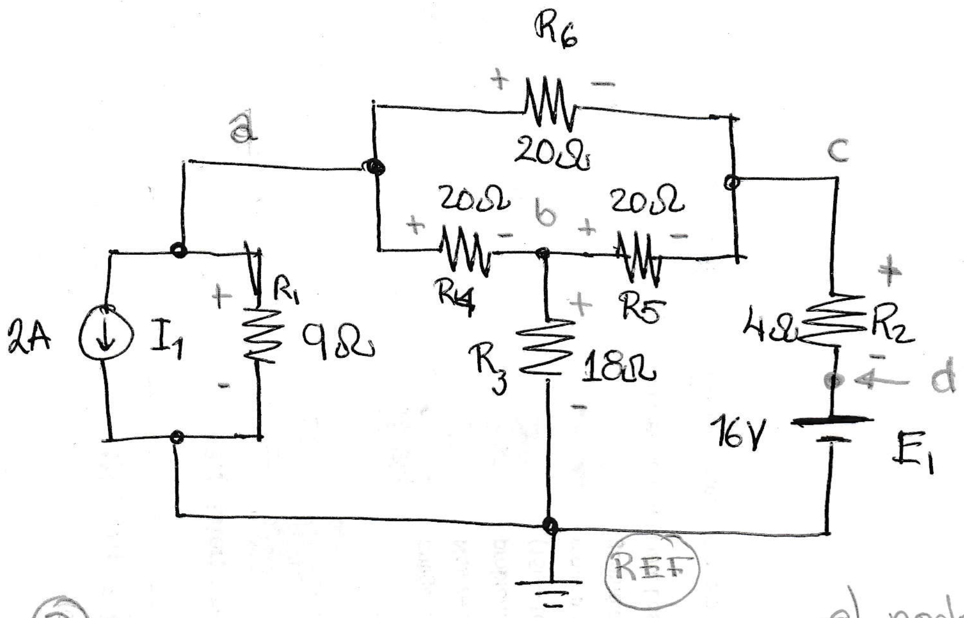
7. Método de nodos

H1



① Definimos los nodos en el circuito

• Como la fuente de tensión E_1 está conectando al nodo 'd' con el nodo de referencia, el voltaje en el nodo $V_d = E_1 = 16V$

② Son asignadas las polaridades al circuito.

③ Se escriben las ecuaciones nodales

Nodo a| $I_1 + \frac{V_a}{R_1} + \frac{V_a - V_b}{R_4} + \frac{V_a - V_c}{R_6} = \emptyset$ (1

Nodo b| $\frac{V_b}{R_3} - \frac{V_a - V_b}{R_4} + \frac{V_b - V_c}{R_5} = \emptyset$

Nodo c| $-\frac{V_a - V_c}{R_6} - \frac{V_b - V_c}{R_5} + \frac{V_c - V_d}{R_2} = \emptyset$

Nodo d| $V_d = E_1$

Aplicando la LCK

④ Agrupando las tensiones en las ecuaciones nodales

H2

i) $V_a\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_6}\right) - V_b\left(\frac{1}{R_4}\right) - V_c\left(\frac{1}{R_6}\right) = -I_1$

ii) $-V_a\left(\frac{1}{R_4}\right) + V_b\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right) - V_c\left(\frac{1}{R_5}\right) = \emptyset$

iii) $-V_a\left(\frac{1}{R_6}\right) - V_b\left(\frac{1}{R_5}\right) + V_c\left(\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_6}\right) - V_d\left(\frac{1}{R_2}\right) = \emptyset$

iv) $V_d = E_1$

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⑤ Se transforma el sistema de ecuaciones a forma matricial, y se cambian las resistencias por conductancias.

$$\frac{1}{R_n} = S_n$$

$$\begin{bmatrix} S_1+S_4+S_6 & -S_4 & -S_6 \\ -S_4 & S_4+S_5 & -S_5 \\ -S_6 & -S_5 & S_2+S_5+S_6 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} -I_1 \\ \emptyset \\ S_2 \cdot E_1 \end{bmatrix}$$

⑥ sustituimos los valores dados (Cumple propiedad de simetría)

$$\begin{bmatrix} 1/9+1/20+1/20 & -1/20 & -1/20 \\ -1/20 & 1/18+1/20+1/20 & -1/20 \\ -1/20 & -1/20 & 1/4+1/20+1/20 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} -2 \\ \emptyset \\ 16/4 \end{bmatrix}$$

$$\begin{bmatrix} 19/90 & -1/20 & -1/20 \\ -1/20 & 7/45 & -1/20 \\ -1/20 & -1/20 & 7/20 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} -2 \\ \emptyset \\ 4 \end{bmatrix}$$

7) Se resuelve el sistema matricial por Regla de Cramer (H3)

- Matriz de conductancias

$$\Delta = \begin{bmatrix} 19/90 & -1/20 & -1/20 \\ -1/20 & 7/45 & -1/20 \\ -1/20 & -1/20 & 7/20 \end{bmatrix} = \frac{19}{90}\begin{vmatrix} 7/45 & -1/20 \\ -1/20 & 7/20 \end{vmatrix} - \left(-\frac{1}{20}\right)\begin{vmatrix} -1/20 & -1/20 \\ -1/20 & 7/20 \end{vmatrix} + \left(-\frac{1}{20}\right)\begin{vmatrix} -1/20 & 7/45 \\ -1/20 & -1/20 \end{vmatrix}$$

$$\Delta = \frac{19}{90}\left(\frac{187}{3600}\right) - \left(-\frac{1}{20}\right)\left(-\frac{1}{50}\right) + \left(-\frac{1}{20}\right)\left(\frac{37}{3600}\right) = \frac{49}{5184}$$

$$V_a = \frac{\begin{bmatrix} -2 & -1/20 & -1/20 \\ \varnothing & 7/45 & -1/20 \\ 4 & -1/20 & 7/20 \end{bmatrix}}{\Delta} = \frac{-2\begin{vmatrix} 7/45 & -1/20 \\ -1/20 & 7/20 \end{vmatrix} - \left(-\frac{1}{20}\right)\begin{vmatrix} \varnothing & -1/20 \\ 4 & 7/20 \end{vmatrix} + \left(-\frac{1}{20}\right)\begin{vmatrix} \varnothing & 7/45 \\ 4 & -1/20 \end{vmatrix}}{49/5184}$$

$$V_a = \frac{-2\left(\frac{187}{3600}\right) - \left(-\frac{1}{20}\right)\left(+\frac{1}{5}\right) + \left(-\frac{1}{20}\right)\left(-28/45\right)}{49/5184} = \frac{-113/1800}{49/5184} = \frac{8136}{1225} V$$

$$\boxed{V_a \approx -6.6416 V}$$

$$V_b = \frac{\begin{vmatrix} 19/90 & -2 & -1/20 \\ -1/20 & \varnothing & -1/20 \\ -1/20 & 4 & 7/20 \end{vmatrix}}{\Delta} = \frac{\frac{19}{90}\begin{vmatrix} \varnothing & -1/20 \\ 4 & 7/20 \end{vmatrix} - (-2)\begin{vmatrix} -1/20 & -1/20 \\ -1/20 & 7/20 \end{vmatrix} + \left(-\frac{1}{20}\right)\begin{vmatrix} -\frac{1}{20} & \varnothing \\ -1/20 & 4 \end{vmatrix}}{49/5184}$$

$$= \frac{\frac{19}{90}\left(\frac{1}{5}\right) - (-2)\left(-\frac{1}{50}\right) + \left(-\frac{1}{20}\right)\left(-\frac{1}{5}\right)}{49/5184} = \frac{11/900}{49/5184} = \frac{1584}{1225} V$$

$$\boxed{V_b \approx 1.2931 V}$$

H4

$$V_c = \frac{\begin{vmatrix} 19/90 & -1/20 & -2 \\ -1/20 & 7/45 & \varnothing \\ -1/20 & -1/20 & 4 \end{vmatrix}}{\Delta} = \frac{\frac{19}{90}\begin{vmatrix} 7/45 & \varnothing \\ -1/20 & 4 \end{vmatrix} + \frac{1}{20}\begin{vmatrix} -\frac{1}{20} & \varnothing \\ -1/20 & 4 \end{vmatrix} - 2\begin{vmatrix} -\frac{1}{20} & 7/45 \\ -\frac{1}{20} & -1/20 \end{vmatrix}}{49/5184}$$

$$= \frac{\frac{19}{90}\left(\frac{28}{45}\right) + \frac{1}{20}\left(-\frac{1}{5}\right) - 2\left(\frac{37}{3600}\right)}{49/5184} = \frac{\frac{1633}{16200}}{49/5184} = \frac{8465472}{793800}$$

$$= \frac{4232736}{396900} = \frac{1058184}{99225}\text{V} \approx 10.6645\text{V}$$

Resultados

Las tensiones en el circuito son

$$V_a = -6.6416\text{V}$$
$$V_b = 1.2931\text{V}$$
$$V_c = 10.6645\text{V}$$