

Ex 1: Find the minimum sum of products for each function using a Karnaugh map.

1. $f_1(a, b, c) = m_0 + m_2 + m_5 + m_6$

2. $f_2(r, s, t) = rt' + r's' + r's$

3. $f_3(x, y, z) = M_0 \cdot M_5$

Ex 2:

1. Plot the following function on a Karnaugh map. (Do not expand to minterm form before plotting.)

$$F(A, B, C, D) = BD' + B'CD + ABC + ABC'D + B'D'$$

2. Find the minimum sum of products.
3. Find the minimum product of sums.

Ex 3: A switching circuit has two control inputs (C_1 and C_2), two data inputs (X_1 and X_2), and one output (Z). The circuit performs one of the logic operations AND, OR, EQU (equivalence), or XOR (exclusive OR) on the two data inputs. The function performed depends on the control inputs:

C_1	C_2	Function performed by circuit
0	0	OR
0	1	XOR
1	0	AND
1	1	EQU

1. Derive a truth table for Z .
2. Use a Karnaugh map to find a minimum AND-OR gate circuit to realize Z .

Ex 4: Find the minimum sum-of-products expression for each function. Underline the essential prime implicants in your answer and tell which minterm makes each one essential.

1. $f(a, b, c, d) = \sum m(0, 1, 3, 5, 6, 7, 11, 12, 14)$

2. $f(a, b, c, d) = \prod M(1, 9, 11, 12, 14)$

$$3. f(a,b,c,d) = \prod M(5,7,13,14,15) \cdot \prod D(1,2,3,9)$$

Ex 5: Find the minimum sum-of-products expression for each function.

$$1. f(a,b,c,d) = \sum m(0,2,3,4,7,8,14)$$

$$2. f(a,b,c,d) = \sum m(1,2,4,15) + \sum d(0,3,14)$$

$$3. f(a,b,c,d) = \prod M(1,2,3,4,9,15)$$

$$4. f(a,b,c,d) = \prod M(0,2,4,6,8) \cdot \prod D(1,12,9,15)$$

Ex 6: Find the minimum sum of products and the minimum product of sums for each function:

$$1. f(a,b,c,d) = \prod M(0,1,6,8,11,12) \cdot \prod D(3,7,14,15)$$

$$2. f(a,b,c,d) = \sum m(1,3,4,11) + \sum d(2,7,8,12,14,15)$$

Ex 7: Given $F = AB'D' + A'B + A'C + CD$.

1. Use a Karnaugh map to find the maxterm expression for F (express your answer in both decimal and algebraic notation).
2. Use a Karnaugh map to find the minimum sum-of-products form for F' .
3. Find the minimum product of sums for F .

Ex:1

①

a	b\c	00	01	11	10
0	1	0	0	0	1
1	0	1	0	0	1

$$f_1 = \underline{a'c' + bc' + ab'c}$$

②

r	s\t	00	01	11	10
0	1	1	1	1	1
1	1	0	0	0	1

$$f_2 = \underline{r' + t'}$$

③

x	y\z	00	01	11	10
0	0	1	1	1	1
1	1	0	1	1	1

$$f_3 = \underline{xz' + x'z + y}$$

Ex:2

①

AB	CD	00	01	11	10
00	1	0	1	1	1
01	1	0	0	0	1
11	1	1	1	1	1
10	1	0	1	1	1

$$F = \underline{D' + AB + B'C}$$

②

AB	CD	00	01	11	10
00	1	0	1	1	1
01	1	0	0	0	1
11	1	1	1	1	1
10	1	0	1	1	1

$$F = \underline{(B+C+D')(A+B'+D')}$$

Ex: 3

	C_1	C_2	X_1	X_2	Z
OR	0	0	0	0	0
			0	1	1
			1	0	1
			1	1	1
XOR	0	1	0	0	0
			0	1	1
			1	0	1
			1	1	0
AND	1	0	0	0	0
			0	1	0
			1	0	0
			1	1	1
EQU	1	1	0	0	1
			0	1	0
			1	0	0
			1	1	1

	$x_1 x_2$	00	01	11	10
$c_1 c_2$	00	0	1	1	1
	01	0	1	0	0
	11	1	0	1	0
	10	0	0	1	0

$$\begin{aligned}
 Z &= C_1 C_2 X_1' X_2' + C_1' X_1' X_2 + \\
 &C_1 X_1 X_2 + C_1' X_1 X_2' + \\
 &+ C_1' C_2' X_2
 \end{aligned}$$

Ex: 4

	cd	00	01	11	10
ab	00	1	1	1	0
	01	0	1	1	1
	11	1	0	0	1
	10	0	0	1	0

$$f = \overline{\overline{a}d} + \overline{\overline{a}\overline{b}d'} + \overline{\overline{b}\overline{c}d} + \overline{\overline{a}\overline{b}'\overline{c}'} + \overline{bcd'}$$

due to m_5 due to m_{11} due to m_{12} Due to m_0

(2)

$ab \backslash cd$	00	01	11	10
00	1	0	1	1
01	1	1	1	1
11	0	1	1	0
10	1	0	0	1

$$f = \underline{\underline{bd}} + \underline{\underline{b'd'}} + \underline{\underline{a'c}} + \underline{\underline{a'b}}$$

Due to m_{13} and m_{15} Due to m_8 and m_{10} Due to m_3

(3)

$ab \backslash cd$	00	01	11	10
00	1	X	X	X
01	1	0	0	1
11	1	0	0	0
10	1	X	1	1

$$f = \underline{\underline{b}} + \underline{\underline{c'd'}} + \underline{\underline{a'd'}}$$

Due to m_{10} , m_{11} due to m_{12} due to m_6

Ex: 5

(1)

$ab \backslash cd$	00	01	11	10
00	1	0	1	1
01	1	0	1	0
11	0	0	0	1
10	1	0	0	0

$$f = a'c'd' + a'cd + b'c'd' + a'b'c + abcd'$$

②

	cd	00	01	11	10
ab	00	X	1	X	1
	01	1	0	0	0
	11	0	0	1	X
	10	0	0	0	0

$$f = \underline{a'b' + abc + a'c'd'}$$

③

	cd	00	01	11	10
ab	00	1	0	0	0
	01	0	1	1	1
	11	1	0	0	1
	10	1	0	1	1

$$f = \underline{ad' + b'c'd' + bc'd + a'bc + ab'c}$$

④

	cd	00	01	11	10
ab	00	0	X	1	0
	01	0	1	1	0
	11	X	1	X	1
	10	0	X	1	1

$$f = \underline{d + ac}$$

Ex: 6

①

	cd	00	01	11	10
ab	00	0	0	X	1
	01	1	1	X	0
	11	0	1	X	X
	10	0	1	0	1

$$f = \underline{a'bc' + ac'd + b'c'd'}$$

	cd	00	01	11	10
ab	00	0	0	X	1
	01	1	1	X	0
	11	0	1	X	X
	10	0	1	0	1

$$f = \underline{(c'+d')(b'+c')(a+b+c)(a'+c+d')}$$

(2)

ab	cd	00	01	11	10
00		0	1	1	X
01		1	0	X	0
11		X	0	X	X
10		X	0	1	0

$$f = cd + a'b'd + bc'd'$$

ab	cd	00	01	11	10
00		0	1	1	X
01		1	0	X	0
11		X	0	X	X
10		X	0	1	0

$$f = (b+d)(b'+d')(b'+c)(a'+c)$$

Ex: 7

(1)

AB	CD	00	01	11	10
00		0	0	1	1
01		1	1	1	1
11		0	0	1	0
10		1	0	1	1

$$F(A, B, C, D) = M_0 M_1 M_9 M_{12} M_{13} M_{14}$$

(2)

$$F'(A, B, C, D) = A'B'C' + AC'D + ABD'$$

(3)

$$F(A, B, C, D) = (A+B+C)(A'+C+D')(A'+B'+D)$$

