BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI - KK BIRLA
G O A C A M P U S
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MATHEMATICS - III

Tutorial–3

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1. Show that $y_1 = e^{-x}$ and $y_2 = e^{2x}$ are solutions of the differential equation $y'' - y' - 2y = 0$. What is the general solution?

2. Show that $y = c_1 x^{-1} + c_2 x^5$ is a general solution of

$$x^2 y'' - 3xy' - 5y = 0$$

on any interval not containing 0.

3. Show that $y = x^2 \sin x$ and $y = 0$ are both solutions of

$$x^2 y'' - 4xy' + (x^2 + 6)y = 0,$$

and that both satisfy the conditions $y(0) = 0$ and $y'(0) = 0$. Does this contradict Theorem A (page 82)? If not, why not?

4. Show that $y = c_1 x + c_2 x^2$ is the general solution of $x^2 y'' - 2xy' + 2y = 0$ on any interval not containing 0 and find the particular solution for which $y(1) = 3$ and $y'(1) = 5$.

5. Are the following pairs of functions linearly independent on the given interval? Justify your answer.
(a) $x^3, x^2|x|; -1 < x < 1$, (b) $x|x|, x^2; 0 \le x \le 1$, (c) $\ln x, \ln x^2; x > 0$.

6. Use the Wronskian to prove that the two solutions of the homogeneous

$$y'' + P(x)y' + Q(x)y = 0$$

on an interval $[a, b]$ are linearly dependent if

   (a) they have a common zero in $[a, b]$.

   (b) they have maxima or minima at the same point in $[a, b]$.

   (c) one solution is tangent to the $x-$axis at a point x_0, $x_0 \in [a, b]$.

7. Without using Wronskian solve Question 6.

8. Find the general solution of the following differential equations, when one of the solution $y_1(x)$ is known

$(i)\ x^2y'' + xy' - 4y = 0,$	$y_1(x) = x^2$
$(ii)\ x^2y'' + xy' + (x^2 - 1/4)y = 0,$	$y_1(x) = x^{-1/2}\sin x$
$(iii)\ x^2y'' - x(x+2)y' + (x+2)y = 0,$	$y_1(x) = x$
$(iv)\ xy'' - (2x+1)y' + (x+1)y = 0,$	$y_1(x) = e^x$
$(v)\ (1 - x^2)y'' - 2xy' + 2y = 0,$	$y_1(x) = x.$

9. If n is a positive integer, find two linearly independent solutions of

$$xy'' - (x+n)y' + ny = 0.$$

10. Find the general solution of the following differential equations

$(i)\ y'' + y' - 6y = 0,$	$(ii)\ y'' - 9y' + 20y = 0,$
$(iii)\ y'' + y' = 0,$	$(iv)\ y'' + 8y' - 9y = 0,$
$(v)\ y^{iv} - y = 0,$	$(vi)\ y''' - 3y'' + 4y' - 2y = 0.$

11. Find the general solution of the following equations by reducing them to constant coefficient equation

$(i)\ x^2y'' + pxy' + qy = 0,$ p and q are constants, $x > 0.$

$(ii)\ x^2y'' + 2xy' - 12y = 0,\ x > 0.$

$(iii)\ x^2y'' - 3xy' + 4y = 0,\ x > 0.$

12. Show that the general solution of $y'' + py' + qy = 0$, where p and q are constants, approaches 0 as $x \to \infty$ if and only if p and q are both positive.

13. Consider the general homogeneous equation

$$y'' + P(x)y' + Q(x)y = 0, \tag{1}$$

and change the independent variable from x to $z = z(x)$, where $z(x)$ is an unspecified function of x. Show that equation (1) can be transformed in this way into an equation with constant coefficients if and only if $\dfrac{Q' + 2PQ}{Q^{3/2}}$ is constant.

14. Using result of Question 13, solve the following differential equation

$$xy'' + (x^2 - 1)y' + x^3y = 0.$$

15. Using the Method of Undetermined Coefficients find particular solution of the following differential equations

$$(i)\ y'' - 2y' + 2y = e^x \sin x, \qquad (ii)\ y'' - 3y' + 2y = 2xe^x + 3\sin x,$$
$$(iii)\ y'' + y = \sin^3 x, \qquad (iv)\ y^{(4)} - 2y''' + 2y'' - 2y' + y = \sin x$$

16. If $y_1(x)$ and $y_2(x)$ are solutions of $y'' + P(x)y' + Q(x)y = R_1(x)$ and $y'' + P(x)y' + Q(x)y = R_2(x)$ respectively. Then show that $y(x) = y_1(x) + y_2(x)$ is a solution of

$$y'' + P(x)y' + Q(x)y = R_1(x) + R_2(x).$$

Use this to find the general solution of

$$y'' + 4y = 4\cos 2x + 6\cos x + 8x^2 - 4.$$