BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI - KK BIRLA
G O A   C A M P U S
FIRST SEMESTER 2018-2019

MATHEMATICS - III

Tutorial Sheet–4

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1. Using the Method of Variation of Parameters find the particular solution of the following differential equations

$$(i)\ y'' + y = \tan x, \qquad (ii)\ y'' - 3y' + 2y = (1 + e^{-x})^{-1},$$
$$(iii)\ y'' + y = x \cos x, \qquad (iv)\ y'' - 6y' + 9y = e^{3x}/x^2.$$

2. Show that the method of variation of parameters applied to the equation $y'' + y = f(x)$ leads to the particular solution

$$y_p(x) = \int_0^x f(t)\ \sin(x - t)dt.$$

3. Find the general solution of the following equations

$$(i)\ (x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2, \qquad (ii)\ x^2y'' - 2xy' + 2y = xe^{-x}$$

4. Find the general solution of the following equations

$$(i)\ y''' - 3y'' + 4y' - 2y = 0. \qquad (ii)\ y^{(4)} + 4y''' + 6y'' + 4y' + y = 0.$$

5. Using Operator method to find particular solution of the following differential equations

$$(i)\ y'' - y = x^2e^{2x}, \qquad (ii)\ y'' - 2y' - 3y = 6e^{5x},$$
$$(iii)\ y'' - 4y = e^{2x}, \qquad (iv)\ y'' + 2y' + y = 2x^2e^{-2x} + 3e^{2x},$$
$$(v)\ y^{(4)} - y = 1 - x^3, \qquad (vi)\ y''' - y'' + y' = 1 + x.$$

6. Using exponential shift rule to find the general solution of the following differential equations

$$(i)\ (D - 2)^3y = e^{2x}, \qquad (ii)\ (D + 1)^3y = 12e^{-x},$$

7. Use the exponential shift rule to show that $(D - r)^k y = 0$ has

$$y = (c_1 + c_2x + c_3x^2 + \cdots + c_kx^{k-1})e^{rx}$$

as its general solution.

8. Use the exponential shift rule to find the general solution of the following equations

$$(i)\ (D - 2)^3y = e^{-2x}, \qquad (ii)\ (D - 2)^2y = e^{2x} \sin x.$$