STA 235H - Binary Outcomes

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Binary Outcomes

- We have been using binary outcomes in regressions, but haven't fully discussed the issues they might bring.

What can we do about them?

How to handle binary outcomes?

Linear Probability Model

Logistic Regression

How to interpret an LPM?

- $\hat{\beta}$'s interpreted as **change in probability**

$$E[Y|X_1,\ldots,X_P] = Pr(Y=0|X_1,\ldots,X_p)\cdot 0 + Pr(Y=1|X_1,\ldots,X_p)\cdot 1$$

$$= Pr(Y=1|X_1,\ldots,X_p)$$

How to interpret an LPM?

- $\hat{\beta}$'s interpreted as change in probability

$$E[Y|X_1,\ldots,X_P] = Pr(Y=0|X_1,\ldots,X_p)\cdot 0 + Pr(Y=1|X_1,\ldots,X_p)\cdot 1$$
$$= Pr(Y=1|X_1,\ldots,X_p)$$

- Example:

$$Pass = \beta_0 + \beta_1 \cdot Study + \varepsilon$$

- $\hat{\beta}_1$ is the estimated change in probability of passing STA 235H if I study one more hour.

Let's look at an example

- Home Mortgage Disclosure Act Data (HMDA) from the `AER` package

```
##   deny pirat hirat     lvrat chist mhist phist unemp selfemp insurance condomin
## 1   no 0.221 0.221 0.8000000     5     2    no   3.9      no        no       no
## 2   no 0.265 0.265 0.9218750     2     2    no   3.2      no        no       no
## 3   no 0.372 0.248 0.9203980     1     2    no   3.2      no        no       no
## 4   no 0.320 0.250 0.8604651     1     2    no   4.3      no        no       no
## 5   no 0.360 0.350 0.6000000     1     1    no   3.2      no        no       no
## 6   no 0.240 0.170 0.5105263     1     1    no   3.9      no        no       no
##   afam single hschool
## 1   no     no     yes
## 2   no    yes     yes
## 3   no     no     yes
## 4   no     no     yes
## 5   no     no     yes
## 6   no     no     yes
```

Probability of someone getting a mortgage loan denied?

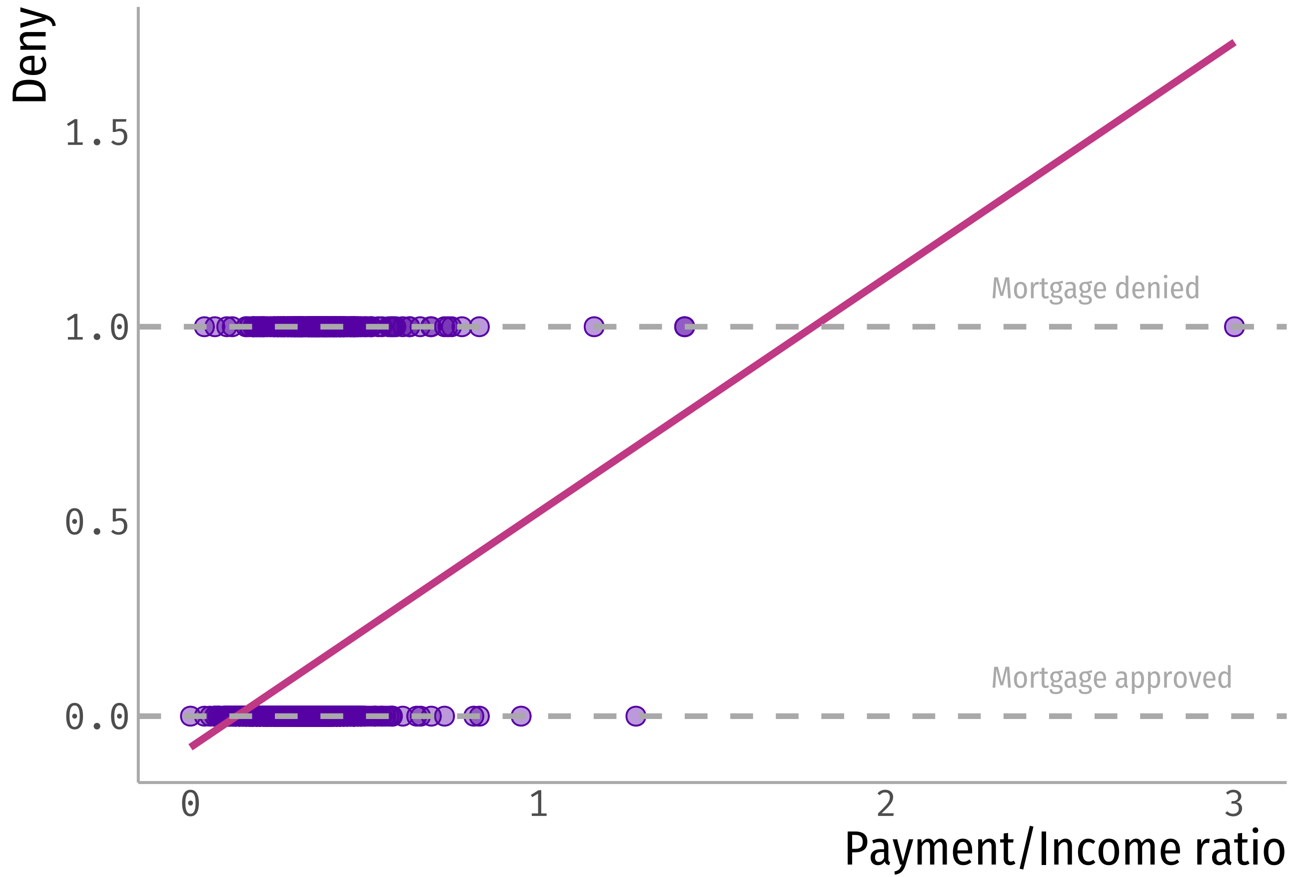
- Getting mortgage denied (1) based on race, conditional on payments to income ratio (`pirat`)

```
hmda <- hmda %>% mutate(deny = as.numeric(deny) - 1)

summary(lm(deny ~ pirat + factor(afam), data = hmda))
```

```
##
## Call:
## lm(formula = deny ~ pirat + factor(afam), data = hmda)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.62526 -0.11772 -0.09293 -0.05488  1.06815
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)     -0.09051    0.02079  -4.354 1.39e-05 ***
## pirat            0.55919    0.05987   9.340  < 2e-16 ***
## factor(afam)yes  0.17743    0.01837   9.659  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3123 on 2377 degrees of freedom
## Multiple R-squared:  0.076,    Adjusted R-squared:  0.07523
## F-statistic: 97.76 on 2 and 2377 DF,  p-value: < 2.2e-16
```

How does this LPM look?



Issues with an LPM?

- Main problems:
  - Non-normality of the error term
  - Heteroskedasticity (i.e. variance of the error term is not constant)
  - Predictions can be outside [0,1]
  - LPM imposes linearity assumption

Issues with an LPM?

- Main problems:
  - Non-normality of the error term $\rightarrow$ Hypothesis testing
  - Heteroskedasticity $\rightarrow$ Validity of SE
  - Predictions can be outside [0,1] $\rightarrow$ Issues for prediction
  - LPM imposes linearity assumption $\rightarrow$ Too strict?

Are there solutions?

- **Don't use small samples**: With the CLT, non-normality shouldn't matter much.
- **Saturate your model**: In a fully saturated model (i.e. include dummies and interactions), CEF is linear.
- **Use robust standard errors**: Package `estimatr` in R is great!
- **Not appropriate for prediction**

Run again with robust standard errors

```r
library(estimatr)

model1 <- lm(deny ~ pirat + factor(afam), data = hmda)
model2 <- lm_robust(deny ~ pirat + factor(afam), data = hmda)
```

	Model 1	Model 2
(Intercept)	-0.091***	-0.091***
	(0.021)	(0.031)
pirat	0.559***	0.559***
	(0.060)	(0.095)
factor(afam)yes	0.177***	0.177***
	(0.018)	(0.025)
R2	0.076	0.076
R2 Adj.	0.075	0.075
se_type		HC2
* p < 0.1, ** p < 0.05, *** p < 0.01		

Logistic Regression

- Typically used in the context of binary outcomes (*Probit is another popular one*)

- Nonlinear function to model the conditional probability function of a binary outcome.

$$Pr(Y = 1|X_1, \ldots, X_p) = F(\beta_0 + \beta_1 X_1 + \ldots + \beta_p X_p)$$

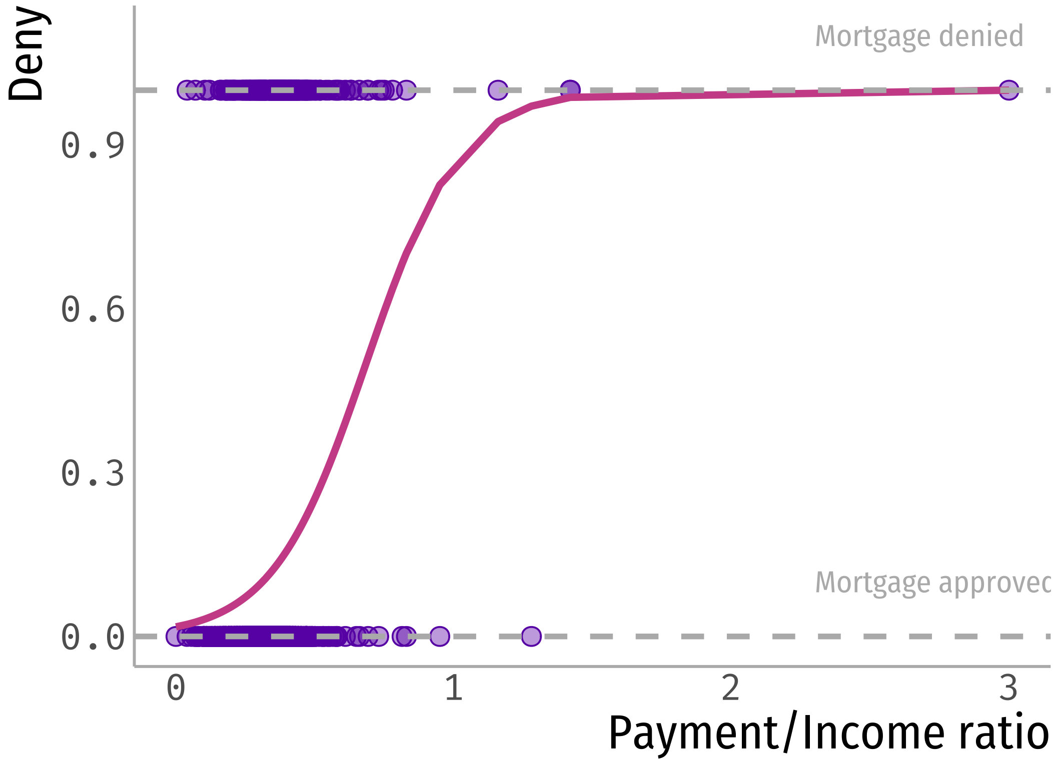
Where in a logistic regression: $F(x) = \frac{1}{1+exp(-x)}$

- *In the LPM,* $F(x) = x$

How does this look in a plot?

```r
logit1 <- glm(deny ~ pirat, family = binomial(link = "logit"),
              data = hmda)

prob <- predict(logit1, type = "response") # probabilities
```



How to interpret the coefficients?

```r
summary(glm(deny ~ pirat + factor(afam), family = binomial(link = "logit"),
              data = hmda))
```

```
##
## Call:
## glm(formula = deny ~ pirat + factor(afam), family = binomial(link = "logit"),
##     data = hmda)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -2.3709  -0.4732  -0.4219  -0.3556   2.8038
##
## Coefficients:
##                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)      -4.1256     0.2684 -15.370  < 2e-16 ***
## pirat             5.3704     0.7283   7.374 1.66e-13 ***
## factor(afam)yes   1.2728     0.1462   8.706  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1744.2  on 2379  degrees of freedom
## Residual deviance: 1591.4  on 2377  degrees of freedom
## AIC: 1597.4
##
## Number of Fisher Scoring iterations: 5
```

How to interpret the coefficients? (cont.)

- No easy way!
  - An odd is the probability of success over probability of failure: $\frac{p}{1-p}$
  - An odds ratio is the odds for scenario 1 over the odds for scenario 2: $\frac{p_1}{1-p_1} \cdot \frac{1-p_2}{p_2}$

How to interpret the coefficients? (cont.)

- No easy way!
  - An odd is the probability of success over probability of failure: $\frac{p}{1-p}$
  - An odds ratio is the odds for scenario 1 over the odds for scenario 2: $\frac{p_1}{1-p_1} \cdot \frac{1-p_2}{p_2}$
  - Coefficients in the output are log odds ratio:

$$\log(\frac{p}{1-p}) = \beta_0 + \beta_1 X_1 + \ldots + \beta_p X_p$$

- $(\exp(\beta_1) - 1) \cdot 100\%$ is the expected increase in the odds of $Y = 1$ for a one unit increase of X_1.

How to interpret the coefficients? (cont.)

- Let's go back to our example:

```
glm(deny ~ pirat + factor(afam), family = binomial(link = "logit"),
          data = hmda)
```

```
##
## Call:  glm(formula = deny ~ pirat + factor(afam), family = binomial(link = "logit"),
##     data = hmda)
##
## Coefficients:
##     (Intercept)            pirat  factor(afam)yes
##          -4.126            5.370            1.273
##
## Degrees of Freedom: 2379 Total (i.e. Null);  2377 Residual
## Null Deviance:        1744
## Residual Deviance: 1591     AIC: 1597
```

- $(\exp(1.27) - 1) \cdot 100\% = 257\%$ more likely to be denied a mortgage if you are African American vs not African American, holding payments to income ratio constant.

How to interpret the coefficients? (cont.)

- Let's look at **probabilities**

- E.g. Choose coefficient of interest, and fix the other variables to their mean or mode:

```
logit1 <- glm(deny ~ pirat + factor(afam), family = binomial(link = "logit"),
              data = hmda)

predictions_afam <- predict(logit1, newdata = data.frame("afam" = c("no", "yes"),
                                                         "pirat" = c(mean(hmda$pirat), mean(hmda$pirat))),
                            type = "response")
predictions_afam
```

```
##          1          2
## 0.08714775 0.25422824
```

How to interpret the coefficients? (cont.)

- Let's look at **probabilities**
- E.g. Choose coefficient of interest, and fix the other variables to their mean or mode:

```
logit1 <- glm(deny ~ pirat + factor(afam), family = binomial(link = "logit"),
              data = hmda)

mean_pirat <- hmda %>% select(pirat) %>% summarize_all(mean) %>% pull()

predictions_afam <- predict(logit1, newdata = data.frame("afam" = c("no", "yes"),
                                                          "pirat" = c(mean_pirat, mean_pirat)),
                            type = "response")
predictions_afam
```

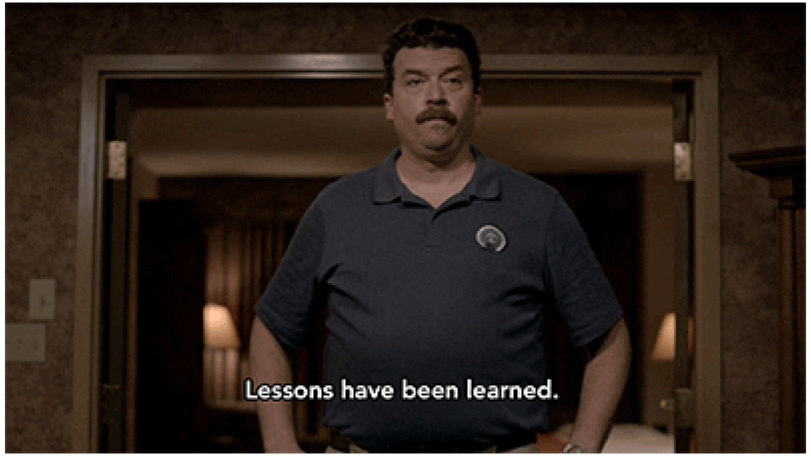
```
##          1          2
## 0.08714775 0.25422824
```

```
diff(predictions_afam)
```

```
##         2
## 0.1670805
```

- Remember that for the LPM model, $\hat{\beta}_{afam} = 0.177$

Main takeaway points



- LPM and Logistic Regression can **both be useful** depending on the context.
  - LPM for explanation (causal inference) and Logistic Regression for prediction.
- **Be careful** with the interpretation!

References

- Hanck, C. et al. (2020). "Econometrics with R". *Regression with a Binary Dependent Variable*
- James, G. et al. (2017). "Introduction to Statistical Learning with Applications in R". *Chapter 4.3*
- Grace-Martin, K. (2018). "Why logistic regression for binary responses?"
- Bellemare, M. (2013) "A Rant on Estimation with Binary Dependent Variables (Technical)"