

Maxwell equations:

$$\begin{aligned}\nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times H &= J + \frac{\partial D}{\partial t}\end{aligned}$$

Vector operations show that

$$H \cdot (\nabla \times E) - E \cdot (\nabla \times H) = \nabla \cdot (E \times H)$$

Plug the Maxwell equations in,

$$-H \cdot \frac{\partial B}{\partial t} - E \cdot \left(J + \frac{\partial D}{\partial t} \right) = \nabla \cdot (E \times H)$$

Integrate over the volume of concern,

$$\int_V \left(H \cdot \frac{\partial B}{\partial t} + E \cdot \left(J + \frac{\partial D}{\partial t} \right) \right) dV = - \int_V (\nabla \cdot (E \times H)) dV$$

Use the divergence theorem,

$$\int_V \left(H \cdot \frac{\partial B}{\partial t} + E \cdot \left(J + \frac{\partial D}{\partial t} \right) \right) dV = - \oint_S (E \times H) \cdot dS$$

For linear, time-invariant media, the formula can be recast into the form

$$\int_V \left(\frac{\partial}{\partial t} \left(\frac{B \cdot H}{2} \right) + \frac{\partial}{\partial t} \left(\frac{D \cdot E}{2} \right) + E \cdot J \right) dV = - \oint_S (E \times H) \cdot dS$$

1. The first term is the time rate of increase of the stored energy in the magnetic field of the region;
2. The second term is the time rate of increase of the stored energy in the electric field of the region;
3. The third term is either the ohmic power loss if J is a conduction current density or the power required to accelerate charges if J is a convection current arising from moving charges;

The rate of energy

$$W = \oint_S P \cdot dS$$

where

$$P = E \times H$$

Maxwell equations:

$$\begin{aligned}\nabla \times E &= -j\omega B \\ \nabla \times H &= J + j\omega D\end{aligned}$$

Vector operations show that

$$H^* \cdot (\nabla \times E) - E \cdot (\nabla \times H^*) = \nabla \cdot (E \times H^*)$$

The general Poynting theorem is

$$\begin{aligned} \int_V \nabla \cdot (E \times H^*) dV &= \oint_S (E \times H^*) \cdot dS \\ &= - \int_V (E \cdot J^* + j\omega(H^* \cdot B - E \cdot D^*)) dV \end{aligned}$$

The electric field in the wire is

$$E = I_z R$$

The magnetic field at radius r outside the wire is

$$H_\phi = \frac{I_z}{2\pi r}$$

The Poynting vector is

$$\begin{aligned} P_r &= -E_z H_\phi \\ &= -\frac{R I_z^2}{2\pi r} \end{aligned}$$

The energy power

$$\begin{aligned} W &= 2\pi r (-P_r) \\ &= I_z^2 R \end{aligned}$$