STA 235 - Prediction II: Classification and Regression Trees (CART)

Spring 2021

McCombs School of Business, UT Austin

Some reminders

Prediction Project has been posted

Remember: Teams of two (max)

Two tasks: Binary outcome (classification) and continuous outcome (regression)

Need to use two (2) different methods for each

Make sure you read the instructions! (and ask questions)

Where we've been...

- Talking about **bias vs variance** trade-off.
- **Model selection and regularization**: Stepwise selection, Ridge and Lasso regression.
- **K-nearest neighbors**

... and where we're going.

- Continue on our **prediction** journey:
  - **Decision Trees**: Classification and Regression Trees (CART)
- **Participation**: Activity in R.

Trees, trees everywhere!

Let's start with a simple example

Remember our Disney+ example?

Predict who will cancel their subscription

We have some **information:**

- `city`: Whether the customer lives in a big city or not
- `female`: Whether the customer is female or not
- `age`: Customer's age (in years)
- `logins`: Number of logins to the platform in the past week.
- `mandalorian`: Whether the person has watched the Mandalorian or not.
- `unsubscribe`: Whether they canceled their subscription or not.

The prediction task: Classification

- Our outcome is **binary**, so this is a **classification task**.
- Let's start looking at **two variables**:

City & Mandalorian

- Which one do you think is a better predictor?

Let's look at the data!

City vs. Mandalorian

```r
disney <- read.csv("https://raw.githubusercontent.com/maibennett/sta235/main/exampleSite/content/Cla

disney.train <- disney %>% dplyr::filter(train==1)

#Whole data
table(disney.train %>% dplyr::select(unsubscribe))

table(disney.train %>% dplyr::select(city, mandalorian))
```

Subscribers vs Unsubscribers

Unsubscribe	Freq
0	2900
1	2100

2x2 Frequency table

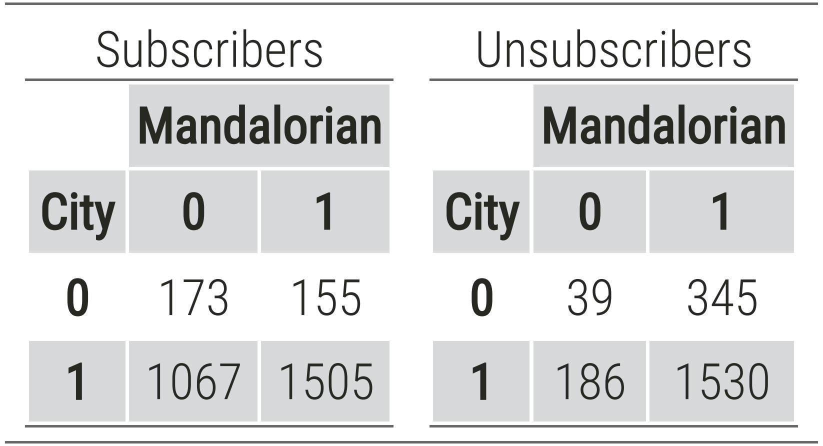
	Mandalorian	
City	**No**	**Yes**
0	212	500
1	1253	3035

City vs. Mandalorian

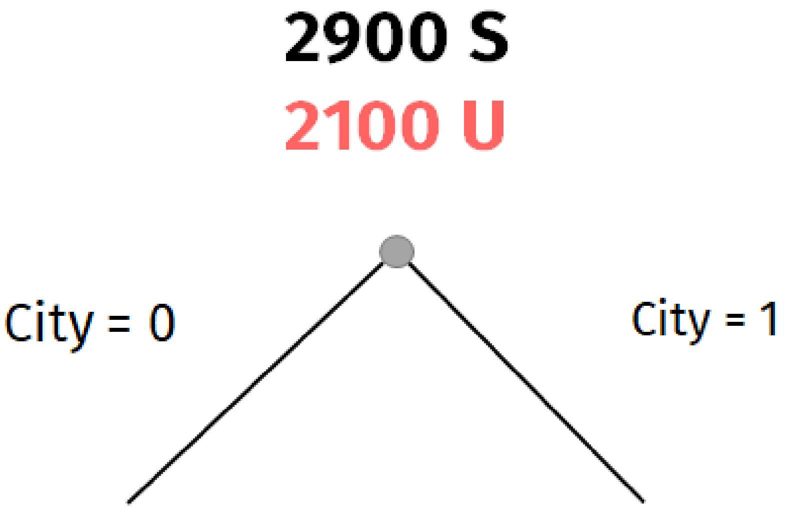
```
#Subscribers
table(disney.train %>% dplyr::filter(unsubscribe==0) %>%
        dplyr::select(city, mandalorian))
#Unsubscribers
table(disney.train %>% dplyr::filter(unsubscribe==1) %>%
        dplyr::select(city, mandalorian))
```

Subscribers			Unsubscribers		
	Mandalorian			Mandalorian	
City	**0**	**1**	**City**	**0**	**1**
0	173	155	**0**	39	345
1	1067	1505	**1**	186	1530

Let's split by `city` first...

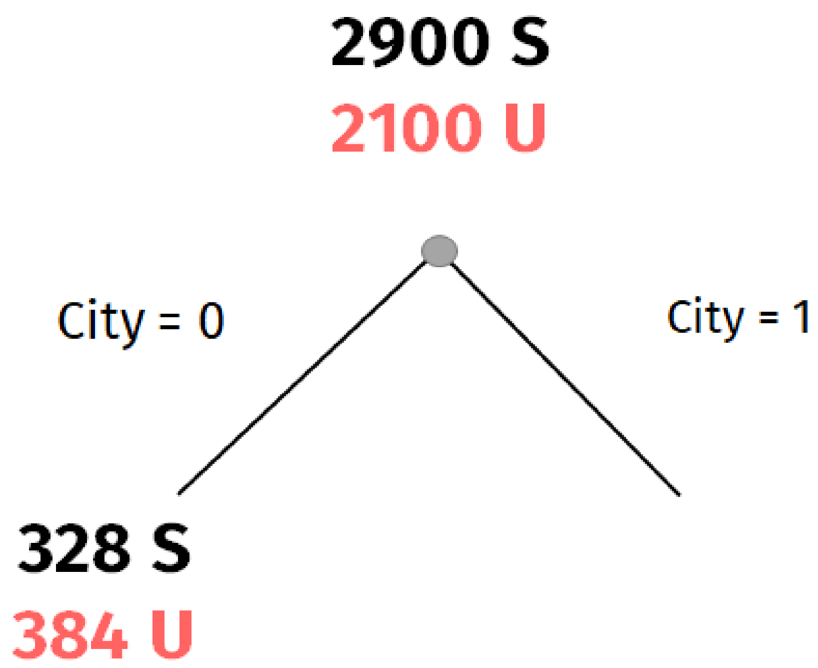


Subscribers			Unsubscribers		
	Mandalorian			Mandalorian	
City	0	1	City	0	1
0	173	155	0	39	345
1	1067	1505	1	186	1530

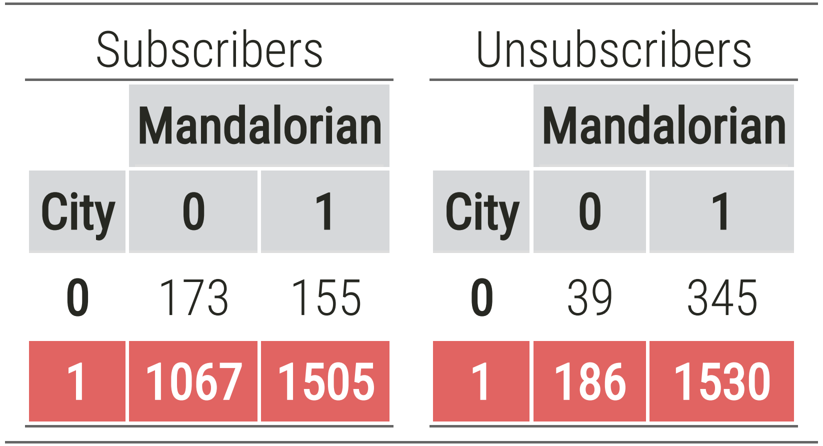


Let's split by `city` first...

Subscribers			Unsubscribers		
	Mandalorian			Mandalorian	
City	0	1	City	0	1
0	173	155	0	39	345
1	1067	1505	1	186	1530



Let's split by `city` first...

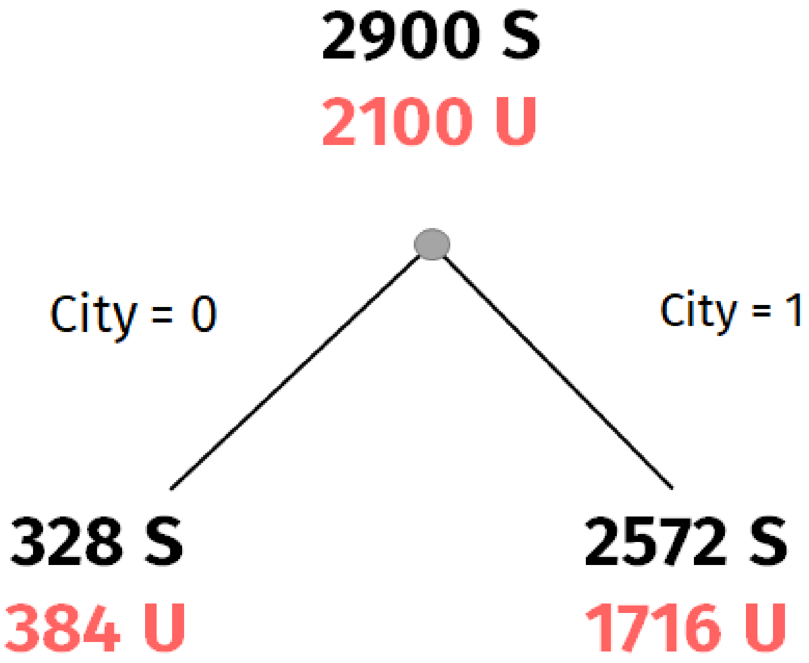


Subscribers

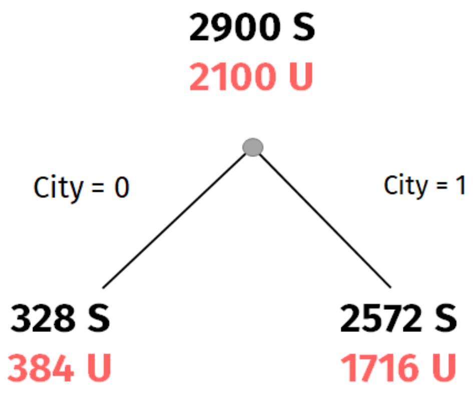
	Mandalorian	
City	0	1
0	173	155
1	1067	1505

Unsubscribers

	Mandalorian	
City	0	1
0	39	345
1	186	1530

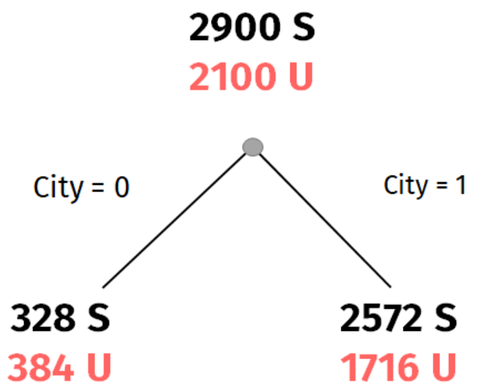


Calculate probabilities for `city`



$$\Pr[\text{Correct} \mid \text{city} = 0] = ?$$

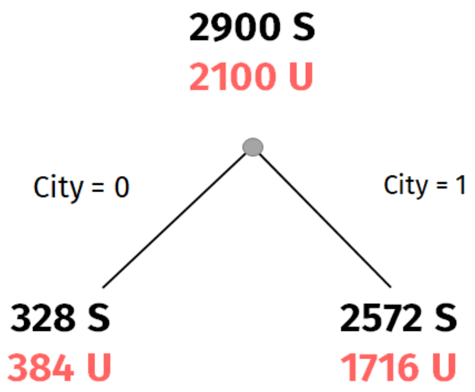
Calculate probabilities for `city`



$$\Pr[\text{Correct} \mid \text{city} = 0] = (\frac{328}{328 + 384})^2 + (\frac{384}{328 + 384})^2$$

$$\Pr[\text{Correct} \mid \text{city} = 0] = 0.503$$

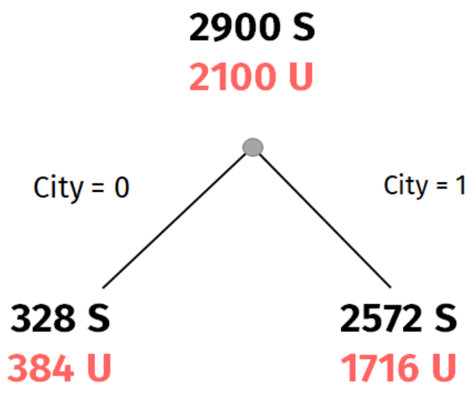
Calculate probabilities for `city`



$$\Pr[\text{Correct} \mid \text{city} = 1] = (\frac{2572}{2572 + 1716})^2 + (\frac{1716}{2572 + 1716})^2$$

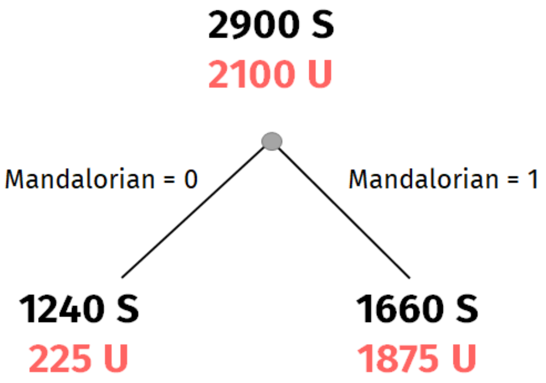
$$\Pr[\text{Correct} \mid \text{city} = 1] = 0.52$$

Calculate probabilities for `city`



$$\begin{aligned}\Pr[\text{Correct}] =& \Pr[\text{Correct}|\text{city} = 0] \times \Pr[\text{city} = 0] + \\ & \Pr[\text{Correct} \mid \text{city} = 1] \times \Pr[\text{city} = 1] = \\ =& 0.5 \times \frac{328 + 324}{5000} + 0.52 \times \frac{2572 + 1716}{5000} = \\ =& 0.518\end{aligned}$$

And we can do the same for `mandalorian`



$$
\begin{aligned}
\Pr[\text{Correct}] =& \Pr[\text{Correct}|\text{mandlr} = 0] \times \Pr[\text{mandlr} = 0] + \\
& \Pr[\text{Correct} \mid \text{mandlr} = 1] \times \Pr[\text{mandlr} = 1] = \\
=& 0.74 \times \frac{1240 + 225}{5000} + 0.502 \times \frac{1660 + 1875}{5000} = \\
=& 0.572
\end{aligned}
$$

Poll Time!

Which variable would you choose for prediction?

Choosing predictors

- From the previous exercise, we can see that **using `mandalorian` yields a higher accuracy** (0.57 vs. 0.52)

But we have more variables

How do we choose?

Decision Trees

- Main idea → **flowchart!**
- We will **stratify** (or segment) the predictor space into regions (ISLR, Ch. 8).
- Similar to KNN, we assign the **mean** or **mode** of the training obs in the region.

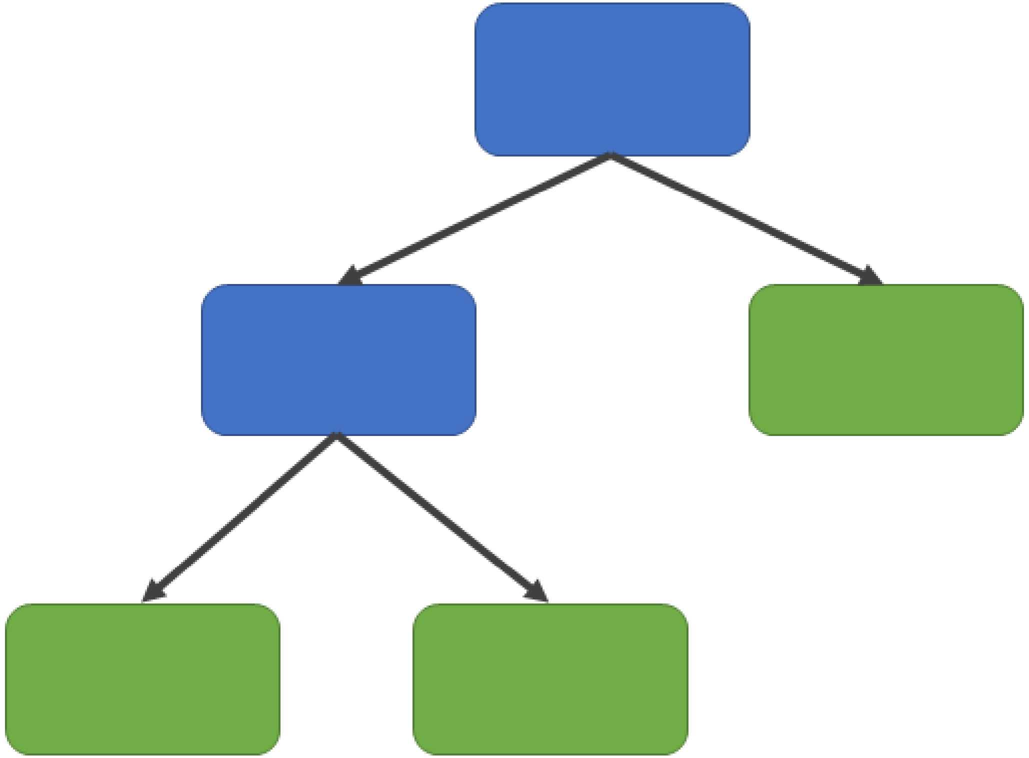
Main advantages

Simple interpretation

Main disadvantages

Overfitting

Structure of Decision Trees



Structure:

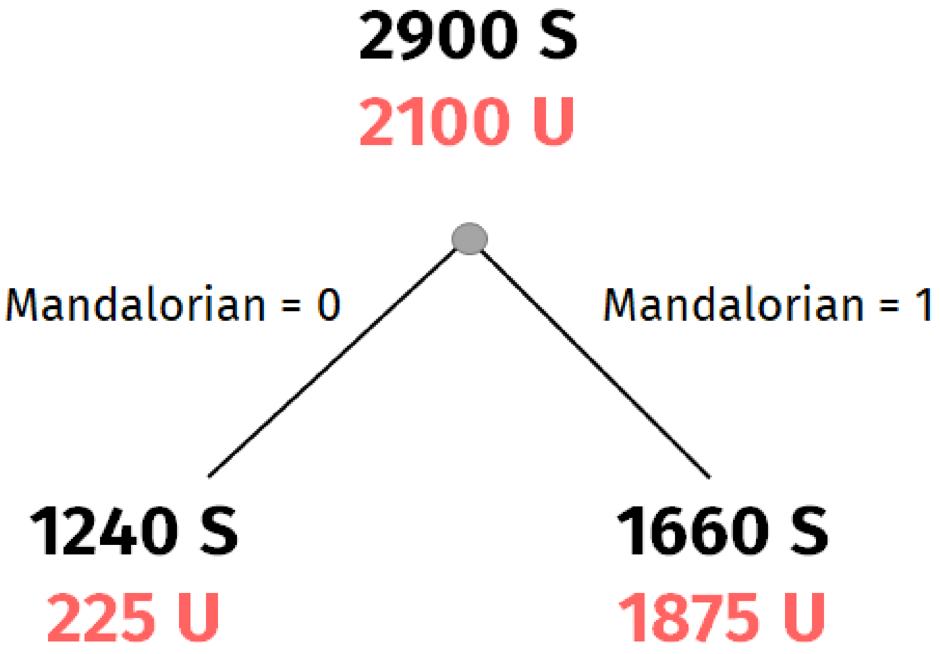
- Root node
- Internal nodes
- Leaves

Classification Trees

Classification Tree

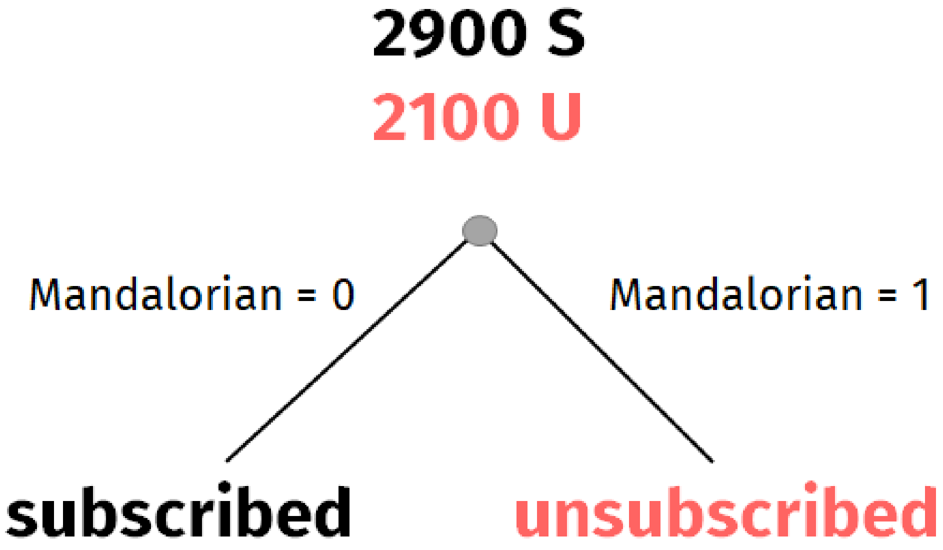
- Outcome is **categorical** (e.g. binary)
- Previous example: Chose splitting variable based on $\text{Pr}(\text{Correct})$
- What if we just assigned **based on the proportion in each leave?** (i.e. similar to KNN)

Let's go back to our drawing



- $\Pr[\mathrm{U} \mid \text{Leaf 1}] = 0.15$
- $\Pr[\mathrm{U} \mid \text{Leaf 2}] = 0.53$

Let's go back to our drawing



- $\Pr[\mathrm{U} \mid \mathrm{Leaf}\ 1] = 0.15$
- $\Pr[\mathrm{U} \mid \mathrm{Leaf}\ 2] = 0.53$

Classification error: 42%

Measures for accuracy

- The classification error rate is **not very sensitive for tree-growing**.

Poll time!

What is the main problem if our measure is not very sensitive for tree-growing?

Measures for accuracy

- The classification error rate is **not very sensitive for tree-growing**.
- Another measure is called **Gini index**:
  - Total variance across classes:

$$G = \sum_{k=1}^{K} \hat{p}_{mk}(1 - \hat{p}_{mk})$$

where $\hat{p}_{mk}$ is the proportion of training obs in region m for class k.

  - In our previous example: `

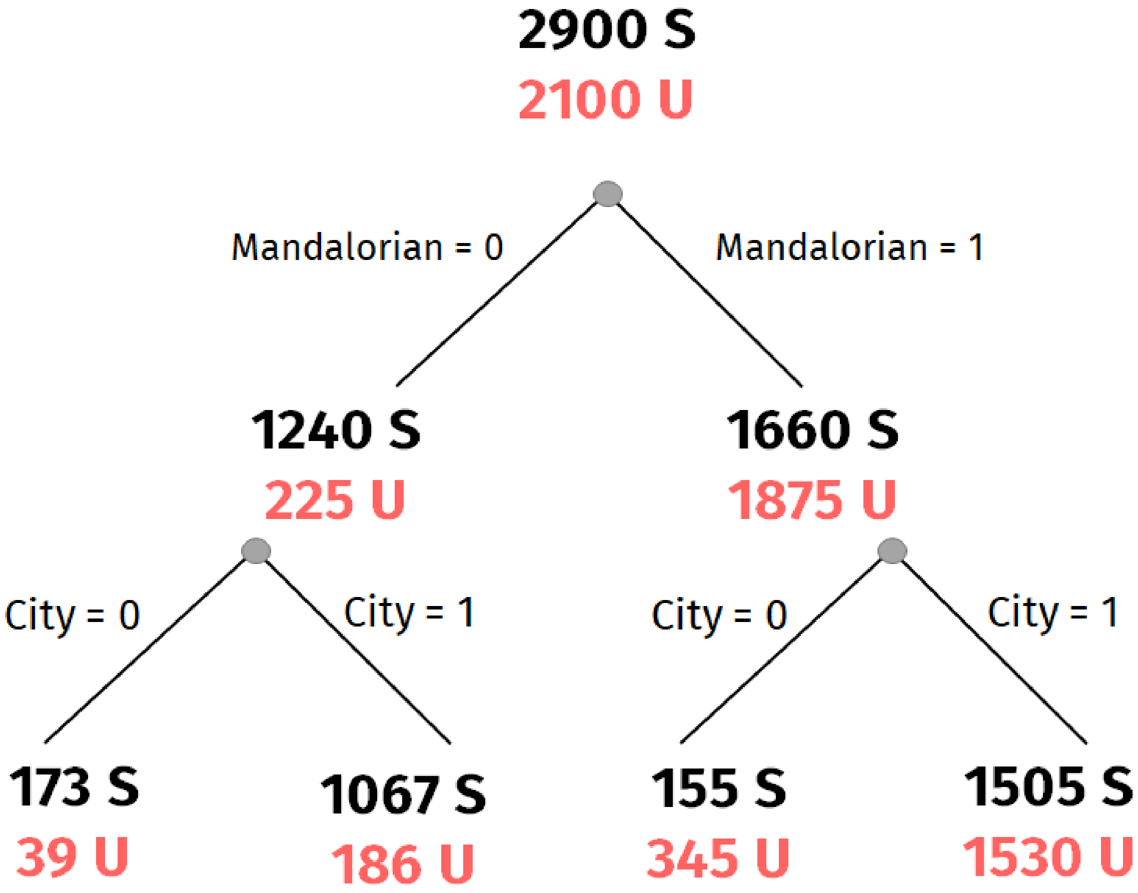
$$G_{mandalorian=0} = \frac{1240}{1240+225}(1 - \frac{1240}{1240+225}) + \frac{225}{1240+225}(1 - \frac{225}{1240+225}) = 0.26$$

Poll time!

According to the Gini Index, is it better or worse to have a high p_{mk}?

$$G = \sum_{k=1}^{K} \hat{p}_{mk}(1 - \hat{p}_{mk})$$

Ok, but how about including other variables?



Let's see how to do it in R!

```r
library(rpart)

d.train <- disney.train %>% dplyr::select(mandalorian, city, unsubscribe)

set.seed(100)

m1 <- rpart(unsubscribe ~., data = d.train, method = "class", cp=-1)
```

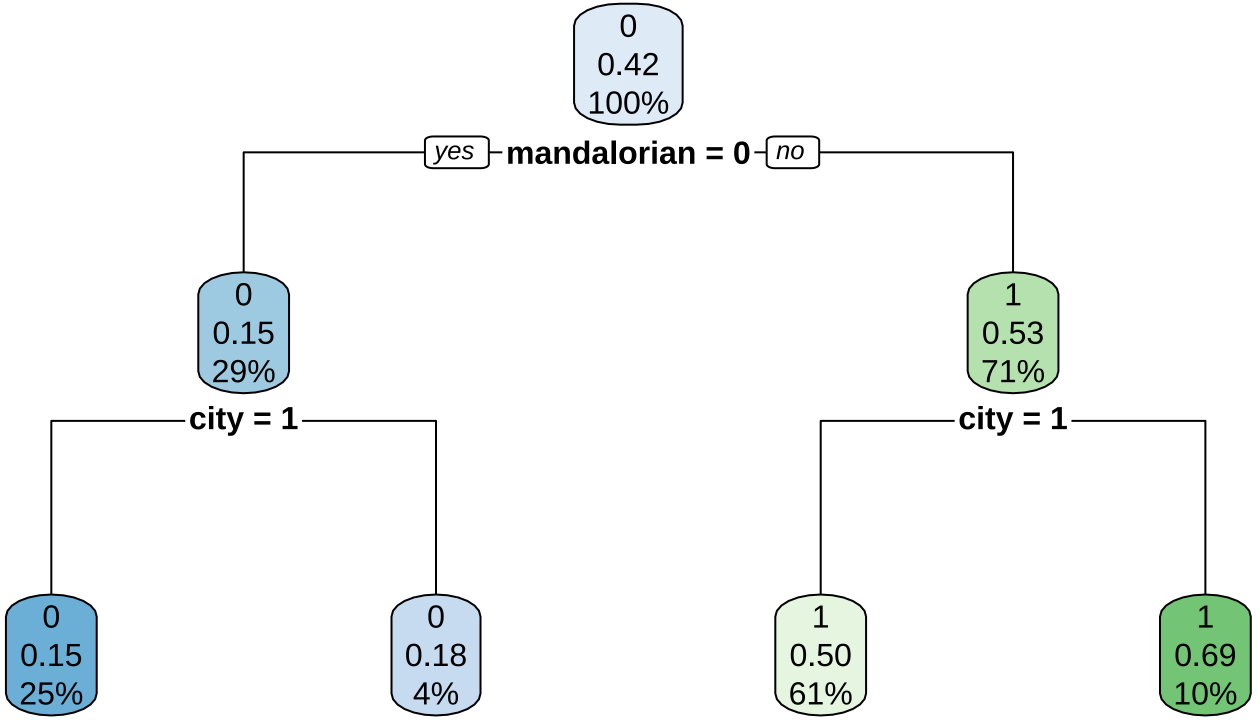
Let's see how to do it in R!

```r
library(rpart)

d.train <- disney.train %>% dplyr::select(mandalorian, city, unsubscribe)

set.seed(100)

m1 <- rpart(unsubscribe ~., data = d.train, method = "class", cp=-1)
```

Let's see how to do it in R!

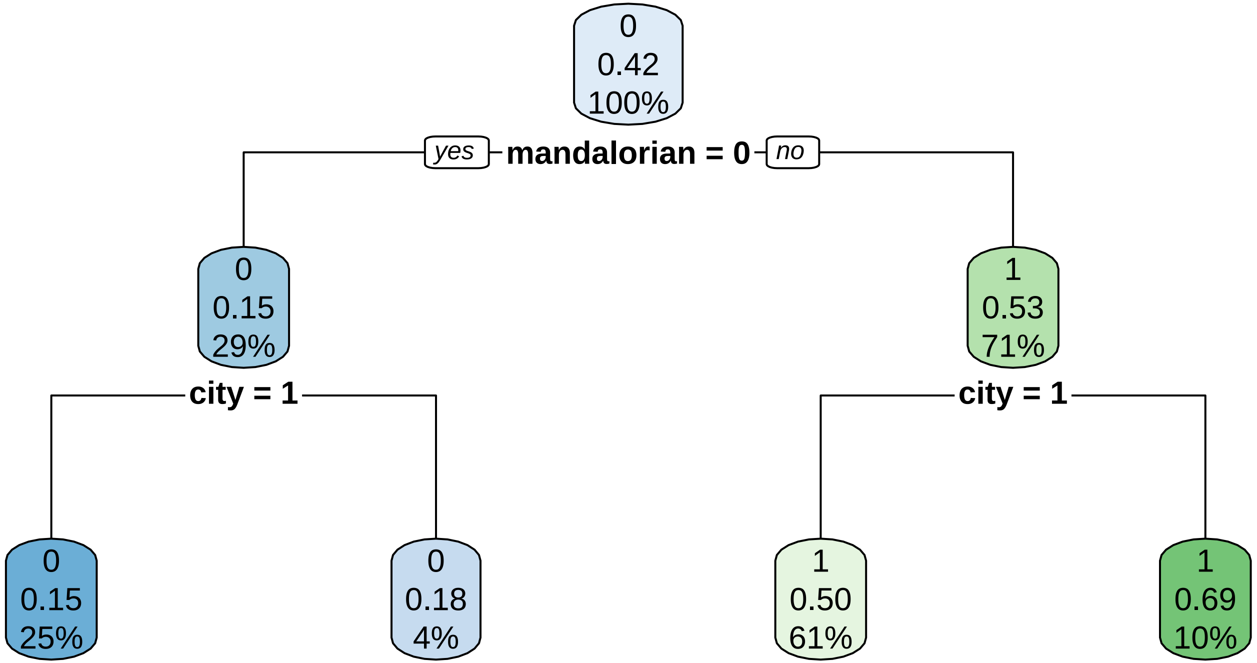
```r
library(rpart)

d.train <- disney.train %>% dplyr::select(mandalorian, city, unsubscribe)

set.seed(100)

m1 <- rpart(unsubscribe ~., data = d.train, method = "class", cp=-1)
```

Tree using all the covariates



Poll time!

What do you think the percentages in the leaves represent?



Some parameters that might be important

```
m1 <- rpart(unsubscribe ~., data = d.train, method = "class",
            control = rpart.control(cp=-1,
                                    minsplit = 20))
```

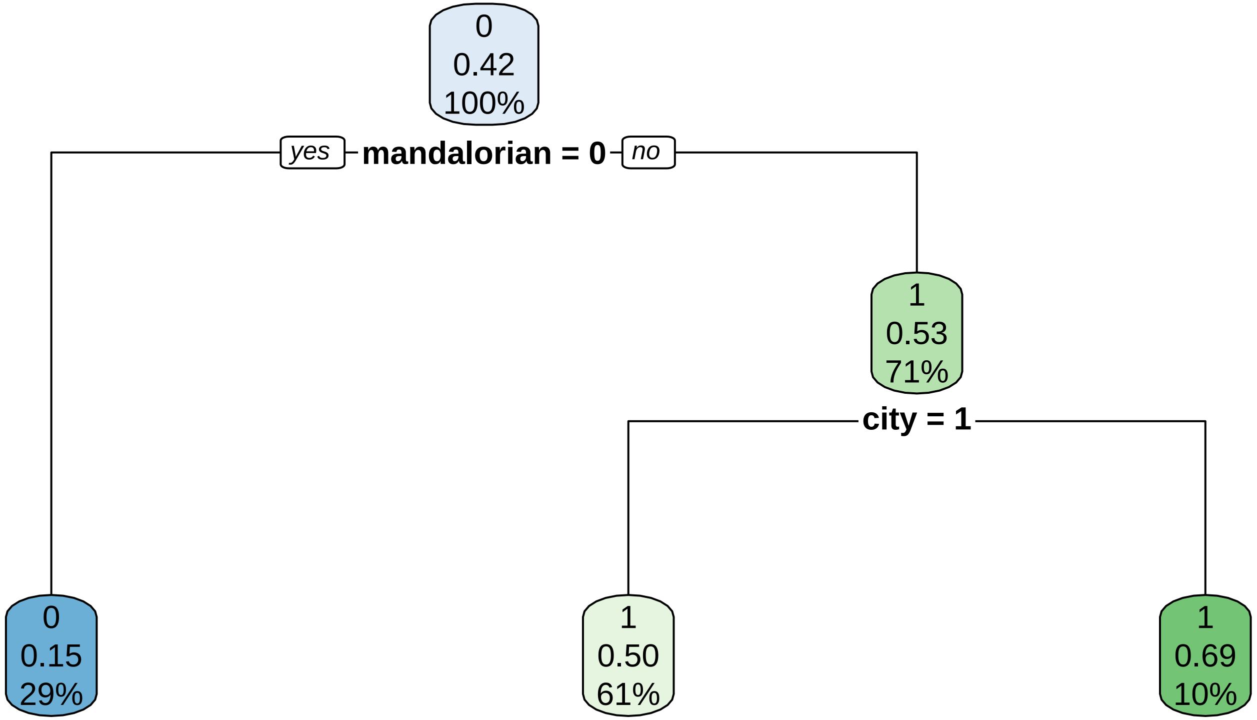
- `cp`: Complexity parameter
  - Split must decrease the overall lack of fit by a factor of `cp`, or is not attempted.
  - Parameter for **pruning the tree**.
  - Higher `cp`, smaller the tree!

Some parameters that might be important

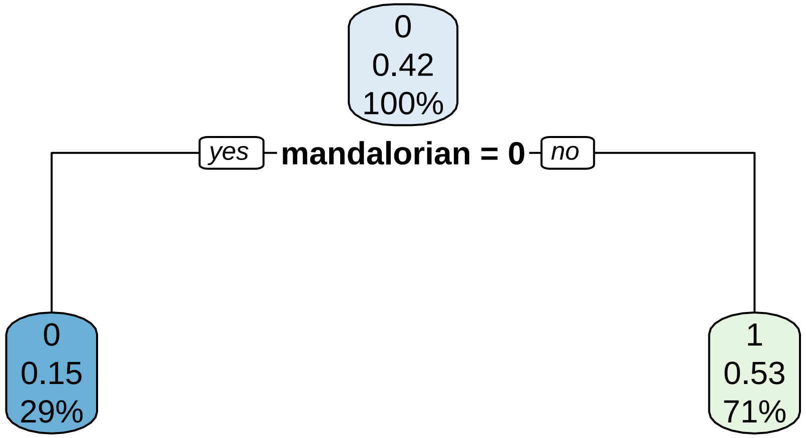
```r
m1 <- rpart(unsubscribe ~., data = d.train, method = "class",
            control = rpart.control(cp=-1,
                                    minsplit = 20))
```

- `minsplit`: Min. number of obs in a node to attempt a split.

If we set `minsplit` to 1500...



If we don't set `cp`...



If we don't set `cp`...

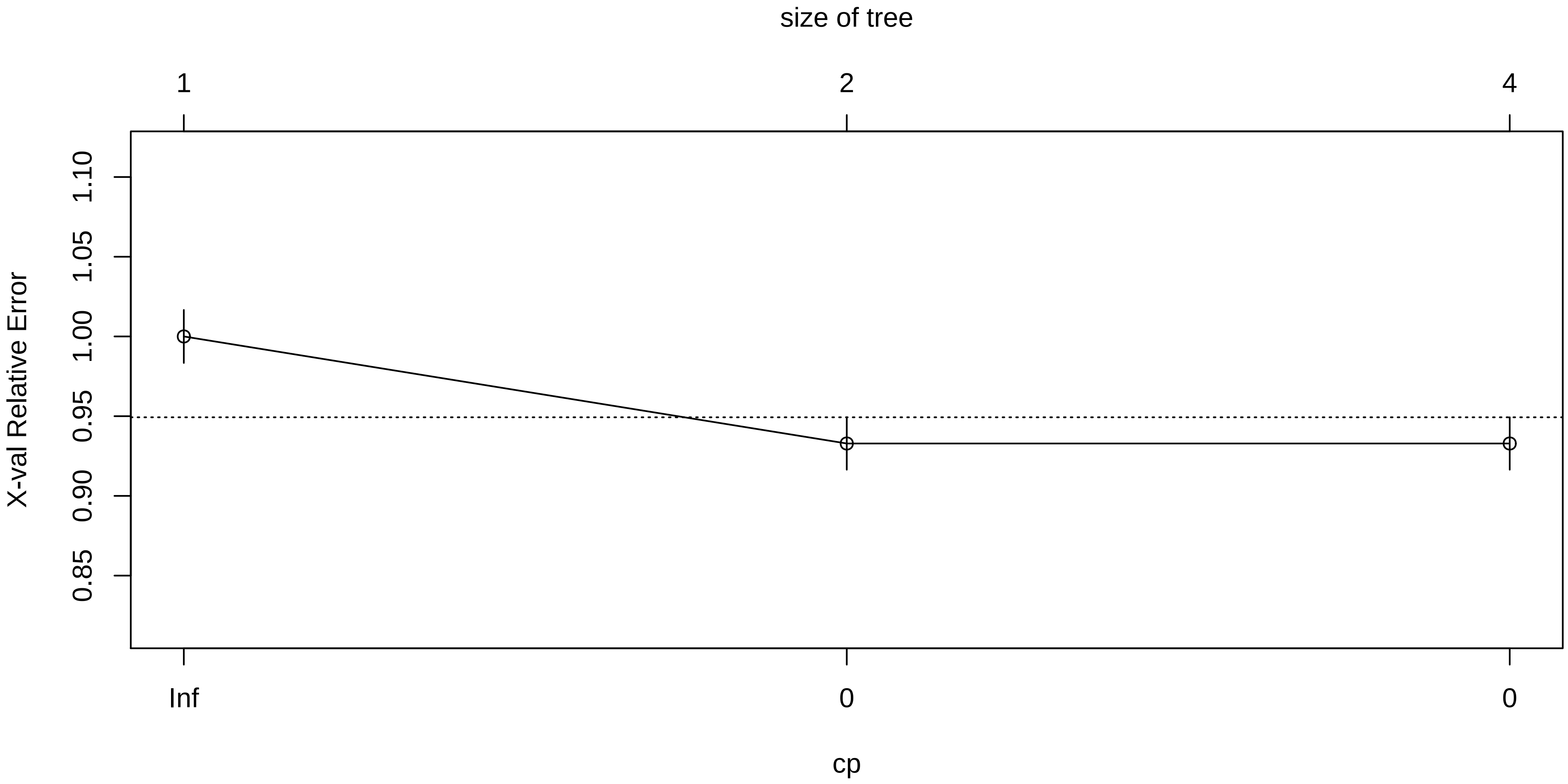
```
m1$cptable
```

```
##           CP nsplit rel error    xerror       xstd
## 1  0.102381      0  1.000000 1.0000000 0.01661898
## 2  0.000000      1  0.897619 0.9328571 0.01643695
## 3 -1.000000      3  0.897619 0.9328571 0.01643695
```

```
m3$cptable
```

```
##          CP nsplit rel error   xerror       xstd
## 1 0.102381      0  1.000000 1.000000 0.01661898
## 2 0.010000      1  0.897619 0.897619 0.01631851
```

How can we use this for selecting the size of our tree?



Basic Algorithm

1) Start at the root node

2) Split the parent node at covariate x_i to minimize the sum of child node impurities

3) Assign training samples to new child nodes

4) Stop if leaves are pure or early stopping criteria is satisfied, else repeat step (1) and (2) for each new child nodes

"Machine Learning FAQs" (Raschka, 2021)

Now it's your turn!

Instructions

- Using the code **provided on the course's website**:
    - Fit a classification tree using **all the covariates**

Play around with cp. What happens?

Interpret your tree!

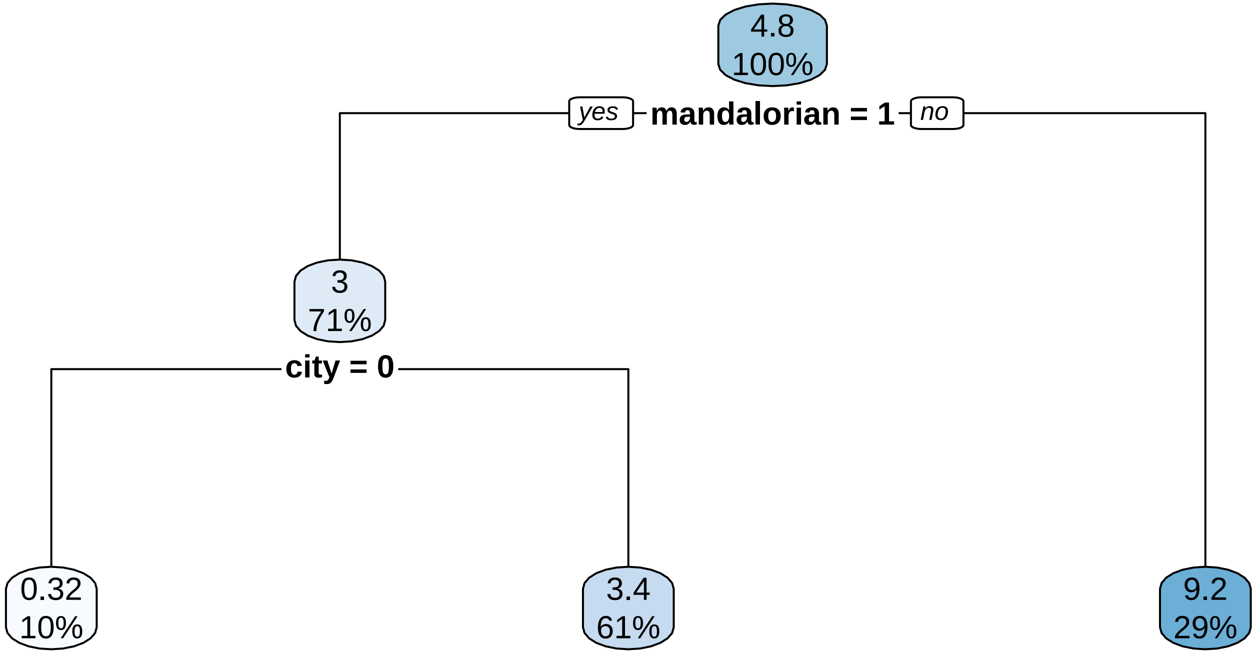
Regression Trees

Regression Trees

- Outcome is **continuous**
- Very similar to what we have seen with **classification trees**:
    - Predicted outcome is the **mean outcome for the leaf/region**.

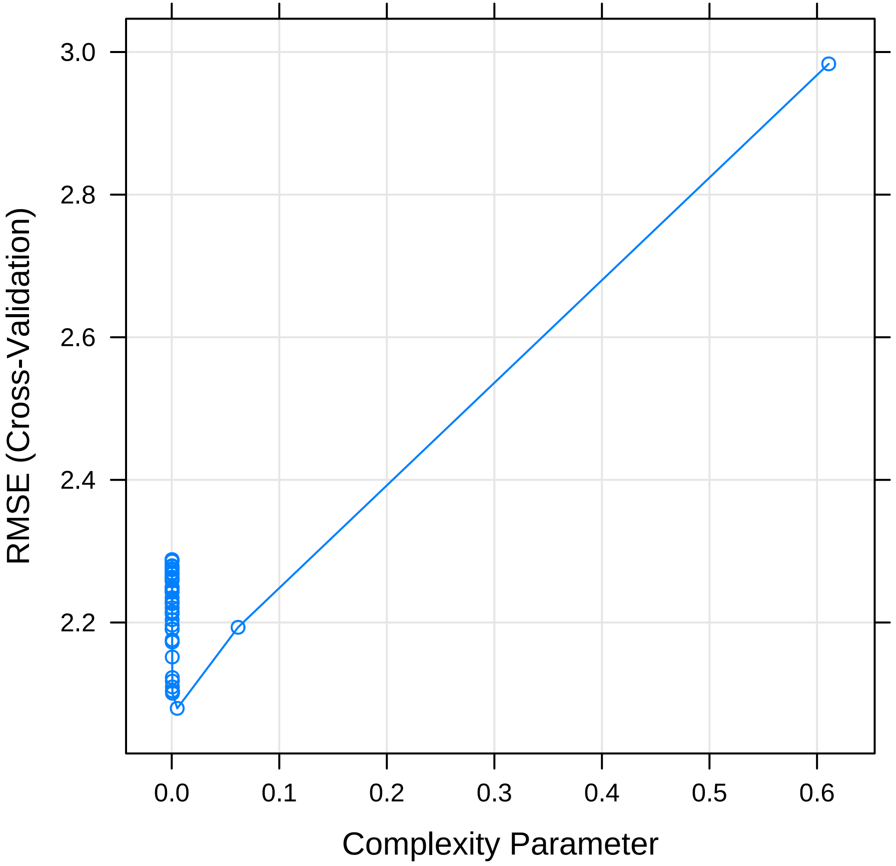
In R is basically the same

```
set.seed(100)
r1 <- rpart(logins ~. - unsubscribe, data = disney.train,
            method = "anova")

rpart.plot(r1)
```



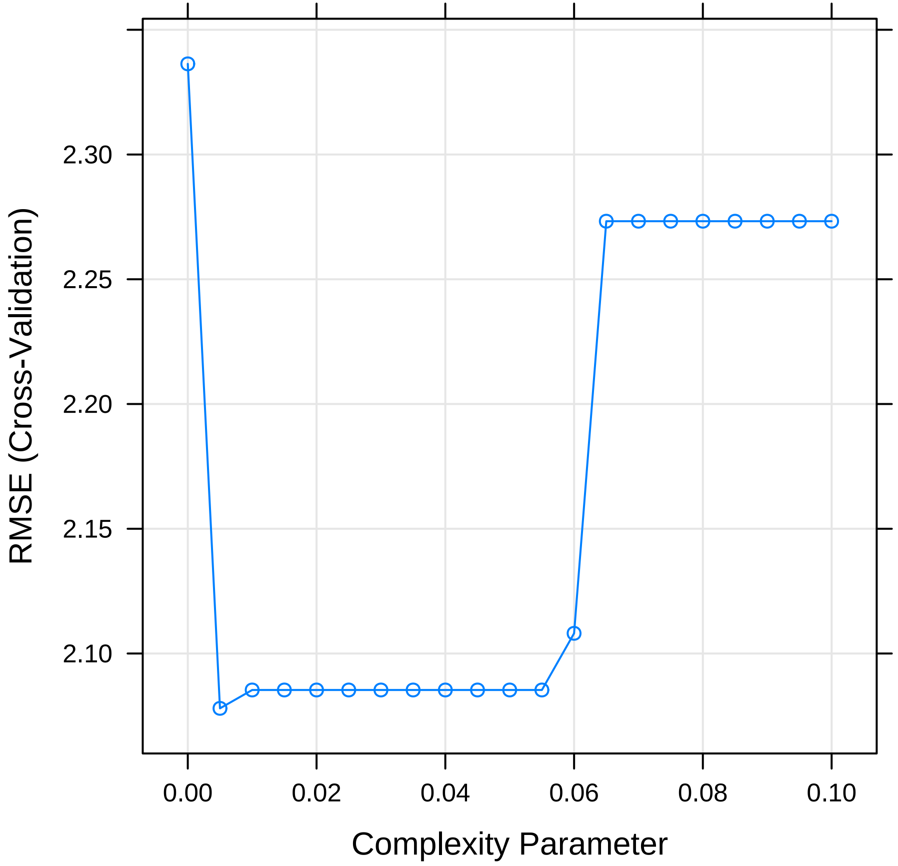
Let's incorporate cross-validation!

```r
library(caret)
set.seed(100)

mcv <- train(
  logins ~. - unsubscribe, data = disney.train
  method = "rpart",
  trControl = trainControl("cv", number = 10),
  tuneLength = 50
  )

plot(mcv)
```

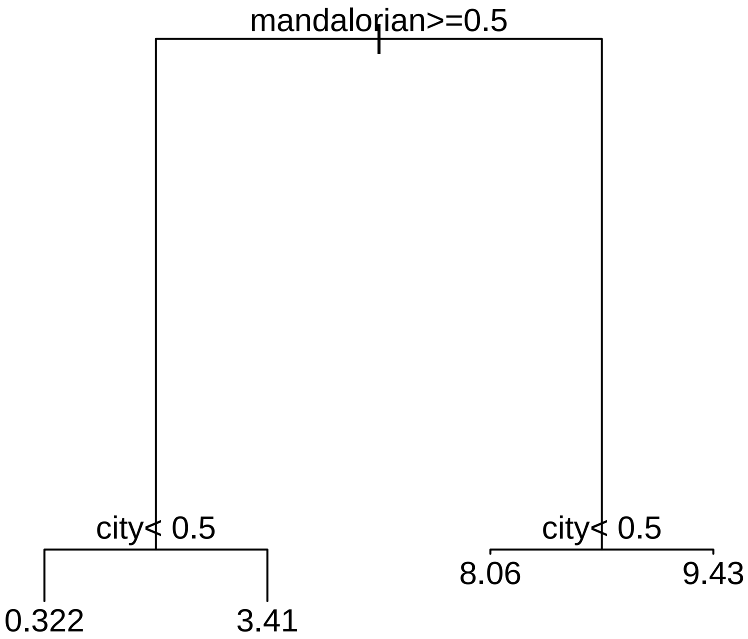


Let's incorporate cross-validation!

```
library(caret)
set.seed(100)

tuneGrid <- expand.grid(cp = seq(0, 0.1, 0.005

mcv <- train(
  logins ~. - unsubscribe, data = disney.train
  method = "rpart",
  trControl = trainControl("cv", number = 10),
  tuneGrid = tuneGrid
  )

plot(mcv)
```



Plot the tree

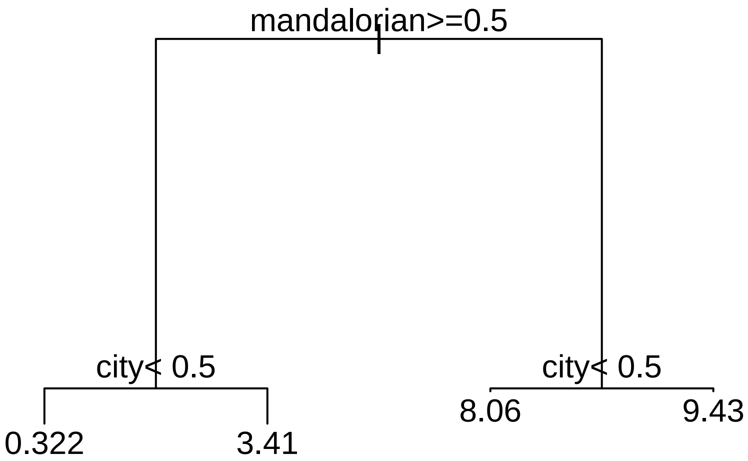


```
mcv$finalModel
```

```
## n= 5000
##
## node), split, n, deviance, yval
##       * denotes terminal node
##
## 1) root 5000 66387.3700 4.806800
##   2) mandalorian>=0.5 3535 24633.5000 2.973409
##     4) city< 0.5 500   517.1580 0.322000 *
##     5) city>=0.5 3035 20022.2800 3.410214 *
##   3) mandalorian< 0.5 1465  1200.0180 9.230717
##     6) city< 0.5 212   132.2028 8.061321 *
##     7) city>=0.5 1253   728.8571 9.428571 *
```

Poll time!

What would the predicted value be for a customer who hasn't watched The Mandalorian and lives in a city?



```
mcv$finalModel
```

```
## n= 5000
##
## node), split, n, deviance, yval
##       * denotes terminal node
##
## 1) root 5000 66387.3700 4.806800
##   2) mandalorian>=0.5 3535 24633.5000 2.973409
##     4) city< 0.5 500   517.1580 0.322000 *
##     5) city>=0.5 3035 20022.2800 3.410214 *
##   3) mandalorian< 0.5 1465  1200.0180 9.230717
##     6) city< 0.5 212   132.2028 8.061321 *
##     7) city>=0.5 1253   728.8571 9.428571 *
```

Remember that we care about predictions outside our training sample

- For predicting **unsubscribe**

```
disney.test <- disney %>% dplyr::filter(train==

mclass <- train(
  factor(unsubscribe) ~., data = disney.train,
  method = "rpart",
  trControl = trainControl("cv", number = 10),
  tuneLength = 50
  )

pred.class <- mclass %>% predict(disney.test)

mean(pred.class==disney.test$unsubscribe)
```

```
## [1] 0.639
```

- For predicting **logins**:

```
pred.reg <- mcv %>% predict(disney.test)

RMSE(pred.reg, disney.test$logins)
```

```
## [1] 2.099631
```

Main takeaways of decision trees

Main advantages:

- Easy to interpret and explain (you can plot them!)
- Mirrors human decision-making.
- Can handle qualitative predictors (without need for dummies).

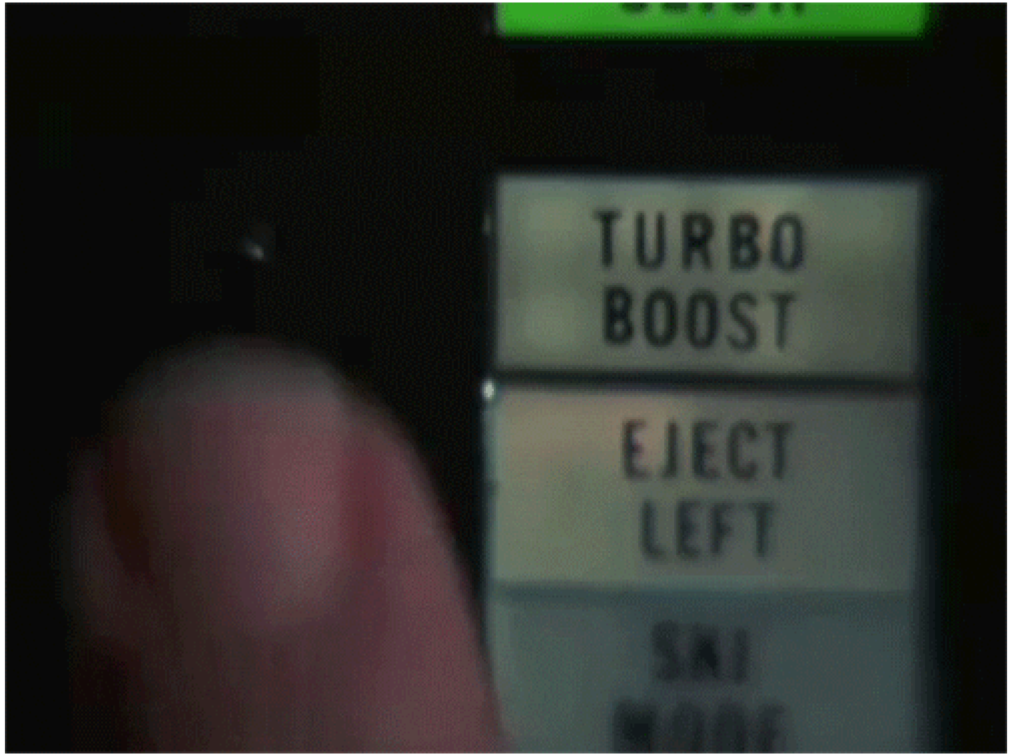
Main disadvantages:

- Accuracy not as high as other methods
- Very sensitive to training data (e.g. overfitting)

Next class

Use of decision trees as building blocks for **more powerful prediction methods!**

- Bagging
- Random Forests
- Boosting



References

- James, G. et al. (2013). "Introduction to Statistical Learning with Applications in R". *Springer. Chapter 8*.
- Ritvik Kharkar. (2019). “Decision Trees”. *Video materials from ritvikmath (YouTube)*.
- Starmer, J.. (2018). "Decision Trees". *Video materials from StatQuest (YouTube)*.
- STDHA. (2018). "CART Model: Decision Tree Essentials"