STA 235 - Multiple Regression: Statistical Adjustment

Spring 2021

McCombs School of Business, UT Austin

Quick reminders

sta235.netlify.app

- Slides are posted on the website before the class.
- Required readings are posted in the *Classes* folder at least a week before.
- Check the code for each class.

Last week

- Quick **multiple regression** review
- Comparing **effect sizes**: Standardizing variables (i.e. all $\hat{\beta}$'s in the same scale)
- **Uncertainty quantification** in regression: Adj-R^2 and RSE.

Today

- **Statistical adjustment in regressions**:
  - How do we interpret coefficients?
  - What are those standard errors?
  - Multicollinearity?
- Regression models with **binary outcomes**

But first... JITTs!

- (Almost) **everyone** answered the JITT.
- Answers are very useful and **we will use them in today's class**.
- People want **more plots!**
  - *Ask and you shall receive.*

Remember to ask questions!

Multiple Regression

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p + \varepsilon$$

Assumptions of the linear regression model:

(i) The conditional mean of Y is **linear** in the X_j variables

(ii) The error terms are:

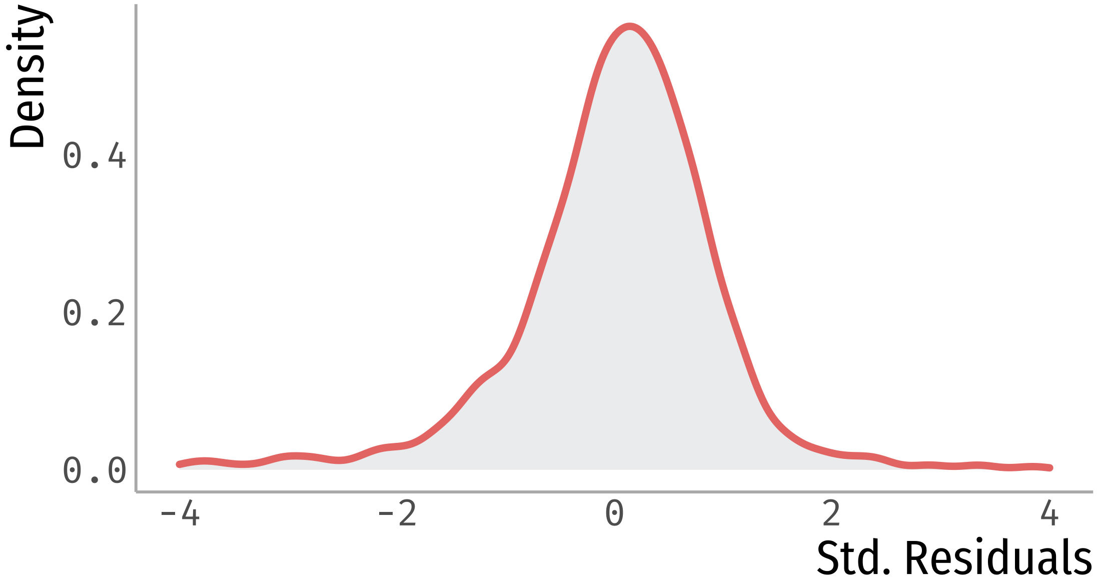
- normally distributed
- independent from each other
- identically distributed (i.e. constant variance)

- Last two assumptions are the ones that we refer to when saying $\varepsilon \sim iid$

How can we check these assumptions?

Remember last week's Bechdel test example:

```
ggplot(data = bechdel_fitted, aes(x = .std.resid)) +
  geom_density()
```



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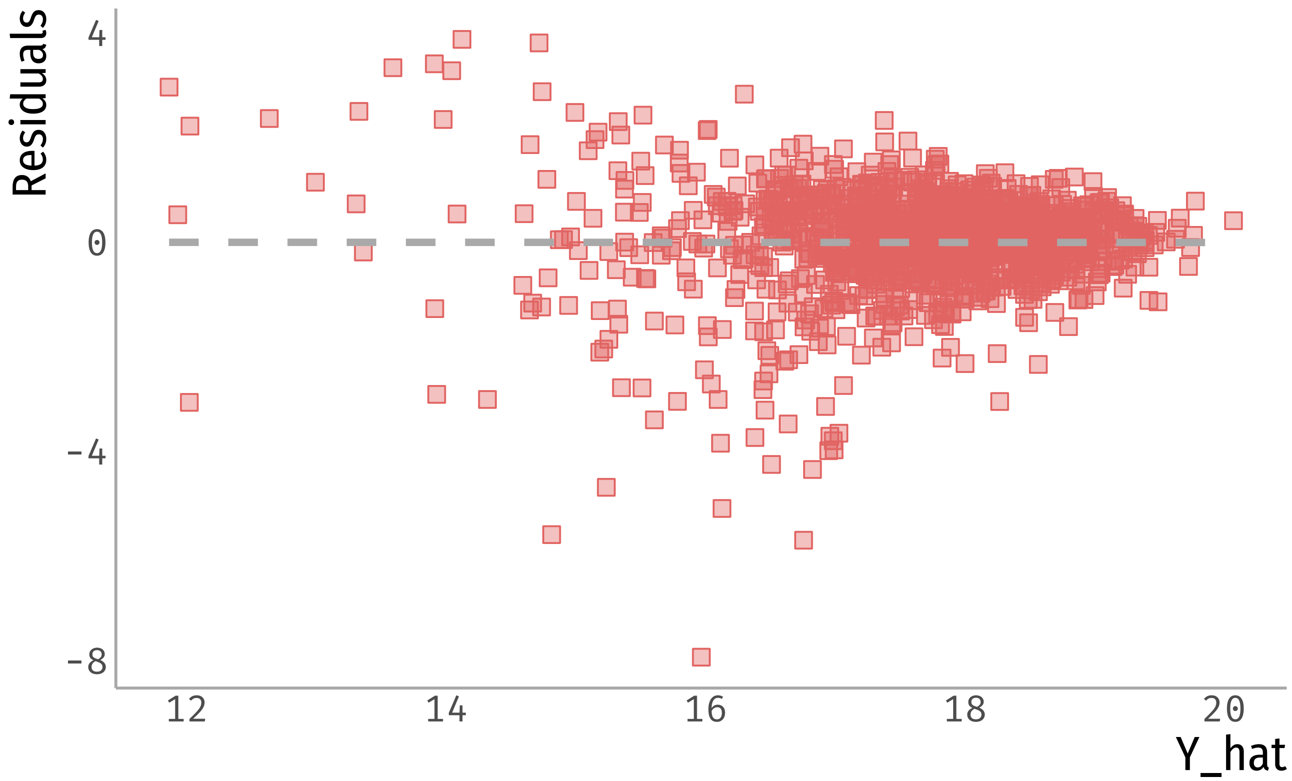

What happens if the assumptions break?

- **Don't panic!** There's still much to be done.
- **Heteroskedasticity** (non-constant variance) does not bias your estimates.
- Can be fixed many times with **robust standard errors**

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- **Don't panic!** There's still much to be done.
- **Heteroskedasticity** (non-constant variance) does not bias your estimates.
- Can be fixed many times with **robust standard errors**:
  - Easy to implement in R!

Statistical Adjusment

- Remember that an important part of our job is to **correctly estimate** the β parameters:
  - Point estimates
  - Standard errors
- **Two ways** to estimate standard errors:
  - Direct estimation: Use probability theory (e.g. $SE(\bar{x}) = \frac{\hat{\sigma}}{\sqrt{n}}$)
  - Simulation: Repeat the sampling process and estimates how much our estimate changes from one sample to the next (e.g. bootstrapping)

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Important to understand R output!

Let's look at some data

- I have some data for price and sales of product 1, but also I'm tracking the prices of its competitor, product 2.
- The data looks like this:

```
##     p1    p2  Sales
## 1 5.14  5.20 144.49
## 2 3.50  8.06 637.25
## 3 7.28 11.68 620.79
## 4 4.66  8.36 549.01
## 5 3.58  2.15  20.43
## 6 5.17 10.15 713.01
```

Let's fit a model

$$Sales_i = \beta_0 + \beta_1 p1_i + \beta_2 p2_i + \varepsilon_i$$

```
summary(lm(Sales ~ p1 + p2, data = sales))
```

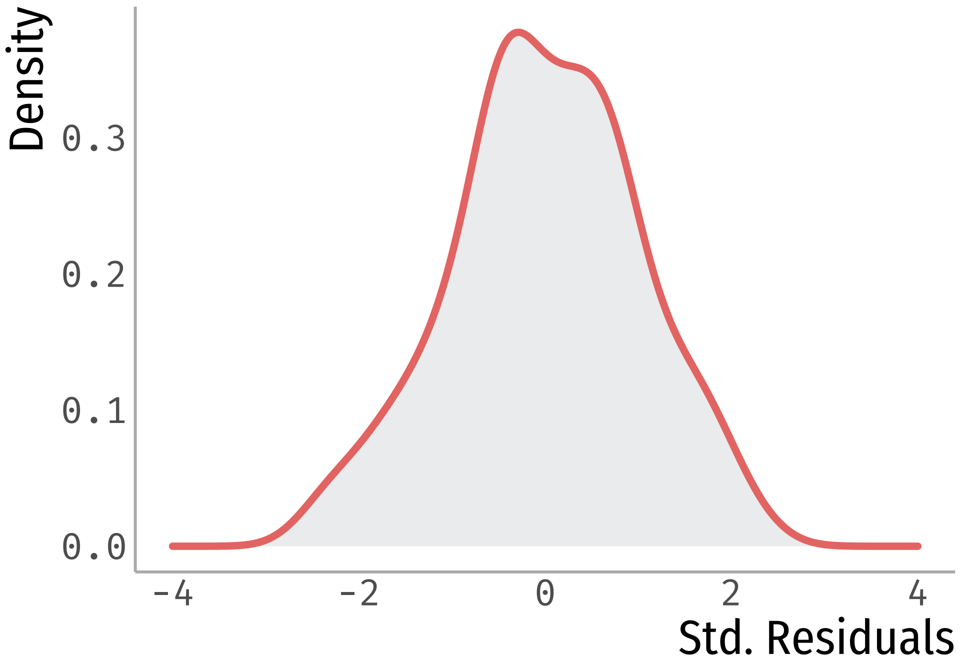
```
##
## Call:
## lm(formula = Sales ~ p1 + p2, data = sales)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -66.916 -15.663  -0.509  18.904  63.302
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  115.717      8.548   13.54   <2e-16 ***
## p1           -97.657      2.669  -36.59   <2e-16 ***
## p2           108.800      1.409   77.20   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 28.42 on 97 degrees of freedom
## Multiple R-squared:  0.9871,    Adjusted R-squared:  0.9869
## F-statistic:  3717 on 2 and 97 DF,  p-value: < 2.2e-16
```

Do assumptions hold?

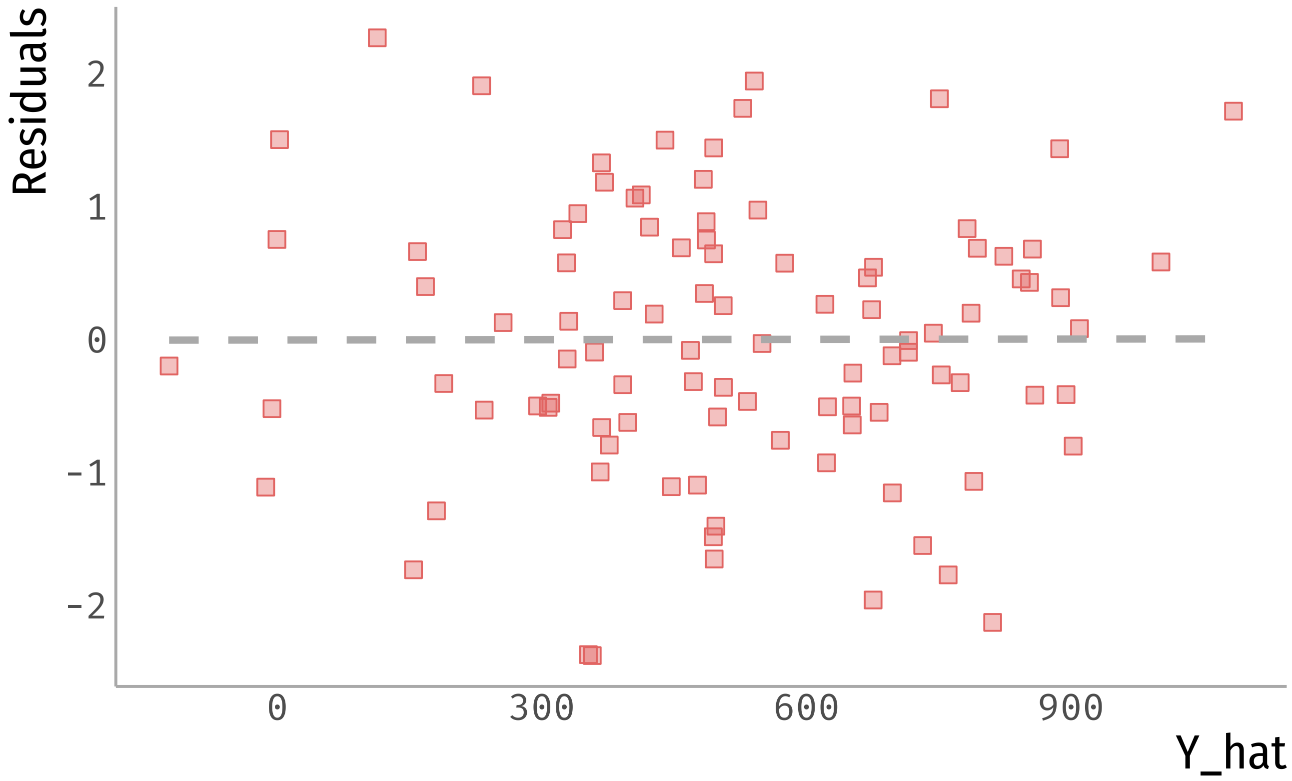
$$\varepsilon_i \sim N(0, \sigma^2)$$

```
sales_fitted <- augment(lm(Sales ~ p1 + p2, data = sales))

ggplot(data = sales_fitted, aes(x = .std.resid)) +
  geom_density()
```



Do assumptions hold? (cont.)



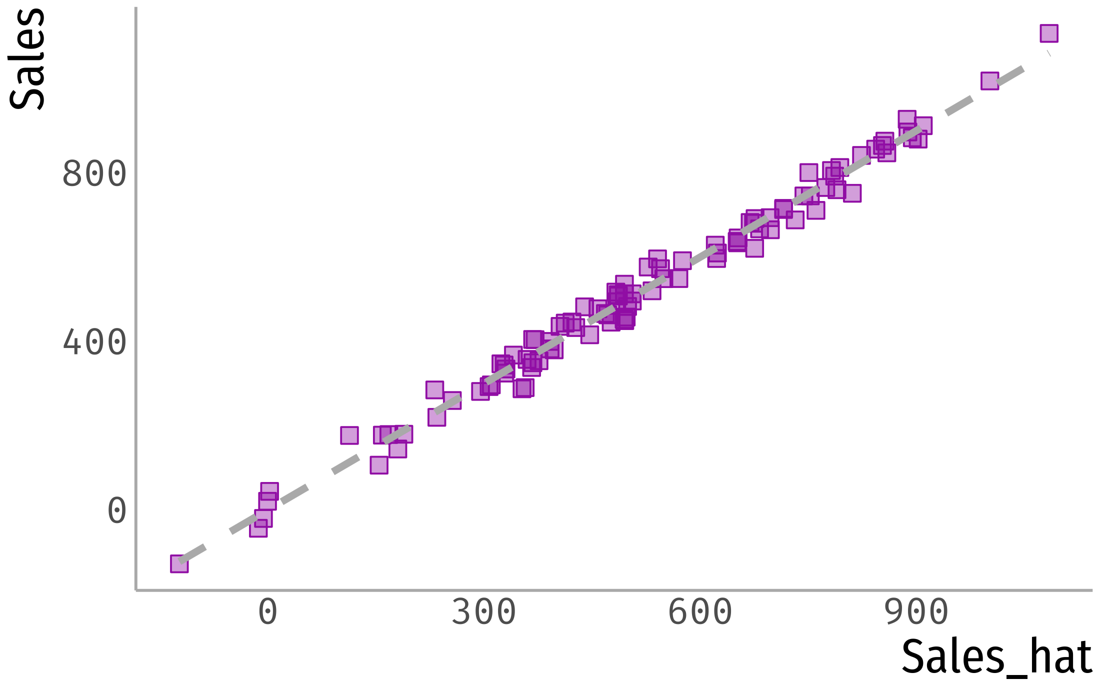
Let's go back to our model

$$Sales_i = \beta_0 + \beta_1 \boldsymbol{p1}_i + \beta_2 \boldsymbol{p2}_i + \varepsilon_i$$

	Model 1
(Intercept)	115.717***
	(8.548)
p1	-97.657***
	(2.669)
p2	108.800***
	(1.409)
Num.Obs.	100
F	3717.292

* p < 0.1, ** p < 0.05, *** p < 0.01

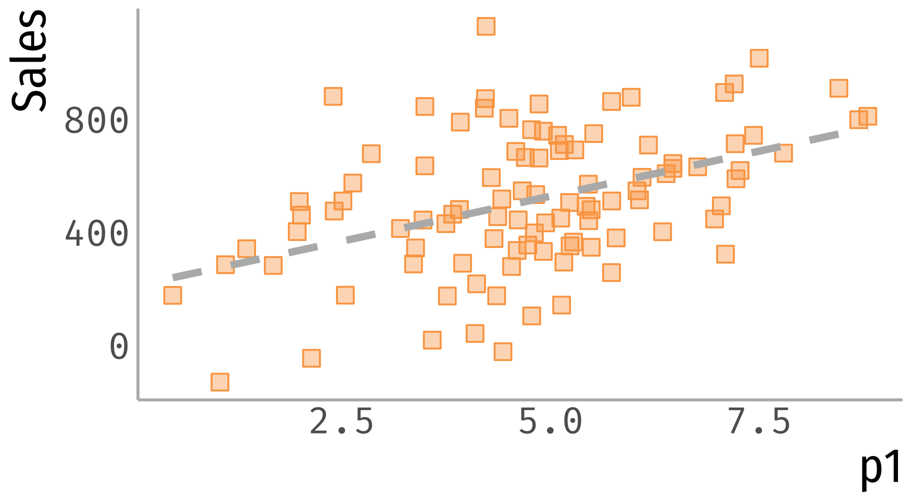
How is the prediction working?



- **Can you guess the slope?**

What if we only had p1 and not p2?

```r
ggplot(data = sales, aes(x = p1, y = Sales)) +
  geom_point() +
  geom_smooth(method = "lm")
```



- If I increase the price sales go up?

What if we only had p1 and not p2?

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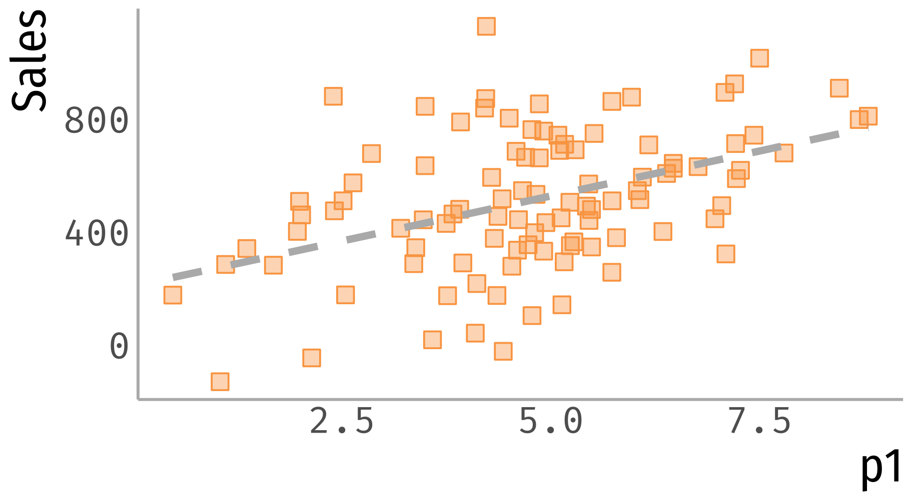
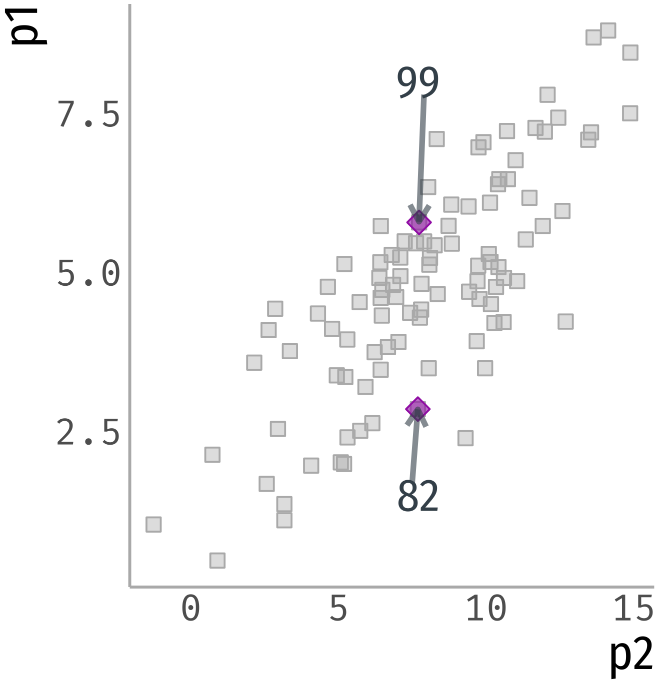


- If I increase the price sales go up? **NO!**

Relationship between p1 and p2

- Let's compare two different observations: Week 82 and week 99.



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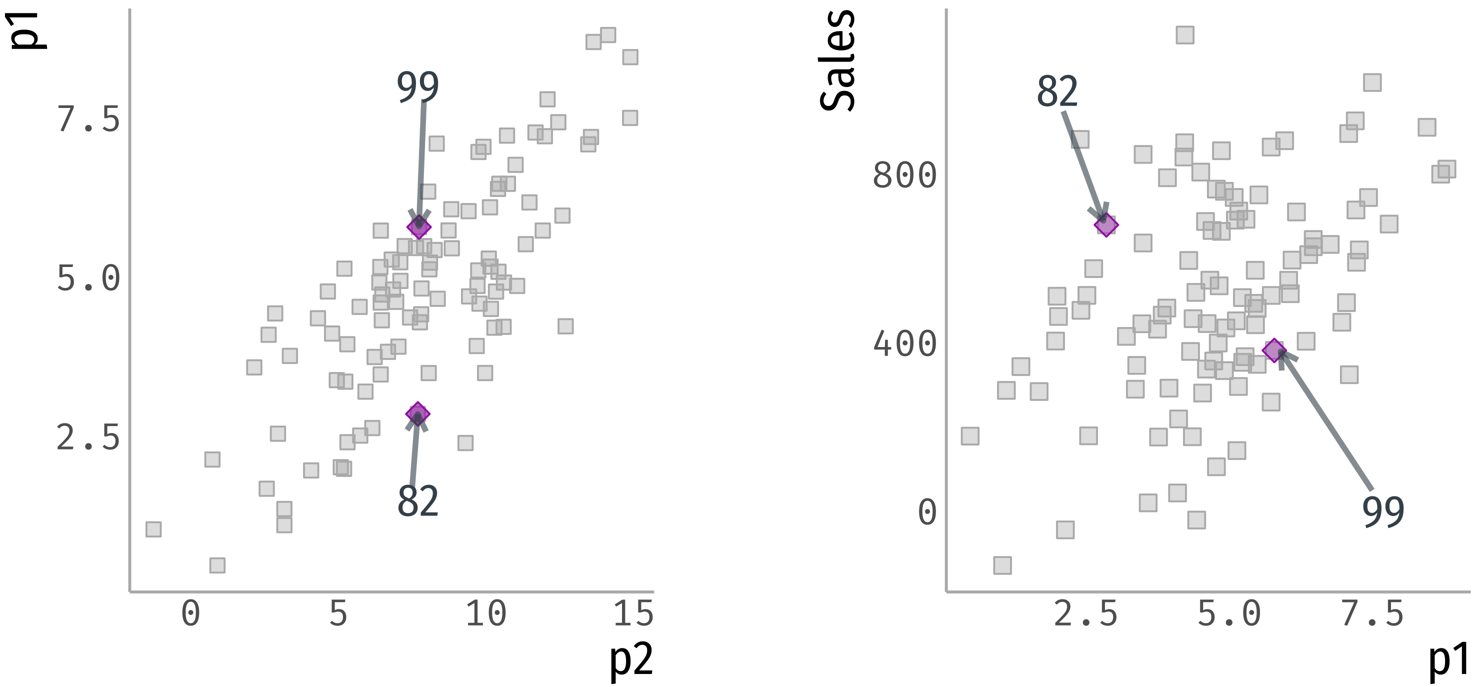
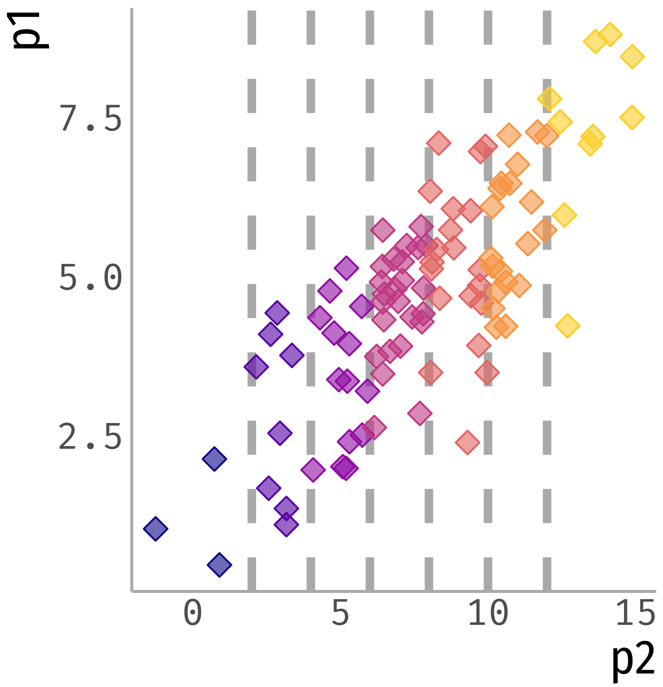


Relationship between p1 and p2 (cont.)

- Same thing happens when looking at a set of obs where $p2 \sim$ constant.



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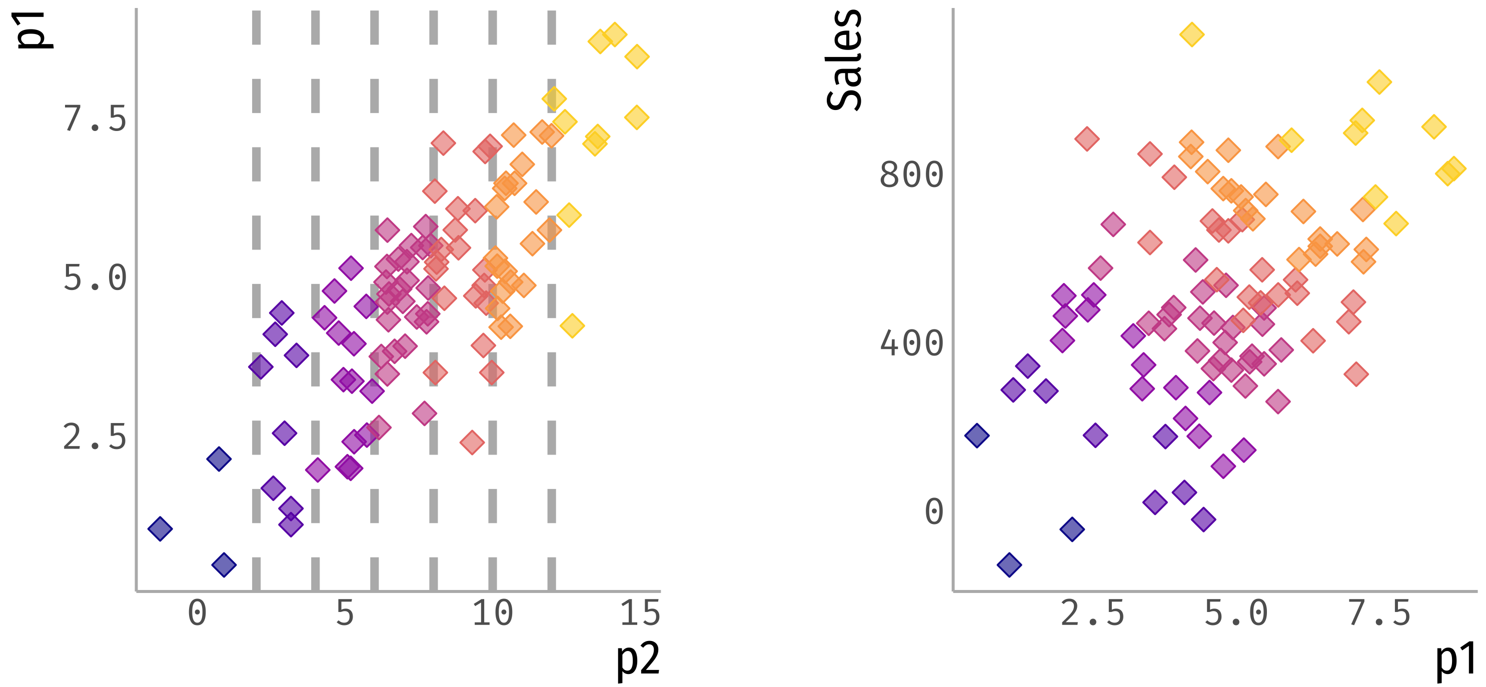

Conclusions?

A larger p1 is associated with a larger p2, and overall, with more sales!

If we keep p2 constant, a larger p1 is associated with lower sales.

Let's look at more data: Beer limit

- From the JITT assignment, we have `beers` data
  - `nbeer`: Number of beers before getting tipsy
  - `height`, `weight`, `age`
  - `female`: Whether the student is female or not

```
##   nbeer weight height age female
## 1    12    192     72  26      0
## 2    12    160     66  27      0
## 3     5    155     65  25      0
## 4     5    120     66  28      0
## 5     7    150     67  28      0
## 6    13    175     71  31      0
```

Is the number of beers related to height

- What model would you run?

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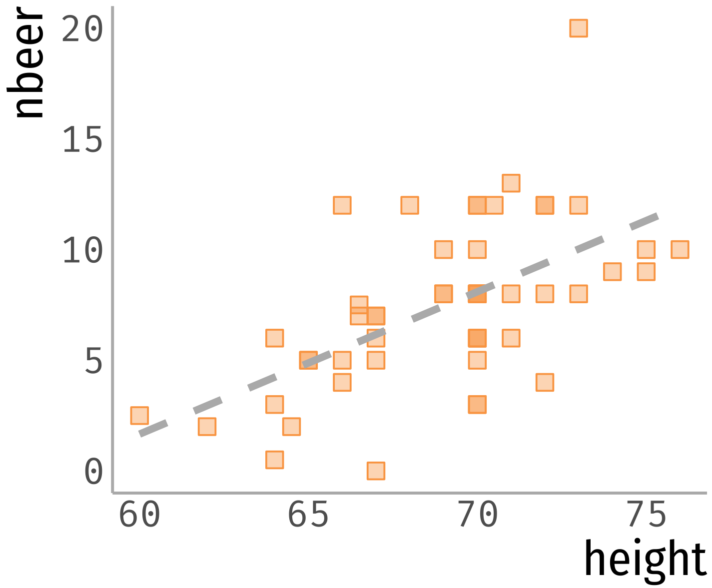

Is the number of beers related to height

- What model would you run?

$$nbeers_i = \beta_0 + \beta_1 \cdot height_i + \varepsilon$$

```
summary(lm(nbeer ~ height, data = beers))
```

```
##
## Call:
## lm(formula = nbeer ~ height, data = beers)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.164 -2.005 -0.093  1.738  9.978
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -36.9200     8.9560  -4.122 0.000148 ***
## height        0.6430     0.1296   4.960 9.23e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.109 on 48 degrees of freedom
## Multiple R-squared:  0.3389,    Adjusted R-squared:  0.3251
## F-statistic:  24.6 on 1 and 48 DF,  p-value: 9.23e-06
```

Is this the explanation that I want?

- Height can be a **proxy** for "bigger" people.

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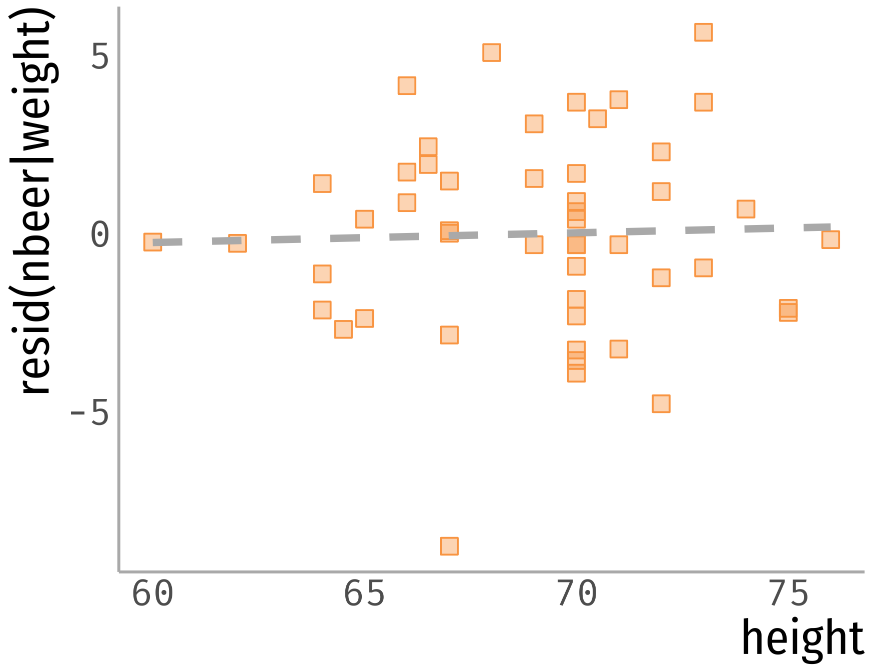
- What do you think will happen when I control for **weight**?

```
beers_fitted_weight <- augment(lm(nbeer ~ weight, data = beers))
```



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- Height can be a **proxy** for "bigger" people.

```
summary(lm(nbeer ~ weight + height, data = beers))
```

```
##
## Call:
## lm(formula = nbeer ~ weight + height, data = beers)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.5080 -2.0269  0.0652  1.5576  5.9087
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -11.18709   10.76821  -1.039 0.304167
## weight        0.08530    0.02381   3.582 0.000806 ***
## height        0.07751    0.19598   0.396 0.694254
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.784 on 47 degrees of freedom
## Multiple R-squared:  0.4807,    Adjusted R-squared:  0.4586
## F-statistic: 21.75 on 2 and 47 DF,  p-value: 2.056e-07
```

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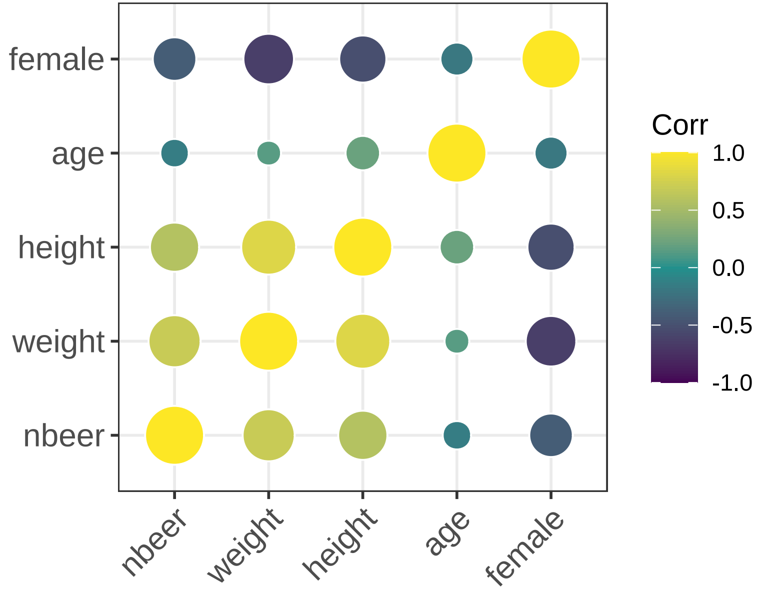
- Height can be a **proxy** for "bigger" people.

```r
M <-cor(beers)

ggcorrplot(M, method = "circle", outline.color = "white", ggtheme = ggplot2::theme_bw,
           colors = viridis(3))
```



Let's look at the two models closer!

	Model 1	Model 2
(Intercept)	-7.021***	-11.187
	(2.213)	(10.768)
weight	0.093***	0.085***
	(0.014)	(0.024)
height		0.078
		(0.196)
Num.Obs.	50	50
R2	0.479	0.481
R2 Adj.	0.468	0.459
F	44.119	21.750
* p < 0.1, ** p < 0.05, *** p < 0.01		

- Which model do you prefer?

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- Which model do you prefer?
- What happened to the SE for `weight`? Why?

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- Limit scenario: $|Cor(x_1, x_2)| = 1$
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Multicollinearity

- If variable `x1` and `x2` are highly correlated, it is difficult to disentangle their effects
  - **Context matters!**
- Limit scenario: $|Cor(x_1, x_2)| = 1$
  - Cannot estimate both parameters: One is dropped!

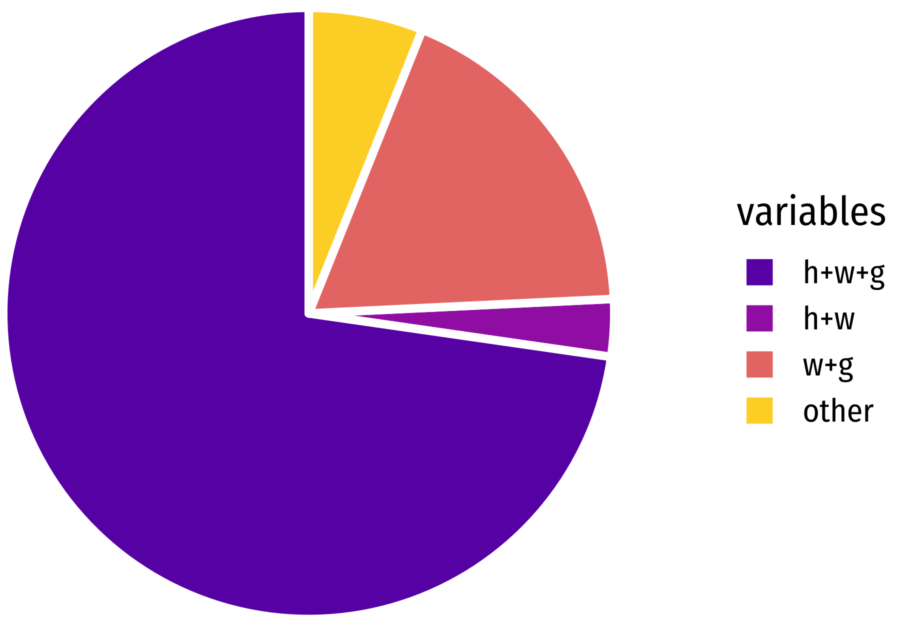
Can I add both binary variables `US_born` and `Foreign_born` to a regression?

Does gender matter?

	Model 1	Model 2	Model 3	Model 4
(Intercept)	-7.021***	-11.187	-7.830**	-12.067
	(2.213)	(10.768)	(3.013)	(11.084)
weight	0.093***	0.085***	0.097***	0.090***
	(0.014)	(0.024)	(0.018)	(0.027)
height		0.078		0.079
		(0.196)		(0.198)
female			0.528	0.536
			(1.320)	(1.333)
R2	0.479	0.481	0.481	0.482
R2 Adj.	0.468	0.459	0.459	0.449

* p < 0.1, ** p < 0.05, *** p < 0.01

Some of your answers in the JITT Assignment



Other ways to control for variables

- **Interactions**:
    - E.g. The relationship between `weight` and `nbeers` is different for males and females.

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```
lm(nbeer ~ weight*female, data = beers)
```

```
##
## Call:
## lm(formula = nbeer ~ weight * female, data = beers)
##
## Coefficients:
##   (Intercept)         weight         female  weight:female
##     -7.790193       0.097234       0.225748       0.002465
```

- How do we interpret these results?

Other ways to control for variables

- **Other polynomial terms**:
    - E,g, The relationship between `weight` and `nbeers` is quadratic.

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- **Other polynomial terms**:
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```
lm(nbeer ~ weight + I(weight^2), data = beers)
```

```
##
## Call:
## lm(formula = nbeer ~ weight + I(weight^2), data = beers)
##
## Coefficients:
## (Intercept)       weight  I(weight^2)
##   0.1078784   -0.0016255    0.0003033
```

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```

- How do we interpret these results?

$$\frac{\partial Y_{beers}}{\partial X_w} = \beta_1 + 2 \cdot \beta_2 X_w$$

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    - E,g, You want to include a factor variable for year the student is in.

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    - E,g, You want to include a factor variable for year the student is in.

```
table(beers$year)
```

```
##
## freshmen    junior    senior sophmore
##         7        14        14        15
```

```
lm(nbeer ~ weight + factor(year), data = beers)
```

```
##
## Call:
## lm(formula = nbeer ~ weight + factor(year), data = beers)
##
## Coefficients:
##          (Intercept)                weight    factor(year)junior
##             -6.85394               0.09237              -0.35244
##   factor(year)senior  factor(year)sophmore
##             -0.03144               0.07474
```

What should I control for?

- If your goal is **prediction**:

Overfitting

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- If your goal is **description**:

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Confidence Intervals

- If your goal is **causality**:

Bias

What should I control for?

- If your goal is **prediction**:

Overfitting

- If your goal is **description**:

Confidence Intervals

- If your goal is **causality**:

Bias

All of them matter!

References

- Hanck, C. et al. (2020). "Econometrics with R". *The Multiple Regression Model*