Introduction to Algorithms

What are algorithms?

algorithm

/ˈalgərɪð(ə)m/

noun

noun: **algorithm**; plural noun: **algorithms**

a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer.
"a basic **algorithm for** division"

Origin

late 17th century (denoting the Arabic or decimal notation of numbers): variant (influenced by Greek *arithmos* 'number') of Middle English *algorism*, via Old French from medieval Latin *algorismus* . The Arabic source, *al-Ḵwārizmī* 'the man of Ḵwārizm' (now Khiva), was a name given to the 9th-century mathematician Abū Ja'far Muhammad ibn Mūsa, author of widely translated works on algebra and arithmetic.

https://www.dictionary.com/browse/algorithm

British Dictionary definitions for algorithm

algorithm

noun

1. a logical arithmetical or computational procedure that if correctly applied ensures the solution of a problem: Compare heuristic
2. **logic maths** a recursive procedure whereby an infinite sequence of terms can be generated

French name: **algorism**

Derived Forms

algorithmic, adjective
algorithmically, adverb

Word Origin

C17: changed from algorism, through influence of Greek *arithmos* number

algorithm in Science

algorithm

[ăl’gə-rĭð’əm]

1. A finite set of unambiguous instructions performed in a prescribed sequence to achieve a goal, especially a mathematical rule or procedure used to compute a desired result. Algorithms are the basis for most computer programming.

Combining definitions…

Algorithm (noun):

A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

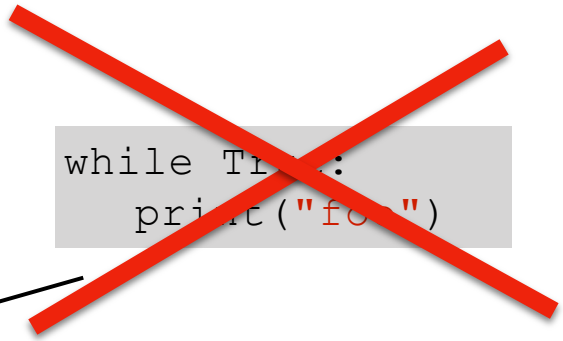
Combining definitions…

```
while True:
   print("foo")
```

Algorithm (noun):

A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Combining definitions…

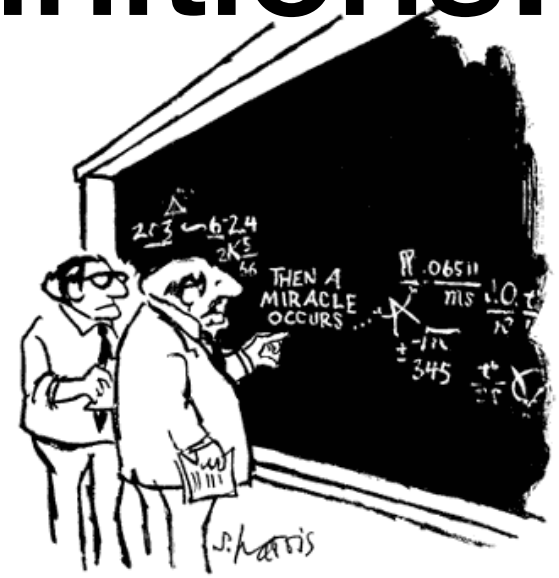


We want our algorithms to **terminate!**

Algorithm (noun):

A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Combining definitions…



"I think you should be more explicit here in step two."

Algorithm (noun):

A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Cartoon from http://www.sciencecartoonsplus.com/gallery/math/math07.gif
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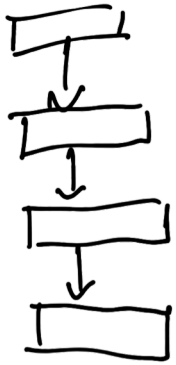
Combining definitions…

Algorithm (noun):

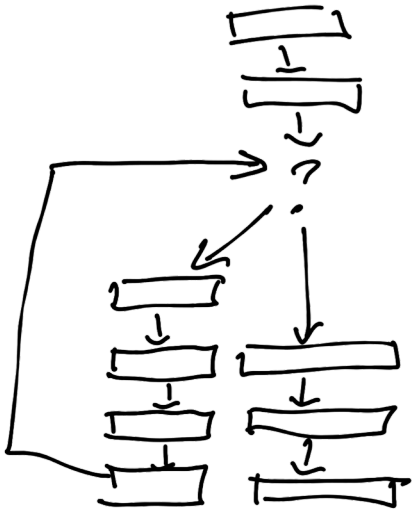
A *finite* sequence of *unambiguous* steps that *solves* a specific problem.

Building-blocks in Programming and in Algorithms

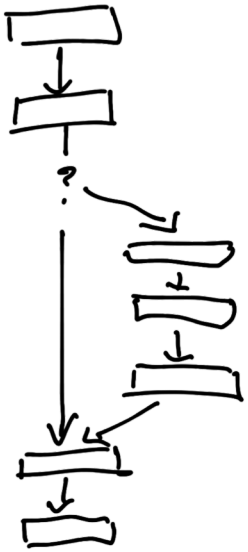
Sequential

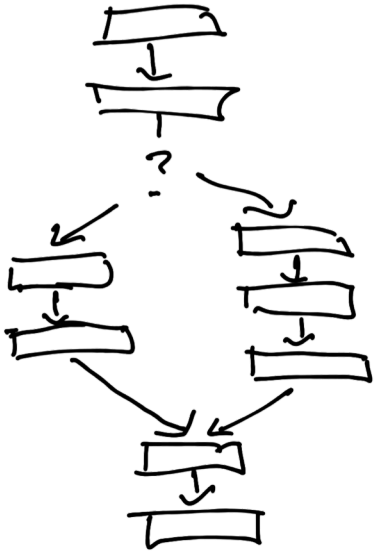


Looping



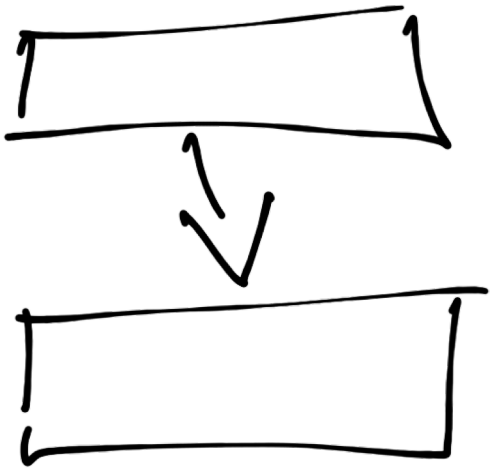
Branching



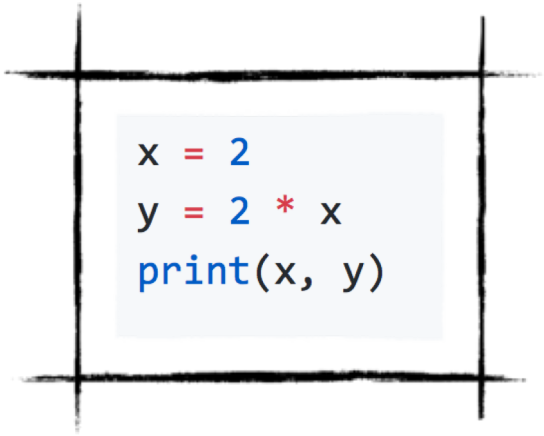


Sequential execution

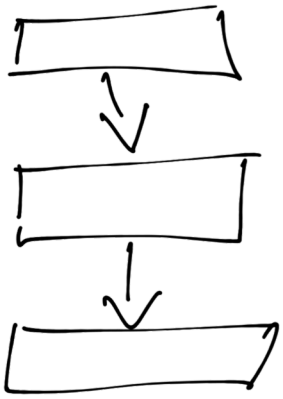
```
print("Hello")
print("World")
```



Variables (program state)



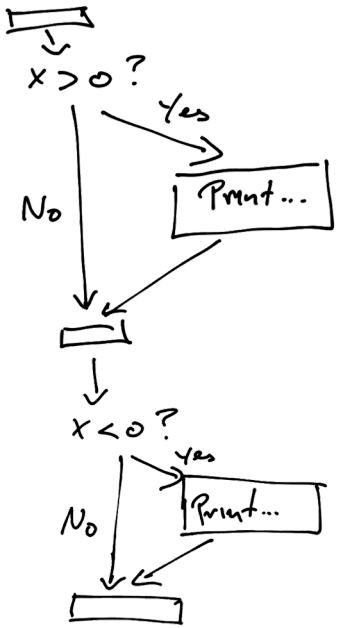
```
x = 2
y = 2 * x
print(x, y)
```

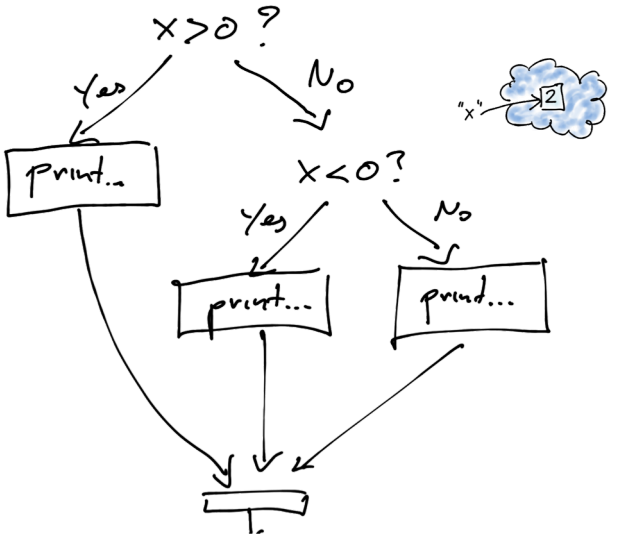


Branching

```
if x > 0:
  print("x is positive")
if x < 0:
  print("x is negative")
```

```
if x > 0:
  print("x is positive")
elif x < 0:
  print("x is negative")
else:
  print("x is zero")
```

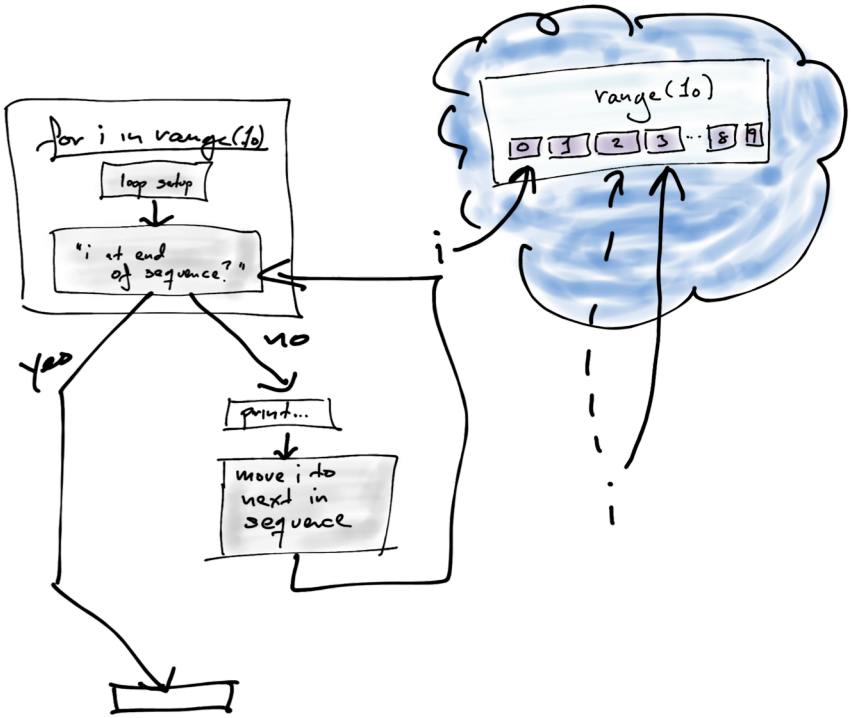


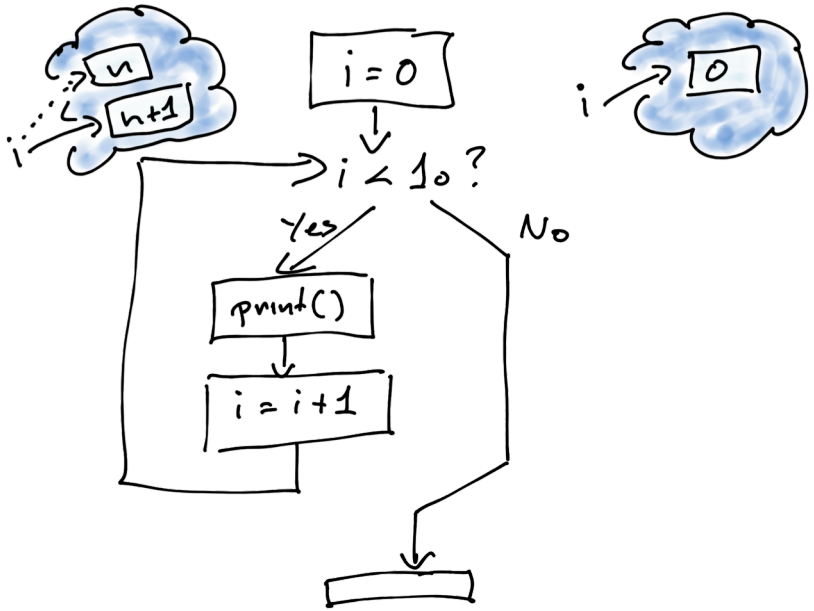


Looping

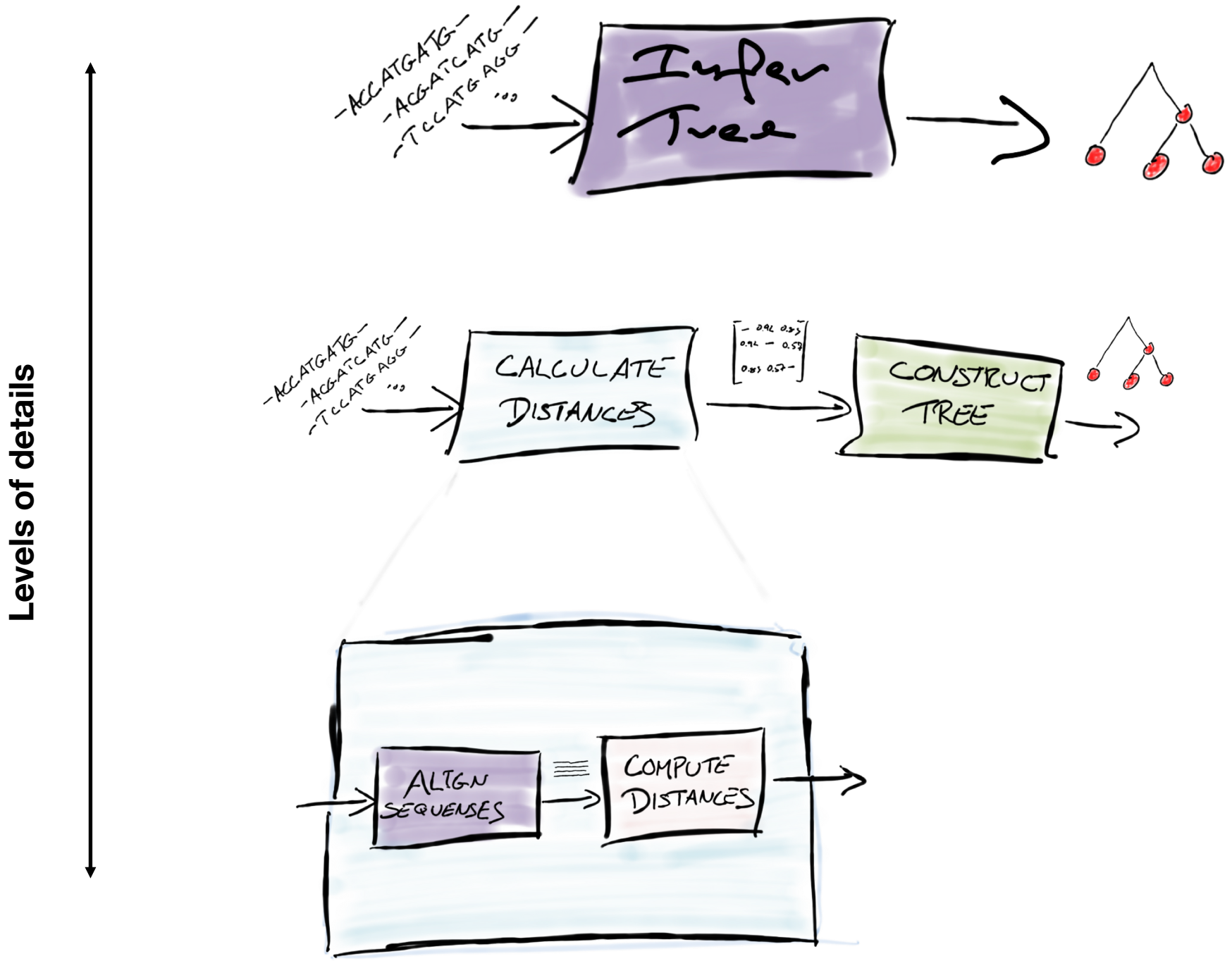
```
for i in range(10):
  print("i =", i)
```

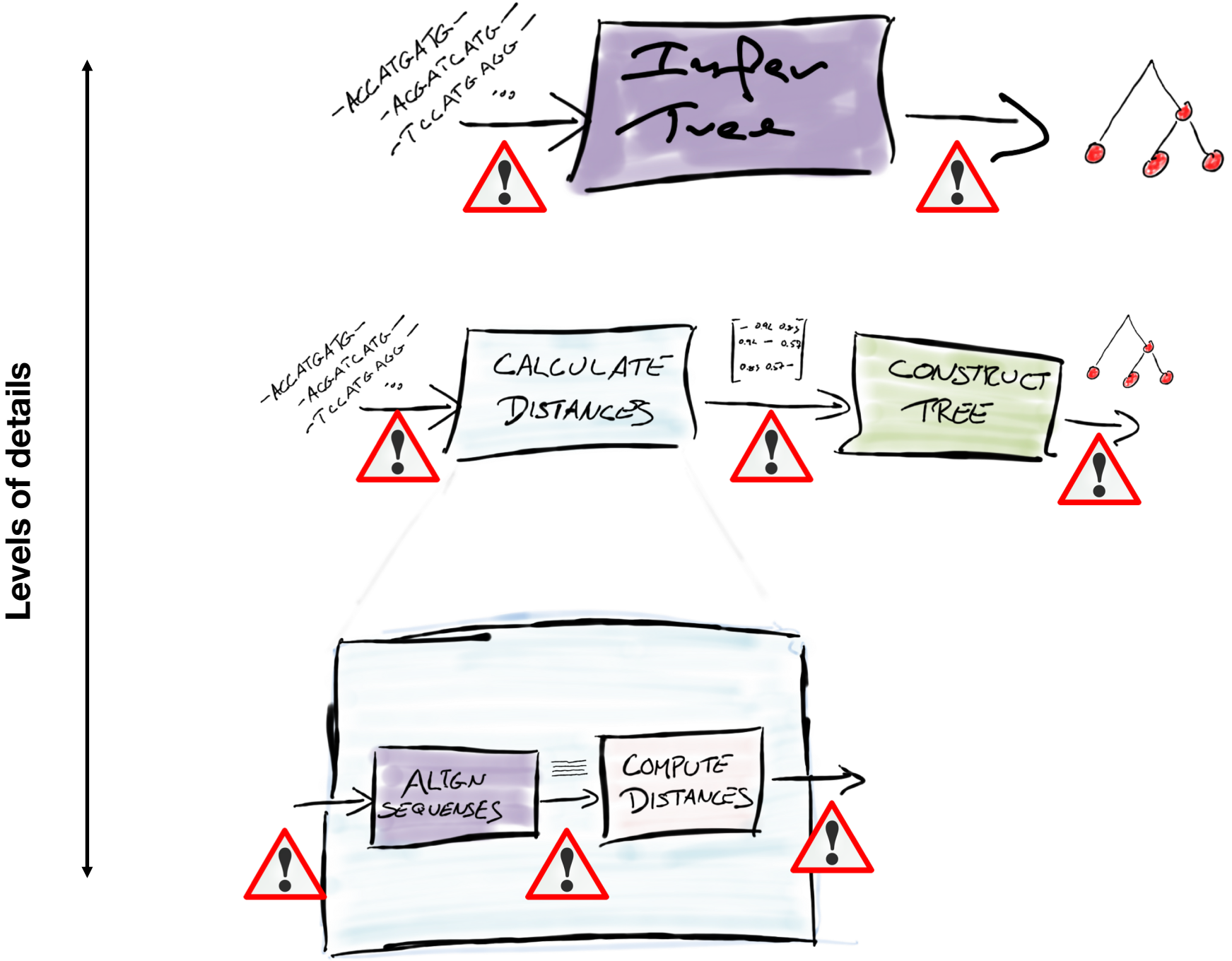
```
i = 0
while i < 10:
  print("i =", i)
  i = i + 1
```

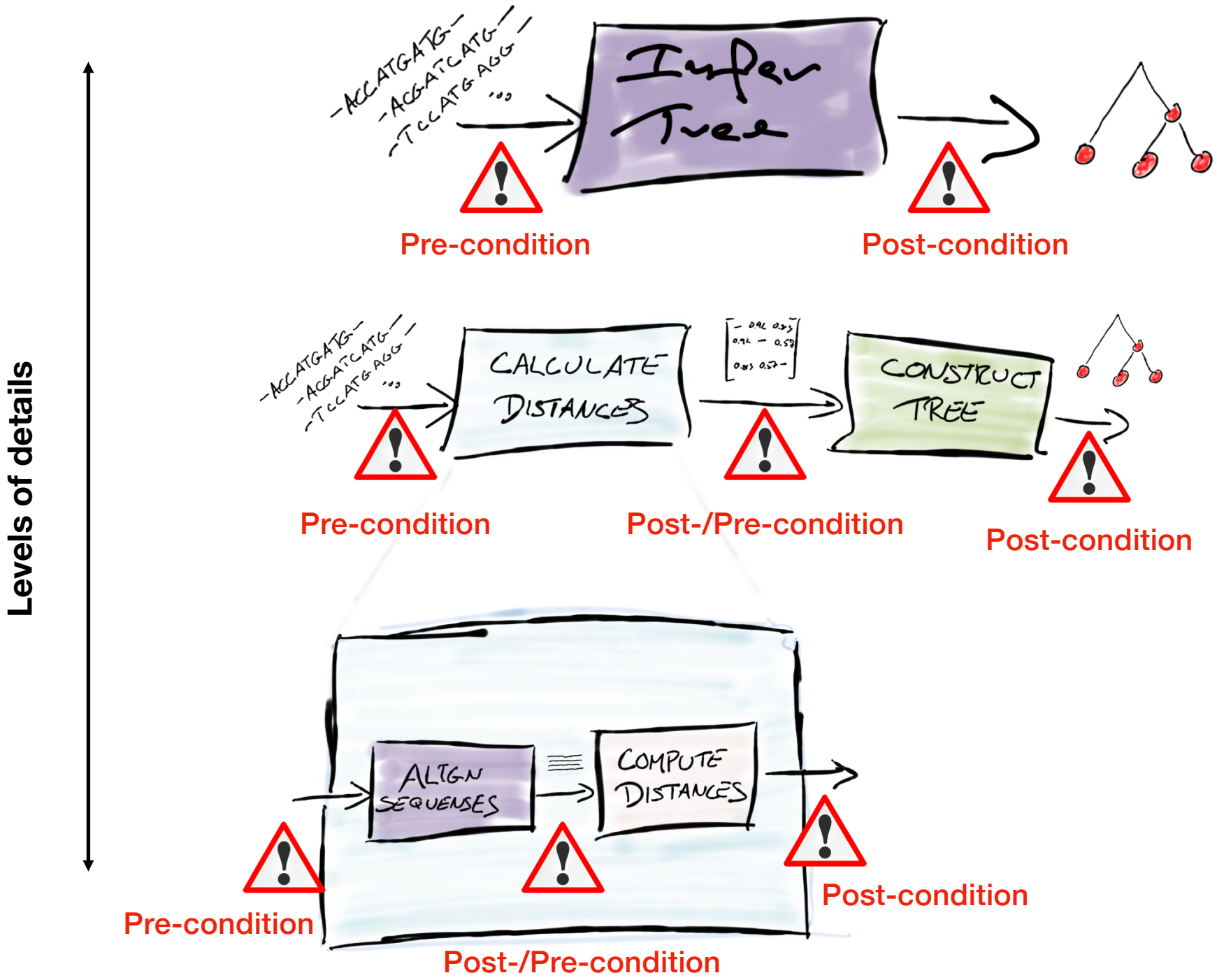


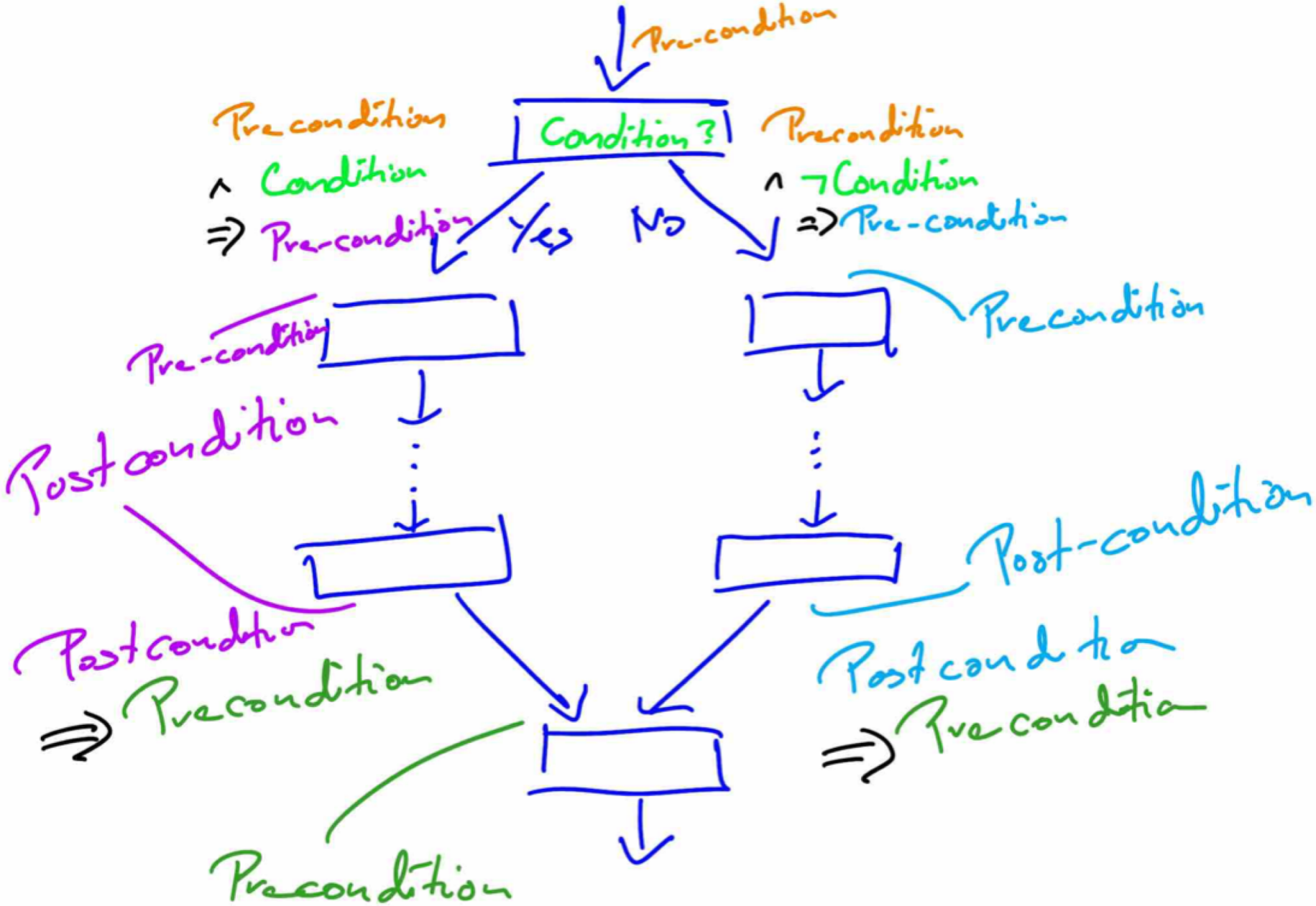


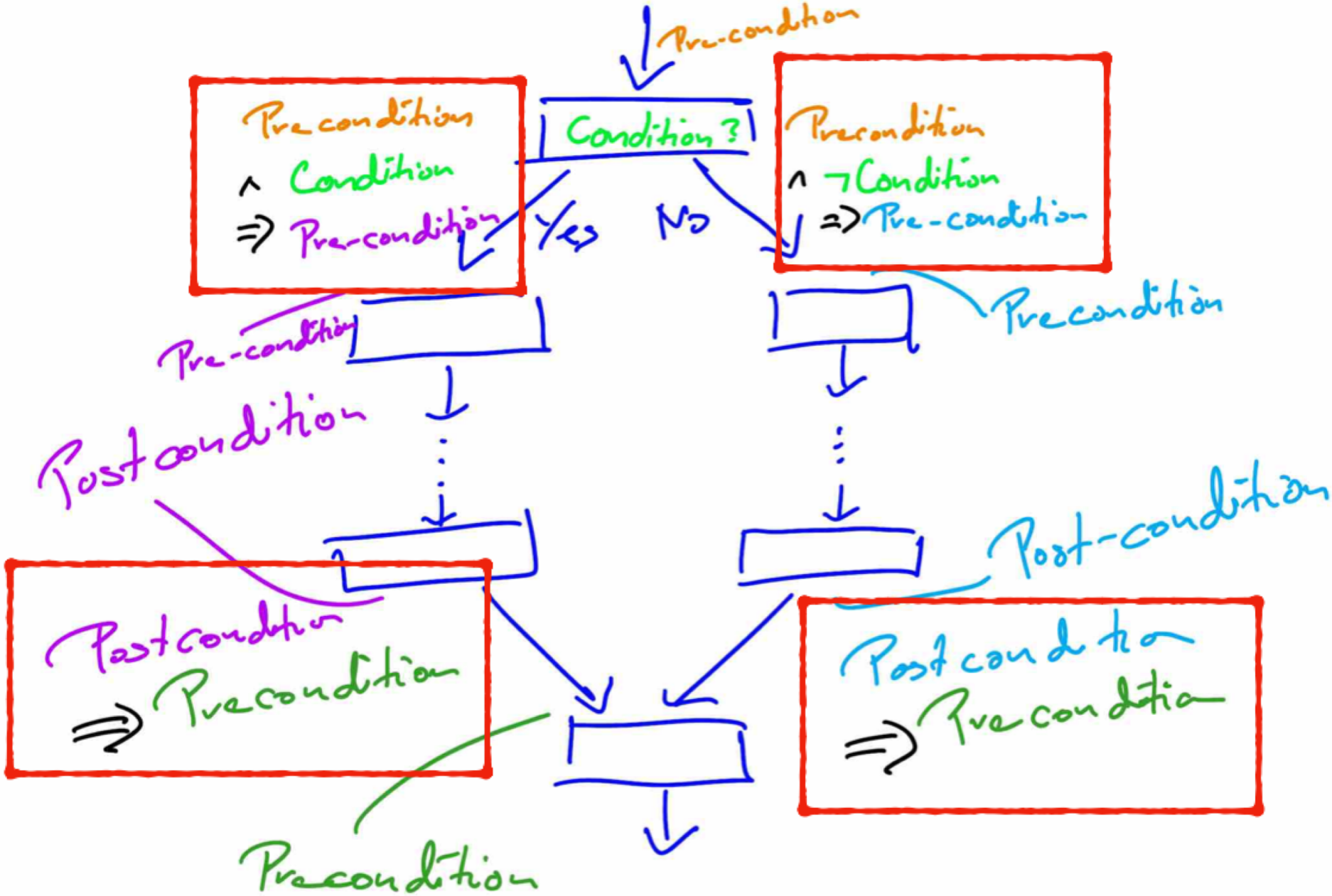
Designing algorithms: Breaking down problems to smaller parts

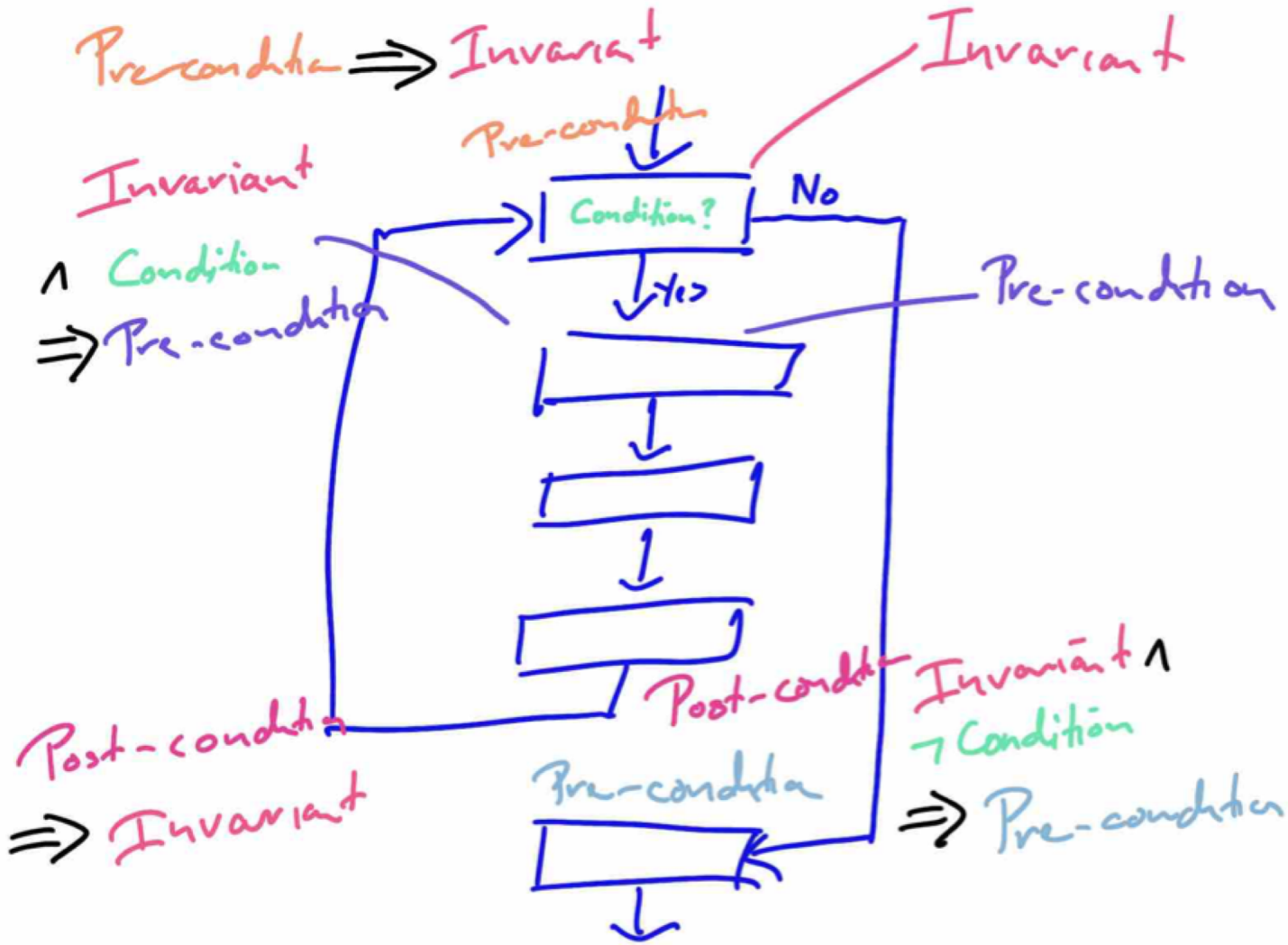

Levels of details
-ACCATGATG-
-ACGATCATG-
-TCCATGAGG-
...
Infer Tree
-ACCATGATG-
-ACGATCATG-
-TCCATGAGG-
...
CALCULATE DISTANCES
CONSTRUCT TREE
ALIGN SEQUENSES
COMPUTE DISTANCES

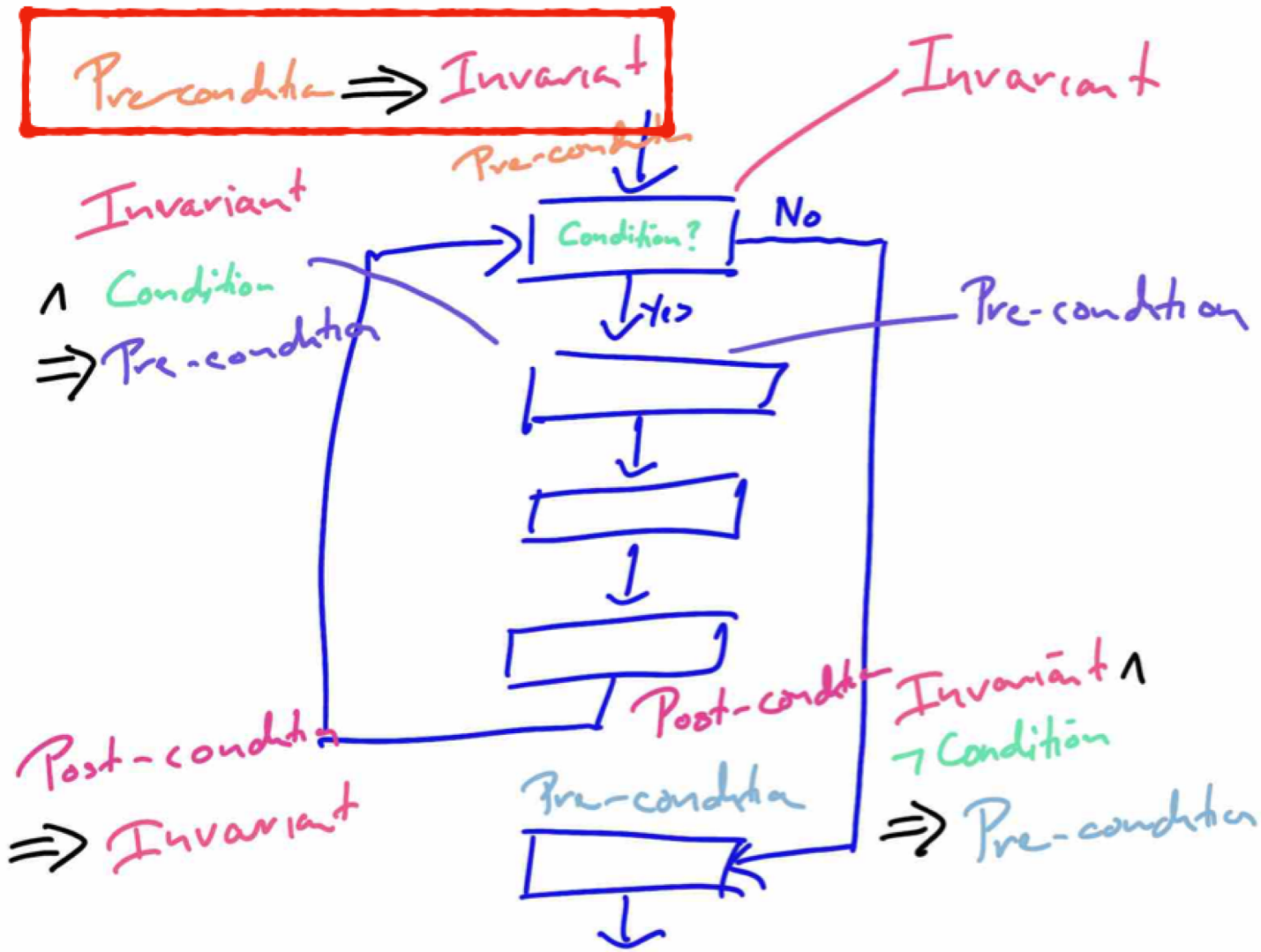

Levels of details
Infer Tree
CALCULATE DISTANCES
CONSTRUCT TREE
ALIGN SEQUENSES
COMPUTE DISTANCES

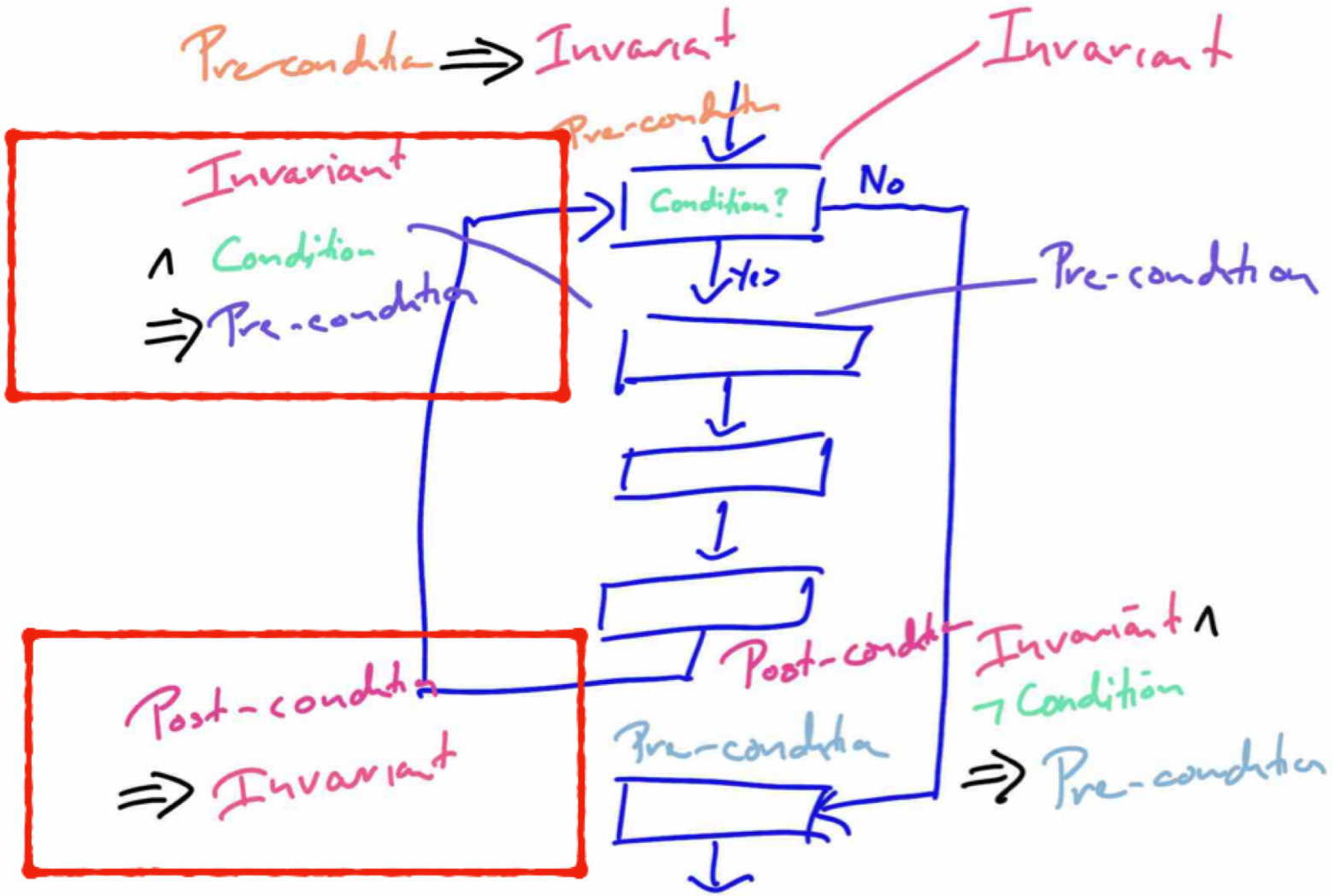

Levels of details
Infer Tree
Pre-condition
Post-condition
CALCULATE DISTANCES
CONSTRUCT TREE
Pre-condition
Post-/Pre-condition
Post-condition
ALIGN SEQUENSES
COMPUTE DISTANCES
Pre-condition
Post-condition
Post-/Pre-condition

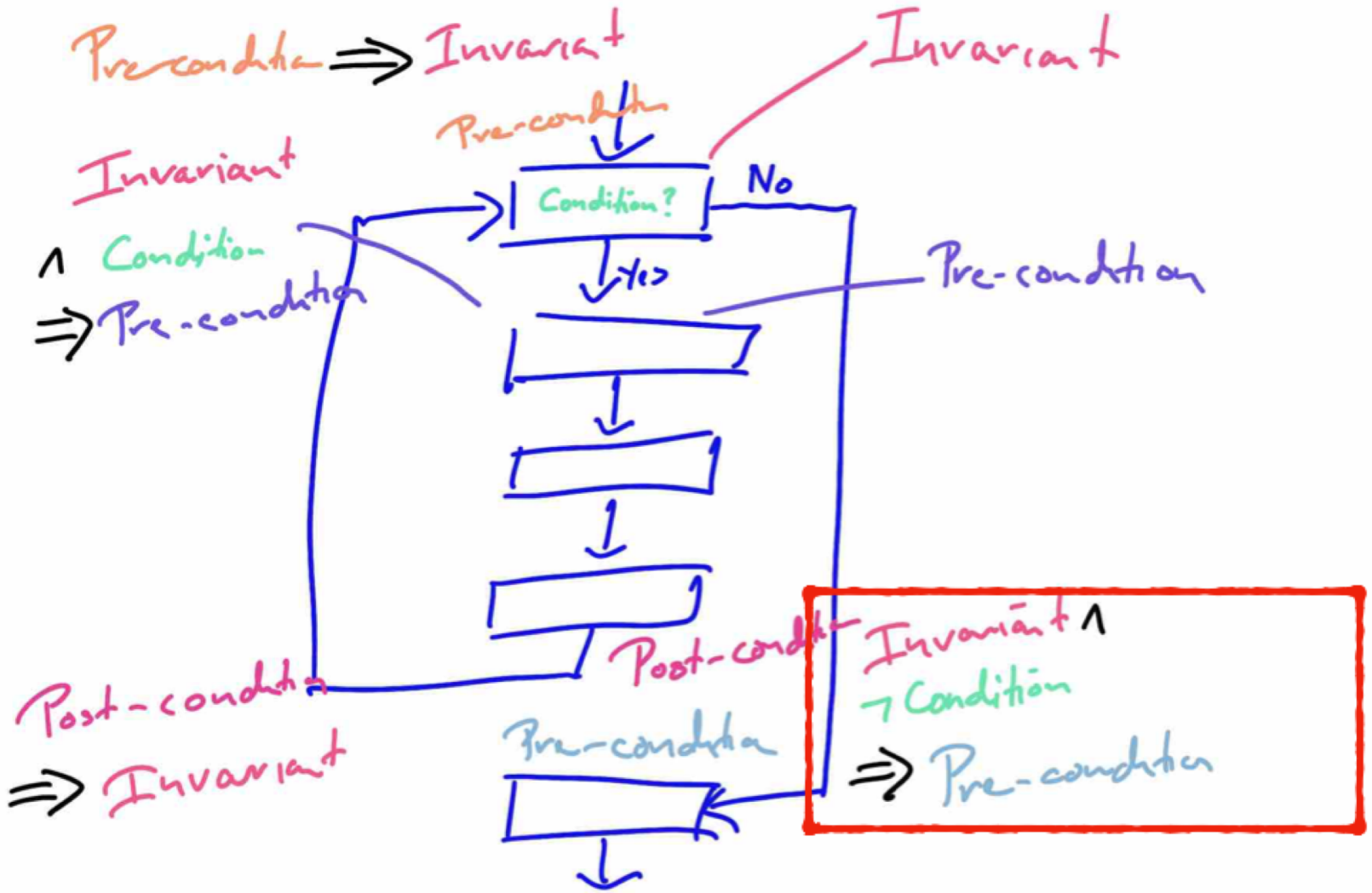

Pre-condition
Condition?
Precondition
∧ Condition
⇒ Pre-condition
Precondition
∧ ¬Condition
⇒ Pre-condition
Yes
No
Pre-condition
Precondition
Postcondition
Post-condition
Postcondition
⇒ Precondition
Postcondition
⇒ Precondition
Precondition


Pre-condition
Condition?
Precondition
∧ Condition
⇒ Pre-condition
Yes
No
Precondition
∧ ¬Condition
⇒ Pre-condition
Pre-condition
Precondition
Postcondition
Post-condition
Postcondition
⇒ Precondition
Postcondition
⇒ Precondition
Precondition

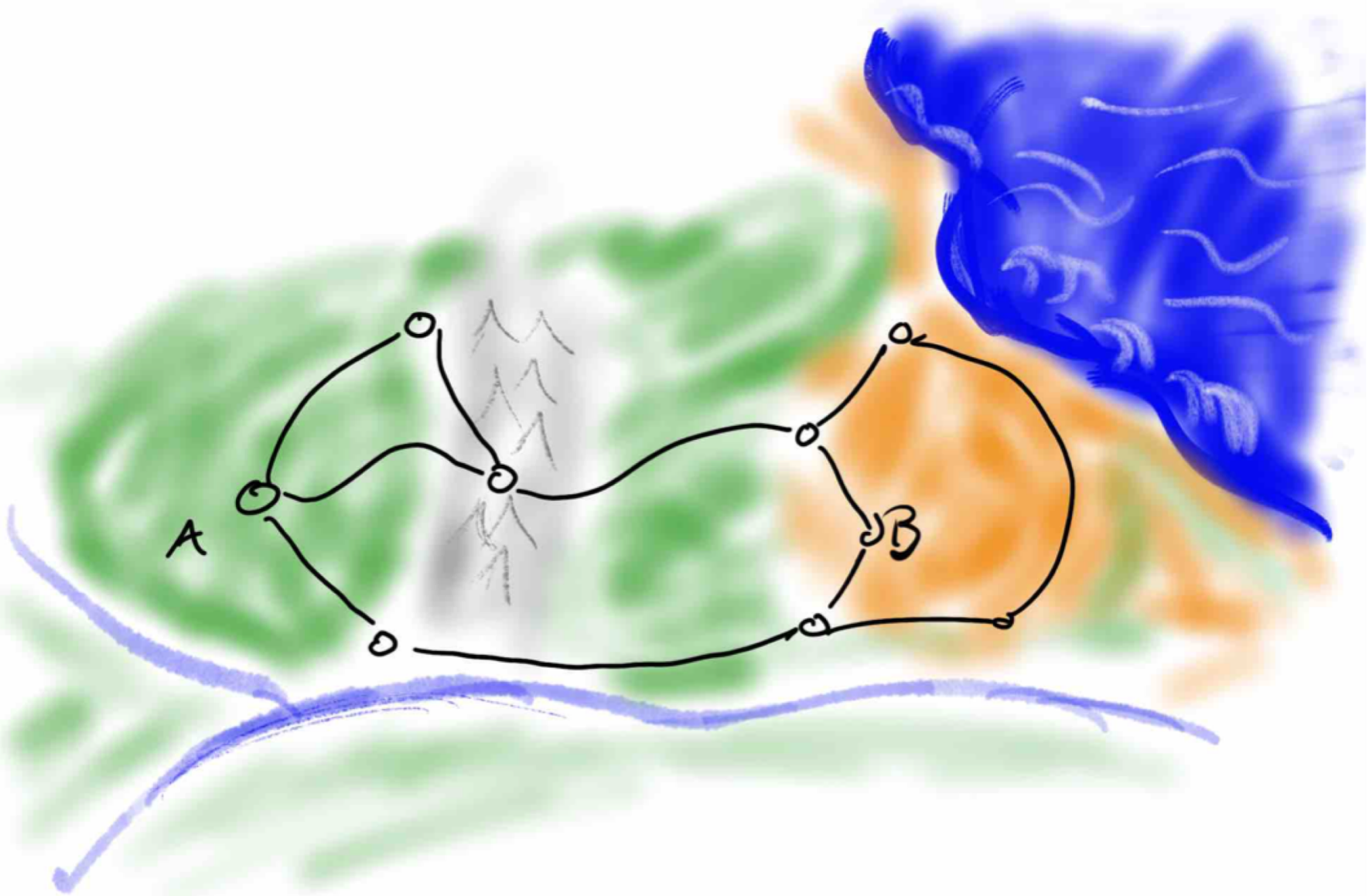

Precondition ⇒ Invariant
Invariant
Pre-condition
Invariant
Condition?
No
Yes
∧ Condition
⇒ Pre-condition
Pre-condition
Post-condition
Post-condition
Invariant ∧
¬ Condition
⇒ Invariant
Pre-condition
⇒ Pre-condition

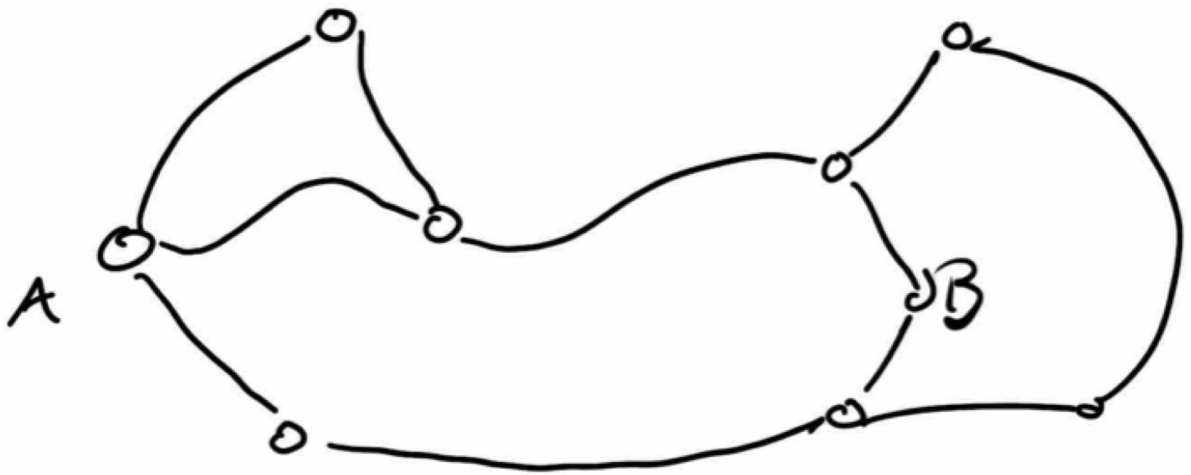

Precondition ⇒ Invariant
Invariant
Pre-condition
Invariant
Condition?
No
Yes
∧ Condition
⇒ Pre-condition
Pre-condition
Post-condition
Post-condition
⇒ Invariant
Invariant ∧
¬ Condition
⇒ Pre-condition
Pre-condition

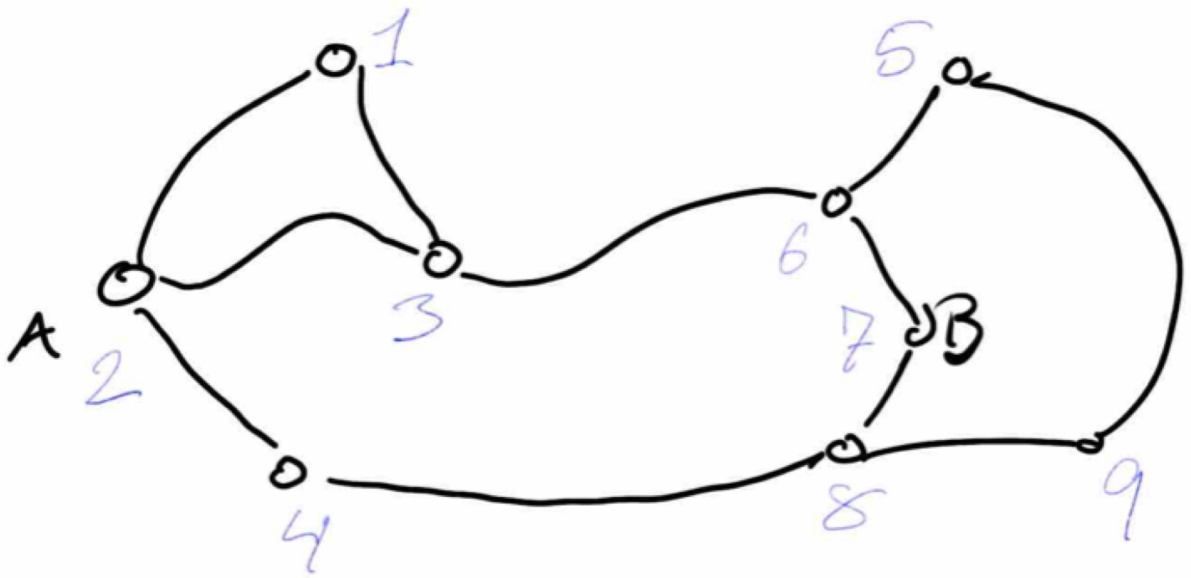

Invariant
Condition?
No
Yes
Invariant ∧
¬ Condition

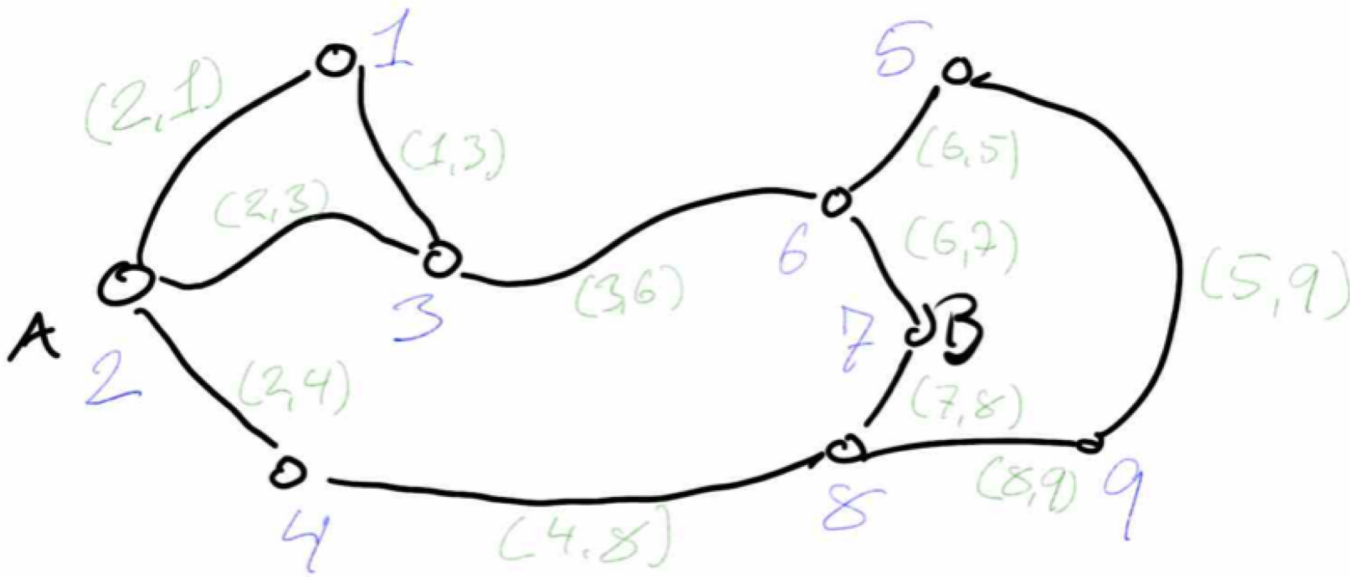

Precondition ⇒ Invariant
Invariant
Pre-condition
Invariant
Condition?
No
Yes
Invariant
∧ Condition
⇒ Pre-condition
Pre-condition
Post-condition
Post-condition
⇒ Invariant
Pre-condition
Invariant ∧
¬ Condition
⇒ Pre-condition

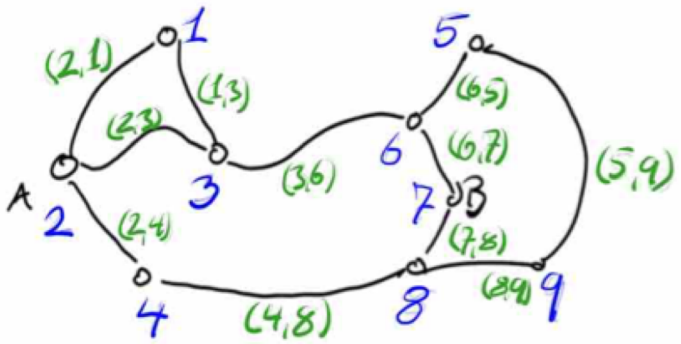
Example: are cities A and B connected?


A
B


A
B

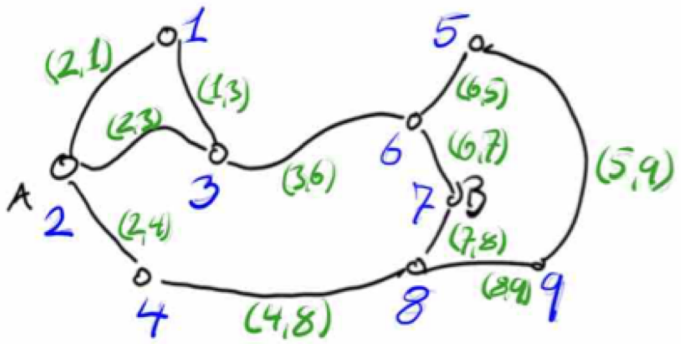

1
5
6
A
3
7
B
2
4
8
9


1
5
(2,1)
(1,3)
(6,5)
(2,3)
6
(6,7)
(5,9)
A
3
(3,6)
7
B
2
(2,4)
(7,8)
(8,9)
9
4
(4,8)
8



	1	0
A	2	0
	3	0
	4	0
	5	0
	6	0
B	7	0
	8	0
	9	0

(2,1)
(2,3)
(2,4)
(1,3)
(3,6)
(4,8)
(6,5)
(6,7)
(7,8)
(8,9)
(5,9)



1 0 → 1

A 2 0 → 2

3 0 → 3

4 0 → 4

5 0 → 5

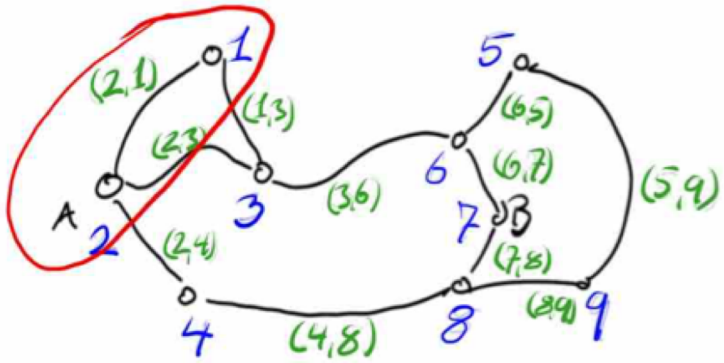
6 0 → 6

B 7 0 → 7

8 0 → 8

9 0 → 9

Seen	Unseen
(2,1)	
(2,3)	
(2,4)	
(1,3)	
(3,6)	
(4,8)	
(6,5)	
(6,7)	
(7,8)	
(8,9)	
(5,9)	



	1	0	→ 1
A	2	0	2
	3	0	→ 3
	4	0	→ 4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

(2,1) Seen

(2,3) Unseen

(2,4)

(1,3)

(3,6)

(4,8)

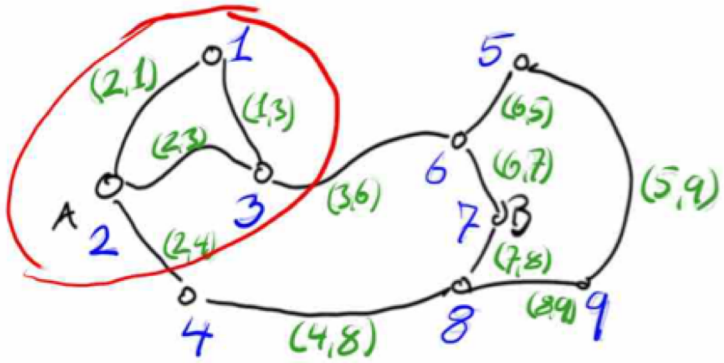
(6,5)

(6,7)

(7,8)

(8,9)

(5,9)



	1	0	→ 1
A	2	0	→ 2
	3	0	3
	4	0	→ 4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

(2,1)

(2,3) Seen

(2,4) Unseen

(1,3)

(3,6)

(4,8)

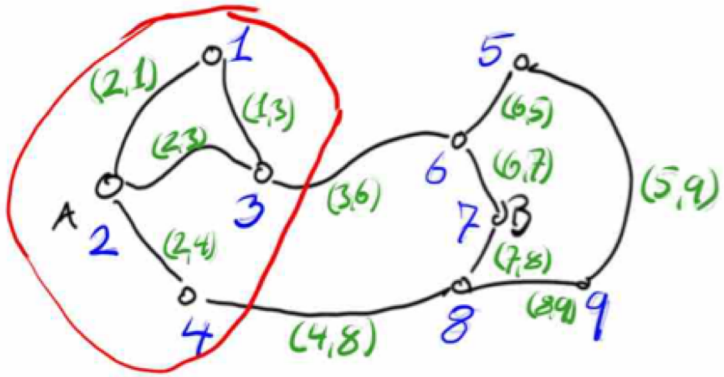
(6,5)

(6,7)

(7,8)

(8,9)

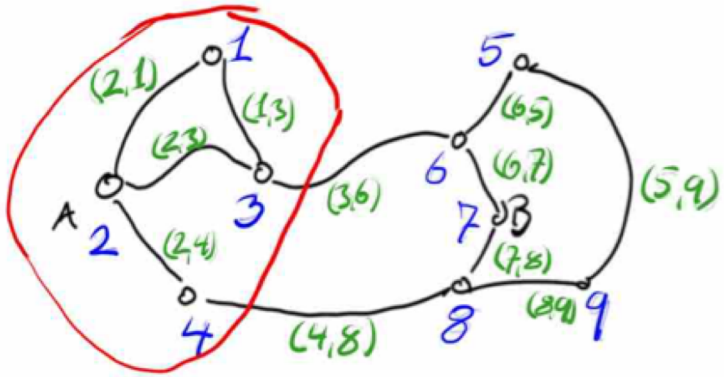
(5,9)



	1	0	→ 1
A	2	0	2
	3	0	3
	4	0	4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

(2,1)
(2,3)
(2,4) Seen

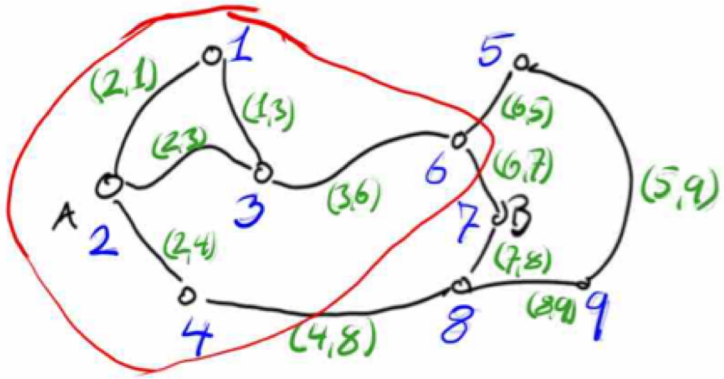
(1,3) Unseen
(3,6)
(4,8)
(6,5)
(6,7)
(7,8)
(8,9)
(5,9)



	1	0	→ 1
A	2	0	→ 2
	3	0	3
	4	0	4
	5	0	→ 5
	6	0	→ 6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

(2,1)
(2,3)
(2,4)
(1,3) Seen

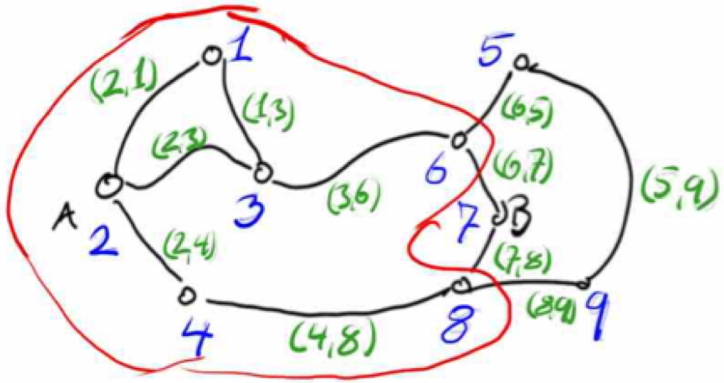
(3,6) Unseen
(4,8)
(6,5)
(6,7)
(7,8)
(8,9)
(5,9)



	1	0	→ 1
A	2	0	→ 2
	3	0	3
	4	0	4
	5	0	→ 5
	6	0	6
B	7	0	→ 7
	8	0	→ 8
	9	0	→ 9

(2,1)
(2,3)
(2,4)
(1,3)
(3,6) Seen

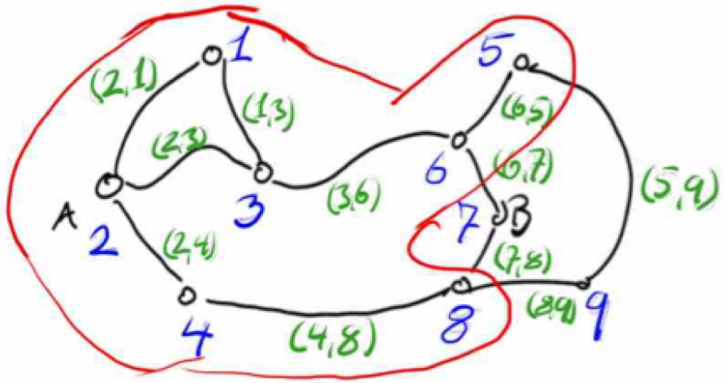
(4,8) Unseen
(6,5)
(6,7)
(7,8)
(8,9)
(5,9)



	1	0	1
A	2	0	2
	3	0	3
	4	0	4
	5	0	5
	6	0	6
B	7	0	7
	8	0	8
	9	0	9

(2,1)
(2,3)
(2,4)
(1,3)
(3,6)
(4,8) Seen

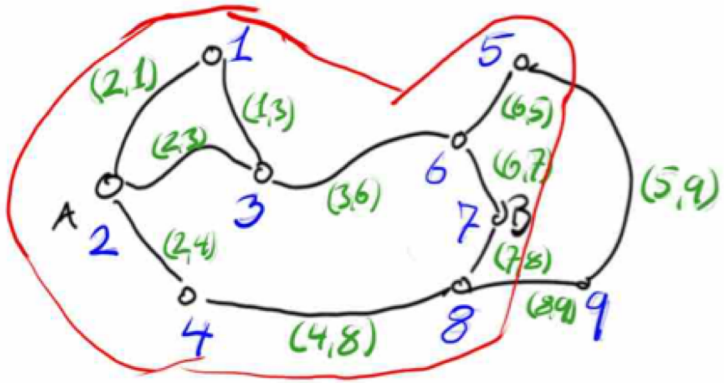
(6,5) Unseen
(6,7)
(7,8)
(8,9)
(5,9)



	1	0	1
A	2	0	2
	3	0	3
	4	0	4
	5	0	5
	6	0	6
B	7	0	7
	8	0	8
	9	0	9

(2,1)
(2,3)
(2,4)
(1,3)
(3,6)
(4,8)
(6,5) Seen

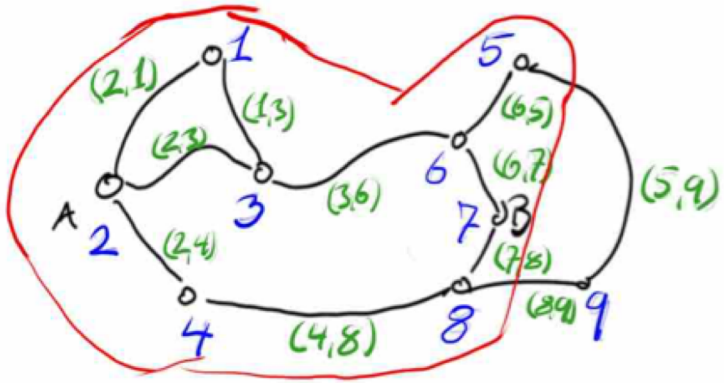
(6,7) Unseen
(7,8)
(8,9)
(5,9)



	1	0	1
A	2	0	2
	3	0	3
	4	0	4
	5	0	5
	6	0	6
B	7	0	7
	8	0	8
	9	0	9

(2,1)
(2,3)
(2,4)
(1,3)
(3,6)
(4,8)
(6,5)
(6,7) Seen

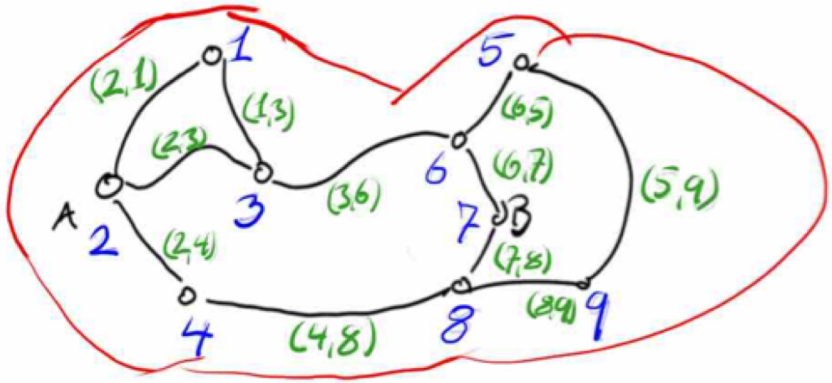
(7,8) Unseen
(8,9)
(5,9)

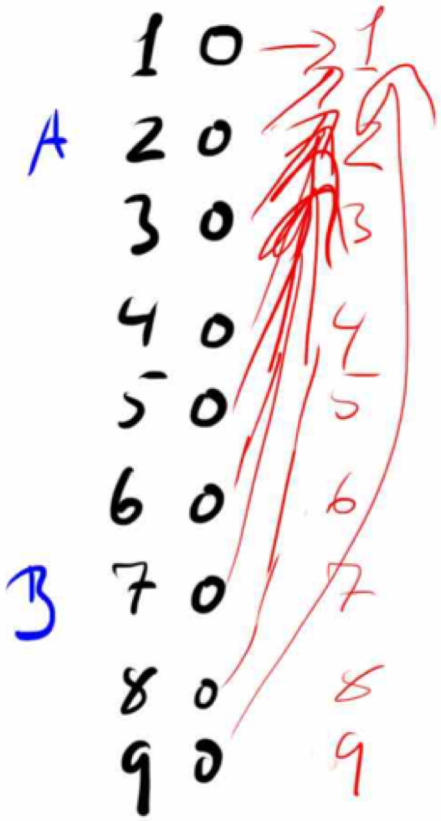


	1	0	1
A	2	0	2
	3	0	3
	4	0	4
	5	0	5
	6	0	6
B	7	0	7
	8	0	8
	9	0	9

(2,1)
(2,3)
(2,4)
(1,3)
(3,6)
(4,8)
(6,5)
(6,7)
(7,8) Seen

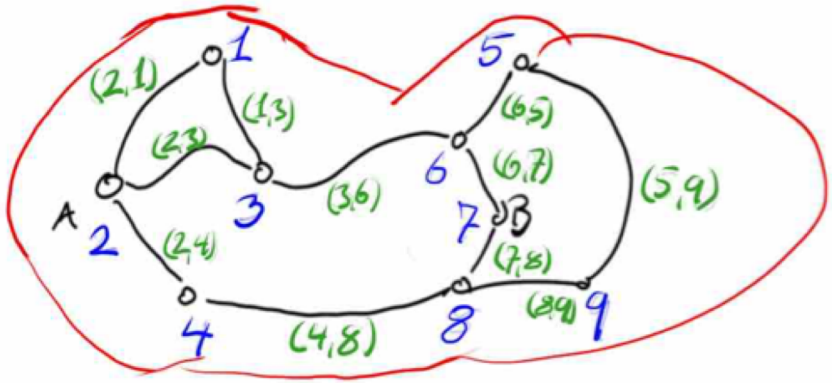
(8,9) Unseen
(5,9)

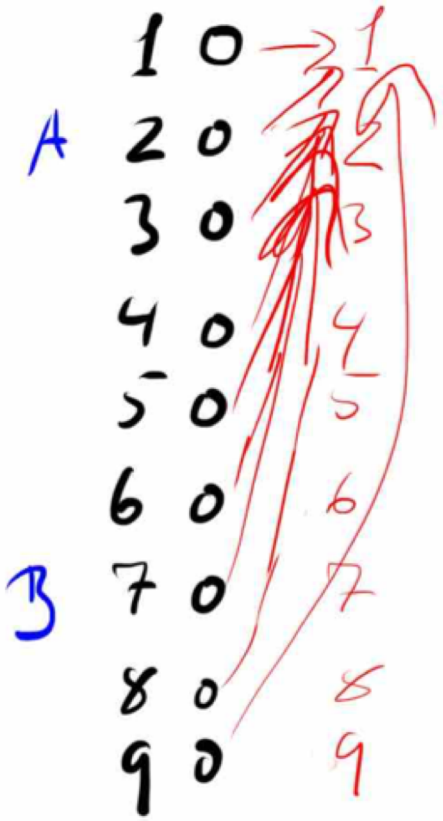




(2,1)
(2,3)
(2,4)
(1,3)
(3,6)
(4,8)
(6,5)
(6,7)
(7,8)
(8,9) Seen

(5,9) Unseen





(2,1)

(2,3)

(2,4)

(1,3)

(3,6)

(4,8)

(6,5)

(6,7)

(7,8)

(8,9)

(5,9) Seen

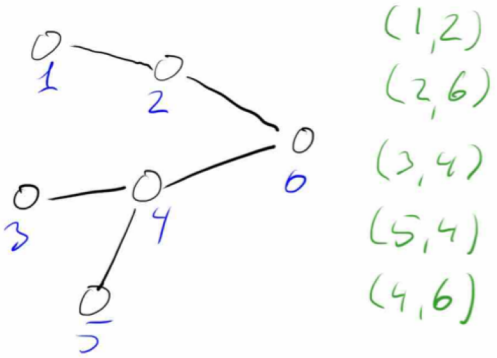
Unseen

Intermezzo

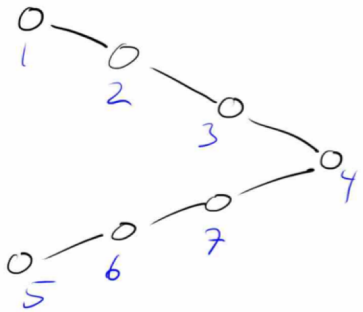
- How would you represent the components in Python?
- How would you formalise the loop invariant to take into account the representation of the components?

Intermezzo

- Run the algorithm on these two graphs:



(1,2)
(2,6)
(3,4)
(5,4)
(4,6)



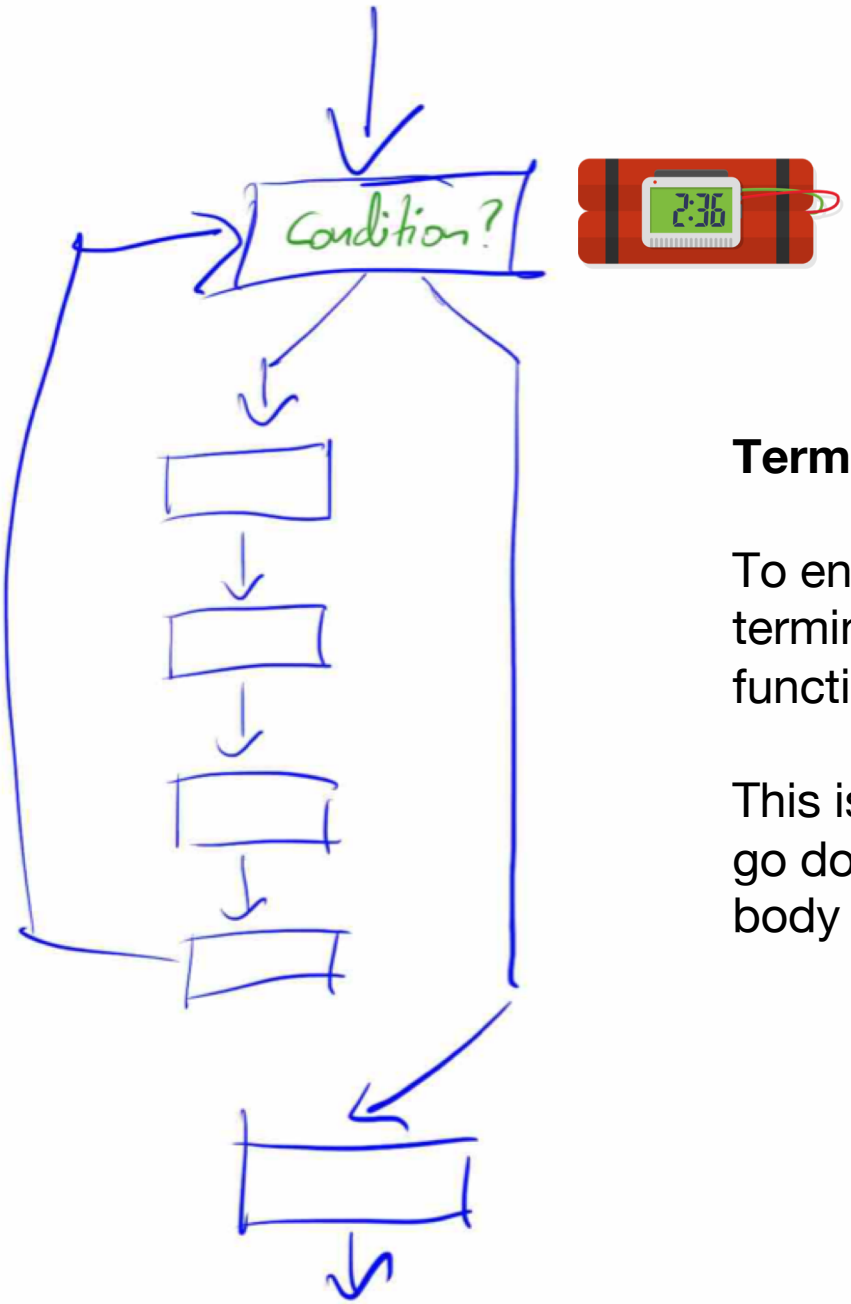
(1,2)
(2,3)
(3,4)
(5,6)
(6,7)
(7,4)

Correctness

How do we prove correctness?

- We prove that all pre- and post-conditions are satisfied through the steps in the algorithm.
- We prove that the post-condition of the last step implies that the overall problem is solved.
- *Correctness is just a special case of post-conditions*

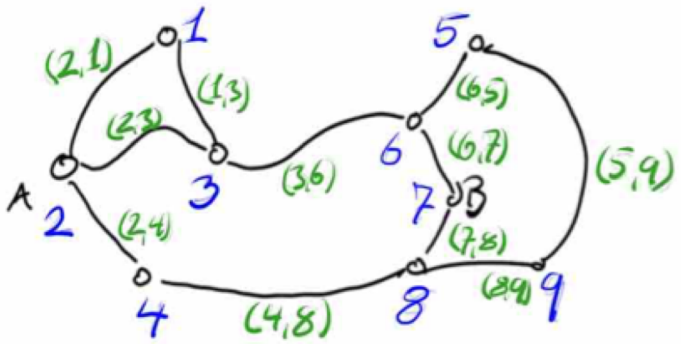
Termination



Termination functions:

To ensure that our algorithm terminations, we add a "termination" function to each loop.

This is a count-down that should go down each time we execute the body of the loop.



1 0 → 1

A 2 0 → 2

3 0 → 3

4 0 → 4

5 0 → 5

6 0 → 6

B 7 0 → 7

8 0 → 8

9 0 → 9

Seen	Unseen
(2,1)	
(2,3)	
(2,4)	
(1,3)	
(3,6)	
(4,8)	
(6,5)	
(6,7)	
(7,8)	
(8,9)	
(5,9)	

Get the binary representation of a number *n*

```
reverse_bits = []
while n > 0:
    reverse_bits.append(n % 2)
    n //= 2
print(reverse_bits[::-1])
```

Termination function:

$t(n) = n$

Thats it!

Now it is time to do the exercises to test that you now know how to construct algorithms