

# STA 235 - Causal Inference: Regression Discontinuity Design

Spring 2021

McCombs School of Business, UT Austin

# Another identification strategy

- We have seen:

**RCTs**

**Selection on observables**

**Natural experiments**

**Differences-in-Differences**

**Regression Discontinuity Designs**

I'm on the edge [of glory?]

# Introduction to Regression Discontinuity Designs

## Regression Discontinuity (RD) Designs

Arbitrary rules determine treatment assignment

E.g.: If you are above a threshold, you are assigned to treatment, and if your below, you are not (or vice versa)

# Key Terms

**Running/ forcing variable**

Index or measure that determines eligibility

**Cutoff/ cutpoint/ threshold**

Number that formally assigns you to a program or treatment

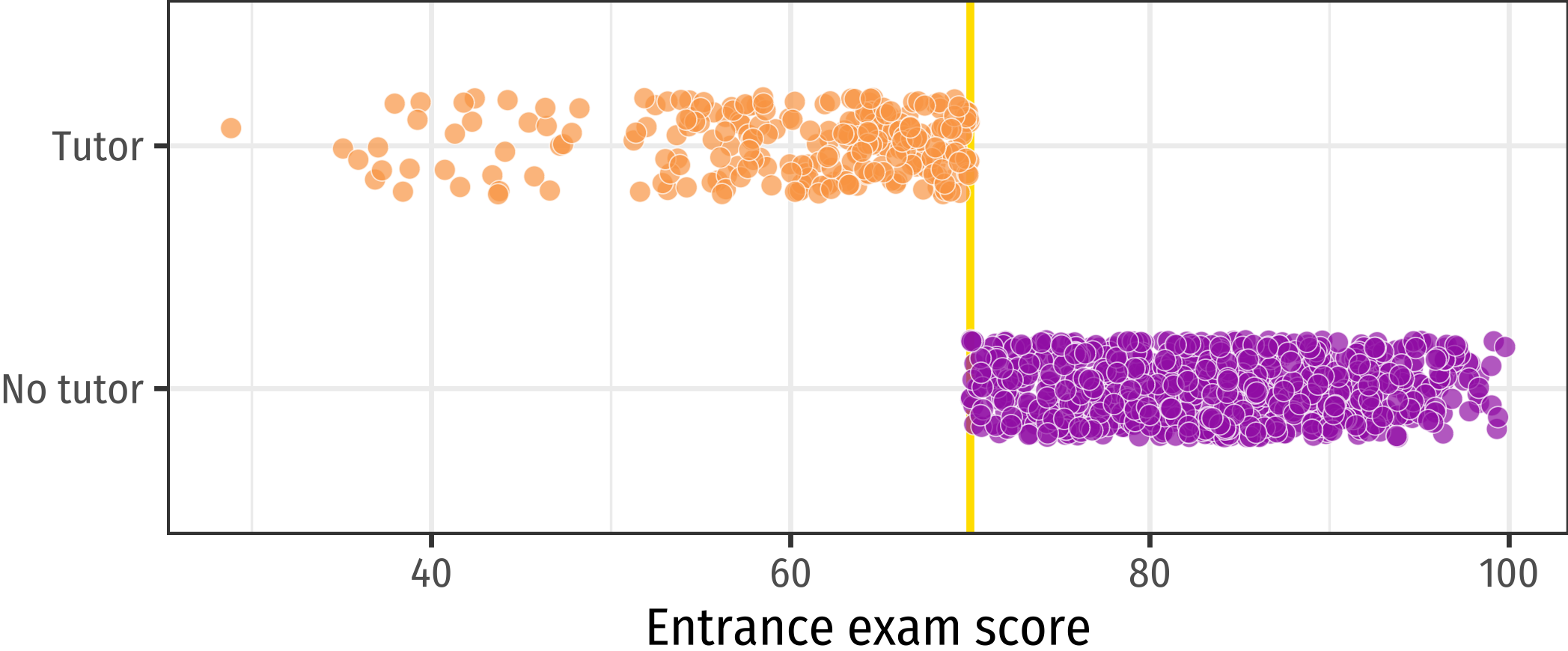
# Hypothetical tutoring program

**Students take an entrance exam**

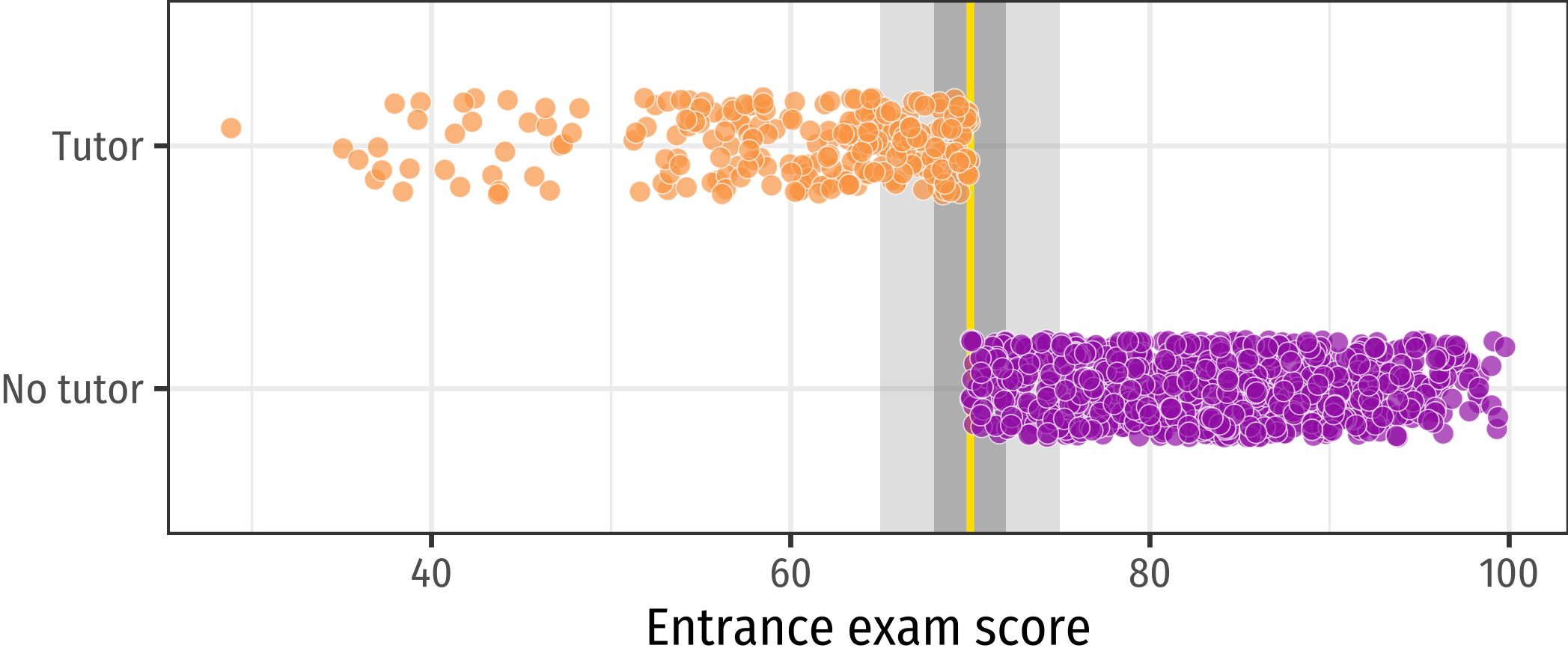
**Those who score 70 or lower  
get a free tutor for the year**

**Students then take an exit exam  
at the end of the year**

# Assignment based on entrance score

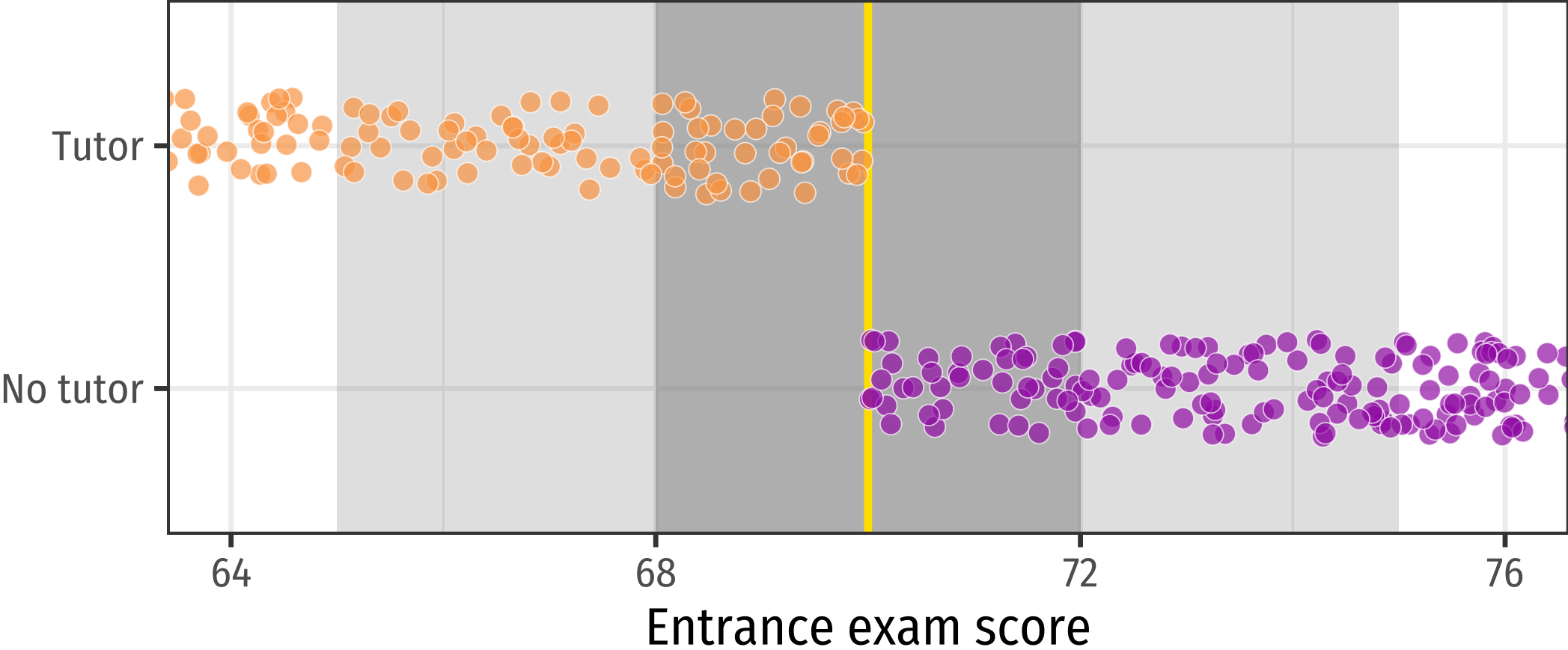


# Let's look at the area close to the cutoff





# Let's get closer



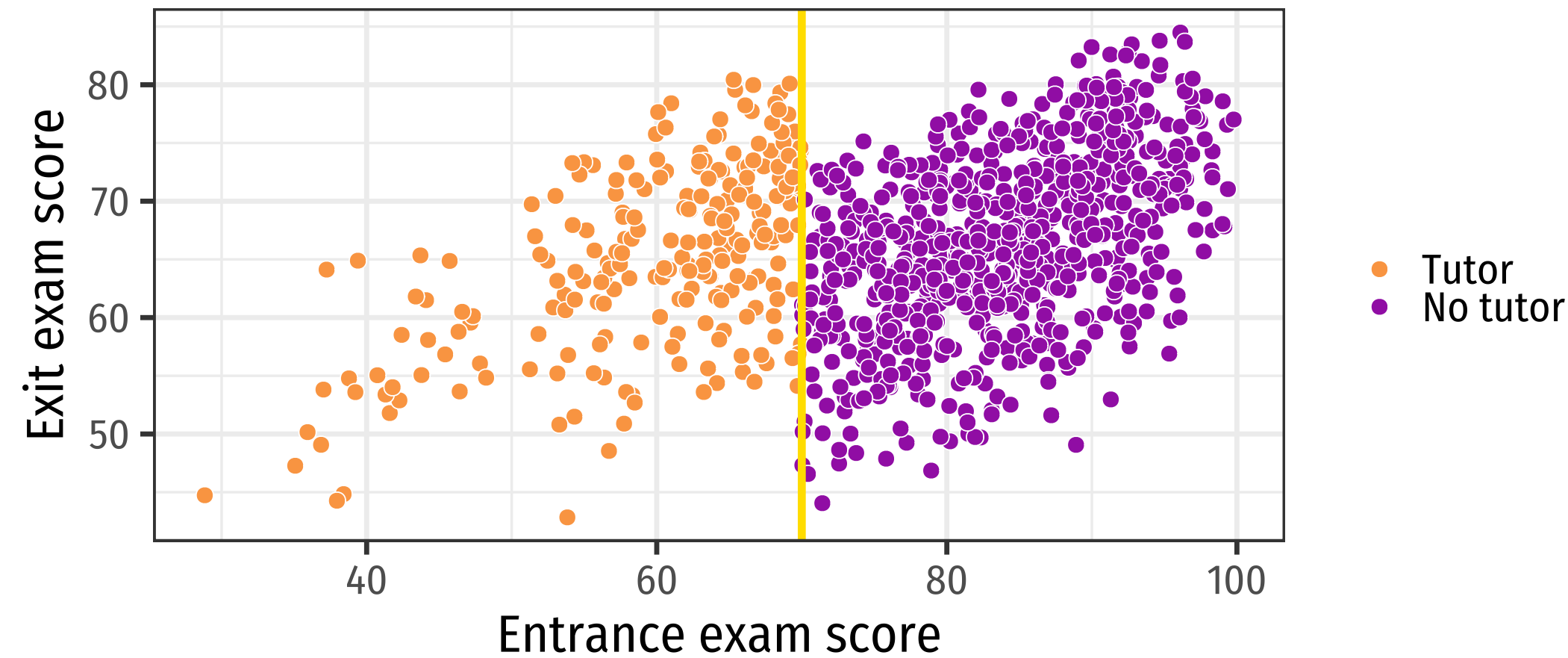
# Causal inference intuition

Observations right before and after the threshold are essentially the same

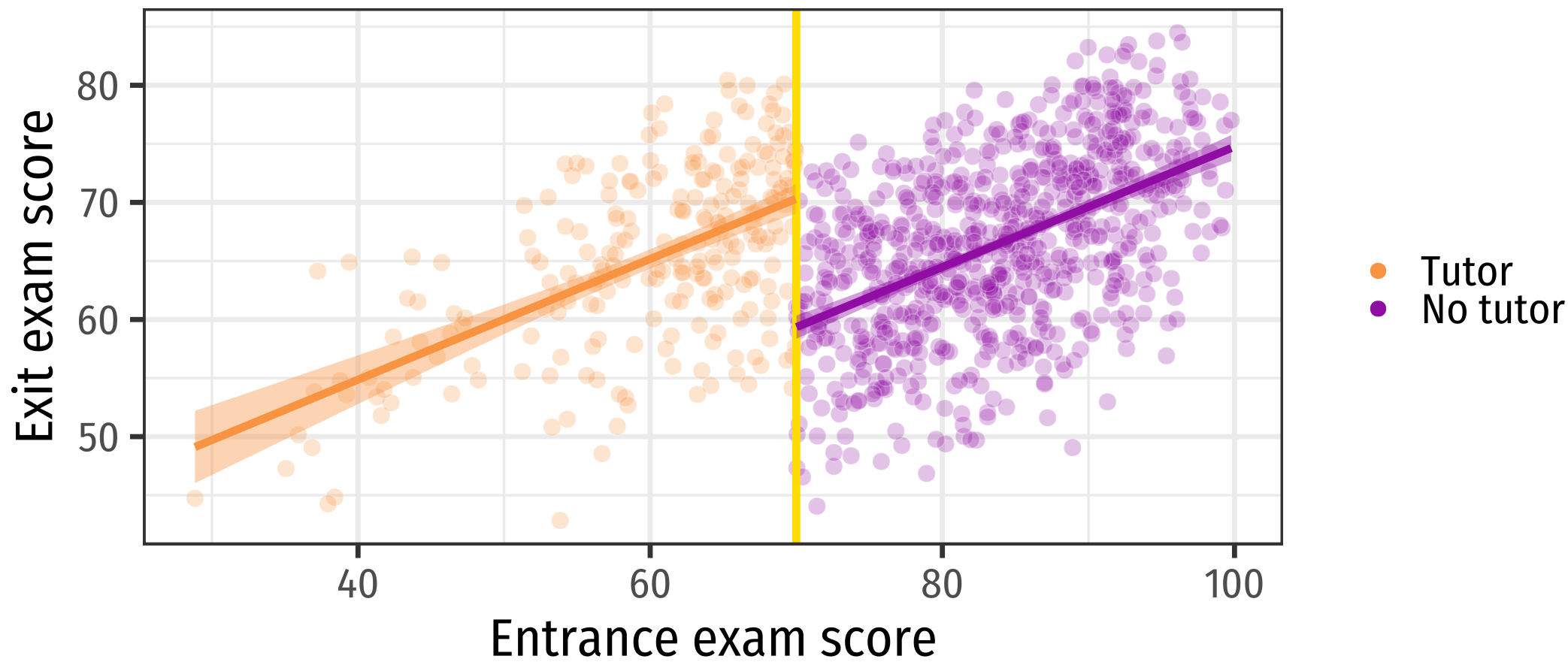
Pseudo treatment and control groups!

Compare outcomes right at the cutoff

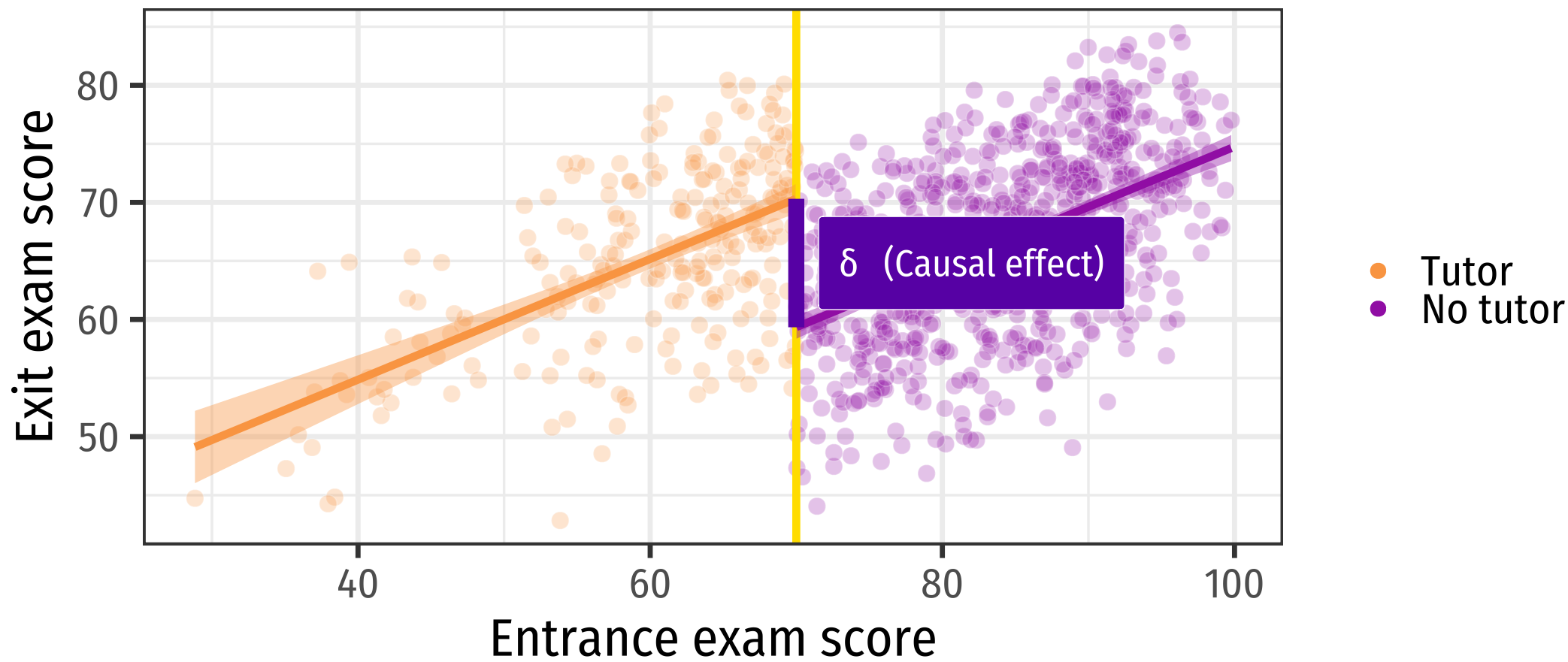
# Exit exam results according to running variable



# Fit a regression at the right and left side of the cutoff

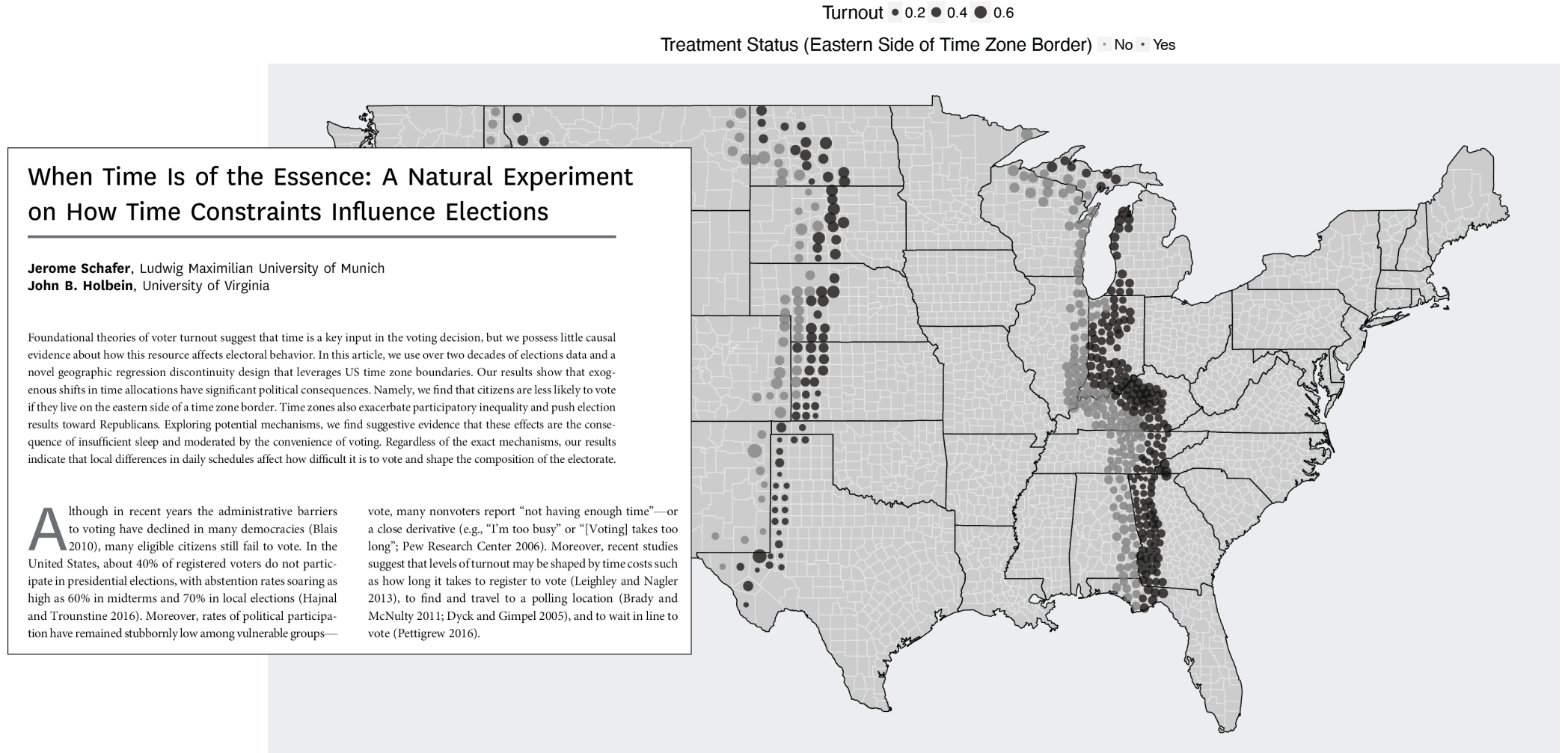


# Fit a regression at the right and left side of the cutoff



**You can find discontinuities  
everywhere!**

# Geographic discontinuities



# Time discontinuities

## After Midnight: A Regression Discontinuity Design in Length of Postpartum Hospital Stays<sup>†</sup>

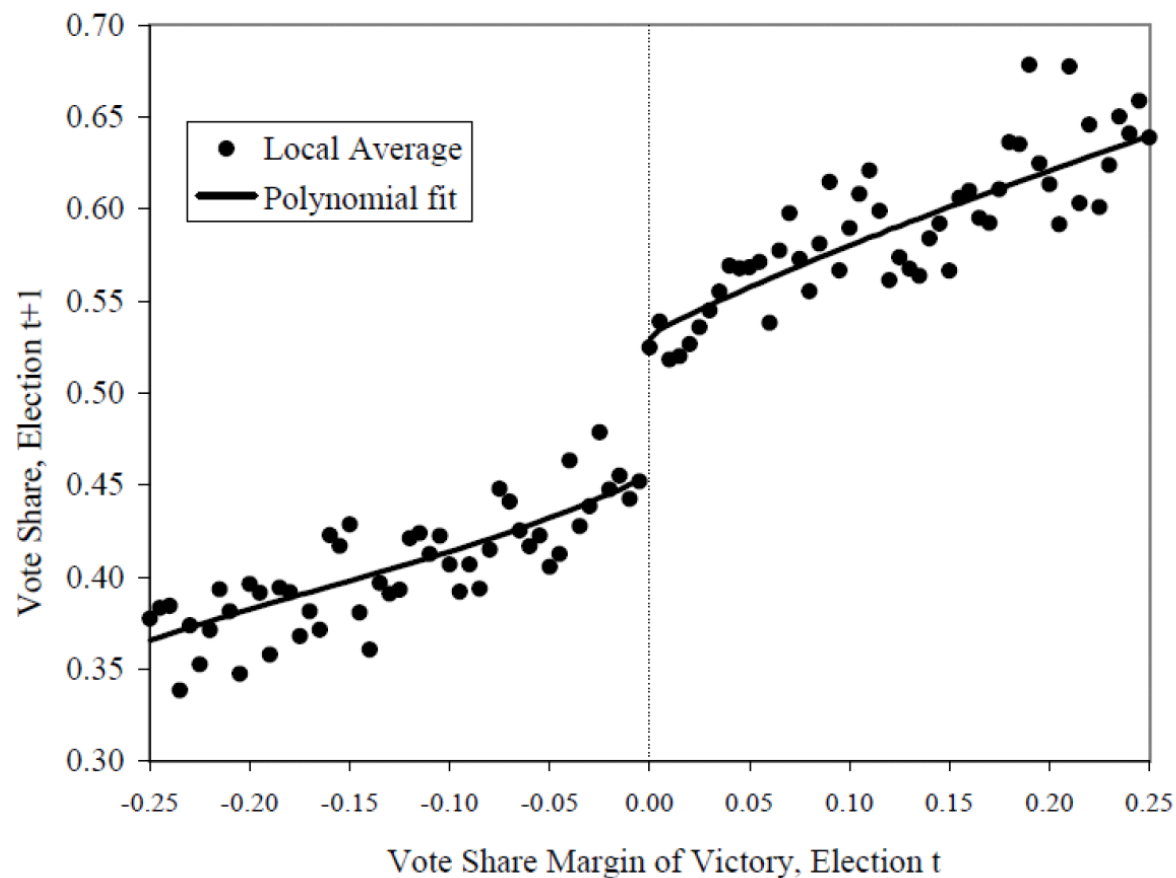
By DOUGLAS ALMOND AND JOSEPH J. DOYLE JR.\*

*Estimates of moral hazard in health insurance markets can be confounded by adverse selection. This paper considers a plausibly exogenous source of variation in insurance coverage for childbirth in California. We find that additional health insurance coverage induces substantial extensions in length of hospital stay for mother and newborn. However, remaining in the hospital longer has no effect on readmissions or mortality, and the estimates are precise. Our results suggest that for uncomplicated births, minimum insurance mandates incur substantial costs without detectable health benefits. (JEL D82, G22, I12, I18, J13)*



# Voting discontinuities

Figure IVa: Democrat Party's Vote Share in Election  $t+1$ , by Margin of Victory in Election  $t$ : local averages and parametric fit



How do we do RDs in practice?

# Behind the scenes of RDs

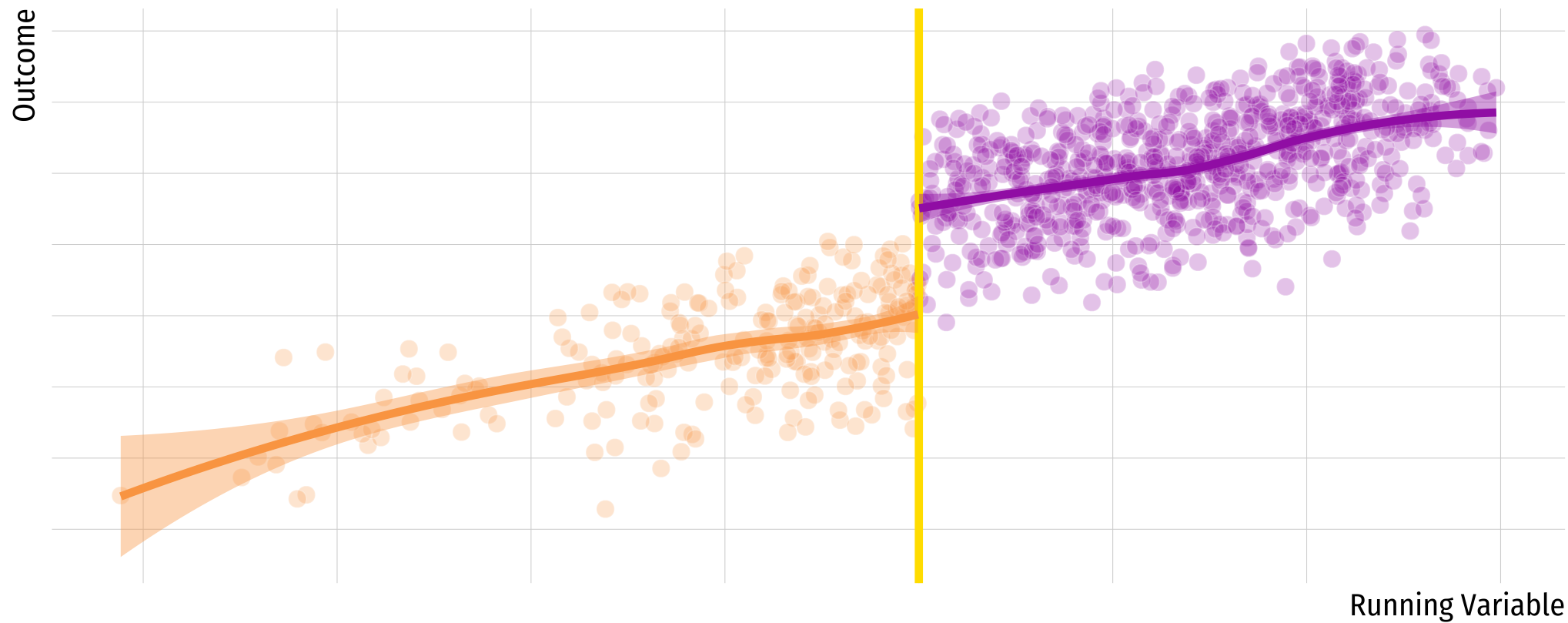
- Basically, regression discontinuities work under an **asymptotic assumption**:
- Let  $Y_i$  be the outcome of interest,  $Z_i$  the treatment assignment,  $R_i$  the running variable, and  $c$  the cutoff score:

$$Z_i = \begin{cases} 0 & R_i \leq c \\ 1 & R_i > c \end{cases}$$

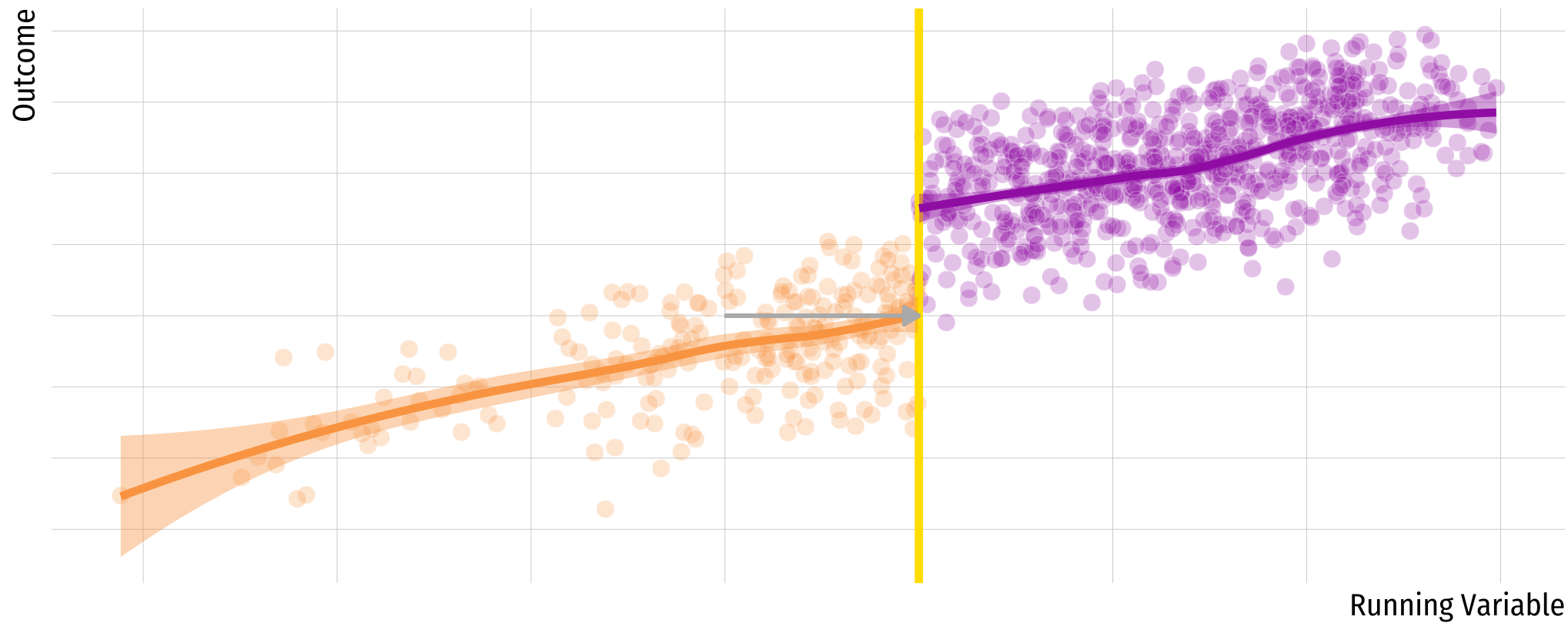
- Then, we can define the treatment effect  $\delta$  as:

$$\delta = \lim_{\epsilon \rightarrow 0^+} E[Y_i | R_i = c + \epsilon] - \lim_{\epsilon \rightarrow 0^-} E[Y_i | R_i = c + \epsilon]$$

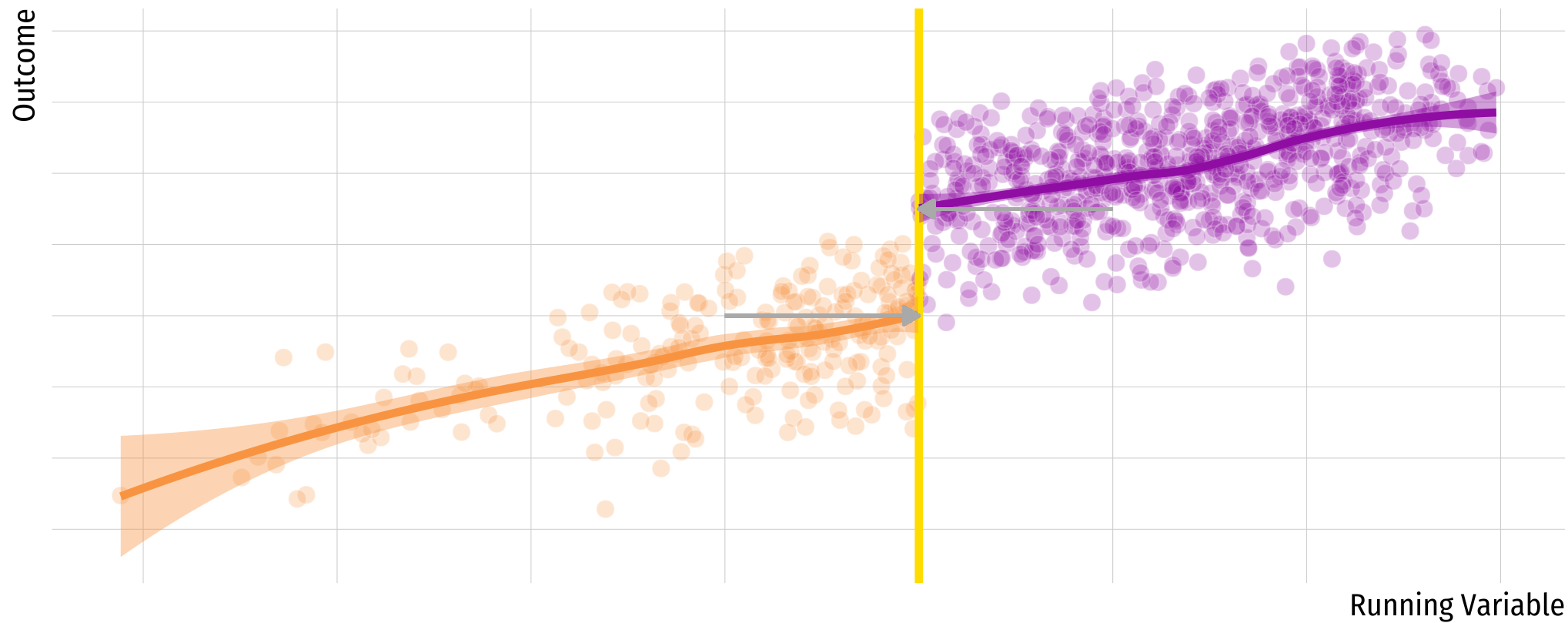
# What does the limit expression mean?



# What does the limit expression mean?



# What does the limit expression mean?



**What is the estimand we are  
estimating?**

**Local Average Treatment Effect  
(LATE) for units at  $R=c$**

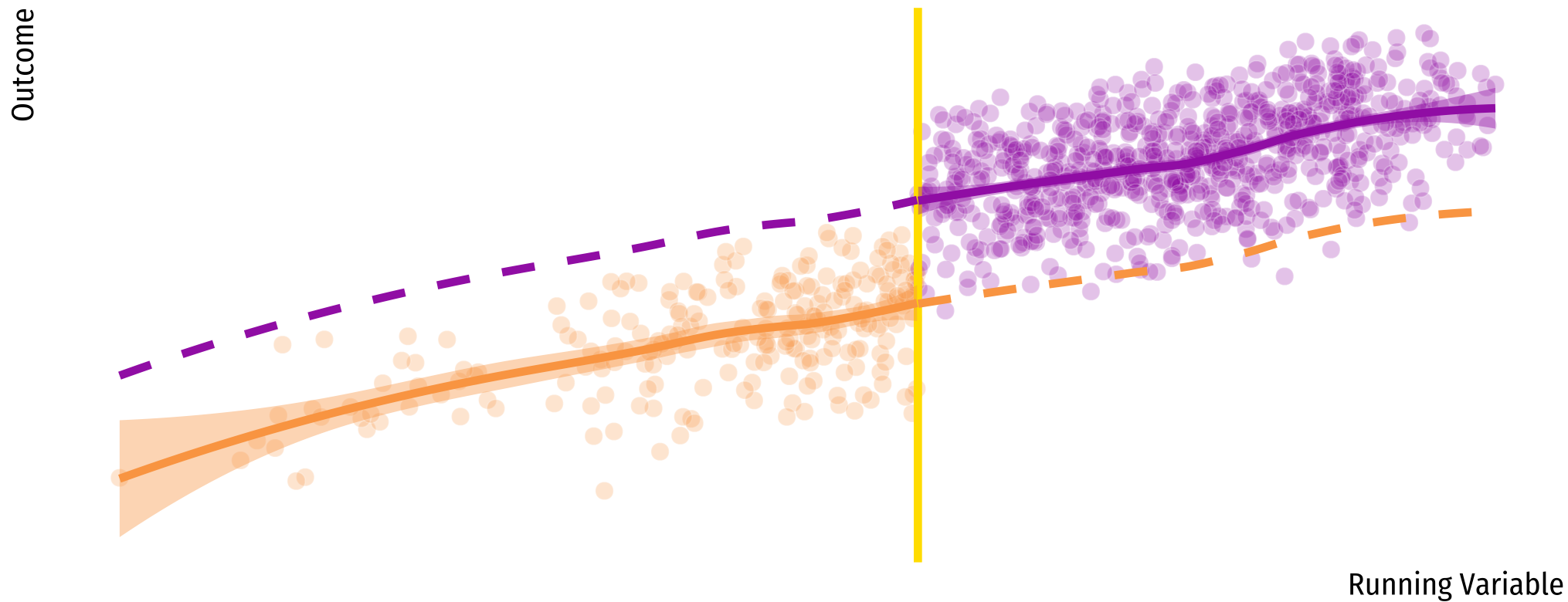


# Conditions required for identification

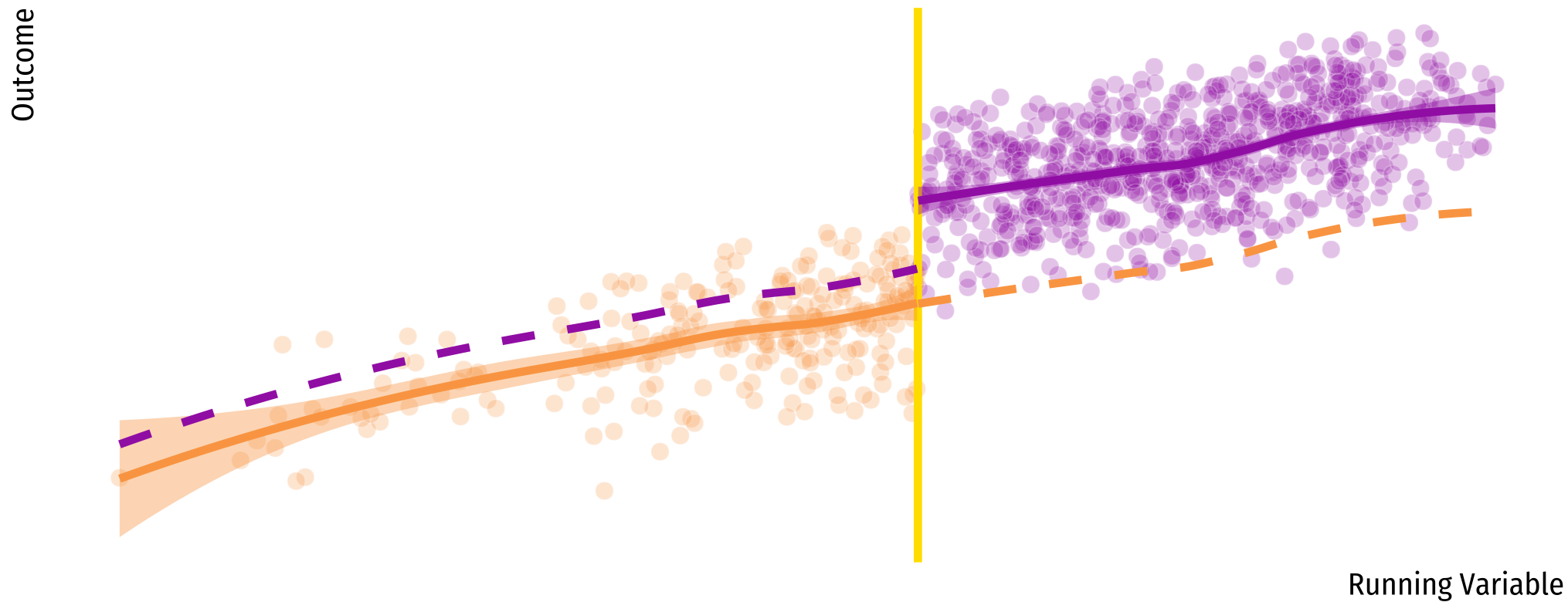
- Threshold rule **exists** and cutoff point is **known**
- The running variable  $R_i$  is **continuous** near  $c$ .
- Key assumption:

Continuity of  $E[Y(1)|R]$  and  $E[Y(0)|R]$  at  $R=c$

# Potential outcomes need to be smooth across the threshold



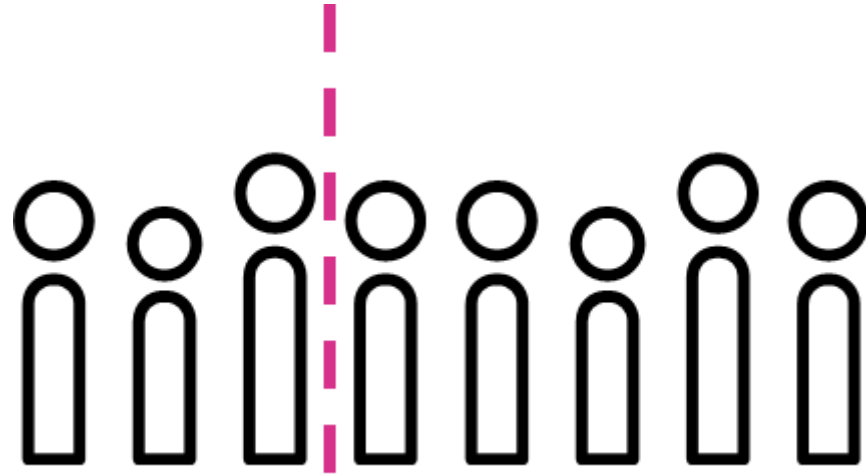
# Potential outcomes need to be smooth across the threshold



**Can you think situations where  
that could happen?**

# Let's go back to our discount example

- Customers are given discounts based on their **order of arrival**



- We could think of this as an **RD in time**, where  $c$  is the time of arrival of customer 1,000.

# Work in groups

1) Each group will be given a task and some code

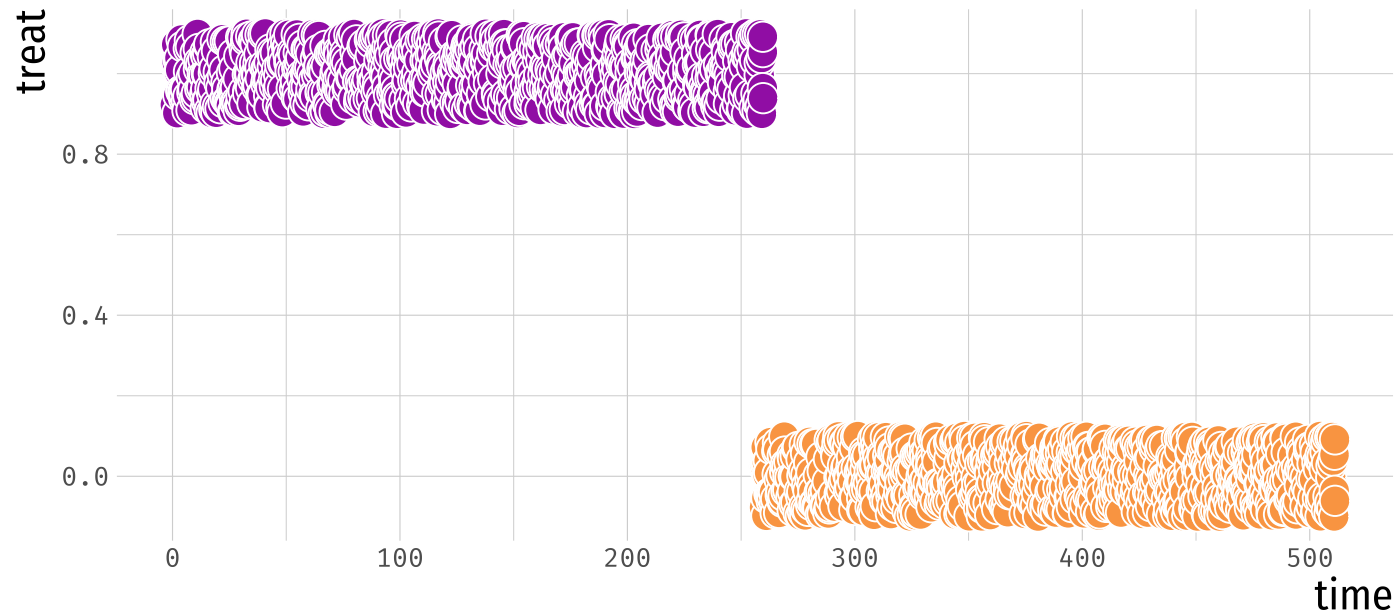
2) You need to complete the code and discuss the results

# Group 1

**What did you have to do?**

# Group 1: Check the treatment assignment

```
c = max(sales$time[sales$treat==1])  
  
ggplot(data = sales, aes(x = time, y = treat)) +  
  geom_point(data = filter(sales, time<=c), pch = 21, color = "white", fill="#900DA4", position = pos_early) +  
  geom_point(data = filter(sales, time>c), pch = 21, color = "white", fill="#F89441", position = pos_late)
```



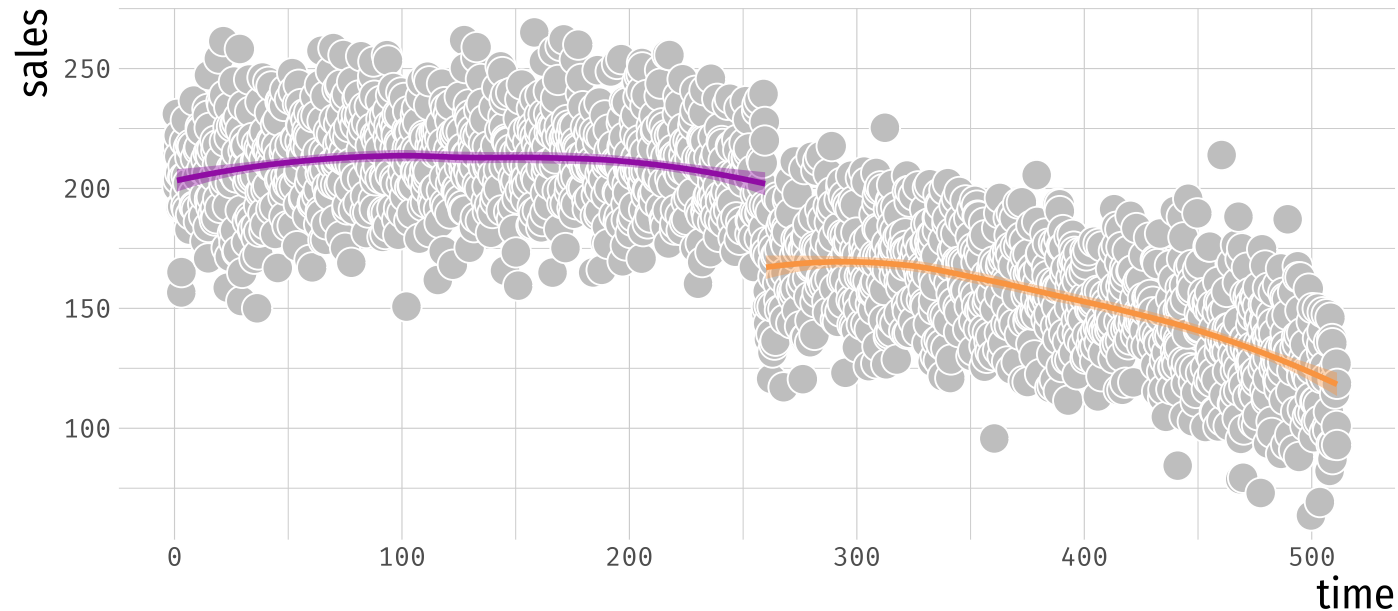


## Group 2

**What did you have to do?**

## Group 2: Check the regression discontinuity on the outcome

```
ggplot(data = sales, aes(x = time, y = sales)) +  
  geom_point(pch = 21, color = "white", fill="grey") +  
  geom_smooth(data = filter(sales, time>c), method = "loess", se=TRUE, color = "#F89441", fill = "#F89441") +  
  geom_smooth(data = filter(sales, time<=c), method = "loess", se=TRUE, color = "#F89441", fill = "#F89441")
```

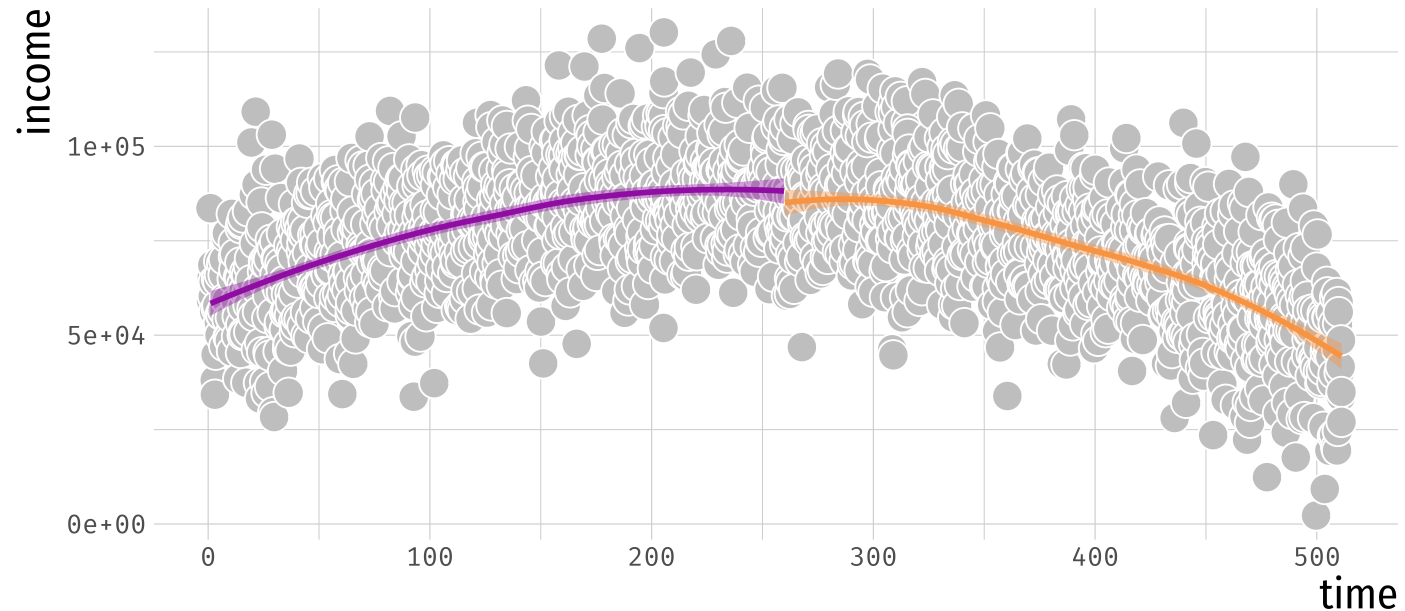


## Group 3

**What did you have to do?**

# Group 3: Check smoothness of income

```
ggplot(data = sales, aes(x = time, y = income)) +  
  geom_point(pch = 21, color = "white", fill="grey") +  
  geom_smooth(data = filter(sales, time>c), method = "loess", se=TRUE, color = "#F89441", fill = "#F89441") +  
  geom_smooth(data = filter(sales, time<=c), method = "loess", se=TRUE, color = "#F89441", fill = "#F89441")
```

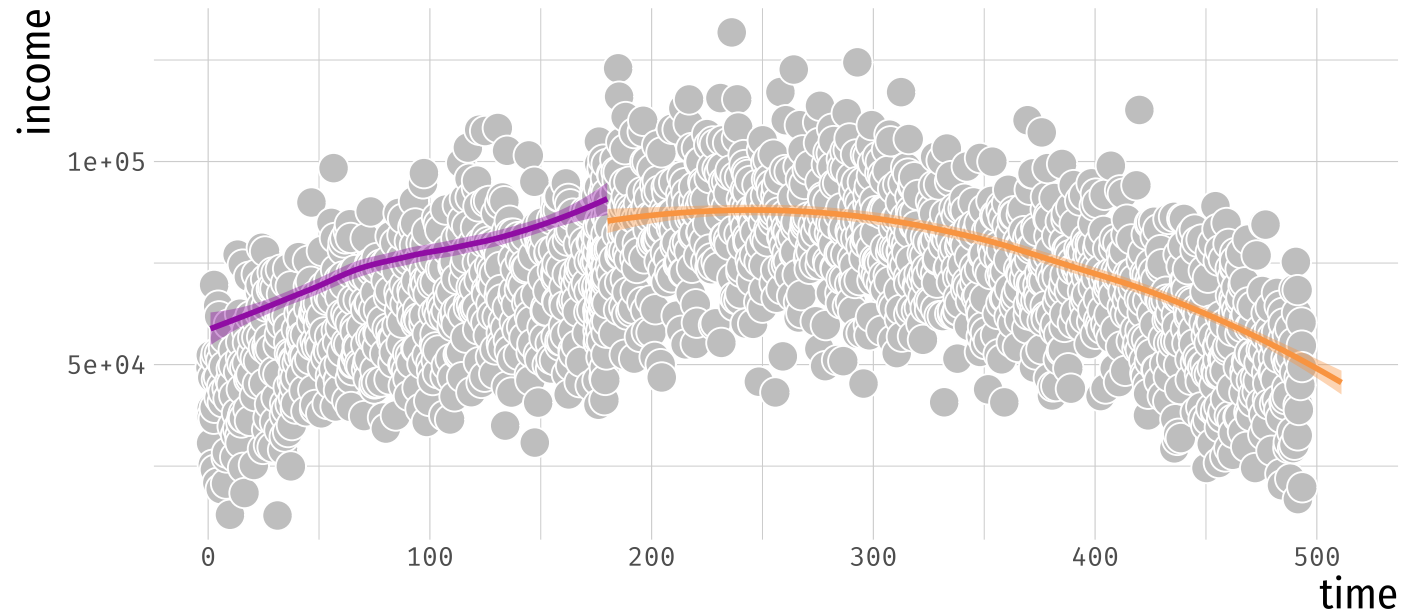


## Group 4

**What did you have to do?**

# Group 4: Check smoothness of income under different assignment

```
ggplot(data = sales_mod, aes(x = time, y = income)) +  
  geom_point(pch = 21, color = "white", fill="grey") +  
  geom_smooth(data = filter(sales, time>c), method = "loess", se=TRUE, color = "#F89441", fill = "#F89441") +  
  geom_smooth(data = filter(sales, time<=c), method = "loess", se=TRUE, color = "#F89441", fill = "#F89441")
```



# How do we actually estimate an RD?

- The simplest way to do this is to fit a regression:

$$Y_i = \beta_0 + \beta_1(R_i - c) + \beta_2\mathbf{I}[R_i > c] + \beta_3(R_i - c)\mathbf{I}[R_i > c]$$

# How do we actually estimate an RD?

- The simplest way to do this is to fit a regression:

$$Y_i = \beta_0 + \beta_1 \underbrace{(R_i - c)}_{\text{Distance to the cutoff}} + \beta_2 \mathbf{I}[R_i > c] + \beta_3 \overbrace{(R_i - c)}^{\text{Distance to the cutoff}} \mathbf{I}[R_i > c]$$



# How do we actually estimate an RD?

- The simplest way to do this is to fit a regression:

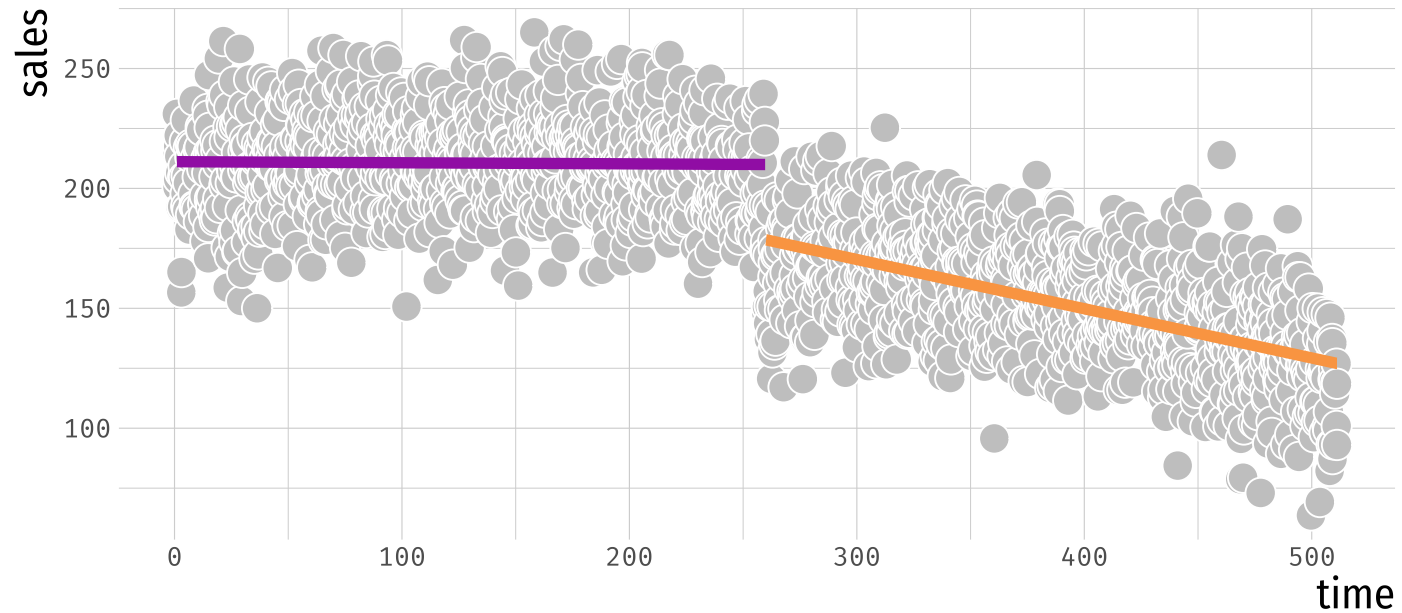
$$Y_i = \beta_0 + \beta_1(R_i - c) + \underbrace{\beta_2 \mathbf{I}[R_i > c]}_{\text{Treatment}} + \beta_3(R_i - c) \underbrace{\mathbf{I}[R_i > c]}_{\text{Treatment}}$$

- You want to add **flexibility** for each side of the cutoff.

Can you identify these parameters in a plot?

# Let's see some examples: Sales using a linear model

```
sales <- sales %>% mutate(dist = c-time)  
lm(sales ~ dist + treat + dist*treat, data = sales)
```



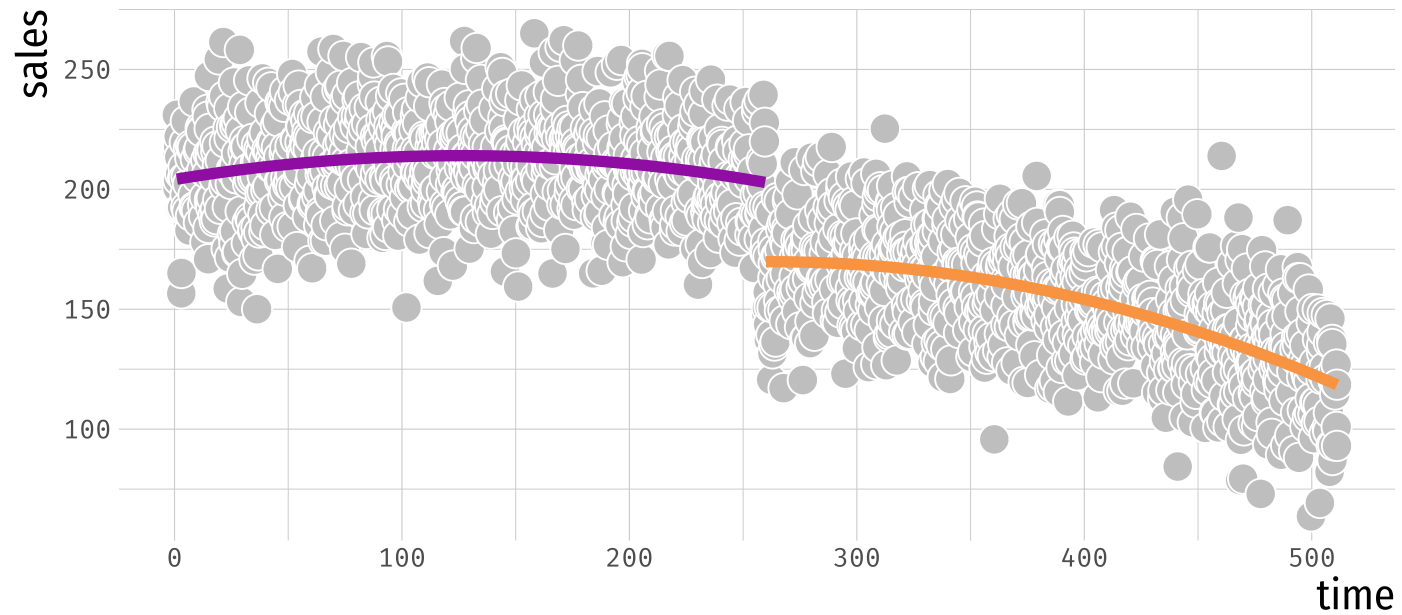
# Let's see some examples: Sales using a linear model

```
summary(lm(sales ~ dist + treat + dist*treat, data = sales))
```

```
##
## Call:
## lm(formula = sales ~ dist + treat + dist * treat, data = sales)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -65.738 -13.940   0.051  13.538  76.515
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 178.640954   1.300314  137.38  <2e-16 ***
## dist         0.205355   0.008882   23.12  <2e-16 ***
## treat        31.333952   1.842338   17.01  <2e-16 ***
## dist:treat   -0.200845   0.012438  -16.15  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.52 on 1996 degrees of freedom
## Multiple R-squared:  0.6939,    Adjusted R-squared:  0.6934
## F-statistic: 1508 on 3 and 1996 DF,  p-value: < 2.2e-16
```

# What happens if we fit a quadratic model?

```
lm(sales ~ dist + I(dist^2) + treat + dist*treat + treat*I(dist^2), data = sales)
```



# What happens if we fit a quadratic model?

```
summary(lm(sales ~ dist + I(dist^2) + treat + dist*treat + treat*I(dist^2), data = sales))
```

```
##
## Call:
## lm(formula = sales ~ dist + I(dist^2) + treat + dist * treat +
##     treat * I(dist^2), data = sales)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -66.090 -13.979   0.239  13.154  76.656
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   1.698e+02  1.937e+00  87.665  < 2e-16 ***
## dist         -4.302e-03  3.556e-02  -0.121  0.903725
## I(dist^2)     -8.288e-04  1.363e-04  -6.083  1.41e-09 ***
## treat         3.308e+01  2.747e+00  12.041  < 2e-16 ***
## dist:treat    1.713e-01  4.964e-02   3.452  0.000569 ***
## I(dist^2):treat 2.034e-04  1.877e-04   1.084  0.278554
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.23 on 1994 degrees of freedom
```

# Next class



- Check how to rely less on **parametric assumptions**
- What is the **optimal bandwidth** to estimate our RD?
- Talk about **fuzzy regression discontinuities**

**Have a good Spring Break!**

# References

- Angrist, J. and S. Pischke. (2015). "Mastering Metrics". *Chapter 4*.
- Heiss, A. (2020). "Program Evaluation for Public Policy". *Class 10: Regression Discontinuity I, Course at BYU*.
- Lee, D. and T. Lemieux. (2010). "Regression Discontinuity in Economics". *Journal of Economic Literature* 48, pp 281-355.