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# Solutions to some exercises
of the book
"Introduction to Perturbation Methods"
by Mark H. Holmes

## 1 Introduction to Asymptotic Approximations

**Exercise 1.32.** Find a two-term asymptotic expansion, for small $\varepsilon$, of the solution of the following problems:

(a) $y'' + \varepsilon y' - y = 1$, where $y(0) = 0$ and $y(1) = 1$.

**Solution.** We assume $y(\tau) \sim y_0(\tau) + \varepsilon y_1(\tau) + o(\varepsilon)$. Substituting this into the ODE yields

$$y_0''(\tau) + \varepsilon y_1''(\tau) + \varepsilon y_0'(\tau) - y_0 - \varepsilon y_1 + o(\varepsilon) = 1 \tag{1}$$

and from the initial values $y(0) = 0$ and $y(1) = 1$ we get

$$y_0(0) + \varepsilon y_1(0) + o(\varepsilon) = 0 \text{ and } y_0(1) + \varepsilon y_1(1) + o(\varepsilon) = 1. \tag{2}$$

Now by equating like powers of $\varepsilon$, we obtain the following problems:

$O(1)$
$y_0''(\tau) - y_0(\tau) = 1$,
$y_0(0) = 0$, $y_0(1) = 1$.
The solution to this problem is given by

$$y_0(\tau) = \frac{1}{2}\tau^2 + \frac{e^{-1}}{2}e^{\tau} - 1. \tag{3}$$