STA 235H - Classification and Regression Trees (CART)

Fall 2023

McCombs School of Business, UT Austin

Announcements

- Next week will be the **last class with new material**.
- The final week of class will be for a **review session**.
  - Final Trivia!
- You need to choose a topic for **Homework 6**
  - Remember that this homework cannot be dropped.
  - All tasks have the same difficulty.
  - You will only be competing with people that choose your same dataset.

Where we've been...

- Talking about **bias vs variance** trade-off.
- **Linear models, model selection and regularization**:
  - Linear regressions.
  - Stepwise selection.
  - Ridge and Lasso regression.

… and where we're going.

- Continue on our prediction journey:
  - Decision Trees: Classification and Regression Trees (CART)
  - Activity in R: Remember to try to complete it before the end of the class!

Before we start... knowledge check!

- Ridge and lasso regression **add bias to a linear model to reduce variance**:
  - Remember that when we fit a ridge or lasso regression, we use all the predictors we have in our data!
- λ represents the **ridge/lasso penalty**: The larger the λ the smaller the (sum of) coefficients, e.g. $\sum_k \beta_k^2$ or $\sum_k |\beta_k|$.
  - We "mute" or decrease the relation between predictors and outcome.

Q1: What is the main difference (in terms of the final model) between Ridge and Lasso regression?

Trees, trees everywhere!

From the videos/readings, how would you explain to someone what a decision tree is?

Idea behind Decision Trees

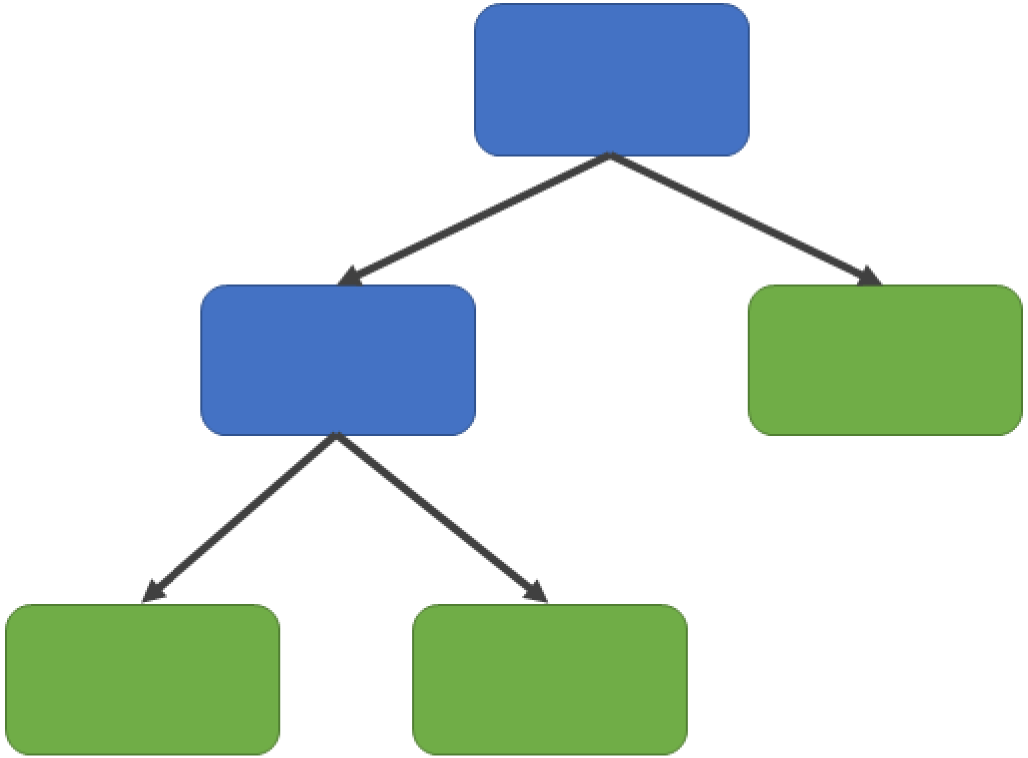
- Create a flow chart for making decisions
  - *How do we classify an individual or what value do we assign to an observation?*
- … But there are many decisions!
  - How many variables do we use?
  - How do we sort them? In what order do we place them?
  - How do we split them?
  - How deep do we go?

Q2: What is the main disadvantage of a shallower tree (compared to a deeper tree)?

a) Higher variance

b) Higher bias

Structure of Decision Trees



Structure:

- Root node
- Internal nodes
- Leaves

Why do we like/not like Decision Trees?

Main advantages

- Simple interpretation
- Mirror human decision-making
- Graphic displays!
- Handle categorical variables

Main disadvantages

- Overfitting
- Not very accurate/not very robust

Let's start with a simple example

Remember our Hbo Max example?

Predict who will cancel their subscription

We have some information:

- `city`: Whether the customer lives in a big city or not
- `female`: Whether the customer is female or not
- `age`: Customer's age (in years)
- `logins`: Number of logins to the platform in the past week.
- `succession`: Whether the person has watched the Succession or not.
- `unsubscribe`: Whether they canceled their subscription or not.

The prediction task: Classification

- Our outcome is **binary**, so this is a **classification task**.
- Let's start looking at **two variables**:

City & Succession

- Which one do you think should be at the top of the tree?

How do we decide?

- Recursive Binary Splitting:
  - Divide regions of covariates in two (recursively).
  - This works both for continuous and categorical/binary variables
- We test out every covariate and see which one reduces the error the most in our predictions
  - In regression tasks, we can use RMSE.
  - In classification tasks, we can use accuracy/classification error rate, Gini Index, or entropy

$$G = \sum_{k=0}^{1} \hat{p}_{mk}(1 - \hat{p}_{mk})$$

where $\hat{p}_{mk}$ is the proportion of obs. in the m region for class k.

How do we decide?

In our HBO Max example:

- k represents the different values that the outcome can take (e.g. $Unsubscribe \in \{0, 1\}$), and
- m represents the values that the predictor takes (e.g. $Succession = 0$).

E.g.:

- $p_{mk} = p_{00}$: The proportion of people who are subscribed (Unsubscribed = 0) and that have not watched Succession (Succession = 0)
- $p_{mk} = p_{01}$: The proportion of people who are unsubscribed (Unsubscribed = 1) and that have not watched Succession (Succession = 0)

- Usually, you want the Gini index to be small!

Q3: According to the Gini Index, is it better or worse to have a high p_{mk} (i.e. closer to 1)?

$$G = \sum^{K} \hat{p}_{mk}(1 - \hat{p}_{mk})$$

Choosing predictors

- From the previous exercise, we can see that using `succession` yields a lower Gini compared to `city` (0.428 vs. 0.482)

But we have more variables

How do we choose?

Basic Algorithm

1) Start at the root node

2) Split the parent node at covariate x_i to minimize the sum of child node impurities

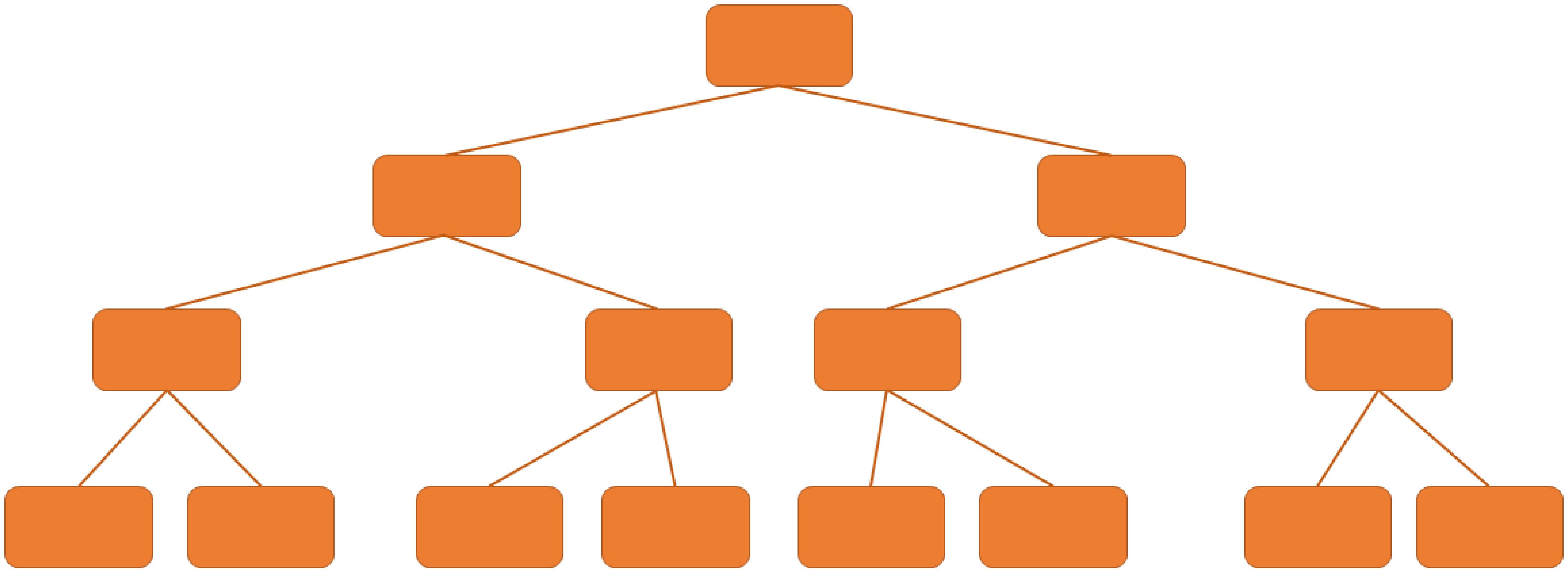
3) Stop if leaves are pure or early stopping criteria is satisfied, else repeat step (1) and (2) for each new child nodes

4) Prune your tree according to a complexity parameter (cp)

5) Assign the average outcome (regression) or the majority (classification) in each leaf.

Adapted from "Machine Learning FAQs" (Raschka, 2021)

Grow full tree and prune it



Hyper-parameter: Complexity parameter

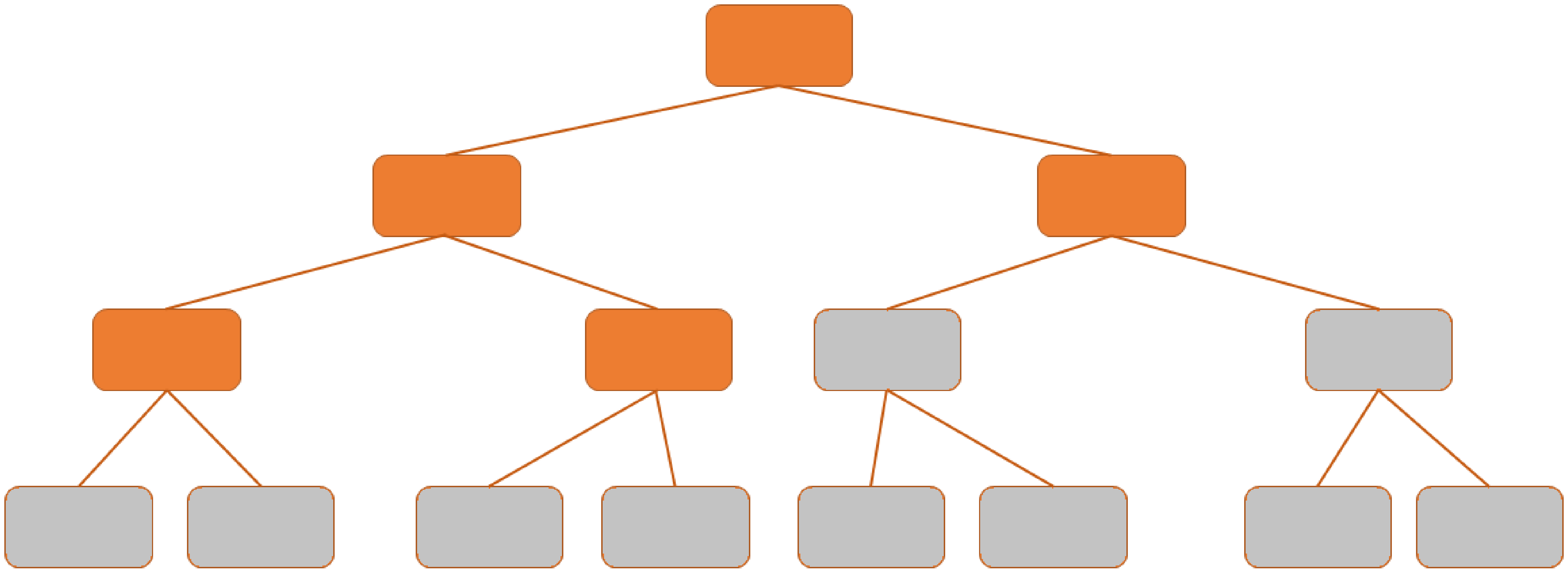
- Measure of how much a split should improve prediction for it to be worth it.

$$\sum_{m=1}^{|T|} \sum_{i:i \in R_m} (y_i - \hat{y}_i)^2 + \alpha |T|$$

- $|T|$: Number of terminal nodes or leaves (e.g. size of the tree)
- R_m: Predictor space of the mth leaf
- α: Tuning parameter

What happens if $\alpha = 0$?

Only attempt a split if it's worth it



Let's see how to do it in R!

```
library(caret)

set.seed(100)

ct = train(
  factor(unsubscribe) ~ . - id, data = hbo.train, #remember your outcome needs to be a factor!
  method = "rpart", # The method is called rpart
  trControl = trainControl("cv", number = 10),
  tuneLength = 15
)
```

Let's see how to do it in R!

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Let's see how to do it in R!

```r
library(caret)

set.seed(100)

ct = train(
  factor(unsubscribe) ~ . - id, data = hbo.train, #remember your outcome needs to be a factor!
  method = "rpart",
  trControl = trainControl("cv", number = 10),
  tuneLength = 15
)
```

- `tuneLength` is useful when you don't want to pass a specific grid (usually it might not be enough though!)

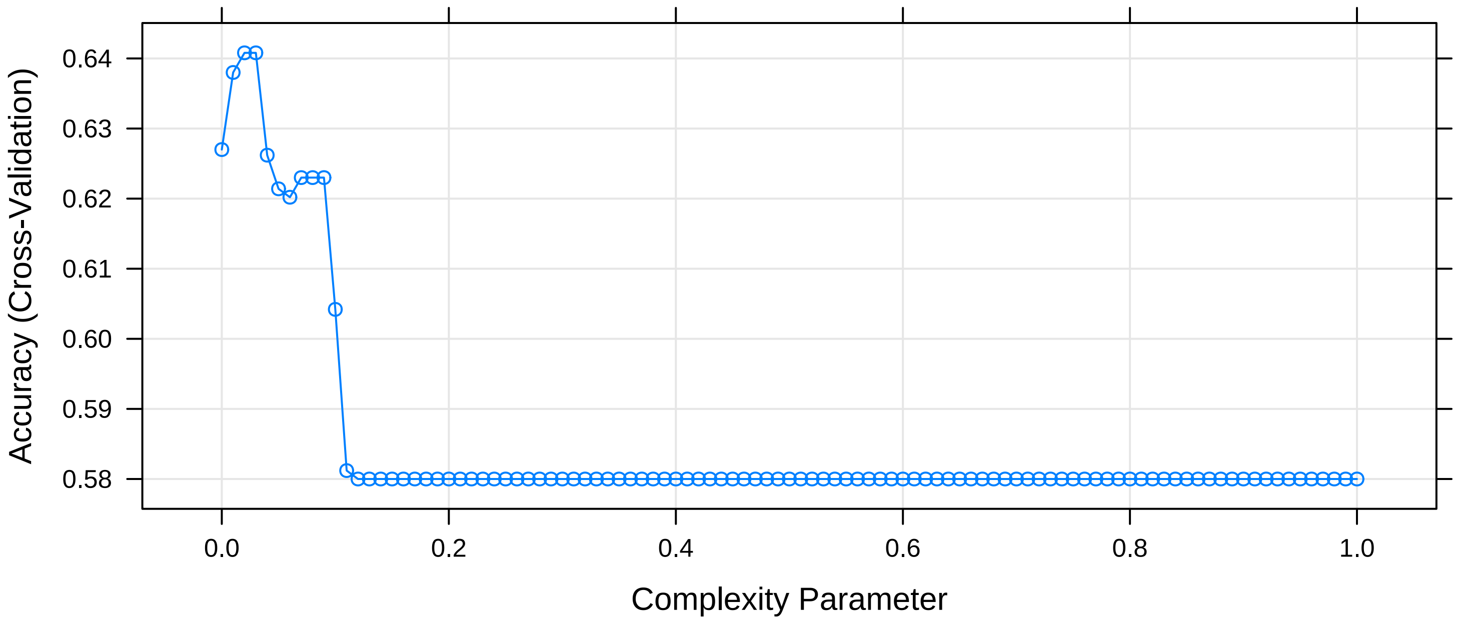
We could also provide a grid of complexity parameters

```r
library(rpart)

set.seed(100)

ct = train(
  factor(unsubscribe) ~ . - id, data = hbo.train,
  method = "rpart",
  trControl = trainControl("cv", number = 10),
  tuneGrid = expand.grid(cp = seq(0,1, by = 0.01)),
  control = rpart.control(minsplit = 20)
)
```

- `cp`: Complexity parameter
  - Split must decrease the overall lack of fit by a factor of `cp`, or is not attempted.
  - Parameter for **pruning the tree**.
  - Higher `cp`, smaller the tree!
- `minsplit`: Min. number of obs in a node to attempt a split.

This works similarly to the penalty term in regularization...

```
plot(ct)
```

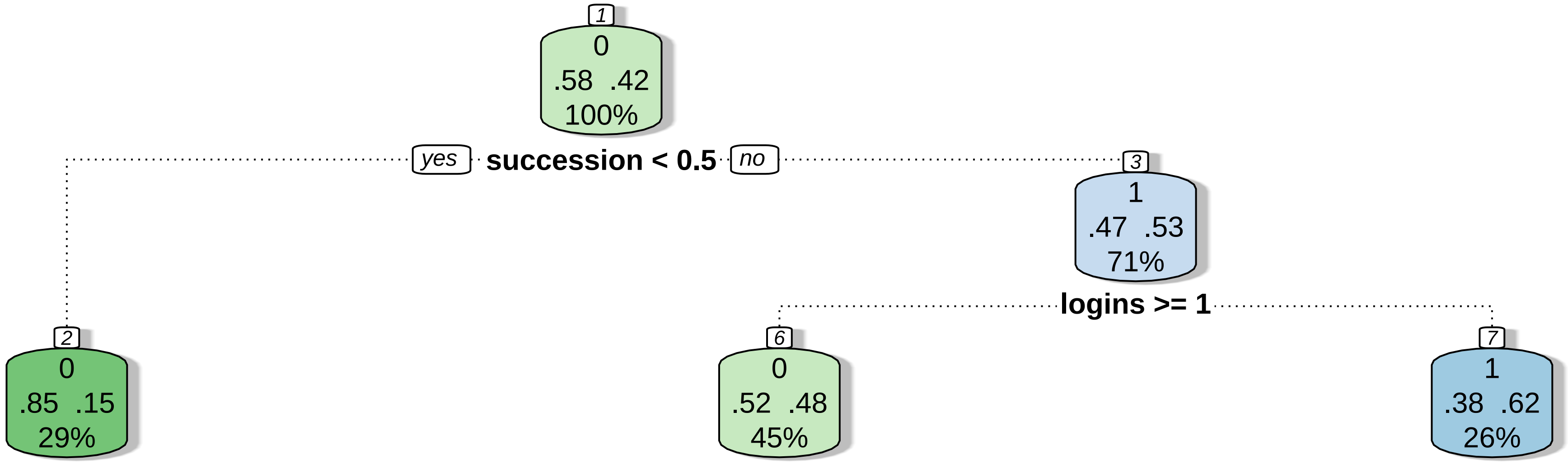



```
ct$bestTune
```

```
##     cp
## 4 0.03
```

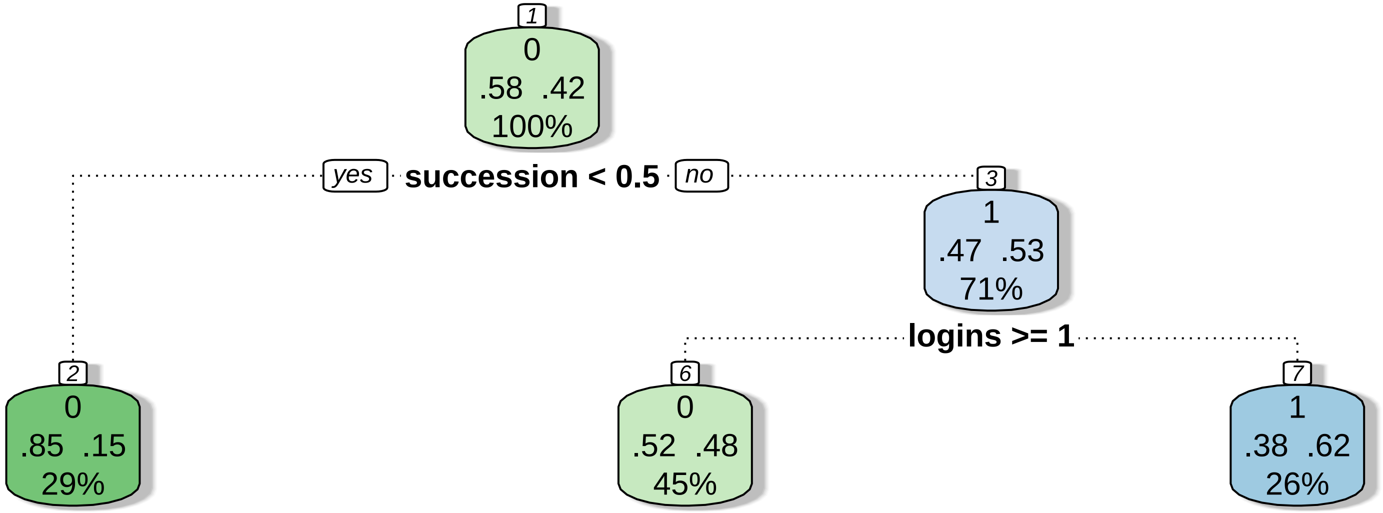
... And we can also plot the tree!

```r
library(rattle)

fancyRpartPlot(ct$finalModel, caption = "Classification tree for Unsubscribe")
```



Classification tree for Unsubscribe

What do you think the percentages in the leaves represent?



Classification tree for Unsubscribe

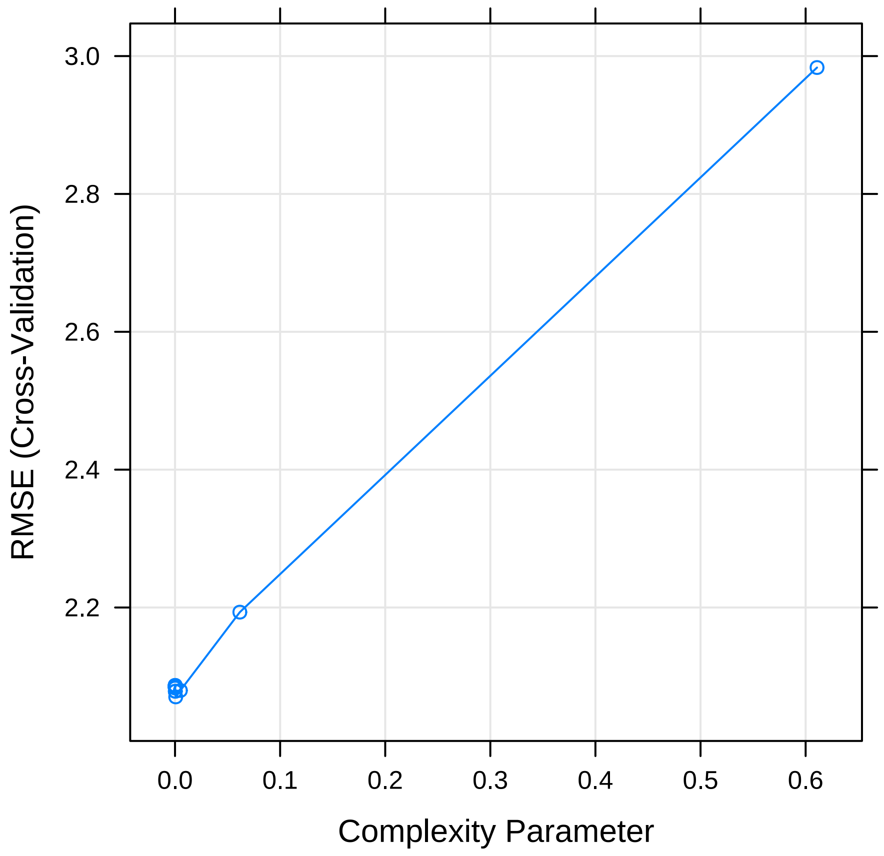
Regression Trees

Regression Trees

- Outcome is **continuous**
- Very similar to what we have seen with **classification trees**:
    - Predicted outcome is the **mean outcome for the leaf/region**.

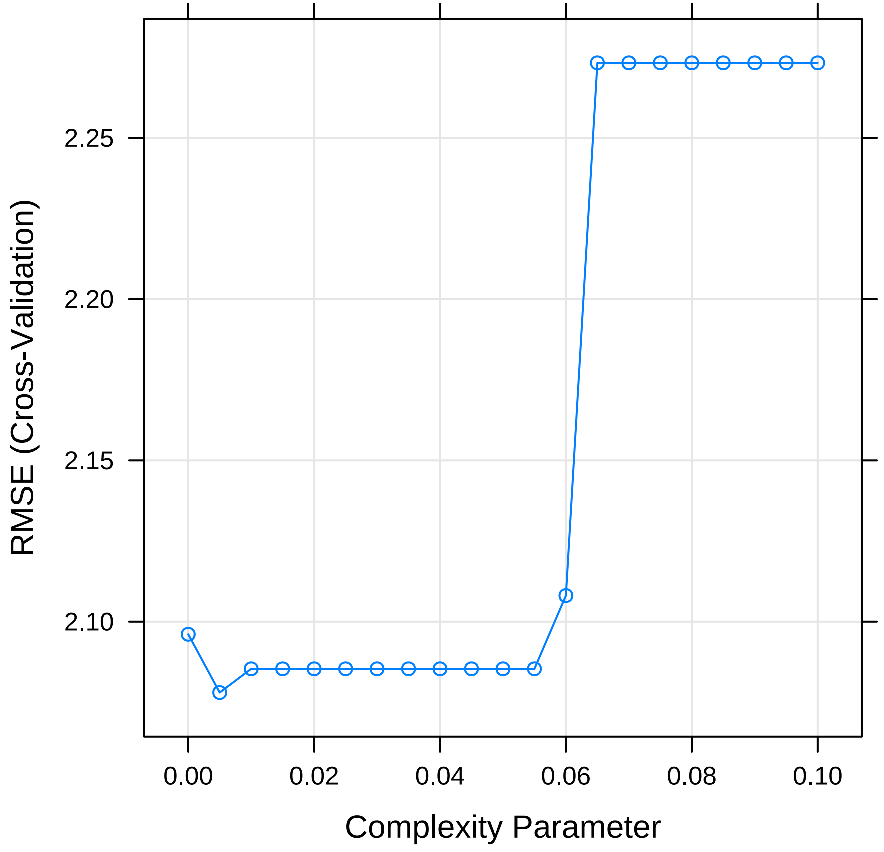
In R is basically the same

```
set.seed(100)

rt = train(
  logins ~. - unsubscribe - id, data = hbo.trai
  method = "rpart",
  trControl = trainControl("cv", number = 10),
  tuneLength = 20
  )

plot(rt)
```



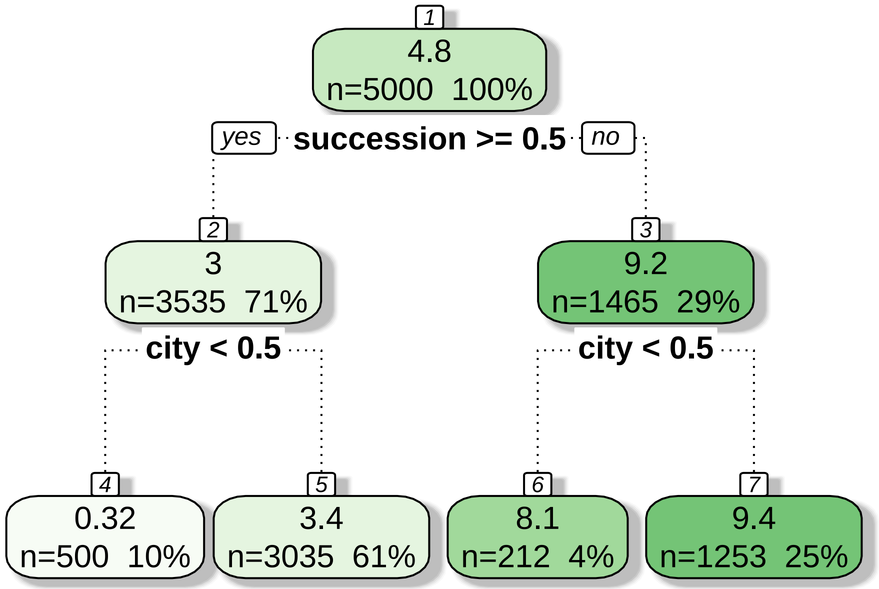
Providing a specific grid for cp

```r
set.seed(100)

tuneGrid = expand.grid(cp = seq(0, 0.1, by = 0

rt = train(
  logins ~. - unsubscribe - id, data = hbo.tra
  method = "rpart",
  trControl = trainControl("cv", number = 10),
  tuneGrid = tuneGrid
  )

plot(rt)
```



Plot the tree

```r
fancyRpartPlot(rt$finalModel, caption="Regress:
```

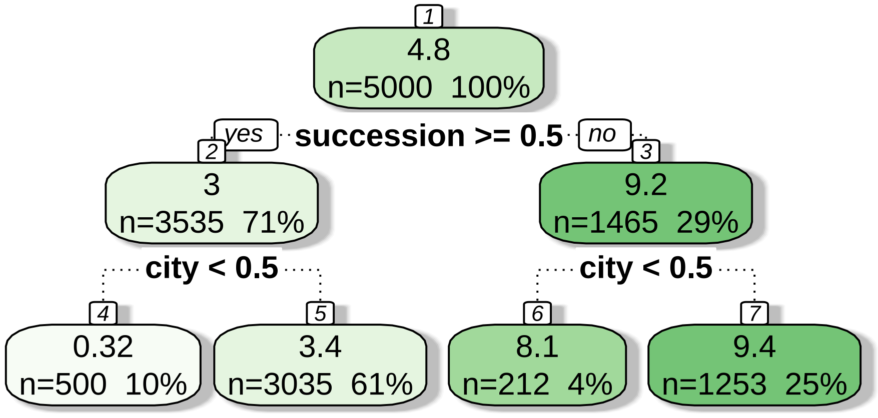



```r
rt$finalModel
```

```
## n= 5000
##
## node), split, n, deviance, yval
##       * denotes terminal node
##
## 1) root 5000 66387.3700 4.806800
##   2) succession>=0.5 3535 24633.5000 2.973409
##     4) city< 0.5 500   517.1580 0.322000 *
##     5) city>=0.5 3035 20022.2800 3.410214 *
##   3) succession< 0.5 1465  1200.0180 9.230717
##     6) city< 0.5 212   132.2028 8.061321 *
##     7) city>=0.5 1253   728.8571 9.428571 *
```

Q4: What would the predicted value be for a customer who hasn't watched GOT and lives in a city?

```
fancyRpartPlot(rt$finalModel, caption="Regress
```



Regression Tree for Login

```
rt$finalModel
```

```
## n= 5000
##
## node), split, n, deviance, yval
##       * denotes terminal node
##
## 1) root 5000 66387.3700 4.806800
##   2) succession>=0.5 3535 24633.5000 2.973409
##     4) city< 0.5 500   517.1580 0.322000 *
##     5) city>=0.5 3035 20022.2800 3.410214 *
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##     6) city< 0.5 212   132.2028 8.061321 *
##     7) city>=0.5 1253   728.8571 9.428571 *
```

Main takeaways of decision trees

Main advantages:

- Easy to interpret and explain (you can plot them!)
- Mirrors human decision-making.
- Can handle qualitative predictors (without need for dummies).

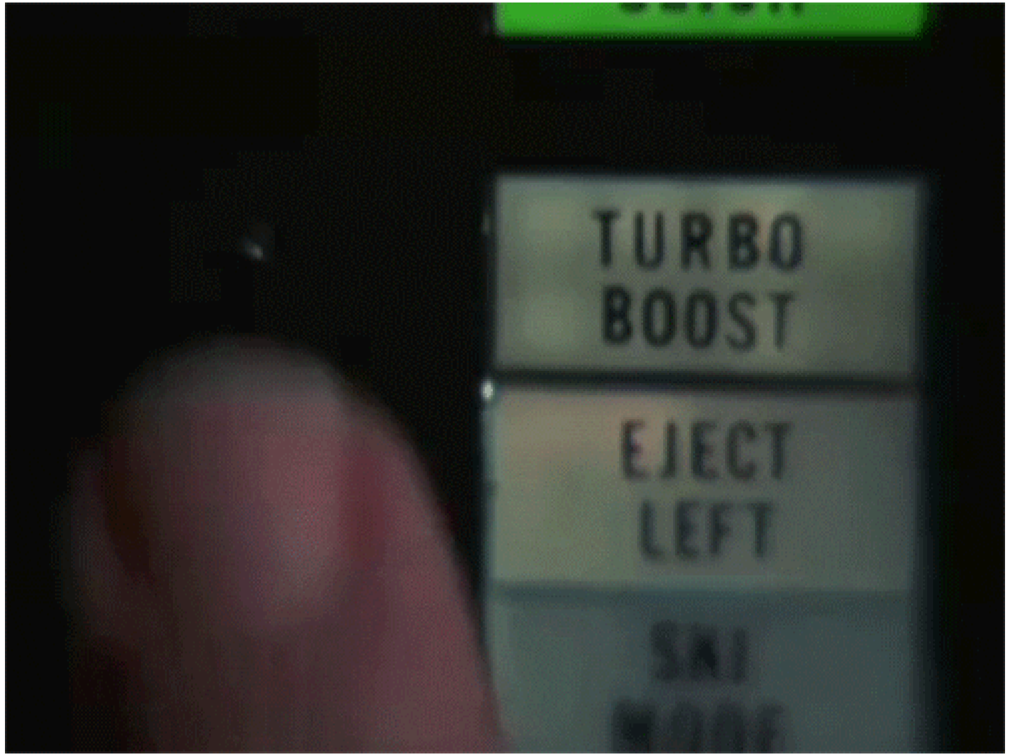
Main disadvantages:

- Accuracy not as high as other methods
- Very sensitive to training data (e.g. overfitting)

Next class

Use of decision trees as building blocks for **more powerful prediction methods!**

- Bagging
- Random Forests
- Boosting



References

- James, G. et al. (2021). "Introduction to Statistical Learning with Applications in R". *Springer. Chapter 8*
- Starmer, J.. (2018). "Decision Trees". *Video materials from StatQuest (YouTube).*
- STDHA. (2018). "CART Model: Decision Tree Essentials"