Homework 1

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Exercise 1.1. Rumor spreading in star graph is the same problem as coupon collector. Starting from each arbitrary node, informing the ith node has probability $p_i = \frac{n-i}{n}$ and expected value $E_i = \frac{1}{p_i}$. The expected time T of informing all nodes is:

$$E[T] = \sum_{i=1}^{n} E_i = n \times (\sum_{i=1}^{n} \frac{i}{n}) = n \ln n + \Theta(n)$$

Exercise 1.2. There are $i = nf$ informed nodes. The probability that a fixed uninformed node remains uninformed after $T = \frac{1}{fg}$ rounds is:

$$(1 - \frac{1}{n})^{iT} = (1 - \frac{1}{n})^{\frac{n}{g}}$$

So the expected number of informed nodes after T rounds is:

$$n - n(1 - f)(1 - \frac{1}{n})^{\frac{n}{g}}$$

The probability that after T rounds at least $X = n(1 - g)$ nodes are informed is:

$$Pr(X \geq n(1 - g)) \leq \frac{E(X)}{n(1-g)} = \frac{1-(1-f)(1-\frac{1}{n})^{\frac{n}{g}}}{1-g} = \frac{1-(1-o(1))(1-\frac{1}{n})^{\omega(n)}}{1-g} \leq 1 - o(1)$$

Exercise 1.3. content...

Exercise 1.4. content...

Exercise 1.5. content...