

# How Far is Too Far?

## Generalization of a Regression Discontinuity Design Away from the Cutoff

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Magdalena Bennett

January 15, 2020

Teachers College, Columbia University

# My Research Agenda

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- Use of new data science + causal inference methods for impact evaluation in experiments and observational studies:
  - Heterogeneity in spillover effects through network of peers (Bennett & Bergman, 2019).
  - Performance prediction on college admission using machine learning.
- Development and improvement of causal inference methods:
  - Representative template matching (Bennett, Vielma, & Zubizarreta, 2019).
  - Generalization of Regression Discontinuity Design (Bennett, 2020).

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  - **Generalization of Regression Discontinuity Design**(Bennett, 2020).

# Today's talk

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Motivation

Generalized Regression Discontinuity Design

Framework

GRD in practice

Simulations

Application: Free Higher Education in Chile

Conclusions

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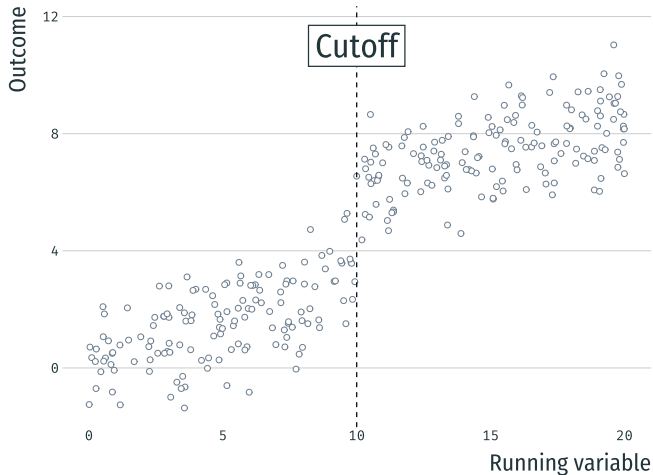
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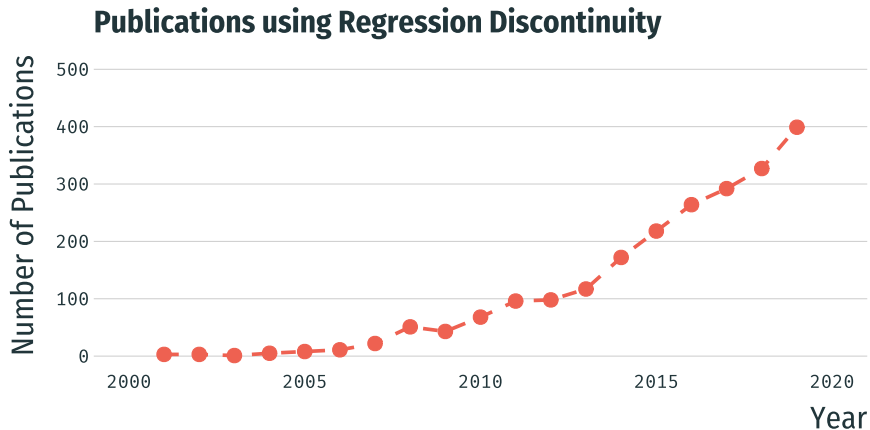
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# Regression discontinuity design

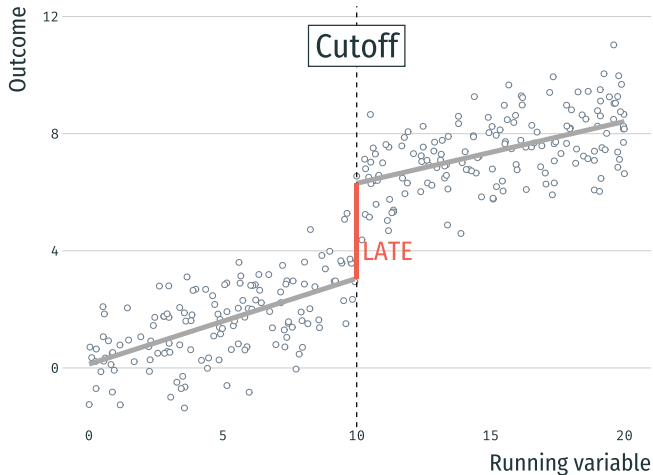


- Treatment assignment based on running variable.
- Many policies use this strategy

# Regression discontinuity design: Increasingly popular



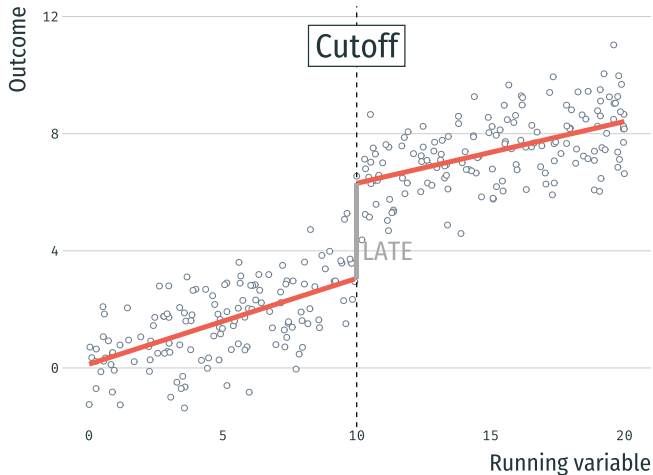
# Regression discontinuity design: Strong internal validity



- Identification of LATE under mild assumptions.
- Effect of intervention at  $R = c$ .

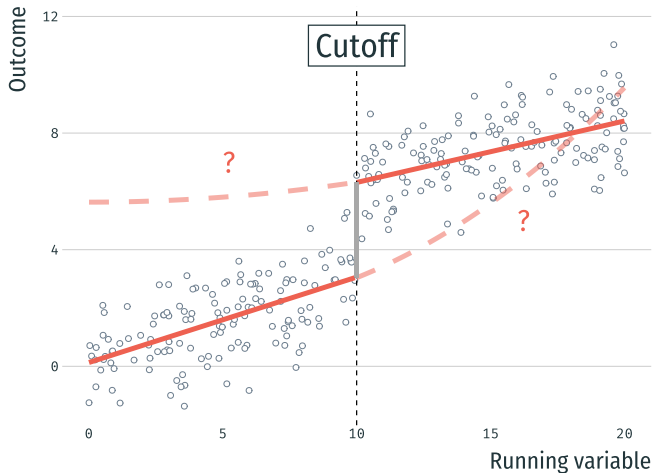


# Regression discontinuity design: Limited external validity



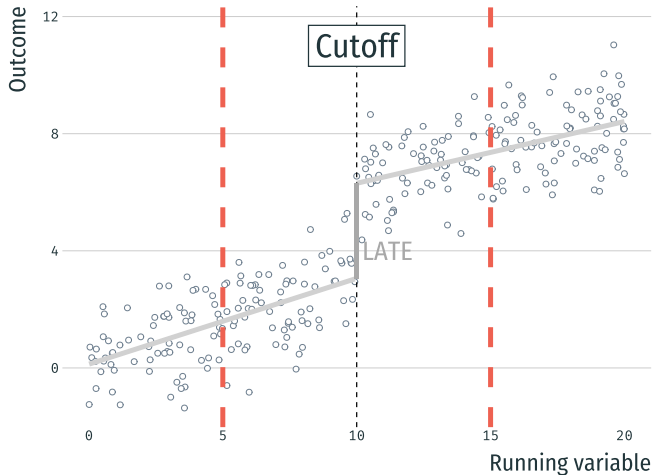
- Correlation between running variable and outcome.
- Need more assumptions to obtain generalized effect.

# Regression discontinuity design: Limited external validity



- Lack of overlap on running variable.
- Importance due to **heterogeneous effects**.

# Regression discontinuity design: Generalization interval?



- Identification of interval to explain away  $\text{Corr}(\text{Outcome}, \text{Running Var})$ .
- Identify effect away from the cutoff.

## Estimation of ATT for population within a generalization interval:

- Pre-intervention period (or other group) informs generalization interval

(Wing & Cook, 2013; Keele, Small, Hsu, & Fogarty, 2019)

- Leverage the use of predictive covariates

(Angrist & Rokkanen, 2015; Rokkanen, 2015; Keele, Titiunik, & Zubizarreta, 2015)

- Based on local randomization near the cutoff

(Lee, 2008; Cattaneo, Frandsen, & Titiunik, 2015)

# This paper

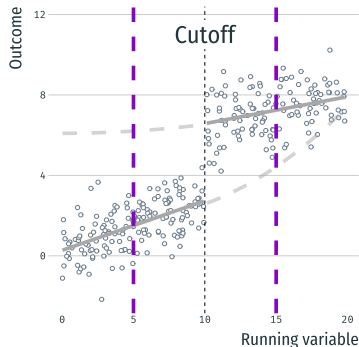
## Main advantages:

- Gradual approach
  - No need for “All or Nothing”
  - Interval informed by the data (Cattaneo et al., 2015)
- No extrapolation of population characteristics
  - Compare like-to-like (Rosenbaum, 1987)
  - Makes overlap region explicit
- Generalization to population of interest
  - Use of representative template matching

(Silber et al, 2014; Bennett, Vielma, & Zubizarreta, 2018)

- Sensitivity analysis to hidden bias (Rosenbaum, 2010;

Keele et al., 2019)



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# Generalized Regression Discontinuity Design (GRD)

## Setup:

- Two periods: pre- and post-intervention ( $t = 0$  and  $t = 1$ )
- $R$  determines assignment to  $Z$  in  $t = 1$ , e.g.:

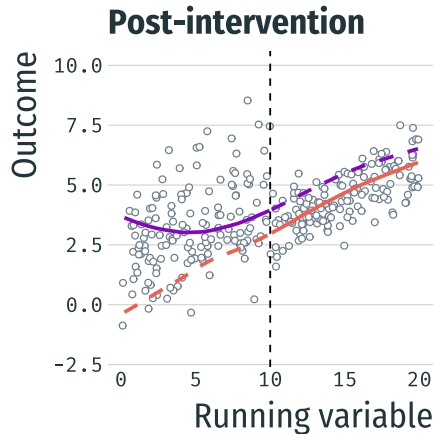
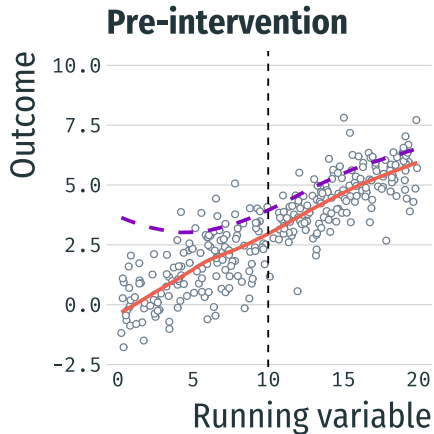
$$Z = \mathbb{I}(R < c)$$

- Potential outcomes under treatment  $z = 0, 1$ :

$$Y_{it}^{(z)} = g(\mathbf{X}_{it}, \mathbf{u}_{it}, r_{it}) + z_{it} \cdot \underbrace{\tau_{it}(\mathbf{X}_{it}, \mathbf{u}_{it}, r_{it})}_{\text{Treat. Effect}} + \underbrace{\alpha_t}_{\text{Period FE}}$$

- $\mathbf{X}$ : Predictive covariates
- $\mathbf{u}$ : Unobserved confounder
- $\tau_i$ : individual causal effect

## Two periods for GRD



—  $Y_0(R)$  —  $Y_1(R)$



## GRD: A gradual approach

- Conditional expectations of potential outcomes:

$$Y_0^{(0)}(R) = \mathbb{E}[Y_{i0}^{(0)}|R] = \mu(R)$$
$$Y_0^{(1)}(R) = \mathbb{E}[Y_{i0}^{(1)}|R] = \underbrace{\mu(R)}_{\text{Avg. Outcome by R}} + \underbrace{\tau(R)}_{\text{Treat. Effect by R}}$$

- Identify generalization interval  $H = [H_-, H_+]$  for  $t = 0$ :

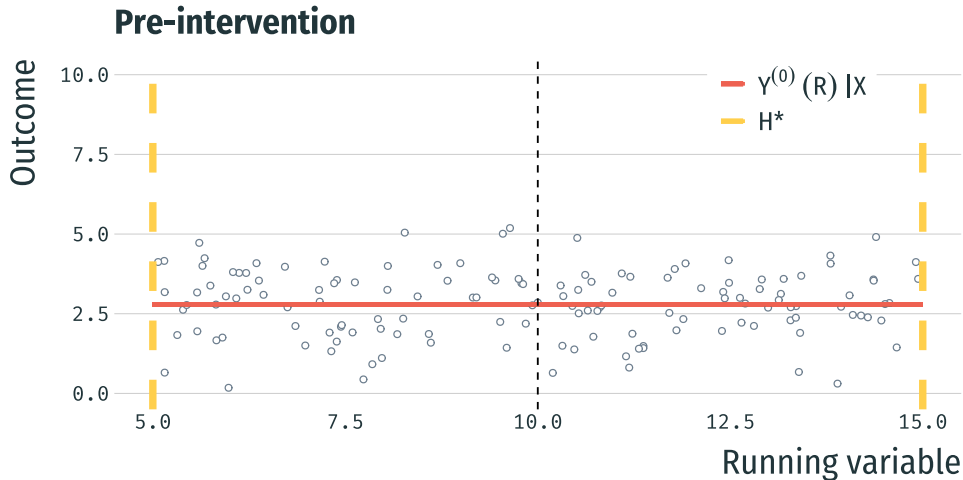
$$R_i = h(X_i) + \eta_i \quad \forall R_i \in H$$

where  $H^* = \max\{|H|\}$ .

- If  $H^*$  exists, then for a set of covariates  $\mathbf{X} = \mathbf{X}_T$ :

$$Y_0^{(0)}(R')|\mathbf{X}_T = Y_0^{(0)}(R'')|\mathbf{X}_T \quad \text{for any } R', R'' \in H^*$$

# Conditional Outcome within Generalization Interval



## GRD: Assumptions for generalization to $t=1$

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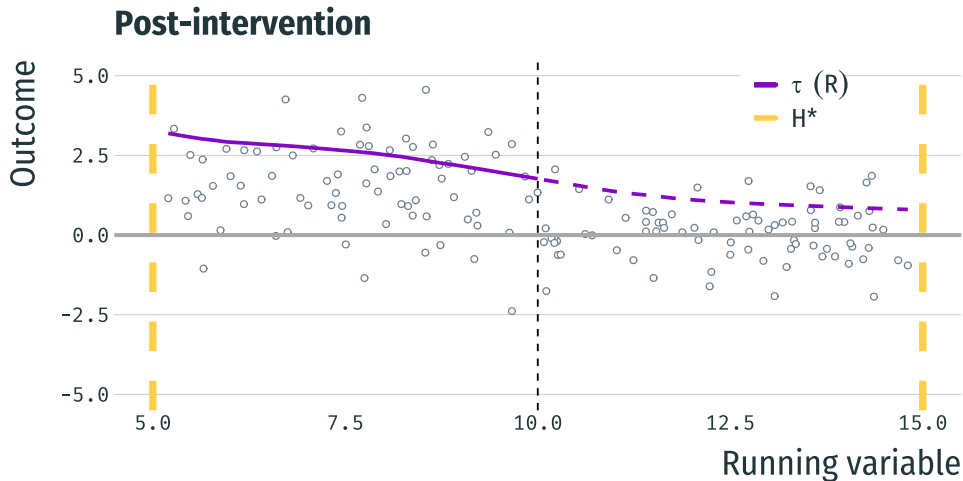
Assumption I: Conditional time-invariance under control

$$Y_0^{(0)}(R|X) = Y_1^{(0)}(R|X) + \alpha_t, \quad \forall R \in H^*$$

Assumption II: Heterogeneity only through  $\tau$

$$Y_1^{(1)}(R|X) \perp \mathbf{u} \quad \forall R \in H^*$$

## GRD: Estimating effects away from the cutoff



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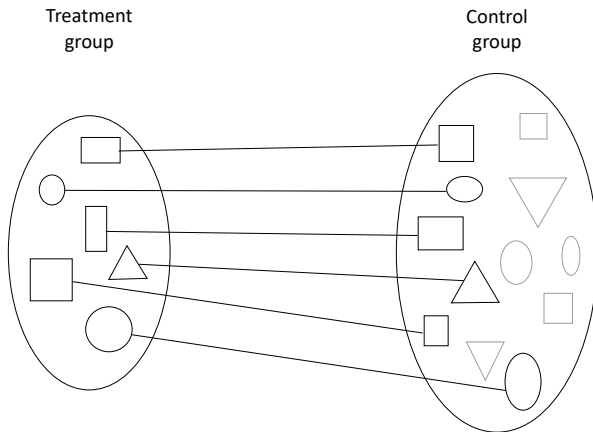
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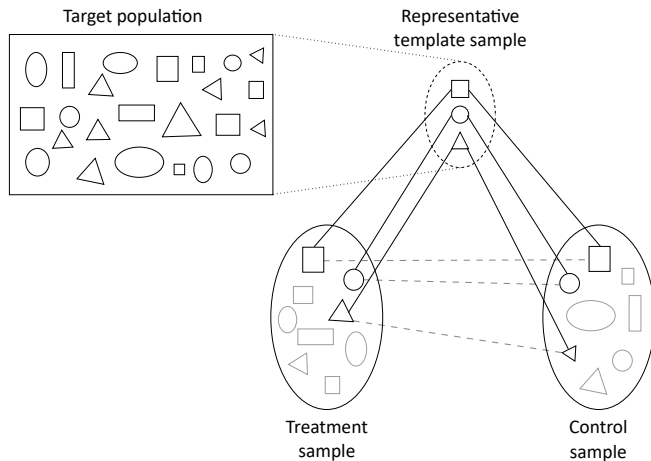
# Overview: Representative Template Matching (Bennett, Vielma, & Zubizarreta, 2019)

## Traditional Matching



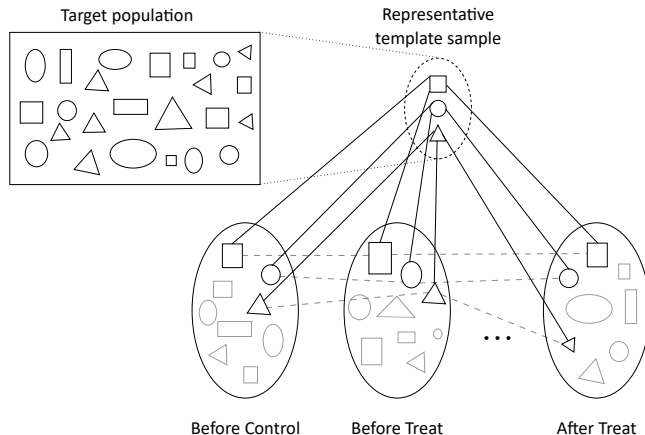
# Overview: Representative Template Matching (Bennett, Vielma, & Zubizarreta, 2019)

## Representative Template Matching for Two Groups



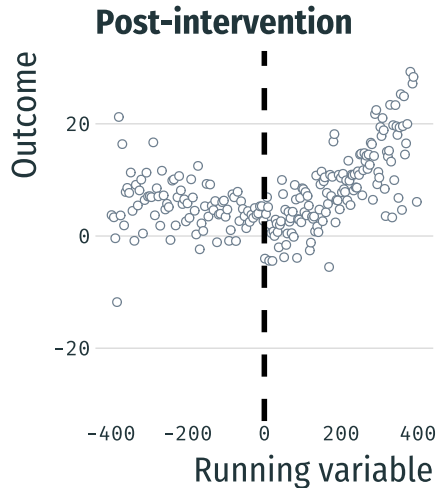
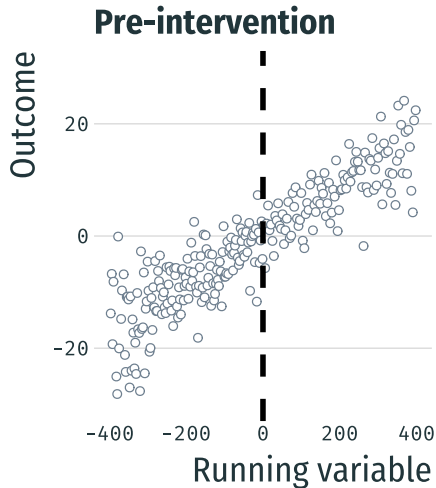
# Overview: Representative Template Matching (Bennett, Vielma, & Zubizarreta, 2019)

## Representative Template Matching for Diff-in-Diff

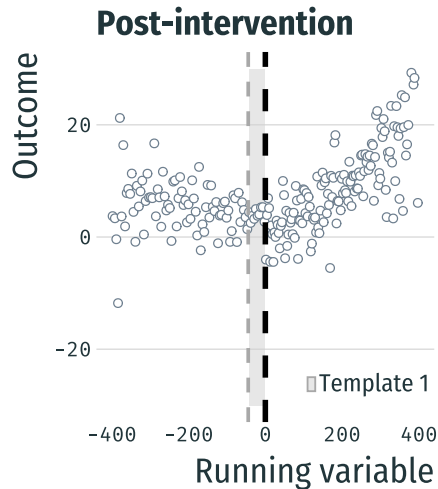
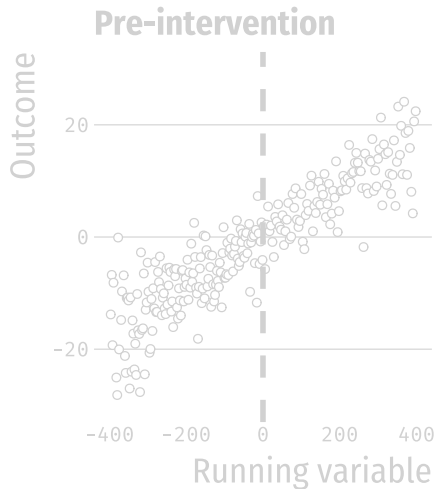




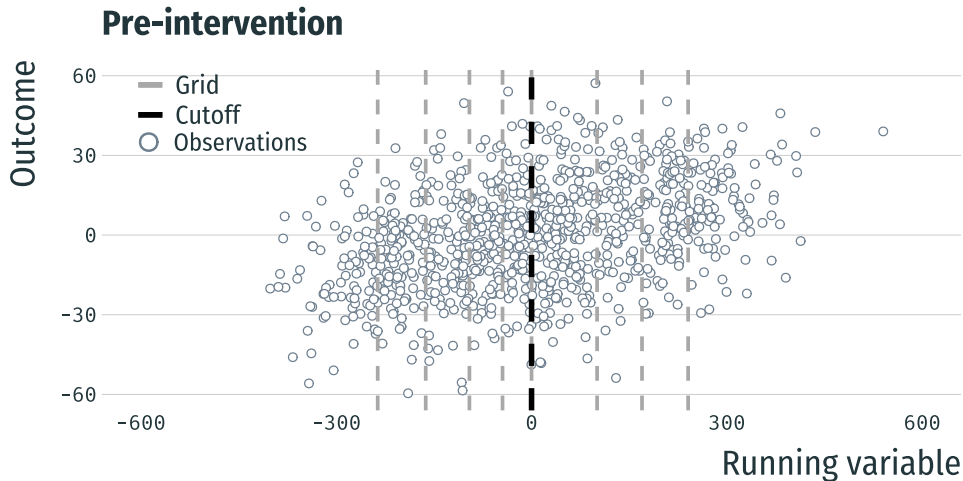
## GRD: Start with two periods



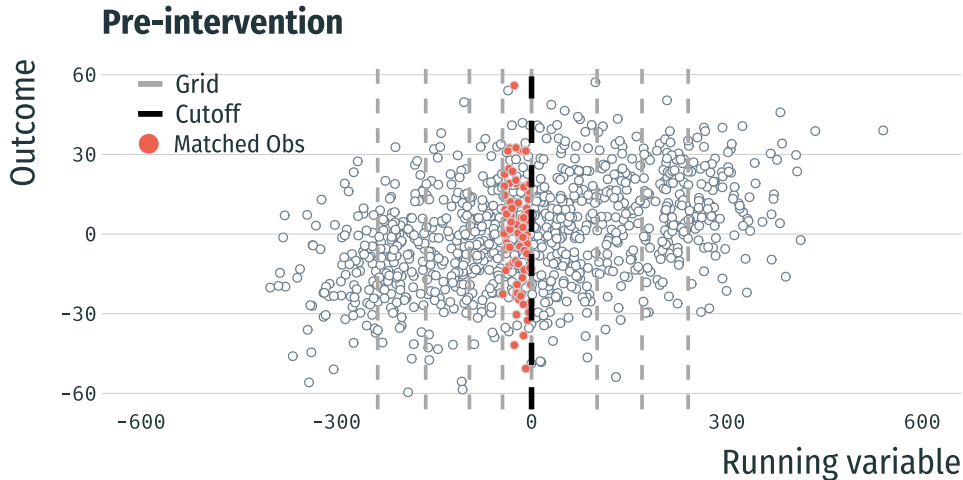
## GRD: Select template sample from post-intervention



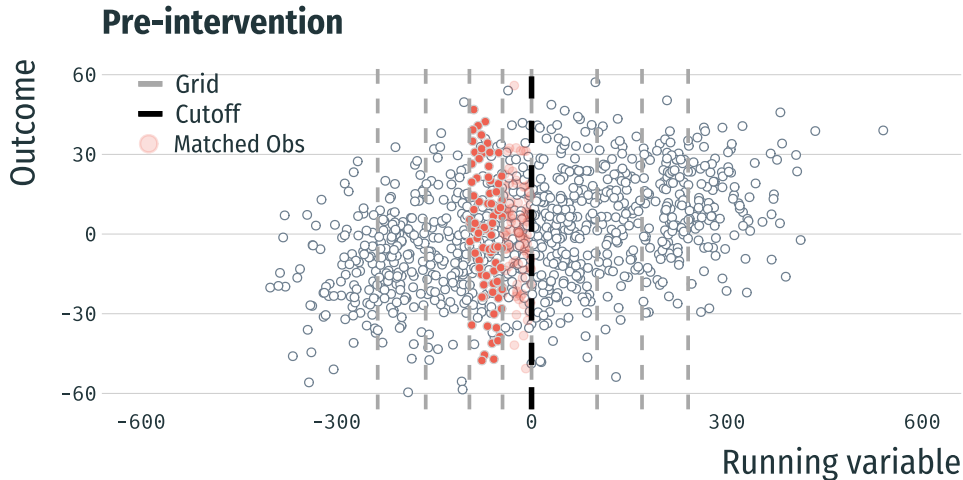
## GRD: Divide pre-intervention into grid



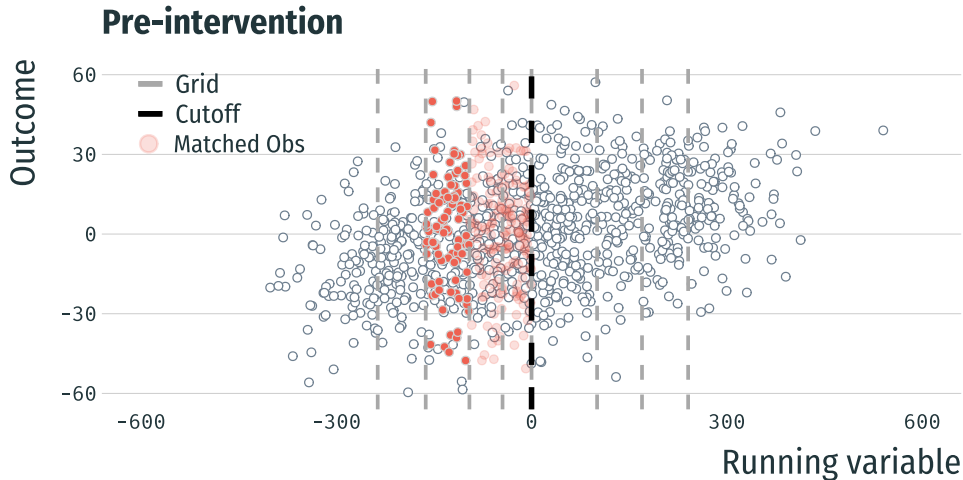
## GRD: Match template to grid



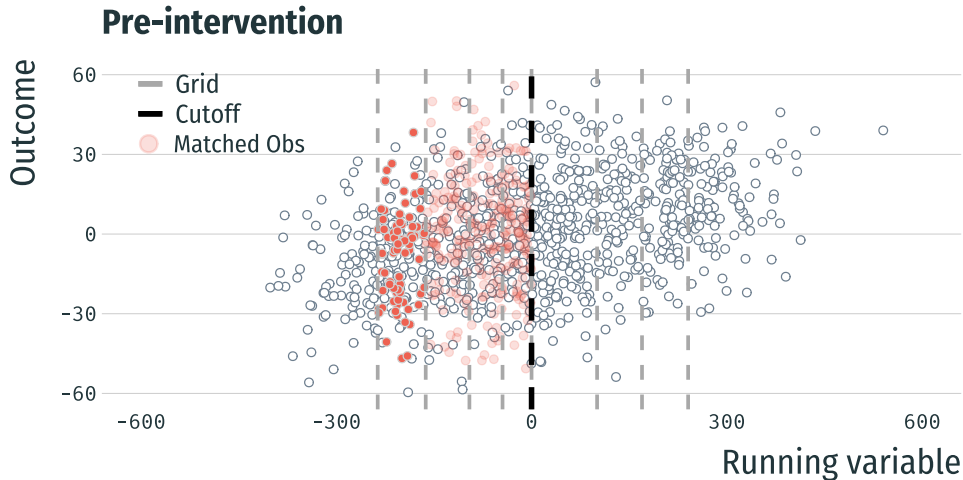
## GRD: Match template to grid



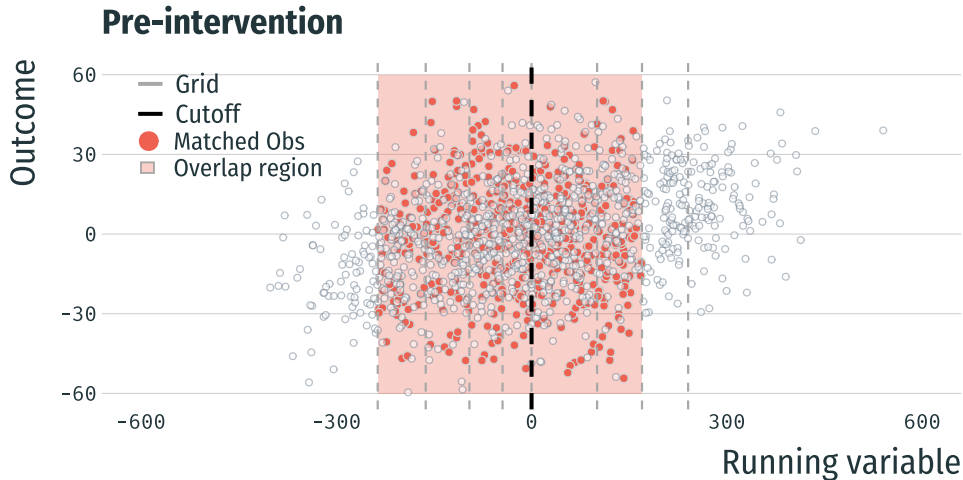
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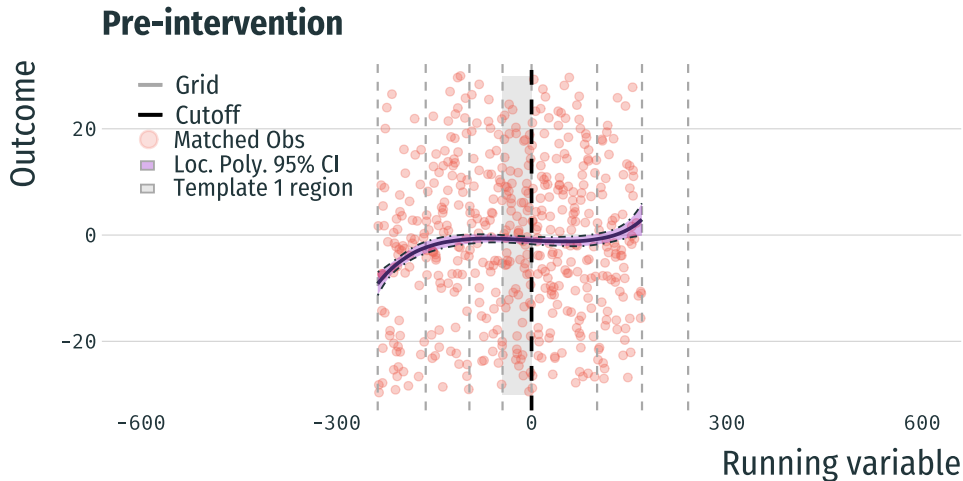


## GRD: Explicit overlap region

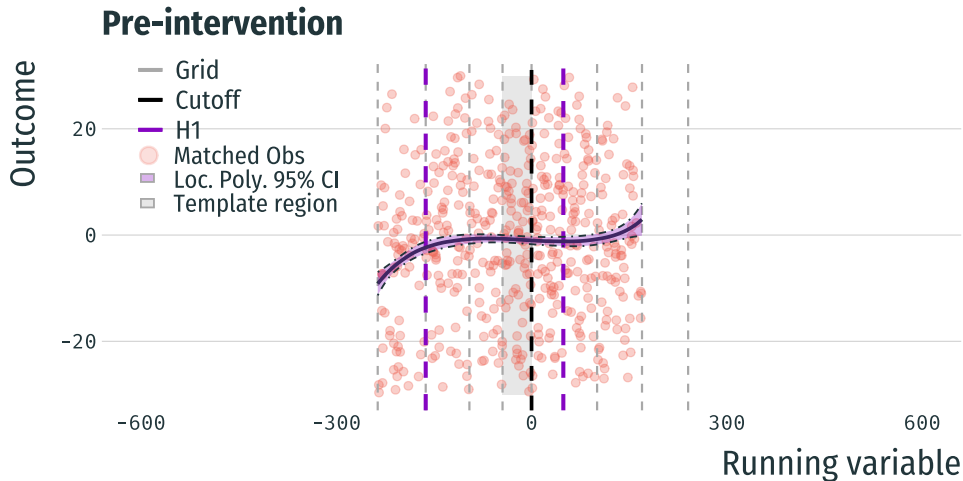




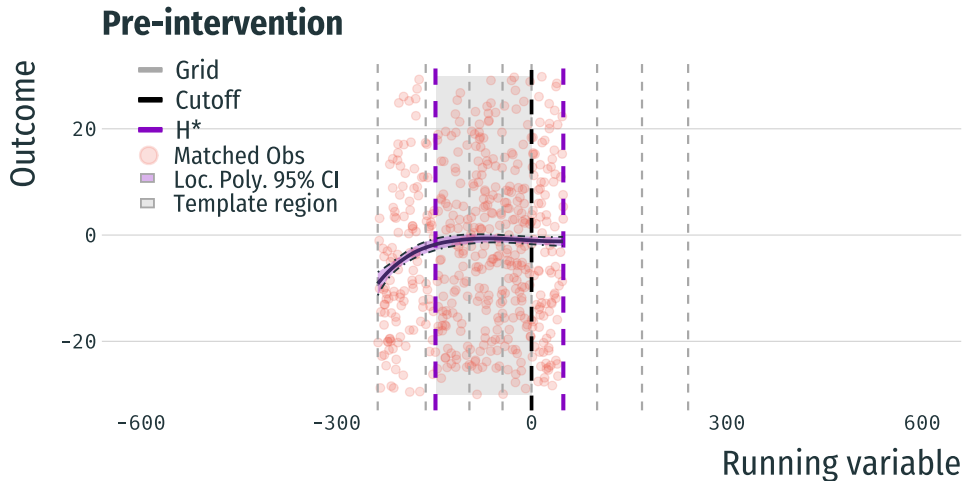
## GRD: Estimate local polynomial on matched sample



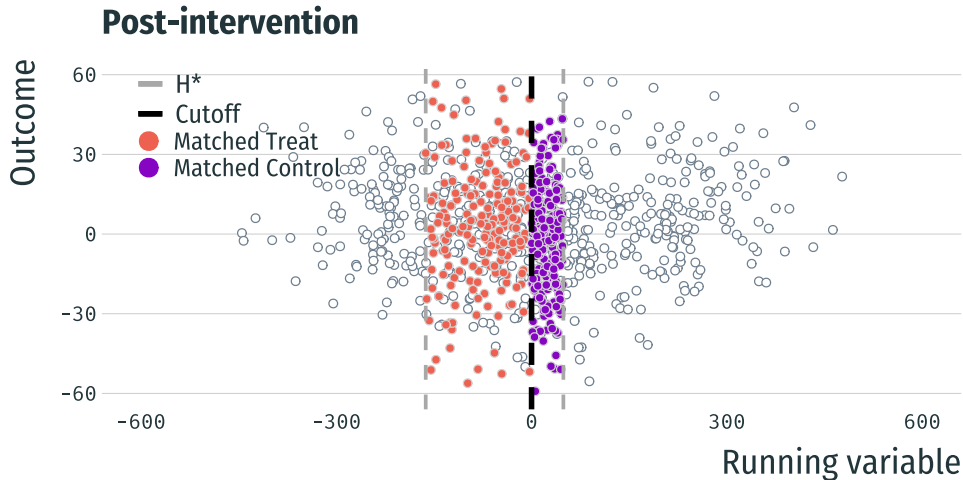
## GRD: Identify generalization interval $H_1$



GRD: Repeat procedure until  $H_j \subseteq T$



## GRD: Match post-intervention period to the template



Straightforward estimation given matched sample:

- E.g. paired t-test:

$$\hat{\tau}_{ATT} = \sum_{k=1}^N \frac{Y_{k(1)1} - Y_{k(0)1} - (Y_{k(1)0} - Y_{k(0)0})}{N} = \sum_{k=1}^N \frac{d_k}{N}$$

$Y_{k(z)t}$ : outcome within matched group  $k$  with treatment  $z = \{0, 1\}$  for period  $t = \{0, 1\}$

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# Simulations: Assess performance of GRD

- Compare GRD performance to `rdrobust()` (LATE) (Calonico et al., 2018) and **A-R RD generalization** (ATT) (Angrist & Rokkanen, 2015)  
→ 500 simulations
- Simulations scenarios:
  - Low vs. high correlation:  
 $\text{Corr}(R, X) = \{0.33, 0.66\}$
  - Constant vs. heterogeneous effects:  
 $\tau_{\text{constant}} = 0.2\sigma$   
 $\tau_{\text{linear}} = 0.2\sigma + 0.0025\sigma \cdot R$   
 $\tau_{\text{quad}} = 0.2\sigma + 0.0025\sigma \cdot R^2$
  - Small vs. large samples:  
2,000 vs 20,000 obs

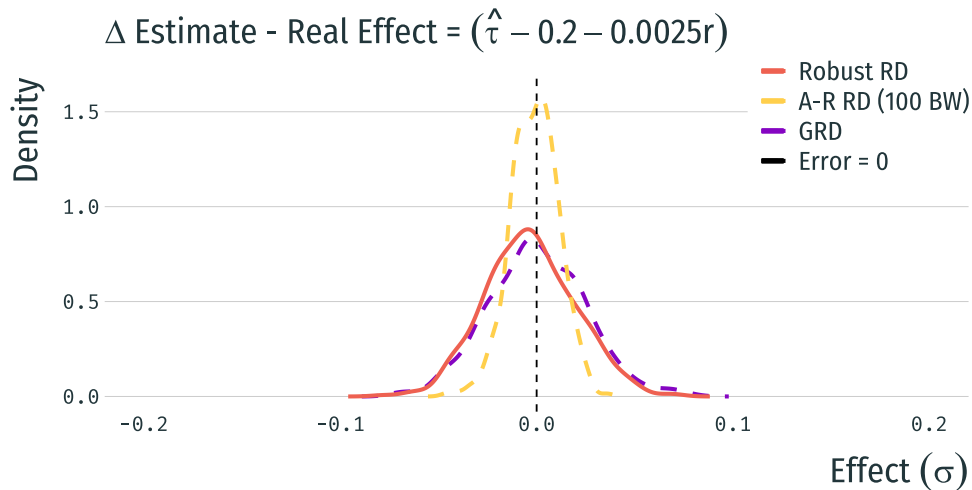
# Simulation Results

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- Similar performance for GRD and RD robust for constant and linear effects.
- A-R RD generalization performs better than GRD in terms of variance **if treatment effect is tested within generalization interval (GI)**.
  - 18% simulations failed residual test in quadratic treatment effect within GI.



## Simulation distribution: $\tau_{linear}$ (s: high corr & large sample)



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# Free Higher Education (FHE) in Chile

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## Higher education in Chile:

- Centralized admission system (deferred admission mechanism)
- Admission score: PSU score + GPA score + ranking score
- Before 2016: Scholarships + government-backed loans

## Free higher education policy:

- Introduced in December 2015 (unanticipated)
- Eligibility: Lower 50% income distribution + admitted to eligible program

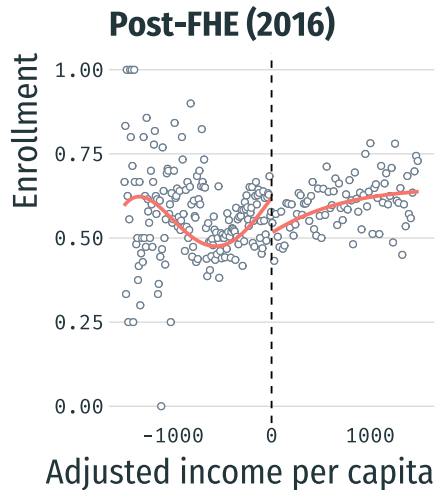
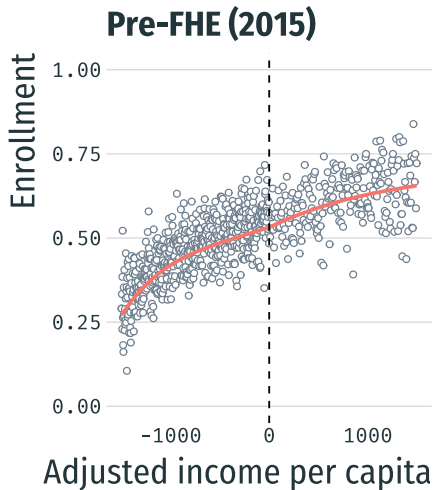
## FHE: Research Question

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- **Treatment:** SE eligibility for FHE
- **Two outcomes:** Application to university and enrollment
  - Lower-income students → financial constraints
  - Salience of policy
- Larger effects for students away from the cutoff?
  - Compare RD and GRD results

- **3 Cohorts:** 2014, 2015, and 2016. ( $\sim$  200,000 students)
- **Rich baseline data:** Demographic and socioeconomic data at student level, 10th (8th) grade standardized scores, school characteristics.
- **Application data:** Scores by subject, application, enrollment.

## FHE: How does the RDs look like?



# GRD for Free Higher Education

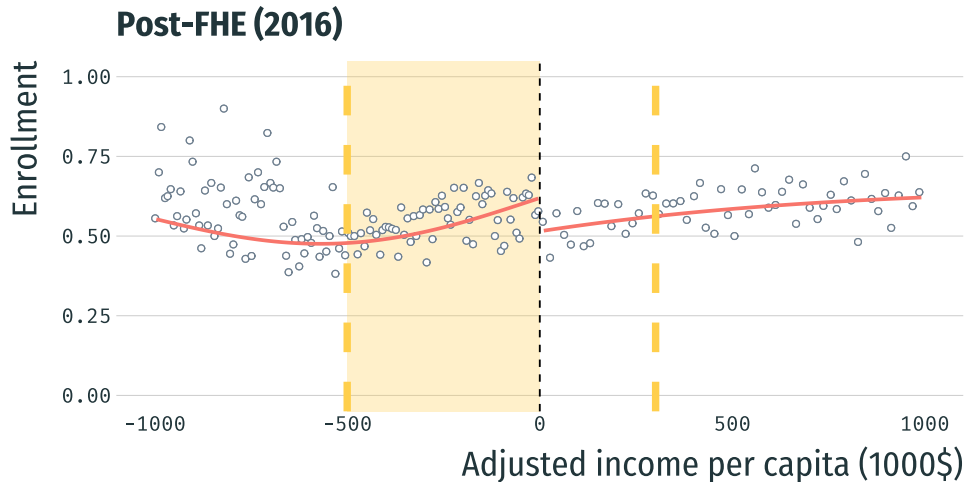
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## Steps for GRD:

- Select template size:  $N = 1,000$
- 20 bins for grid
- MIP matching → Variable selection using ML
  - Restricted mean balance (0.05 SD):
    - Academic performance, school characteristics, demographic/socioeconomic variables.
  - Fine balance:
    - Gender, mother's and father's education (8 cat), PSU Language score (deciles), PSU math score (deciles), HS GPA (quintiles).
- Generalization interval: [-M\$500.3, M\$300.9]

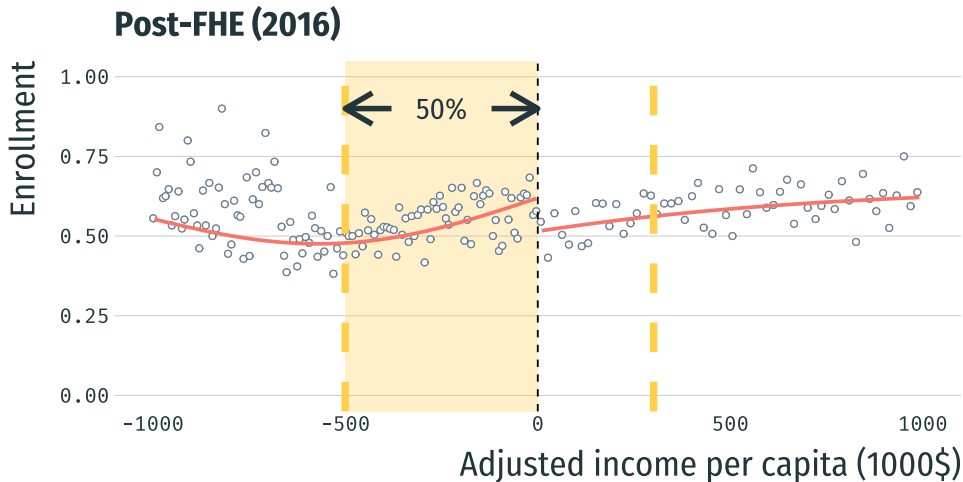
► Sample characteristics

For what population are we generalizing for?

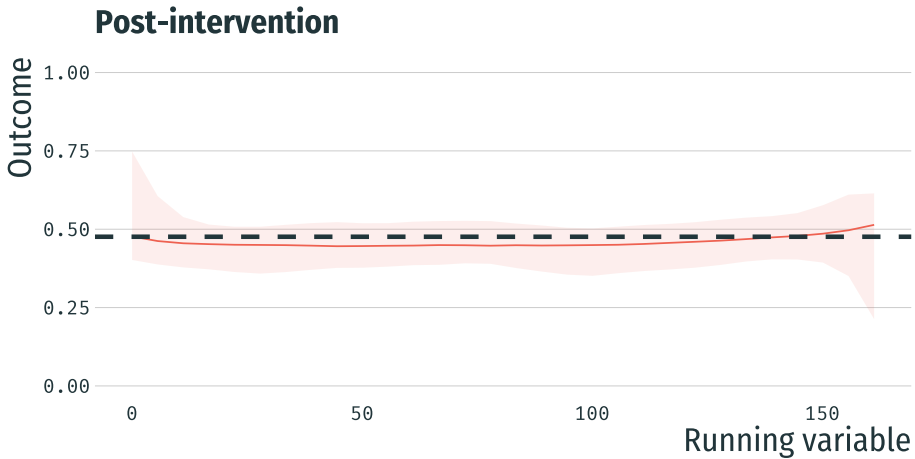




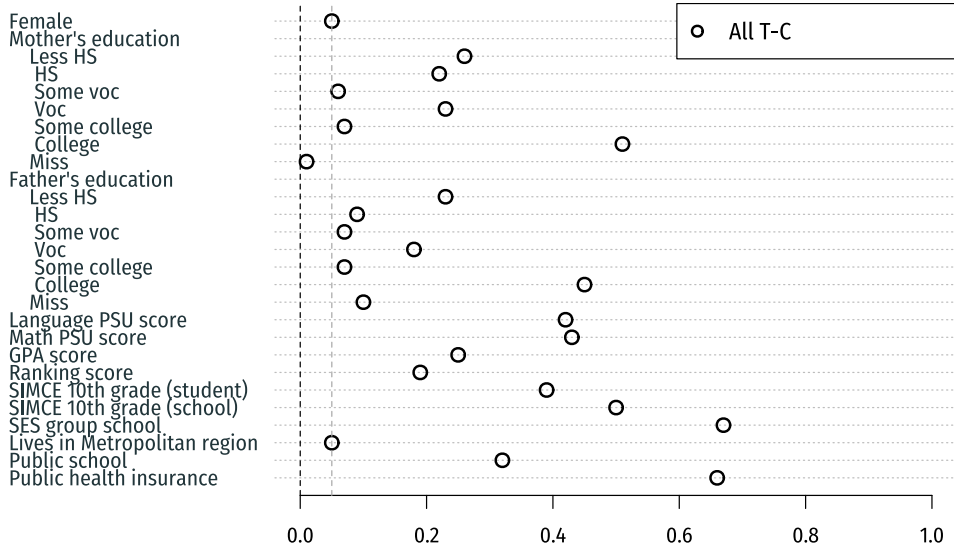
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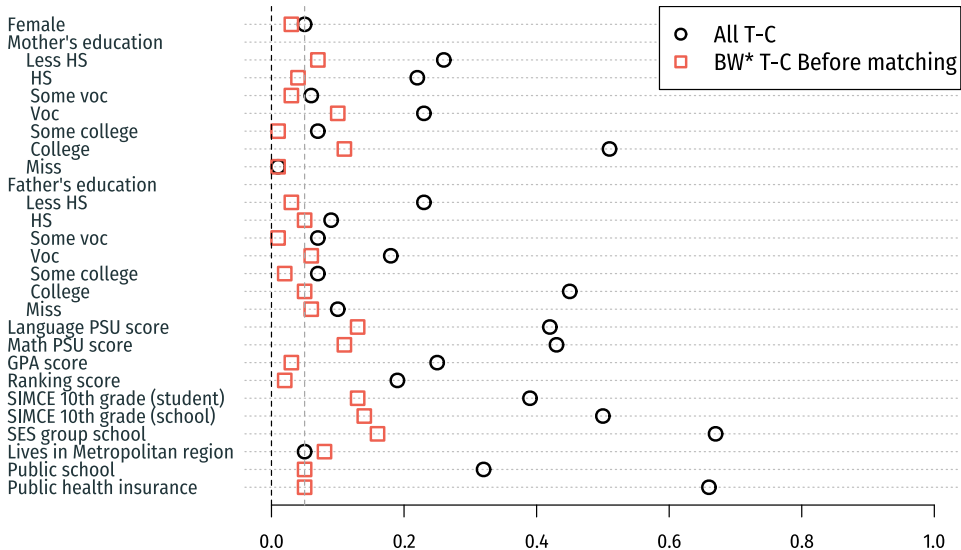
## Local Polynomial for Control Outcome in $t=1$



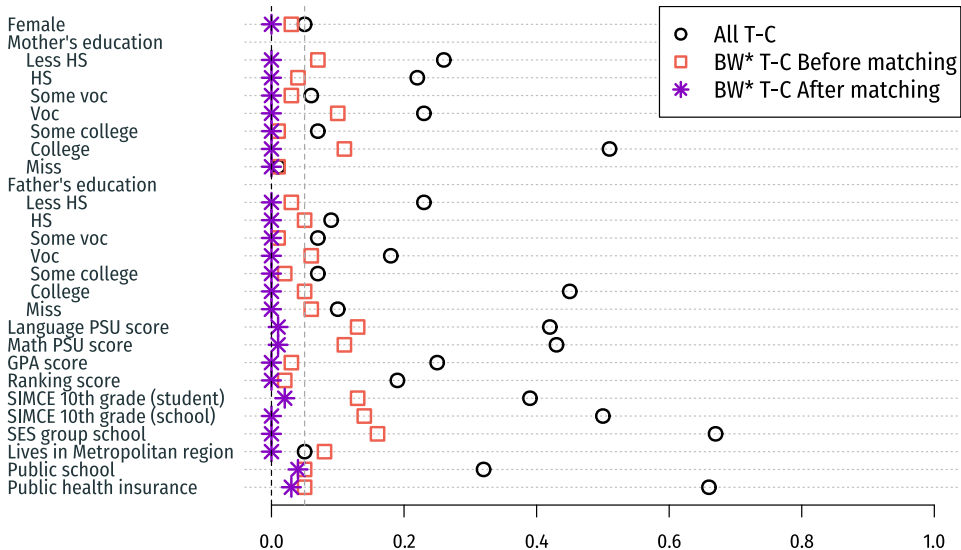
## Balance: Entire sample



## Balance: Within $H^*$ before matching



## Balance: Within $H^*$ after matching



## Effects of introduction of FHE: RD and GRD

	Robust RD results		GRD results	
	Application	Enrollment	Application	Enrollment
Effect	0.035 [-0.007, 0.077]	0.069** [0.026, 0.112]	0.052** [0.008, 0.096]	0.077*** [0.029, 0.125]
Effective N Obs	6,588	6,458	2,000	2,000
Control Mean	0.606	0.515	0.568	0.472

Generalization interval [-M\$500, M\$301]

95% CI in brackets

## Effects of introduction of FHE: Application

	Robust RD results		GRD results	
	Application	Enrollment	Application	Enrollment
Effect	0.035 [-0.007, 0.077]	0.069** [0.026, 0.112]	0.052** [0.008, 0.096]	0.077*** [0.029, 0.125]
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## Effects of introduction of FHE: Enrollment

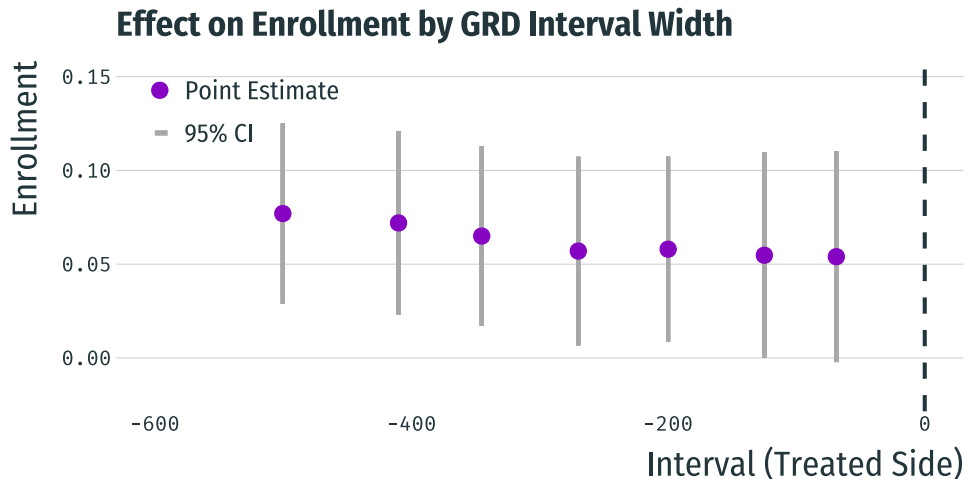
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## How does the effect change with interval width?



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# Conclusions

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- GRD as a gradual approach for generalization (not “all or nothing”)
- Use data to inform interval for generalization
- Use of matching to avoid extrapolation
- Limitations
  - More data: two periods
  - Conditional time invariance assumption for  $t = 1$
- Multiple applications for DD-RD: e.g. geographic RDs.
- Heterogeneous treatment effects targeting?

# How Far is Too Far?

## Generalization of a Regression Discontinuity Design Away from the Cutoff

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Magdalena Bennett

January 15, 2020

Teachers College, Columbia University

# Sensitivity Analysis to Hidden Bias

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- Quantify bias of unobserved confounder to change qualitative results of the study
- Adaptation of Keele et al. (2019) sensitivity analysis for Diff-in-Diff.
- Moderately sensitive to hidden bias:  $\Gamma=1.6$

$$\rightarrow \Pr(Z_{i1} = 1) = 0.62 \wedge \Pr(Z_{i1} = 0) = 0.38$$

# GRD for Fuzzy Regression Discontinuity

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- Wald-type IV estimand:

$$\tau_{Fuzzy} = \frac{\sum_{k=1}^N (Y_{k(1)1} - Y_{k(0)1} - (Y_{k(1)0} - Y_{k(0)0}))}{\sum_{k=1}^N (D_{k(1)1} - D_{k(0)1} - (D_{k(1)0} - D_{k(0)0}))}$$

- $Y_{k(z)t}$ : Outcome for unit in matched group  $k$  under treatment assignment  $z$  in period  $t$ .
- $D_{k(z)t}$ : Actual treatment for unit in matched group  $k$  under treatment assignment  $z$  in period  $t$ .

# Data Generating Processes for Simulations

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- Observed covariate:  $X \sim \mathcal{N}(0, 10)$
- Unobserved confounder:  $U \sim \mathcal{N}(0, 10)$
- Running variable for scenario  $s$ :

$$r_{it} = \alpha_{s,x}x_{it} + \alpha_{s,u}u_{it} + \varepsilon_{it}$$

- Observed outcome for scenario  $s$ :

$$y_{it} = \beta_{s,x}x_{it} + \beta_{s,u}u_{it} + \beta_{s,r}r_{it} + Z_{it}\tau_s + \nu_{it}$$

- True  $H = [-200, 200]$

## Simulations: Setup for GRD

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- Distributional (fine) balance for  $X$  deciles
- Template size: 1,000 and 100
- Grid: Equally sized bins (20)
- Significance level for detecting GRD interval: 0.1

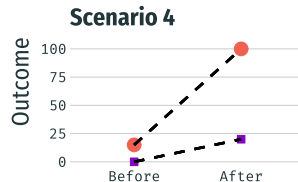
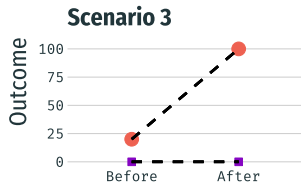
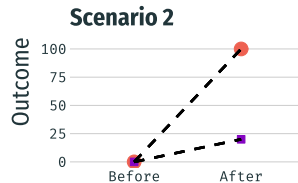
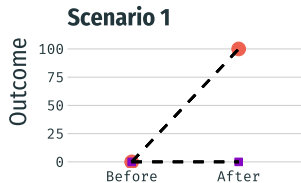
◀ Go back



## Comparison between treatment groups

	Treat group (All)	Treat group within H*
Female	0.55	0.55
Mother's education (years)	11.37	11.57
Father's education (years)	11.52	11.67
Language PSU score	504.08	510.20
Math PSU score	507.69	513.30
GPA score	554.88	558.11
Ranking score	579.84	583.11
SIMCE 10th grade (student)	274.90	276.95
SIMCE 10th grade (school)	266.91	268.52
SES group school	2.68	2.73
Lives in Metropolitan region	0.40	0.42
Public school	0.35	0.34
Public health insurance	0.82	0.79

# Different Diff-in-Diff Scenarios



● Treat ● Control