情報連携学部 INIAD

Statistics and data analysis I

Week 10

"Probability Distribution (II): Normal and Exponential Distributions"

Takashi Sano and Hirotada Honda

Lecture plan

Week1: Introduction of the course and some mathematical preliminaries
Week2: Overview of statistics, One dimensional data(1): frequency and histogram
Week3: One dimensional data(2): basic statistical measures
Week4: Two dimensional data(1): scatter plot and contingency table
Week5: Two dimensional data(2): correlation coefficients, simple linear regression and concepts of Probability /
Probability(1):randomness and probability, sample space and probabilistic events
Week6:Probability(2): definition of probability, additive theorem, conditional probability and independency
Week7:Review and exam(i)
Week8: Random variable(1): random variable and expectation
Week9: Random variable(2): Chebyshev's inequality, Probability distribution(1):binomial and Poisson distributions
Week10: Probability distribution(2): normal and exponential distributions
Week11: From descriptive statistics to inferential statistics -z-table and confidwncw interval-
Week12: Hypothesis test(1) -Introduction, and distributions of test statistic (t-distribution)-
Week13: Hypothesis test(2) -Test for mean-
Week14: Hypothesis test(3) -Test for difference of mean-
Week15: Review and exam(2)

※ Might be changed!

Agenda

1. Normal distribution
2. Exponential distribution

1. Normal distribution

Examples of probability distribution / density

- Well known probability distributions for natural or socio-physical phenomena

- Pip of dice: Uniform distribution
- Resource survey:Hypergeometric distribution
- Summarization of questionnaire : Binomial distribution
- Accidents of airplanes

Discrete

- Measurements of something: Normal distribution
- Waiting time:Exponential distribution
- Lifetime of a system: Gamma distribution
- Income and saving : Logarithmic normal distribution

Continuous

Normal distribution

- Applied to very various kinds of phenomena in natural and social physics.
- The most important distribution in statistics.

  It's density is $$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

  - $E(X) = \mu$
  - $V(X) = \sigma^2$

- Denoted as $N(\mu, \sigma^2)$

【Reference】Gaussian distribution

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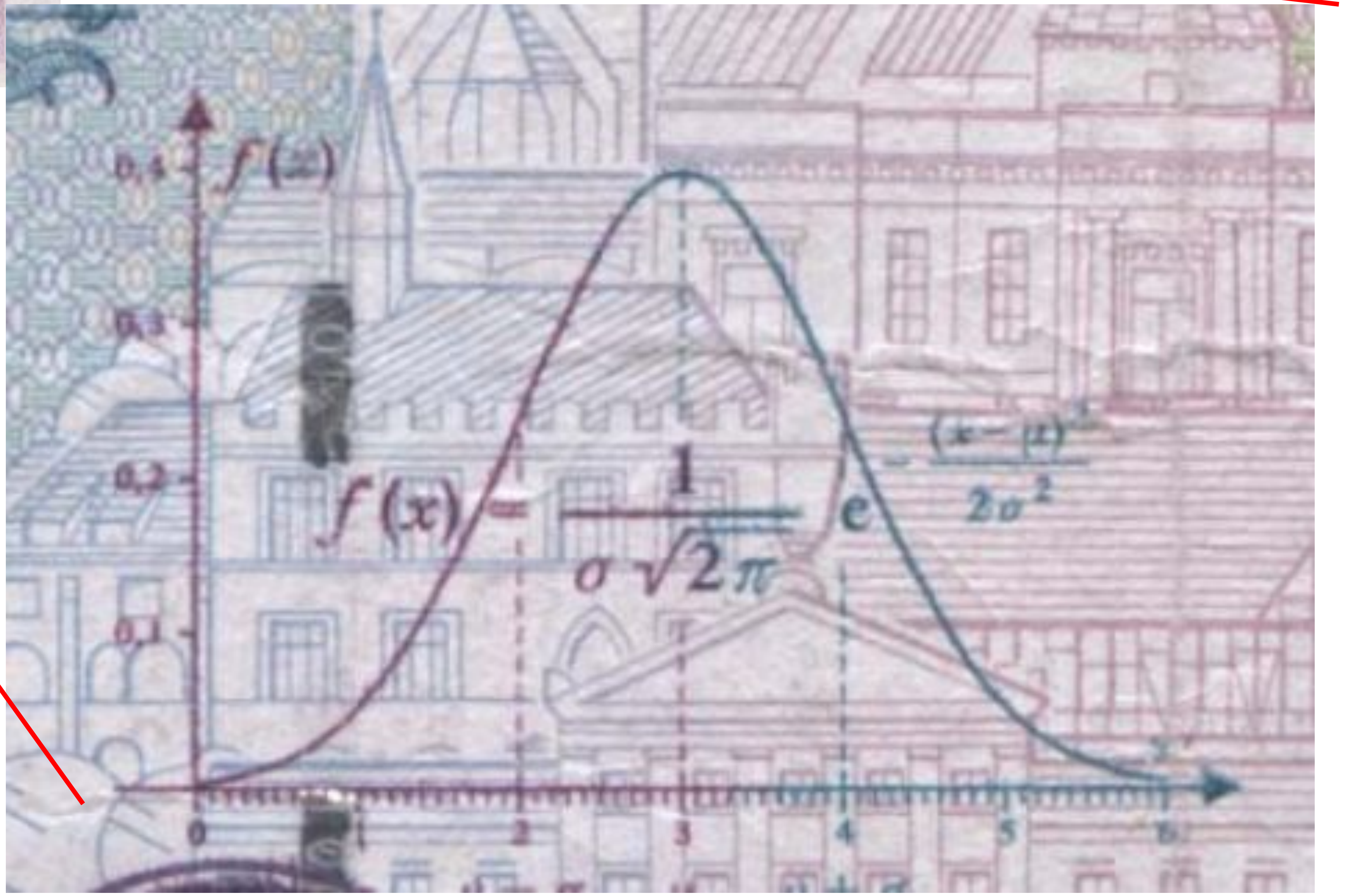


Standard normal distribution (z-distribution)

- Normalized r.v. Z＝(X－μ)/σ, where x ～ N(μ,σ)
- N(0, 1) is the z-distribution. It's density is

$$f(x) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{x^2}{2}}$$

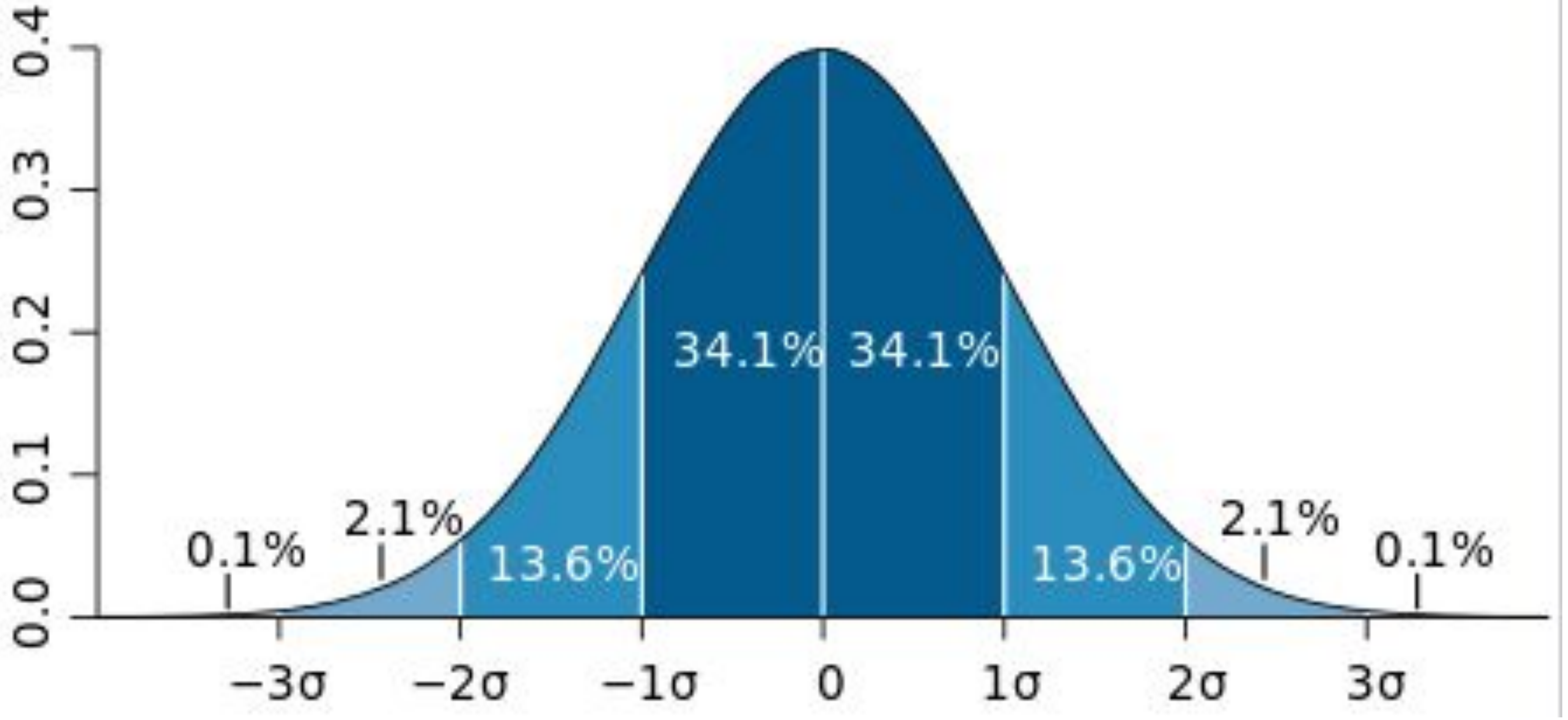
- Cumulative distribution of z-distribution

$$\phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{x^2}{2}}\, \mathrm{d}x$$

▲Frequently used, but cannot be calculated explicitly.

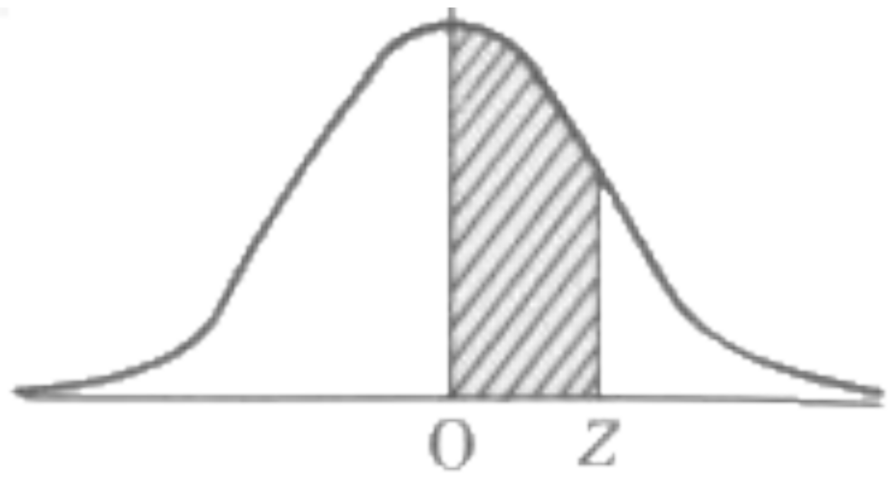
Z-distribution

- Symmetric with expected value of z=0.
- You can discuss the peculiarity of data by using the distance from z=0.
- The integral from z=0 is known (z-table)



Z-table

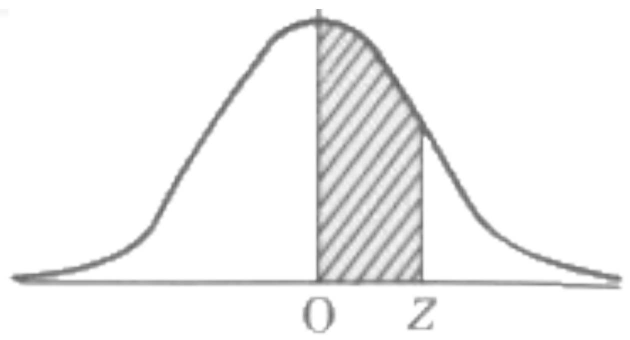
Stands for the integral from z=0. Of course, the integral over the whole real umber should be 1.



For instance, for z=1.00,the corresponding value is 0.3413, the hatched area occupies 34.13%.

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

To find probability of rv following z-dist.



①Z-table

http://www.koka.ac.jp/morigiwa/sjs/standard_normal_distribution.htm

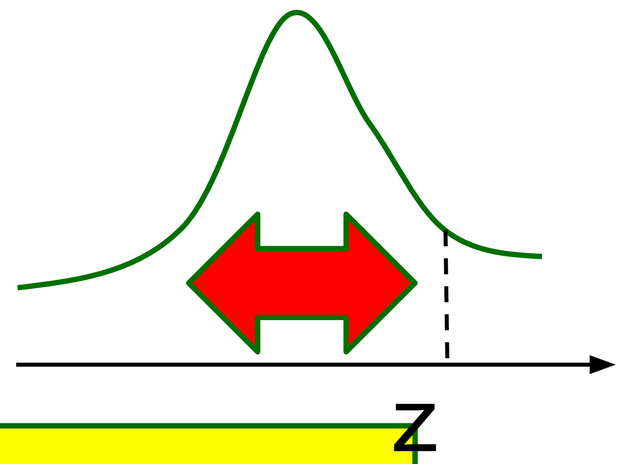
Note that the ways of presentation depend on softwares.

Python

- Assume X follows the z-dist. Then, find $P(-\infty<X<1.281552)$.

```
from scipy.stats import norm
norm.cdf(1.281552)
```

```
0.9000000762461767
```



The integral of pdf of z-dist. from -∞ to1.281552 is 0.9.

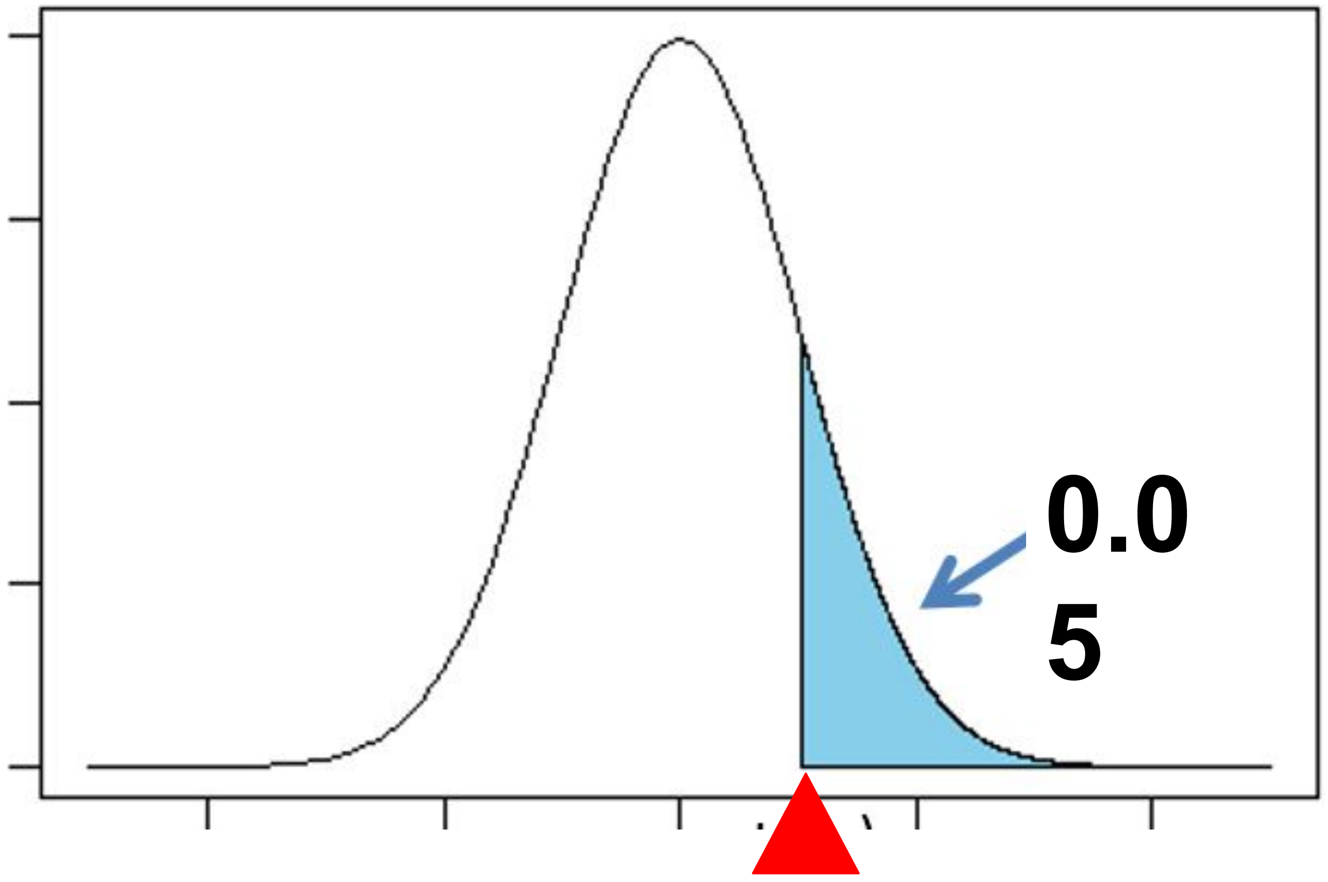
Upper O-percentile

Upper percentile

Frequently used hereafter.

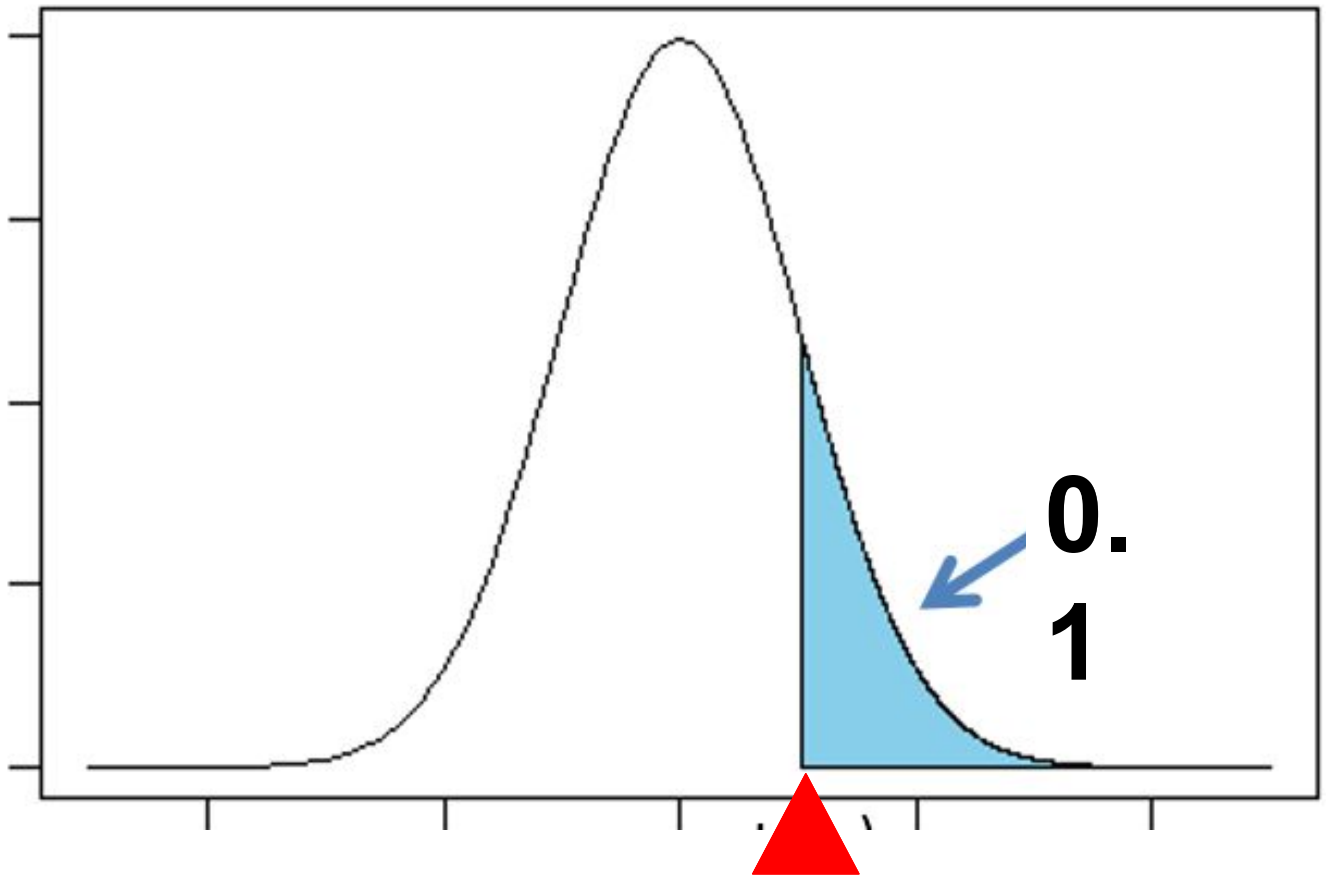
A point the size of which amounts to the value of the percentile.

In the figure below, for instance, the point ▲ is the upper 5-percentile. The size of the region above it is 0.05.



Upper percentile(2)

In the figure below, for instance, the point ▲ is the upper 10-percentile. The size of the region above it is 0.10.



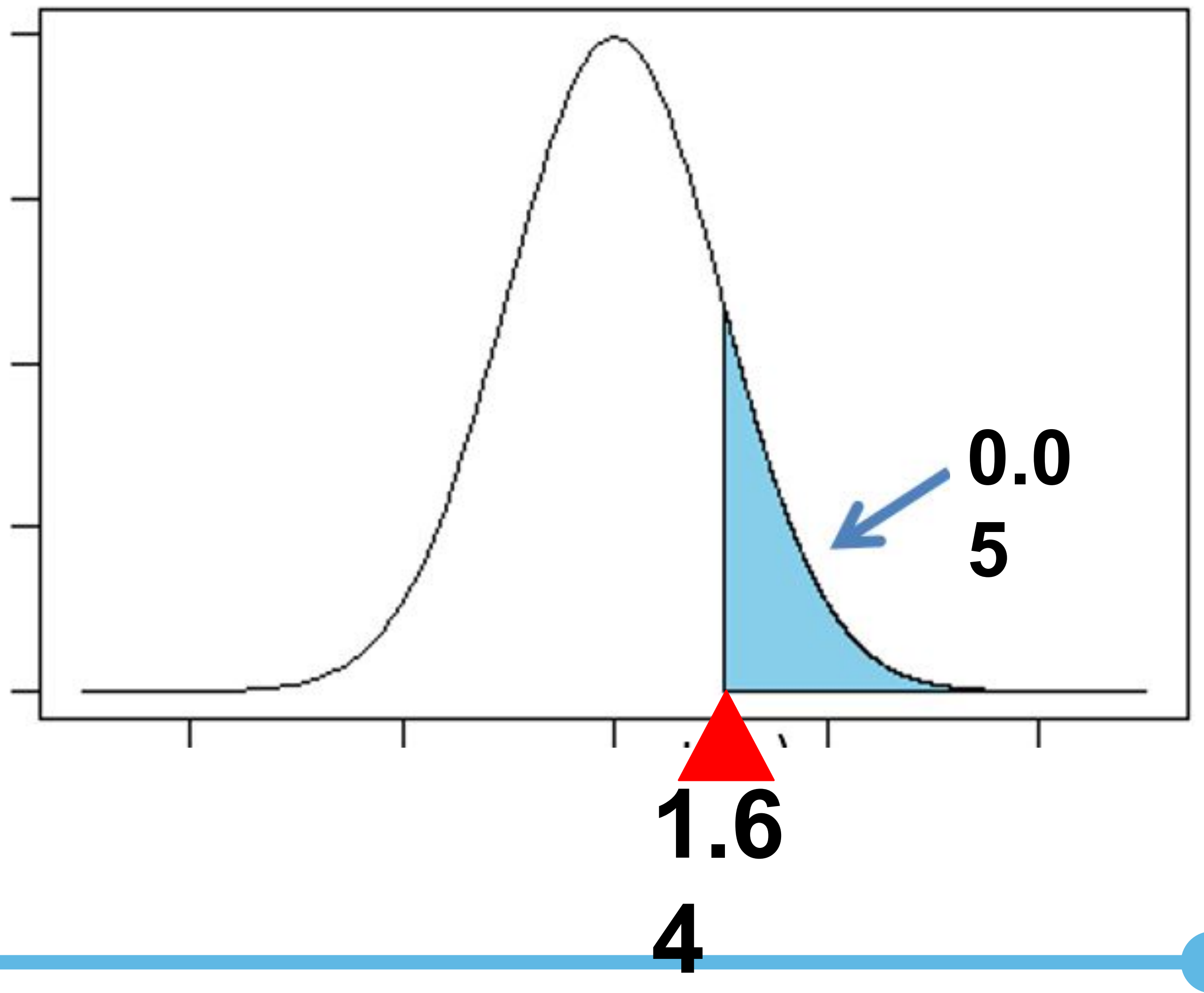
Upper percentile by python

- Scipy's stats module.
- "norm.ppf".

- For the upper 5-percentile:

```
from scipy.stats import norm
norm.ppf(0.95)
```

1.6448536269514722

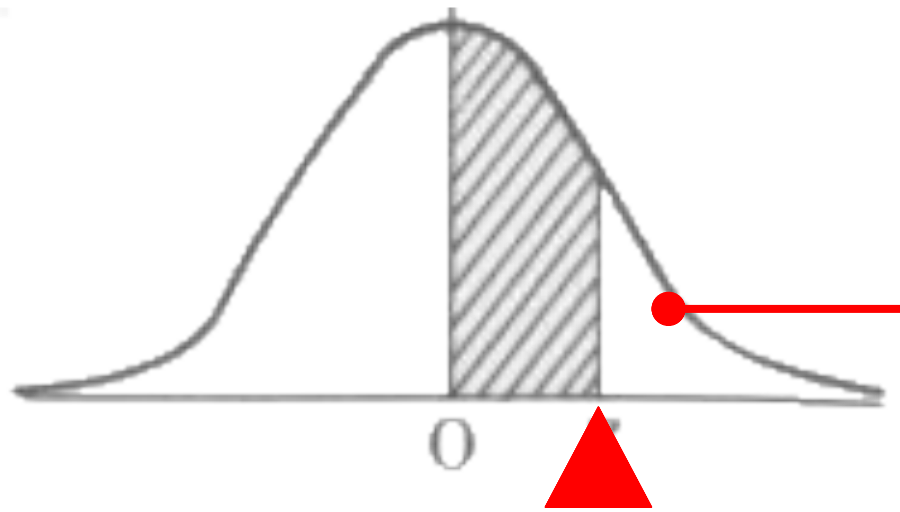


Example-1

Find the upper 10-percentile of z-dist.。

Example-1【Ans】

Method① Z-table.



Find a point here. The size of this Point amounts to 0.1.

Example-1【Ans】

Method① Z-table.

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
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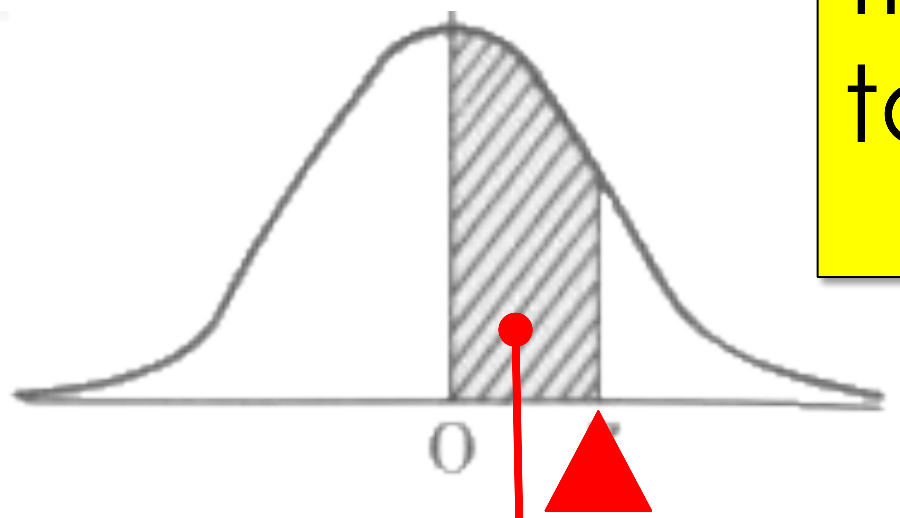
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Which is the closest to 0.4?



Then, the size of this hatched region is 0.4.

Example-1【Ans】

Method① Z-table.

Here

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
[illegible]	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.3980	.3997	.4015
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
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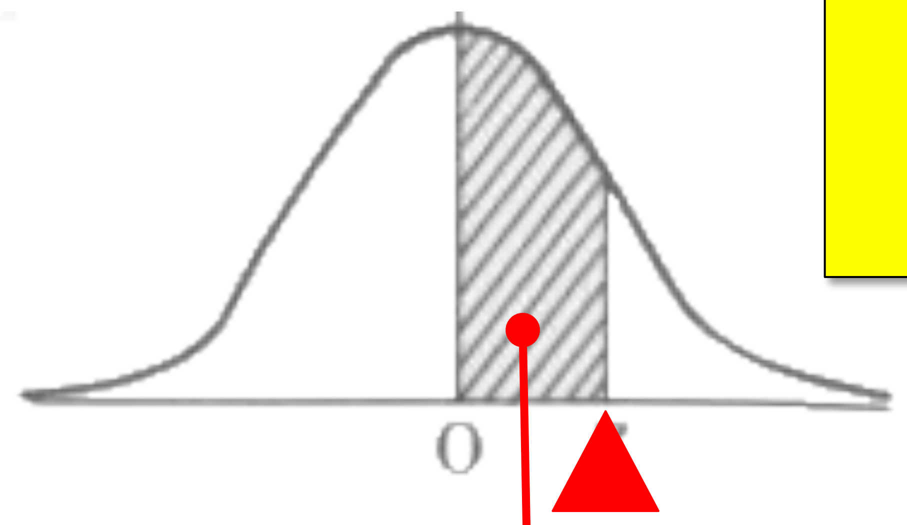
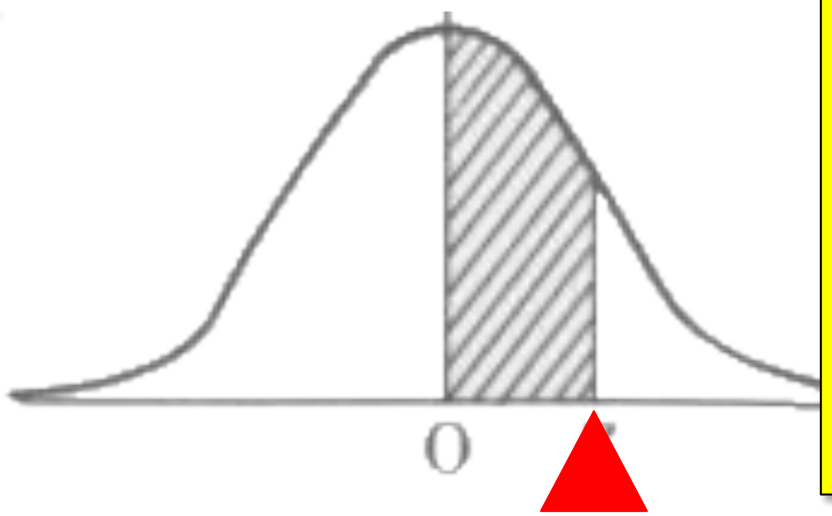
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990



Then, the size of this hatched region is 0.4.

Example-1【Ans】

Method① Z-table.



The value of z here is 1.28

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
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1.5	.4332	.4345	.4357	[illegible]	[illegible]	[illegible]	.4406	.4418	.4429	.4441
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4495	.4505	.4515	.4525	.4535	.4545
[illegible]	[illegible]	[illegible]	[illegible]	.4582	.4591	.4599	.4608	.4616	.4625	.4633
[illegible]	[illegible]	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
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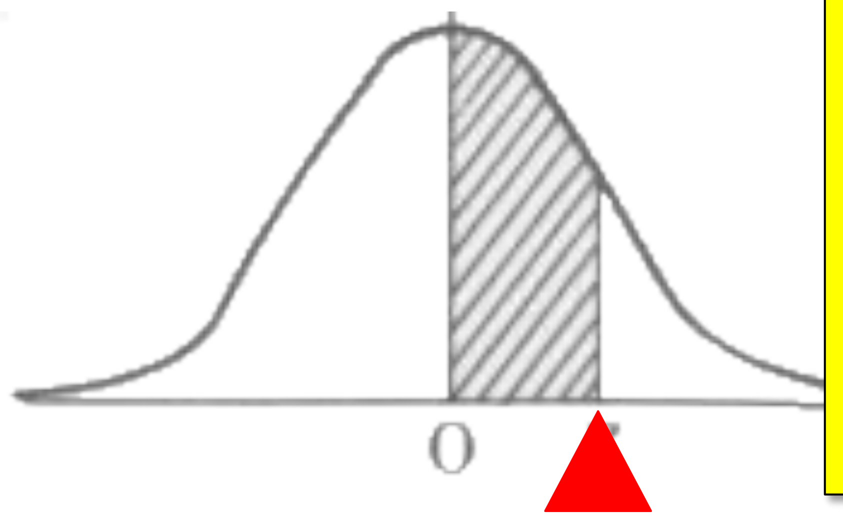
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Example-1【Ans】

Method① Z-table.



1.28

The value of z here is 1.28

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	[illegible]	[illegible]	[illegible]	.4406	.4418	.4429	.4441
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.4495	.4505	.4515	.4525	.4535	.4545
[illegible]	[illegible]	[illegible]	[illegible]	.4582	.4591	.4599	.4608	.4616	.4625	.4633
[illegible]	[illegible]	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Example-1【Ans】

Method② Python

```
from scipy.stats import norm
norm.ppf(0.9)
```

```
1.2815515655446004
```

For normal dist. in general

The z-dist. means the normal dist. with the expected value and SD of 0 and 1, resp.

Now, how about the normal dist. in general, whose expected value and SD are, say, μ and σ?

Probability of normal dist. in general

Assume a r.v. X follows the normal dist. With expected value and SD of 0 and 3, resp. Then, find the probability $P(-\infty<X<0)$

Python

- For normal dist. in general: loc =μ, scale =σ

Integrate the pdf of $N(0,3^2)$ on $(-\infty,0)$.

```
In [5]: from scipy.stats import norm
        norm.cdf(0,loc=0,scale=3)

Out[5]: 0.5
```

Probability of normal dist. in general

Now, you can find the probability on arbitrary intervals: $P(a<X<b)$.

Ex)

Assume a r.v. X follows the normal dist. With expected value and SD of 5 and 2, resp. Then, find the probability $P(-10<X<2)$

Probability of normal dist. in general

Now, you can find the probability on arbitrary intervals: P(a<X<b).

Ex)
Assume a r.v. X follows the normal dist. With expected value and SD of 5 and 2, resp. Then, find the probability P(-10<X<2)

```
from scipy.stats import norm
norm.cdf(2, loc=5,scale=2)-norm.cdf(-10, loc=5,scale=2)
```

```
0.06680720126882617
```

Probability of normal dist. in general

Assume a r.v. X follows the normal dist. With expected value and SD of 0 and 3, resp. Then, find the upper 5-percentile of the pdf of this dist.

```
from scipy.stats import norm
norm.ppf(0.95,loc=0,scale=3)
```

```
4.934560880854416
```

Try
‘2019_StatI_Week10_DL
_0’.

標準正規分布の上側5%点を求めよ。Find the upper 5-percentile of z-dist.

1. 1.64
2. 1.98
3. 2.23
4. 1.28
5. 1.45

標準正規分布の上側5%点を求めよ。Find the upper 5-percentile of z-dist.

1. ○ 1.64
2. ○ 1.98
3. ○ 2.23
4. ○ 1.28
5. ○ 1.45

```
from scipy.stats import norm
norm.ppf(0.95)
```

1.6448536269514722

標準正規分布の上側10%点を求めよ。Find the upper 10-percentile of z-dist.

1. 1.28
2. 1.31
3. 1.45
4. 1.64
5. 1.98

標準正規分布の上側10%点を求めよ。Find the upper 10-percentile of z-dist.

1. 1.28
2. 1.31
3. 1.45
4. 1.64
5. 1.98

```
from scipy.stats import norm
norm.ppf(0.9)
```

1.2815515655446004

正規分布 $N(3,3^2)$ の上側5%点を求めよ。

Find the upper 5-percentile of $N(3,3^2)$.

1. 7.93
2. 6.22
3. 5.66
4. 8.12
5. 8.67

正規分布 $N(3, 3^2)$ の上側5%点を求めよ。

Find the upper 5-percentile of $N(3, 3^2)$.

1

1. 7.93
2. 6.22
3. 5.66
4. 8.12
5. 8.67

```
from scipy.stats import norm
norm.ppf(0.95,loc=3,scale=3)
```

7.934560880854416

正規分布 $N(10, 1^2)$ の上側10%点を求めよ。

Find the upper 10-percentile of $N(10, 1^2)$.

1. ○ 11.28
2. ○ 10.35
3. ○ 9.68
4. ○ 1.28
5. ○ 6.97

正規分布 $N(10, 1^2)$ の上側10%点を求めよ。

Find the upper 10-percentile of $N(10, 1^2)$.

1

1. 11.28
2. 10.35
3. 9.68
4. 1.28
5. 6.97

```python
from scipy.stats import norm
norm.ppf(0.9,loc=10,scale=1)
```

```
11.281551565546
```

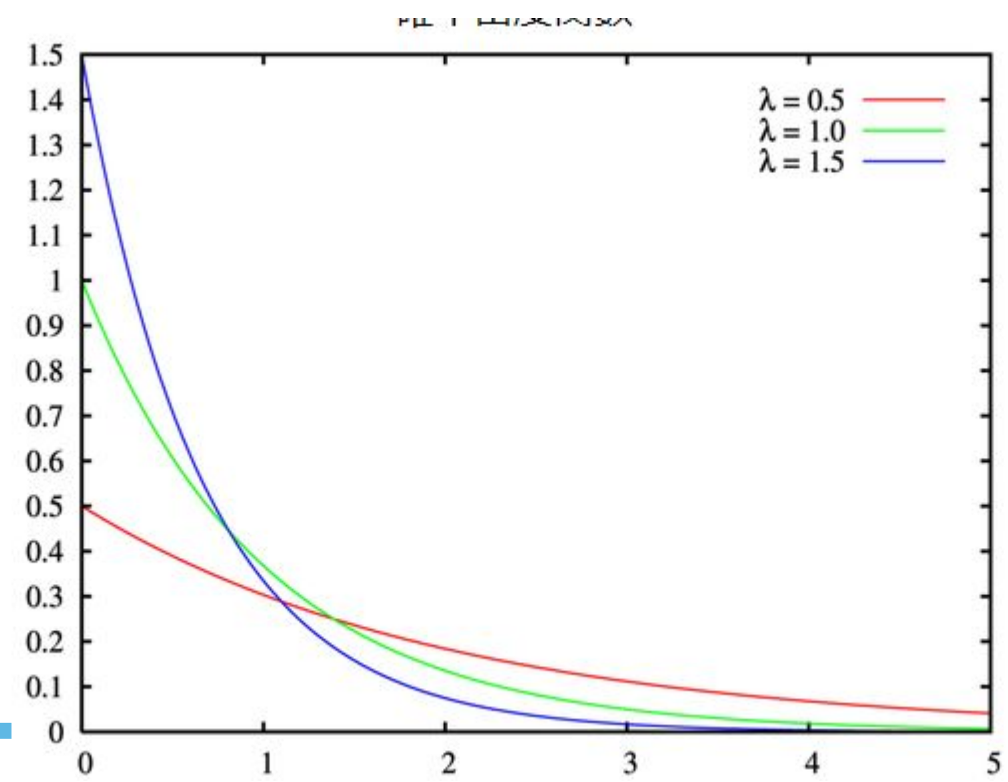
2. Exponential distribution

Exponential distribution

- Let a certain event happen λ times on average within an unit time interval.
- Then, the probability that the interval of events, say, x, is subject to the exponential distribution.

  $f(x) = \lambda e^{-\lambda x} \quad (x \geqq 0)$
- The cumulative distribution is:

  $F(x) = P(X \leqq x) = 1 - e^{-\lambda x} \quad (x \geqq 0)$



- $E(X) = 1/\lambda$
- $V(X) = 1/\lambda^2$

Exponential distribution

Applied to repeated events:

- Customer arrival
- Intervals of phone calls
- Lifetime of a system

Relationship between exponential and Poisson distributions

- Poisson distribution concerns the *numbers* of events within an unit time interval.
- Exponential distribution concerns the *interval* between the events.
- You observe the same events from different point of view.
  - Poisson distribution：numbers, Exponential：interval
  - The expected value of exponential distribution is $1/\lambda$.
  - Variance is $1/\lambda^2$.

CDF of exp. dist. With python

The cdf of exp. dist. , P(X≦x), is found by the command As below.
For instance, for exp. Dist. with λ=10, P(X≦0.25) can be found by

```
from scipy.stats import expon
lamb=10
expon.cdf(0.25,scale=1/lamb)
```

```
0.9179150013761012
```

Try
‘2019_StatI_Week10_DL_
2a’.

$\lambda = 5$ の指数分布に従う確率変数Xについて、累積確率 $P(X \leq 3)$ を求めよ。

Find the cumulative distribution $P(X \leq 3)$ for X following the exp. dist. of $\lambda = 5$.

1. 1.00
2. 0.87
3. 0.91
4. 0.23
5. 0.65

$\lambda = 5$ の指数分布に従う確率変数Xについて、累積確率 $P(X \leq 3)$ を求めよ。
Find the cumulative distribution $P(X \leq 3)$ for X following the exp. dist. of $\lambda = 5$.

1. ○ 1.00
2. ○ 0.87
3. ○ 0.91
4. ○ 0.23
5. ○ 0.65

```
from scipy.stats import expon
lamb=5
expon.cdf(3,scale=1/lamb)
```

```
0.9999996940976795
```

$\lambda = 4$ の指数分布に従う確率変数Xについて、累積確率 $P(X \leq 0.1)$ を求めよ。
For a r.v. X that follows the exp.dist. of $\lambda = 4$, find $P(X \leq 0.1)$.

1. 0.33
2. 0.23
3. 0.15
4. 0.41
5. 0.48

$\lambda = 4$ の指数分布に従う確率変数Xについて、累積確率 $P(X \leq 0.1)$ を求めよ。
For a r.v. X that follows the exp.dist. of $\lambda = 4$, find $P(X \leq 0.1)$.

1. 0.33
2. 0.23
3. 0.15
4. 0.41
5. 0.48

```
from scipy.stats import expon
lamb=4
expon.cdf(0.1,scale=1/lamb)
```

0.3296799539643607

$\lambda = 6$ の指数分布に従う確率変数Xについて、累積確率 $P(X \leq 0.5)$ を求めよ

For a r.v. X that follows the exp. dist. of $\lambda = 6$, find $P(X \leq 0.5)$.

1. ○ 0.95
2. ○ 0.91
3. ○ 0.84
4. ○ 0.75
5. ○ 0.79

「$\lambda = 6$ の指数分布に従う確率変数Xについて、累積確率 $P(X \leq 0.5)$ を求めよ」

For a r.v. X that follows the exp. dist. of $\lambda = 6$, find $P(X \leq 0.5)$.

1. 0.95
2. 0.91
3. 0.84
4. 0.75
5. 0.79

```
from scipy.stats import expon
lamb=6
expon.cdf(0.5,scale=1/lamb)
```

```
0.950212931632136
```

Exercise 1

- In a certain shop, 5 customers arrive during one hour (i.e., the interval of customer arrival is 12 minutes).

Then, find the probability that next customer arrives within 15 minutes after the current customer arrives.

Exercise 1【Answer】

- In a certain shop, 5 customers arrive during one hour (i.e., the interval of customer arrival is 12 minutes).

Then, find the probability that next customer arrives within 15 minutes after the current customer arrives.

$\lambda = 5$[customers／hour], $x = 0.25$[ours].

Thus,

$$P(X \leqq 0.25) = 1 - e^{-\lambda x} = 1 - e^{-1.25}$$

$$= 1 - 0.2865 = 0.7135$$

```python
from scipy.stats import expon

lamd=5
res = expon.cdf(0.25,scale=1/lamd)
print(res)
```

```
0.7134952031398099
```

Exercise (Exp. dist.)

Many e-mails arrive at Mr.K, a soft-ware developer.
He replies each mail within 15 mins for each on average, and the intervals follow the exp. dist.
Now, find the probability that he replies an e-mail within 30 mins.

Exercise (Exp. dist.)【Ans】

Take 'one minute' as an unit time.

Then, $\lambda = 1/15$ [replies/min], so

$$P(X \leqq 30) = 1 - e^{-\lambda x} = 1 - e^{-30*1/15}$$

$$= 1 - 0.135 = \underline{0.865}$$

```python
from scipy.stats import expon
lamb=1/15
expon.cdf(30,scale=1/lamb)
```

```
0.8646647167633873
```

Exercise② (Exp. dist.)

In a certain shop, the intervals of customer-arrival follow The exp. dist., and the average interval is one minute.

Now, find the probability that the next customer will arrive within 2 mins.

Exercise② 【Ans】

Take ‘one minute’ as the time unit. Then,

λ= 1[customer／min], so

$P(X \leqq 2) = 1 - e^{-\lambda x} = 1 - e^{-1*2}$

$= 1 - 0.135 = \underline{0.865}$

```python
from scipy.stats import expon
lamb=1
expon.cdf(2,scale=1/lamb)
```

0.8646647167633873

Summary(Check list)

- You can explain the normal/exp. dist.?
- You can find the probability of normak dist.?
- You can find the CDF of exp. Dist.