EE225E Project Proposal: Metabolism mapping with prior information

John Maidens and Kamin Kahrizi

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0.1 Dynamic metabolic MRI using hyperpolarized carbon-13 pyruvate

Hyperpolarized carbon-13 magnetic resonance imaging (MRI) has enabled the real-time observation of perfusion and metabolism in preclinical and clinical studies. This technology is made possible by techniques for dynamic nuclear polarization (DNP) that have led to signal-to-noise ratio (SNR) increases of four to five orders of magnitude compared with endogenous signal in dissolved ^{13}C-labelled molecules [1, 2]. Injected [1-^{13}C] pyruvate is frequently used as a substrate in metabolism experiments and its rate of conversion to [1-^{13}C] lactate has been shown to distinguish between healthy and diseased tissues in animal [3], and recently human [4], studies.

In contrast with conventional MRI, hyperpolarized experiments are inherently dynamic as images must be acquired as the injected substrate spreads through the body and is metabolized. This necessitates dynamical system modelling and estimation for quantifying metabolic reaction rates.

0.2 Low SNR degrades parameter map quality

Even with dynamic nuclear polarization, carbon-13 MRI suffers from low SNR relative to conventional MRI. This makes it difficult to acquire high-resolution images, as large voxel sizes are required to achieve sufficient SNR.

In Figure 1 we show the effect of increased resolution on the quality of parameter maps computed by an independent maximum-likelihood fit to each voxel. We generate data from the model given in equation (2) of [5], assuming that SNR of the trajectories of the model is proportional to voxel volume. We see that when voxels are treated independently (*i.e* no spatial structure is assumed in the parameter maps) increased resolution can be detrimental to parameter map quality.

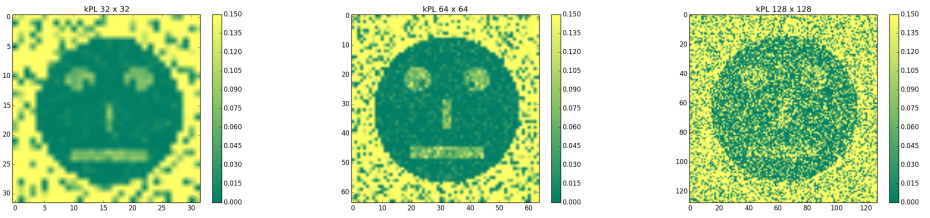
0.3 A Douglas-Rachford splitting algorithm for spatial regularization

In order to overcome this limitation, we propose to investigate incorporating prior information about parameter maps by including regularization in the mapping. This will allow spatial structure in the data to be exploited, which was ignored in the previous analysis. We hope that this will help to solve the problem of low SNR in high-resolution carbon-13 images.

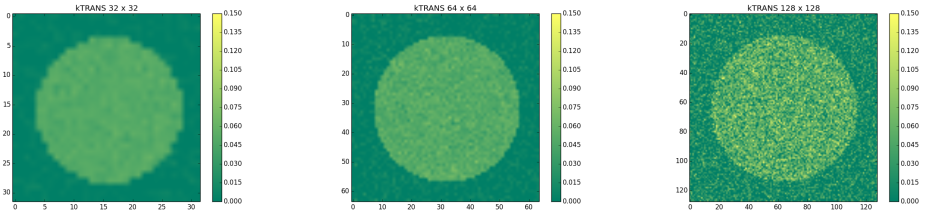
Let y_i denote a dynamic dataset generated by the model given in equation (2) of [5] corresponding to some voxel i from the set of all voxels $\mathcal{V}$. We wish to solve the optimization problem

$$\textbf{minimize} \quad \sum_{i \in \mathcal{V}} -\ell(\theta_i | y_i) + \lambda r(\theta) \tag{1}$$

where $\ell(\theta_i|y_i)$ denotes the log likelihood function corresponding to the model and r is a regularizer that corresponds to prior information we have about the parameter map. For example, for the parameter map in Figure 1 we might use a total-variation regularization term in order to exploit the fact that we know that the map is piecewise-constant.



(a) k_{PL}: 32 × 32 (b) k_{PL}: 64 × 64 (c) k_{PL}: 128 × 128



(d) k_{TRANS}: 32 × 32 (e) k_{TRANS}: 64 × 64 (f) k_{TRANS}: 128 × 128

Figure 1: Parameter maps of varying resolution fit to simulated data sets. The first row shows parameter maps for the metabolic rate parameter k_{PL}; the second row shows parameter maps for the perfusion parameter k_{TRANS}.

0.4 Project goals:

- Explore whether similar things have been done before (*e.g.* in dynamic contrast enhanced MRI).
- Implement a Douglas-Rachford iteration to try to solve (1) and investigate convergence properties (since the negative log likelihood in this case in non-convex).
- Investigate applying to more realistic simulated data sets (*e.g.* subsampled k-space).

References

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