STA 235H - Multiple Regression: Overview and Analysis

Fall 2021

McCombs School of Business, UT Austin

Today

- Quick multiple regression review
  - How does OLS work?
- Inspecting your data
  - What to do with outliers?
- Comparing effect sizes in regressions

Remembering Regressions

- Linear Regression is a very useful tool.
  - Simple supervised learning approach.
  - Many fancy methods are generalizations or extensions of linear regression!
- It's a way to (partially) describe a data generating process (DGP).

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$$

- What are β's?

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- What are β's?
  - β's are the population parameters we want to estimate.
  - $\hat{\beta}$ are the estimates of those parameters.

Essential Parts of a Regression

Y

Outcome Variable

Response Variable

Dependent Variable

Thing you want to explain or predict

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- Outcome Variable
- Response Variable
- Dependent Variable

Thing you want to explain or predict

X

- Explanatory Variable
- Predictor Variable
- Independent Variable

Thing you use to explain or predict Y

Identify the variables

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Fantasy football fanatics predict the performance of a player based on past performance, health status, and characteristics of the opposite team

Netflix uses your past viewing history, the day of the week, and the time of the day to guess which show you want to watch next

Two Purposes of Regression

Prediction

Forecast the future

Focus is on Y

Netflix trying to guess your next show

Explanation

Explain the effect of X on Y

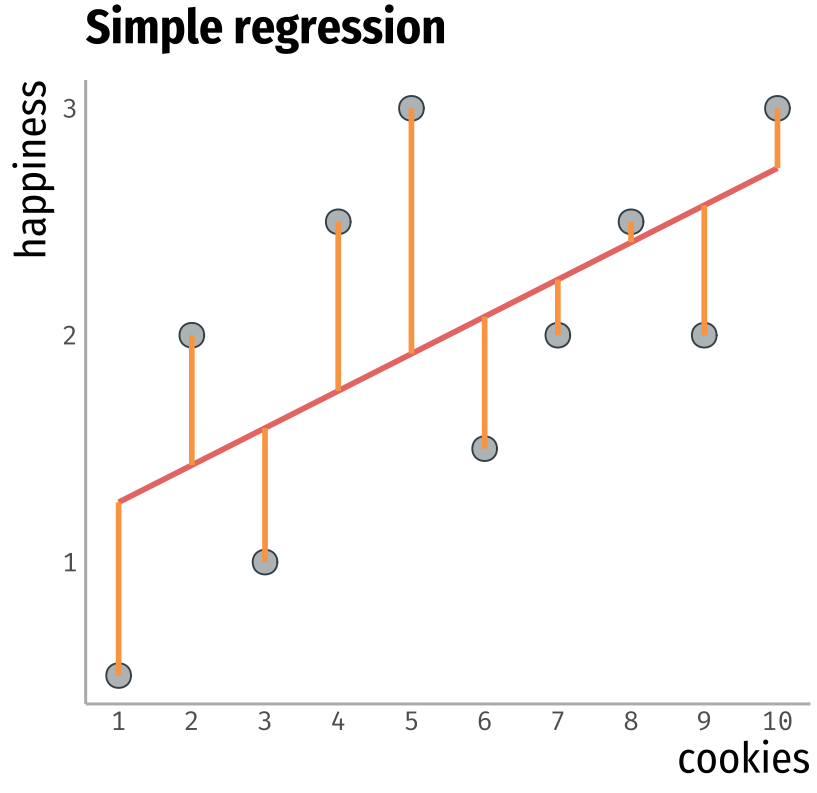
Focus is on X

Netflix looking at the effect of time of the day on show selection

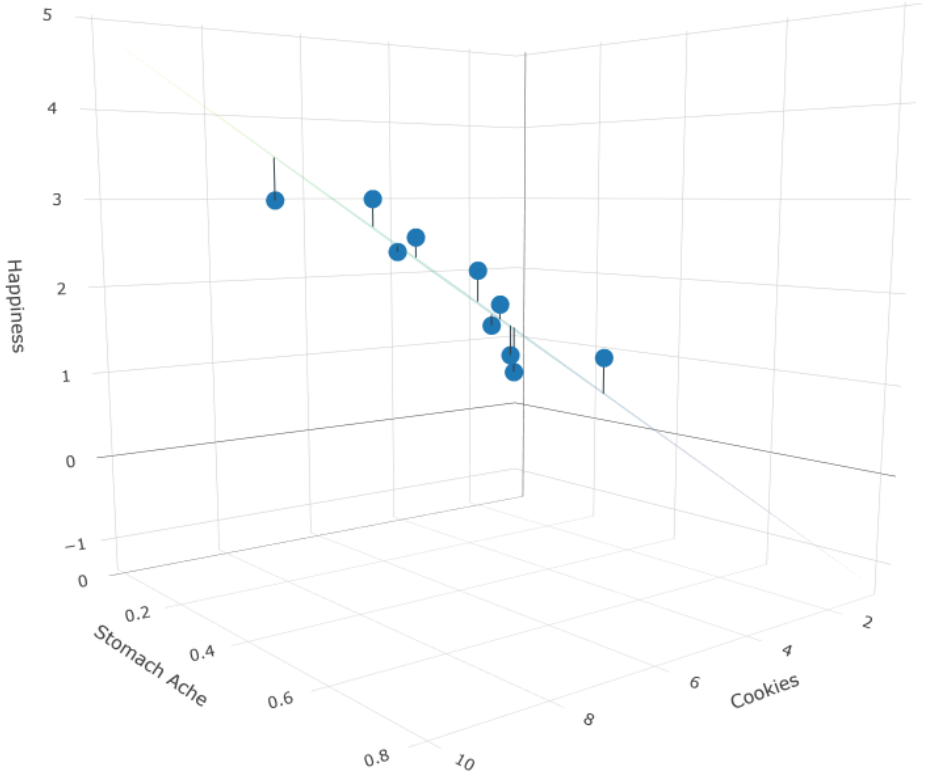
How do we estimate the coefficients in a regression ?

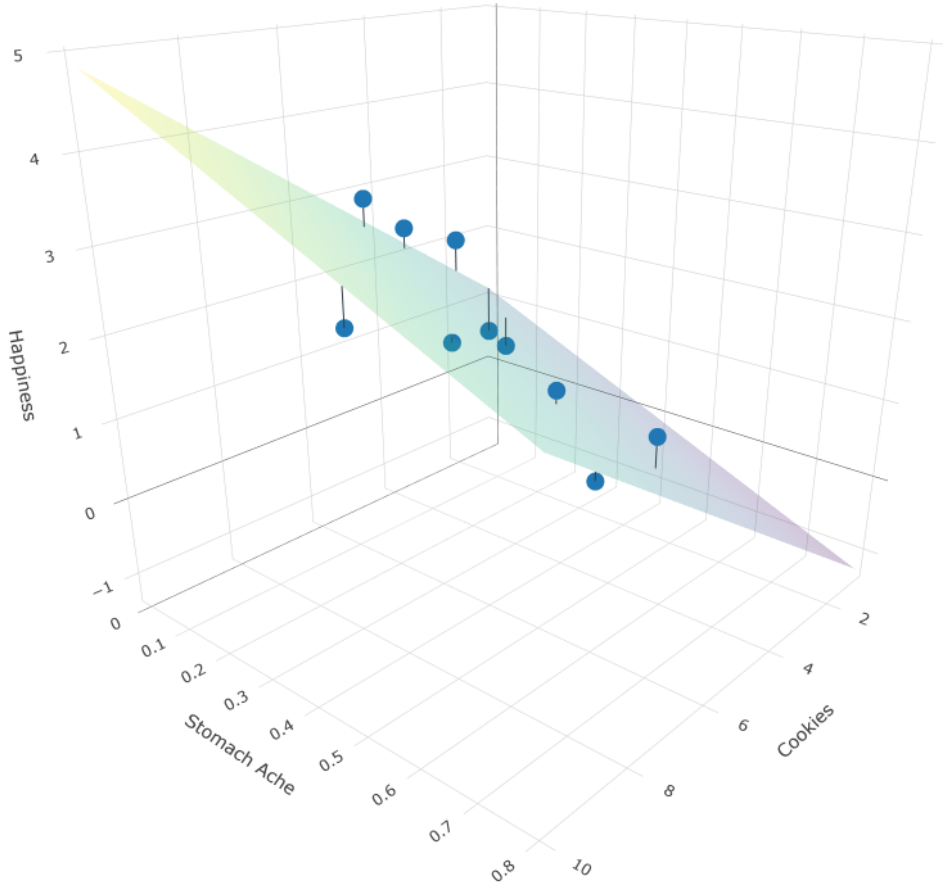
- Ordinary Least Squares is the most popular way.

$$\min_{\beta} \sum [Y_i - (\sum_{j=1}^{p} \beta_j X_{ij})]^2$$



How do we estimate the coefficients in a regression ? (cont.)

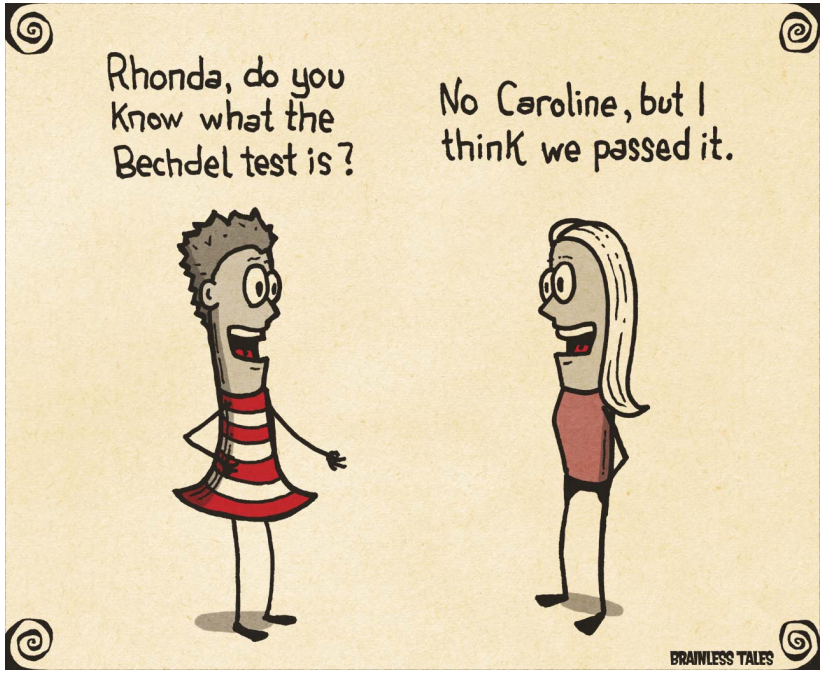




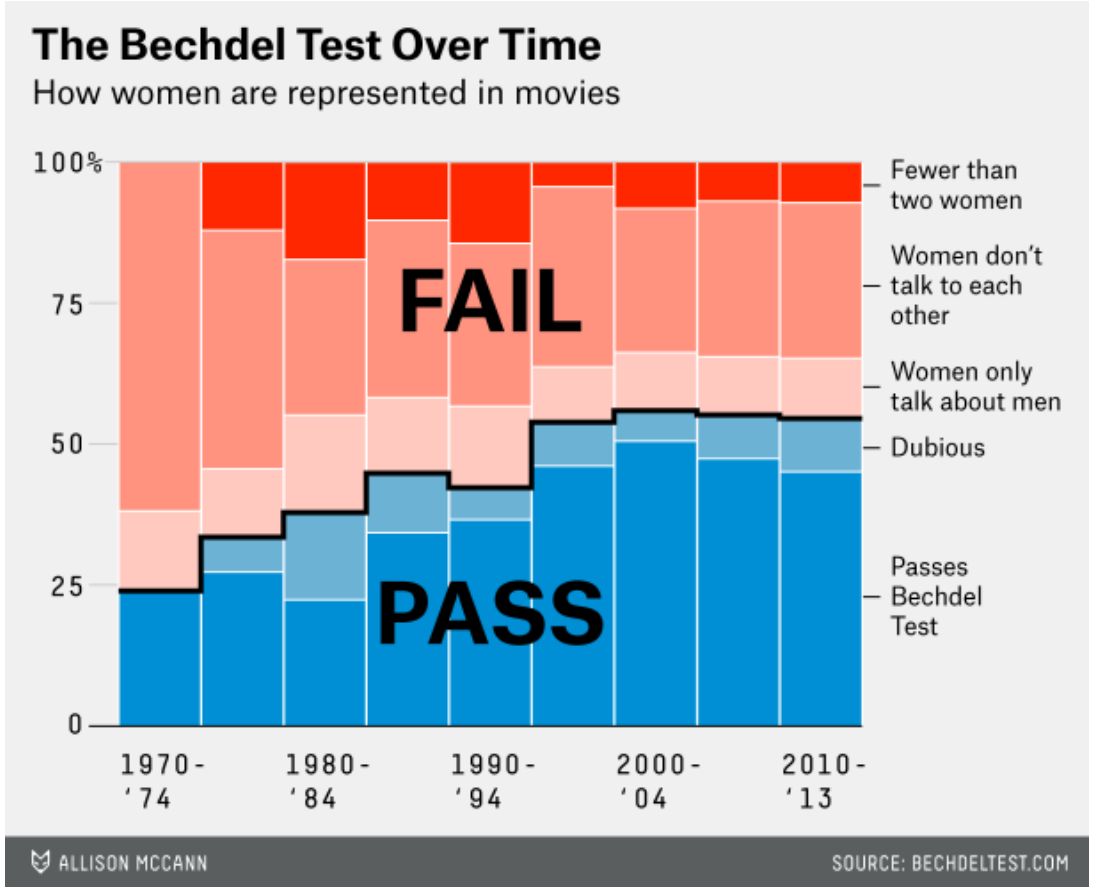
Let's introduce an example: The Bechdel Test

- Three criteria:

    1. At least two named women
    2. Who talk to each other
    3. About something besides a man



Do movies pass the test?



Is it convenient for my movie to pass the Bechdel test?

- I'm a profit-maximizing investor and want to know whether it's in my best interest to switch a male for a female character.
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$$Revenue = \alpha + \beta Bechdel + \varepsilon$$

Is this right?

What should we do before we ran any model?

Inspect your data!

vtable() can be of help

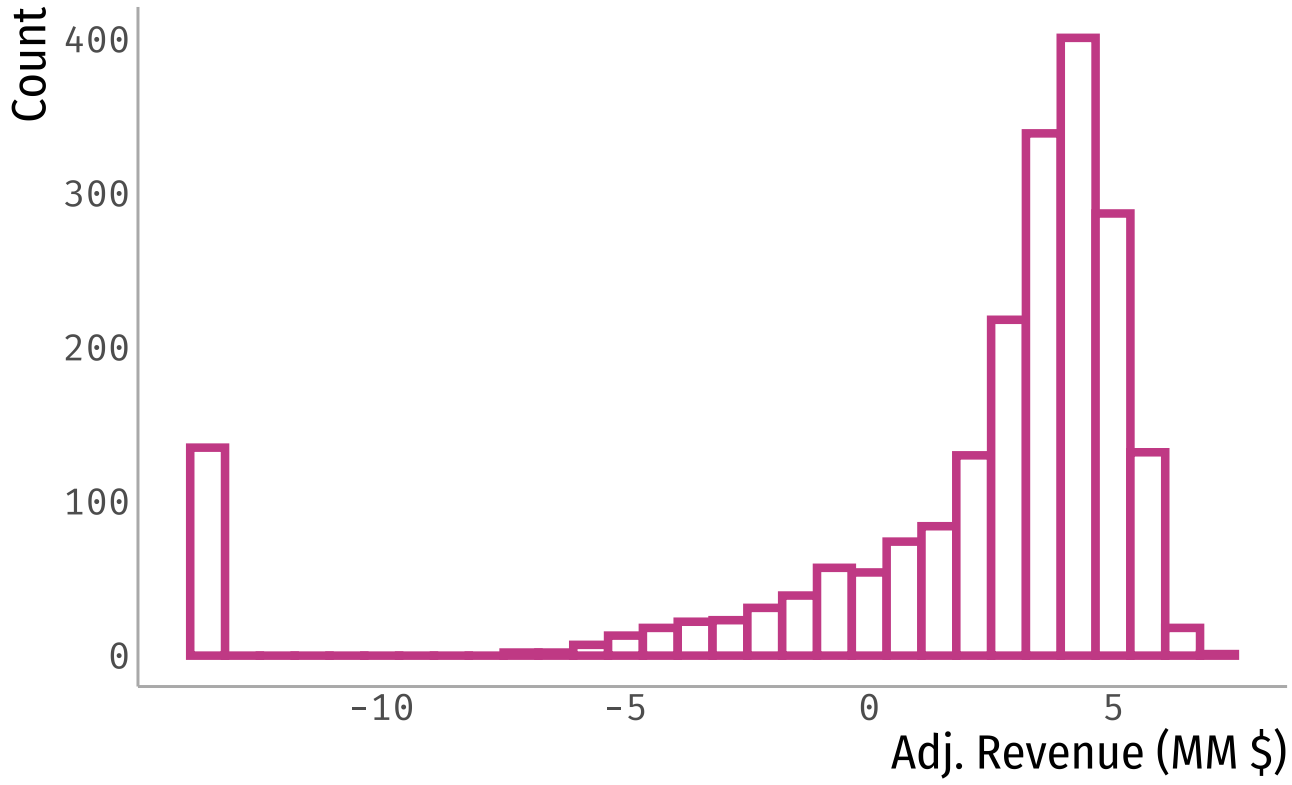
```
library(tidyverse)
library(vtable)

rawData <- read.csv("https://raw.githubusercontent.com/maibennett/sta235/main/exampleSite/content/Cl

# Select movies post 1990
rawData <- rawData %>% filter(Year>1989)

# Create return on Investment (ROI) measures
# Passes Bechdel test:
rawData <- rawData %>% mutate(ROI = Revenue/Budget, #  Total ROI
                              pass_bechdel = ifelse(rating==3, "PASS", "FAIL"))

vtable(rawData)
```

Look at the data

Look at the data

What can you say about this variable?

Logarithms to the rescue?



What to do with outliers?

1. Check them!
    - Make sure there's no coding error; try to understand what's happening there.

2a. If they are wrongly coded:

- You can remove them, always adding a note of why you did so. Issues with the analysis will come from sample selection.

2b. If they are correctly coded:

- Run analysis both with and without outliers (don't just drop them!). E.g. Results do not depend exclusively on a few observations.

Let's analyze some models

```
summary(lm(log(Adj_Revenue) ~ bechdel_test, data = bechdel))
```

```
##              Estimate Std. Error  t value Pr(>|t|)
## (Intercept)   17.0321     0.0808 210.9100        0
## bechdel_test  -0.4418     0.1079  -4.0954        0
```

- How do you interpret these results?
- What are the units for the dependent variable?

A side note on log-transformed variables...

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$$\log(\frac{y_1}{y_0}) = \hat{\beta}_1$$

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$$\rightarrow \frac{\Delta y}{y} = \exp(\hat{\hat{\beta}}_1) - 1$$

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- $(e^{\beta} - 1) \cdot 100 \rightarrow$ A movie that passes the Bechdel test is associated with a 36% decrease in Revenue

Negative effect of including more women?

What gives?

FiveThirtyEight

Politics Sports Science Podcasts Video

APR. 1, 2014, AT 1:52 PM

The Dollar-And-Cents Case Against Hollywood's Exclusion of Women

By Walt Hickey

Filed under Movies

Get the data on GitHub

A Walmart employee puts Lionsgate's "The Hunger Games: Catching Fire" Blu-ray Combo Pack and DVD on the rack prior to the midnight release at Walmart on March 6, 2014 in Orange, California. JEROD HARRIS / GETTY IMAGES

More variables

- Bechdel test could be capturing the effect of other variables:
  - What type of movies are the ones that pass the test?
  - What is their budget?

More variables

```
lm(log(Adj_Revenue) ~ bechdel_test + log(Adj_Budget) + Metascore + imdb, data=bechdel)
```

```
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)       1.3798     0.5126  2.6921   0.0072
## bechdel_test      0.2275     0.0665  3.4229   0.0006
## log(Adj_Budget)   0.8594     0.0256 33.6160   0.0000
## Metascore         0.1012     0.0293  3.4512   0.0006
## imdb              0.0864     0.0517  1.6716   0.0948
```

Positive and significant!

Comparing effect sizes

- Another investor says that it's better to bring in a better actor because it will increase ratings.
- How do you compare effect sizes?
    - How does one more point on IMDB compare to passing/failing the Bechdel test?

Standardized Partial Coefficients

- Main idea: Transform everything to the same scale (standard deviations)

$$X$$

$$\frac{X - \bar{X}}{\sigma_X}$$

- Will this change our estimates? How?

Transform the data

```
scale2 <- function(x, na.rm = FALSE) (x - mean(x, na.rm = na.rm)) / sd(x, na.rm)

bechdel_std <- bechdel %>% select(log_Adj_Revenue,log_Adj_Budget,
                                  bechdel_test, Metascore, imdb) %>%
  mutate_at(c("log_Adj_Revenue","log_Adj_Budget", "bechdel_test",
                  "Metascore", "imdb"),~scale2(.,na.rm=TRUE))

lm(log_Adj_Revenue~bechdel_test+log_Adj_Budget + Metascore + imdb,
   data=bechdel_std)
```

```
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.3384     0.0138 24.5385   0.0000
## bechdel_test     0.0476     0.0139  3.4229   0.0006
## log_Adj_Budget   0.4683     0.0139 33.6160   0.0000
## Metascore        0.0706     0.0205  3.4512   0.0006
## imdb             0.0342     0.0205  1.6716   0.0948
```

- What are the units on `bechdel_test` now? Does it make sense?

Main takeaway points

- Data can tell different stories depending on how you handle it.
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"If you torture the data long enough, it will confess to anything"

- Assumptions and measures matter.
- Plot your data!

Next class

- Finishing with multiple regression models:
    - Statistical adjustment and collinearity
- Assumptions for OLS
- In the following weeks, we will start with Causal Inference.

References

- Heiss, A. (2020). "Course: Program Evaluation for Public Service". *Slides for Regression and Inference*.
- Ismay, C. & A. Kim. (2021). “Statistical Inference via Data Science”. Chapter 10.
- Keegan, B. (2018). "The Need for Openess in Data Journalism". *Github Repository*