

STA 235H - Potential Outcomes

Fall 2021

McCombs School of Business, UT Austin

Let's do a short exercise

Take three pieces of papers: One of each color

Don't look!

The problem

- Imagine **everyone** here has a headache
- Everyone takes an aspirin to mitigate the headache
- How many students **still have a headache** (look at your **orange** paper)

Wow, did the aspirin work?

How? Potential Outcomes Framework

What? Causal Estimands

Why? Causal Questions and Study Design

The "How": Potential outcomes framework



Geoffrey Supran
@GeoffreySupran



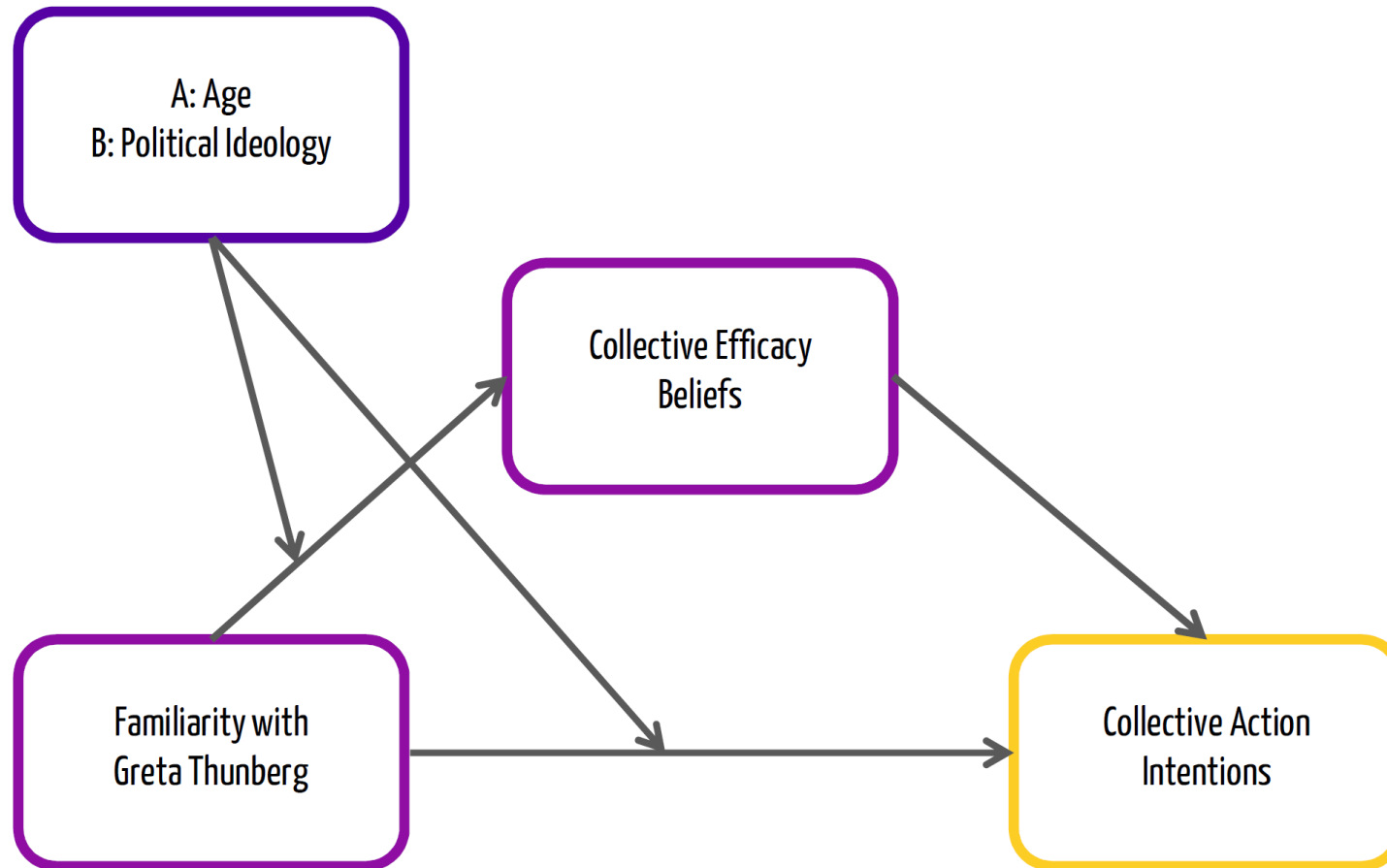
"The @GretaThunberg Effect" is now an empirically demonstrated, peer-reviewed phenomenon:

"We find that those who are more familiar with Greta Thunberg have higher intentions of taking collective actions to reduce global warming."

Open access: onlinelibrary.wiley.com/doi/epdf/10.1111/jasp.12727



the Year by some, and asked to “work on her anger management issues” by others (Alter et al., 2019; McCarthy, 2019). The present study, to date, is one of the first to present empirical evidence supporting the “Greta Thunberg Effect,” and to offer a potential explanation of why a young leader could be a powerful influence on collective action. We find that familiarity with Greta Thunberg is



**What do you think are the biggest
issues here?**



Khoa Vu

@KhoaVuUmn



"The Greta effect" Effect: Your misuse of causal language is never too wrong to make famous people retweet your study.



Hillary Clinton  @HillaryClinton · Jan 28

Data proving @GretaThunberg right—"you are never too small to make a difference." twitter.com/GeoffreySupran...

11:22 AM · Jan 29, 2021 · Twitter Web App

What about other topics?

Before we start...

Be clear about your language

Before we start...

Be clear about your language

Be clear about your data

Before we start...

Be clear about your language

Be clear about your data

Be clear about your assumptions

What is Causal Inference?

Inferring the effect of one thing on another thing

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- "Providing students support when filling out FAFSA forms improves college access and completion."

A world of potential (outcomes)

- Under a binary treatment or intervention, there are **two potential worlds**:

A world of potential (outcomes)

- Under a binary treatment or intervention, there are **two potential worlds**:
- **World 1**: You take the pill
- **World 2**: You don't take the pill



A world of potential (outcomes)

- A **potential outcome** is the outcome under each of these scenarios or "worlds".
 - *There will be one for each path!*

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Fundamental Problem of Causal Inference

**What are the potential outcomes
for our previous example?**

Potential Outcomes Examples

- "My headache went away because I took an aspirin".

Outcome if I take an aspirin/ Outcome if I don't take an aspirin

- "The new marketing campaign increased our sales by 20%"

Sales with a marketing campaign/ Sales without a marketing campaign

- "Providing students support when filling out FAFSA forms improves college access and completion."

College access and completion with support/ College access and completion without support

What are some causal questions you might be interested in?

Potential Outcomes Framework

Let's introduce some notation:

- Let Y_i be the observed outcome for unit i (e.g. whether I have a headache or not in an hour).
- Let Z_i be the treatment or intervention (e.g. taking a pill).

Then,

$$Y_i | (Z_i = 1) \triangleq Y_i(1)$$

where $Y_i(1)$ is the **potential outcome under treatment**.

In the same fashion,

$$Y_i | (Z_i = 0) \triangleq Y_i(0)$$

where $Y_i(0)$ is the **potential outcome under control**.

Potential Outcomes Framework

This means that we can write the observed outcome as a function of the *potential outcomes*:

$$\rightarrow Y_i = Z_i \cdot Y_i(1) + (1 - Z_i) \cdot Y_i(0)$$

- This definition will be useful because we can see this as a **missing data problem**.

Causal Effects

Individual Causal Effect

$$ICE_i = Y_i(1) - Y_i(0)$$

Causal Effects

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Can we ever observe individual causal effects?

Causal Effects

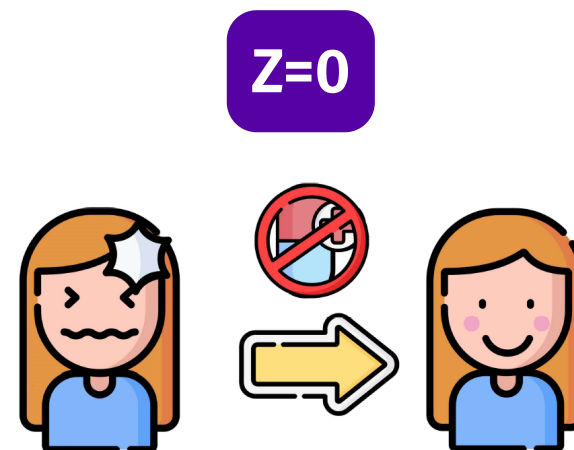
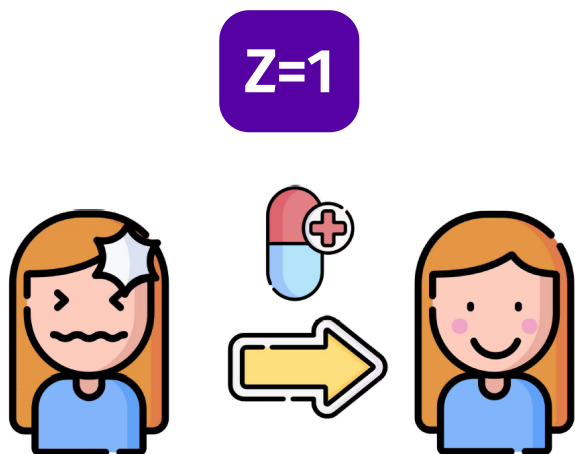
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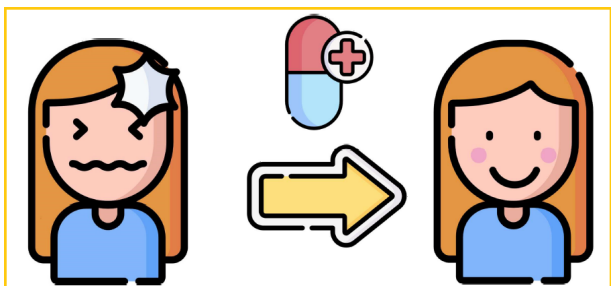
No!*

Only one realization

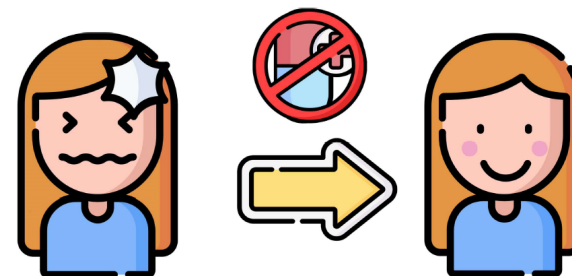


Only one realization

$Z=1$



$Z=0$



The "What": Causal estimands, estimates,
and estimators

Estimands vs Estimates vs Estimators

Estimand

A quantity we want to estimate

Estimator

A rule for calculating an estimate based on data

Estimate

The result of an estimation

Estimands vs Estimates vs Estimators

Estimand

A quantity we want to estimate

E.g.: Population mean

μ

Estimator

A rule for calculating an estimate based on data

E.g.: Sample mean

$$\frac{1}{n} \sum_i Y_i$$

Estimate

The result of an estimation

E.g.: Result of the sample mean for a given sample S

$\hat{\mu}$

Estimands vs Estimates vs Estimators



estimand

Ingredients	Method
150g unsalted butter, plus extra for greasing	1. Heat the oven to 160C/140C fan/gas 3. Grease and base line a 1 litre heatproof glass pudding basin and a 450g loaf tin with baking parchment.
150g plain chocolate, broken into pieces	
150g plain flour	
1/2 tsp baking powder	2. Put the butter and chocolate into a saucepan and melt over a low heat, stirring. When the chocolate has all melted remove from the heat.
1/2 tsp bicarbonate of soda	
200g light muscovado sugar	
2 large eggs	

estimator



estimate

Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

Average Treatment Effect (ATE)

Average Treatment Effect on the Treated (ATT)

Conditional Average Treatment Effect (CATE)

Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

$$ATE = E[Y(1) - Y(0)]$$

$$ATT = E[Y(1) - Y(0) | Z = 1]$$

$$CATE = E[Y(1) - Y(0) | X]$$

Getting around the fundamental problem of causal inference

- Let's go back to our original example: Does a pill help reduce headaches?

i	z	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	0	1	?	1	?
2	1	1	1	?	?
3	1	0	0	?	?
4	0	0	?	0	?
5	0	1	?	1	?
6	1	0	0	?	?

Getting around the fundamental problem of causal inference

- We have a **missing data problem**

i	z	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	0	1	?	1	?
2	1	1	1	?	?
3	1	0	0	?	?
4	0	0	?	0	?
5	0	1	?	1	?
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Getting around the fundamental problem of causal inference

- Compare those who **took the pill** to the ones **did not take it**.

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- What is the **estimand**?

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Difference in sample means

Getting around the fundamental problem of causal inference

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Difference in sample means

- What is the **estimate** and *how do we interpret it*?

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Difference in sample means

- What is the **estimate** and *how do we interpret it?*

33.3 percentage point decrease in probability of having a headache

What could be the problem with comparing the sample means?

The "Why": Causal questions and study designs

Under what assumptions is our estimate causal?

We are using:

$$\hat{\tau} = \frac{1}{3} \left(\sum_{i \in Z=1} Y_i - \sum_{i \in Z=0} Y_i \right)$$

to estimate:

$$\tau = E[Y_i(1) - Y_i(0)]$$

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Let's do some math

Under what assumptions is our estimate causal?

$$\begin{aligned}\tau &= E[Y_i(1) - Y_i(0)] \\ &= E[Y_i(1)] - E[Y_i(0)]\end{aligned}$$

Key assumption:

Ignorability

- Ignorability means that the potential outcomes $Y(0)$ and $Y(1)$ are independent of the treatment, e.g. $(Y(0), Y(1)) \perp\!\!\!\perp Z$.

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 - Remember that if $A \perp\!\!\!\perp B \rightarrow E[A|B] = E[A]$

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$$\tau = E[Y_i(1)] - E[Y_i(0)] = \underbrace{E[Y_i(1)|Z = 1]}_{\text{Obs. Outcome for T}} - \overbrace{E[Y_i(0)|Z = 0]}^{\text{Obs. Outcome for C}}$$

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Ignorability Assumption

We can just "ignore" the missing data problem:

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6	1	0	0		
			1/3	2/3	

Let's do a little exercise

Look at your **green** piece of paper and go to the following website



Would you go to a physician/urgent care?

Now let's assume I randomly allocate whether you go or not go to the hospital



Do the previous results hold?

Randomization is an awesome tool for causal inference

Main takeaway points

Causal Inference is hard

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- Check **validity** of assumptions (*Is ignorability plausible? Am I controlling for the right covariates?*)

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Causal Inference is hard

- Think about the **causal problem**
- Always **look at your data**
- Check **validity** of assumptions (*Is ignorability plausible? Am I controlling for the right covariates?*)
- Most of this chapter will be spent on looking for **exogenous variation** to make the ignorability assumption happen.

Next week

- **Randomized Controlled Trials:**
 - Pros and Cons
 - Concept of validity
 - A/B Testing



References

- Angrist, J. & S. Pischke. (2015). "Mastering Metrics". *Chapter 1*.
- Cunningham, S. (2021). "Causal Inference: The Mixtape". *Chapter 4: Potential Outcomes Causal Model*.
- Neil, B. (2020). "Introduction to Causal Inference". *Fall 2020 Course*