

# STA 235H - Potential Outcomes

Fall 2021

McCombs School of Business, UT Austin

**Let's do a short exercise**

**Take three pieces of papers: One of each color**

**Don't look!**

# The problem

- Imagine **everyone** here has a headache
- Everyone takes an aspirin to mitigate the headache
- How many students **still have a headache** (look at your **orange** paper)

**Wow, did the aspirin work?**

**How? Potential Outcomes Framework**

**What? Causal Estimands**

**Why? Causal Questions and Study Design**

The "How": Potential outcomes framework



Geoffrey Supran  
@GeoffreySupran



"The @GretaThunberg Effect" is now an empirically demonstrated, peer-reviewed phenomenon:

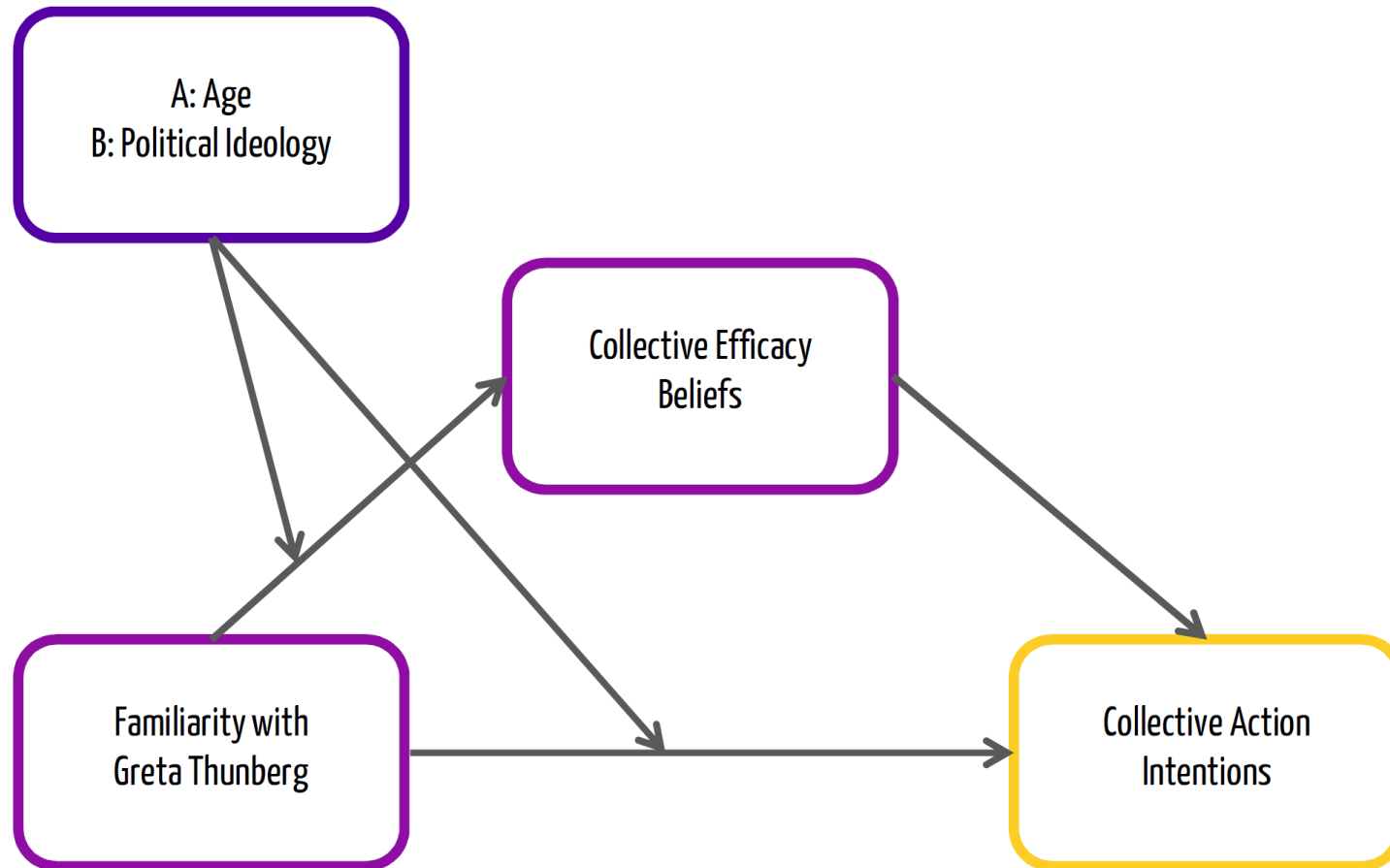
"We find that those who are more familiar with Greta Thunberg have higher intentions of taking collective actions to reduce global warming."

Open access: [onlinelibrary.wiley.com/doi/epdf/10.1111/jasp.12727](https://onlinelibrary.wiley.com/doi/epdf/10.1111/jasp.12727)



the Year by some, and asked to “work on her anger management issues” by others (Alter et al., 2019; McCarthy, 2019). The present study, to date, is one of the first to present empirical evidence supporting the “Greta Thunberg Effect,” and to offer a potential explanation of why a young leader could be a powerful influence on collective action. We find that familiarity with Greta Thunberg is





**What do you think are the biggest  
issues here?**



**Khoa Vu**  
@KhoaVuUmn



"The Greta effect" Effect: Your misuse of causal language is never too wrong to make famous people retweet your study.



**Hillary Clinton**  @HillaryClinton · Jan 28

Data proving @GretaThunberg right—"you are never too small to make a difference." [twitter.com/GeoffreySupran...](https://twitter.com/GeoffreySupran...)

11:22 AM · Jan 29, 2021 · Twitter Web App

**What about other topics?**

**Before we start...**

**Be clear about your language**

# Before we start...

**Be clear about your language**

**Be clear about your data**

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**Be clear about your language**

**Be clear about your data**

**Be clear about your assumptions**

# What is Causal Inference?

**Inferring the effect of one thing on another thing**



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- "Providing students support when filling out FAFSA forms improves college access and completion."

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- Under a binary treatment or intervention, there are **two potential worlds**:
- **World 1**: You take the pill
- **World 2**: You don't take the pill



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- A **potential outcome** is the outcome under each of these scenarios or "worlds".
  - *There will be one for each path!*

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- A **potential outcome** is the outcome under each of these scenarios or "worlds".
  - *There will be one for each path!*
- A priori, each of these scenarios has a *potential outcome*
- A posteriori, I can only observe **at most one of the potential outcomes**

**Fundamental Problem of Causal Inference**



**What are the potential outcomes  
for our previous example?**

# Potential Outcomes Examples

- "My headache went away because I took an aspirin".

*Outcome if I take an aspirin/ Outcome if I don't take an aspirin*

- "The new marketing campaign increased our sales by 20%"

*Sales with a marketing campaign/ Sales without a marketing campaign*

- "Providing students support when filling out FAFSA forms improves college access and completion."

*College access and completion with support/ College access and completion without support*

**What are some causal questions you might be interested in?**

# Potential Outcomes Framework

Let's introduce some notation:

- Let  $Y_i$  be the observed outcome for unit  $i$  (e.g. whether I have a headache or not in an hour).
- Let  $Z_i$  be the treatment or intervention (e.g. taking a pill).

Then,

$$Y_i | (Z_i = 1) \triangleq Y_i(1)$$

where  $Y_i(1)$  is the **potential outcome under treatment**.

In the same fashion,

$$Y_i | (Z_i = 0) \triangleq Y_i(0)$$

where  $Y_i(0)$  is the **potential outcome under control**.

# Potential Outcomes Framework

This means that we can write the observed outcome as a function of the *potential outcomes*:

$$\rightarrow Y_i = Z_i \cdot Y_i(1) + (1 - Z_i) \cdot Y_i(0)$$

- This definition will be useful because we can see this as a **missing data problem**.

# Causal Effects

## Individual Causal Effect

$$ICE_i = Y_i(1) - Y_i(0)$$

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**Can we ever observe individual causal effects?**

# Causal Effects

## Individual Causal Effect

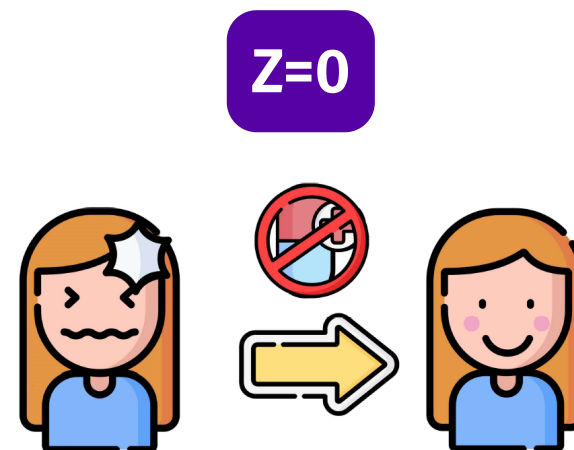
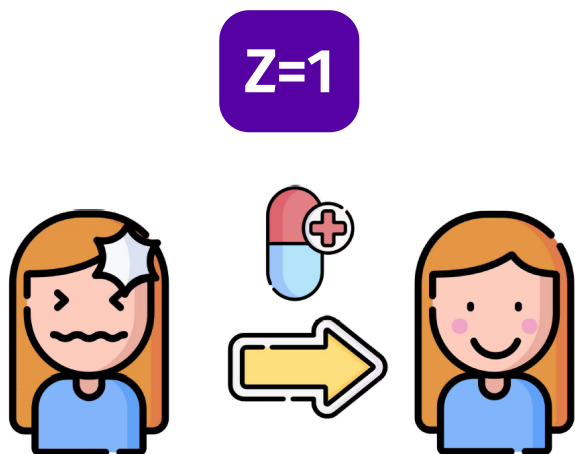
$$ICE_i = Y_i(1) - Y_i(0)$$

Can we ever observe individual causal effects?

No!\*

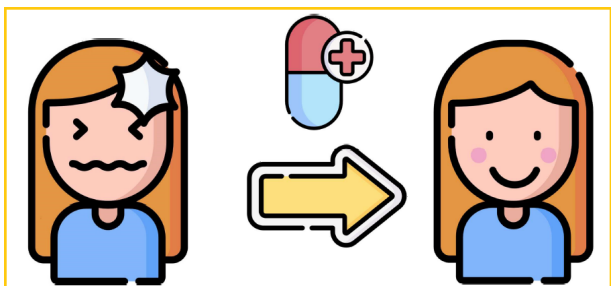


# Only one realization

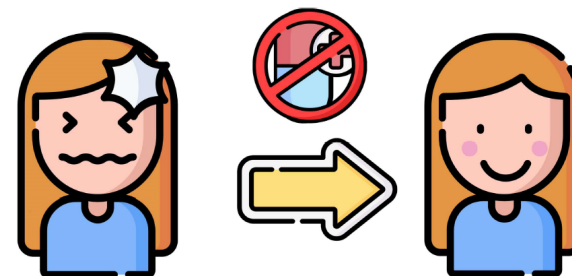


# Only one realization

$Z=1$



$Z=0$



The "What": Causal estimands, estimates,  
and estimators

# Estimands vs Estimates vs Estimators

## Estimand

A quantity we want to estimate

## Estimator

A rule for calculating an estimate based on data

## Estimate

The result of an estimation

# Estimands vs Estimates vs Estimators

## Estimand

A quantity we want to estimate

E.g.: Population mean

$\mu$

## Estimator

A rule for calculating an estimate based on data

E.g.: Sample mean

$$\frac{1}{n} \sum_i Y_i$$

## Estimate

The result of an estimation

E.g.: Result of the sample mean  
for a given sample  $S$

$\hat{\mu}$

# Estimands vs Estimates vs Estimators



estimand

Ingredients	Method
150g unsalted butter, plus extra for greasing	1. Heat the oven to 160C/140C fan/gas 3. Grease and base line a 1 litre heatproof glass pudding basin and a 450g loaf tin with baking parchment.
150g plain chocolate, broken into pieces	
150g plain flour	
½ tsp baking powder	2. Put the butter and chocolate into a saucepan and melt over a low heat, stirring. When the chocolate has all melted remove from the heat.
½ tsp bicarbonate of soda	
200g light muscovado sugar	
2 large eggs	

estimator



estimate

# Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

Average Treatment Effect (ATE)

Average Treatment Effect on the Treated (ATT)

Conditional Average Treatment Effect (CATE)

# Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

$$ATE = E[Y(1) - Y(0)]$$

$$ATT = E[Y(1) - Y(0) | Z = 1]$$

$$CATE = E[Y(1) - Y(0) | X]$$



# Getting around the fundamental problem of causal inference

- Let's go back to our original example: Does a pill help reduce headaches?

i	z	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	0	1	?	1	?
2	1	1	1	?	?
3	1	0	0	?	?
4	0	0	?	0	?
5	0	1	?	1	?
6	1	0	0	?	?

# Getting around the fundamental problem of causal inference

- We have a **missing data problem**

i	z	Y	Y(1)	Y(0)	Y(1)-Y(0)
1	0	1	?	1	?
2	1	1	1	?	?
3	1	0	0	?	?
4	0	0	?	0	?
5	0	1	?	1	?
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# Getting around the fundamental problem of causal inference

- Compare those who **took the pill** to the ones **did not take it**.

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Difference in sample means

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Difference in sample means

- What is the **estimate** and *how do we interpret it*?

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Difference in sample means

- What is the **estimate** and *how do we interpret it*?

33.3 percentage point decrease in probability of having a headache

**What could be the problem with comparing the sample means?**

# The "Why": Causal questions and study designs

# Under what assumptions is our estimate causal?

We are using:

$$\hat{\tau} = \frac{1}{3} \left( \sum_{i \in Z=1} Y_i - \sum_{i \in Z=0} Y_i \right)$$

to estimate:

$$\tau = E[Y_i(1) - Y_i(0)]$$

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**Let's do some math**

# Under what assumptions is our estimate causal?

$$\begin{aligned}\tau &= E[Y_i(1) - Y_i(0)] \\ &= E[Y_i(1)] - E[Y_i(0)]\end{aligned}$$

Key assumption:

**Ignorability**

- Ignorability means that the potential outcomes  $Y(0)$  and  $Y(1)$  are independent of the treatment, e.g.  $(Y(0), Y(1)) \perp\!\!\!\perp Z$ .



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  - Remember that if  $A \perp\!\!\!\perp B \rightarrow E[A|B] = E[A]$

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# Ignorability Assumption

We can just "ignore" the missing data problem:

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1	0	1		1	
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			1/3	2/3	

**Let's do a little exercise**



Look at your **green** piece of paper and go to the following website



Would you go to a physician/urgent care?

**Now let's assume I randomly allocate whether you go or not go to the hospital**



**Do the previous results hold?**

**Randomization is an awesome tool for causal inference**

# Main takeaway points

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# Main takeaway points

## Causal Inference is hard

- Think about the **causal problem**
- Always **look at your data**
- Check **validity** of assumptions (*Is ignorability plausible? Am I controlling for the right covariates?*)
- Most of this chapter will be spent on looking for **exogenous variation** to make the ignorability assumption happen.



# Next week

- **Randomized Controlled Trials:**
  - Pros and Cons
  - Concept of validity
  - A/B Testing



# References

- Angrist, J. & S. Pischke. (2015). "Mastering Metrics". *Chapter 1*.
- Cunningham, S. (2021). "Causal Inference: The Mixtape". *Chapter 4: Potential Outcomes Causal Model*.
- Neil, B. (2020). "Introduction to Causal Inference". *Fall 2020 Course*