DIGITAL DESIGN

CS/ECE/EEE/INSTR F215

Lecture 7
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Quine-McCluskey method (QM)

- Suitable for computer solution
- Uses Tabular method

QM Method

- D = f(a, b, c, d) = Σ (0,1,2,3,6,7,8,9,14,15)

D = f(a, b, c, d) = Σ (0,1,2,3,6,7,8,9,14,15)

Index	Decimal Number	Binary representation			
0	0	0	0	0	0
1	1	0	0	0	1
	2	0	0	1	0
	8	1	0	0	0
2	3	0	0	1	1
	6	0	1	1	0
	9	1	0	0	1
3	7	0	1	1	1
	14	1	1	1	0
4	15	1	1	1	1

0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0
3	0	0	1	1
6	0	1	1	0
9	1	0	0	1
7	0	1	1	1
14	1	1	1	0
15	1	1	1	1

(0,1)	0	0	0	-
(0,2)	0	0	-	0
(0,8)	-	0	0	0
(1,3)	0	0	-	1
(1,9)	-	0	0	1
(2,3)	0	0	1	-
(2,6)	0	-	1	0
(8,9)	1	0	0	-
(3,7)	0	-	1	1
(6,7)	0	1	1	-
(6,14)	-	1	1	0
(7,15)	-	1	1	1
(14,15)	1	1	1	-

(0,1)	0	0	0	-
(0,2)	0	0	-	0
(0,8)	-	0	0	0
(1,3)	0	0	-	1
(1,9)	-	0	0	1
(2,3)	0	0	1	-
(2,6)	0	-	1	0
(8,9)	1	0	0	-
(3,7)	0	-	1	1
(6,7)	0	1	1	-
(6,14)	-	1	1	0
(7,15)	-	1	1	1
(14,15)	1	1	1	-

(0,1,2,3)	0	0	-	-
(0,1,8,9)	-	0	0	-
(2,3,6,7)	0	-	1	-
(6,7,14,15)	-	1	1	-

PI	0	1	2	3	6	7	8	9	14	15
a' b'	×	×	×	×						
b' c'	×	×					×	×		
a' c			×	×	×	×				
b c					×	×			×	×

PI	0	1	2	3	6	7	8	9	14	15
a' b'	×	×	×	×						
b' c'	×	×					⊗	⊗		
a' c			×	×	×	×				
b c					×	×			⊗	⊗

PI	0	1	2	3	6	7	8	9	14	15
a' b'	×	×	×	×						
b' c'	×	×					⊗	⊗		
a' c			×	×	×	×				
b c					×	×			⊗	⊗

f(a, b, c, d) =b' c'+ b c+ a' c

$S = f(w, x, y, z) = \Sigma (1, 3, 13, 15) + \Sigma_d (8,9,10,11)$

1	0	0	0	1
8	1	0	0	0
3	0	0	1	1
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
13	1	0	1	1
15	1	1	1	1

(1,3)	0	0	-	1
(1,9)	-	0	0	1
(8,9)	1	0	0	-
(8,10)	1	0	-	0
(3,11)	-	0	1	1
(9,11)	1	0	-	1
(9,13)	1	-	0	1
(10,11)	1	0	1	-
(11,15)	1	-	1	1
(13,15)	1	1	-	1

(1,3)	0	0	-	1
(1,9)	-	0	0	1
(8,9)	1	0	0	-
(8,10)	1	0	-	0
(3,11)	-	0	1	1
(9,11)	1	0	-	1
(9,13)	1	-	0	1
(10,11)	1	0	1	-
(11,15)	1	-	1	1
(13,15)	1	1	-	1

(1,3,9,11)	-	0	-	1
(8,9,10,11)	1	0	-	-
(9,13,11,15)	1	-	-	1

PI	1	3	13	15
x' z	×	×		
w x'				
w z			×	×

f(w, x, y, z) = x' z +w z

D = f(a, b, c, d) = Σ (0,5,7,8,9,10,11,14,15)

(0,8) E	-	0	0	0
(8,9)	1	0	0	-
(8,10)	1	0	-	0
(5,7) D	0	1	-	1
(9,11)	1	0	-	1
(10,11)	1	0	1	-
(10,14)	1	-	1	0
(7,15) C	-	1	1	1
(11,15)	1	-	1	1
(14,15)	1	1	1	-

(8,9,10,11) B	1	0	-	-
(10,11,14,15) A	1	-	1	-

PI	0	5	7	8	9	10	11	14	15
A						×	×	⊗	×
B				×	⊗	×	×		
C			×						×
D		⊗	×						
E	⊗			×					

F(w,x,y,z) = A+B+D+E = wy+wx'+w'xz+x'y'z'

f (a,b,c,d) = Σ m(1,3,4,5,6,7,10,12,13)+ Σ d(2,9,15)

(1,3)	0	0	-	1
(1,5)	0	-	0	1
(1,9)	-	0	0	1
(2,3)	0	0	1	-
(2,6)	0	-	1	0
(2,10)	-	0	1	0
(4,5)	0	1	0	-
(4,6)	0	1	-	0
(4,12)	-	1	0	0
(3,7)	0	-	1	1
(5,7)	0	1	-	1
(5,13)	-	1	0	1
(6,7)	0	1	1	-
(9,13)	1	-	0	1
(12,13)	1	1	0	-
(7,15)	-	1	1	1
(13,15)	1	1	-	1

(1,3,5,7) F	0	-	-	1
(1,5,9,13) E	-	-	0	1
(2,3,6,7) D	0	-	1	-
(4,5,6,7) C	0	1	-	-
(4,12,5,13) B	-	1	0	-
(5,13,7,15) A	-	1	-	1

Don't care's (2,9,15) not added here.

PI	1	3	4	5	6	7	10	12	13
A				×		×			×
B			×	×				×	×
C			×	×	×	×			
D		×			×	×			
E	×			×					
F	×	×		×		×			
G									

EPI

EPI

PI	1	3	6	7
A				×
C			×	×
D		×	×	×
E	×			
F	×	×		×

Column

7 ⊃ 6

7 ⊃ 3

Row

C ⊃ A

D ⊃ C

D ⊃ A

F ⊃ E

Remove dominating column (here 7)

Remove dominated Rows (Here A, C and E)

PI	1	3	6
D		×	×
F	×	×	

Solution is B+G+D+F

= bc'+b'cd'+a'c+a'd

PI	1	3	6
D		×	×
F	×	×	

EPI

$f(a,b,c,d) = \Sigma m(3,4,6,7,8,9,11,13,14) + \Sigma d(2,5,15)$

(2,3)	0	0	1	-
(2,6)	0	-	1	0
(4,5)	0	1	0	-
(4,6)	0	1	-	0
(8,9)	1	0	0	-
(3,7)	0	-	1	1
(3,11)	-	0	1	1
(5,7)	0	1	-	1
(5,13)	-	1	0	1
(6,7)	0	1	1	-
(6,14)	-	1	1	0
(9,11)	1	0	-	1
(9,13)	1	-	0	1
(7,15)	-	1	1	1
(11,15)	1	-	1	1
(13,15)	1	1	-	1
(14,15)	1	1	1	-

(2,3,6,7) F	0	-	1	-
(4,5,6,7) E	0	1	-	-
(3,7,11,15) D	-	-	1	1
(5,7,13,15) C	-	1	-	1
(6,7,14,15) B	-	1	1	-
(9,11,13,15) A	1	-	-	1

G

PI	3	4	6	7	8	9	11	13	14
A						X	X	X	
B			X	X					X
C				X				X	
D	X			X			X		
E		X	X	X					
F	X		X	X					
G					X	X			

Only terms

PI	3	11	13
A		X	X
C			X
D	X	X	
F	X		

- P = (F+D)(D+A)(C+A)
- = (D+AF)(C+A)
- = (CD+AD+ACF+AF)

- Use CD or AD or AF (Decision based on no. of literals)

- Solution is F = B+E+G+ C+D
- Or F = B+E+G+ A+D
- Or F = B+E+G+ A+F

Various examples of PI table

- Consider following table after removing EPI terms

PI	1	5	7	9	11	15
A						X
B					X	X
C			X			X
D				X	X	
E		X	X			
F	X			X		
G	X	X				

- P = (F+G)(E+G)(C+E)(D+F)(B+D)(A+B+C)
- = (G+EF)(D+BF)(C+AE+BE)
- = (GD+GBF+DEF+BEF)(C+AE+BE)
- = (CGD+GDAE+GDBE+CGBF+AEGBF+GBFE+CDEF
- +ADEF+BDEF+BCEF+ABEF+BEF

- Use Either CGD or BEF (Decision based on no. of literals)

- Solution is F = (EPI terms) + C+G+D
- Or F = (EPI terms) + B+E+F