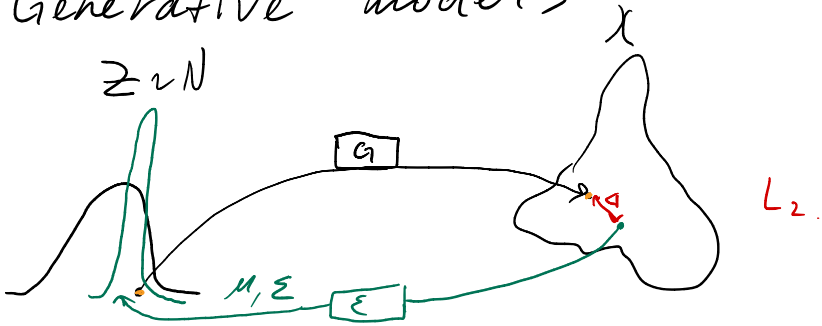
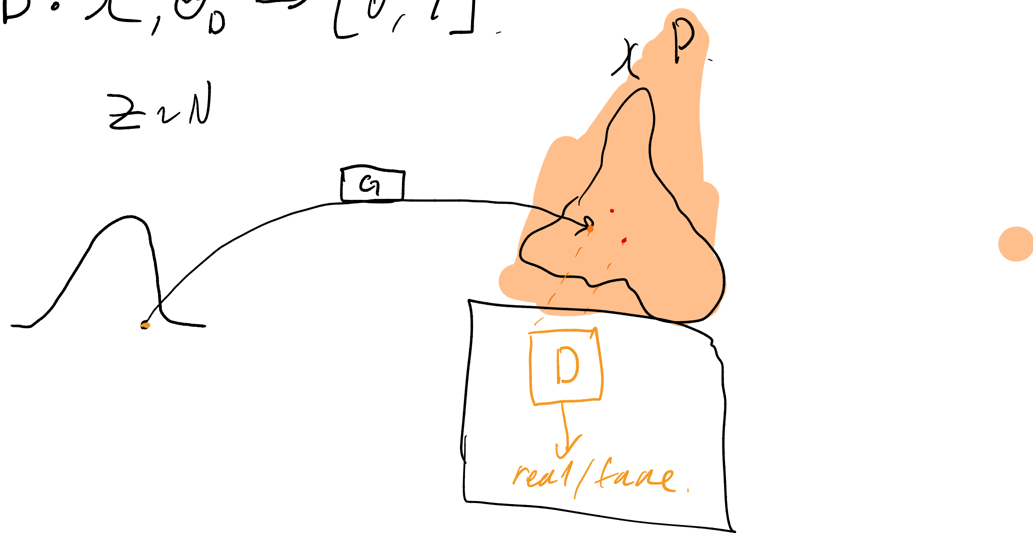
Generative models



$G : z, \theta_G \to X$

VAE

$D : X, \theta_D \to [0, 1]$



$$\underbrace{-\mathbb{E}_{x\sim p}\log_2(D(x)) - \mathbb{E}_{z\sim N}\log_2(1-D(G(z)))}_{L_D(\theta_D)}$$

$$L_G(\theta_G) = \mathbb{E}_{z\sim N}\log_2(1-D(G(z))) = \mathbb{E}_{x\sim q}\log_2(1-D(x))$$

$$V(\theta_G, \theta_D) = \mathbb{E}_{x \sim p} \log_2(D(x)) + \mathbb{E}_{z \sim N} \log_2(1 - D(G(z))).$$

$$\boxed{\min_{\theta_G} \max_{\theta_D} V(G, D).}$$

$$\int_x p(x) \log_2 D(x)\, dx + \int_z p_z(z) \log_2(1 - D(G(z)))\, dz$$

q - s $G(z)$, $z \sim N$

$$= \int_X \left[p(x) \log_2 D(x) + q(x) \log_2(1 - D(x)) \right] dx$$

$$\underbrace{a \log m + b \log_2(1-m)}$$

$$\boxed{D_G^*(x) = \frac{p(x)}{p(x) + q(x)}.}$$

$$\boxed{\mathbb{E}_{x \sim p} \log_2 \frac{p(x)}{p(x)+q(x)} + \mathbb{E}_{x \sim q} \log_2 \frac{q(x)}{p(x)+q(x)}.}$$

$$-\log_2 2 - \log_2 2 = -\log_2 4.$$

$$\underbrace{\log_2 4 + KL\left(P \,\|\, \frac{P+q}{2}\right) + KL\left(q \,\|\, \frac{P+q}{2}\right)}_{\text{Jensen - Shannon divergence}}$$

$$KL(P \| q) = \mathbb{E}_{x \sim P} \log_2 \frac{P(x)}{q(x)}$$

$$KL(q \| P) = \mathbb{E}_{x \sim q} \log_2 \frac{q(x)}{P(x)},$$

$$JS(P, q) = \underbrace{\left(\log_2 4 + KL\left(P \,\|\, \frac{P+q}{2}\right) + KL\left(q \,\|\, \frac{P+q}{2}\right)\right)}_{4} \frac{1}{2}.$$

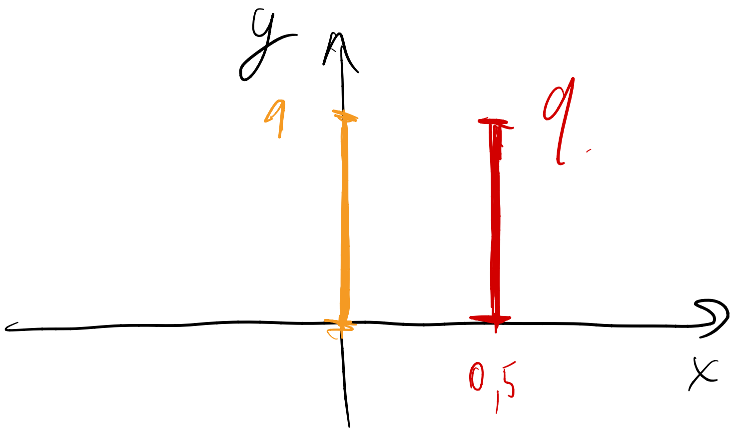
$$\boxed{\theta \neq 0 \quad \log 2.}$$

$z \sim U[0, 1]$

$p = (0, z)$

$q = (\theta, z)$

$$\boxed{\begin{array}{c}\theta \neq 0 \\ \theta = 0\end{array}}$$



$$\frac{1}{2}\left(\log_2 4 + \underset{x\sim p}{\mathbb{E}} \underbrace{\log_2 \frac{2p(x)}{p(x)+\cancel{q(x)}^{=0}}}_{=1} + \underset{x\sim q}{\mathbb{E}} \underbrace{\log_2 \frac{2q(x)}{\cancel{p(x)}_{0}+q(x)}}_{=1}\right) =$$

$$= \frac{1}{2} \log_2 4 = \log_2 2 = 1$$

$p, q \qquad \forall \gamma \in \Pi \quad \int_x \gamma(x,y)\,dx = q(y)$

$\int_y \gamma(x,y)\,dy = p(x)$

$$W(p,q) = \inf_{\gamma \in \Pi} \mathbb{E}_{(x,y)\sim\gamma} \|x - y\|_2$$

$$W(p,q) = \sup_{\|f\|_L \le 1} \left(\mathbb{E}_{x\sim p} f(x) - \mathbb{E}_{x\sim q} f(x) \right)$$

$$|f(x) - f(y)| \le L^{1} \cdot |x - y|$$

$$\big| \|\nabla_x f\| - 1 \big|$$

$D: X, \theta_D \to \mathbb{R}$

$$L_D(\theta_D) = -\mathbb{E}_{x \sim p} D(x) + \mathbb{E}_{x \sim q} D(x) + \| \|\nabla_x D\| - 1 \|$$

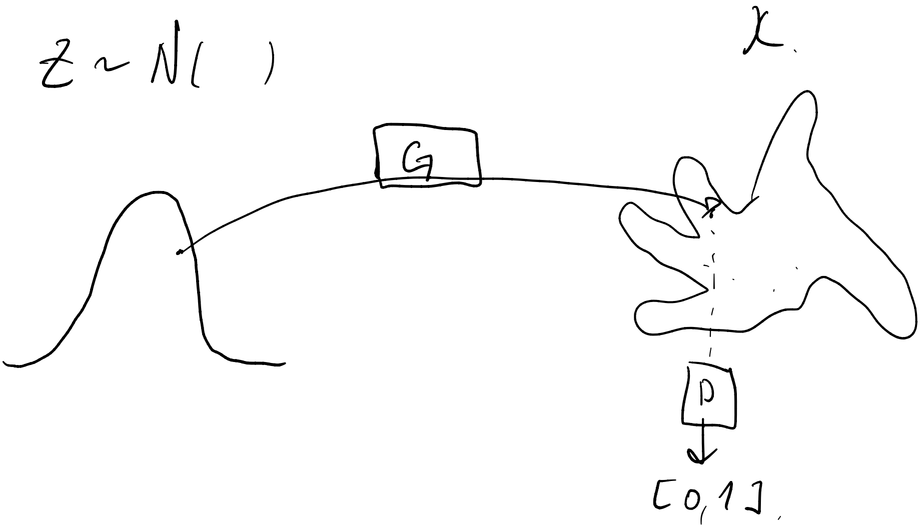
$$L_G(\theta_G) = -\mathbb{E}_{x \sim q} D(x)$$

R1. $\boxed{\nabla_x D(x)}$

$\boxed{\text{spectral norm}}$

$\theta = P \Sigma V^T$ SVD.

$\boxed{\max(\Sigma) = 6}$ $\theta / 6$.



$$G : Z \to X$$

$$D : X \to [0, 1]$$

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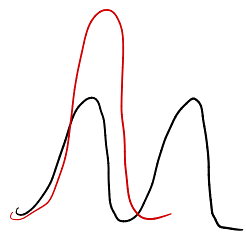
q

$$L_g = -\mathbb{E}_{z \sim N} \log(D(G(z))) =$$

$$\boxed{= -\mathbb{E}_{x \sim q} \log(D(x))} = ?$$

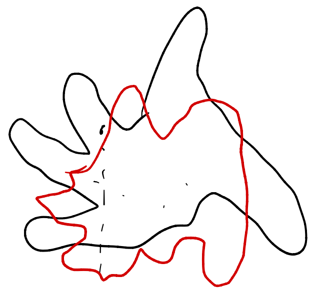


$\mathcal{X}$ $\qquad\qquad$ $\mathcal{X}$

$x^{(1)} \in \mathbf{X}_1$ $\qquad\qquad$ $x^{(2)} \in \mathbf{X}_2$

$f : \mathcal{X} \to \mathcal{X}$

$f(x^1) \in D_2$



$\boxed{\text{cycle gan}}$

$G_{1\to2} : \mathcal{X}, z \to \mathcal{X}$ $\qquad$ $D_2 : \mathcal{X} \to \mathbb{R}$

$G_{2\to1} : \mathcal{X}, z \to \mathcal{X}$ $\qquad$ $D_1 : \mathcal{X} \to \mathbb{R}$

$x^{(1)} \sim p_1$

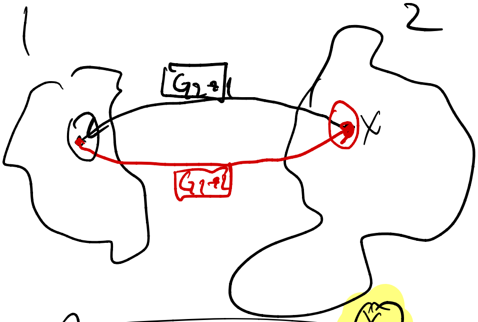
$\hat{x}^{(2)} = G_{1\to2}(x^{(1)})$ $\qquad$ $D_2(\hat{x}^{(2)}) \to \max$

$\leftarrow \max$

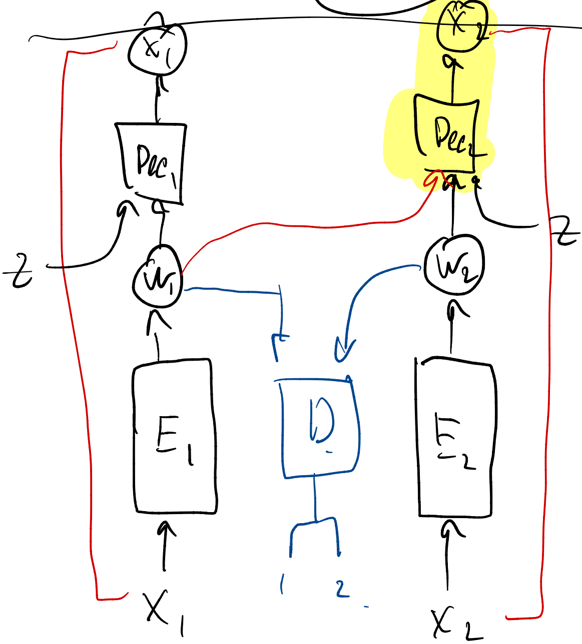
$\hat{x}^{(1)} = G_{2\to1}(\hat{x}^{(2)})$

$\boxed{\|\hat{x}^{(1)} - x^{(1)}\| \to \min}$

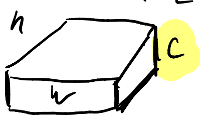
cycle consistency loss



$$x = G_{2\to1}(G_{1\to2}(x))$$



$\mathbb{R}^{512}$



$\in \mathbb{R}^C$

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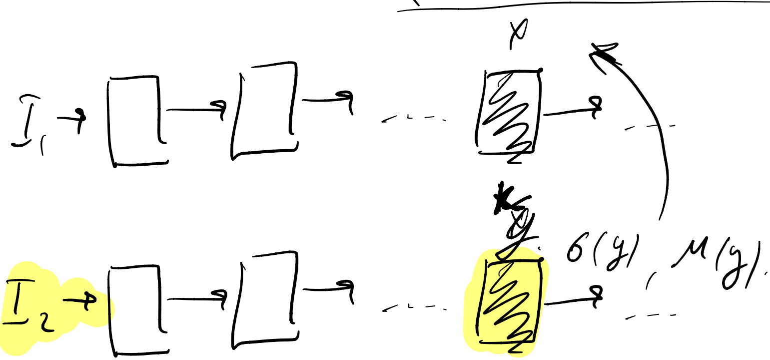
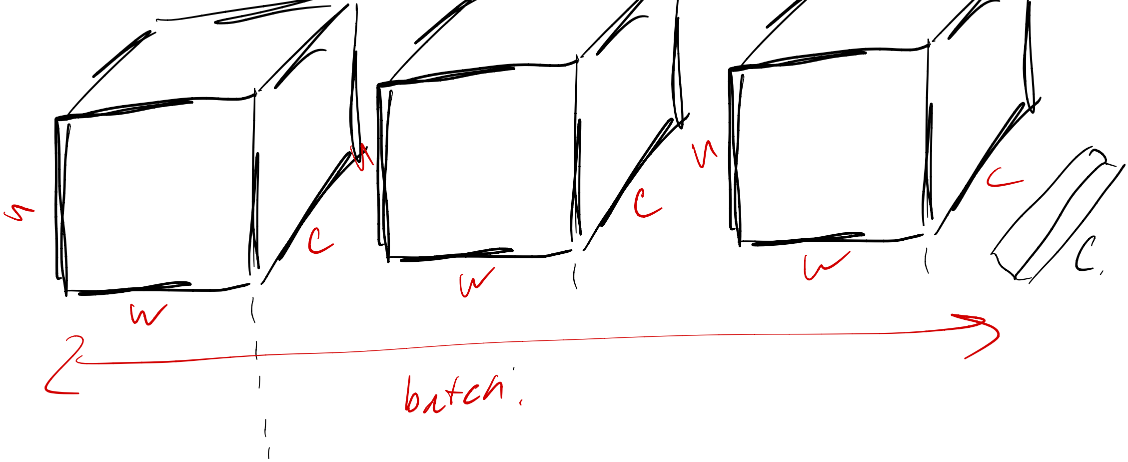
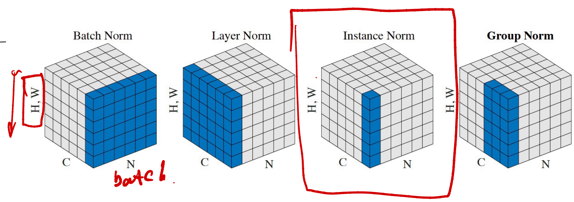
$$\mathrm{adain}(x) = \boxed{\left(\frac{x-\mu(x)}{\sigma(x)}\right)\cdot\sigma(y)+\mu(y)}$$

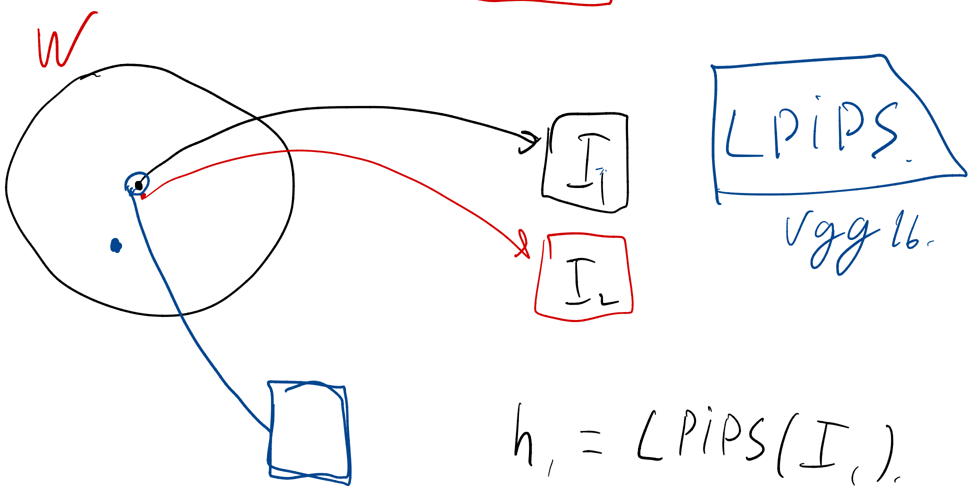




$X \quad (b, c, h, w).$

$$\mu = \frac{1}{b} \sum_{i,j,k} X[i, :, j, k]. \qquad \mu \quad (c).$$





$$h_1 = LPIPS(I_1).$$

$$|h_1 - h_2|$$

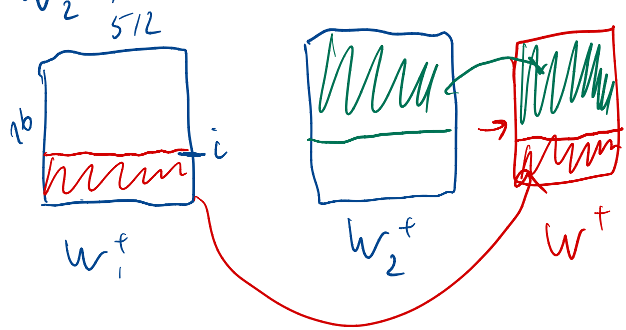
$$h_1 \quad (128, 16, 16).$$

$w = m(z), \quad z \sim \mathcal{N}(0, \mathbb{1})$ h_2.

$$\underset{(b,\,512)}{w} \to \underset{(b,\,16,\,512)}{w^+}$$

w_1^+ $\quad i \sim U[0, 16]$. $\quad \hat{w}^+ = \mathrm{cat}(w_1^+[:, :i, :],$

w_2^+ $\quad\quad w_2^+[:, i:, :], \mathrm{dim}=1)$



$(b, 16, 512)$.

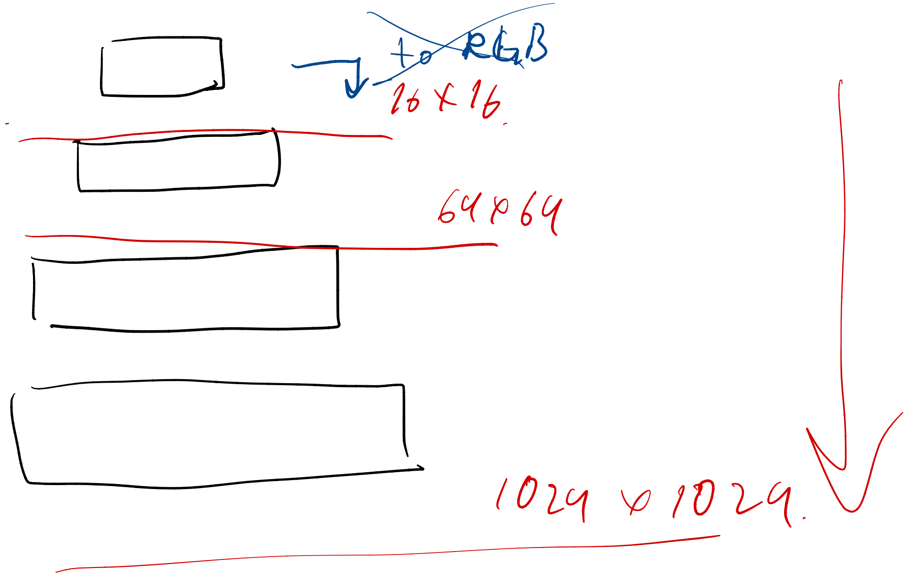
modulated conv.

modulation

$$k' = \underset{(C_{out},\, C_{in},\, h,\, w)}{k} * \underset{(1,\, C_{in},\, 1,\, 1)}{s}$$

de-modulation

$$\sigma = \sigma_{dim=0}(k') \quad \boxed{k''} = k'/\sigma.$$



Normalizing flows.

X

P

$z \underset{p_z}{\sim} N(0, 1)$.

$f: X \to Z$.

$$p_z(f(X)) = p\left(f^{-1}(z)\right) \cdot \left|\det Df(x)\right|$$

word2vec.

$\boxed{64}$

E

$$\Theta_E^T \cdot X = h \quad (64)$$
$$L \times 64$$

D

$$\Theta_D^T h = \hat{y}_L$$

$$X \to \boxed{E} \to h \to \boxed{D} \to y.$$

y_1 (k_1)	~~X~~	y_2 (k_2)
мама	мыла	раму

раму.

мыла маму.

$$\begin{cases} i = k_1, & 1 \\ i = k_2, & 1 \\ 0 & \end{cases}$$

$$\hat{y} = \text{softmax}(\hat{y}).$$

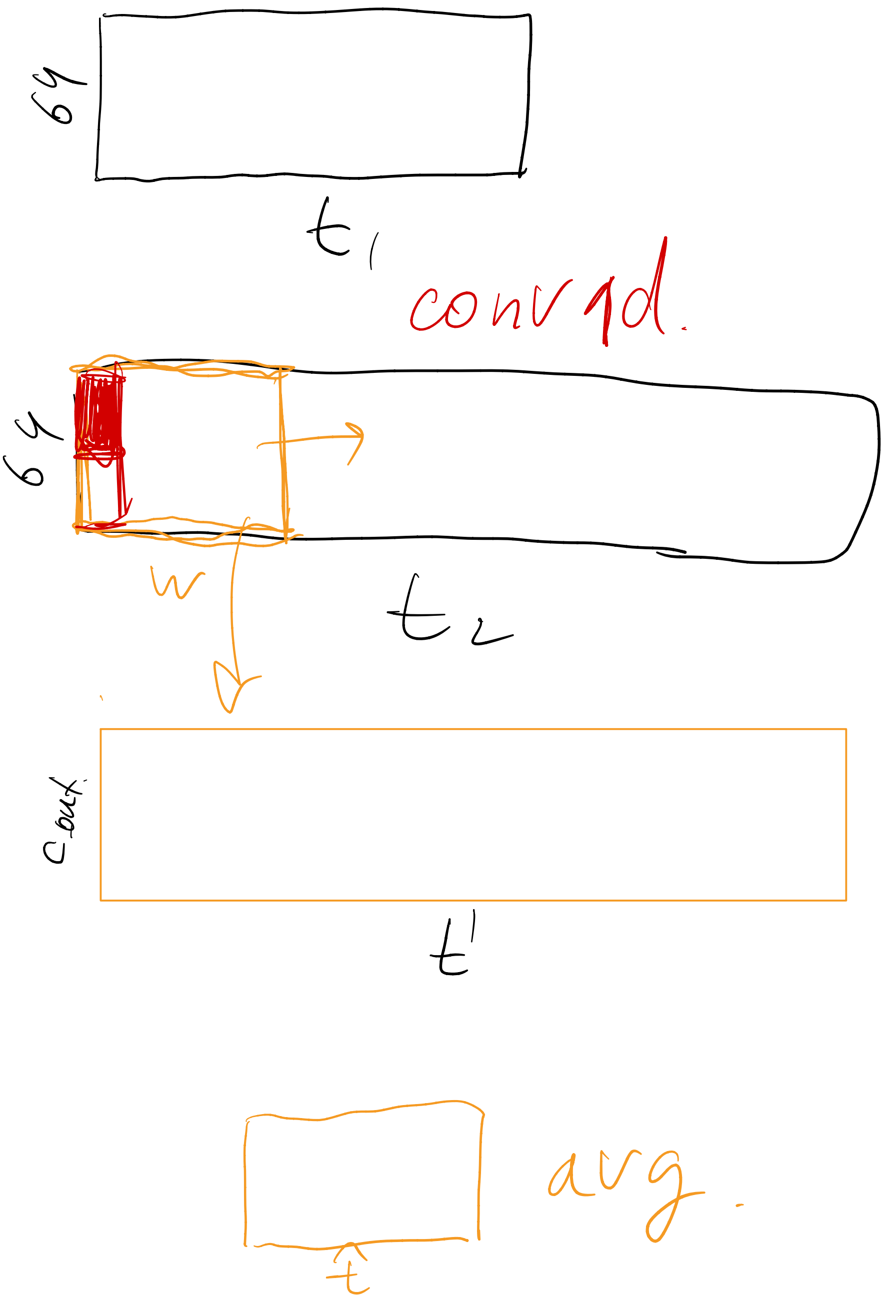
NLL$(\hat{y}, y_1)$

NLL$(\hat{y}, y_2)$.

$$-\log \frac{e^{\hat{y}[k_1]}}{\sum_j e^{\hat{y}[i]}} - \log \frac{e^{\hat{y}[k_2]}}{\sum_j e^{\hat{y}[i]}}$$

w2v

GLOVE.



RNN. LSTM.