Introduction to the modeling and simulation of Cellular Wireless Communication Systems

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1 Introduction

This text is a study program aimed at students of the 2nd through 4th semesters of telecommunications and computer engineering. It does not require in-depth prior knowledge of wireless communications. Accessible language, examples and analogies are used as much as possible. Mathematical formality is kept to minimum and used only when strictly necessary. Rigor will sometimes be compromised for the sake of easier understanding of concepts and phenomena.

The general objective of the program is to put the student in contact with simple models that allow the understanding of the fundamental trade offs of cellular wireless communication systems through simple analyzes and computer simulations.
This is a study program divided into phases, with a gradual increase in the complexity of the models, the dimensionality of the problems and the sophistication of the experiments. This document specifies the first phase of this study program.

2 Specific Objectives

This study program aims at making the student familiar with the following concepts and equipped with the following skills:

- Basic models and quantities involved in the various aspects of cellular wireless communications systems such as radio access network, propagation law, power ratios, computer simulations and performance indicators.
- Monte Carlo simulation technique and the practical use of random variables in computer simulations.
- Development of a simple system simulator that allows studying the basic performance trade offs in wireless systems by varying system configuration parameters and physical deployment scenarios.This entails a basic understanding of radio resource management using the most

fundamental resources in a radio access network that is: spectrum, transmit power, access points and user equipments.

- Present, analyze and interpret results reaching sound conclusions and inferring general principles involved in the design and dimensioning of wireless systems considering not only performance but also the costs involved in each configuration choice.

3 System Model

The student must get used and understand the following models. Our complete wireless communication system model is based on the concatenation of each of these individual models. Wireless systems are complex and composed of many parts. Some of the most important components are modeled below. As you progress, in the next phases, more models and/or more complex versions of the models below will be presented.

As an introductory reading, the student must read and generally comprehend Chapter 1 of the book [1]. You will get a free copy of this book that you can keep as long as you stay enrolled in this program.

3.1 General System View

We will be studying a cellular wireless communication system composed of Access Points (AP) and User Equipments (UEs). APs are antennas deployed by the telecom operator. A typical example of UE is a smartphone. We will be studying the radio communications link from UEs to the APs, that is, the UEs transmit and the APs receive radio signals that carry information. This direction of information flow is known in radio communications as the "uplink".There also exists simultaneously the opposite link (the "downlink") in which the AP transmits and the UE receives. It is because of these 2 simultaneous radio links that you are able to talk and isten at the same time when calling a friend using your smart phone. Usually, the uplink and the downlink operate in different frequencies to avoid interference.

APs, UEs and the corresponding radio links form the Radio Access Network (RAN). The RAN is a critical part of the wireless systems because it is through the RAN that your smartphone gets connected to to the operator's antenna (i.e the APs) and then to the Internet. The RAN also allows you to move with your smartphone while connected.

As said above, our focus for now will be the uplink. By studying the uplink, our objective is to predict the experience of the users when e.g. transmitting a live video stream via Youtube or uploading files to a cloud server such as Google Drive. We will seek to analyze how this experience varies with the number of APs and UEs in a given coverage area. It will be a really exciting study and in the end you will be able to understand the basic factors that influence how a wireless cellular communication system is designed to meet quality of service expectations from users.

In the following we will address the several parts of our system model, the building blocks that will be integrated to allow you to study the performance of the system in different scenarios.

3.2 Modeling of the Coverage Area, positions of APs and UEs

The coverage area for our system is a square with side L. The coverage area can be considered a 2D plane as seen from above (top view) as illustrated in Fig.1.

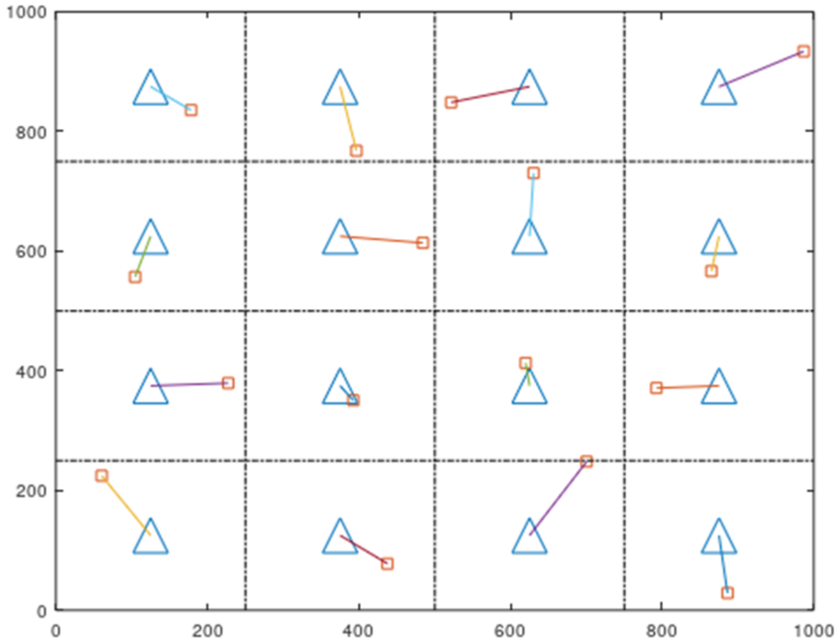

Figure 1: Example of coverage area for the system under study.

In the illustration in Fig.1 the coverage area is of 1km2 since $L = 1000$ meters. There are 16 Access Points (triangles) regularly spaced defining hypothetical cell borders (delimited by dotted lines). In each cell there is one User Equipment (small squares) which communicates with its AP via a radio link (symbolically illustrated by solid lines) .UEs are uniformly distributed over the coverage area meaning they can be positioned anywhere. As a consequence, notice the varying distances among UEs and APs, some UEs closer, some far away from their serving AP.

As commented before this is a top view as seen from above the Earth. For the moment we will confine ourselves to this 2 dimensional (2D) model of the coverage area. This 2D assumption is reasonable for relatively large areas, but in the future 3D modeling will be necessary for more sophisticated analysis.

3.3 Propagation Model

One of the most fundamental models necessary to study wireless systems is the channel or propagation model. This model relates the power received by an AP from one transmitting UE at a given position. The received power is a function of the transmitter-receiver distance and is given by:

$$p_r^{i,j}(d_{i,j}) = p_t^j \frac{c}{d_{i,j}^n} \tag{1}$$

$$d \geqq d_0 \tag{2}$$

in which $p_r^{i,j}$ is the power received by the i-th AP transmitted from the j-th UE; $d_{i,j}$ is the physical distance between the i-th AP and j-th UE; p_t^j is the transmit power form the j-th UE; n is the path loss exponent and defines how fast the received power decreases with distance; c is a constant; both n and c depend on the actual physical propagation environment.

The constant c can be determined for a particular propagation environmente by measuring the received power for a known transmit power at a fixed reference distance d_0. This also means that, for modeling purposes, the received power is not defined for $d < d_0$. The value for d_0 is typically small (e.g. 1 meter) with respect to the dimensions of the coverage area, so that this assumption does not impact significantly the results. As for the path-loss exponent n, it is estimated also by making measurements for a fixed transmitt power and varying transmit-receive distances.Typical values of n are in the range 2 to 4.

Notice that the model in eq.1 is very simple and do not capture all the relevant phenomena involved in mobile radio transmission and reception. In the future we will make this model more realistic (and complex). Nonetheless it serves well the purposes of our current study.

3.4 Limiting Factors: bandwidth and noise

There are two fundamental limiting factors in any communication system: receiver noise and interference. Let us deal with noise first.

Noise is a physical perturbation that is inherent to any communications receiver. The important thing regarding noise is to measure its power so that you can calculate the desired signal power to noise power ratio (signal-to-noise ratio, or SNR, for simplicity). Noise power depends on several factors but for the purposes of the present study the factor we will consider most important is the bandwidth of the radio channel allocated to each radio link. We know from intuition that the larger the bandwidth of a channel, the higher the capacity of the radio link (i.e., its data rate). This will be mathematically expressed further on. But larger bandwidths come at a cost in terms of noise power.

Consider that a total bandwidth of B_T [Hertz] is available for the entire system. This total bandwidth can be divided into $N \geqslant 1$ non-overlapping (and therefore non-interfering) channels, that is $B_c = B_T/N$ in which B_c [Hertz] is the bandwidth of each non-overlapping channel. The non-overlapping property of the N channels also means that mathematically these channels are orthogonal (hence, the non-interfering nature). In practice it means that each channel has a different central carrier frequency and all information transmitted and received in that channel does not interfere with information being transmitted and received in another channel at the same time. This approach for dealing with multiple simultaneous transmissions is called frequency division multiple access (FDMA) and is the most classic way of allowing multiple information sources to co-exist in a wireless system. For example, FDMA is applied to broadcast radio (AM/FM) and TV resulting in the usual user experience of tuning to different stations (which equates changing to different channels).

Notice, however, that you can also work with one single channel ($N = 1$) for all radio links. The consequences of choosing a higher or lower value for N will become apparent further on. But for the moment it suffices to say that a key relationship between channel bandwidth and noise power is that noise power at the receiver is directly proportional to channel bandwidth, that is:

$$p_n = K_0 B_c = K_0 \frac{B_T}{N} \tag{3}$$

in which K_0 is a constant to be defined later on.

We are now in position to express the signal-to-noise ratio (SNR) in AP i when the signal of interest comes from UE j as:

$$\gamma_{i,j} = \frac{p_r^{i,j}}{p_n} \tag{4}$$

Finally we can relate the signal-to-noise ratio to the quality (stability and reliability) of the radio link. For a radio link to be established between an AP and an UE it must have a minimum SNR that is: $\gamma_{i,j} \geqslant \gamma_{\min}$. If the SNR in a radio link falls below $\gamma_{\min}$ your equipment loses its connection to the network (i.e. you will be out of coverage). In practice we want the SNR to be well above $\gamma_{\min}$ so that we not only are connected to the network but have a received signal well above the noise floor. A high SNR means less errors in the detection process at the receiver which translates into higher capacity (i.e. data rate). The mathematical relationship between capacity (i.e. data rate) and SNR will become clear further on.

3.5 Limiting factors: interference

The second fundamental limiting factor in wireless communications is interference. There are many types of interference, but here we will focus on the specific case of co-channel interference or CCI. This is the interference that results when 2 or more UEs share the same channel and transmit at the same time. The effect of such simultaneous transmission is that the received signal will be a mix of the 2 or more transmitted signals. This means that there will be errors when detecting the received signal at the AP and the capacity (data rate) will decrease.

Ideally we would like to have zero co-channel interference but in practice this is not possible. As the number of UEs in the system grows, given a fixed total bandwidth B_T, CCI will inevitably show up. The important question is what is the number of UEs that the system supports while keeping the CCI in a relatively low level (such that signal detection is possible with reasonable reliability). Put in another way, we want to measure the system performance versus the number of UEs in the system (i.e. “the load” in the system). The performance in its turn will be affected by the level of CCI which depends, among other factors, in such “load”.

To mathematically express the level of CCI in a radio link we also use a power ratio analogous to the signal-to-noise ratio. The ratio in this case is called signal-to-interference ratio (SIR). Let us express the SIR for a particular UE (j-th) as seen by the AP (i-th) which intends to detect its signal being transmitted in a particular channel:

$$\gamma_{i,j} = \frac{p_r^{i,j}}{\sum\limits_{k=1,k\neq j}^{K} p_r^{i,k}} \tag{5}$$

Notice that we used the same notation for the SIR as for the SNR (the greek letter γ).The SIR expression above is simply the ratio of the power of the desired signal (from j-th UE) to the sum of all other interfering signal powers transmitting in the same channel at the same time assuming there are a total of K simultaneous transmissions from all UEs using a particular channel. If you have more than one channel (i.e. $N > 1$) you should consider only the UEs transmitting in the same channel for computing the SIR.

Exercise 1 *In this exercise you will calculate the SIR for the situation illustrated in Fig.2. Consider a segment of a highway road in which there are 2 APs and 2 UEs. The APs are space by 200 meters. The positions and distances of APs and UEs are shown in the figure. UE1 and UE2 transmit in the same channel. UE1 is connected do AP1 while UE2 is connected to AP2. UEs transmit with 1 W of power. The received power follows eq.1 assuming $c = 10^{-4}$ and $n = 4$. Notice that there are 4 radio links involved: the 2 “desired” ones carrying the signals of interest from UE1 and UE2 to AP1 and AP2 respectively; and the 2 interfering links that are “undesired” but also reach AP1 (from UE2) and AP2 (from UE1). Given this situation calculate the SIR for UE1 at AP1 and for UE2 at AP2. Compare both and conclude which UE experiments better quality of service. Question: before calculations, by simply looking at Fig.2 could you anticipate your conclusion ?*

So far we presented two performance indicators, SNR (when only noise is present) and SIR (when only interference is present). We can combine the SNR and SIR power ratios into one single metric for the situation in which both noise and interference are present in the system, which is typical. The combination of both metrics leads us to the signal to interference plus noise ratio (SINR) that, following same conventions in eqs.4 and 5, is given by:

$$\gamma_{i,j} = \frac{p_r^{i,j}}{\sum\limits_{k=1,k\neq j}^{K} p_r^{i,k} + p_n} \tag{6}$$

3.6 Interpreting noise and interference: the cocktail party analogy

You may be wondering how to rightfully interpret the concepts of noise and interference in wireless communications. The key to understand “noise” is that its effects are perceived in the receiver.

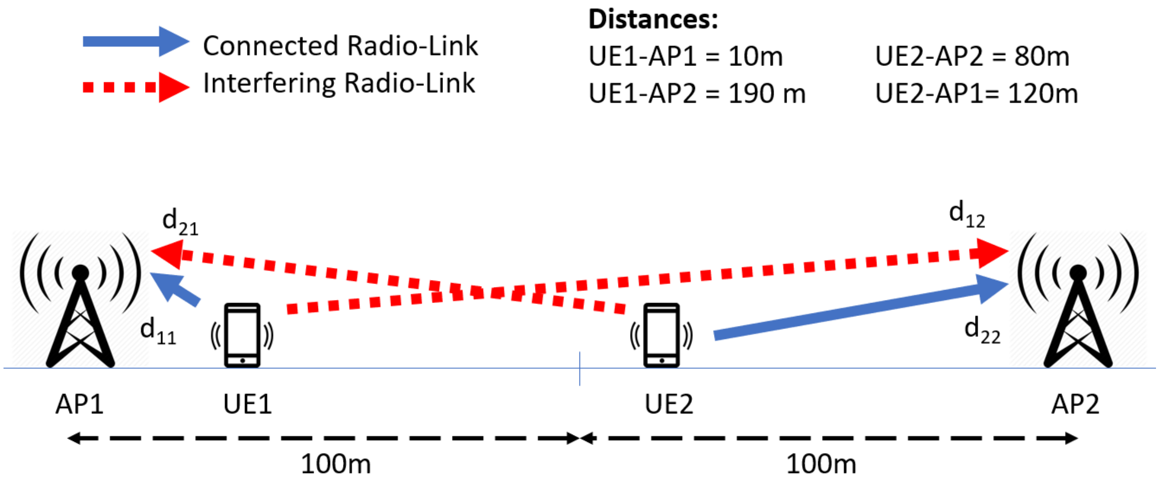

Figure 2: Illustration of the wireless system for Exercise 1

Interference (CCI in particular), in its turn, depends on other transmitters and is due to the inherent broadcast nature of the radio communications spectrum. An analogy might help at this point.

Suppose you and a friend are in an evening cocktail party. Suppose also that there many other people in this party, always in pairs, exactly as you and your friend. Every pair is seated in small tables close by one from another.

Initially, everybody is silent. The only sound anyone can hear is the “background noise” of people drinking and eating appetizers, chewng food, moving silverware, of waiters dispensing food and beverages and so on. The key point here is that even if everybody is silent, everyone can hear this background noise.That is also true in telecommunications: even if no signal is transmitted, noise is present at the receiver (i.e. your ears in the analogy).

Now suppose you start talking to your friend which is close by. You are the “transmitter”. Only you is talking in the whole party, and your friend hears with attention (your friend, or more precisely, your friend’s ears, is the “receiver”). In that situation, information (your speech) can be transmitted to your friend with excellent fidelity: that is, your friend has no problem in hearing and, most importantly, understanding you, since the only impairment is the background noise. You just have to adjust the proper “volume” of your voice so that your friend can understand you clearly. Adjusting your voice’s volume is analogous to setting the transmitter’s transmit power. In other words you just have to set the proper “signal-to-noise ratio” for your conversation to flow smoothly. Notice that in this case (in which only you in the whole party is talking) you don’t need to speak very loud since the only impairment against your friend´s understanding you is the “noise”.

But then, other people start talking in the party. Gradually more and more people talk, while you try to keep talking to your friend. As more people talk, it becomes more and more difficult for your friend to understand what you say. This is because there is more and more “interference” (i.e. the speech of other people). Notice how interference is different from noise: interference depends on other “transmissions” happening at the same time.

As your friend mimics to you that he can’t understand what you say anymore because of so much “interference” (i.e. many other people talking at the same time) you decide that you should speak louder (i.e. increase your “transmission power”). Momentarily this helps and your friend can better understand you. However, the person in the table just beside you will feel more interference from you as you talk louder. In response, this person also starts talking louder so her pair can better understand their conversation. This in turn entails that the person close to that person also starts talking louder, and so on. In a few seconds, everybody is talking louder and nobody can understand each other! That is, speaking louder (i.e. increasing transmit power) does not solve the “interference” problem, it might turn it even worse!

What has happened is that the “signal to interference ratio” has degraded to a point in which conversation is impossible. What could people in the party do instead of speaking louder to make every person to understand its respective pair ? Think a little bit.

Actually there could be several possibilities for improving the situation, but as the say goes “there is no free lunch”. In general there will be some cost or inconvenience to improve the situation.

For example, one solution is that every person talks for one minute while everybody else is silent. Then this person shuts up and the next person starts talking for another minute and so on. Since only one person is talking at the same time, you solve the “interference” problem. But what is the cost involved ? If there are only 2 pairs in the party, you will talk every other minute and this might be acceptable. But if there are 60 pairs in the party, you will have to wait 1 hour before given the opportunity to talk again! This clearly is not acceptable. This solution is an example of Time-Division Multiple-Access or TDMA.

What could be an analogy to the FDMA, i.e. creating separate orthogonal channels as explained before ? Well, this will sound a bit complicated but it would work. In FDMA people talk at the same time but somehow they only hear their intended transmitter (i.e. your own friend). One way to see this is to construct thick concrete walls around every table. With such thick walls separating every pair of people, everybody can talk freely without hearing each other. Background noise is also reduced. Notice however the enormous complexity involved in building walls around every table!

Even if you disregard the complexity involved in the above analogy of FDMA there are other inconveniences in “building walls”. Suppose that the party organizers plan to give a general speech to everybody in the saloon at some moment during the evening. They would step up over a stage in the middle of the saloon and give a welcoming and thanksgiving speech to everybody. Now, if you build walls, the organizers would have to go into every cubicle and repeat the speech to every pair of people. With FDMA you lose flexibility and the ability to transmit information to all party goers at the same time in a simple way (although this is possible by addressing each channel separately).

If you think a bit more you might suspect that there could be easier ways to overcome the cocktail party syndrome. Let us suppose the saloon in which the party is taking place is very large. You could then suggest to the party organizers to space out the tables as much as needed so that “interference”, although not eliminated, becomes weak and in practice does not disturb your conversation anymore. Every pair of talking people will be far apart from on one another. The drawback of this option is that you need more space and this is not always available.Question: how to interpret this solution in terms of our wireless system ?

A second easier solution is that you and your friend move closer as much as needed to improve the “signal to interference ratio”: as an extreme, think of talking very close to your friend´s ear. This will surely improve his/her understanding of your speech. This however might be quite inconvenient for a sustained conversation. Question: how to interpret this solution in terms of our wireless system ?

Could there be other more convenient and/or efficient solutions?

As imperfect as our analogy is, it helps you understand some of the fundamental impacts of noise and interference in wireless communications. Keep in mind that, according to our analogy, we want that as many people as possible talk as much as they want, whenever and wherever they want, with perfect understanding of the spoken messages and without disturbing each other. Can you translate this objective into the reality of an actual wireless communication system?

3.7 Measuring Performance: Link Capacity

So far we have presented one key performance indicator (KPI), i.e. the SINR (notice that the SNR and the SIR metrics are particular cases of the SINR). However, this KPI is not meaningful to the user, that is, it does not tell the user what is the capacity (or data rate) of the radio link as a function of its SINR. The mapping from SINR to capacity can be approached using a famous equation from information theory which allows for an approximation of the capacity of channels impaired by noise. This is the Shanon equation and for our purposes it can be written as:

$$C_{i,j} = B_c(1 + \gamma_{i,j}) \tag{7}$$

$C_{i,j}$ [bits per second] is the expected capacity of the radio link from transmitting UE j to receiver AP i using a channel of bandwidth B_c; $\gamma_{i,j}$ is the SINR as defined by eq. 6[1]. We will not delve into the theory behind this equation now. For the moment it suffices that you accept this equation as practical way of mapping the SINR into a meaningful KPI that is comprehensible by anyone who uses a smartphone.

With this we can start to make assertions about "quality of service" (QoS) for our system.QoS is a term that can mean many things. In general it is expressed by a key performance indicator that in turn must reflect some degree of satisfaction of the telecom service user. For instance, if the user uses the mobile broadband service, then the experienced data rate in a given time frame is a reasonable KPI. Higher data rates are correlated with a degree of user satisfaction for such service. Notice that there are other possible measures of QoS by KPIs such as network latency (or delay) and service interruption (i.e. connection stability), among others. The QoS of a given service might be expressed by several different KPIs, and in general different services might require a different set of KPIs to represent its QoS. But these are topics for future studies.

Exercise 2 *Sticking to the definition of Qos based on the data rate KPI, consider the following situation in which a person is deciding among two telecom operators that offer mobile broadband services. Telecom operator A establishes that its QoS target is to provide an* ***average*** *capacity of 100 Mbps for all of its clients. Telecom operator B aims at providing* ***at least*** *100 Mbps for 90% of their clients. Question: which operator would you like to subscribe to ? Another question: why is it difficult for a telecom operator of a cellular wireless systems to guarantee that their clients will always get good QoS ?*

3.8 UE-to-AP Association

A final piece of our system model needs to be put in place. Let us define in more details how is it that UEs get attached to specific APs. Have a new look at Fig.1. That figure is a bit idealistic since only one UE is attached per AP, that is, the number of UEs and APs is the same. In a more

[1] The Shanon formula is defined using the SNR. This is an approximation that takes into account the interference as well.

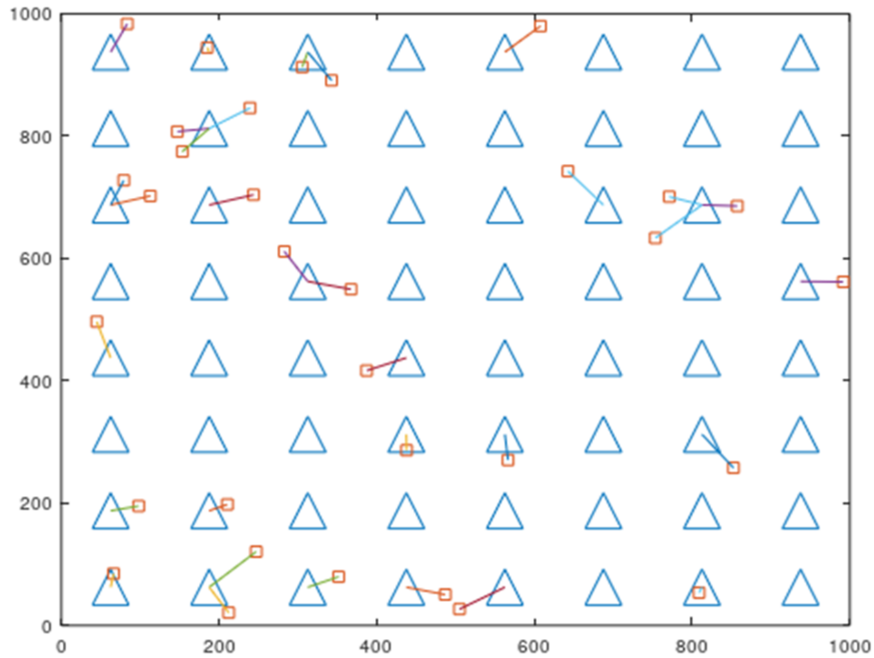

Figure 3: A general deployment scenario with 64 APs and 32 UEs (same conventions as in Fig. 1).

realistic situation there could be more or fewer UEs than APs. In this case one AP could have 2 or more UEs attached to it, while another one could be idle without any UE attached at all. Let us now have a look at Fig.3 with a more general deployment scenario.

Notice now that APs may have 1, 2 or even 3 connected UEs while others have none. You may then ask: how do we decide to which AP an UE must connect to ? It is a very important question and technically we call this decision the initial access procedure. It happens when you switch on you smartphone or when you “wake” it after a long time without using it. In that moment, your phone will search for a nearby AP and ask it for connection. For that to happen, each AP transmits continuously a special signal called “pilot” in a specific channel called control channel. Such control channel is only for control purposes, as its name implies, and no user data is carried over it. This special “pilot” signal is analogous to a strong light beam that a lighthouse emits from the shore to orient ships in the ocean.

Every UE is able to detect pilots coming from several APs in its neighborhood and measure the power of each received pilot signal. Then the UE chooses the AP with the strongest signal and asks it to connect. If everything is alright (e.g. if you have paid your phone bill) the AP will permit the UE to connect and start communications of useful data.

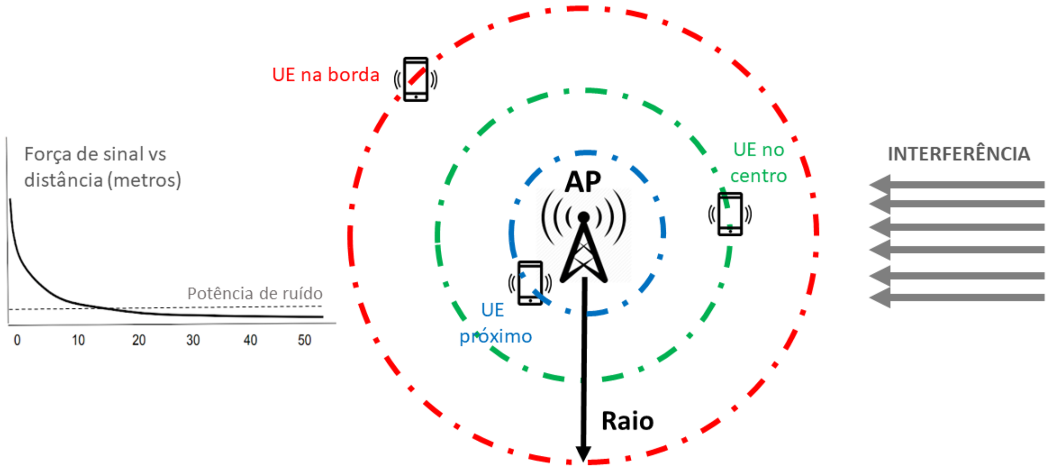

Figure 4: Key factors that define performance in a wireless system

3.9 Summary of relevant factors in modeling wireless systems

A summary of the relevant factors in modeling wireless communication systems is shown schematically in Fig.4. A single cell is shown and the respective coverage from the corresponding AP. The circles indicate relevant distances from the AP pointing to UEs close to the AP and those located in the cell center and close to the cell border. The indicated radius of the cell is just a hypothetical quantity (that can be estimated anyway based on the number of APs and the total coverage area). As you should have already perceived the AP-UE distance is crucial for performance: UEs closer to the AP provide stronger received signal and potentially better quality of service (QoS).

Besides UE-AP distance, the figure also shows other key quantities that define performance in the radio access network: the received power (falling as distance grows), the noise power (also caleed noise "floor") and the interference power. The figure suggests that interference comes from other cells, that is, it is external and not internal to the cell shown in the figure. Many models for wireless systems assume that there is no "intra-cell" interference, only "inter-cell" interference as suggested by the figure, meaning that any 2 UEs connected to the same cell will necessarily be allocated to different channels.

However, modern systems do not make this assumption an obligation. In fact UEs can share the same channel within one cell. This of course has the potential to greatly increase interference but there are other factors at play that may compensate for such increase. If by any means you are able to control the extra interference within the cell you achieve higher "spectral efficiency" since you have more UEs using the same amount of bandwidth. For the moment it suffices for you to understand that in our model we do not require that all interference is external. The pros and cons of this strategy will make itself clear as you progress in your studies.

4 Methodology: Monte Carlo Simulation

Given the system model of the previous section, the next obvious question is how to create a computer simulation of the entire system. With such simulator we would be equipped to address issues regarding the design and dimensioning of the radio access network, e.g. estimating QoS for a given population of clients in the coverage area. In order to help answer this question we turn to a famous computer simulation methodology known as Monte Carlo simulation.

Since you will start using computer simulations, before moving on you must get used to a computer programming language. I suggest that you learn the basics of Matlab (or its freeware version Octave). Matlab is both a high level language, a programming and a simulation environment. It has many built in functions that are helpful in engineering studies and makes quite simple to manipulate vectors and matrices, for example. It is also good for creating charts and graphics to show your results. There is plenty of online training about Matlab (e.g. in Youtube) and you can find documentation and usage examples in their official website: www.mathworks.com. If you are entirely new to computer programming, look for basic tutorials for begginners and move slowly towards more complex topics.

Other programming languages that you can use for developing our system simulations are Python and C++. These might require a bit more of programming skills but, on the other hand, usually run simulations much faster than Matlab. If you are into computer programming then you can alternatively consider using one of these two languages.

4.1 Area Estimation using the Monte Carlo method

As an example of the application of the Monte Carlo method we will employ it to numerically estimate the area of a circle of diameter $D = 1$[2]. For that, let us first define a square of side $L = 1$. Now let us randomly place N_p points inside this square. It is important that these points are uniformly distributed within the square. Fig.5 shows a plot of the square and a total of $N_p = 500$ randomly placed points within it (represented by crosses):

The Matlab code for generating Fig.5 is:

```
x=rand(1,500)
y=rand(1,500)
plot(x,y,"k+")
axis('square')
```

Now let us draw our circle of diameter $D = 1$ within the square as shown in Fig.6.

In order to estimate the area of the circle, using simple intuition and basic probability theory, one can simply count the number of points that falls within a distance $d \leqslant 0,5$ from the center of the circle and divide it by the total number of points within the square. This quotient must then multiply the area of the square (which in this case is 1 anyway). In other words, the proportion of points within the circle to the total number of points in the square is our area estimate.

[2]This example is adapted from: Tranter et. al; Principles of Communication System Simulation with Wireless Aplications, Chapter 9.

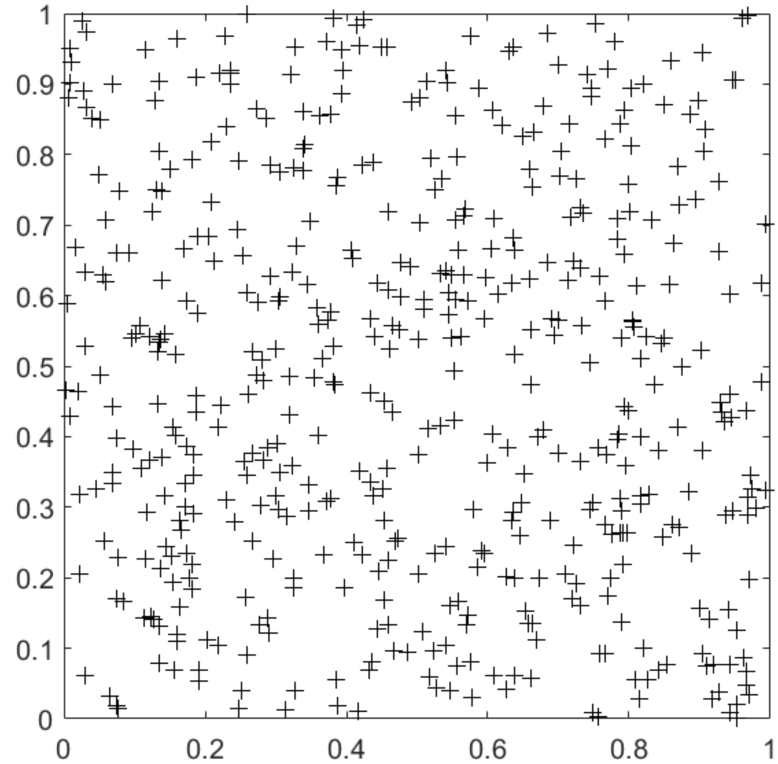

Figure 5: Randomly placed points within a square of side $L = 1$

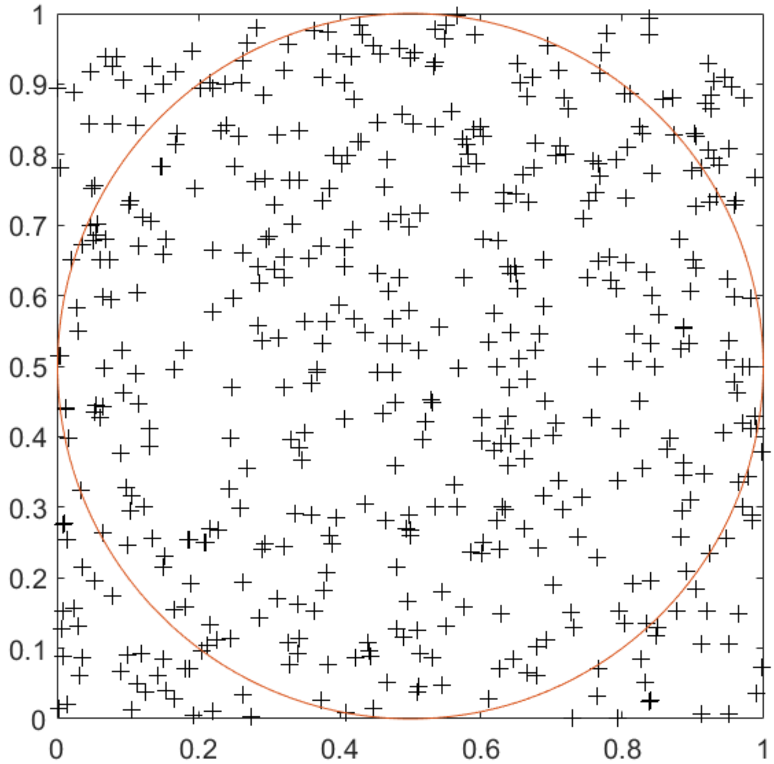

Figure 6: Circle of unit diameter inscribed on the square of side $L = 1$

In order to demonstrate the veracity of this statement let us first recall from simple geometry that the area of this circle is:

$$A = \pi R^2 = \pi(0,5)^2 = \pi/4 = 0,7854.$$

Now we will use Matlab to count the number of points falling within the circle (points whose coordinates place them at a distance $d \leqslant 0,5$ from the center of the circle).

A single Matlab run for counting the crosses within the circle in Fig.6 returned 382 points (if you run your own simulation, the numbers will be slightly different each time you run). This number gives an area estimate of $382/500 = 0,7640$. That is relatively close to the real number.

Now, we can improve the estimate by repeating this experiment many times and taking the average of all individual estimates. This way we average out the natural fluctuations that is inherent in one single snapshot as the one shown in Figs.5 and 6. For showing the effect of averaging the estimate from many repetitions of this experiment the plot in Fig.7 shows the evolution of the averaged estimate for increasing number of repetitions starting with one single experiment up to 1000 repetitions.

It is clear from this figure that the precision of the estimate improves as the the number of repetitions increase. We see the estimate converging to the true value $(0,7854)$ as shown by the horizontal line in the chart.

Exercise 3 *Repeat yourself the computer simulation of this section.*

What conclusions can we draw from this experiment ? First, that the Monte Carlo method is based on creating random experiments. Second, that by repeating many times a single experiment we can obtain an estimation of an unknown quantity or parameter.Third that the more repetitions you run, the better the precision of the estimate.

You may think at first that this example is not that much useful since you knew exactly what is the area of a circle of unit diameter anyway. What would then be the point for using such a complicated method to estimate a quantity you already knew ? The answer is that this method could be used to estimate the area of *any arbitrary 2 dimensional shape*, for example one that you could not calculate its area using known formulas. And you can trust the method since you validated it by applying it to a geometrical shape whose area you knew precisely in advance (in this case the area of a circle).Therefore you can be confident to use the same method to estimate the unknown area of an arbitrary geometrical shape following the same steps.

By the same token, we will apply this method to estimate the performance of wireless communication systems. Of course, the code for simulating a communication system is more complex than the one shown above for placing crosses within a square area to estimate the area of a circle. But the principles of the Monte Carlo method apply equally to more complex problems.

4.2 Statistics from Repeated Experiments

Let us continue analyzing the experiment of the previous section. Consider that we repeated 1000 times the circle area estimate experiment with $N_p = 500$ points in each repetition. One way to express the result of the repeated experiments is by calculating the average of the 1000 individual estimates. This is exactly what Fig.7 shows. We can see from this figure that the average estimate of 1000 repetitions is very close to the true value of the area.

Now, there are other possible ways to analyze these results. One such way that is quite useful in analyzing results from repeated experiments is the so called empirical cumulative distribution

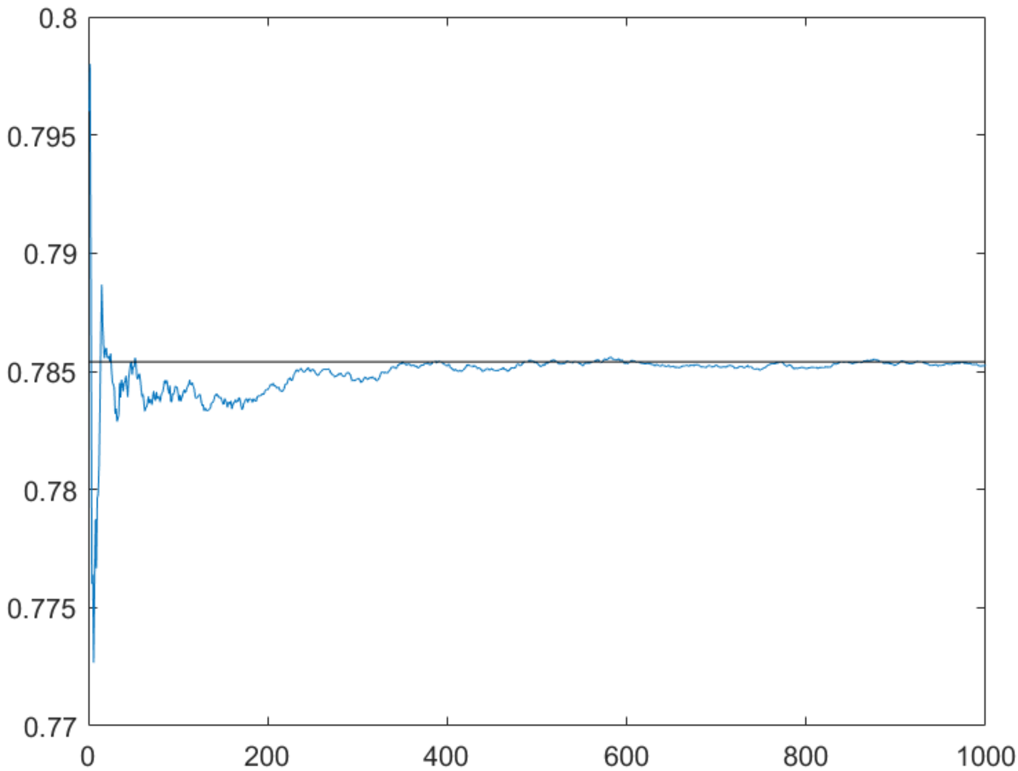

Figure 7: Evolution of the area estimate as the number of repetitions increases

function or CDF. The CDF is not a single number as the average, but a distribution (i.e. a curve) of the estimates observed along all repetitions.

For obtaining the CDF in this example you should first obtain the estimated area for the 1000 repetitions but, instead of computing the average of all estimates, you will create a vector cotaining all individual estimates from each run. The next setp is to reorder the elements in the vector from the lowest to the highest value. For example, if you run 1000 repetitions of the circle area estimate and plot the obtained individual values ordered from the lowest to the highest value, you will obtain a curve similar as the one shown in Fig.8. Notice that in your own experiments you will find slightly different curves each time you run, since we are dealing with a random model.

From inspecting the curve in Fig.8 we find from one round of 1000 repetitions that the minimum estimate (i.e. the first element in the vector of ordered estimates) is $0,722$ while the maximum one (i.e. the last element in such vector) is $0,83$. Interesting values to obtain from a CDF are the ones corresponding to the 10% lowest and the 10% highest values: in the case of a vector with 1000 individual estimates, these correspond to vector elements number 100 and number 900 respectively. These are also called the 10-th and 90-th percentiles respectively. By inspecting results in Fig.8 we find that the 10-th percentile is $0,76$ and the 90-th percentile is $0,808$. We can say of the 10-th percentile that 90% of the observations are above this value. As for the 90-th percentile we can say that 90% of the observations are below it. Also of interest is the 50-th percentile also known as the median: the value exactly in the middle of the observations vector, for which half of observations are below it and half are above it.

Bear in mind this analysis approach based on CDFs since they are useful for gaining insights from results obtained via Monte Carlo simulations. Besides the arithmetic average, the 10th, 50th and 90th percentiles allow for a better understanding about how the individual observations behave.

Exercise 4 *Continue the simulation from Exercise 2 and construct the CDF of your results. Find the 10-th, 50-th and 90-th percentile from your results.*

A simple analogy might help understand how it can be more important to understand the behavior of individual observations than the arithmetic average itself. For example, if you travel abroad to a city for the first time, you usually are interested in knowing how the weather looks like: you want to know if it is hot, cold or mild throughout a typical day during your stay, so that you dress appropriately. A friend of yours that lives in such a place might tell you that the *average* temperature throughout the day is 15 degrees Celsius. What your friend did not tell you is that the temperature varies a lot throughout the day: in early morning it is 5 degrees while early afternoon the temperature rises to 30 degrees and then drops again to 10 degrees in late evening. Although the average is 15 degrees, you probably will be better prepared to your trip if you understand how the temperature varies, from the minimum to the maximum value.

5 Analysis of Wireless Communication Systems via Monte Carlo Simulation

We are now in conditions to start the actual analysis of wireless cellular communication systems via computer simulations using the Monte Carlo method. For that you will build a system simulator using and connecting the various models presented in the previous sections. It is up to you to actually program the simulator and verify the correctness of the results before reaching any definitive conclusions from the results you observe.

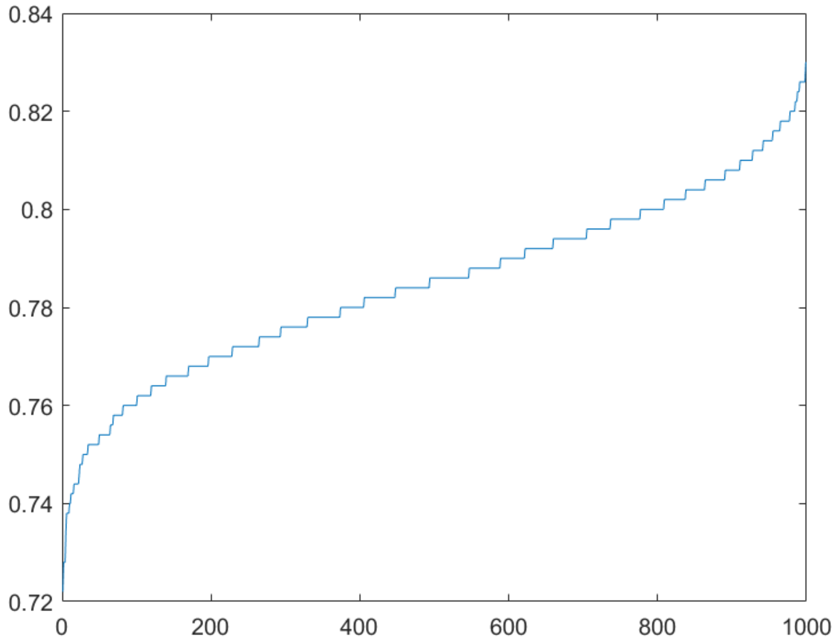

Figure 8: Individual estimates ordered from lowest to highest

Exercise 5 *Draw a block diagram of your simulator before starting to build it and discuss it with your advisor. Then write a step-by-step algorithm that represents your block diagram. You can also think about the general organization of the computer program for your simulation before actually starting to write it.*

5.1 Main simulation scenario

We start by defining a main scenario that represents a fixed set of parameters and configurations valid for all cases, as follows:

- Coverage area of 1000x1000m (1 km2).
- Total available bandwidth: $B_T = 100$ MHz.
- UE Transmission power (the same for all UEs): $p_t = 1$ W.
- Constants for the propagation model (eq. 1): $c = 10^{-4}$; $n = 4$; $d_0 = 1$ meter.
- Constant for the noise power (eq. 3): $K_0 = 10^{-17}$.
- QoS target: 100 Mbps per UE at cell border (10th percentile of the capacity CDF).

5.2 Design Parameters

For designing the radio access network (RAN) you have the following parameters to choose and vary:

- Number of orthogonal channels for multiple access in each cell: you can choose any integer value $N \geqq 1$. All channels are available in all cells, that is, the whole set of channels is repeated in every cell meaning that any channel is available for UEs to freely select in every cell. For $N > 1$ you must allocate each UE to a randomly chosen channel in a particular AP.
- Number of Access Points (regularly spaced as in figs.1 and 3): the radio access network can be configured with following number of APs: $M = 1$, 4, 9, 16, 25, 36, 49 or 64 APs. Do not use other values out of this set. Notice that the number of APs is the most important "cost" factor for the RAN. The more APs you install the costlier is the network.
- Number of UEs: you can select any integer number of UEs $K \geqq 1$, but pay attention to your QoS target.

5.3 Key performance indicators

Your simulator must measure the following two KPIs:

- SINR according to eq. 6(or SNR according to eq. 4 in case of K=1).
- Capacity according to eq. 7.

5.4 Running Simulations and Collecting Results

You define a scenario by choosing the triplet (M, K, N) that is the number of APs, of UEs and of channels. In order to evaluate the performance of a specific scenario you have to repeat many times a "snapshot" of the system (i.e. a static "photography") by randomly positioning the UEs in the coverage area and then calculating and storing the KPIs. We suggest you repeat each scenario for at least 10^4 times before collecting results for that scenario.

Results for a scenario are collected by extracting the 10-th and the 50-th percentiles of SINR and Capacity after the Monte Carlo repetitions. These would represent the typical system performance at cell border and at cell center. You will then compare the performance of different scenarios also having in mind if the QoS target has been achieved.Beware also as how the cell center and cell border performance are affected by the different choices of (M, K, N).

5.4.1 Experiment #1: coverage analysis

The first analysis you should do is a coverage analysis. It is the most basic design objective of any wireless system, that is, to provide adequate signal coverage. For this analysis you will consider only one user ($K = 1$) randomly placed in the coverage area. Now you can gradually increase the number of APs and analyze the impact on SNR and Capacity. Is it possible to achieve the QoS target ? What is the minimum number of APs to achieve this target ? Notice that for this case you must use $N = 1$ (actually it does not make much sense to use $N > 1$; why?).

5.4.2 Experiment #2: capacity Analysis

Now you should gradually increase the number of UEs (K). Observe and interpret the impact of increased demand for capacity at the edge and center of the cell. For a given number of UEs, the 2 degrees of freedom that the RAN designer has to achieve the QoS target are: the AP density (M) and the number of orthogonal channels (N). The student is free to experiment with combinations of these factors and observe how the capacity and QoS behave for different combinations of M and N as K increases. Remember that when 2 or more UEs happen to be transmitting using different channels they do not interfere.

Now the most important practical question that you should answer is: what is the maximum number of UEs supported by the system while respecting the minimum QoS target ?

5.5 Present your final report

At this point you should have evaluated several different scenarios. You have lots of data to analyze. It is now the moment to organize your results and create beautiful and meaningful graphics to summarize your main findings. Prepare a slide presentation including charts showing the numerical results you obtained for several scenarios and KPIs. Look for trends and variations that illustrate the fundamental trade offs inherent to wireless systems. Explain your methodology and obstacles during your studies. State your final conclusions based on the results you obtained.

Exercise 6 *Prepare a 30-minute slide presentation divided into the following parts: 1-Introduction and Objectives; 2-System Model; 3-Characteristics of the System Simulator; 4-Simulation Results; 5-Discussion and Conclusions. Present it to your advisor and be prepared to answer questions.*

6 Conclusion and Next Steps

At this point you should have grasped a rough understanding about how a wireless communication system works and what are the design objectives of the telecom engineer when dimensioning the radio access network. You should also be attentive to the costs involved with the different solutions. For example, the more APs the higher the overall cost of the network.

You can now ask your advisor: what if I want to have even more UEs in the system ? That is, more UEs than the number you found in Experiment #2 above while keeping the same maximum number of APs and same QoS target as before. What are your options to increase capacity beyond that point ? Do you have any ideas about how to increase the system capacity ? Maybe there is one or two obvious ways but don´t be shy: think of other clever ways you can improve the "spectral efficiency" of the system. Discuss your ideas with your advisor.

References

[1] F. R. P. Cavalcanti, T. F. Maciel, W. C. Freitas Jr., Y. C. B. Silva; Comunicação Móvel Celular; Elsevier 2018.