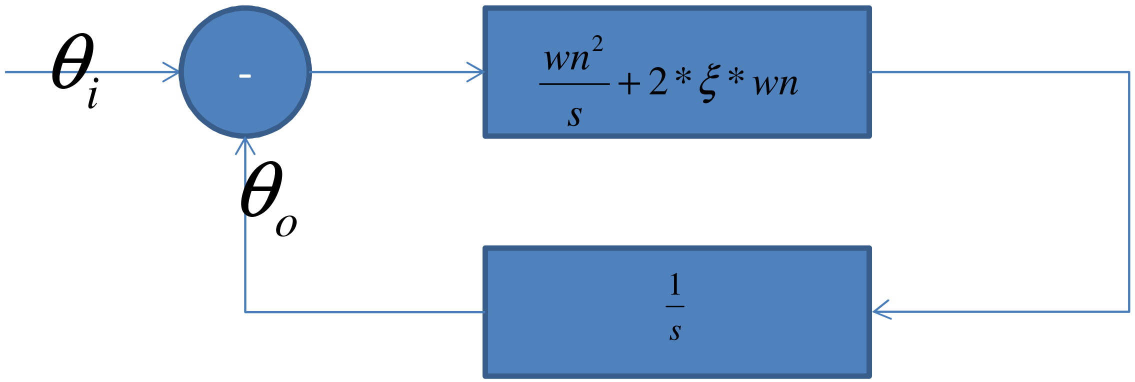
Low pass filtering (LPF) with PI control

Jan 2013

Basics 1

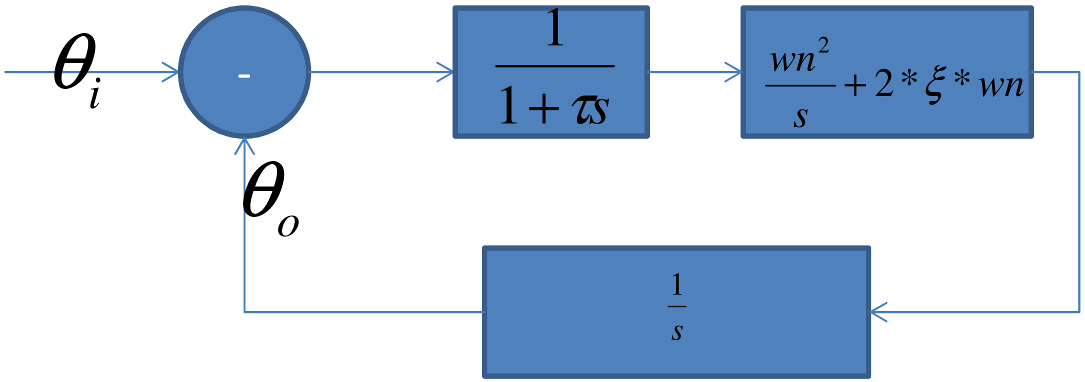
- classical PI control



$$H_1(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{2 * \xi * wn * s + wn^2}{s^2 + 2 * \xi * wn * s + wn^2}$$

Basics 2

- PI control with a LPF



$$H_2(s)=\frac{\theta_o(s)}{\theta_i(s)}=\frac{2*\xi*wn*s+wn^2}{\tau s^3+s^2+2*\xi*wn*s+wn^2}$$

Simple observations

- Our carrier control loop used in the Buffacc.cpp is not really a classical PI control due to the 1ms integration which acts as an low pass filter with $\tau = 1ms$
- For small τ, $H_2(s) \approx H_1(s)$ because $\tau s^3 \approx 0$ for frequency of interest.

Impact of low pass time constant τ

- To keep it within context of Buffacc, the following control parameters are used and kept constant.

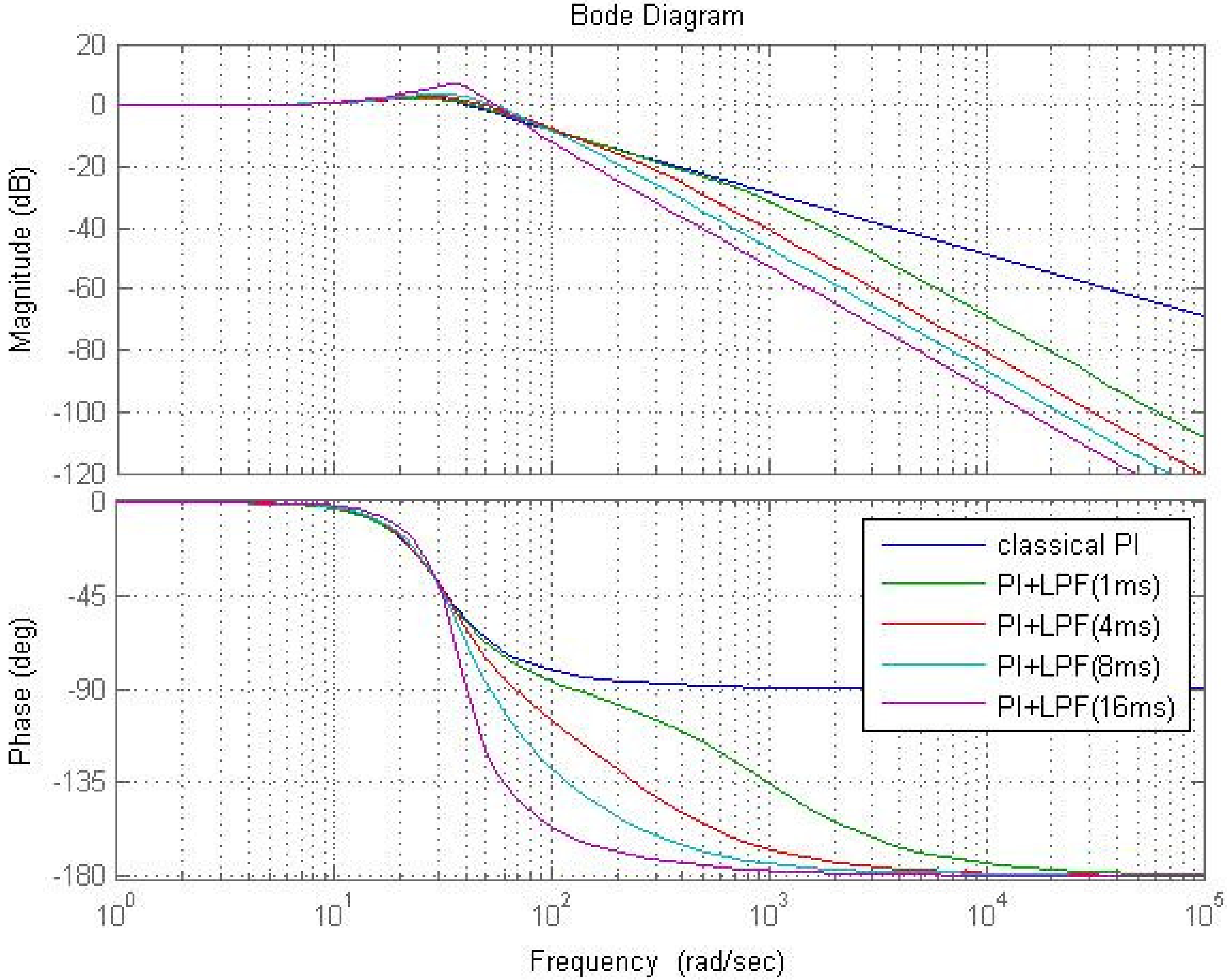
$$wn = 2 * \pi * 18.88Hz$$

$$\xi = 0.638$$

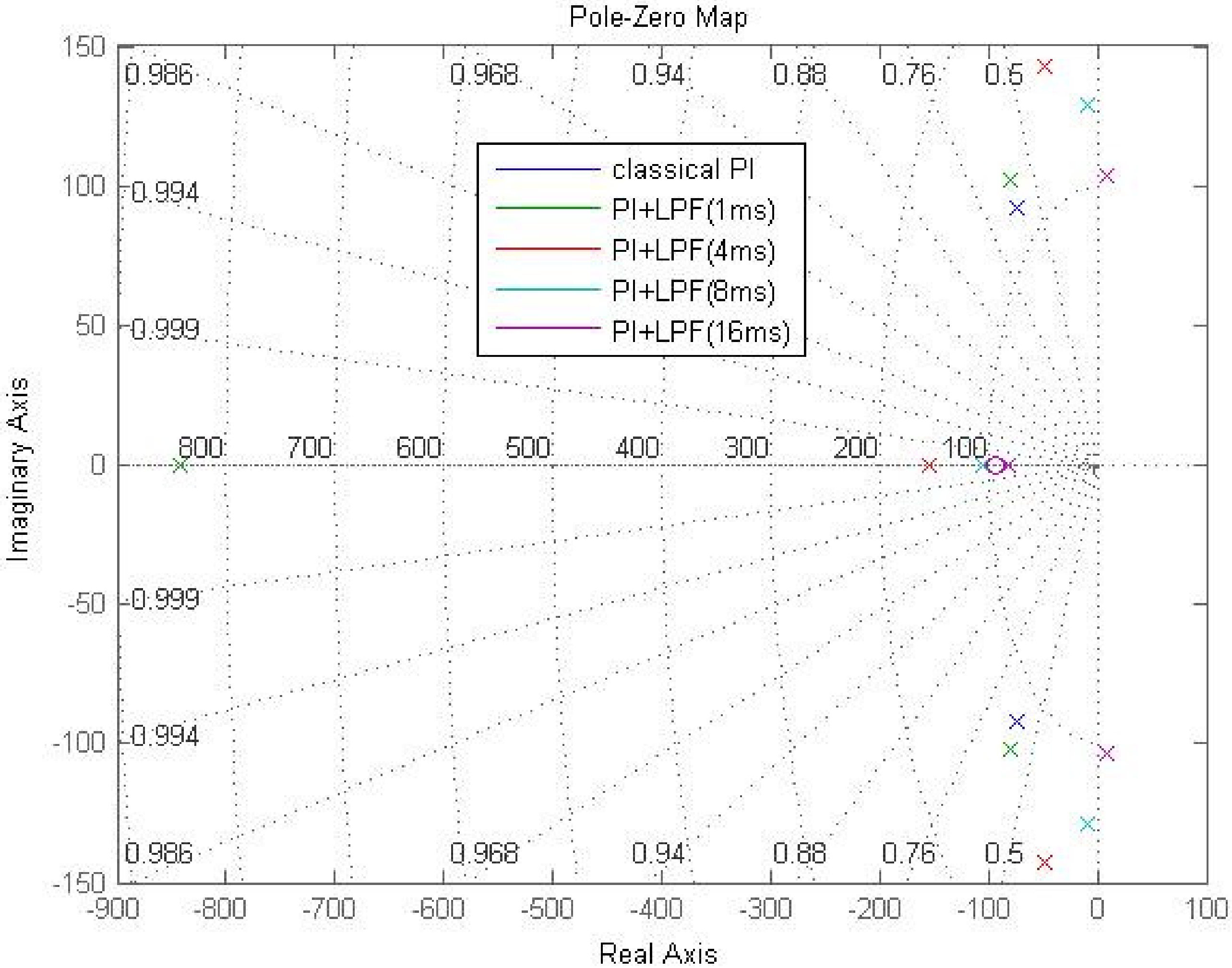
- The value of low pass time constant is varied with 1,4,8,16 ms to analyze the frequency response, pole zeros, and step response of the following transfer function.

$$H_2(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{2 * \xi * wn * s + wn^2}{\tau s^3 + s^2 + 2 * \xi * wn * s + wn^2}$$

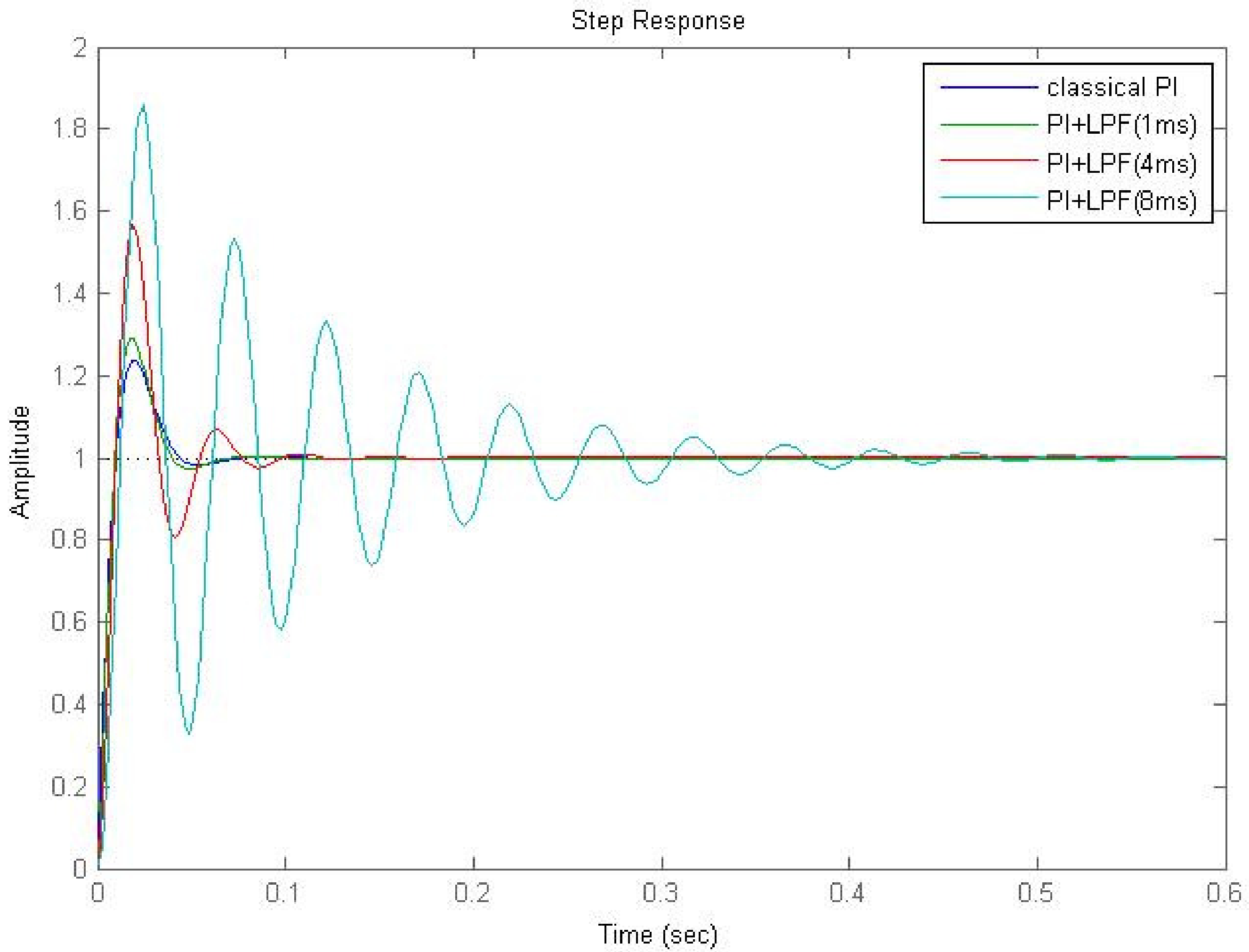
Bode diagram



Pole and zeros



Step Response



Simulation Summary 1

- With 16ms low pass filtering, the transfer function has positive poles which means the system is unstable, so no longer than 16 ms low pass filtering should ever be employed.
- With 8 ms low pass filtering, the transfer function has negative poles which are very close to the imaginary axis, indicating possible oscillations.

Simulation Summary 2

- With 1ms low pass filtering, the step response is very similar to the classical PI control with no low pass filtering, around 4% higher peak transient error.
- With 4 ms low pass filtering, the transient error starts to pick up, around 25% higher than the classical PI. The time to settle (around 85ms) is also increased by almost 100% compared to the classical PI control (40 ms).

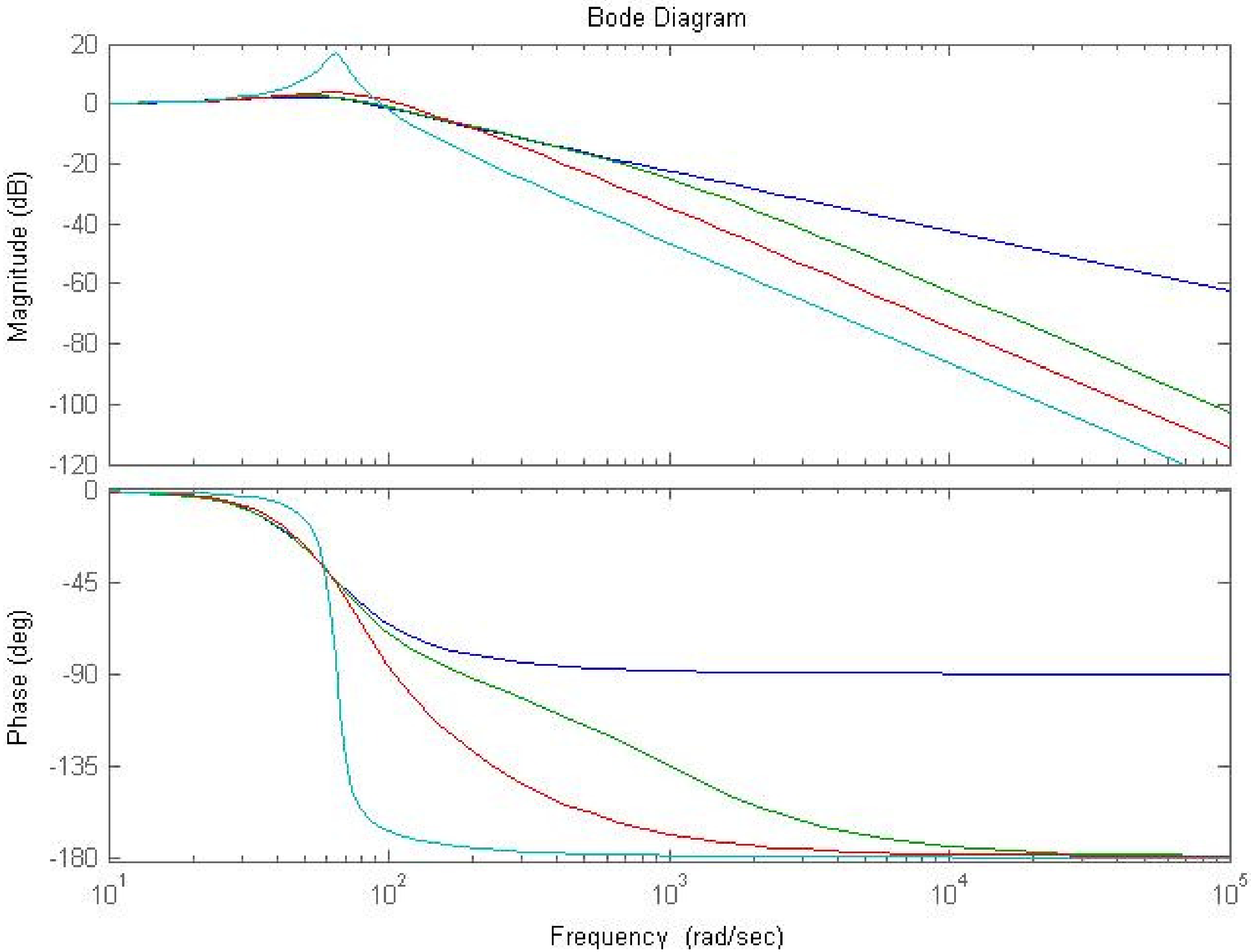
Reduction of the peak transient error

- Reduce the bandwidth
- Increase the damping
- Simulation
  - $wn = 2 * \pi * \frac{18.88}{2} Hz$

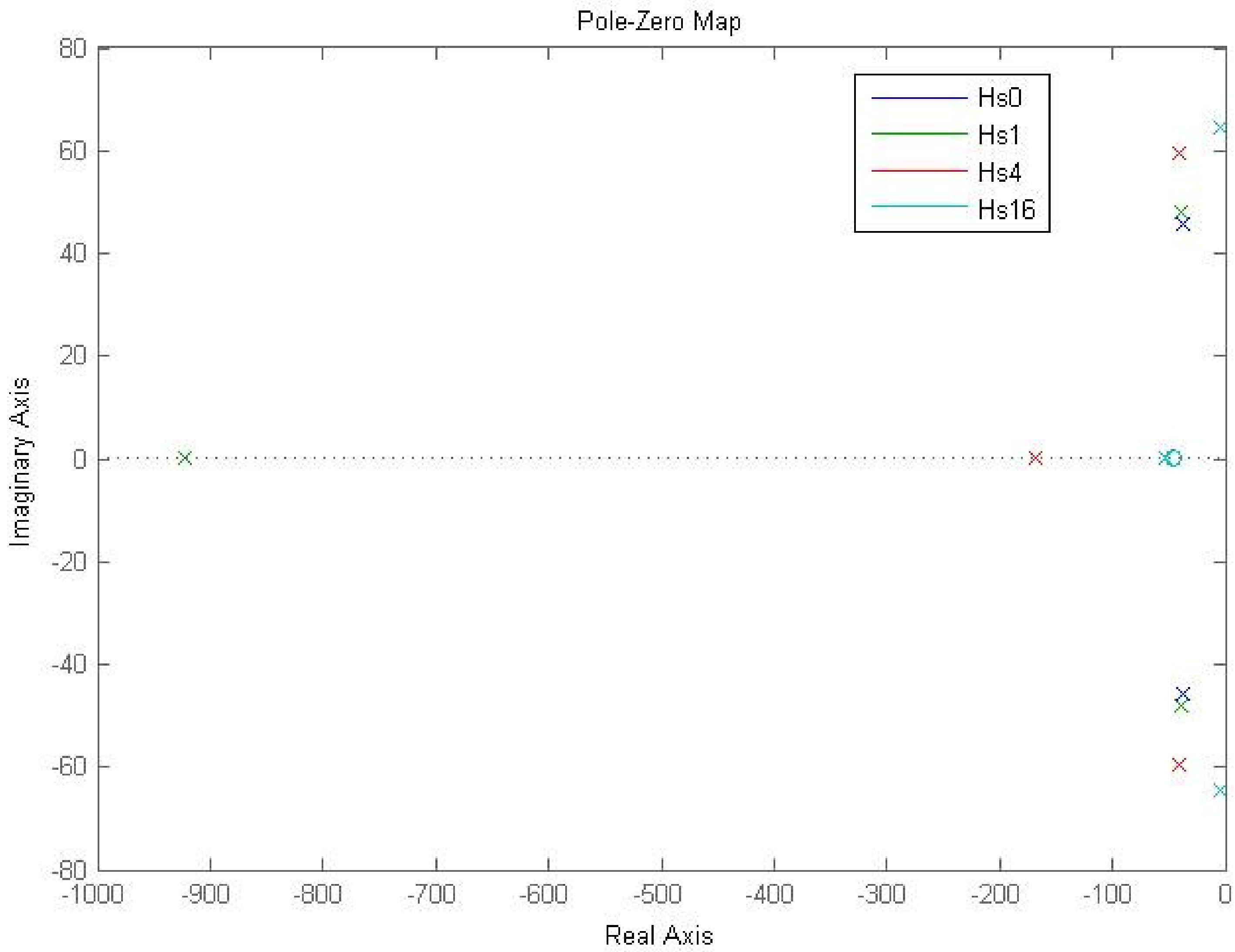
    $\xi = 0.638$

    $\tau = 0,1,4,16$

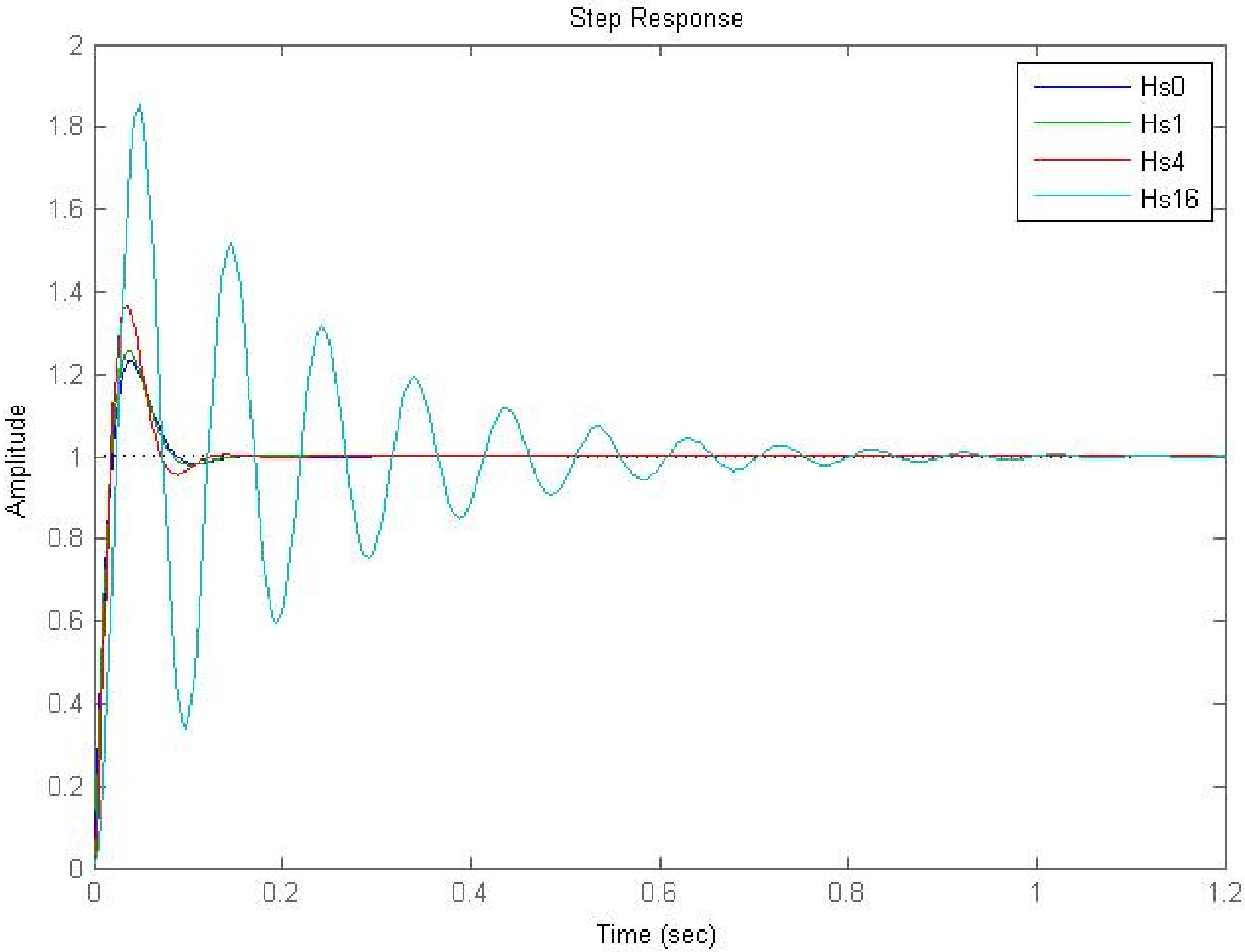
bode


Bode Diagram
Magnitude (dB)
20
0
-20
-40
-60
-80
-100
-120
Phase (deg)
0
-45
-90
-135
-180
10^1
10^2
10^3
10^4
10^5
Frequency (rad/sec)

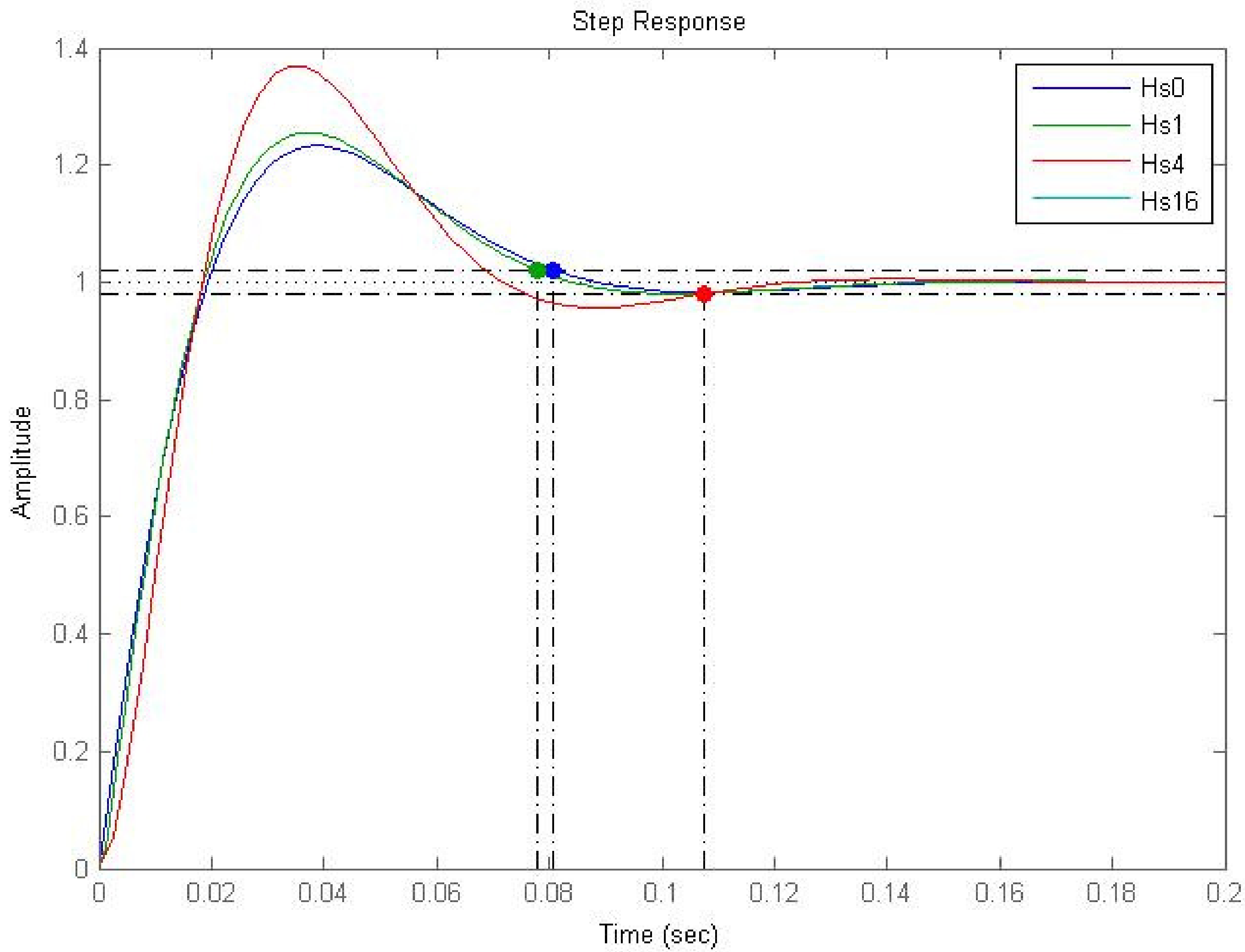
Poles and zeros



step


Step Response
Amplitude
Time (sec)
Hs0
Hs1
Hs4
Hs16
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Settling time in step response



Simulation Summary with reduced bandwidth

- The positive poles with 16ms low pass time disappear when the bandwidth is reduced half
  - With longer low pass filtering, the bandwidth must be reduced to maintain stability.
- The peak transient error has dropped. With 4 ms low pass filtering, the transient error dropped from 1.56 to 1.37, around 10% higher than the classical PI. The time to settle (around 108ms) is longer than the 85 ms in the original bandwidth, however with the halved bandwidth, the settling time with classic PI control also increased to 80 ms, a lesser difference.

Reduction of the peak transient error

- Now increase the damping from 0.638 to 1
- Simulation
  –

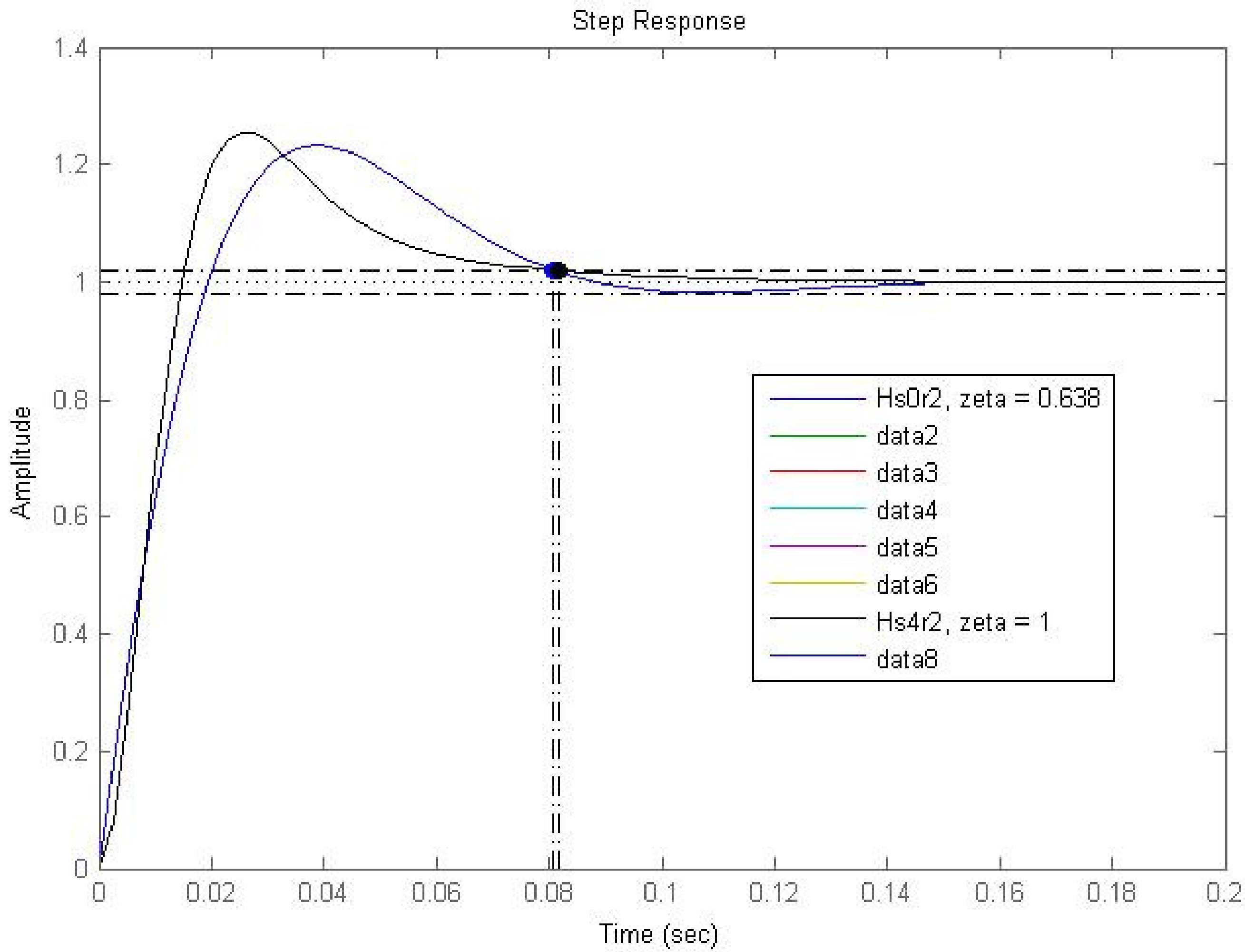
$$wn = 2 * \pi * \frac{18.88}{2} Hz$$

$$\xi = 1$$

$$\tau = 4$$

- The next slide shows settling time and peak transient error of a step response with the above parameters compared to a classic PI control with zeta=0.638 and the same bandwidth as wn (18.88/2 Hz), it can be seen that now the settling time of the PI with a low pass filtering time of 4ms is almost the same as the a classic PI with zeta=0.638, the peak transient error is almost the same, a good choice.

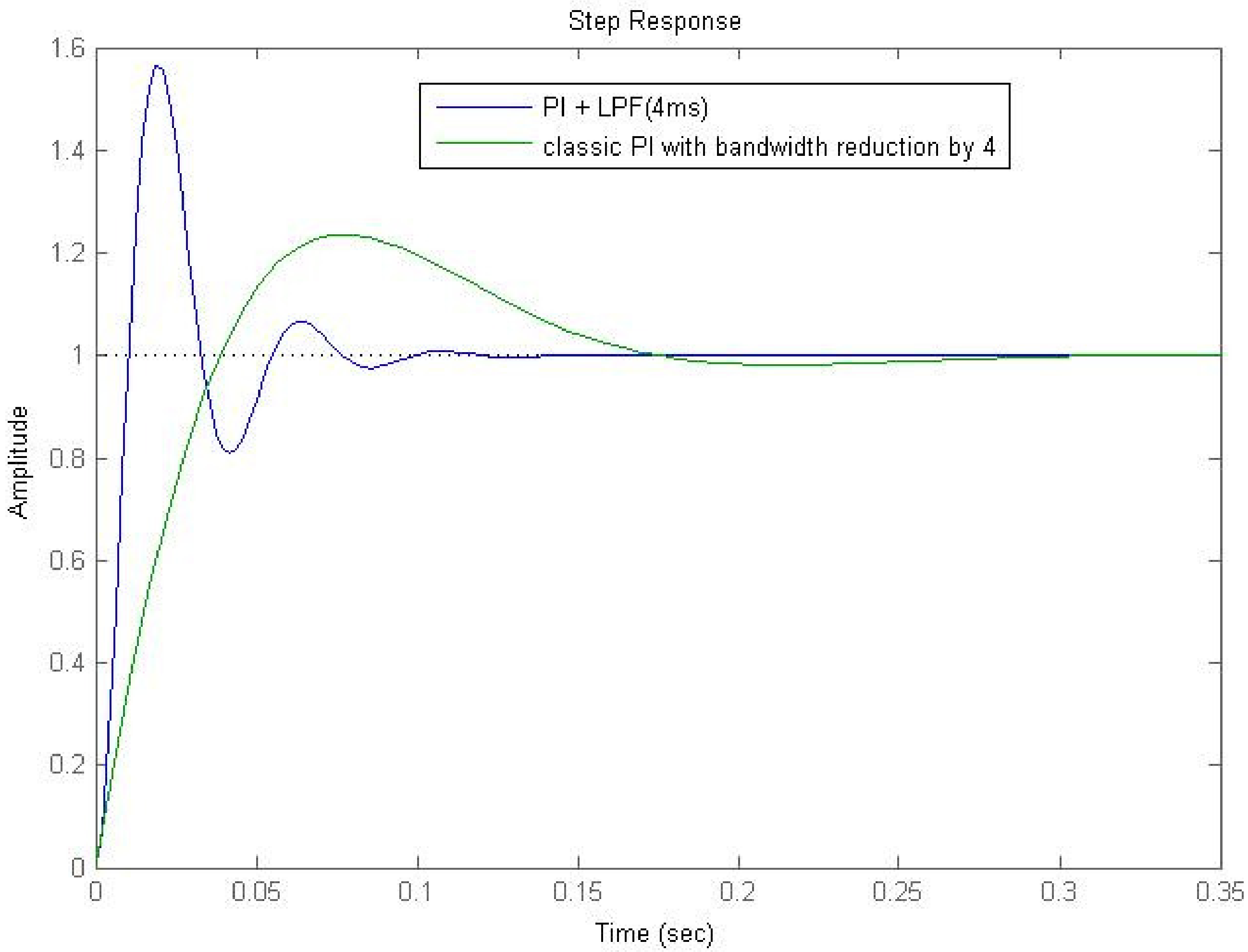
Reduction of the peak transient error 2



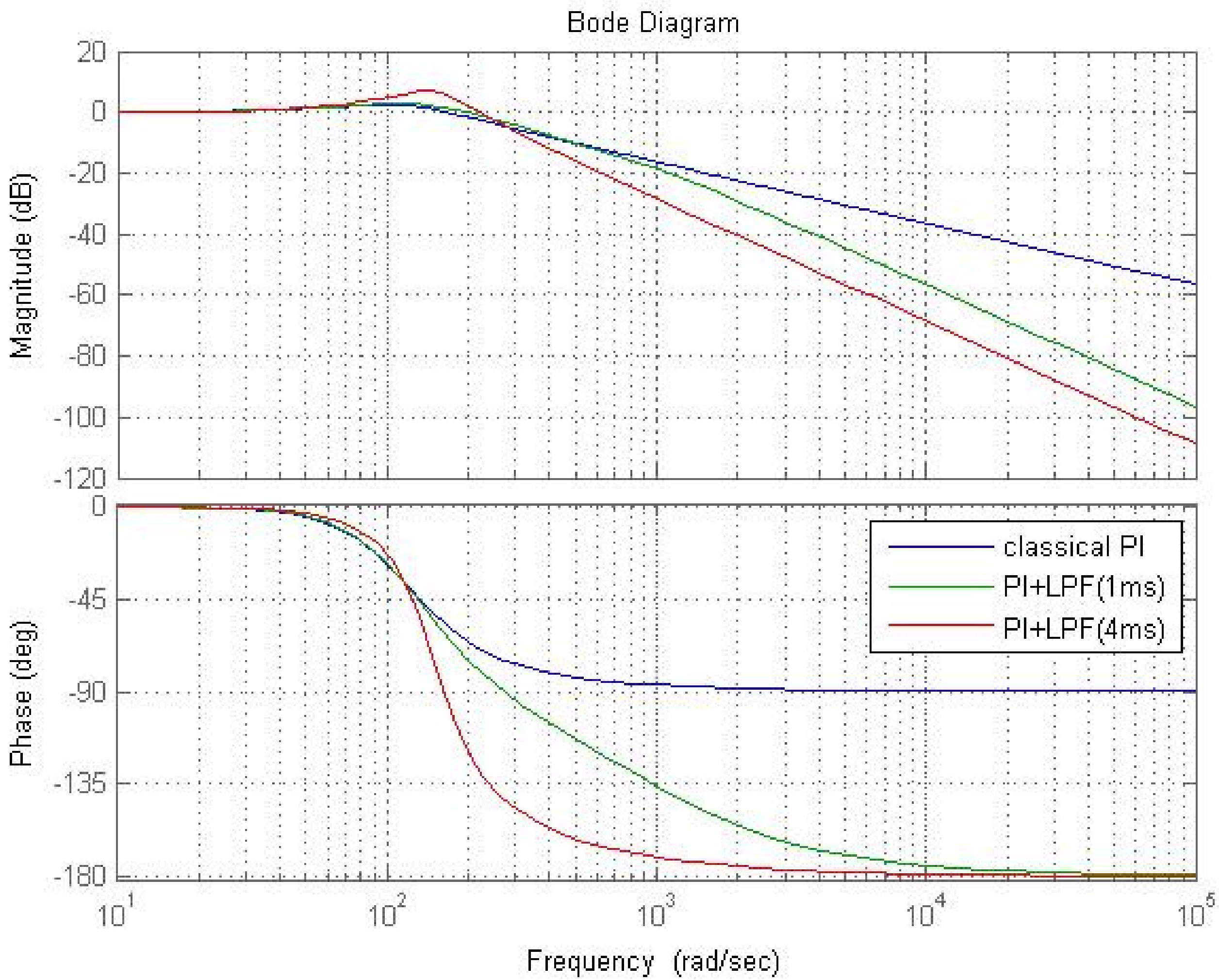
LPF or gain/bandwidth reduction?

- What is the difference between low pass filtering and gain/bandwidth reduction?
  - Assume that instead of low pass filtering with a 4 ms LPF, the classical PI control is used and the bandwidth wn is reduced by a factor of 4, from 18.88 Hz to 18.88/4 Hz. The step response is plotted in the next slide. The time to settle is around 162 ms, which could be problematic in dynamic environment.
- The bode diagram which follows shows that the PI with 4ms LPF has almost the same bandwidth as the classic PI control.

Step response comparison



Bode diagram



Conclusions

- It has been simulated earlier that 4ms LPF has a 6dB sensitivity advantage.
- It seems the only penalty PI with 4ms LPF pays is the higher transient error and longer step response settling time, which is only a one time penalty though in the pull in stage, and can be reduced further with a proper selection of damping(increase). However increasing the damping will also increase the noise bandwidth.
- When use 16 ms LPF with PI, bandwidth must be reduced, otherwise, positive poles (as with 18.88 Hz).