MATHEMATICS-III

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Course information

- ❖ What is for?
  - ❖ This course provides an elementary introduction to classical methods for solving differential equations which arises in various branch of science and engineering.

Topics in the Course

- First order and second order ODE.
- Special functions: Legendre and Bessel.
- Higher order ODE and system of Diff. equations.
- Laplace transform and its application to ODE.
- Fourier Series and its application.
- Classical methods for solving PDE.

Course Goals

- Students at the end of course should be able to do the following:
  - Solve first and 2^{nd} order linear differential equations and use these techniques to solve applied problems.
  - Solve higher order diff. equations, system of equations and its use in applied problems.
  - Find power series solution and use in physical problems.
  - Find Fourier series of function and can use in power series.

Books

- **Textbooks** (required):

  G. F. Simmons: Differential Equations with Applications and Historical Notes, 2^{nd} Edition, Tata MacGraw Hill.

- **References:**

- Erwin Kreyszig, *Advanced Engineering Mathematics*, John Wiley & sons, 8th Ed., 2005.
- M.D. Raisinghania, Ordinary & Partial Differential Equation, S Chand Publication, 2005
- E. A. Coddington, *An Introduction to Ordinary Differential Equations* Prentice Hall, 1961.
- For more details refer the handout

Grading

- Grades for the course will be based on the Handout
- **Note: This time other than tests, 80 marks will be open book Quizzes.**

Checking web page

- I am highly recommend that each student check this web page at least once in a day for new announcements.

  http://photon.bits-goa.ac.in/lms/

Outline

- What is Differential Equation ?
- Why we need ?
- How to solve ?

Differential equations

Definition:

An equation involving one dependent variable and its derivative with respect to one or more independent variables is called differential equations.

Examples:

$$\frac{dy}{dx} = 2x + 3$$

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + ay = 0$$

y is dependent variable and ***x*** is independent variable, and these are ordinary differential equations

Partial Differential Equation

Examples:

1. $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

u is dependent variable and ***x and y*** are independent variables, and is partial differential equation.

2. $$\frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial t^4} = 0$$

3. $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} - \frac{\partial u}{\partial t}$$

u is dependent variable and ***x and t*** are independent variables

In Applications

- Differential equations arise when we can relate the rate of change of some quantity back to the quantity itself.

$dy/dx \longleftrightarrow y$

Example (#1)

For falling objects(freely falls):

According to Newton's 2nd law of motion,

$$F = ma = m\frac{dv}{dt} = m\frac{d^2y}{dt^2}$$

Since the only for acting on it is *mg*, *g* is the acceleration due to gravity

$$\Rightarrow \frac{d^2y}{dt^2} = g.$$

Example (#2)

-- with air resistance, the total force acting on the body is *mg-kv*. For such an object we have the differential equation:

$$m\frac{d^2y}{dt^2} = mg - k\frac{dy}{dt}.$$

Example (#3)

In a different field:

Radioactive substances decompose at a rate proportional to the amount present.

Suppose y(t) is the amount present at time t.

rate of change of amount **is**

proportional to **the amount** **(and decreasing)**

$$\frac{dy}{dt} = -k\,y$$

Other problems that yield the same equation:

In the presence of abundant resources (food and space), the organisms in a population will reproduce as fast as they can --- this means that

the rate of increase of the population will be proportional to the population itself:

$$\frac{dP}{dt} = k\,P$$

..and another

The balance in an interest-paying bank account increases at a rate (called the interest rate) that is proportional to the current balance. So

$$\boxed{\frac{dB}{dt} = kB}$$

and for the Interest Problem...

For annuities: **Some accounts pay interest but at the same time the owner intends to withdraw money at a constant rate (think of a retired person who has saved and is now living on the savings).**

Order of Differential Equation

The **order** of the differential equation **is order of the highest derivative** in the differential equation.

Differential Equation	**ORDER**
$\frac{dy}{dx} = 2x + 3$	**1**/2/3
$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^4 + 6y = 3$	1/2/**3**/4
$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 9y = 0$	1/**2**/3

Degree of Differential Equation

The **degree** of a differential equation is **degree/integral power of the highest order derivative term** in the differential equation.

Differential Equation	**Degree**
$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + ay = 0$	**1**/2/3
$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^4 + 6y = 3$	**1**/2/3/4
$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^5 + 3 = 0$	1/2/**3**/4

Linear Differential Equation

A differential equation is **linear,** if

1. dependent variable and its derivatives are of degree one,
2. coefficients of a term does not depend upon dependent variable.

Example: 1. $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 9y = 0.$$

is linear.

Example: 2. $$\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^4 + 6y = 3$$

is non - linear because in 2nd term is not of degree one.

Example: 3. $$x^2 \frac{d^2 y}{dx^2} + y \frac{dy}{dx} = x^3$$

is non - linear because in 2nd term coefficient depends on *y*.

Example: 4. $$\frac{dy}{dx} = \sin y$$

is non - linear because

$$\sin y = y - \frac{y^3}{3!} + -$$ is non – linear

1st – order differential equation

1. Differential form:

$$M(x, y)dx + N(x, y)dy = 0$$

3. General form:

$$\frac{dy}{dx} = f(x, y) \text{ or } y' = f(x, y)$$

1^{st} order differential equation

Q1. How to justify existence of the Solution ?

Q2. Is the solution Unique ?

Q3. How to find the solution ?

Existence of Solution

$$\boxed{\text{Peano Theorem: } \left(\text{For } \frac{dy}{dx} = f(x, y) \right)}$$

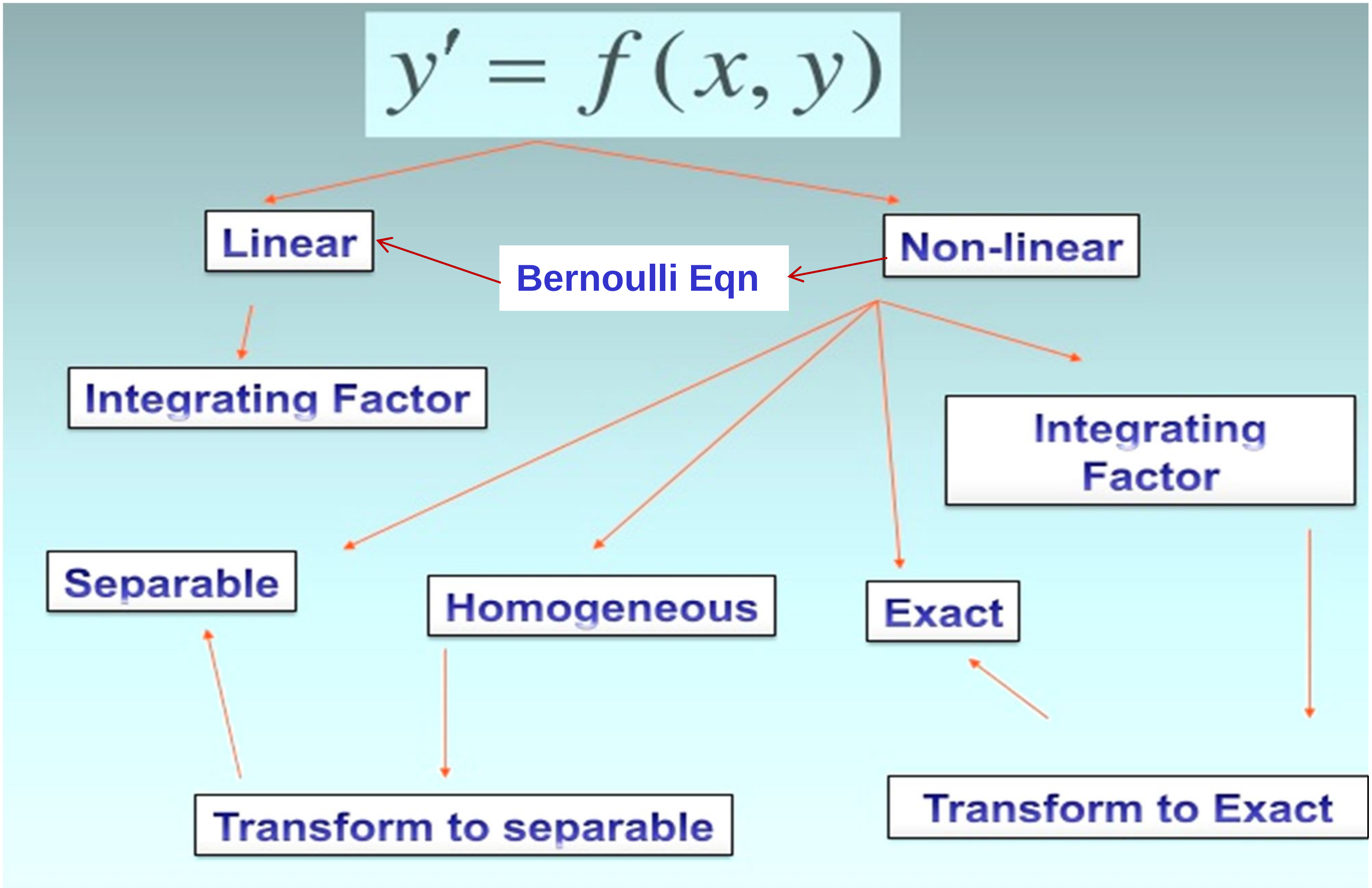
If f is continuous then the differential equation has a solution.

Uniqueness of the Solution

Picard's Theorem: $\left(\dfrac{dy}{dx} = f(x, y),\ y(x_0) = y_0 \right)$

If f and $\dfrac{\partial f}{\partial y}$ are continuous on a closed rectangle R then the differential equation has a unique solution in the domain R.

Methods to Solve 1st order ODE



THANK YOU