STA 235H - Potential Outcomes II

Fall 2022

McCombs School of Business, UT Austin

Housekeeping

- Homework 2 is due this Friday
  - Note on Chatter: In Task 2, use *price000s* from 2.2 onwards.
  - Send your questions until Friday 5.00pm.
- Remember to send re-grading requests for Homework 1 until Thursday
- No OH this Thursday (changed to Wed and Friday; check OH calendar).

Homework 3 will be posted this Friday

(Half the length of a homework)

Last week

- Finished our chapter on multiple regression.
  - How to handle binary outcomes: Linear Probability Models.
  - Video posted online about heteroskedasticity.
- Introduced Causal Inference

Today

- Continue with **causal inference**:
  - Potential outcomes
  - Ignorability assumption
- **Introduction to Randomized Controlled Trials**

Before we start, let's check this week's JITT

"You are trying to analyze what variables contribute to someone donating or not to a charity. You are running the following linear probability mode to analyze the association between different covariates and a binary response of whether someone responds with a charitable gift or not."

"The variables used in this model are the following:

- *respond*: Binary variable of whether the person responded with a gift (1) or not (0).
- *mailsyear*: number of mailings per year
- *propresp*: response rate to mailings. Continuous variable, measured from 0 to 1.
- *avggift*: average amount of past gifts (in US$)."

```
Call:
lm_robust(formula = respond ~ mailsyear + propresp + avggift,
    data = charity)

Standard error type:  HC2

Coefficients:
              Estimate Std. Error t value   Pr(>|t|)   CI Lower  CI Upper   DF
(Intercept) -0.1355169  2.294e-02  -5.908  3.742e-09 -1.805e-01 -0.090544 4264
mailsyear    0.0600391  1.064e-02   5.642  1.788e-08  3.918e-02  0.080901 4264
propresp     0.8449998  2.430e-02  34.771 1.939e-233  7.974e-01  0.892644 4264
avggift      0.0001718  5.826e-05   2.948  3.214e-03  5.754e-05  0.000286 4264

Multiple R-squared:  0.204 ,    Adjusted R-squared:  0.2034
F-statistic: 460.5 on 3 and 4264 DF,  p-value: < 2.2e-16
```

TRUE OR FALSE: "For one additional dollar donated in the past, the probability of donating increases by 0.0002, on average, holding other variables constant."

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TRUE OR FALSE: "Holding mailings per year and average past gift constant, increasing the response rate of mailings from 0% to 100% is associated with an average 84 percentage point increase in the probability of donating."

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TRUE OR FALSE: "Holding mailings per year and average past gift constant, a 1% increase in the response rate of mailings is associated with an average 84% increase in the probability of donating."

Causal Inference: Terminology and Notation

Potential Outcomes

- Last week we discussed potential outcomes, (e.g. $Y_i(1)$ and $Y_i(0)$):

"The outcome that *we would have observed* under different scenarios"

- Potential outcomes are related to your choices/possible conditions:
  - One for each path!
  - Do not confuse them with the **values** that your outcome variable can take.
  - Q: "You have to choose between three portfolios (P_1, P_2, P_3) for investing your money. What are your potential outcomes?"

- Definition of **Individual Causal Effect**:

$$ICE_i = Y_i(1) - Y_i(0)$$

What was the Fundamental Problem of Causal Inference?

Estimands vs Estimates vs Estimators

Estimand

A quantity we want to estimate

Estimator

A rule for calculating an estimate based on data

Estimate

The result of an estimation

Estimands vs Estimates vs Estimators

Estimand

A quantity we want to estimate

E.g.: Population mean

μ

Estimator

A rule for calculating an estimate based on data

E.g.: Sample mean

$$\frac{1}{n}\sum_i Y_i$$

Estimate

The result of an estimation

E.g.: Result of the sample mean for a given sample S

$\hat{\mu}$

Estimands vs Estimates vs Estimators

Source: Deng, 2022

Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

Average Treatment Effect (ATE)

Average Treatment Effect on the Treated (ATT)

Conditional Average Treatment Effect (CATE)

Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

$$\boxed{ATE = E[Y(1) - Y(0)]}$$

$$\boxed{ATT = E[Y(1) - Y(0)|Z = 1]}$$

$$\boxed{CATE = E[Y(1) - Y(0)|X]}$$

Getting around the fundamental problem of causal inference

- Let's go back to our original example: **Does a pill help reduce headaches?**

i	Z	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	0	1	?	1	?
2	1	1	1	?	?
3	1	0	0	?	?
4	0	0	?	0	?
5	0	1	?	1	?
6	1	0	0	?	?

Getting around the fundamental problem of causal inference

- We have a **missing data problem**

i	Z	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	0	1	?	1	?
2	1	1	1	?	?
3	1	0	0	?	?
4	0	0	?	0	?
5	0	1	?	1	?
6	1	0	0	?	?

Getting around the fundamental problem of causal inference

- Compare those who took the pill to the ones did not take it.

i	Z	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	0	1	?	1	?
2	**1**	**1**	**1**	**?**	**?**
3	**1**	**0**	**0**	**?**	**?**
4	0	0	?	0	?
5	0	1	?	1	?
6	**1**	**0**	**0**	**?**	**?**

Getting around the fundamental problem of causal inference

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Getting around the fundamental problem of causal inference

- Compare those who **took the pill** to the ones **did not take it.**

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6	1	0	0	?	?

$$\hat{\tau} = \frac{1}{3}(\sum_{i \in Z=1} Y_i - \sum_{i \in Z=0} Y_i) = -0.333$$

Getting around the fundamental problem of causal inference

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- What is the **estimand**?

Getting around the fundamental problem of causal inference

$$\hat{\tau} = \frac{1}{3}\left(\sum_{i \in Z=1} Y_i - \sum_{i \in Z=0} Y_i\right) = -0.333$$

- What is the **estimand**?

Average Treatment Effect

Getting around the fundamental problem of causal inference

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- What is the estimand?

Average Treatment Effect

- What is the estimator?

Getting around the fundamental problem of causal inference

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Average Treatment Effect

- What is the **estimator**?

Difference in sample means

Getting around the fundamental problem of causal inference

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- What is the estimand?

Average Treatment Effect

- What is the estimator?

Difference in sample means

- What is the estimate and *how do we interpret it*?

Getting around the fundamental problem of causal inference

$$\hat{\tau} = \frac{1}{3}(\sum_{i \in Z=1} Y_i - \sum_{i \in Z=0} Y_i) = -0.333$$

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Average Treatment Effect

- What is the estimator?

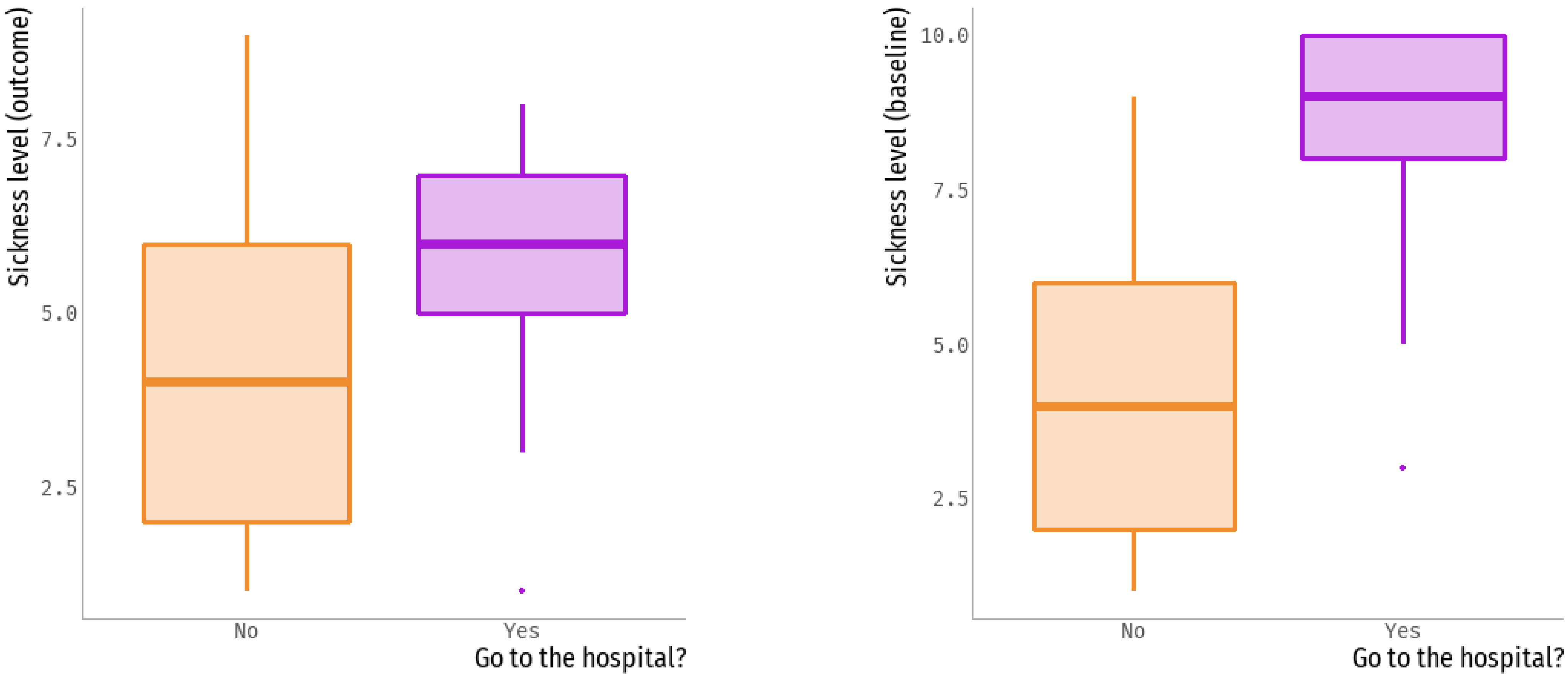
Difference in sample means

- What is the estimate and *how do we interpret it*?

33.3 percentage point decrease in probability of having a headache

What could be the problem with comparing the sample means?

Remember our exercise last week!



Under what assumptions is our estimate causal?

We are using:

$$\hat{\tau} = \frac{1}{3}(\sum_{i \in Z=1} Y_i - \sum_{i \in Z=0} Y_i)$$

to estimate:

$$\tau = E[Y_i(1) - Y_i(0)]$$

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Let's do some math

Under what assumptions is our estimate causal?

$$\tau = E[Y_i(1) - Y_i(0)]$$

$$= E[Y_i(1)] - E[Y_i(0)]$$

Key assumption:

Ignorability

- Ignorability means that the potential outcomes $Y(0)$ and $Y(1)$ are independent of the treatment, e.g. $(Y(0), Y(1)) \perp\!\!\!\perp Z$.

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- Ignorability means that the potential outcomes $Y(0)$ and $Y(1)$ are independent of the treatment, e.g. $(Y(0), Y(1)) \perp\!\!\!\perp Z$.
  - *Remember that if* $A \perp\!\!\!\perp B \rightarrow E[A|B] = E[A]$
  - *Remember that if* $Z = 1$, *then* $Y_i = Y_i(1)$, *and if* $Z = 0$, *then* $Y_i = Y_i(0)$

Under what assumptions is our estimate causal?

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$$= E[Y_i(1)] - E[Y_i(0)]$$

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$$\tau = E[Y_i(1)] - E[Y_i(0)] = E[Y_i(1)|Z=1] - E[Y_i(0)|Z=0]$$

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$$\tau = E[Y_i(1)] - E[Y_i(0)] = \underbrace{E[Y_i(1)|Z=1]}_{\text{Obs. Outcome for T}} - \overbrace{E[Y_i(0)|Z=0]}^{\text{Obs. Outcome for C}}$$

Under what assumptions is our estimate causal?

$$\tau = E[Y_i(1) - Y_i(0)]$$

$$= E[Y_i(1)] - E[Y_i(0)]$$

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$$= E[Y_i|Z = 1] - E[Y_i|Z = 0]$$

References

- Angrist, J. & S. Pischke. (2015). "Mastering Metrics". *Chapter 1*.
- Cunningham, S. (2021). "Causal Inference: The Mixtape". *Chapter 4: Potential Outcomes Causal Model*.
- Neil, B. (2020). "Introduction to Causal Inference". *Fall 2020 Course*