BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE PILANI - KK BIRLA
G O A C A M P U S
FIRST SEMESTER 2018-2019

MATHEMATICS - III

Tutorial Sheet–1

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1. Find degree and order of the following differential equations. Also classify (linear/nonlinear) ?

   (i) $\frac{d^3y}{dx^3}+4\left(\frac{dy}{dx}\right)^2=\sin y$, (ii) $\frac{dy}{dx}+2y=\sin x$, (iii) $y\frac{d^2y}{dx^2}+2x\sqrt{\frac{dy}{dx}}+y=0$,
   (iv) $\frac{d^4y}{dx^4}+\sin x\frac{dy}{dx}+x^2y=0$, (v) $(1+y^2)\frac{d^2y}{dx^2}+x\frac{d^6y}{dx^6}+y=e^x$.

2. Solve the following differential equations

   (i) $2\sqrt{x}\frac{dy}{dx}=\sqrt{1-y^2}$, (ii) $(x^2+1)(\tan y)\frac{dy}{dx}=x$
   (iii) $x\frac{dy}{dx}+y=y^2,\ y(1)=2$, (iv) $\frac{dy}{dx}=(y-x)^2$.

3. Verify that the following equations are homogeneous, and solve them:

   (i) $(x^2-2y^2)dx+xydy=0$, (ii) $x\sin\left(\frac{y}{x}\right)\frac{dy}{dx}=y\sin\left(\frac{y}{x}\right)+x$,
   (iii) $x^2y'=y^2+2xy$, (iv) $(x^2+xy)dy=(y^2-xy)dx$,
   (v) $x^2\frac{dy}{dx}=y^2+xy+x^2$,

4. If $ae\neq bd$, show that constants h and k can be chosen in such a way that the substitutions $x=z-h$, $y=w-k$ reduce

   $$\frac{dy}{dx}=F\left(\frac{ax+by+c}{dx+ey+f}\right)$$

   to a homogeneous equation.

5. Show that the differential equation $\frac{dy}{dx}=\frac{ax+by+m}{cx+dy+n}$, where a,b,m,c,d,n are constants, can be reduced to homogeneous or separable form. Also find the general solution of the following problems:

   (i) $(1+x-2y)+(4x-3y-6)y'=0$, (ii) $y'=\frac{y-x+1}{y-x+5}$, (iii) $(x+2y+3)+(2x+4y-1)y'=0$.

6. Solve the following equations

   (i) $\frac{dy}{dx}+\frac{y-x}{y+x}=0$, (ii) $x\frac{dy}{dx}=y-xe^{\frac{y}{x}}$, (iii) $\frac{dy}{dx}=\cos(x+y+1)$
   (iv) $\frac{dy}{dx}=(x+y-1)^2$, (v) $\frac{dy}{dx}=2\sqrt{2x+y+1}$.

7. Determine which of the following equations are exact, and solve the ones that are:

(i) $\left(x + \frac{2}{y}\right) dy + y dx = 0$,

(ii) $-\frac{1}{y} dx + \frac{x}{y^2} \sin\left(\frac{x}{y}\right) dy = 0$,

(iii) $dx = \frac{y}{1 - x^2 y^2} dx + \frac{x}{1 - x^2 y^2} dy$,

(iv) $2x \sin y \, dx + x^2 \cos y \, dy = 0$.

8. Solve $\frac{y dx - x dy}{(x + y)^2} + dy = dx$ as an exact equation.