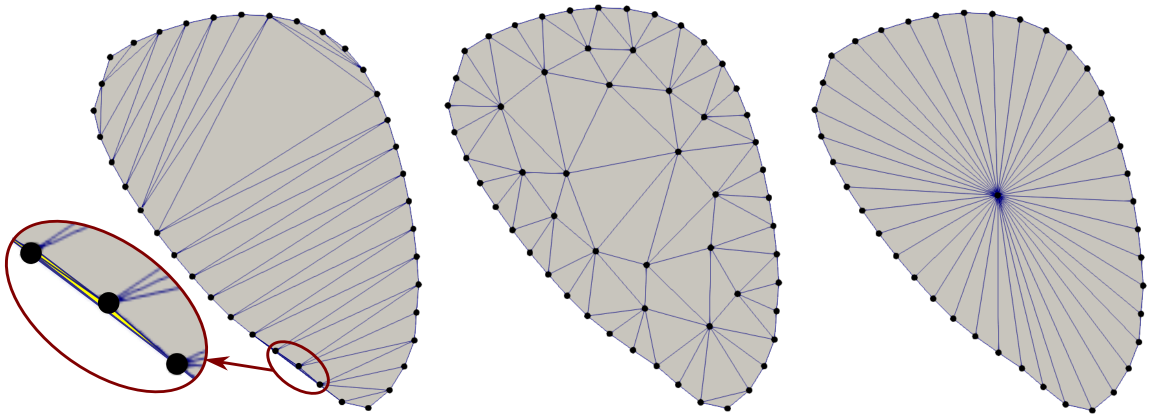

(a) First triangulation method (b) Second triangulation method (c) Third triangulation method

Figure 1.13: Intermediate step of 3D mesh generation: triangulation of slices, result of different triangulation methods for a slice in the center of the biceps muscle.

In our algorithm, harmonic maps are also used for the purpose of generating high quality meshes. In contrast to the literature, the mapping is based on the muscle slices instead of the surface. Also, different parameter domains are investigated.

A function $u : \Omega \to \mathbb{R}$ on a domain $\Omega \in \mathbb{R}^d$ is *harmonic* if it is a solution of the Laplace equation $\Delta u = 0$. From variational calculus it is known that harmonic functions are extremals of the *Dirichlet energy functional* [Wey40],

$$E[u] = \frac{1}{2} \int_\Omega |\nabla u|^2 \,\mathrm{d}\mathbf{x}.$$

For an intuitive understanding, the map u can be seen as deforming an elastic material that is initially located tension-free in the domain Ω. Then, the Dirichlet energy $E[u]$ describes the total amount of squared stretch or elastic energy resulting from the tension that occurs in the deformed state. A harmonic map minimizes this total tension. Qualitatively, the map deforms neighborhoods of all points in Ω by a similar amount, thus, preserving geometrical structures in Ω, e.g., given by a mesh. The idea of our approach is that applying a harmonic map on a mesh with good quality preserves the mesh quality also in the image under the map.

In Alg. 1, computing the harmonic maps u and v is done in line 4. For a given slice S_M, the functions u and v map from points $\mathbf{x} \in S_M$ to coordinates $u(\mathbf{x}), v(\mathbf{x}) \in \mathbb{R}$ of a parameter domain $\Omega_P \subset \mathbb{R}^2$. The parameter domain is either a unit circle or a unit square.

The vector $\mathbf{y}(\mathbf{x}) := (u(\mathbf{x}), v(\mathbf{x}))^\top$ for $\mathbf{x} \in S_M$ is interpreted as position in Ω_P. The maps