Sumatorias

- Calcule la suma:

1. $\sum_{K=1}^{5}(3K-10) = \sum_{K=1}^{5} 3K - \sum_{K=1}^{5} 10 \Leftarrow \cdot \sum[f(x)+g(K)] = \sum f(K) + \sum g(x)$

$= 3\sum_{K=1}^{5} K - \sum_{K=1}^{5} 10 \Leftarrow \cdot \sum C\cdot f(K) = C\sum f(K)$

$= 3\,\frac{n(n+1)}{2} - 10n$

$\left\{ \begin{array}{l} \cdot \sum_{K=1}^{n} K = \frac{n(n+1)}{2} \\ \cdot \sum_{K=1}^{n} C = C\cdot n \quad C\text{-Constante} \end{array} \right.$

Sustituye $n=5$

$\sum_{K=1}^{5}(3K-10) = 3\,\frac{5(5+1)}{2} - 10(5)$

$\sum_{K=1}^{5}(3K-10) = \frac{3}{2}(5)(6) - 50 = \frac{3}{2}\cdot 30 - 50 = 3(15) - 50$

$\sum_{K=1}^{5}(3K-10) = 45 - 50$

$\boxed{\sum_{K=1}^{5}(3K-10) = -5}$

2. $\sum_{K=1}^{9}(9-2K) = \sum_{K=1}^{9} 9 - \sum_{K=1}^{9} 2K \Rightarrow \cdot \sum[f(K)+g(K)] = \sum f(K) + \sum g(K)$

$= \sum_{K=1}^{9} 9 - 2\sum_{K=1}^{9} K \Rightarrow \cdot \sum C\cdot f(K) = C\sum f(K)$

C- constante

$= 9\cdot n - 2\frac{n(n+1)}{2}$

$\left[\begin{array}{l} \cdot \sum_{K=1}^{n} C = nC \\ \cdot \sum_{K=1}^{n} K = \frac{n(n+1)}{2} \end{array}\right.$

Sustituimos $n=9$

$\sum_{K=1}^{9}(9-2K) = 9\cdot(9) - \cancel{2}\cdot\frac{9(9+1)}{\cancel{2}}$

$= 81 - 90$

$$\boxed{\sum_{K=1}^{9}(9-2K) = -9}$$

3. $\sum_{j=1}^{4}(j^2+1) = \sum_{j=1}^{4} j^2 + \sum_{j=1}^{4} 1 \Rightarrow \cdot \sum[f(K)+g(K)] = \sum f(K) + \sum g(K)$

$= \frac{n(n+1)(2n+1)}{6} + n \Rightarrow \cdot \sum_{K=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$

$\cdot \sum C = C\cdot n$

Sustituimos $n=4$

$\sum_{j=1}^{4}(j^2+1) = \frac{4(5)(9)}{6} + 4 = \frac{180}{6} + 4 = 30+4$

$$\boxed{\sum_{j=1}^{4}(j^2+1) = 34}$$

4. $$\sum_{n=1}^{10}\left[1+(-1)^n\right] = \sum_{n=1}^{10} 1 + \sum_{n=1}^{n} (-1)^n$$

$$\bullet \sum_{i=1}^{n} c^i = \frac{c^{n+1}-1}{c-1} - 1$$

$$= n + \frac{c^{n+1}-1}{c-1} - 1$$

Sustituimos $n=10$

$$\sum_{n=1}^{10}\left[1+(-1)^n\right] = 10 + \frac{(-1)^{10+1}-1}{(-1)-1} - 1$$

$$* \quad (-1)^n = \begin{cases} -1 & \text{para } n \text{ impar} \\ 1 & \text{para } n \text{ par} \end{cases}$$

$$\sum_{n=1}^{10}\left[1+(-1)^n\right] = 10 + \frac{-1-1}{-1-1} - 1$$

$$= 10 + \frac{-2}{-2} - 1$$

$$= 10 + 1 - 1$$

$$\boxed{\sum_{n=1}^{10}\left[1+(-1)^n\right] = 10}$$

*Nota.- También está la fórmula $\sum_{i=1}^{n} c^{i-1} = \frac{c^n-1}{c-1}$

$5.\ \sum_{k=0}^{5} k(k-1) = \sum_{k=0}^{5} (k^2 - k)$ $\Longleftarrow$

• Aplica la propiedad Distributiva

$a(b+c) = ab + ac$

$$\sum_{k=0}^{5} k(k-1) = \sum_{k=0}^{5} k^2 - \sum_{k=0}^{5} k$$

• $\sum_{k=0}^{n} k^2 = \sum_{k=1}^{n} k^2$

• $\sum_{k=0}^{n} k = \sum_{k=1}^{n} k$

*Ya que suma cero

$$= \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

sustituye $n=5$

$$\sum_{k=0}^{5} k(k-1) = \frac{(5)\cancel{(6)}(11)}{\cancel{6}} - \frac{(5)(6)}{2} = 5(11) - 15 = 55 - 15$$

$$\sum_{k=0}^{5} k(k-1) = 40$$

$6.\ \sum_{k=0}^{4} (k-2)(k-3) = \sum_{k=0}^{4} (k^2 - 5k + 6)$ $\Longleftarrow$

• Aplica el producto de binomios con término común

$(a+b)(a+c) = a^2 + (b+c)\cdot a + b\cdot c$

se suman

se multiplican

$$\sum_{k=0}^{4} (k-2)(k-3) = \sum_{k=0}^{4} k^2 - \sum_{k=0}^{4} 5k + \sum_{k=0}^{4} 6$$

$$\sum_{k=0}^{4} (k-2)(k-3) = \sum_{k=0}^{4} k^2 - 5\sum_{k=0}^{4} + \sum_{k=0}^{4} 6$$

Ojo

Sustituye $n=4$

$$= \frac{n(n+1)(2n-1)}{6} - 5\cdot\frac{n(n+1)}{2} + 6n$$

$$= \frac{4(5)(9)}{6} - 5\frac{(4)(5)}{2} + 6(4) = \frac{(36)(5)}{6} - \frac{(20)(5)}{2} + 24$$

$$\sum_{k=0}^{4} (k-2)(k-3) = (6)(5) - (10)(5) + 24 = 30 - 50 + 24 = 4$$

7. Calcule $\sum_{k=1}^{n}(k^2+3k+5) = \sum_{k=1}^{n} k^2 + 3\sum_{k=1}^{n} k + \sum_{k=1}^{n} 5$

$$\sum_{k=1}^{n}(k^2+3k+5) = \frac{n(n+1)(2n+1)}{6} + 3\,\frac{n(n+1)}{2} + 5n$$

• nótese que se repite 'n' por lo que se puede factorizar

$$\sum_{k=1}^{n}(k^2+3k+5) = n\left[\frac{(n+1)(2n+1)}{6} + 3\,\frac{(n+1)}{2} + 5\right]$$

• En los primeros dos términos se repite el 2, 2x3=6 y (n+1)

$$\sum_{k=1}^{n}(k^2+3k+5) = n\left\{\frac{(n+1)}{2}\left[\frac{(2n+1)}{3} + 3\right] + 5\right\}$$

• Realiza la suma $\underbrace{(2n+1)/3}_{\text{Tercios}} + 3$ ← Enteros a tercios

3x3=9

$$\sum_{k=1}^{n}(k^2+3k+5) = n\left\{\frac{n+1}{2}\left[\frac{(2n+1)}{3} + \frac{9}{3}\right] + 5\right\}$$

$$= n\left\{\frac{n+1}{2}\left[\frac{2n+1+9}{3}\right] + 5\right\}$$

$$= n\left\{\frac{n+1}{2}\left[\frac{2n+10}{3}\right] + 5\right\}$$

multiplicamos fracciones $\frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d}$

$$= n\left\{\frac{2n^2+10n+2n+10}{6} + 5\right\}$$

* Recuerda $(a+b)(c+d) = a\cdot c + a\cdot d + b\cdot c + b\cdot d$

$$\sum_{K=1}^{n}(K^2+3K+5)=n\left\{\frac{2n^2+10n+2n+10}{6}+5\right\}$$

Simplifica y suma sextos con 5 enteros

$6\times5=30$

$$\sum_{K=1}^{n}(K^2+3K+5)=n\left\{\frac{2n^2+12n+10}{6}+\frac{30}{6}\right\}$$

$$=n\left\{\frac{2n^2+12n+10+30}{6}\right\}$$

$$=n\left\{\frac{2n^2+12n+40}{6}\right\}$$

Los coeficientes 2, 12, 40 y el denominador 6 tienen mitad, por lo que se simplifican

$$\sum_{K=1}^{n}(K^2+3K+5)=n\left\{\frac{n^2+6n+20}{3}\right\}$$

Finalmente se multiplica por n a la expresión entre paréntesis

$$\boxed{\sum_{K=1}^{n}(K^2+3K+5)=\frac{n^3+6n^2+20n}{3}}$$

8. Determine $\sum_{K=1}^{n} (2K-3)^2$ ⟸

• Aplica el desarrollo del binomio al cuadrado

$(a+b)^2 = a^2+2ab+b^2$

$$\sum_{K=1}^{n} (2K-3)^2 = \sum_{K=1}^{n} (4K^2-12K+9)$$

• Continua con las reglas de los ejemplos anteriores

$$\sum_{K=1}^{n} (2K-3)^2 = 4\sum_{K=1}^{n} K^2 - 12\sum_{K=1}^{n} + \sum_{K=1}^{n} 9$$

Ahora aplicamos fórmulas

$$\sum_{K=1}^{n} (2K-3)^2 = 4\,\frac{n(n+1)(2n+1)}{6} - 12\,\frac{n(n+1)}{2} + 9n$$

$$= n\left[2\,\frac{(n+1)(2n+1)}{3} - 6(n+1) + 9\right]$$

$$= n\left\{2(n+1)\left[\frac{(2n+1)}{3} - 3\right] + 9\right\}$$

$$= n\left\{2(n+1)\left[\frac{(2n+1)-9}{3}\right] + 9\right\}$$

$$= n\left\{2(n+1)\left[\frac{2n-8}{3}\right] + 9\right\}$$

$$= n\left\{\frac{(2n+2)}{1}\cdot\left[\frac{2n-8}{3}\right] + 9\right\}$$

$$= n\left\{\frac{4n^2-16n+4n-16}{3} + 9\right\}$$

$$\sum_{K=1}^{n} (2K-3)^2 = n\left\{ \frac{4n^2 - 16n + 4n - 16}{3} + 9 \right\}$$

$$\sum_{K=1}^{n} (2K-3)^2 = n\left\{ \frac{4n^2 - 12n - 16}{3} + \frac{27}{3} \right\}$$

$$\sum_{K=1}^{n} (2K-3)^2 = n\left\{ \frac{4n^2 - 12n + 11}{3} \right\}$$

Finalmente el resultado:

$$\sum_{K=1}^{n} (2K-3)^2 = \frac{4n^3 - 12n^2 + 11n}{3}$$

9. Calcule $\sum_{k=1}^{n}(k^3+2k^2-k+4)$

$$\sum_{k=1}^{n}(k^3+2k^2-k+4)=\sum_{k=1}^{n}k^3+2\sum_{k=1}^{n}k^2-\sum_{k=1}^{n}k+\sum_{k=1}^{n}4$$

$$=\frac{n^2(n+1)^2}{4}+2\,\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}+4n$$

$$=n\left[\frac{n(n+1)^2}{4}+\frac{(n+1)(2n+1)}{3}-\frac{(n+1)}{2}+4\right]$$

$$=n\left\{(n+1)\left[\frac{n(n+1)}{4}+\frac{2n+1}{3}-\frac{1}{2}\right]+4\right\}$$

* Para sumar la fracción usamos el mínimo común múltiplo m.c.m.

$$\begin{array}{ccc|c} 4 & 3 & 2 & 2 \\ 2 & 3 & 1 & 2 \\ 1 & 3 & & 3 \\ & 1 & & \end{array}$$

m.c.m. $=2\times2\times3$

m.c.m. $=12$

$$\sum_{k=1}^{n}(k^3+2k^2-k+4)=n\left\{\frac{(n+1)}{1}\left[\frac{3n(n+1)+4(2n+1)-6}{12}\right]+4\right\}$$

$3=12\div4$ $\quad 4=12\div3$ $\quad 6=12\div2$ Denominadores

$$=n\left\{\frac{n+1}{1}\left[\frac{3n^2+3n+8n+4-6}{12}\right]+4\right\}$$

$$=n\left\{\frac{n+1}{1}\left[\frac{3n^2+11n-2}{12}\right]+4\right\}$$

$$=n\left\{\frac{3n^3+11n^2-2n+3n^2+11n-2}{12}+4\right\}$$

$$\sum_{K=1}^{n}(K^3+2K^2-K+4) = n\left\{\frac{3n^3+11n^2-2n+3n^2+11n-2}{12}+4\right\}$$

$$= n\left\{\frac{3n^3+14n^2+9n-2}{12}+\frac{48}{12}\right\}$$

$$= n\left\{\frac{3n^3+14n^2+9n+46}{12}\right\} \quad * \; \underline{48-2=46}$$

Resultado

$$\boxed{\sum_{K=1}^{n}(K^3+2K^2-K+4) = \frac{3n^4+14n^3+9n^2+46n}{12}}$$

10. Calcule el área bajo la gráfica de f entre a y b usando a) Rectángulos inscritos b) rectángulos circunscritos. Trace la gráfica y unos rectángulos típicos y marque las dimensiones con los nombres adecuados considere n = 100

i) $f(x)=2x+3;\ a=0,\ b=4,\ n=8$ - Rectángulos Inscritos

- Encuentra el valor de las bases (Δx) $n=8$

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{8} = \frac{4}{8} = \frac{1}{2} = 0.5$$

- Aplica la ecuación para rectángulos inscritos

$$A_n = \sum_{K=1}^{8} f(x_{K-1})\Delta x = \Delta x\left[f(x_0)+f(x_1)+f(x_2)+\cdots+f(x_7)\right]$$

- Calcula los valores de $f(x_{K-1})$

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
0	0.5	1	1.5	2	2.5	3	3.5

$f(x_0) = 2(0)+3 = 0+3 = 3$

$f(x_1) = 2(0.5)+3 = 1+3 = 4$

$f(x_2) = 2(1)+3 = 2+3 = 5$

$f(x_3) = 2(1.5)+3 = 3+3 = 6$

$f(x_4) = 2(2)+3 = 4+3 = 7$

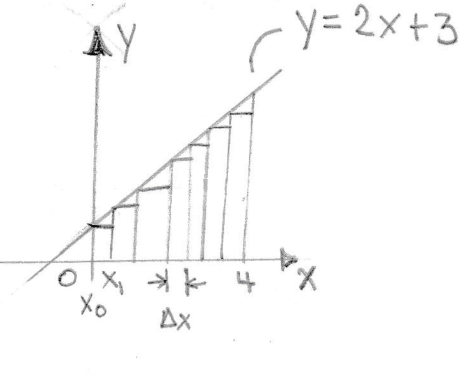
$f(x_5) = 2(2.5)+3 = 5+3 = 8$

$f(x_6) = 2(3)+3 = 6+3 = 9$

$f(x_7) = 2(3.5)+3 = 7+3 = 10$

- Sustituimos los valores en la suma explícita

$$A_n = 0.5\left[3+4+5+6+7+8+9+10\right] = \underline{21u^2}*$$



ii) $f(x)=2x+3$; $a=0$, $b=4$, $n=8$ - Rectángulos Circunscritos

- Encuentra el valor de las bases (Δx)

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{8} = \frac{4}{8} = \frac{1}{2} = 0.5$$

- Aplica la ecuación para rectángulos Circunscritos

$$A_n = \sum_{K=1}^{8} f(x_K)\,\Delta x = \Delta x\,[f(x_1)+f(x_2)+\cdots+f(x_8)]$$

- Calcula los valores de $f(x_K)$

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
0.5	1	1.5	2	2.5	3	3.5	4

$f(x_1)=2(0.5)+3=1+3=4$

$f(x_2)=2(1)+3=2+3=5$

$f(x_3)=2(1.5)+3=3+3=6$

$f(x_4)=2(2)+3=4+3=7$

$f(x_5)=2(2.5)+3=5+3=8$

$f(x_6)=2(3)+3=6+3=9$

$f(x_7)=2(3.5)+3=7+3=10$

$f(x_8)=2(4)+3=8+3=\underline{11}$

- Sustituimos los valores en la suma explícita

$$A_n = 0.5[4+5+6+7+8+9+10+11] = \underline{30u^2}$$

iii) Aproxima el área para $f(x) = 2x+3$, $a=0$, $b=4$, $n=100$ con rectángulos inscritos

- Se determina $\Delta x = \frac{b-a}{n} = \frac{4-0}{100} = \frac{4}{100} = \frac{2}{50} =$

- Dado que $A_n = \sum_{k=1}^{n} f(x_{k-1})\Delta x$, $x_{k-1} = (k-1)\Delta x$

por lo tanto $f(x_{k-1}) = 2x_{k-1} + 3$

$$f(x_{k-1}) = 2(k-1)\Delta x + 3$$

- El área por lo tanto es:

$$A_n = \sum_{k=1}^{n} [2(k-1)\Delta x + 3]\Delta x$$

$$A_n = \sum_{k=1}^{n} [2k\Delta x - 2\Delta x + 3]\Delta x$$

$$A_n = \sum_{k=1}^{n} [2k\Delta x^2 - 2\Delta x^2 + 3\Delta x]$$

$$A_n = 2\Delta x^2 \sum_{k=1}^{n} k - \Delta x^2 \sum_{k=1}^{n} 2 + \Delta x \sum_{k=1}^{n} 3$$

$$A_n = \cancel{2}\Delta x^2 \frac{n(n+1)}{\cancel{2}} - \Delta x^2 \cdot 2n + \Delta x \cdot 3n$$

$$A_n = \Delta x^2 [n(n+1) - 2n] + \Delta x \cdot 3n \qquad n = 100,\ \Delta x = \frac{4}{100}$$

$$A_n = \left(\frac{4}{100}\right)^2 [100(101) - 2(100)] + \frac{4}{\cancel{100}} \cdot 3(\cancel{100})$$

$$A_n = \frac{4^2}{100^{\cancel{2}}} \cdot \cancel{100}(101-2) + 12 = \frac{16}{100} \cdot 99 + 12$$

$$A_n = 15.84 + 12 \qquad \boxed{A_n = 27.84}$$

Si $n = 1000$

$$A_n = \left(\frac{4}{1000}\right)^{\cancel{2}} \left[\cancel{1000}(1001) - 2(\cancel{1000})\right] + \frac{4}{\cancel{1000}} \cdot 3(\cancel{1000})$$

$$A_n = \frac{4^2}{1000}\left[1001-2\right] + 12 = 15.984 + 12 = 27.984$$

También se puede sintetizar

$$A_n = \underline{\Delta x^2}\, \underline{n}(n+1) - \underline{\Delta x^2} \cdot 2\underline{n} + \Delta x\, 3n$$

Como $\Delta x = \frac{4}{n}$

$$A_n = \Delta x^2 n\left[n+1-2\right] + 3n\,\Delta x$$

$$A_n = \frac{4^2}{n^{\cancel{2}}} \cancel{n}\left[n-1\right] + 3\cancel{n} \cdot \frac{4}{\cancel{n}}$$

$$A_n = 4^2\left[n-1\right] + 12$$

$n = 1000$

$$A_n = \frac{4^2}{1000}(1000-1) + 12 = \frac{16}{1000}(999) + 12$$

$$A_n = 15.984 + 12 = \underline{27.984}$$