1. Suppose that a card is chosen from a standard 52-card deck of playing cards. Count the number of ways in which each of the following results could happen.

a) a black, non-face card is drawn

b) an even card or a red card is drawn

c) a heart or a face card is drawn

2. A three-reel slot machine is a machine consisting of three thin cylinders that rotate about their centers on a common axis. The cylinders have various symbols along their edges, each in a position called a *stop*, and a window in the machine makes part of the edges visible to the player. The player puts a coin into a slot and then pulls an arm to activate the machine. The cylinders rotate until they come to rest, and if the front row, called the *payline*, shows one of several designated combinations of symbols, the player wins an amount of money dependent on what combination shows. (Modern machines are usually electronic representations of a mechanical device, and often take money in forms other than coins. They are also often button-activated rather than having an arm to pull.)

The Fey "Liberty Bell" slot machine from 1899 had the following symbols on its reels.

stop #	reel 1	reel 2	reel 3
1	bell	bell	bell
2	Ω	Ω	♢
3	♠	♠	★
4	Ω	Ω	♠
5	♢	♢	bell
6	Ω	Ω	♢
7	♠	♠	♡
8	Ω	Ω	★
9	♡	♡	♠
10	Ω	Ω	♢

a) In how many ways can the reels come to rest? (Assume that each reel always comes to rest with a single stop facing front.)

b) How many three-symbol sequences are possible?

The payouts are as follows. They are payouts in the number of coins for a one-coin bet.

reel 1	reel 2	reel 3	payout
bell	bell	bell	20
♡	♡	♡	16
♢	♢	♢	12
♠	♠	♠	8
Ω	Ω	★	4
Ω	Ω	not ★	2

c) What is the probability that a player will win money by using this slot machine once?

d) Does this payout table make sense? Why or why not?

3. In blackjack, the players and the dealer are dealt two cards each. The suits of the cards have no role in blackjack. Each face card is given the value of 10, aces have the value of 1 or 11, and all other cards have the value indicated by the cards themselves. Each player has the option of *hitting* (taking more cards) or *standing* (not taking any more cards). Once each player has had a turn, the dealer hits until reaching a total of 17 or higher. The players' goal is to beat the dealer. This occurs when the player has a total higher than the dealer's without *busting* (having a total of over 21), or when the dealer busts and the player does not. Thus, if the dealer busts, then all players who have not busted win. A *natural*, or *blackjack*, is an ace-10 pair dealt before anyone hits. (Remember that all face cards have the value 10, so an ace-queen is a natural, as is an ace-10 where the 10 is actually a card numbered 10.) Naturals always win except against a dealer's natural, in which case it is a tie.

a) What is the probability of being dealt a natural?

b) What is the probability of winning by getting a natural?

c) What is the probability of the dealer winning by getting a natural?