



STA 235 - Causal Inference: Regression Discontinuity Design (Cont.)

Spring 2021

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Reminders

In-class midterm March 29th

- **What will the midterm look like?**
 - Shorter version of a homework: Examples/cases with data (conceptual + R code)
- **Where should I study from?**
 - R code posted for class (includes conceptual questions), R code example questions for midterm, examples/questions seen in class, JITTs, etc.
 - Other resources posted on the bookmark section (websites have *a lot* of data exercises)
 - **Don't memorize anything.**

Today

- Quick recap and finish with **regression discontinuity design**:
 - How do we estimate an effect in an RD?
- **Instrumental variables**:
 - Noncompliance in RCTs.
 - Fuzzy regression discontinuity designs.



Let's recap

Behind the scenes of RDs

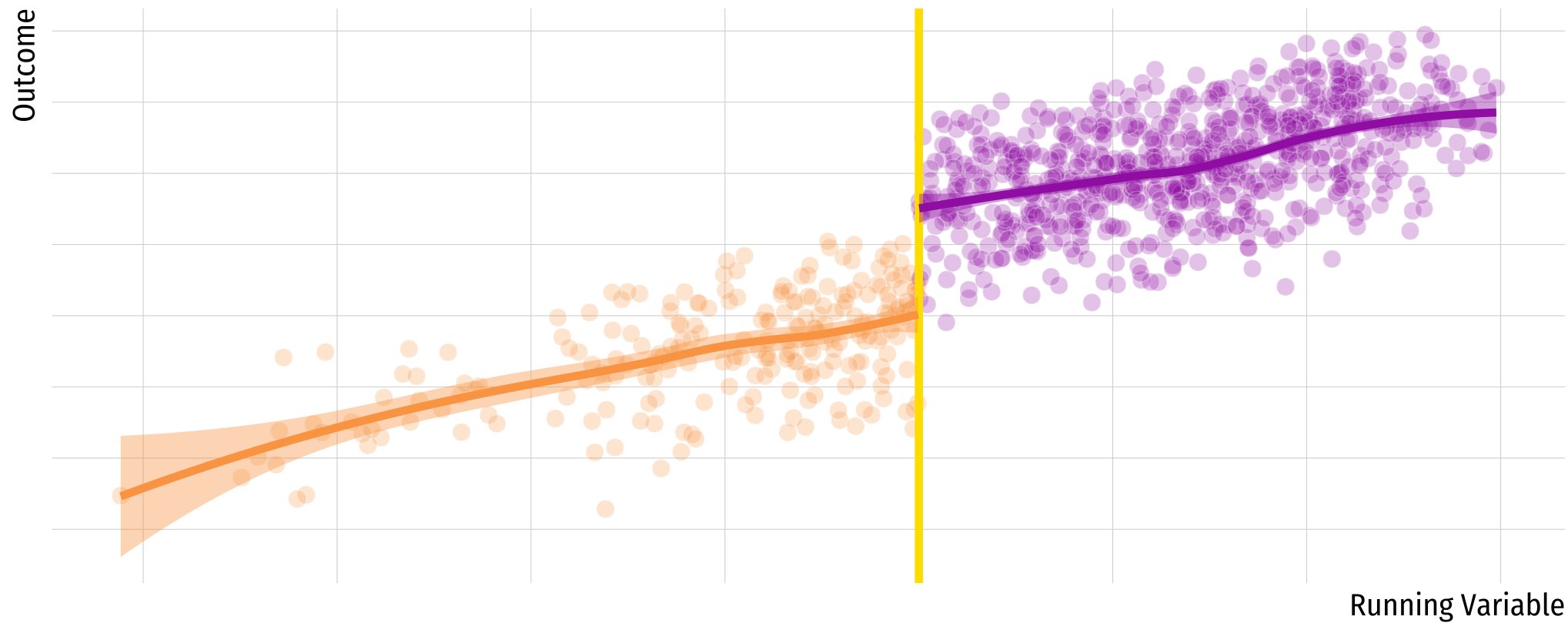
- Basically, regression discontinuities work under an **asymptotic assumption**:
- Let Y_i be the outcome of interest, Z_i the treatment assignment, R_i the running variable, and c the cutoff score:

$$Z_i = \begin{cases} 0 & R_i \leq c \\ 1 & R_i > c \end{cases}$$

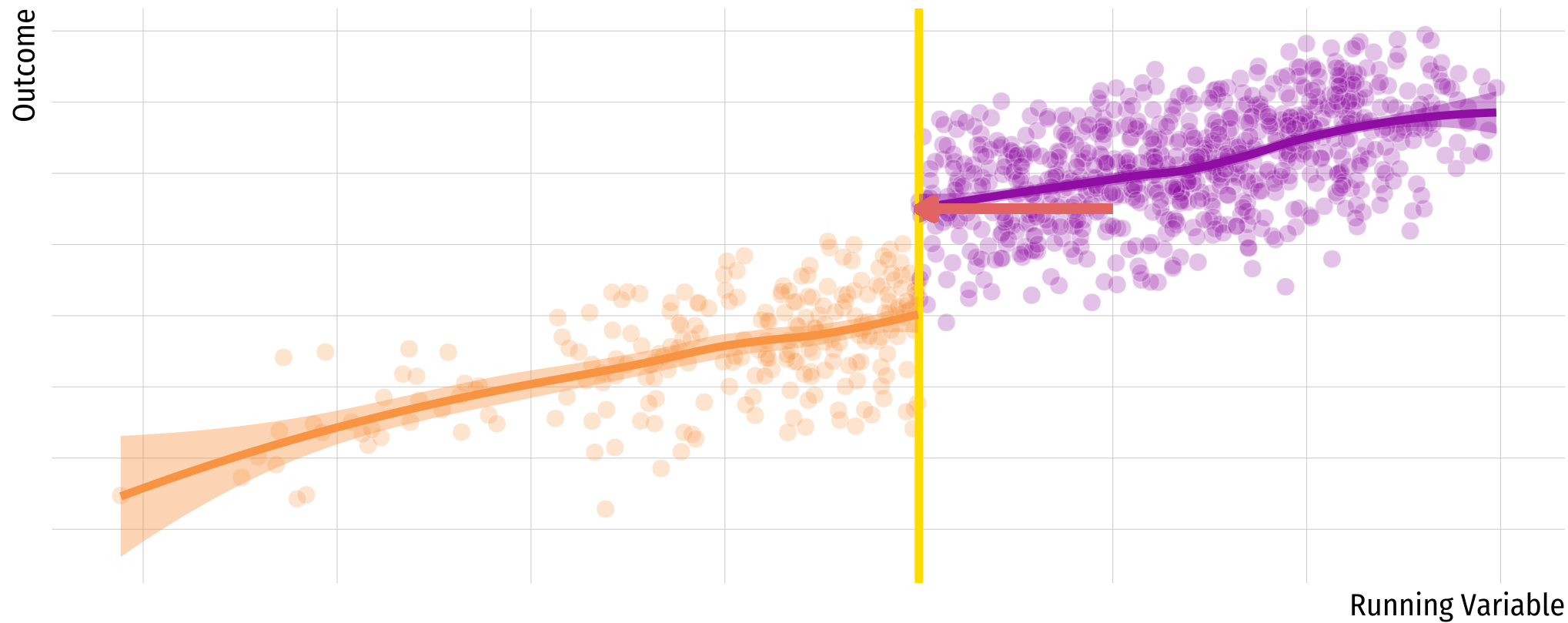
- Then, we can define the treatment effect δ as:

$$\delta = \lim_{\epsilon \rightarrow 0^+} E[Y_i | R_i = c + \epsilon] - \lim_{\epsilon \rightarrow 0^-} E[Y_i | R_i = c + \epsilon]$$

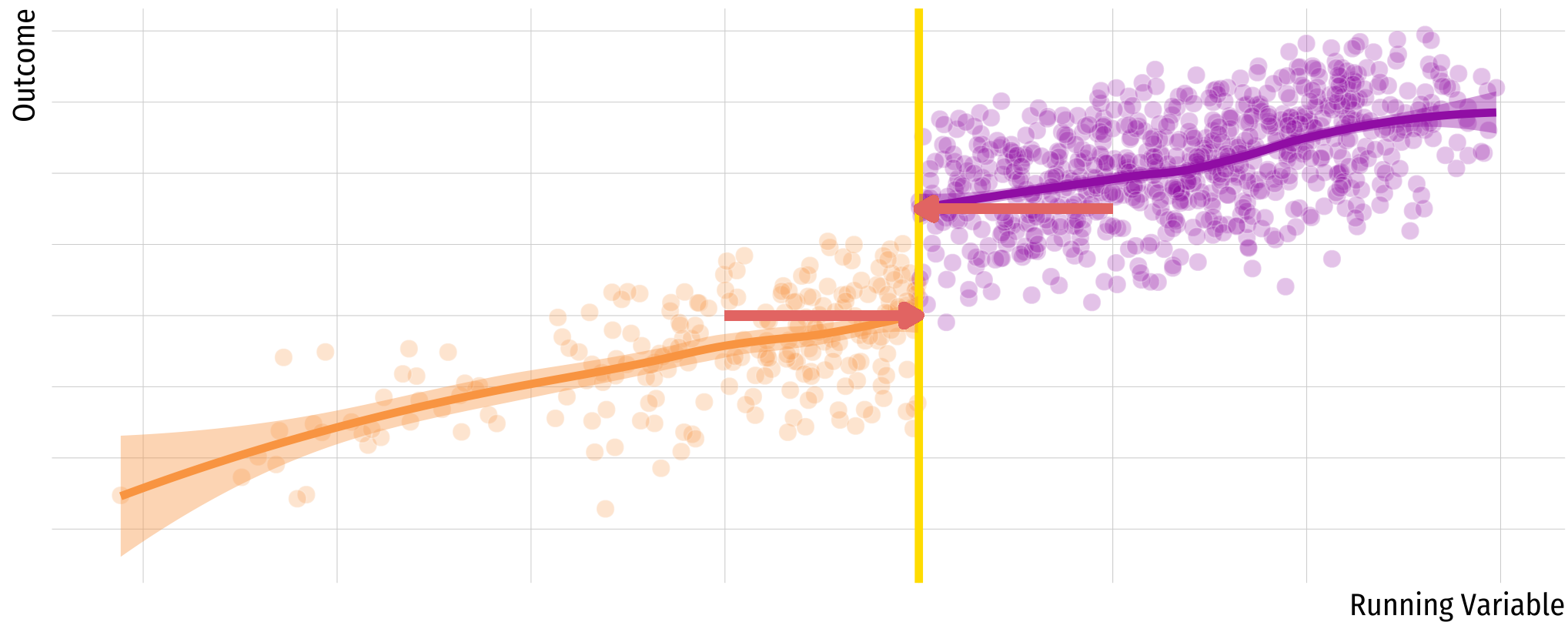
What does the limit expression mean?



What does the limit expression mean?



What does the limit expression mean?

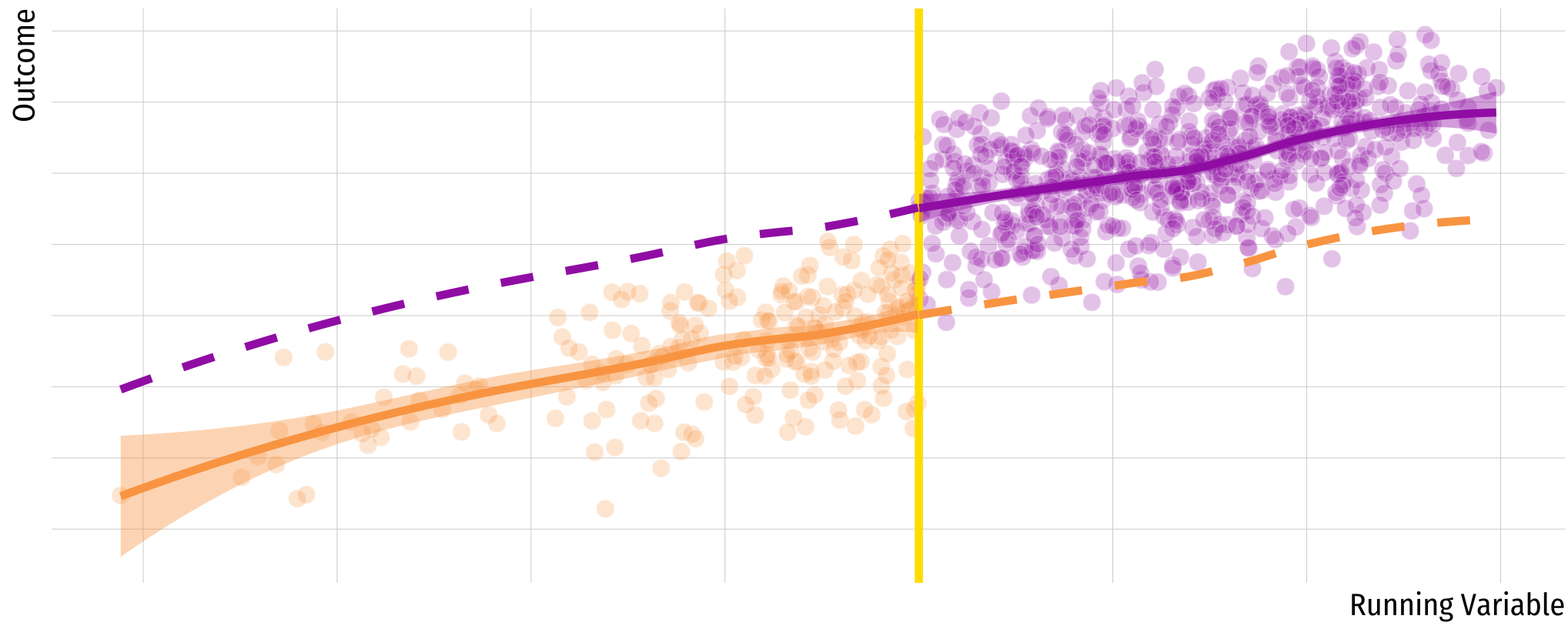


Conditions required for identification

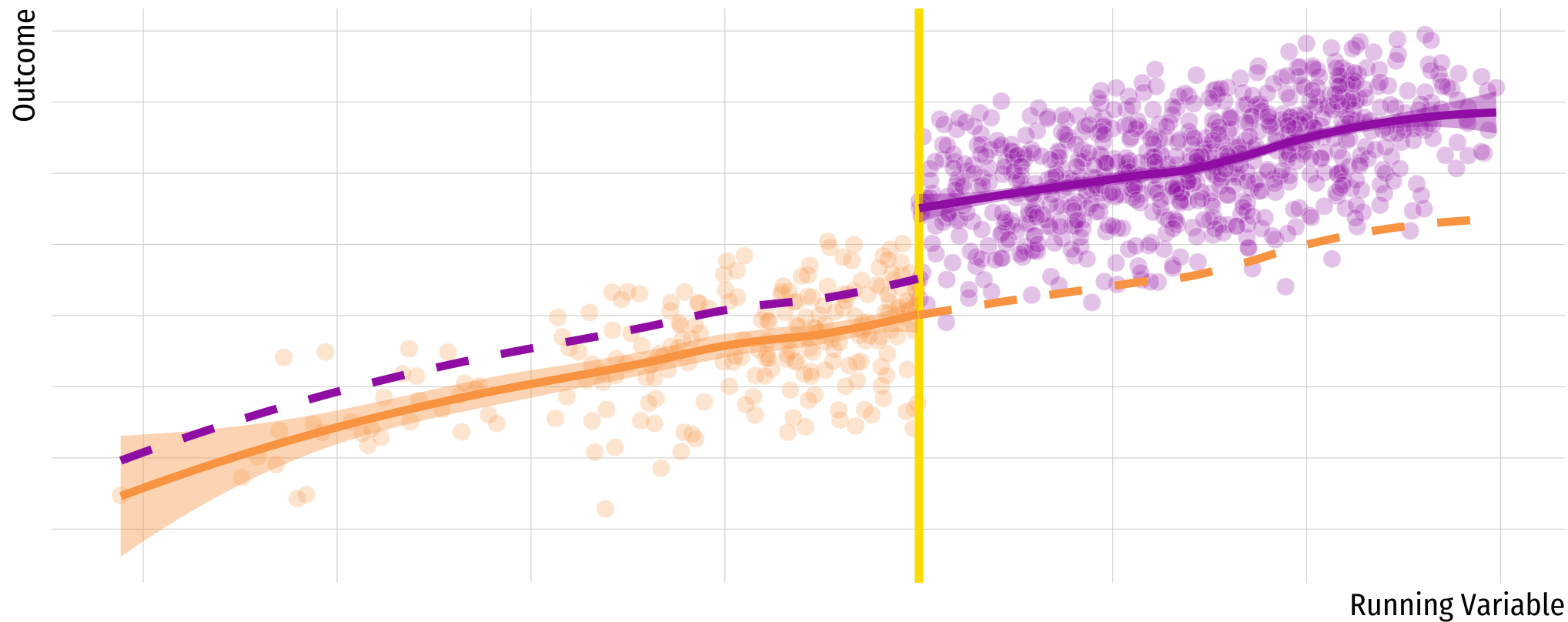
- Threshold rule **exists** and cutoff point is **known**
- The running variable R_i is **continuous** near c .
- Key assumption:

Continuity of $E[Y(1)|R]$ and $E[Y(0)|R]$ at $R=c$

Potential outcomes need to be smooth across the threshold



Potential outcomes need to be smooth across the threshold



How can I check if this assumption holds?

You can't! (it's an assumption)

Robustness checks:

- Check density across the cutoff
- Check RD for covariates

Estimation in practice

How do we actually estimate an RD?

- The simplest way to do this is to fit a regression:

$$Y_i = \beta_0 + \beta_1(R_i - c) + \beta_2\mathbf{I}[R_i > c] + \beta_3(R_i - c)\mathbf{I}[R_i > c]$$

How do we actually estimate an RD?

- The simplest way to do this is to fit a regression:

$$Y_i = \beta_0 + \beta_1 \underbrace{(R_i - c)}_{\text{Distance to the cutoff}} + \beta_2 \mathbf{I}[R_i > c] + \beta_3 \overbrace{(R_i - c)}^{\text{Distance to the cutoff}} \mathbf{I}[R_i > c]$$

How do we actually estimate an RD?

- The simplest way to do this is to fit a regression:

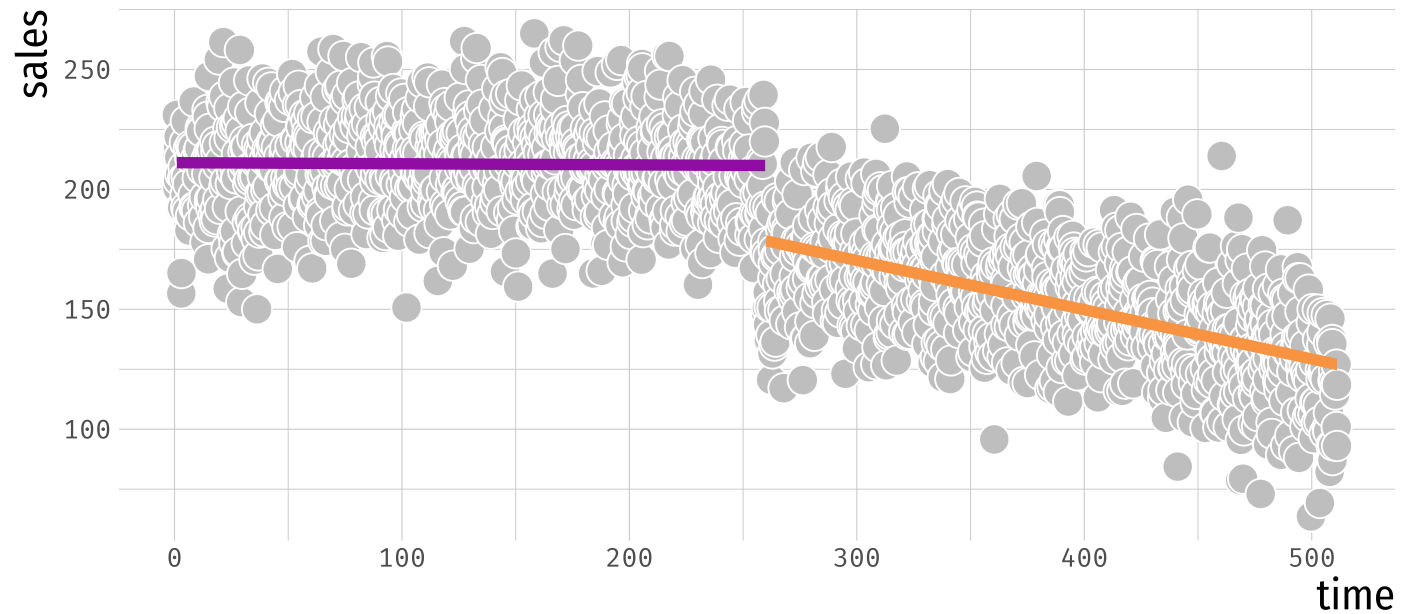
$$Y_i = \beta_0 + \beta_1(R_i - c) + \underbrace{\beta_2 \mathbf{I}[R_i > c]}_{\text{Treatment}} + \beta_3(R_i - c) \underbrace{\mathbf{I}[R_i > c]}_{\text{Treatment}}$$

- You want to add **flexibility** for each side of the cutoff.

Can you identify these parameters in a plot?

Let's see some examples: Sales using a linear model

```
sales <- sales %>% mutate(dist = c-time)  
lm(sales ~ dist + treat + dist*treat, data = sales)
```



Let's see some examples: Sales using a linear model

```
summary(lm(sales ~ dist + treat + dist*treat, data = sales))
```

```
##
## Call:
## lm(formula = sales ~ dist + treat + dist * treat, data = sales)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -65.738 -13.940   0.051  13.538  76.515
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 178.640954   1.300314  137.38  <2e-16 ***
## dist         0.205355   0.008882   23.12  <2e-16 ***
## treat        31.333952   1.842338   17.01  <2e-16 ***
## dist:treat   -0.200845   0.012438  -16.15  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.52 on 1996 degrees of freedom
## Multiple R-squared:  0.6939,    Adjusted R-squared:  0.6934
## F-statistic: 1508 on 3 and 1996 DF,  p-value: < 2.2e-16
```

We can be more flexible

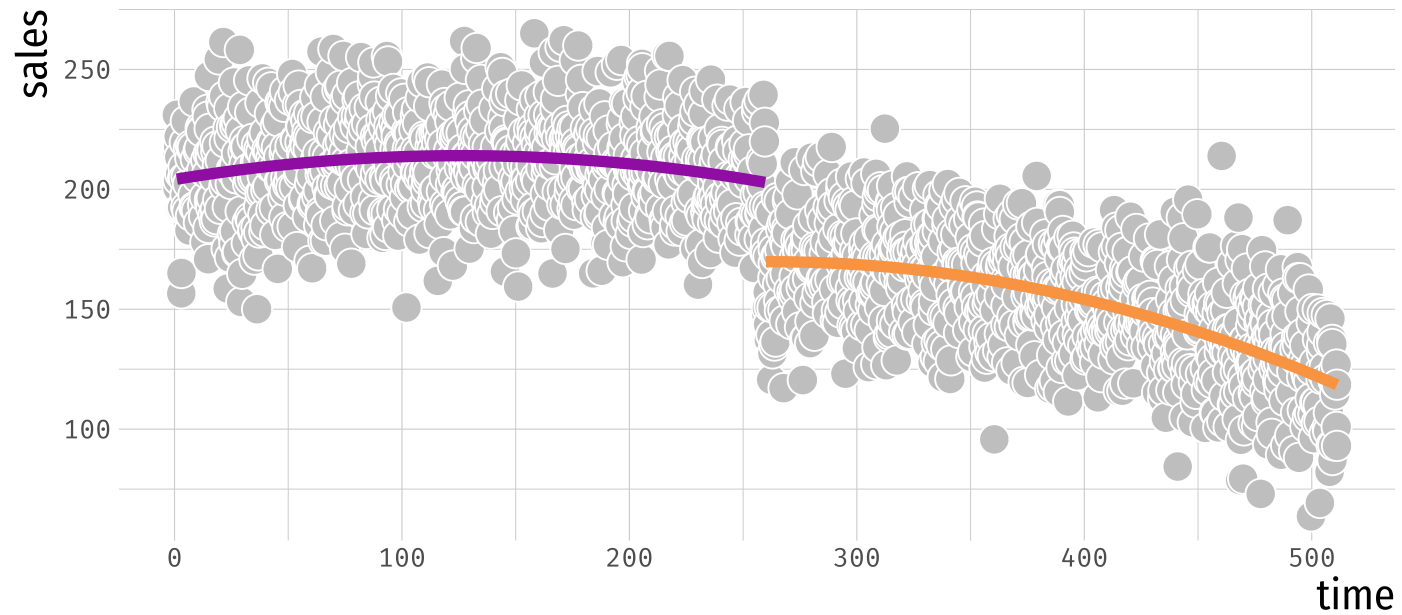
- The previous example just included linear terms, but you can also be more flexible:

$$Y_i = \beta_0 + \beta_1 f(R_i - c) + \beta_2 \mathbf{I}[R_i > c] + \beta_3 f(R_i - c) \mathbf{I}[R_i > c]$$

- Where f is any function you want.

What happens if we fit a quadratic model?

```
lm(sales ~ dist + I(dist^2) + treat + dist*treat + treat*I(dist^2), data = sales)
```



What happens if we fit a quadratic model?

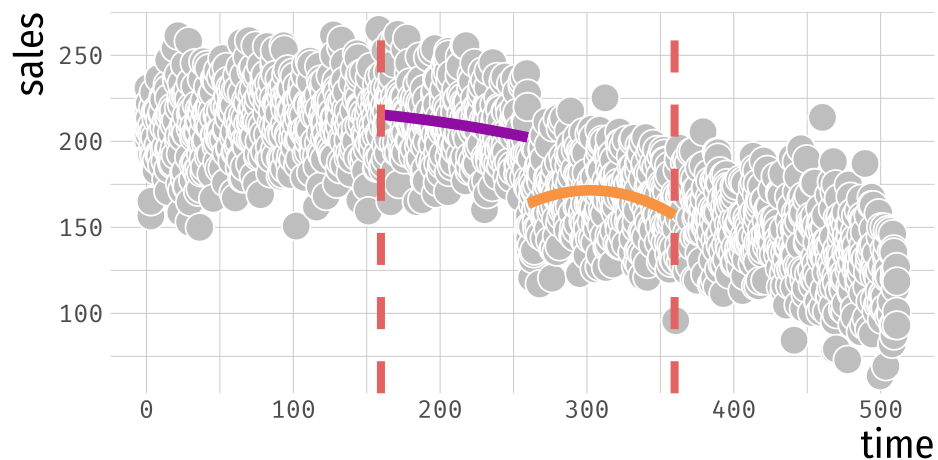
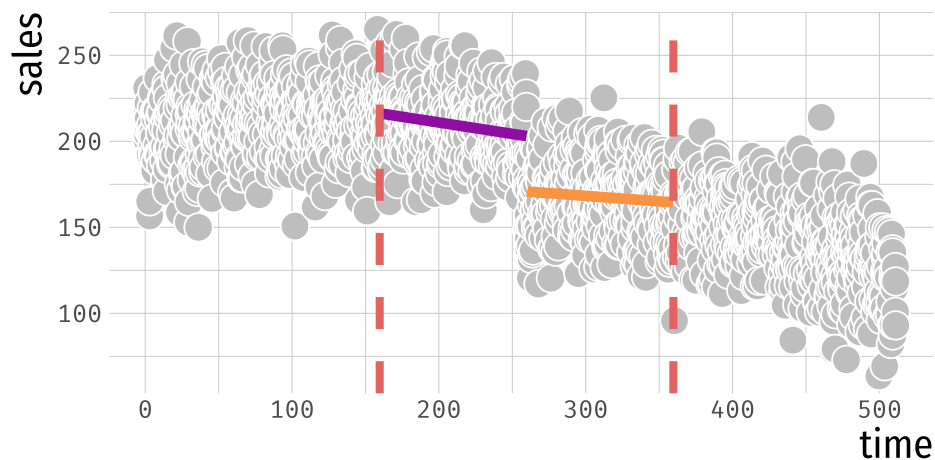
```
summary(lm(sales ~ dist + I(dist^2) + treat + dist*treat + treat*I(dist^2), data = sales))
```

```
##
## Call:
## lm(formula = sales ~ dist + I(dist^2) + treat + dist * treat +
##     treat * I(dist^2), data = sales)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -66.090 -13.979   0.239  13.154  76.656
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.698e+02  1.937e+00  87.665  < 2e-16 ***
## dist         -4.302e-03  3.556e-02  -0.121  0.903725
## I(dist^2)     -8.288e-04  1.363e-04  -6.083  1.41e-09 ***
## treat         3.308e+01  2.747e+00  12.041  < 2e-16 ***
## dist:treat    1.713e-01  4.964e-02   3.452  0.000569 ***
## I(dist^2):treat 2.034e-04  1.877e-04   1.084  0.278554
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.23 on 1994 degrees of freedom
## Multiple R-squared:  0.7029,    Adjusted R-squared:  0.7021
## F-statistic: 943.5 on 5 and 1994 DF,  p-value: < 2.2e-16
```

What happens if we only look at observations close to c ?

```
sales_close <- sales %>% filter(dist>-100 & dist<100)

lm(sales ~ dist + treat + dist*treat + treat, data = sales_close)
lm(sales ~ dist + I(dist^2) + treat + dist*treat + treat*I(dist^2), data = sales_close)
```



How do they compare?

```
summary(lm(sales ~ dist + treat + dist*treat + treat, data = sales_close))
```

```
##
## Call:
## lm(formula = sales ~ dist + treat + dist * treat + treat, data = sales_close)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -53.241 -14.764   0.268  12.938  57.811
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 170.84457    2.05528  83.125  <2e-16 ***
## dist         0.06345    0.03542   1.791   0.0736 .
## treat        32.21243    2.93614  10.971  <2e-16 ***
## dist:treat    0.06909    0.05047   1.369   0.1714
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.25 on 782 degrees of freedom
## Multiple R-squared:  0.5261,    Adjusted R-squared:  0.5243
## F-statistic: 289.4 on 3 and 782 DF,  p-value: < 2.2e-16
```

How do they compare?

```
summary(lm(sales ~ dist + I(dist^2) + treat + dist*treat + treat*I(dist^2), data = sales_close))
```

```
##
## Call:
## lm(formula = sales ~ dist + I(dist^2) + treat + dist * treat +
##     treat * I(dist^2), data = sales_close)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -50.080 -14.238  -0.463  12.740  54.231
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   163.550012    3.001833   54.483  < 2e-16 ***
## dist          -0.375526    0.136936   -2.742  0.006240 **
## I(dist^2)     -0.004415    0.001331   -3.317  0.000951 ***
## treat         38.757140    4.316684    8.978  < 2e-16 ***
## dist:treat     0.552254    0.195847    2.820  0.004927 **
## I(dist^2):treat 0.003975    0.001894    2.099  0.036121 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.13 on 780 degrees of freedom
## Multiple R-squared:  0.5328,    Adjusted R-squared:  0.5298
## F-statistic: 177.9 on 5 and 780 DF,  p-value: < 2.2e-16
```


Potential problems

- There are **many potential problems** with the previous examples:
 - Which polynomial function should we choose? Linear, quadratic, other?
 - What bandwidth should we choose? Whole sample? $[-100, 100]$?



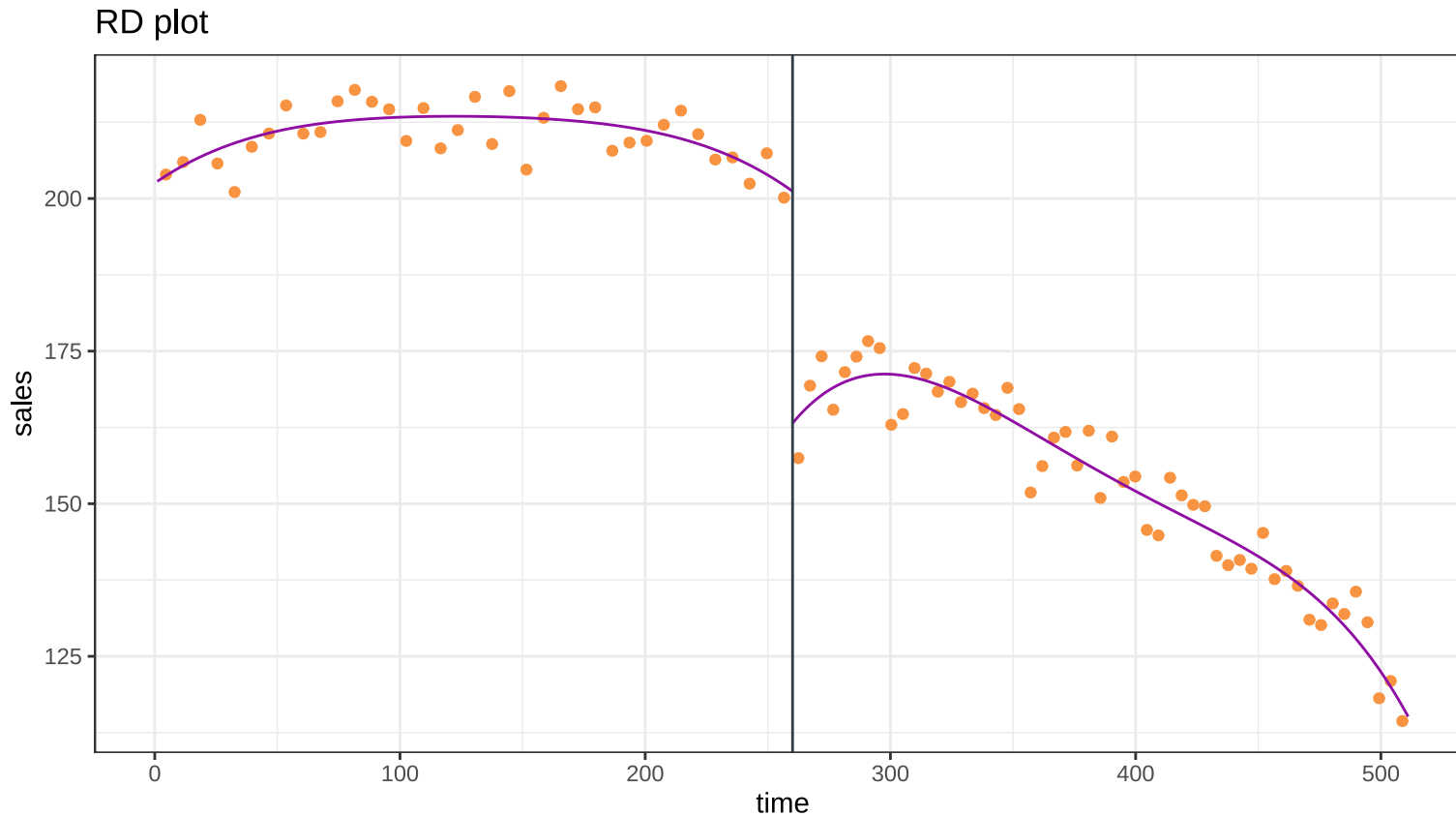
- There are some ways to address these concerns.

Package `rdrobust`

- Robust Regression Discontinuity introduced by Cattaneo, Calonico, Farrell & Titiunik (2014).
- Use of **local polynomial** for fit.
- **Data-driven optimal bandwidth** (bias vs variance).
- `rdrobust`: Estimation of LATE and opt. bandwidth
- `rdplot`: Plotting RD with nonparametric local polynomial.

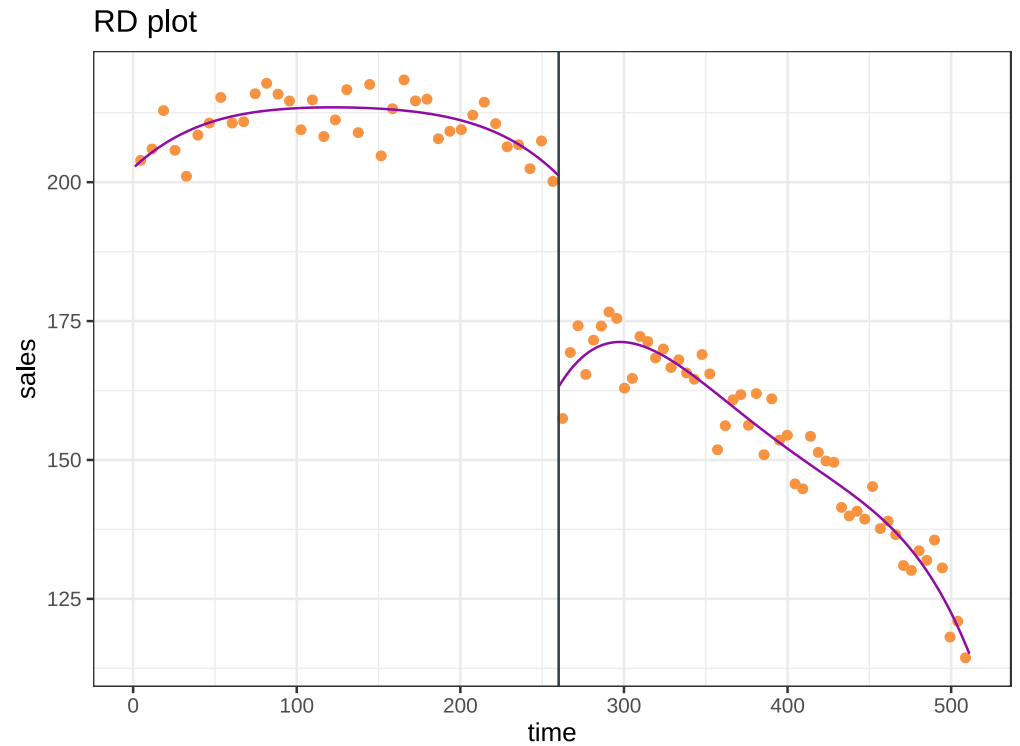
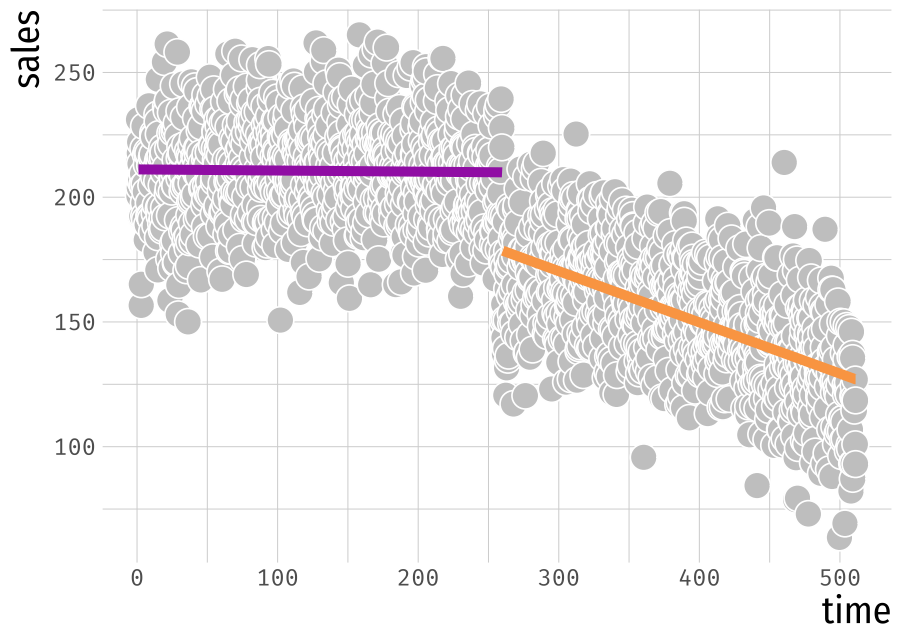
Let's compare with previous parametric results

```
rdplot(y = sales$sales, x = sales$time, c = c,  
       title = "RD plot", x.label = "time", y.label = "sales")
```



Let's compare with previous parametric results

```
rdplot(y = sales$sales, x = sales$time, c = c,  
       title = "RD plot", x.label = "time", y.label = "sales")
```



Let's compare with previous parametric results

```
summary(rdrobust(y = sales$sales, x = sales$time, c = c))
```

```
## Call: rdrobust
##
## Number of Obs.          2000
## BW type              mserd
## Kernel              Triangular
## VCE method              NN
##
## Number of Obs.          1000      1000
## Eff. Number of Obs.      202      213
## Order est. (p)            1         1
## Order bias (q)            2         2
## BW est. (h)             54.304     54.304
## BW bias (b)             87.787     87.787
## rho (h/b)              0.619     0.619
## Unique Obs.             1000     1000
##
## =====
##      Method      Coef. Std. Err.      z    P>|z|      [ 95% C.I. ]
## =====
## Conventional  -37.434    4.344   -8.618   0.000  [-45.948 , -28.921]
## Robust         -         -    -7.610   0.000  [-48.596 , -28.691]
## =====
```

How do we weight observations?

- `rdrobust` uses `rdbwselect()` function (by default) to estimate a data-driven bandwidth (i.e. what observations we are going to use for estimation).
 - If we use a bandwidth, does this mean that the RD is estimating an effect for that population within the bandwidth?
- **Kernels** are also important in this context:
 - How do I weight observations within the bandwidth (e.g. uniform, triangle)

Observing kernels

Takeaway points

- RD designs are **great** for causal inference!
 - Strong internal validity
 - Number of robustness checks
- **Limited** external validity.
- Make sure to check your data:
 - Discontinuity in treatment assignment
 - Density across the cutoff
 - Smoothness of covariates



References

- Angrist, J. and S. Pischke. (2015). "Mastering Metrics". *Chapter 4*.
- Calonico, Cattaneo and Titiunik. (2015). "rdrobust: An R Package for Robust Nonparametric Inference in Regression-Discontinuity Designs". *R Journal* 7(1): 38-51.
- Heiss, A. (2020). "Program Evaluation for Public Policy". *Class 10: Regression Discontinuity I, Course at BYU*.
- Lee, D. and T. Lemieux. (2010). "Regression Discontinuity in Economics". *Journal of Economic Literature* 48, pp 281-355.