

# STA 235H - Ignorability Assumption and Randomized Controlled Trials

Fall 2023

McCombs School of Business, UT Austin

# Housekeeping

- **Homework 2** is due this Friday.
  - Remember to ask questions **in advance**! (Discussion board for general q's/clarifications; email/OH if it shows your work)
  - **Check your full code** before submission: Instructions on the course website.
- About **JITT feedback**:
  - Thanks for questions/suggestions!
  - Currently ~90% ok with pace.
  - Additional support is available.
- **No OH next Thursday (09/28)** (changed to Tuesday; check OH calendar).

# Last week

- Finished our chapter on **multiple regression**.
  - **How to add flexibility to our model:**  
Regressions with polynomial terms.
  - For small changes in  $X$  (e.g. one-unit increase), we can approximate  $\Delta Y$  with the derivative!
- Introduced **Causal Inference**



# Today



- Continue with **causal inference**:
  - Ignorability assumption
- **Introduction to Randomized Controlled Trials**:
  - Why do we randomize?
  - How to analyze RCTs?
  - Are there any limitations?

**Similar to last week: Let's do a  
little exercise**

Look at your **green** piece of paper and go to the following website



<https://sta235h.rocks/week5>

I will now decide whether you go to the hospital or not!

Causal Inference: Things we can "ignore"

# Potential Outcomes

- Last week we discussed potential outcomes, (e.g.  $Y_i(1)$  and  $Y_i(0)$ ):

*"The outcome that we would have observed under different scenarios"*

- Potential outcomes are related to your choices/possible conditions:
  - One for each path!
  - Do not confuse them with the **values** that your outcome variable can take.
- Definition of **Individual Causal Effect**:

$$ICE_i = Y_i(1) - Y_i(0)$$



# Estimands vs Estimates vs Estimators

## Estimand

A quantity we want to estimate

## Estimator

A rule for calculating an estimate based on data

## Estimate

The result of an estimation

# Estimands vs Estimates vs Estimators

## Estimand

A quantity we want to estimate

E.g.: Causal effect of attending college on income

$\tau$

## Estimator

A rule for calculating an estimate based on data

E.g.: Difference in means

$$\frac{1}{n_1} \sum_{Z=1} Y_i - \frac{1}{n_0} \sum_{Z=0} Y_i$$

## Estimate

The result of an estimation

E.g.: Result of the difference in means for a given sample  $S$

$\hat{\tau}$

# Estimands vs Estimates vs Estimators



estimand

Ingredients	Method
150g unsalted butter, plus extra for greasing	1. Heat the oven to 160C/140C fan/gas 3. Grease and base line a 1 litre heatproof glass pudding basin and a 450g loaf tin with baking parchment.
150g plain chocolate, broken into pieces	
150g plain flour	
½ tsp baking powder	2. Put the butter and chocolate into a saucepan and melt over a low heat, stirring. When the chocolate has all melted remove from the heat.
½ tsp bicarbonate of soda	
200g light muscovado sugar	
2 large eggs	

estimator



estimate

# Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

Average Treatment Effect (ATE)

Average Treatment Effect on the Treated (ATT)

Conditional Average Treatment Effect (CATE)

# Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

ATE: E.g. Average Treatment Effect for all customers

ATT: E.g. Average Treatment Effect for customers that received the email

CATE: E.g. Average Treatment Effect for customer under 25 years old

# Estimands vs Estimates vs Estimators

- Some important **estimands** that we need to keep in mind:

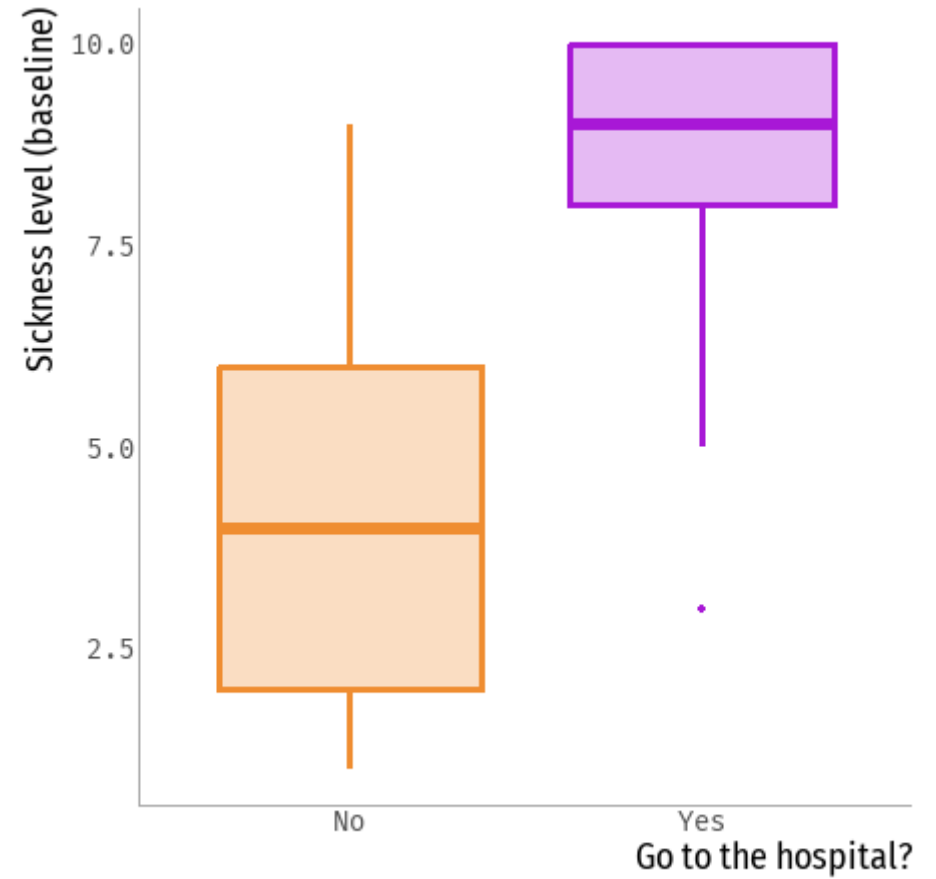
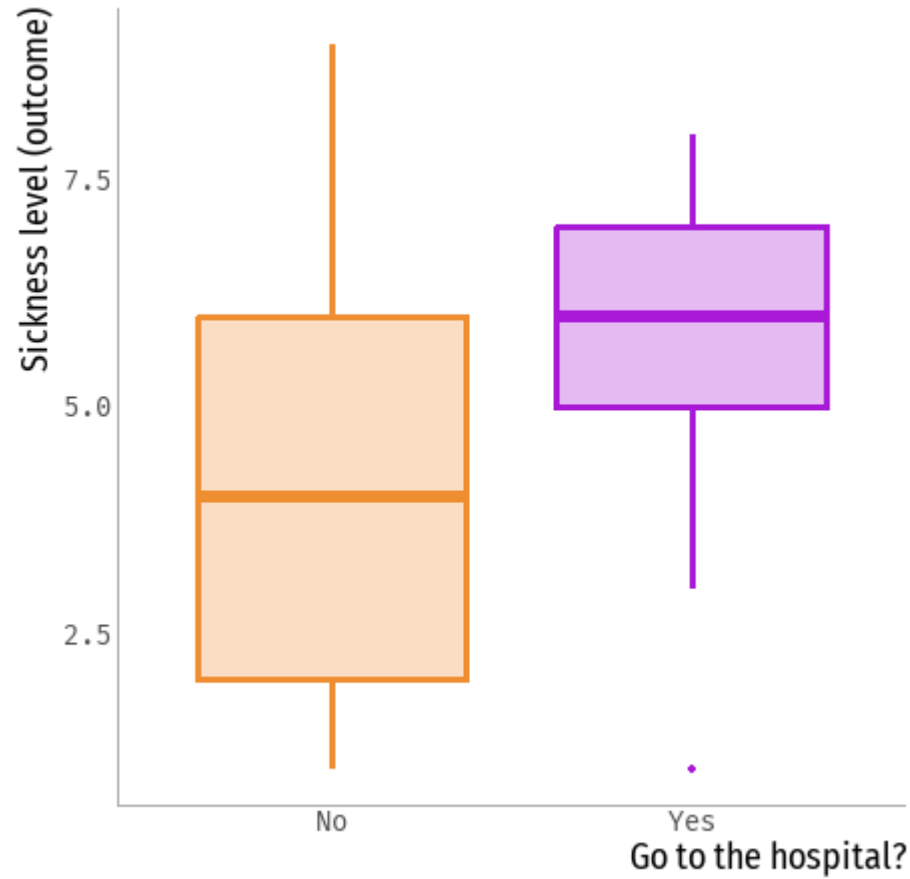
$$ATE = E[Y(1) - Y(0)]$$

$$ATT = E[Y(1) - Y(0) | Z = 1]$$

$$CATE = E[Y(1) - Y(0) | X]$$

**What was the problem with comparing the sample means to get a causal effect?**

# Remember our exercise last week!





# Under what assumptions is our estimate causal?

We are using a difference in means:

$$\hat{\tau} = \frac{1}{n_1} \sum_{i \in Z=1} Y_i - \frac{1}{n_0} \sum_{i \in Z=0} Y_i$$

to estimate:

$$\tau = E[Y_i(1) - Y_i(0)]$$

# Under what assumptions is our estimate causal?

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$$\hat{\tau} = \frac{1}{n_1} \sum_{i \in Z=1} Y_i - \frac{1}{n_0} \sum_{i \in Z=0} Y_i$$

to estimate:

$$\tau = E[Y_i(1) - Y_i(0)]$$

Let's do some math

# Under what assumptions is our estimate causal?

$$\begin{aligned}\tau &= E[Y_i(1) - Y_i(0)] \\ &= E[Y_i(1)] - E[Y_i(0)]\end{aligned}$$

Key assumption:

**Ignorability**

Ignorability means that the potential outcomes  $Y(0)$  and  $Y(1)$  are independent of the treatment, e.g.  $(Y(0), Y(1)) \perp\!\!\!\perp Z$ .

$$E[Y_i(1) | Z = 0] = E[Y_i(1) | Z = 1] = E[Y_i(1)]$$

and

$$E[Y_i(0) | Z = 0] = E[Y_i(0) | Z = 1] = E[Y_i(0)]$$

# Under what assumptions is our estimate causal?

$$\tau = E[Y_i(1) - Y_i(0)]$$

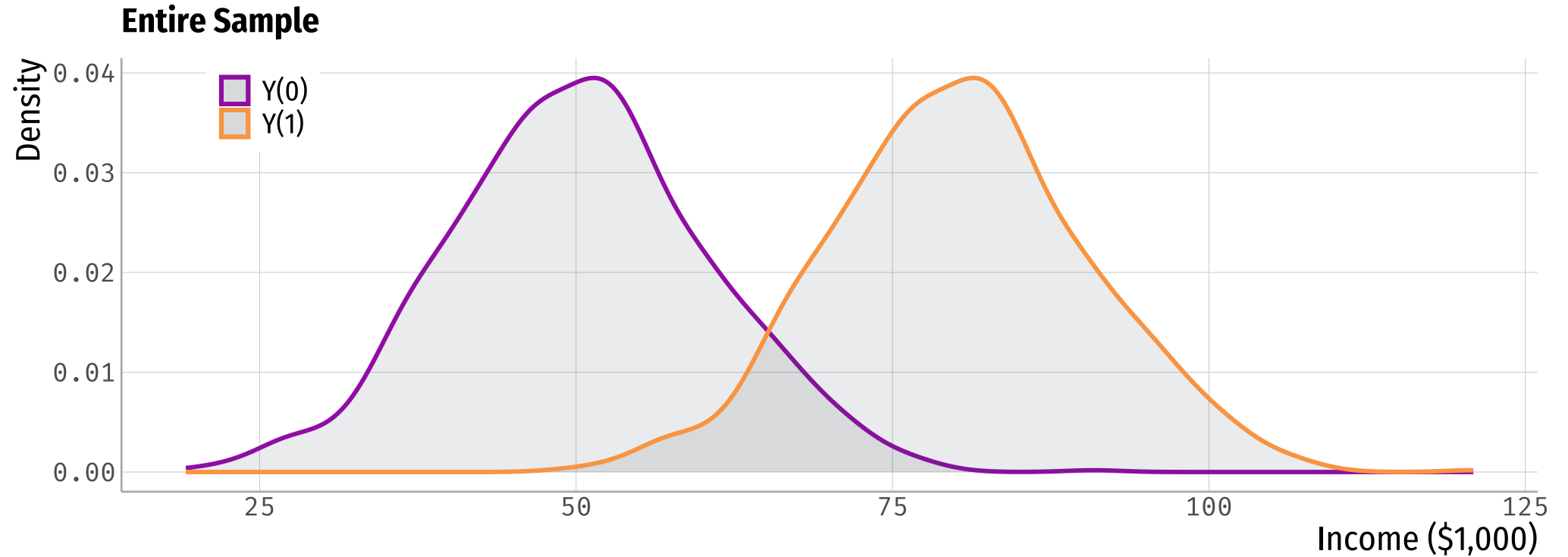
$$= E[Y_i(1)] - E[Y_i(0)]$$

- Under ignorability (see previous slide),  $E[Y_i(1)] = E[Y_i(1) | Z = 1] = E[Y_i | Z = 1]$  and  $E[Y_i(0)] = E[Y_i(0) | Z = 0] = E[Y_i | Z = 0]$ , then:

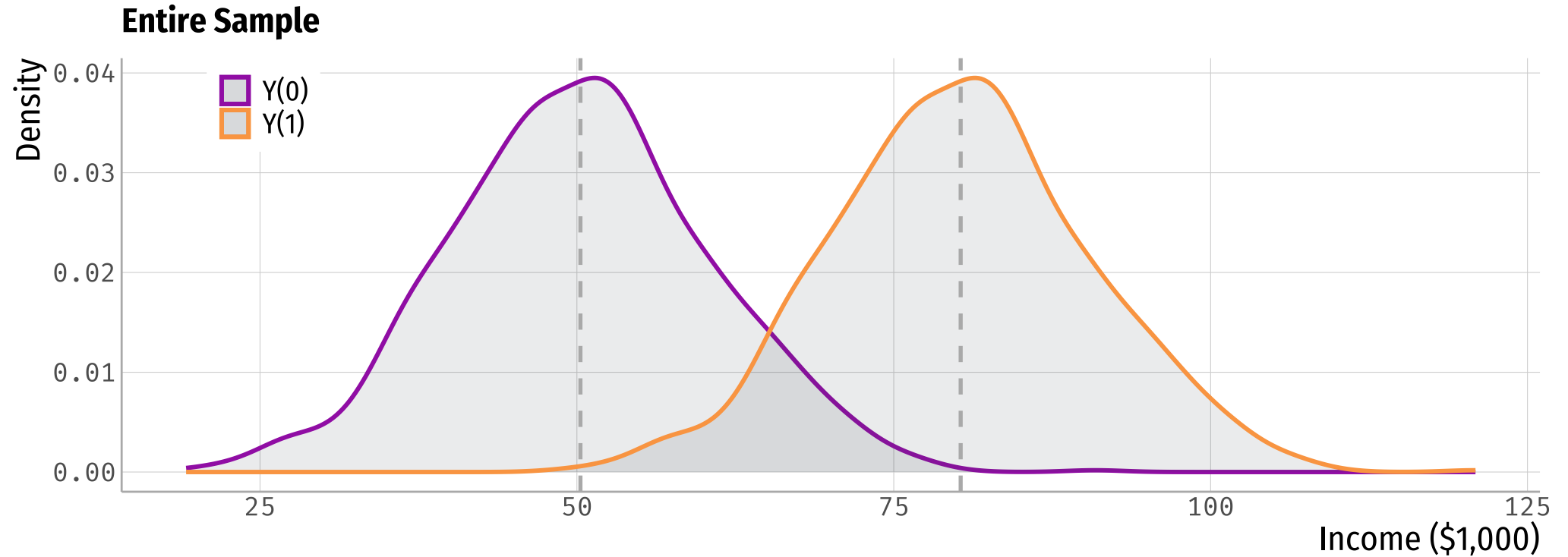
$$\tau = E[Y_i(1)] - E[Y_i(0)] = \overset{\text{Obs. Outcome for C}}{E[Y_i(1) | Z = 1]} - \overset{\text{Obs. Outcome for T}}{E[Y_i(0) | Z = 0]}$$

**Let's see an example: Why did you enroll in the Honors program?**

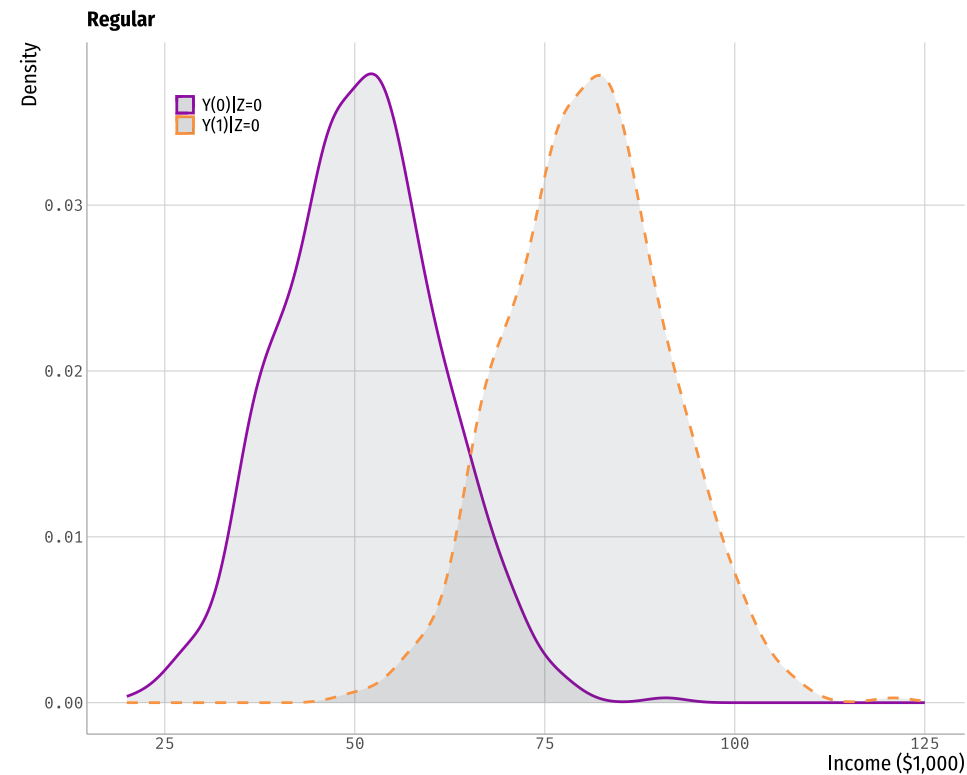
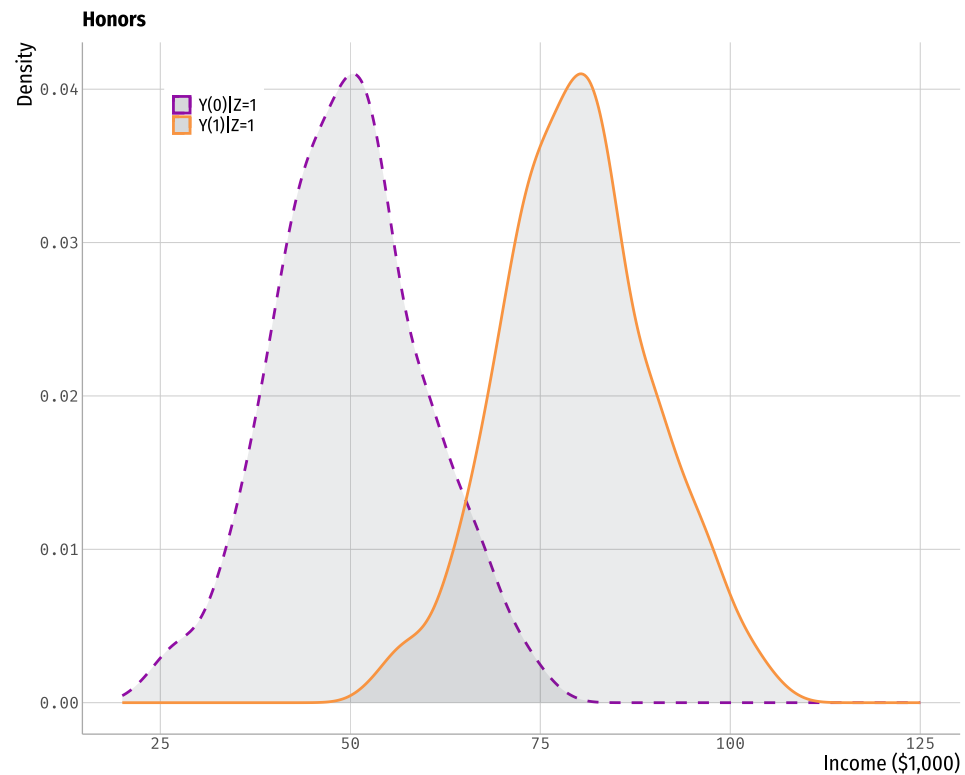
# Let's see the distributions of potential outcomes



# Let's see the distributions of potential outcomes



# We can only observe one distribution per group!

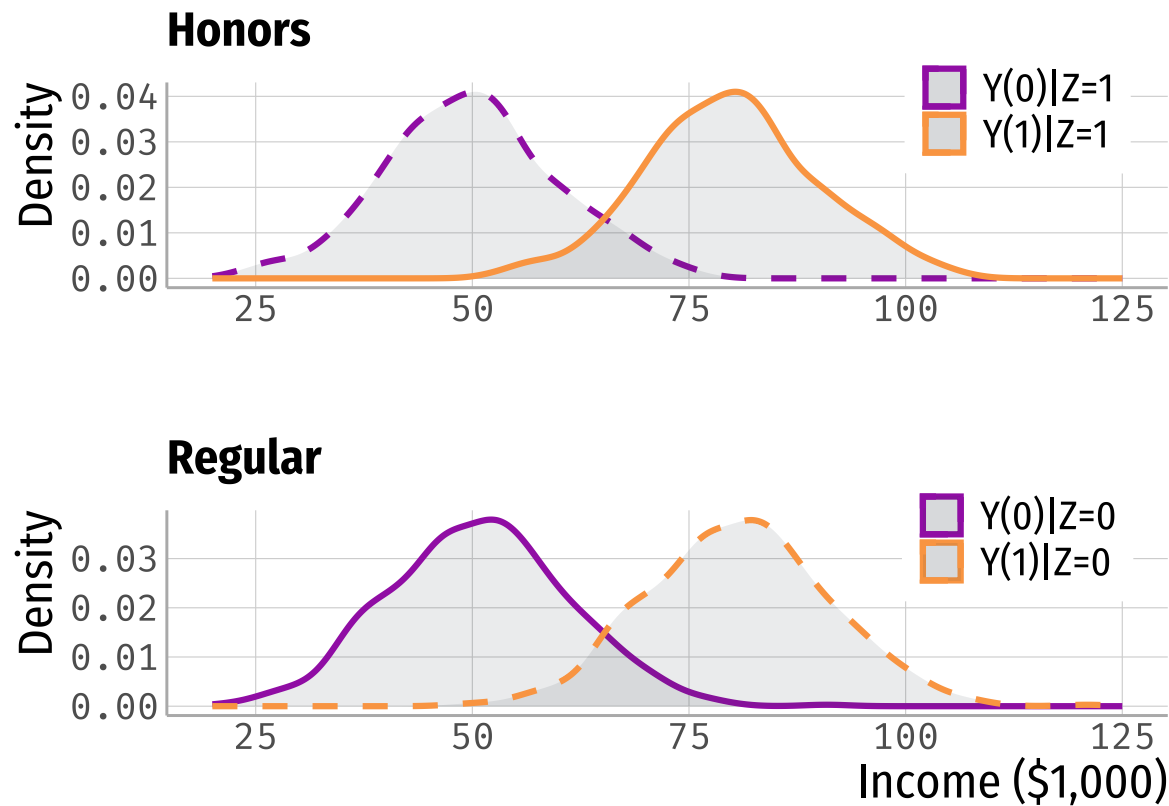




# Under Ignorability Assumption

$$Y(0), Y(1) \perp\!\!\!\perp Z$$

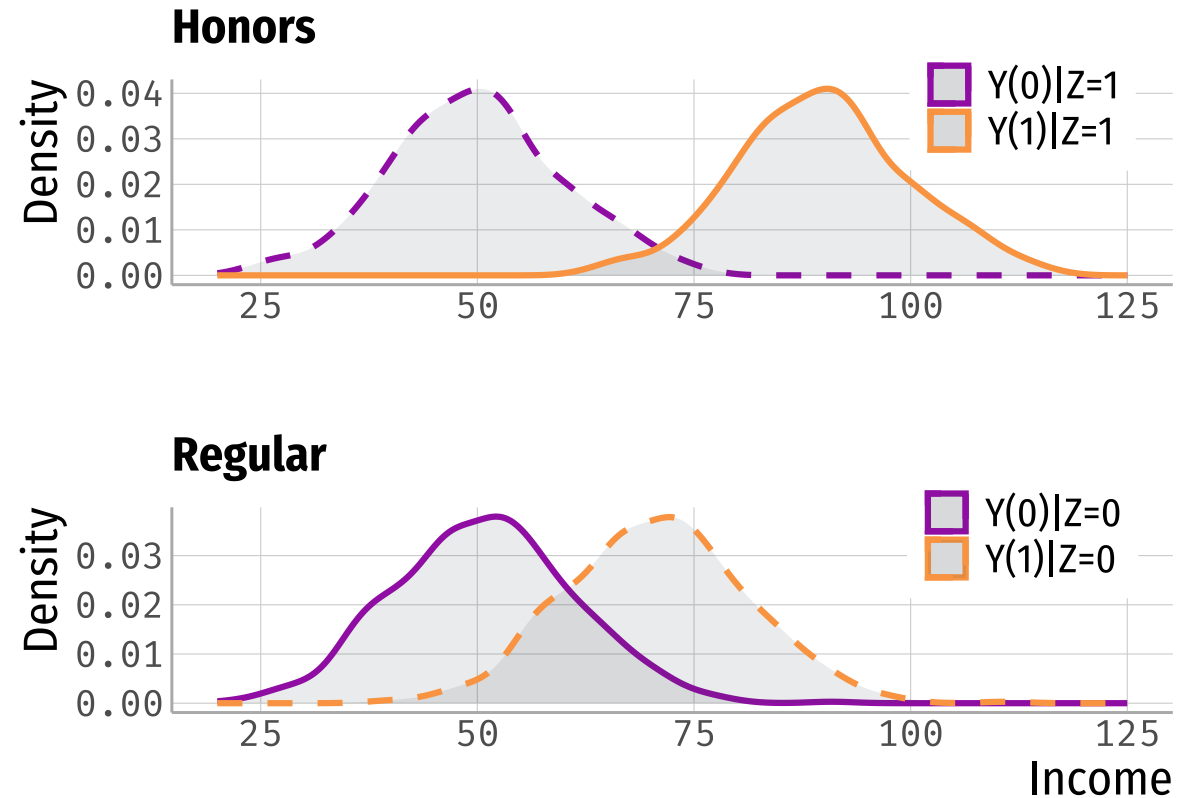
$$Income(0), Income(1) \perp\!\!\!\perp Honors$$



# What about if the ignorability assumption doesn't hold?

$$Y(0), Y(1) \not\perp Z$$

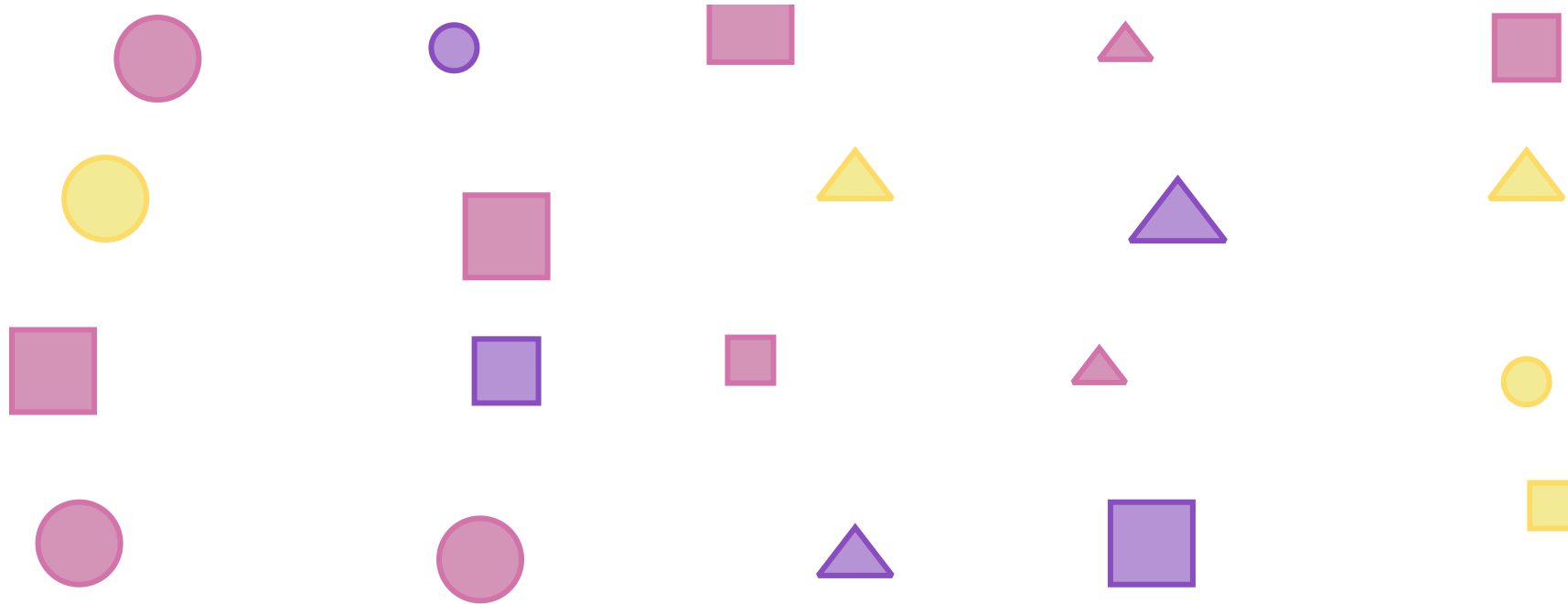
E.g. Individuals that can take  
**more advantage from honors  
program (in terms of income)**  
are more likely to go.



**What can we do to make the ignorability assumption hold?**

# The Magic of Randomization

# The problem with self-selection



Play

# The power of randomization

- One way to make sure the ignorability assumption holds is to do it by design:

**Randomize the assignment of  $Z$**

i.e. Some units will **randomly** be chosen to be in the treatment group and others to be in the control group.

**What does randomization buy us?**

# The power of randomization

- One way to make sure the ignorability assumption holds is to do it by design:

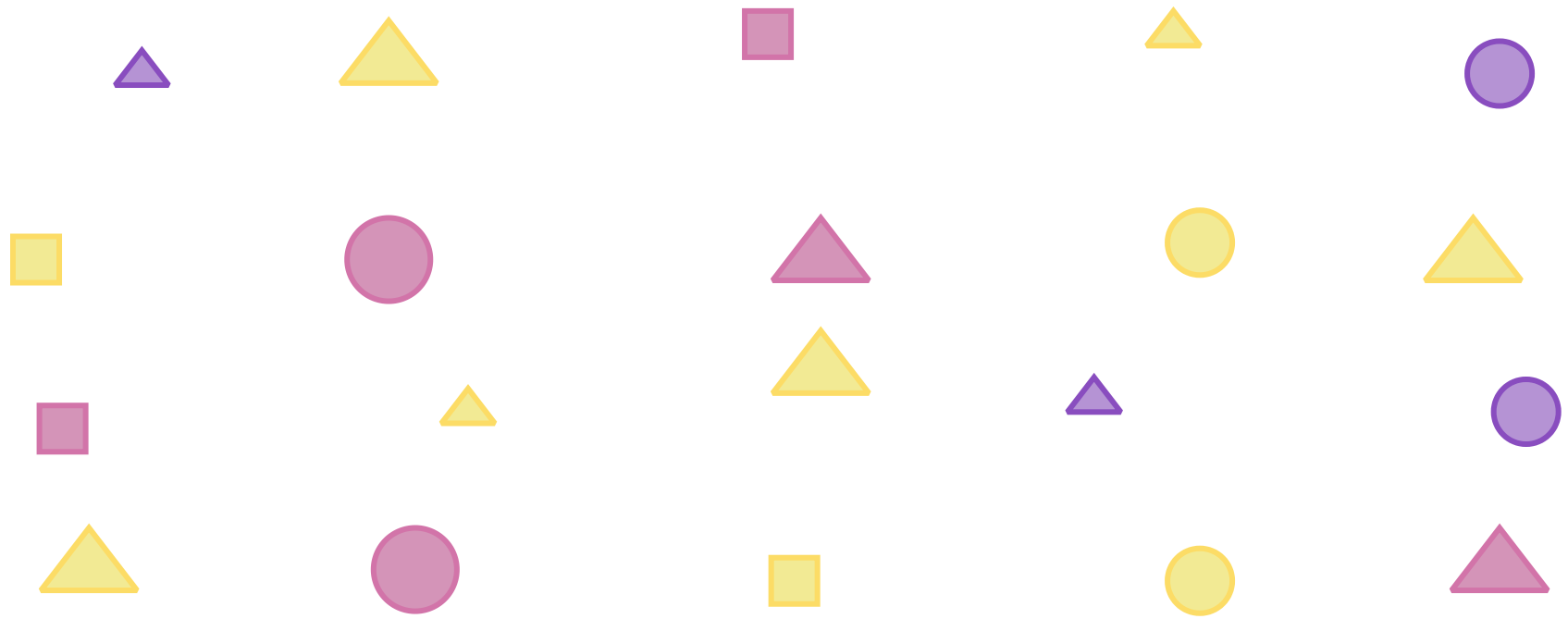
**Randomize the assignment of Z**

i.e. Some units will **randomly** be chosen to be in the treatment group and others to be in the control group.

**What does randomization buy us?**

**No (systematic) selection on observables OR unobservables**

# Randomization of z

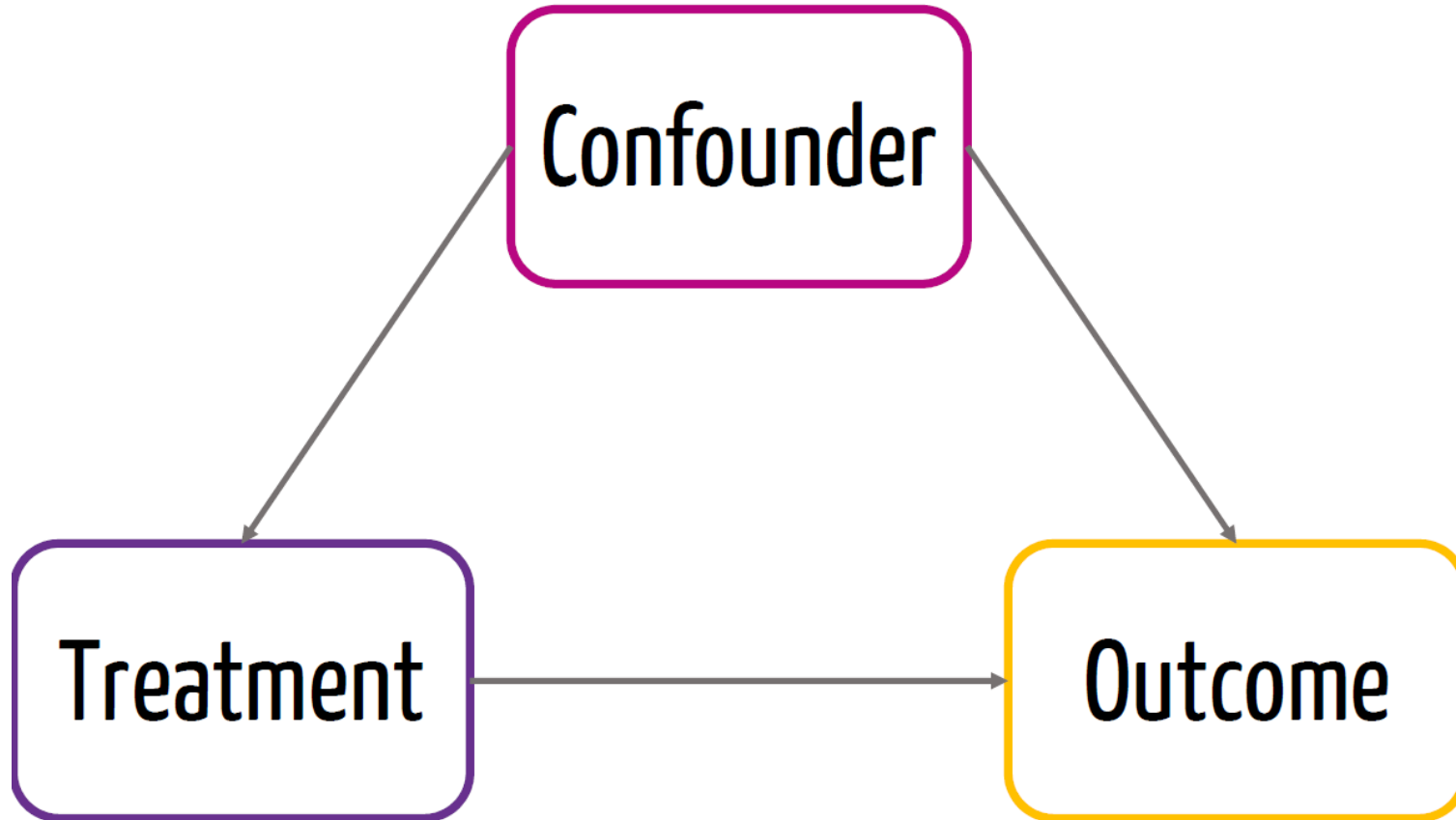


Play



# Non-Experimental Causal Graph

- Confounder is a variable that **affects both the treatment AND the outcome**

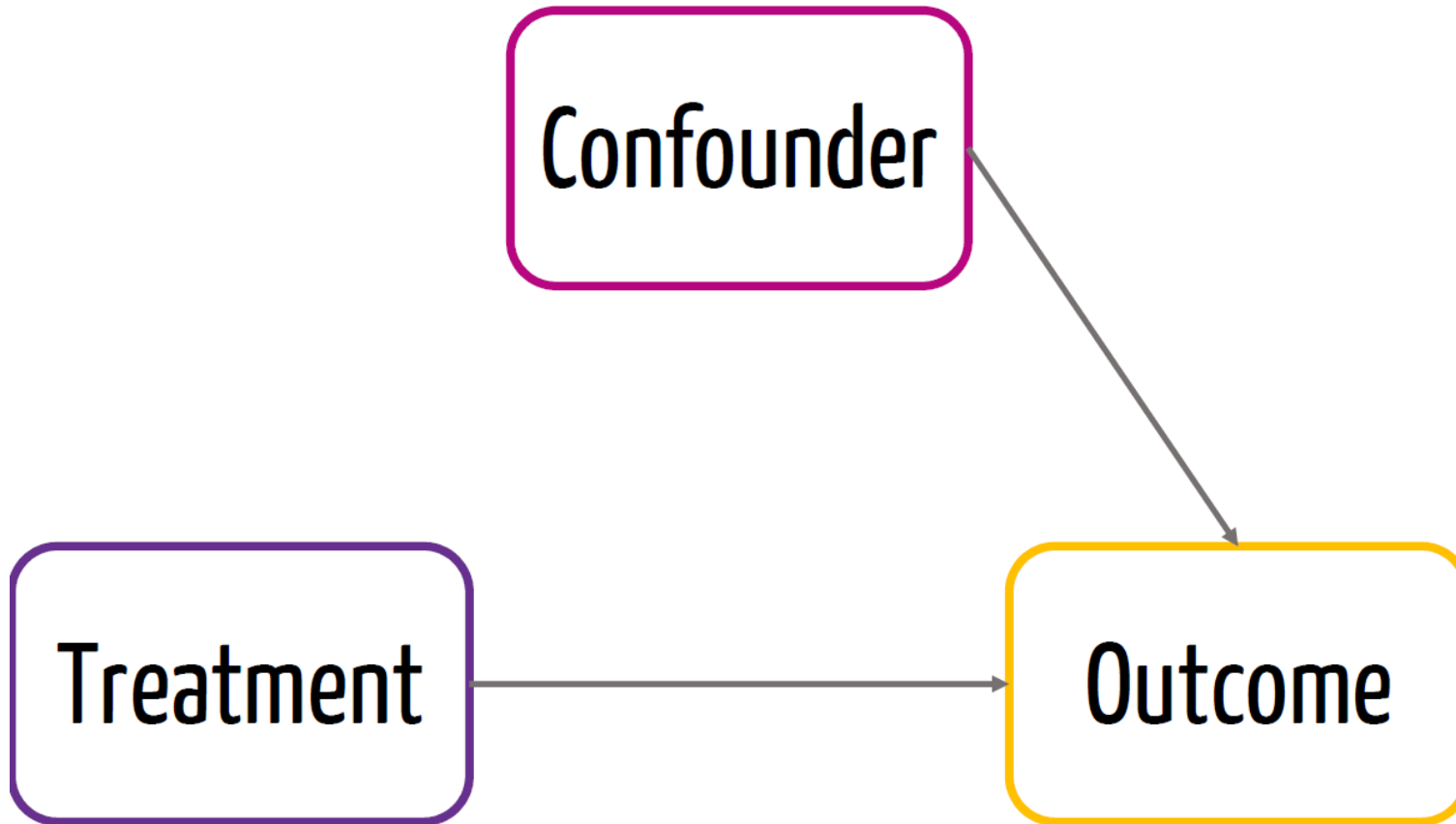


# Let's identify some confounders

- Estimate the effect of insurance vs no insurance on number of accidents → Compare people with insurance vs people without insurance.
- Estimate the effect of attending office hours vs not attending on your grade → Compare people who attend OH vs people who don't.

# Experimental Causal Graph

- Due to randomization, we know that **the treatment is not affected\*** by a confounder

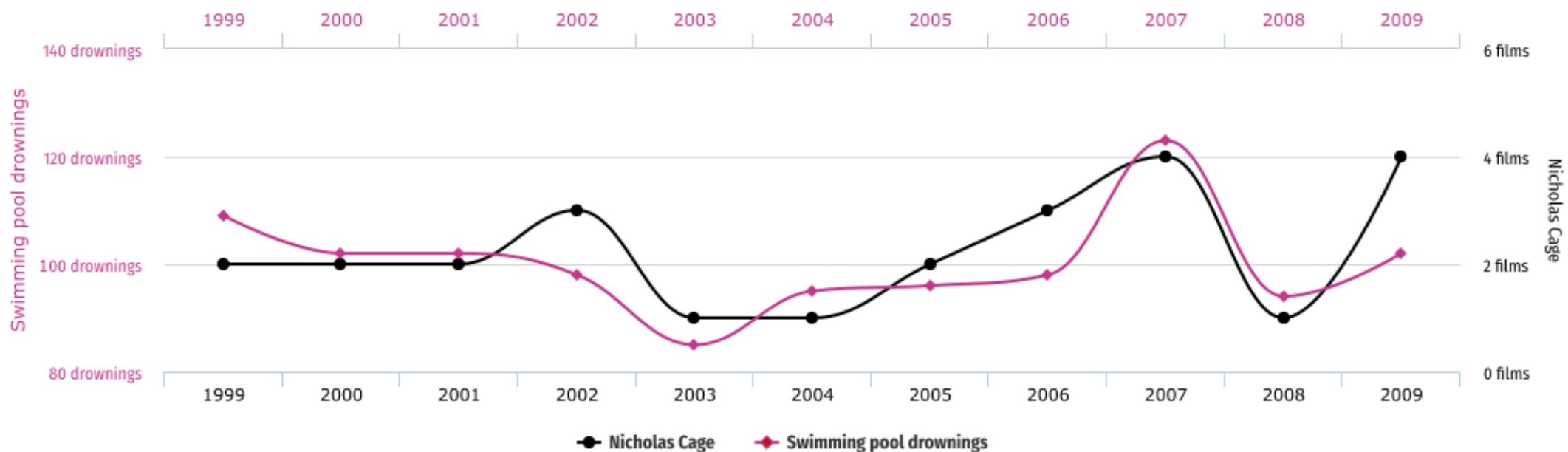


**If I randomize treatment allocation...**

**Can the treatment be potentially correlated with a confounder?**

# Just by chance!

**Number of people who drowned by falling into a pool**  
correlates with  
**Films Nicolas Cage appeared in**



# RCTs: The Gold Standard

The New York Times

## *Nobel Economics Prize Goes to Pioneers in Reducing Poverty*

Three professors, Abhijit Banerjee and Esther Duflo, both of M.I.T., and Michael Kremer of Harvard, were honored.



Abhijit Banerjee and Esther Duflo, both of M.I.T., and Michael Kremer of Harvard University won the Nobel Memorial Prize in Economic Sciences. Jonathan Nackstrand/Agence France-Presse — Getty Images

## **The Nobel went to economists who changed how we help the poor. But some critics oppose their big idea.**

Randomized controlled trials and the debate over them, explained.

By Kelsey Piper | Dec 11, 2019, 9:00am EST



The Laureates of The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel (L-R) Michael Kremer, Esther Duflo and Abhijit Banerjee pose after their Nobel Lectures at Stockholms University in Stockholm, Sweden, on December 8, 2019. | Photo by CHRISTINE OLSSON/TT News Agency/AFP via Getty Images

# How to analyze RCTs?

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**Easy! (Statistically speaking)**



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**Easy! (Statistically speaking)**

**1) Check for balance**

# How to analyze RCTs?

**Easy! (Statistically speaking)**

**1) Check for balance**

**2) Calculate difference in sample means between treatment and control group**

**Let's see an example**

# Are Emily and Greg More Employable Than Lakisha and Jamal?

- Actual **field experiment** conducted in Boston and Chicago.
- Send out resumes with **randomly assigned names**:
  - Female- and male-sounding names.
  - White- and African American-sounding names
- Measure whether **applicant was called back**

# Are Emily and Greg More Employable Than Lakisha and Jamal?

Variable	Description
education	0 = not reported; 1 = High school dropout (HSD); 2 = High school graduate (HSG); 3 = Some college; 4 = college +
ofjobs	Number of jobs listed on resume
yearsexp	Years of experience
computerskills	Applicant lists computer skills
sex	gender of the applicant (according to name)
race	race-sounding name
h	high quality resume
l	low quality resume
city	c = chicago, b = boston
call	applicant was called back

**Let's go to R**

When we assume...

# Other potential issues to have in mind

## Generalizability of our estimated effects

- Where did we get our sample for our study from? Is it representative of a larger population?

## Spillover effects

- Can an individual in the control group be affected by the treatment?

## General equilibrium effects

- What happens if we scale up an intervention? Will the effect be the same?



# Next class

- **Limitations** of RCTs
- Selection on **observables**
- The wonderful world of **matching!**



# References

- Angrist, J. and S. Pischke. (2015). "Mastering Metrics". *Chapter 1*.
- Heiss, A. (2020). "Program Evaluation for Public Policy". *Class 7: Randomization and Matching, Course at BYU*
- Imbens, G. and D. Rubin. (2015). "Causal Inference for Statistics, Social, and Biomedical Sciences: An Introduction". *Chapter 1*

