LINEAR PREDICTABILITY: A SEA SURFACE HEIGHT CASE STUDY

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1. Abstract

A benchmark of linear predictictability of sea surface height (SSH) globally is presented, complementing more complicated studies of SSH predictability. Twenty years of the Estimating the Circulation and Climate of the Ocean (ECCOv4) state estimate (1992-2011) are used, fitting autoregressive moving average (ARMA(n,m)) models where the order of the coefficients is chosen by the Akaike Information Criteria (AIC). Up to 50% of the ocean SSH variability is dominated by the seasonal signal. The variance accounted for by the non-seasonal SSH is particularily distinct in the Southern and Pacific Ocean, containing >95% of the total SSH variance and the expected prediction error growth taking a few months to reach a threshold of 1 cm. Isolated regions take twelve months or more to cross an accuracy threshold of 1 cm. Including the trend, significantly increases the time taken to reach the threshold, particularly in the South Pacific. Annually averaging has expected prediction error growth of a few years to reach a threshold of 1 cm. Including the trend mainly increases the time taken to reach the threshold, but the timeseries is short and noisy.

2. Motivation and methods

- SSH (η) prediction involves projection and space-time integration of diverse physics on months to thousands of years.
- **Prediction error** (PE) growth is not well established for statistical or complex prediction efforts.
- **AIM**:This study presents a benchmark of linear η predictability using autoregressive moving average models (ARMA(n,m)).

- **Ocean model:** ECCOv4 1° global bi-decadal state estimate, adjusting a free-running MITgcm to observations.
- We split the signal into sesonal (η) and non-seasonal (η'):

$$\eta = \bar{\eta} + \eta'$$

- We use:

$$AR(n): \eta'(t) = a_1\eta'(t-1) + a_2\eta'(t-2) + \ldots + \overbrace{a_n}^{\text{coeff.}}\eta'(t-n)$$

$$MA(m): \eta'(t) = \underbrace{e(t)}_{\text{noise}} + b_1e(t-1) + b_2e(t-2) + \ldots + \underbrace{b_m}_{\text{coeff.}}e(t-m)$$

ARMA(n,m):

$$\eta'(t)=a_1\eta'(t-1)+a_2\eta'(t-2)+..+a_n\eta'(t-n)+e(t)+b_1e(t-1)+b_2e(t-2)+..+b_me(t-m)$$

- PE for τ-ahead is most explicit in its MA(m) form:

$$\langle(\eta'(1+\tau)-\eta'(\tau+\tau))^2\rangle = \sigma_\varepsilon^2\sum_{p=0}^{\tau}b_p^2,\ b_0=1.$$

- PE varies by region (Fig.1), and assess performance in terms of time taken for the error to grow beyond a threshold of 1 cm.
- The **Akaike Information Criterion** is used to determine the ARMA(n,m) order:

$$AIC = 2k - 2\ln(L)$$

where L is the likelihood, and k the order.

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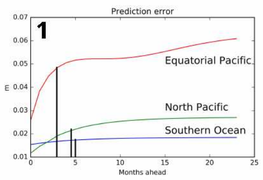

3.1 Seasonal and non-seasonal variance

- Fig.2 shows the variance of η (m^2), with seasonal (Fig.3) and non-seasonal (Fig.4) component.
- >50% of the ocean variance is dominated by seasonal $\bar{\eta}$, but the Pacific and Southern Ocean have strong non-seasonal contributions.

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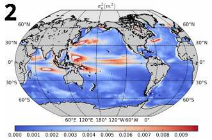

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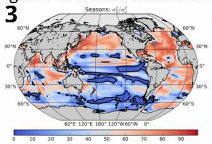

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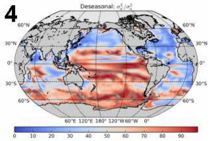

3.2 Detrended η' prediction accuracy (months) to reach 1cm

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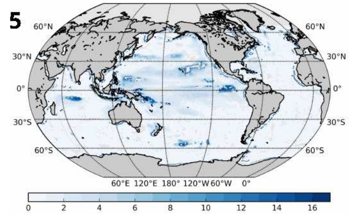

3.3 Non-detrended η' prediction accuracy (months) to reach 1cm threshold

- Trend included as an unresolved component of a rednoise process. Fig.6 shows increased time taken to cross 1cm from Fig.5.
- Fig.6 has large areas exceeding 12 months to exceed the threshold of 1 cm. Particularly bands in the subtropical Atlantic and Pacific, Drake passage, South Pacific and the Kuroshio path.

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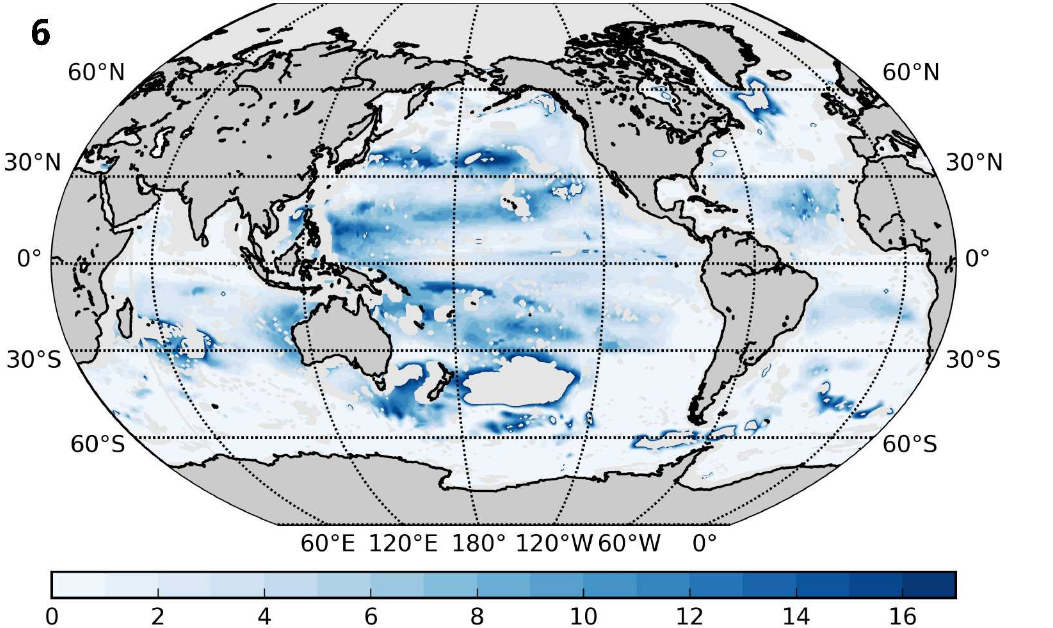

3.4 ARMA (n,m) coefficients and assumption of normally distributed data

- The Shapiro-Wilk test for detrended η' normality (Fig.7). This is important in interpretation.
- Fig. 8 and 9 show n and m from ARMA(n,m). n is higher with regional coherence.

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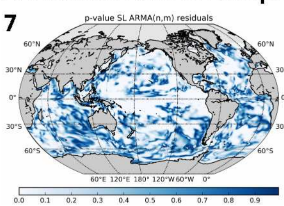

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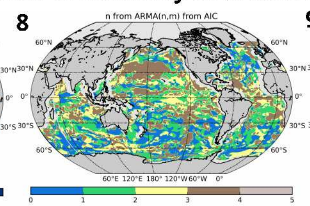

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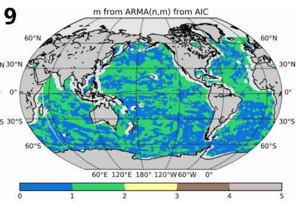

3.5 Annually averaged η'

- Interannual and monthly physics are distinct, which is reflected in the ARMA(n,m) in Fig.10 (for n) and 11 (for m).
- Fig.12 shows time (years) for the PE to reach the threshold of 1cm for detrended, annually averaged, η'. Fig.13 shows the same with the trend. A general increase is seen retaining the trend, but this is not as strong as with monthly averaging.
- The assumptions of a correctly estimated covariance and a Gaussian distribution are problematic.

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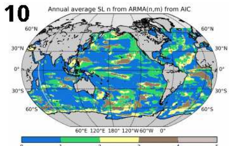

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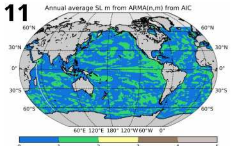

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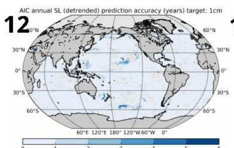

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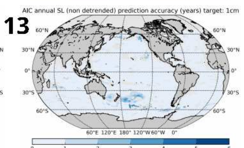

5. Main conclusions:

- **A benchmark of linear univariate predictability of η is presented to complement more complex studies.**
- **Any more sophisticated attempt needs to do significantly better to be justified. Identified dynamically coherent regions encourage such work.**
- **In >50% of the ocean the seasonal cycle is sufficient to estimate the $\bar{\eta}$ variability indefinitely, as defined.**
- **The stochastic residual, η', is important in large areas of the Southern Hemisphere.**
- **Including the linear trend as part of the continuum increases the predictive performance, taking >12 months to reach a threshold of 1 cm in large regions of the the ocean; South Pacific, Southern Ocean, Irminger and labrador Sea.**
- **Annually averaged η' shows different physical mechanisms, but is likely prone to noise.**
- **Moving forwards, extending global measurements of η and understanding the underlying physical processes are key to progress.**

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