DIGITAL DESIGN

CS/ECE/EEE/INSTR F215

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GATE LEVEL MINIMIZATION

Karnaugh Map or K-Map method

K-Map

A \ B	0	1
0	A'B'	A'B
1	AB'	AB

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

A \ B	0	1
0	0	1
1	2	3

A \ B	0	1
0	00	01
1	10	11

Examples of simplification

A \ B	0	1
0	0	1
1	0	0

F = A'B

A \ B	0	1
0		1
1		1

F = A'B + AB =B

A \ B	0	1
0		
1	1	1

F = AB' + AB =A

A \ B	0	1
0	1	
1	1	

F = A'B' + AB' =B'

- F = A'B'+AB'+AB

A \ B	0	1
0	1	
1	1	1

A

B'

- F = A+B'

Three variable K-map

A \ BC	00	01	11	10
0	A'B'C'	A'B'C	A'BC	A'BC'
1	AB'C'	AB'C	ABC	ABC'

A \ BC	00	01	11	10
0	000	001	011	010
1	100	101	111	110

Three variable K-map

A \ BC	00	01	11	10
0			1	1
1	1	1		

A \ BC	00	01	11	10
0	1	1		
1	1	1		

Three variable K-map

A \ BC	00	01	11	10
0	1	1	1	1
1				

A \ BC	00	01	11	10
0	1	1	1	
1	1	1		

Three variable K-map

A \ BC	00	01	11	10
0			1	
1	1		1	1

A \ BC	00	01	11	10
0	1		1	1
1	1			1

Three variable K-map

A \ BC	00	01	11	10
0		1		1
1	1		1	

A \ BC	00	01	11	10
0	1	1	1	1
1	1	1	1	1

Four Variable K-map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Four Variable K-map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

0000	1100
0001	1101
0011	1111
0010	1110
0110	1010
0111	1011
0101	1001
0100	1000

K-map simplification

- Pair – Group of 2 adjacent minterms - eliminates 1 variable
- Quad - Group of 4 adjacent minterms - eliminates 2 variables
- Octet - Group of 8 adjacent minterms - eliminates 3 variables
- Redundant group – Is the one in which all the elements of the group are covered by some other group.

Simplify – F(w,x,y,z) =∑(0,1,2,4,5,6,8,9,12,13,14)

F=y'+w'z' +xz'

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

Simplify –

F=B'D'+B'C'+A'CD'

AB \ CD	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

For simplification

- Prime implicants – A product term obtained by combining the maximum possible number of adjacent squares in the map.
- Essential prime implicant – If a minterm in the square is covered by only one prime implicant, that prime implicant is termed as "Essential".

Simplify – F(A,B,C,D) = ∑(0,2,3,5,7,8,9,10,11,13,15)

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

Simplify – F(A,B,C,D) = ∑(0,2,3,5,7,8,9,10,11,13,15)

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

B'D'

EPI

Simplify –
F(A,B,C,D) = ∑(0,2,3,5,7,8,9,10,11,13,15)

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

BD

EPI

Simplify –
F(A,B,C,D) = ∑(0,2,3,5,7,8,9,10,11,13,15)

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

AB’
or
AD

PI

Simplify –
F(A,B,C,D) = ∑(0,2,3,5,7,8,9,10,11,13,15)

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

B’C
or
CD

PI

Simplify –
F(A,B,C,D) = ∑(0,2,3,5,7,8,9,10,11,13,15)

- F(A,B,C,D) = BD+B’D’+CD+AD
- F(A,B,C,D) = BD+B’D’+CD+AB’
- F(A,B,C,D) = BD+B’D’+B’C+AD
- F(A,B,C,D) = BD+B’D’+B’C+AB’

F(A,B,C) =
A’B’C’+A’B’C+A’BC+AB’C’+ABC

A’B’ A’C

A \ BC	00	01	11	10
0	1	1	1	
1	1		1	

B’C’ BC

F=B’C’+BC+A’B’

Using k-map for obtaining POS form

- Squares in k-map not marked with 1’s gives the complement function.
- Mark empty squares with 0’s →combine them to obtain expression of complement of Function.
- Complement of F’ is F. (Using De-Morgan’s theorem)

Ex. F(A,B,C,D) =∑(0,1,2,5,8,9,10)

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

CD

BD'

AB

Map-Entered Variable (MEV) Or Variable Entered Mapping (VEM)

- Allows smaller map to handle greater number of variables.
- In K-map, for n variables, it requires 2^n=m squares.
- In MEV, map dimensions can be compressed.
- In K-map, each square represents a minterm, maxterm or a don't care term.
- In MEV, K-map cell is permitted to contain single variable (x) or a complete switching expression (xy'+z).

P= f(a,b,c) = Σ(0,1,4,5,7)

A \ BC	00	01	11	10
0	1	1		
1	1	1	1	

F = B'+AC

Variable c -MEV

MEV	Std.	A	B	C	P	P
0	0	0	0	0	1	1
0	1	0	0	1	1	
1	2	0	1	0	0	0
1	3	0	1	1	0	
2	4	1	0	0	1	1
2	5	1	0	1	1	
3	6	1	1	0	0	C
3	7	1	1	1	1	

For a simplified function from MEV K-map

- Determine the EPI's consisting of only 1's along with any don't care terms that may exist. (cover all 1's)
- Consider the 1's as don't care terms once step 1 is completed, because all 1's have been covered.
- Group all identical MEV terms with 1's or don't care terms to maximize the MEV EPI size.
- Determine the MEV EPIs by reading k-map in normal fashion, then AND the MEV variable with remaining k-map variable.

A \ B	0	1
0	1	0
1	1	C

AC

B'

F = B'+AC

P= f(w,x,y,z) = Σ(2,4,5,10,11,14) +D(7,8,9,12,13,15)

WX \ YZ	00	01	11	10
00	0	0	0	1
01	1	1	X	0
11	X	X	X	1
10	X	X	1	1

xy'

w

x'yz'

MEV	Std	W	X	Y	Z	O/p
0	0	0	0	0	0	0
0	1	0	0	0	1	0
1	2	0	0	1	0	1
1	3	0	0	1	1	0
2	4	0	1	0	0	1
2	5	0	1	0	1	1
3	6	0	1	1	0	0
3	7	0	1	1	1	X
4	8	1	0	0	0	X
4	9	1	0	0	1	X
5	10	1	0	1	0	1
5	11	1	0	1	1	1
6	12	1	1	0	0	X
6	13	1	1	0	1	X
7	14	1	1	1	0	1
7	15	1	1	1	1	X

o/p
0
Z'
1
0
X
1
X
1

W \ XY	00	01	11	10
0	0	Z'	0	1
1	X	1	1	X

F = w+xy'+x'yz'

P= f(a,b,c,d) = Σ(2,9,10,11,13,14,15)

MEV	Std	W	X	Y	Z	O/p
0	0	0	0	0	0	0
0	1	0	0	0	1	0
1	2	0	0	1	0	1
1	3	0	0	1	1	0
2	4	0	1	0	0	0
2	5	0	1	0	1	0
3	6	0	1	1	0	0
3	7	0	1	1	1	0
4	8	1	0	0	0	0
4	9	1	0	0	1	1
5	10	1	0	1	0	1
5	11	1	0	1	1	1
6	12	1	1	0	0	0
6	13	1	1	0	1	1
7	14	1	1	1	0	1
7	15	1	1	1	1	1

o/p
0
Z'
0
0
Z
1
Z
1

W \ XY	00	01	11	10
0	0	Z'	0	0
1	Z	1	1	Z

W \ XY	00	01	11	10
0	0	Z'	0	0
1	Z	1	1	Z

WY

W \ XY	00	01	11	10
0	0	Z'	0	0
1	Z	1	1	Z

WY WZ

x'yz'

W \ XY	00	01	11	10
0	0	Z'	0	0
1	Z	1	1	Z

WY WZ

P= f(x,y,z) = Σ(0,4,5,7)

x \ yz	00	01	11	10
0	1			
1	1	1	1	

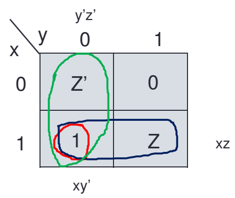
P= f(x,y,z) = Σ(0,4,5,7)

x \ yz	00	01	11	10
0	1			
1	1	1	1	

F = y'z'+xz

Variable Z -MEV

MEV	Std.	X	Y	Z	o/p	o/p
0	0	0	0	0	1	Z'
0	1	0	0	1	0	
1	2	0	1	0	0	0
1	3	0	1	1	0	
2	4	1	0	0	1	1
2	5	1	0	1	1	
3	6	1	1	0	0	Z
3	7	1	1	1	1	



F = y'z'+xz

Note – Always covering 1's by themselves or in combination with don't care terms may lead to non-optimal solutions.

Quine-McCluskey method (QM)

- Suitable for computer solution
- Uses Tabular method

QM Method

- D = f(a, b, c, d) = Σ (0,1,2,3,6,7,8,9,14,15)

D = f(a, b, c, d) = Σ (0,1,2,3,6,7,8,9,14,15)

Index	Decimal Number	Binary representation			
0	0	0	0	0	0
1	1	0	0	0	1
	2	0	0	1	0
	8	1	0	0	0
2	3	0	0	1	1
	6	0	1	1	0
	9	1	0	0	1
3	7	0	1	1	1
	14	1	1	1	0
4	15	1	1	1	1

0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
8	1	0	0	0
3	0	0	1	1
6	0	1	1	0
9	1	0	0	1
7	0	1	1	1
14	1	1	1	0
15	1	1	1	1

(0,1)	0	0	0	-
(0,2)	0	0	-	0
(0,8)	-	0	0	0
(1,3)	0	0	-	1
(1,9)	-	0	0	1
(2,3)	0	0	1	-
(2,6)	0	-	1	0
(8,9)	1	0	0	-
(3,7)	0	-	1	1
(6,7)	0	1	1	-
(6,14)	-	1	1	0
(7,15)	-	1	1	1
(14,15)	1	1	1	-

(0,1)	0	0	0	-
(0,2)	0	0	-	0
(0,8)	-	0	0	0
(1,3)	0	0	-	1
(1,9)	-	0	0	1
(2,3)	0	0	1	-
(2,6)	0	-	1	0
(8,9)	1	0	0	-
(3,7)	0	-	1	1
(6,7)	0	1	1	-
(6,14)	-	1	1	0
(7,15)	-	1	1	1
(14,15)	1	1	1	-

(0,1,2,3)	0	0	-	-
(0,1,8,9)	-	0	0	-
(2,3,6,7)	0	-	1	-
(6,7,14,15)	-	1	1	-

PI	0	1	2	3	6	7	8	9	14	15
a' b'	×	×	×	×						
b' c'	×	×					×	×		
a' c			×	×	×	×				
b c					×	×			×	×

PI	0	1	2	3	6	7	8	9	14	15
a' b'	×	×	×	×						
b' c'	×	×					⊗	⊗		
a' c			×	×	×	×				
b c					×	×			⊗	⊗

PI	0	1	2	3	6	7	8	9	14	15
a' b'	×	×	×	×						
b' c'	×	×					⊗	⊗		
a' c			×	×	×	×				
b c					×	×			⊗	⊗

f(a, b, c, d) =b' c'+ b c+ a' c