Electronic Devices

Lecture 8

23-08-2018

Density of states function

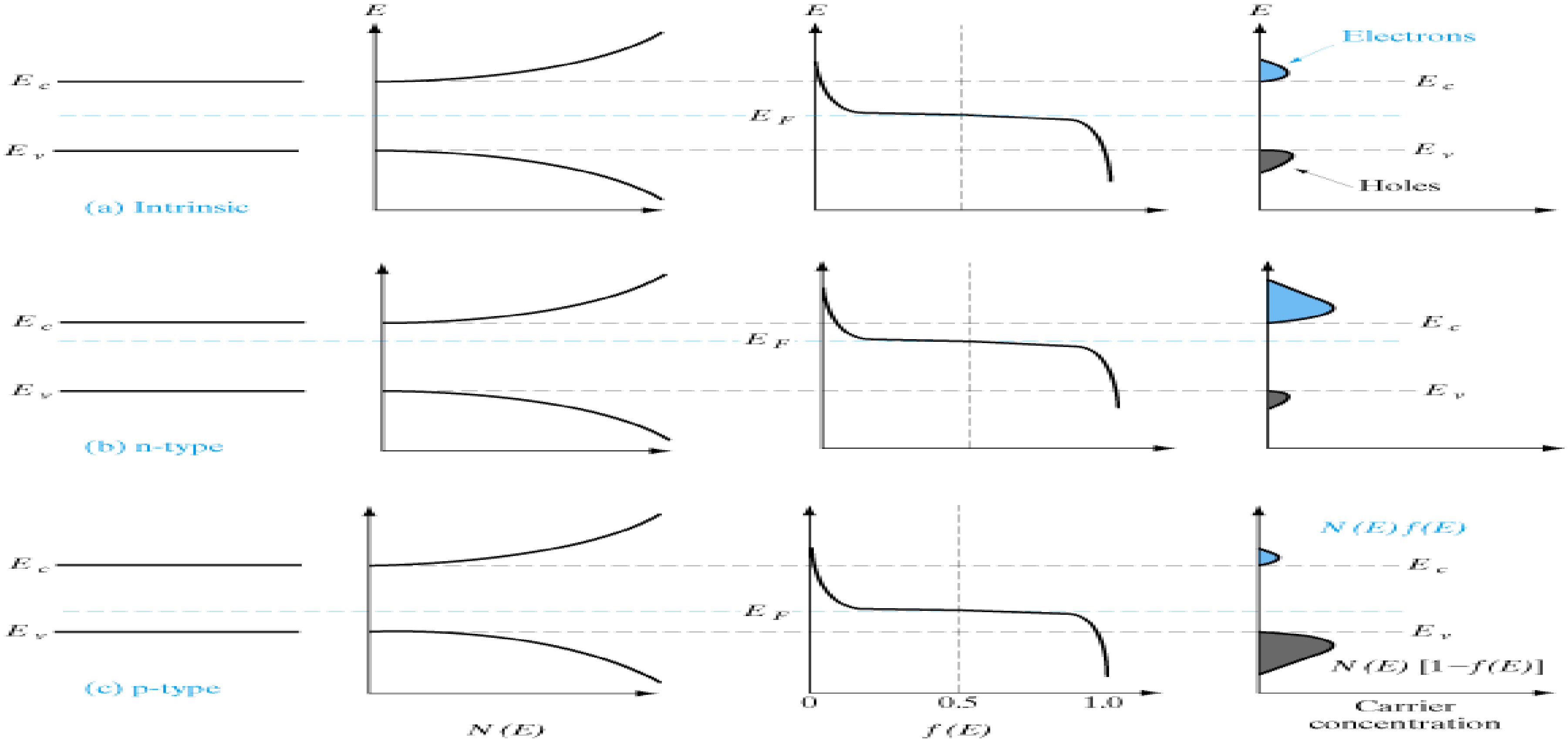
Table 3.1 | Effective density of states function and effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

Schematic Band Diagram



Schematic band diagram, density of states, Fermi–Dirac distribution, and the carrier concentrations for (a) intrinsic, (b) n-type, and (c) p-type semiconductors at thermal equilibrium.

- Recall these two equations :

$$n_0 = N_C f(E_C) = N_C e^{-(E_C - E_F)/kT}$$

$$p_0 = N_V [1 - f(E_V)] = N_V e^{-(E_F - E_V)/kT}$$

Intrinsic electron and hole concentrations where almost $\boldsymbol{E_F = E_i}$:

$$p_i = N_V e^{-(E_i - E_V)/kT} \qquad n_i = N_C e^{-(E_C - E_i)/kT}$$

$$n_0 p_0 = (N_C e^{-(E_C - E_F)/kT})(N_V e^{-(E_F - E_V)/kT}) =$$

$$N_C N_V e^{-(E_C - E_V)/kT} = N_C N_V e^{-E_g/kT}$$

$$n_i p_i = (N_C e^{-(E_C - E_i)/kT})(N_V e^{-(E_i - E_V)/kT}) =$$

$$N_C N_V e^{-(E_C - E_V)/kT} = N_C N_V e^{-E_g/kT}$$

Thus, intrinsic electron and hole concentrations are equal since the carriers are created in pairs : $n_i = p_i$:

Thus, intrinsic concentration :

Also,

Note: n_i of Si at RT = 1.5 x 10^{10} cm^{-3} & $E_C - E_i = E_g/2$ if $N_C = N_V$

$$n_0 p_0 = n_i^2$$

$$n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}$$

Two convenient expressions :

$$n_0 = N_C e^{-(E_C - E_F)/kT} = N_C e^{-(E_C - E_i)/kT} e^{(E_F - E_i)/kT} = n_i e^{(E_F - E_i)/kT}$$

Thus,

$$p_0 = N_V e^{-(E_F - E_V)/kT} = N_V e^{-(E_i - E_V)/kT} e^{(E_i - E_F)/kT} = n_i e^{(E_i - E_F)/kT}$$

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

Find the equilibrium electron and hole concentrations and the location of the Fermi level (with respect to the intrinsic Fermi level E_i) in silicon at 300K if the silicon contains 8×10^{16} cm^{-3} Arsenic (As) and 2×10^{16} cm^{-3} boron (B) atoms.

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

$$\mathrm{n} = 6 \times 10^{16} \ \mathrm{cm}^{-3}$$

$$p = \frac{n_i^2}{n} = 3.5 \times 10^{3} \mathrm{cm}^{-3}$$

$$E_f - E_i = kT \ln(n / n_i) = 0.393 eV$$

$$E_c - E_f = kT \ln(N_c / n) = 0.0258 \ln(2.8 \times 10^{19} / 6 \times 10_{16})$$

$$= 0.159 eV$$

Temperature Dependence of Carrier Concentrations

$$n_i = \sqrt{N_C N_V}\, e^{-E_g/2kT}$$

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$$

Take $\boldsymbol{n_i}$ to calculate $\boldsymbol{n_0}$ and $\boldsymbol{p_0}$ in the following equations:

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

Temperature dependence of carrier concentrations

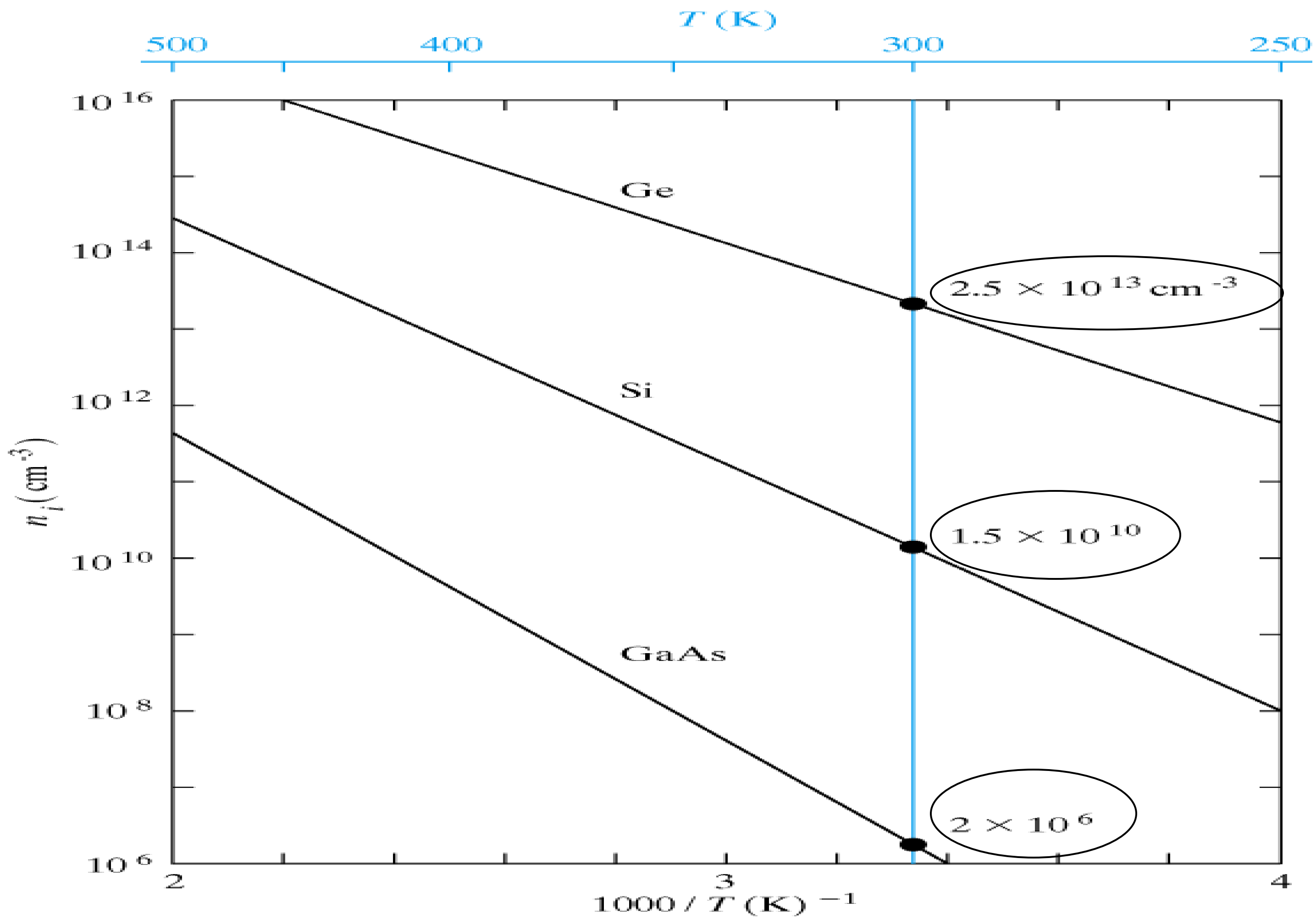


Figure 3—17
Intrinsic carrier concentration for Ge, Si, and GaAs as a function of inverse temperature. The room temperature values are marked for reference.

Carrier Concentration vs. Inverse Temperature

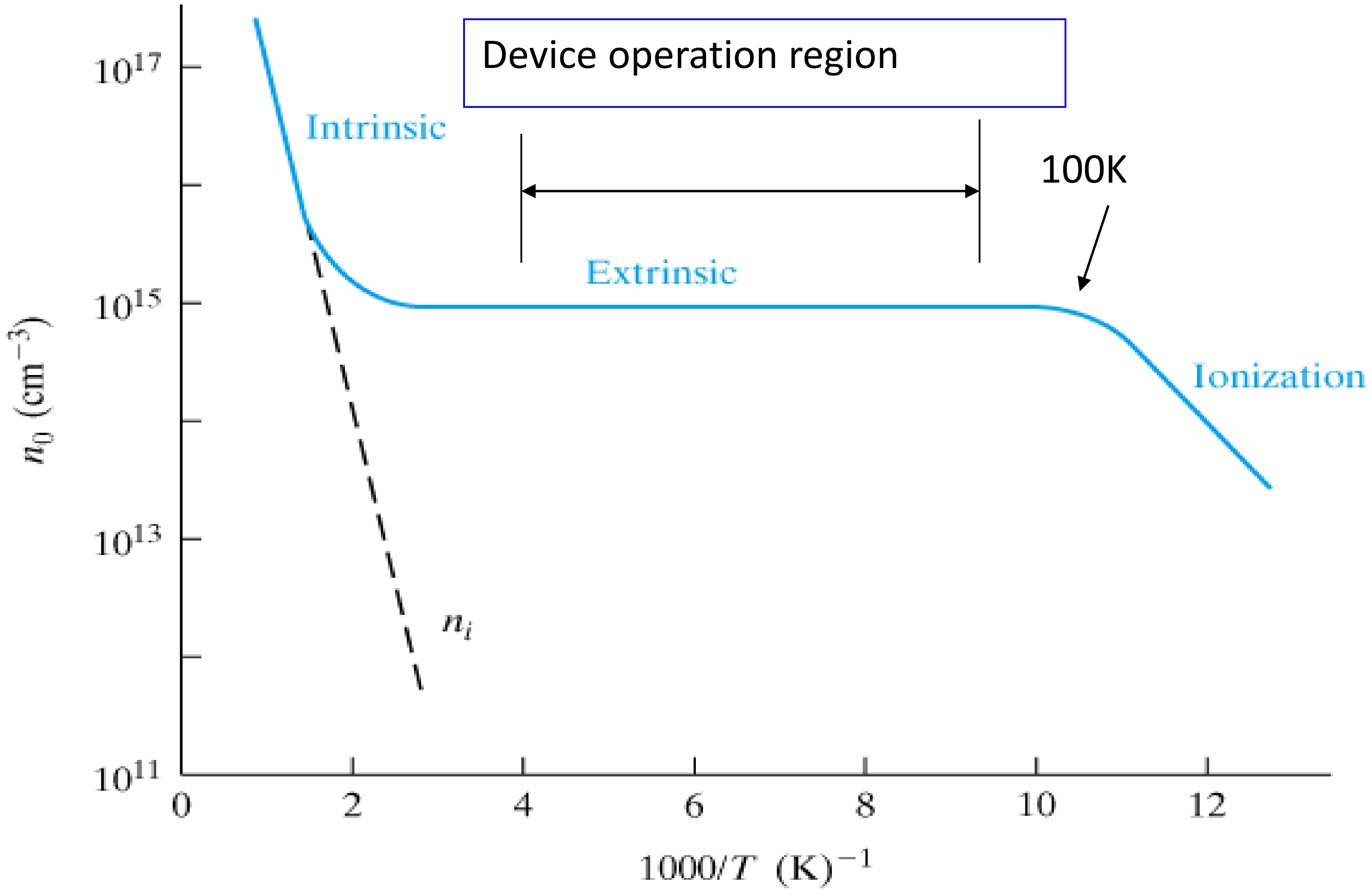
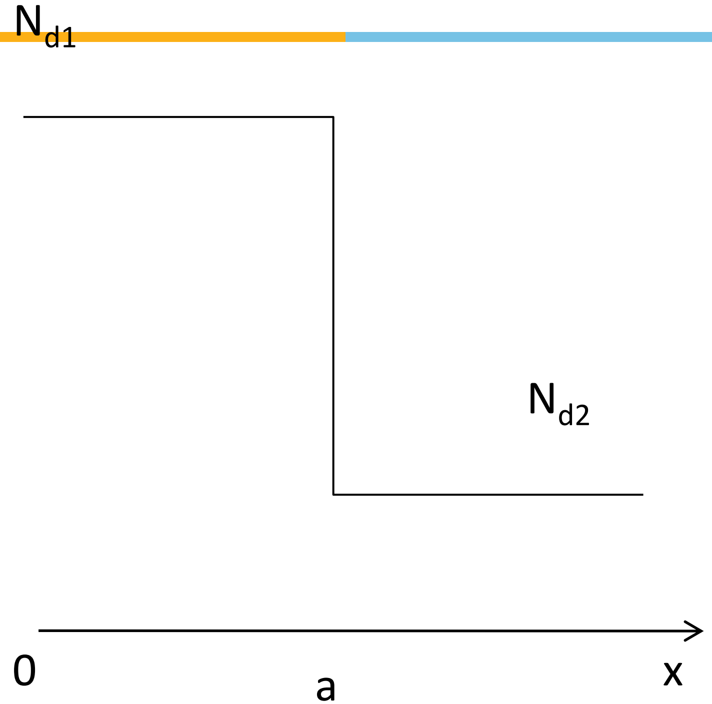


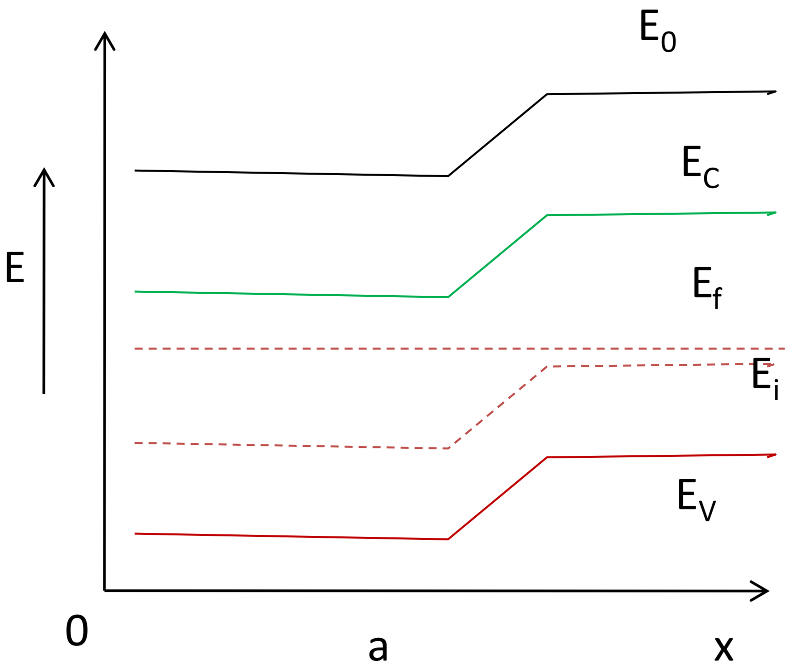
Figure 3—18
Carrier concentration vs. inverse temperature for Si doped with 10^{15} donors/cm^3.

A silicon crystal is known to contain 10-4 atomic percent of arsenic (As) as an impurity. It then receives a uniform doping 3 ×1016 cm-3 phosphorous (P) atoms and a subsequent uniform doping of 1018cm-3 boron (B) atoms. A thermal annealing treatment then completely activates all impurities.

(a) What is the conductivity type of this silicon sample?
(b) What is the density of the majority carriers?

Inhomogeneously Doped Semiconductor





innovate achieve lead

Electronic Devices

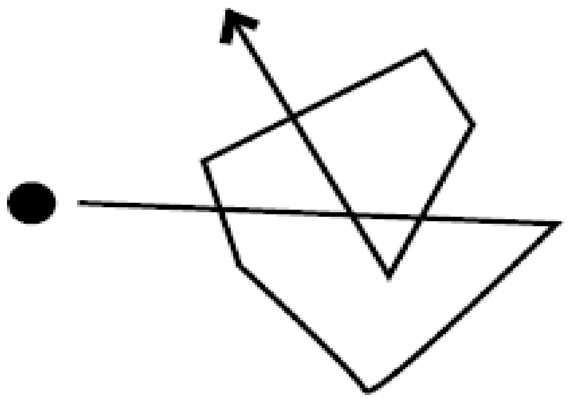
Lecture 9

28-08-2018

Thermal Motion

In thermal equilibrium, carriers are not sitting still:

- undergo collisions with vibrating Si atoms (*Brownian motion*)
- electrostatically interact with charged dopants and with each other



Characteristic time constant of thermal motion - mean free time between collisions:

$$\tau_c \equiv \textit{collision time}\,[s]$$

In between collisions, carriers acquire high velocity:

$$v_{th} \equiv \textit{thermal velocity}\,[cm/s]$$

Characteristic length of thermal motion:

$$\lambda \equiv \ mean\ free\ path\ [cm]$$

$$\lambda = v_{th}\tau_c$$

Put numbers for Si at 300 K:

$$\tau_c \simeq 10^{-14} \sim 10^{-13}\ s$$

$$v_{th} \simeq 10^7\ cm/s$$

$$\Rightarrow \lambda\ \simeq 1 \sim 10\ nm$$

For reference, state-of-the-art MOSFETs today:

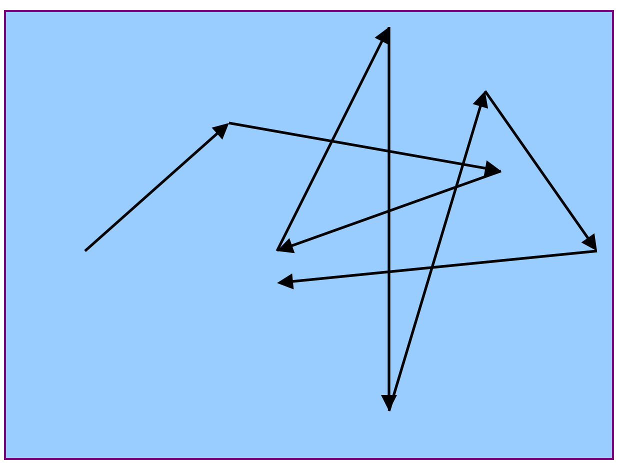
$$L_g \simeq 50\ nm$$

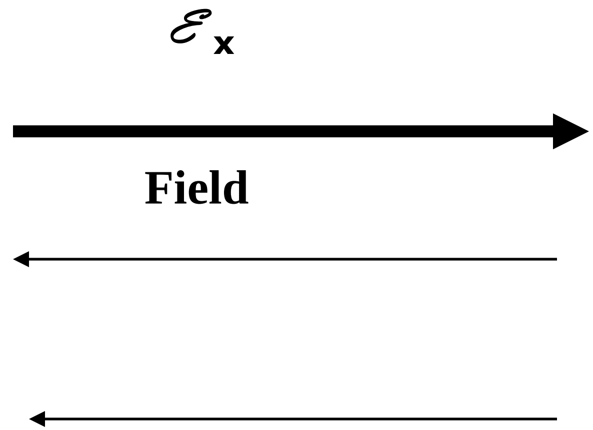
$\Rightarrow$ carriers undergo many collisions in modern devices

CONDUCTIVITY AND MOBILITY

To calculate current flow in presence of electric and magnetic field we have to take into account the collisions of the charge carriers with the lattice and impurities.

Ease with which electrons and holes can flow through the crystal determines their mobility within the solid.





$$v_x = -\mu_n \mathcal{E}_x$$

CONDUCTIVITY AND MOBILITY

Net force experienced by each electron –q$\mathcal{E}_x$

$$-nq\mathrm{E}_x = \left.\frac{dp_x}{dt}\right|_{field}$$

Let N(t) is the number of electrons that have not undergone collision by time t.

The rate of decrease in N(t) at any time t is

$$-\frac{dN(t)}{dt} = \frac{1}{\bar{t}}N(t)$$

$$N(t) = N_o e^{-t/\bar{t}}$$

CONDUCTIVITY AND MOBILITY

The probability that any electron has a collision in time dt is - dt/ $\bar{t}$

The differential change is p_x due to collisions is

$$dp_x = -p_x \frac{dt}{\bar{t}}$$

$$\left.\frac{dp_x}{dt}\right|_{collision} = -\frac{p_x}{\bar{t}}$$

$$-\frac{p_x}{\bar{t}} - nq\mathscr{E}_x = 0$$

-The average acceleration and deceration must be zero for steady state

$$\langle p_x \rangle = \frac{p_x}{n} = -q\bar{t}\mathscr{E}_x$$

-Average momentum

$$\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = -\frac{q\bar{t}}{m_n^*}\mathscr{E}_x$$

-Average velocity

m_n* - conductivity effective mass for electrons

CONDUCTIVITY AND MOBILITY

for electrons: $v_{dn} = -\mu_n E$ for holes: $v_{dp} = \mu_p E$

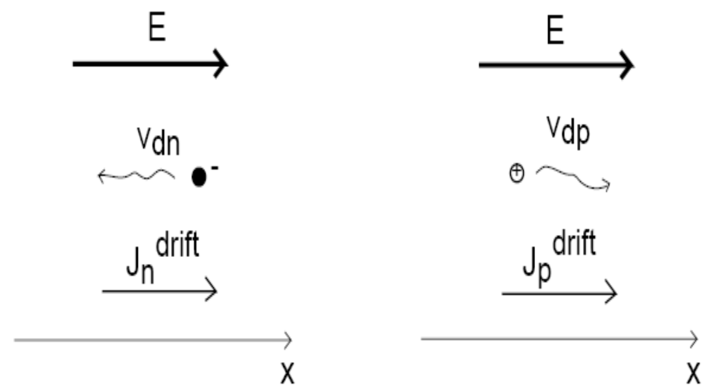
Drift current

Net velocity of charged particles $\Rightarrow$ electric current:

$$
\begin{aligned}
\textit{Drift current density} &\propto \textit{carrier drift velocity} \\
&\propto \textit{carrier concentration} \\
&\propto \textit{carrier charge}
\end{aligned}
$$

$$J_n^{drift} = -qnv_{dn} = qn\mu_n E$$

$$J_p^{drift} = qpv_{dp} = qp\mu_p E$$



CONDUCTIVITY AND MOBILITY

Total drift current:

$$J^{drift} = J_n^{drift} + J_p^{drift} = q(n\mu_n + p\mu_p)E$$

Has the shape of *Ohm's Law*: $J = \sigma E = \dfrac{E}{\rho}$

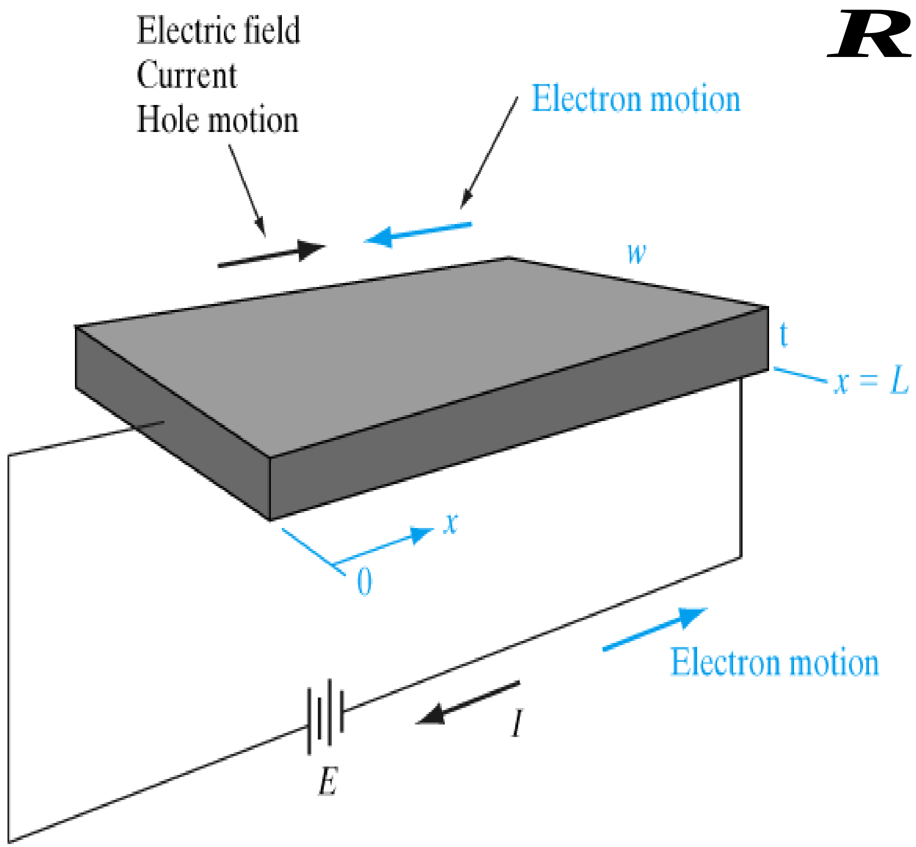
$$\sigma \equiv conductivity\,[\Omega^{-1} \cdot cm^{-1}]$$

$$\rho \equiv resistiviy\,[\Omega \cdot cm]$$

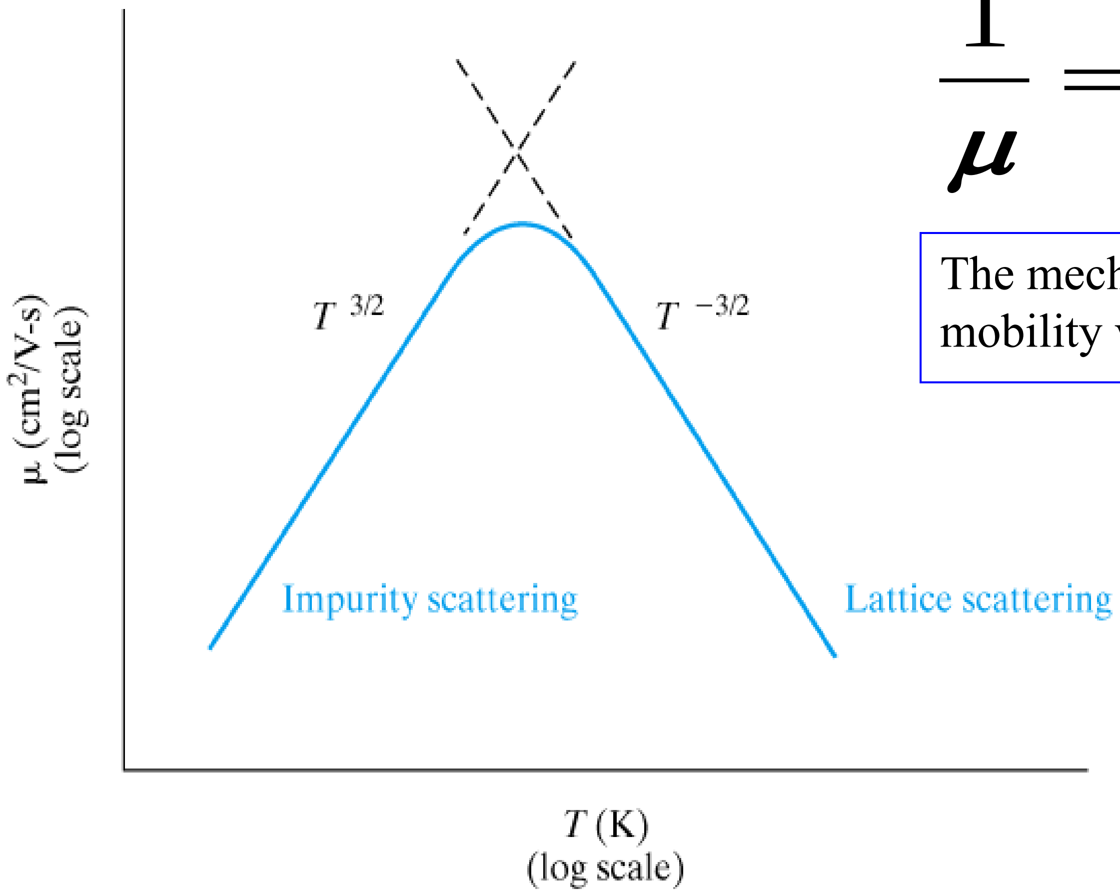
$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$

Drift of electrons and holes in a semiconductor bar

$$R = \frac{\rho L}{wt} = \frac{L}{wt}\frac{1}{\sigma}$$



Impurity & Lattice Scatterings



$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_\ell}$$

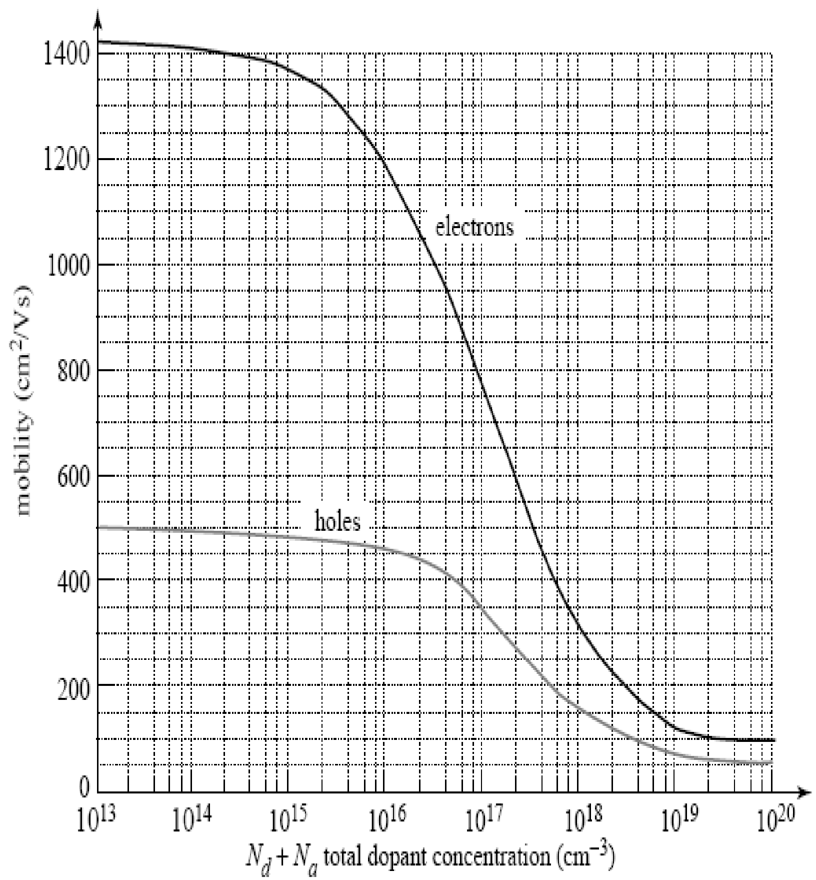
The mechanism causing the lower mobility value dominates !

Figure 3—21
Approximate temperature dependence of mobility with both lattice and impurity scattering.

Effect of temperature and doping on mobility

$\mu \quad \alpha \quad T^{-3/2}$ lattice scattering

$\mu \quad \alpha \; T^{3/2}$ impurity scattering



Mobility depends on doping. For Si at 300K:

Mobility vs. doped impurity concentration

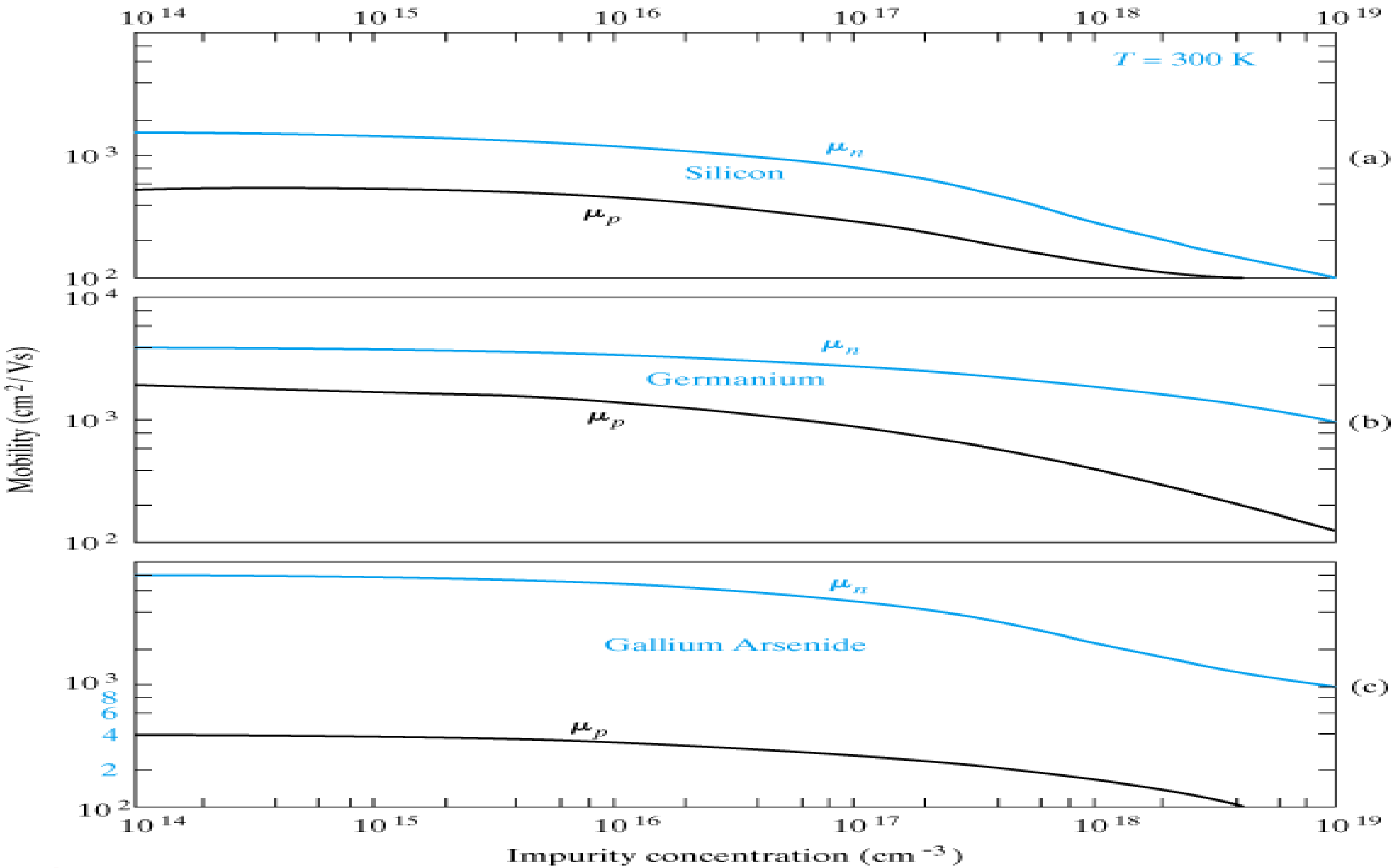
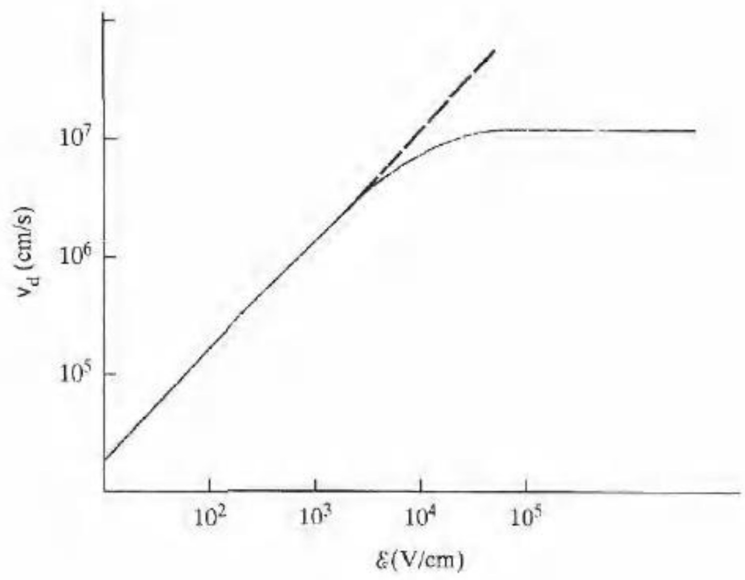


Figure 3—23

Variation of mobility with total doping impurity concentration ($N_a + N_d$) for Ge, Si, GaAs at 300 K.

Saturation of electron drift velocity at high electric field

Velocity reaches a saturation value of the mean thermal velocity.



Field required to saturate velocity:

$$\mathcal{E}_{sat} = \frac{v_{sat}}{\mu}$$

Since μ depends on doping, $\mathcal{E}_{sat}$ depends on doping too.

Electrical conductivity

- $\sigma_i = en_i(\mu_n + \mu_p)$ is the intrinsic conductivity of a semiconductor material
- For extrinsic semiconductors assuming complete ionization, $N_d or\ N_a \gg n_i$

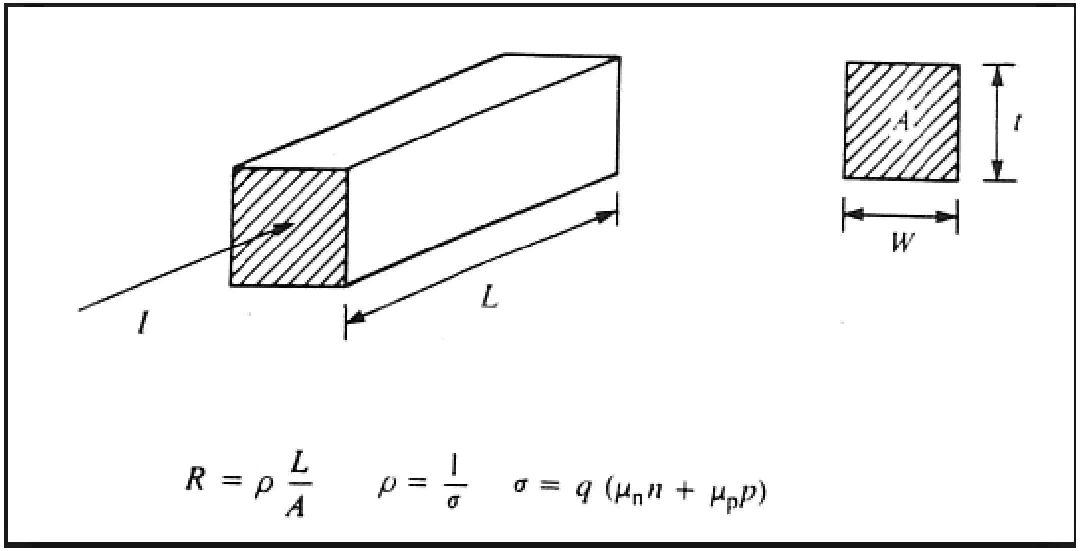
  Hence the conductivity reduces to

  $\sigma_n \approx eN_d\mu_n\ and\ \sigma_p \approx e\ N_a\mu_p$

  thus the conductivity is purely dependent on the majority carrier concentration

Electrical resistance

- $J = \sigma E = \sigma \frac{V}{L}$ where L is the length of the SC material
- $I = JA = \frac{\sigma VA}{L}$ and the elctrical resistance is given by
- $R = \frac{V}{I} = \frac{1}{\sigma}\frac{L}{A} = \frac{\rho L}{A}$



Electrical resistivity

- $\rho = \frac{1}{\sigma} = \frac{1}{e(n\mu_n + p\mu_p)}$ is the electrical resistivity
- Resistance, conductance, resistivity and conductivity, depend only on the majority carrier concentration and not on the minority carrier concentration