Anomaly Detection in Temporal Multilayer Networks using GLMs and Tensor Decomposition

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Outline

- Overview and Problem Definition
- Literature review
- Proposed Method
    - Tensor-based State Space Model
    - Extended Kalman Filter
- Experiments and Results
- Summary and Future Works

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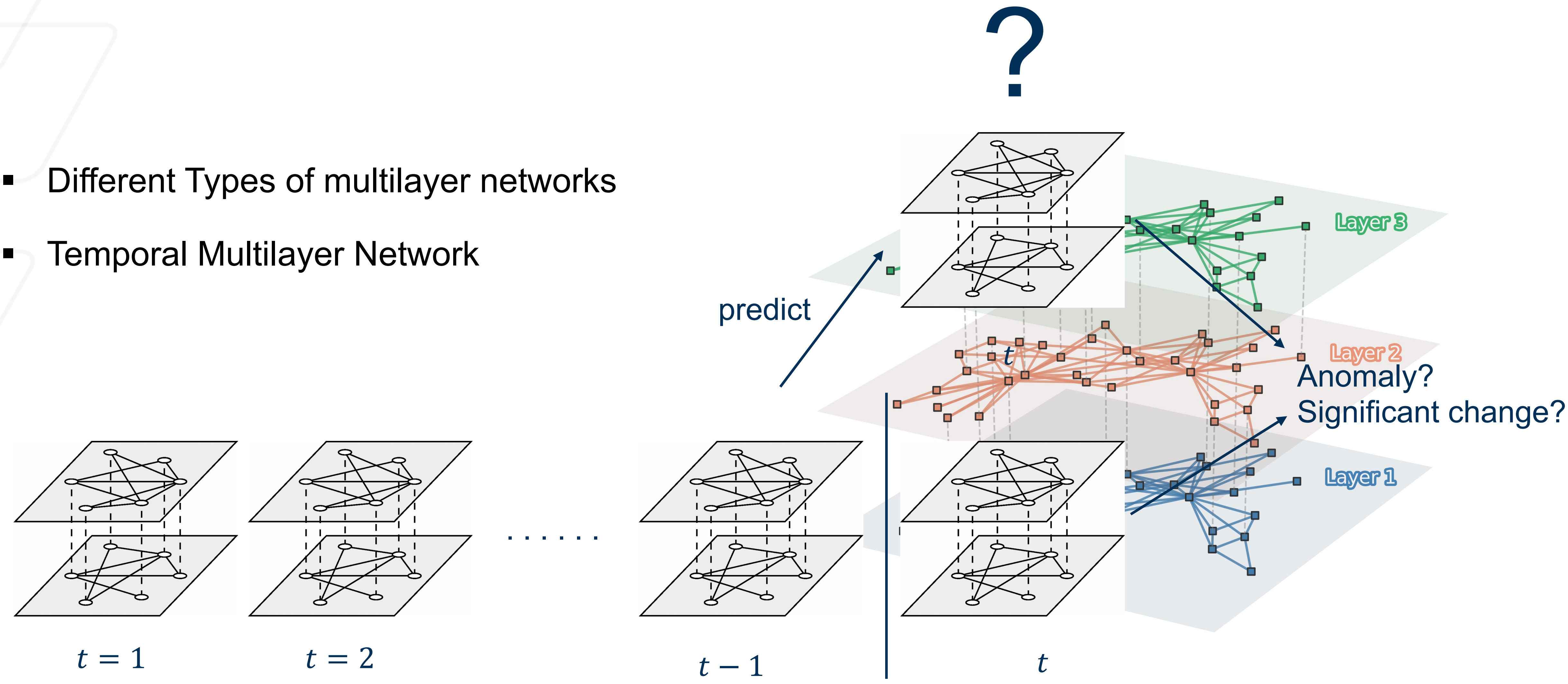
Overview and Problem Definition

- Different Types of multilayer networks
- Temporal Multilayer Network



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Literature Review

Paper	Year	Method
Change Detection in a Dynamic Stream of Attributed Networks Mostafa Reisi Gahrooei, Kamran Paynabar	2017	They combined state space model, and Kalman filter to detect change points in one-layer networks. They consider nodes' attributes.
Modeling and Change Detection for Count-Weighted Multilayer Networks Hang Dong, Nan Chen & Kaibo Wang	2019	They use SBM models to detect changes in community behavioral patterns. The community memberships cannot be different across all layers. they do not consider nodes' features!
Latent Evolution Model for Change Point Detection in Time-varying Networks Yongshun Gong, Xue Dong, Jian Zhang, Meng Chen	2022	They treat a network as a graph and used Graph-based change point detection algorithms. They work on one-layer networks.
Multi-view change point detection in dynamic networks Yingjie Xie, Wenjun Wang, Minglai Shao, Tianpeng Li, YandoNg Yu	2023	They detect changes between and within communities. And they do not work with attributed networks.

Problem Definition

- **Objective**

  Detect anomalies in temporal attributed multilayer networks

- **Challenges**

  High dimensional data

  Attributes

  Dynamics of networks

- **Proposed Method**

  Anomaly detection method using tensor based Generalized linear models and Extended Kalman filter.

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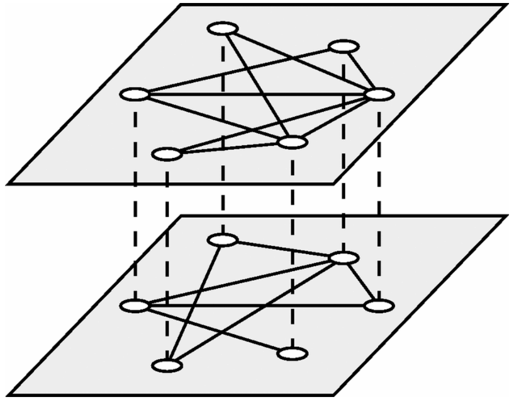
Tensor-based State Space Model

l: number of layers
m: number of features
n: number of nodes in each layer
W_t: Adjacency tensor at time t, dimension: $(l * l * n * n)$
x_i: features of node i, dimension: $(1 * m)$
X: Tensor of features, dimension: $(n * n * m)$
β_t: Coefficent of features, dimension at time t: $(l * l * m)$
θ_t: Parameters of function $f(\theta_t)$, dimension at time t: $(l * l * n * n)$



$$W_t = f(\theta_t)$$
$$\theta_t = g(X\beta_t)$$

$W_t \sim Bernoulli\ (\theta_t)$, f produces a realization of Bernoulli

$g(.)$ is the logit inverse function

- We observe the network (W_t) and need to estimate β_t.
- To reduce dimensions, we use tensor tucker decomposition.

β Estimation

$$W_t = f(\theta_t)$$
$$\theta_t = g(X\beta_t)$$

$W_t \sim Bernoulli\ (\theta_t)$, f produces a realization of Bernoulli

$g(.)$ is the logit inverse function

- To deal with high-dimensionality, β_t is decomposed via Tucker decomposition

$$\beta = C \times_1 U_1 \times_2 V_1 \times_3 V_2$$

$$\beta : m * l * l$$
$$C : \widetilde{m} * \widetilde{l} * \widetilde{l}$$
$$U_1 : m * \widetilde{m}$$
$$V_1 : l * \widetilde{l}$$
$$V_2 : l * \widetilde{l}$$

- Hence, the log likelihood function can be optimized by block coordinate descent

$$L = \sum_{i=1}^{n} (-w_i \log (1 + e^{-x_i (C \times_1 U_1 \times_2 V_1 \times_3 V_2)}) + (w_i - 1) \log (1 + e^{x_i (C \times_1 U_1 \times_2 V_1 \times_3 V_2)}))$$

β Estimation

- Log likelihood function

$$L = \sum_{i=1}^{n} \left(-w_i \log\left(1 + e^{-x_i (C \times_1 U_1 \times_2 V_1 \times_3 V_2)}\right) + (w_i - 1) \log\left(1 + e^{x_i (C \times_1 U_1 \times_2 V_1 \times_3 V_2)}\right)\right)$$

- We use tucker decomposition of W to estimate V_1 and V_2. (1)

$$\bar{W} = Q \times_1 V_1 \times_2 V_2$$

- Block coordinate descent to estimate C and U_1.

nodes	layers	features	error
20	5	6	0.07
50	5	6	0.07
100	5	6	0.06
100	5	12	0.09
100	6	12	0.10

1) Mostafa Reisi Gahrooei, Hao Yan, Kamran Paynabar & Jianjun Shi (2020): Multiple Tensor-on-Tensor Regression: An Approach for Modeling Processes With Heterogeneous Sources of Data, Technometrics.

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Extended Kalman Filter (EKF)

- To include the dynamic of the system in our estimation, we use EKF.
- Recursive mathematical algorithm used for estimating the state of a system from a series of noisy measurements.
- The relationships between the state, measurements, and process noise are not linear.

$\beta_{t|t}$: estimation of β at time t
$\beta_{t|t-1}$: prediction of β at time t
$\hat{\beta}_t$: observation of β at time t
K_t: kalman gain at time t
P_t: covariance matrix of β at time t
R_t: covariance matrix of $\hat{\beta}$ at time t

Estimation

$$\beta_{t|t} = \beta_{t|t-1} + K_t(\hat{\beta}_t - \beta_{t|t-1})$$

$$P_{t|t} = (I - K_t)P_{t|t-1}$$

$$K_t = P_{t|t-1}(P_{t|t-1} + R_t)^{-1}$$

Prediction

$$\beta_{t|t-1} = F\beta_{t-1|t-1} + \epsilon$$

$$P_{t|t-1} = F^T P_{t-1|t-1} F + Q_t$$

Proposed Monitoring Method

MLNs for time steps 1 to T-1
$W_1, \dots, W_{T-1}$

Estimate $\hat{\beta}_1, \dots, \hat{\beta}_{T-1}$ using β estimation algorithm

Calculate Covariance matrices of $\widehat{\beta_t}$

Use EKF to predict β_T and corresponding $\widehat{W}_T$ using state space model

Calculate residuals between $\widehat{W}_t$ and W_t

Establish EWMA chart to detect change points

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Experiments and Results

- Synthetic data
- Time: 45 timesteps (In control data: first 20 timesteps)
- Time of change = 20
- Numbers of layers = 5
- Numbers of nodes = 35
- Numbers of features = 4

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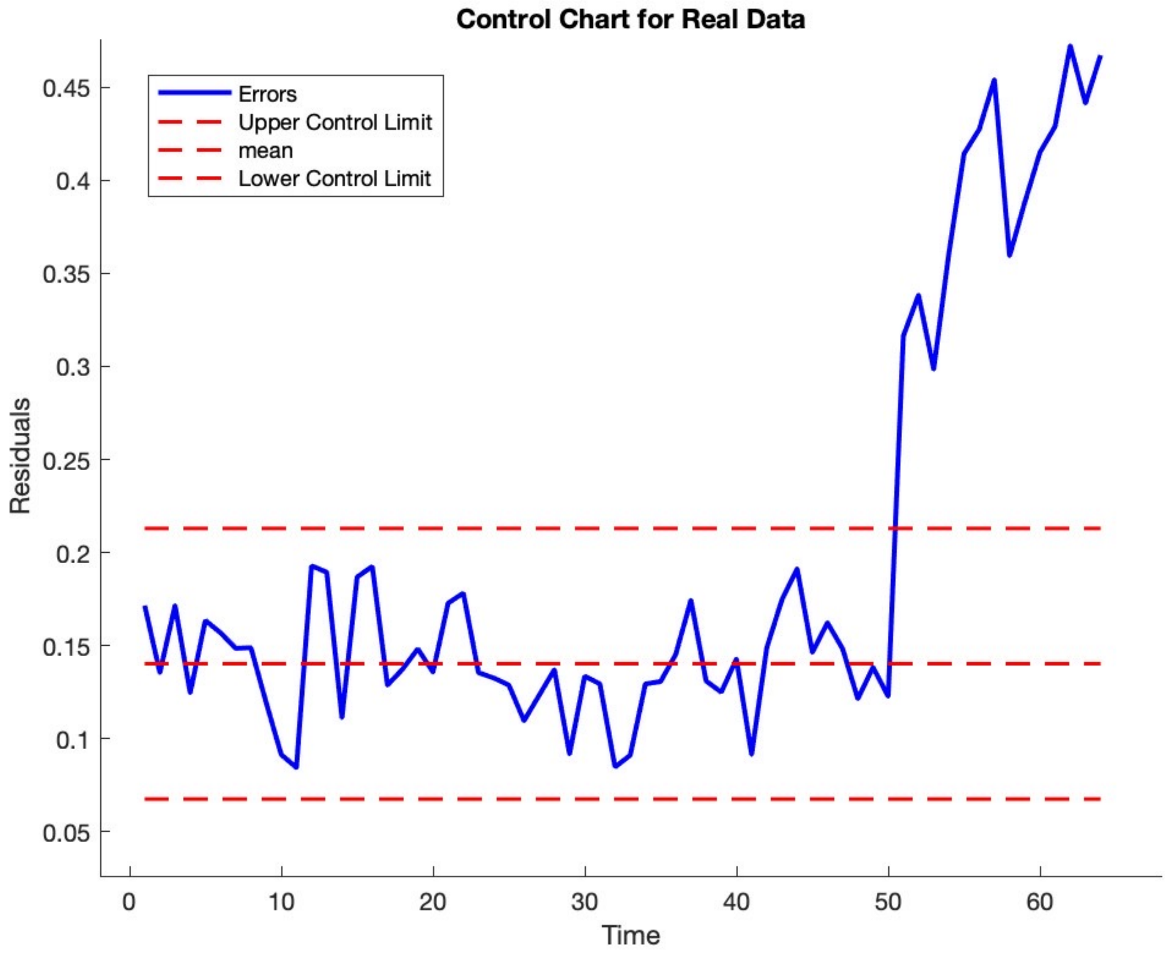
Experiments and Results

- Real data: Enron dataset
- Time: 64 weeks (In control data: first 49 weeks)
- Numbers of layers = 2
- Numbers of nodes = 28
- Numbers of features = 7



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▪ Summary

- We introduced a new method for change detection in temporal multilayer networks.
- We utilized tensor tucker decomposition along GLMs to estimate the parameters.
- We used extended Kalman filter for recursive estimation and prediction of adjacency tensors.
- We used EWMA control charts to detect anomalies.
- Our method is scalable and can be used for directional networks as well.

▪ Future Directions

ARL analysis	Benchmarks	New datasets
We want to do ARL analysis to find the average run length on synthetic data to find the best point to add shift to the data.	Find other methods which are applicable in this concept and compare their performance with the proposed method.	Apply the proposed method on different real dataset and see how it performs.

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Thank you

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