This print-out should have 16 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Suppose that you want to use induction to prove a statement $P(n)$ for all natural numbers n. What is the inductive hypothesis?

1. $P(n)$ implies $P(n+1)$ for all n

2. $P(n)$ holds for some n **correct**

3. $P(n)$ holds for all n

4. $P(n)$ implies $P(n+1)$ for some n

Explanation:

002 10.0 points

Suppose that you want to prove a statement $P(x)$ for all $x \in X$. Induction is most likely to be useful if X is which of the following sets?

1. rational numbers

2. even integers

3. positive integer powers of 3 **correct**

4. positive real numbers

Explanation:

003 10.0 points

Consider the following claim:

There is some positive integer k such that $n^2 < 2^n$ for all $n \geq k$.

If you want to prove this statement by induction, how many of the elements of the set $\{3, 4, 5, 6\}$ could be base cases?

1. 4

2. 0

3. 2 **correct**

4. 3

5. 1

Explanation:

004 (part 1 of 5) 10.0 points

Let A and B be sets. Which of the following is equivalent to $A \cap (A \cup B)$?

1. $\emptyset$

2. $A \cap B$

3. $A \setminus B$

4. B

5. $A \setminus B$

6. $A \cup B$

7. A **correct**

Explanation:

005 (part 2 of 5) 10.0 points

Which is equivalent to $A \cup (A \cap B)$?

1. B

2. $\emptyset$

3. $B \setminus A$

4. $A \cup B$

5. A **correct**

6. $A \cap B$

7. $A \setminus B$

Explanation:

006 (part 3 of 5) 10.0 points

Which is equivalent to $A \setminus (B \setminus A)$?

1. $A \cup B$

2. A

3. $\emptyset$

4. $A \cap B$

5. B

6. $A \setminus B$ **correct**

7. $B \setminus A$

Explanation:

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007 (part 4 of 5) 10.0 points

Which is equivalent to $A \setminus (A \setminus B)$?

1. $\emptyset$

2. B

3. $A \setminus B$

4. $A \cup B$

5. $B \setminus A$

6. A

7. $A \cap B$ **correct**

Explanation:

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008 (part 5 of 5) 10.0 points

Which is equivalent to $B \setminus (A \cap B)$?

1. A

2. $\emptyset$

3. $B \setminus A$ **correct**

4. $A \cup B$

5. B

6. $A \cap B$

7. $A \setminus B$

Explanation:

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009 10.0 points

Consider a function $f : A \to B$, and let $C \subseteq B$. Which of the following conditions is necessary and sufficient to guarantee that $f[f^{-1}[C]] = C$?

1. f is surjective.

2. C is in the range of f. **correct**

3. f is injective on C.

4. f is injective.

Explanation:

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010 10.0 points

Suppose that $g \circ f$ is surjective. Then g ...

1. must be surjective.

2. cannot be surjective.

3. might be surjective. **correct**

Explanation:

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011 (part 1 of 3) 10.0 points

For this problem, consider all functions defined on their natural (that is, largest possible) domains and let their co-domains be all real numbers.

The set of all rational functions contains ...

1. neither surjections nor injections.

2. injections but not surjections.

3. surjections and injections. **correct**

4. surjections but not injections.

Explanation:

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012 (part 2 of 3) 10.0 points

The set of all polynomials contains ...

1. neither of the other two types of functions.

2. surjections that are not injections. **correct**

3. injections that are not surjections.

Explanation:

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013 (part 3 of 3) 10.0 points

Which of the following classes of functions contains bijections?

1. logarithmic functions **correct**

2. trigonometric functions

3. inverse trigonometric functions

4. exponential functions

Explanation:

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014 10.0 points

The function $f : N \to Q$ defined by $f(x) = x$ verifies that ...

1. $|N| < |Q|$

2. $|N| = |Q|$

3. $|Q| < |N|$

4. $|N| \leq |Q|$ **correct**

Explanation:

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015 (part 1 of 2) 10.0 points

Let $A = \{1/n \mid n \in N\}$. Let $f : Q \to A$ be defined by

$$f(p/q) = 1/q,$$

where p/q is expressed in lowest terms. The function f is ...

1. injective but not surjective.

2. injective and surjective.

3. neither injective nor surjective.

4. surjective but not injective. **correct**

Explanation:

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016 (part 2 of 2) 10.0 points

Which of the following is true?

1. $|A| > |Q|$

2. $|A| = |Q|$ **correct**

3. $|A| < |Q|$

Explanation: