BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI - KK BIRLA
G O A   C A M P U S
FIRST SEMESTER 2018-2019

MATHEMATICS - III

Tutorial Sheet–2

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1. If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{Ny - Mx}$ is a function $g(z)$ of the product $z = xy$, then show that $\mu = e^{\int g(z)dz}$ is an integrating factor for the equation $M(x, y)dx + N(x, y)dy$.

2. Verify that the equation $Mdx + Ndy = 0$ can be expressed in the form

$$\frac{1}{2}(Mx + Ny)d(\ln xy) + \frac{1}{2}(Mx - Ny)d\left(\ln\left(\frac{x}{y}\right)\right) = 0$$

   Hence, show that if $Mx + Ny = 0$, then $\dfrac{1}{Mx - Ny}$ is an integrating factor of $Mdx + Ndy = 0$.

3. If $Mx - Ny \neq 0$ and the equation $Mdx + Ndy = 0$ if of the form $f(xy)ydx + g(xy)xdy = 0$, then $\frac{1}{Mx-Ny}$ is an integrating factor.

4. Using Question 3, solve $x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy$.

5. Solve each of the following equations by finding an integrating factor

   (a) $(xy - 1)dx + (x^2 - xy)dy = 0$, (b) $ydx + (x - 2x^2y^3)dy = 0$,
   (c) $(x^3 + xy^3)dx + 3y^2dy = 0$, (d) $xdy + ydx + 3x^3y^4dy = 0$.

6. Under what circumstances will equation $M(x, y)dx + N(x, y)dy$ have an integrating factor that is a function of the sum $z = x + y$?

7. Solve the following linear equations

   (i) $y' + y = \dfrac{1}{1 + e^x}$, (ii) $\dfrac{dx}{dy} + 2yx = e^{-y^2}$,
   (iii) $y' + y = 2xe^{-x} + x^2$, (iv) $L\dfrac{di}{dt} + Ri = E\sin kt$ (Simple Electric Circuit).
   (v) $f(y)^2\frac{dx}{dy} + 3f(y)f'(y)x = f'(y)$.

8. Reduce the following equations to linear differential equations and hence find the solution.

   (i) $xdy + ydx = xy^2dx$, (ii) $y' + xy = \dfrac{x}{y^3}, y \neq 0$, (iii) $(e^y - 2xy)y' = y^2$.

9. Solve the following equations (using reduction of order)

   (i) $yy'' + (y')^2 = 0$, (ii) $xy'' + y' = 4x$,
   (iii) $y'' = 1 + (y')^2$, (iv) $y'' + (y')^2 = 1$.