BPT_MattPort

June 1, 2016

0.1 Making the BPT calculation work for Python

```
In [386]: # Loading some stuff...
          from __future__ import print_function
          import numpy as np
          from scipy import stats, io
          import scipy
          import pandas as pd
          import matplotlib.pyplot as plt
          %matplotlib inline
          from netCDF4 import Dataset
          from mpl_toolkits.basemap import Basemap, cm, shiftgrid
          import h5py
          import base64
          from IPython.display import Image
```

So Matthew generously sent some code to calculate the BPT in ECCO-GODAE version 3. This is calculated as:

$$w_b = u_b \cdot \nabla(H) = \frac{1}{\rho_0 f} J(p, H)|_b.$$

I like to think of it in terms of vortex retching and squishing... This isn't strictly correct, but it's a nice place to start assuming no normal flow at the bottom:

$$\begin{aligned}
\frac{1}{\rho_0} J(p_b, H) &= \frac{1}{\rho_b}\left[\frac{\partial p_b}{\partial x}\frac{\partial H}{\partial y} - \frac{\partial p_b}{\partial y}\frac{\partial H}{\partial x}\right] \\
&= f\left[v_{gb}\frac{\partial H}{\partial y} + u_{gb}\frac{\partial H}{\partial x}\right] \\
&= f\mathbf{v}_{gb} \cdot \nabla H \\
&= -f w_{gb}
\end{aligned} \tag{1}$$

If you want to be more finicky:

$$\left(\frac{D\mathbf{u}_b}{Dt} - \frac{\partial \tau}{\partial z}\right) \cdot |\nabla H|\mathbf{t}_H + f\mathbf{u} \cdot \nabla H = \underbrace{-k \cdot \nabla H \times \nabla p_b}_{\| \atop J(p_b, H)}.$$

We arrive at this from the depth intragrated transport. See Highes and de Cuevas 2001 for a full derivation... (Or Matthew's paper, or my draft...)

I basically just go through what Matthew did step by step. Add some figures and verify at the end.

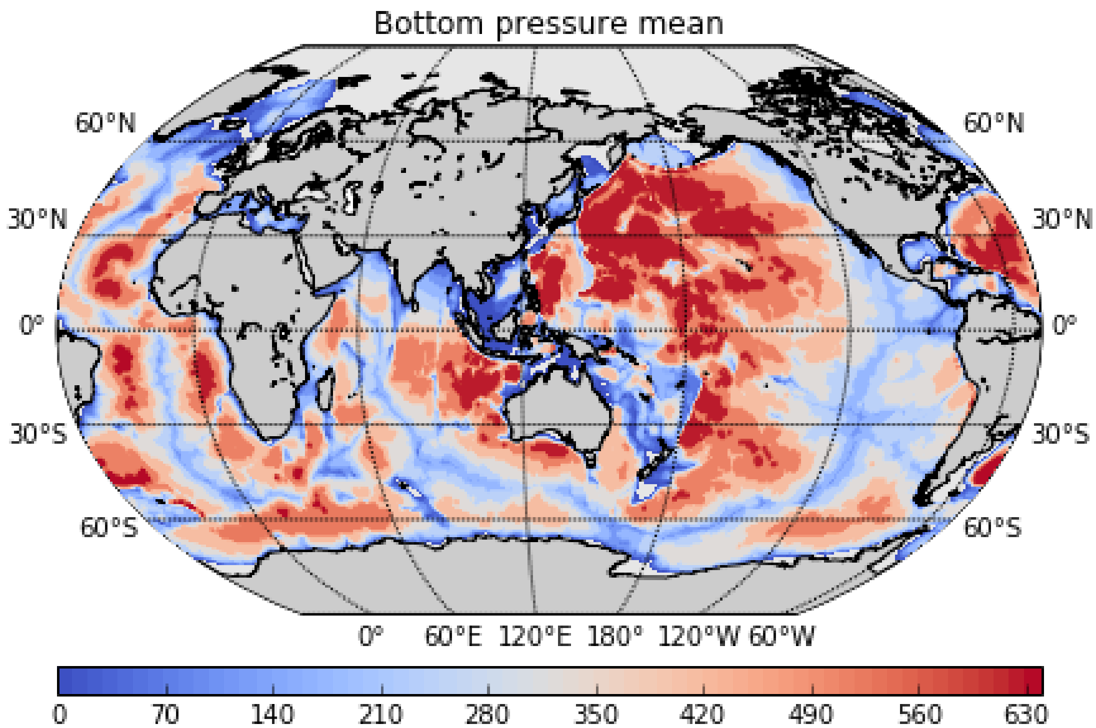
```
In [1]: #Bundle some plotting stuff..
        def makeFig(data, cMin, cMax, cStep, title, ColMap, saveName):
                dataIn = data
                lons = np.roll(np.linspace(0, 360, 360),100)
                lons = lon_cMatt
                dataIn,lons = shiftgrid(lon_cMatt[0],np.flipud(dataIn),lons,start=True, cyclic=360)
                lats = lat_cMatt
                llons, llats = np.meshgrid(lons,lats)
                fig = plt.figure()
                ax = fig.add_axes([0.05,0.05,0.9,0.9])
                # create Basemap instance.
                m = Basemap(projection='kav7',lon_0=130,resolution='l')
                m.drawmapboundary(fill_color='0.9')
                im1 = m.contourf(llons,llats,np.fliplr(np.fliplr(dataPlot)),np.arange(cMin, cMax, cStep
                m.drawmapboundary(fill_color='0.9')
                m.drawparallels(np.arange(-90.,99.,30.),labels=[1,1,0,1])
                m.drawmeridians(np.arange(-180.,180.,60.),labels=[1,1,0,1])
                m.drawcoastlines()
                m.fillcontinents()
                cb = m.colorbar(im1,"bottom", size="5%", pad="9%")
                # add a title.
                ax.set_title(title)
                plt.savefig(saveName, format='png', dpi=500)
                plt.show()
```

So I used the data Matthew gave us. Below I just load the data, and plot for a ballpark sanity check.

```
In [99]: b_prMattF=io.loadmat('/home/maike/Documents/BPT/BPT_calc/time_mean_pressbot.mat')
         b_pr=b_prMattF['botpress_mean']
         lat_c=b_prMattF['lat_c']
         lon_c=b_prMattF['lon_c']
```

```
In [160]: ecco_paramsF=io.loadmat('/home/maike/Documents/BPT/BPT_calc/ecco_params.mat')
          time_mean_pressF=io.loadmat('/home/maike/Documents/BPT/BPT_calc/time_mean_press.mat')
          depth_l=ecco_paramsF['depth_l']
          lat_v=ecco_paramsF['lat_v']
          hydpress_mean=time_mean_pressF['hydpress_mean']
```

```
In [376]: makeFig(b_prMatt, 0,650,10, 'Bottom pressure mean', plt.cm.coolwarm,'bprMatt.png')
```



Right, so now I'll just translate this step by step from what Matthew sent. I'm double checking using Octave (read: Matlab)

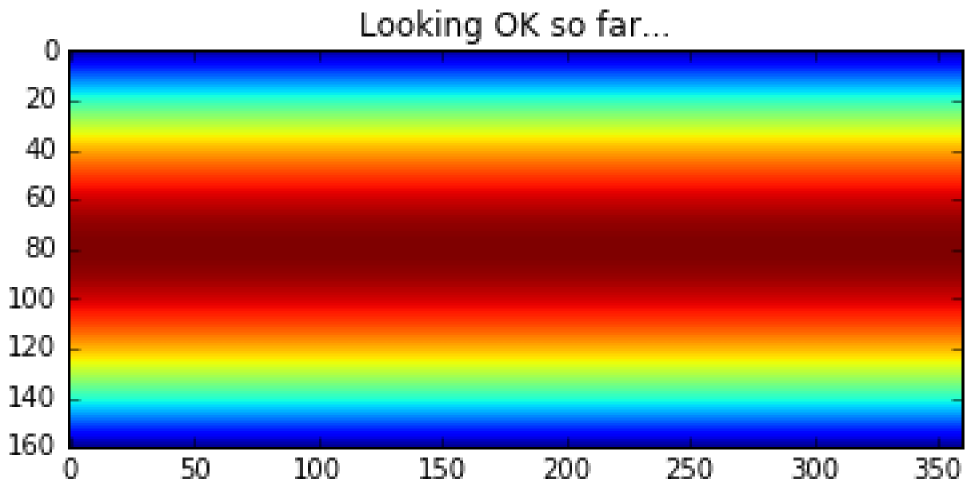
0.1.1 Establish some standard values

In [42]:
```
lon_diff=1.111774765625000e+05
Omega=7.2921e-5 #2*np.pi/T

f=np.array(2*Omega*np.sin((np.arange(np.min(lat_c),np.max(lat_c)+abs(lat_c[2]-lat_c[1]),abs(la

beta_fgrid=np.zeros(lat_c.shape[0]);beta_fgrid[:]=np.nan
beta_fgrid[1:160]=((f[1:160]-f[0:159])/(lon_diff)) # Gives beta at the cell corner (the vortic
beta_fgrid=beta_fgrid.repeat(360).reshape(160,360)   # Just makes it 2D
```

In [295]:
```
# grid cell widths and heights etc.
grid_width_u=(lon_diff*np.cos(lat_c*np.pi/180))*np.ones(360)
grid_width_v=(lon_diff*np.cos(lat_v*(np.pi/180)))*np.ones(360)
depth_l_diff=np.append(10,np.diff(np.abs(depth_l[:,0])))
depth_l_diff_glob=np.repeat(depth_l_diff,(160*360)).reshape(23,160,360)
grid_width_u_glob=np.tile(grid_width_u,(23,1,1))#.reshape(23,160,360)
grid_width_v_glob=np.tile(grid_width_v,(23,1,1))#.reshape(23,160,360)
```

In [377]:
```
plt.imshow(grid_width_v_glob[0,...])
plt.title('Looking OK so far...')
```

Out[377]:
```
<matplotlib.text.Text at 0x7f0f56666050>
```



0.1.2 Determine the pressure gradient term in the depth-integrated momentum equation.

In [298]:
```
# calculate gradients.
gradp_i=np.zeros(grid_width_v_glob.shape);gradp_i[:]=np.nan
gradp_j=np.zeros(grid_width_v_glob.shape);gradp_j[:]=np.nan

gradp_i[:,:,1:]=-(hydpress_mean[:,:,1:]-hydpress_mean[:,:,0:-1])
gradp_i[:,:,0]=(-(hydpress_mean[:,:,0]-hydpress_mean[:,:,-1]))
gradp_i=gradp_i/grid_width_u_glob
gradp_j[:,1:,:]=(-(hydpress_mean[:,1:,:]-hydpress_mean[:,0:-1,:]))
gradp_j=gradp_j/lon_diff
```

In [299]:
```
# vertically integrate grad(p) to bottom.
gradp_int_i=np.nansum(gradp_i[:,:,:]*depth_l_diff_glob[:,:,:],0)
gradp_int_j=np.nansum(gradp_j[:,:,:]*depth_l_diff_glob[:,:,:],0)
```

0.1.3 Calculate the form stress term in the depth-integrated momentum equation.

In [300]:
```
# vertically integrate p to bottom.
p_int=np.nansum(hydpress_mean[:,:,:]*depth_l_diff_glob[:,:,:],0)
```

In [301]:
```
# take gradient
grad_intp_i=np.zeros(p_int.shape);grad_intp_i[:]=np.nan
grad_intp_j=np.zeros(p_int.shape);grad_intp_j[:]=np.nan

grad_intp_i[:,1:]=-(p_int[:,1:]-p_int[:,0:-1])
grad_intp_j[1:,:]=-(p_int[1:,:]-p_int[0:-1,:])
grad_intp_i[:,0]=-(p_int[:,0]-p_int[:,-1])
grad_intp_i=grad_intp_i/grid_width_u
grad_intp_j=grad_intp_j/lon_diff
```

In [302]:
```
# The form stress term:
pb_gradH_i=gradp_int_i-grad_intp_i
pb_gradH_j=gradp_int_j-grad_intp_j
```

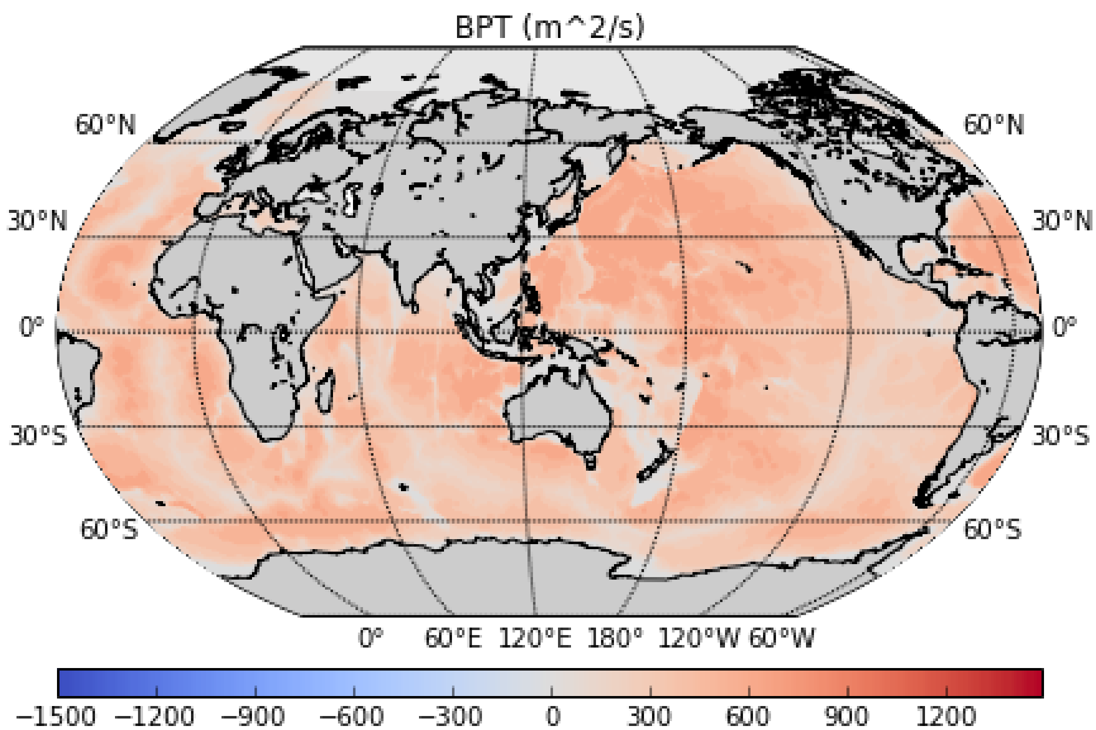
0.1.4 Compute curl (in spherical polar coordinates) to get the depth-integrated vorticity equation.

In [303]:
```
np.rollaxis(np.tile(depth_l_diff,(160,360,23)),2,0)

coslatc_glob=np.tile(np.cos(lat_c*np.pi/180)*np.ones(360),(23,1,1)) # cosine of latitude (on
coslatv_glob=np.tile(np.cos(lat_v*np.pi/180)*np.ones(360),(23,1,1)) # cosine of latitude (on

# first scale the zonal terms with cos(lat). Needed for the spherical coordinate calculation.
# Spherical coordinate calculations are based on this page: https://en.wikipedia.org/wiki/Del
gradp_i_scale=gradp_i*coslatc_glob
gradp_int_i_scale=gradp_int_i*coslatc_glob[0,:,:]
grad_intp_i_scale=grad_intp_i*coslatc_glob[0,:,:] #OK
pb_gradH_i_scale=pb_gradH_i*coslatc_glob[0,:,:]
```

In [304]:
```
# Find the curl of the depth integrated pressure gradient. This is the term used for BPT.
NansToPad=np.zeros((1,360)); NansToPad[:]=np.nan
curl_gradp_int_kcomp=(np.insert(gradp_int_i_scale[1:,:]-gradp_int_i_scale[0:-1,:], 0, NansToPa
#and calculate the first longitude index
#curl_gradp_int_kcomp(:,1)=(padarray((gradp_int_i_scale(2:end,1)-gradp_int_i_scale(1:end-1,1)
#    -((gradp_int_j(:,1)-gradp_int_j(:,end))./grid_width_v(:,1));
```

In [324]:
```
#This is it!
BPT=curl_gradp_int_kcomp/beta_fgrid
```

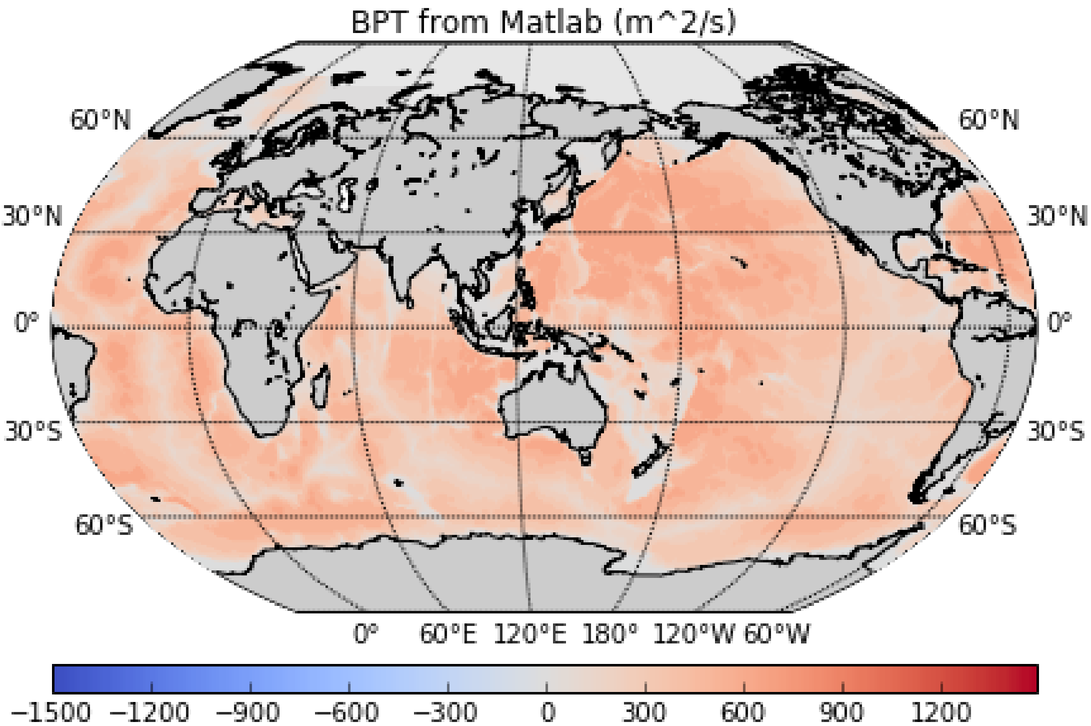
In [357]:
```
makeFig(BPT, -1500,1500,10, 'BPT (m^2/s)', plt.cm.coolwarm,'BPT_Matt.png')
#Tada!!
```



Right, now we load the data from using Matt's matlab code to compare. . .

In [336]:
```
BPT_MattF=h5py.File('/home/maike/Documents/BPT/BPT_calc/bpt.h5', 'r')
bpt_matt = BPT_MattF['BPT']
bpt_matt = bpt_matt['value']
```

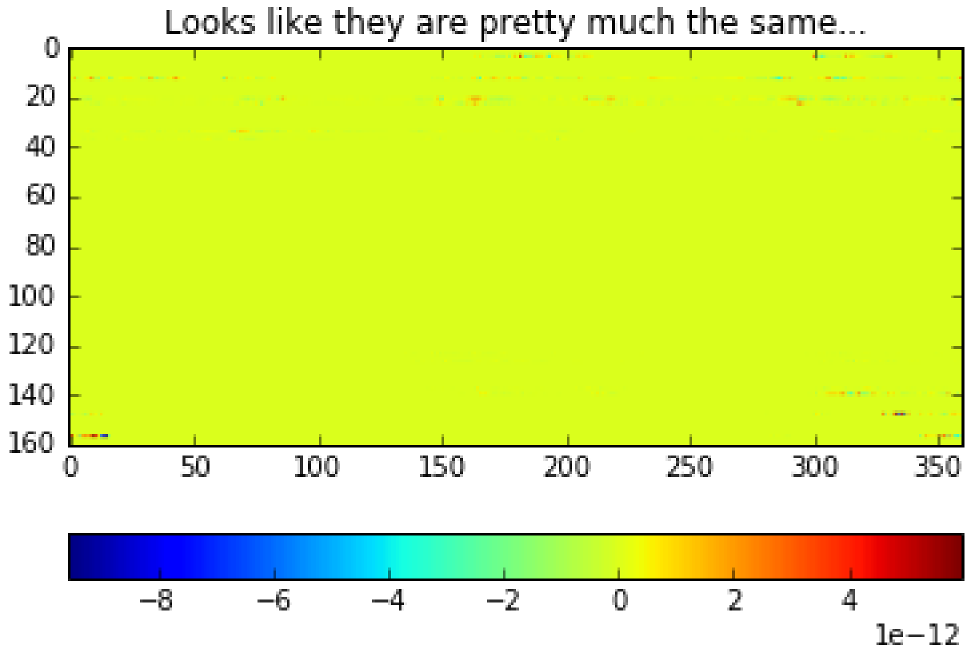
In [356]:
```
makeFig(bpt_matt, -1500,1500,10, 'BPT from Matlab (m^2/s)', plt.cm.coolwarm,'BPT_Matt_matlab.p
```



Just to demonstrate, it's machine precision, so I'm OK saying the above python code does what the matlab code Matt developed does!

In [375]:
```
plt.imshow(BPT+np.flipud(np.rot90(bpt_matt[:],1)))
plt.colorbar(orientation='horizontal')
plt.title('Looks like they are pretty much the same...' )
```

Out[375]:
```
<matplotlib.text.Text at 0x7f0fc1c81c90>
```



0.1.5 Right; fabulous! Just some thought's to round off:

1: Next step is to make this work for ECCOv4! Then onwards in the vorticity budget!

2: Would be cool just to check if the whole:

$$w_b = u_b \cdot \nabla(H) = \frac{1}{\rho_0 f} J(p, H)|_b.$$

I'm curious mainly as ECCO doesn't do the partial cells. Wells and de Cuevas 1995 had some interesting figures showing that you had to ignore the points where the change in bathymetry were too abrupt. This would likely be exagerated in ECCO as it's quite coarse.

3: Overall, I guess my main interest is playing with this is how ECCO measures up against other models. I really like the thought of recreating something like the figure below from Schonoover (2016), but adding info from the sensitivities. The below is just for the North Atlantic gyre, so I'd have to think of a good way to graphically present it, but I think you could start comenting on how the sensitivities express themselves on the vorticity budget. That's my goal anyhows!

```
In [382]: Image(filename="/home/maike/Documents/BPT/schoonover2016Fig3.png")
```

```
Out[382]:
```

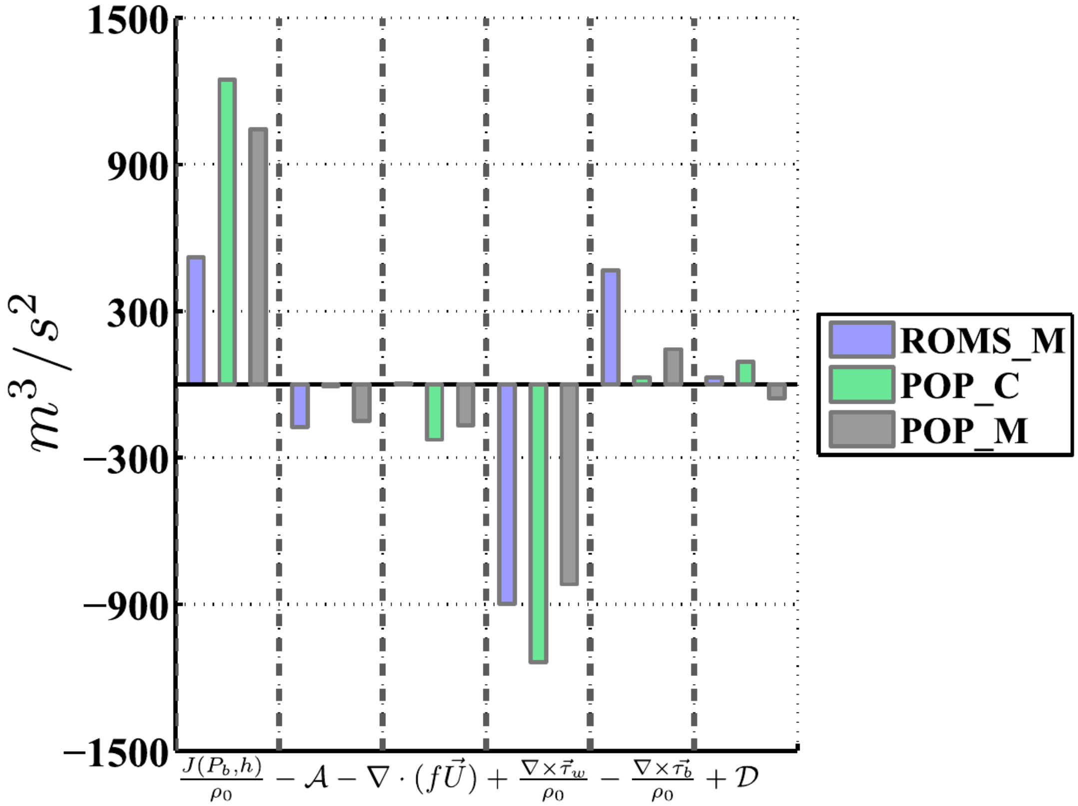


FIG. 3. Time-averaged and gyre-integrated barotropic vorticity budget [Eq. (1)] is shown for ROMS_M, POP_C, and POP_M. The boundary of the gyre for each simulation is shown in Fig. 2. The units are $\mathrm{m^3\,s^{-2}}$.

0.1.6 Finally: General thoughts on looking at the impact on the gyre circulation and the overturning streamfunction

To assess the impact of the BPT and other terms, it would be useful to know their impact on the overturning. Here, is a bit on the theoretical foundation of this link. It's a bit pie-in-the-sky, but worth hashing out!

If we use the barotropic vorticity relation, and keep the (horizontal) non-divergence in mind implied by:

$$\nabla\cdot\mathbf{U} = \nabla\cdot\mathbf{U}_g + \nabla\cdot\mathbf{U}_{ag} = -\partial_t h \tag{2}$$

Where "g" is the geostrophic component and "ag" is the ageostrophic component. We can recast the equation as a streamfunction equation for the barotropic gyre circulation:

$$\begin{aligned}
\beta\partial_x\psi_G &= \frac{1}{\rho_0}J(p_b,H) + \frac{1}{\rho_0}\nabla\times\Delta\tau \\
\psi_G &= -\frac{1}{\beta}\int_{x_w}^{x_e}\left[\frac{1}{\rho_0}J(p_b,H) + \frac{1}{\rho_0}\nabla\times\Delta\tau\right]dx \\
\psi_G &= \psi_G^{BPT} + \psi_G^{\tau}
\end{aligned} \tag{3}$$

Where $\mathbf{U} = \mathbf{k} \times \nabla\psi_G$ the "G" is for gyre. Here we follow Yeager (2015), looking at the gyre circulation in the North Atlantic first (We leave out the non-linear and lateral friction terms, which can easily be added). A closure at the western boundary is implied ($\psi_G(x_w) = 0$) with the integral balance of terms:

$$0 = \int_{x_w}^{x_e} \left[\frac{1}{\rho_0} J(p_b, H) + \frac{1}{\rho_0} \nabla \times \Delta\tau \right] dx \tag{4}$$

In depth-space, where the overturning ("o") streamfunction is defined in the y-z plane ($-\partial_z \psi_o = \int_{x_w}^{x_e} v dx$ and $\partial_y \psi_o = \int_{x_w}^{x_e} w dx$) this translates to:

$$\begin{aligned} \psi_o &= -\frac{1}{\beta} \int_{x_w}^{x_e} \left[\frac{1}{\rho_0} J(p_b, H)(z) + \frac{1}{\rho_0} \nabla \times \Delta\tau(z) \right] dx \\ \psi_o &= \psi_o^{BPT} + \psi_o^{\tau} \end{aligned} \tag{5}$$

Or in density-space:

$$\begin{aligned} \psi_\rho &= -\frac{1}{\beta} \int_{x_w}^{x_e} \left[\frac{1}{\rho_0} J(p_b, H)(\rho) + \frac{1}{\rho_0} \nabla \times \Delta\tau(\rho) \right] dx \\ \psi_\rho &= \psi_\rho^{BPT} + \psi_\rho^{\tau} \end{aligned} \tag{6}$$

Here the partial z (or ρ) implies the closure $\psi_0(\nu) = 0$ (and a statically stable water column for ρ), and highlights the importance of abyssal flows and the bathymetric control on the deep meridional flow closing the overturning circulation.

In this manner we could assess the relative contributions to the vorticity balance in density space. This would be particularily interesting in terms of the long term mean and the more transient monthly means, where the importance of terms such as the BPR etc terms will likely reveal changes in the mecanisms active to realise the overturning and gyre circulation...

Comments?

0.1.7 PDF aside:

I thought it would be nice just to have a quick look at how the probability distribution functions look. In sum, they look **really** smooth. I think this is probably because we're working directly on the time mean data. I added a plot from NEMO to illustrate what sort of shape I was expecting..

Comments?

```
In [399]: def makeMaskSection(data,upper,lower,section):
              nx,ny=data.shape
              mask = np.fromfunction(lambda i, j: ((data[section,:]<upper) & (data[section,:]>=lower)),
              return mas

          def makeMask(data,upper,lower):
              nx,ny=data.shape
              mask = np.fromfunction(lambda i, j: ((data[:,:]<=upper) & (data[:,:]>lower)), (nx, ny), d
              return mask

          def maskArray(Array,mask):
              maskedArray = ma.masked_array(Array, mask[:])
              return maskedArray

          def pdfCalc(data,area,maskf,steps):
              bins=np.linspace(-2000, 2000, steps)#np.min(data), np.max(data), steps)
              pdf=np.empty(bins.shape)
```

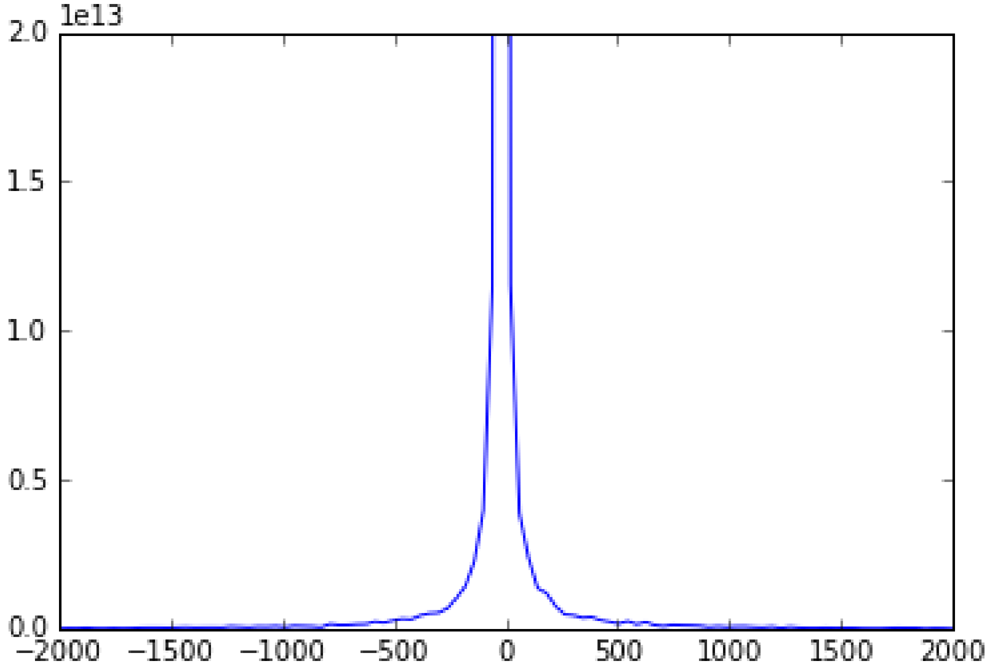
```
        #counter = 1
        for nr in xrange(len(bins)-1):
            #print nr, nr+1,bins[nr+1], bins[nr]
            upper = bins[nr+1]
            lower = bins[nr]
            maskNr = makeMask(data,upper,lower)
            mA = maskArray(area, ~maskNr)
            pdf[nr] = np.nansum(mA*maskf)#[0,0,...])
            if nr%1==10: print(upper,lower)
            #counter=counter+1
            #print nr
        return pdf, bins

    def pdfCalcLat(data,area,maskf,steps):
        nx,ny=data.shape
        bins=np.linspace(-2000, 2000, steps)
        pdf=np.empty([nx,steps])
        counter = 1
        for nr in xrange(len(bins)-1):
            #print nr
            upper = bins[nr+1]
            lower = bins[nr]
            maskNr = makeMask(data,upper,lower)
            mA = maskArray(area, ~maskNr)
            pdf[:,nr] = np.nansum(mA*maskf[...],axis=1)
            if nr%1==10: print(upper,lower)
            counter=counter+1
        return pdf, bins
```

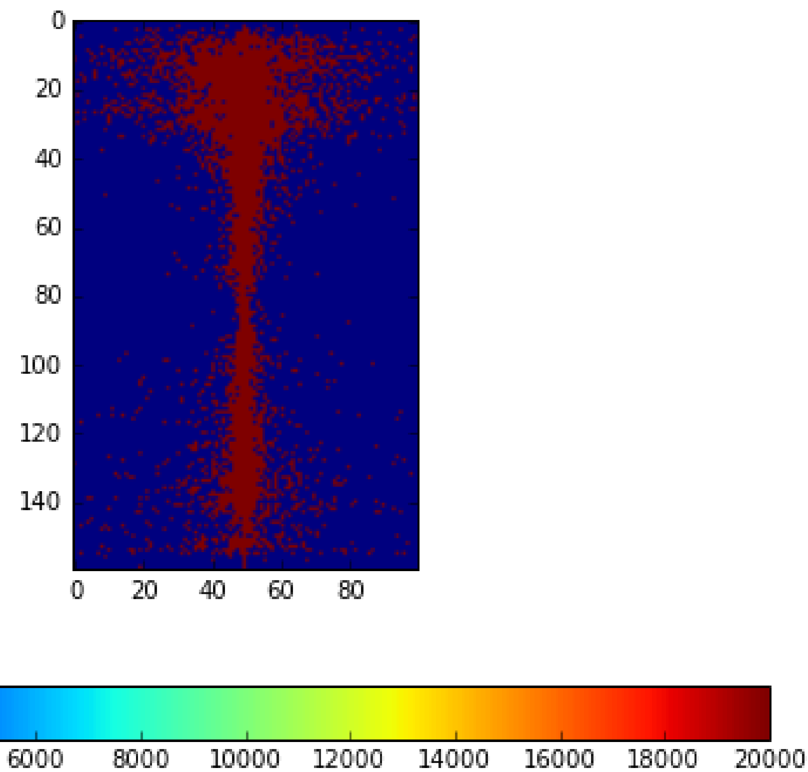
In [390]:
```
area = grid_width_v_glob[0,...]*grid_width_u_glob[0,...]
```

In [417]:
```
mask=np.ones(BPT.shape)
index = np.isnan(b_prMatt)
mask[index]=np.nan

pdfBPT, bins = pdfCalc(BPT,area,mask,100)
pdfBPTLat, bins = pdfCalcLat(BPT,area,mask,100)
```

In [428]:
```
plt.plot(bins,pdfBPT)
plt.ylim(0,0.2e14)
plt.figure(figsize=(8, 6))
plt.imshow(pdfBPTLat)
plt.clim(0,20000)
plt.colorbar(orientation='horizontal')
```

Out[428]:
```
<matplotlib.colorbar.Colorbar at 0x7f0fc1f13550>
```

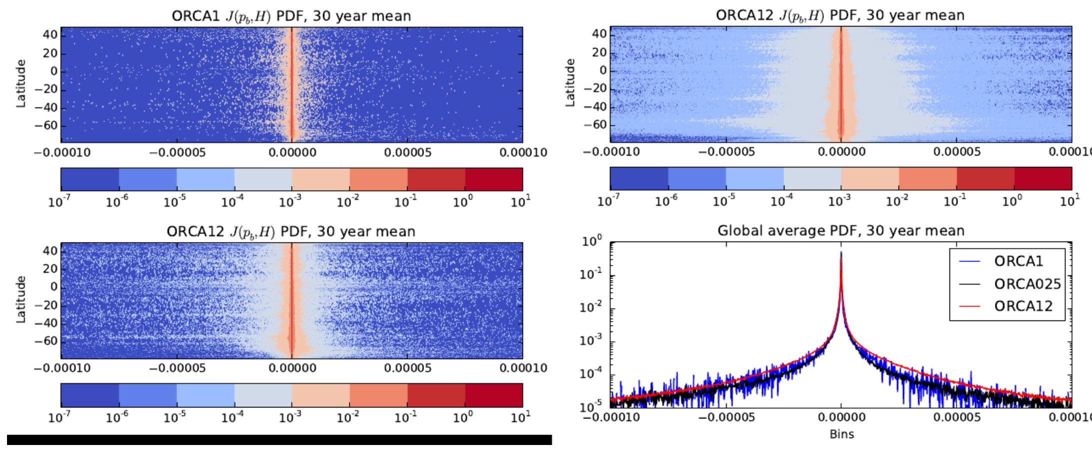

1e13
2.0
1.5
1.0
0.5
0.0
−2000 −1500 −1000 −500 0 500 1000 1500 2000


0
20
40
60
80
100
120
140
0 20 40 60 80
0 2000 4000 6000 8000 10000 12000 14000 16000 18000 20000

For comparison from NEMO. Things that stand out is how smoth ECCO is. I was expecting something like ORCA1 (1°)? Also, ECCO seems to cluster around the northern Hemisphere, rather than more uniformly/around the Southern Ocean? Can we think of a reason for this? Again I wonder about working on the time mean data...

```
In [427]: Image(filename="/home/maike/Documents/BPT_draft/talkPDFmeanBPT.png")
```

Out[427]:




```
In [ ]:
```