STA 235H - Multiple Regression: Outliers, Multicollinearity, and Heteroskedasticity

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Why should we inspect our data before doing anything else?

Identifying outliers

- How do we identify outliers?
  - Visual inspection (e.g. plots, tables)
  - Creating thresholds (e.g. z-scores, IQ)
- There is no definite way to identify outliers
  - Like the characterization of pornography, "I know it when I see it" (P. Stewart, 1964)

Let's go to R

What to do with outliers?

1. Check them!
    - Make sure there's no coding error; try to understand what's happening there.

2a. If they are wrongly coded:

- You can remove them, always adding a note of why you did so
- Be aware of sample selection!

2b. If they are correctly coded:

- Run analysis both with and without outliers (don't just drop them!).
- Robust results: Do not depend exclusively on a few observations.

What about multicollinearity?

What's the problem with multicollinearity?

- What do you think would happen if I ran this regression?

$$Cancer = \beta_0 + \beta_1 PackCigarettes + \beta_2 NCigarettes + \varepsilon$$

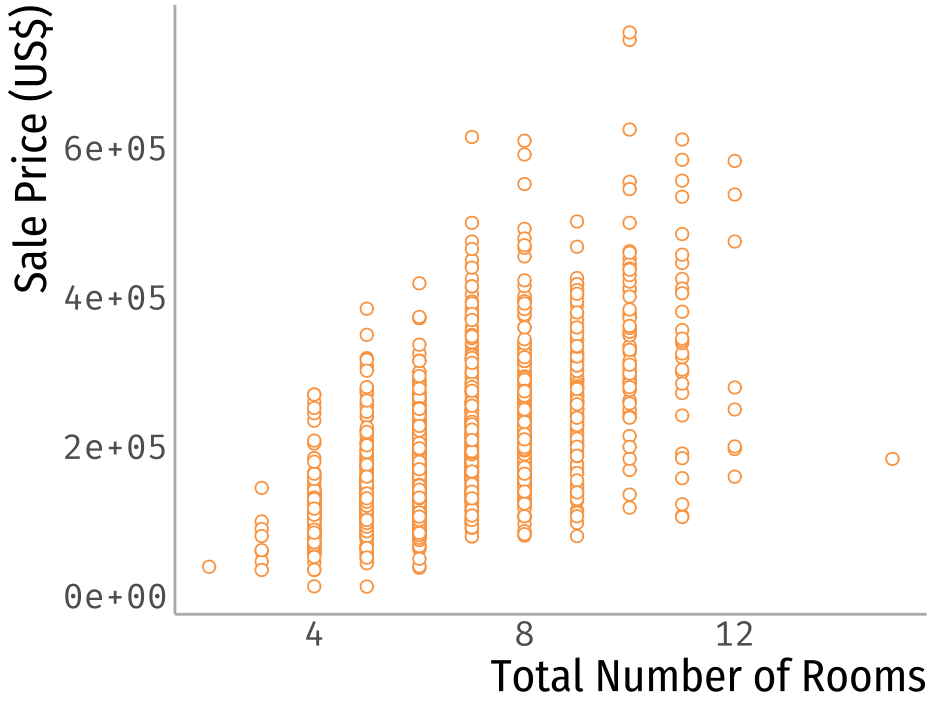
- If two covariates are perfectly collinear, we can't run the regression
    - R will drop one of them!
- But what happens when they are highly correlated but not *perfectly collinear*?

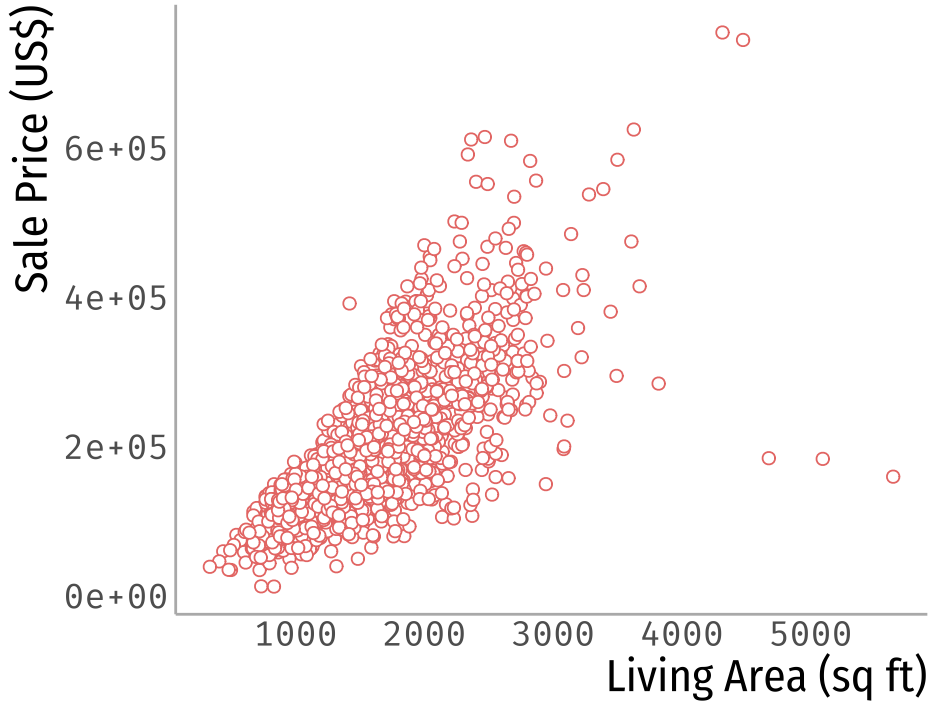
Let's look at some housing data

- Housing data from Ames, Iowa:

```
library(tidyverse)
housing <- read.csv("https://raw.githubusercontent.com/maibennett/sta235/main/exampleSite/content/Classes/Week3/2_OLS_Issu
housing <- housing %>% filter(Bldg.Type=="1Fam")
```

How do number of rooms and living area relate to sale price?





Let's run a model

```
lm_full <- lm(SalePrice ~ TotRms.AbvGrd + Gr.Liv.Area + Garage.Cars + Fireplaces + Total.Bsmt.SF, data = housing)

summary(lm_full)
```

```
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -41874.0627  4464.9400 -9.3784   0.0000
## TotRms.AbvGrd   1314.4670  1079.3420  1.2178   0.2234
## Gr.Liv.Area       62.9742     3.4525 18.2403   0.0000
## Garage.Cars    33846.9897  1452.9358 23.2956   0.0000
## Fireplaces      8029.2652  1558.3305  5.1525   0.0000
## Total.Bsmt.SF     53.8685     2.4248 22.2154   0.0000
```

- What can you say about total rooms?

Multicollinearity in Regressors

- If two covariates are highly correlated, it is difficult to separate the contribution of each!
    - E.g. They move together
- Be careful with interpretations
    - The same with extrapolation zones!

Can we do something about it?

- Exclude one variable
  - Depends on what you are analyzing.
- Create a new variable that captures clean information
  - E.g. Residuals
- Aggregate both variables into one:
  - Linear combination (e.g. add them)
  - Create an index

What about binary responses?

Binary Outcomes

- You have probably used **binary outcomes** in regressions, but do you know the issues that they may bring to the table?

What can we do about them?

How to handle binary outcomes?

Linear Probability Model

Logistic Regression

How to interpret a LPM?

- A Linear Probability Model is just a traditional regression with a binary outcome
- $\hat{\beta}$'s interpreted as change in probability

$$E[Y|X_1, \ldots, X_P] = Pr(Y = 0|X_1, \ldots, X_p) \cdot 0 + Pr(Y = 1|X_1, \ldots, X_p) \cdot 1$$

$$= Pr(Y = 1|X_1, \ldots, X_p)$$

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- Example:

$$GradeA = \beta_0 + \beta_1 \cdot Study + \varepsilon$$

- $\hat{\beta}_1$ is the average change in probability of getting an A if I study one more hour.
- Studying one more hour is associated with an increase in the probability of getting an A of $\hat{\beta}_1 \times 100$ **percentage points**.

Let's look at an example

- Home Mortgage Disclosure Act Data (HMDA) from the AER package

```
library(AER)

data("HMDA")

hmda <- data.frame(HMDA)

head(hmda)
```

```
##   deny pirat hirat     lvrat chist mhist phist unemp selfemp insurance condomin
## 1   no 0.221 0.221 0.8000000     5     2    no   3.9      no         no       no
## 2   no 0.265 0.265 0.9218750     2     2    no   3.2      no         no       no
## 3   no 0.372 0.248 0.9203980     1     2    no   3.2      no         no       no
## 4   no 0.320 0.250 0.8604651     1     2    no   4.3      no         no       no
## 5   no 0.360 0.350 0.6000000     1     1    no   3.2      no         no       no
## 6   no 0.240 0.170 0.5105263     1     1    no   3.9      no         no       no
##   afam single hschool
## 1   no     no     yes
## 2   no    yes     yes
## 3   no     no     yes
## 4   no     no     yes
## 5   no     no     yes
## 6   no     no     yes
```

Probability of someone getting a mortgage loan denied?

- Getting mortgage denied (1) based on race, conditional on payments to income ratio (`pirat`)

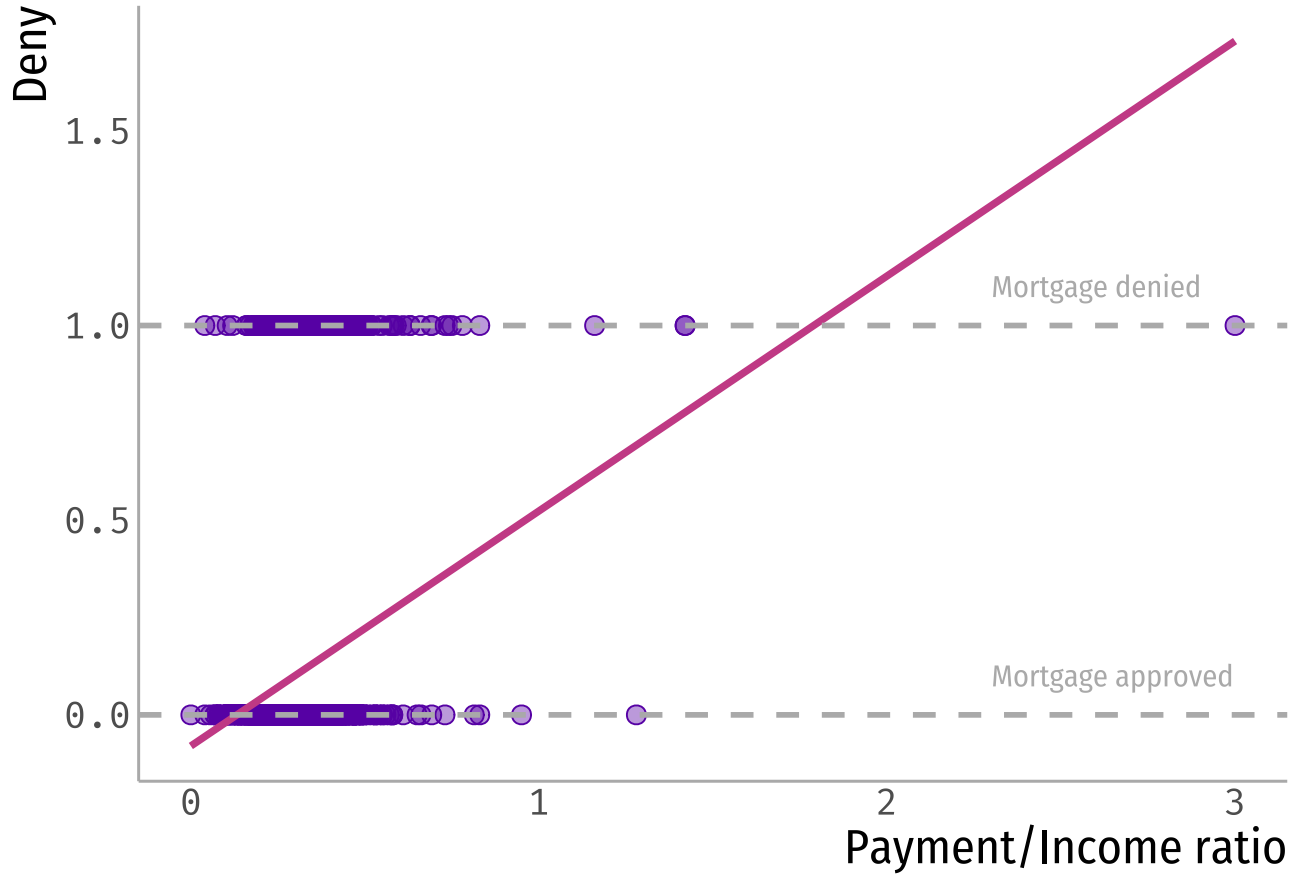
```
hmda <- hmda %>% mutate(deny = as.numeric(deny) - 1)

summary(lm(deny ~ pirat + factor(afam), data = hmda))
```

```
##
## Call:
## lm(formula = deny ~ pirat + factor(afam), data = hmda)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.62526 -0.11772 -0.09293 -0.05488  1.06815
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)     -0.09051    0.02079  -4.354 1.39e-05 ***
## pirat            0.55919    0.05987   9.340  < 2e-16 ***
## factor(afam)yes  0.17743    0.01837   9.659  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3123 on 2377 degrees of freedom
## Multiple R-squared:  0.076,    Adjusted R-squared:  0.07523
## F-statistic: 97.76 on 2 and 2377 DF,  p-value: < 2.2e-16
```

- Holding payment-to-income ratio constant, an AA client has a probability of getting their loan denied that is 18 pp higher, on average, than a non AA client.
- Being AA is associated to an average increase of 0.177 in the probability of getting a loan denied compared to a non AA, holding payment-to-income ratio constant.

How does this LPM look?



Issues with a LPM?

- Main problems:
  - Non-normality of the error term
  - Heteroskedasticity (i.e. variance of the error term is not constant)
  - Predictions can be outside [0,1]
  - LPM imposes linearity assumption

Issues with a LPM?

- Main problems:
  - Non-normality of the error term → Hypothesis testing
  - Heteroskedasticity → Validity of SE
  - Predictions can be outside [0,1] → Issues for prediction
  - LPM imposes linearity assumption → Too strict?

Are there solutions?

- Don't use small samples: With the CLT, non-normality shouldn't matter much.
- Saturate your model: In a fully saturated model (i.e. include dummies and interactions), CEF is linear.
- Use robust standard errors: Package `estimatr` in R is great!

Run again with robust standard errors

```
library(estimatr)

model1 <- lm(deny ~ pirat + factor(afam), data = hmda)
model2 <- lm_robust(deny ~ pirat + factor(afam), data = hmda)
```

	Model 1	Model 2
(Intercept)	-0.091***	-0.091**
	(0.021)	(0.031)
pirat	0.559***	0.559***
	(0.060)	(0.095)
factor(afam)yes	0.177***	0.177***
	(0.018)	(0.025)
Std.Errors		HC2
+ p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001		

- Can you interpret these parameters? Do they make sense?

Most issues are solvable, but...

What about prediction?

Logistic Regression

- Typically used in the context of binary outcomes (*Probit is another popular one*)
- Nonlinear function to model the conditional probability function of a binary outcome.

$$Pr(Y = 1|X_1, \ldots, X_p) = F(\beta_0 + \beta_1 X_1 + \ldots + \beta_p X_p)$$

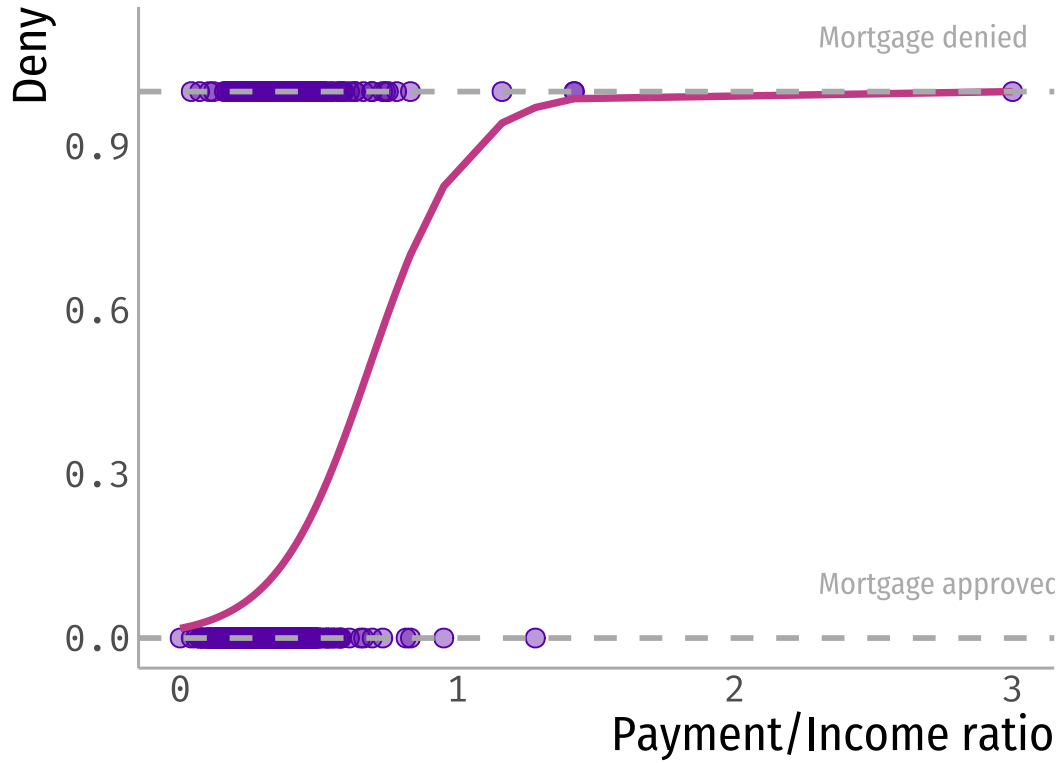
Where in a logistic regression: $F(x) = \frac{1}{1+exp(-x)}$

- *In the LPM,* $F(x) = x$

How does this look in a plot?

```
logit1 <- glm(deny ~ pirat, family = binomial(link = "logit"),
              data = hmda)

prob <- predict(logit1, type = "response") # probabilities
```



When will we use logistic regression?

- As you discovered in the readings, logit is great for prediction (much better than LPM).
- For explanation, however, LPM simplifies interpretation.

Use LPM for explanation and logit for prediction

(but remember robust SE!)

Takeaway points

- Always make sure to check your data:
    - What are analyzing? Does the data behave as I would expect? Should I exclude observations?
- Is it a problem if my covariates are highly correlated?
    - Sometimes yes, sometimes no.
- For LPM, always include robust standar errors!

References

- Ismay, C. & A. Kim. (2021). “Statistical Inference via Data Science”. Chapter 6 & 10.