1. (4pt) Let the matrix A be

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

a. Factor the matrix A into $A = LU$ where L is a lower triangular matrix and U is an upper triangular matrix.

b. Solve the linear system $LU\boldsymbol{x} = \boldsymbol{b}$ for $\boldsymbol{x}$, with $\boldsymbol{b} = [2, -1, 1]^T$.

2. (Note: This problem needs you to implement the Gaussian Elimination algorithm for part b) A Fredholm integral equation of the second kind is an equation of the form

$$u(x) = f(x) + \int_a^b K(x,t)u(t)dt,$$

where a and b and the functions f and K are given. To approximate the function u on the interval $[a, b]$, a partition $x_0 = a < x_1 < \cdots < x_{N-1} < x_N = b$ is selected and the equations

$$u(x_i) = f(x_i) + \int_a^b K(x_i,t)u(t)dt, \quad \text{for each } i = 0, \cdots, N\,.$$

are solved for $u(x_0), u(x_1), \cdots, u(x_N)$. The integrals are approximated using quadrature formulas based on the nodes $x_0, \cdots, x_N$. In our problem, $a = 0, b = 1, h = (b-a)/N, f(x) = x^2$, and $K(x,t) = e^{|x-t|}$.

a. (2pt) Show that the linear system

$$\begin{aligned} u(0) &= f(0) + \frac{1}{2}[K(0,0)u(0) + K(0,1)u(1)] \\ u(1) &= f(1) + \frac{1}{2}[K(1,0)u(0) + K(1,1)u(1)] \end{aligned}$$

must be solved when the Trapezoidal rule ($h = 1$) is used.

b. (4pt) Set up and solve the linear system that results when the Composite Trapezoidal rule is used with $h = 0.1$.