QUESTION 1

Theorem $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$ is false.

proof: By contradiction.

Assume there exist some $m, n \in \mathbb{N}$ that satisfy $3m + 5n = 12$.

Rearranging and grouping the terms, $5n = 12 - 3m = 3(4 - m)$, i.e. $3|5n$. By Euclid's Lemma, since 3 and 5 are coprime, n is divisible by 3.

Let $n = 3q$ where $q \in \mathbb{N}$. Substituting $3q$ into n then rearranging the terms,

$$5(3q) = 12 - 3m$$

$$5q = 4 - m$$

$$m = 4 - 5q$$

Since $q \in \mathbb{N}$, so $q \geq 1$, so $m \leq -1$. But that contradicts the fact that $m \in \mathbb{N}$.

Hence there do not exist $m, n \in \mathbb{N}$ that satisfy $3m + 5n = 12$.

$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$ is false. The proof is complete.