DISCOVERY THROUGH GOSSIP

MOHAMMAD MAHZOUN

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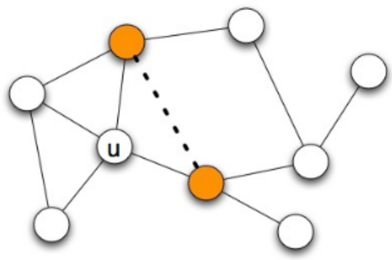
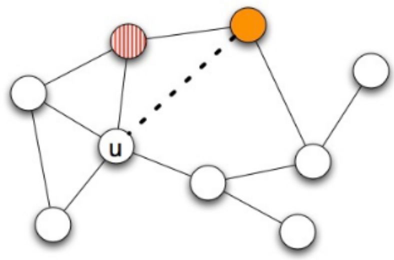


(a) Triangulation



(b) Two-hop walk

Figure 1: Discovery Methods

1 INTRODUCTION

We want to study Gossip based discovery processes in both directed and undirected graphs using push discovery (triangulation) and pull discovery (two-hop walk process).

We are interested in studying the time taken by process to converge to the transitive closure of the graph.

1.1 Notation

Table 1: Table of Notations

Notation	Description
δ_t	Minimum degree of G_t
$N_t^i(u)$	Set of nodes at distance i from u in G_t
$d_t(u)$	Degree of u in G_t
$d_t(u, S)$	Degree induced on S

1.2 Useful lemmas

***Lemma* 1.** $|\cup_{i=1}^{4} N_t^i(u)| \geqslant \min\{2\delta_t, n-1\}$ for all $u \in G_t$.

Proof. If $N_t^3 \neq \emptyset$, then $|\cup_{i=2}^{4} N_t^i(u)| \geqslant \delta_t$ and $|N_t^1(u) \geqslant \delta_t$. So $|\cup_{i=1}^{4} N_t^i(u)| \geqslant 2\delta_t$ since the two sets are disjoint.
If $N_t^3 = \emptyset$, $N_t^1(u) \cup N_t^2(u) = n-1$ since G_t is connected. □

***Lemma* 2.** Consider k Bernoulli experiments in which the success probability of the ith experiment is at least i/m where $m \geqslant k$. If X_i denotes the number of trials needed for experiment i to output a success and $X = \sum_{i=1}^{k} X_i$, then $\Pr[X > (c+1)m \ln m] < \frac{1}{m^c}$.

Proof. w.l.o.g assume that $k = m$. The problem can be seen as *coupon collector problem* where X_{m+i-1} is the number of steps to collect ith coupon. Consider the probability of not obtaining the ith coupon after $(c+1)m \ln m$ steps, we have: $(1 - \frac{1}{m})^{(c+1)m \ln m} < e^{-(c+1)\ln m} = \frac{1}{m^{c+1}}$ By union bound, the probability that some coupon has not been collected after $(c+1)m \ln m$ steps is less than $\frac{1}{m^c}$. □

2 TRIANGULATION RESULTS

2.1 Upper Bound

2.2 Lower Bound

Theorem 3 (Upper bound for triangulation process)**.** *For any connected undirected graph, the triangulation process converges to a complete graph in* $O(n \log^2 n)$ *rounds with high probability.*

In order to prove Theorem 3, we prove that the minimum degree of the graph increases by a constant factor (or equals to $n-1$) in $O(n \log n)$ steps. We say that a node v is **weakly tied** to a set of nodes S if $d_t(v, S) < \delta_0/2$, and **strongly tied** to a set of nodes S if $d_t(v, S) \geqslant \delta_0/2$.

***Lemma* 4.** If $d_t(u) < \min\{n-1, (1 + \frac{1}{4}\delta_0)\}$ and $w \in N_t^1(u)$ has at least $\frac{\delta_0}{4}$ edges to $N_t^2(u)$, then the probability that u connects to a node in $N_t^2(u)$ through w in round t is at least $\frac{1}{6n}$.

Proof. The probability that u connects to a node in $N_t^2(u)$ through w in round t is:

$$\frac{d_t(w,N_t^2(u))}{D_t(w)} \times \frac{1}{d_t(w)} \geqslant \frac{d_t(w,N_t^2(u))}{D_t(w)} \times \frac{1}{n} \geqslant \frac{d_t(w,N_t^2(u))}{|N_t^1(u)|+d_t(w,N_t^2(u))} \times \frac{1}{n} \geqslant \frac{d_t(w,N_t^2(u))}{(1+\frac{1}{4})\delta_0+d_t(w,N_t^2(u))} \times \frac{1}{n} \geqslant \frac{\frac{\delta_0}{4}}{(1+\frac{1}{4})\delta_0+\frac{\delta_0}{4}} \times \frac{1}{n} = \frac{1}{6n}$$

□

***Lemma* 5.** If $d_t(u) < \min\{n-1, (1 + \frac{1}{4}\delta_0)\}$ and $w \in N_t^1(u)$ is weakly tied to $N_t^2(u)$, and $v \in N_0^2(u) \cap N_0^1(w)$, then u connects to v through w in round t with probability at least $\frac{1}{4\delta_0^2}$.

Proof. Since w is weakly tied to $N_t^2(u)$ and $d_t(w)$ is at most $|N_t^1(u)| + d_t(w, N_t^2(u))$, we obtain that $d_t(w)$ is at most $(1 + \frac{1}{4})\delta_0 + \frac{\delta_0}{2}$. Therefore, the probability that u connects to v through w in round t equals:

$$\frac{1}{d_t(w)^2} \geqslant \frac{1}{((1+\frac{1}{4})\delta_0+\frac{\delta_0}{2})^2} \geqslant \frac{1}{\frac{7\delta_0}{4}^2} \geqslant \frac{1}{4\delta_0^2}.$$

□

To analyze the growth in the degree of a node u, we consider two overlapping cases. The first case is when more than $\delta_0/4$ nodes of $N_t^1(u)$ are strongly tied to $N_t^2(u)$, and the second is when less than $\delta_0/3$ nodes of $N_t^1(u)$ are strongly tied to $N_t^2(u)$.

***Lemma* 6** (Several nodes are strongly tied to two-hop neighbors)**.** There exists a $T = O(n \log n)$ such that if more than

2.3 Paragraphs

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2.4 Math

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$$\cos^3\theta = \frac{1}{4}\cos\theta + \frac{3}{4}\cos 3\theta \tag{1}$$

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Definition 1 (Gauss)**.** To a mathematician it is obvious that $\int_{-\infty}^{+\infty} e^{-x^2}\,dx = \sqrt{\pi}$.

Theorem 7 (Pythagoras)**.** *The square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.*

Proof. We have that $\log(1)^2 = 2\log(1)$. But we also have that $\log(-1)^2 = \log(1) = 0$. Then $2\log(-1) = 0$, from which the proof. □

3 RESULTS AND DISCUSSION

Reference to Figure 2 on the following page.

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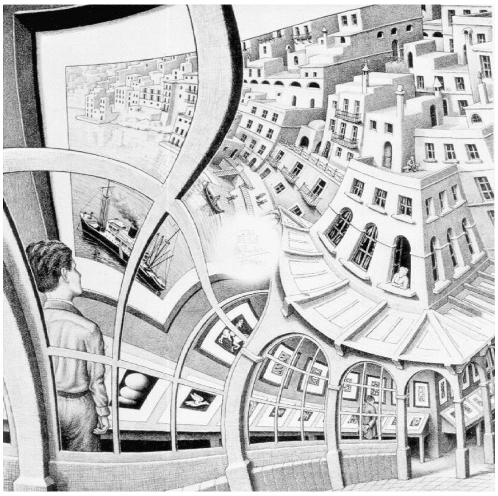

Figure 2: An example of a floating figure (a reproduction from the *Gallery of prints*, M. Escher, from `http://www.mcescher.com/`).

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3.1 Subsection

3.1.1 *Subsubsection*

WORD Definition

CONCEPT Explanation

IDEA Text

- First item in a list
- Second item in a list
- Third item in a list

Reference to Table 1 on page 2.

3.2 Figure Composed of Subfigures

Reference the figure composed of multiple subfigures as Figure 3 on the following page. Reference one of the subfigures as Figure 3b on the next page.

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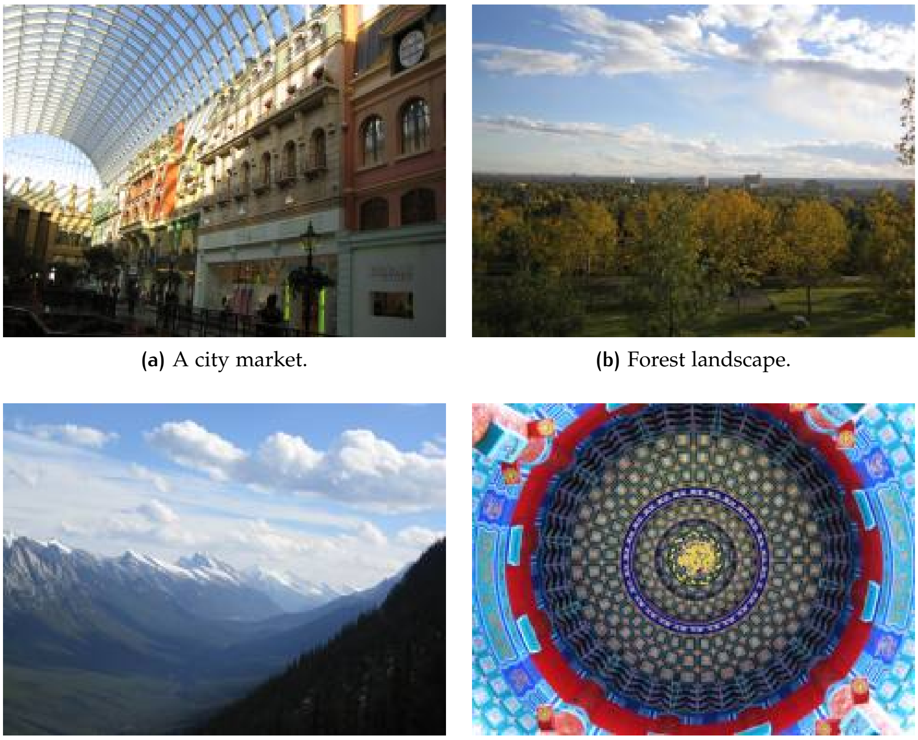

(a) A city market.

(b) Forest landscape.

(c) Mountain landscape.

(d) A tile decoration.

Figure 3: A number of pictures with no common theme.

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REFERENCES

[1] A. J. Figueredo and P. S. A. Wolf. Assortative pairing and life history strategy - a cross-cultural study. *Human Nature*, 20:317–330, 2009.