STA 235 - Causal Inference: Regression Discontinuity Design

Spring 2021

McCombs School of Business, UT Austin

Another identification strategy

- We have seen:

RCTs

Selection on observables

Natural experiments

Differences-in-Differences

Regression Discontinuity Designs

I'm on the edge [of glory?]

Introduction to Regression Discontinuity Designs

Regression Discontinuity (RD) Designs

Arbitrary rules determine treatment assignment

E.g.: If you are above a threshold, you are assigned to treatment, and if your below, you are not (or vice versa)

Key Terms

Running/ forcing variable

Index or measure that determines eligibility

Cutoff/ cutpoint/ threshold

Number that formally assigns you to a program or treatment

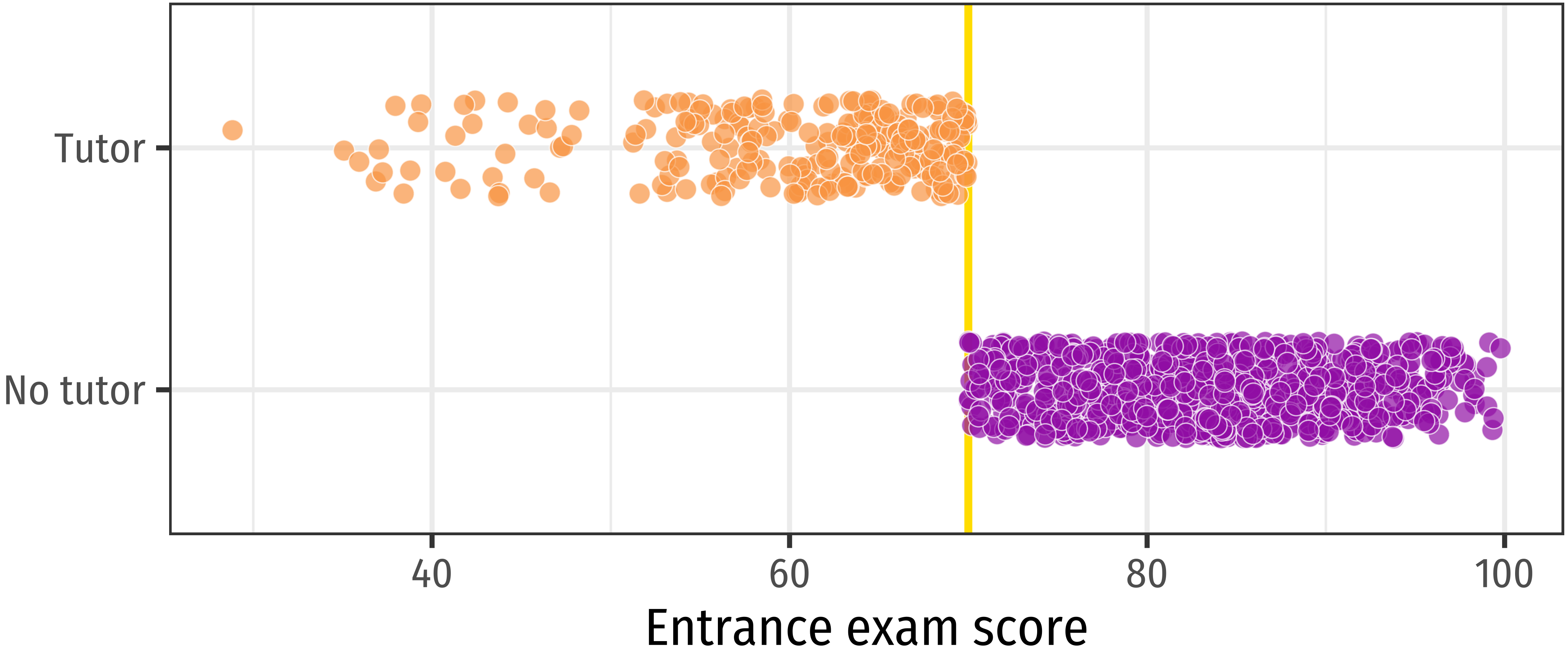
Hypothetical tutoring program

Students take an entrance exam

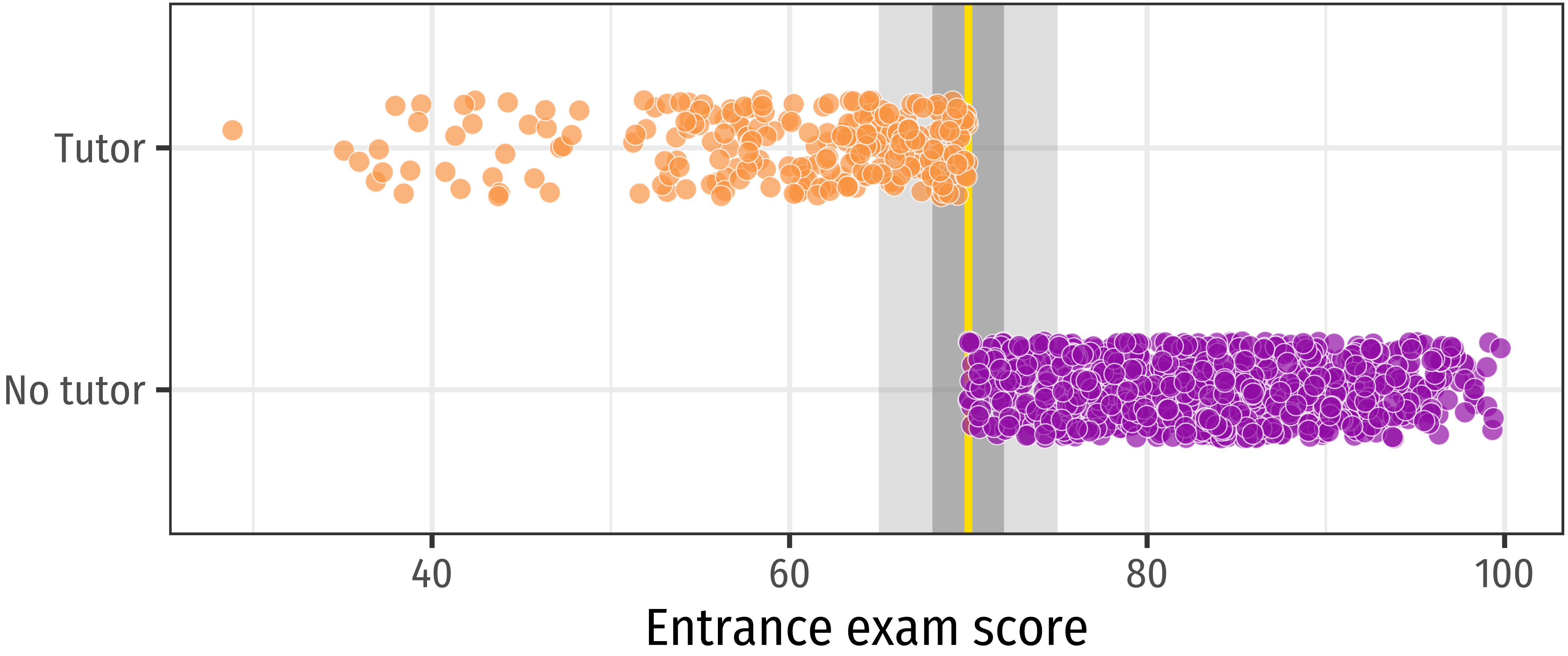
Those who score 70 or lower get a free tutor for the year

Students then take an exit exam at the end of the year

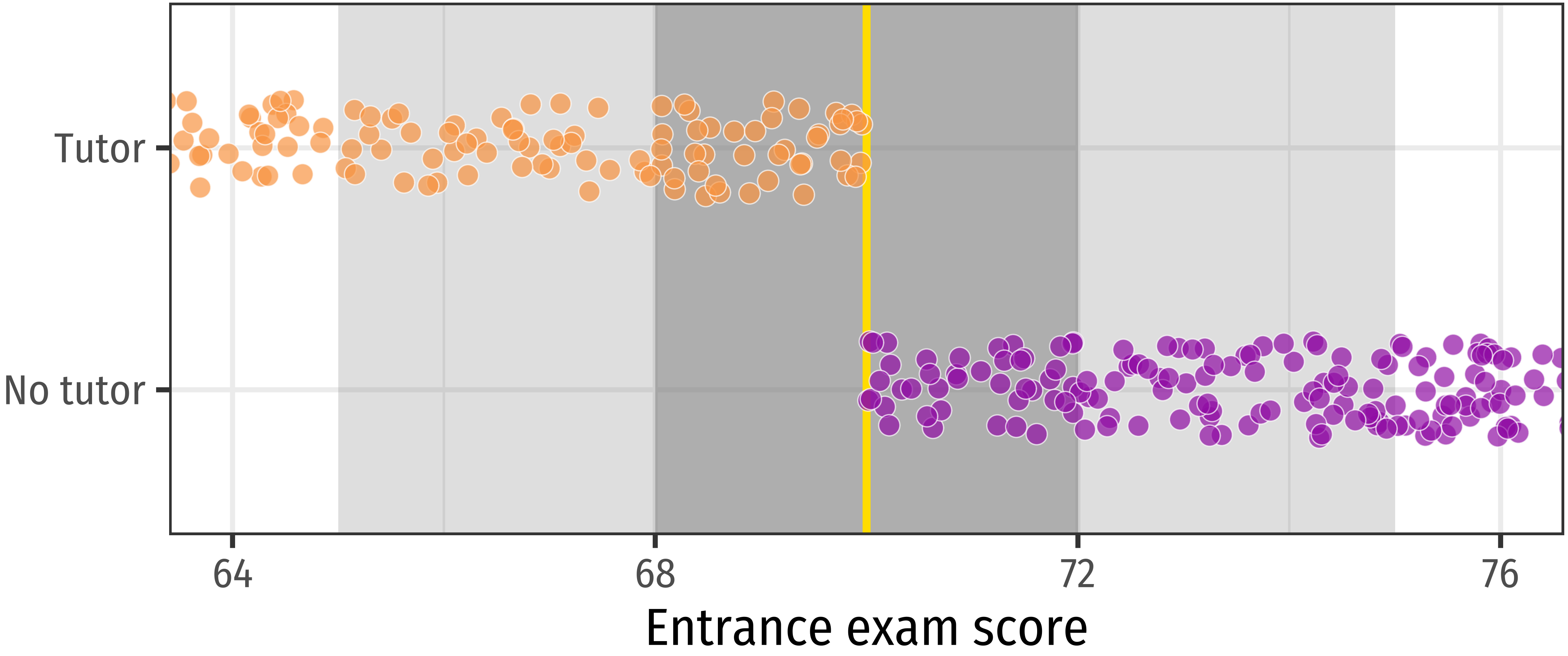
Assignment based on entrance score



Let's look at the area close to the cutoff



Let's get closer



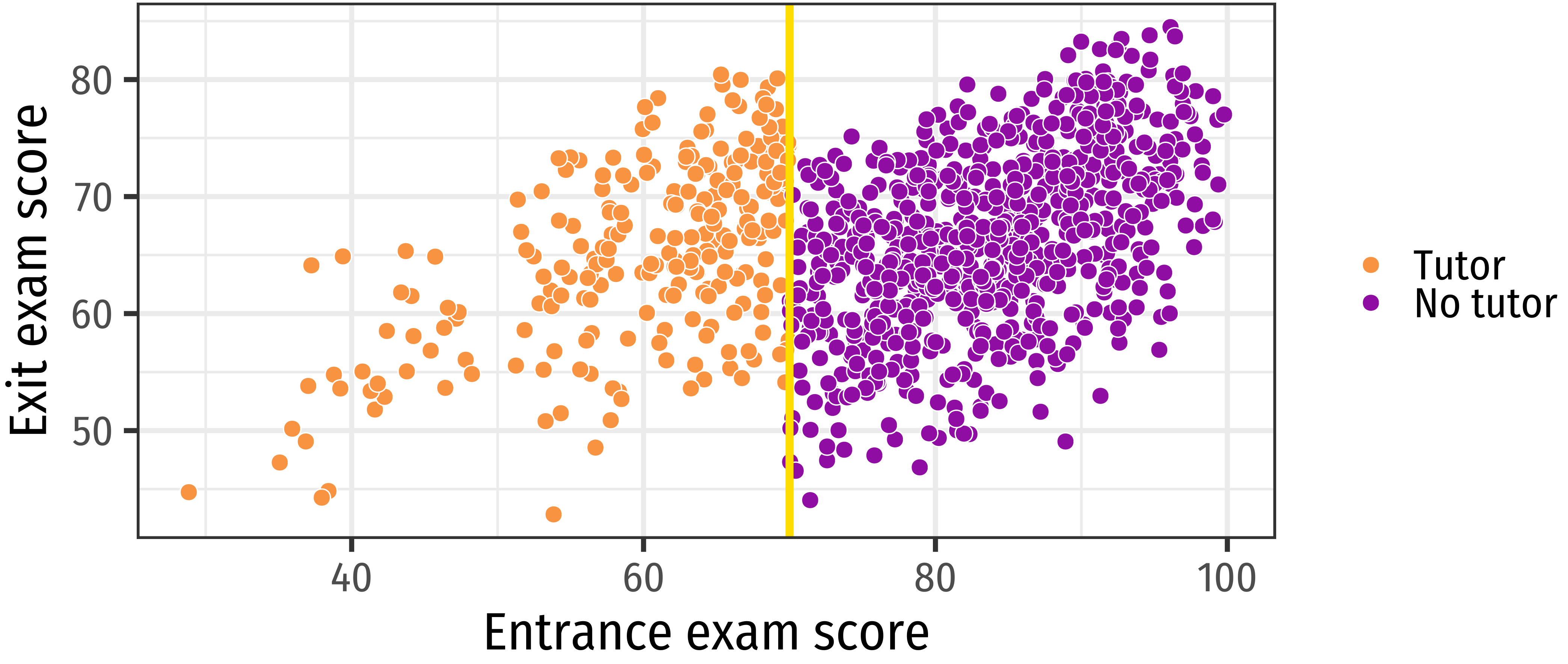
Causal inference intuition

Observations right before and after the threshold are essentially the same

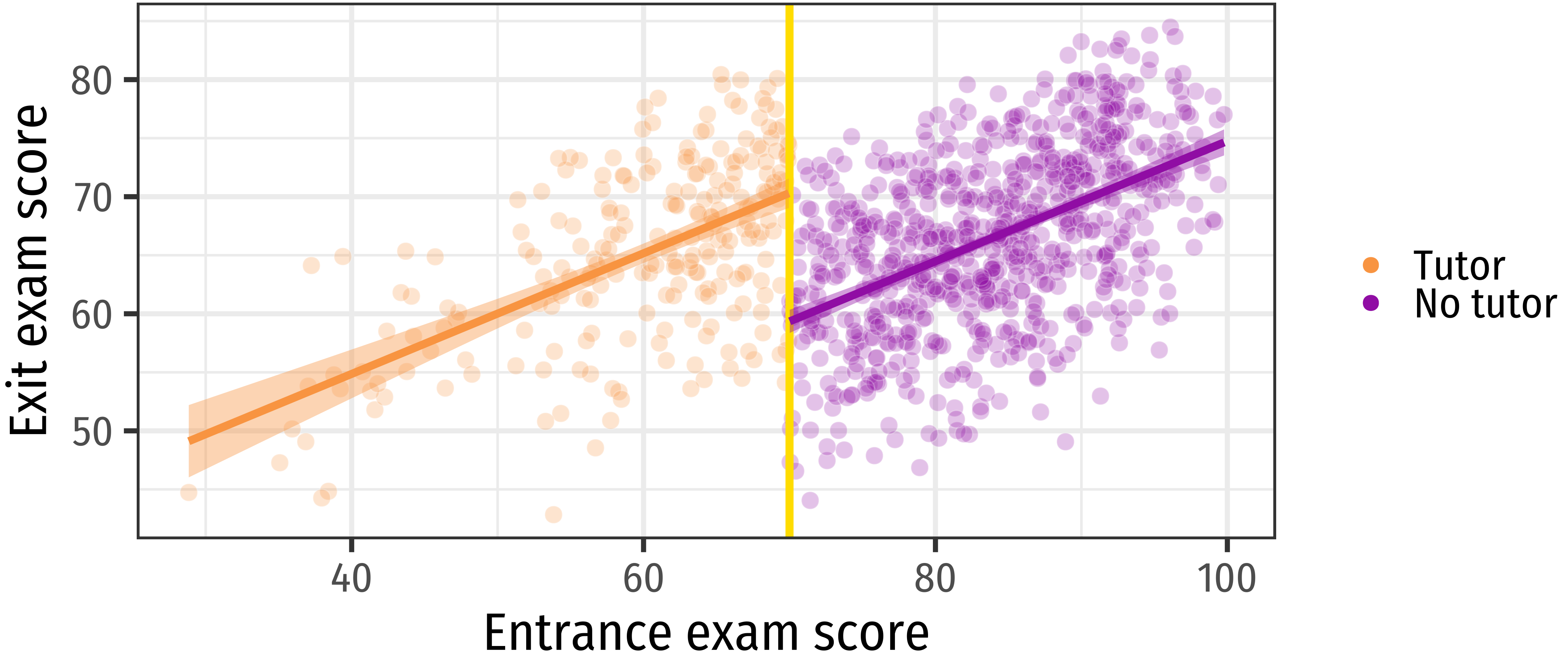
Pseudo treatment and control groups!

Compare outcomes right at the cutoff

Exit exam results according to running variable



Fit a regression at the right and left side of the cutoff



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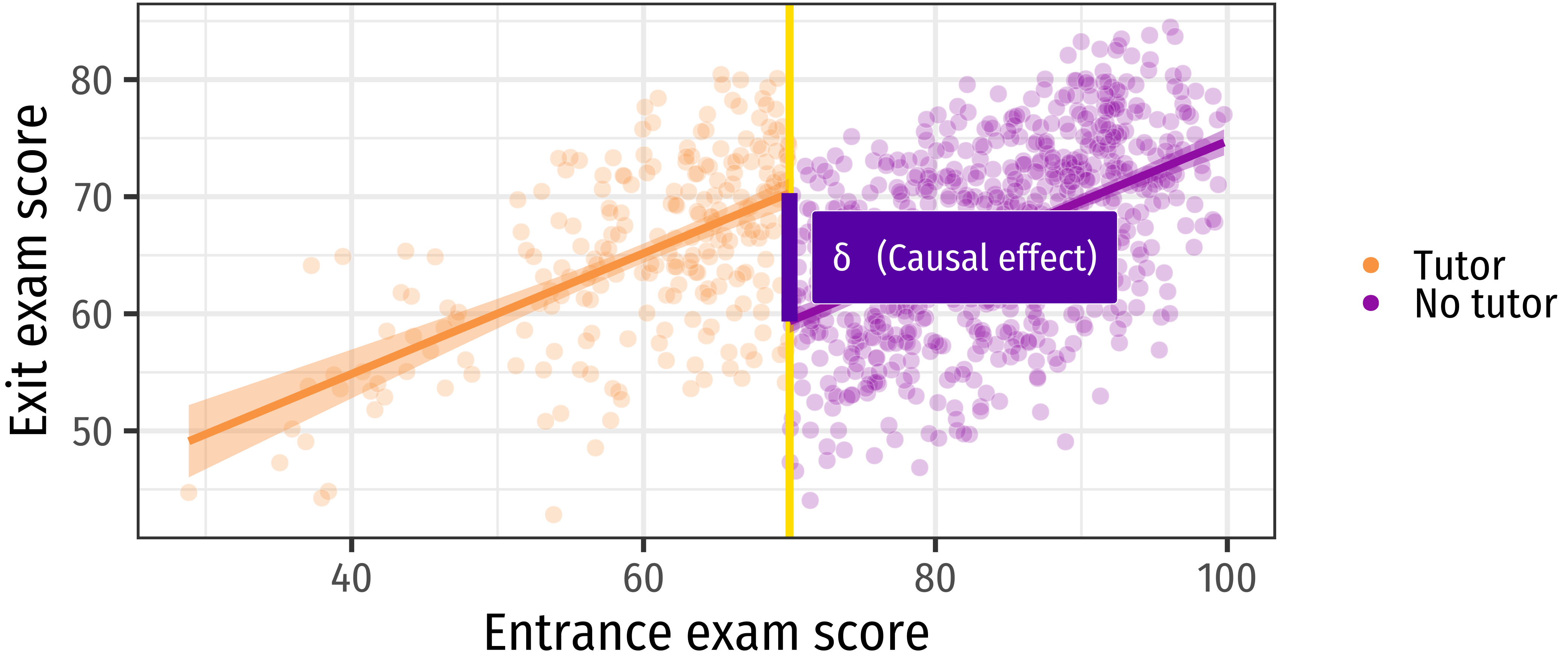


You can find discontinuities everywhere!

Geographic discontinuities

Time discontinuities

Voting discontinuities

How do we do RDs in practice?

Behind the scenes of RDs

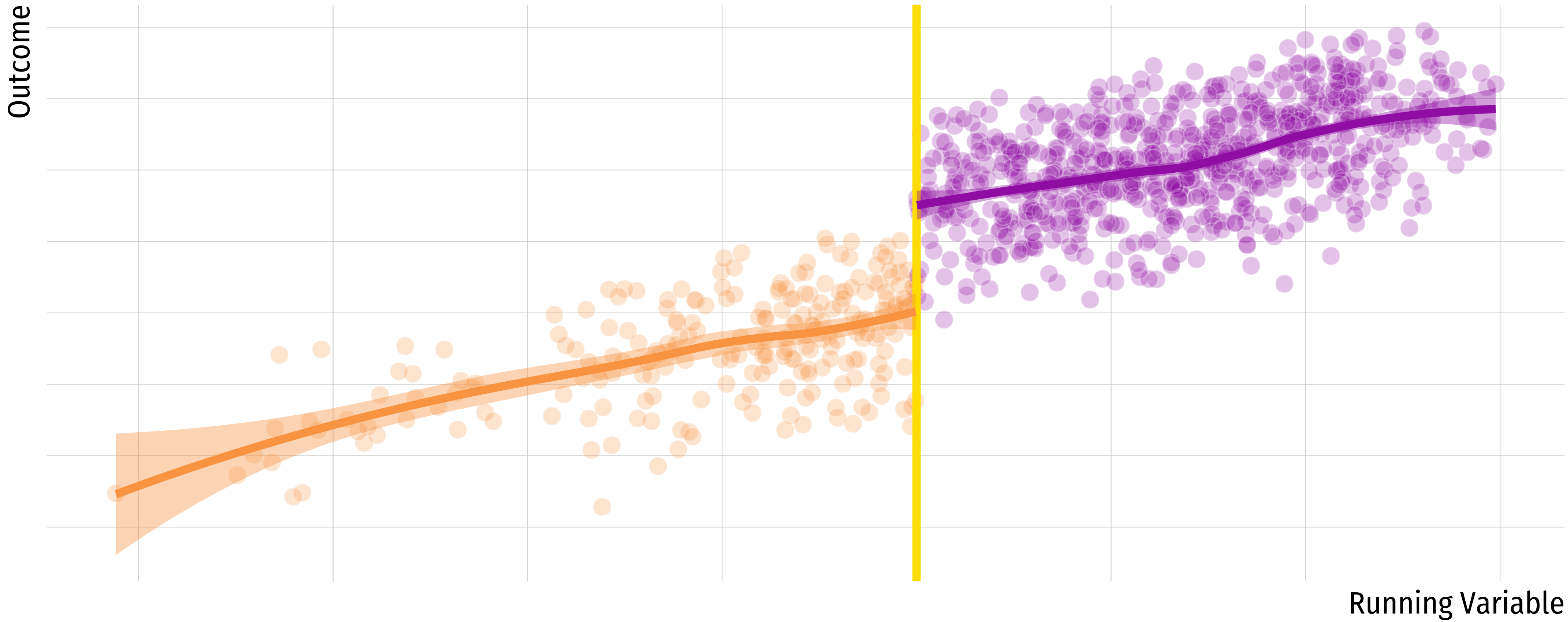
- Basically, regression discontinuities work under an **asymptotic assumption**:
- Let Y_i be the outcome of interest, Z_i the treatment assignment, R_i the running variable, and c the cutoff score:

$$Z_i = \begin{cases} 0 & R_i \leq c \\ 1 & R_i > c \end{cases}$$

- Then, we can define the treatment effect δ as:

$$\delta = \lim_{\epsilon \to 0^+} E[Y_i | R_i = c + \epsilon] - \lim_{\epsilon \to 0^-} E[Y_i | R_i = c + \epsilon]$$

What does the limit expression mean?



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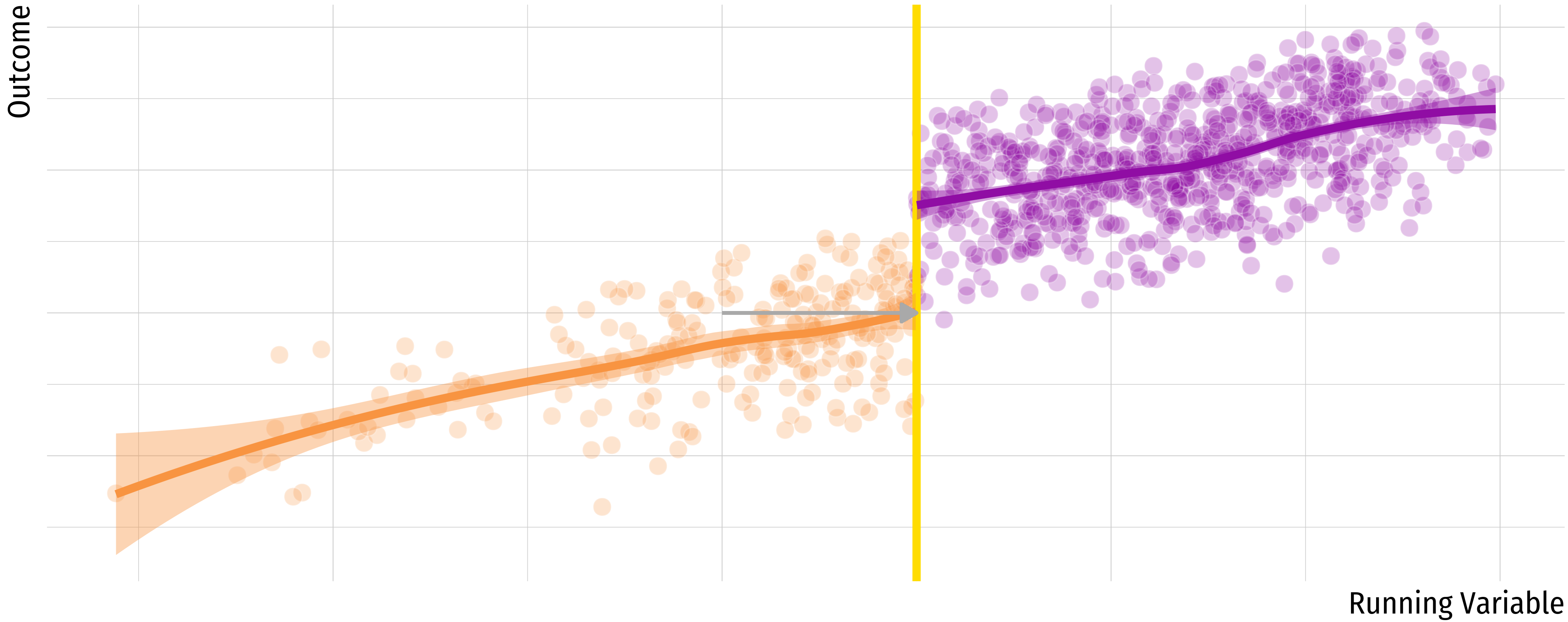

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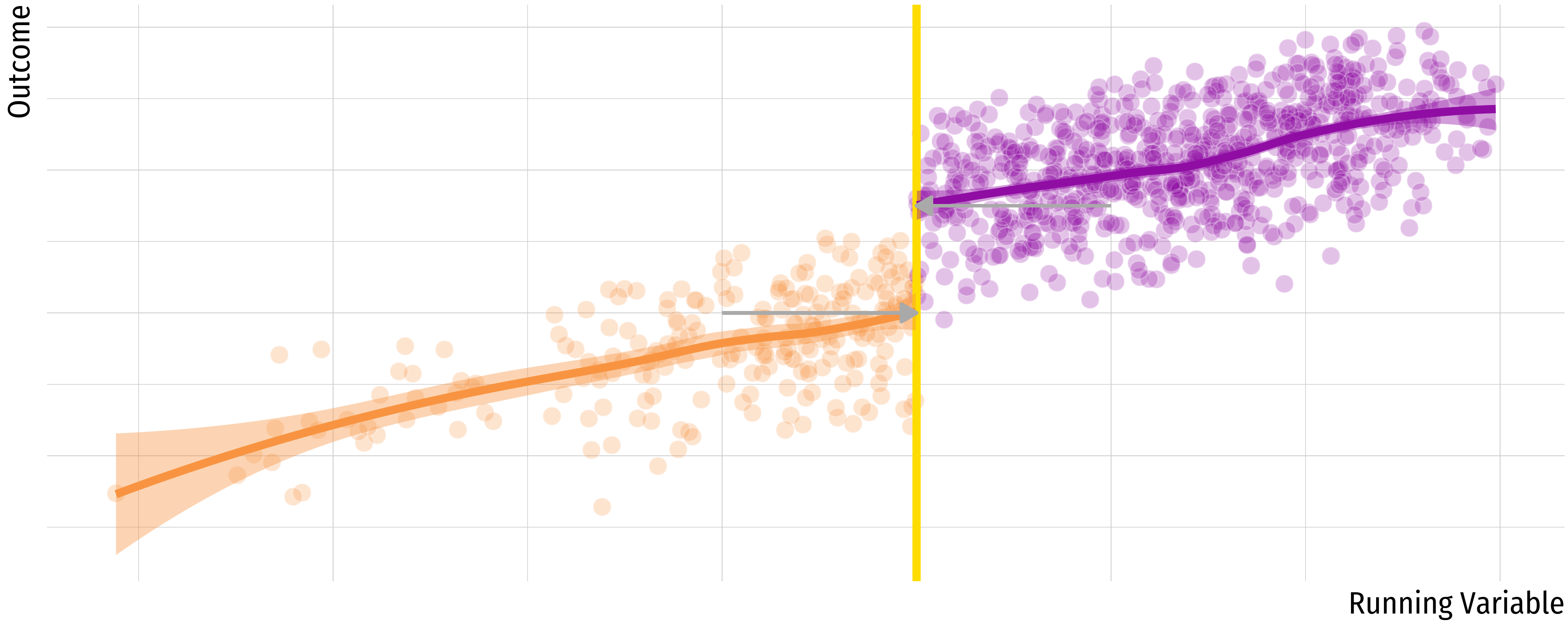


What is the estimand we are estimating?

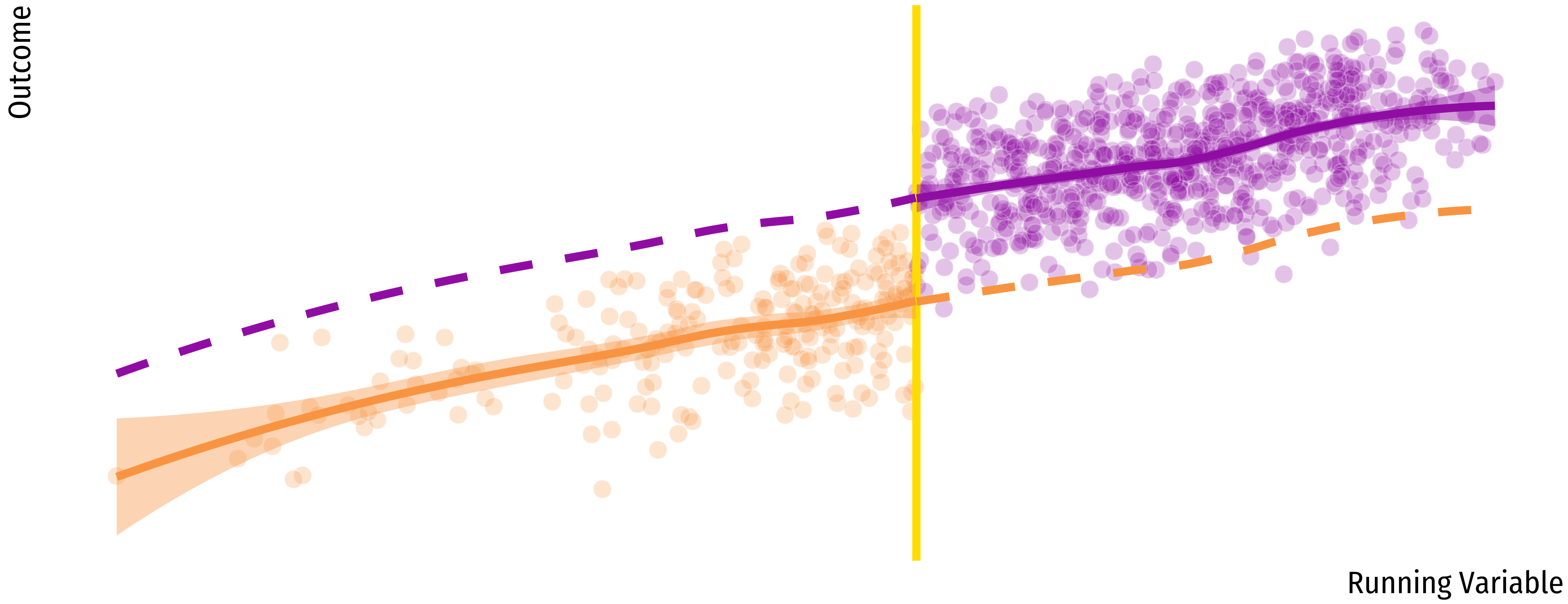
Local Average Treatment Effect (LATE) for units at R=c

Conditions required for identificaton

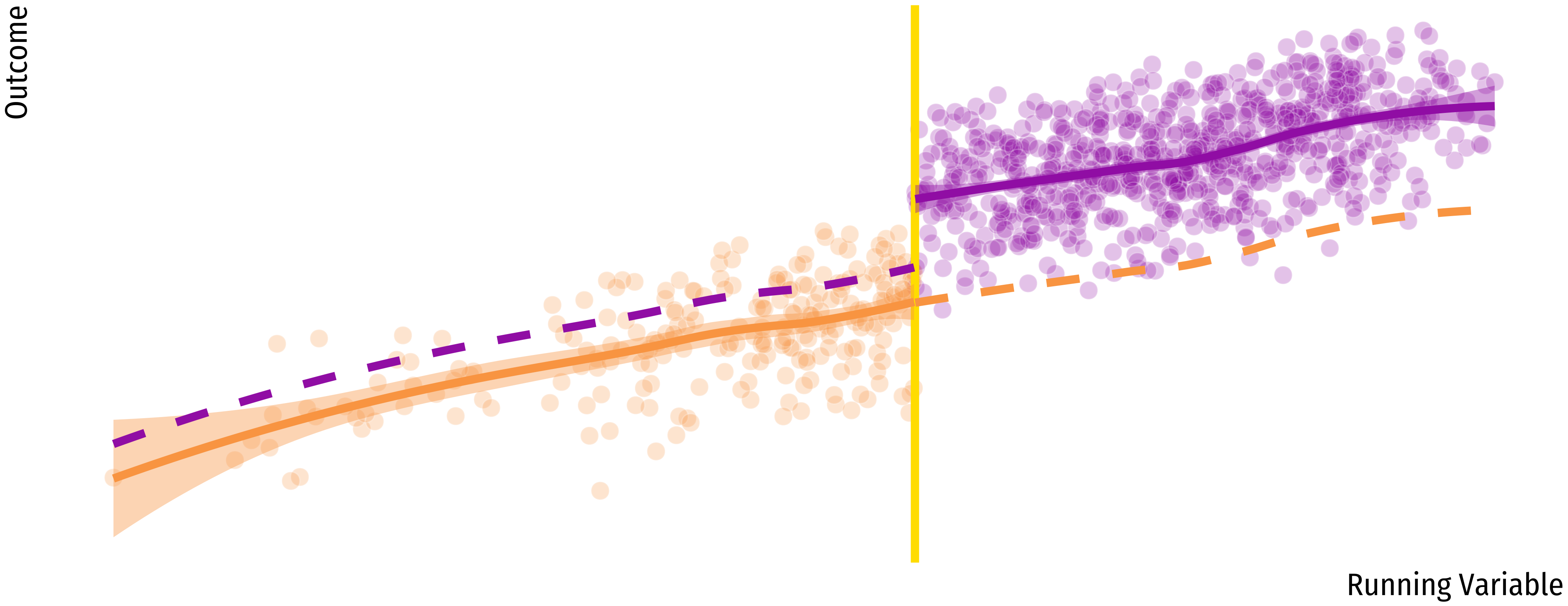
- Threshold rule **exists** and cutoff point is **known**
- The running variable R_i is **continuous** near c.
- Key assumption:

Continuity of E[Y(1)|R] and E[Y(0)|R] at R=c

Potential outcomes need to be smooth across the threshold



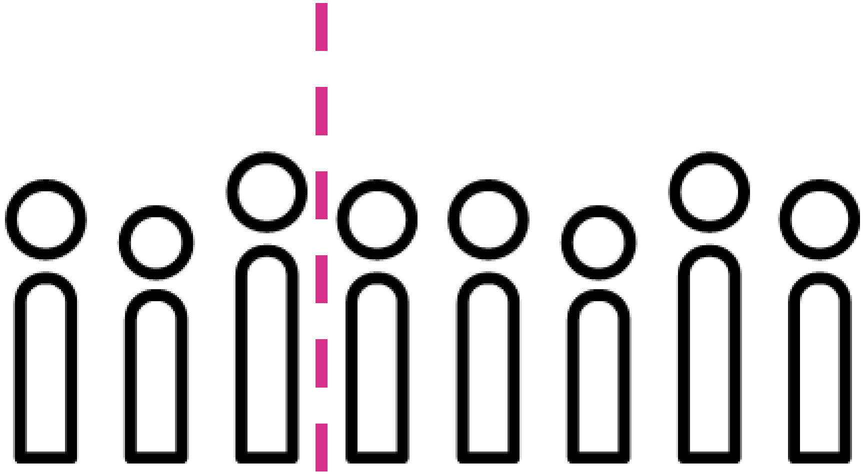
Potential outcomes need to be smooth across the threshold



Can you think situations where that could happen?

Let's go back to our discount example

- Customers are given discounts based on their **order of arrival**



- We could think of this as an **RD in time**, where c is the time of arrival of customer 1,000.

Work in groups

1) Each group will be given a task and some code

2) You need to complete the code and discuss the results

Group 1

What did you have to do?

Group 2

What did you have to do?

Group 3

What did you have to do?

Group 4

What did you have to do?

Let's get into estimation

How do we actually estimate an RD?

- The simplest way to do this is to fit a regression:

$$Y_i = \beta_0 + \beta_1(R_i - c) + \beta_2 \mathrm{I}[R_i > c] + \beta_3(R_i - c)\mathrm{I}[R_i > c]$$

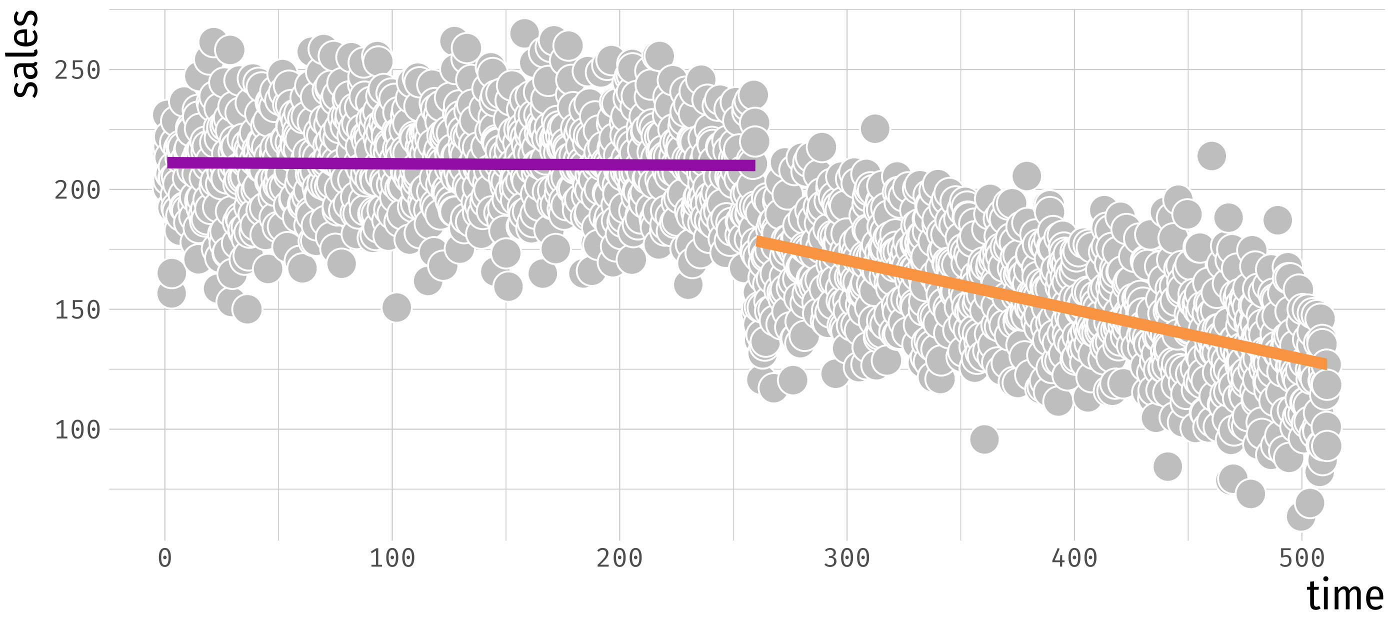
- You want to add **flexibility** for each side of the cutoff.

Can you identify these parameters in a plot?

Let's see some examples: Sales using a linear model

```
sales <- sales %>% mutate(dist = c-time)

lm(sales ~ dist + treat + dist*treat, data = sales)
```



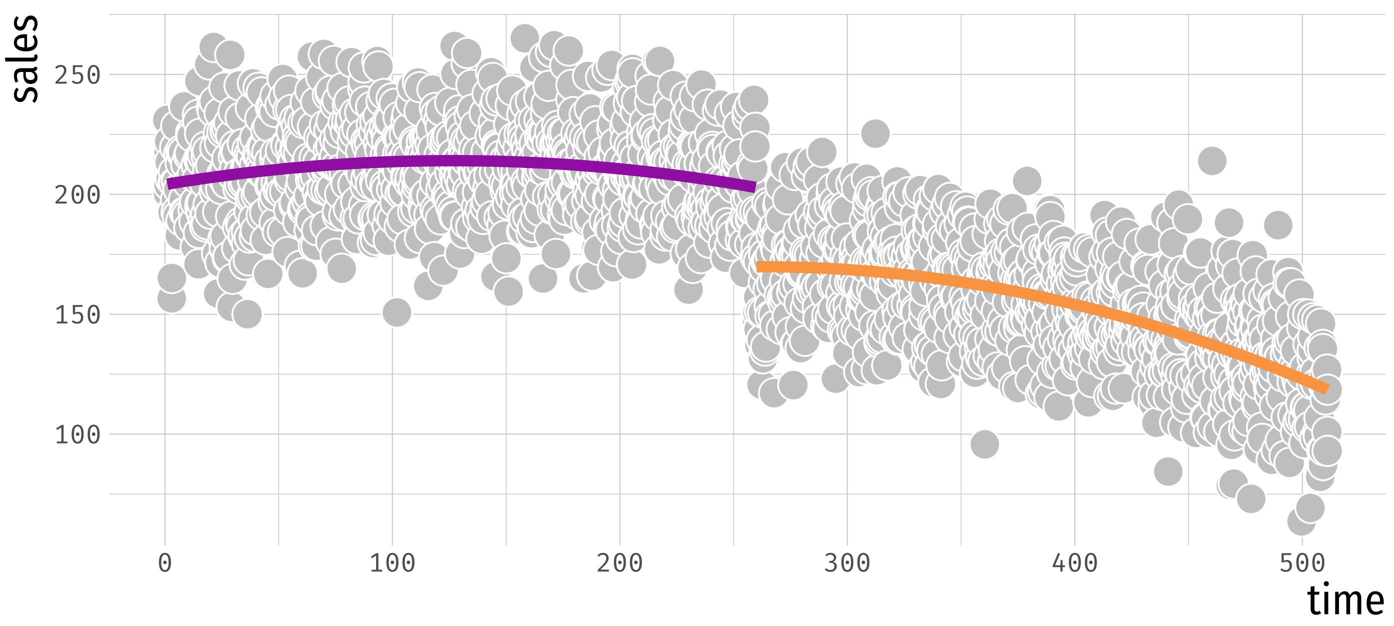
Let's see some examples: Sales using a linear model

```
summary(lm(sales ~ dist + treat + dist*treat, data = sales))
```

```
##
## Call:
## lm(formula = sales ~ dist + treat + dist * treat, data = sales)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -65.738 -13.940   0.051  13.538  76.515
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 178.640954   1.300314  137.38   <2e-16 ***
## dist          0.205355   0.008882   23.12   <2e-16 ***
## treat        31.333952   1.842338   17.01   <2e-16 ***
## dist:treat   -0.200845   0.012438  -16.15   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.52 on 1996 degrees of freedom
## Multiple R-squared:  0.6939,    Adjusted R-squared:  0.6934
## F-statistic:  1508 on 3 and 1996 DF,  p-value: < 2.2e-16
```

What happens if we fit a quadratic model?

```
lm(sales ~ dist + I(dist^2) + treat + dist*treat + treat*I(dist^2), data = sales)
```

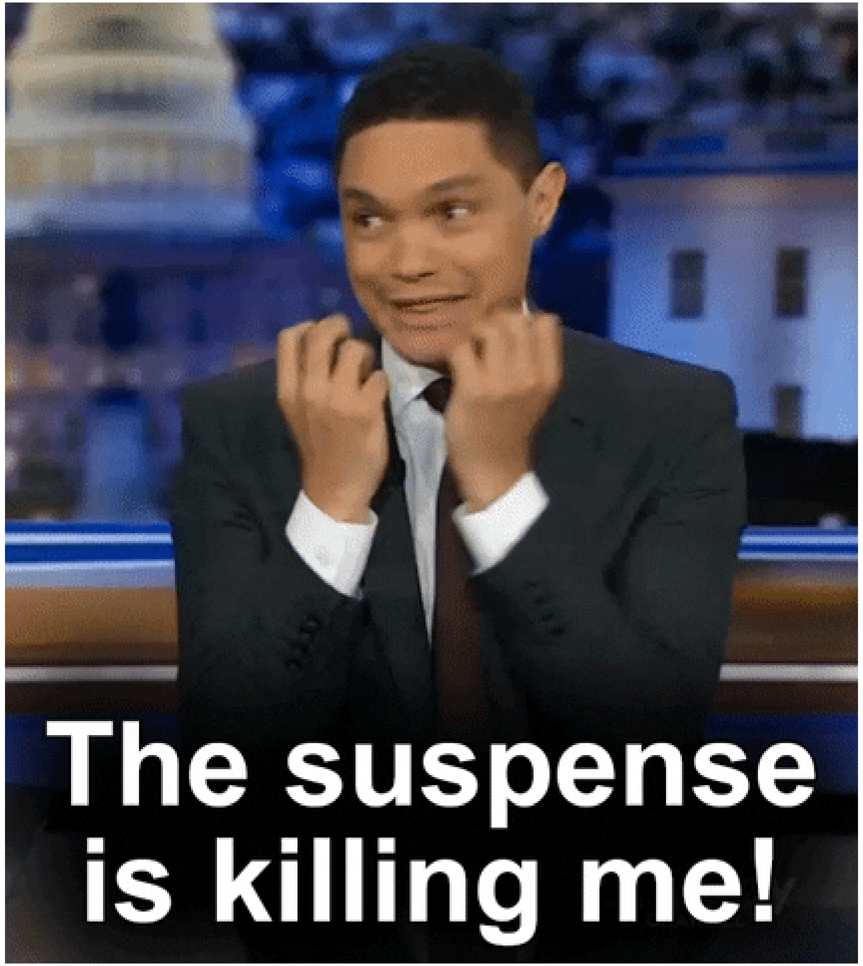


What happens if we fit a quadratic model?

```
summary(lm(sales ~ dist + I(dist^2) + treat + dist*treat + treat*I(dist^2), data = sales))
```

```
##
## Call:
## lm(formula = sales ~ dist + I(dist^2) + treat + dist * treat +
##     treat * I(dist^2), data = sales)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -66.090 -13.979   0.239  13.154  76.656
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)      1.698e+02  1.937e+00  87.665  < 2e-16 ***
## dist            -4.302e-03  3.556e-02  -0.121 0.903725
## I(dist^2)       -8.288e-04  1.363e-04  -6.083 1.41e-09 ***
## treat            3.308e+01  2.747e+00  12.041  < 2e-16 ***
## dist:treat       1.713e-01  4.964e-02   3.452 0.000569 ***
## I(dist^2):treat  2.034e-04  1.877e-04   1.084 0.278554
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.23 on 1994 degrees of freedom
```

Next class



- Check how to rely less on **parametric assumptions**
- What is the **optimal bandwidth** to estimate our RD?
- Talk about **fuzzy regression discontinuities**

Have a good Spring Break!