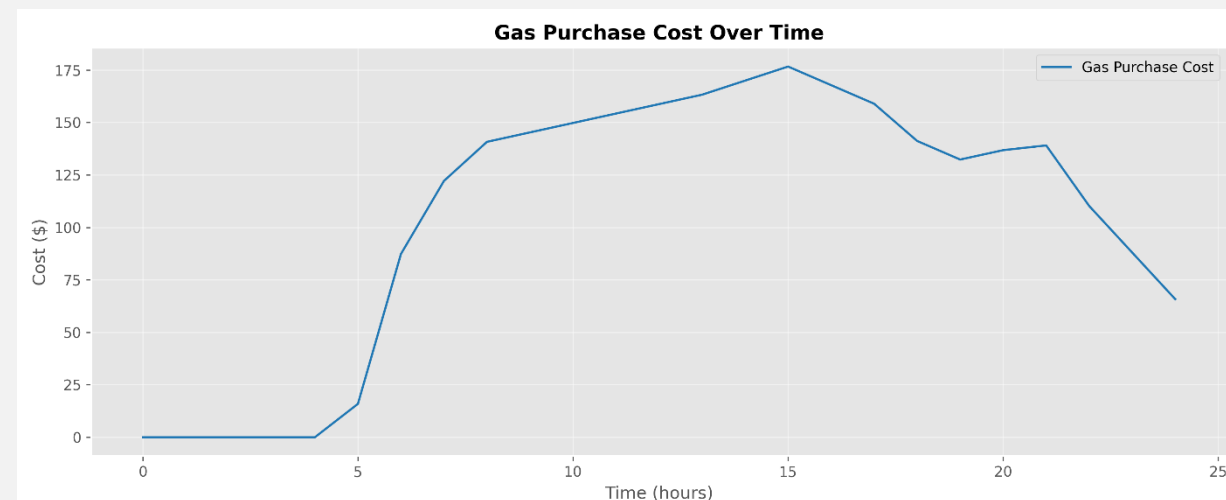

Figure 13. Energy Cost Breakdown

Optimization Results

Figure 14. Hourly variations of electricity and gas prices procured from the upstream network

Figure 15. Energy Demand

Figure 16. Electricity Cost Over Time

Figure 17. Gas Cost Over Time

Conclusion

- Optimal charging/discharging and energy trading reduced total system cost.

- Peak shaving: storages charged during low demand, discharged at peak.

- Renewable energy (solar) fully utilized; no surplus sold to grid.

- CHP and absorption chiller rarely used due to high cost / low efficiency.

- Energy storage crucial for load balancing and dynamic response.

- **Optimization strategy:** peak shaving, max use of renewables, integrated resource management, cost reduction without compromising supply.

Limitation and Future Work

Limitations:

- Simplified linear models for CHP and absorption chiller.

- No uncertainty considered (renewables, prices, demand).

- Grid dynamics (frequency/voltage) ignored.

- Equipment operational limits simplified (wear, efficiency, maintenance).

Future Directions:

- Use data-driven models (e.g., Gaussian Processes) for uncertainty prediction.

- Expand objectives beyond cost: emission reduction, storage lifetime.

- More accurate dynamic models for nonlinear units.

- Study multiple hubs and grid interactions at local/regional scale.

References

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- [2] Ding et al., Review of Optimization Methods for Energy Hub Planning, 2022

- [3] Lasemi et al., Optimization challenges of smart energy hubs under uncertainty, 2022

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