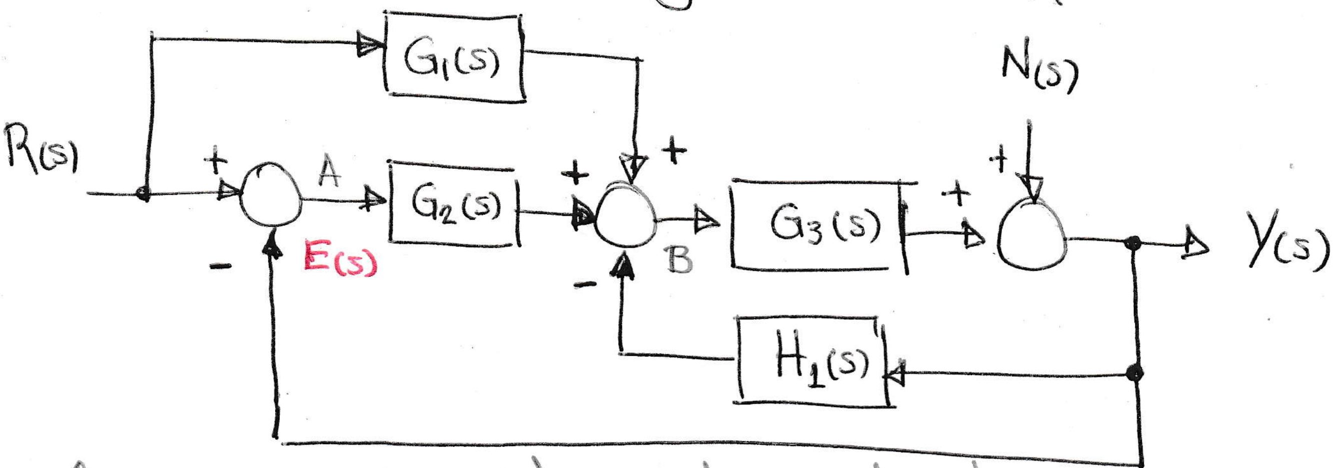
① Reducción del diagrama de bloques



- Considerando que el sistema está inicialmente relajado, con error cero, solo la entrada es la perturbación N

$A = \emptyset - Y$

$B = G_1 \cdot \emptyset + G_2 A - H_1 \cdot Y_n$

$B = G_2(-Y_n) - H_1 Y_n$

$= -G_2 Y_n - H_1 Y_n$

$B = -Y_n(G_2 + H_1)$

$Y_n = N + G_3 B$

$Y_n = N + G_3[-Y_n(G_2+H_1)]$

$Y_n = N - G_3 Y_n (G_2 + H_1)$

$Y_n[1 + G_3(G_2+H_1)] = N$

Función de transferencia solo con disturbio $\longrightarrow$ $\dfrac{Y_n}{N} = \dfrac{1}{1 + G_3(G_2+H_1)}$

- Ahora solo consideramos la referencia con disturbio nulo

$A = R - Y_R$

$B = RG_1 + G_2 A - H_1 Y_R$

$B = RG_1 + G_2(R - Y_R) - H_1 Y_R$

$B = RG_1 + G_2 R - G_2 Y_R - H_1 Y_R$

$B = R(G_1 + G_2) - Y_R(G_2 + H_1)$

$Y_R = BG_3$

$Y_R = [R(G_1+G_2) - Y_R(G_2+H_1)]G_3$

$Y_R = RG_3(G_1+G_2) - Y_R G_3(G_2+H_1)$

$Y_R[1 + G_3(G_2+H_1)] = RG_3(G_1+G_2)$

Función de transferencia sin disturbio $\dfrac{Y_R}{R} = \dfrac{G_3(G_1+G_2)}{1 + G_3(G_2+H_1)}$

Aplicando el teorema de la superposición a sistemas lineales

$$Y = Y_n + Y_R = \frac{N}{1 + G_3(G_2+H_1)} + \frac{RG_3(G_1+G_2)}{1 + G_3(G_2+H_1)} = [N + RG_3(G_1+G_2)] \cdot \frac{1}{1 + G_3(G_2+H_1)}$$

Sea $G_1(s) = \frac{1}{s+2}$, $G_2(s) = \frac{1}{s^2+1}$, $G_3(s) = \frac{1}{s+3}$, $H_1(s) = \frac{1}{s+0.5}$

$N(s) = \frac{1}{s+1}$

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$$1 + G_3(G_2 + H_1) = 1 + \frac{1}{s+3}\left(\frac{1}{s^2+1} + \frac{1}{s+0.5}\right)$$

$$= 1 + \frac{1}{s+3} \cdot \frac{(s+0.5)+(s^2+1)}{(s^2+1)(s+0.5)}$$

$$= 1 + \frac{1}{s+3} \cdot \frac{s^2+s+1.5}{(s^2+1)(s+0.5)}$$

$$\frac{-\frac{1}{2} \pm \sqrt{1-6.0}}{2}$$

$$-\frac{1}{2} \pm \frac{\sqrt{5}}{2}j$$

$$\boxed{1 + G_3(G_2+H_1) = 1 + \frac{(s+1/2+\sqrt{5}/2\,j)(s+1/2-\sqrt{5}/2\,j)}{(s+0.5)(s+3)(s^2+1)}}$$ *

$$G_3(G_1+G_2) = \frac{1}{s+3}\left(\frac{1}{s+2} + \frac{1}{s^2+1}\right)$$

$$\pm \frac{\sqrt{1-12}}{2} = \frac{\sqrt{-11}}{2} = \frac{\sqrt{11}}{2}j$$

$$= \frac{1}{s+3}\left(\frac{s^2+1+s+2}{(s+2)(s^2+1)}\right)$$

$$= \frac{1}{s+3} \cdot \frac{s^2+s+3}{(s+2)(s^2+1)}$$

$$\boxed{G_3(G_1+G_2) = \frac{(s+1/2+\sqrt{11}/2\cdot j)(s+1/2-\sqrt{11}/2\,j)}{(s+2)(s+3)(s^2+1)}}$$ *

Para

$$\frac{Y_R}{R} = \frac{\dfrac{s^2+s+3}{(s+2)(s+3)(s^2+1)}}{1 + \dfrac{s^2+s+1.5}{(s+0.5)(s+3)(s^2+1)}} = \frac{\dfrac{s^2+s+3}{(s+2)(s+3)(s^2+1)}}{\dfrac{(s+0.5)(s+3)(s^2+1) + s^2+s+1.5}{(s+0.5)(s+3)(s^2+1)}}$$

$$\frac{Y_R}{R} = \frac{(s+0.5)\cancel{(s+3)}\cancel{(s^2+1)}(s^2+s+3)}{(s+2)\cancel{(s+3)}\cancel{(s^2+1)}\left[(s+0.5)(s+3)(s^2+1) + s^2+s+1.5\right]}$$

$$\frac{Y_R}{R} = \frac{(s+0.5)(s^2+s+3)}{(s+0.5)(s+2)(s+3)(s^2+1) + (s+2)(s^2+s+1.5)}$$ → Diverge

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Función de transferencia de lazo cerrado para el sistema dado

$$\boxed{\frac{Y_R}{R} = \frac{s^3+1.5s^2+3.5s+1.5}{s^5+5.5s^4+10.5s^3+11.5s^2+12s+6}}$$

Salida de Y cuando $R(s) = \frac{1}{s}$ y $N(s) = \frac{1}{s+1}$

$$Y = \underbrace{\left[\frac{1}{s+1} + \frac{1}{s} \cdot \frac{s^2+s+3}{(s+2)(s+3)(s^2+1)}\right]}_{N \; + \; R \cdot \; G_3(G_1+G_2)} \cdot \frac{1}{1 + \frac{s^2+s+1.5}{(s+0.5)(s+3)(s^2+1)}}$$

$$Y = \left[\frac{1}{(s+1)} + \frac{s^2+s+3}{s(s+2)(s+3)(s^2+1)}\right] \cdot \frac{(s+0.5)(s+3)(s^2+1)}{[(s+0.5)(s+3)(s^2+1) + s^2+s+1.5]}$$

$$Y = \frac{[s(s+2)(s+3)(s^2+1) + s^2+s+3]}{s(s+1)(s+2)\cancel{(s+3)}\cancel{(s^2+1)}} \cdot \frac{(s+0.5)\cancel{(s+3)}\cancel{(s^2+1)}}{[(s+0.5)(s+3)(s^2+1) + s^2+s+1.5]}$$

$$Y = \frac{(s+0.5)[s(s+2)(s+3)(s^2+1) + (s^2+s+3)]}{s(s+1)(s+2)\,[(s+0.5)(s+3)(s^2+1) + (s^2+s+1.5)]}$$

salida del sistema en términos de 's'

NOTA: Falta determinar la salida con respecto al tiempo y la estimación de error pero diverge.