UNIT - 1

Ordinary Differential Equation of Higher Order

Differential Equation :-

An Equation consisting of dependent variable, independent variable and derivative of dependent with respect to independent variable.

dependent variable ← $y = f(x)$ ← independent variable

$$\frac{dy}{dx} = f'(x) = y'(x)$$

$$\frac{d^2y}{dx^2} = f''(x) = y''(x)$$

Eg:- $\frac{dy}{dx} + 2y = e^x$

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x^2$$

Types of differential Equation

According to linearity

① Linear diff. Equation

② Non- linear diff Eqn.

According to No. of independent variable in equation

① Ordinary diff. Eqn

② Partial diff Eqn.

Linear differential Equation

An Eqn is said to be linear if y and its derivative have their power 1 and there is no term is product of y and its derivative. Otherwise it is said to be Non-Linear.

Ordinary differential Equation

A differential Eqn having only one independent variable is called ordinary diff. Eqn

$y = f(x)$

Eg: $y\frac{dy}{dx} + 2y = e^x$

Partial differential Equation

A diff. Eqn having more than one independent variable is called partial diff. Equation.

$z = f(x, y)$

Eg: $y\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} = e^{x+y}$

Solution of differential Equation

$$\frac{d^2y}{dt^2} \propto -y$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

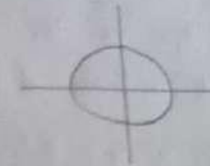
$y = a\sin\omega t$

$$\frac{dy}{dt} = a\omega\cos\omega t$$

$$\frac{d^2y}{dt^2} = -a^2\omega^2\sin\omega t$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$



$y = b\cos\omega t$

$$\frac{dy}{dt} = -b\omega\sin\omega t$$

$$\frac{d^2y}{dt^2} = -b\omega^2\cos\omega t$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

$y = a\sin\omega t + b\cos\omega t$

General Soln = ?

$$\frac{dy}{dt} = \omega a\cos\omega t - \omega b\sin\omega t$$

$$\frac{d^2y}{dt^2} = -\omega^2 a\sin\omega t - \omega^2 b\cos\omega t$$

$$= -\omega^2(a\sin\omega t + b\cos\omega t)$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

General Solution

Arbitrary const = Order of diff Eqn

Particular Solution

If we put any particular value of arbitrary const. in given solution.

No. of particular Soln = Infinite.

Order of a differential Eqn

Order of a diff. Eqn is the order of highest order derivative present in the differential Eqn.

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = e^x$$

order = 2

Degree of a differential Equation

Degree of a diff. Eqn is the highest exponent of highest order derivative present in the diff Eqn when the equation is free from radicals, fraction and transcedental terms

linear $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = e^x$

Degree = 1

Non linear $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y = e^x \rightarrow$ order = 2, degree = 1

Eg: $\rho = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$ order = 2, degree = 2

$$\rho\frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

$$\rho^2\left(\frac{d^2y}{dx^2}\right)^2 = \left[1+\left(\frac{dy}{dx}\right)^2\right]^3$$

$$\rho^2\left(\frac{d^2y}{dx^2}\right)^2 = 1+\left(\frac{dy}{dx}\right)^3 + 3\left(\frac{dy}{dx}\right)^4 + 3\left(\frac{dy}{dx}\right)^2$$

$$\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^6 + \frac{dy}{dx} + y = \cos x$$

order = 3

Degree = 4

$\cos\left(\frac{dy}{dx}\right) + \frac{d^2y}{dx^2} = \tan x$ order = 2, Degree = 1

$\frac{dy}{dx} + \cos\left(\frac{d^2y}{dx^2}\right) = \tan x$ order = 2, Degree = undefined

Ordinary diff. Eqn of first order & first degree

Standard form $\frac{dy}{dx} = f(x, y)$

① Variable separable

$$\frac{dy}{dx} = f(x,y) \Rightarrow \int d_1(y)\,dy = \int d_2(x)\,dx$$

Eg: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$$\int\frac{1}{1+y^2}\,dy = \int\frac{1}{1+x^2}\,dx$$

$$\tan^{-1}y = \tan^{-1}x + \tan^{-1}c$$

$$\tan^{-1}y - \tan^{-1}x = \tan^{-1}c$$

$$\tan^{-1}\frac{y-x}{1+xy} = \tan^{-1}c$$

$$\boxed{\frac{y-x}{1+xy} = c}$$

$$\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$$

$$\int(\sin y + y\cos y)\,dy = \int x(2\log x + 1)\,dx$$

$$-\cos y + y\sin y - \int \sin y\,dy = 2\left[\frac{x^2}{2}\log x - \int \frac{1}{x}\cdot\frac{x^2}{2}\,dx\right] + \frac{x^2}{2} + c$$

$$-\cos y + y\sin y + \cos y = x^2\log x - \frac{x^2}{4} + \frac{x^2}{2} + c$$

$$y\sin y = x^2\log x + \frac{x^2}{4} + c$$

$$\frac{dy}{dx} = (x+y)^2 \qquad \text{let } x+y = v$$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} - 1 = v^2$$

$$\frac{dv}{dx} = 1 + v^2$$

$$\int \frac{dv}{1+v^2} = \int dx$$

$$\tan^{-1} v = x + c$$

$$\tan^{-1}(x+y) = x + c$$

(2) Homogeneous Equation

$$\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)} \qquad (y = vx)$$

$$x^2 dy + y(x+y)\,dx = 0$$

$$\frac{dy}{dx} = -\frac{y(x+y)}{x^2} \qquad y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = -\frac{vx(x+vx)}{x^2}$$

$$= -v(1+v)$$

$$v + x\frac{dv}{dx} = -v - v^2$$

$$x\frac{dv}{dx} = -2v - v^2$$

$$\int \frac{1}{2v+v^2}\,dv = -\int \frac{1}{x}\,dx$$

$$\int \frac{1}{v(v+2)}\,dv = \int -\frac{1}{x}\,dx$$

Partial fraction

$$\frac{1}{2}\int\left(\frac{1}{v} - \frac{1}{v+2}\right)dv = -\int \frac{1}{x}\,dx$$

$$\frac{1}{2}\left[\log v - \log(v+2)\right] = -\log x + \log c$$

$$\frac{1}{2}\log\frac{v}{v+2} = \log\frac{c}{x}$$

$$\left(\frac{v}{v+2}\right)^{1/2} = \frac{c}{x}$$

$$\left(\frac{y/x}{y/x + 2}\right)^{1/2} = \frac{c}{x}$$

$$\frac{y}{y+2x} = \frac{c^2}{x^2}$$

$$\boxed{\frac{y}{y+2x} = \frac{c'}{x^2}} \qquad \{c^2 = c'\}$$

Reducible to Homogeneous

Standard form $\dfrac{dy}{dx} = \dfrac{ax+by+c}{a'x+b'y+c'} \quad \left(\dfrac{a}{a'} \neq \dfrac{b}{b'}\right)$

$$\frac{dy}{dx} = \frac{ax + bvx + c}{a'x + b'vx + c'} \Rightarrow \text{Not Homogeneous}$$

let $x = X + h \Rightarrow dx = dX$

$y = Y + K \Rightarrow dy = dY$

$$\frac{dY}{dX} = \frac{a(X+h) + b(Y+K) + c}{a'(X+h) + b'(Y+K) + c'}$$

$$= \frac{aX + bY + (ah + bk + c)}{a'X + b'Y + (a'h + b'k + c')}$$

homogeneous diff eqn

$$\left.\begin{array}{l} ah + bk + c = 0 \\ a'h + b'k + c' = 0 \end{array}\right] h, k = ?$$

$$\boxed{\frac{dY}{dX} = \frac{aX + bY}{a'X + b'Y}}$$

$$Y = vX$$

$$\frac{dY}{dX} = v + X\frac{dv}{dX}$$

$$v + X\frac{dv}{dX} = \frac{aX + bvX}{a'X + b'vX} = \frac{a + bv}{a' + b'v}$$

$$\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3} \qquad \left(\frac{1}{2} \neq \frac{2}{1}\right)$$

$$x = X + h, \quad y = Y + k$$

$$dx = dX, \quad dy = dY$$

$$\frac{dY}{dX} = \frac{X + h + 2(Y + k) - 3}{2(X + h) + Y + k - 3}$$

$$= \frac{X + 2Y + (h + 2k - 3)}{2X + Y + (2h + k - 3)}$$

$$\left.\begin{array}{l} h + 2k - 3 = 0 \\ 2h + k - 3 = 0 \end{array}\right\} \begin{array}{l} h = 1 \\ k = 1 \end{array}$$

$$\left.\frac{dY}{dX} = \frac{X + 2Y}{2X + Y}\right]$$ homogeneous

$$Y = vX$$

$$\frac{dY}{dX} = v + X\frac{dv}{dX}$$

$$v + X\frac{dv}{dX} = \frac{X + 2vX}{2X + vX} = \frac{1 + 2v}{2 + v}$$

$$X\frac{dv}{dX} = \frac{1 + 2v}{2 + v} - v = \frac{1 + 2v - 2v - v^2}{2 + v}$$

$$X\frac{dv}{dX} = \frac{1 - v^2}{2 + v}$$

$$\int \frac{2 + v}{1 - v^2}\, dv = \int \frac{1}{X}\, dX$$

$1 - v^2 = t$
$-2v\, dv = dt$
$v\, dv = -\frac{1}{2} dt$

$$\int \frac{2}{1 - v^2}\, dv + \int \frac{v}{1 - v^2}\, dv = \int \frac{1}{X}\, dX$$

$$2 \cdot \frac{1}{2} \log \frac{1 + v}{1 - v} - \frac{1}{2} \int \frac{dt}{t} = \int \frac{1}{X}\, dX$$

$$\log \frac{1 + v}{1 - v} - \frac{1}{2} \log t = \log X + \log C$$

$$\frac{1 + v}{1 - v} \cdot \frac{1}{\sqrt{t}} = CX$$

$$\frac{1 + Y/X}{1 - Y/X} \cdot \frac{1}{\sqrt{1 - Y^2/X^2}} = CX$$

$$\frac{X + Y}{X - Y} \cdot \frac{X}{\sqrt{X^2 - Y^2}} = CX$$

$$\Rightarrow \frac{\sqrt{X + Y}}{(X - Y)^{3/2}} = C$$

$X = x - h = x - 1$
$Y = y - k = y - 1$

$$\frac{\sqrt{x - 1 + y - 1}}{(x - 1 - y + 1)^{3/2}} = C$$

$$\boxed{\frac{\sqrt{x + y - 2}}{(x - y)^{3/2}} = C}$$

Linear differential Equation

Standard form – $\frac{dy}{dx} + Py = Q$ (linear in y)

P, Q function of x

$\frac{dx}{dy} + Px = Q$ (linear in x)

P, Q function of y

Integrating factor (IF) = $e^{\int P dx}$

Soln → $y(IF) = \int Q(IF)\,dx + c$

Similarly, $I.F = e^{\int P dy}$

Soln → $x(IF) = \int Q(IF)\,dy + c$

Solve $(1+x^2)\frac{dy}{dx} + 2yx - 4x^2 = 0$ type → first order, first degree

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$$

Linear in y

$$P = \frac{2x}{1+x^2}, \quad Q = \frac{4x^2}{1+x^2}$$

$$I.F = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1+x^2$$

Soln $y.(IF) = \int Q(IF)\,dx + c$

$$y(1+x^2) = \int \frac{4x^2}{1+x^2}\cdot 1+x^2\,dx + c$$

$$= \int 4x^2 dx + c$$

$$\boxed{y(1+x^2) = \frac{4x^3}{3} + c}$$

Ques: $(x+y+1)\frac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{x+y+1}$$

$$x+y+1 = \frac{dx}{dy}$$

$$\frac{dx}{dy} - x = y+1 \quad \text{(linear in } x\text{)}$$

$$IF = e^{\int(-1)dy} = e^{-y}$$

$$xe^{-y} = \int(y+1)e^{-y}dy + c$$

$$xe^{-y} = -(y+1)e^{-y} + \int e^{-y}dy + c$$

$$= -(y+1)e^{-y} - e^{-y} + c$$

$$x = -y-1-1+ce^{y}$$

$$x = -y-2+ce^{y}$$

Reducible to linear form

Bernoulli form

$$\frac{dy}{dx} + Py = Qy^n$$

$$\frac{1}{y^n}\frac{dy}{dx} + P\frac{y}{y^n} = Q$$

$$\frac{1}{y^n}\frac{dy}{dx} + P\frac{1}{y^{n-1}} = Q$$

$$\frac{1}{y^{n-1}} = v$$

$$-(n-1)y^{-n+1-1}\frac{dy}{dx} = \frac{dv}{dx}$$

$$-(n-1)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{1}{y^n}\frac{dy}{dx} = -\frac{1}{(n-1)}\frac{dv}{dx}$$

$$-\frac{1}{(n-1)}\frac{dv}{dx} + Pv = Q$$

$$\frac{dv}{dx} + \underbrace{P(1-n)}_{\text{const of } x}{}^{P'}v = (1-n)Q^{\,Q'}$$

$$\boxed{\frac{dv}{dx} + P'v = Q'}$$ Linear in v

$$3\frac{dy}{dx} + \frac{2}{x+1}y = \frac{x^3}{y^2}$$

$$\frac{dy}{dx} + \frac{2}{3(x+1)}y = \frac{x^3}{3y^2}$$

$$\frac{dy}{dx} + \frac{2}{3(x+1)}y = \frac{x^3}{3}y^{-2}$$

$$\frac{1}{y^{-2}}\frac{dy}{dx} + \frac{2}{3(x+1)}\frac{y}{y^{-2}} = \frac{x^3}{3}$$

$$y^2\frac{dy}{dx} + \frac{2}{3(x+1)}y^3 = \frac{x^3}{3}$$

$$y^3 = v$$

$$3y^2\frac{dy}{dx} = \frac{dv}{dx}$$

$$y^2\frac{dy}{dx} = \frac{1}{3}\frac{dv}{dx}$$

$$\frac{1}{3}\frac{dv}{dx} + \frac{2}{3(x+1)}v = \frac{x^3}{3}$$

$$\frac{dv}{dx} + \frac{2}{x+1}v = x^3 \quad] \text{ linear in } v$$

$$P = \frac{2}{x+1} \qquad Q = x^3$$

$$IF = e^{\int\frac{2}{x+1}dx} = e^{2\log(1+x)} = (1+x)^2$$

$$v(1+x)^2 = \int x^3(1+x^2)\,dx + c$$

$$= \int x^3(1+x^2+2x^2)\,dx + c$$

$$v(1+x)^2 = \frac{x^6}{6} + \frac{2x^5}{5} + \frac{x^3}{3} + c$$

$$\frac{dy}{dx}(x^2y^3 + xy) = 1$$

$$\frac{dx}{dy} = x^2y^3 + xy$$

$$\frac{dx}{dy} - yx = x^2y^3 \qquad \left(\frac{dx}{dy} + Px = Qx^n\right)$$

P, Q → function of y

$$\frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3$$

$$-\frac{1}{x} = v$$

$$\frac{1}{x^2}\frac{dx}{dy} = \frac{dv}{dy}$$

$$\frac{dv}{dy} + yv = y^3 \quad] \text{ linear in } v \text{ independent variable } y$$

$$P = y \qquad Q = y^3$$

$$(y\log x - 1)\,y\,dx = x\,dy$$

$$(y^2\log x - y)\,dx = x\,dy$$

$$x\frac{dy}{dx} = y^2\log x - y$$

$$x\frac{dy}{dx} + y = y^2\log x$$

$$\frac{dy}{dx} + \frac{y}{x} = y^2\frac{\log x}{x}$$

$$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{xy} = \frac{\log x}{x}$$

let $\frac{1}{y} = v$

$$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{1}{y^2}\frac{dy}{dx} = -\frac{dv}{dx}$$

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$$-\frac{dv}{dx} + \frac{v}{x} = \frac{\log x}{x}$$

$$\frac{dv}{dx} - \frac{1}{x} v = -\frac{\log x}{x}$$

$$P = -\frac{1}{x} \qquad Q = -\frac{\log x}{x}$$

$$I.F = e^{\int P dx}$$

$$= e^{\int -1/x \, dx} = e^{-\log x} = \frac{1}{x}$$

$$v \cdot \frac{1}{x} = \int -\frac{\log x}{x^2} dx + c$$

Exact differential Equation

standard form

$Mdx + Ndy = 0$

M, N are function of x, y

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$] condn of exact diff eqn

Soln $\int_{y \text{ const}} M dx + \int (\text{term of } N \text{ free from } x)\, dy = c$

$$(x^2 - ay)dx = (ax - y^2)dy$$

$$\underbrace{(x^2 - ay)}_{M}dx + \underbrace{(y^2 - ax)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = -a \qquad \frac{\partial N}{\partial x} = -a$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \text{exact diff. eqn}$$

$$\int_{y \text{ const}} (x^2 - ay)dx + \int y^2 dy = c$$

$$\frac{x^3}{3} - \ldots \frac{y^3}{3} = c$$

$$x^3 + y^3 - 3axy = 3c$$

$$x^3 + y^3 - 3axy = c'$$

Reducible to Exact diff. Eqn

If $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ → not exist

Condn → If the Equation $Mdx + Ndy = 0$ is of the form $[f_1(x,y)]y\,dx + [f_2(x,y)]x\,dy = 0$ ① $\frac{1}{Mx - Ny}$ is an integrating factor, $Mx \neq Ny$

we multiply integrating factor in eqn ① then diff. Eqn will become exact.

Ex: $(x^3y^3 + x^2y^2 + xy + 1)y\,dx + (x^3y^3 - x^2y^2 - xy + 1)x\,dy$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x^3y^4 + x^2y^3 + xy^2 + y)$$

$$= 4x^3y^3 + 3x^2y^2 + 2xy + 1$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^4y^3 - x^3y^2 - x^2y + x)$$

$$= 4x^3y^3 - 3x^2y^2 - 2xy + 1$$

$$\frac{1}{Mx - Ny} = \frac{1}{(x^3y^3 + x^2y^2 + xy + 1)xy - (x^3y^3 - x^2y^2 - xy + 1)xy}$$

$$= \frac{1}{x^4y^4 + x^3y^3 + x^2y^2 + xy - x^4y^4 + x^3y^3 + x^2y^2 - xy}$$

$$= \frac{1}{2x^3y^3 + 2x^2y^2}$$

$$= \frac{1}{2x^2y^2(xy+1)} = I.F$$

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$$(x^3y^3+x^2y^2+xy+1)y\,dx+(x^3y^3-x^2y^2-xy+1)x\,dy=0$$

$$\frac{(x^3y^3+x^2y^2+xy+1)y\,dx}{2x^2y^2(xy+1)}+\frac{(x^3y^3-x^2y^2-xy+1)x\,dy}{2x^2y^2(xy+1)}=0$$

$$\frac{[x^2y^2(xy+1)+(xy+1)]y\,dx}{2x^2y^2(xy+1)}+\frac{[x^3y^3+1-xy(xy+1)]x\,dy}{2x^2y^2(xy+1)}=0$$

$$\frac{(x^2y^2+1)(xy+1)y\,dx}{2x^2y^2(xy+1)}+\frac{(xy+1)(x^2y^2+1-xy)-xy(xy+1)x\,dy}{2x^2y^2(xy+1)}=0$$

$$\frac{(x^2y^2+1)y\,dx}{2x^2y^2}+\frac{(x^2y^2+1-xy-xy)x\,dy}{2x^2y^2}=0$$

$$\left[\frac{1}{2}+\frac{1}{2x^2y^2}\right]y\,dx+\frac{(x^2y^2+1-2xy)x\,dy}{2x^2y^2}=0$$

$$\left[\frac{1}{2}+\frac{1}{2x^2y^2}\right]y\,dx+\left[\frac{2}{2}-\frac{2}{xy}+\frac{1}{2x^2y^2}\right]x\,dy=0$$

$$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}\left[\frac{y}{2}+\frac{y}{2x^2y^2}\right]$$

$$=\frac{1}{2}-\frac{1}{2x^2y^2}$$

$$\frac{\partial N}{\partial x}=\frac{\partial}{\partial x}\left[\frac{2x}{2}-\frac{2x}{xy}+\frac{x}{2x^2y^2}\right]$$

$$=\frac{1}{2}-\frac{1}{2x^2y^2}$$

$\Rightarrow \int\limits_{y\,\text{const}} M\,dx+\int \text{term of } N \text{ free from } x\; dy = C$

$$\int \frac{y}{2}+\frac{1}{2x^2y}\,dx + \int -\frac{2}{y}\,dy = C$$

$$xy-\frac{1}{xy}-2\log y = C$$

Case II: When $Mx+Ny\neq 0$ and the eqn $M\,dx+N\,dy$ is Homogeneous then Integrating factor is $\frac{1}{Mx+Ny}$

Solve $\quad x^2y\,dx-(x^3+y^3)\,dy=0$

$$\frac{dy}{dx}=\frac{x^2y}{x^3+y^3}\qquad y=vx \leftarrow \text{Homogeneous}$$

$$\frac{\partial M}{\partial y}=x^2\qquad \frac{\partial N}{\partial x}=-3x^2$$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}\leftarrow \text{Not exact}$$

$$\text{I.F}=\frac{1}{Mx+Ny}=\frac{1}{(x^2y)x+(-x^3+y^3)y}=\frac{1}{x^3y-x^3y-y^4}=-\frac{1}{y^4}$$

$$\Rightarrow \frac{-x^2y}{y^4}\,dx-\frac{x^3+y^3}{-y^4}\,dy=0$$

$$\underbrace{-\frac{x^2}{y^3}}_{M}\,dx+\underbrace{\left(\frac{x^3}{y^4}+\frac{1}{y}\right)}_{N}dy=0$$

$$\frac{\partial M}{\partial y}=\frac{3x^2}{y^4}\qquad \frac{\partial N}{\partial x}=\frac{3x^2}{y^4}$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}\rightarrow \text{exact}$$

$$\int\limits_{y\,\text{cons}} -\frac{x^2}{y^3}\,dx+\int\frac{1}{y}\,dy=C$$

$$-\frac{x^3}{3y^3}+\log y=C$$

Case III: If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$, then $e^{\int f(x)dx}$ is

I.F of $M dx + N dy = 0$

Case IV: If $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = f(y)$, then $e^{\int f(y) dy}$ is

I.F of $M dx + N dy = 0$

Solve $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \frac{1}{4}(x + xy^2) dy = 0$

$\frac{\partial M}{\partial y} = 1 + y^2 \qquad \frac{\partial N}{\partial x} = \frac{1}{4}(1+y^2)$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \leftarrow$ Not Exact

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{\frac{1}{4}(x+xy^2)}\left(1+y^2 - \frac{1}{4}(1+y^2)\right)$

$= \frac{\frac{3}{4}(1+y^2)}{\frac{1}{4}x(1+y)^2} = \frac{3}{x}$

I.F $= e^{\int f(x) dx} = e^{\int 3/x\, dx} = e^{\log x^3} = x^3$

$x^3\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \frac{x^3}{4}(x + xy^2) dy = 0$

$\frac{\partial M}{\partial y} = x^3 + \frac{3x^3y^2}{3} = x^3 + x^3y^2$

$\frac{\partial N}{\partial x} = \frac{4x^3}{4} + \frac{4x^3y^2}{4} = x^3 + x^3y^2$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow$ Exact

$\int_{y\,\text{const}} x^3\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \int 0\, dy = 0$

$\frac{x^4 y}{4} + \frac{x^4y^3}{12} + \frac{x^6}{12} = c$

$x^4 y + \frac{x^4 y^3}{3} + \frac{x^6}{3} = 4c$

Solve $(xy^3 + y) dx + 2(x^2y^2 + x + y^4) dy$

$\frac{\partial M}{\partial y} = 3xy^2 + 1 \qquad \frac{\partial N}{\partial x} = 2(2xy^2 + 1)$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \leftarrow$ not Exact

If $= \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{2(x^2y^2+x+y^4)}(3xy^2 + 1 -$

$= \frac{1}{2(x^2y^2+x+y^4)}(-1-xy^2)$

$\Rightarrow \frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{xy^3+y}(3xy^2 + 1 - 2(2x$

$= \frac{(-1)}{xy^3+y}(-1-xy^2)$

$= \frac{1(1+xy^2)}{y(1+xy^2)} = +\frac{1}{y} =$ I.F

$\Rightarrow e^{\int f(y) dy} = e^{\int +\frac{1}{y}} = e^{\log(y)}$

$\Rightarrow y(xy^3 + y) dx + 2y(x^2y^2 + x + y^4$

$\frac{\partial M}{\partial y} = 4xy^3 + 2y \qquad \frac{\partial N}{\partial x} = 4xy^2 +$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \leftarrow$ exact

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$$\int_{y\,const} (xy^4 + y^2)\,dx + \int 2y^5\,dy = c$$

$$\frac{x^2y^4}{2} + xy^2 + \frac{2y^6}{6} = c$$

$$3x^2y^4 + 6xy^2 + 2y^6 = 6c$$

$$3x^2y^4 + 6xy^2 + 2y^6 = c'$$

linear diff Eqn with constant coefficient

standard form

$$a_n\frac{d^ny}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^{n-1}} + a_{n-2}\frac{d^{n-2}y}{dx^{n-2}} + \dots + a_1\frac{dy}{dx} + a_0y = f(x)$$

$a_n, a_{n-1}, a_{n-2} \dots a_1$ & a_0 are constant.

Soln y = complementary function + Particular integral

$= CF + P.I$

(LHS) (RHS) $\left(\frac{d}{dx} = D\right)$

$$a_nD^ny + a_{n-1}D^{n-1}y + a_{n-2}D^{n-2}y + \dots + a_1Dy + a_0y = f(x)$$

$$[a_nD^n + a_{n-1}D^{n-1} + a_{n-2}D^{n-2} + \dots + a_1D + a_0]\,y = f(x)$$

$$\boxed{F(D)\,y = f(x)}$$

Auxialary Eqn (AE)

$F(m) = 0$

$$a_nm^n + a_{n-1}m^{n-1} + a_{n-2}m^{n-2} + \dots + a_1m + a_0 = 0$$

we get n roots of m

Case I: When roots are distinct say $m_1, m_2 \dots m_n$

$$C.F = C_1e^{m_1x} + C_2e^{m_2x} + C_3e^{m_3x} + \dots + C_ne^{m_nx}$$

$C_1, C_2 \dots C_n$ arbitary const.

Case II: If two roots are repeated $m_1 = m_2$

$$C.F = (C_1 + C_2x)e^{m_1x} + C_3e^{m_3x} + \dots + C_ne^{m_nx}$$

If 3 roots are repeated $m_1 = m_2 = m_3$

$$C.F = (C_1 + C_2x + C_3x^2)e^{m_1x} + C_4e^{m_4x} + \dots + C_ne^{m_nx}$$

Case III: If roots are in complex form $\alpha \pm i\beta$

$$C.F = C_1e^{(\alpha+i\beta)x} + C_2e^{(\alpha-i\beta)x}$$

$$= e^{\alpha x}(C_1e^{i\beta x} + C_2e^{-i\beta x})$$

$$= e^{\alpha x}[C_1(\cos\beta x + i\sin\beta x) + C_2(\cos\beta x - i\sin\beta x)]$$

$$= e^{\alpha x}[(C_1+C_2)\cos\beta x + i(C_1-C_2)\sin\beta x]$$

$$= e^{\alpha x}[A\cos\beta x + B\sin\beta x]$$

$$C.F = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x)$$

Case IV: $\alpha \pm \sqrt{\beta}$

$$C.F = C_1e^{(\alpha+\sqrt{\beta})x} + C_2e^{(\alpha-\sqrt{\beta})x}$$

$$= e^{\alpha x}[C_1e^{\sqrt{\beta}x} + C_2e^{-\sqrt{\beta}x}]$$

Hyperbolic function

$$e^x = \cosh x + \sinh x$$

$$= e^{\alpha x}[C_1(\cosh\sqrt{\beta}x + \sinh\sqrt{\beta}x) + C_2(\cosh\sqrt{\beta}x - \sinh\sqrt{\beta}x)]$$

$$= e^{\alpha x}[(C_1+C_2)\cosh\sqrt{\beta}x + (C_1-C_2)\sinh\sqrt{\beta}x]$$

$$CF = e^{\alpha x}[A\cosh\sqrt{\beta}x + B\sinh\sqrt{\beta}x]$$

Q Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$

$y = CF + PI$

$PI = 0$

$CF = (D^2 - 6D + 13)y = 0$

$F(D)y = f(x)$

A.E $\Rightarrow$ $F(m) = 0$

$m^2 - 6m + 13 = 0$

$m = \frac{6 \pm \sqrt{36-52}}{2} = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2}$

$= 3 \pm 2i$

$C.F = e^{3x}[C_1 \cos 2x + C_2 \sin 2x]$

$y = e^{3x}[C_1 \cos 2x + C_2 \sin 2x]$

Solve $\frac{d^4y}{dx^4} + a^4 y = 0$

$y = CF + PI$

$PI = 0$

$CF = (D^4 + a^4)y = 0$

$F(D)y = f(x)$

A.E $\Rightarrow$ $F(m) = 0$

$m^4 + a^4 = 0$

$m^4 + a^4 - 2m^2a^2 + 2m^2a^2 = 0$

$(m^2 + a^2)^2 - 2m^2a^2 = 0$

$(m^2 + a^2 + \sqrt{2}ma)(m^2 + a^2 - \sqrt{2}ma) = 0$

$m^2 + \sqrt{2}ma + a^2 = 0$

$m = \frac{-\sqrt{2}a \pm \sqrt{2a^2 - 4a^2}}{2}$

$= \frac{-\sqrt{2}a \pm i\sqrt{2}a}{2}$

$m^2 - \sqrt{2}ma + a^2 = 0$

$m = \frac{\sqrt{2}a \pm \sqrt{2a^2 - 4a^2}}{2}$

$m = \frac{a}{\sqrt{2}} \pm \frac{a}{\sqrt{2}}$

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$CF = e^{-\frac{a}{\sqrt{2}}x}\left[C_1 \cos \frac{a}{\sqrt{2}}x + C_2 \sin \frac{a}{\sqrt{2}}x\right] + e^{\frac{a}{\sqrt{2}}x}\left[C_3 \cos \frac{a}{\sqrt{2}}x + C_4 \sin \frac{a}{\sqrt{2}}x\right] = y$

$\Rightarrow$ $F(D)y = f(x)$

$PI = \frac{f(x)}{F(D)}$

Case I: RHS $= f(x) = e^{ax}$

$PI = \frac{e^{ax}}{F(D)} = \frac{e^{ax}}{F(a)}$, $F(a) \neq 0$

if $F(a) = 0$

$= x\frac{e^{ax}}{F'(D)}$, $F'(a) \neq 0$

$= x\frac{e^{ax}}{F'(a)}$

if $F'(a) = 0$

$= x^2\frac{e^{ax}}{F''(D)}$

$= x^2\frac{e^{ax}}{F''(a)}$, $F''(a) \neq 0$

Que $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$

$y = CF + PI$

$\underbrace{(D^2 - 3D + 2)}_{F(D)}y = \underbrace{e^{5x}}_{f(x)}$

A.E $\Rightarrow$ $m^2 - 3m + 2 = 0$

$(m-2)(m-1) = 0$

$m = 1, 2$

$CF = C_1 e^x + C_2 e^{2x}$

$PI = \frac{f(x)}{F(D)} = \frac{e^{5x}}{D^2 - 3D + 2} = \frac{e^{5x}}{5^2 - 3\times 5 + 2}$

$= \frac{e^{5x}}{12}$

$y = C_1 e^x + C_2 e^{2x} + \frac{e^{5x}}{12}$

Ques $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^{2x}$

$[D^3 - 6D^2 + 11D - 6]y = e^{2x}$

$F(D)y = f(x)$

A.E $\Rightarrow m^3 - 6m^2 + 11m - 6 = 0$

$m^3 - 2m^2 - 4m^2 + 8m + 3m - 6 = 0$

$m^2(m-2) - 4m(m-2) + 3(m-2) = 0$

$(m-2)(m^2 - 4m + 3) = 0$

$(m-2)(m-1)(m-3) = 0$

$m = 1, 2, 3$

$CF = C_1 e^x + C_2 e^{2x} + C_3 e^{3x}$

$PI = \frac{e^{2x}}{D^3 - 6D^2 + 11D - 6} = \frac{e^{2x}}{2^3 - 6\times 4 + 11\times 2 - 6}$

$= \frac{e^{2x}}{8 - 24 + 22 - 6} = \frac{e^{2x}}{0}$

$\Rightarrow x\frac{e^{2x}}{3D^2 - 12D + 11} = \frac{x e^{2x}}{3\times 4 - 12\times 2 + 11}$

$= x\frac{e^{2x}}{12 - 24 + 11}$

$= -x e^{2x}$

$y = CF + PI$

$C_1 e^x + C_2 e^{2x} + C_3 e^{3x} - x e^{2x}$

Ques $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 2\sinh x$ ↓ exponential formula

$(D^2 + 4D + 4)y = 2\left[\frac{e^x - e^{-x}}{2}\right]$

A.E $\Rightarrow m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$m = -2, -2$

$CF = (C_1 + C_2 x)e^{-2x}$

$PI = \frac{e^x - e^{-x}}{D^2 + 4D + 4} = \frac{e^x}{D^2 + 4D + 4} - \frac{e^{-x}}{D^2 + 4D + 4}$

$= \frac{e^x}{1 + 4 + 4} - \frac{e^{-x}}{1 + 4 + 4} = \frac{e^x}{9} - \frac{e^{-x}}{1 - 4 + 4}$

$= \frac{e^x}{9} - \frac{e^{-x}}{1}$

$y = (C_1 + C_2 x)e^{-2x} + \frac{1}{9}e^x - e^{-x}$

Case 2: RHS $f(x) = \sin ax$ or $\cos ax$

$PI = \frac{f(x)}{F(D)} = \frac{\sin ax}{F(D)} = \frac{\sin ax}{\text{const} \mid D\text{term} \mid \text{zero}}$

$D^2 \to -a^2$

$D^4 \to D^2 \cdot D^2$

$= (-a)^2 (-a)^2$

$D^3 = D^2 \cdot D = (-a^2)D$

Eg: $\frac{\sin ax}{5D + 3}$

$= \frac{\sin ax}{(5D+3)}\frac{(5D-3)}{(5D-3)}$

$= \frac{(5D-3)\sin ax}{25D^2 - 9}$

$= \frac{5a\cos ax - 3\sin ax}{\text{const}}$

Ques $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos 3x$

$(D^2 - 3D + 2)y = \cos 3x$

$F(D)\,y = f(x)$

AE $\Rightarrow$ $m^2 - 3m + 2 = 0$

$(m-1)(m-2) = 0$

$m = 1, 2$

CF $= C_1 e^x + C_2 e^{2x}$

PI $= \frac{\cos 3x}{D^2 - 3D + 2} = \frac{\cos 3x}{-(3)^2 - 3D + 2}$

$= \frac{\cos 3x}{-9 - 3D + 2} = \frac{\cos 3x}{-7 - 3D}$

PI $= -\frac{\cos 3x}{3D + 7}$

$= -\frac{\cos 3x}{(3D+7)} \frac{(3D-7)}{(3D-7)} = \frac{-(3D-7)\cos 3x}{9D^2 - 49}$

$= \frac{-(3D-7)\cos 3x}{9(-9) - 49} = -\frac{(3D-7)\cos 3x}{+130}$

$= \frac{1}{130}(3(-3\sin 3x) - 7\cos 3x)$

$= \frac{-1}{130}(9\sin 3x + 7\cos 3x)$

$y = C_1 e^x + C_2 e^{2x} - \frac{1}{130}(9\sin 3x + 7\cos 3x)$

Ques $y'' + y = \sin 3x * \cos 2x$

$\frac{d^2y}{dx^2} + y = \frac{2}{2}(\sin 3x * \cos 2x)$

$(D^2 + 1)y = \frac{1}{2}[\sin 5x + \sin x]$

AE $\Rightarrow$ $m^2 + 1 = 0$

$m = \pm i \quad (\alpha \pm i\beta)$

CF $= e^{0x}[C_1\cos x + C_2 \sin x]$

$= C_1\cos x + C_2 \sin x$

PI $= \frac{\frac{1}{2}[\sin 5x + \sin x]}{D^2 + 1}$

$= \frac{1}{2}\left[\frac{\sin 5x}{D^2+1} + \frac{\sin x}{D^2+1}\right]$

$= \frac{1}{2}\left[\frac{\sin 5x}{-25+1} + \frac{\sin x}{-1+1}\right]$

$= \frac{1}{2}\left[\frac{\sin 5x}{-24} + x\frac{\sin x}{2D}\right]$

$= \frac{1}{2}\left[-\frac{1}{24}\sin 5x + \frac{1}{2}x \frac{\sin x}{D}\frac{D}{D}\right]$

$= \frac{1}{2}\left[-\frac{1}{24}\sin 5x + \frac{1}{2}x \frac{\cos x}{-1}\right]$

$= \frac{1}{2}\left[-\frac{1}{24}\sin 5x - \frac{1}{2}x\cos x\right]$

in some book

$\frac{1}{2}\frac{D(x\sin x)}{D^2}$

$\frac{1}{2}\frac{\sin x + x\cos x}{(-1)}$

$-\frac{1}{2}(\sin x + x\cos x)$

soln

$y = C_1\cos x + C_2\sin x - \frac{1}{48}\sin 5x - \frac{x}{4}\cos x$

$y = C_1\cos x + C_2\sin x - \frac{1}{48}\sin 5x - \frac{1}{4}x\cos x - \frac{1}{4}\sin x$

$= C_1\cos x + (C_2 - \frac{1}{4})\sin x - \frac{1}{48}\sin 5x - \frac{1}{4}x\cos x$

$= C_1\cos x + C_2'\sin x - \frac{1}{48}\sin 5x - \frac{1}{4}x\cos x$

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Case 3 : RHS $f(x) = x^m$

$$PI = \frac{f(x)}{F(D)} = \frac{x^m}{F(D)}$$

$$= \frac{x^m}{\text{take common min degree of } x \text{ term}}$$

$$= \frac{x^m}{[1 + \cdots]}$$

$$= [1 + \cdots]^{-1} x^m$$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \ldots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots$

eg : $\frac{x^3}{D^2+4D+3}$

$$= \frac{x^3}{3\left[1 + \frac{D^2+4D}{3}\right]}$$

$$\Rightarrow \frac{1}{3}\left[1 + \frac{D^2+4D}{3}\right]^{-1} x^3$$

Binomial expansion

$$\frac{1}{3}\left[1 - \left(\frac{D^2+4D}{3}\right) + \left(\frac{D^2+4D}{3}\right)^2 + \left(\frac{D^2+4D}{3}\right)^3\right] x^3$$

$$\frac{1}{3}\left[1 - \frac{D^2}{3} - \frac{4}{3}D + \frac{1}{9}(8D^3 + 16D^2) - \frac{1}{27}(64D^3)\right] x^3$$

$$= \frac{1}{3}\left(x^3 - \frac{1}{3}6x - \frac{4}{3}3x^2 + \frac{8}{9}\times 6 + \frac{16}{9}6x - \frac{64}{27}\cdot 6\right)$$

Ques $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$

$(D^3 - D^2 - 6D)y = 1 + x^2$

AE $\Rightarrow$ $m^3 - m^2 - 6m = 0$

$m(m^2 - m - 6) = 0$

$m(m-3)(m+2) = 0$

$m = 0, -2, 3$

$$C.F = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{m_3 x}$$

$$= C_1 + C_2 e^{-2x} + C_3 e^{3x}$$

$$PI = \frac{1+x^2}{D^3 - D^2 - 6D} = \frac{1+x^2}{-6D\left[1 - \frac{D^3 - D^2}{6D}\right]}$$

$$= \frac{1+x^2}{-6D\left[1 - \frac{D^2 - D}{6}\right]}$$

$$= -\frac{1}{6D}\left[1 - \frac{D^2-D}{6}\right]^{-1}(1+x^2)$$

$$= -\frac{1}{6D}\left[1 + \frac{D^2-D}{6} + \left(\frac{D^2-D}{6}\right)^2\right](1+x^2)$$

$$= -\frac{1}{6D}\left[1 + \frac{D^2}{6} - \frac{D}{6} + \frac{D^2}{36}\right](1+x^2)$$

$$= -\frac{1}{6D}\left[1 + x^2 + \frac{1}{6}\times 2 - \frac{1}{6}2x + \frac{1}{36}\times 2\right]$$

$$= -\frac{1}{6D}\left[1 + x^2 + \frac{1}{3} - \frac{x}{3} + \frac{1}{18}\right]$$

$$= -\frac{1}{6D}\left[\frac{25}{18} + x^2 - \frac{x}{3}\right]$$

$$= -\frac{1}{6}\left[\frac{25}{18}x + \frac{x^3}{3} - \frac{x^2}{6}\right]$$

$D \leftarrow$ Differentiate

$\frac{1}{D} \leftarrow$ Integration

$$y = C_1 + C_2 e^{-2x} + C_3 e^{3x} - \frac{1}{6}\left[\frac{25}{18}x + \frac{x^3}{3} - \frac{x^2}{6}\right]$$

Eg : $(xD+5)(xD+2)y$

$$\left(x\frac{d}{dx} + 5\right)\left(x\frac{d}{dx} + 2\right)y$$

$$\left(x\frac{d}{dx} + 5\right)\left(x\frac{dy}{dx} + 2y\right)$$

$$x\frac{d}{dx}\left(x\frac{dy}{dx} + 2y\right) + 5\left(x\frac{dy}{dx} + 2y\right)$$

$$x\left[x\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2\frac{dy}{dx}\right] + 5x\frac{dy}{dx} + 10y$$

$$x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} + 10y$$

$$(x^2D^2 + 8xD + 10)y$$

Ques $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + \cos 2x$

$(D^2 - 4D + 4)y = x^2 + e^x + \cos 2x$

A.E $\Rightarrow$ $m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$m = 2, 2$

$C.F = (C_1 + C_2 x)e^{2x}$

$PI = \frac{x^2 + e^x + \cos 2x}{D^2 - 4D + 4}$

$= \underset{(I)}{\frac{x^2}{D^2 - 4D + 4}} + \underset{(II)}{\frac{e^x}{D^2 - 4D + 4}} + \underset{(III)}{\frac{\cos 2x}{D^2 - 4D + 4}}$

(I) $\rightarrow$ $\frac{x^2}{D^2 - 4D + 4} = \frac{x^2}{4\left[1 + \frac{D^2 - 4D}{4}\right]}$

$= \frac{1}{4}\left[1 + \frac{D^2 - 4D}{4}\right]^{-1} x^2$

$= \frac{1}{4}\left[1 - \frac{D^2 - 4D}{4} + \left(\frac{D^2 - 4D}{4}\right)^2\right] x^2$

$= \frac{1}{4}\left[1 - \frac{D^2}{4} + D + D^2\right] x^2$

$= \frac{1}{4}\left[1 + D + \frac{3D^2}{4}\right] x^2$

$= \frac{1}{4}\left[x^2 + 2x + \frac{3 \times 2}{4}\right]$

$= \frac{1}{4}\left[x^2 + 2x + \frac{3}{2}\right]$

(II) $\rightarrow$ $\frac{e^x}{D^2 - 4D + 4} = \frac{e^x}{1^2 - 4 + 4} = e^x$

(III) $\rightarrow$ $\frac{\cos 2x}{D^2 - 4D + 4} = \frac{\cos 2x}{-4 - 4D + 4} = \frac{\cos 2x}{-4D}$

$D^2 \rightarrow -a^2$

$D^2 \rightarrow -4$

$= -\frac{1}{4} \frac{\sin 2x}{2} = -\frac{1}{8}\sin 2x$

$y = CF + PI$

$= (C_1 + C_2 x)e^{2x} + \frac{1}{4}\left[x^2 + 2x + \frac{3}{2}\right] + e^x - \frac{1}{8}\sin 2x$

Case III : RHS $\rightarrow$ $e^{ax} V(x)$

$PI = \frac{RHS}{F(D)} = \frac{e^{ax} V}{F(D)}$; $(D \rightarrow D + a)$

$= e^{ax} \frac{V(x)}{F(D+a)}$

Ques : Solve $(D^2 - 2D + 5)y = e^{2x} \sin x$

A.E $\Rightarrow$ $m^2 - 2m + 5 = 0$

$m = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$

$CF = e^x (C_1 \cos 2x + C_2 \sin 2x)$

$PI = \frac{e^{2x} \sin x}{D^2 - 2D + 5}$ $D \rightarrow D + 2$

$= e^{2x} \cdot \frac{\sin x}{(D+2)^2 - 2(D+2) + 5}$

$= e^{2x} \cdot \frac{\sin x}{D^2 + 4D + 4 - 2D - 4 + 5}$

$= e^{2x} \cdot \frac{\sin x}{D^2 + 2D + 5}$ $D^2 \rightarrow -a^2$

$= e^{2x} \frac{\sin x}{-1 + 2D + 5}$ $D^2 \rightarrow -1$

$= e^{2x} \frac{\sin x}{2D + 4} \Rightarrow \frac{1}{2} e^{2x} \frac{(D-2)\sin x}{(D+2)(D-2)}$

$= \frac{1}{2} e^{2x} \frac{(D-2)\sin x}{D^2 - 4} = \frac{1}{2} e^{2x} \frac{(D-2)\sin x}{-5}$

$= -\frac{1}{10} e^{2x} (\cos x - 2\sin x)$

General Case

RHS is any function of x

$$PI = \frac{RHS}{F(D)} = \frac{f(x)}{F(D)} \leftarrow \text{linear factor } (D-\alpha)$$

$$= \frac{f(x)}{D-\alpha}$$

$$= \boxed{e^{\alpha x}\int e^{-\alpha x} f(x)\,dx}$$

$$y = \frac{f(x)}{D-\alpha} \Rightarrow (D-\alpha)\,y = f(x)$$

$$\left(\frac{d}{dx}-\alpha\right)y = f(x)$$

$$\frac{dy}{dx} - \alpha y = f(x)$$

$$IF = e^{-\int \alpha\,dx}$$

$$= e^{-\alpha x}$$

$$y\,e^{-\alpha x} = \int f(x)\,e^{-\alpha x}\,dx$$

$$y = e^{\alpha x}\int f(x)\,e^{-\alpha x}\,dx$$

Ques $\frac{d^2y}{dx^2} + a^2 y = \sec ax$

$$AE = m^2 + a^2 = 0$$

$$m = \pm ai \quad (\alpha \pm i\beta)$$

$$CF = C_1\cos ax + C_2 \sin ax$$

$$PI = \frac{\sec ax}{D^2+a^2} = \frac{\sec ax}{(D+ia)(D-ia)}$$

$$= \frac{1}{2ia}\left[\frac{1}{D-ia} + \frac{1}{D+ia}\right]\sec ax$$

$$= \frac{1}{2ia}\left[\frac{\sec ax}{D-ia} - \frac{\sec ax}{D+ia}\right]$$

I $\quad$ II

$$= \frac{1}{2ia}\left[e^{iax}\int e^{-iax}\sec ax\,dx\right]$$

$$= \frac{1}{2ia}\left[e^{iax}\int(\cos ax - i\sin ax)\sec ax\,dx\right]$$

$$= \frac{1}{2ia}\left[e^{iax}\int(1 - i\tan ax)\,dx\right]$$

$$= \frac{1}{2ia}\left[e^{iax}\left(x + \frac{i}{a}\log\cos ax\right)\right]$$

II $\quad \frac{1}{2ia}\left[e^{iax}\int e^{iax}\sec ax\,dx\right]$

$$\frac{1}{2ia}\left[e^{iax}\int(\cos ax + i\sin ax)\sec ax\,dx\right]$$

$$\frac{1}{2ia}\left[e^{iax}\int(1 + i\tan ax)\,dx\right]$$

$$\frac{1}{2ia}\left[e^{iax}\left(x - \frac{i}{a}\log\cos ax\right)\right]$$

Ques $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\sin x$

$$m^2 - 2m + 1 = 0$$

$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$CF = (C_1 + C_2 x)e^x$$

$$PI = \frac{x\sin x}{D^2 - 2D + 1}$$

$$e^{ix}.x = (\cos x + i\sin x)x$$

$$= \underset{R.P}{x\cos x} + \underset{I.P}{ix\sin x}$$

$$= \text{IP of } \frac{e^{ix}.x}{D^2-2D+1}$$

$$= \text{IP of } e^{ix}\frac{x}{(D+i)^2 - 2(D+i)+1}$$

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$$\text{I.P of } e^{ix} \frac{x}{D^2 - 1 + 2iD - 2D - 2i + 1}$$

$$= \text{I.P of } e^{ix} \frac{x}{D^2 + (2i - 2)D - 2i}$$

$$= \text{I.P of } e^{ix} \frac{x}{D^2 + 2(i-1)D - 2i}$$

$$= \text{I.P of } e^{ix} \frac{x}{-2i\left[1 - \frac{D^2 + 2(i-1)D}{2i}\right]}$$

$$= \text{I.P of } e^{ix} \times \frac{-1}{2i}\left[1 - \frac{D^2 + 2(i-1)D}{2i}\right]^{-1} x$$

$$= \text{I.P of } e^{ix} \left(\frac{-1}{2i}\right)\left[1 - \frac{2(i-1)D}{2i}\right] x$$

$$= \text{I.P of } e^{ix} \left(\frac{-1}{2i}\right)\left[x - \frac{(i-1)}{i}\right]$$

$$= \text{I.P of } \cos x + i \sin x$$

Homogeneous linear diff Eqn with constant coefficient

General Form

$$a_n x^n \frac{d^n y}{dx^n} + a_{n-1} x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_{n-2} x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} - - - - -$$

$$- - - a_1 x \frac{dy}{dx} + a_0 y = Q(x)$$

$a_n, a_{n-1}, a_{n-2} - - a_1$ and a_0 are const.

Substitute $x = e^z \Rightarrow \log x = z \Rightarrow \frac{dz}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{x} \frac{dy}{dz}$$

$$\left[\frac{d}{dz} = D'\right]$$

$$\boxed{x \frac{dy}{dx} = \frac{dy}{dz} = D'y}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{x} \frac{dy}{dz}\right)$$

$$= -\frac{1}{x^2} \frac{dy}{dz} + \frac{1}{x} \frac{d}{dx}\left(\frac{dy}{dz}\right)$$

$$= -\frac{1}{x^2} \frac{dy}{dz} + \frac{1}{x} \frac{d}{dz}\left(\frac{dy}{dz}\right) \frac{dz}{dx}$$

$$= -\frac{1}{x^2} \frac{dy}{dz} + \frac{1}{x} \frac{d^2y}{dz^2} \frac{1}{x}$$

$$x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz}$$

$$\boxed{x^2 \frac{d^2y}{dx^2} = (D'^2 - D')y = D'(D'-1)y}$$

$$x^3 \frac{d^3y}{dx^3} = D'(D'-1)(D'-2)y$$

$$\boxed{x^n \frac{d^ny}{dx^n} = D'(D'-1)(D'-2) - - - (D'-(n-1))y}$$

$$a_n D'(D'-1)(D'-2) - - - (D'-(n-1))y + a_{n-1} D'(D'-1) - -$$
$$- - + a_1 D'y + a_0 y = f(z)$$

Ques $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - 3y = x^2 + x$

let $x = e^z \Rightarrow z = \log x$ $\quad D' = \frac{d}{dz}$

$$D'(D'-1)(D'-2)y + 2D'(D'-1)y + 3D'y - 3y = e^{2z} + e$$

$$[D'(D'^2 - 3D' + 2) + 2D'^2 - 2D' + 3D' - 3]y = e^{2z} + e$$

$$[D'^3 - 3D'^2 + 2D' + 2D'^2 - 2D' + 3D' - 3]y = e^{2z}$$

$[D'^3 - D'^2 + 3D' - 3]y = e^{2z} + e^z$

$\frac{d^3y}{dz^3} - \frac{d^2y}{dz^2} + 3\frac{dy}{dz} - 3y = e^{2z} + e^z$ ⇒ linear diff Eqn with const coeff.

A.E ⇒ $m^3 - m^2 + 3m - 3 = 0$

$m^2(m-1) + 3(m-1) = 0$

$(m-1)(m^2+3) = 0$

$m = 1, \pm i\sqrt{3}$

C.F $= c_1 e^z + e^{0z}(c_2 \cos\sqrt{3}z + c_3 \sin\sqrt{3}z)$

$$PI = \frac{e^{2z} + e^z}{D'^3 - D'^2 + 3D' - 3} = \frac{e^{2z}}{D'^3 - D'^2 + 3D' - 3} + \frac{e^z}{D'^3 - D'^2 + 3D' - 3}$$

$$= \frac{e^{2z}}{8-4+6-3} + \frac{e^z}{1-1+3-3=0}$$

$$= \frac{e^{2z}}{7} + z\frac{e^z}{3D'^2 - 2D' + 3} = \frac{e^{2z}}{7} + \frac{ze^z}{3-2+3}$$

$$= \frac{e^{2z}}{7} + \frac{ze^z}{4}$$

$$y = c_1 e^z + c_2\cos\sqrt{3}z + c_3\sin\sqrt{3}z + \frac{e^{2z}}{7} + \frac{ze^z}{4}$$

$$y = c_1 x + c_2\cos\sqrt{3}(\log x) + c_3\sin\sqrt{3}(\log x) + \frac{x^2}{7} + \frac{x\log x}{4}$$

Ques $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2\log x$

let $x = e^z \Rightarrow z = \log x$ $\quad D' = \frac{d}{dz}$

$D'(D'-1)y - D'y - 3y = e^{2z}z$

$[D'^2 - D' - D' - 3]y = e^{2z}.z$

$\frac{d^2y}{dz^2} - 2\frac{dy}{dz} - 3y = ze^{2z}$ LD eqn with const coeff

A.E ⇒ $m^2 - 2m - 3 = 0 \Rightarrow (m-3)(m+1) = 0$

$m = 3, -1$

C.P $= c_1 e^{3z} + c_2 e^{-z}$

$[D' \to D'+2]$

$$PI = \frac{e^{2z}.z}{D'^2 - 2D' - 3} = e^{2z}\frac{z}{(D'+2)^2 - 2(D'+2) - 3}$$

$$= e^{2z}.\frac{z}{D'^2 + 4D' - 2D' + 4 - 4 - 3}$$

$$= e^{2z}.\frac{z}{D'^2 + 2D' - 3} = e^{2z}\frac{z}{-3\left[1 - \frac{D'^2 + 2D'}{3}\right]}$$

$$= e^{2z}\left(-\frac{1}{3}\right)\left[1 - \frac{D'^2 + 2D'}{3}\right]^{-1} z$$

$$= -\frac{1}{3}e^{2z}\left[1 + \frac{D'^2}{3} + \frac{2}{3}D'\right] z$$

$$= -\frac{1}{3}e^{2z}\left[z + \frac{2}{3}\right]$$

$$y = c_1 e^{3z} + c_2 e^{-z} - \frac{1}{3}e^{2z}\left(z + \frac{2}{3}\right)$$

$$= c_1 x^3 + \frac{c_2}{x} - \frac{1}{3}x^2\log\left(x + \frac{2}{3}\right)$$

Cauchy Homogeneous LD Eqn with const. coeff.

$$a_n(ax+b)^n\frac{d^ny}{dx^n} + a_{n-1}(ax+b)^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + a_{n-2}(ax+b)^{n-2}\frac{d^{n-2}y}{dx^{n-2}}$$

$$- \cdots + a_1(ax+b)\frac{dy}{dx} + a_0 y = Q(x)$$

Substitute → $ax + b = e^z \Rightarrow z = \log(ax+b)$

$(ax+b)\frac{dy}{dx} = aD'y$

$(ax+b)^2\frac{d^2y}{dx^2} = a^2 D'(D'-1)y$

$(ax+b)^n\frac{d^ny}{dx^n} = a^n D'(D'-1)(D'-2)\cdots(D'-(n-1))y$

Solve $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2+4x+1$

let $3x+2 = e^z \Rightarrow z = \log(3x+2)$

$3^2 D'(D'-1)y + 3\cdot 3D'y - 36y = 3\left(\frac{e^z-2}{3}\right)^2 + 4\left(\frac{e^z-2}{3}\right) + 1$

$[9(D'^2 - D') + 9D' - 36]y = 3\left[\frac{e^{2z}+4-4e^z}{9}\right] + 4\left[\frac{e^z-2}{3}\right] + 1$

$[9D'^2 - 36]y = \frac{e^{2z} - 4e^z + 4 + 4e^z - 8 + 3}{3}$

$[9D'^2 - 36]y = \frac{1}{3}[e^{2z} - 1]$

$(D'^2 - 4)y = \frac{1}{27}(e^{2z} - 1)$

$A.E \Rightarrow m^2 - 4 = 0 \Rightarrow m = \pm 2$

$C.F = C_1 e^{2z} + C_2 e^{-2z}$

$P.I = \frac{\frac{1}{27}[e^{2z} - 1]}{D'^2 - 4}$

$= \frac{1}{27}\left[\frac{e^{2z}}{D'^2-4} - \frac{e^{0z}}{D'^2-4}\right]$

$= \frac{1}{27}\left[\frac{e^{2z}}{2^2-4=0} - \frac{e^{0z}}{0-4}\right]$

$= \frac{1}{27}\left[z\cdot\frac{e^{2z}}{2D'} + \frac{1}{4}\right]$

$= \frac{1}{27}\left[\frac{ze^{2z}}{4} + \frac{1}{4}\right]$

$y = C_1 e^{2z} + C_2 e^{-2z} + \frac{1}{27}\left[\frac{z}{4}e^{2z} + \frac{1}{4}\right]$

$= C_1(3x+2)^2 + C_2(3x+2)^{-2} + \frac{1}{27}\left[\frac{\log(3x+2)}{4}(3x+2)^2 + \frac{1}{4}\right]$

Ordinary Simultaneous Differential Eqn

$f_1(D)x + f_2(D)y = T_1(t)$

$\phi_1(D)x + \phi_2(D)y = T_2(t)$

$(D+1)x + Dy = e^t$ $\qquad D = \frac{d}{dt}$

$Dx + (D+2)y = t$

$\frac{dx}{dt} + x + \frac{dy}{dt} = e^t$

$\frac{dx}{dt} + \frac{dy}{dt} + 2y = t$

Methods to solve Equation

→ Substitution

→ Elimination

→ Cramer's

Ques Solve the simultaneous Eqn

$\frac{dx}{dt} - 7x + y = 0, \quad \frac{dy}{dt} - 2x - 5y = 0$

$Dx - 7x + y = 0, \quad Dy - 2x - 5y = 0$

$(D-7)x + y = 0 \quad (1), \qquad -2x + (D-5)y = 0 \quad (2)$

multiply $(D-5)$

$(D-5)(D-7)x + (D-5)y = 0$

$-2x + (D-5)y = 0$

$+ \qquad -$

$[(D-5)(D-7)+2]x = 0$

$[D^2 - 12D + 35 + 2]x = 0$

$(D^2 - 12D + 37)x = 0$

$\frac{d^2x}{dt^2} - 12\frac{dx}{dt} + 37x = 0$

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A.E $\Rightarrow m^2 - 12m + 37 = 0$

$$m = \frac{12 \pm \sqrt{144-148}}{2} = \frac{12 \pm 2i}{2}$$
$$= 6 \pm i$$

$CF = e^{6t}[C_1 \cos t + C_2 \sin t]$

$PI = 0$

$\cancel{y} = x = e^{6t}[C_1 \cos t + C_2 \sin t]$

$\Rightarrow \frac{dx}{dt} - 7x + y = 0$

$y = 7x - \frac{dx}{dt}$

$y = 7[e^{6t}(C_1 \cos t + C_2 \sin t)] - \frac{d}{dt}[e^{6t}(C_1 \cos t + C_2 \sin t)]$

$y = 7e^{6t}[C_1 \cos t + C_2 \sin t] - 6e^{6t}(C_1 \cos t + C_2 \sin t) - e^{6t}(-C_1 \sin t + C_2 \cos t)$

$y = 7e^{6t}C_1 \cos t + 7e^{6t}C_2 \sin t - 6e^{6t}C_1 \cos t - 6e^{6t}C_2 \sin t + e^{6t}C_1 \sin t - e^{6t}C_2 \cos t$

$y = e^{6t}[(C_1 - C_2)\cos t + (C_2 + C_1)\sin t]$

Ques $\frac{d^2x}{dt^2} - 3x - 4y = 0$, $\frac{d^2y}{dt^2} + x + y = 0$

$\cancel{(D^2-4)y - 3x = 0}$, $\cancel{(D^2+1)y + x = 0}$

$(D^2-3)x - 4y = 0$ $\quad (D^2+1)y + x = 0$

$$\Delta = \begin{vmatrix} D^2-3 & -4 \\ 1 & D^2+1 \end{vmatrix} = (D^2-3)(D^2+1)+4$$
$$= D^4 - 2D^2 - 3 + 4$$
$$= D^4 - 2D^2 + 1$$

$$\Delta x = \begin{vmatrix} 0 & -4 \\ 0 & D^2+1 \end{vmatrix} = 0$$

$$\Delta y = \begin{vmatrix} D^2-3 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$x = \frac{\Delta x}{\Delta} \Rightarrow \Delta \cdot x = \Delta x$ $\quad\Big|\quad \frac{\Delta y}{\Delta} = y$

$(D^4 - 2D^2 + 1)x = 0$ $\quad\Big|\quad \Delta \cdot y = \Delta y$

$(D^4 - 2D^2 + 1)y = 0$

A.E $\Rightarrow m^4 - 2m^2 + 1 = 0$

$(m^2 - 1)^2 = 0$

$m = \pm 1$ (Repeated)

$C.F = (C_1 + C_2 t)e^t + (C_3 + C_4 t)e^{-t}$

$PI = 0$

$x = (C_1 + C_2 t)e^t + (C_3 + C_4 t)e^{-t}$

$y = (C_1' + C_2' t)e^t + (C_3' + C_4' t)e^{-t}$ ← but we cannot write this because there should be only 4 arbitrary constant

$\Rightarrow 4y = \frac{d^2x}{dt^2} - 3x = C_1 e^t + C_2(C_1 t e^t + 2e^t) + C_3 e^{-t} + C_4 t e^{-t} - 3[e^t(C_1 + C_2 t) + e^{-t}(C_3 + C_4 t)]$

Ques $\frac{d^2x}{dt^2} + 4x + y = te^{3t}$, $\frac{d^2y}{dt^2} + y - 2x = \cos^2 t$

$(D^2+4)x + y = te^{3t}$ —— (i)

$-2x + (D^2+1)y = \cos^2 t$ —— (ii)

(i) (D^2+1) − (ii) $\Rightarrow$

$(D^2+1)(D^2+4)x + (D^2+1)y = (D^2+1)te^{3t}$

$-2x + (D^2+1)y$

$[(D^2+1)(D^2+4)+2]x = (D^2+1)te^{3t} - \cos^2 t$

$[D^4+5D^2+4+2]x = D^2(te^{3t}) + te^{3t} - \cos^2 t$

$[D^4+5D^2+6]x = 6e^{3t}+9te^{3t} + te^{3t} - \cos^2 t$

$[D^4+5D^2+6]x = 10te^{3t} + 6e^{3t} - \cos^2 t$

$A.E \Rightarrow m^4 - 5m^2 + 6 = 0$

$(m^2+3)(m^2+2) = 0$

$m = \pm\sqrt{3}i, \pm\sqrt{2}i$

$C.F = e^{0t}[C_1\cos\sqrt{3}t + C_2\sin\sqrt{3}t) + e^{0\cdot t}(C_3\cos\sqrt{2}t + C_4\sin\sqrt{2}t)$

$P.I = \dfrac{10te^{3t} + 6e^{3t} - \cos^2 t}{D^4 - 5D^2 + 6}$

$= \dfrac{10te^{3t}}{D^4-5D^2+6} + \dfrac{6e^{3t}}{D^4-5D^2+6} - \dfrac{\cos^2 t}{D^4-5D^2+6}$

(I) (II) (III)

$I = \dfrac{10te^{3t}}{D^4-5D^2+6} = 10e^{3t}\dfrac{t}{(D+3)^4 - 5(D+3)^2+6}$

$= \dfrac{10e^{3t}\cdot t}{D^4+12D^3+54D^2+108D+81+5(D^2+9+6D)+6}$

$= \dfrac{10e^{3t}\cdot t}{D^4+12D^3+54D^2+138D+132}$

$= 10e^{3t}\dfrac{t}{132}\left[1+\dfrac{138D+54D^2+12D^3+D^4}{132}\right]^{-1}$

$= \dfrac{10}{132}e^{3t}\left[1-\dfrac{138}{132}D\right]t$

$= \dfrac{10}{132}e^{3t}\left[t-\dfrac{138}{132}\right]$

$= \dfrac{5}{66}e^{3t}\left[t-\dfrac{23}{22}\right]$

$II \rightarrow \dfrac{6e^{3t}}{D^4+5D^2+16} = \dfrac{6e^{3t}}{3^4+5\cdot3^2+16}$

$= \dfrac{6e^{3t}}{81+45+16} = \dfrac{6e^{3t}}{132} = \dfrac{e^{3t}}{22}$

$III \rightarrow \dfrac{-\cos^2 t}{D^4+5D^2+6} = -\dfrac{\left(\frac{\cos 2t - 1}{2}\right)}{D^4+5D^2+6}$

$= -\dfrac{1}{2}\left[\dfrac{\cos 2t}{D^4+5D^2+6} + \dfrac{1}{D^4+5D^2+6}\right]$

$= -\dfrac{1}{2}\left[\dfrac{\cos 2t}{16-20+6} + \dfrac{e^{0t}}{0+0+6}\right]$

$= -\dfrac{1}{2}\left[\dfrac{\cos 2t}{2} + \dfrac{1}{6}\right]$

$x = C_1\cos\sqrt{3}t + C_2\sin\sqrt{3}t + C_3\cos\sqrt{2}t + C_4\sin\sqrt{2}t + \dfrac{5}{66}e^{3t}\left[t-\dfrac{23}{22}\right] + \dfrac{1}{22}e^{3t} - \dfrac{1}{2}\left[\dfrac{\cos 2t}{2}+\dfrac{1}{6}\right]$

$\Rightarrow \dfrac{d^2x}{dt^2} + 4x + y = te^{3t}$

$y = te^{3t} - 4x - \dfrac{d^2x}{dt^2}$

$y = te^{3t} - 4[\{C_1\cos\sqrt{3}t + C_2\sin\sqrt{3}t + C_3\cos\sqrt{2}t + C_4\sin\sqrt{2}t + \dfrac{5}{66}e^{3t}\left(t-\dfrac{23}{22}\right) + \dfrac{1}{22}e^{3t} - \dfrac{1}{2}\left(\dfrac{\cos 2t}{2}+\dfrac{1}{6}\right)] - 3C_1\cos\sqrt{3}$

$- 3C_2\sin\sqrt{3}t - 2C_3\cos\sqrt{2}t - 2C_4\sin\sqrt{2}t + \dfrac{5}{66}[9te^{3t}+6$

$- \dfrac{115\times 9e^{3t}}{22\times 16} + \dfrac{9}{22}e^{3t} -$

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Ques $\frac{dx}{dt} + \frac{2}{t}(x-y) = 1$, $\frac{dy}{dt} + \frac{1}{t}(x+5y) = t$

$t\frac{dx}{dt} + 2(x-y) = t$, $t\frac{dy}{dt} + x + 5y = t^2$

$(tD+2)x - 2y = t$ — (i), $x + (tD+5)y = t^2$ — (ii)

(i) $(tD+5)$ + (ii) $\times 2$

$(tD+5)(tD+2)x - 2(tD+5)y = (tD+5)t$

$2x + 2(tD+5)y = 2t^2$

$[(tD+5)(tD+2)+2]x = (tD+5)t + 2t^2$

$(tD+5)(tD+2)x + 2x = \left(t\frac{d}{dt}+5\right)t + 2t^2$

$\left(t\frac{d}{dt}+5\right)\left(t\frac{dx}{dt}+2x\right) + 2x =$ "

$t\frac{d}{dt}\left(t\frac{dx}{dt}+2x\right) + 5\left(t\frac{dx}{dt}+2x\right) + 2x = t + 5t + 2t^2$

$t\left[t\frac{d^2x}{dt^2} + \frac{dx}{dt} + 2\frac{dx}{dt}\right] + 5t\frac{dx}{dt} + 12x = 2t^2 + 6t$

$t^2\frac{d^2x}{dt^2} + \cancel{t\frac{dx}{dt}} + 3t\frac{dx}{dt} + 5t\frac{dx}{dt} + 12x = 2t^2 + 6t$

$t^2\frac{d^2x}{dt^2} + 8t\frac{dx}{dt} + 12x = 2t^2 + 6t$

Let $t = e^z \Rightarrow z = \log t$, $D' = \frac{d}{dz}$

$[D'(D'-1)x + 8D' + 12]x = 2e^{2z} + 6e^z$

$[D'^2 - D'x + 8D' + 12]x =$ "

$[D'^2 + 7D' + 12]x = 2e^{2z} + 6e^z$

$\frac{d^2x}{dz^2} + 7\frac{dx}{dz} + 12x = 2e^{2z} + 6e^z$

$m^2 + 7m + 12 = 0$, $m = -4, -3$

$CF = C_1e^{-4z} + C_2e^{-3z}$

$= C_1/t^4 + C_2/t^3$

$PI = \frac{2e^{2z} + 6e^z}{D^2 + 7D + 12}$

$= \frac{2e^{2z}}{D^2+7D+12} + \frac{6e^z}{D^2+7D+12}$

$= \frac{2e^{2z}}{4+14+12} + \frac{6e^z}{1+7+12}$

$= \frac{2e^{2z}}{30} + \frac{6e^z}{20}$

$= \frac{e^{2z}}{15} + \frac{3e^z}{10} = \frac{t^2}{15} + \frac{3t}{10}$

$x = CF + PI$

$= \frac{C_1}{t^4} + \frac{C_2}{t^3} + \frac{t^2}{15} + \frac{3t}{10}$

$\Rightarrow t\frac{dx}{dt} + 2x - 2y = t$

$2y = t\frac{dx}{dt} + 2x - t$

$= t\frac{d}{dt}\left(\frac{C_1}{t^4} + \frac{C_2}{t^3} + \frac{t^2}{15} + \frac{3t}{10}\right) + 2\left(\frac{C_1}{t^4} + \frac{C_2}{t^3} + \frac{t^2}{15} + \frac{3t}{10}\right) - t$

$= t\left[-\frac{4C_1}{t^5} - \frac{3C_2}{t^4} + \frac{2t}{15} + \frac{3}{10}\right] + \frac{2C_1}{t^4} + \frac{2C_2}{t^3} + \frac{2t^2}{15} + \frac{6t}{10} - t$

$= -\frac{4C_1t}{t^5} - \frac{3C_2t}{t^4} + \frac{4t^2}{15} + \frac{9t}{10} - t + \frac{2C_1}{t^4} + \frac{2C_2}{t^3}$

$= -\frac{2C_1}{t^4} - \frac{C_2}{t^3} + \frac{4t^2}{15} - \frac{t}{10}$

$$y = -\frac{c_1}{t^4} + \frac{c_2}{2t^3} + \frac{x^2}{15} - \frac{t}{20}$$

Ques $\frac{dx}{dt} = y, \quad \frac{dy}{dt} = z, \quad \frac{dz}{dt} = x$

$$\frac{d^2x}{dt^2} = \frac{dy}{dt} \Rightarrow \frac{d^2x}{dt^2} = z$$

$$\frac{d^3x}{dt^3} = \frac{dz}{dt} = x$$

$$\boxed{\frac{d^3x}{dt^3} - x = 0}$$

$$m^3 - 1 = 0$$

$$(m-1)(m^2+m+1) = 0$$

$$m = 1, \quad m = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$m = -\frac{1}{2} \pm \frac{\sqrt{3}i}{2}$$

$$C.F = c_1 e^t + e^{-\frac{1}{2}t}\left[c_2 \cos\frac{\sqrt{3}}{2}t + c_3 \sin\frac{\sqrt{3}}{2}t\right]$$

$$PI = 0$$

$$x = c_1 e^t + e^{-\frac{1}{2}t}\left[c_2 \cos\frac{\sqrt{3}}{2}t + c_3 \sin\frac{\sqrt{3}}{2}t\right]$$

$$y = \frac{dx}{dt}$$

$$= c_1 e^t + c_2\left[e^{-1/2t}\left(-\frac{1}{2}\right)\cos\frac{\sqrt{3}}{2}t - e^{-1/2t}\sin\frac{\sqrt{3}}{2}t \cdot \frac{\sqrt{3}}{2}\right]$$

$$+ c_3\left[e^{-1/2t}\left(-\frac{1}{2}\right)\sin\frac{\sqrt{3}}{t} + e^{-1/2t}\frac{\sqrt{3}}{2}\cos\frac{\sqrt{3}}{2}t\right]$$

$$= c_1 e^t + e^{-1/2t}\left[-\frac{c_2}{2}\cos\frac{\sqrt{3}}{2}t + \frac{\sqrt{3}c_3}{2}\cos\frac{\sqrt{3}}{2}\right.$$

$z = ?$

Linear differential Eqn with variable coeff.

Standard form $\rightarrow$

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{—(1)}$$

P, Q, R function of x

Method (1) $\rightarrow$ when part of C.F is known.

Let u is the part of CF of eqn (1)

Let $y = uv$ is the complete soln of diff Eqn (1)

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d^2y}{dx^2} = u\frac{d^2v}{dx^2} + \frac{du}{dx} * \frac{dv}{dx} + \frac{dv}{du} + \frac{du}{dx} + v\frac{d^2u}{dx^2}$$

$$= u\frac{d^2v}{dx^2} + 2\frac{du}{dx}\frac{dv}{dx} + v\frac{d^2u}{dx^2}$$

Put the value of y, $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ in eqn (1)

$$u\frac{d^2v}{dx^2} + 2\frac{du}{dx}\frac{dv}{dx} + v\frac{d^2u}{dx^2} + P\left[u\frac{dv}{dx} + v\frac{du}{dx}\right]$$

$$+ Quv = R$$

$$u\frac{d^2v}{dx^2} + \left[2\frac{du}{dx} + Pu\right]\frac{dv}{dx} + \left[\frac{d^2u}{dx^2} + P\frac{du}{dx} + Qu\right]$$

Since u is the part of CF of Eqn (1)

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0 \quad \text{—(II)}$$

$y = CF +$

$y =$

Hence part of CF u satisfy the diff eqn

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$$

$$\frac{d^2u}{dx^2} + P\frac{du}{dx} + Qu = 0$$

$$u\frac{d^2v}{dx^2} + \left[2\frac{du}{dx} + Pu\right]\frac{dv}{dx} = R$$

$$\frac{d^2v}{dx^2} + \underbrace{\left[\frac{2}{u}\frac{du}{dx} + P\right]}_{P'}\frac{dv}{dx} = R/u$$

$$\frac{dv}{dx} = \omega$$

$$\frac{d\omega}{dx} + P'\omega = R/u$$

If $m^2 + Pm + Q = 0$ then $u = e^{mx}$ is part of C.F

$(m=1)$ $1+P+Q=0$ then $u = e^{x}$ " "

$(m=-1)$ $1-P+Q=0$ then $u = e^{-x}$ " "

If $m(m-1) + Pmx + Qx^2 = 0$
then $u = x^m$ is part of CF

$m=1$, $Px + Qx^2 = 0 \Rightarrow P + Qx = 0$

then $u = x$ is part of C.F

$m=2$, $2 + 2Px + Qx^2 = 0$ then $u = x^2$
is a part of CF.