Section 5.3: Example 3

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Rosen states exactly how the functions are recursively defined.
1. Basis step specify the value of the function at zero.
2. Recursive step give a rule for finding its value at an integer from its values at smaller integers (Rosen).
"Recursively defined functions are well defined meaning that for every positive integer, the function at this integer is determined in an unambiguous way"(Rosen).
I thought that this statement is more clearly written and understandable when he explains the above statement further.
"This means that given any positive integer, we can use the two parts of the definition to find the value of the function at that integer, and that we obtain the same value no matter how we apply the two parts of the definition" (Rosen). This paragraph is in page 346 of the textbook after example 1.
Example 3 definitely defines the function in a clear way for every positive integer.
Here is the explanation of how the example makes sense along with explanation of how to read the summation notation.

$$\sum_{k=0}^{n} a_k$$

-basis step-

$$\sum_{k=0}^{0} a_k = a_0$$

plug in 0 for n to find the value of the function at 0.

Here, recall how to read sigma notation:
1. Top: This is where 0 is in this case, it means to stop at number 0.
2. Bottom: k=0 This is the starting point of the sequence
3. The actual function on the side: a_k tells us what to add for the sequence. We plug in the number in k.
For the basis step, the starting point and the ending point of the sequence is the same 0, so we stop after we plug in 0 to a_k in this case.
The basis step is "well defined" because the solution to this sequence is clearly a_0, nothing else.

$$\sum_{k=0}^{n+1} a_k = (\sum_{k=0}^{n} a_k) + a_{n+1}$$

In the recursive step, we need to find the rule or a function that will always be true no matter what positive integers we plug in the function. In this case, we need to find the rule for finding $\sum_{k=0}^{n+1} a_k$ from $\sum_{k=0}^{n} a_k = a_0$. (a_{n+1} from a_n).
To understand the above equation better, we can rewrite the original function as:

$$\sum_{k=0}^{n} a_k = a_0 + a_1 + a_2 + ...a_n \text{ (whatever n value would be at)}$$

And to represent $\sum_{k=0}^{n+1} a_k$, we just need one more element which is the value one more than a_n. That would be a_{n+1}.
So the rule we are looking for will be:

$$a_0 + a_1 + a_2 + ...a_n + a_{n+1}$$

Which we can rewrite this in a clearer way as:

$$(\sum_{k=0}^{n} a_k) + a_{n+1}$$

Now that we have found the recursive definition, we can use these basis and recursive step to find the value of the function at a specific integer, given any positive integer. This definition should always give us the same answer no matter what.