University of Tehran
School of Electrical and Computer Engineering

Formal Methods in Software Engineering
Assignment 1
Due Date: Aban 7th

Please be sure to upload your solution (typed or hand-written) as a PDF to the course website before the due date.

In case of any questions, you can contact me at ykamkar99@gmail.com.

Problem	1	2	3	4	5	6	7	Total
Point	10	12	12	12	18	18	18	100

Problem 1

Express the following declarative sentences in propositional logic. In each case, state the meaning of all atomic propositions [10 points].

  a) You can pass this course if and only if you get at least 10 points.

  b) If you think that you're the richest and smartest person in the world, you are dreaming.

  c) I will not play ping pong unless you play with me.

  d) This isn't true that he is slow but not smart.

  e) It is neither hot nor cold today.

Problem 2

Write proofs using truth tables for the following inferences (p, q, r, and s are proposition symbols) [12 points].

  a) $(p \leftrightarrow q) \land (p \lor r) \vdash q \lor r$

  b) $((p \land q) \lor (\neg p \land q)) \vdash ((p \lor q) \land (\neg p \lor q))$

  c) $(p \rightarrow q) \land (p \rightarrow r) \vdash p \rightarrow (q \land r)$

Problem 3

Write proofs using truth tables for each formula to determine whether they are tautologies or contradictions (p, q, and r are proposition symbols) [12 points].

  a) $((p \rightarrow q) \land (q \rightarrow r) \land (p \lor q)) \rightarrow r$

  b) $(p \land q) \land ((p \leftrightarrow r) \leftrightarrow \neg(q \leftrightarrow r))$

  c) $((p \rightarrow q) \lor (\neg q \rightarrow \neg p)) \rightarrow p$

Problem 4

Determine and prove whether or not $\{\uparrow\}$ is an adequate set of connectives [12 points].

p	q	$p \uparrow q$
0	0	1
0	1	1
1	0	1
1	1	0

Problem 5

Write proofs using Hilbert System for the following inferences (p, q, and r are proposition symbols). You can only use the Hilbert System's basic rules and laws that have been proven using the Hilbert System in the course slides [18 points].

    a) $p \rightarrow q \vdash_H (r \rightarrow p) \rightarrow (r \rightarrow q)$

    b) $\vdash_H (p \rightarrow (q \rightarrow r)) \rightarrow (q \rightarrow (p \rightarrow r))$

    c) $\vdash_H (\neg q \rightarrow p) \rightarrow (\neg p \rightarrow q)$

Problem 6

Write natural deduction proofs for the following inferences (p, q, and r are proposition symbols) [18 points].

    a) $q \rightarrow r \vdash_N (p \rightarrow q) \rightarrow (p \rightarrow r)$

    b) $p \wedge q \vdash_N \neg(\neg p \vee \neg q)$

    c) $((p \wedge q) \vee (q \wedge r)) \rightarrow (q \wedge (p \vee r))$

Problem 7

Write proofs using resolution for each one of the following formulas after transforming them to the conjunctive normal form to determine whether they are satisfiable or not (p, q, r, and s are proposition symbols) [18 points].

    a) $(p \leftrightarrow q) \wedge (q \rightarrow r) \wedge \neg r \wedge (\neg p \vee q)$

    b) $((p \rightarrow q) \rightarrow q) \wedge ((p \rightarrow p) \rightarrow r) \wedge ((r \rightarrow s) \rightarrow \neg(s \rightarrow q))$

    c) $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \wedge \neg r$