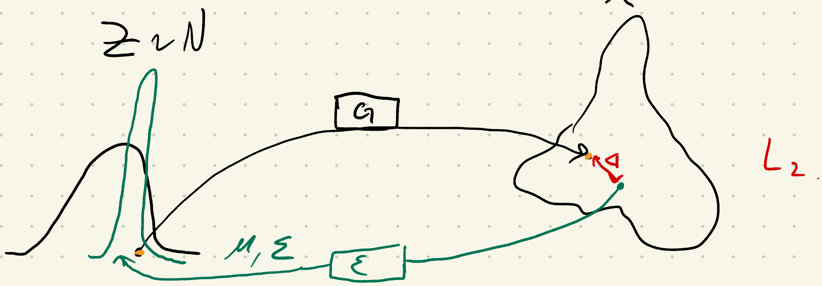
Generative models

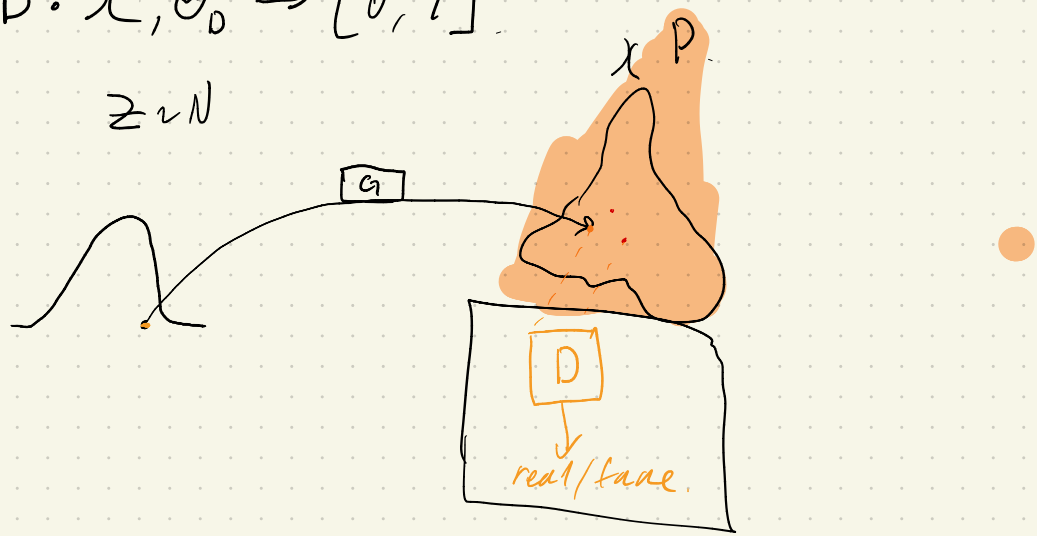
from Lect11



$G: z, \theta_G \to x$

VAE

$D: x, \theta_D \to [0, 1]$



$$\underbrace{-\mathbb{E}_{x\sim p} \log_2(D(x)) - \mathbb{E}_{z\sim N} \log_2(1-D(G(z)))}_{L_D(\theta_D)}$$

$$L_G(\theta_G) = \mathbb{E}_{z\sim N} \log_2(1-D(G(z))) = \mathbb{E}_{x\sim q} \log_2(1-D(x))$$

from Lect11

$$V(\Theta_G, \Theta_D) = \mathbb{E}_{x\sim p} \log_2(D(x)) + \mathbb{E}_{z\sim N} \log_2(1 - D(G(z))).$$

$$\boxed{\min_{\Theta_G} \max_{\Theta_D} V(G, D).}$$

$$\int_x p(x) \log_2 D(x)\, dx + \int p_z(z) \log_2(1 - D(G(z)))\, dz$$

$d. \; s\; G(z), \; z \sim N$

$$= \int_X \left[p(x) \log_2 D(x) + q(x) \log_2(1 - D(x)) \right] dx$$

$$\underbrace{a \log m + b \log_2(1-m)}$$

$$D^*_G(x) = \frac{p(x)}{p(x) + q(x)}.$$

$$\mathbb{E}_{x\sim p} \log_2 \frac{p(x)}{p(x)+q(x)} + \mathbb{E}_{x\sim q} \log_2 \frac{q(x)}{p(x)+q(x)}.$$

$$-\log_2 2 - \log_2 2 = -\log_2 4.$$

$$\underbrace{\log_2 4 + KL\left(p \,\|\, \frac{p+q}{2}\right) + KL\left(q \,\|\, \frac{p+q}{2}\right)}_{\text{Jensen-Shannon divergence.}}$$

$$KL(p\|q) = \mathbb{E}_{x\sim p} \log_2 \frac{p(x)}{q(x)}$$

$$KL(q\|p) = \mathbb{E}_{x\sim q} \log_2 \frac{q(x)}{p(x)},$$

$$JS(p,q) = \underbrace{\left(\log_2 4 + KL\left(p \,\|\, \frac{p+q}{2}\right) + KL\left(q \,\|\, \frac{p+q}{2}\right)\right)}_{4} \frac{1}{2}.$$

$$\boxed{\theta \neq 0 \quad \log 2.}$$

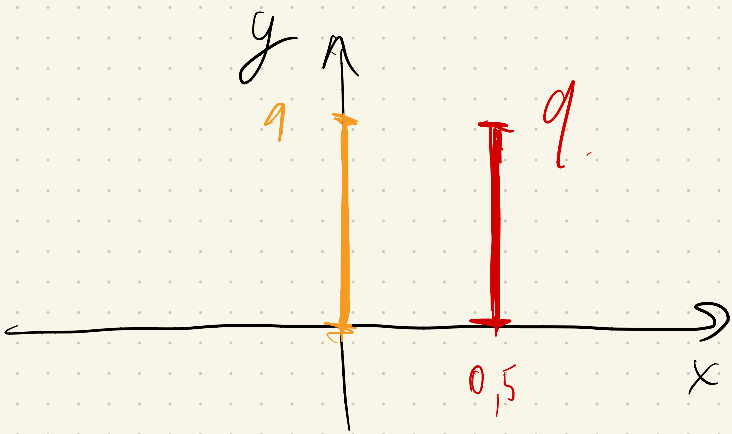
from Lect11

$z \sim U[0, 1]$

$p = (0, z)$

$q = (\theta, z)$

$$\boxed{\begin{array}{c} \theta \neq 0 \\ \theta = 0 \end{array}}$$



$$\frac{1}{2}\left(\log_2 4 + \mathbb{E}_{x\sim p} \log_2 \frac{2p(x)}{p(x)+\underbrace{\cancel{q(x)}}_{=0}} + \mathbb{E}_{x\sim q} \log_2 \frac{2q(x)}{\underbrace{\cancel{p(x)}}_{0}+q(x)}\right) =$$

$$\underbrace{}_{=2} \qquad \underbrace{}_{=1}$$

$$= \frac{1}{2} \log_2 4 = \log_2 2 = 1$$

from Lect11

$p, q \qquad \forall \gamma \in \underline{\Pi} \quad \int_x \gamma(x,y)\,dx = q(y)$

$$\int_y \gamma(x,y)\,dy = p(x).$$

$$W(p,q) = \inf_{\gamma \in \Pi} \mathbb{E}_{(x,y)\sim\gamma} \|x - y\|_2$$

$$W(p,q) = \sup_{\|f\|_L \leq 1} \left(\mathbb{E}_{x\sim p} f(x) - \mathbb{E}_{x\sim q} f(x) \right)$$

$$|f(x) - f(y)| \leq \cdot \overset{1}{L} \cdot |x - y|$$

$$\left| \|\nabla_x f\| - 1 \right|$$

$D: X, \theta_D \to \mathbb{R}.$

from Lect11

$$L_D(\theta_D) = -\mathop{\mathbb{E}}_{x\sim p} D(x) + \mathop{\mathbb{E}}_{x\sim q} D(x) + \| \|\nabla_x D\| - 1 \|$$

$$L_G(\theta_G) = -\mathop{\mathbb{E}}_{x\sim q} D(x).$$

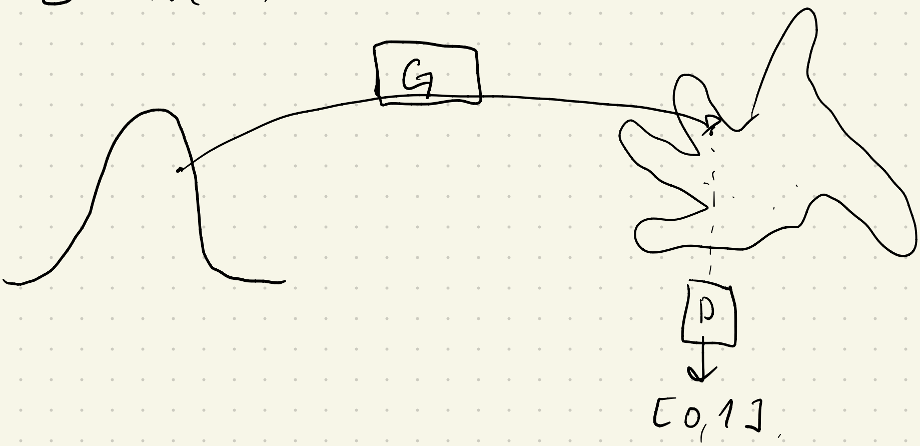
R 1. $\boxed{\nabla_x D(x)}$

$\boxed{\text{spectral norm}}$

$\Theta = P \Sigma V^T$ SVD.

$\boxed{\max(\Sigma) = 6}$ $\Theta / 6.$

$z \sim N(\ )$ $\mathcal{X}$



$G: Z \to \mathcal{X}$

$D: \mathcal{X} \to [0, 1]$

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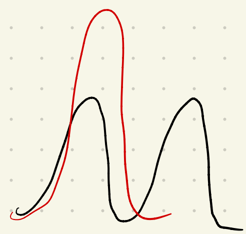
q

$$L_g = -\mathbb{E}_{z \sim N} \log(D(G(z))) =$$

$$\boxed{= -\mathbb{E}_{x \sim q} \log(D(x))} = ?$$



$\mathcal{X}$ $\mathcal{X}$

$x^{(1)} \in \mathbf{X}_1$ $x^{(2)} \in \mathbf{X}_2$

$f: \mathcal{X} \to \mathcal{X}$

$f(x^1) \in D_2$

$\boxed{\text{cycle gan}}$

$G_{1\to2}: \mathcal{X}, z \to \mathcal{X}$ $D_2: \mathcal{X} \to \mathbb{R}$

$G_{2\to1}: \mathcal{X}, z \to \mathcal{X}$ $D_1: \mathcal{X} \to \mathbb{R}$

$x^{(1)} \sim p_1$

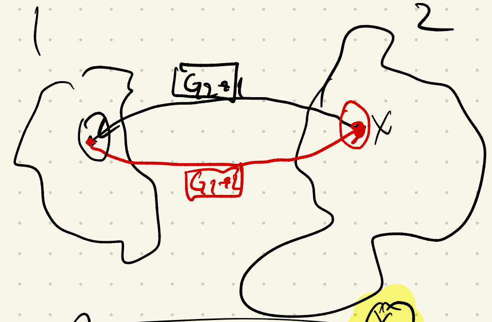
$\hat{x}^{(2)} = G_{1\to2}(x^{(1)})$ $D_2(\hat{x}^{(2)}) \to \max$

$\leftarrow \max$

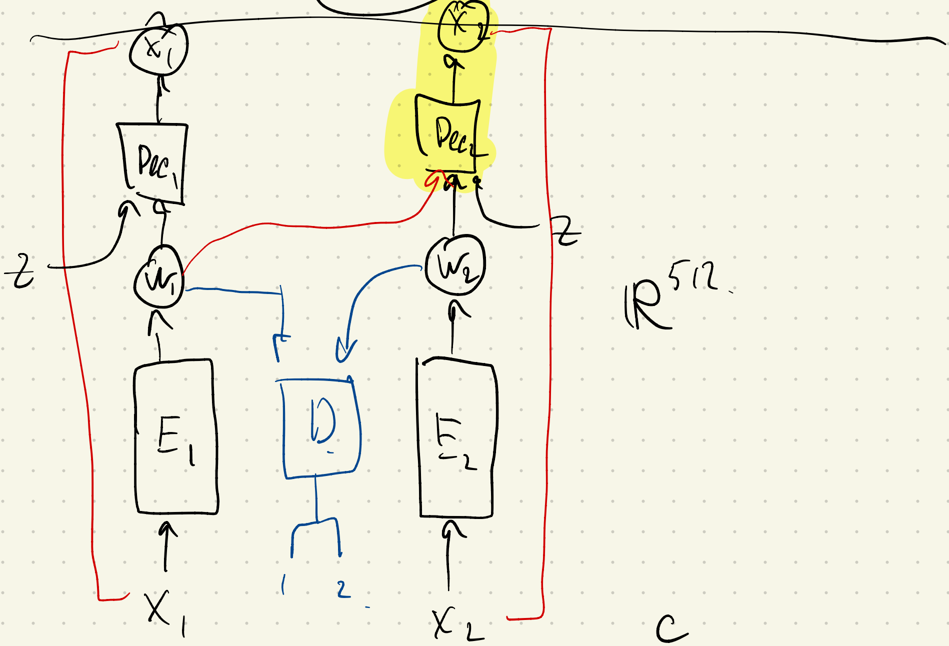
$\hat{x}^{(1)} = G_{2\to1}(\hat{x}^{(2)})$

cycle consistency loss

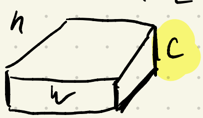
$\boxed{\|\hat{x}^{(1)} - x^{(1)}\| \to \min}$



$$x = G_{2\to 1}(G_{1\to 2}(x))$$



$\mathbb{R}^{512}$



$\in \mathbb{R}^c$ $\in \mathbb{R}^c$

$$\text{adain}(x) = \boxed{\left(\frac{x-\mu(x)}{\sigma(x)}\right)\cdot\sigma(y) + \mu(y)}$$