1. (2pt) Approximate the integral $\int_1^{1.5} x^2 \ln x dx$ using Gaussian quadrature. Compute the error for this approximation and compare it with the error of the Trapezoidal rule from Homework 6.

2. (2pt) The first column $\{R_{k,1}\}_{k\geq 1}$ of Romberg's integral scheme is given by the Composite Trapezoidal rule. Show that the second column $\{R_{k,2}\}_{k\geq 2}$ is the same as the Composite Simpson's rule.

3. (3pt) Adapt your code of Problem 4 from Homework 6 to implement Romberg's integration scheme. Print the first few columns in Romberg's scheme for the case $[-3\sigma, 3\sigma]$. Take a large enough n and use the computed value $R_{n,1}$ as a good enough approximation to the exact solution, and then compute the errors of each $R_{k,j}$. What are your observations of convergence orders?

4.(3pt) Implement the forward Euler's method to compute the initial value problem:

$$y'(t) = \frac{2}{t}y + t^2 e^t, \ 1 \leq t \leq 2, \ y(1) = 0,$$

with exact solution $y(t) = t^2(e^t - e)$.

a. Use $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

b. Use the answers generated in part (a) and the linear interpolation code from Homework 4 to approximate the following values of y, and compare them to the actual values.
(i) $y = 1.04$ (ii) $y = 1.55$ (iii) $y = 1.97$