1. Find explicit formulas for the sequences given below.

a) $1/5, -1/10, 1/15, -1/20, 1/25, \ldots$

b) $1/6, 4/12, 9/18, 16/24, 25/30, \ldots$

c) $2, 7/6, 1, 13/14, 8/9, \ldots$

2. Combine the following expressions into single summations.

a) $3(n+1)^2 - 2(n+1) + 4 + \sum_{i=1}^{n} \left(3i^2 - 2i + 4\right)$

b) $1 + \sum_{i=1}^{n} 2^i$

c) $4\sum_{i=1}^{n} \left(i^3 - 8\right) + 3\sum_{i=1}^{n} \left(2i^3 + i^2 + 7\right)$

3. Simplify the following as much as possible. Many can (and should) be expressed well without the benefit of factorials.

a) $\dfrac{n!}{(n-1)!}$

b) $\dfrac{n!}{n}$

c) $\dfrac{n!}{(n-2)!}$

d) $\dfrac{(n+1)!}{(n-1)!}$

e) $\dfrac{\big(2(n-1)\big)!}{(2n)!}$

f) $\dfrac{(2n)!}{n!}$

4. Consider the sequence defined by

$$a_n := \frac{2n + (-1)^n - 1}{4}$$

for $n \in \mathbb{N}$. Find an alternative expression for a_n using the floor function defined by

$$\lfloor x \rfloor := \max\{n \in \mathbb{Z} \mid n \le x\}.$$

5. Prove that

$$\binom{n}{r+1} = \frac{n-r}{r+1}\binom{n}{r}$$

for all $n, r \in \mathbb{N}$ where $n \geq r + 1$.

6. Are there arbitrarily long sequences of consecutive composite numbers?